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FINITE ELEMENT ANALYSIS OF EFFECTIVE MECHANICAL PROPERTIES OF HIERARCHICAL HONEYCOMB STRUCTURES

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FINITE ELEMENT ANALYSIS OF EFFECTIVE MECHANICAL PROPERTIES OF
HIERARCHICAL HONEYCOMB STRUCTURES

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Ninad Nutankumar Gandhi
August 2015

Accepted by:
Dr. Lonny Thompson, Committee Chair
Dr. Joshua Summers
Dr. Gang Li

ABSTRACT

Honeycomb structures are widely used in engineering applications mainly due to their high strength to weight ratio. By changing the base material and geometry of the repeating unit cell structure, target effective properties can be achieved. Hierarchical honeycomb structures are known to have enhanced mechanical properties when compared to regular honeycomb structures. Therefore, it is important to understand and quantify the mechanical properties and the variation of these properties with the presence of hierarchy. This investigation builds upon prior work and considers the mechanical properties of two dimensional hierarchical honeycomb structures.

Previous research of hierarchical honeycomb structures studied replacing the homogeneous cell walls with truss lattices, or by replacing the cell walls by composite layers. Another hierarchy was examined by replacing the vertices of hexagon by smaller hexagons. However, in contrast to these previous studies, reiterated hierarchy is studied in this work, where a first order hierarchy structure is created by placing smaller honeycombs inside the conventional honeycombs such that midpoints of edges of the base level-0 honeycomb are shared vertices of the smaller level-1 honeycomb. In this work, the in-plane effective mechanical properties of these reiterated hierarchical honeycomb structures are studied with both regular and auxetic honeycombs. Effective elastic moduli and Poisson's ratio properties are determined and compared for a range of different cell wall thickness ratios between the base level-0 and smaller level-1 hierarchy. For comparisons, the mass was kept constant in all cases. Given the total mass and thickness ratio of the level-0 to level-1 hierarchy, the mass distribution is varied. The

mechanical properties are determined from finite element analysis of a patch of honeycombs in both uni-axial tension and shear loading conditions.

By changing the thickness ratio of level-0 to level-1 hierarchy, a nonlinear variation in mechanical properties is observed showing maximum and minimum values at specific ratios. From the results of first order regular hierarchical honeycomb structures, it can be said that for the same mass, the effective Young's modulus for thickness ratio of 0.1 between level-0 divided by level-1 is maximum and is about 1.45 times that of the zeroth order. Maximum effective shear modulus occurs for the special case with thickness ratio of zero, corresponding to a special level-1 honeycomb structure with the level-0 structure removed, and is 1.57 times that of the zeroth order.

From the results of first order auxetic hierarchical honeycomb structures, it can be said that the effective relative Young's modulus, and shear modulus of first order is higher for any thickness ratio than that of the zeroth order auxetic honeycomb structure of the same mass. The maximum effective Young's modulus occurs for thickness ratio 9 and is about 2.8 times that of the zeroth order. The maximum effective shear modulus of first order structure is maximum at ratio 0.1 and is 2.6 times that of the zeroth order.

DEDICATION

I dedicate this work to my parents, Nutankumar Gandhi and Nisha Gandhi, and my brothers Nimish and Nitant for their unconditional love, faith and support.

ACKNOWLEDGMENTS

I would like to offer my deepest appreciation and sincere gratitude to my advisor Dr. Lonny Thompson for his continuous guidance and support throughout my Master's degree. His expertise in the field of finite element analysis was immensely helpful during my research. His suggestions and comments have helped me to successfully complete my thesis. I would also like to thank advisory committee Dr. Joshua Summers and Dr. Gang Li for their precious time to be a part of my committee. I would like to thank Dr. Oliver Myers and Department of Mechanical Engineering, Clemson University for providing me Grading Assistantship position. Finally, I would like to thank all my friends who helped me through the highs and lows at graduate school.

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CHAPTER ONE INTRODUCTION

The selection of materials is an important factor in design of optimal structures. Material properties can directly affect the performance and form of the final design. In many applications, a component is designed to have a minimum mass without failure under certain loading conditions. Homogeneous materials have a fixed set of material properties [1]. This leaves the designer with only a limited number of discrete options. Properties of cellular structures depend on fixed properties of the base material and geometry of the structure [1,2,3,4]. Cellular materials offer broad range of overall effective properties with modification of geometry of the cells. This allows the designer to select the material and geometry of the structure to optimize the design.

Hexagonal honeycomb structures are part of broader class of cellular materials [1]. Hexagonal honeycomb structures are popular as they possess properties substantially different from the base material. By varying the geometry of the structure, properties can be adjusted for a suitable application without changing the base material. This offers the designer flexibility to meet multiple requirements simultaneously. These flexible structures allows the designer to adjust the geometry of the structure to get the required effective properties with the specific material.

Honeycomb structures have high out-of-plane stiffness to weight ratio [1]. In addition to these desirable properties, honeycomb structures offer a major advantage that their overall effective properties can be tailored depending on the application. These

materials are used in a variety of engineering applications as a core material sandwiched between two homogeneous face-sheets [1, 5].

There are numerous materials (natural and man-made) that demonstrate structural hierarchy. This is represented when the structures themselves contain structural elements. Many natural hierarchical materials have displayed very high damage tolerances from impact loading. The main objective of introducing hierarchy to cellular structures is to further enhance the mechanical behavior of the structures without compromising the elastic properties of the material. Hierarchical structures are obtained by adding material where it is needed either to occupy areas of high stress or transfer load gradually. This process maximizes the efficiency of the resulting product and the load bearing component.

The way in which cells are organized or stacked together in a hierarchical structure plays a significant role in identifying the mechanical properties of the solid. Research has shown that the hierarchical cell organization of sandwich panels with cores made of composite lattice structures or foams can result in enhanced mechanical behavior and superior elastic properties [6-13]. It has also been proven that increasing the levels of hierarchy in cellular structures produces better performing structures that are lighter weight.

1.1 Previous research on hierarchical honeycomb structures

Several studies have been done in the past regarding hierarchical honeycomb structures. The incorporation of hierarchy in the conventional honeycombs can be achieved using several different techniques.

Kooistra [6] defined the hierarchical honeycomb structure as shown in the Figure

1.1.

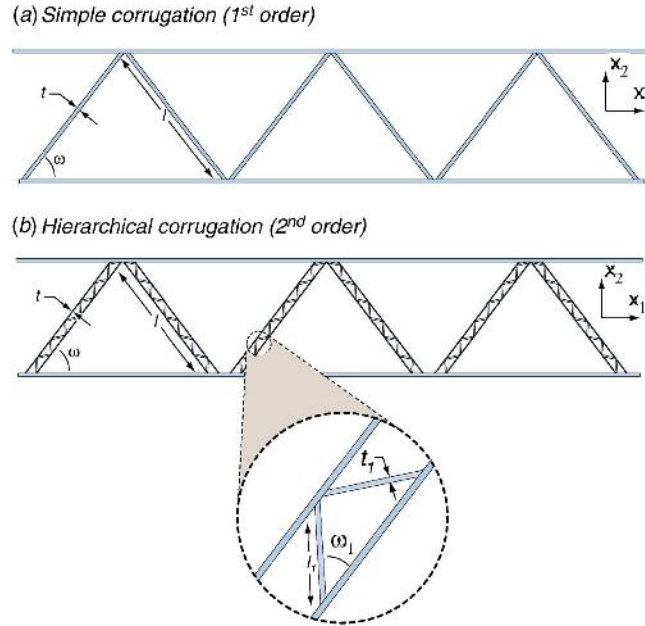


Figure 1.1. Hierarchical structure suggested by Kooistra

He suggested to replace homogeneous cell walls of the honeycomb structure by trusses. He derived analytical expressions for compressive and collapse mechanism of the second order structure and used these expressions to select design for the second order. He optimized the second order design to maximize the collapse strength for the same mass. He found from these analytical expressions that the second order truss demonstrate collapse strength about ten times greater than that of a first order truss of the same relative density which he verified with experimental investigation. But there was no enhancement in the stiffness to weight ratio with the hierarchy.

Fan [7] suggested hierarchical structures with sandwich walls. His proposed design of hierarchical structure is shown in the Figure 1.2.

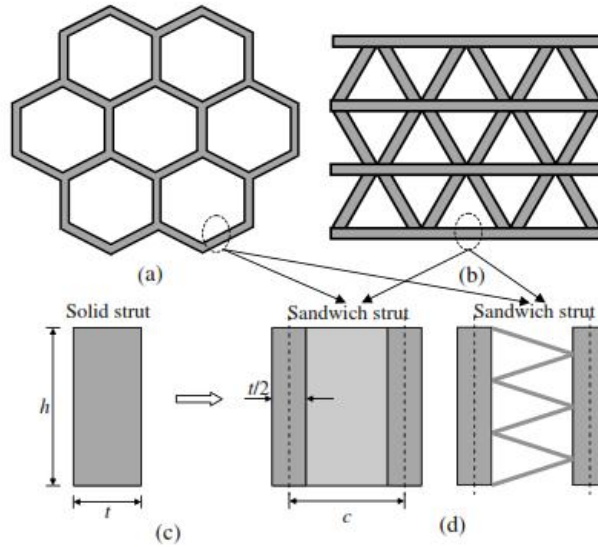


Figure 1.2. Hierarchical structure suggested by Fan

He suggested to replace homogeneous cell walls of the honeycomb structure by different material or trusses similar to Kooistra. He deduced the relations for the stiffness, buckling strength, plastic collapse strength, brittle failure strength and fracture toughness. He found that enhancement in mechanical properties (stiffness, Euler buckling strength, plastic collapse strength, brittle failure strength) of second order hierarchical honeycomb is substantial.

Ajdari [8,9] created the hierarchical by replacing every three-edge vertex of a regular hexagonal lattice with a smaller hexagon and named it as self-similar hierarchical structures. The structure is shown in Figure 1.3.

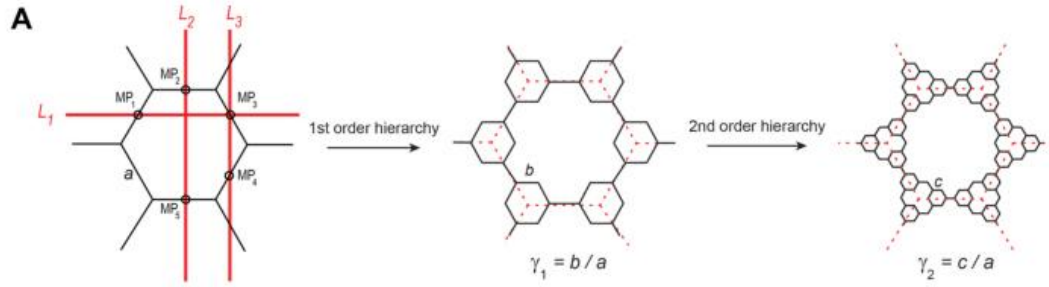


Figure 1.3. Hierarchical structure suggested by Ajdari

He found that such structures result in isotropic in-plane elastic properties (effective Young's modulus and Poisson's ratio). These properties can be tailored with the different ratios for different hierarchical orders. The result with hierarchical honeycombs of first and second order was up to 2.0 and 3.5 times stiffer than regular honeycomb at the same mass.

Oftdeh [10] has carried out further analysis on these self-similar hierarchical honeycombs. His results show that anisotropic hierarchical honeycombs of first to fourth order can be 2.0–8.0 times stiffer and at the same time up to 2.0 times stronger than regular honeycomb at the same wall angle and the same overall average density. Plastic collapse analysis showed that anisotropic hierarchical honeycomb has the larger plastic collapse strength compared to regular hierarchical honeycomb of the same order at certain oblique wall angles.

Oftdeh's results also show that the effective elastic modulus of the self-similar cellular material can be increased significantly by increasing the hierarchical order while preserving the structural density. His studies indicate that there can be significant enhancement of performance by adding the structural hierarchy. His work provides

insight into how incorporating hierarchy into the structural organization can play a substantial role in improving the properties and performance of materials and structural systems and introduces scope for development of new metamaterials with tailorable properties.

Babak [11] also carried out research on these self-similar hierarchical structures. He explored over a range of loadings and iteration parameters using analytical and numerical techniques to investigate both elastic and plastic properties. His results indicate that a wide variety of specific stiffness and specific strengths (up to fourfold increase) can be achieved. The results offer insights into the potential value of iterative structural refinement for creating low-density materials with desired properties and function.

Mousanezhad [12] considered spiderweb type hierarchical structures. Figure 1.4 shows typical spiderweb hierarchical structure with hierarchical parameters.

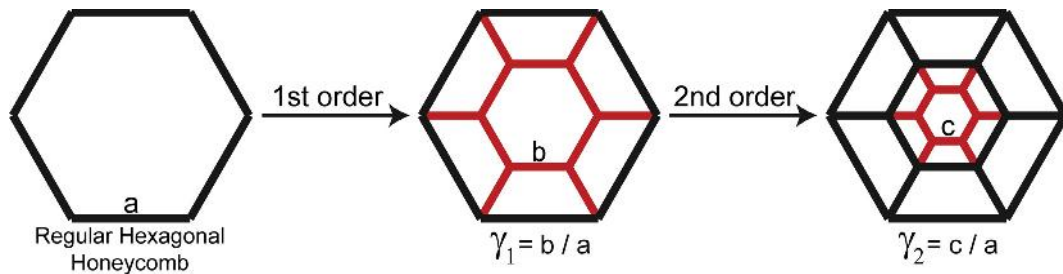


Figure 1.4. Spiderweb hierarchical structure suggested by Mousanezhad

He carried out analytical modeling, numerical simulations, and mechanical testing of these structures. He found that the isotropic in-plane properties (Young's modulus and Poisson's ratio) of the structures are controlled by dimension ratios in the

hierarchical pattern of spiderweb. He says that the main feature of these structures is combination of high stiffness and toughness.

Taylor et al [13] has proposed hierarchical structures by adding sub structures to honeycombs. He investigated the effects of adding hierarchy into a structure, at the exact same density, on the elastic properties especially elastic modulus. The structure analyzed by him is shown in the Figure 1.5.

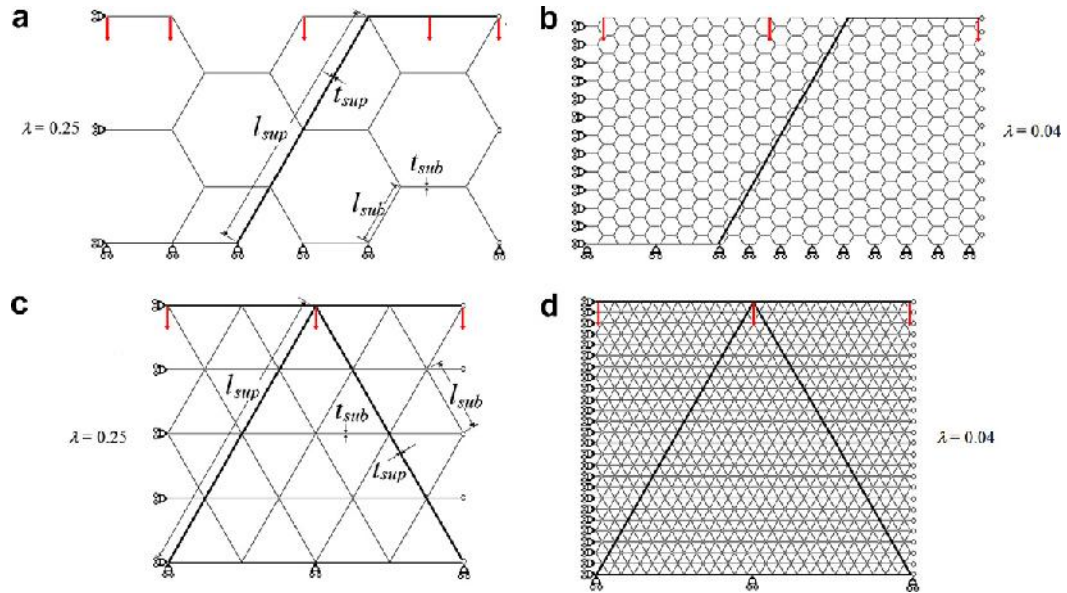


Figure 1.5. Hierarchical structure suggested by Taylor

He explored the effects of adding such hierarchy in honeycomb with hexagonal, triangular or square geometry via simulation using finite element analysis. He found that the introduction of a hierarchical sub-structure into a honeycomb, in most cases, has a deleterious effect upon the in-plane density specific elastic modulus, typically a reduction of 40 to 50% vs a conventional honeycomb. He further suggested that with careful design of functionally graded unit cells it is possible to exceed, by up to 75%, the density

specific modulus of conventional versions. Also, negative Poisson's ratio sub-structure also engenders substantial increases to the density modulus versus regular honeycombs.

Dag Lukkassen [19,20] studied reiterated honeycombs with different micro-levels. The micro-levels are formed by subdividing edges by 3, creating a symmetric interior with 6 cells surrounding a center cell. Further levels are achieved in the hierarchy using the same subdivision of the previous level cells. He found bounds for effective thermal properties of these structures using homogenization theory. While there have been some bounds for mechanical properties, there has not been an extensive study of the effects of changing the thickness ratio between the base honeycomb and interior honeycombs.

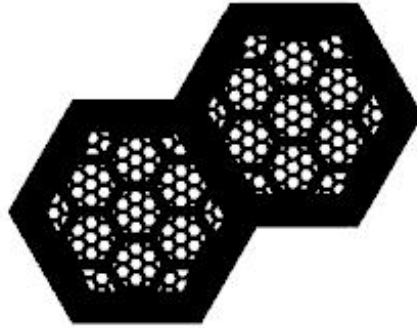


Figure 1.6. Iterated hierarchical honeycomb structure suggested by Lukkassen

1.2 Motivation for present work

Another application of honeycomb structures is in the field of sound transmission, sound scattering. The major components that affect the sound transmission capabilities of a panel are its in plane properties and weight.

Griese [2] analyzed the sound transmission loss through honeycomb structure by varying the geometry of the honeycomb structure. Galgalikar [14] performed the optimization of honeycomb sandwich panel for maximum sound transmission loss and found that honeycomb structure with negative Poisson's ratio has better sound transmission loss characteristics. Joshi [3] carried out the effective properties analysis of chiral honeycomb structure and also carried out analysis of sound transmission through these structures. Iyer [15] carried out the acoustic scattering and radiation response analysis of circular hexagonal and auxetic honeycomb structures. Mor [16] carried out acoustic scattering response analysis of hierarchical honeycomb structures for cylindrical and spherical structures.

