Formulation of an Interactive Ruled-based Design Envelope for Ensuring Aftermarket Vehicle Dynamics Compliance

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FORMULATION OF AN INTERACTIVE RULED-BASED DESIGN ENVELOPE FOR ENSURING AFTERMARKET VEHICLE DYNAMICS COMPLIANCE

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Automotive Engineering

by
Xianjie Zhou
December 2016

Accepted by:
Bashah Ayalew, Committee Chair
Yunyi Jia
Robert Prucka
Paul Venhovens
ABSTRACT

The objective of this research is to develop an integrated system engineering methodology for the customization design to maximize vehicle performance upgrade freedom while ensuring vehicle dynamics compliance.

A post-delivery modification framework, which is led by an aftermarket umbrella organization and involve various stakeholders has been established. The umbrella organization will be in charge of developing the design envelope and distribute to various aftermarket kit suppliers to generate specific products according to their brand essence.

A generic mathematical representation of a (proprietary) ESC system has been developed for virtual certification purposes. This approach is a cost-effective alternative to physical on-road testing and hardware-in-the-loop (HiL) simulations. Furthermore, based on the stability control model, the modification impacts on the vehicle dynamics and stability performance was assessed using the Taguchi design of experiment (DOE) method. DOE results provide three distinct ways for supporting aftermarket modifications. First, main effects help customizers to understand which modification bring benefits or risks. Second, a regression model of the lateral offset metrics helps suppliers to predict closed-loop performances with open-loop testing information which require much less time and cost. Finally, the pass/fail criteria regarding federally mandated ESC compliance (FMVSS 126) brought on the ‘Pass Region’ which consisted of feasible configurations such that customizers may configure their options within a safe
zone. Each of these methods complements others for supporting the aftermarket modification.

In order to improve the computation efficiency, two lower fidelity models were developed: A linear model and a surrogate model. The linear model is derived from the high fidelity model with reduced degrees of freedom (DOF) and linearized parameters. Tire cornering stiffness is treated as constants for gentle maneuvers, and varying parameters for high-dynamic driving maneuver. The linear system is either a linear time-invariant (LTI) system or a linear parameter-varying (LPV) system depending on the application context. The PD yaw stability control algorithm, which is inherited from the high fidelity model, was simplified but retained with critical nonlinear features. A quadratic regression model that was dedicated for compliance metrics was developed as a surrogate model incorporated in an interactive rule-based design envelope.

An interactive design envelope has been created incorporating the rules established using computational efficient linear and surrogate models. The constraint satisfaction problem is described in the nonlinear programming context and solved using sequential quadratic programming. The quasiconvexity of the design space, which is the necessary condition for the proposed approach, is also investigated by inspecting the constraint functions. Finally, two case studies were developed to demonstrate the framework developed which was validated against the high fidelity co-simulation model.
DEDICATION

I would like to dedicate this research to my beloved parents, Yongjun Zhou and Shuhua Liang, who have been very supportive for my pursuing of the degree aboard. I also want to dedicate this dissertation to my dear wife, Qinglin (Fiona) Xu, and my sweet heart, Ariana Zhou, who have sacrificed so much for the time being to back my research and give me the warmest embrace whenever I need them. Without my family’s strongest encouraging and inspiring, this dissertation will not be success.
ACKNOWLEDGMENTS

I would like to express my wholehearted gratitude to my doctoral advisor, Dr. Paul Venhovens, who has guided me through the journey of the research using his wisdom, vision, and experience. He has imparted not only his knowledge but also a very professional attitude towards the research. He has provided abundant opportunities for me to reach the frontier of the industries and access the state-of-the-art researches.

I also appreciate Dr. Ayalew laying the critical foundation of knowledge in vehicle dynamics and stability controls, which has been one of the most fundamental topics in my research. I would also acknowledge him for providing constructive advises for the final stage of the doctoral study and the completion of the dissertation.

I would also show my appreciation to my committee members, Dr. Prucka and Dr. Jia, for their time and effort dedicated to reviewing and refining my research work with their expertise and experience.

This research was supported by Specialty Equipment Market Association (SEMA) under grant number 2099738. I would like to thank John Waraniak, Vice President of Vehicle Technology at SEMA, for his tremendous support and effort to coordinate the project activities with various SEMA member companies involved. I would also like to express their gratitude to Dr. Thomas D. Gillespie, co-founder of Mechanical Simulation Corporation, for sharing his expertise and wisdom in the domain of vehicle dynamics to make this research happened.
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<tr>
<td>a</td>
<td>m</td>
<td>Distance from center of gravity to front axle</td>
</tr>
<tr>
<td>b</td>
<td>m</td>
<td>Distance from center of gravity to rear axle</td>
</tr>
<tr>
<td>ay</td>
<td>m/s²</td>
<td>Lateral acceleration at center of gravity</td>
</tr>
<tr>
<td>aySen</td>
<td>m/s²</td>
<td>Lateral acceleration at measuring point</td>
</tr>
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<td>Bφs</td>
<td>N·m·s/rad</td>
<td>Sprung mass roll damping</td>
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<tr>
<td>C₁</td>
<td>N/rad</td>
<td>Front axle cornering stiffness</td>
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<tr>
<td>C₂</td>
<td>N/rad</td>
<td>Rear axle cornering stiffness</td>
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<td>CPD</td>
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<tr>
<td>Cxy</td>
<td>-</td>
<td>Covariance function</td>
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<td>m</td>
<td>Transversal distance of springs</td>
</tr>
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<td>DA</td>
<td>m</td>
<td>Transversal distance of ARB</td>
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<tr>
<td>g</td>
<td>m/s²</td>
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<tr>
<td>Gr Mz</td>
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<td>erₜ</td>
<td>rad/s</td>
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<td>er</td>
<td>rad/s</td>
<td>Yaw rate error</td>
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<td>Fₓ</td>
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<td>Longitudinal tire force</td>
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<td>Fₓ−est</td>
<td>N</td>
<td>Estimated longitudinal force</td>
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<td>Fₓf, Fₓr</td>
<td>N</td>
<td>Front/Rear tire brake force</td>
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<td>Fzf, Fzr</td>
<td>N</td>
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<td>Fyf, Fyr</td>
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<td>H₁</td>
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<td>$H_{cgS}$</td>
<td>m</td>
<td>Current CG height of sprung mass</td>
</tr>
<tr>
<td>$H'_{cgS}$</td>
<td>m</td>
<td>OE CG height of sprung mass</td>
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<td>$H_{lift}$</td>
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<td>Chassis lift height</td>
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<tr>
<td>$I_{XS}$</td>
<td>kg m$^2$</td>
<td>Roll moment of inertia of sprung mass</td>
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<td>$I_{XZS}$</td>
<td>kg m$^2$</td>
<td>Product of sprung mass inertia in XZ plane</td>
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<td>$I_z$</td>
<td>kg m$^2$</td>
<td>Yaw moment of inertia of full vehicle</td>
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<td>Derivative gain</td>
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<td>$K_p$</td>
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<td>$K_{\phi S}$</td>
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<tr>
<td>$K_{sp}$</td>
<td>N/m</td>
<td>Spring stiffness</td>
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<td>$K_{ARB}$</td>
<td>N/m</td>
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<td>$k_{perc}$</td>
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<td>kg</td>
<td>Vehicle total mass</td>
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<td>$M_S$</td>
<td>kg</td>
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<td>MPa</td>
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<td>Front/Rear tire dynamic rolling radii</td>
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<td>$r$</td>
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<td>rad/s</td>
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<td>rad/s</td>
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<td>$r_d$</td>
<td>rad/s</td>
<td>Desired yaw rate</td>
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<td>$r_{unlimited}$</td>
<td>rad/s</td>
<td>Unlimited desired yaw rate</td>
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<td>Upper bound of desired yaw rate</td>
<td>$rad/s$</td>
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<td>Axle cornering stiffness scale factor</td>
<td>-</td>
</tr>
<tr>
<td>$S_{Ctire}$</td>
<td>Tire cornering stiffness scale factor</td>
<td>-</td>
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<td>$T$</td>
<td>Vehicle track width</td>
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<td>$t_{pb}$</td>
<td>Brake dynamics model peak response time</td>
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<td>$V$</td>
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<tr>
<td>$\delta_f$</td>
<td>Road wheel steering angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>Time constant for the yaw rate saturation model</td>
<td>$s$</td>
</tr>
<tr>
<td>$\phi_S$</td>
<td>Sprung mass roll angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\phi_U$</td>
<td>Unsprung mass roll angle</td>
<td>$rad$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Road-tire contract friction coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>Longitudinal tire-road coefficient of friction</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Longitudinal slip ratio</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_{roll}$</td>
<td>Wheel rolling speed</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\zeta_b$</td>
<td>Brake dynamics model damping ratio</td>
<td>-</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Reference model time constant</td>
<td>$s$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Brake dynamics model natural frequency</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Closed-loop stability control system natural frequency</td>
<td>$rad/s$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Closed-loop stability control system damping ratio</td>
<td>-</td>
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ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>SWD</td>
<td>Sine-with-dwell</td>
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<tr>
<td>FMVSS</td>
<td>Federal Motor Vehicle Safety Standards</td>
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<tr>
<td>CG</td>
<td>Center of gravity</td>
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<tr>
<td>LM</td>
<td>Linear model</td>
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<tr>
<td>LFM</td>
<td>Low fidelity model</td>
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<tr>
<td>HFM</td>
<td>High fidelity model</td>
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<tr>
<td>SM</td>
<td>Surrogate model</td>
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<tr>
<td>COS</td>
<td>Complete of steering</td>
</tr>
<tr>
<td>BOS</td>
<td>Beginning of steering</td>
</tr>
<tr>
<td>OEM</td>
<td>Original equipment manufacturer</td>
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<tr>
<td>GDP</td>
<td>Gross domestic product</td>
</tr>
<tr>
<td>ESC</td>
<td>Electronic stability control</td>
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<tr>
<td>SiL</td>
<td>Software-in-the-loop</td>
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<tr>
<td>HiL</td>
<td>Hardware-in-the-loop</td>
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<tr>
<td>UI</td>
<td>User interface</td>
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<tr>
<td>CODP</td>
<td>Customer order decoupling point</td>
</tr>
<tr>
<td>ETO</td>
<td>Engineer-to-order</td>
</tr>
<tr>
<td>MTO</td>
<td>Made-to-order</td>
</tr>
<tr>
<td>ATO</td>
<td>Assemble-to-order</td>
</tr>
<tr>
<td>MTS</td>
<td>Make-to-stock</td>
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<tr>
<td>MC</td>
<td>Mass customization</td>
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<tr>
<td>MP</td>
<td>Mass personalization</td>
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<tr>
<td>CSP</td>
<td>Constraint satisfaction problem</td>
</tr>
<tr>
<td>QFD</td>
<td>Quality function deployment</td>
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<tr>
<td>DOF</td>
<td>Degree-of-freedom</td>
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<tr>
<td>DOE</td>
<td>Design of experiment</td>
</tr>
<tr>
<td>ARB</td>
<td>Anti-roll bar</td>
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<tr>
<td>NHTSA</td>
<td>National Highway Traffic Safety Administration</td>
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List of Abbreviations (Continued)

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>NLP</td>
<td>Nonlinear programming</td>
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<tr>
<td>SQP</td>
<td>Sequential quadratic programming</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear time-invariant</td>
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<tr>
<td>LPV</td>
<td>Linear parameter-varying</td>
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CHAPTER ONE
INTRODUCTION

The automotive industry can be classified into two main sections, the original equipment manufacturer (OEM) and aftermarket industry. The United States automotive aftermarket is estimated to be worth $318.2 billion (2013), contributing more than 2.3% to gross domestic product (GDP) [1]. Vehicle modification is one of the most popular topics in the aftermarket. User design customization enables users ‘partially design’ their product by specifying the product properties according to their preferences/requirement [2]. It exists not only in automotive industry but also in other areas such as personal computer, clothing, and housing.

There are two major challenges with vehicle customization. First, users usually have little or no background in the underlying technical detail, so they might not be able to determine the best options that fit their specific needs. For example, a user who looks for improving cornering ability may have very little idea about which component should be upgraded, and which should not.

Aftermarket suppliers can give suggestions based on the sales guidelines, but are rarely able to quantify the improvement that the user will get from each component upgrade. Second, users’ modification preference towards one aspect (such as appearance), may cause other aspects to degrade or may even violate compliance with motor vehicle regulation. The application of Electronic stability control (ESC) has been mandatory as part of FMVSS126 since 2012 in the United States for original equipment
manufacturers (OEM) as well as for aftermarket suppliers to provide vehicle stability in critical driving conditions [3]. OEMs and final stage manufacturers usually have full access to ESC hardware and software which enables them to calibrate the controller for validity purposes. In the case of customized vehicles, aftermarket suppliers providing chassis modification components also need to assure that their products remain in compliance with FMVSS126 (even though these manufacturers/vendors are not able to gain access to the ESC units). Evaluating aftermarket components performance and regulation compliance can become very cumbersome since it requires an in-depth assessment of the vehicle’s stability behavior (with ESC engaged), considering every possible permutation and combination of suspension, brakes, wheels, and tires upgrades.

1.1 Motivation

Vehicle product design and development processes and tools have been studied for decades and have become very mature. However, aftermarket product development process and configuration management have evolved less due to the lack of resources and data available. The design procedures and development tools are less elaborate, and it focuses mostly around improving the performance of one or two subsystem and does not deal with the design task on a full vehicle system integration level. A typical aftermarket application is the installation of a lift kits for light duty trucks that will increase the overall ride height of the vehicle either for appearance reasons or improved off-roading capability. The installation of such a lift kit may severely affect general vehicle handling and emergency maneuver stability. Currently, customers and aftermarket suppliers are the
main stakeholders involved in configuring, purchasing and installation of the modification kits. These stakeholders normally do not fully comprehend the impact of such modifications. It is impractical and costly to certify individual and/or combinations of aftermarket products with physical testing following OE processes. Therefore an efficient tool to support this design and assessment task for aftermarket modification is required.
1.2 Problem Statement

Ensuring compliance with aftermarket performance components can become very cumbersome since it requires an in-depth understanding and evaluation of the vehicle’s stability behavior (with ESC engaged), considering every possible permutation and combination of suspension, brake, wheel, and tire upgrades. Up until now most suppliers are conducting product design and verification in a ‘trial and error’ way for only parts of their own product portfolio. These suppliers install their product in an OEM vehicle and conduct simple road test often not following any established standards or procedures using objective metrics. If the product dissatisfies subjectively or fails, the aftermarket supplier may modify the component and run another trial with a different setup. This process may be very inefficient in terms of time and cost and may not comprehend the entire scope of testing required as part of the OE homologation process.

The existence of virtual simulation tools has significantly reduced the time and cost for product design and validation. The usability and quality of simulation tools and models very much depends on the validation (or lack thereof) with physical test result. With mechatronic systems (such as a vehicle using Electronic Stability Control (ESC) to improve its performance) it is generally more difficult to validate and obtain agreement between simulation and physical testing due to the interaction of mechanical behavior and software control. There are currently two ways to simulate vehicle dynamics integrated with ESC control: 1) HiL (hardware-in-the-loop) and 2) SiL (software-in-the-loop). HiL integrates the exact physical controller equipped in the vehicle into a simulation model. It requires an elaborate computation platform (real-time control and
I/O between hardware and software) and access to (proprietary) communication protocols that are often not accessible for aftermarket suppliers or customizers. SiL integrates a controller model built on software platform with the vehicle dynamics simulation models. Often, the software controller may containing similar (simplified) control objectives compared to the software embedded in the physical Electronic Control Unit (ECU) installed in the vehicle. Such a simplified (first-order) model may only represent the physical vehicle control behavior partially and determination/validation of the application range is therefore very important.

The aftermarket modification process differs significantly from the OE vehicle development stages. The design targets can be highly variable and depend on individual customer preferences and needs. Some customers lift their vehicle for aggressive aesthetics, others desire to improve handling only, while some may want to improve ride comfort. Therefore, aftermarket suppliers may need to develop and provide specific solutions for a variety of customer segments.

There is no single optimal solution for the range of customer needs. Usually, customers are unable to quantify their performance requirements, and they often express their needs qualitatively (simply better than OE or 20% more aggressive than OE). Moreover, customers expect that the modified vehicle setup remains in the safe zone for stability without elaborating what this exactly means.

The process for defining design factors can also be unpredictable. Some customers will request a specific ride height and don’t care about other (secondary) aspects/impacts while other customers may insist a specific suspension kit that is
indifferent to the weight increase. Such arbitrariness requires new a new approach and tools to assist the design and tuning of aftermarket chassis modifications.
1.3 Objectives

The purpose of this research is to develop a constraint-based design framework (tool) that supports the aftermarket chassis industry to maximize vehicle performance design freedom while ensuring vehicle dynamics compliance (with a focus on ESC compliance).

The design tool requirements are:

• Regulating the aftermarket modification problem in the product life cycle scheme and explicating the stakeholders’ engagement

• Study of the chassis modification impacts on the vehicle dynamics compliance

• Determination of the design constraints to meet different requirements in vehicle dynamics and stability aspects.

• Providing rule-based interactive means for the aftermarket manufacturers to determine solutions according to brand and customer preferences.

• Developing a computationally efficient interface for non-expert users
CHAPTER TWO
BACKGROUNDs

In this chapter, research related to the state-of-the-art of vehicle design for unique customer needs and vehicle dynamics stability will be discussed. Subsequently, the research gaps and opportunities are derived based on the background research.

2.1 Literature Review

Some fundamentals and methodologies of traditional mass customization (MC) are also applicable for aftermarket customization, even though aftermarket customization is conducted post-manufacturing by consumers compared to product development carried out by engineers.

2.1.1 Meeting Unique Customer Needs

Due to more and more market diversity being expected, products are developed for the purpose of meeting unique customer needs. One of the most predominant topics is mass customization. Methodologies for mass customization have been proposed and studied for different perspectives of product life cycle: general strategy, sales and marketing, design for solution space, manufacture, supply chain management and end-user delivery. This literature research below is focused on topics of strategy and solution space that support the achievement of personalization.

Mass customization was development in the 1980s aiming at "developing, producing, marketing and delivering affordable goods, and services with enough variety and customization that nearly everyone finds exactly what they want." [4] Tseng and Jiao
[5] emphasized that ‘mass customization should also characterize by ‘near mass production efficiency.’ Mass customization is a paradigm that is looking forward to achieving an optimum trade-off between product variety and cost.

