Computational Engineering Analysis of Materials and Structural Aspects of Gas Turbine Engine Ceramic Matrix Composite Components

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COMPUTATIONAL ENGINEERING ANALYSIS OF MATERIALS AND STRUCTURAL ASPECTS OF GAS TURBINE ENGINE CERAMIC MATRIX COMPOSITE COMPONENTS

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Mechanical Engineering

by
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Accepted by:
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ABSTRACT

Ever increasing world energy need and growing environmental concerns have resulted in rising efficiency and reduced emissions requirements from the energy industry. Current gas turbines, widely used for power generation, have reached a plateau in efficiency. To further boost their efficiency and reduce emissions it is imperative to increase the operating temperatures. This necessitates the advent of new materials which have higher temperature capability than the existing super alloys, used to manufacture current gas turbines. Ceramic Matrix Composites (CMCs) are such a class of material, which have very high melting points and are extremely light weight in comparison to the superalloys. The CMCs are made from ceramic constituents that are inherently brittle; however, the CMCs show metal-like ductile behavior.

The present work focuses on a non-oxide class of CMCs which are made SiC fibers and SiC matrix. A room temperature multi-length scale constitutive material model has been developed by homogenization at two characteristic microstructural Length Scales (LS), fiber/tow LS and ply/lamina LS. The results obtained from virtual mechanical tests on representative volume elements for the two LS are homogenized to generate a component length scale material model which exhibits the characteristic elastic and inelastic behavior of CMCs. This material model is implemented as a user subroutine for a commercial finite element package ABAQUS. Being a relatively new class of material, the CMCs are targeted initially for manufacturing low stress bearing stationary components in the hot-section of the gas turbines. Hence, the material model is tested by conducting a foreign object impact test on a typical stationary gas turbine hot-section component, namely the inner shroud. The effect of fiber architecture (cross-ply vs. plain weave) and strength of the fiber-matrix bond on the impact resistance of the inner shroud is demonstrated.
In the hot-section of the gas turbine, the CMC components experience significant in-service high temperature environmental degradation. To capture this degradation four environmental effects: (a) grain growth and porosity growth; (b) creep; (c) dry oxidation; and (d) wet oxidation, have been identified. Using experimental data reported in open literature, the component length scale CMC material model properties are modified to be a function of the nature, duration and extent of the environmental exposure. Again, foreign object impact tests are conducted to measure the CMC material degradation after exposing it to the four environmental conditions. Out of the four environmental effects considered the wet oxidation results in highest material degradation, at a given time and temperature exposure.

After the commercial success of stationary CMC components is established, more hot-section components like turbine blades are expected to be made from CMCs to further extend the efficiency benefits offered by the use of CMCs in gas turbines. Creep is a primary failure mechanism for rotating components like blade, which experience high in-service temperature. A generalized anisotropic 3-D creep deformation and creep rupture model is developed for SiC/SiC CMCs subjected to multi-axial stresses. Experimental results from open literature are used to parameterize and validate the creep deformation and rupture model for the SiC/SiC CMCs. This model is then used in a finite element package ABAQUS to predict the gas turbine operation time associated with the first blade-tip rub and eventual creep rupture (at the root) of a CMC blade used in the low pressure turbine of a gas turbine engine. The gas turbine engine maintenance schedule and life time of CMC blades, which are governed by the engine operation time associated with blade-tip rub and creep rupture events, are predicted using the results of this analysis.
Lastly, the issue of attaching the stationary CMC component (inner shroud) to the metallic components in the gas turbine has been addressed. Traditional fastening techniques are not suitable since the CMCs have a very low thermal expansion coefficient in comparison to the surrounding metallic components. Hence, a floating type assembly is used to attach the inner shroud to the outer casing. It consists of pre-compressed spring to provide clamping force to the inner shroud. The metallic spring undergoes creep and oxidation since it is located in the hot-section of the gas turbine, resulting in a loss of clamping force. This is a potential life limiting mechanism for the CMC inner shroud. Material selection procedures are developed for the metallic spring using rigorous finite element method and relatively simplified analytical technique. The objective is to minimize the loss in spring clamping force, subjected to geometric constraints (spring dimensions are limited by the size of the cavity that houses it) and functional constraint (maximum allowable drop in spring clamping force over the expected inner shroud life time). Both the procedures generate consistent ordering of candidate materials for the spring in the case of creep. However, consideration of oxidation alters the results among the two procedures.

The computational procedures and the results from this dissertation are intended to complement the ongoing and future experimental CMC development efforts by reducing the associated time and cost.
Keywords: Ceramic Matrix Composites, Gas Turbines, multi-length scale material constitutive model, creep, high-temperature oxidation, finite element method.
DEDICATION

This dissertation is dedicated to my parents Mr. Ram Galgalikar and Mrs. Vibha Galgalikar, my uncle Mr. Nitin Bendre and aunt Mrs. Charuta Bendre, who have always supported me through my thick and thin.
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CHAPTER 1: INTRODUCTION

1.1 Introduction and background

The ever increasing world energy requirements and environmental concerns have led to stricter emissions norms and increased efficiency requirements from the energy industry. Along with the development of cleaner and renewable energy generation methods like solar power and wind power, there is a significant research effort dedicated towards improving the efficiency of the existing methods and technologies used in power generation. An important class of devices used in the energy industry is gas turbines. They are used extensively for electricity generation and aviation propulsion. Along with component design improvements there is a need to increase the operating temperature in the gas turbines to attain higher efficiency and lower emissions. A major obstacle to increase the operating temperature in the gas turbine is the melting point of the superalloys which are used to manufacture them. Various internal cooling technologies are being used in current gas turbines to operate them at temperatures within 50°C of the material melting point. Thus advent of new materials which are capable of handling higher temperatures is imperative for further increase in gas turbine operating temperatures and efficiency.

Ceramic materials are known to have higher melting points than most of the metals and metallic alloys e.g. SiC (ceramic) melts at approximately 2700°C in contrast to Inconel (Nickel-Chromium based superalloy) which melts at approximately 1400°C. However, brittle and stochastic failure of ceramics has prevented their use in the gas turbines. Composite material made from ceramic constituents, which shows metal-like ductile behavior and have very high melting points would be extremely desirable for use in gas turbine application. Ceramic Matrix Composites (CMCs) are such a class of material which is being developed in the last 20 years [1]. The CMCs are made from ceramic fibers which are embedded in a ceramic matrix. The ceramic
fibers are coated with thin ceramic and/or non-ceramic layers to achieve a weak bonding with the ceramic matrix. This results in the CMCs having their characteristic metal-like ductile behavior. One important distinction between the CMCs and the metals is the physical origin of the ductile behavior. In metals the ductility results from the plastic deformation. However, in CMCs, the ductile behavior is due to progressive non-catastrophic crack propagation through the ceramic constituents, which is enabled by the weak fiber-matrix bond.

The development of CMCs over years has focused on experiments, which are costly [2-4]. The latest computation methods can complement the experimental efforts to reduce the CMC development cost and time. In the current work finite element analysis based computational technique has been used to develop multi-length scale constitutive material model for a class of non-oxide CMCs which are made from SiC fibers and SiC matrix.

1.2 Dissertation outline

The introduction and background about the CMCs is presented in chapter 1. In the second chapter, three characteristic length scales are identified for the SiC/SiC CMCs. The coarsest length scale represents the structural component, while two finer length scales represent characteristic microstructural features. Homogenization is carried out at two microstructural length scales to derive a constitutive model, capturing elastic and inelastic behavior of CMCs at room temperature. This material model is implemented as a user defined subroutine for a commercial finite element software ABAQUS and its utility is demonstrated by applying it to problem of foreign object impact on shroud which is a typical hot-section stationary CMC component.

In the third chapter the room temperature material model is extended to account for the in-service high temperature environmental degradation experienced by the CMC components
when they are used in the hot-section of the gas turbines. Four environmental effects have been
identified: (a) grain growth and porosity growth; (b) creep; (c) dry oxidation; and (d) wet
oxidation. The component length scale CMC properties are made function of the nature, duration
and extent of environmental exposure, to determine the residual material properties.

In the fourth chapter, a generalized 3-D creep model is developed for CMCs. Initially the
CMCs are expected to be used for low stress experiencing stationary components in the hot-
section of the gas turbine. Once the commercial success of the stationary CMC components is
established, higher stress experiencing moving components e.g. turbine blades will be made from
CMCs. For the moving components, which experience high temperature and stress, creep is a
primary life-limiting phenomenon. A generalized 3-D creep model is intended to help in
development of such moving CMC components. Experimental results from open literature are
used to parameterize the 3-D creep model and it is used to predict the life time of CMC blade
used in low pressure turbine.

The chapter 5 focuses on attachment of stationary CMC component (inner shroud) to the
metallic components in the gas turbine. Since the CMCs have a very low thermal expansion
coefficient in comparison to the surrounding metallic components, traditional connections cannot
be used. Floating type assembly is used to attach the inner shroud to the outer casing. A pre-
compressed spring is used in the floating assembly to provide clamping force to the inner shroud.
The metallic spring undergoes creep and oxidation since it is located in the hot-section of the gas
turbine, resulting in a loss of clamping force. This is a potential life limiting phenomenon for the
CMC inner shroud. A material selection procedure is developed for the metallic spring based on
finite element method and an analytical procedure. The order of candidate materials generated by
both the techniques is compared for the case of creep and oxidation induced spring force relaxation.

A general summary and conclusion with suggestions of future work is presented in chapter 6.

1.3 References


CHAPTER 2: MULTI-LENGTH-SCALE DERIVATION OF THE ROOM-TEMPERATURE MATERIAL CONSTITUTIVE MODEL FOR SiC/SiC CERAMIC-MATRIX COMPOSITES (CMCs)

2.1 Abstract

In the present work, multi-length-scale physical and numerical analyses are used to derive an SiC/SiC ceramic matrix composite (CMC) material model suitable for use in a general room-temperature, finite-element-based, structural/damage analysis of gas-turbine engine components. Due to its multi-length-scale character, the material model incorporates the effects of fiber/tow (e.g. the volume fraction of the filaments, thickness of the filament coatings, decohesion properties of the coating/matrix interfaces, quality, as quantified by the Weibull distribution parameters, of the filament, coating, and matrix materials, etc.) and ply/lamina (e.g. the 0°/90° cross-ply vs. plain-weave architectures, the extent of tow crimping in the case of the plain-weave plies, cohesive properties of the inter-ply boundaries, etc.) length-scale microstructural/architectural parameters on the mechanical response of the CMCs. To identify and quantify the contribution of the aforementioned parameters on the material response, detailed numerical procedures involving the representative volume elements and the virtual mechanical tests are developed and utilized. The resulting homogenized turbine-engine component-level material model is next integrated into a user-material subroutine and used, in conjunction with a commercial finite element program, to analyze the foreign object damage experienced by a toboggan-shaped turbine shroud segment. The results obtained clearly revealed the role different fiber/tow and ply/lamina microstructural parameters play in the structural/damage response of the gas-turbine CMC components.
2.2. Introduction

The present work addresses the problem of multi-length-scale derivation, parameterization and validation of a room-temperature material constitutive model for continuous SiC-filament reinforced SiC-matrix ceramic-matrix composites (CMCs). Consequently, the concepts most relevant to the present work are: (a) the basics of ceramic-matrix composites; (b) microstructure hierarchy in CMCs; (c) room-temperature mechanical properties of CMCs; and (d) prior work on the development of material constitutive models for CMCs. In the remainder of this section, a brief description is provided for each of these concepts.

2.2.1 The Basics of Ceramic-Matrix Composites

High-temperature metallic materials such as nickel-, cobalt- or iron-based superalloys used in gas-turbine engines have been pushed to their thermal-stability limit since they are often made to operate at temperatures which are within 50 degrees of their melting point. To increase power density and energy efficiency of the gas-turbine engines, new materials are needed which can operate at temperatures as high as 1400 K. The main candidate materials currently identified for use in the next generation of gas-turbine engines are (monolithic) ceramics and CMCs. Since these materials can withstand extremely high temperatures, their use in hot sections of gas-turbine engines can yield a number of benefits such as: (i) improvements in thrust and fuel efficiency; (ii) lower pollutant emissions; (iii) reduced cooling requirements; (iv) simplification of the engine-component design; and (v) reduced requirements for the strength/weight of the supporting structure.

However, due to their relatively low fracture toughness, tensile strength and damage tolerance, monolithic ceramics are not being perceived as respectable candidate materials for use in critical turbine-engine structural applications (e.g. turbine blades). On the other hand, CMCs
consisting of a ceramic matrix and ceramic fibers possess superior structural properties relative to their monolithic-ceramic counterparts, while retaining their high-temperature stability and integrity. This is the reason that the CMCs are being aggressively researched and developed for use in future gas-turbine engines, [e.g. 1-7]. The potential of the CMCs in revolutionizing the performance of the gas-turbine engines is shown schematically in Figure 2-1 [8]. In this figure, the x-axis represents the approximate period of dominance in usage of the particular class of high-temperature materials (and associated cooling technologies), while the y-axis denotes the temperature capability of the material class in question. It is seen that the temperature capability of CMCs lies above the fitting line for the temperature capabilities of the past and present gas-turbine engine materials.

![Figure 2-1](image)

**Figure 2-1** Temperature capability (i.e. maximum service temperature) of various gas-turbine engine materials as a function of the time period of their dominant usage
Presently, CMCs are being perceived as candidate materials in gas-turbine engine components which experience low in-service loads such as nozzles, combustor liners, turbine-shrouds and exhaust components. However, it is hoped that with future improvements in the processing and performance of CMCs, these materials could be used in the manufacture of more critical gas-turbine engine components such as turbine blades, which experience higher in-service loads. Clearly, this will require major advances in the CMC synthesis/manufacture processes to ensure high quality/performance, high yield and reduced cost of these materials.

2.2.2 Microstructure Hierarchy in CMCs

Following the methodology developed in our prior work [9-12], one can identify a number of characteristic length scales and the associated material microstructures within CMCs. These length scales span from the finest nanometer size (the scale at which the local material microstructure consists of an assembly of interacting and bonded atoms/ions/molecules) to the coarsest centimeter-to-meter size engine-component-level scale (the material at this length scale is treated as being featureless and homogeneous). In the present work, only the three coarsest length scales are explicitly considered. The remaining length scales will be the subject of our future work. The three length scales considered, in ascending order, include: (a) the 10–100 micrometer characteristic-length size fiber/tow length-scale; (b) the 500–2000 micrometer characteristic-length size single-ply (or a two-ply stack) length-scale; and (c) the centimeter-to-meter characteristic-length size engine-component length-scale. Labeled schematics of the three length scales analyzed are depicted in Figures 2-2 (a)–(c). In the remainder of this subsection, brief descriptions will be provided for each of the three length scales under consideration.
Figure 2-2 Schematics of three length scales analyzed: (a) fiber/tow; (b) single-ply (or a two-ply stack); and (c) engine-component.
Fiber/Tow Length-Scale: The SiC/SiC CMCs considered in the present work are based on multi-filament SiC reinforcing tows (like the Hi-Nicalon™ fibers, typically containing 500 filaments with an average filament and tow diameters of ca. 10 micron and 150 micron, respectively [12]). Generally, CMCs based on multi-filament SiC fibers, like Hi-Nicalon™, possess superior properties and are more suitable for the fabrication of geometrically complex (curved, small-fillet radius) gas-turbine engine components. The filaments within the tows are coated (typically with BN or Si₃N₄, 2–4 micron thick), Figure 2-2 (a), for protection against oxidation and for attainment of relatively weak matrix/coated-fiber interfaces [13]. Typically, using either the reactive melt infiltration (RMI) [14] or chemical vapor infiltration (CVI) process [15], the inter-filament and inter-tow spaces are filled with the SiC matrix. The infiltration process is generally not perfect and the inter-filament/inter-tow space is either not completely filled with SiC (results in the formation of voids) or during the RMI process, conversion of molten silicon into SiC is not complete (results in the presence of residual/unreacted carbon and silicon). Consideration of the effect of these deviations from the void-free fully-SiC matrix microstructure is beyond the scope of the present work.

Ply/Lamina Length-Scale: While there are various ply architectures used in the fabrication of CMCs, the following two are most frequently employed: (a) 0°/90° cross-ply; and (b) plain-weave. Both of these architectures are depicted in Figure 2-2 (b). Material microstructure at this length scale, Figure 2-2(b), is characterized by various geometrical details of the two architectures such as tow diameter, tow-crimping wavelength and amplitude, etc., as well as by the fiber-reinforcement volume fraction.

Engine-Component/Laminate Length-Scale: As mentioned earlier, the CMC material is fully homogenized at this length scale. On the other hand, since the material behavior at this length-
scale is affected by the two finer microstructures, the material is not isotropic, but could initially be considered to be in-plane isotropic. Furthermore, after the material is subjected to a general state of loading and undergoes deformation/damage, the material tends to exhibit orthotropic or even general anisotropic behavior. In the present work, the CMC material at this length scale will be considered as being orthotropic and, hence, at each material point one must define the orientation of the three mutually orthogonal material directions before specifying the material constitutive model. Figure 2-2 (c) shows a schematic of a gas-turbine engine shroud component consisting of 12 toboggan-shaped segments. The main function of the shroud is to increase the efficiency of the gas-turbine engine by preventing the leakage of hot exhaust gases around the blade tips, while retaining its structural integrity under harsh (high-temperature, oxidative and high-water vapor) environments encountered within a gas-turbine engine. Modeling of the structural behavior of such a component requires the availability of a fully-homogenized laminate-/component-scale high-fidelity material model.

2.2.3 Room-Temperature Mechanical Properties of CMCs

While in general one is concerned with the mechanical properties of CMCs when subjected, over a long period of time, to multi-axial/cyclic/sustained loading under a high-temperature, oxidizing, high water-vapor-content environment, the present work will be limited to the room-temperature mechanical properties of these materials under low-rate monotonic loading/unloading conditions. During the development of a new class/grade of CMCs, it is a common practice to validate the success of the fabrication process by first checking the CMCs room-temperature mechanical properties. Extension of this work to include the effects of cyclic and sustained loading, and the oxidizing/high-humidity environment is underway.
The room-temperature mechanical behavior of laminated (cross-ply or plain-weave) CMCs is exemplified by the in-plane uniaxial-tension stress-strain curve shown in Figure 2-3. Careful examination of the stress-strain curve shown in this figure reveals the presence of four distinct regions (labeled I–IV), each associated with the characteristic mechanical response of the loaded material. The four regions of the stress-strain curve can be explained as follows:

(i) Region I is characterized by the linear elastic response of the (virgin) composite material. This region extends up to the stress level commonly referred to either as first matrix-cracking stress or proportional-limit stress, and does not induce any noticeable damage into the material;

(ii) In region II, the material deformation involves an inelastic component resulting from the continuing matrix cracking. Concomitantly, fibers bridging the matrix cracks are shear-debonded from the matrix. These damage processes are accompanied by a reduction in the material stiffness and a gradual increase in the material strength (as flaws of progressively lower potency have to be activated to produce additional matrix cracks);

(iii) In region III, continuation of the matrix cracking would require the activation of very weak flaws while the extent of load transfer from the fibers to the fractured-matrix fragments is severely decreased. Consequently, matrix-crack density begins to saturate while the material response to the applied loading begins to include fiber cracking. These microstructural-evolution processes are accompanied by further material-stiffness degradation and by “strain hardening” (the rate of which is higher than that in region II); and

(iv) As the loading proceeds, progressively more fibers are broken and although the remaining unbroken fibers are stronger, the number of unbroken fibers decreases (causing the stress within these fibers to increase). Consequently, as loading continues, the material passes through the state
of critical stability (corresponding to a critical fraction of the broken fibers) beyond which the increase in the average fiber strength is exceeded by an increase in the average stress experienced by the unbroken fibers (Region IV). As a result, the material begins to fail. Since at this point some of the fibers may still be bonded to the matrix, their sliding and the final pullout is associated with an additional strain and, consequently, the material fails gradually rather than abruptly.

Figure 2-3 Prototypical in-plane uniaxial-tension stress-strain curve, exemplifying the room-temperature mechanical behavior of laminated (cross-ply or plain-weave) CMCs.
2.2.4 Prior Work on Development of Material Constitutive Models for CMCs

A detailed review of the public-domain literature carried out as part of the present work revealed that the majority of the published modeling/computation-based work is concerned with addressing various aspects of the room-temperature mechanical response of the CMCs, and with providing an in-depth insight into the complex phenomena and processes accompanying material deformation, progressive damage and ultimate failure. However, only a few of the reported studies are concerned with the development of the continuum-type CMC material constitutive models (the subject of the present work) suitable for use in the finite-element structural analysis of various gas-turbine engine components. In the remainder of this sub-section, a brief overview is provided of the few published studies which are deemed most relevant to the present subject matter.

Marshall et al. [16] were among the first to attempt to model complex deformation, damage and ultimate failure of unidirectionally-reinforced CMCs. While accounting for the contribution and the stochastic nature of the constituent-material (filament-coating/matrix/interphase) properties to the CMC mechanical response, they explicitly modeled the bridging of matrix-cracks by the unbroken filaments and the accompanying closure tractions acting on the crack faces. Although the analysis of Marshall et al. [16] provided a lot of useful insight into the complex phenomena and processes accompanying deformation and damage of the CMC during loading, no attempt was made to derive a general continuum-type material model for use in three-dimensional finite-element structural analyses.

Talreja [17] proposed a homogenized three-dimensional material model for unidirectionally fiber-reinforced ceramic matrix composites. Within the model, the elastic response of the material is assumed to be transversely isotropic while the inelastic response is
assumed to be controlled by a combination and interaction of several (distributed) material-damage mechanisms such as matrix cracking, fiber/matrix interfacial slip, fiber/matrix debonding and filament failure. The extent of various damage modes is quantified using the appropriate second-order tensors while the evolution of the damage modes was modeled using the appropriate rate equations. To derive the basic stress vs. strain functional relationships and to describe damage-induced material-stiffness degradation, a thermodynamics-based strain-energy density formalism is developed within which the material internal energy is assumed to be a function of its strain and damage states. The general utility of the material model developed has not been validated through its application in a finite-element-based structural analysis of a CMC component.

Curtin et al. [18] developed a micromechanics-based uniaxial-stress vs. uniaxial-strain material model for unidirectionally-reinforced SiC/SiC CMCs as a function of the stochastic properties of the constituent (matrix and fiber) materials and the associated phase-interfaces. The model clearly revealed the effect of the constituent-material properties on the CMC uniaxial mechanical response. Specifically, it was demonstrated that, depending on the relative strength of the matrix phase, the CMC can possess different levels of tensile strength, ductility and toughness. For example, in the case of a higher-strength matrix, matrix-cracking is immediately followed by fiber-cracking, resulting in a limited ductility/toughness of the CMC. In sharp contrast, when matrix-cracking occurs at lower stresses, fiber-sliding and pullout precede fiber-cracking, giving rise to an increase in the material ductility/toughness. While the model enabled a deeper insight into the role and interaction of various material-deformation and damage processes, it was never formulated as a general three-dimensional material constitutive model suitable for use in finite-element-based structural analysis of the CMC components.
Camus [19] proposed a three-dimensional continuum damage mechanics constitutive model for plain-woven SiC/SiC CMCs. As in the case of Talreja\textsuperscript{17}, the model was based on an elastic-strain-energy thermodynamics framework and employs a number of internal damage variables, one or more of which controls the specific component of the orthotropic elastic stiffness tensor of the material. The evolution equations for the damage variables include coupling of different damage modes and the contributions of different energy-conjugate thermodynamics forces to the evolution of a given damage mode. The unique feature of the model is the introduction of an “effective compliance tensor” which accounts for the fact that only tensile normal and shear loads result in material degradation, and that compressive normal loads lead to the deactivation of the material-damage processes. After parameterization, the model is validated by demonstrating a relatively good agreement between the in-plane experimentally measured and computationally predicted mechanical properties of the plain-woven SiC/SiC CMCs. However, the utility of the material model in more general multiaxial loading scenarios was not established.