As discussed earlier, honeycombs are two dimensional cellular structures that are used for many applications including energy absorption and thermal insulation. The stiffness and strength of honeycombs is controlled by the bending of the cell walls when exposed to loading [1]. This means that if load is gradually transferred from the cell walls deformation and bending can be minimized. Thus, increasing the energy that can be absorbed.

The main properties that define the in-plane behavior of cellular materials are effective Young's modulus, effective shear modulus and effective Poisson's ratio. As discussed in Section 1.1, some prior work has been done to study hierarchical structure. The way of incorporating hierarchy in the honeycomb structure adopted by researchers is different. While these studies have considered primarily Young's modulus of the hierarchical structures, the effective shear modulus and Poisson's ratio has not been

studied carefully, especially for different thickness ratios in the hierarchy. In this work, all of the mechanical properties the reiterated hierarchic honeycomb structures are considered with a wide range of cell wall thickness ratio for level 1 hierarchy, including the limiting configuration off completely removing the underlying base cell structure walls, leaving only the interior cells.

Furthermore, Taylor showed that auxetic honeycomb substructures engender substantial increase to the density modulus versus regular honeycomb substructures in the hierarchic structures he studied. In this work, the idea of reiterative hierarchy is generalized to a novel multi-level hierarchical structure for auxetic honeycomb structures. The effective properties of these structures are compared with non-hierarchical, conventional honeycomb structures.

1.3 Objective of thesis

The objectives of this thesis are to study the effective mechanical properties of regular and new auxetic first order reiterative honeycomb structures using finite element analysis. The specific objectives are the following:

- (1) Develop hierarchical geometry for regular and auxetic honeycomb structures.
- (2) Develop a finite element model using commercially available software to study the effective properties of hierarchical honeycomb structures.
- (3) Investigate the key effective mechanical properties of these structures.
- (4) Compare these effective properties with non-hierarchical, conventional honeycombs of the same mass.

1.4 Outline of thesis

First order regular and auxetic honeycomb structures are modeled using the finite element solver ABAQUS 6.11. An approximately square patch of honeycomb structure is modeled with repetitive unit cells to obtain the effective in-plane mechanical properties (Young's modulus, Poisson's ratio and Shear modulus).

Chapter 1 consists of the introduction to hierarchical honeycomb structures. Different geometries of hierarchical structure are discussed in the brief literature survey. The main findings regarding the effective properties of these structures are also discussed. Based on the literature review, the gaps in the previous work are identified. The motivation and objectives for present work are stated.

Chapter 2 describes the geometry of the first order regular and auxetic honeycomb structures. A detailed unit cell representation for first order honeycomb structures and basic geometric parameters that make up the hierarchical structures are discussed. Analytical formulation of effective mechanical properties of zeroth order honeycomb structure suggested by Gibson and Ashby [1] is also discussed.

Chapter 3 consists of the detailed analysis of in-plane effective mechanical properties for zeroth order and first order regular honeycomb structures by using finite element solver - ABAQUS 6.11. An approximately square overall dimensions are maintained with sufficient number of unit cells along x and y-direction to give accurate results. First zeroth order structure is analyzed and model setup, boundary conditions are verified and then similar conditions are applied to first order hierarchical structure. A parametric study of thickness ratio is then carried out to obtain the effects of mass shared

between different levels of hierarchy. Results of these first order regular honeycomb structures are compared with zeroth order regular honeycomb structure of same mass.

Chapter 4 consists of the detailed analysis of in-plane effective mechanical properties for zeroth order and first order auxetic honeycomb structures by using finite element solver - ABAQUS 6.11. Same procedure is followed as that of the regular honeycomb structure discussed in Chapter 3. Similar parametric study is then carried out to obtain the effects of mass shared between different levels of hierarchy. Results of these first order auxetic honeycomb structures are compared with zeroth order auxetic honeycomb structure of same mass.

In Chapter 5, key results of the present research work are summarized and recommendations for future work are also made based on this research.

CHAPTER TWO

GEOMETRY OF FIRST ORDER HONEYCOMB STRUCTURES

The mass density of honeycomb structures allows the design of light and stiff sandwich panels. Honeycomb structures find applications in various fields like sound transmission, sound scattering, thermal insulation, crash testing of vehicles, aerospace lightweight construction etc [1-5]. Due to the special geometry, honeycomb structures exhibit effective properties which are different than the material of which they are made up of. Thus, it is vital to study the overall stiffness and mass properties for use in detailed analysis. This makes important to determine the effective elastic moduli of structures under different loading conditions. In the present work, in-plane stiffness properties of the honeycomb structure are studied. In this chapter, detailed geometry of first order reiterative honeycomb structures and parameters that define the geometric configuration are discussed.

2.1. Introduction to hierarchical structures

The concept of structural hierarchy in materials is developed in different fields. Hierarchical structures are represented when structures themselves contain the same underlying structural elements. The main objective of introducing hierarchy in honeycomb structures is to further enhance the mechanical behavior of structure. It is believed that by introducing the hierarchy, the efficiency of resulting structure in terms of load bearing capacity can be enhanced [6-13].

The arrangement of the cells in the hierarchical structures play an important role in identifying the effective properties of the overall structure. As discussed in chapter one, different ways of incorporating hierarchy in honeycomb structures are studied. From this previous research, it is understood that by introducing the hierarchy structures can be made more efficient [6-13]. In the present work, properties of first order regular and auxetic honeycomb structures are studied.

As discussed earlier, honeycomb structures are used in many applications. The stiffness and strength of honeycombs is controlled by bending of the cell walls when load is applied. If we introduce hierarchy such that loads from the cell walls are transferred gradually, we can minimize the deformation and bending. Hence, making the overall structure stiff. Geometry of the first order hierarchical structures is discussed in the next few sections of this chapter. The effect of mass shared by zeroth order and first order honeycomb structures on the overall properties is studied in this work by varying the thickness ratio. Performance of these structures is studied in further chapters.

2.2. Geometry of first order regular honeycomb structure

The unit cell of zeroth order regular honeycomb structure is highlighted in Figure 2.1 (a). The geometry of honeycomb structure can be completely defined by the following parameters: horizontal member length (H), slant edge length (L), cell angle (θ). These parameters can be used to determine the effective mechanical properties of the overall structure. The unit cell shown in the Figure 2.1 (a) has $\theta = 30^\circ$ and $H = L$, corresponding to the standard hexagonal shape. In this work, this shape is defined as

zeroth order regular honeycomb structure. The main characteristic of these regular zeroth order honeycomb structures is that they are transversely isotropic.

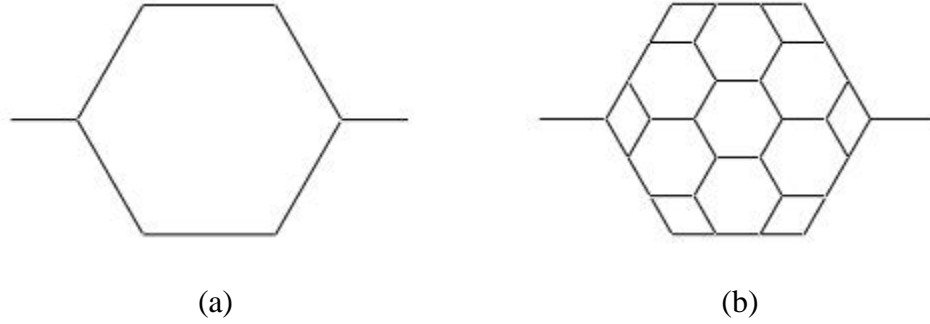


Figure 2.1. Regular zeroth order and first order hierarchical honeycomb structures

In this study, the first order regular reiterative honeycomb structure is created by introducing six smaller honeycombs having $(h = H / 3)$, $(l = L / 3)$ and $\alpha = 30^\circ$. Each of these smaller honeycombs are placed in the conventional honeycombs, zeroth order honeycomb, such that midpoints of the edges shared by them coincide with each other. The unit cell of first order regular honeycomb structure is highlighted in Figure 2.1 (b).

2.3. Geometry of first order auxetic structure

The unit cell of zeroth order auxetic honeycomb structure is highlighted in Figure 2.2 (a). The geometry of honeycomb structure can be completely defined by the following parameters: cell angle, slant edge length, horizontal member length. These parameters can be used to find out the effective mechanical properties of the overall structure. The unit cell shown in the figure has $\alpha = -30^\circ$ and $H = 2L$, corresponding to the standard hexagonal shape. In this work, this shape is defined as zeroth order auxetic

honeycomb structure. The main characteristic of these zeroth order auxetic honeycomb structures is that they are transversely isotropic and have negative Poisson's ratio.

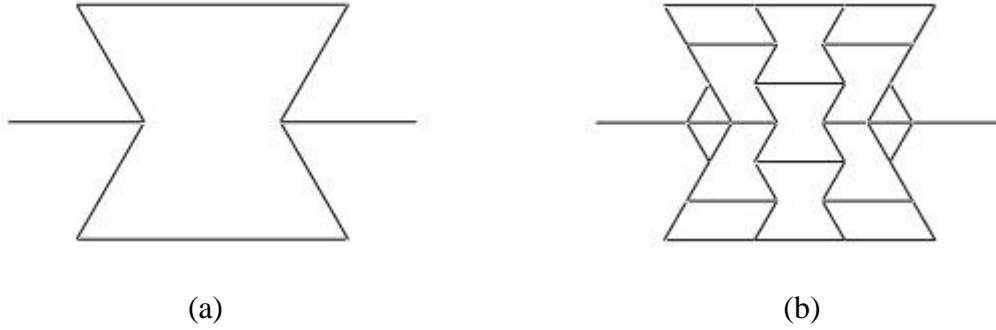


Figure 2.2. Auxetic zeroth order and first order hierarchical honeycomb structures

In this study, the first order auxetic honeycomb structure is created in the same way as of the first order regular honeycomb structure by introducing six smaller honeycombs having $(h = H/3)$, $(l = L/3)$ and $\alpha = -30^\circ$. Each of these smaller honeycombs are placed in the conventional honeycomb, zeroth order, such that midpoints of the edges shared by them coincide with each other. The unit cell of first order auxetic honeycomb structure is highlighted in Figure 2.2 (b).

2.4. Effective mechanical properties analytical solution for zeroth order

The main advantage of honeycomb structures is that the overall properties of the structure can be tailored easily with the geometrical parameters horizontal member length (H), slant edge length (L), cell angle (α). Based on Euler-Bernoulli beam theory, Gibson and Ashby's proposed cellular material theory [1]. This can be used to find out the overall effective properties of the structure. The effective properties studied in the present study are focused on in-plane behavior of the structure. In the next chapters, these

effective properties of the structure are compared with numerical simulation. The effective properties of the honeycomb structure based on the geometrical parameters are as follows. The effective Young's moduli of zeroth order honeycomb structure made of material having Young's modulus, E_s , is given by

$$E_1 = \left(\frac{t}{L} \right)^3 \left(\frac{H/L + \sin \theta}{\cos^3 \theta} \right) E_s$$

$$E_2 = \left(\frac{t}{L} \right)^3 \frac{\cos \theta}{(H/L + \sin \theta) \sin^2 \theta} E_s$$

The effective shear modulus is given by

$$G_{12} = \left(\frac{t}{L} \right)^3 \frac{H/L + \sin \theta}{(H/L)(1 + 2H/L) \cos \theta} E_s$$

The effective Poisson's ratio is given by,

$$\nu_{12} = \frac{(H/L + \sin \theta) \sin \theta}{\cos^2 \theta}$$

$$\nu_{21} = \frac{\cos^2 \theta}{(H/L + \sin \theta) \sin \theta}$$

It is important to note that the effective Poisson's ratio of the honeycomb structures does not depend on the base material. It only depends on the geometrical parameters.

Honeycomb structures follow the reciprocal theorem,

$$E_1 \nu_{21} = E_2 \nu_{12}$$

Regular and auxetic honeycomb structures considered in this work have special nature of transversely isotropy. Due to the geometry of the structure and formulae of effective properties, Poisson's ratio of regular and auxetic honeycomb structures is 1 and -1

respectively. This makes these structures to have same effective Young's modulus in both directions.

2.5. Mass properties

Mass is an important factor that decides the overall property of the structure. By adding the structural elements of hierarchy, the mass of the structure will increase if the cell wall thickness is kept constant. The effective properties of all the first order structures are compared with the zeroth order structure having same mass. Mass of the first order structure, m_1 , is given by,

$$\begin{aligned} m_1 &= m_0 + m_h \\ &= r_0 v_0 + r_h v_h \\ &= r_0 d_0 (t_0 l_0 + t_h l_h) \end{aligned}$$

In this equation, $r = (t_0 / t_h)$ is the ratio of thicknesses of zeroth order edges to first order edges. Material used in all the analysis is aluminum thus making ρ_0 constant. Depth of the structure, d_0 , is considered as 1 m. The values of thicknesses of zeroth order t_0 and hierarchical structure t_h are calculated at specific thickness ratios, r , making the total mass equal to the mass of zeroth order structure. By this, the mass shared by zeroth order and hierarchical structure is varied.

In this analysis thickness are assigned to zeroth order structure and hierarchical structure keeping the mass constant. By this, the mass shared by different levels of hierarchy is varied. The main aim of the present study is to analyze the effect of mass shared by different levels of hierarchy on the overall properties of the structure. The in-

plane properties of the structure studied are effective Young's modulus, effective Poisson's ratio and effective shear modulus. These properties are compared with the conventional i.e. zeroth order honeycomb structure in the upcoming chapters.

CHAPTER THREE

EFFECTIVE MECHANICAL PROPERTIES FOR FIRST ORDER REGULAR HIERARCHICAL HONEYCOMB

In this chapter, the behavior of first order hierarchical regular hexagonal honeycomb structure under in-plane loading condition is investigated to obtain effective mechanical properties Young's modulus, Poisson's ratio and shear modulus. Comparison of these properties of first order structure with zeroth order regular hexagonal honeycomb of the same mass is made in the Section 3.4.

3.1 Previous studies

As discussed in chapter 1, Taylor [13] has considered introducing hierarchy in the regular honeycomb structure. He considered the effects of using hexagonal, triangular, rectangular substructures. He considered the effect of mass distribution between the substructure and superstructure on the effective elastic modulus. He also considered non-uniform distribution of mass in substructure. He found that in most cases introducing the hierarchy has deleterious effect with typically reduction of 40% to 50% in the in-plane density specific elastic modulus. But in some cases the density specific elastic modulus was increased up to 75% of the conventional honeycombs. His work was mainly focused on effective Young's modulus. To completely define the effective properties of cellular material effective Young's modulus, effective Poisson's ratio and effective shear modulus are important. In this work, all these properties are analyzed for varying distribution of mass between hierarchical structure and regular structure.

3.2 Model setup

In the present work, the first order regular hierarchical honeycomb structure has been modeled by using commercial finite element solver ABAQUS 6.11. Different models were created by varying the number of unit cells in x and y direction and keeping the overall dimension of the structure square. This was done in order to avoid the boundary effects and to make sure that effective properties are not dependent on the number of unit cells. 5 cells along x-direction and 8 cells along y-direction as shown in Figure 3.1 give a considerable accuracy in obtaining the effective mechanical properties of the structure. The difference between the effective properties of this structure and structures having more number of unit cells was less than 2%. The properties obtained from this model are in accordance with the theoretical formulae discussed in the earlier chapter suggested by Gibson and Ashby [1].

The zeroth order regular honeycomb structure investigated by Griese [2] is considered as a reference. Figure 3.1 shows zeroth order regular honeycomb structure.

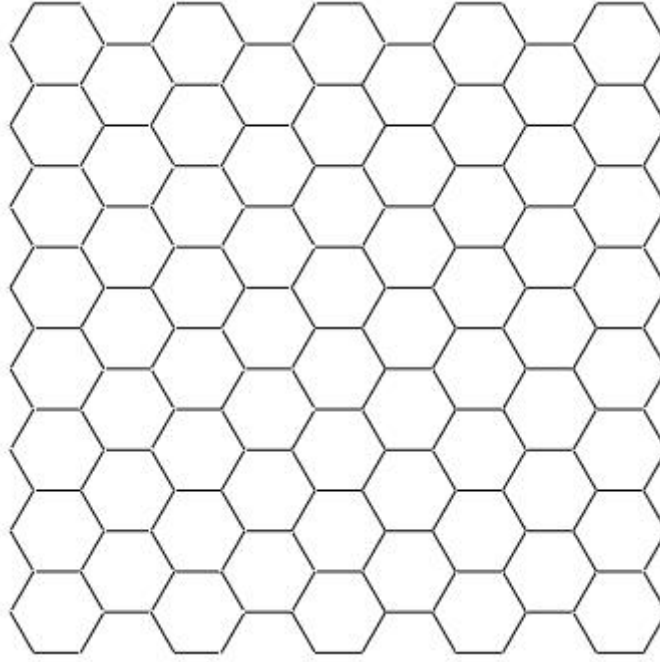


Figure 3.1 Model used to investigate effective mechanical properties of zeroth order regular honeycomb structure

On the basis of overall dimensions of the sandwich panel with hexagonal core, Griesse obtained the dimensions for the unit cell of regular honeycomb structure as: $H = L = 28.87$ mm for the case of one unit cell of hexagonal core. A 2D planar deformable part is created using ABAQUS version 6.11. As mentioned earlier, the structure created consists of 5 cells in x-direction and 8 cells along y-direction.

Linear 2 node cubic beam elements (B23) are used to mesh the structure. To use Euler-Bernoulli elements the ratio of length to thickness should be greater than 10 [17]. Smallest length of the beam in the level-0 structure is 28.87 mm. Thickness assigned to the structure is 1mm, making the ratio 28.87. Hence, beam 23 elements having Euler-Bernoulli formulation can be used. Seed size is selected such that there are at least 4

elements along the edge. Mesh convergence study is also carried out to make sure the seed size is not affecting results. Generated mesh is shown in the Figure 3.2.