*Customization in the product life cycle*

Duray [6] identified and classified mass customization with the point of customer involvement during the production cycle. There are four phases, design, fabrication, assembly, and use. Da Silverira [7] concluded more detailed classification with eight levels. (See Figure 1) Rudberg adopted the concept of customer order decoupling point (CODP) and aligned four most frequently used CODPs to the mass customization levels [8]. CODP is defined as the point in the product lifecycle, after which the decisions made are toward certain customer demand. Obviously, CODP is very comparable to the concept of point of customer involvement defined by Duray. There are four most frequently used CODPs: engineer-to-order (ETO), make-to-order (MTO), assemble-to-order (ATO), and make-to-stock (MTS).

![Customization level](image)

**Figure 1** Mass customization levels and CODPs
Examples of each MC levels are:

a) Customized design is the top level of mass customization. Customer preferences were inputted to the design team in the early stage of product life cycle. The product is designed (and engineered) to the orders. A concept car designed and made for a specific client is a typical example of customized design.

b) Customized fabrication means to tailor (make) products (usually regarding shape, size, material, and other properties) to meet customer requirements (orders) following the basic, predefined designs. For example, tailoring a suit to fit individual body shape is a customized fabrication process, and the CODP is MTO. Some luxury automobile companies offer MTO products enabling customization of interior’s shapes and materials to meet unique demands.

c) The most widely adopted strategy is customization in the assembly stage. The strategy is regarded as ATO (though sometimes it is referred to MTO or built-to-order (BTO)). Most vehicle manufacturers deploy ATO to configure different combinations of modules (powertrain, interior, exterior, wheel, tire, safety feature, etc.) to satisfy a large variety of customer needs. The customization application is postponed to the assembly stage and requires flexible production approach [9].

d) Additional custom work and services are achieved by adding light customization (personalization) and personal service at the point of delivery,
such as auto dealer provide a decal or sticker to personalize the appearance of the vehicle to identify individual membership of associations.

e) Customized packaging in different sizes and ways normally happens at the point of delivery. It is typically deployed in the food industry or while wrapping gift.

f) Customized usage is a product that can function in different modes to adapt to various circumstances and user preferences. It is widely applied with automotive products, e.g., a vehicle provides different operation options and allow the customer to switch modes between comfort, sporty, and racing using predefined setups/maps for powertrain and chassis components. There is no order from customers involved in the mass customization level, so it is regarded as MTS.

g) The lowest level of mass customization is standardization which means there is no customization with active customer involvement. Various product variants may still be designed, fabricated, and assembled, based on demand forecasts and market research. Standardization is also known as MTS.

The concept of modern product life cycle can be abstracted in five domains (see Figure 2): Customer domain, functional domain, design domain, process domain, and logistic domain [10] [11]. Mass customization approaches are developed to relay each step throughout the life cycles [12].
Figure 2. Mass customization in product life cycle

(CA: customer attribute, FR: functional requirement, DP: design parameter, PV: process variable, LV: logistic variable)

*Product family design*

Design for mass customization is emphasized with elevation from the practice of designing an individual product to designing a product family platform [13]. A Product family platform is required to identify target customer preferences and needs and try to fulfill these demands with various modular configurations based on a common platform. It has been well recognized as an effective means to achieve the economies of scale across diverse market niches [14] [15].

In the paradigm of design domains, product family design solutions are generated in the physical domain by mapping FRs to design parameters (DPs) based on the shared product platform [16]. Baldwin defined modularity for a product platform design with three aspects: modular architecture, interfaces, and the standards that the modules must conform [17]. Modularity allows manufacturers/consumer to mix and match modules to
come up with a configurational product that suits their preferences and needs. Other than modular customization, Simpson also define another common approach which is called scalable (namely parametric) product family design, whereby scaling variables are used to “stretch” or “shrink” the product platform in one or more dimensions to satisfy a variety of customer needs [18] [19].

Change management

Product family design has predefined a target customer and their demands. However, there may be situations where customers wish to have extra customization or the market changes in fast pace and thereby additional changes for a new option have to be made [20]. Hence, companies have to deliver systems incorporating an architecture, which supports changes throughout its entire lifecycle [21]. Change favorable representation (C-FAR) [22] aims at identifying the effects of changes using data-driven change representation and a propagation mechanism.

Change prediction method, as described by Clarkson [23], analyzed change processes and change propagation mechanism using likelihood, impact and risk design structure matrix (DSM).

Fricke proposed the design for changeability (DFC) [21], which is different from product platform design that it is enabled to deal with foreseen, and unexpected changes in the architecture throughout the life cycle. The DFC architecture is deployed considering four aspects of changes: robustness, flexibility, agility, and adaptability using principles including simplicity, independence, and modularity.

Mass personalization
In the recent years, information technology and pervasive internet connectivity entitled the development of mass personalization (MP). MP takes further steps forward to meet unique demands of individuals rather than demands of the market niches that has been realized by MC. Kumar elaborated the approaches to transform the mass customization to mass personalization [24]. Tseng proposed the design for mass personalization framework, suggesting latent customer needs are translated into functional requirements in the functional domain that formulates the specification of a product ecosystem. Mapping from functional requirements to design parameters is based on the shared product and value chain platform [25].

2.1.2 Exploration of Solution Space

One of the key issue of design for unique needs (DFUN) problem is mapping the FRs to DPs incorporating diverse customer needs. The aggregate feasible design candidates formulate a solution space. The design space has been visualized using graphical user interface by the early researcher. [26] [27]

Robust Design

Orthogonal arrays for the robust design aimed to reduce the solution space so as to maximize product platform profit [28]. Hernandez applied compromised decision support problem method for platform design by using a network of response surface [29]. Robust design principles were also used for product platform concept exploration method (PACEM) incorporated with metamodeling techniques [18]. Physical programming was
integrated with PACEM for the product platform definition [30]. Combined design spaces were developed to determine product family configurations [31].

**Optimization**

The optimization algorithm is one of the most powerful tools for solution determination from the design space. Linear and nonlinear programming algorithms, such as SLP, SQP [32], NLP and GRG, are employed in many studies, in addition to derivative-free methods including genetic algorithms, simulated annealing, pattern search, and branch and bound techniques [19]. Multilevel multidisciplinary design optimization was also proposed for a family of reconfigurable vehicles [33]. Hazare applied a genetic algorithm to generate concept design solutions in simulation-based engineering framework integrating customer preferences, and brand essence in customer domain [34] proposed.

**Design Space Exploration**

The procedure of identifying the feasible design space is referred to as design space exploration. Various methods have been developed for exploring the design space [35]. Generation of possible design points is often a procedure of sampling candidates, followed by verification of constraint satisfaction. Monte Carlo sampling method was adopted by many researchers [36] [35] [37]. Another author used the design of experiment as the sampling method to explore the design space of a parameterized system [38].

However, it is very inefficient for highly constraint problem that most candidates sampled might turn out to be unfeasible. Yannou proposed to use a primary constraint
programming to prune the solution space before using sampling method [35]. Later on, design space exploration is generalized into the constraint satisfaction problem (CSP). Hu modeled the searching of feasible design space as quantified constraint satisfaction problem (QCSP) and solve it using a branch-and-prune algorithm [37]. Interval arithmetic was integrated to define the acceptable region [39]. Recent researcher proposed an approach to reduce design space by coupling multi-fidelity models in a sequential decision process [40].

2.1.3 Electronic Stability Control Systems

The core know-how of ESC control algorithms was proposed in 1992 [41] and has been further developed for more than 20 years. The first commercial implementation of ESC in a production vehicle was in 1995. Despite this long history, the basic control philosophy has not changed much. The foundation of the ESC control logic is based on a simplified ideal vehicle dynamics model (usually a single-track model) to determine the desired vehicle behavior regarding yaw rate and sideslip angle upon driver steering input at a given vehicle speed [42]. Desired yaw rate and sideslip angle will be limited by the ESC controller for stabilization and controllability through individual wheel brake interventions by comparing the expected values with the instantaneously measured values. [41] [43] [44] [45].

In the last decade, many researchers have proposed alternative approaches to generating yaw moment to either stabilize the vehicle or improve the vehicle’s agility. These include active steering [45] [46] [47] [48], torque vector control with active
differential [49] [50] and drive/brake torque control of the electric motor in the emerging electric vehicle and hybrid vehicle [51] [52]. Different types of actuator are designed to function independently or coupled via optimal control strategy to allocate the gain for the various actuators [47] [48].

This dissertation focuses on conventional brake force control based ESC system behavior to comply with federal safety regulation requirement. Nevertheless, the proposed process will also be applicable to other types of stability/agility control systems.

ESC systems have been modeled to study driver/vehicle behavior study using driving simulator. Denoual integrated a simplified generic ESC model with the simulator to predict the driver subjective assessment of loss of adherence [53]. Pan integrated a supplier ESC software into powertrain and brake systems of a real-time simulation model to evaluate the effectiveness of ESC system [54]. The model was validated for straight-line acceleration/braking, pulse steer, and double lane change. By using this model Watson designed scenarios for ESC effectiveness study with the simulator [55]. Papelis also evaluated the effectiveness of ESC using simulator equipped with a Bosch ESC system [56]. Kinjawadekar [57] developed a simple ESC model and co-simulated with CarSim model which was validated regarding suspension characteristics and a sine-with-dwell (SWD) maneuver. Picot [58] introduced OEM process for specifying compliance boundary of CG location for final stage manufacturer using HiL. Kwon [59] also exploited the HiL approach to analyze the robustness of ESC system with a single-factor-multi-level methodology which included spring, stabilizer bar, damper, and tires as factors and yaw velocity change ratios as responses. Mcnaull validated a tractor-trailer
model, which was equipped with roll stability controller (as an extension of the ESC algorithm), regarding steady state and rapid ramp steer handling [60]. All researchers have shown moderate correlations between physical hardware test and virtual simulations. However, many were not able to elaborate on the following topics: firstly, they did not describe the structure of the controller in great detail due to the proprietary nature; secondly, they did not elaborate on the calibration process to adapt the control behavior to different types of vehicle; thirdly the models were insufficiently validated for their particular application purposes.
2.2 Research Opportunities (Gaps) from Literature Review

By reviewing the current state-of-the-art literature in the area of product customization design, there are no specific solutions explored for meeting the unique needs in the post-delivery product usage stage.

2.2.1 Product Engineering Scheme for Post-Delivery Modification

From the state-of-the-art research, different solutions that meet the requirements of variability scale of customization have been addressed. Standard product development considers a minimum level of customization; product family design creates a set of predefined products according to market research, the variability increases but is usually a trade-off with profitability concerns; change management can deal with predictable changes that go beyond the predefined product family and prepare the solution beforehand. Mass personalization develops a framework to satisfy a higher level of unique customer needs. In the aftermarket user modification domain, the variability of customer needs seems to be infinite and unpredictable due to the large numbers of consumers and available aftermarket options.

Many existing theories and approaches are established based on the context that original equipment manufacturers lead the customization process with a focus on the domains before the point of delivery in the product lifecycle timeline. In the aftermarket customization stage (which is after the point of delivery) aftermarket supplier and end-users (not original equipment manufacturers) take on the main role of development, selection, installation and validation. In some cases, original equipment manufacturers
may share a subset of specifications related to interfaces (geometrical and dimensional information, assembling standard, electronic protocol, etc.) to third party manufacturers. For example, body builder books/websites provide vehicle body interface specifications for intermediate and final stage manufacturers) [61]. However, a comprehensive framework that bridges the OE and aftermarket and is applicable for end-users does not exist yet. Such a framework also needs to tackle the difficulties to access (non-geometric) vehicle data that is typically of proprietary nature within the OE. In the current situation, aftermarket suppliers and end-users are unable to design, select and evaluate product performance parts on a system integration level like the OEs, who have full access to qualified engineering professional and engineering data.

2.2.2 Efficient Interactive Approach for Feasible Solution Determination

A core aspect of an aftermarket modification framework is the task of determining the feasible solution characterized by following attributes: First, it should provide the feasible ranges of the design variables; second, it should be user-interactive which obtains suppliers/customers inputs of preference and filters the feasible candidates accordingly; third, it should be computational efficient to ensure the interactive operation in a timely manner. Robust design and optimization methods provide the set of configurations which are robust or in the Pareto front, but they are not able to be simply represented by range information. Existing design space exploration methods focus on discovery all viable solutions through the design space, which also indicate the range
information. However, their speed of solution finding is too inefficient for a user-
interactive operation.

2.2.3 Precise Virtual Validation Approach for Vehicle Dynamics Compliance

To ensure vehicle dynamics compliance using virtual tools, a precise, easy-access,
and efficient simulation model is required. The model should represent the real vehicle
performances that are relevant to the chassis modification. Furthermore, the model should
reflect sensitivities of component variations accurately and appropriately. Existing
research either apply complex simulation platform using HiL technique or driving
simulators for high-fidelity studies which are complex to setup and operate. Alternatively
simplified simulation models have been used for control algorithm research and
qualitative studies, however, often without proper validation of the baseline model or
outcome proposed.
CHAPTER THREE
DEFINITION OF AFTERMARKET MODIFICATION METHOD

In this chapter, the proposed aftermarket personalization framework is discussed by comparing with conventional OEM led assemble-to-order personalization framework. The critical tasks and challenges for the implementation are elaborated from a higher level point of view.

3.1 Conventional Automotive Customization Framework

OEMs typically lead the traditional automotive mass customization process involving customer, legislation and component suppliers (Figure 3). OEMs normally own all the knowledge and resources that are required to develop such a product platform. Customers are involved in the early stage by introducing preference information through market research and clinics and captured as the voice of customer (VOC). Furthermore, legislation requirements, also known as the voice of legislation (VOL), are captured throughout the definition stage. OEMs incorporate the VOC and the VOL with their brand essence to define high-level product targets, which are described in qualitative terms and transformed into quantitative values of engineering functions. Quality function deployment (QFD) [62] is a popular method to translate these targets in a structural manner. As part of mass customization, there will be ranges of functional requirements to satisfy a diverse customers segment. OEMs have well-developed product family design method that generates multiple design configurations satisfying these ranges of requirements. Component suppliers are also involved during the product family design
stage with information about component properties (range of specifications), cost, weight and availability. This range of customized vehicles belong to the same product family, are manufactured and assembled at the same OE location in a flexible assembly line and delivered to a variety of customers at branded retail locations.

Figure 3 Traditional vehicle mass customization process
3.2 Post-Delivery Modification Framework

In the proposed aftermarket design process (Figure 4), there are two additional stakeholders involved: one is an umbrella organization, e.g., the Specialty Equipment Market Association (SEMA), who does not own the parts or vehicles and is responsible for defining product development guidelines and the formulation of the rule-based design envelope. The other stakeholder is the aftermarket kit supplier, who develops, manufactures, sells (and sometimes installs) the aftermarket modification parts for customers. Kit in this context means a set of components that replace the original parts or add-on the original vehicle.

The umbrella organization is responsible for setting up the guidelines incorporating VOC and VOL for a specific aftermarket segment, e.g., light truck modification. Those guidelines entail creating a general understanding of performance attributes that customers desire (to change) and capturing legislative aspects that are affected by vehicle modifications. These guidelines act as the outer layer in the design envelope, under which there is the supporting layer consists of mathematical (engineering) interrelationships between the attributes and design variables, evaluating and quantifying the rules. A design envelope is the outer boundary of a set of design alternatives, within which all designs are feasible. Rules are generic for a specific aftermarket segment, but design envelope is unique for a specific vehicle. OE vehicle data (functional as well as geometric) acquired directly from OE manufacturers or via reverse engineering methods, is fed into the rule-based system to determine the design envelope.
The rule-based design envelope becomes a general guidance for kit suppliers who will generate configurations for various kits. Kit suppliers define functional attribute set according to their brand essence for rule-based design envelope, allowing them to develop a catalog of configurations derived from the component design space. The component design space is a set of available specifications of components (e.g., spring stiffness, stabilizer bar stiffness). The kit supplier is responsible for collecting the component specifications either from component suppliers or measurement.

For marketing purpose, kit suppliers usually provide options for diverse needs; some configurations emphasize lifting vehicle ride height, and some focus on improving handling performance. The rule-based design envelope assures freedom for kit suppliers to configure kits that maximize the required performance while ensuring vehicle dynamics compliance. It is integrated with the attribute requirements and design space to generate configuration family for varied aftermarket modification demands.

In the final stage, components that picked for the kit configurations will be outsourced or manufactured in-house after which they get sold to customers as packages. Retrofitting is the last step in the aftermarket modification cycle. Modification kits will either replace the original vehicle components (spring, damper, control arms, etc.) or add to the vehicle (extra-subframe, for example).
Figure 4 Post-delivery modification framework
3.3 Definition of Rules and Compliances

A rule-based design envelope is a tool that is rooted in set-based design methodologies. The fundamental idea is to integrate the rules to determine feasible solutions sets.

Aftermarket chassis modifications, which fundamentally change the parameters in suspension system, will greatly impact vehicle dynamics behaviors. Those characteristics under routine operations such as steady state and transient state handling are not regulated by the government legislation, even though inappropriate steady state and transient state performance could lead to potentially unexpected accidents. The U.S. government regulates vehicle stability control by specifying a standardized counter steering maneuver and inspecting the agility and stability criteria as part of FMVSS126. In general, the stability controller, or specifically ESC, will automatically apply the brakes to individual wheels according to the vehicle handling states thereby providing a corrective yaw moment to stabilize the vehicle. It is known that a controller needs to be calibrated for a specific plant to have the most robust performances. Changing the plant only without recalibrating the controller might cause improper system response. For example, using poor quality tires in the rear axle may deteriorate the rear cornering stiffness and cause excessive oversteer that exceeds the stability controller’s capability to stabilize the vehicle promptly.

Vehicle stability control behavior is measured and quantified using criteria elaborated in FMVSS126 (similar with ECE R13H in Europe [63] and ADR 35-03 in Australia [64]). A SWD maneuver is used to apply perturbation to the vehicle from the
steering wheel and provokes the vehicle into an unstable condition; then the vehicle response is observed to examine the ESC performances. The perturbation is created by the SWD steering wheel input with different steering scalar from 1.5 to 6.5 (or 270 degrees) times the steering wheel angle at which the vehicle lateral acceleration reaches 0.3g in steady state [65].

There are two requirements for compliance, agility and stability requirements. The vehicle is required to dodge an obstacle with sufficient lateral offset when the steering scalar is more or equal to five, using lateral displacement at 1.07s from beginning of steering (BOS) as pass criterion; stability criterion comprises yaw rate ratio at 1s and 1.75s from complete of steering (COS) for all steering wheel scalars. While yaw rate ratio and lateral displacement are primary criteria to determine pass/fail in FMVSS126, wheel-lift-off-ground is also monitored to as part of the passing condition. FMVSS126 does not literally specify wheel-lift-off-ground situation as fail. However, in the rollover stability documentation The Fishhook maneuver Test Procedure [66], the termination condition ‘two wheel lift’ is defined as two wheels on one side lift off the ground at the same time for more than two inches. Therefore, in this research the lift heights of each wheel is recorded and the maximum value during each trial is used for data analysis.