Pailler and Lamon [20] developed a micromechanics-based uniaxial-stress – uniaxial-strain material constitutive model for Hi-Nicalon tow-reinforced, chemical vapor infiltrated SiC/SiC minicomposites (composites containing a single filament tow). The model includes the stochastic aspects of the filament and matrix strengths, as well as the presence of the processing-induced residual stresses. While the model was extended to include high-temperature matrix/fiber interface degradation and matrix-crack healing effects, as well as the creep character of the material deformation, it was never formulated as a general three-dimensional material constitutive model for the CMCs.
Zhang and Hayhurst [21] developed a homogenized, continuum-type, tow length-scale material model capable of capturing the mechanical response of cross-ply and plain-weave CMCs under multiaxial loading conditions. The resulting material model was first implemented in a material-model subroutine, and then linked with a finite-element code to investigate in-plane mechanical properties of the CMCs with the two basic (cross-ply and plain-weave ply architectures). During the derivation of the material constitutive model, filaments, filament coatings, inter-filament matrix and inter-tow matrix, as well as the associated interface boundaries are all modeled explicitly and the stochastic nature of their properties accounted for. To reduce the computational cost, the non-linear, matrix-cracking-dominated, inelastic response of the material is simplified using a piecewise linear approximation. As in the work of Camus [19], the continuum damage mechanics formalism was employed in order to describe the contribution and interaction of various damage mechanisms to the material-stiffness degradation and to the final failure.

2.2.5 Main Objective

The main objective of the present work is to develop a multi-length-scale based CMC material model which can be used in a general room-temperature, finite-element-based, structural/damage analysis of gas-turbine engine components. The model should incorporate two main contributions: (a) that associated with various structural constituents (e.g. multi-filament reinforcements, filament coatings, ceramic-matrix composites, phase interfaces, etc.); and (b) that associated with the laminated-CMC (0°/90° cross-ply vs. plain-weave) ply architecture. Consequently, the model will involve two levels of material homogenization. Within the first level, the effective response of the material consisting of filaments, filament coatings, inter-filament matrix and inter-tow matrix will be determined and formulated as a material constitutive
model. Within the second level, the material model derived within the first level will be used in conjunction with the two laminated-CMC architectures to determine the effective response of the fully-homogenized and featureless CMC material. This response will also be cast in the form of a continuum material model suitable for linking with commercial finite element programs, enabling finite-element structural analysis of the gas-turbine engine components.

2.2.6. Chapter Organization

Development of the Level 1 (i.e. tow length-scale) material constitutive model is presented in Section 2-3. In Section 2-4, Level 2 constitutive material model development is presented. An example of a transient, non-linear dynamics, finite-element structural analysis of a prototypical gas-turbine engine component (turbine shroud) being impacted by a high-speed foreign object (ingested and accelerated by the engine) is presented in Section 2-5. A summary of the main conclusions resulting from the present work is given in Section 2-6.
2.3 Fiber/Tow Length-Scale Material-Model Development

The problem analyzed in this portion of the present work deals with the development of a (homogenized) fiber/tow length-scale material model for the SiC/SiC CMCs (based on multi-filament tow reinforcements). The procedure to be used will involve: (a) the construction of the corresponding representative volume element (RVE); (b) a series of virtual mechanical tests on the RVE; and (c) post-processing of the mechanical test data in order to assemble and parameterize the corresponding homogenized material constitutive model. In the remainder of this section, the necessary details are provided regarding the aforementioned three-step procedure, as well as of the key results yielded by the current approach.

2.3.1 Construction of the Fiber/Tow Length-Scale CMC RVE

An example of the fiber/tow length-scale CMC RVE is shown in Figure 2-4 (a). For clarity, different constituents of the RVE are labeled in this figure and the associated dimensions are specified. The RVE consists of 19 circular cross-section, 10 micron diameter SiC filaments, each coated with a 0.125 micron thick layer of BN. The center-to-center inter-filament spacing within the tow is 15 micron, and the volume of the tow-surrounding SiC matrix is adjusted to make the reinforcement volume fraction approximately 32%. The BN coating is assumed to be perfectly bonded (i.e. tied kinematically) to the SiC filament, and relatively weakly bonded to the matrix. Consequently, the BN-coating/SiC-matrix interface acts as an interphase (i.e. an initially zero-thickness region possessing unique cohesive damage and failure properties) and has to be considered explicitly.
Figure 2-4 (a) Schematic of the fiber/tow length-scale CMC representative volume element (RVE); and (b) a close-up of the finite element mesh used
2.3.2 Virtual Mechanical Tests on the CMC RVE

Virtual mechanical testing of the CMC RVE is carried out using the conventional displacement-based finite-element analysis (FEA). Within a typical FEA, the following should be specified: (a) geometrical model (defined above as the fiber/tow length-scale RVE, Figure 2-4(a)); (b) meshed model; (c) computational algorithm; (d) initial conditions; (e) boundary/loading conditions; (f) contact interactions; (g) material models; and (h) computational tool. Items (b)–(h) are briefly overviewed in the remainder of this sub-section.

**Meshed Model:** Due to the regular geometry of the SiC filaments, each filament is meshed identically using 24 six-node triangular-prism, first-order, continuum finite elements per cross-section and 100 such elements along the filament length (aligned in the x-direction). For the same reason, the BN interphase/coating is meshed using 24 and 100 eight-node hexahedral, first-order, continuum finite elements, circumferentially and longitudinally, respectively. To ensure perfect bonding between the coating and the filament, the two meshes are made to share nodes along their contact surfaces. As far as the matrix is concerned, to gain regularity in its meshing the transverse y-z cross-section of the RVE is assumed to be composed of regular hexagonal tiles with the center of the tile containing the fiber with the circular cross-section and coating with the circular-ring cross-section. A close-up of the finite-element mesh over a transverse section of the RVE is shown in Figure 2-4(b). The RVE mesh for the matrix consists of 2700 eight-node hexahedral, first-order, continuum finite elements per cross-section and 100 such elements longitudinally. To model the weak matrix-coated filament interface, the matrix mesh and the coating mesh are allowed to contain coincident, but separate, nodes, and the two sets of nodes are used to construct eight-node cohesive traction-separation interfacial elements.
Computational Algorithm: To ensure robustness of the computational analysis, all the calculations carried out in the present work utilized a transient, displacement-based, purely-Lagrangian, conditionally-stable, dynamic (low loading rate) explicit finite-element algorithm.

Initial Conditions: While, in general, CMCs in their as-fabricated condition may possess processing-induced residual stresses, such stresses were not considered in the present work (due to the lack of their knowledge). Instead, at the beginning of each virtual mechanical test (VMT), the computational domain was assumed to be stationary and stress-free.

Boundary/Loading Conditions: Different types of velocity-based boundary/loading conditions are employed depending on the nature of the VMT. The VMTs employed in the present work are discussed in greater detail in Section 2.3.3. The details regarding the justification for the choice of the employed boundary/loading conditions used can be found in our prior work [22, 23].

Contact Interactions: As mentioned earlier, initial bonding across the matrix/coating interfaces is modeled by prescribing cohesive interfacial forces. However, once the interfacial cohesion is compromised/cracked, and two free surfaces are formed, subsequent interactions across the matrix/coating interfaces are described using the “Hard Contact Pair” algorithm. Within this algorithm, contact pressures between two bodies are not transmitted unless the nodes on the “slave (node-based) surface” contact the “master (element-based) surface.” No penetration/over-closure between the slave and master surfaces is allowed, and there is no limit to the magnitude of the contact pressure that can be transmitted when the surfaces are in contact. Transmission of shear stresses across the contact interfaces is assumed to be controlled by a modified Coulomb friction law. This law utilizes a static, $\mu_{st}$, and a kinetic, $\mu_{kin}$, friction coefficient and an upper-bound shear stress limit, $\tau_{slip}$ (a maximum value of shear stress which can be transmitted before shearing within the softer material rather than interfacial sliding begins to take place).
As far as the friction coefficient is concerned, it is generally assumed that this contact parameter is controlled by the roughness/asperity height of the contacting surfaces. Furthermore, it is recognized that the friction coefficient is a function of a number of factors such as the contact surface traction, contact surfaces’ roughness/topology, etc. To assign the appropriate value to the friction coefficient, functional relationships derived in our recent work [24, 25] were used.

**Material Models:** Based on the previous discussion, it is seen that the tow length-scale RVE contains three (SiC filament, BN coating, and SiC matrix) continuum materials and one (matrix/coating) cohesive-interface. Constitutive models for these materials are described in the remainder of this subsection.

**SiC-filament Material Model:** Elastically, SiC filaments are treated as transversely isotropic materials with a unique direction ($x=x_1$) along the filament axis. The five independent elastic constants used are as follows [13, 20]: $E_{11} = 370\, \text{GPa}$; $E_{22} = E_{33} = 200\, \text{GPa}$; $\nu_{12} = \nu_{13} = 0.25$; $\nu_{23} = 0.22$; and $G_{12} = G_{13} = 80\, \text{GPa}$; with $G_{23} = E_{22}/(2(1+\nu_{23}))$. Strength-wise, the same material is assumed to be brittle under axial tension. Since fibers (as well as the coating) mainly experience tensile loads, no other mode of filament failure is considered. The filament axial tensile strength is treated as a stochastic quantity, so that the probability, $P_f$, for a filament volume element $V_f$ to contain a crack-generating flaw at an applied axial stress $\sigma$, is defined by the two-parameter Weibull distribution as:

$$P_f = 1 - \exp\left[-\frac{V_f}{V_{0f}} \left(\frac{\sigma}{\sigma_{0f}}\right)^{m_f}\right]$$

(2.1)

where the three (two of which are independent) material parameters are given the following values [13]: $V_{0f} = 1\, \text{m}^3$, $m_f = 5$, $\sigma_{0f} = 6\, \text{MPa}$. The strength portion of the filament material
model is depicted graphically in Figure 2-5, using the standard log-log vs. log Weibull plot. When constructing Figure 2-5, \( V_f \) was set to the value corresponding to the volume of a single filament within the RVE.

Figure 2-5 Log-log Weibull axial-strength distribution plot for SiC-filament, BN-coating and SiC-matrix materials.

**BN-coating Material Model:** Since the BN coating is deposited using chemical vapor deposition (CVD), and possesses sub-micron sized equiaxed grains, it is considered elastically isotropic with the two elastic moduli \( E = 110 \text{ GPa} \) and \( \nu = 0.25 \). As far as the strength portion of the material model is concerned, it is also assumed to involve only axial tensile failure, and to be governed by
Eq. (1) with the following parameterization [26]: $V_{oc} = 1 \text{ m}^3$, $m_c = 4.95$, $\sigma_{oc} = 7.5 \text{ MPa}$. The strength portion of the BN-coating material model is also depicted graphically in Figure 2-5.

**SiC-matrix Material Model:** The same material model as that used for the BN coating is used for the SiC matrix, but with the following parameterization [20, 27]: $E = 350 \text{ GPa}$, $\nu = 0.17$, $V_{om} = 1 \text{ m}^3$, $m_c = 4.2$, $\sigma_{oc} = 5.7 \text{ MPa}$. The strength portion of the SiC-matrix material model is also depicted graphically in Figure 2-5. Matrix cracking (as well as fiber and coating cracking) are handled in the following way:

(a) under the given mode of loading, a failure probability of 0.5 was selected arbitrarily and Eq. (1) used to compute the corresponding fracture-strength level;

(b) once the applied stress has reached the fracture strength level within the given CMC component, the appropriate crack is created within the component in question;

(c) the largest fragment within the given component is identified and its volume is subsequently used within Eq. (1); and

(d) Eq. (1) is reapplied to determine the new increased fracture-strength level and the procedure is repeated beginning with step (b).

**Matrix/Coating Cohesive-Interface Material Model:** The initial cohesion across the SiC-matrix/BN-coating interfaces is described using a traction/separation constitutive law defined by the following components: (a) normal/tangential uncoupled elastic response – normal ($K_{nn} = 75 \text{ GPa}$) and two tangential ($K_{ss} = K_{tt} = 50 \text{ GPa}$) stiffness parameters$^{28}$; and (b) linear-elastic fracture-mechanics-based interfacial failure – defined using the three corresponding critical energy-release rates – $G_{nc} = 15 \text{ J/m}^2$ and $G_{sc} = G_{tc} = 10 \text{ J/m}^2$ [28].

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Size/Scale Effects: It should be noted that, due to use of Eq. (1) the strength properties of the filament, coating and matrix are all scale/size dependent. However, the properties of the CMCs at fiber/tow length-scale are found, in the present work, to be only weakly scale/size dependent. In other words, when the longitudinal dimension of the RVE was increased by a factor of 100% the mechanical response of the CMC under different VMTs was found to change by no more than 3% relative to its counterpart obtained for the case of the RVE described earlier in Section 2.3.1. This finding can be rationalized in the following way: in the case of monolithic ceramics the material displays the so-called “weakest link” behavior. That is, formation of the first crack leads to the complete failure of the material/component. In sharp contrast, matrix cracking is not catastrophic in the case of CMCs, since the interaction of these cracks with the weak matrix/coating interfaces results in their deflection. In addition, failure of the individual filaments is not catastrophic since the load carried by the broken filaments is redistributed to the remaining unbroken filaments.

Computational Tool: All the calculations carried out in the present work were done using ABAQUS/Explicit, a general purpose finite-element program [29]. Within this tool, the problem at hand (formulated in terms of a set of mass, momentum and energy conservation differential equations along with the material constitutive relations, initial, boundary and contact/interaction conditions) is solved numerically using the aforementioned finite-element algorithm. A typical transient non-linear dynamics analysis of a VMT required 4 hours of (wall-clock) time on a 12-core, 3.0 GHz machine with 12 GB of memory.

2.3.3 Derivation of the Homogenized Material Model

To derive the homogenized material model for the SiC/SiC CMCs at the fiber/tow length-scale, the following three-step procedure is used: (a) first, a set of predetermined VMTs is carried
out and the mechanical-response results analyzed to identify the defining features of this response; (b) next, by accounting for the key mechanical response features identified in (a), the homogenized material constitutive model is constructed as a set of governing equations (suitable for implementation into a user-material subroutine which can be linked with a commercial finite element program); and (c) the material constitutive model constructed in (b) is used in conjunction with the boundary/loading conditions of the VMTs within an optimization analysis in order to identify optimal values of the homogenized material-model parameters, which yield the best agreement with the VMT results.

**VMT Results:** As will be discussed in detail below, three uniaxial tensile stress and three pure shear VMTs are carried out. An example of the results obtained is depicted in Figures 2-6(a)-(b). The use of the results, displayed in these figures, in the construction of the fiber/tow scale CMC material model will be discussed in a later section, after details of the VMTs are introduced.
Figure 2-6 Stress-strain curves obtained from: (a) the three uniaxial tensile stress; and (b) the three pure shear VMTs.
Construction of the Homogenized Material Model: Since, in SiC/SiC CMCs at the fiber/tow length-scale, material elastic properties control the stress levels during loading while matrix-/fiber-cracking induced inelasticity controls material ductility, these two aspects of the homogenized material model will be discussed separately.

Elastic Properties: Elastically, the CMC at the fiber/tow length-scale can be considered as an (nearly transversely isotropic) orthotropic material. The elastic response of an orthotropic material is defined by the three Young’s moduli \((E_{11}, E_{22} \text{ and } E_{33})\), three Poisson’s ratios \((\nu_{12}, \nu_{13} \text{ and } \nu_{23})\) and three shear moduli \((G_{12} = G_{21}, G_{13} = G_{31} \text{ and } G_{23} = G_{32})\). To determine these elastic moduli, the RVE is subjected to the following VMTs:

(a) *Axial (x-direction) tension* – to conduct this test, constant axial velocity is applied to all the nodes on one \(yz\) end-face of the test specimen, a zero axial velocity is applied to all the nodes on the opposite end-face and no other boundary conditions are imposed on the test specimen (i.e. RVE) faces. This test yields, at an axial stress \(\sigma_1\), the three normal strains \(\varepsilon_1, \varepsilon_2\) and \(\varepsilon_3\) so that the following elastic moduli can be determined: \(E_{11} = \sigma_1/\varepsilon_1, \nu_{12} = -\varepsilon_2/\varepsilon_1\) and \(\nu_{13} = -\varepsilon_3/\varepsilon_1\). \(\sigma_1\) and \(\varepsilon_1\) are taken directly from the curve labeled “x-direction Tension” in Figure 2-6(a), while \(\varepsilon_2\) and \(\varepsilon_3\) yielded by the test are not shown for brevity.

When the procedure described above is applied to the virgin CMC test specimens, it yields the initial \(E_{11} = E_{11}^0, \nu_{12} = \nu_{12}^0 \text{ and } \nu_{13} = \nu_{13}^0\). However, once matrix cracking initiates, \(E_{11}\) begins to degrade and the extent of this degradation scales with the extent of matrix cracking. Since
matrix cracking results in an irreversible/residual strain $\varepsilon_{1,\text{irr}}$, $E_{11}$ becomes a function of $\varepsilon_{1,\text{irr}}$.

Such a function can be inferred by completely unloading the test specimen (to quantify $\varepsilon_{1,\text{irr}}$ and $E_{11}(\varepsilon_{1,\text{irr}})$), reloading the test specimen to resume matrix cracking and repeating this unloading/reloading procedure. As far as the Poisson’s ratios $\nu_{12}$ and $\nu_{13}$ (as well as the remaining Poisson’s ratios) are concerned, they are found not to be significantly affected by matrix cracking and, hence, this effect has been neglected in the remainder of this manuscript.

It should be noted that the elastic stiffness degradation process is of a coupled nature, i.e. damage produced in one deformation mode (i.e. VMT) can affect the characteristic stiffness modulus associated with another VMT. Specifically, $xy$- and $xz$-shears can produce cracks orthogonal to the $x$-direction, in the same way as $x$-tension does. Consequently, one expects that $E_{11}$ is a function of not only $\varepsilon_{1,\text{irr}}$, but also $\gamma_{12,\text{irr}}$ and $\gamma_{31,\text{irr}}$. Under the assumption that there are no interaction effects between different components of the irreversible strain, the effect of $\gamma_{12,\text{irr}}$ and $\gamma_{31,\text{irr}}$ can be determined by subjecting the test specimen to the shear in question (to cause matrix cracking), unloading the specimen (to determine the irreversible shear strain), and subjecting the specimen to tension in the $x$-direction (to determine the corresponding degraded $E_{11}$);

An example of the results obtained in this portion of the work is shown in Figures 2-7(a)-(d). In these figures, the status of material degradation/cracking over three matrix crack planes (labeled “MC Plane 1, 2, 3”) and over nineteen matrix/coating cylindrical interfaces (colored white) is depicted, as a function of the applied $x$-tension strain. Degraded/cracked regions are colored red, while the in-tact regions are shown in blue. Examination of the results displayed in
Figures 2-7(a)-(d) clearly reveals: (a) the stochastic nature of the matrix crack initiation; (b) progression of matrix cracking during subsequent loading; and (c) matrix/coating interfacial decohesion triggered by the arrival of the matrix cracks to the interface. As mentioned earlier, this phenomena is associated with matrix crack deflection and it is one of the main contributors to the improved tensile strength ductility and fracture toughness of the CMCs.

Figure 2-7 Temporal evolution and spatial distribution of material degradation/cracking, as a function of applied strain, for the x-direction tension VMT.
Figure 2-7 Continued.
(b) **Transverse (y-direction) tension** – to conduct this test, constant axial velocity is applied to all the nodes on one \(xz\) end-face of the test specimen, a zero axial velocity is applied to all the nodes on the opposite end-face and no other boundary conditions are imposed on the test specimen faces. In an analogous manner as the axial tension test, this test yields the following elastic moduli: \(E_{22}, \nu_{21}\) and \(\nu_{23}\).

By employing the loading/unloading/reloading procedures described above, the \(E_{22}(\varepsilon_{2,irr}, \gamma_{12,irr}, \gamma_{23,irr})\) relationship can be inferred;

(c) **Transverse (z-direction) tension** – to conduct this test, constant axial velocity is applied to all the nodes on one \(xy\) end-face of the test specimen, a zero axial velocity is applied to all the nodes on the opposite end-face and no other boundary conditions are imposed on the test specimen faces. This test yields: \(E_{33}, \nu_{31}\) and \(\nu_{32}\).

By employing the procedures described above, the \(E_{33}(\varepsilon_{3,irr}, \gamma_{31,irr}, \gamma_{23,irr})\) relationship can be determined;

(d) **(xy-plane) pure shear** – to conduct this test, positive \(x\)-displacements are prescribed on the \(x\)-min and \(x\)-max faces of the test specimen. The magnitude of the displacement is proportional to the \((y-y\text{-min})\) distance. Likewise, positive \(y\)-displacements are prescribed on the \(y\)-min and \(y\)-max faces of the test specimen. The magnitude of this displacement is proportional to the \((x-x\text{-min})\) distance. Lastly, \(z\)-min and \(z\)-max faces are constrained in the \(z\)-direction. This test yields, at a shear stress \(\tau_{12} = \sigma_4\), the engineering shear strain \(\gamma_{12} = 2\varepsilon_4\) (defined as the total change of angle between the \(x\)-min and \(y\)-min faces) so that the following elastic moduli can be determined:

\[
G_{12} = \frac{\tau_{12}}{\gamma_{12}}.
\]

\(\tau_{12}\) and \(\gamma_{12}\) are taken directly from the curve labeled “xy Pure Shear” in Figure 2-6(b). The procedure mentioned above can also be used here to evaluate the effect of \(xy\)-shear
induced matrix cracking, as characterized by the irreversible strain $\gamma_{12,\text{irr}}$, on $G_{12}$. Likewise, the coupling effects can be assessed to quantify $G_{12}$ dependence on $\varepsilon_{1,\text{irr}}$ and $\gamma_{12,\text{irr}}$:

(e) **(yz-plane) pure shear** – to conduct this test, positive $y$-displacements are prescribed on the $y$-min and $y$-max faces of the test specimen. The magnitude of the displacement is proportional to the $(z - z\text{-min})$ distance. Likewise, positive $z$-displacements are prescribed on the $z$-min and $z$-max faces of the test specimen. The magnitude of this displacement is proportional to the $(y - y\text{-min})$ distance. Lastly, $x$-min and $x$-max faces are constrained in the $x$-direction. In a similar manner to the $xy$-plane shear test, this test, associated with $\tau_{23} = \sigma_6$ and $\gamma_{23} = 2\varepsilon_6$, yields $G_{23}$.