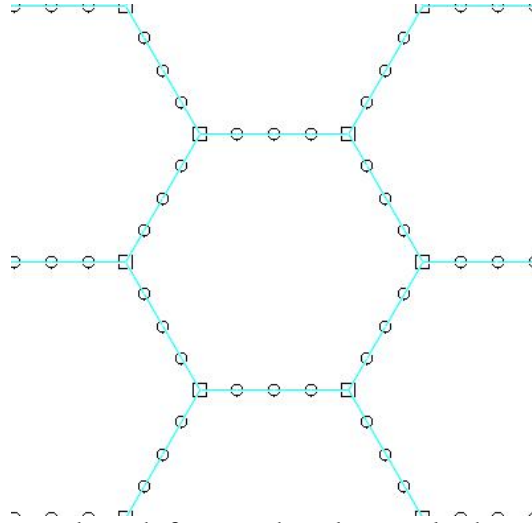


Figure 3.2. Generated mesh for zeroth order regular honeycomb structure

A rectangular beam section with aluminum is created with unit depth in z-direction and thickness as 1mm. The material properties of aluminum specified in the analysis are density () 2700 kg/m^3 , Young's modulus 71.9 Mpa and Poisson's ratio 0.33 [18].

After creating the mesh, different node sets are created to apply boundary conditions and to calculate different properties of the structure from the field output request.

3.3. Step and output analysis

A static analysis of the structure is carried out to investigate the properties. Static General Step is created and default option is selected. The analysis is carried out in the

elastic region and structure obeys Hooke's law. All the deformations are small and NLGeom option is turned off in Abaqus.

The average stresses are calculated from the nodal reaction forces, so nodal reaction forces are requested on the left hand side extreme nodes. To calculate Poisson's ratio, displacements at 4 interior points shown in the fig. are requested. The interior points are selected so as not to have any boundary effects.

The mass of all the structures, zeroth order and first order structures, is made same by adjusting the thicknesses of the edges of the honeycomb structure. The thicknesses of the various structures of zeroth and first order edges are calculated from the MATLAB code attached in the appendix. To confirm the mass of all the models is same, Current mass of the model or region option is selected in the history output request.

3.4. Boundary conditions

To calculate the effective mechanical properties displacement boundary condition is applied x direction loading at extreme left and extreme right nodes. For this two node sets are created consisting of the end vertices at extreme right and extreme left end cells. Along with this boundary condition, y-direction displacement and rotation degrees of freedom at the point of symmetry in y direction at the ends is also applied so that the structure does not move in the y direction as shown in the Figure 3.3.

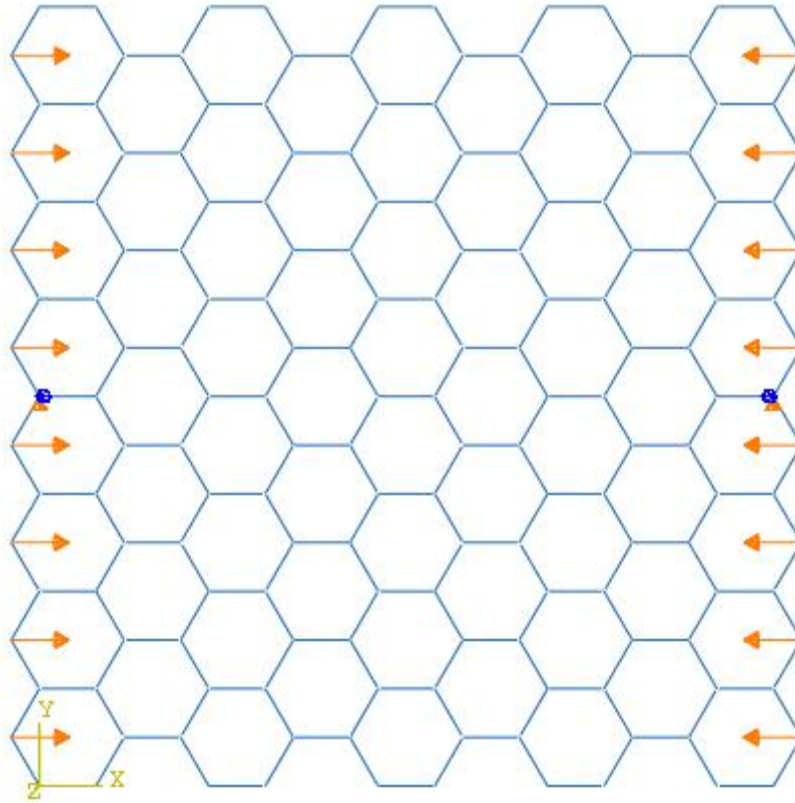


Figure 3.3. Boundary conditions applied to calculate effective Young's modulus

To calculate effective shear modulus, loading in y direction is applied. Displacement boundary condition is applied as it gives better results. The same model is used to calculate the shear modulus. Displacement equivalent of 2% strain in y-direction is applied at left most nodes. All degrees of freedom of extreme right nodes are constrained as shown in the Figure 3.4.

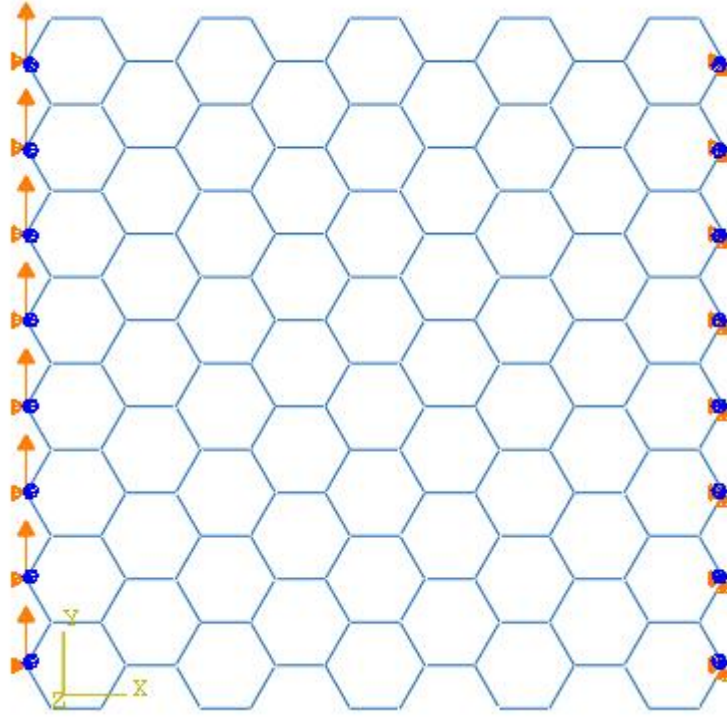


Figure 3.4. Boundary conditions applied to calculate effective shear modulus

3.5. First order regular hierarchical honeycomb structure

To study the effect of distribution of mass between different levels of hierarchy, different thicknesses to different orders of hierarchy are assigned. Different models of first order hierarchical honeycomb structures with different ratios of thicknesses of zeroth order to first order having the same mass as that of the zeroth order regular honeycomb were created. Two different sets were created containing zeroth order and first order edges as shown in the Figure 3.5 and Figure 3.6. In Figure 3.5, highlighted structure represents zeroth order structure. And in Figure 3.6, highlighted structure represents first order hierarchical honeycomb structure. It is expected that due to the symmetry, the

structure is transversely isotropic similar to zeroth order structure and has same effective elastic modulus in x and y direction.

Two sets were used to assign specific thicknesses to the specific level of hierarchy. By this the mass shared by each hierarchical order is varied. The total mass of all these first order hierarchical regular honeycomb structures have the exact same mass as that of the zeroth order regular honeycomb structure. The total lengths of first order and zeroth order edges are calculated. For a specific ratio r , ratio of thicknesses of zeroth order edges to first order hierarchical edges, thicknesses are calculated such that total mass remains the same.

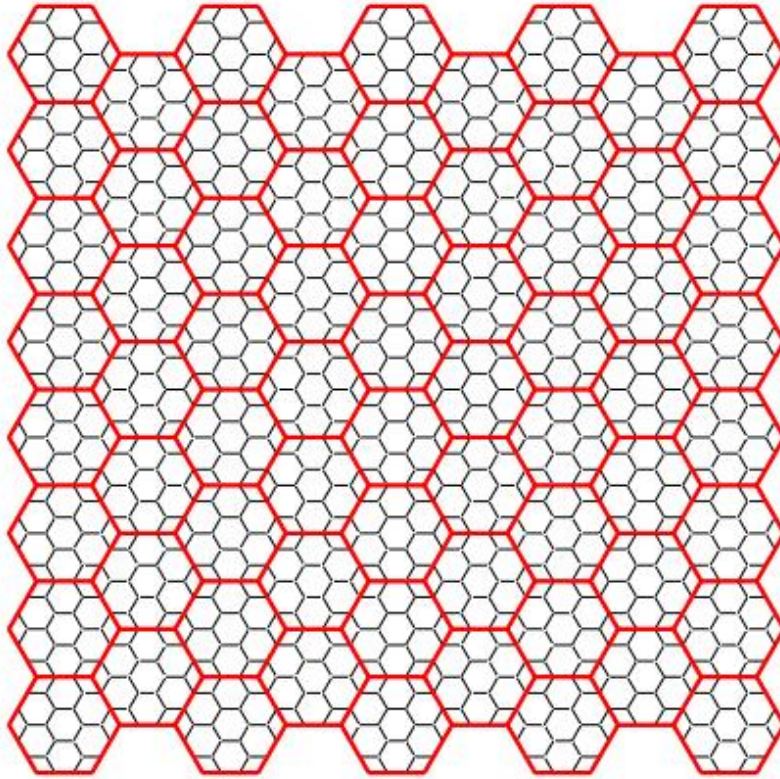


Figure 3.5. Highlighted structure representing zeroth order edges

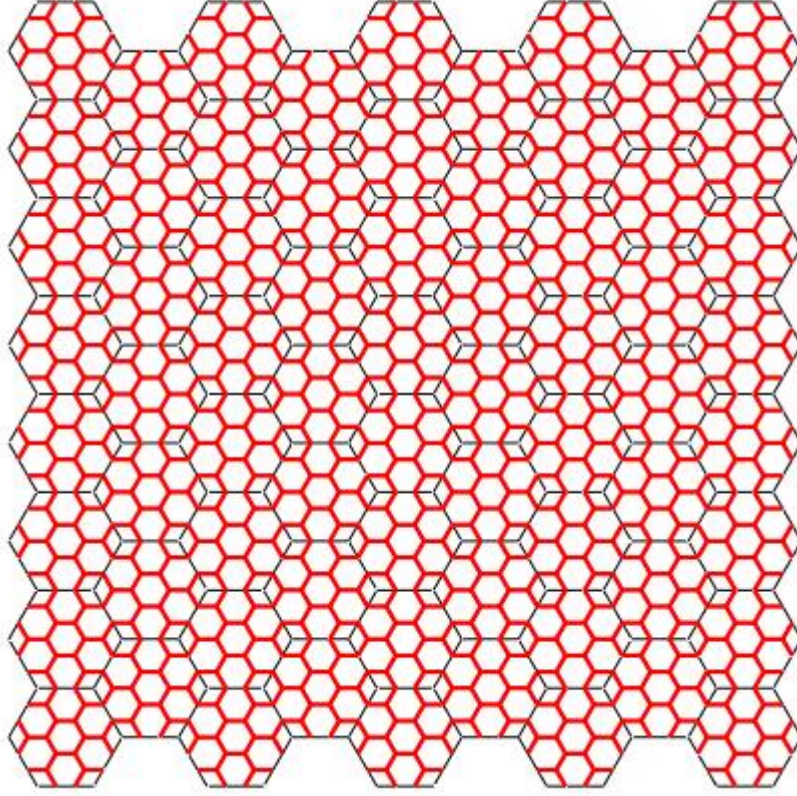


Figure 3.6. Highlighted structure representing first order hierarchical edges

Ratio, $r = (t_0 / t_h)$	Thickness of zeroth order edges, t_0 (mm)	Thickness of first order edges, t_1 (mm)
0	0	0.35443
0.1	0.0041747	0.41747
0.2	0.0080149	0.40074
0.3	0.11559	0.38530
0.4	0.14840	0.37101
0.5	0.17887	0.35774
0.6	0.20723	0.34538
0.7	0.23369	0.33385
0.8	0.25845	0.32306
0.9	0.28166	0.31295
1	0.30346	0.30346
2	0.46562	0.23281

3	0.56653	0.18884
4	0.63539	0.15885
5	0.68537	0.13707
6	0.72330	0.12055
7	0.75306	0.10758
8	0.77705	0.097131
9	0.79679	0.088532
10	0.81332	0.0081332

Table no. 3.1. Thicknesses of zeroth order and first order hierarchical edges

To use Euler-Bernoulli elements, ratio of length of the beam to thickness of the beam should be greater than 10. Smallest length of the beam in the structure is 9.6233 mm. For first order regular honeycomb case, largest thickness assigned is 0.41747 mm. Thus, ratio (l/t) is always greater than 23. Hence, Euler-Bernoulli elements can be used for the analysis.

Following Figure 3.7 represents the difference in thickness assigned to zeroth order and first order edges. To recognize the difference scaling factor is chosen 2 for thicknesses.

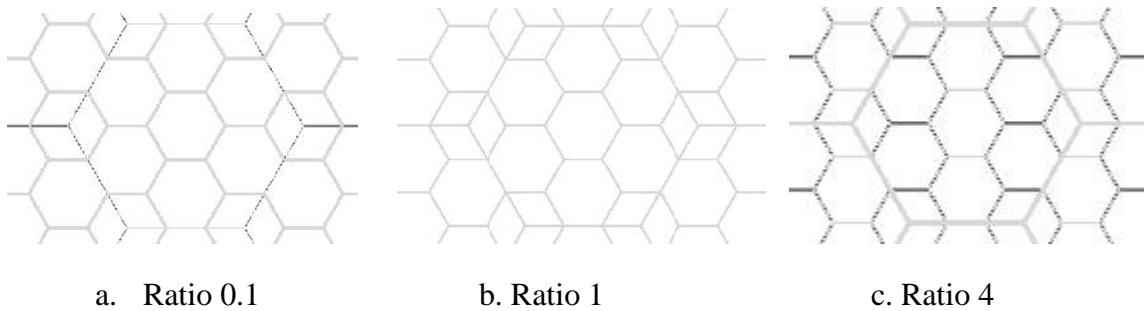


Figure 3.7. Schematic showing difference in thicknesses assigned to first and zeroth order edges

It can be seen that when the ratio is 0.1, thickness of first order edges is 10 times that of zeroth order edges. When the thickness ratio is 1, thicknesses of zeroth order and first order edges is same. When the thickness ratio is 4, thickness of zeroth order edges is 4 times that of first order. In short, when thickness ratio increases, thickness of zeroth order edges approach to 1 mm and that of first order edges approach to 0.

A similar setup has been used to investigate the effective mechanical properties of first order regular honeycomb structures as that of the zeroth order structure. To maintain the mass of all the structures same, a MATLAB code is used to find out the correct thicknesses of zeroth order and first order edges in the structure for different ratios. Thicknesses calculated from the MATLAB code are tabulated in Table no.3.1. These thicknesses are assigned to the respective sets using the python script. In the current analysis various thickness ratios from 0.1 to 10 on logarithmic scale are studied.

The basic model setup and the procedure followed to calculate effective mechanical properties is exactly the same as described in the section 3.2. The boundary conditions are shown in the Figure 3.8 and Figure 3.9.

Displacement boundary condition is applied x-direction loading at extreme left and extreme right nodes. For this two node sets are created consisting of the end vertices at extreme right and extreme left end cells. Along with this boundary condition, y-direction displacement and rotation degrees of freedom at the point of symmetry in y direction at the ends is also applied so that the structure does not move in the y direction. Applied boundary conditions are shown in Figure 3.8.

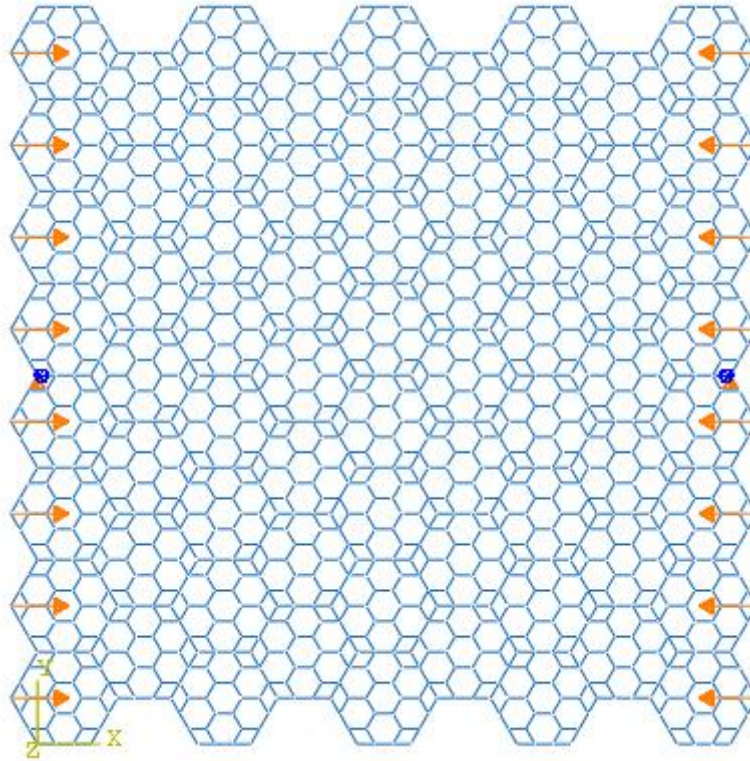


Figure 3.8. Boundary conditions applied to calculate effective Young's modulus of first order hierarchical honeycomb structure

To calculate effective shear modulus, loading in y direction is applied. Displacement boundary condition is applied as it gives better results. The same model is used to calculate the shear modulus. Displacement equivalent of 2% strain in y-direction is applied at left most nodes. All degrees of freedom of extreme right nodes are constrained. Applied boundary conditions are shown in Figure 3.9.