The FMVSS126 pass criteria discussed above are mandatory rules that apply to the design envelope to ensure regulatory compliances. In the same time, additional non-regulatory requirement (such as ride comfort and non-emergency handling) can impact the design envelopes as additional constraints. In the vehicle dynamics field, the ride and
handling metrics have been well defined objectively to quantify various subjective aspect of customer perceivable quality. The umbrella organization mentioned above will need to determine upfront which metrics to incorporate in the design envelope according to specific fleet market.
CHAPTER FOUR
HIGH FIDELITY VEHICLE DYNAMICS AND CONTROL MODEL

Physical assessment of the ESC performance for every possible combination of chassis modifications would be impractical, time-consuming and very expensive. Therefore, the authors have developed a virtual evaluation process using high fidelity vehicle dynamics and control model to support the products development of aftermarket products regarding vehicle handling and FMVSS 126 compliance. Besides understanding the vehicle dynamics behavior using mathematical models, knowledge of the ESC algorithm plays a significant role in the closed-loop vehicle performance assessment process. The control code inside ESC systems developed by the various tier suppliers is proprietary (black-box) and unknown to the aftermarket community. However, the fundamental theories of vehicle stability control are widely studied and published by both academic researchers and commercial product developers. So it is viable to construct a generic (white-box) model, which is built upon this knowledge with elementary and essential functionality, to provide a conservative assessment of specific vehicle stability performances (e.g. FMVSS126). This chapter will elaborate on the development, calibration, and application of such a model.
4.1 Purpose of the High Fidelity Modeling

This research developed a process to create and calibrate a vehicle dynamics and ESC co-simulation model aiming to satisfy the application of SWD defined in FMVSS126 and other dry road emergency maneuvers (e.g. double lane change, brake-in-turn). At present, the ESC systems have become quite sophisticated regarding functionality and interaction with other vehicle control systems. The ESC controller must stabilize the vehicle for different road conditions (dry/wet, slip mu, banked, sloped) during different cornering operation (brake in turn/accelerate in turn, oversteer/understeer). The system must resist various perturbations and must be robust enough to tolerate system noise and deal with sensor fault condition [43] [67]. In this research, the stability control model is designed to be simple yet efficient to represent primary vehicle behavior using a generic algorithm. The purpose is to have a generic parameterized stability control model, which can be combined with various types of light duty vehicles, e.g. passenger cars, SUVs, and light trucks.
4.2 Vehicle Dynamics Modeling and Calibration

A vehicle dynamics model describes vehicle motion characteristics during different driving maneuvers. Various types of mathematical model, ranging from 2 to 15 degrees of freedom (DOF), have been developed by different researchers specifically for ESC related research [68]. Furthermore, commercially available vehicle dynamics simulation tools such as CarSim are available to deliver accurate, detailed, and efficient methods for simulating the vehicle dynamics performance. To avoid manually derive equations of motion for new tools, CarSim was selected as the primary simulation platform. CarSim vehicle dynamics model is based on a 15-DOF math model including: six DOFs (linear and angular motions) for the sprung mass as a rigid body; two DOFs (bounce and roll) for each suspension system; one spin DOF for each wheel; one DOF for the steering system.

![15-DOF vehicle dynamics model](image)

Figure 5. 15-DOF vehicle dynamics model
4.2.1 Specific Modeling Consideration

Details regarding modelling in CarSim are not focus of this dissertation; however, there are various critical model setup and parameter aspects that require meticulous consideration for best representation of vehicle behavior such as:

1) Mass and inertia adjustment. When conducting simulations for FMVSS 126 related maneuvers and the subject vehicle is a truck, multi-purpose vehicle (MPV) or bus, vehicle mass and inertia parameters must be adjusted for outriggers which are required to be mounted on the body in the front and rear during the FMVSS 126 SWD test for safety (roll over) concern. Vehicles with a baseline weight under 2,722 kg (6,000 lbs.) must be equipped with ‘standard’ outriggers; vehicles with a baseline weight equal to, or greater than 2,722 kg (6,000 lbs.) shall be fitted with ‘heavy’ outriggers [65]. Outriggers account for about 10% of the vehicle’s yaw moment of inertia and will alter the lateral response, which is a primary concern for calibrating and validating ESC behavior.

2) Brake system calibration. In CarSim, a simplified brake system model transforms brake pressure at the wheel cylinder to brake torque acted on the wheels. To accurately define the brake system it requires two key parameters: front and rear brake torque/pressure coefficients that need calibration. They are defined as the ratio of brake output torque over the pressure applied to the wheel brake cylinder. This ratio is a final result of various factors including cylinder size, brake pad size, position and material, and brake disc diameter. There are two ways to identify the ratio: one way is to acquire all these parameter values and derive the ratio using their mathematical relationship; another way is to reverse-identify the ratio if braking test data are available. The author used the
latter approach. As part of FMVSS 126, braking tests are required for different decelerations resulting in various front/rear brake pressure distributions. The front/rear braking force ratio tends to be greater under hard braking than that in the moderate braking. The front and rear brake coefficients are identified respectively by importing brake pressure signals from physical test into the CarSim model and correlating the simulation results with the ground test result. This methodology was proved to be efficient and precise.

(3) Specify motion sensor location. In the CarSim model, simulative accelerometers and angular rate sensors on a reference point generate the vehicle motion information such as lateral acceleration and yaw rate. A higher location of the reference point may cause higher longitudinal and lateral acceleration values due to the pitch and roll motion of the vehicle body. Therefore, the reference point location needs to be specified by the position of the motion sensor mounted on the real vehicle during testing.

(4) Tire input data requirement. CarSim supports semi-empirical tire models (Pacejka 5.2 and TNO Delft-tire.) and tabular data tire models. During ESC activation, the tire vertical load can increase to almost double of its nominal value when one side of the vehicle is marginally lifted off the ground. It is known that tire cornering stiffness increases along with vertical load in a nonlinear way. The relationship can be approximately described using empirical magic formula [69]. In case this data for such a range is not available, CarSim will extrapolate the cornering force values using its internal algorithm which might not represent the real tire behavior. We, therefore, have to
ensure that the tire data (in lookup table format) will cover a broad range of vertical loads.

### 4.2.2 Vehicle Dynamics Model Validation

A mathematic model can only be an approximation of the actual system; therefore, the calibration/validation process is essential to determine an accurate representation of the system for particular objectives of the study [70]. In this study, the model is validated to ensure its credibility for the purpose of FMVSS126 simulation. The SWD test, which is the core maneuver to determine compliance, is a transient state handling with the input of steering wheel angle and braking force and output of lateral dynamics metrics (yaw rate, sideslip angle, lateral acceleration and lateral displacement).

In order to validate the vehicle dynamics model, the following three aspects are proposed: steady state handling, transient state handling, brake intervention handling.

1. **Steady state handling.** The steady state handling is about the basic vehicle dynamics characteristics such as understeer gradient, yaw rate gradient, and roll gradient. The result can be used to validate the configuration accuracy of vehicle load distribution, the center of gravity height, tire cornering stiffness, vehicle rolling stiffness, steering system, and suspension kinematics characteristics. There are various types of steady state handling maneuver defined in ISO 4138 [71] for validating the steady state handling.

2. **Transient state handling.** The transient state handling is about vehicle dynamic responses under unsteady inputs. This performance highly depends on information like moments of inertia, suspension damping, and transient tire characteristics, steering
system compliance. The ISO7401 [72] standard introduces five different types of test for validating transient state dynamics.

(3) Brake intervention maneuver. Since ESC utilizes braking force to provide a stabilizing yaw moment, accurate braking responses of vehicle and wheels are the most important factors to determine the quality of ESC-involved simulations. In the previous chapter, the brake system coefficient has been identified using straight line braking information. A next stage is to validate handling and braking coupled performance by applying recorded brake signals along with other driver commands to determine the model responses. Since the model has been validated for transient handling and straight line braking, more modeling attention can be devoted regarding combined slip effect [69] of tires, which is regarded as one of the most critical factors for brake intervention handling simulation.
4.3 Stability Control Modeling

The generic (white box) stability control model proposed has been developed with following requirements:

- It is derived from conventional theory and based on simple control logic;
- It has the demanding functionality for particular application circumstance;
- It holds certain flexibility which enables calibration by users;
- It is manifest such that users do not need profound prerequisite knowledge.

4.3.1 Stability Control Strategy

In general, stability controller calculates the necessary corrective yaw moment and activates the brakes at specific vehicle wheels to achieve the required stabilizing yaw moment [43] [44] [67]. The proposed ESC model is dedicated for CarSim co-simulation applications. Various system states are directly accessible from the CarSim model, so there is no need of states observers to determine sideslip angle, slip ratio, tire force and road friction coefficient [43] [73] [74] [75] [76] [77] [78].

Figure 6 shows the basic block diagram of the ESC model control strategy. The stability control system usually consists of an upper-level controller and a lower-level controller. The upper-level controller determines the yaw moment by different ways depending on types of control state variables and control method. The yaw rate and sideslip angle are the most common used state variables. Sideslip angle control benefits from applications on low friction road conditions where the vehicle sideslips substantially more than on low friction road surfaces [79]. Considering that our research mainly
focuses on dry road maneuvers, the controller only uses yaw rate feedback. A PID type control method has been used to determine the yaw moment correction gain since it has been widely applied in commercial products [80] [81] [82].

The lower-level controller determines individual wheel brake force distribution based on the yaw moment demand. A slip ratio controller function is integrated as part of the ESC system to avoid locking a wheel (basic ABS functionality).

![Figure 6. ESC model controls strategy](image-url)
4.3.2 Reference Model

A single-track vehicle dynamics model is used as a reference model for tracking control. The vehicle lateral dynamics are described through the following state space equation (1): [83] [84]

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{\beta} \\
\dot{\gamma}
\end{bmatrix} +
\begin{bmatrix}
h_1 \\
h_2
\end{bmatrix} \delta_f
\]  

(1)

where:

\[a_{11} = -\frac{c_1 + c_2}{M V}, \quad a_{12} = -1 - \frac{(a c_1 - b c_2)}{M V}, \quad a_{21} = \frac{-a c_1 + b c_2}{l_z}, \quad a_{22} = -\frac{a^2 c_1 + b^2 c_2}{l_z V}\]

\[b_2 = \frac{1}{l_z}, \quad h_1 = \frac{c_1}{M V}, \quad h_2 = \frac{a c_1}{l_z}\]

The lateral tire load transfer (during cornering), which is a consequent of lateral acceleration, will deteriorate the effective cornering stiffness due to the tire characteristics [69] [83]. For the reference model, it is simplified that the effective tire cornering stiffness is a linear function of the lateral acceleration:

\[C_1 = d_1 a_y + d_2 \quad C_2 = d_3 a_y + d_4\]

(2)

Since a steady-state simulation model has been formerly constructed and validated, so the coefficients \(d_1, d_2, d_3,\) and \(d_4\) can be identified using linear regression of the tire cornering stiffness and lateral acceleration data from steady-state simulations. This description is more realistic because the effective cornering stiffness takes into account factors such as tire properties, suspension kinematic and compliance (K&C) characteristics and lateral load transfer [84].
During steady state the desired yaw rate can be derived from (1):

\[ r_d = \frac{a_{11} h_2 - a_{21} h_1}{a_{21} a_{12} - a_{11} a_{22}} \delta_f \]  

(3)

The desired yaw rate response \( r_d \) represents the desired vehicle response for model tracking control algorithm [46] [68] [85].

### 4.3.3 Yaw Rate Boundary

The desired yaw rate only considers the ideal vehicle response to steering wheel input generated by the driver regardless of the limit of tire-to-road friction. However, in reality, the yaw rate is constrained by the maximal acceleration the vehicle can achieve. Therefore, from the relationship of lateral acceleration and yaw rate, an upper boundary for the target yaw rate could be defined as [67]:

\[ a_y = V r + \dot{V} \beta + V \dot{\beta} \leq \mu g \Rightarrow r_{\text{ubound}} = k_{\text{perc}} \frac{\mu g}{V} \]  

(4)

where \( k_{\text{perc}} \) denotes percentage of centripetal acceleration \( (V r) \) counts for total lateral acceleration. Usually \( k_{\text{perc}} \) is between 0.85 and 1.00. For example, if the vehicle is cornering at constant speed and constant radius, \( k_{\text{perc}} \) is equal to 1.

When the desired yaw rate is higher than the boundary value, a yaw rate correction \( \Delta r \) can be calculated using following equations:

\[
\begin{align*}
\Delta \dot{r} &= \frac{1}{\tau_r} \left( |r_{\text{unlimited}}| - |r_{\text{ubound}}| - \Delta r \right), \text{if } |r_{\text{unlimited}}| > |r_{\text{ubound}}| \\
\Delta r &= 0, \text{ if } |r_{\text{unlimited}}| \leq |r_{\text{ubound}}|
\end{align*}
\]

(5)

where \( \tau_r \) is the time constant and \( r_{\text{unlimited}} \) is the unbounded desired yaw rate.

The target yaw rate is given by:
\[ r_d = (|r_{unlimited}| - \Delta r) \cdot sgn(r_{unlimited}) \]  

4.3.4 Yaw Moment Controller

The yaw moment controller determines the magnitude of corrective yaw moment based on the error between the target value and measured value.

By setting state variable \( X = [\beta \quad r]^T \) and taking the yaw moment \( M_z \) as the system input, the system equation can be obtained:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{r}
\end{bmatrix} = 
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
\beta \\
r
\end{bmatrix} + 
\begin{bmatrix}
0 \\
b_2
\end{bmatrix} M_z
\]  

The transfer function can be derived as:

\[
G_{r,M_z}(s) = \frac{r(s)}{M_z(s)} = \frac{b_2(s - a_{11})}{s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}}
\]

A PD-controller can be used to close the loop and force the actual yaw rate to follow the desired yaw rate:

\[
C_{PD}(s) = K_p + K_d s
\]

The yaw moment can be expressed with:

\[
M_z(s) = (K_p + K_d s)(r_d - r)
\]

4.3.5 Brake Pressure Allocation

The brake force is determined from the desired the yaw moment generated by the yaw moment controller using the following equation:

\[
F_x = \frac{2M_z}{T}
\]
where \( F_x \) denotes total braking force required to generate the desired yaw moment and \( T \) is the average vehicle track width (assuming the front and rear track width are identical).

Next, the brake force needs to be allocated to front and rear wheels. To produce a yaw moment efficiently, it is assumed that the front or rear braking force are proportional to the vertical tire loads. The front and rear coefficient of friction \( \mu_x \) is assumed identical [84] according to:

\[
\frac{F_{xf}}{F_{zf}} = \frac{F_{xr}}{F_{zr}} = \mu_x \quad (12)
\]

In the simulation model, the vertical tire loads can be determined by CarSim such that the values can directly be used to calculate the front and rear brake force distribution given by:

\[
F_{xf} = \frac{F_{zf}F_x}{F_{zf}+F_{zr}} \quad F_{xr} = \frac{F_{zr}F_x}{F_{zf}+F_{zr}} \quad (13)
\]

Now the brake pressure at the wheel can be determined by:

\[
P_{bf} = \frac{F_{xf}R_{tire,f}}{C_{bf}} \quad P_{br} = \frac{F_{xr}R_{tire,r}}{C_{br}} \quad (14)
\]

where \( R_{tire,f} \) and \( R_{tire,r} \) are front and rear tire dynamic rolling radii, \( C_{bf} \) and \( C_{br} \) are brake torques – brake pressure ratio coefficient.

In most cases, the brake pressure is applied on one side of the wheels determined by the vehicle understeer/oversteer condition. Positive yaw rate error, which means desired yaw rate is larger than the current yaw rate (the vehicle is understeer), leads to positive desired yaw moment, resulting in brake pressure application to the inner wheels. Negative yaw rate error, which means the vehicle is oversteer, leads to negative desired yaw moment and consequential outer wheels braking.
The brake forces determined by the ESC controller will be merged with the brake command from the driver brake pedal. The ESC system will select the brake force whichever is larger such that the vehicle can be slowed down according to the driver’s command while remaining stable.

### 4.3.6 Slip Ratio Controller

The wheel slip ratio has been applied as the control objective in [42] [73] [74] by regulating the slip ratio according to nominal (demanding) value via brake torque manipulation. The slip ratio controller functions like an anti-lock brake (ABS) controller. The input of this module is the tire longitudinal slip ratio, which is defined as:

\[
\lambda = 1 - \frac{\omega_{\text{roll}} R_{\text{tire}}}{V}, \quad 0 < \lambda < 1
\]

where \( \omega_{\text{roll}} \) denotes the wheel rolling speed.

The target of slip ratio controller is to regulate the slip ratio around the operation point where the friction coefficient peak is located, providing the wheel for generating the braking force most efficiently. For example, the tire data shown in Figure 7 indicates that the operation points range from 0.15 to 0.25 where the friction coefficient is around 1. A simple logic of the slip ratio controller is shown in Figure 8. When the slip ratio is less than 0.15, the brake action is turned on; when the slip ratio is between 0.15 and 0.25, the brake holds the status; and when the slip ratio is larger than 0.25, the brake action is turned off.
Figure 7. Friction coefficient peaks around longitudinal slip ratio=0.2

Figure 8. Slip ratio control logic

4.3.7 Brake Actuator Model

The dynamic behavior of the hydraulic brake actuator system has been simplified using relay switches in combination with a second-order transfer function. The relay switches will turn the slip ratio signal into on/off command for the brake actuator. A second-order transfer function is used to simulate the pressure build-up process of the hydraulic system according to:
\[
\frac{P_{bo}}{P_{bi}} = \frac{\omega_b^2}{s^2 + 2\zeta_b \omega_b s + \omega_b^2}
\]  \hspace{1cm} (16)

where \( P_{bi} \) and \( P_{bo} \) are input and output pressure signal; \( \omega_b \) denotes system natural frequency; \( \zeta_b \) denotes brake system damping ratio.

The damping ratio is set to 0.8 to assure no overshoot and oscillations occur in the system response. Natural frequency \( \omega_b \) is determined by the response speed of the system. With a high damping ratio, the desired time that response achieves the steady-state value is approximated to that of peak response. The system peak response time characterize the response delay of the ESC system. It is related to natural frequency and damping ratio according to the equation (17). By choosing appropriate natural frequency and damping values the system can be calibrated to respond with desired response speed.

\[
t_{pb} = \frac{\pi}{\omega_b \sqrt{1 - \zeta_b^2}}
\]  \hspace{1cm} (17)
4.4 Stability Model Co-Simulation and Calibration

The ESC model is designed to work with CarSim by exchanging required signals during the simulation with CarSim vehicle dynamics model. Furthermore, various parameters inside the ESC model need to be properly defined for the model to run appropriately.