The aforementioned procedure can similarly be used here to evaluate the effect of $yz$-shear induced matrix cracking, as characterized by the irreversible strain $\gamma_{23,\text{irr}}$, on $G_{23}$. Also, as mentioned above, the coupling effects can be assessed to quantify $G_{23}$ dependence on $\varepsilon_{2,\text{irr}}$ and $\gamma_{12,\text{irr}}$; and

(f) **(xz-plane) pure shear** – to conduct this test, positive $x$-displacements are prescribed on the $x$-min and $x$-max faces of the test specimen. The magnitude of the displacement is proportional to the $(z - z\text{-min})$ distance. Likewise, positive $z$-displacements are prescribed on the $z$-min and $z$-max faces of the test specimen. The magnitude of this displacement is proportional to the $(x - x\text{-min})$ distance. Lastly, $y$-min and $y$-max faces are constrained in the $y$-direction. This test, associated with $\tau_{13} = \sigma_5$ and $\gamma_{13} = 2\varepsilon_5$, yields $G_{31}$.

The above procedure can be used here to determine the effect of $xz$-shear induced matrix cracking, as characterized by the irreversible strain $\gamma_{31,\text{irr}}$, on $G_{31}$. Likewise, the coupling effects can be assessed to quantify $G_{31}$ dependence on $\varepsilon_{3,\text{irr}}$ and $\gamma_{23,\text{irr}}$. 

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Summary of the Results: A summary of the damage (i.e. irreversible strain) dependent material stiffness constants for SiC/SiC CMC at the fiber/tow length-scale, obtained using the aforementioned VMTs is given in Table 2-1. The expressions listed in Table 2-1 have been verified to ensure that they meet the restrictions associated with the requirement that the sum of work done by all stress components is positive and, hence, there is no unphysical energy generation. Specifically, it was confirmed that: (a) each $E_{ij}$ and $G_{ij}$ is positive; (b) the determinant of the elastic stiffness matrix is positive; and (c) $\nu_{ij} < \sqrt{E_{ii}/E_{jj}}$ $(i, j = 1 – 3)$.

Table 2-1 The damage (i.e. irreversible strain) dependent material stiffness constants for SiC/SiC CMC at the fiber/tow length-scale

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>GPa</td>
<td>$363 - 2400 \varepsilon_{1,irr} - 4200 \gamma_{12,irr} - 4600 \gamma_{31,irr}$</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>GPa</td>
<td>$261 - 2600 \varepsilon_{2,irr} - 3800 \gamma_{12,irr} - 3600 \gamma_{23,irr}$</td>
</tr>
<tr>
<td>$E_{33}$</td>
<td>GPa</td>
<td>$245 - 2700 \varepsilon_{3,irr} - 3700 \gamma_{23,irr} - 3700 \gamma_{31,irr}$</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>GPa</td>
<td>$82.3 - 850 \varepsilon_{1,irr} - 1600 \gamma_{12,irr} - 1100 \gamma_{31,irr}$</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>GPa</td>
<td>$84.5 - 770 \varepsilon_{3,irr} - 1200 \gamma_{23,irr} - 1800 \gamma_{31,irr}$</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>GPa</td>
<td>$89.6 - 730 \varepsilon_{2,irr} - 1900 \gamma_{12,irr} - 2300 \gamma_{23,irr}$</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
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</tr>
<tr>
<td>$\nu_{13}$</td>
<td>N/A</td>
<td>0.255</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>N/A</td>
<td>0.225</td>
</tr>
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</table>
**Inelastic Response:** In order to fully describe the mechanical response of the SiC/SiC CMC at the fiber/tow length-scale, both the elastic (dominated by matrix cracking) and inelastic portions of this response must be defined. The elastic portion of the response directly controls the stress state within the material, while the inelastic portion directly controls the evolution of the inelastic-strain components and, via the damage-dependency of the elastic stiffness constants, indirectly affects the stress state. The overall mechanical response requires specification of the following four functional relationships: (a) stress vs. elastic strain; (b) inelastic-deformation onset/continuation criterion; (c) inelastic-deformation flow rule; and (d) a constitutive law relating material strength and the overall extent of inelastic deformation. These four functional relationships will be defined, and their parameterization discussed, in the remainder of this subsection.

(a) **Stress vs. Elastic Strain:** This relationship is defined by using the generalized Hooke’s law in the form:

$$\{\sigma\} = [C(\{\varepsilon_{irr}\})] \{\varepsilon_{el}\} = [C(\{\varepsilon_{irr}\})] \{\varepsilon_{el}^0\} + \{\Delta\varepsilon_{el}\} = [C(\{\varepsilon_{irr}\})] \{\varepsilon_{el}^0\} + \{\Delta\varepsilon_{el}\}$$

\[ (2.2) \]

where symbols {…} and [... ] are used to denote a 6x1 vector and 6x6 matrix, respectively, \{\sigma\} and \{\varepsilon\} are respectively stress and strain vectors, \([C]\) is elastic stiffness matrix, \(\Delta\) designates an increment in a dependent variable over a time increment of duration \(\Delta t\), superscript \(0\) defines the state of a quantity at the end of the previous time increment and subscripts \(el\) and \(irr\) denote an elastic and an irreversible/inelastic quantity respectively. Since \([C(\{\varepsilon_{irr}\})]\) are known functions, Table 2-1, then Eq. (2) represents a set of six equations with twelve (six \{\sigma\} and six \{\Delta\varepsilon_{irr}\}) unknowns.
(b) **Inelastic-Deformation Onset/Continuation Criterion:** To account for the fact that only tensile normal and shear stresses can cause matrix/coated-tow cracking, and that the material inelastic behavior is of an orthotropic character, this criterion is defined by the following function:

\[ f = \left( H(\sigma_{11})m_{11}^2 \sigma_{11}^2 + H(\sigma_{22})m_{22}^2 \sigma_{22}^2 + H(\sigma_{33})m_{33}^2 \sigma_{33}^2 + m_{12}^2 \sigma_{12}^2 + m_{13}^2 \sigma_{13}^2 + m_{23}^2 \sigma_{23}^2 \right)^{1/2} \geq 1 \]  \hspace{1cm} (2.3)

where \( H(\ldots) \) denotes the Heaviside function, \( m_{ij}^2 = \frac{1}{S_{ij}(\bar{\varepsilon}_{irr})^2} \) \((i,j = 1-3)\), \( S_{ij}(\bar{\varepsilon}_{irr}) \) is the material “yield” (i.e. first matrix cracking) strength associated with pure \( \sigma_{ij} \) loading and \( \bar{\varepsilon}_{irr} \) is the equivalent irreversible strain, defined as

\[ \bar{\varepsilon}_{irr} = (n_{11}^2 \varepsilon_{11,irr}^2 + n_{22}^2 \varepsilon_{22,irr}^2 + n_{33}^2 \varepsilon_{33,irr}^2 + n_{12}^2 \varepsilon_{12,irr}^2 + n_{13}^2 \varepsilon_{13,irr}^2 + n_{23}^2 \varepsilon_{23,irr}^2)^{1/2} \]  \hspace{1cm} (2.4)

where parameters \( n_{ij}^2 \) \((i,j = 1-3)\) are obtained using the non-linear regression analysis and the VMT results displayed in Figures 2-6(a)-(b). This was necessary since, in contrast to the conventional metal plasticity, matrix-cracking dominated inelastic deformation is associated with a positive volume change and the inelastic Poisson’s effects are relatively weak.

(c) **Flow Rule:** Following the common practice the associated flow rule, which equates the flow potential (controls the direction of the irreversible deformation) to \( f \) given by Eq. (3) (controls the onset and continuation of the irreversible flow), is defined as:

\[ \{ \Delta \varepsilon_{irr} \} = \{ \dot{\varepsilon}_{irr} \} \Delta t = \dot{\lambda}_{irr} \frac{\partial f}{\partial \sigma} \Delta t \]  \hspace{1cm} (2.5)

where \( \dot{\lambda}_{irr} \) is a scalar-type irreversible deformation rate parameter and a raised dot is used to denote the time derivative of a quantity.

It should be noted that Eq. (5) represents a set of six equations with only one additional unknown, \( \dot{\lambda}_{irr} \).
(d) **Constitutive Law:** By recognizing the fact that additional matrix cracking, and ultimately filament and coating cracking, requires the activation of flaws of continually decreasing potency (a strain-hardening effect), the following functional relationship is chosen for the constitutive law:

\[ S_y(\varepsilon_{irr}) = S_y(\varepsilon_{irr} = 0) + \alpha_y \varepsilon_{irr}^{\beta_y} \]

(2.6)

where \( \alpha_{ij} \) and \( \beta_{ij} \) are material parameters obtained using the same aforementioned nonlinear regression analysis and the VMT data. Eqs. (3) and (6) together define the missing functional relationship needed to evaluate the thirteen unknowns, i.e. six components of \( \{\sigma\} \), six components of \( \{\Delta\varepsilon_{irr}\} \) and \( \dot{\lambda}_{irr} \).

**Fracture Criterion:** By analyzing the VMT results, it was established that, to a first-order approximation, the fracture criterion can be defined by the condition when the sum of squares of the ratios of the maximum irreversible strain components and their critical (i.e. fracture) counterparts (extracted from Figures 2-6(a)–(b)) becomes equal to or exceeds 1.0.

**2.3.4 Homogenized Material Model Optimization and Parameterization**

The homogenized material model as defined by Eqs. (2.2)-(2.6) is implemented as a VUMAT user-material subroutine and linked with ABAQUS/Explicit. The homogenized material model is next combined with the boundary/loading conditions of the six VMTs and used within a single cubic element FEA to produce the corresponding stress-strain curves. However, before this could be done, values for the 18 remaining unknown material parameters (i.e. six \( n_{ij}^2 \)’s, six \( \alpha_{ij} \)’s and six \( \beta_{ij} \)’s) had to be determined, using the aforementioned nonlinear regression analysis. Towards that end, ABAQUS/Explicit simulations of the six VMTs based on
the homogenized material model are integrated with an optimization routine (constructed within MATLAB, a general-purpose mathematical package [30]). This enabled the identification of optimal values of the aforementioned 18 material parameters which minimize, in the least-squares sense, the differences between the mechanical response of the fiber/tow length-scale RVE and the corresponding homogenized material-model one-element FEA results, under identical VMT boundary/loading conditions. The optimum value of the two material parameters \( n_{ij}^2, \alpha_{ij} \) and \( \beta_{ij} \) as well as the six values of the initial material strengths, \( S_y(\varepsilon_{irr} = 0) \), are given in Table 2-2.

<table>
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<tr>
<th>Parameter</th>
<th>( i j )</th>
<th>( n_{ij}^2 )</th>
<th>( S_y(\varepsilon_{irr} = 0) ) (MPa)</th>
<th>( \alpha_{ij} ) (MPa)</th>
<th>( \beta_{ij} )</th>
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<td>1.96</td>
<td>50.0</td>
<td>442.7</td>
<td>0.53</td>
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<td>2 3</td>
<td>2.04</td>
<td>51.2</td>
<td>797.3</td>
<td>0.61</td>
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</table>
2.4 PLY/Lamina Length-Scale Material-Model Development

The problem analyzed in this portion of the present work deals with the development of a (homogenized) ply/lamina length-scale material model for the SiC/SiC CMCs with cross-ply and plain-weave architectures. As in the previous case, the procedure used involves: (a) the construction of the corresponding ply/lamina length-scale RVE; (b) a series of VMTs on the RVE; and (c) post-processing of the mechanical test data in order to assemble the corresponding homogenized material constitutive model. In the remainder of this section, the necessary details are provided regarding the aforementioned three-step procedure, as well as of the key results yielded by the current approach. Since some of the details are effectively analogous to their counterparts described in the previous section, they will not be covered.

2.4.1 Construction of the PLY/Lamina Length-Scale CMC RVE

The two RVEs, each used to represent one of the analyzed (cross-ply and plain-weave) CMC architectures, are shown in Figures 2-8(a)–(b), respectively. It should be noted that the ply architecture used within the RVEs differs from the one shown previously in Figure 2-2(b). The reason for this is that the “enriched tows” (i.e. the fiber/tow scale RVEs) used within the ply/lamina scale RVEs contain coated filaments, inter-filament matrix and inter-tow matrix and, hence, must completely fill the RVE volume (i.e. no “filament-free” matrix phase or CMC-matrix pores are assumed to exist in the present case). Due to these constraints, the architecture of the cross-ply CMC RVE, Figure 2-8(a), is quite simplified while that of the plain-weave CMC RVE, Figure 2-8(b), had to be made somewhat more complicated. It should be also noted that the cross-ply RVE, Figure 2-8(a), contains one 0° ply and one 90° ply, while the plain-weave RVE, Figure 2-8(b), contains two plain-weave plies (and is, hence, twice as thick as the cross-ply RVE).
Since matrix and filament fracture and interphase debonding are all accounted for by the tow length-scale material model, individual “enriched tows” within the same ply of the ply-scale RVE shown in Figures 2-8(a)–(b) are assumed to be perfectly bonded. On the other hand, a provision is made for inter-ply separation/delamination by treating ply/plly interfaces not as being perfectly bonded, but as being capable of undergoing normal/tangential-type decohesion. Potential delamination surfaces are labeled in Figures 2-8(a)-(b).

![Figure 2-8 Ply/lamina RVEs for the SiC/SiC CMCs with: (a) cross-ply; and (b) plain-weave architectures.](image-url)
2.4.2 Virtual Mechanical Tests on the CMC RVE

The same set of finite-element-based VMTs is employed as in the previous section. As far as the finite-element procedure employed is concerned, it had the same steps (a)–(h). Since most of these steps are identical to the ones presented in the previous section, only the ones which had to be modified/augmented are discussed in the remainder of this subsection.

Meshed Model: $0^\circ/90^\circ$ enriched tows in the cross-ply case, Figure 2-8(a), and warp/weft enriched tows in the plain-weave case, Figures 2-8(b) are all meshed using eight-node hexahedron and six-node triangular-prism first-order, continuum finite elements with a characteristic length of 0.1 mm. Typically, the cross-ply and plain-weave RVEs contain ca. 30,000 and 80,000 finite elements, respectively.

Material Models: Each enriched tow is assigned the material model derived in the previous section. However, since the material in question is orthotropic, the local material coordinate system had to be defined for each finite element before the material model is assigned. Inter-ply delamination has been modeled using the same type (with different parameterization) traction/separation constitutive law as that used for modeling matrix/coating decohesion. The following parameterization is used here: (a) stiffness parameters – $K_{nn} = 110$ GPa and $K_{ss} = K_{tt} = 70$ GPa; and (b) energy-release rates – $G_{tn} = 21$ J/m$^2$ and $G_{tc} = G_{sc} = 14$ J/m$^2$.

2.4.3 Derivation of the Homogenized Material Model

To derive the homogenized material model for the SiC/SiC CMCs at the ply/lamina length-scale, the same three-step procedure used in Section 2.3 is employed here.

Analysis of the VMT Results: The same six VMTs described in Section 2.3.3 are used here. The results obtained (not shown for brevity) are again analyzed to identify the defining features of the
material’s response at the current length-scale. These results clearly demonstrated that the material is effectively transversely isotropic with a unique through-the-thickness direction.

Construction of the Homogenized Material Model: Since the ply/lamina architecture is expected only to affect, but not completely alter, the materials’ response relative to that at the fiber/tow length-scale, the same homogenized material model as that developed in Section 2.3 (but with different parameterization and material symmetry) was used here. The new parameterization has been obtained by subjecting the RVEs displayed in Figures 2-8(a)-(b) to the same set of VMTs and by applying the same post-processing/optimization procedure as that described in Section 2.3. Due to space limitations, the results of this procedure will only be briefly reviewed in the remainder of this section. A comprehensive description of the ply/lamina-scale material model derivation through the use of the VMTs will be provided in a future communication.

Cross-ply Architecture: As in the fiber/tow length-scale case, elastic and inelastic responses of the material are considered separately.

Elastic Properties: The same procedure employed in Section 2.3.3 yielded five independent transversely-isotropic material elastic constants: $E_{11} = E_{22}$, $E_{33}$, $\nu_{12} = \nu_{21}$, $\nu_{13} = \nu_{23}$, and $G_{13} = G_{23}$. The effect of the irreversible-strain components on these constants was determined using the same procedure as that described in Section 2.3.3.

Inelastic Properties: As mentioned above, the inelastic-deformation onset criterion, flow rule and constitutive relations used in Section 2.3.3 are utilized here, but with recognition of the transverse isotropy of the material at this length scale. An example of this recognition is that the material possesses two (not three) unique normal, $S_{11} = S_{22}$ and $S_{33}$, and two (not three) unique shear, $S_{13} = S_{23}$ and $S_{12}$, strengths. Consequently, the number of the \((n_{ij}^2, \alpha_{ij} \text{ and } \beta_{ij})\) model...
parameters which had to be determined using the aforementioned optimization procedure decreased from 18 to 12. As far as the fracture criterion is concerned, the same type was used here (and for the plain-weave architecture) as in the case of the fiber/tow length-scale. The critical (fracture) irreversible strain components at the ply/lamina length-scale were again obtained from the VMT results analogous to those displayed in Figures 2-6(a)–(b).

Plain-weave Architecture:

Elastic Properties: The same five elastic constants as in the cross-ply case, and the effect of the irreversible-strain components on these constants for this architecture, have been determined. The results showed minor through-the-thickness stiffening due to the out-of-plane weaving of the enriched tows.

Inelastic Properties: The same optimization procedure is employed here to determine 12 ($n_{ij}^2$, $\alpha_{ij}$ and $\beta_{ij}$) material model parameters. Examination of the results obtained revealed noticeable through-the-thickness strengthening which could be attributed to the smaller number of ply/ply interfaces per unit thickness of the lamina in the case of the plain-weave architecture.

Material Model Implementation: The homogenized material model for both architectures of the SiC/SiC CMC at the ply/lamina length-scale is also implemented into the VUMAT subroutine and linked with ABAQUS/Explicit.

2.4.4 Prototypical Results

As mentioned earlier, a detailed account of the procedures used and the results obtained at the ply/lamina scale will be reported in our future communication. In this subsection, an example of the results is shown to showcase the utility of the material model within the material laboratory-testing environment.
Temporal evolution of the von Mises stress in one quarter of the plain-weave ply-scale RVE, subjected to a double cantilever bending test, is depicted in Figures 2-9(a)-(d). The results displayed clearly reveal progressive ply/ply interfacial decohesion (starting at the face of the RVE opposite to the one being clamped). In addition, the effect of the plain-weave ply architecture on the spatial distribution of the von Mises stress within the two plies displayed is apparent.

Figure 2-9 Temporal evolution of the von Mises stress in one quarter of the plain-weave ply/lamina length-scale RVE, subjected to a double cantilever bending test.
2.4.4 Prototypical Results

In this subsection, an attempt is made to validate the present ply/lamina material model by comparing its predictions for an in-plane tensile test against the corresponding experimental results reported in Ref [13]. The stress-strain curve obtained in Ref. [13] are depicted in Figure 2-10, the curve labeled “Corman and Luthra (2005)”. These results are obtained under the following conditions:
(a) the CMC contained approximately 32 vol. % of Hi-Nicalon multi-filament fibers, to which a 1 micron thick 20% Si-doped BN coating and a 0.25 micron thick C coating was applied;
(b) the CMC contained a 0°/90° cross-plied architecture; and
(c) tensile testing was done at room temperature with the load applied in one of the in-plane fiber directions.

Also depicted in Figure 2-10 is the stress-strain curve yielded by the corresponding virtual mechanical tensile test, the curve labeled “Present model”. A comparison between the two sets of results depicted in Figure 2-10 shows an acceptable agreement, suggesting that the present ply/lamina SiC/SiC CMC material model can reasonably well account for the observed mechanical response of this material.

Figure 2-10 Validation of the ply/lamina material model developed in the present work using the model predictions for the virtual mechanical in-plane tensile test and comparing them with the corresponding experimental results reported in Ref. [13].
2.5. Foreign Object Damage (FOD) Of CMC Turbine Shrouds

To further demonstrate the full utility of the newly developed ply/lamina length-scale CMC material model (referred to hereafter as the component-level material model), the material model developed in Section 2.4 is used in the current section to analyze, using a transient non-linear dynamics finite-element analysis, the structural/damage response of a prototypical gas-turbine engine component, a turbine shroud, to impact by a foreign object [31-37]. Such an analysis involves the same finite-element steps (a)–(h) identified in Section 2.3. Since most of these steps are identical to the ones presented in Section 2.3, only the ones which had to be modified/augmented are discussed in the following subsection.

2.5.1 Finite Element Analyses of the FOD

Geometrical Model: The computational model employed in this portion of the present work comprises a 1.59 mm-diameter solid-spherical projectile (made of AISI H13 steel) and a 15° toboggan-shaped turbine-shroud CMC segment, Figure 2-11. The remaining geometrical details of the shroud segment are marked in this figure.

![Figure 2-11 Schematic of the component-level model containing a 1.59 mm-diameter solid-spherical projectile and a 15° toboggan-shaped turbine-shroud CMC segment.](image)
Meshed Model: The shroud-segment is meshed using (100,000) eight-node, first-order, continuum finite elements with a characteristic edge length of 0.33 mm. As far as the solid spherical projectile is concerned, it was discretized using (typically ca. 6,000) four-node tetrahedron continuum elements, with a characteristic edge length of 0.25 mm. The Cartesian coordinate system employed in the present analysis is shown in Figure 2-11. It is seen that \( x \) is the axial direction of the turbine shroud, \( y \) runs circumferentially while \( z \) is oriented generally in the shroud radial direction.

Initial Conditions: At the beginning of an analysis, both the impactor and the shroud-segment are assumed to be stress free. In addition, the shroud-segment is assumed to be stationary/quiescent while the impactor is assumed to be moving in the \( x-z \) plane at an angle of 45° with respect to the \( x \)-axis, with a total velocity of 600 m/s.

Boundary Conditions: The bottom-most portions of the toboggan-shaped shroud-segment, which are normally connected to the turbine casing, are subjected to the Encastre boundary conditions.

Material Models: As mentioned earlier, the material model developed in Section 2.4 is assigned to each finite element of the shroud-segment after the local material coordinate system is defined.

2.5.2 FEA Results and Discussion

The unique feature of the component-level material model used in the FEA of the FOD of the toboggan-shaped single segment of the turbine shroud is that its formulation and parameterization are affected by: (a) fiber/tow material microstructural parameters such as the volume fraction of the filaments, thickness of the filament coatings, decohesion properties of the coating/matrix interfaces, quality (as quantified by the Weibull distribution parameters) of the filament, coating, and matrix material, etc.; and (b) ply/lamina material architectural and structural parameters, such as 0°/90° cross-ply vs. plain-weave architectures, the extent of tow
crimping in the case of the plain-weave plies, cohesive properties of the inter-ply boundaries, etc. Thus, by specifying, as input parameters, the items listed in (a) and (b), a customized parameterization of the component-level material model can be obtained. In the present section, a couple of FEAs are carried out to demonstrate the effect of several fiber/tow and ply/lamina length-scale parameters on the structural/damage response of the shroud-segment to the given incident-velocity foreign-object impact conditions.

Case A—Cross-ply Architecture/Reference: In this case, the fiber/tow and ply/lamina microstructural/architectural material parameters are kept at their values determined in Sections 2.3 and 2.4 and, hence, the component-level material model parameters are set equal to the values derived for the cross-ply architecture in Section 2.4.