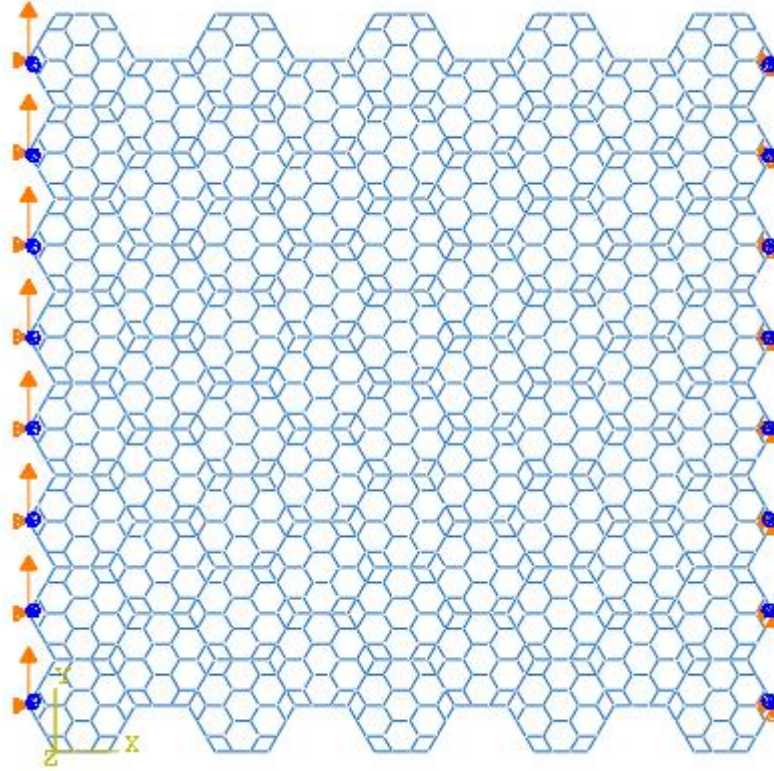


Figure 3.9. Boundary conditions applied to calculate effective shear modulus of first order hierarchical honeycomb structure

3.6. Effective mechanical properties calculation

3.6.1. Young's modulus and Poisson's ratio

Figure 3.3 and 3.8 shows the model setup for zeroth order and first order regular honeycomb structure for the investigation of Young's modulus and Poisson's ratio in x-direction loading conditions. As shown in the Figure 3.3, displacement equivalent of 2% strain of is applied on the extreme left and right hand side nodes of the structure. This displacement will generate the reaction forces at the corresponding nodes. The summation of the reaction forces on the nodes on one side (either right or left) of the structure divided by the cross sectional area will give us the average stress induced on the structure. Young's modulus is then calculated by dividing the average stress obtained

from the above calculation by the applied strain on the structure. Here, reaction forces at four points are taken into consideration. Two points on either side of the point of symmetry are chosen. These points are selected so as to avoid boundary effects. Sum of these forces is then divided by the corresponding area gives average stress.

$$\dagger_x = \frac{\sum_{i=1}^4 F_i}{l_y \times b}$$

where i = number of nodes

The effective Young's modulus is given by,

$$E_x = \frac{\dagger_x}{v_x}$$

Poisson's ratio of the material is given by the negative ratio of lateral strain to linear strain.

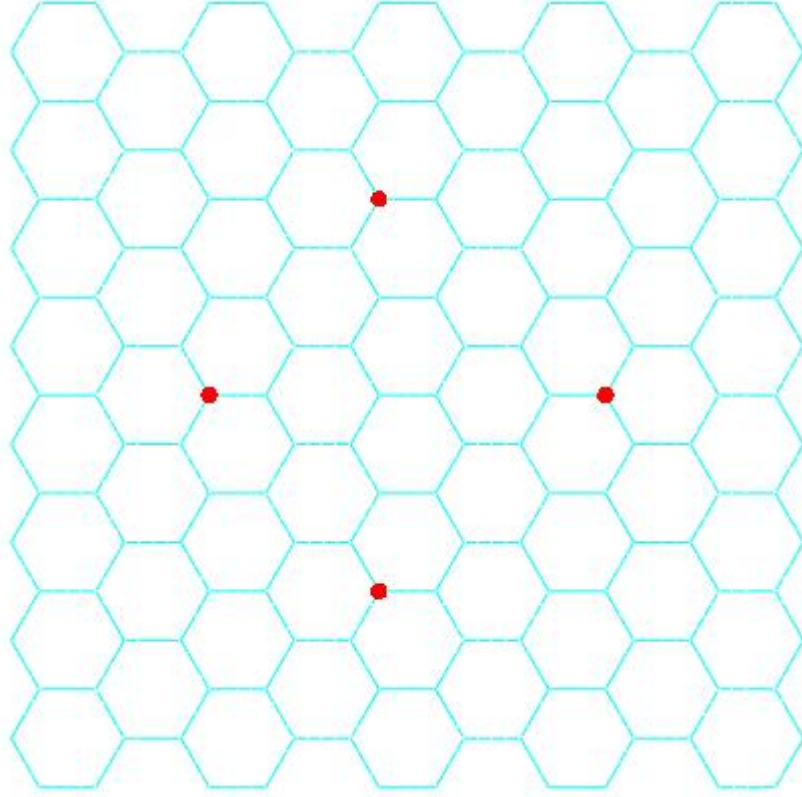


Figure 3.10. Nodes selected to investigate effective Poisson's ratio

$$\epsilon_{xy} = -\frac{V_y}{V_x}$$

For the structure shown in the figure the strain in x or y direction is calculated from the relative displacement in that direction of the pair of nodes represented by red dots in the inner region of the structure. Red dots in the Figure 3.10 represent the nodes which are used to calculate the Poisson's ratio. Nodes are selected from the inner region so as to eliminate the boundary effects.

3.6.2. Shear modulus

Figure 3.4 and 3.9 shows the model setup for the analysis of effective shear modulus of zeroth order and first order regular honeycomb structure respectively. All degrees of freedom on extreme right side nodes are constrained. Displacement equivalentl of 2% strain is applied at the left end extreme nodes and other degrees of freedom are constrained. The average shear stress can be calculated from the nodal reaction forces generated at these nodes divided by the area. Shear modulus is given by the ratio of average shear stress to shear strain. Here, reaction forces at four points are taken into consideration. Two points on either side of the point of symmetry are chosen. These points are selected so as to avoid boundary effects. Sum of these forces is then divided by the corresponding area gives average stress.

$$\ddagger = \frac{\sum_{i=1}^4 (F_y)_i}{l_y \times b}$$

where i = number of nodes

The effective shear modulus is given by,

$$G = \frac{\ddagger}{\chi}$$

3.7. Results and discussion

3.7.1 Zeroth order regular results

Deformed shape is shown in the Figure 3.11.

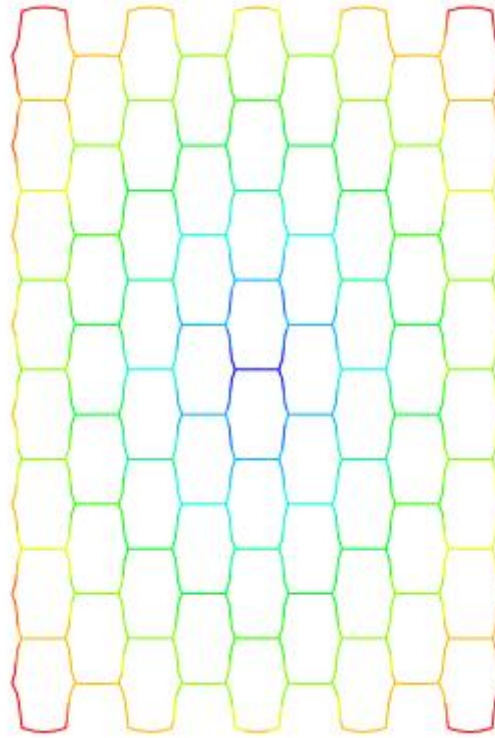


Figure 3.11. Deformed shape of zeroth order regular honeycomb structure for effective Young's modulus

The Figure 3.11 shows the deformed shape of the zeroth order regular honeycomb structure in x-direction loading case. Displacements equivalent to 2% strain are applied on both the extreme ends. Y-direction displacement and rotation at the point of symmetry at the ends are constrained so that the structure does not move.

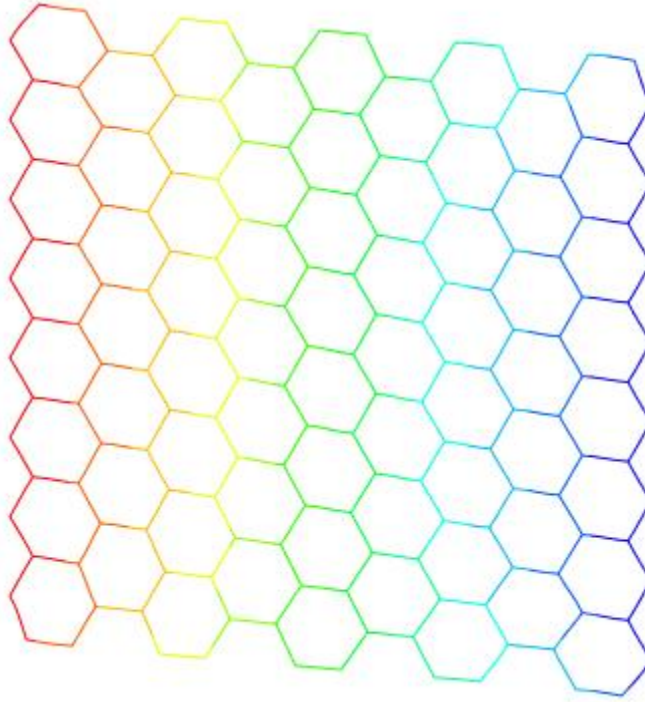


Figure 3.12. Deformed shape of zeroth order regular honeycomb structure for effective shear modulus

The Figure 3.12 shows the deformed shape of the zeroth order regular honeycomb structure in shear loading case. Displacement equivalent of 2% shear strain is applied in y-direction at extreme left vertices and other degrees of freedom are constrained.

The effective properties are tabulated in Table 3.2. Effective properties of the regular honeycomb structure are compared with theoretical values obtained from analytical values Gibson and Ashby formulae.

	FEA result level-0 (%diff)	FEA result level- 0 refined (%diff)	Theoretical
Young's modulus (MPa)	6.485 (6.02%)	6.762 (1.8%)	6.900
Shear modulus (MPa)	1.568 (9.07%)	1.679 (3%)	1.731
Poisson's ratio	1.158 (15.9%)	1.07 (7%)	1

Table no. 3.2 Results of zeroth order regular honeycomb structure

Level-0 refined structure is the structure with the same mass and overall dimensions with more number of cells. This structure is discussed in detail in Section 3.7.2.4. As more number of cells are considered, the values of the effective properties of this structure match more closely with theoretical values. These formulae are based on the assumption of infinite number of cells in x and y direction. This can be the reason of variation in the effective properties obtained from FEA simulation.

3.7.2 First order regular honeycomb structure results

3.7.2.1. Effective Young's modulus

Figure 3.13 shows the deformation shape of first order regular hierarchical honeycomb structure with thickness ratio, r , of 0.1 when x-direction loading is applied to calculate effective modulus and Poisson's ratio.

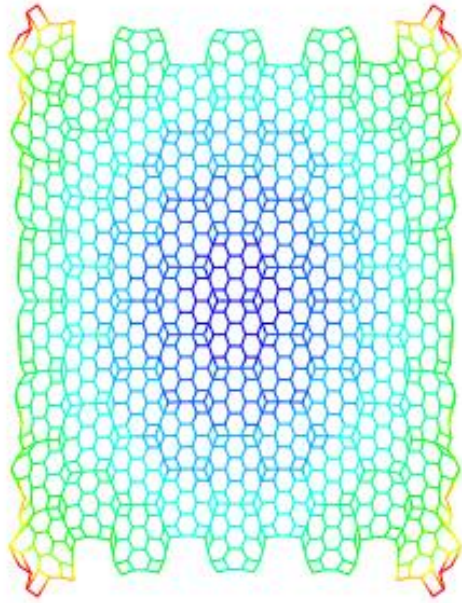


Figure 3.13. Deformed shape of first order regular hierarchical structure with thickness ratio 0.1

Following Figures 3.14 are the plots of Von Misses stresses for key values of ratio thickness, r , when uni-axial load is applied. The plots are of central patch in order to understand the stress distribution at cellular level.

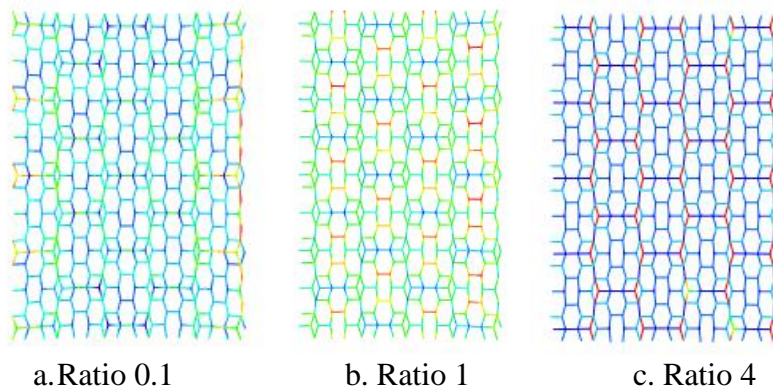


Figure 3.14. Von Misses stresses of first order regular hierarchical structure at key thickness ratios for elastic loading

Figure 3.15 shows the plot of relative effective Young's modulus of the first order regular honeycomb structure with respect to effective Young's modulus of zeroth order honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges.

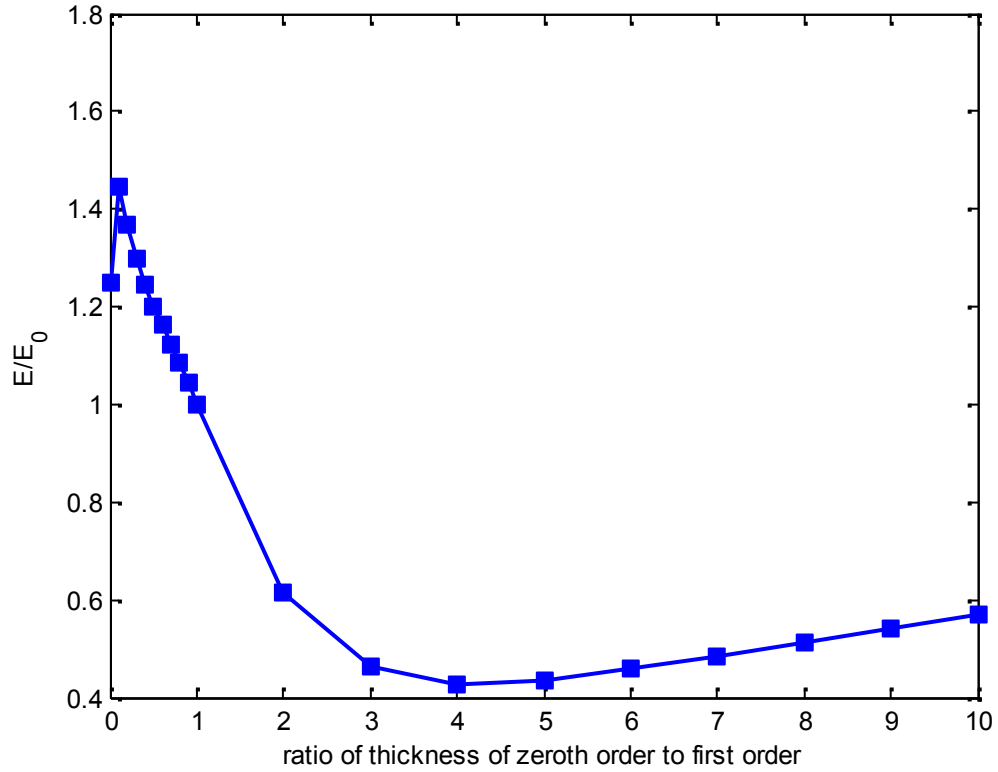


Figure 3.15. Plot of normalized effective Young's modulus of first order regular honeycomb structure vs thickness ratio

By changing the thickness ratio of level-0 to level-1 hierarchy, a nonlinear variation in mechanical properties is observed showing maximum and minimum values at specific ratios. From the plot it can be seen that when the ratio is less than 1 that is when the thickness of zeroth order edges is less than first order edges the effective Young's modulus is higher than that of the zeroth order regular honeycomb structure of same mass. Effective Young's modulus is highest at ratio 0.1 and is about 1.45 times that of the

zeroth order regular honeycomb structure of same mass. The Young's modulus decreases with the increase in the ratio till ratio 4. Effective relative Young's modulus of first order regular honeycomb structure is minimum between the ratio 4 to 5 and is about 0.42 times that of the zeroth order regular honeycomb structure of the same mass. After the ratio 5, effective Young's modulus again increases.

As the ratio increases thickness of zeroth order edges increases and approaches to 1mm which is the thickness taken for zeroth order regular honeycomb structure for the calculation of effective mechanical properties. So it is expected that the effective relative Young's modulus will increase till it reaches zeroth order effective Young's modulus. As the ratio gets lower and lower, effective Young's modulus of the structure increases. So as a special case of first order structure the results for ratio zero that is with first order structure and without zeroth order structure are discussed in the section 3.7.2.3.

3.7.2.2. Effective Poisson's ratio

To calculate effective Poisson's ratio, same models that were used to calculate effective Young's modulus are used.

Figure 3.16 shows the plot of effective Poisson's ratio of the first order regular honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges. This represents the behavior of the first order structure with different mass distribution between different levels of hierarchy.