4.4.1 Import/Export Channel Definition

The import channels from the ESC model to CarSim are the brake pressure at each wheel. They will replace the internal braking signal in CarSim and manipulate vehicle behaviors. The export channels from CarSim to ESC model are a series of driver command and vehicle response information. Detailed channel information is shown in Figure 9.

![Figure 9. Import/Export channels for CarSim-ESC co-simulation](image-url)
4.4.2 Stability Controller Calibration

Calibration is a process to refine the controller model by configuring parameters according to vehicle model information and ultimately create a simulation model which performs as close to the physical vehicle test results as possible for specific handling maneuvers. The stability controller model proposed allows flexible calibration for different types of vehicles. Several aspects can be calibrated in the ESC model: reference model response, yaw moment PD controller gain and brake actuator response delay.

4.4.3 Reference Model Calibration

As introduced in Chapter Three, the reference model is a single-track vehicle model, which describes the desired yaw rate using steady state values. This method is inherently inadequate because the desired yaw rate responds simultaneously to the steering wheel input, which is not a realistic phenomenon and will lead to an incorrect yaw rate error for the desired yaw moment calculation. The dynamic characteristics of the steering system, suspension system, tires in conjunction with the vehicle’s moment of inertia lead to a delay in the yaw rate response. Therefore, first-order dynamics was applied for the calibration of the reference model time delay. The desired yaw rate now becomes:

\[ r_d(s) = \left( \frac{a_{11}h_2 - a_{21}h_1}{a_{21}a_{12} - a_{11}a_{22}} \right) \left( \frac{1}{\tau s + 1} \right) \delta_f(s) \]  

(18)

where \( \tau \) denotes time constant of first-order system.

To determine a precise number for the time constant is difficult due to complex factors involved. However, since a validated CarSim model is already available, it can
provide necessary information to calibrate the reference model in moderate handling behavior (when the vehicle should be stable with no ESC intervention needed). Figure 10 shows that, for a sine steer (slalom) at 60km/h maneuver, the reference model behaves very similarly to the CarSim model. Other maneuvers (such as steady state cornering or moderate transient state handling) can also be used to calibrate the reference model as long as the ESC functionality is not engaged.

![Figure 10. Sine steers response comparison between reference model and CarSim model](image)

**4.4.4 Yaw Moment PD Controller Gain Calibration**

The yaw moment PD controller gains ($K_p$ and $K_d$) vary depending on the vehicle dynamic behavior. The closed-loop yaw rate control is shown in the block diagram in Figure 11:
Figure 11. PD control based ESC system block diagram

The closed-loop transfer function of response yaw rate respect to desired yaw rate is:

\[
\frac{r(s)}{r_d(s)} = \frac{C_{PD}(s)G_{r,Mz}(s)}{1 + C_{PD}(s)G_{r,Mz}(s)}
\] (19)

The characteristics equation can be described as a standard second order equation form:

\[
s^2 + 2\zeta \omega_n s + \omega_n^2 = 0
\] (20)

where:

\[
2\zeta \omega_n = \frac{K_p b_2 - a_{11}b_2 K_d - a_{11} - a_{22}}{1 + b_2 K_d}
\] (21)

\[
\omega_n^2 = \frac{a_{11}a_{22} - a_{12}a_{21} - a_{11}K_p b_2}{1 + b_2 K_d}
\] (22)

By selecting a proper damping ratio and system settling time, the controller gain can be initially determined using the following description:

\[
K_p = \frac{a_{22}}{b_2} - \frac{(2\zeta \omega_n + a_{11})a_{12}a_{21}}{b_2(\omega_n^2 + 2\zeta \omega_n a_{11} + a_{11}^2)}
\] (23)

\[
K_d = -\frac{1}{b_2} - \frac{a_{12}a_{21}}{b_2(\omega_n^2 + 2\zeta \omega_n a_{11} + a_{11}^2)}
\] (24)
\[ T_s = \frac{4}{\zeta \omega_n} \] (25)

It is required to accurately define the vehicle effective cornering stiffness, system damping ratio, and system settling time so as to have reasonable controller gain values. In SWD maneuver, the vehicle behaves between minimal to maximal lateral acceleration. So it is suggested to recalculate the cornering stiffness under average lateral acceleration of the maneuver (approximate 0.5g), using the formerly defined linear function. The damping ratio of a system can be set to 0.7 to reduce response oscillations while maintaining the response speed, and system settling time \( T_s \) can be set to 0.5s which is fast enough to stabilize the vehicle. Subsequently \( \omega_n \) is obtained to be around 11Hz. The gains of the ESC controller can be determined from the damping ratio \( \zeta \) and natural frequency \( \omega_n \) in conjunction with equation (23) and (24).

Since these control gains are based on a simplified single-track vehicle model, slight adjustments might be required because the CarSim model has more degrees of freedom. In some cases, the gains (especially \( K_d \)) can be reduced to mitigate the braking response, which may affect the passenger’s subjective feeling.

### 4.4.5 Calibration of Activation Threshold

The abrupt activation of the stability control system may startle the driver due to its intrusive intervention [67]. For this reason, activation dead-band is incorporated to enable/disable the system. Two thresholds are proposed to active the stability control system using a minimum yaw rate \( r_t \) and minimum yaw rate error \( e_{rt} \). The yaw rate threshold is applied to deactivate the system during low speed maneuvers resulting in
high yaw rate error. The yaw rate error threshold is applied to accommodate for small discrepancies between reference model and CarSim model; on the other hand, to deactivate the system until emergency situations occur. Two thresholds need to be satisfied at the same time to trigger the controller. Therefore, the output yaw moment is defined by the following equation:

$$
\begin{align*}
M_z(s) &= (K_p + K_d s)e_r & \text{if } r > r_t & \text{ and } e_r > e_{rt} \\
M_z(s) &= 0 & \text{else}
\end{align*}
$$

(26)
4.5 Calibration Case Study

Two case studies regarding the ESC model calibration were conducted. The first one comprises the FMVSS126 ESC behavior of the 2014 Chevrolet Silverado (light duty truck), the second case deals with the double lane change maneuver (with ESC activation) of 2012 Ford Focus (passenger car).

The correlation coefficient $R_{xy}$ is used to numerically evaluate the quality of validation. It is defined as:

$$R_{xy} = \frac{C_{xy}}{\sqrt{(C_{xx}C_{yy})}}$$  \hspace{1cm} (27)

where $C_{xy}$ as the covariance function according to:

$$C_{xy} = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})$$  \hspace{1cm} (28)

where $X_i$ denotes test data and $Y_i$ denotes simulation data. The correlation coefficient ranges from 0 to 1. A coefficient value close to 1 indicates that the two data sets are very similar, in other words that the simulation model represents the physical vehicle well.

4.5.1 Case Study 1: 2014 Chevrolet Silverado

Vehicle model calibration

In the first case study, the 2014 Chevrolet Silverado vehicle model was validated with following maneuvers:

(1) Slowly increase steer (13.5 deg/s) at 80km/h for steady state handling.

(2) 1 Hz sine wave steering (130 deg) at 60 km/h for transient state handling.
(3) SWD with wheel brake input for handling with braking.

The steering wheel angle, vehicle speed, lateral acceleration, yaw rate, roll rate and roll angle are recorded for a side-by-side comparison of simulation and test result. The responses for lateral acceleration, yaw rate, and roll angle were assessed concerning the correlation coefficients. All correlation coefficients were greater than 0.93, which meant the simulation model is high fidelity. (Figure 12, Figure 13, and Figure 14)

Figure 12. Responses of Silverado slowly-increase-steering at 80km/h
Figure 13. Responses Silverado 1Hz sine wave steering at 60km/h
Figure 14. Responses of Silverado in SWD maneuver (brake signal input)
Table 1 Silverado model correlation coefficients

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Lateral acceleration</th>
<th>Yaw rate</th>
<th>Roll angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slowly-increase-steering</td>
<td>0.9957</td>
<td>0.9954</td>
<td>0.9956</td>
</tr>
<tr>
<td>1Hz sine wave</td>
<td>0.939</td>
<td>0.924</td>
<td>0.934</td>
</tr>
<tr>
<td>Brake signal input SWD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>136 deg</td>
<td>0.995</td>
<td>0.994</td>
<td>0.987</td>
</tr>
<tr>
<td>171 deg</td>
<td>0.993</td>
<td>0.993</td>
<td>0.980</td>
</tr>
<tr>
<td>205 deg</td>
<td>0.986</td>
<td>0.995</td>
<td>0.968</td>
</tr>
<tr>
<td>239 deg</td>
<td>0.967</td>
<td>0.992</td>
<td>0.930</td>
</tr>
<tr>
<td>270 deg</td>
<td>0.975</td>
<td>0.990</td>
<td>0.939</td>
</tr>
</tbody>
</table>

**ESC model validation for SWD maneuver**

ESC integrated co-simulation model was validated with SWD maneuvers listed in Table 3, including minimum and maximum steering input cases. Though there are other maneuvers with different steering scale factors and different steering direction in FMVSS126 certificate test, it is believed that these maneuvers can sufficiently validate a symmetrically built simulation model.

Table 2. SWD simulation cases list

<table>
<thead>
<tr>
<th>Steering angle requirement</th>
<th>Steering angle target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left to Right 1.5X</td>
<td>51 deg</td>
</tr>
<tr>
<td>Left to Right 2.0X</td>
<td>68 deg</td>
</tr>
<tr>
<td>Left to Right 3.0X</td>
<td>102 deg</td>
</tr>
<tr>
<td>Left to Right 4.0X</td>
<td>136 deg</td>
</tr>
<tr>
<td>Left to Right 5.0X</td>
<td>171 deg</td>
</tr>
<tr>
<td>Left to Right 6.0X</td>
<td>205 deg</td>
</tr>
<tr>
<td>Left to Right 7.0X</td>
<td>239 deg</td>
</tr>
<tr>
<td>Left to Right 8.0X</td>
<td>270 deg</td>
</tr>
</tbody>
</table>

The appropriate model parameters listed in Table 13 were determined using the procedures explained in the previous chapter.
Table 3. ESC parameters configuration for Silverado

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>m</td>
<td>1.593</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>3.645</td>
</tr>
<tr>
<td>$T$</td>
<td>m</td>
<td>1.77</td>
</tr>
<tr>
<td>$m$</td>
<td>kg</td>
<td>2366</td>
</tr>
<tr>
<td>$I_x$</td>
<td>kg $\cdot$ m$^2$</td>
<td>5800</td>
</tr>
<tr>
<td>$R_{tire}$</td>
<td>m</td>
<td>0.385</td>
</tr>
<tr>
<td>$C_1$</td>
<td>N$\cdot$deg$^{-1}$</td>
<td>$-114600a_y + 194820$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>N$\cdot$deg$^{-1}$</td>
<td>$-25327a_y + 150930$</td>
</tr>
<tr>
<td>$t_{pb}$</td>
<td>s</td>
<td>0.2</td>
</tr>
<tr>
<td>$K_p$, $K_d$</td>
<td>-</td>
<td>58000, 50</td>
</tr>
<tr>
<td>$\tau$</td>
<td>s</td>
<td>0.22</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>rad$\cdot$s$^{-1}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$e_{rt}$</td>
<td>rad$\cdot$s$^{-1}$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

A subset of the co-simulation results on vehicle states and brake pressure signals are shown in Figure 15 and Figure 16. The correlation coefficients for the maneuvers investigated were all above 0.9, which means a good simulation quality considering the complexity of the model itself (Table 4). The brake pressure at each wheel shows that the ESC model performs quite well following the desired logic: not engaged or marginally engaged while the vehicle is stable and activated during evasive handling maneuvers.

Table 4. ESC model co-simulation correlation coefficient

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Lateral acceleration</th>
<th>Yaw rate</th>
<th>Roll angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>51 deg</td>
<td>0.997</td>
<td>0.979</td>
<td>0.990</td>
</tr>
<tr>
<td>68 deg</td>
<td>0.997</td>
<td>0.983</td>
<td>0.992</td>
</tr>
<tr>
<td>102 deg</td>
<td>0.995</td>
<td>0.990</td>
<td>0.988</td>
</tr>
<tr>
<td>136 deg</td>
<td>0.989</td>
<td>0.994</td>
<td>0.976</td>
</tr>
<tr>
<td>171 deg</td>
<td>0.987</td>
<td>0.994</td>
<td>0.968</td>
</tr>
<tr>
<td>205 deg</td>
<td>0.972</td>
<td>0.986</td>
<td>0.939</td>
</tr>
<tr>
<td>239 deg</td>
<td>0.947</td>
<td>0.979</td>
<td>0.906</td>
</tr>
<tr>
<td>270 deg</td>
<td>0.956</td>
<td>0.980</td>
<td>0.922</td>
</tr>
</tbody>
</table>
Figure 15. Silverado SWD simulation and test data comparison (ESC model co-simulation)
Figure 16. Silverado braking pressure simulation and test data comparison (ESC model co-simulation)
4.5.2 Case Study 2: 2012 Ford Focus

The second case study was conducted for a passenger car. The vehicle model was validated with steady (constant radius cornering), transient (step steer) and brake intervention (double lane change with brake signal input) maneuvers (Figure 17 and Figure 18). Table 5 indicates that the minimum correlation coefficient is 0.919, which acknowledges the model fidelity. The ESC model was calibrated to adapt to the vehicle configuration with parameters configuration listed in Table 6.

Three different co-simulations of double lane change maneuvers were carried out at three different speeds: 90km/h, 97km/h, and 104km/h. The correlation coefficient results (Table 7) show that the simulation model replicated the test vehicle behavior very well. A subset of the simulation results is presented in Figure 17 -Figure 19.

Table 5. Vehicle model correlation coefficients

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Lateral Acceleration</th>
<th>Yaw rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant radius cornering</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>Step steer</td>
<td>0.998</td>
<td>0.999</td>
</tr>
<tr>
<td>Brake signal input double lane change</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90 km/h</td>
<td>0.962</td>
<td>0.977</td>
</tr>
<tr>
<td>97 km/h</td>
<td>0.993</td>
<td>0.986</td>
</tr>
<tr>
<td>104 km/h</td>
<td>0.919</td>
<td>0.932</td>
</tr>
</tbody>
</table>
Table 6. ESC model parameters configuration for Focus

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>m</td>
<td>0.9</td>
</tr>
<tr>
<td>(L)</td>
<td>m</td>
<td>2.651</td>
</tr>
<tr>
<td>(T)</td>
<td>m</td>
<td>1.55</td>
</tr>
<tr>
<td>(m)</td>
<td>kg</td>
<td>1384</td>
</tr>
<tr>
<td>(I_z)</td>
<td>kg(\cdot)m(^2)</td>
<td>1500</td>
</tr>
<tr>
<td>(R_{tire})</td>
<td>m</td>
<td>0.321</td>
</tr>
<tr>
<td>(C_1)</td>
<td>N(\cdot)deg(^{-1})</td>
<td>(-69587.412 \cdot Ay + 137520)</td>
</tr>
<tr>
<td>(C_2)</td>
<td>N(\cdot)deg(^{-1})</td>
<td>(-31744.2 \cdot Ay + 74490)</td>
</tr>
<tr>
<td>(t_{pb})</td>
<td>s</td>
<td>0.48</td>
</tr>
<tr>
<td>(K_p, K_d)</td>
<td>-</td>
<td>13000, 50</td>
</tr>
<tr>
<td>(\tau)</td>
<td>s</td>
<td>0.22</td>
</tr>
<tr>
<td>(r_e)</td>
<td>rad(\cdot)s(^{-1})</td>
<td>0.1</td>
</tr>
<tr>
<td>(e_{rt})</td>
<td>rad(\cdot)s(^{-1})</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 7. Double lane change correlation coefficients (ESC model co-simulation)

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Lateral acceleration</th>
<th>Yaw rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 km/h</td>
<td>0.944</td>
<td>0.959</td>
</tr>
<tr>
<td>97 km/h</td>
<td>0.993</td>
<td>0.982</td>
</tr>
<tr>
<td>104 km/h</td>
<td>0.955</td>
<td>0.947</td>
</tr>
</tbody>
</table>
Figure 17. Focus vehicle model validation
(a) Steady state; (b) Transient state; (c) Brake intervention handling
Figure 18. Focus double lane change @97km/h (ESC model co-simulation)
Figure 19. Focus double lane change @103km/h (ESC model co-simulation)
CHAPTER FIVE
SAMPLING BASED DESIGN ENVELOPE DEVELOPMENT

This chapter will elaborate on the application of the validated simulation model introduced in Chapter Four to study the impact of major design parameter variations on regulatory compliance. A DOE using the Taguchi method was carried out to analyze component variation impact to vehicle dynamics (open-loop) and stability control (closed-loop) performance. The main effect of factors indicated determined components corresponded to different performance metrics. The interrelations between open-loop metrics and closed-loop metrics were studied, and the feasibility of predicting closed-loop behaviors with open-loop responses is discussed. Finally, preliminary design boundaries are projected to the two-dimensional plane to form a guideline for aftermarket suppliers.

5.1 Establish the Design of Experiment

Among vehicle categories currently available, light-duty trucks and sports utility vehicles (SUV) attract more attention regarding ESC regulations due to their high center of gravity. Aftermarket kits that lift the center of gravity could potentially increase the risk of non-compliance or heighten the risk of roll-over. Therefore, in the following section various lift kit configurations were studied using DOE. Lift kit modifications usually combine several components such as subframe, spring, anti-roll bar (ARB), damper and oversized tires. The Taguchi method [86] DOE method was used as an efficient means to study the influence of different vehicle modification combinations.
5.1.1 Determination of the Design Factors

Considering the popularity of light-truck lift kits currently existing in the market, five control factors were selected: sprung mass lift height, front and rear spring stiffness, front and rear ARB roll stiffness (equivalent at the wheel). Damper alterations will cause damping force changes at multiple working points, and tires alteration also causes multiple characteristic changes (lateral, longitudinal and aligning directions) at the same time. To reduce computational effort and post process, they are not considered as factors for this DOE study.

Each factor is sampled with five levels to capture potential systematic nonlinearity and approximately identify the pass/fail margin. Lift height varies from 0 to 12 inches which cover most of the lift kit products available in the market today. Front and rear spring stiffness, and front ARB range from 50% to 150% of nominal value. While a rear ARB is not equipped in the OE vehicle, so its incremental step between each level is set to 300Nm/deg (see Table 8). Orthogonal array L25 (Taguchi et al., 1989) is used to generate experiment matrix for five factors with five levels design. There are total 25 trials, each level of every factor occurs for five times (see Table 9).