Temporal evolution and spatial distribution of the shroud segment SiC/SiC CMC and of the AISI H13 steel impactor materials during the segment/impactor interactions is depicted in Figures 2-12(a)-(c). It should be noted that, for improved clarity, the CMC shroud component is made transparent in these, as well as in the subsequent figures. The results depicted in these figures will be used as the baseline with which the subsequent results will be compared to reveal the effect of fiber/tow and ply/lamina length-scale parameters on the structural/damage response of the shroud-segment to the constant incident-velocity foreign-object impact conditions.
Figure 2-12 Temporal evolution and spatial distribution of the shroud segment and the impactor materials during the segment/impactor interactions for the case of the cross-ply architecture.
**Case B – Plain-weave Architecture/Reference:** In this case, the fiber/tow and ply/lamina microstructural/architectural material parameters are also kept at their values determined in Sections 2.3 and 2.4 and, hence, the component-level material model parameters are again set equal to the values found for the plain-weave architecture in Section 2.4.

Temporal evolution and spatial distribution of the shroud-segment and of the impactor materials during the segment/impactor interactions is depicted in Figures 2-13(a)-(c). It should be noted that the three sets of results displayed in Figures 2-12(a)-(c) and those displayed in Figures 2-13(a)-(c) (as well as in the remaining figures) are generated at the same post-impact times. A comparison of the results displayed in Figures 2-13(a)-(c) with those in Figures 2-12(a)-(c) reveals that in the case of the plain-weave architecture the extent of shroud-component damage is somewhat lower and so is the residual velocity of the projectile, than in the cross-ply case.
Figure 2-13 Temporal evolution and spatial distribution of the shroud segment, containing matrix at the reference strength, and the impactor materials during the segment/impactor interactions for the case of the plain-weave architecture.
Case C – Plain-weave Architecture/Defective Matrix: Except for the matrix strength, Case C is identical to Case B. The matrix, in Case C, is assumed to contain a larger concentration of processing-induced flaws and, hence, inferior strength (achieved by properly selecting the Weibull probability parameters for the matrix).

Temporal evolution and spatial distribution of the shroud-segment and of the impactor materials during the segment/impactor interactions, for Case C, is depicted in Figures 2-14(a)-(c). A comparison of the results displayed in Figures 2-14(a)-(c) with those in Figures 2-13(a)-(c) reveals that a decrease in the as-fabricated quality (i.e. an increase in the extent of processing induced flaws) can significantly increase both the extent of damage experienced by the CMC shroud component and the impactor exit velocity.
Figure 2-14 Temporal evolution and spatial distribution of the shroud segment, containing matrix of inferior strength, and the impactor materials during the segment/impactor interactions for the case of the plain-weave architecture.
Case D – Plain-weave Architecture/ Stronger Matrix/Coating Cohesion: Except for the strength of the matrix/coating cohesion, Case D is identical to Case B. The matrix/coating cohesion strength, in Case D, is increased by 100% to impede the matrix/crack deflection at the matrix/coating interface.

Temporal evolution and spatial distribution of the shroud-segment and of the impactor materials during the segment/impactor interactions, for Case D, is depicted in Figures 2-15(a)-(c). A comparison of the results displayed in Figures 2-15(a)-(c) with those in Figures 2-13(a)-(c) reveals that an increase in the matrix/coating cohesion strength promotes localization of the damage within the CMC shroud component. In other words, the extent of damage spreading due to crack deflection along the matrix/coating interfaces is limited. As a consequence, the CMC material behaves more as a brittle monolithic ceramic, than as a toughened CMC, as evidenced by the highest residual velocity of the impactor, Figure 2-15(c).
Figure 2-15 Temporal evolution and spatial distribution of the shroud segment, with increased matrix/coating cohesion strength, and the impactor materials during the segment/impactor interactions for the case of the plain-weave architecture.
2.6 Summary And Conclusions

Based on the results obtained in the present work, the following main summary remarks and conclusions can be drawn:

1. A fully homogenized gas-turbine engine component-level room-temperature material model for SiC/SiC ceramic matrix composites (CMCs) has been derived using a multi-length scale approach.

2. The model enables incorporation of the fiber/tow (e.g. the volume fraction of the filaments, thickness of the filament coatings, decohesion properties of the coating/matrix interfaces, quality (as quantified by the Weibull distribution parameters) of the filament, coating, and matrix materials, etc.) and ply/lamina (e.g. the 0°/90° cross-ply vs. plain-weave architectures, the extent of tow crimping in the case of the plain-weave plies, cohesive properties of the inter-ply boundaries, etc.) length-scale microstructural/architectural parameters on the mechanical response of the CMCs.

3. The model is integrated into a user-material subroutine and linked with a commercial finite element program in order to investigate the structural/damage response of a prototypical turbine-engine component (turbine shroud) to impact by a foreign object (ingested and accelerated by the engine).

4. The finite element results obtained clearly revealed the role different fiber/tow and ply/lamina length-scale microstructural and architectural parameters play in the structural survivability of the turbine engine components.
2.7 References


CHAPTER 3: MULTI-LENGTH-SCALE MATERIAL MODEL FOR SiC/SiC CERAMIC-MATRIX COMPOSITES (CMCs): INCLUSION OF IN-SERVICE ENVIRONMENTAL EFFECTS

3.1 Abstract

In our recent work, a multi-length-scale room-temperature material model for SiC/SiC ceramic-matrix composites (CMCs) was derived and parameterized. The model was subsequently linked with a finite-element solver so that it could be used in a general room-temperature, structural/damage analysis of gas-turbine engine CMC components. Due to its multi-length-scale character, the material model enabled inclusion of the effects of fiber/tow (e.g. the volume fraction, size and properties of the fibers; fiber-coating material/thickness; decohesion properties of the coating/matrix interfaces; etc.) and ply/lamina (e.g. the 0°/90° cross-ply vs. plain-weave architectures, the extent of tow crimping in the case of the plain-weave plies, cohesive properties of the inter-ply boundaries, etc.) length-scale microstructural/architectural parameters on the mechanical response of the CMCs. One of the major limitations of the model is that it applies to the CMCs in their as-fabricated conditions (i.e. the effect of prolonged in-service environmental exposure and the associated material ageing-degradation is not accounted for). In the present work, the model is upgraded to include such in-service environmental-exposure effects. To demonstrate the utility of the upgraded material model, it is used within a finite-element structural/failure analysis involving impact of a toboggan-shaped turbine shroud segment by a foreign object. The results obtained clearly revealed the effects that different aspects of the in-service environmental exposure have on the material degradation and the extent of damage suffered by the impacted CMC toboggan-shaped shroud-segment.
3.2 Introduction

The present work addresses the problem of inclusion of the in-service environmental effects on the room-temperature mechanical response (as predicted by the appropriate material constitutive model) of ceramic-matrix composites (CMCs) used in gas-turbine hot-section component application. Consequently, the concepts most relevant to the present work are: (a) the basics of ceramic-matrix composites; (b) room-temperature mechanical properties of CMCs; and (c) degradation of CMCs mechanical properties due to their exposure to the gas-turbine engine in-service environment. In the remainder of this section, a brief description is provided for each of these concepts. Since the inclusion of the environmental effects will be made into our recently developed multi-length-scale CMC material model [1], this model will be briefly overviewed in Section 3.3.

3.2.1 The Basics of Ceramic-Matrix Composites

High-temperature metallic materials such as nickel-, cobalt- or iron-based superalloys used in gas-turbine engines have been pushed to their thermal-stability limit since they are often made to operate at temperatures which are within 50 K of their melting point. To increase power density and energy efficiency of the gas-turbine engines, new materials are needed which can operate at temperatures as high as 1400 K. The main candidate materials currently identified for use in the next generation of gas-turbine engines are (monolithic) ceramics and CMCs. Since these materials can withstand extremely high temperatures, their use in hot sections of gas-turbine engines can yield a number of benefits such as: (i) improvements in thrust and fuel efficiency; (ii) lower pollutant emissions; (iii) reduced cooling requirements; (iv) simplification of the engine-component design; and (v) reduced requirements for the strength/weight of the supporting structure.
However, due to their relatively low fracture toughness, tensile strength and damage tolerance, monolithic ceramics are not being perceived as respectable candidate materials for use in critical turbine-engine structural applications (e.g. turbine blades). On the other hand, CMCs consisting of a ceramic matrix and ceramic fibers possess superior structural properties relative to their monolithic-ceramic counterparts, while retaining their high-temperature stability and integrity. This is the reason that the CMCs are being aggressively researched and developed for use in future gas-turbine engines. The potential of the CMCs in revolutionizing the performance of the gas-turbine engines is shown schematically in Figure 3-1 [2]. In this figure, the x-axis represents the approximate period of dominance in usage of the particular class of high-temperature materials (and associated cooling technologies), while the y-axis denotes the temperature capability of the material class in question. It is seen that the temperature capability of CMCs lies above the fitting line for the temperature capabilities of the past and present gas-turbine engine materials.

![Figure 3-1 Temperature capability (i.e. maximum service temperature) of various gas-turbine engine materials as a function of the time period of their dominant usage.](image-url)
Presently, CMCs are being perceived as candidate materials in gas-turbine engine components which experience low in-service loads such as nozzles, combustion liners, airfoils and exhaust components. However, it is hoped that with future improvements in the processing and performance of CMCs, these materials could be used in the manufacture of more critical gas-turbine engine components such as turbine blades, which experience higher in-service loads. Clearly, this will require major advances in the CMC synthesis/manufacture processes to ensure high quality/performance, high yield and reduced cost of these materials.

3.2.2 Room-Temperature Mechanical Properties of CMCs

In general, one is concerned with the mechanical properties of CMCs when subjected, over a long period of time, to multi-axial/cyclic/sustained loading under a high-temperature, oxidizing, high water-vapor-content environment. However, mechanical tests involving such complex thermo-mechanical and environmental test conditions are quite expensive and time-consuming. Consequently, after exposure to the simulated gas-turbine in-service thermo-mechanical and environmental conditions, CMC materials are often tested at room temperature (the testing procedure which is significantly less expensive and faster). While this approach cannot fully reveal the mechanical response of the CMCs under in-service loading conditions, it can provide a good insight into the extent of material-property degradation due to exposure of the material to such conditions.

The room-temperature mechanical behavior of laminated (cross-ply or plain-weave) CMCs is exemplified by the in-plane uniaxial-tension stress-strain curve shown in Figure 3-2. Careful examination of the stress-strain curve shown in this figure reveals the presence of four distinct regions (labeled I–IV), each associated with the characteristic mechanical response of the loaded material. The four regions of the stress-strain curve can be explained as follows:
Figure 3-2 Prototypical in-plane uniaxial-tension stress-strain curve, exemplifying the room-temperature mechanical behavior of laminated (cross-ply or plain-weave) CMCs.

(i) Region I is characterized by the linear elastic response of the (virgin) composite material. This region extends up to the stress level commonly referred to either as first matrix-cracking stress or proportional-limit stress, and does not induce any noticeable damage into the material;

(ii) In region II, the material deformation involves an inelastic component resulting from the continuing matrix cracking. Concomitantly, fibers bridging the matrix cracks are shear-debonded from the matrix. These damage processes are accompanied by a reduction in the material stiffness and a gradual increase in the material strength (as flaws of progressively lower potency have to be activated to produce additional matrix cracks);

(iii) In region III, continuation of the matrix cracking would require the activation of very weak flaws while the extent of load transfer from the fibers to the fractured-matrix fragments is
severely decreased. Consequently, matrix-crack density begins to saturate while the material response to the applied loading begins to include fiber cracking. These microstructural-evolution processes are accompanied by further material-stiffness degradation and by “strain hardening” (the rate of which is higher than that in region II); and (iv) As the loading proceeds, progressively more fibers are broken and although the remaining unbroken fibers are stronger, the number of unbroken fibers decreases (causing the stress within these fibers to increase). Consequently, as loading continues, the material passes through the state of critical stability (corresponding to a critical fraction of the broken fibers) beyond which the increase in the average fiber strength is exceeded by an increase in the average stress experienced by the unbroken fibers (Region IV). As a result, the material begins to fail. Since at this point some of the fibers may still be bonded to the matrix, their sliding and the final pullout is associated with an additional strain and, consequently, the material fails gradually rather than abruptly.

3.2.3 Degradation of CMC Mechanical Properties in Gas-Turbine Engine Environment

CMC gas-turbine engine components, once installed, are expected to last for a few years while retaining their structural integrity and functionality. This is generally quite challenging for hot-section engine components, which are exposed to highly aggressive thermal-mechanical and environmental in-service conditions, the conditions which may severely degrade the load-bearing capacity and environmental resistance of the material.

While in-service material degradation is the result of a very complex interaction between thermo-mechanical and environmental effects on the one hand, and the material/component on the other, it is customary to identify and analyze key basic “pure” modes of material degradation. These modes include:
(a) prolonged high-temperature exposure in the absence of significant mechanical and aggressive-environmental effects – Mere prolonged exposure of CMCs to high temperatures can cause (primarily mechanical) property-degrading microstructural changes such as grain growth (e.g. [3]), porosity development (e.g. [3]), CMC-constituent interfacial reactions (e.g. [4]), etc.;

(b) prolonged exposure to thermo-mechanical effects in the absence of aggressive environment – The combination of high temperature and sustained loading gives rise to creep, a deformation process which can severely compromise the mechanical properties of the CMCs. This compromise is caused by the fact that the CMC constituents possess different creep tendencies, and in order for them to acquire a common creep rate, undesirable load transfer typically occurs between fibers and the matrix. As a result, the first matrix-cracking stress (a measure of the CMC strength) is lowered [5]. In addition, if the reduction in the CMC strength is large enough to cause material cracking, stiffness degradation ensues [6]. Also, excessive creep deformation can compromise dimensional tolerances of the components and result in the development of increased mounting and contact stresses with the adjoining components, excessive friction and wear in the case of moving components, etc. [7];

(c) prolonged exposure to high-temperature and dry-oxidizing environment, in the absence of sustained loading – In a gas-turbine hot-section, the combustion products typically consist of burnt and unburnt hydrocarbons, oxygen and water vapor. Here, only the effect of oxygen on the degradation of material properties is considered. Once matrix cracking has occurred, oxygen present in the gas-turbine engine can enter the CMC component, and attack and oxidize its interior constituents (like coatings and fibers). The resulting oxides act as potent flaws within the CMC constituents, causing a substantial reduction in the strength of these relatively brittle
materials. While oxidation also degrades the constituent stiffness, these effects are relatively small compared to the ones concerning the strength [8, 9];

(d) prolonged exposure to high-temperature and wet-oxidizing environment, in the absence of sustained loading – The presence of water vapor leads to wet-oxidizing conditions, which may result in more severe and accelerated degradation of material properties, in comparison to the dry-oxidation effects discussed above. Specifically, the oxide formed can be hydrolyzed through interaction with water with the formation of a volatile hydroxide. This process continuously exposes a fresh surface for oxidation and material removal. Thus, oxidation is not slowed down by oxygen diffusion through the oxide layer, and the overall rate of oxidation is increased.

3.2.4 Main Objective

The main objective of the present work is to extend our recently-developed multi-length-scale based CMC material model [1] in order to include the effects of the prolonged material exposure to in-service thermo-mechanical and environmental conditions, the conditions which can result in material-property degradation. The upgraded material model will then be linked with a commercial finite-element program, allowing a structural analysis of various gas-turbine engine hot-section CMC components as a function of component exposure to in-service thermo-mechanical and environmental conditions of different degrees and durations.

3.2.5 Chapter Organization

A brief overview of the CMC material constitutive model developed in Ref. [1] is presented in Section 3.3. Details of various thermo-mechanical and environmental conditions and their effect on the material-property degradation are presented in Section 3.4. The upgrading of the CMC material model through inclusion of these effects is explained in Section 3.5. In Section
3.6, results are presented and discussed for a series of transient, non-linear dynamics, finite-element structural analyses of a prototypical gas-turbine engine component (turbine shroud) being impacted by a high-speed foreign object (ingested and accelerated by the engine). In these analyses, the conditions of the impact are identical but the nature, degree and duration of exposure of the turbine shroud to the in-service engine environments is varied. A summary of the main conclusions resulting from the present work is given in Section 3.7.
3.3 Multi-Length-Scale CMC Material-Model

As mentioned earlier, the main objective of the present work is to include the in-service exposure effects into our recently developed multi-length-scale-based room-temperature CMC material model [1]. In the remainder of this section, a brief description is given of the multi-length-scale-based room-temperature CMC material model. It should be noted that this description will only be limited to the physical foundation of the model, and that the constitutive relations governing elastic and inelastic (including damage and failure) behavior of the subject material will not be repeated here, since they are available in our recently published work, Ref. [1]. The inclusion of the in-service effects considered in the present work does not alter the material constitutive model developed in [1], but only makes its parameters dependent on the nature, intensity and duration of the environmental effects.

In short, the model incorporates two main material microstructure/macrostructure contributions: (a) that associated with various structural constituents (e.g. multi-filament reinforcements, filament coatings, ceramic-matrix composites, phase interfaces, etc.); and (b) that associated with the laminated-CMC (0°/90° cross-ply vs. plain-weave) ply architecture. Consequently, the model involves two levels of material homogenization. Within the first level, the effective response of the material consisting of filaments, filament coatings, inter-filament matrix and inter-tow matrix is determined and formulated as a material constitutive model. Within the second level, the material model derived within the first level is used in conjunction with the two laminated-CMC architectures to determine the effective response of the fully-homogenized and featureless CMC material. This response is also cast in the form of a continuum material model suitable for linking with commercial finite element programs, enabling finite-element structural analysis of the gas-
turbine engine components. In the remainder of this section, more details are given for the two microstructural length-scales and for the material-model derivation at each length-scale.

It should be noted that porosity was not modeled explicitly at either of the two microstructural length-scales analyzed. The reason for this is that the original CMC material model developed in Ref. [1] dealt with CMCs in which the matrix was synthesized using the so-called Reactive Melt Infiltration (RMI) process. Within this process, SiC-fiber carbonaceous preform is infiltrated with molten silicon to form an SiC matrix. This process typically does not generate a significant amount of porosity. It should be further noted that if an alternative matrix-synthesis process such as Chemical Vapor Infiltration (CVI) is used, the extent of matrix porosity is generally substantially larger and a CMC material model like the one developed here must account for the effect of porosity.

3.3.1 Fiber/Tow Length-Scale

Development of the continuum-type material model at this length-scale involved a three-step procedure described below.

Step 1: Construction of the Fiber/Tow Length-Scale CMC Representative Volume Element: An example of the fiber/tow length-scale CMC representative volume element (RVE) is shown in Figure 3-3. For clarity, different constituents of the RVE are labeled in this figure and the associated dimensions are specified. The RVE shown in Figure 3-3 consists of 19 circular cross-section, 10-micron diameter SiC filaments, each coated with a 0.125 micron thick layer of BN. The center-to-center inter-filament spacing within the tow is 15 micron, and the volume of the tow-surrounding SiC matrix is adjusted to make the reinforcement volume fraction approximately 32%. The BN coating is assumed to be perfectly bonded (i.e. tied kinematically) to the SiC filament, and relatively weakly bonded to the matrix. Consequently, the BN-coating/SiC-matrix
interface acts as an interphase (i.e. an initially zero-thickness region possessing unique cohesive damage and failure properties) and has to be considered explicitly. It should be noted that the matrix cracking is assumed to take place on planes orthogonal to the filament axis. While in Figure 3-3, seven such planes have been explicitly shown, the RVE contains a much larger number of such planes. Matrix-crack planes parallel to the filament axis are not considered explicitly. Instead, cracking along the axis of the filaments is accommodated through the matrix/coating decohesion.

![Figure 3-3 Schematic of the fiber/tow length-scale CMC representative volume element (RVE).](image)

**Step 2: Virtual Mechanical Tests (VMTs) on the CMC RVE:** Virtual mechanical testing of the CMC RVE is carried out using a series of conventional displacement-based finite-element analyses (FEA). Six types of VMTs are employed: three uniaxial tensile stress and three pure shear (each test associated with a unique set of the RVE edge directions). In each case, tests were carried out in a load/unload/reload/unload/reload … fashion in order to assess the extent of material permanent-degradation caused by loading. In these analyses, the mechanical response of
the CMC constituents was accounted for using the appropriate material constitutive models. Except for the parameterization and the degree of material symmetry (i.e. fibers – transversely isotropic, coating – isotropic, matrix – isotropic), the same material models are used for the three constituents. These models treat the material as being linearly elastic and brittle (i.e. failure occurs when a total-stress invariant reaches a critical value, which follows the Weibull distribution). In addition to the CMC-constituent continuum-type material models, a cohesive-type traction-separation material model was specified for the weak/debondable matrix-coating interface. According to this model, complete debonding occurs when the cumulative contribution of the properly weighted normal and tangential interfacial tractions reaches a critical value [10, 11].

**Step 3: Derivation of the Homogenized Material Model:** The VMT results obtained are next analyzed to identify the defining features of the mechanical response. Next, by accounting for the key mechanical response features identified, the homogenized material constitutive model is constructed as a set of governing equations (suitable for implementation into a user-material subroutine which can be linked with a commercial finite element program). Lastly, the material constitutive model constructed is used in conjunction with the boundary/loading conditions of the VMTs within an optimization analysis in order to identify optimal values of the homogenized material-model parameters, which yield the best agreement with the VMT results.

The procedure just described established that elastically, the CMC at the fiber/tow length-scale can be considered as an (nearly transversely isotropic) orthotropic material. The elastic response of such a material is then defined by the three Young’s moduli \( E_{11}, E_{22} \) and \( E_{33} \), three Poisson’s ratios \( \nu_{12} = \nu_{21} = \frac{E_{11}}{E_{22}}, \nu_{13} = \nu_{31} = \frac{E_{11}}{E_{33}}, \) and \( \nu_{23} = \nu_{32} = \frac{E_{22}}{E_{33}} \) and three shear moduli
\( G_{12} = G_{21}, \ G_{13} = G_{31} \text{ and } G_{23} = G_{32}. \) Since loading associated with the VMTs induces damage into the material, the Young’s moduli and shear moduli are not constant but decreasing functions of the damage incurred by the material during loading. A summary of the damage (i.e. irreversible strain) dependent material stiffness constants for SiC/SiC CMC at the fiber/tow length-scale, obtained using the aforementioned VMTs is given in Table 3-1. The expressions listed in Table 3-1 have been verified to ensure that they meet the restrictions associated with the requirement that the sum of work done by all stress components is positive and, hence, there is no unphysical energy generation.

### Table 3-1 The damage (i.e. irreversible strain) dependent material stiffness constants for SiC/SiC CMC at the fiber/tow length-scale

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{11} )</td>
<td>GPa</td>
<td>( 363 - 2400 \varepsilon_{1,irr} - 4200 \gamma_{12,irr} - 4600 \gamma_{31,irr} )</td>
</tr>
<tr>
<td>( E_{22} )</td>
<td>GPa</td>
<td>( 261 - 2600 \varepsilon_{2,irr} - 3800 \gamma_{12,irr} - 3600 \gamma_{23,irr} )</td>
</tr>
<tr>
<td>( E_{33} )</td>
<td>GPa</td>
<td>( 245 - 2700 \varepsilon_{3,irr} - 3700 \gamma_{23,irr} - 3700 \gamma_{31,irr} )</td>
</tr>
<tr>
<td>( G_{12} )</td>
<td>GPa</td>
<td>( 82.3 - 850 \varepsilon_{1,irr} - 1600 \gamma_{12,irr} - 1100 \gamma_{31,irr} )</td>
</tr>
<tr>
<td>( G_{13} )</td>
<td>GPa</td>
<td>( 84.5 - 770 \varepsilon_{3,irr} - 1200 \gamma_{23,irr} - 1800 \gamma_{31,irr} )</td>
</tr>
<tr>
<td>( G_{23} )</td>
<td>GPa</td>
<td>( 89.6 - 730 \varepsilon_{2,irr} - 1900 \gamma_{12,irr} - 2300 \gamma_{23,irr} )</td>
</tr>
<tr>
<td>( v_{12} )</td>
<td>N/A</td>
<td>0.250</td>
</tr>
<tr>
<td>( v_{13} )</td>
<td>N/A</td>
<td>0.255</td>
</tr>
<tr>
<td>( v_{23} )</td>
<td>N/A</td>
<td>0.225</td>
</tr>
</tbody>
</table>
Since CMCs acquire irreversible strain during deformation, the inelastic response of this material had to be defined. This was done by defining and parameterizing the following five governing equations:

(a) stress vs. elastic strain – a generalized Hooke’s law;
(b) inelastic-deformation onset/continuation criterion – a modified Hill’s yield criterion;
(c) inelastic-deformation flow rule – the associated flow rule;
(d) fracture criterion – a criterion based on the sum of squares of the irreversible-strain components normalized by the corresponding fracture-strain components; and
(e) a constitutive law relating material strength and the overall extent of inelastic deformation – a power-law strain-hardening relationship.