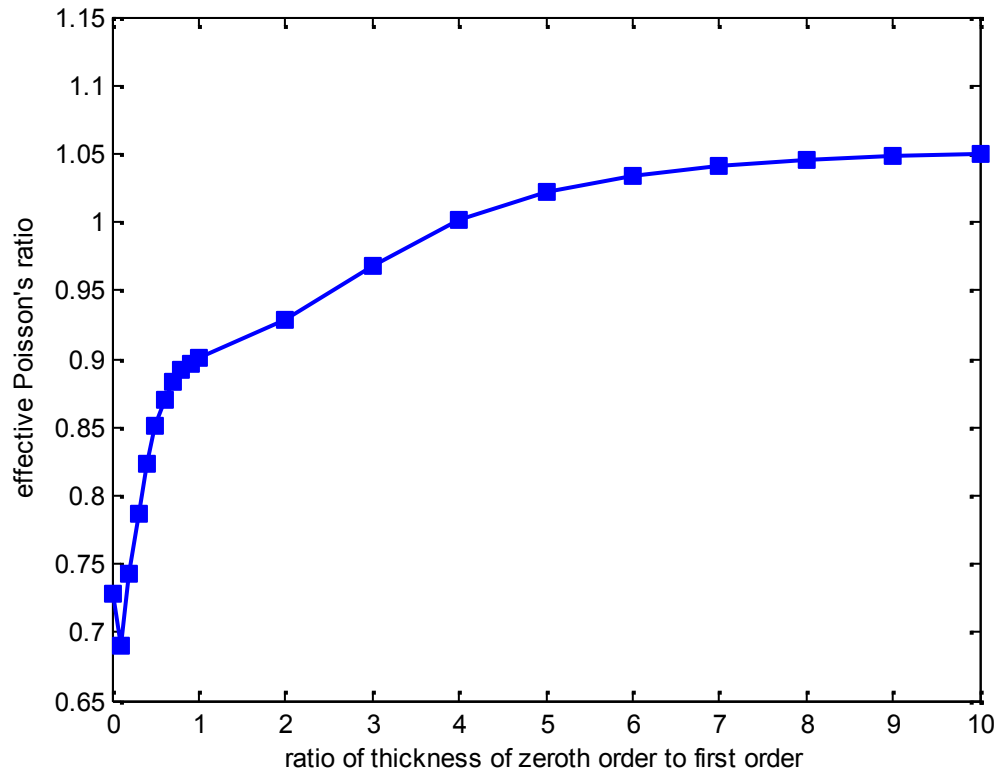


Figure 3.16. Plot of effective Poisson's ratio of first order regular honeycomb structure vs thickness ratio

The values of Poisson's ratio are of the values of the effective structure. Higher values of Poisson's ratio are characteristic of regular hexagonal honeycombs structures. The values of the Poisson's ratio are seen to be increasing from 0.67 to 1.05. The values of the effective Poisson's ratio are constantly increasing with the increase in thickness ratio. As the ratio increases, the slope of the line decreases. At ratio 10, the value of effective Poisson's ratio almost matches that of the estimated effective Poisson's ratio of zeroth order structure.

3.7.2.3. Effective shear modulus

Figure 3.17 shows the deformation shape of first order regular hierarchical honeycomb structure with thickness ratio, r , of 0.1 when subjected to shear loading.

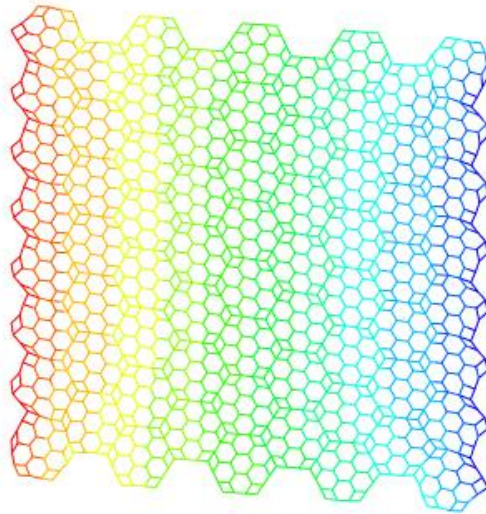


Figure 3.17. Deformed shape of first order regular hierarchical structure with thickness ratio 0.1

Following Figures 3.18 are the plots of Von Mises stresses for key values of ratio thickness, r when shear load is applied. The plots are of central patch in order to understand the stress distribution at cellular level.

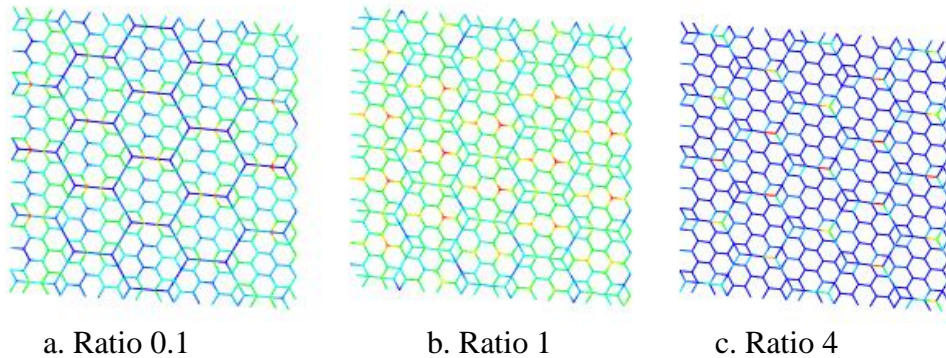


Figure 3.18. Von Mises stresses of first order regular hierarchical structure at key thickness ratios

Figure 3.19 shows the plot of relative effective shear modulus of first order regular honeycomb structure with respect to effective shear modulus of zeroth order regular honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges.

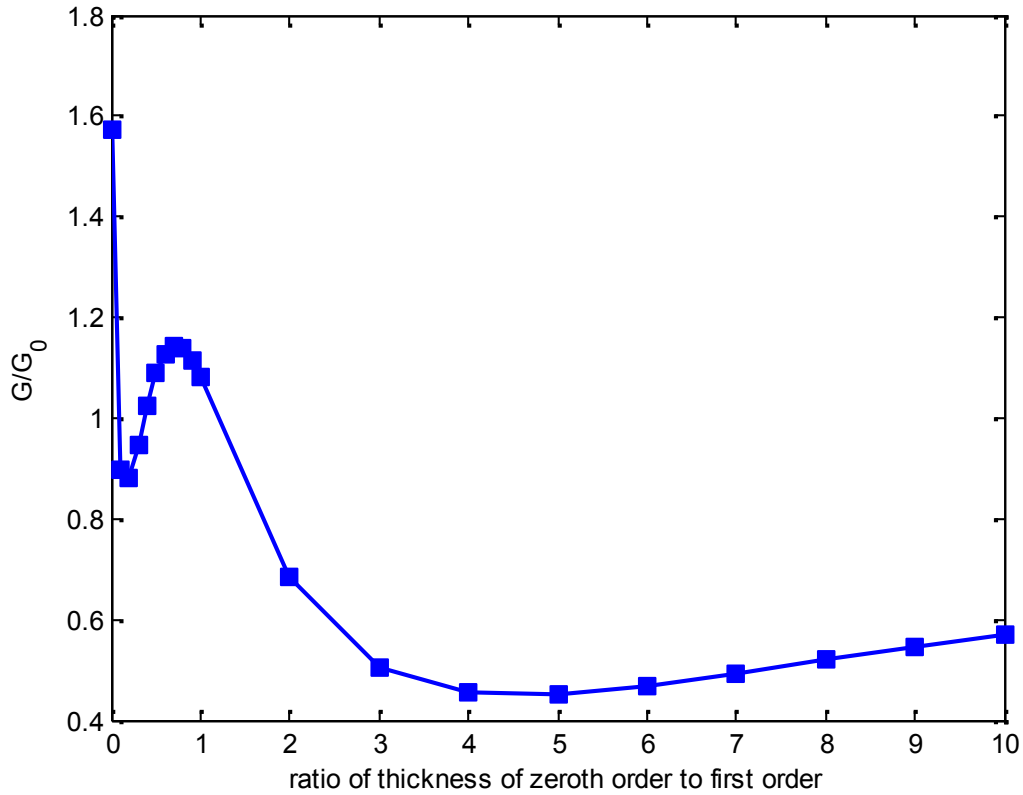


Figure 3.19. Plot of normalized effective shear modulus of first order regular honeycomb structure vs thickness ratio

From the plot it can be seen that effective shear modulus of the first order regular honeycomb structure is maximum between at ratio 0.7 and is about 1.15 times that of the zeroth order regular honeycomb structure of the same mass. It can also be seen that in the ratio 0.4 to 1 the shear modulus is higher than that of the zeroth order regular honeycomb structure. As the ratio increases, the shear modulus of the structure decreases drastically

till the ratio 5. At ratio 5, shear modulus is minimum and is 0.45 times that of the zeroth order regular honeycomb structure of the same mass. After ratio 5, shear modulus of the first order regular honeycomb structure increases. But the increase in the shear modulus with the increase in the ratio is very gradual. As the ratio increases, the thickness of the zeroth order structure will approach to 1mm (that of the zeroth order structure analyzed) and that of first order structure will approach to 0. So it is expected that the first order structure with higher thickness ratio for the same mass will approach that of the only zeroth order structure.

3.7.2.4. Special case of First order structure

A special case of first order regular honeycomb structure with ratio of thickness of zeroth order edges to first order edges as zero is studied. During the study, it was seen that as the thickness ratio gets smaller and smaller, the effective Young's modulus of the structure increases.

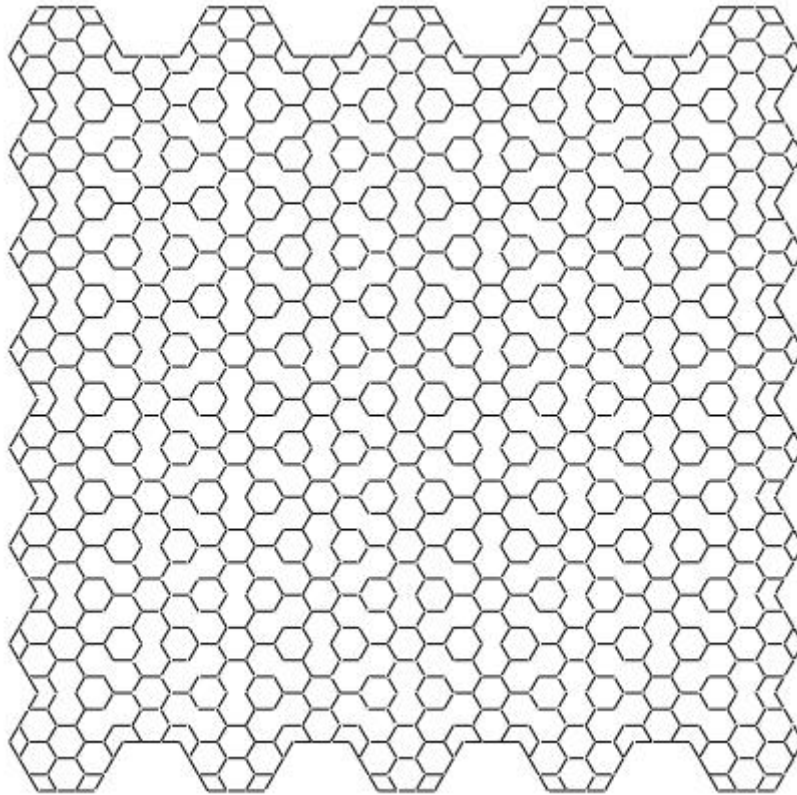


Figure 3.20. Special case of first order regular hierarchical honeycomb structure

Thus, this special case was studied and its properties are compared with the regular zeroth order honeycomb structure with smaller hexagon length. The plots of Von Misses stresses of this structure when uni-axial and shear loading are applied are shown in the Figure 3.21.

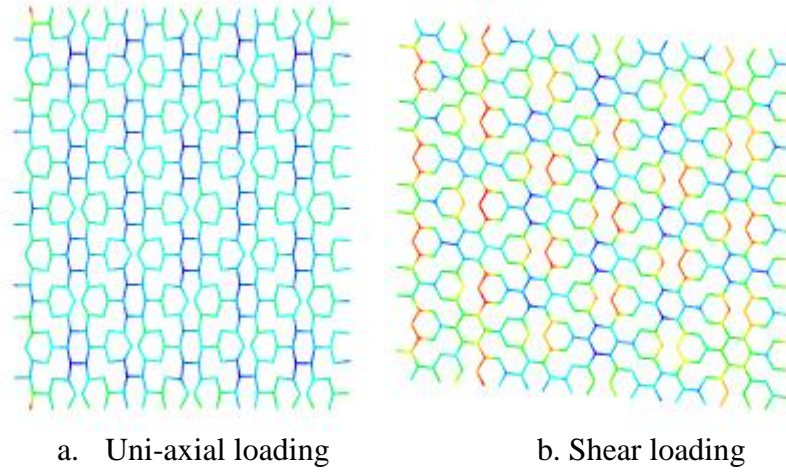


Figure 3.21. Von-Mises stresses for special case of first order regular honeycomb structure

This structure has effective modulus of 1.25 times that of the zeroth order regular honeycomb structure of same mass. This structure has shear modulus of 1.57 times that of the zeroth order regular honeycomb structure.

The properties of this special case of first order regular honeycomb structure is compared with zeroth order regular honeycomb structure having $h = l = 9.6233\text{mm}$ and $\theta = 30^\circ$. The structure is shown in the Figure 3.22.

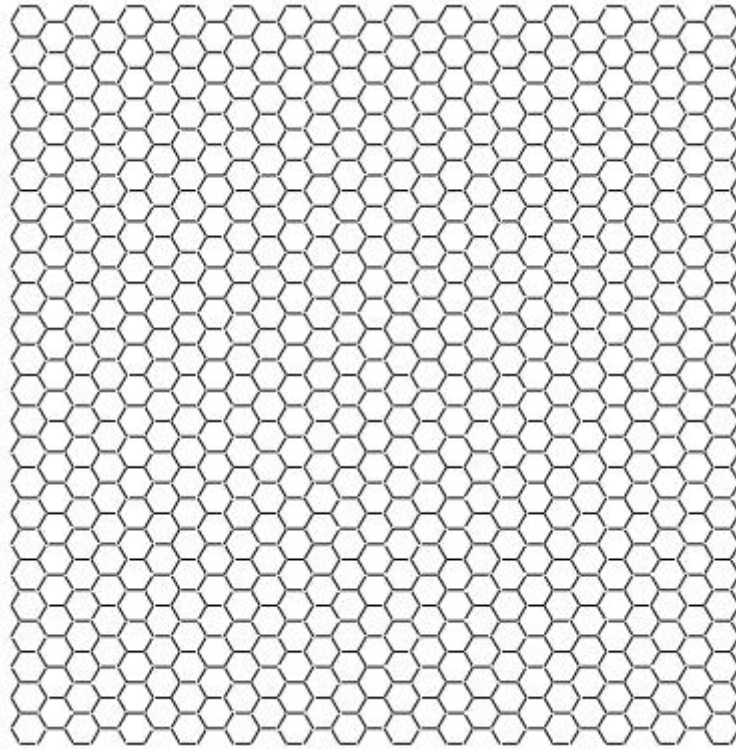


Figure 3.22. Regular zeroth order honeycomb structure with smaller edge length

It is seen that the effective Young's modulus of this special case of first order structure is higher than that of the zeroth order structure. As compared with theoretical values effective properties of this zeroth order structure with smaller edge length match more closely. Effective Young's modulus is about 98.2% of the theoretical value and effective shear modulus is about 97% of the theoretical. Effective Poisson's ratio is about 1.07.

From this all analysis, it can be said that effective Young's modulus for the same mass in descending order is 1. thickness ratio 0.1 (1.45 times), 2. Special case of First order regular hierarchical honeycomb structure with ratio 0 (1.25 times), 3. Minimum at ratio 4 (0.45).

And the descending order sequence for effective shear modulus for the same mass is 1. Special case of first order regular hierarchical honeycomb structure (1.57 times) 2. First order regular hierarchical honeycomb structure with thickness ratio 0.7 (1.15 times), 3. Minimum at ratio 4 (0.45 times).

Thus, from this analysis it can be said that there is a trade-off between the effective Young's modulus and effective shear modulus of the structure. The designer has to make the decision depending on the requirement.

CHAPTER FOUR

EFFECTIVE MECHANICAL PROPERTIES FOR FIRST ORDER AUXETIC HIERARCHICAL HONEYCOMB

In this chapter, the behavior of first order auxetic hexagonal honeycomb structure under in-plane loading condition is investigated to obtain effective mechanical properties (Young's modulus, Poisson's ratio and shear modulus) in the same way as that of the first order regular honeycomb structure. Comparison of the properties of first order structure with zeroth order auxetic hexagonal honeycomb of the same mass is made in the Section 4.4.

There has been very few research done on hierarchical structure of auxetic honeycomb. Taylor [13] studied effect of using structures having negative Poisson's ratio as substructure in regular honeycomb structures. He found that structures having negative Poisson's ratio endangers substantial increase in density modulus versus conventional honeycombs. His main focus was on effective Young's modulus. For cellular materials, to define its in-plane behavior, effective Young's modulus, effective shear modulus and effective Poisson's ratio are required. Auxetic materials studied in this work have characteristic property of negative Poisson's ratio and they are transversely isotropic.

4.1 Model setup

In the present work, the first order auxetic hierarchical honeycomb structure has been modeled by using commercial finite element solver ABAQUS 6.11. Different models were created by varying the number of unit cells in x and y direction and keeping

the overall dimension of the structure square. This was done in order to avoid the boundary effects and to make sure that effective properties are not dependent on the number of unit cells. 5 cells along x-direction and 8 cells along y-direction as shown in Figure 4.1 give a considerable accuracy in obtaining the effective mechanical properties of the structure. The difference between the effective properties of this structure and structures having more number of unit cells was less than 2%. The properties obtained from this model are in accordance with the theoretical formulae discussed in the earlier chapter suggested by Gibson and Ashby [1].

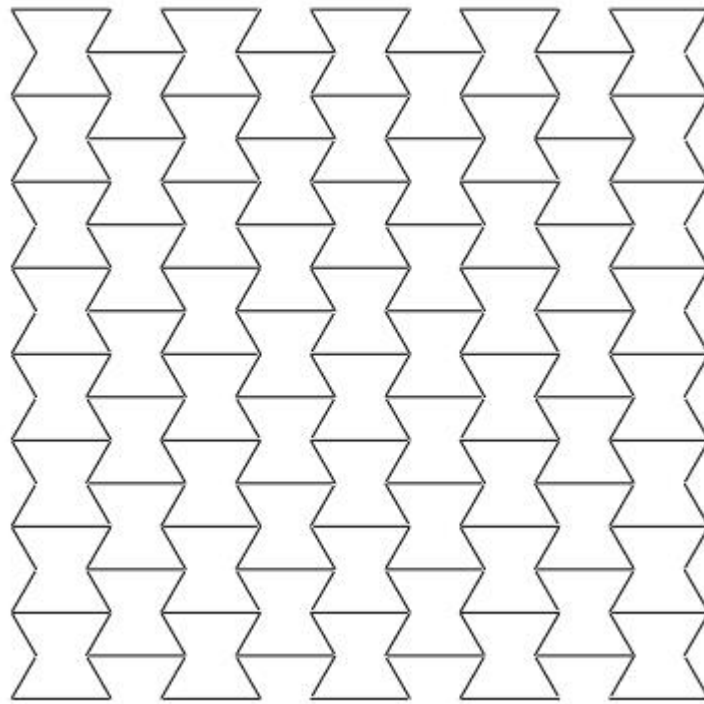


Figure 4.1. Model used to investigate effective mechanical properties of zeroth order auxetic honeycomb structure

The zeroth order auxetic honeycomb structure investigated by Griese [2] is considered as a reference. Figure 4.1 shows zeroth order auxetic honeycomb structure.