Table 8. Factors determination for DOE

<table>
<thead>
<tr>
<th>factors</th>
<th>Description</th>
<th>Unit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>vehicle sprung mass lift height</td>
<td>inch</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>X2</td>
<td>front spring stiffness</td>
<td>%</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
<td>150</td>
</tr>
<tr>
<td>X3</td>
<td>front anti-roll bar stiffness</td>
<td>Nm/deg</td>
<td>675</td>
<td>1012.5</td>
<td>1350</td>
<td>1687.5</td>
<td>2025</td>
</tr>
<tr>
<td>X4</td>
<td>rear spring stiffness</td>
<td>%</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
<td>150</td>
</tr>
<tr>
<td>X5</td>
<td>rear anti-roll bar stiffness</td>
<td>Nm/deg</td>
<td>0</td>
<td>300</td>
<td>600</td>
<td>900</td>
<td>1200</td>
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</table>
Table 9. Orthogonal array L25 for design of experiment

<table>
<thead>
<tr>
<th>Trial</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
</tr>
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<tbody>
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<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

5.1.2 Determination of the Responses

Though the focus in this study is ESC performance and FMVSS126 certify test, open-loop (without ESC intervention) maneuver are also included to provide a full picture of chassis modification impacts on the vehicle dynamics performances. It also supports the exploration of the relationships between different performance metrics. Since yaw motion and roll motion attract more concerns for vehicle stability due to their tight association with regulation criteria, so the responses listed in Table 10 will be used for further analysis in the DOE. Understeer gradient is the fundamental characteristics showing vehicle steady state lateral response to steering wheel input; roll gain defines
vehicle rolling resistance to the roll moment caused by lateral acceleration; yaw rate rise time labels the vehicle yaw response rapidity; yaw rate overshoot reflects vehicle yaw damping, same to roll overshoot. Procedure and metrics definitions of constant radius cornering and step steer are described in ISO4138 and ISO7401.

The physical certifying test consists of a series of trials with different steering wheel angle, which is defined by steering wheel angle at 0.3g (in slowly-increase-steer maneuver) times the scalars that range from 1.5 to 6.5 (maximum steering wheel angle is 270 degrees). Moreover, the tests are repeated for both left and right cornering. However, in this simulation-based study, the model is built symmetrically, cornering to the left is equivalent to the right. Moreover, lateral offset criterion only applies to trials with steering scalar bigger than 5. So for the DOE four trials are selected that steer from left to right with different magnitudes: 171 deg (5x), 205 deg (6x), 239 deg (7x) and 270 deg.

Table 10. Responses for DOE study

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Radius (100m) Cornering</td>
<td>Understeer Gradient (hand wheel)</td>
</tr>
<tr>
<td></td>
<td>Roll Gain</td>
</tr>
<tr>
<td>One Radian Step Steer @80km/h</td>
<td>Yaw Rate Rise Time</td>
</tr>
<tr>
<td></td>
<td>Yaw Rate Overshoot</td>
</tr>
<tr>
<td></td>
<td>Roll Overshoot</td>
</tr>
<tr>
<td>SWD</td>
<td>Yaw Rate Ratio @1s from COS</td>
</tr>
<tr>
<td></td>
<td>Yaw Rate Ratio @1.75s from COS</td>
</tr>
<tr>
<td></td>
<td>Lateral Offset @1.07 from BOS</td>
</tr>
<tr>
<td></td>
<td>Max Wheel Lift Height From Ground</td>
</tr>
</tbody>
</table>
5.2 Result Analysis for Experimental Design

The experimental design data provide supportive information for aftermarket modification in three different ways. Firstly, main effects of each performance metrics were calculated to indicate the impact of each factor; results will help modifiers to understand which component benefits most and which induce more risks. Secondly, the interconnection between open-loop performance and closed-loop metrics were studied so as to discover a predictive methodology for estimating closed-loop stability performances using open-loop metrics; thirdly with pass/fail criteria applied to the experiments, the passed samples composed the boundary hypercube of feasible design space. The hypercube is projected to two-dimensional graphs which became a design guideline for aftermarket modification customers.

5.2.1 Result of the Main Effect

Several observations were obtained from the DOE results of open-loop maneuvers (Figure 20). Vehicle understeer gradient tends to raise by lifted height and stiffened front spring. Softer rear spring and rear ARB also lead to a reduction of understeer gradient mildly. All five factors cause impacts on steady state roll gain: stiffer springs and ARBs limit the vehicle roll behaviors, while higher CG position tends to encourage more roll motion. Yaw rate rise time characterizes the vehicle yaw response speed. Lifted CG, stiffer rear spring or rear ARB gain most improvement in response speed. Adding stiffness to spring and ARB, especially in the front, brings more yaw rate overshoot. Roll overshoot in step steer maneuver is mostly influenced by front spring and front ARB. In
general, high spring/ARB stiffness cause lower steady state roll gain, but conversely more roll overshoot.
Figure 20. Main effect study for open loop maneuver
The main effect analysis for SWD is depicted in Figure 21 with following observations:

(1) The Lateral offset characterizes vehicle agility to avoid an obstacle in an emergency situation. The average lateral offset increases along with steering wheel angle input in a nonlinear manner: The increase from 171deg to 205deg is a bigger leap than that from 239deg to 270deg, which is in consistence with the vehicle’s understeer property. The lower center of gravity and stiffer rear ARB elevate the lateral offset most.

(2) In all experiments, yaw rate ratios with highest 6.2% @1s COS and 3.2% @1.75s COS are far less than the fail threshold: 35% and 20% respectively. With the intervention of control system, all the vehicles with different configurations appeared to be very stable in terms of yaw motion. The main effect investigations are not able to evidently reveal dominant factors that influence yaw rate ratio.
Figure 21. Main effect study for SWD maneuver

The main effect plots of wheel lift height from the ground shows that center of gravity height dominates the influence of all wheel lifts. The rear axle is more sensitive to the factor variations than front axle. The stiffer front spring and front ARB cause fewer wheel lifts, conversely stiffer rear counterparts lift the rear wheels even higher, and in other words, reinforcing the rear spring and ARB might not always help vehicle stability.
Figure 22. Main effect study for wheel-lift-off-ground in SWD maneuver
5.2.2 Correlation between the Open-Loop and the Closed-Loop

Closed-loop and open-loop maneuvers differ in that the closed-loop involves a feedback controller which intervenes the vehicle behavior automatically, and open-loop behavior characterizes vehicle with intrinsic metrics that potentially associated with closed-loop behaviors. In this section, five open-loop metrics and two closed-loop metrics (lateral offset and wheel lift height) are evaluated to demonstrate their correlation. As discussed before, yaw rate ratio variations were so small that no definite trend could be derived. Therefore it is not considered in the following correlation studies.

Analyses of the relationship between wheels lift height and open-loop metrics are plotted in Figure 23. Each Y-axis represents different open-loop metrics, and each X-axis represents lift heights of different wheels. Even though there are not obvious linear correlation, several trends can be recognized. Trials with higher steady state roll gain are more likely resulting in higher lift height. For example, trials with roll gain around 3 deg/g have about 0~0.06m rear wheel lift height, while those with 5 deg/g have wheel lift height up to 0.12m. This trend also applies to yaw rate overshoot and roll overshoot. Trials with smaller yaw overshoot and roll overshoot are more likely to have larger wheel lift height.
Figure 23. Wheel lifts versus open loop metrics
In Figure 24, results of all trials are plotted with the lateral offset in the Y-axis, and five open-loop metrics in the X-axis. Obvious trends exist among steady state roll gain, understeer gradient, yaw rate rise time and lateral offset. Higher steady state roll gain, higher understeer gradient, and shorter yaw rate rise time cause reduction of lateral offset. However, all experiments satisfied the minimum 1.83m lateral offset requirement.

Figure 24. Lateral displacement versus open loop metrics

The open-loop and closed-loop associate more tightly regarding lateral offset than that of wheel-lift-off-ground, so in the next step the correlation coefficients is evaluated and a regression model is constructed (Table 11). Yaw rate rise time is strongly tied with the lateral offset criterion; understeer gradient and roll gain are also important clues that can reflect the lateral offset level.

Table 11. Correlation coefficients for open-loop metrics and closed-loop metrics

<table>
<thead>
<tr>
<th></th>
<th>171</th>
<th>205</th>
<th>239</th>
<th>270</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understeer Gradient</td>
<td>-0.75</td>
<td>-0.80</td>
<td>-0.71</td>
<td>-0.60</td>
</tr>
<tr>
<td>Roll Gain</td>
<td>-0.79</td>
<td>-0.67</td>
<td>-0.58</td>
<td>-0.51</td>
</tr>
<tr>
<td>Yaw Rate Rise Time</td>
<td>0.93</td>
<td>0.93</td>
<td>0.85</td>
<td>0.75</td>
</tr>
<tr>
<td>Yaw Rate Overshoot</td>
<td>0.47</td>
<td>0.27</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>Roll Overshoot</td>
<td>0.35</td>
<td>0.20</td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>
A regression model is proposed to describe the lateral offset as a function of these three open-loop metrics and steering wheel angle magnitude. Natural logarithm operation aims to the normalization of different steering wheel angle. A first order linear function is designed to represent the linear trends between open-loop metrics and lateral offset criterion. The regression function for lateral offset is defined as below:

\[
y = ax_1 + b\ln(x_2 - c) + d
\]

Where:
- \( y \): Lateral offset (m)
- \( x_1 \): Open-loop metrics
- \( x_2 \): Steering wheel angle magnitude
- \( a, b, c, d \): Regression coefficient

By using nonlinear regression technique, the coefficients of equation (42) can be identified for the best fitting outcome (Table 12). R-square and root of mean square error (RMSE) are two metrics to determine the quality of regression function. The resulting regression models are compared with original data using Figure 25, which indicated a quasi-linearity between the two. With the regression function, it is possible to predict the lateral offset with the input of steering wheel angle magnitude and open-loop maneuver performance values; results error is within \( \pm 0.04m \). The ground tests to measure open-loop characteristics require much less time and cost compared to a full set of FMVSS126 test. With the prediction model a preliminary assessment for the modification of vehicle could help shrink down the scale of physical ground test.
Table 12. Regression model coefficients

<table>
<thead>
<tr>
<th></th>
<th>Understeer Gradient</th>
<th>Roll Gain</th>
<th>Yaw Rate Rise Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.00646</td>
<td>-0.0229</td>
<td>4.24</td>
</tr>
<tr>
<td>b</td>
<td>0.0476</td>
<td>0.0476</td>
<td>0.0476</td>
</tr>
<tr>
<td>c</td>
<td>167.7</td>
<td>167.7</td>
<td>167.7</td>
</tr>
<tr>
<td>d</td>
<td>2.82</td>
<td>2.62</td>
<td>1.87</td>
</tr>
<tr>
<td>R-square</td>
<td>0.76</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.038</td>
<td>0.034</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Figure 25. Regression models for lateral displacement
5.3 Design Envelop for Pass/Fail

For a certification test, the final delivery is passing or failing the regulation. For the criteria of lateral offset and yaw rate ratio, from above DOE results, all the trials can pass the regulatory requirements. The wheel-lift-off-ground situation is ambiguously defined in the regulation, so in this study 2 inches lift of wheels on one side at the same time is set to be the failure threshold (adopted from the NHTSA rollover test). The pass/fail results are listed in Table 13.

According to FMVSS 126, all cases in different steering scalars must satisfy the criteria to authorize the trial a pass. Results in Table 13 indicates that when the controller is “on” only trial 10 failed when the controller is “off” only trial eight pass. The strong contrast unveils that yaw stability controller permits a high degree of freedom for altering a vehicle while remaining safe within the legislative framework.
Table 13. Pass/fail result of design of experiments

<table>
<thead>
<tr>
<th>Trials</th>
<th>Stability Control On</th>
<th>Stability Control Off</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>171 205 239 270</td>
<td>171 205 239 270</td>
</tr>
<tr>
<td>1</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>2</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>3</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>4</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>5</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>6</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>7</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>8</td>
<td>p p p p</td>
<td>p p p p</td>
</tr>
<tr>
<td>9</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>10</td>
<td>p f f f</td>
<td>f f f f</td>
</tr>
<tr>
<td>11</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>12</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>13</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>14</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>15</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
<tr>
<td>16</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>17</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>18</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>19</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>20</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>21</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>22</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>23</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>24</td>
<td>p p p p</td>
<td>f f f f</td>
</tr>
<tr>
<td>25</td>
<td>p p p p</td>
<td>p f f f</td>
</tr>
</tbody>
</table>

Trial 10 failed because the excessive rolling happened. Factor levels in the failed trial ten are shown in Table 14. The vehicle sprung mass is lifted up by 12 inches which largely raises the center of gravity. However, the front spring and front ARB are in low levels thus fail to provide sufficient roll stiffness to resist the rolling motion. Even though the stability controller is very robust in most cases, incorrect configuration of chassis component will jeopardize the vehicle’s stability.
Table 14. Configuration detail of the failed trial

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Level</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>vehicle sprung mass lift height</td>
<td>5</td>
<td>12</td>
<td>inch</td>
</tr>
<tr>
<td>x2</td>
<td>front spring stiffness</td>
<td>1</td>
<td>50</td>
<td>%</td>
</tr>
<tr>
<td>x3</td>
<td>front ARB stiffness</td>
<td>2</td>
<td>1012.5</td>
<td>Nm/deg</td>
</tr>
<tr>
<td>x4</td>
<td>rear spring stiffness</td>
<td>3</td>
<td>100</td>
<td>%</td>
</tr>
<tr>
<td>x5</td>
<td>rear ARB stiffness</td>
<td>2</td>
<td>600</td>
<td>Nm/deg</td>
</tr>
</tbody>
</table>

Since there is a limit to the modification to ensure the vehicle will pass the regulation, a more intuitive tool to visualize the limit is proposed. Factor values in the configurations that pass the test compose a ‘Pass Region.’ The ‘Pass Region’ is a hypercube (5-cube) in five-dimension space. It is not able to demonstrate a 5-cube in a three dimensional world. However, two dimensional projections can outline the pass region from different angles (Figure 26). The ‘Pass Region’ projection is a design envelope which could become a powerful tool to guide designers or customers how to play with their modification within the safety zone. Users of the tool can navigate the design space and define the variables step by step. By picking a value for a variable, for example, the sprung mass lift height in level 5 (12 inches), the user can identify that front springs at level 2-5 will result in a pass, front ARB at level 3-5 will result in a pass. By repeating the process, one can determine a configuration that is in compliance with the regulatory stability requirement.
Figure 26. Pass region projection in two dimensions coordinate
CHAPTER SIX
LOW FIDELITY MODELING

6.1 Purpose for Low Fidelity Modeling

The sampling-based design envelope determination discussed in previous chapter require significant computational effort to achieve a good representation of design space. The higher the model fidelity, the more computational resources are requires solving the simulation problem.

The physical real-world vehicle is a ‘property’ that has 100% fidelity. However, using a physical vehicle to validate every configuration option is unrealistic. A high fidelity model like described in Chapter Four describes all relevant details of the vehicle behavior and is developed using complex modeling tools (software) to devise this approximation.

Low fidelity model (LFM) is a highly simplified model representing only a few degrees of freedom that are mostly concerned. Most parameters are linearized and usually is implemented using generic tools that can solve the problem with fewer computational resource needed compared to the HFM. However, the precision of LFM is limited to specific application scenarios; a significant error may occur when LFM is used out of the designated scenarios. The LFM benefits us with not only its faster-solving speed but also its easier operation compared to HFM. LFM could be embedded underneath the user-friendly interface that is dedicated to non-expert users.

In this research, there are two types of LFM: a linearized model (LM), and the surrogate model (SM). Compared to HFM, the LM has less DOFs (Usually 2-4,
compared to 15 in HFM) and its parameters are mostly linearized. Surrogate model (SM) is a model that can mimic the response of the system by mapping the input to the output while being computationally cheaper to evaluate. It is usually constructed using data-driven, bottom-up approaches. The most popular approaches include polynomial response surfaces, kriging, support vector machines, space mapping, and artificial neural networks [87].

HFM, LM and SM are derived and applied to serve different purposes. The HFM is applied in two stages: in the early (exploration) stage it is used to determine baseline knowledge and assess the impact of aftermarket modification on vehicle dynamics compliances; in the last stage of the design envelope (and tool) formulation process, the HFM is used to validate design envelope and tool (closing the loop). Various vehicle dynamics LMs are developed for different usage scenarios (steady state, transient state, or close-loop maneuver). The models could be explicit analytical functions that directly describe input/output relationships or a set of integrated functions that work together to determine the input/output relationships. The SM is the most efficient model that is focus on the searching algorithm (optimization and design space exploration) application. The use of LFM benefits from not only having better computational efficiency but also allows for easier implementation in an off the shelf software environment compared to HFMs that require dedicated (expense) software packages for solving the problem. LFM may potentially be embedded as dedicated code into a user-friendly interface that is developed for non-expert users. Even though the application of LFM will require much less computational effort than the application of HFM, the LFM for FMVSS126 SWD
simulation still may be too computationally inefficient (several seconds to solve the problem) and may not meet real-time requirements as part of the interactive design envelope tool. Ultimately, the aftermarket configuration process requires an end-user to configure “on the fly” various aftermarket performance kits with the tool providing immediate feedback on performance change and compliance without the need to wait for extensive (time-consuming) analyses for each kit configuration. Therefore, surrogate models may need to be derived and applied to assure that the interactive aspect of the approach can be implemented.
6.2 Linear Model for Vehicle Dynamics and Stability Control

This section will introduce the formation of the linearized vehicle dynamics model integrated with stability control logic. Compared to the 15-DOF HFM, the linearized model only considers three DOFs that are mostly related to lateral stability. The PD control strategy is simplified and incorporated into the linear model.