Since a detailed explanation of the governing-equation functional forms, material-model parameters and their identification can be found in Ref. [1], the same will not be provided here.

3.3.2 Ply/Lamina Length-Scale

In Ref. [1], two SiC/SiC CMC architectures, i.e. cross-ply, Figure 3-4(a), and plain-weave, Figure 3-4(b), were investigated and, to derive the corresponding continuum-type material models, the same three-step procedure as that for the fiber-tow length-scale material model was employed: (a) the construction of the corresponding ply/lamina length-scale RVE; (b) a series of VMTs on the RVE; and (c) post-processing of the mechanical test data in order to assemble the corresponding homogenized-material constitutive model.
Figure 3-4 Schematics of ply-lamina length-scale architectures analyzed: (a) cross-ply; and (b) plain-weave; ply/lamina RVEs for the SiC/SiC CMCs with: (c) cross-ply; and (d) plain-weave architectures.
Figure 3-4 Continued.

Step 1: Construction of the Ply/Lamina Length-Scale CMC RVE: The two RVEs, each used to represent one of the analyzed (cross-ply and plain-weave) CMC architectures, are shown in Figures 3-4(c)–(d), respectively. It should be noted that the ply architecture used within the RVEs differs from the one shown previously in Figures 3-4(a)–(b). The reason for this is that the
“enriched tows” (i.e. the fiber/tow scale RVEs) used within the ply/lamina scale RVEs contain coated filaments, inter-filament matrix and inter-tow matrix and, hence, must completely fill the RVE volume (i.e. no “filament-free” matrix phase or CMC-matrix pores are assumed to exist in the present case). Due to these constraints, the architecture of the cross-ply CMC RVE, Figure 3-4(c), is quite simplified while that of the plain-weave CMC RVE, Figure 3-4(d), had to be made somewhat more complicated. It should also be noted that the cross-ply RVE, Figure 3-4(c), contains one 0° ply and one 90° ply, while the plain-weave RVE, Figure 3-4(d), contains two plain-weave plies (and is, hence, twice as thick as the cross-ply RVE).

Since matrix and filament fracture and interphase debonding are all accounted for by the tow length-scale material model, individual “enriched tows” within the same ply of the ply-scale RVE shown in Figures 3-4(c)–(d) are assumed to be perfectly bonded. On the other hand, a provision is made for inter-ply separation/delamination by treating ply/plly interfaces not as being perfectly bonded, but as being capable of undergoing normal/tangential-type decohesion. Potential delamination surfaces are labeled in Figures 3-4(c)–(d).

Step 2: VMTs on the RVE: The same set of six finite-element-based VMTs is employed as for the fiber/tow length-scale. In this case, each enriched tow is assigned the material model derived in the previous section. The local orientation of the material coordinate system had to be defined throughout the RVE. Inter-ply delamination has been modeled using the same type (with different parameterization) traction/separation constitutive law as that used at the fiber/tow length-scale to model matrix/coating decohesion.

Step 3: Derivation of the Homogenized Material Model: The VMT results obtained are next analyzed to identify the defining features of the mechanical response of the material at this length-scale. Next, by accounting for the key mechanical-response features identified, the
A homogenized-material constitutive model is constructed as a set of governing equations (suitable for implementation into a user-material subroutine which can be linked with a commercial finite element program). Since the ply/lamina architecture was found only to affect, but not completely alter, the materials’ response relative to that at the fiber/tow length-scale, the same homogenized material model as that developed in Section 3.3.1, but with different parameterization and material symmetry (orthotropic, at the fiber/tow level and transversely isotropic, at the ply/lamina level), was used here. Lastly, an optimization procedure is employed to determine the optimal values of the cohesive parameters for the interlaminar boundaries. As in the fiber/tow length-scale, the resulting material models (one for each of the two CMC architectures considered) are implemented into the VUMAT subroutines and linked with ABAQUS/Explicit.
3.4 Thermo-Mechanical And Environmental Effects

In this section, a detailed analysis of the four types of thermo-mechanical and environmental effects identified in the subsection entitled “Degradation of CMCs Mechanical Properties in Gas-Turbine Engine Environment” of Section 3.2 is carried out. Towards that end, a series of multi-physics computational analyses will be conducted to identify, model and quantify the effect of various thermo-mechanical/environmental conditions which, when present for a sufficient amount of time, may cause significant changes in the CMC material microstructure and properties. In Section 3.5, the outcome of these analyses will be incorporated into the CMC multi-scale material model [1] by making the material-model parameters dependent on the type, intensity and duration of the in-service environmental exposure.

3.4.1 Prolonged High-Temperature Exposure in the Absence of Significant Mechanical and Aggressive-Environmental Effects

The effect of prolonged high-temperature exposure is considered separately for different SiC/SiC CMC constituents.

**Matrix:** In addition to SiC the typical chemical constituents of CMC matrix, being considered in the present work, are residual carbon (i.e. the infiltrated carbon which did not react with molten Si infiltrant), excess/unreacted silicon (often introduced intentionally in order to reduce matrix stiffness), and in some cases reactively-formed SiB₆ (formed by chemical reaction between molten Si and pre-infiltrated B₄C; after oxidation of SiB₆, matrix-crack-sealing B₂O₃ is formed). The effect of prolonged high-temperature exposure on each of these CMC-matrix constituents is investigated in the present work, and it was found that none of the three additional matrix-constituents is expected to experience substantial changes. That is, the amount of residual carbon is expected to be minimal, considering the fact that the multi-filament fibers used for CMCs
necessitated the use of highly-reactive submicron carbon particles in the pre-infiltration slurry. Excess Si is found to be in thermodynamic equilibrium with SiC at temperatures as high as 1700 K. Also, Si, SiC and SiB$_6$ are found to coexist in thermal equilibrium up to the same temperatures. While the matrix can experience grain-growth during prolonged high-temperature exposure, this is not found to be of a serious concern since the as-fabricated average grain size is relatively large (ca. 1 μm).

**Fiber Strength Degradation:** Degradation of fiber strength due to prolonged high-temperature exposure is mainly the result of grain growth and porosity development. As will be discussed below, grain growth occurs in all classes of SiC-based fibers while porosity development typically occurs only in those fiber classes containing relatively high levels of oxygen.

Since the average grain size of the multi-filament fibers analyzed here is relatively small (ca. 5–10 nm), one of the main microstructural changes brought into the fibers during prolonged high-temperature exposure is grain-coarsening. In addition, fibers like Nicalon which possess amorphous, thermally-unstable Si$_{0.72}$O$_{0.26}$C$_{0.87}$ tend to develop porosity during prolonged high-temperature exposure. As part of the present work, grain-coarsening equations are derived for two fiber types, Hi-Nicalon and Nicalon. This was done by fitting the grain-size data resulting from the experimental investigations of fibers exposed to high temperatures, as reported by Takeda *et al.* [6] and Bodet *et al.* [3], to the conventional Lifshitz-Slyozov functional relationship. This procedure yielded the following relationships for the Hi-Nicalon and Nicalon fibers, respectively:
\[ D^3(T,t) = \left(3.26 \times 10^{-9}\right)^3 + \left(2.42 \times 10^{-18}\right)t \cdot \exp\left(-\frac{3.42 \times 10^5}{8.314 T}\right) \] (3.1)

\[ D^3(T,t) = \left(2.7 \times 10^{-9}\right)^3 + \left(1.13 \times 10^{-24}\right)t \cdot \exp\left(-\frac{1.68 \times 10^5}{8.314 T}\right) \] (3.2)

where \( D \) is the mean grain size (m), \( T \) is the temperature (K), and \( t \) is the time (sec). The first terms on the right-hand-sides of Eqs. (3.1) and (3.2), i.e. \( \left(3.26 \times 10^{-9}\right)^3 \) and \( \left(2.7 \times 10^{-9}\right)^3 \), represent cubes of the corresponding initial grain sizes (in meters).

Graphical representation of the functional relationships given by Eqs. (3.1) and (3.2) are depicted, as contour plots, in Figures 3-5(a)-(b), respectively. Also, the time-temperature experimental conditions utilized in Refs. [6] and [3] are respectively superposed, as white-filled circular symbols, on the contour plots shown in Figures 3-5(a)-(b). To show the degree of experiment/theory agreement for the average grain-size in these figures, two numbers (in nm, separated by a slash) are given for each point. The first one corresponds to the experimental value and the second to the computed value, both corresponding to the same \( (T,t) \) pair. The same approach is used in Figures 3-6(a)-(c), and Figures 8(a) and (c) discussed below. Examination of the results depicted in Figures 3-5(a)-(b) reveals a good agreement between the experimental and computed results (the degree-of-freedom-adjusted coefficient of determination, \( R^2_{adj} = 0.998 \) for Hi-Nicalon and \( R^2_{adj} = 0.951 \) for Nicalon).
Figure 3-5 Mean grain-size (nm) contour plots showing functional dependence on temperature and time of exposure for: (a) Hi-Nicalon; and (b) Nicalon fibers. The experimental results from Refs. [6] and [3] are respectively superposed on the contour plots. Please see text for details.
Next, the effect of grain coarsening on the strength is expressed using the standard Hall-Petch functional relationship, which is parameterized for the case of Hi-Nicalon fibers using the experimental results reported in Ref. [6]. Then the parameterized Hall-Petch relationship is combined with Eq. (3.1) in order to obtain a functional relationship between the fiber strength and its (isothermal) exposure to temperature $T$ for the duration of $t$ as:

$$\sigma(T,t) = 13.35 \times 10^6 (D(T,t))^{-0.27}$$  \hspace{1cm} (3.3)

where $D(T,t)$ on the right-hand side of this equation is given by Eq. (3.1).

Graphical representation of the functional relationship given by Eq. (3.3) is depicted, as a contour plot, in Figure 3-6(a). Also, the experimental results from Ref. [6] are superposed on the contour plot shown in Figure 3-6(a). Examination of the results depicted in Figure 3-6(a) reveals a good ($R^2_{adj} = 0.953$) agreement between the experimental and computed results.

In the case of Nicalon fibers, high-temperature exposure also gives rise to development of porosity. To quantify the temporal evolution of porosity at different temperatures, experimental data reported by Bodet et al. [3] are fitted to the Kolmogorov-Johnson-Mehl-Avrami equation to yield:

$$P(T,t) = 38 \left[ 1 - \exp \left( -1.36 \times 10^{30} \exp \left( -0.935 \times 10^6 / 8.314T \right) t^{1.08} \right) \right]$$ \hspace{1cm} (3.4)

where $P(T,t)$ is the thermal-exposure-induced porosity volume percent and the first term on the right-hand side (38) is the porosity percentage at the completion of $\text{Si}_{0.72}\text{O}_{0.26}\text{C}_{0.87}$ thermal decomposition.

Graphical representation of the functional relationship given by Eq. (3.4) is depicted as a contour plot in Figure 3-6(b). Also, the experimental results from Ref. [3] are superposed on the contour plot shown in Figure 3-6(b). Examination of the results depicted in Figure 3-6(b) reveals a good ($R^2_{adj} = 0.985$) agreement between the experimental and computed results.
Figure 3-6 Contour plots showing functional dependence on temperature and time of exposure for: (a) strength (GPa) of Hi-Nicalon fibers; (b) porosity (%) of Nicalon fibers; (c) strength (GPa) of Nicalon fibers; and (d) axial Young’s modulus (GPa) of Nicalon fibers. The experimental results from Refs. [6] and [3] are superposed on the contour plots. Please see text for details.
Figure 3-6 Continued.
Since in the case of Nicalon fibers, both grain growth and porosity degrade material strength, a procedure was devised to assess individual contributions of these two effects. This was done by identifying a portion of the high-temperature exposure regime within which fiber microstructural and, thus, property changes are dominated by grain growth. This enabled assessment of the grain-coarsening contribution to the material-strength degradation. Then the experimental results for the effect of temperature and time of exposure on the material strength as reported in Ref. [3] are used to assess the contribution of porosity to the material-strength degradation. This procedure yielded the following functional relationship for the effect of grain growth and porosity on the material strength:

\[
\sigma(T,t) = 10.77 \times 10^6 (D(T,t))^{-0.26} \left[2 - \frac{1}{1 - \left(\frac{P(T,t)}{100}\right)^{1.32}}\right]
\]  

(3.5)

where \( D(T,t) \) and \( P(T,t) \) on the right-hand side of Eq. (3.5) are given by Eqs. (3.2) and (3.4), respectively.

Graphical representation of the functional relationship given by Eq. (3.5) is depicted, as a contour plot, in Figure 3-6(c). Also, the experimental results from Ref. [3] are superposed on the contour plot shown in Figure 3-6(c). Examination of the results depicted in Figure 3-6(c) reveals a good (\( R^2_{adj} = 0.916 \)) agreement between the experimental and computed results.

It should be noted that Eqs. (3.3) and (3.5) and the results presented in Figures 3-6(a) and (c) pertain to the fiber-strength degradation and not to the CMC-strength degradation. To obtain the CMC-strength degradation results corresponding to the fiber-strength degradation results displayed in Figures 3-6(a) and (c), the following simple procedure is employed:
(a) at the first matrix-cracking stress, it is assumed that prolonged high-temperature exposure does not degrade the CMC;
(b) at the CMC ultimate tensile strength, it is assumed that the relative extent of the CMC-strength degradation is identical to that of the fiber; and
(c) at an intermediate stress, $\sigma$, the extent of CMC-strength degradation is assumed to be proportional to a product of the fiber-strength degradation and $\left(\sigma - \sigma_{\text{first}}\right) / \left(\sigma_{\text{UTS}} - \sigma_{\text{first}}\right)$, where $\sigma_{\text{first}}$ and $\sigma_{\text{UTS}}$ denote the first matrix-cracking stress and the ultimate tensile strength, respectively.

Stiffness Degradation: While grain-coarsening does not significantly affect material stiffness, porosity can severely compromise this material property. To assess the effect of porosity on the axial Young’s modulus of the Nicalon fibers, the experimental results reported in Ref. [3] are fitted to a power-law relationship to yield:

$$E(T,t) = 228 - 2.42 \times 10^{-6} (P(T,t))^{4.63}$$

where $E$ (GPa) is the fiber axial Young’s modulus and the first term on the right-hand side of Eq. (3.6) corresponds to the axial Young’s modulus in the as-fabricated condition of the Nicalon fiber.

Graphical representation of the functional relationship given by Eq. (3.6) is depicted, as a contour plot, in Figure 3-6(d). Also, the experimental results from Ref. [3] are superposed on the contour plot shown in Figure 3-6(d). Examination of the results depicted in Figure 3-6(d) reveals a reasonable ($R^2_{\text{adj}} = 0.873$) agreement between the experimental and computed results.

It should be noted that Eq. (3.6) and the results displayed in Figure 3-6(d) pertain to the fiber-stiffness degradation and not to the CMC-stiffness degradation. To obtain the CMC-
stiffness degradation results corresponding to the fiber-stiffness degradation results displayed in Figure 3-6(d), the following procedure was adopted:

(a) In the fiber axial direction, in which the matrix and the fibers are arranged in a parallel configuration, CMC stiffness is assumed to be a volume-fraction-weighted rule of mixtures of the fibers and matrix stiffnesses;

(b) In the fiber transverse directions, in which the matrix and fibers are arranged in a series configuration, CMC inverse stiffness is assumed to be a volume-fraction-weighted rule of mixtures of the inverse stiffnesses of the constituents;

(c) Since with respect to the shear stresses, the two CMC constituents are arranged in series, the inverse-stiffness relationship described in (b) was also used to describe degradation of the CMC shear moduli; and

(d) For both (a) and (b), the results for the degraded fiber stiffness are included in the appropriate CMC stiffness or inverse-stiffness relationship.

Coating: Fiber coating can be present as a single layer or a multi-layer coating. In general, the number of potential problems associated with prolonged high-temperature exposure is expected to be greater in the case of multi-layer coatings, due to the larger number of interfaces and the higher probability of the existence of a significant thermal-expansion-coefficient mismatch. It should be noted that due to mismatches in the thermal expansion coefficients between the coating (or coating layers), matrix and the fibers, the coating (as well as the matrix and fibers) are subjected to thermal stresses (even in the absence of in-service mechanical stresses). Excessively high tensile stresses within the coating can cause it to fracture while excessively high compressive stresses can cause its debonding from the fiber. Both of these scenarios are undesirable, leading to the CMC-property degradation, but the first of the two is generally
expected to be more detrimental since cracks in fully-bonded coating act as stress-concentrating flaws in the fibers. Also, since fiber-coatings are typically deposited using chemical vapor deposition (CVD) processes and, hence, contain relatively small as-synthesized grain-size, coating-grain coarsening can also lead to property degradation. Examination of the open-domain literature did not reveal any comprehensive investigation of the effect of prolonged high-temperature exposure on either single-layer BN or five-layer carbon-BN-carbon-Si$_3$N$_4$-carbon coatings commonly employed in conjunction with SiC filaments. Consequently, while recognizing that prolonged thermal degradation can cause coating fracture/debonding and, in turn, CMC-property degradation, this effect has not been accounted for in the upgraded material model.

**Interfaces:** Two types of interfaces are encountered in the CMCs under consideration: (a) coating/matrix interface; and (b) coating/fiber interface. Coating/matrix interfaces are initially designed to be weak and to promote the matrix crack deflection and fiber pull-out. On the other hand, since coating acts as environmental protection to the fibers, the coating/fiber interfaces are initially designed to be strong and tough. However, interfaces are the place where dissimilar materials are in contact, and particularly at high temperatures dissimilar materials tend to chemically react. This may result in the formation of interfacial phases, commonly referred to as interphases, and may significantly alter the properties of the coating/matrix and coating/fiber interfaces. In addition, thermal-expansion-mismatch stresses are typically the largest at or in the vicinity of the interfaces. An overview of the literature did not reveal any comprehensive investigation of the effect of prolonged high-temperature exposure on the coating/matrix and coating/fiber interfaces. This finding suggests that the potential problems identified above are not of life-limiting importance, which can be justified by the fact that one of the coating-material
selection criteria involves a low reactivity with the matrix and the fibers. Consequently, the effect of prolonged high-temperature exposure on the CMC-material interfaces will not be considered in the upgraded material model.

3.4.2 Prolonged Exposure to Thermo-Mechanical Effects in the Absence of Aggressive Environment

In contrast to the consideration of the effect of high-temperature exposure, the effect of thermo-mechanical exposure and the accompanying creep deformation is not analyzed here at the constituent level but rather at the CMC-material level. This was done because the CMC-creep-rate is not only affected by the tendencies of its constituents to creep but also by the extent of mismatch of these tendencies.

To analyze creep at the CMC level, a simple rectangular-parallelepiped-shaped computational model, shown in Figure 3-7(a), is constructed. This model is partitioned by an x-z plane into two regions, one filled with fiber and the other with the matrix materials, and the fiber axis is aligned with the x-direction. The model is then converted into its structural equivalent, so that in each of the three principal directions for each of the two materials/constituents, an elastic spring element and an inelastic creep element are introduced. Thus, in the x- and z-directions, the model consists of two parallel branches, each containing an elastic element and a creep element. On the other hand, in the y-direction, the model contains two sets of elastic/creep pairs in series.

It should be noted that the effect of shear stress was not included in the creep analysis presented in this subsection. The reason for this is that in the case of shear loading, each material constituent experiences the same shear stress (equal to the applied shear stress). In this case, differences in creep rates between matrix and fiber will result in differences in shear strain, but this will not result in redistribution of the stresses between the constituents (i.e. the condition
of shear-stress equality between the two materials is retained). For the same reason, creep caused
by normal stress in the \( y \)-direction does not result in stress redistribution. Furthermore, although
this total applied stress produces elastic strain in the other two principal directions due to the
Poisson’s effect, the total stresses (which are the root cause of the creep) in these two directions
remain unaltered.

For the remaining two directions (\( x \) and \( z \)), it is then recognized that the applied load is
partitioned between the two branches while the total (elastic+creep) strain rates are identical. By:
(a) writing the equality of the total strain rates in two branches in each of these two directions; (b)
expressing the elastic strain rate using the rate form of the generalized Hooke’s Law; (c) taking
advantage of the fact that the sum of the matrix and fiber stresses in each direction is constant
(under conventional creep-loading conditions), and the total stress rate in each of the two
directions is zero; and (d) utilizing a generalized power-law creep relationship in the form

\[
\dot{\epsilon}_{i,creep} = A_i \left( \sigma_{eq}^j \right)^{n_i} \frac{\partial f}{\partial \sigma_{i}^{j}} \frac{\sigma_{i}^{j}}{S_i^{j}}
\]

(3.7)
a system of two first-order nonlinear coupled ordinary differential equations for the matrix stress
in directions \( x \) and \( z \) is obtained. By numerically solving this equation system using the fourth-
order Runge-Kutta method, temporal evolution of the matrix (and thus, the fiber) stresses in the \( x \)-
and \( z \)-directions are obtained. A detailed description of the creep model and analysis presented
above will be given in a future manuscript in which the results of the present analysis will be
compared with those obtained using more elaborate CMC-creep models (e.g. [7, 12]).

In Eq. (3.7), the following nomenclature is used: the superscript \( j \) refers to the material
constituent, matrix or the fiber and subscript $i$ refers to the component (1 and 3) of the creep strain rate or stress, the term $S_{ij}^i$ is defined as the material strength for the $j^{th}$ constituent in the $i^{th}$ direction and $\sigma_{eq}$ is an equivalent stress for anisotropic material (equivalent to the von Mises stress for an isotropic material) and is defined as $\sigma_{eq} = S_1 f$, and Hill’s yield potential $f$ is defined as:

$$f = \sqrt{F\left(\sigma_2 - \sigma_3\right)^2 + G\left(\sigma_3 - \sigma_1\right)^2 + H\left(\sigma_1 - \sigma_2\right)^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2} = 1$$

where

$$F = \frac{1}{2} \left[ \frac{1}{(S_2^i)^2} + \frac{1}{(S_3^i)^2} - \frac{1}{(S_1^i)^2} \right], \quad G = \frac{1}{2} \left[ \frac{1}{(S_3^i)^2} + \frac{1}{(S_1^i)^2} - \frac{1}{(S_2^i)^2} \right], \quad H = \frac{1}{2} \left[ \frac{1}{(S_1^i)^2} + \frac{1}{(S_2^i)^2} - \frac{1}{(S_3^i)^2} \right], \quad L = \frac{1}{2(S_{23}^i)^2}, \quad M = \frac{1}{2(S_{31}^i)^2} \text{ and } N = \frac{1}{2(S_{12}^i)^2}.$$

Strength Degradation: Solution of the two aforementioned differential equations revealed that the (tensile) matrix stress increases with creep time and ultimately approaches asymptotically a constant value, the magnitude of which is a function of the fiber/matrix creep-rate mismatch. That is, the larger is this mismatch (occurs at lower temperatures), the higher is the extent of fiber-to-matrix (steady state) stress redistribution. On the other hand the time required to reach the steady state decreases with an increase in the exposure temperature. It should also be noted that the stress transfer is permanent, that is, upon the removal of the applied stress, the matrix contains tensile stress while the fiber contains a balancing compressive residual stress. This finding then indicates that the fiber-to-matrix load transfer results in an effective matrix-strength loss equal to the magnitude of the corresponding residual stress within the matrix.
Figure 3-7 (a) Schematic of the material constituent arrangement in the three dimensions; (b) fractional retained strength of the CMC, with a purely normal stress in the $x$-direction of 80 MPa; and (c) fractional retained axial stiffness as a function of the exposure temperature and time, with a purely normal stress in the $x$-direction of 100 MPa.
Figure 3-7 Continued.