Dimensions of the unit cell are: $H = 28.87 \text{ mm}$, $H = 2L$ and $\theta = -30^\circ$. A 2D planar deformable part is created using ABAQUS version 6.11. As mentioned earlier, the structure created consists of 5 cells in x-direction and 8 cells along y-direction.

Linear 2 node cubic beam elements (B23) are used to mesh the structure. Smallest length of the beam in the zeroth order auxetic structure is 14.435mm and thickness assigned is 1mm. Hence, beam 23 elements having Euler-Bernoulli formulation can be used. Seed size is selected such that there are at least 4 elements along the horizontal edge. Generated mesh is shown in the Figure 4.2.

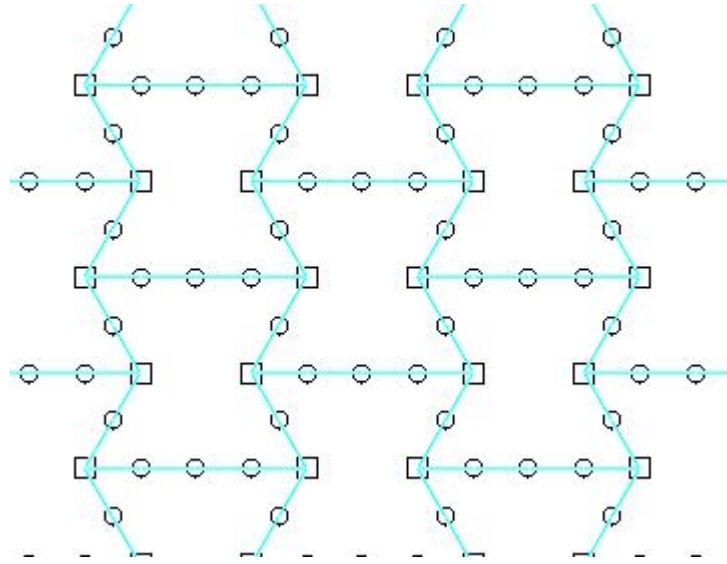


Figure 4.2. Generated mesh for zeroth order auxetic honeycomb structure

A rectangular beam section with aluminum is created with unit depth in z-direction and thickness as 1mm. The material properties of aluminum specified in the analysis are density () 2700 kg/m^3 , Young's modulus 71.9 Mpa and Poisson's ratio 0.33 [18].

After creating the mesh, different node sets are created to apply boundary conditions and to calculate different properties of the structure from the field output request.

4.2. Step and output analysis

A static analysis of the structure is carried out to investigate the properties. Static General Step is created and default option is selected. The analysis is carried out in the elastic region and structure obeys Hooke's law. All the deformations are small and NLGeom option is turned off in Abaqus.

The average stresses are calculated from the nodal reaction forces, so nodal reaction forces are requested on the left hand side extreme nodes. To calculate Poisson's ratio, displacements at 4 interior points shown in the Figure 4.1 are requested. The interior points are selected so as not to have any boundary effects.

The mass of all the structures, zeroth order and first order structures, is made same by adjusting the thicknesses of the edges of the honeycomb structure. The thicknesses of the various structures of zeroth and first order edges are calculated from the MATLAB code attached in the appendix. To confirm the mass of all the models is same, Current mass of the model or region option is selected in the history output request.

4.3. Boundary conditions

To calculate the effective mechanical properties displacement boundary condition is applied x direction loading at extreme left and extreme right nodes. For this two node sets are created consisting of the end vertices at extreme right and extreme left end cells.

Along with this boundary condition, y-direction displacement and rotation degrees of freedom at the point of symmetry in y direction at the ends is also applied so that the structure does not move in the y direction as shown in the Figure 4.3.

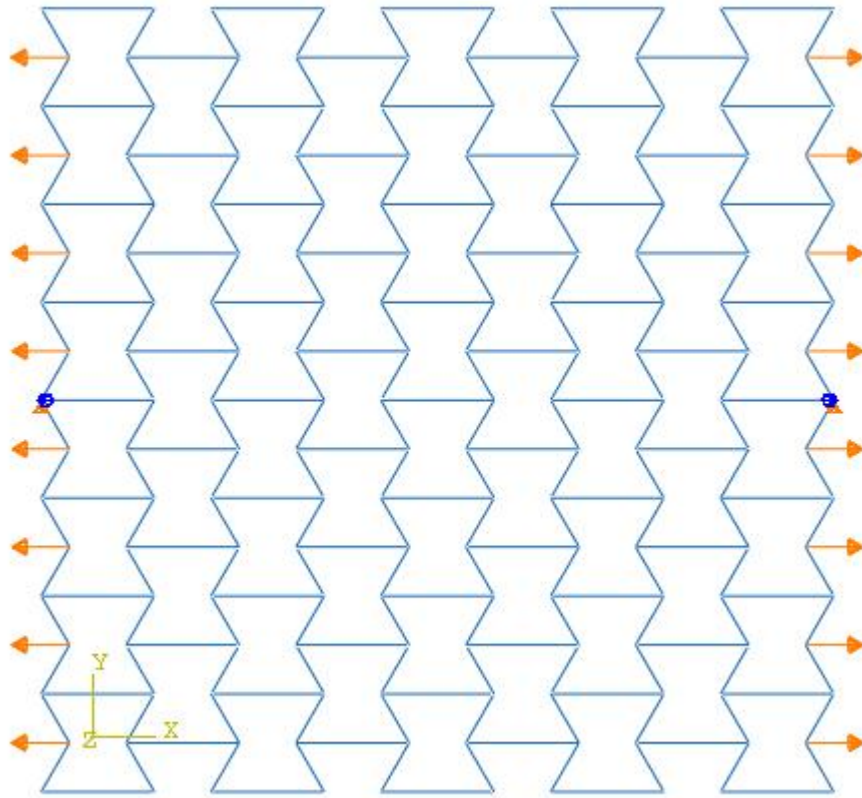


Figure 4.3. Boundary conditions applied to calculate effective Young's modulus

To calculate effective shear modulus, loading in y direction is applied. Displacement boundary condition is applied as it gives better results. The same model is used to calculate the shear modulus. Displacement equivalent of 2% strain in y-direction is applied at left most nodes. All degrees of freedom of extreme right nodes are constrained as shown in Figure 4.4.

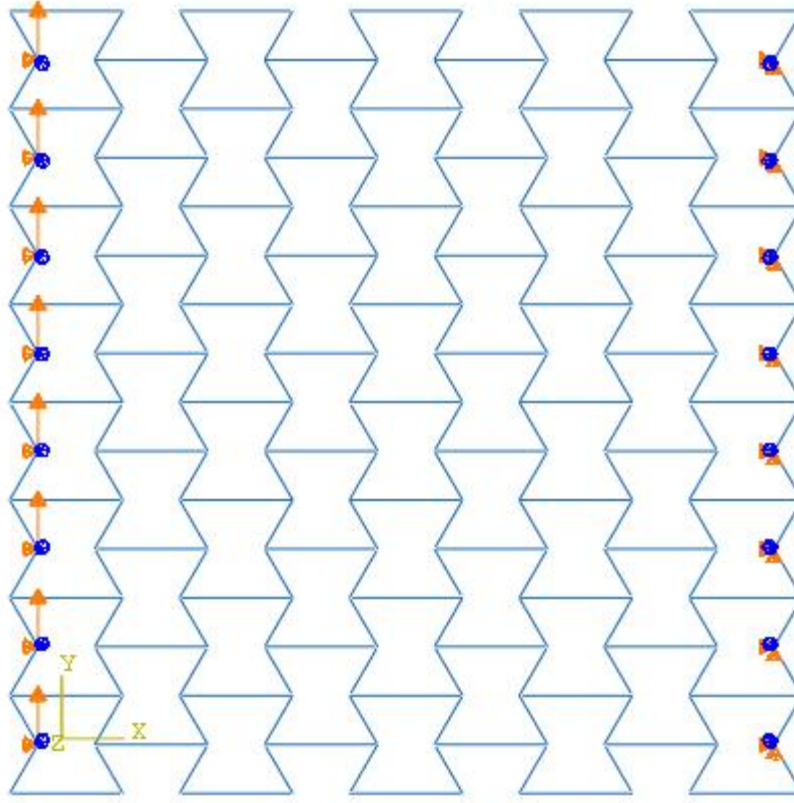


Figure 4.4. Boundary conditions applied to calculate effective shear modulus

4.4. First order auxetic hierarchical honeycomb structure

To study the effect of distribution of mass between different levels of hierarchy, different thicknesses to different orders of hierarchy are assigned. Different models of first order hierarchical honeycomb structures with different ratios of thicknesses of zeroth order to first order having the same mass as that of the zeroth order auxetic honeycomb were created. Two different sets were created containing zeroth order and first order edges as shown in the Figure 4.5 and Figure 4.6. In Figure 4.5, highlighted structure represents zeroth order structure. And in Figure 4.6, highlighted structure represents first order hierarchical honeycomb structure. It is expected that due to the symmetry, the

structure is transversely isotropic similar to zeroth order structure and has same effective elastic modulus in x and y direction.

Two sets were used to assign specific thicknesses to the specific level of hierarchy. By this the mass shared by each hierarchical order is varied. The total mass of all these first order hierarchical auxetic honeycomb structures have the exact same mass as that of the zeroth order auxetic honeycomb structure. The total lengths of first order and zeroth order edges are calculated. For a specific ratio r , ratio of thicknesses of zeroth order edges to first order hierarchical edges, thicknesses are calculated such that total mass remains the same.

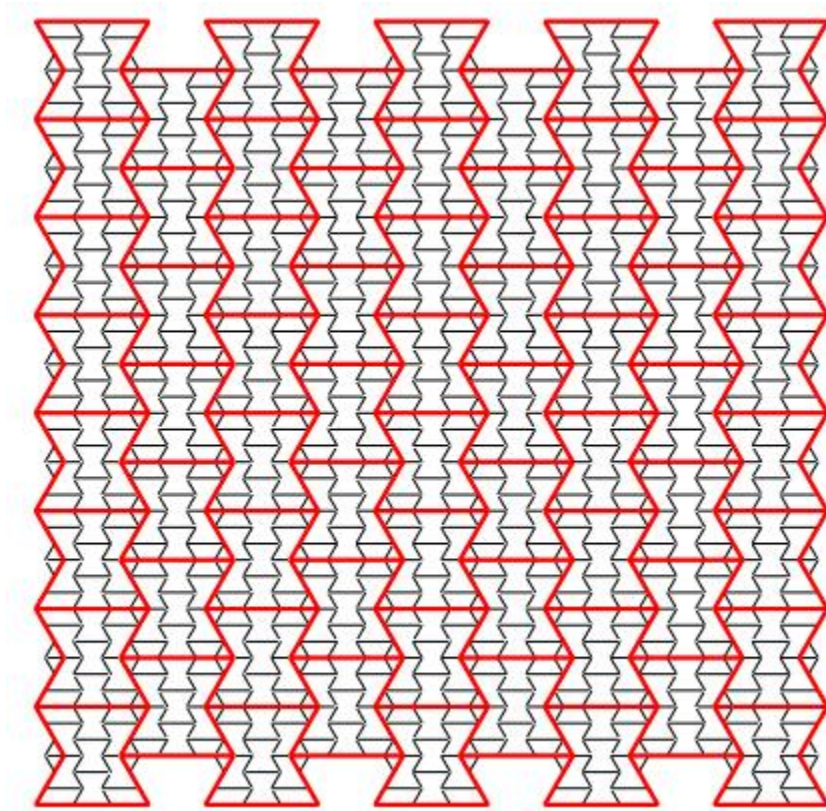


Figure 4.5. Highlighted structure representing zeroth order edges

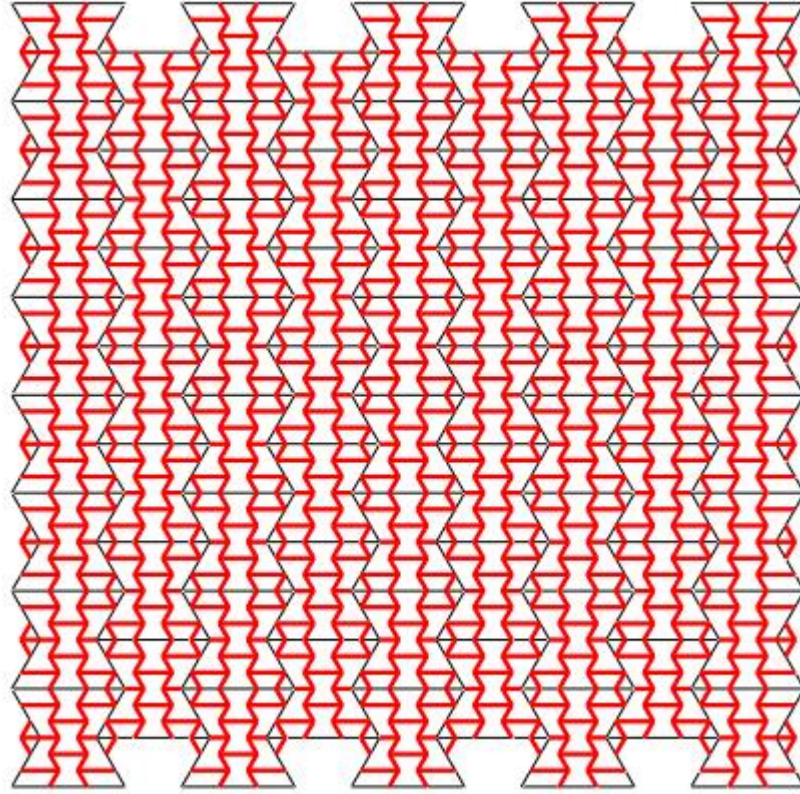


Figure 4.6. Highlighted structure represents first order edges

Ratio, $r = (t_0 / t_h)$	Thickness of zeroth order edges, t_0 (mm)	Thickness of first order edges, t_1 (mm)
0.1	0.043810	0.43810
0.2	0.083942	0.41971
0.3	0.12084	0.40281
0.4	0.15488	0.38721
0.5	0.18639	0.37277
0.6	0.21563	0.35938
0.7	0.24284	0.34691
0.8	0.26822	0.33528
0.9	0.29196	0.32440
1	0.31421	0.31421
2	0.47817	0.23909

3	0.57886	0.19295
4	0.64698	0.16174
5	0.69613	0.13923
6	0.73326	0.12221
7	0.76231	0.10890
8	0.78565	0.098207
9	0.80482	0.0089425
10	0.82084	0.0082084

Table no. 4.1. Thicknesses of zeroth order and first order hierarchical edges

Following Figure 4.7 represents the difference in thicknesses assigned to zeroth order and first order edges. To recognize the difference scaling factor is chosen 2 for thicknesses.

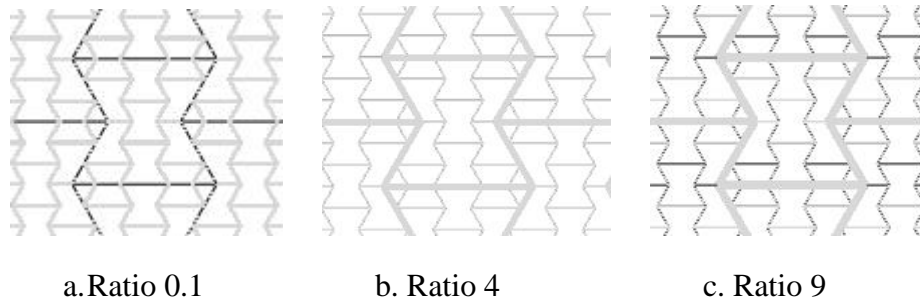


Figure 4.7. Schematic showing difference in thicknesses assigned to first and zeroth order edges

It can be seen that when the ratio is 0.1, thickness of first order edges is 10 times that of zeroth order edges. When the thickness ratio is 4, thickness of zeroth order edges is 4 times that of first order. When the thickness ratio is 9, thickness of zeroth order edges is 9 times that of first order. In short, when thickness ratio increases, thickness of zeroth order edges approach to 1 mm and that of first order edges approach to 0.

A similar setup has been used to investigate the effective mechanical properties of first order auxetic honeycomb structures as that of the zeroth order. To maintain the mass of all the structures same, a MATLAB code is used to find out the correct thicknesses of zeroth order and first order edges in the structure for different ratios. Thicknesses calculated from the MATLAB code are tabulated in Table no.4.1. These thicknesses are assigned to the respective sets using the python script. In the current analysis various thickness ratios from 0.1 to 10 on logarithmic scale are studied.

The basic model setup and the procedure followed to calculate effective mechanical properties is exactly the same as described in the section 4.1. The boundary conditions are shown in the Figure 4.8 and Figure 4.9.

Displacement boundary condition is applied x direction loading at extreme left and extreme right nodes. For this two node sets are created consisting of the end vertices at extreme right and extreme left end cells. Along with this boundary condition, y-direction displacement and rotation degrees of freedom at the point of symmetry in y direction at the ends is also applied so that the structure does not move in the y direction. Applied boundary conditions are shown in Figure 4.8.