6.2.1 Equations of motion

There are four DOFs considered in the linear model: yaw, sideslip, roll angle, and roll rate. A fourth order state space model is the core model with four state variables: vehicle yaw rate, sideslip angle, sprung mass roll angle, and sprung mass roll rate. The state-space representation of the system is shown in equation (30). More details about the derivation process are discussed in Appendix A.

\[
\begin{bmatrix}
MV & 0 & -M_S H_2 & 0 \\
0 & l_Z & l_{XZS} & 0 \\
-M_S H_2 V & l_{XZS} & l_{XS} + 2M_S H_2^2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\beta} \\
\dot{r} \\
\dot{\phi}_S \\
\phi_S \\
\end{bmatrix}
= 
\begin{bmatrix}
C_1 + C_2 & \frac{aC_1 - bC_2}{V} - MV & 0 & 0 \\
(aC_1 - bC_2) & \frac{(a^2 C_1 + b^2 C_2)}{V} & 0 & 0 \\
0 & M_S H_2 V & -B_{\phi_S} & -(K_{\phi_S} - M_S g H_2) \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\beta \\
r \\
\phi_S \\
\end{bmatrix}
+ 
\begin{bmatrix}
-C_1 & 0 \\
-aC_1 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta_f \\
M_f \\
\end{bmatrix}
\]

For steady state and mild transient maneuvers, most of the subsystems behave with linear characteristics so the gain parameters are treated as constants (e.g. roll rate,
roll damping, cornering stiffness), and thus it is a linear time-invariant system (LTI). But for high-dynamic handling such as sine-with-dwell, in which the radical steering input and braking force cause the tires to behave nonlinearly, the system is regarded as linear parameter-varying (LPV) system. Equation (30) can be transformed into the standard form of LTI and LPV system as below:

**LTI:** \( \dot{X} = AX + BU \) \( (31) \)

**LPV:** \( \dot{X} = A(p)X + B(p)U \) \( (32) \)

Two varying parameters are evaluated using following approaches:

*Vehicle speed*

The vehicle speed is evaluated using simplified longitudinal dynamics in the form of recursive formula:

\[
V_{j+1} = V_j - \frac{F_{x-est}}{M} \Delta t \tag{33}
\]

where \( F_{x-est} \) denotes the estimated longitudinal force which consist of two portions: one is due to the stabilizing yaw moment; another one is due to the front wheel steer angle. It is described as:

\[
F_{x-est} = \frac{2|M_z|}{T} + \frac{1}{2}|F_{yf} \sin \delta_f| \tag{34}
\]

*Tire characteristics*
The tire lateral characteristics are in tabular form based upon actual tire measurement, showing the relationship between tire lateral force and sideslip angle under various vertical tire loads. The cornering stiffness, which is defined as the derivative of lateral force with respect to sideslip angle, is highly nonlinear for high dynamic handling maneuvers: it increases along with vertical load but might decrease to a negative value along with the sideslip angle causing a loss in vehicle stability. To retain this nonlinearity, the tire force is modeled as piecewise linear function of tire sideslip angle:

\[ F_{yi} = k_i(\Delta F_z)\alpha + f_i(\Delta F_z), \text{when } \alpha \in [\alpha_i, \alpha_{i+1}] \]  

\[(i = 1,2, ... n)\]

The coefficient \(k_i\) and \(f_i\) are the also the functions of lateral loads transfer.

Rewriting equation (30) results in:

\[
\begin{bmatrix}
MV & 0 & -M_s H_2 & 0 \\
0 & I_z & I_{ZXS} & 0 \\
-M_s H_2 V & I_{ZXS} & I_{XS} + 2M_s H_2^2 & 0 \\
0 & 0 & 0 & 11
\end{bmatrix}
\begin{bmatrix}
\ddot{\beta} \\
\dot{r} \\
\ddot{\phi}_S \\
\ddot{\psi}_S
\end{bmatrix}
= 
\begin{bmatrix}
k_{1i} + k_{2i} & \frac{a k_{1i} - b k_{2i}}{V} - MV & 0 & 0 \\
0 & \left(\frac{a^2 k_{1i} + b^2 k_{2i}}{V}\right) & 0 & 0 \\
0 & M_s H_2 V & -B_{\phi S} & -(K_{\phi S} - M_s g H_2) \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\beta \\
r \\
\phi_S \\
\psi_S
\end{bmatrix}
\]

\[\text{(36)}\]

6.2.2 Stability Controls Integration

The PD controller generates the stabilizing yaw moment \(M_Z\) based on the yaw rate error as shown in equation (37)
\[
M_z = K_p (r_d - r) + K_d (r_d - \dot{r})
\]  

(37)

Other controller features are preserved. Controller triggering condition and yaw moment saturation (due to braking force limits) are considered using a sigmoid function to approximate the on/off switch and saturation function without affecting the mathematical solver speed (due to non-derivability).

A sigmoid function (38) is used to approximate the switching function. When the yaw rate error is below the threshold, it outputs a switch state close to zero, and when the yaw rate error is above the threshold, it outputs a switch state close to one. (See Figure 27)

\[
S_{\text{switch}} = \frac{1}{\pi} \arctan(1000(e_r - e_{rt}) + \frac{\pi}{2})
\]  

(38)

Figure 27 Sigmoid approximation for switching function

The sigmoid function (39) is used to correct the output yaw moment. The scale factor \(W_s\) (usually around 2~3) offers flexibility of adjusting the shape of sigmoid function to reach the best estimation (see Figure 28).
\[ M_{Zout} = M_{sat} \left( \frac{2}{1 + e^{-W_{SM} M_Z}} - 1 \right) \]  

\[ (39) \]

Figure 28 Sigmoid approximation for saturation function

6.2.3 Output of the Model

Besides state variable outputs (yaw rate, sideslip angle, roll angle, roll rate) additional outputs that assess the closed-loop stability in the previous discussion need to be considered. The lateral offset of the vehicle at 1.07s after BOS remains one of the key responses as the agility criteria in the SWD maneuver.

The lateral offset is calculated using double integral of lateral acceleration.

\[ D_y = \iiint a_y dt = V \iint (\dot{\beta} + r) dt \]  

\[ (40) \]

The wheel lift criterion is adjusted to be more conservative: instead of assessing the maximum wheel lift of both front and rear wheels, only the maximum rear wheel lift
is used because the losing rear axle cornering ability will more likely induce loss of stability [83].

The unsprung mass roll angle can be estimated by incorporating the system outputs (lateral acceleration and sprung mass roll angle) and the equilibrium relationship as shown in Equation (41):

\[
\phi_{Ur} = \frac{(M_r a_y (H_U + H_1) + M_{Ur} a_y H_1 + K_{\phi \text{Sr}} \phi_S)}{K_{\phi \text{Sr}} + K_{\phi \text{Ur}}} \tag{41}
\]

If the wheel loses contact with the road, the wheel lift height can be calculated using following equation:

\[
L_{WC-r} = \max \left(0, \frac{\phi_{Ur} T}{2} + R_{\text{tire-r}} - R_{\text{free}} \right) \tag{42}
\]

6.2.4 Validate the Linearized Model in the Response Space

The LM will need to be validated and calibrated using the HFM before it can be implemented in the design envelope determination framework. The design space, which is the set of sampled design variables, are the same for both LM and HFM. The response space is the set of results that mapped the design space with the vehicle dynamics models. The idea is to apply the same sampled design configurations and identify the deviation between results of two models. The sampling method used is full factorial so as to represent the whole response space thoroughly. A three-level, four-factor full factorial experiment is used to generate the response surface. The design variables and respective levels are determined by typical variants associated with typical aftermarket
modifications: front and rear axles roll stiffness variation of +/- 25%, center of gravity height variation of +/- 3 inches, and tire cornering stiffness variation of +/- 10%. (See Table 15)

**Table 15 Design variables of full factorial DOE**

<table>
<thead>
<tr>
<th>Factors</th>
<th>Description</th>
<th>Unit</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Front axle rolling stiffness</td>
<td>%</td>
<td>75</td>
</tr>
<tr>
<td>X2</td>
<td>Rear axle rolling stiffness</td>
<td>%</td>
<td>75</td>
</tr>
<tr>
<td>X3</td>
<td>CG height</td>
<td>inch</td>
<td>-3</td>
</tr>
<tr>
<td>X4</td>
<td>Tire cornering stiffness</td>
<td>%</td>
<td>90</td>
</tr>
</tbody>
</table>

The lateral offset and maximum rear wheel lift relative to the road surface are the metrics to examine for the response space. The normalized root-mean-square deviation (NRMSD), which is defined in equation (43) is used to determine the deviation between LM and HFM model responses:

\[
NRMSD = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_L - Y_H)^2} \div Y_{H_{max}} - Y_{L_{min}}
\]

In the equation \( Y_H \) denotes HFM model response and \( Y_L \) denotes LM model responses.

A comparison of the results is shown in Figure 29. The NRMSD of the rear wheel maximum lift height is 8.78% while the NRMSD of the lateral offset is 5.95%. With that it can be concluded that the application of a LFM can still retain 92.6% of the precision of a HFM.
Figure 29 Comparison of max rear wheel lift response

Figure 30 Comparison of lateral offset responses
6.3 Surrogate Model for Closed-loop Stability Criteria

Even though the LM provides a faster and more efficient way to generate solutions than the HFM, it is still too time-consuming for the application of an interactive feasible solution application. Metamodels can be used to further approximate and improve the speed of solution exploration of complex and non-explicit systems [88]. A full quadratic regression function is selected to drive the response surface regression analysis. The regression function with four design variables shown in equation (44) includes constant, linear, interactive, and quadratic terms. Compared to the linear model, the quadratic model additionally considers the interaction of variables and higher (second) order dynamics. The results of the application shown later also indicate that the quadratic model is of sufficient accuracy and therefore higher order terms are not necessary.

\[ \hat{y} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j=i}^{n} \beta_{ij} x_i x_j \]  

(44)

The surrogate models are built for the nonlinear closed-loop simulations that require long computational time. Lateral offset and maximum rear wheel center height are modeled as linear-quadratic functions of four design variables: the front axle roll stiffness, the rear axle roll stiffness, the center of gravity height, and tire cornering stiffness scale factor.

A full factorial experiment with five levels, four variables generate 625 samples for each regression analysis. The quality of regression is assessed by coefficients of determination (also called R-square) which is regarded as one of the most important metrics to evaluate the regression model [89].
The results of two regression models are shown in Table 16, Figure 31 and Figure 32. The coefficients of determination are both close to 1, meaning that the regression models can accurately represent the original LM.

Table 16 Regression model fitting results

<table>
<thead>
<tr>
<th></th>
<th>Lateral offset</th>
<th>Wheel center height</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>8.43E-01</td>
<td>-1.33E-02</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>9.75E-02</td>
<td>7.93E-01</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>4.36E-02</td>
<td>-3.10E-01</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>8.52E-03</td>
<td>-4.96E-02</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>2.67E+00</td>
<td>3.36E-01</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>4.87E-03</td>
<td>-1.65E-01</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>7.93E-02</td>
<td>2.99E-01</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>-3.64E-01</td>
<td>1.02E-01</td>
</tr>
<tr>
<td>$\beta_{23}$</td>
<td>-2.35E-02</td>
<td>-5.76E-02</td>
</tr>
<tr>
<td>$\beta_{24}$</td>
<td>2.64E-02</td>
<td>-4.02E-02</td>
</tr>
<tr>
<td>$\beta_{34}$</td>
<td>2.72E-03</td>
<td>-2.62E-02</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
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<tr>
<td>$\beta_{22}$</td>
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<td>2.06E-01</td>
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<td>$\beta_{33}$</td>
<td>-1.51E-02</td>
<td>-6.42E-02</td>
</tr>
<tr>
<td>$\beta_{44}$</td>
<td>-6.13E-01</td>
<td>-1.17E-01</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9997</td>
<td>0.9951</td>
</tr>
</tbody>
</table>

Figure 31 Regression model for lateral offset
As discussed before, this research uses the HFM as the best representation of a physical vehicle, so it is necessary to evaluate the fidelity of using the SM to represent HFM. The NRMSDs between LM and HFM and coefficients of determination between SM and LM can be regarded as the fidelity index (FI). It is proposed to evaluate the fidelity indexes using equation (45) so as to reflect the lumped fidelity of the SM.

\[
FI = (1 - NRMSD) \cdot R^2
\]  

(45)

The FI of the lateral offset SM is 0.94, and the FI of the wheel center lift height is 0.91. In other words, the SM can preserve more than 90% of fidelity of HFM.

Figure 32 Regression model for maximum rear wheel center height
CHAPTER SEVEN
CREATING THE INTERACTIVE DESIGN ENVELOPE

Previous chapters discussed and addressed the limitations of the sampling-based design envelope determination. This chapter will introduce the development of a design envelope that is dynamic and interactive, incorporating rules established by computationally efficient models. The problem is described in nonlinear programming context and solved using sequential quadratic programming. A case study in the last section demonstrates a prototype of the UI. Application examples are validated using a high fidelity co-simulation model.
7.1 Nonlinear Programming Application for Constraint Problem

The proposed approach determines feasible solutions in two steps: The first step is to generate the design envelope, which initially shows the feasible ranges of design variables; the second step is to determine solutions with user inputs. The input of user selections is equivalent to adding a constraint and removing the inconsistent options accordingly. It helps to target the solution much faster in a continuous design space where there are infinite numbers of solutions.

The concept of aftermarket product modification is a design process driven by aftermarket kit suppliers and end-users. In the process, initial inputs, which consist of customer needs, engineering constraints and regulatory requirements are translated into performances constraints. After that the feasible design envelope, which represents the modification guideline ensuring compliance within the constraints, is identified with the rules connecting the performances attributes and design parameters.

7.1.1 Constraint Satisfaction Problem Description

The problem can be described by adopting conventional constraint satisfaction problem theory [90]. The standard form of CSP is defined as triple \(< X, D, C >\), where

\[ X = \{X_1, X_2, ..., X_n\}, \text{ a set of variables} \]

\[ D = \{D_1, D_2, ..., D_n\}, \text{ a set of corresponding domains} \]

\[ C = \{C_1, C_2, ..., C_m\}, \text{ a set of constraints} \]

Define \( V \) as an instantiation of the problem.
\[ V = \{(v_1, v_2, ..., v_n) | v_1 \in D_1, v_2 \in D_2, ..., v_n \in D_n\} \quad (46) \]

Constraints \( C_j \) \((j = 1,2, \ldots, n)\) describe the relationship between variables and performances and requirements of performances.

\[ C_j = \{G_j(V) \in Y_j\} \quad (47) \]

\[ Y_j = \{y_j | y_j \in [y_j^{lb}, y_j^{ub}], (j = 1,2, \ldots, m)\} \quad (48) \]

So the feasible design space is defined by:

\[ F = \{V | V \in C_j, j = 1,2, \ldots, m\} \quad (49) \]

If the design space is a continuous convex set, all the feasible values of each variable compose a continuous set, which can be called feasible design range of a design variable \( x_i \) and can be define as:

\[ B_i = \{v_i | v_i \in [\text{min}(v_i), \text{max}(v_i)]\} \quad (50) \]

Obviously, the feasible design space is a subset of Cartesian product of the feasible ranges:

\[ F \subset (B_1 \times B_2 \times \ldots \times B_n) \quad (51) \]

It is not difficult to prove that: for all values assigned to variable \( X_s \) within the feasible design range, at least one feasible solution \( V \) exists:
\[(\forall s \in B_s, \exists V = (v_1, v_2, ..., v_s, ..., v_n): G_j(V) \in Y_j)\]  (52)

By defining the value of \(X_s\), the design space to be explored collapse by one dimension. If \(p\) variables have been defined as \((v_{s_1}, v_{s_2}, ..., v_{s_p})\), then the remained design space is a \((n-p)\)-dimension space, in which the undefined variable ranges can be described as:

\[B_{i}^{(p)}: = \{v_i | v_i \in [min(v_i), max(v_i)], i \neq s, s = \{s_1, s_2, ..., s_p\}\} \]  (53)

The design space will collapse into single solutions if the process is repeated for \((n-1)\) steps. Figure 33 shows a simple example of the design process for a two-dimension design space. The left side shows the graphical representation of the design problem. There are two design variables and two attribute requirements. The performances contour plot is projected to the 2-D design space (plane) of variable 1 and variable 2. The box which is bounded with \(X_1\) and \(X_2\) is the initial design space. The shaded area is the feasible design space constrained by \(Y_1\) and \(Y_2\). In step one \(B_1\) and \(B_2\) indicate the range of all possible design values that can satisfy performances requirements. In step two, the user decides the value \(x_1\) for variable 1 from the candidate pool represented with \(B_1\), and then the range of variable 2 is consequentially narrowed into \(B_2^{(1)}\). In step 3, variable 2 is defined with a specific value, which is selected from the feasible space \(B_2^{(1)}\) by the customers. So that the design variables are fully determined by the user within the boundaries.
Figure 33 Example of interactive design envelope determination
The right side in Figure 33 is a sample of the user interface. It displays the up-to-date candidate range at each step with end-users understanding immediately what their options are. Furthermore, end-users can select a variable value corresponding to their preferences from the candidates. The concept is designed for non-expert users, who do not need to explore the full design space in a visualized model. The feasible design space is expressed using ranges of each design variable rather than visualized design space projection [35] [91] [92]. It does not require to explore all feasible solutions in the design space, only the ranges (extremum values) are adequate for the customers to support making decision for the next steps; it featured with flexibility of constraints adjustment on the design variables so that the users can either narrow or widen the constraint of any of the variables in a convenient way. The sequence of variables determination is also flexible upon the customer’s preferences. For example, some customers may attach higher value to upgrading springs and do care less about others components; other customers only require an optical lift of the vehicle and do not desire to upgrade springs. Last but not least, all customer decisions are bounded and ensure vehicle dynamics compliance.

The key in the process is to determine the feasible ranges so that the user can select a value within the allowable space until the solution is fully defined. It needs $n$ steps to define a feasible solution of an $n$-dimensional design space. In the first step, no variable is assigned with specific values, and it is required to determine the ranges $B_i^1$ for $n$ design variables. In the $p$-th step, $(p - 1)$ variables have been assign with specific values, and it is required to determine ranges $B_i^p$ for $n - p + 1$ variables. In the $nth$ step,
n − 1 variables have been fixed, and range $B^n_n$ for 1 remaining variable need to be determined. In the $(n + 1)th$ step, the last variable will be assigned with a specific value, composing a final feasible solution together with the formerly defined variables.