An example of the results obtained in this portion of the work is displayed in Figure 3-7(b), in which the contour plot of the fractional retained strength of the CMC is plotted as a function of the exposure temperature and time at a constant applied normal stress in the $x$-direction of 80 MPa (this level of stress is sufficiently low so that the load transfer to the matrix, in the range of exposure temperatures and times examined, does not cause matrix cracking) and zero values of the other applied-stress components. Other parameters used in the construction of Figure 3-7(b) include: (a) first matrix-cracking stress = 100 MPa; (b) matrix-creep parameter

$$A^m = 1.045 \times 10^{-7} \exp\left(-\frac{428.3 \times 10^3}{RT}\right)\left(\frac{1}{\text{Pa}^{n^m}}\right)$$

where $n^m = 1.43$ [13]; (c) fiber-creep parameter

$$A^f = 1.16 \times 10^{-9} \exp\left(-\frac{229.9 \times 10^3}{RT}\right)\left(\frac{1}{\text{Pa}^{n^f}}\right)$$

where $n^f = 1.29$ [14]; (d) Young’s moduli of fiber and matrix $E_x^f = 370 \text{ GPa}$, $E_y^f = E_z^f = 200 \text{ GPa}$, and $E^m = 350 \text{ GPa}$ [1]; (e)
Poisson’s ratios of fiber and matrix \( \nu_{xy}^f = \nu_{xz}^f = 0.25, \ \nu_{yz}^f = 0.22, \) and \( \nu^m = 0.17 \) [1]; and (f) volume fractions of fiber and matrix \( \nu^f = 0.35 \) and \( \nu^m = 0.65 \) [1].

Examination of the results displayed in Figure 3-7(b) reveals that at the exposure times greater than ca. 40 days, there is an intermediate exposure-temperature range (centered around 1400 K) which is associated with a minimum retention of the material strength. At lower temperatures, while the steady-state strength retention is lower, the observed strength retention is higher since the steady-state condition is not reached (due to the fact that the exposure time is lower than the steady-state time). Contrary, at higher temperatures, the steady-state and, hence, the actual strength retention is higher.

**Stiffness Degradation:** When stress redistribution does not cause the matrix stress to exceed the matrix strength, cracking within this constituent does not take place and, hence, there is no first-order change in the material stiffness. Contrary, when creep-induced stress redistribution gives rise to matrix cracking and, thus, irreversible strain, stiffness degradation takes place. To evaluate the extent of stiffness degradation, the inelastic deformation analysis mentioned earlier is utilized to assess the extent of creep-induced damage/inelastic deformation. Then, functional relationships given in Table 3-1 are used to assess the degree of creep-induced stiffness degradation.

An example of the results obtained in this portion of the work is displayed in Figure 3-7(c), in which the contour plot of the fractional retained axial stiffness of the CMC is plotted as a function of the exposure temperature and time at a constant applied normal stress in the \( x \)-direction of ca. 100 MPa (equal to the first matrix-cracking stress, so that any fiber-to-matrix load transfer results in matrix cracking). The remaining parameters used are identical to the ones stated in the subsection entitled “Strength Degradation”.

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Examination of the results displayed in Figure 3-7(c) and their comparison with the results displayed in Figure 3-7(b) reveals a close qualitative similarity between the two sets of the results. This is to be expected since the same phenomenon, creep-induced fiber-to-matrix load transfer, is the root cause for both the strength and stiffness degradation of the CMC material.

**Excessive Creep Strain**: As mentioned earlier, excessive creep deformation can compromise dimensional tolerances of the components and result in the development of increased mounting and contact stresses with the adjoining components, excessive friction and wear in the case of moving components, etc. The solution of the two differential equations given above can also be used to determine the temporal evolution of the overall creep rate (as a function of the constituent creep rates and their mismatch) and thus, to assess the extent to which CMC-component creep can become a problem. However, this effect is not considered here since it does not affect the formulation of the CMC-constituent material model being upgraded.

**3.4.3 Prolonged Exposure to High-Temperature and Dry-Oxidizing Environment, in the Absence of Sustained Loading**

The analysis presented in this subsection deals with: (a) the mechanism and the kinetics of the CMC (internal) dry oxidation; and (b) quantification of the effect of dry oxidation on material degradation.

**Mechanism and Kinetics of CMC (Internal) Dry Oxidation**: Oxidation of the CMC components, i.e. fibers and matrix, occurs when presence of matrix cracks, which are bridged by the intact fibers, lead to ingress of oxygen into the CMC material. The SiC fibers react with the oxygen to produce SiO$_2$ (a solid oxide which surrounds the intact fiber through which the oxygen has to diffuse to continue further oxidation of the fiber) and CO$_2$ (a gaseous oxide that escapes out
through the matrix cracks). While continued oxidation of SiC could be either an (a) oxidation-reaction-or (b) oxygen-diffusion-controlled process, it is generally observed [e.g. 9] that except for the very short oxidation times, the oxidation process is diffusion-controlled. This finding is reasonable since as the time increases, the oxygen has to diffuse through a growing thickness of SiO$_2$, which slows down the oxygen diffusion and thus further oxidation of the fiber.

The oxidation reaction results in an increase in mass of the fiber (50% increase of mass when the reaction is complete). By analyzing and fitting the experimental data from Ref. [15] pertaining to the effect of oxygen partial pressure, temperature and time on the extent of 15-$\mu$m diameter fiber-weight gain, and taking into account the fact that the oxidation process is in the diffusion-controlled regime, the following functional relationship is derived:

$$X(P_o, T, t) = \sqrt{2128561 e^{\frac{-159.721 \times 10^3}{8.3147 T}} \left(P_o\right)^{0.7} t}$$

(3.8)

where $X$ is the percent weight gain, $P_o$ is the oxygen partial pressure in atm, $T$ and $t$ are the exposure temperature in K and time in hours, respectively.

Graphical representation of the functional relationship given by Eq. (3.8), at an oxygen partial pressure set to 0.25 atm (the level used in Ref. [13]), is depicted as a contour plot in Figure 3-8(a). It is seen that, as expected, the largest extent of weight gain occurs at the highest exposure temperature and longest exposure time analyzed. Also in Figure 3-8(a), the experimental results from Ref. [13] are superposed on the contour plot. Examination of the results depicted in Figure 3-8(a) reveals a reasonable ($R^2_{adj} = 0.857$) agreement between the experimental and computed results.
Figure 3-8 (a) Percent gain in fiber weight (experimental results from Ref. [13] are superposed on the contour plot); and (b) fractional retained fiber strength of the CMC at an oxygen partial pressure of 0.25 atm, as a function of the temperature and time of exposure to dry-oxidation conditions.
Quantification of Effect of Dry Oxidation on Material-Strength Degradation: The oxidation product SiO$_2$ is brittle and readily fractures under the in-service tensile loads. The SiO$_2$ cracks act as flaws in the fibers, and when the length of these flaws exceeds the maximum length of the fiber preexisting flaws, fiber strength begins to degrade. This causes some fibers to fail, leading to load being transferred to the remaining unbroken fibers, which undergo accelerated failure.

Next, a functional relationship is derived between the percentage weight increase $X$ and oxide thickness $w$ as: $w(r_{\text{core}}) = \sqrt{(r_{\text{init}}^2 - r_{\text{core}}^2) R_{\text{MV}} + r_{\text{core}}^2 - r_{\text{core}}}$ where

$$r_{\text{core}}(X) = r_{\text{init}} \sqrt{\frac{\rho_{\text{SiC}} (1 + X/100) - \rho_{\text{SiO}_2} R_{\text{MV}}}{\rho_{\text{SiC}} - \rho_{\text{SiO}_2} R_{\text{MV}}} }$$

and $X = X(P_{O_2}, T, t)$ as given by Eq. (3.8). This relationship is then evaluated for the fiber initial radius $r_{\text{init}} = 7.5 \text{ \mu m}$ and the SiO$_2$-to-SiC molar volume ratio, $R_{\text{MV}}$, is computed using the molecular weights of SiC (40 g/mol) and SiO$_2$ (60 g/mol), and the corresponding mass densities (3.21 g/cm$^3$ for SiC and 2.65 g/cm$^3$ for SiO$_2$). Then, by: (a) utilizing the newly-derived functional relationship between the fiber weight gain and the oxide thickness; (b) setting the oxidation-flaw size equal to the oxide thickness; and (c) invoking the concept of brittle-fracture-controlled material strength, as quantified by the linear-elastic fracture mechanics, the strength of the fiber (in Pa) is given as:

$$\sigma(P_{O_2}, T, t) = \min \left\{ \frac{K_f}{\sqrt{\pi w(P_{O_2}, T, t)}}, \sigma_{\text{init}} \right\}$$

(3.9)

where the SiC-fiber stress-intensity factor is set as $K_f = 3.5 \text{ MPa.m}^{1/2}$, $X(P_{O_2}, T, t)$ is given by Eq. (8), and $\sigma_{\text{init}} = 2 \text{ GPa}$.
Graphical representation of the functional relationship given by Eq. (3.9), after normalization by $\sigma_{\text{init}}$ and at an oxygen partial pressure set to 0.25 atm, is shown as a contour plot in Figure 3-8(b). Examination of the results from Figure 3-8(b) shows that:
(a) higher exposure temperatures and longer exposure times result in larger strength-degradation of the fiber;
(b) at the shortest exposure times, the strength degradation has not occurred, as the oxide flaw thickness is still smaller than the most potent preexisting fiber flaw; and
(c) the time of onset of oxidation-induced strength degradation decreases as the exposure temperature increases.

It should be noted that Eq. (3.9) and the results presented in Figure 3-8(b) pertain to the fiber-strength degradation and not to the CMC-strength degradation. To obtain the CMC-strength degradation results corresponding to the fiber-strength degradation results displayed in Figure 3-8(b), the same procedure as that used in the case of fiber-strength degradation due to prolonged high-temperature exposure is employed here.

**Quantification of Effect of Dry Oxidation on Axial-Stiffness Degradation:** Axial-stiffness degradation of the fiber is generally related to the oxidation-induced reduction in the non-oxidized fiber cross-sectional area. Since, under dry-oxidation conditions, such cross-sectional area reductions are very small ($< 0.1\%$, under the temperature/time/oxygen partial pressure conditions investigated), the effect of dry oxidation on the axial stiffness of the fiber, and therefore on the axial stiffness of the CMC, is assumed to be negligible.
3.4.4 Prolonged Exposure to High-Temperature and Wet-Oxidizing Environment, in the Absence of Sustained Loading

As in the case of dry oxidation, two aspects of the case of wet oxidation are considered: (a) the mechanism and the kinetics of the CMC (internal) wet oxidation; and (b) quantification of the effect of wet oxidation on material-property degradation.

Mechanism and Kinetics of CMC (Internal) Wet Oxidation: Under the wet oxidizing condition, oxidation of SiC and the formation of SiO$_2$ can take place via two chemical reactions: (a) oxidation of SiC with O$_2$ (the reaction which also takes place under dry-oxidizing conditions); and (b) oxidation of SiC with H$_2$O. Furthermore, the SiO$_2$ layer reacts with H$_2$O to form Si(OH)$_4$, a gaseous product that escapes leading to recession of the SiO$_2$ layer.

The experimental investigation carried out in [16, 17] established the following in regard to the two oxidation and one hydroxide-formation reactions:

(a) the rate of hydroxide-formation and volatilization reaction is quite small in comparison to the rates of the oxidation reaction. Consequently, to a first order of approximation, the effect of recession can be neglected;

(b) except at very low SiO$_2$-thickness levels the two oxidation reactions appear to be diffusion-controlled; and

(c) the oxidation reaction involving H$_2$O is substantially higher than that involving O$_2$. This is the result of substantially higher H$_2$O solubility limit (and, thus the concentration gradient) in SiO$_2$ than that of O$_2$. Consequently, despite the fact that H$_2$O diffusion coefficient through SiO$_2$ is significantly lower than that of O$_2$, the oxidation reaction involving H$_2$O is still substantially higher.
Based on these observations, it was found appropriate to simplify the apparently complex process of SiC chemical degradation under wet-oxidizing conditions as a process dominated by diffusion-controlled SiC oxidation with H₂O [16, 17].

By analyzing and fitting the experimental data [16] pertaining to the effect of water-vapor partial pressure, temperature and time on the extent of 15-μm diameter fiber-weight gain, and taking into account the fact that the oxidation process is in the diffusion-controlled regime, the following functional relationship is derived:

$$X(P_{H₂O}, T, t) = \sqrt{\left(43003.85 - 515\right) e^{-0.33P_{H₂O}} + 515} e^{146.91 \times 10^3 - 55.44 \times 10^3} e^{-0.111H₂O} + 55.44 \times 10^3} \frac{8.1347}{t}$$

Graphical representation of the functional relationship given by Eq. (3.10), at a water vapor partial pressure of 0.2 atm (the level used in Ref. [16]), is depicted as a contour plot in Figure 3-9(a). It is seen that, as expected, the largest extent of weight gain occurs at the highest exposure temperature and longest exposure time analyzed. Also in Figure 3-9(a), the experimental results from Ref. [16] are superposed on the contour plot. Examination of the results depicted in Figure 3-9(a) reveals a very good ($R^2_{adj} = 0.957$) agreement between the experimental and computed results.

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Figure 3-9 (a) Percent gain in fiber weight (experimental results from Ref. [16] are superposed on the contour plot); and (b) fractional retained fiber strength of the CMC at a water vapor partial pressure of 0.2 atm, as a function of the temperature and time of exposure to wet-oxidation conditions.
Quantification of the Effect of Wet Oxidation on Material-Strength Degradation: As in the case of
dry oxidation, a functional relationship is first derived between the oxide thickness and the weight
gain, and then an analogous three-step procedure is employed in order to correlate the fiber
weight gain and the strength of the fibers. This procedure established that the strength of the fiber
is defined as:

\[
\sigma(P_{H_2O}, T, t) = \min \left\{ \frac{K_c}{\sqrt{\pi w(P_{H_2O}, T, t)}}, \sigma_{ini} \right\}
\]

where the SiC-fiber stress-intensity factor is set as \( K_c = 3.5 \text{ MPa.m}^{1/2} \), \( X(P_{H_2O}, T, t) \) is given
by Eq. (3.10), and \( \sigma_{ini} = 2 \text{ GPa} \).

Graphical representation of the functional relationship given by Eq. (3.11), after
normalization by \( \sigma_{ini} \) and at a water vapor partial pressure of 0.2 atm, is shown as a contour plot
in Figure 3-9(b). Examination of the results from Figure 3-9(b) and their comparison with the
results shown in Figure 3-8(b) shows that:

(a) as in the case of dry oxidation:

(i) higher exposure temperatures and longer exposure times result in larger strength-degradation
of the fiber;

(ii) at the shortest exposure times, strength degradation has not occurred, as the oxide flaw
thickness is still smaller than the most potent preexisting fiber flaw; and

(iii) the time of onset of oxidation-induced strength degradation decreases as the exposure
temperature increases.

Points (ii) and (iii) are hard to discern from the figure due to very short times of onset of fiber-
strength-degradation induced by wet oxidation; and
(b) over the entire temperature-time exposure domain, the extent of strength degradation is larger in the case of wet oxidation relative to that observed in the case of dry oxidation.

It should be noted that Eq. (3.11) and the results presented in Figure 3-9(b) pertain to the fiber-strength degradation and not to the CMC-strength degradation. To obtain the CMC-strength degradation results corresponding to the fiber-strength degradation results displayed in Figure 3-9(b), the same procedure as in the case of dry oxidation was employed.

Quantification of Effect of Wet Oxidation on Axial-Stiffness Degradation: Under wet-oxidation conditions, the reduction in the non-oxidized fiber cross-sectional area is also quite small (< 0.3%, under the temperature/time/water-vapor partial pressure conditions investigated). Hence, as in the case of dry oxidation, the effect of wet oxidation on the axial stiffness of the fiber, and therefore on the axial stiffness of the CMC, is neglected.
3.5 Upgrade Of The CMC Material Model

In this section, the procedures for incorporating, into the CMC material constitutive model [1], the environmental effects (described and quantified in the previous section) on the CMC strength and stiffness are presented. As mentioned earlier, the model is of a linear-elastic (with degradable elastic moduli) and inelastic (with inelastic deformation being dominated by matrix and fiber damage and cracking) type. Consequently, separate procedures are developed for including the effects of in-service environment on stiffness and on strength (i.e. inelastic response). It should be recalled that the inclusion of the in-service effects considered in the present work does not alter the material constitutive model developed in [1], but simply makes its elasticity and strength parameters dependent on the nature, intensity and duration of the environmental effects.

3.5.1 Stiffness Degradation

At the ply/lamina length-scale, CMCs can be considered, after homogenization, as being transversely isotropic and, hence, their elastic response is defined by five elastic moduli. For the case of \( x_3 \) being the unique material-direction, the five elastic moduli are defined as: \( E_{11} = E_{22} \), \( E_{33} \), \( \nu_{12} = \nu_{21} \), \( \nu_{13} = \nu_{23} \), and \( G_{13} = G_{23} \) (and \( G_{12} = \frac{E_{11}}{2(1+\nu_{12})} \), \( \nu_{31} = \nu_{32} = \frac{\nu_{13}}{E_{11}} E_{33} \)). The work presented in Ref. [1], as well as the results given in Table 3-1, demonstrate that the degradation of the elastic moduli \( \Phi = E, G \) can be defined by the following general relationship:

\[
\Phi_{ij} = \Phi_{ij,0} - a_{ijkl} \varepsilon_{kl,irr}
\]  

(3.12)

where \( a \) are the (positive) linear stiffness-degradation coefficients, \( \varepsilon \) is the true strain, subscript 0 denotes the initial (damage-free) state of the material, subscript irr denotes the
irreversible/inelastic part of a strain component, and summation is implied over the repeated indices.

In the previous section, two main sources of stiffness degradation were identified:

(a) porosity development within oxygen-rich fibers like Nicalon during prolonged high-temperature exposure. Since this microstructural change does not, in the absence of stress, lead to any matrix/fiber cracking (and thus, to irreversible strain), to include the effect of porosity on CMC stiffness, the magnitude of the $\Phi_{ij,0}$ term in Eq. (3.12) is reduced by an amount dependent on the severity and duration of high-temperature exposure (as quantified by the analysis presented in Section 3.4.1); and

(b) matrix cracking due to excessive-creep-induced fiber-to-matrix load transfer. In this case, depending on the extent of the load transfer, the CMC will acquire different values of the $\varepsilon_{kl,irr}$ term. Consequently, to include the effect of creep-induced matrix cracking onto the CMC-stiffness degradation, the second term on the right-hand side of Eq. (3.12) is evaluated using the original values of $\alpha_{ijkl}$ and the acquired values of $\varepsilon_{kl,irr}$.

3.5.2 Strength Degradation

Before the effects of the in-service environment on the material strength can be incorporated into the material constitutive model, Ref. [1], the inelastic portion of the model had to be carefully examined. As mentioned earlier, the inelastic response of the CMC, which is controlled not only by the applied-stress state but also by the magnitudes of the instantaneous strength measures, was modeled in Ref. [1] by defining and parameterizing the following governing equations:
(a) inelastic-deformation onset/continuation criterion – Except that the values of the material-strength parameters appearing in this equation are affected by the in-service environmental effects, the functional form of the inelastic-deformation onset/continuation criterion remains unchanged;

(b) inelastic-deformation flow rule – As in the case of the inelastic-deformation onset criterion, only the values of the material strength parameters, but not the functional form of the flow rule, are affected;

(c) a constitutive law relating material strength parameters, \( S_{ij} \), and the overall extent of inelastic deformation, \( \bar{\varepsilon}_{irr} \) – The following exponential hardening law was employed in Ref. [1]:

\[
S_{ij}(\bar{\varepsilon}_{irr}) = S_{ij}(\bar{\varepsilon}_{irr} = 0) + \alpha_{ij} \bar{\varepsilon}_{irr}^\beta_{ij}
\]

where \( \alpha_{ij} \) and \( \beta_{ij} \) are material constitutive parameters; and

(d) fracture criterion – a strain-based relationship defining the condition for a complete loss of load-bearing capacity by the CMC. In the current upgraded rendition of the CMC material model, neither the functional form nor parameterization of this criterion has been affected by the in-service exposure conditions.

The analysis presented in the previous section established that all four environmental effects analyzed affect material strength (but in different ways, and to different extents). Consequently, inclusion of the contribution of the four environmental effects on the material-strength degradation is analyzed separately, in the remainder of this subsection.
Prolonged High-Temperature Exposure in the Absence of Significant Mechanical and Aggressive-Environmental Effects: According to the analysis presented in Section 3.4.1, this environmental effect affects only the strain-hardening behavior (i.e. the second term on the right-hand side of Eq. (3.13)) but not the initial material strength (i.e. the first term on the right-hand side of Eq. (3.13)). Based on the analysis presented in Section 3.4.1: (a) only the $\alpha_{ij}$ coefficient within the strain-hardening term is affected by prolonged high-temperature exposure; and (b) to include the effect of high-temperature exposure, $\alpha_{ij}$ is modified as follows:

$$\alpha_{ij}(\bar{\varepsilon}_{irr}, \bar{\varepsilon}_{ij, irr}^{\text{crit}}) = \alpha_{ij,0} - \delta_{ij}(\bar{\varepsilon}_{irr}, \bar{\varepsilon}_{ij, irr}^{\text{crit}})(\Delta S_{ij,US})$$

(3.14)

where $\alpha_{ij,0}$ is the initial constant value of $\alpha_{ij}$ as given in Ref. [1],

$$\delta_{ij}(\bar{\varepsilon}_{irr}, \bar{\varepsilon}_{ij, irr}^{\text{crit}}) = \left( S_{ij}(\bar{\varepsilon}_{irr}) - S_{ij}(\bar{\varepsilon}_{irr} = 0) \right) \left( S_{ij,US}(\bar{\varepsilon}_{ij, irr}^{\text{crit}}) - S_{ij}(\bar{\varepsilon}_{irr}) \right)$$

is a scaling parameter, $S_{ij,US}(\bar{\varepsilon}_{ij, irr}^{\text{crit}})$, as operationally defined in Ref. [1], is the ultimate strength associated with $i, j$ material directions, $\bar{\varepsilon}_{ij, irr}^{\text{crit}}$ is a $i, j$-dependent value of the equivalent irreversible strain at the corresponding ultimate-strength level, and $\Delta S_{ij,US}$ is the reduction of the $(i, j)$ ultimate strength caused by high-temperature exposure of the given duration as quantified in Section 3.4.1.