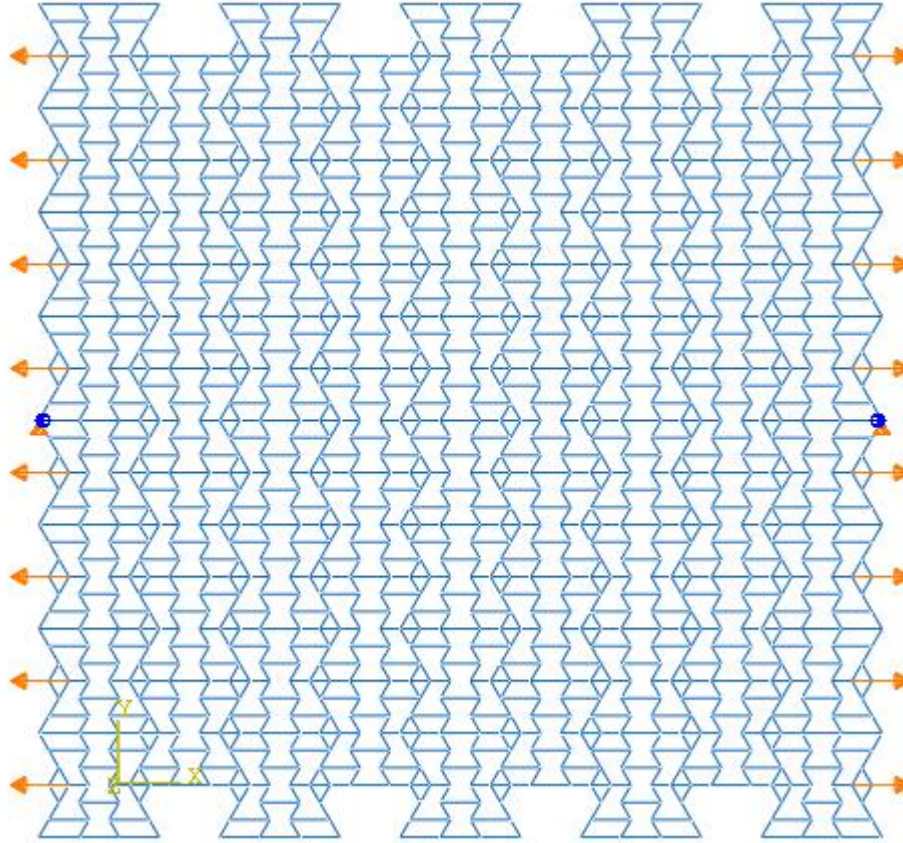


Figure 4.8. Boundary conditions applied to calculate effective Young's modulus of first order hierarchical honeycomb structure

To calculate effective shear modulus, loading in y direction is applied. Displacement boundary condition is applied as it gives better results. The same model is used to calculate the shear modulus. Displacement equivalent of 2% strain in y-direction is applied at left most nodes. All degrees of freedom of extreme right nodes are constrained. Applied boundary conditions are shown in Figure 4.9.

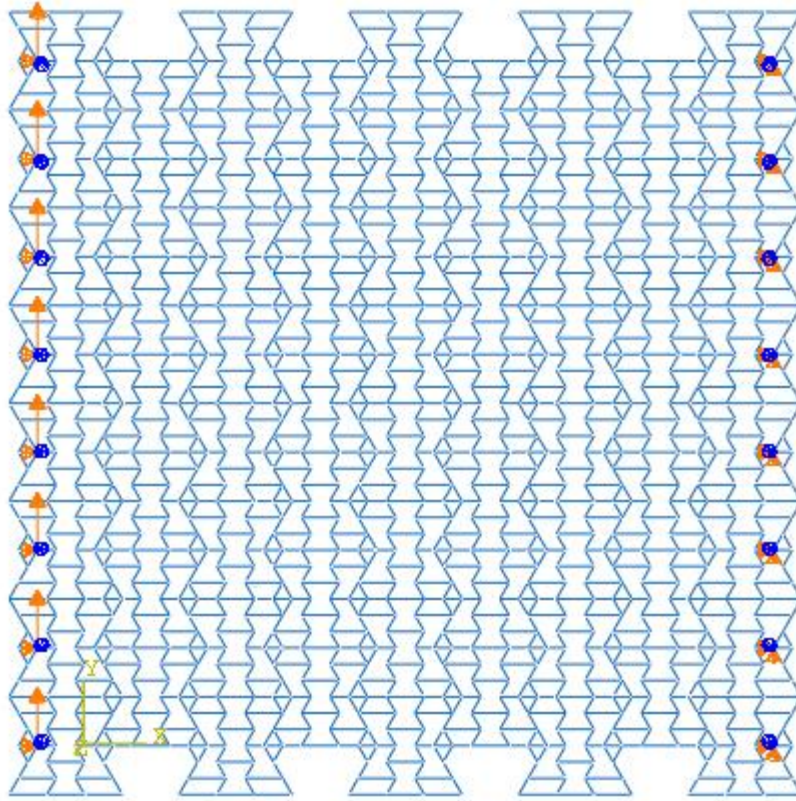


Figure 4.9. Boundary conditions applied to calculate effective shear modulus of first order hierarchical honeycomb structure

4.5. Results and discussion

Effective properties of the structure (Young's modulus, Poisson's ratio and shear modulus) are calculated using the same formulae discussed in the section 3.6.

4.5.1. Zeroth order auxetic results

Deformed shape of the structure is shown in the Figure 4.10.

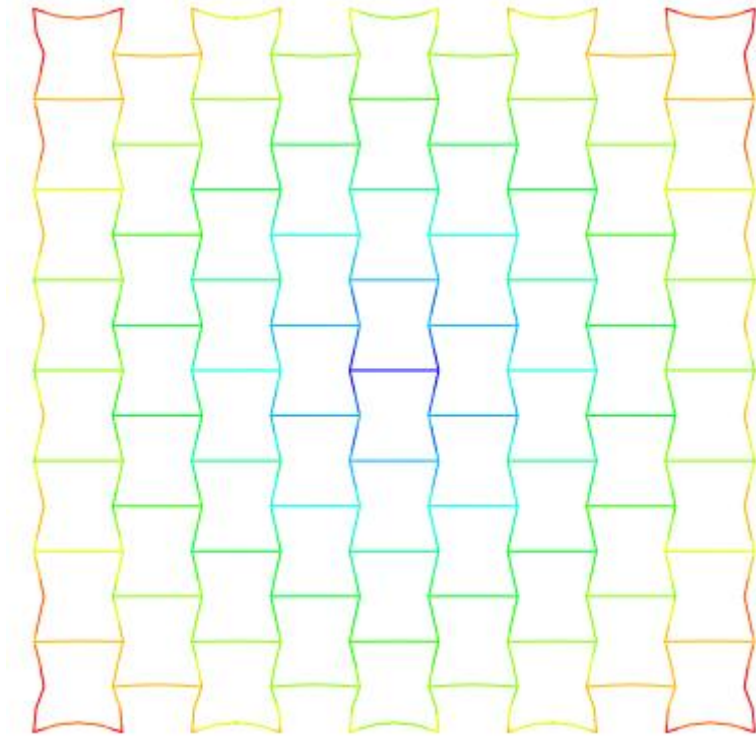


Figure 4.10. Deformed shape of zeroth order regular honeycomb structure for effective Young's modulus

The Figure 4.10 shows the deformed shape of the zeroth order auxetic honeycomb structure in x-direction loading case. Displacements equal to 2% strain are applied on both the extreme ends. Y-direction displacement and rotation at the point of symmetry at the ends are constrained so that the structure does not move.

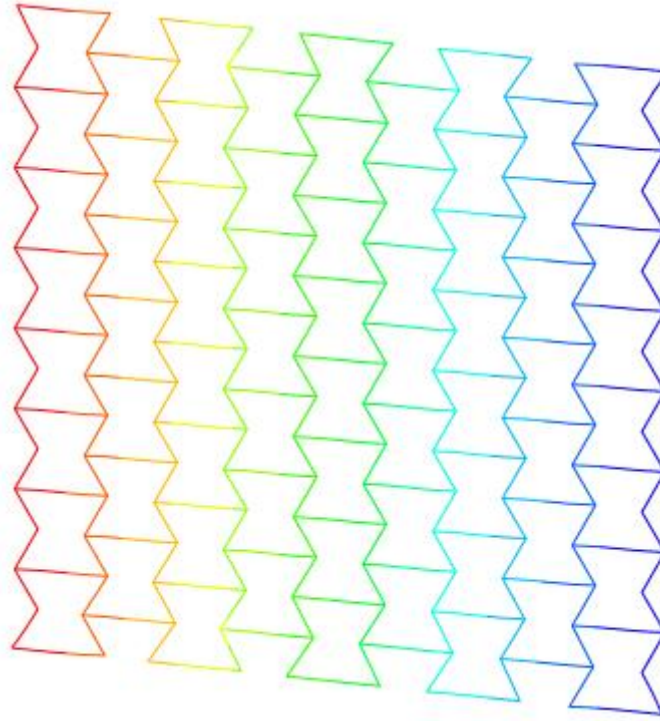


Figure 4.11. Deformed shape of zeroth order auxetic honeycomb structure for effective shear modulus

The Figure 4.11 shows the deformed shape of the zeroth order auxetic honeycomb structure in shear loading case. Displacement equal to 2% shear strain is applied in y-direction at extreme left vertices and other degrees of freedom are constrained.

	FEA result level-0 (%diff)	FEA result level-0 refined (%diff)	Theoretical
Young's modulus (MPa)	52.29 (5%)	54.94 (1%)	55.20
Shear modulus (MPa)	2.10 (1.3%)	2.06 (0.5%)	2.07
Poisson's ratio	-1.20 (20%)	-1.09 (9%)	-1

Table no. 4.2. Results of zeroth order auxetic honeycomb structure

The effective properties are tabulated in Table 3.2. Effective properties of the auxetic honeycomb structure are compared with theoretical values obtained from analytical values Gibson and Ashby formulae.

Level-0 refined structure is the structure with the same mass and overall dimensions with more number of cells. This structure is discussed in detail in Section 4.5. As more number of cells are considered, the values of the effective properties of this structure match more closely with theoretical values. These formulae are based on the assumption of infinite number of cells in x and y direction. This can be the reason of variation in the effective properties obtained from FEA simulation.

4.5.2 First order regular honeycomb structure results

4.5.2.1. Effective Young's modulus

Figure 4.12 shows the deformation shape of first order auxetic hierarchical honeycomb structure with thickness ratio, r , of 0.1 when x-direction loading is applied to calculate effective modulus and Poisson's ratio.

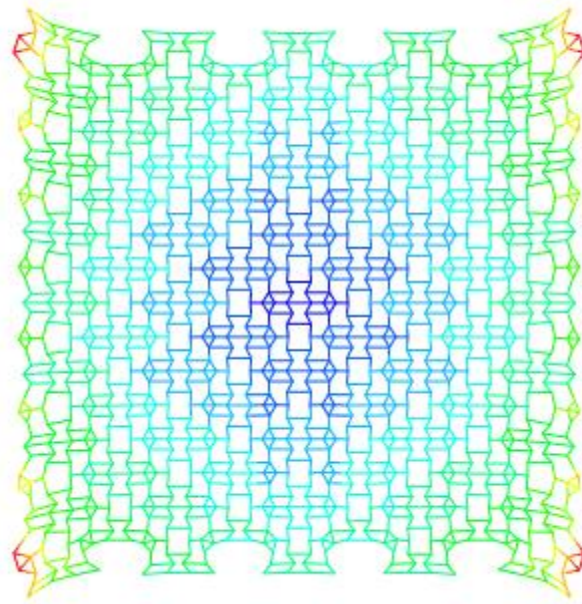


Figure 4.12 Deformed shape of first order auxetic hierarchical structure with thickness ratio 0.1

Following Figure 4.13 show the plots of Von Misses stresses for key values of ratio thickness, r when uni-axial load is applied. The plots are of central patch in order to understand the stress distribution at cellular level.

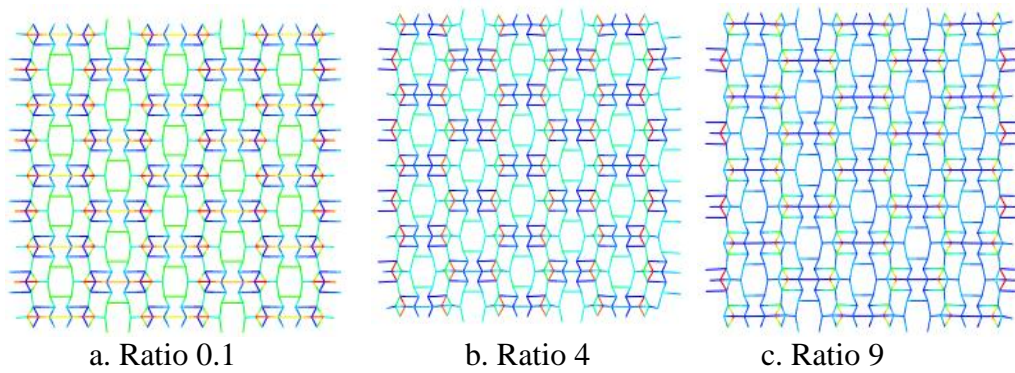


Figure 4.13. Von Misses stresses of first order auxetic hierarchical structure at key thickness ratios

Figure 4.14 shows the plot of relative effective Young's modulus of the first order auxetic honeycomb structure with respect to effective Young's modulus of zeroth order honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges.

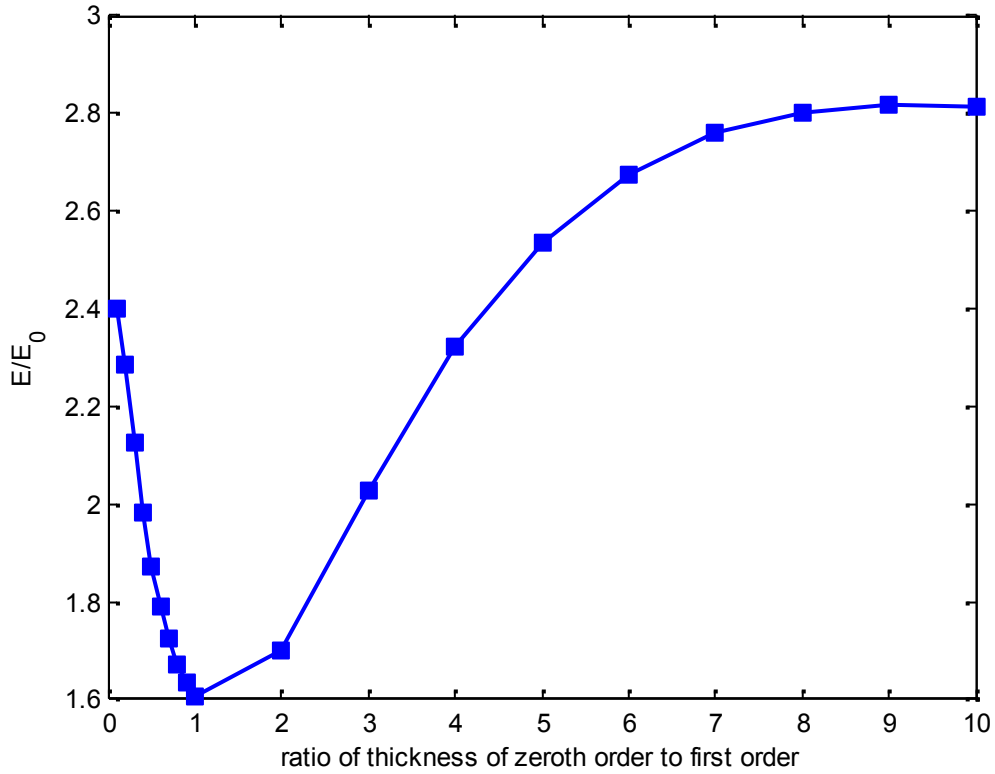


Figure 4.14. Plot of normalized effective Young's modulus of first order auxetic honeycomb structure vs thickness ratio

From the plot, it can be seen that the effective relative Young's modulus of first order is higher for any thickness ratio than that of the zeroth order auxetic honeycomb structure of the same mass. In the ratio 0.1 to 1, relative effective Young's modulus decreases with the increase in the ratio of thicknesses of zeroth order edges to first order edges. At ratio 1, relative effective Young's modulus is the minimum in the range of ratios studied. Even though relative effective Young's modulus is minimum at ratio 1, in

the range of ratios studied, it is still about 1.60 times the effective Young's modulus of the zeroth order auxetic honeycomb structure. After ratio 1, relative effective Young's modulus increase. At ratio 9, the relative effective Young's modulus is highest and is about 2.8 times the effective Young's modulus of the zeroth order auxetic honeycomb structure. After ratio 9, effective Young's modulus again decreases.

4.5.2.2. Effective Poisson's ratio

To calculate effective Poisson's ratio, same models that were used to calculate effective Young's modulus are used.

Figure 4.15 shows the plot of effective Poisson's ratio of the first order auxetic honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges. It represents the behavior of the first order structure with different mass distribution between different levels of hierarchy.

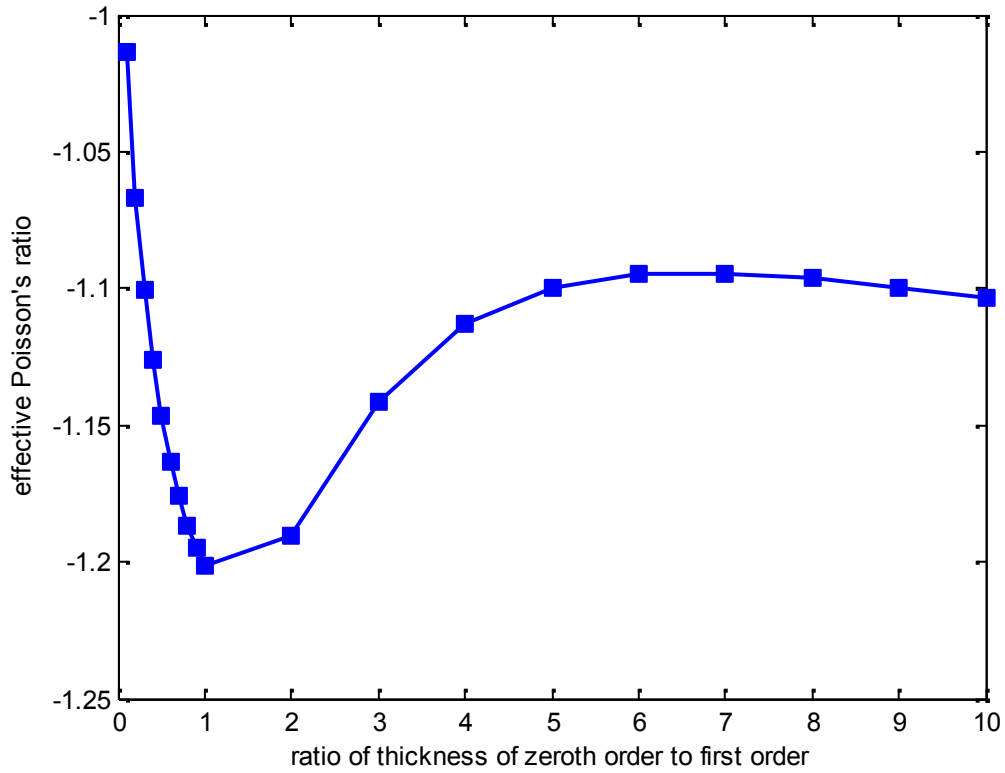


Figure 4.15. Plot of effective Poisson's ratio of first order auxetic honeycomb structure vs thickness ratio

The values of Poisson's ratio are of the values of the effective structure. Negative values of Poisson's ratio are characteristic of auxetic hexagonal honeycombs structures. The values of the Poisson's ratio are seen to be decreasing from -1.01 to -1.20 in the ratio 0.1 to 1. The values of the effective Poisson's ratio are increasing with the increase in thickness ratio after ratio 1. Effective Poisson's ratio increases till ratio 7 to -1.09 then again decreases. Change is very minute after ratio 6.