The constraint-based design envelope can be solved using nonlinear programming approach. In the $p$-th step, lower bound (upper bound) of feasible range $B^p_k$ can be found by solving following NLPs:

$$\min f_k^{lb}(x) = x_k \quad \text{(Upper Bound } \min f_k^{ub}(x) = -x_k) \), k = p, p + 1, \ldots n$$

Subject to: $H_j(x_1, x_2 \ldots x_n) \leq 0, j = 1, 2 \ldots m$

$$x_k \in B^{p-1}_k, k = p, p + 1 \ldots, n, (B^0_k = D_k)$$

$$x_s = v_s, s = 1, 2, \ldots, p - 1$$

Variable domains are updated as

$$B_k^{(p)} = \{ f_k^{lb}, f_k^{ub} \}$$

The problems addressed in this research are constrained NLP problems, which means that the objective function or constraint functions are general functions that are difficult to express by explicit equations. Primary methodologies for constrained NLP include augmented Lagrangian method, interior point method, and Sequential quadratic programming (SQP) [93] [94] [95]. The SQP is among the most popular for general purpose applications. SQP is an iterative procedure that solves the quadratic sub-problems to determine the search direction. At each major iteration, it will update the Hessian of the Lagrangian function using a quasi-Newton updating method. The quadratic sub-problem can be solved with most of the quadratic optimization algorithms.
7.1.2 Identification of Quasiconvexity

To guarantee that the solutions of the NLPs represent the upper bound and lower bound of the feasible ranges, and all design variables within the boundaries are feasible, it is necessary that the design space constraint functions are quasiconvex with respect to all variables in the studied domain. If the constraint function is not quasiconvex (e.g., concave constraint $C_1$ in Figure 34), which causes the design space become non-convex, when value $v_1$ is assigned to $X_1$, the feasible space $B_2'$ will consists of two separate portions, and it cannot be defined simply with the extremums as boundaries.

$$B_2' \neq [x_2^{min}, x_2^{max}] \quad (56)$$

Figure 34 Not quasiconvex constraint leads to noncontinuous feasible ranges

The definition of quasiconvex function is [96] [97]: A function $f$ is quasiconvex if for all $x, y \in S \rightarrow \mathbb{R}$ ($S$ is convex set) and $\lambda \in [0,1]$:
\[ f(\lambda x + (1 - \lambda)y) \leq \max\{f(x), f(y)\} \]  

(57)

According to the process defined in previous chapter, one of the variable 
(e.g., \(X_1\)) will be assigned with a value \(v_1\), then for any two given points on the 
hyperplane \(x_1 = v_1, A(v_1, v_{21})\) and \(B(v_1, v_{22})\), the assumption can be described as:

\[
\text{if: } f(A) \leq y, f(B) \leq y, \text{then: } f(\epsilon A + (1 - \epsilon)B) \leq y
\]

Prove:

\[
\therefore f(x) \text{ is quasiconvex respect to } v_{22}
\]

\[
\therefore f(v_1, \epsilon v_{21} + (1 - \epsilon)v_{22}) \leq \max\{f(v_1, v_{21}), f(v_1, v_{22})\}
\]

\[
\therefore f(A) \leq y, f(B) \leq y
\]

\[
\therefore \max\{f(A), f(B)\} \leq y
\]

\[
\therefore f(\epsilon A + (1 - \epsilon)B) = f(\epsilon(v_1, v_{21}) + (1 - \epsilon)(v_1, v_{22}))
\]

\[
= f(v_1, \epsilon v_{21} + (1 - \epsilon)v_{22}) \leq y
\]

Before solving the nonlinear programming problem, it is necessary to study the 
constraint quasiconvexity. From the definition, it can easily be seen that the convex and
monotonic functions are also quasiconvex. If the Hessian matrix of the function is
positive semi-definite, then the function is convex [98]. If the Hessian matrix is negative
definite, then it is concave. However, the quasiconvexity of a concave function depends
on the design space location. In Figure 35, if the entire design space (shaded area) is
located within the feasible area \((H_1 < 0)\), then the design space is still quasiconvex. To
justify this condition, it is proposed to find the maximum \(H_1(x^*)\) within the design space,
if \(H_1(x^*) \leq 0\), then the quasiconvex condition holds.
If $H_1(x^*) > 0$ (Figure 36), the feasible design space (shaded area) can still be quasiconvex if the constraint function is monotonic in this domain. By examining the sign change of the first-order derivative throughout the design space, one can determine the monotonicity of the constraint function.
If the constraint function has multiple local extrema, or it does not have explicit analytical form, then it will be hard to use the Hessian matrix to determine its convexity in the design space. In this case, one can estimate the quasiconvexity with the observation of the graphical representation generated by sampling methods (Monte-Carlo, Orthogonal, and factorial).

If the constraint (e.g. \( C_1 \)) exists which is quasi-concave and nonmonotonic, there will be at least one local minimum of variables exists \((v_1^*)\) on the hypersurface of constraint boundary in the inner domain (exclude the boundaries). The value, at which the minimum exists, is called a pivot value \((v_2^*)\). Then the design space will be divided (e.g. \( S_1 \) and \( S_2 \)) using this pivot point \((v_1^*, v_2^*)\) in Figure 37.

![Figure 37. Dividing the design space into quasiconvex subspace](image)

If \( q_i \) local minimum points exist overall in the domain of variable \( X_i \), the design space is divided into \( Q \) subspaces whose boundaries are quasiconvex. The number of subspaces \( Q \) can be calculated using following equation:
\[ Q = \prod_{1}^{n} (q_i + 1) \]  

(58)

The feasible ranges will be solved within each of subspace and joined together.

But practically, most applications response in vehicle dynamics are similar to lower order polynomial. So there will not be excessively numerous subspace.

In this research, two compliance constraint functions are constructed using quadratic regression models which have explicit and concise form. It provides another benefit that it is easy to come up with the Hessian matrix and investigate the convexity.

As introduced in previous section, the compliances criteria can be described with following constraint functions. The inequality constraint functions are described with following equations:

\[
\begin{cases}
H_{ineq1} = f_{Dy} - y^{ub}_{Dy} \leq 0 \\
H_{ineq2} = -f_{Dy} + y^{lb}_{Dy} \leq 0 \\
H_{ineq3} = f_{WH} - y^{ub}_{WH} \leq 0 \\
H_{ineq4} = -f_{WH} + y^{lb}_{WH} \leq 0
\end{cases}
\]  

(59)

Where \( f_{Dy} \) and \( f_{WH} \) are the surrogate models defined in Section 6.3 using following quadratic regression functions:

\[
\begin{align*}
f_{Dy} &= \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \sum_{j=i}^{n} \beta_{ij} x_i x_j \\
f_{WH} &= \gamma_0 + \sum_{i=1}^{n} \gamma_i x_i + \sum_{i=1}^{n} \sum_{j=i}^{n} \gamma_{ij} x_i x_j
\end{align*}
\]  

(60)  

(61)
The expression of Hessian matrix of the quadratic constraint functions can be derived using the definition. The determinant of the Hessian matrix gives a clue of convexity of the constraint functions.

Following example demonstrates the process to examine the quasiconvexity of the constraint function $H_{ineq1}$.

The Hessian matrix of $H_{ineq1}$ consists of partial second derivatives:

\[
H_{Dy} = \begin{bmatrix}
\frac{\partial^2 f_{Dy}}{\partial x_1^2} & \frac{\partial^2 f_{Dy}}{\partial x_1 x_2} & \frac{\partial^2 f_{Dy}}{\partial x_1 x_3} & \frac{\partial^2 f_{Dy}}{\partial x_1 x_4} \\
\frac{\partial^2 f_{Dy}}{\partial x_2 x_1} & \frac{\partial^2 f_{Dy}}{\partial x_2^2} & \frac{\partial^2 f_{Dy}}{\partial x_2 x_3} & \frac{\partial^2 f_{Dy}}{\partial x_2 x_4} \\
\frac{\partial^2 f_{Dy}}{\partial x_3 x_1} & \frac{\partial^2 f_{Dy}}{\partial x_3 x_2} & \frac{\partial^2 f_{Dy}}{\partial x_3^2} & \frac{\partial^2 f_{Dy}}{\partial x_3 x_4} \\
\frac{\partial^2 f_{Dy}}{\partial x_4 x_1} & \frac{\partial^2 f_{Dy}}{\partial x_4 x_2} & \frac{\partial^2 f_{Dy}}{\partial x_4 x_3} & \frac{\partial^2 f_{Dy}}{\partial x_4^2}
\end{bmatrix}
\]

(62)

With the coefficients values from the Table 16, the determinant of the Hessian matrix is solved:

\[
Det \left( H_{Dy} \right) = -1.9E - 4 < 0
\]

A negative Hessian means the constraint function $H_{ineq1}$ is concave. A concave function is quasiconvex if and only if it is monotonic, but it is obvious that the quadratic function is non-monotonic. So the next step is to examine whether the design space locates within the feasible region $H_{ineq1} < 0$. The following optimization problem can be composed:
\[
\max f_1 = f_{D_y}(x_i) - y_{ub}^{D_y} \\
\text{s. t. } x_i \in B_i
\]

The upper bound of lateral offset should be greater than the requirement (>1.83m). It is found that the maximum value of \( f \) is equal to -0.25 when the upper bound is set to 1.83, which means with upper bound that is greater than 1.83, the maximum value of objective function is always negative. From the proposition presented in the former section, the design space is within the feasible region of constraint function \( H_{ineq1} \).

\[
f_{1\max} = f_{D_y} - y_{ub}^{D_y} < f_{D_y} - 1.83 = -0.25 < 0
\]

Another constraint function \( H_{ineq2} \) that defines the lower bound is the inverse of \( H_{ineq1} \), so it is a convex function.

In the same way, the Hessian matrix determinant of \( H_{ineq3} \) is -0.0041<0, therefore \( H_{ineq3} \) is a concave constraint. An optimization problem is constructed to examine whether the design space satisfies \( H_{ineq3} \). The upper bound of the wheel center height is set to be 0.5m, the maximized value of objective function is -0.30<0, so the design space is within the feasible region.

\[
f_{3\max} = f_{WH} - y_{ub}^{WH} = f_{WH} - 0.5 < -0.30 < 0
\]

Similarly the other constraint function \( H_{ineq4} \) is the inverse of \( H_{ineq3} \), so it is convex.

Examination on the compliance constraint proves that the feasible design spaces being explored are quasiconvex with respect to all variables. Since the intersection of the
spaces does not change the quasiconvexity, as long as the new constraints are quasiconvex, the nonlinear programming approach can be applied to solve the range of design variables.

7.1.3 Summary of the Problem Workflow

The workflow depicted in Figure 38 illustrates the procedure of problem-solving incorporating the approach previously described. In the first steps, the users will be asked to input the initial design space (design variable intervals) and constraints (performance attribute intervals). In the second step, the constraints quasiconvexity will be examined to define the working space. In the third step, an NLP problem is constructed and solved using SQP method, in the meantime, the feasible ranges of design variables will be updated with the NLP solution. In step four, users are required to decide a value for one of the variables from the feasible ranges. Finally, the system will repeat the third and fourth steps until all variables are determined.
Figure 38 Problem-solving workflow
7.2 Application and Validation

An application example based on 2014 Chevrolet Silverado will demonstrate the approach of generating feasible configurations through the interactive design envelope method.

7.2.1 Interface Introduction

A user interface (UI) has been developed to implement the interactive design envelope approach (Figure 39). It will solicit the end-user for the information which includes requirements for vehicle attributes as initial design input and component preferences. It will also display the available candidates dynamically according to the user's operation. There are three groups of data clustered in the UI. On the left side a group of attributes regarding vehicle dynamics attributes and stability compliance metrics are displayed. In the middle a cluster of intermediate parameters indicating the feasible ranges in accordance with the rules are shown. On the right side of the interface a group of available component candidates are presented. The attributes and intermediate parameters interact with the user using sliders and the components are listed as discrete values. Two layers of design envelope have been developed: in the first layer, the continuous attribute space is mapped to a continuous intermediate parameter space by the solving complex constraint problem; in the second layer the continuous space is mapped to a discrete component space taking advantage of simpler interrelationships. The components are decoupled across the intermediate parameters. That means, each component is only connected with one intermediate parameter. For example, the variation
of the front spring stiffness only affects the front axle roll stiffness. This approach reduces the coupled interactions among parameters to improve the problem-solving efficiency. More importantly, it translates the technical terms that are used for intermediate parameters into straightforward component parameters/descriptions that are meaningful to aftermarket suppliers. Additional details of the mapping rule from continuous space to discrete space are discussed in Appendix B.

There are four major steps to identify a configuration that meets the user needs (Figure 39):

a) The user is requested to select constraints to attributes along with their inputs (e.g. from brand essence);

b) By clicking the ‘Solve’ button, the rule-based constraint problem will be solved using the algorithm introduced in the former section. The UI will display the range of intermediate design parameters using the slider.

c) The ranges of intermediate parameters are mapped to the relevant component candidates by solving a simpler constraint problem.

d) From the available component candidates, the user can compose various configurations that all satisfy the rules defined upfront.
7.2.2 Specification of the Rules

The motivation of this research is to maximize the design freedom of aftermarket modifications while ensuring vehicle dynamics compliance. As an instantiated case study, the handling performances improvement are regarded as the objective of the chassis modification, using explicit handling metrics of steady state and transient state, and compliance criteria formerly discussed. Therefore, the rules in the context mean the connection or interrelationship among the performance attributes (metrics) and design parameters. These attributes are described using ranges and applied to the rule-based system to generate the design envelope. Many researchers have elaborated the meaning of these attributes. Hazare made a comprehensive discussion regarding the physical implication and mathematical equations [34].
### Table 17. Attribute demands of the vehicle design envelope

<table>
<thead>
<tr>
<th>Handling domain</th>
<th>Objective metrics</th>
<th>Description of metrics</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state handling</td>
<td>Understeer Gradient</td>
<td>Steering wheel angle gain of the vehicle</td>
<td>deg/g</td>
</tr>
<tr>
<td></td>
<td>Yaw rate gain</td>
<td>Ratio of vehicle yaw rate over steering wheel inputs</td>
<td>1/s</td>
</tr>
<tr>
<td></td>
<td>Sideslip gain</td>
<td>Vehicle sideslip angle gains per lateral acceleration input</td>
<td>deg/g</td>
</tr>
<tr>
<td></td>
<td>Roll gain</td>
<td>Roll angle gain per lateral acceleration</td>
<td>deg/g</td>
</tr>
<tr>
<td>Transient state handling</td>
<td>Yaw rate time constant</td>
<td>Characterize the yaw rate response speed under steering wheel input</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>Lateral acceleration gains @ 1Hz</td>
<td>The gain of frequency response of lateral acceleration under steering wheel input, specifically at 1Hz</td>
<td>g/deg</td>
</tr>
<tr>
<td></td>
<td>Lateral acceleration delay @ 1Hz</td>
<td>The lag of frequency response of lateral acceleration under steering wheel input, specifically at 1Hz</td>
<td>s</td>
</tr>
<tr>
<td>Regulatory compliance</td>
<td>Max wheel center height</td>
<td>Maximum wheel center height from the ground in the rear axle during the SWD maneuver</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Lateral offset</td>
<td>Vehicle lateral offset at 1.07s after BOS</td>
<td>m</td>
</tr>
</tbody>
</table>

#### 7.2.3 Collection of Component Candidates

Component candidates are defined by the kit suppliers, who develop a set of configurations for the aftermarket vehicle modification. The numbers and characteristics of these components depend on the availability in the marketplace and the kit suppliers’ preference. An example candidate pool is shown in Table 18. It consists of four lift kits various lift heights, six springs and anti-roll bars with different stiffness for front and rear
suspensions respectively and four tires with different cornering stiffness. These parameters will be listed on the right side of the user interface in Figure 39.

### Table 18 Component candidates

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Unit</th>
<th>OE</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift Kit</td>
<td>Height</td>
<td>Inch</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Front spring</td>
<td>Stiffness</td>
<td>N/mm</td>
<td>68</td>
<td>51</td>
<td>57</td>
<td>63</td>
<td>75</td>
<td>79</td>
<td>95</td>
</tr>
<tr>
<td>Front ARB</td>
<td>Stiffness</td>
<td>N/mm</td>
<td>46</td>
<td>35</td>
<td>41</td>
<td>50</td>
<td>61</td>
<td>67</td>
<td>82</td>
</tr>
<tr>
<td>Rear spring</td>
<td>Stiffness</td>
<td>N/mm</td>
<td>48</td>
<td>37</td>
<td>40</td>
<td>46</td>
<td>53</td>
<td>56</td>
<td>60</td>
</tr>
<tr>
<td>Rear ARB</td>
<td>Stiffness</td>
<td>N/mm</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>Tires</td>
<td>Cornering stiffness</td>
<td>%</td>
<td>100</td>
<td>75</td>
<td>86</td>
<td>109</td>
<td>121</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 7.2.4 Performance Attribute Requirement

The performance attributes listed in Table 19 consist of two case studies. One case study aims at improving the on-road handling with lower understeer gradient and less roll gain; the other seeks to improve the vehicle responsiveness during high-dynamic driving maneuvers without deteriorating the sideslip gain and lateral acceleration gain. In both cases, federal FMVSS 126 compliances must be ensured.

### Table 19 Performance attribute requirement

<table>
<thead>
<tr>
<th>Case</th>
<th>Objective metrics</th>
<th>Baseline</th>
<th>Range of attributes</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Understeer Gradient</td>
<td>8.5</td>
<td>6–8.26</td>
<td>deg/g</td>
</tr>
<tr>
<td></td>
<td>Roll gain</td>
<td>5.2</td>
<td>3–4.9</td>
<td>deg/g</td>
</tr>
<tr>
<td></td>
<td>Max wheel center height</td>
<td>0.45</td>
<td>0.4–0.5</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Lateral offset</td>
<td>2.40</td>
<td>2.4–2.7</td>
<td>m</td>
</tr>
<tr>
<td>2</td>
<td>Sideslip gain</td>
<td>4.59</td>
<td>4–4.59</td>
<td>deg/g</td>
</tr>
<tr>
<td></td>
<td>Yaw rate time constant</td>
<td>0.13</td>
<td>0.1–0.148</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>Lateral acceleration @1Hz</td>
<td>0.25</td>
<td>0.25–0.4</td>
<td>g/deg</td>
</tr>
<tr>
<td></td>
<td>Max wheel center height</td>
<td>0.45</td>
<td>0.4–0.6</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Lateral offset</td>
<td>2.40</td>
<td>2.3–2.7</td>
<td>m</td>
</tr>
</tbody>
</table>
7.2.5 Generation of Configurations

The output of the interactive ruled-based design framework is a configuration catalog which consists of multiple coherent combinations of different component. As introduced in previous sections, four steps to identify the feasible solutions are required (Figure 39). In step (a) and (b), an initial result of the ranges of intermediate parameters are solved according to the performance requirements. Figure 40 demonstrates the case study in which four performance requirements are checked resulting in the update of feasible ranges of the intermediate parameters.

Figure 40. Solving the initial ranges of intermediate parameters
In steps (c) and (d), the intermediate parameter space is mapped to the component parameter space for user selection by determining the intermediate parameters consequently. There are three minor steps required to determine the intermediate parameters. In the first step, the candidates of corresponding components that are not in conformity with correspondent intermediate parameter ranges will be eliminated/hidden. In Figure 41, the initial range of sprung mass CG height (m) is mapped to the lift height (in), resulting 0 or 2 inches lift kits as a feasible choice. In the second step, the user will be requested to choose from the feasible candidates for all corresponding components. (a lift height of 0 inches was selected in this example). In the last step, a value will be calculated and assigned to the intermediate parameter based on the chosen components. In case of a 0-inch lift kit the sprung mass CG height corresponds to 0.829m.