Prolonged Exposure to Thermo-Mechanical Effects in the Absence of Aggressive Environment:

According to the analysis presented in Section 3.4.2, creep-induced fiber-to-matrix load transfer gives rise to the reduction in the first matrix-cracking stress, $S_{ij}(\bar{\varepsilon}_{irr} = 0)$, the first term on the right-hand side of Eq. (3.13). The extent of this reduction is a function of the exposure
temperature and time, as well as on the magnitude and nature of the applied stress field, and is quantified here using the procedure described in Section 3.4.2.

**Prolonged Exposure to High Temperature and Dry-Oxidizing Environment, in the Absence of Sustained Loading:** According to the analysis presented in Section 3.4.3, this type of in-service environment affects only the strain-hardening behavior but not the initial material strength. Consequently, the effect of this type of in-service environment on the material strength has been handled in the same way as that for the case of prolonged high-temperature exposure, except that the extent of material-strength degradation is now a function of not only the exposure temperature and time, but also the oxygen partial pressure.

**Prolonged Exposure to High Temperature and Wet-Oxidizing Environment, in the Absence of Sustained Loading:** According to the analysis presented in Section 3.4.4, this type of in-service environment affects only the strain-hardening behavior but not the initial material strength. Consequently, the effect of this type of in-service environment on the material strength has been handled in the same way as that for the case of dry oxidation, except that the extent of material-strength degradation is now a function of the partial pressure of water vapor rather than that of oxygen.

### 3.5.3 Model Implementation

The upgraded CMC material model is next implemented in the material user subroutine, VUMAT, of the commercial finite element program ABAQUS/Explicit [18]. This subroutine is compiled and linked with the finite element solver and enables ABAQUS/Explicit to obtain the needed information regarding the state of the material and the material mechanical response (i.e. stress) during each time step, for each integration point of each element. The effect of the in-service environment is included
into the upgraded material model by properly modifying the values of the stiffness, strength and strain-hardening parameters.

The essential features of the coupling between the ABAQUS/Explicit finite-element solver and the VUMAT material user subroutine at each time increment at each integration point of each element can be summarized as follows:

(a) The corresponding previous time-step stresses and material state variables (i.e. the equivalent inelastic/irreversible strain, the components of inelastic strain, the components of material strength, and the material-failure state, all at the end of the previous time-step) as well as the previous time-step logarithmic strains and current time-step logarithmic-strain increments (both in the current material co-rotational frame) are provided by the ABAQUS/Explicit finite-element solver to the material subroutine; and

(b) Using the information provided in (a), and the upgraded CMC material model, the material stress state as well as values of the material state variables at the end of the current time-step are determined within the VUMAT and returned to the ABAQUS/Explicit finite-element solver. In addition, the changes in the total internal and the inelastic energies are computed and returned to the solver. The information regarding the stress state of the element is then used by the finite-element solver to compute the global equilibrium.
3.6 Foreign Object Damage (FOD) Of CMC Turbine Shrouds

To demonstrate the full utility of the newly developed CMC material model, the material model developed in the previous section is used in the current section to analyze, using a transient non-linear dynamics finite-element analysis, the structural/damage response of a prototypical gas-turbine engine component, a turbine shroud, to impact by a foreign object [19, 20, 23]. Since the same type of problem was analyzed in Ref. [1], but using the original version of the material model [1], only the most important details of the finite-element modeling and analysis will be presented here.

3.6.1 Finite Element Analyses of the FOD

Geometrical Model: The computational model employed in this portion of the present work comprises a 1.59 mm-diameter solid-spherical projectile (made of AISI H13 steel) and a 15° toboggan-shaped turbine-shroud CMC segment, Figure 3-10(a). The remaining geometrical details of the shroud segment are marked in this figure.

Meshed Model: A close-up of the meshed model used is depicted in Figure 3-10(b). The shroud-segment is meshed using ca. 100,000 eight-node, first-order, continuum finite elements with a characteristic edge length of 0.33 mm. As far as the solid spherical projectile is concerned, it was discretized using (typically ca. 6,000) four-node tetrahedron continuum elements, with a characteristic edge length of 0.25 mm. The Cartesian coordinate system employed in the present analysis is shown in Figure 3-10(a). It is seen that \( x \) is the axial direction of the turbine shroud, \( y \) runs circumferentially while \( z \) is orientated generally in the shroud radial direction.
Figure 3-10 (a) Schematic of the component-level model containing a 1.59 mm-diameter solid-spherical projectile and a 15° toboggan-shaped turbine-shroud CMC segment; and (b) close-up of the meshed model of the toboggan-shaped shroud segment.
Initial Conditions: At the beginning of an analysis, both the impactor and the shroud-segment are assumed to be stress free. In addition, the shroud-segment is assumed to be stationary/quiescent while the impactor is assumed to be moving in the $x$-$z$ plane at an angle of 45° with respect to the $x$-axis, with a total velocity of 300 m/s.

Boundary Conditions: The bottom-most portions of the toboggan-shaped shroud-segment, which are normally connected to the turbine casing, are subjected to the Encastré boundary conditions.

Material Models: As mentioned earlier, the material model developed in Section 3.5 is assigned to each finite element of the shroud-segment after the local material coordinate system is defined. As far as the AISI H13 steel is concerned, its constitutive behavior is represented by: (a) an isotropic Hooke’s law; (b) a Mie-Grüneisen equation of state; (c) a Johnson-Cook strength model; and (d) a Johnson-Cook failure model. Since details regarding the functional relationships and parameterization of the AISI H13 model can be found in Refs. [21, 22], they will not be repeated here.

3.6.2 FEA Results:

The unique feature of the upgraded material model used in the FEA of the FOD of the toboggan-shaped single segment of the turbine shroud is that its formulation and parameterization are affected by the nature, intensity and duration of the four in-service environmental effects analyzed in Section 3.4. In the present section, a series of FEAs are carried out to demonstrate the effect of the in-service environmental exposure on the ability of the shroud to withstand impact by the foreign object. In all the cases analyzed, the plain-woven architecture of the CMC was assumed with the volume fraction of Hi-Nicalon fibers being 0.32.
Case A – No In-Service Environmental Exposure, the Reference Case: In this case, the original CMC material model [1] was utilized within the FEA. Temporal evolution and spatial distribution of the shroud segment SiC/SiC CMC and of the AISI H13 steel impactor materials during the segment/impactor interactions is depicted in Figures 3-11(a)–(d). It should be noted that, for improved clarity, the CMC shroud component is made transparent in these, as well as in the subsequent figures. Examination of the results depicted in Figures 3-11(a)–(d) reveals that the foreign object penetrates about one-quarter through the thickness of the shroud before bouncing back. A close-up of the shroud damaged region for the reference case is displayed in Figure 3-12(a). The size and the topology of the foreign-object-induced damage shown in this figure will be used as the baseline with which the subsequent corresponding results will be compared, to reveal the effect of various in-service environmental effects on the ability of the same toboggan-shaped shroud component to withstand oblique impact by the same foreign object propelled at the same initial velocity and in the same incident direction. It should be noted that in Figure 3-12(a), as well as in the remaining figures, the projectile is not shown to help better reveal the damage sustained by the shroud component.
Figure 3-11 Temporal evolution and spatial distribution of the shroud segment and the impactor materials during the segment/impactor interactions for the plain-weave architecture in the absence of in-service environmental effects (the reference case). Please see text for additional details.
Figure 3-11 Continued.
Case B – Prolonged High-Temperature Exposure in the Absence of Significant Mechanical and Aggressive-Environmental Effects: In this case, the shroud was pre-exposed to a temperature of 1400 K for two years (the expected minimum life of the shroud segment). As a result, the ultimate normal strength in the two $x$, $y$ in-plane material directions as well as the three shear strengths are lowered by 24% and the $z$-direction normal strength by 6%. In accordance with the analysis presented in Section 3.4.1, no degradation of the initial CMC strength and stiffness takes place. A close-up of the shroud damaged region for this case is displayed in Figure 3-12(b). A comparison of the results displayed in Figures 3-12(a) and (b) shows that in Case B, the extent of shroud damage is noticeably higher than in Case A. Specifically, the maximum penetration depth has increased to 40% of the shroud thickness and the crater footprint has grown by approximately 15%. The latter finding is consistent with the fact that all components of the material ultimate strength have been degraded by the imposed in-service thermal conditions.

Case C – Prolonged Exposure to Thermo-Mechanical Effects in the Absence of Aggressive Environment: In this case, the shroud was pre-exposed to a temperature of 1400 K for two years, and to a constant tensile stress of 80 MPa in the nominal directions of one of the (weft or warp) fiber tows. This direction is aligned in the global $x$-direction (i.e. shroud axis) in the impacted top and flat portion of the shroud. Due to the resulting creep, the initial CMC strength in the direction of loading ($x$-direction, in this case) has decreased by 55%, while no significant degradation of the other components of the initial strength takes place. It should be noted that all components of the material strength have already degraded due to the prolonged high-temperature exposure (Case B), and that creep additionally degrades the $x$-component of the strength. Also, no matrix cracking has occurred and, thus, the stiffness has not been degraded, Section 3.4.2.
A close-up of the shroud damaged region for this case is displayed in Figure 3-12(c). A comparison of the results displayed in Figures 3-12(a)–(c) shows that in Case C, complete penetration of the shroud has occurred over a relatively small portion of the crater footprint, while over the remaining portion of the crater footprint, the penetration depth is about 50% of the shroud thickness. As far as the crater-footprint size is concerned, it is further increased, particularly in the region where the projectile exits the damaged shroud. These findings are consistent with the fact that the material ultimate strength in the general direction of projectile motion has been further degraded by the imposed in-service thermo-mechanical conditions.

**Case D – Prolonged Exposure to High Temperature and Dry-Oxidizing Environment, in the Absence of Sustained Loading:** In this case, the shroud was pre-cracked by loading it in the $x$-direction to an irreversible strain of 0.1%. Then the shroud is exposed to a temperature of 1400 K for six months (it is assumed that the shroud develops matrix cracks 1.5 years after installation), and to oxygen at a partial pressure of 0.25 atm. As a result of the prescribed pre-cracking, Section 3.4.3, the CMC elastic moduli have degraded as follows: $E_{11}$ by 0.66%, $G_{12}$ by 1.03%, $G_{13}$ by 0.99%, while the reduction in the values of the other elastic moduli is negligibly small. Additionally, as a result of the imposed thermal and dry-oxidizing environmental conditions, the in-plane (i.e. the $x$-$y$ material plane) ultimate strengths have been degraded by 29.6% (without degradation of the initial strengths), while the degradation of the corresponding flow stress is prorated accordingly, Section III.3. Again, it should be recalled that the material has already experienced strength degradation due to prolonged thermal exposure (Case B).

A close-up of the shroud damaged region for this case is displayed in Figure 3-12(d). A comparison of the results displayed in Figures 3-12(a)–(d) shows that in Case D, no complete penetration of the shroud takes place. However, over the major portion of the crater footprint, the
depth of penetration takes on the largest values in Case D. As far as the crater-footprint size for Case D is concerned, it is the largest and elongated in the general direction of projectile motion.

Case E – Prolonged Exposure to High Temperature and Wet-Oxidizing Environment, in the Absence of Sustained Loading: In this case, as in Case D, the shroud was pre-cracked by loading it in the $x$-direction to an irreversible strain of 0.1% (causing the same stiffness reductions as those identified in Case D). Then the shroud is exposed to a temperature of 1400 K for 6 months, and to water vapor at a partial pressure of 0.20 atm. As a result of the prescribed wet-oxidizing environmental conditions, the in-plane (i.e. the $x$-$y$ material plane) ultimate strengths have degraded, in addition to the degradation caused by the prolonged thermal exposure, by 40.4% (without degradation of the initial strengths), while the degradation of the corresponding flow stress is prorated accordingly, Section 3.4.4.

A close-up of the shroud damaged region for this case is displayed in Figure 3-12(e). A comparison of the results displayed in Figures 3-12(a)–(e) shows that in Case E, complete penetration of the shroud takes place over the major portion of the crater footprint. As far as the crater-footprint size for Case E is concerned, it is the largest among the five cases considered. It should be noted that despite the significant extent of damage suffered by the shroud, the projectile did not penetrate it, and was rejected. The largest extent of damage experienced by the shroud observed in Case E is consistent with the fact that wet oxidation and the accompanying material recession give rise to the largest extent of fiber-strength degradation.
Figure 3-12 Two views of the final damage produced during the same foreign-object-impact scenario for the cases of: (a) no in-service environmental exposure, the reference case; (b) prolonged high-temperature exposure in the absence of significant mechanical and aggressive-environmental effects; (c) prolonged exposure to thermo-mechanical effects in the absence of aggressive environment; (d) prolonged exposure to high temperature and dry-oxidizing environment, in the absence of sustained loading; and (e) prolonged exposure to high temperature and wet-oxidizing environment, in the absence of sustained loading. Please see text for additional details.
Figure 3-12 Continued.
3.6.3 Discussion

The results presented in the previous section clearly revealed that prolonged exposure of the gas-turbine CMC components like the shroud to the in-service thermo-mechanical and environmental conditions can seriously compromise their ability to withstand impact by foreign objects (ingested and propelled by the engine). This finding suggests that when designing CMCs for gas-turbine applications, one must address the ability of these materials to withstand thermo-mechanical and environmental in-service conditions without severe degradation. In other words, the short-term mechanical properties like strength, stiffness, fracture toughness, etc. (typically addressed in the design of the advanced structural materials) must be balanced against the environmental resistance of the same materials. Otherwise, the initial performance of the gas-turbine engine components may be quite high, but this performance may relatively quickly degrade, yielding a lower service life of the associated engine-component. In addressing these issues, the so-called materials-by-design methodology should be used. Within this methodology, a material-system-level approach is employed in order to design and fully develop new materials through the use of computer-aided engineering analyses, predictive tools and available material databases [24–30]. The material model developed in Ref. [1] and extended in the present work, as well as the associated databases, could play a major role in the application of the materials-by-design methodology to the CMCs for demanding gas-turbine engine applications. This is being demonstrated in our ongoing work, in which the materials-by-design methodology is being employed in order to significantly enhance the performance, durability and reliability of the CMCs.
3.7 Summary and Conclusions

Based on the results obtained in the present work, the following main summary remarks and conclusions can be drawn:

1. A recently-developed multi-length-scale material model for SiC/SiC ceramic-matrix composites (CMCs) has been upgraded to include the effects of prolonged exposure of the CMC gas-turbine engine components to aggressive thermo-mechanical and oxidizing in-service environments.

2. The effect of the type, intensity and duration of these aggressive thermo-mechanical and oxidizing environments was cast into the appropriate material-property-degradation model and quantified by carrying out a detailed investigation of the prior experimental findings reported in the open literature.

3. The upgraded material model was subsequently implemented into a user-material subroutine suitable for linking with a finite-element solver.

4. The utility of the upgraded material model was demonstrated by carrying out a series of finite-element analyses of a gas-turbine engine CMC shroud impacted by a foreign object (ingested and accelerated by the engine). In these analyses, the same toboggan-shaped shroud component was subjected to oblique impact by the same foreign object propelled at the same initial velocity and in the same incident direction, and the effect of prolonged exposure of the toboggan to various in-service environments on its ability to withstand the impact was investigated. The results obtained clearly revealed the role different in-service thermo-mechanical and oxidizing environments play in degrading CMCs.

5. In particular, it was found that once matrix cracking has initiated, and oxygen and water vapor can enter the interior of a CMC component, the rate of material degradation increases
sharply, resulting in a higher probability for in-service failure of the component. Also, it was found that, since fibers possess finer grains, they tend to creep more readily than the matrix, and this differential-creep phenomenon results in fiber-to-matrix load transfer and an increased likelihood for in-service matrix cracking.
3.8 References


30. M. Grujicic, V. Chenna, R. Yavari, R. Galgalikar, J.S. Snipes, S. Ramaswami, “Multi-
CHAPTER 4: MATERIAL CONSTITUTIVE MODELS FOR CREEP AND RUPTURE OF SiC/SiC CERAMIC-MATRIX COMPOSITES (CMCs) UNDER MULTI-AXIAL LOADING CONDITIONS.

4.1 Abstract

Material constitutive models for creep-deformation and creep-rupture of the SiC/SiC ceramic-matrix composites (CMCs) under general three-dimensional stress states have been developed and parameterized using one set of available experimental data for the effect of stress magnitude and temperature on the time-dependent creep deformation and rupture. To validate the models developed, another set of available experimental data was utilized for each model. The models were subsequently implemented in a user material subroutine and coupled with a commercial finite element package in order to enable computational analysis of the performance and durability of CMC components used in high-temperature high-stress applications, such as those encountered in gas-turbine engines. In the last portion of the work, the problem of creep-controlled contact of a gas-turbine engine blade with the shroud is investigated computationally. It is assumed that the blade is made of the SiC/SiC CMC, and that the creep behavior of this material can be accounted for using the material constitutive models developed in the present work. The results clearly show that the blade-tip/shroud clearance decreases and ultimately becomes zero (the condition which must be avoided) as a function of time. In addition, the analysis revealed that if the blade is trimmed at its tip to enable additional creep-deformation before blade-tip/shroud contact, creep-rupture conditions can develop in the region of the blade adjacent to its attachment to the high-rotational-speed hub.
4.2 Introduction

The present work addresses the problem of creep-deformation and creep-rupture under multiaxial loading conditions in SiC-reinforcement/SiC-matrix ceramic-matrix composites (CMCs). The main objective of the present work is to develop the appropriate creep-deformation and creep-rupture material models which, when implemented in the appropriate user-defined material subroutines and coupled with commercial, open-source or proprietary finite-element solvers, can be used to analyze creep-/creep-rupture-controlled performance and durability of various gas-turbine engine-based CMC components. Consequently, the concepts most relevant to the present work are: (a) the basics of ceramic-matrix composites; (b) creep and creep-rupture of high-temperature structural materials; (c) prior experimental work concerning creep behavior of SiC/SiC CMCs; and (d) prior modeling work concerning creep behavior of SiC/SiC CMCs. In the remainder of this section, a brief description is provided for each of these concepts.

4.2.1 The Basics of Ceramic-Matrix Composites

High-temperature metallic materials such as nickel-, cobalt- or iron-based superalloys used in gas-turbine engines have been pushed to their thermal-stability limit since they are often made to operate at temperatures which are within 50 K of their melting point. To increase power density and energy efficiency of the gas-turbine engines, new materials are needed which can operate at temperatures as high as 1500 K. The main candidate materials currently identified for use in the next generation of gas-turbine engines are (monolithic) ceramics and CMCs. Since these materials can withstand extremely high temperatures, their use in hot sections of gas-turbine engines can yield a number of benefits such as: (i) improvements in thrust and fuel efficiency; (ii) lower pollutant emissions; (iii) reduced cooling requirements; (iv) simplification of the engine-
component design; and (v) reduced requirements for the strength/weight of the supporting structure.

Unfortunately, due to their relatively low fracture toughness, tensile strength and damage tolerance, monolithic ceramics are not being perceived as respectable candidate materials for use in critical turbine-engine structural applications (e.g. turbine shroud segments, vanes and blades). On the other hand, CMCs consisting of a ceramic matrix and ceramic fibers possess superior structural properties (including non-catastrophic nature of failure) relative to their monolithic-ceramic counterparts, while retaining their high-temperature stability and integrity. This is the reason that the CMCs are being aggressively researched and developed for use in future gas-turbine engines. The potential of the CMCs in revolutionizing the performance of the gas-turbine engines is shown schematically in Figure 4-1 [1]. In this figure, the $x$-axis represents the approximate period of dominance in usage of the particular class of high-temperature materials (and associated cooling technologies), while the $y$-axis denotes the temperature capability of the material class in question. It is seen that the temperature capability of CMCs lies above the fitting line for the temperature capabilities of the past and present gas-turbine engine materials.

Initially, CMCs were perceived as candidate materials in gas-turbine engine components such as nozzles, combustion liners, shroud segments and exhaust components, which experience low in-service loads. Currently, these components have reached the commercialization stage, and are being employed in the engines of 200-seater aircraft [2]. In addition, CMCs have been successfully tested for use in low-pressure turbine blades in military aircraft engines [3]. In the case of rotating/revolving components like blades, the sustained presence of the centrifugal forces combined with high temperatures can lead to CMC excessive creep-deformation and/or creep-
rupture. Consequently, detailed understanding of the CMC-creep behavior is critical in designing such components and ensuring their required durability and reliability.

**Figure 4-1** Temperature capability (i.e. maximum service temperature) of various gas-turbine engine materials as a function of the time period of their dominant usage.

4.2.2 Creep-Deformation and Creep-Rupture of High-Temperature Structural Materials

Creep deformation is a time-dependent inelastic deformation mechanism which occurs as a result of material exposure to sustained loading and elevated temperature. Typically, creep occurs at homologous temperatures greater than approximately 0.5 (the homologous temperature is defined as the ratio of the exposure temperature and material melting point, both expressed in kelvin) and at stress levels which are lower than those required to induce inelastic deformation under monotonic loading conditions. Excessive creep deformation is typically followed by material fracture, commonly referred to as “creep rupture.” Figure 4-2 shows typical variation of
the creep strain with time. In Figure 4-2, two creep-strain vs. time curves are depicted. The lower one is associated with stress level \( \sigma_1 \) and temperature \( T_1 \), while the upper curve is associated with one of the following three conditions: (i) \( \sigma_2 > \sigma_1, T_2 = T_1 \); (ii) \( \sigma_2 = \sigma_1, T_2 > T_1 \); and (iii) \( \sigma_2 > \sigma_1, T_2 > T_1 \). Examination of Figure 4-2 reveals that a creep-strain vs. time curve consists of three segments, each reflecting a different stage of creep deformation:

(a) in the primary creep regime, Section I, the creep strain rate (numerically equal to the slope of the curve) decreases monotonically with time due to the operation of strain-hardening processes;

(b) in the steady-state or secondary creep regime, Section II, the material creeps at a constant rate due to a balance of the strain-hardening and strain-softening mechanisms; and

(c) in the final or tertiary creep regime, Section III, the creep rate increases significantly due to the operation of material-degradation processes such as cavity formation, growth and coalescence. When the material has sustained a threshold level of damage, it ruptures.

It should be noted that the x-axis in Figure 4-2 is not drawn to scale, since the steady-state regime of creep typically comprises the major portion of the total creep time before rupture (rupture time). In design applications in which excessive creep deformation is of primary concern, the steady-state or minimum creep rate at the operating stress and temperature conditions is used as the primary creep-resistance material parameter. In other applications in which creep-rupture is of key concern, rupture time is used as the primary creep-resistant material parameter. Examination of Figure 4-2 shows that as the stress and/or temperature are increased, the steady-state creep rate increases while the rupture time decreases. Thus, excessive creep deformation and creep-rupture problems are associated with higher stresses and/or higher temperatures.
As mentioned earlier, gas-turbine engine designs are driven by the demands for greater power, reduced weight, improved fuel economy, and lower life cycle costs. Higher combustion temperatures are a key to meeting these requirements. It is generally accepted that the maximum attainable performance of the gas-turbine engine is controlled by the maximum allowable temperature of the turbine rotor/blades, and that excessive creep-deformation and potential creep-rupture of the turbine blades control the maximum allowable temperature. As far as the stresses experienced by the blades are concerned, they primarily originate from three sources: (i) centrifugal forces due to high rotation speeds; (ii) bending stresses due to the momentum of the
high-pressure high-velocity combustion gases; and (iii) thermal stresses due to non-uniformity in temperature and mismatches in thermal expansion coefficients with the adjoining components.