4.5.2.3. Effective shear modulus

Figure 4.16 shows the deformation shape of first order auxetic hierarchical honeycomb structure with thickness ratio, r , of 0.1 when subjected to shear loading.

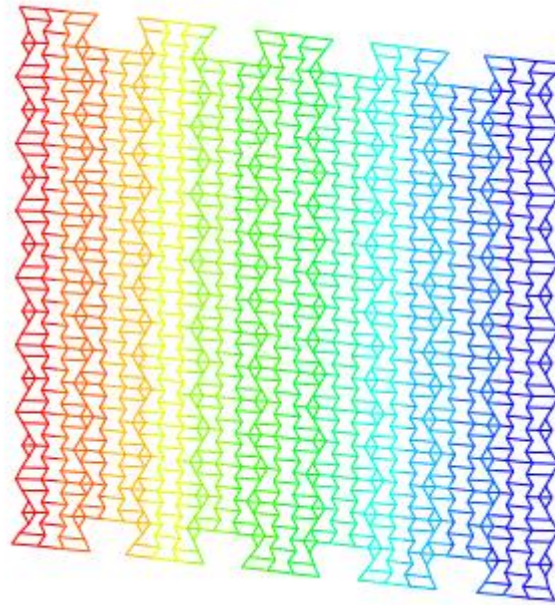


Figure 4.16. Deformed shape of first order auxetic hierarchical structure with thickness ratio 0.1

Following Figure 4.17 are the plots of Von Misses stresses for key values of ratio thickness, r when shear load is applied. The plots are of central patch in order to understand the stress distribution at cellular level.

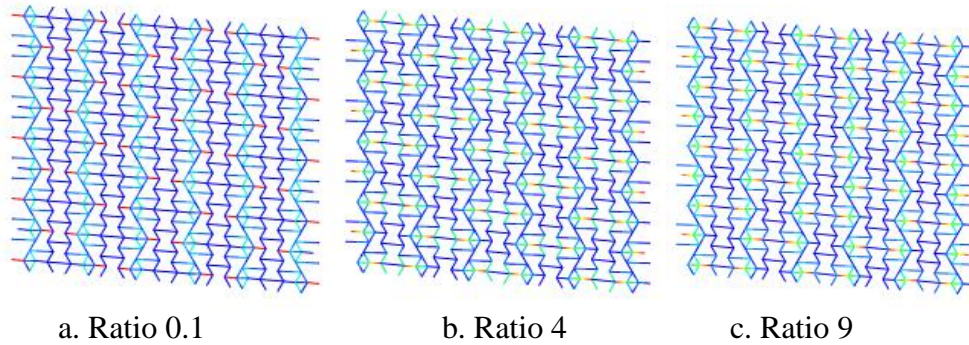


Figure 4.17. Von Misses stresses for central patch of first order auxetic hierarchical structure at key thickness ratios

Figure 4.18 shows the plot of relative effective shear modulus of the first order auxetic honeycomb structure with respect to effective shear modulus of zeroth order

auxetic honeycomb structure vs. ratio of thicknesses of zeroth order edges to first order edges.

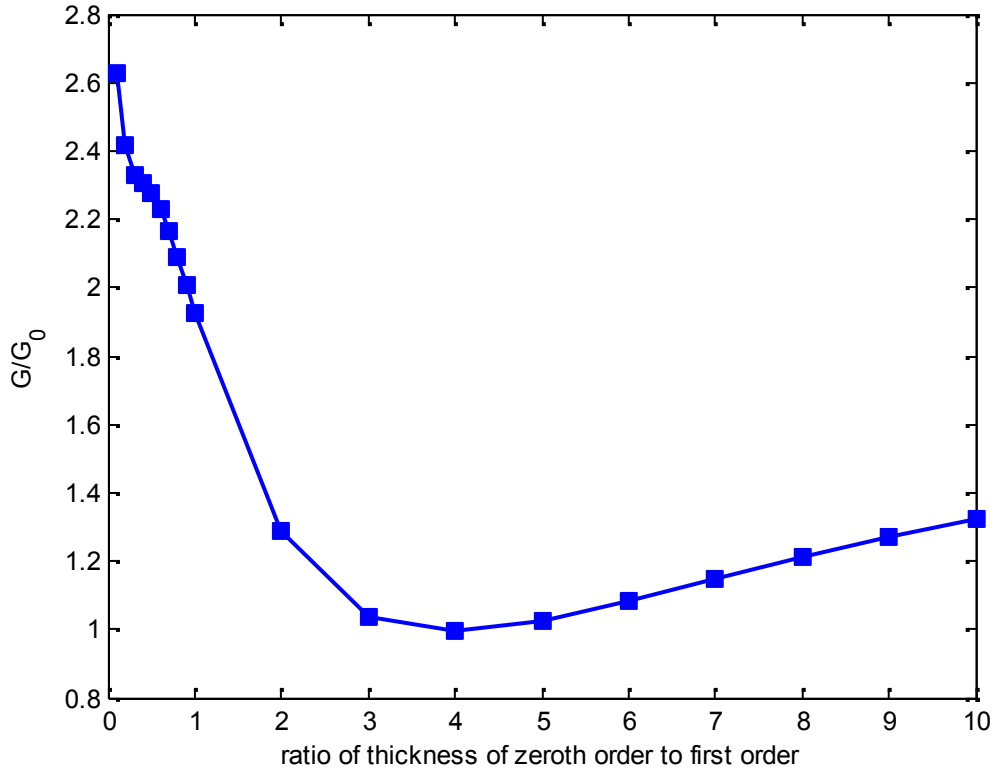


Figure 4.18. Plot of normalized effective shear modulus of first order auxetic honeycomb structure vs thickness ratio

From the plot, it can be seen that the effective relative shear modulus of first order is higher for any thickness ratio than that of the zeroth order auxetic honeycomb structure of the same mass. Relative effective shear modulus of first order structure is maximum at ratio 0.1 and is about 2.6 times that of the zeroth order structure of the same mass. As the ratio increases, the relative effective shear modulus of the first order auxetic honeycomb structure decreases till ratio 4. At ratio 4, the relative effective shear modulus is about the

same as that of the zeroth order auxetic honeycomb structure of the same mass. After ratio 4 as the ratio increases, the relative effective shear modulus increases.

4.5.2.4. Special case of First order structure

A special case of first order auxetic honeycomb structure with ratio of thickness of zeroth order edges to first order edges as zero is studied. In this case zeroth order edges are not completely eliminated. All the zeroth order edges are eliminated except the ones at ends and horizontal members that connect the columns.

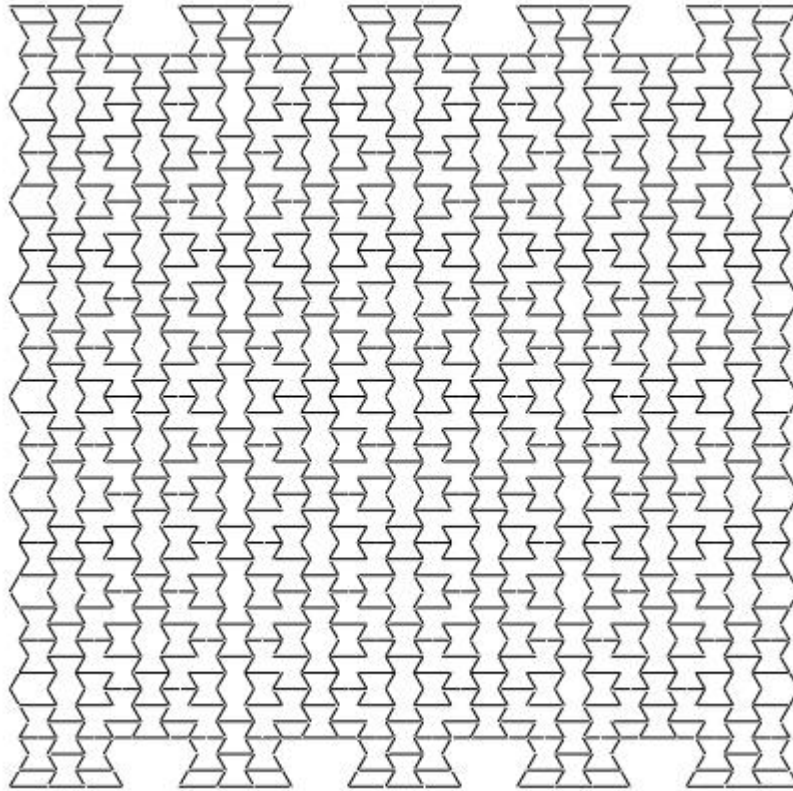


Figure 4.19. Special case of first order auxetic hierarchical honeycomb structure

This special case was studied and its properties are compared with the regular zeroth order honeycomb structure with smaller hexagon length. The plots of Von Misses stresses of this structure when uni-axial and shear loading are applied are shown in the Figure 3.

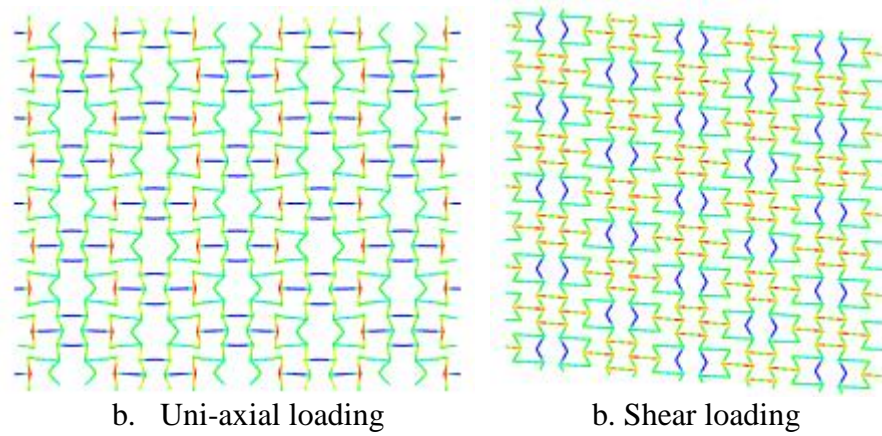


Figure 4.20. Von-Misses stresses for special case of first order regular honeycomb structure

This structure has effective modulus of 0.75 times that of the zeroth order auxetic honeycomb structure of same mass. This structure has shear modulus of 1.5 times that of the zeroth order auxetic honeycomb structure. This structure has Poisson's ratio of -0.36. Thus, overall effective properties of this structure are very much different.

The properties of this special case of first order auxetic honeycomb structure is compared with zeroth order auxetic honeycomb structure having $h = 9.6233\text{mm}$, $l = 4.8117\text{ mm}$ and $\theta = -30^\circ$.

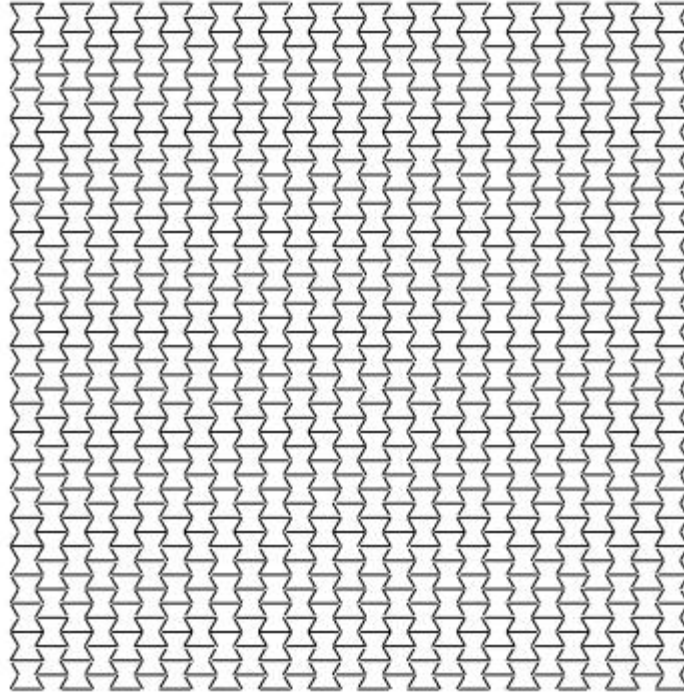


Figure 4.21. Auxetic zeroth order honeycomb structure with smaller edge length

It is seen that the effective Young's modulus of this special case of first order structure is higher than that of the zeroth order structure. As compared with theoretical values effective properties of this zeroth order structure with smaller edge length match more closely. Effective Young's modulus is about 99% of the theoretical value and effective shear modulus is about 98.5% of the theoretical. Effective Poisson's ratio is about -1.09.

From this all analysis, it can be said that the effective properties of first order auxetic structure are always higher than that of the zeroth order structure of same mass. The effective Young's modulus is maximum at ratio 9 (2.8 times) whereas effective shear modulus is maximum at ratio 0.1 (2.6 times). Minimum effective Young's modulus is at ratio and is still 1.6 times that of the zeroth order structure. Minimum effective shear

modulus is at ratio 4 and is about the same as that of the zeroth order structure. Thus, it can be said that first order auxetic structures are superior to zeroth order structure. Decision of the ratio of thicknesses of level-0 to level-1 can be taken based on the basis of the combination of effective properties required.

CHAPTER FIVE

CONCLUSION AND FUTURE WORK

5.1 Conclusions

The main purpose of introducing hierarchy to cellular structures is to further enhance the mechanical behavior of the structures without compromising the elastic properties of the material. From previous research it has been proved that increasing the levels of hierarchy in cellular structures produces better performing structures that are lighter in weight. The stiffness and strength of honeycombs is controlled by bending of the cell walls when load is applied. If hierarchy is introduced such that loads from the cell walls are transferred gradually, the deformation and bending can be minimized, hence, making the overall structure stiff.

In this work, in-plane effective mechanical properties of first order reiterated honeycomb structures are analyzed in both regular and auxetic configurations. First order hierarchical structure is created by placing smaller honeycombs inside the conventional honeycombs, zeroth order honeycomb, such that midpoints of the edges shared by them coincide with each other.

A finite element model was developed to study the effective mechanical properties (effective Young's modulus, effective Poisson's ratio and effective shear modulus) of the first order structures as described in Chapters 3 and 4. First properties of zeroth order structures are analyzed and same boundary conditions are applied to first order structures of same mass. Different thicknesses to different levels of hierarchy are determined to keep the mass the same for different cell wall thickness ratios. By this mass

distribution between different levels of hierarchy was adjusted. This effect on the overall properties of the structure were analyzed and were compared with zeroth order structure.

From the results of first order regular hierarchical honeycomb structures, it was found that effective Young's modulus for thickness ratio 0.1 is maximum and is about 1.45 times that of the zeroth order. Furthermore, the maximum effective shear modulus is for special case of first order regular honeycomb structure and is 1.57 times that of the zeroth order.

From the results of first order auxetic hierarchical honeycomb structures, it can be said that the effective relative Young's modulus, shear modulus of first order is higher for any thickness ratio than that of the zeroth order auxetic honeycomb structure of the same mass. Effective Young's modulus for thickness ratio 9 is maximum and is about 2.8 times that of the zeroth order. And maximum effective shear modulus of first order structure is maximum at ratio 0.1 and is 2.6 times that of the zeroth order.

Analysis of the results showed that there is a trade-off between the effective Young's modulus and effective shear modulus of the structure. The designer has to make the decision depending on the requirement.

5.2 Future work

In this thesis, effective mechanical properties of first order regular and auxetic structures are studied using finite element analysis. Results depend on various factors: mesh size chosen, number of cells considered, boundary effects etc. Experimental validation and analytical formulation of these structures can be considered. There are few

manufacturers of honeycomb structures and <http://indyhoneycomb.com/> can be good place to look for manufacturing of honeycombs required for such analysis.

In this work, main focus is on in-plane effective properties of first order honeycomb structure. It would be interesting to see the behavior of these structures with out-of-plane loading conditions.

In this work, only first order reiterative hierarchical structures are considered. The level of hierarchy can be increased and the effect of the mass shared by different levels of hierarchy can be studied. From this study, optimum design satisfying combination of effective properties can be derived. Equations that can be used to calculate thicknesses to be assigned to different levels of hierarchy, in order to understand mass distribution between different levels of hierarchy, are as follows

$$m_1 = \dots_0 d_0 (t_0 l_0 + t_1 l_1 + t_2 l_2)$$

$$r_1 = (t_0 / t_1)$$

$$r_2 = (t_1 / t_2)$$

Performance and behavior of these structures for various design applications, especially where regular honeycombs are used, can be considered. One of the application is studied by Naveen [21] in case of acoustics. He studied the effect of hierarchy in sound transmission loss analysis through sandwich panels.

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APPENDICES

Appendix A: MATLAB code to calculate thicknesses of first order regular honeycomb

structures

```
clear all
clc
L=28.87/1000; %side of regular hexagon in mm

total_length_regular=L*(5*9+10*16+4*8);

first_order_length=L*24*(40+28)/3;

T=0.001; %thickness of regular zeroth order
hexagonal honeycomb

mass=total_length_regular*T*1; %unit depth

thickness_ratio=input('desired ratio of thicknesses of zeroth order to
first order ');

t0=mass/(total_length_regular+(first_order_length/thickness_ratio))

t1=t0/thickness_ratio

mass1=total_length_regular*t0+first_order_length*t1;
```

Appendix B: MATLAB code to calculate thicknesses of first order auxetic honeycomb

structures

```
clear all
clc
h=28.87/1000;           %dimensions of auxetic honeycomb in mm
l=14.435/1000;

length_zeroth_order=h*(45+32)+l*(10*16);

length_first_order=(h*6*(40+28)/3)+(h*2*8/3)+(h*8*2/6)+(h*(28+24)/3)+(l*16*(40+28)/3);

T=0.001;                %thickness of regular zeroth order hexagonal
honeycomb

mass=length_zeroth_order*T*l;

thickness_ratio=input('desired ratio of thicknesses of zeroth order to
first order ');

t0=mass/(length_zeroth_order+(length_first_order/thickness_ratio))

t1=t0/thickness_ratio

mass1=t0*length_zeroth_order+t1*length_first_order;
```