Figure 41. User selection input
After the first intermediate parameter, the sprung mass CG height, is determined, the feasible ranges of the other three intermediate parameters will be updated. Figure 42 indicates that the initial ranges for the front roll stiffness and rear roll stiffness remain feasible. However, the minimal feasible cornering stiffness is changed from 0.979 to 0.99 of the nominal value indicating that the feasible range has shrunk.

**Figure 42. Update other intermediate parameters**

In the next phase, another intermediate parameter (front roll stiffness) will be determined in four minor steps (Figure 43). The first step is similar to the previous parameter selection (sprung mass CG height). The range of front roll stiffness is mapped to the front spring and front ARB using the second layer design envelope; thereafter the
unfeasible component is unchecked (82 N/mm ARB). In the second step, the user selects the 79N/mm front spring based on his preference. Simultaneously additional unfeasible ARBs are unchecked (35 and 61, 67, 82 N/mm) which indicates that the feasible range of front ARB has shrunk. In the third step, the user selects the 50N/mm front ARB from the feasible candidates. All component parameters that are relevant to the front roll stiffness have been assigned with specific values, therefore, in the last step, these component values are propagated back to the intermediate parameter. The 79N/mm front spring and 50N/mm front ARB result in an overall front roll stiffness of 151,255 N/rad.
Figure 43. Determination of the second intermediate parameter
The second layer design envelope is also interactive and dynamic. Assigning a value to either component parameter will concurrently change the feasible ranges of other parameters. For example, the user can select the front ARB prior to the front spring (the feasible front spring candidates depend on which front ARB was selected). If the 61N/mm is selected, the feasible front springs become 51, 57, 63, 68, and 75 N/mm. If a softer ARB is selected (e.g. 41N/mm), a stiffer front spring (79 or 95N/mm) will be necessary. The design framework’s core value is to provide an interface to explore the envelope rather than providing a specific value, which allows more freedom for the users to make decisions based on their consideration.

![Image](image.png)

Figure 44. Demonstration of the dynamic second layer design envelope

Being an example application, the configurations are compiled to demonstrate the capability of generating diverse kits product. It is assumed that the user’s objective is to pre-compose lift kit packages for a variety of customers using various lift heights combined with springs, ARB, and tires from various suppliers. For demonstration purposes, four configurations have been generated covering all feasible lift kit height options (see Table 20 and Table 21). With configuration 1, the design envelope only
allows for a lift height of no more than two inches due to the performance requirements of roll gain. While with configuration 2, the design envelope allows for up to six inches of lift height in combination with stiffer springs and ARBs (Configuration 8).

Table 20 Component configuration for case 1

<table>
<thead>
<tr>
<th>Unit</th>
<th>OE</th>
<th>Conf.1</th>
<th>Conf.2</th>
<th>Conf.3</th>
<th>Conf.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift Kit</td>
<td>Inch</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Front spring</td>
<td>N/mm</td>
<td>68</td>
<td>79</td>
<td>63</td>
<td>75</td>
</tr>
<tr>
<td>Front ARB</td>
<td>N/mm</td>
<td>46</td>
<td>50</td>
<td>61</td>
<td>50</td>
</tr>
<tr>
<td>Rear spring</td>
<td>N/mm</td>
<td>48</td>
<td>40</td>
<td>48</td>
<td>40</td>
</tr>
<tr>
<td>Rear ARB</td>
<td>N/mm</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Tires</td>
<td>%</td>
<td>100</td>
<td>109</td>
<td>109</td>
<td>109</td>
</tr>
</tbody>
</table>

Table 21 Component configuration for case 2

<table>
<thead>
<tr>
<th>Unit</th>
<th>OE</th>
<th>Conf.5</th>
<th>Conf.6</th>
<th>Conf.7</th>
<th>Conf.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift Kit</td>
<td>Inch</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Front spring</td>
<td>N/mm</td>
<td>68</td>
<td>75</td>
<td>79</td>
<td>63</td>
</tr>
<tr>
<td>Front ARB</td>
<td>N/mm</td>
<td>46</td>
<td>50</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>Rear spring</td>
<td>N/mm</td>
<td>48</td>
<td>40</td>
<td>46</td>
<td>48</td>
</tr>
<tr>
<td>Rear ARB</td>
<td>N/mm</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Tires</td>
<td>%</td>
<td>100</td>
<td>100</td>
<td>109</td>
<td>109</td>
</tr>
</tbody>
</table>

The total operation of determining the interactive design envelope for each case takes less than 3 minutes. The more constraints are applied at the same time the more computational effort is required to generate the feasible ranges. In the application example, it took no more than 5 minutes to determine the feasible ranges that incorporate all constraints.

The performance results are plotted on the 2D response planes (see Figure 45). The boxes in the plots show the performance requirements inputs defined using the sliders on the left side of the user interface. In general, the case study results are scattered within the corresponding box. For example, in the first subplot, the Understeer gradient –
Roll gain plot, the constraints [6, 8.26] and [3, 4.9] only apply to case one, all the results of the case one locates inside the boundary box. In the second subplot, the Sideslip gain – Yaw rate time constant plot, the constraints [4, 4.6] and [0.1, 0.148] are only applied to case two, and all the results of the case two locate within the boundary box.

According to the proposed aftermarket vehicle modification framework, these options can be listed in the kit suppliers’ product catalog as recommended configurations ensuring vehicle dynamics compliance. However, the presented results are just a subset of all feasible configurations. Different kit suppliers may develop different configurations dependent on factors such as baseline vehicle selection, brand essence or cost.
Figure 45. Performance result of the case studies
7.2.6 Compliance Validation

The ultimate objective of this research is to guide the aftermarket vehicle modification in the real world, however, the rule-based interactive design envelope is based on efficient by simplified SMs to generate the configuration suggestions, so it is necessary to validate the result using the best available representation of a physical vehicle. In this research, the HFM will be the reference model for the validation process.

The vehicle (CarSim) model is updated with the parameters of the proposed configurations generated by the interactive rule-based design envelope tool and simulated with the stability control model. Results of the compliance metrics are shown in Table 22 and are also visualized in a scatter plot (Figure 46) which show the relationship between the results (scatter dots) and the user’s initial requirement inputs (the boxes). The two boxes represent different requirements in two cases. Both case results are all located within respective boxes, which indicate that the vehicle retrofitted with proposed kit configurations will be in compliance with the regulatory requirements.

Table 22 Compliance results of different configurations

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Baseline</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral Offset</td>
<td>m</td>
<td>2.52</td>
<td>2.63 2.61 2.61 2.47 2.50 2.59 2.63 2.48</td>
</tr>
<tr>
<td>Max Rear Wheel Center Height</td>
<td>m</td>
<td>0.479</td>
<td>0.505 0.521 0.537 0.491 0.459 0.510 0.597 0.440</td>
</tr>
</tbody>
</table>
Figure 46 Configuration Results Validation
CHAPTER EIGHT
CONCLUSION

8.1 Summary of the Dissertation

This dissertation is focused on the development of a constraint-based design framework (tool) to support the aftermarket industry to maximize the design freedom related to vehicle dynamics performance while ensuring compliance with electronic stability control regulations.

A post-delivery modification framework, which is led by an umbrella market organization and involves various stakeholders (OE, customer, legislator, aftermarket kit supplier, and component suppliers) has been established. The umbrella organization is responsible for the design envelope determination based on the input of VOC, VOL, and OE data. Kit suppliers, as members of the umbrella organization, have access to the design envelope to generate specific kits according to their business objectives and strategic brand essence. The design envelope incorporates rules that are derived from the vehicle performance attributes and regulation criteria relevant to the chassis modifications. The tool enables kit suppliers to configure their vehicle according to their preference while ensuring product compliance with confidence.

A generic mathematical representation of a (proprietary) ESC system has been developed to be used in a pure virtual simulation studies (a.k.a. Software-in-the-Loop or SiL). This approach is a cost-effective alternative to physical on-road testing or hardware-in-the-loop (HiL) simulations. The method can be used to assess the dynamic vehicle behaviors under limit handling (with ESC activation) including the assessment of
FMVSS126 compliance purely based on vehicle dynamics simulations. Once implemented, the SiL approach allows the vehicle system designer to evaluate multiple vehicle configurations (wheel/tires, lift kits, suspension components, and loading conditions) in a very cost effective way without building physical hardware, without requiring access to an actual vehicle and without conducting on-road testing.

The foundation of the ESC control logic is based on a simplified two-degree vehicle dynamics reference model which determines the desired vehicle behavior regarding yaw rate and sideslip angle upon driver steering input at a given vehicle speed. During evasive maneuvers, the brakes at individual wheels will be applied to stabilize the vehicle using a yaw moment controller in combination with the virtual reference model. A PD-controller was developed to close the loop and force the actual yaw rate to follow the desired yaw rate.

Based on the stability control model, a simulation-based methodology was introduced for certification of chassis-modified vehicle stability control performances to create guidelines for customizers. A set of variables with different lift heights, springs, and ARBs combinations were studied using Taguchi DOE method regarding the response of open-loop and closed-loop maneuvers. DOE results provide three distinct ways for supporting the aftermarket modification. First, main effects help customizers to understand which modification bring benefits or risks. Second, a regression model of the lateral offset metrics helps suppliers to predict closed-loop performances with open-loop testing information which require less time and cost. Finally, the compliance pass/fail criteria brought on the ‘Pass Region’ which consisted of feasible configurations so that
customizers can configure their options within the compliance zone. These three methods complement each other. The first one identifies the sensitive components/parameters; the second one provides an efficient way to evaluate the complex testing result by using simple tests, the third one offers a guideline for selecting a chassis upgrade configuration.

Two lower fidelity models, the linear model and surrogate model, are developed to improve computational efficiency. The linear model is derived from the high fidelity model in two steps. The first step is to reduce the number of DOFs and only consider three main DOFs: yaw rate, sideslip angle, and roll angle, which are the dominant DOFs regarding vehicle lateral stability; the second step is linearization of the component characteristics including spring rate, damping, and steering ratio. Tire cornering stiffness is treated as constant for gentle maneuvers (steady state and non-radical transient state handlings), and as varying parameters for extreme driving behaviors (SWD). The linear system is either a LTI system or a LPV system depending on the application circumstance. The PD control algorithm, which is inherited from the high fidelity model, was simplified while retaining critical nonlinear features. A metamodelling technique was applied to develop a quadratic regression surrogate model that is dedicated for compliance metrics. It is formed from the full factorial samples of the LPV model. Both the linear model and surrogate model are validated in the response spaces with the high fidelity model.

Finally, a dynamic and interactive design envelope was created, incorporating the rules that are established by the computationally efficient models formerly introduced. The constraint satisfaction problem is described in the nonlinear programming context.
and solved using sequential quadratic programming. The quasi-convexity of the design space, which is the necessary condition for the proposed approach, is also investigated by inspecting the constraint functions in three stages: function convexity, design space feasibility, and function monotonicity. Two case studies in the last section demonstrate a prototype of the UI and the procedure to generate diverse configurations for various market preferences. The UI decomposes the design problem into two layers of design envelope to simplify the problem complexity and decouple the parameters. In the end, the output configurations are validated regarding the vehicle dynamics and stability compliance.
8.2 Research contribution

1. Developed a vehicle modification framework for the post-delivery stage of product life cycle, which is led by an umbrella organization who will develop the guideline in the form of design envelope for different kit suppliers to compile their configurations according to their brand essence.

2. Developed an interactive rule-based design tool that couples design parameters on a user interface level. It allows users to define the chassis component configurations upon their performance preferences while guarantees the compliance with stability regulations.

3. Developed an efficient, interactive feasible solution determination approach based on nonlinear programming technique incorporated quasi-convexity investigation of constraint functions.

4. Develop a procedure (strategy) of virtual validation by utilizing the generic, high fidelity stability control model which best represents the physical vehicle system.

5. Determined the chassis modification sensitivities and impacts on the typical vehicle dynamics behaviors including steady state, transient state, and intense handling.
8.3 Future Work

This research has provided an approach that turns the design process from functional requirement to design variables into an interactive package. However, there are several aspects that are worth of further research in future:

- The current approach is initiated with the input of attribute requirements which are labeled with the technical terms, so users should have sufficient engineering knowledge to define the requirements. In future research, the interpretation of end-customer demands to technical terms should be abstracted into the UI so as to extent the application to aftermarket customers.

- The second layer of design envelope in this study is constructed using decoupled parameters. However, in the practices applications the components parameters are coupled: changing a component will affect several intermediate parameters concurrently. For example, replacing a tire means to change all the characteristics which will modify the cornering stiffness, the roll rate, and the braking force limit. Further study in future will be needed to deal with the coupling issue.

- The current approach has managed to solve the design/modification problem for a specific gray box mechatronic system (ESC). As more control units are being integrated into the vehicle system, e.g. advanced driver assistant and autonomous driving system, modification of vehicle may induce more risks of impairing the functionality or even safety. Extensive works will be needed in
future to promote the approach for the application of general gray box systems.
APPENDICES

Appendix A.
Linearized Three DOFs System Modeling

The linearized vehicle dynamics model for lateral and roll stability usually consider motions in three degrees of freedom: lateral, yaw, and roll. Figure 47 and Figure 48 depict the bicycle modeling concepts. For lateral dynamics which consists of two DOFs, the vehicle is treated as a single track model. Axles are simplify as wheels with lumped cornering stiffness of inner wheels and outer wheels, taking into account the lateral load transfer effect. For roll dynamics, the roll rates are also lumped parameters that combine the suspension spring stiffness, ARB stiffness, and tire vertical stiffness; the roll damping effect only take the suspension dampers into accounts. Both the roll rate and roll damping are regarded as constant by assuming the linear characteristics of springs and dampers.

Figure 47 Four DOFs bicycle model for lateral dynamics
Figure 48 Dynamics of roll motion

The equations of motion are derived as follows:

Lateral dynamics:
\[ MV(\dot{\phi} + r) = F_{yf} + F_{yr} \]  \hspace{1cm} (63)

Yaw dynamics:
\[ I_Z \dot{\phi} = a_{yf} - b_{yf} + M_Z \]  \hspace{1cm} (64)

Roll dynamics equation refer to rolling center:
\[ I_{xs} \ddot{\phi}_s + M_s H_2 \ddot{\phi}_s = M_s H_2 a_{ys} - B_{\phi_s} \dot{\phi}_s - k_{\phi_s} \phi_s + M_s g H_2 \phi_s \]  \hspace{1cm} (65)

Combining the equations from (63) to (65), the fourth order state space representation of the system is derived as below:
\[
\begin{bmatrix}
MV & 0 & -M_s H_2 & 0 \\
0 & I_z & I_{zz} & 0 \\
-M_s H_2 V & I_{xz} & I_{xs} + 2M_s H_2^2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\hat{\beta} \\
\dot{\beta} \\
\phi_s \\
\phi_s \\
\end{bmatrix} = 
\begin{bmatrix}
C_1 + C_2 & \frac{a C_1 - b C_2}{V} - MV & 0 & 0 \\
(a C_1 - b C_2) & \frac{a^2 C_1 + b^2 C_2}{V} & 0 & 0 \\
0 & M_s H_2 V & -B_{\phi S} & -\left( K_{\phi S} - M_s g H_2 \right) \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\beta \\
r \\
\phi_s \\
\phi_s \\
\end{bmatrix} + 
\begin{bmatrix}
-C_1 \\
0 \\
-aC_1 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\delta \\
M_z \\
\end{bmatrix}
\]
Appendix B. 
Rules to map the lumped variable to component parameter

The continuous space of intermediate parameters is mapped to discrete component space taking advantage of the simpler interrelationship. The components are decoupled across the intermediate parameters, so each of them is only related to one intermediate parameters. The relationships are shown in Figure 49. The lift kit height only affects the sprung mass center of gravity height. The variation of spring stiffness and ARB only determine the axle roll stiffness. The vehicle cornering stiffness is only defined by the tire lateral characteristics.

![Diagram showing the relationships between the intermediate parameters and component parameters](image)

Figure 49. Relationship between the intermediate parameters and component parameters

The translation between two types of parameters utilizes straightforward equations that describe the relationships of the parameters. The lift height is calculated by
simply minoring the original CG height with the new CG height and transforming the unit from millimeter to inch:

$$H_{lift} = (H_{cgS} - H_{cgS}^*) \times 39.37$$  \hspace{1cm} (67)

The equation indicates linear relationship between two variables, so by using the interval arithmetic, the range $\overline{H_{lift}}$ can be calculated directly with following equations:

$$\overline{H_{lift}} = (\overline{H_{cgS}} - \overline{H_{cgS}^*}) \times 39.37$$

The axle roll stiffness ($N \cdot m/rad$) is determined using the kinematic relationships among spring stiffness ($N/m$), ARB ($N/m$) and their transverse distance ($m$):

$$K_{\phi S} = \frac{1}{2} K_{sp} D_S^2 + K_{ARB} D_A^2$$  \hspace{1cm} (68)

And it is easy to derive following expressions of spring stiffness and ARB stiffness:

$$K_{sp} = \frac{2(K_{\phi S} - K_{ARB} D_A^2)}{D_S^2} = f(K_{\phi S}, K_{ARB})$$  \hspace{1cm} (69)

$$K_{ARB} = \frac{K_{\phi S} - \frac{1}{2} K_{sp} D_S^2}{D_A^2} = g(K_{\phi S}, K_{sp})$$  \hspace{1cm} (70)

According to the interval arithmetic, their boundaries can be solved as:

$$K_{sp}^{ub} = f(K_{\phi S}^{ub}, K_{ARB}^{lb}), K_{sp}^{lb} = f(K_{\phi S}^{lb}, K_{ARB}^{ub})$$  \hspace{1cm} (71)

$$K_{ARB}^{ub} = f(K_{\phi S}^{ub}, K_{sp}^{lb}), K_{ARB}^{lb} = f(K_{\phi S}^{lb}, K_{sp}^{ub})$$  \hspace{1cm} (72)
The axle cornering stiffness only takes into account the contributions from the tires, so the scale factor of axle cornering stiffness is directly equal to the scale factor of tire cornering stiffness:

\[ S_{\text{Caxle}} = S_{\text{Ctire}} \]  \hspace{1cm} (73)

Similarly the range is solved directly as:

\[ \overline{S_{\text{Caxle}}} = \overline{S_{\text{Ctire}}} \]  \hspace{1cm} (74)
REFERENCES


[63] UN/ECE, 13-H, Uniform provisions concerning the approval of passenger cars with regard to braking, Revision 3, 2014.