Under normal operating conditions, creep deformation manifests itself as blade elongation. Blade elongation is monitored during operation using various sensors mounted on the engine casings, including eddy current, discharging probe, capacitive, inductive, microwave, and infrared sensors. As part of the regular inspection, blade elongation is measured and the tip of each blade trimmed to maintain the correct tip clearance. Retention of the correct tip clearance is critical since insufficient clearance may lead to the blade tip making contact with the non-rotating shroud, causing a “tip rub” which necessitates dismantling of the engine for repair and quite probably replacement of both blades and shroud. On the other hand, excessive clearance compromises gas-turbine engine efficiency since combustion gases escape from the main flow path. Following several blade-tip trimming operations, after the blades have experienced a predetermined total elongation, they are replaced in order to avoid creep-induced blade rupture (typically in the region where the blade is mounted to the high-speed rotating hub). In the case of gas-turbine engines used in aviation applications, blade inspection is carried out typically after every 2000 hours of operation, while the rotor overhaul is conducted typically after every 5000 hours. In the case of ground power or marine applications, the inspection and overhaul hours are substantially longer.

4.2.3 Prior Experimental Work Concerning Creep Behavior of SiC/SiC CMCs

An overview of the public-domain literature identified a large number of studies dealing with creep behavior of oxide-based CMCs (not the subject of the present work), but only four studies dealing with the creep behavior of SiC/SiC CMCs (the subject of the present work). In the remainder of this subsection, the latter four studies are briefly overviewed.
Corman and Luthra [1] carried out a comprehensive experimental investigation of the creep behavior of prepreg melt-infiltrated SiC/SiC CMCs under uniaxial-stress loading conditions. The results obtained are summarized in Table 4-1 and in Figure 4-3. Table 4-1 contains six columns, with each column providing the following information: (i) sample identification; (ii) test temperature (°C); (iii) applied uniaxial stress (MPa); (iv) rupture time; (v) total strain at failure or at creep run-out time of 1000 hours; and (vi) creep rate at 1000 hours (1/s), if applicable. Examination of this table reveals that:

(a) at 816 °C and applied uniaxial stresses of 125 MPa and 140 MPa, creep deformation cannot be detected. The specimen 704A-2 tested at 816 °C and 140 MPa failed not due to creep but rather due to the presence of a potent pre-existing flaw;

(b) the measured creep rates at 1000 hours are quite small, and if these creep rates are multiplied by the run-out time of 1000 hours, the resulting creep strains are substantially lower than their counterparts, obtained by subtracting from the total strains reported in the fifth column of Table 4-1 the corresponding elastic strains. This finding suggests that within the time period of 1000 hours, the subject CMC was in the primary regime of creep, the regime in which the creep rate continuously decreases as a function of time; and

(c) at the highest temperatures employed, 1093 °C and 1204 °C, the tests conducted did not conclusively identify a lower bound of stress below which creep rupture does not take place.

Figure 4-3 shows the effect of the magnitude of the applied uniaxial stress, at 1093 °C and 1204 °C, on the rupture time. As expected, it is seen that an increase in stress and/or temperature results in a lower rupture time.
Table 4-1 Summary of the tensile creep rupture testing results on GE prepreg MI composites made in July September, 2000 with GEGR optimized Configuration C fiber coating on Hi-Nicalon fiber. All testing was performed in air [1].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Temp. (°C)</th>
<th>Creep Stress (MPa)</th>
<th>Rupture time § (h)</th>
<th>Failure strain * (%)</th>
<th>Creep rate at 1000 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>730-1</td>
<td>816</td>
<td>125</td>
<td>&gt;1000</td>
<td>0.07</td>
<td>&lt;10^{-11}</td>
</tr>
<tr>
<td>704A-2a</td>
<td>816</td>
<td>140</td>
<td>460.2</td>
<td>0.05</td>
<td>–</td>
</tr>
<tr>
<td>732-2</td>
<td>816</td>
<td>140</td>
<td>&gt;1000</td>
<td>0.08</td>
<td>&lt;10^{-11}</td>
</tr>
<tr>
<td>717-2</td>
<td>816</td>
<td>140</td>
<td>&gt;1000</td>
<td>0.08</td>
<td>&lt;10^{-11}</td>
</tr>
</tbody>
</table>

| 704A-1a    | 1093       | 125                 | >1000               | 0.03                 | 2.6 \times 10^{-11}       |
| 704A-4a    | 1093       | 140                 | >1000               | 0.16                 | 1.9 \times 10^{-11}       |
| 732-3      | 1093       | 140                 | >1000               | 0.14                 | 1.1 \times 10^{-10}       |
| 732-7      | 1093       | 140                 | >1000               | 0.13                 | 8.0 \times 10^{-11}       |
| 717-1      | 1093       | 150                 | 394                 | 0.20                 | –                           |
| 732-4      | 1093       | 150                 | 190                 | 0.14                 | –                           |
| 704B-2a    | 1093       | 150                 | 18.1                | 0.12                 | –                           |
| 730-8      | 1093       | 160                 | 15.4                | 0.12                 | –                           |
| 704B-1a    | 1093       | 160                 | 1.45                | 0.10                 | –                           |

a Panel 704 samples were slightly warped, possibly contributing to the relatively low fatigue lives.

§ A value of >1000 indicates a run-out, e.g. the sample survived the full 1000 hours without rupture.
* Strain at failure or strain at 1000 hr if sample was a run-out.

Table 4-1. Continued

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Temp. (°C)</th>
<th>Creep Stress (MPa)</th>
<th>Rupture time § (h)</th>
<th>Failure strain * (%)</th>
<th>Creep strain at 1000 hours</th>
</tr>
</thead>
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<td>&gt;1000</td>
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<td>$3.0 \times 10^{-11}$</td>
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<tr>
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<td>&gt;1000</td>
<td>0.20</td>
<td>$8.0 \times 10^{-11}$</td>
</tr>
<tr>
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<td>140</td>
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<tr>
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<td>0.18</td>
<td>–</td>
</tr>
</tbody>
</table>

* Panel 704 samples were slightly warped, possibly contributing to the relatively low fatigue lives.

§ A value of >1000 indicates a run-out, e.g. the sample survived the full 1000 hours without rupture.

* Strain at failure or strain at 1000 hr if sample was a run-out.
Morscher and Pujar [4] carried out a comprehensive experimental creep testing of SiC/SiC CMCs at 1315 °C, at uniaxial stresses ranging between 100 and 170 MPa, and at a creep run-out time of 100 h. The results obtained revealed a superior creep resistance of the subject materials relative to the competing grades of the CMCs. In addition, post-creep fast-fracture testing of the CMCs revealed that creep deformation within the CMCs slightly increased the first matrix-cracking stress. This finding was attributed to the effect of the stress redistribution between the matrix and the fibers, which reduces the tensile-stress level within the matrix and, hence, delays the onset of matrix cracking.
Morscher et al. [5] also conducted a comprehensive set of creep as well as dwell fatigue and cyclic fatigue experiments on SiC/SiC CMCs consisting of Sylramic-iBN SiC fibers, BN fiber interphase coating, and slurry-cast melt-infiltrated (MI) SiC-based matrix at 1204 °C, in a 180–220 MPa stress range in air. Specimens which did not fail during fatigue testing were subsequently fast-fractured in order to assess the extent of creep-induced material degradation. The results obtained revealed losses of ca. 20% in strength and 10% in stiffness after creep for 100 hours at the highest stress levels applied and after 2000 hours at the lowest stresses applied. The previously mentioned creep-induced increase in the first matrix-cracking stress was again observed.

van Roode et al. [6] investigated creep behavior of two grades of SiC/SiC CMCs, one based on the Hi-Nicalon fibers (possess inferior creep resistance due to nanometer grain size and lack of stoichiometry) and the other based on Sylramic-iBN fibers (possess superior creep resistance due to substantially larger grain size and higher degrees of stoichiometry and crystallinity). The results obtained revealed that in the absence of strong oxidizing-environmental conditions the latter grade of CMCs possesses superior creep-rupture properties, while in the presence of such conditions, creep-rupture properties of the two CMC grades are quite comparable. In addition, it is found that if the CMCs are protected using environmental barrier coatings, the CMC grade based on Sylramic-iBN fibers again displays superior creep-rupture properties even in the presence of strong oxidizing environmental conditions.

4.2.4 Prior Modeling Work Concerning Creep Behavior of SiC/SiC CMCs

An overview of the public-domain literature identified four studies dealing with the modeling of the creep behavior of SiC/SiC CMCs. These four studies are briefly reviewed in the
remainder of this subsection, and the main contributions and shortcomings of these studies have
been identified.

Rospars et al. [7] experimentally and computationally investigated creep behavior of
plain-woven SiC/SiC CMCs, and tried to formulate a continuum damage-mechanics based
material model for creep-induced material damage. To quantify the extent and diversity of local
damage, damage variables are introduced for each mode of material damage. Creep deformation
is modeled as a time-dependent inelastic deformation process, and particular attention is devoted
to identifying the appropriate creep-deformation potential, the gradient of which is used within
the flow-rule functional relationship to model the progress of creep. The main emphasis in the
work of Rospars et al. [7] was on deriving and parameterizing the creep-induced damage-
parameter evolution equations. However, no attempt was made to either predict the creep rupture
as a function of the imposed multiaxial stress and temperature conditions, or to derive a creep-
deformation material constitutive model for the general case of multiaxial loading.

Rugg et al. [8] investigated the creep behavior of SiC/SiC CMC microcomposites
consisting of a single SiC fiber coated with carbon and surrounded with SiC matrix, in the
absence and presence of matrix cracking. Creep behavior of the microcomposite was directly
compared with that of the fiber and the matrix. In order to relate the creep behavior of the
microcomposite to that of the fiber and the matrix, simple phenomenological models have been
developed. While the work of Rugg et al. [8] provides a new insight into the creep behavior of the
CMCs, which is not generally gained in the case of bulk CMCs, the models developed could not
be readily applied to bulk CMCs, even under uniaxial-loading conditions.

Chermant et al. [9] carried out a combined experimental and theoretical investigation of
the creep mechanism(s) in CMCs with both crystalline-ceramic and glassy-ceramic matrices. The
results obtained suggest that in chemical vapor infiltrated (CVIed) CMCs with a crystalline-ceramic matrix like SiC/SiC CMCs, the creep deformation and rupture involve the operation of the following time-dependent processes: (a) matrix cracking; (b) matrix/fiber interfacial shear and debonding; and (c) conventional creep of the fibers bridging the matrix cracks. The experimental results clearly established that the tertiary regime of creep is absent in SiC/SiC CMCs, i.e. creep rupture occurs in the secondary regime of creep. To model the creep-deformation and ultimate creep-rupture processes, a damage-mechanics framework was utilized, within which damage variables involved during creep, are utilized to quantify the extent of the local creep-induced material degradation. While the work of Chermant et al. [9] provides a good insight into the mechanisms responsible for creep deformation and rupture in SiC/SiC CMCs, no attempt was made to develop a continuum-type creep material model which can be used within a finite element computational framework to analyze the performance and durability of CMC components operating under aggressive high-temperature high-stress conditions such as those encountered in gas-turbine engines. An example of the experimental results pertaining to the effect of temperature and stress on the temporal evolution of the creep strain as reported in Ref. [9] is depicted in Figure 4-4. Close examination of the results depicted in this figure reveals that the transition from the primary to secondary creep takes place at a time around 20 hours, and this transition time is not a sensitive function of either the temperature or stress.
Grujicic et al. [10] investigated computationally creep-induced degradation of the mechanical properties of CMCs. This degradation is caused by the fact that the CMC constituents possess different creep tendencies, and in order for them to acquire a common creep rate, undesirable load transfer typically occurs between fibers and the matrix. As a result, for the case of the SiC/SiC CMCs in which the matrix possesses superior creep resistance due to a larger grain size and high degree of stoichiometry and crystallinity, the first matrix-cracking stress (a measure of the CMC strength) is lowered [11]. In addition, if the reduction in the CMC strength is large enough to cause material cracking, stiffness degradation ensues [12]. The results obtained are used to enrich the SiC/SiC CMC material constitutive model suitable for the analysis of the
response of this material under monotonic-loading conditions [13]. In other words, the effect of creep-induced material degradation was incorporated into the material constitutive model. However, the model was of a time-invariant character, and hence, it is not suitable for modeling either creep deformation or creep rupture.

4.2.5 Main Objective

The main objective of the present work is to formulate, parameterize and validate the SiC/SiC-based CMC material models for: (a) creep-deformation; and (b) creep-rupture, under multiaxial loading conditions. The models to be developed will be implemented in a material user subroutine, coupled with a commercial finite element solver, and used in a finite element analysis of a prototypical gas-turbine engine creep deformation/rupture problem, i.e. creep-induced undesirable contact of a turbine-blade tip with the shroud.

4.2.6 Chapter Organization

The creep-deformation and creep-rupture models are formulated, parameterized and validated in Section 4.3. Details regarding the finite-element analysis of the problem involving turbine-blade tip contact with the shroud, including the main results, are presented and discussed in Section 4.4. A summary of the main conclusions resulting from the present work is given in Section 4.5.
4.3 CMC Creep-Models Development

In this section, details are presented regarding the formulation, parameterization and validation of the creep-deformation and creep-rupture material models for SiC/SiC CMCs. For clarity, the creep-deformation and creep-rupture models are presented separately.

4.3.1 Creep-Deformation Material-Model

The results previously presented in Figure 4-4 established that the transition between the primary and secondary creep regimes takes place at a time \( t^* \) which is fairly insensitive to the test temperature and applied stress. For simplicity, \( t^* \) is assumed to be constant and set to its average value of 20 h. Furthermore, it was previously established that in the case of SiC/SiC CMCs (the material under investigation), the tertiary stage of creep is generally not observed [9].

Taking this into consideration, it was decided to first develop a time-dependent creep-deformation model for the primary regime of creep, and then to use the creep-strain-rate predictions of this model at \( t^* \) as a measure of the steady-state creep rate. In other words, unlike the case of the primary-creep regime, the secondary creep regime will not be modeled separately from the first stage of creep.

**Creep-Deformation Material-Model Formulation:** For isotropic materials tested under uniaxial-stress \( \sigma \) creep conditions, the primary-creep time-dependent axial-strain rate \( \dot{\varepsilon}^{cr} \) is generally represented by the following function:

\[
\dot{\varepsilon}^{cr} = A e^{-Q/RT} \left( \frac{\sigma}{\sigma_0} \right)^n t^m
\]

(4.1)
where \( A, Q \) (the activation energy), \( \sigma_0, m \) and \( n \) are material-specific parameters, \( R \) is the universal gas constant, \( T \) is the absolute temperature, and \( t \) is the time.

Under multiaxial-loading conditions, which yield a multidimensional creep-strain state, Eq. (4.1) is expanded for isotropic materials as:

\[
\dot{\varepsilon}^{cr} = A e^{-Q/RT} \left( \frac{q}{\sigma_0} \right)^n t^m
\]  
(4.2)

where \( \dot{\varepsilon}^{cr} \) is the equivalent creep strain rate defined as

\[
\dot{\varepsilon}^{cr}(\dot{\varepsilon}^\sigma) = \sqrt{\frac{2}{3}} \dot{\varepsilon}^\sigma : \dot{\varepsilon}^{cr}
\]  
(4.3)

Time integration of Eq. (4.2) yields

\[
\varepsilon^{cr} = A e^{-Q/RT} \left( \frac{q}{\sigma_0} \right)^n \frac{t^{m+1}}{m+1}
\]  
(4.4)

and the equivalent stress \( q \) is given by the von Mises relation:

\[
q(\sigma) = \sqrt{\frac{1}{2} \left[ (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) \right]}
\]  
(4.5)

It should be noted that boldface symbols are used to denote second-order tensorial quantities, while the : is used to denote an inner-product operator.

In the case of orthotropic materials, or more specifically transversely isotropic materials like CMCs, Eq. (4.2) is still generally accepted but \( \dot{\varepsilon}^{cr} \) and \( q \) are redefined to account for the reduced degree of material isotropy. Specifically, in the case of the CMCs analyzed here, the von Mises equivalent stress is replaced with the Hill’s potential in the form:
\[ q(\sigma) = \sqrt{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2} \quad (4.6) \]

where the dimensionless constants \( F \), \( G \), \( H \), \( L \), \( M \), and \( N \) are functions of the uniaxial and pure-shear material strengths, and are determined by carrying out three uniaxial-tensile tests and three pure-shear tests (each associated with a unique material direction/plane). If a nominal strength measure \( \sigma^0 \) is introduced and the ratios of the six individual material strengths (\( \overline{\sigma}_y \)) and \( \sigma^0 \) are defined as \( R_{11} = \frac{\overline{\sigma}_{11}}{\sigma^0}, R_{22} = \frac{\overline{\sigma}_{22}}{\sigma^0}, R_{33} = \frac{\overline{\sigma}_{33}}{\sigma^0}, R_{23} = \frac{\overline{\sigma}_{23}}{\sigma^0/\sqrt{3}}, R_{31} = \frac{\overline{\sigma}_{31}}{\sigma^0/\sqrt{3}}, \) and
\[ R_{12} = \frac{\overline{\sigma}_{12}}{\sigma^0/\sqrt{3}}, \]
then the Hill’s equivalent stress potential becomes:

\[ q(\sigma) = \frac{1}{2} \left[ \frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2} \right] (\sigma_{22} - \sigma_{33})^2 + \frac{1}{2} \left[ \frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2} \right] (\sigma_{33} - \sigma_{11})^2 \]
\[ + \frac{3}{R_{23}^2}\sigma_{23}^2 + \frac{3}{R_{31}^2}\sigma_{31}^2 + \frac{3}{R_{12}^2}\sigma_{12}^2 \]

\[ \gamma^{1/2} \quad (4.7) \]

For transversely isotropic materials like CMCs with a unique material direction \( x_3 \), only four independent strength ratios are required since \( R_{11} = R_{22} \) and \( R_{23} = R_{31} \). However, since the material is isotropic in the \( x_1-x_2 \) plane and, thus, \( q \) must be invariant for a rotation about the \( x_3 \)-axis, one can easily show that \( R_{12} \) is not an independent strength ratio but rather defined as


\[ R_{12} = \frac{3}{\left( \frac{3}{R_{22}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{33}^2} \right)^{1/3}} \]. Thus, for a transversely isotropic material, there are only three independent strength ratios \((R_{11}, R_{33}, R_{23})\) and, if \(\sigma^0\) is set to \(\bar{\sigma}_{11}\), then \(R_{11} = 1\).

As far as the equivalent creep-strain rate for a transversely-isotropic material is concerned, its functional form is constructed by employing the work-rate equivalency as:

\[
\frac{1}{2} \sigma : \dot{\varepsilon}^c = \frac{1}{2} q \cdot \dot{\varepsilon}^c
\]

This procedure results in the following general functional form:

\[
\dot{\varepsilon}^c(\varepsilon^c, \sigma, R_{11}, R_{33}, R_{23}) = \sqrt{f(\sigma, R_{11}, R_{33}, R_{23})} \dot{\varepsilon}^c : \dot{\varepsilon}^c
\]

Examination of Eq. (4.2) and the recognition that a transversely isotropic material has three unique strength ratios, suggests that the complete definition of a time-dependent first-stage creep-deformation law for a transversely isotropic material requires specification of the following seven material-dependent parameters: \(A, Q, n, m, R_{11}, R_{33}\) and \(R_{23}\) (\(\sigma^0\) is not a material parameter and is set arbitrarily to 1 MPa, but it should be noted that the chosen value of \(\sigma^0\) affects the value of the parameter \(A\)). In the next subsection, a procedure will be described for evaluating these parameters using the experimental results reported in Ref. [9] and replicated in Figure 4-4. This will be followed by a procedure used to validate the formulated and parameterized model against the experimental results reported in Refs. [9,1].

**Creep-Deformation Material-Model Parameterization:** To parameterize the creep-deformation material model formulated in the previous subsection, the portion of the results depicted in Figure 155.
pertaining to the primary regime of creep are utilized. It should be noted that the creep-strain
data given in Figure 4-4 pertain only to the strain component in the direction of the applied
uniaxial stress. However, the material model formulated in the previous section, Eq. (4.4),
requires the knowledge of the equivalent creep strain, i.e. the knowledge of the lateral creep
strains. To overcome this problem, the lateral creep strains were obtained by first computing the
two plastic Poisson’s ratios \( \nu_{12}^p \) and \( \nu_{13}^p \). The two plastic Poisson’s ratios are obtained from the
following two conditions: (i) \( \nu_{12}^p + \nu_{13}^p = 1.0 \) (the volume constancy condition); and (ii)
\( \nu_{12}^p R_{22} = \nu_{13}^p R_{33} \) (the lateral strain scales inversely with the corresponding material strength).
Once the two plastic Poisson’s ratios are obtained, the corresponding lateral creep strains are
computed as \( \dot{\varepsilon}_{22}^{cr} = -\nu_{12}^p \dot{\varepsilon}_{11}^{cr} = -\frac{1}{1 + \frac{R_{11}}{R_{33}}} \dot{\varepsilon}_{11}^{cr} \) and as \( \dot{\varepsilon}_{33}^{cr} = -\nu_{13}^p \dot{\varepsilon}_{11}^{cr} = -\frac{1}{1 + \frac{R_{33}}{R_{11}}} \dot{\varepsilon}_{11}^{cr} \). It
should be noted that analogous relationships exist between the two lateral creep rates and the
axial creep rate, and without loss of generality, \( R_{11} \) can be set to 1.0.

Thus, at a given level of the axial creep stress, once the three strength ratios are
guessed/assumed, the corresponding equivalent creep strain can be determined using the
integrated form of Eq. (4.9), provided the functional form for \( f(\sigma, R_{11}, R_{33}, R_{23}) \) is known. To
determine the functional form for \( f(\sigma, R_{11}, R_{33}, R_{23}) \) for the special case of uniaxial-stress
loading in the \( x_1 \)-direction, Eq. (4.8) is first invoked in conjunction with Eq. (4.7) to yield
\( \dot{\varepsilon}_{11}^{cr} = \dot{\varepsilon}_{11}^{cr} R_{11} \) or in the integrated form, \( \varepsilon_{11}^{cr} = \varepsilon_{11}^{cr} R_{11} \). If the left-hand side of Eq. (4.9) is written
as \( \dot{\varepsilon}_{11}^{cr} R_{11} \), and the following substitution is carried out on the right-hand side of Eq. (4.9):
\[
\dot{\varepsilon}_{22}^{cr} = -\frac{1}{1 + \frac{R_{11}}{R_{33}}} \dot{\varepsilon}_{11}^{cr}, \quad \dot{\varepsilon}_{33}^{cr} = -\frac{1}{1 + \frac{R_{33}}{R_{11}}} \dot{\varepsilon}_{11}^{cr}, \quad \dot{\varepsilon}_{12}^{cr} = \dot{\varepsilon}_{23}^{cr} = \dot{\varepsilon}_{13}^{cr} = 0,
\]
\[ f(\sigma, R_{11}, R_{33}, R_{23}) = \frac{(R_{11})^2}{1 + \left(\frac{v_{12}^p}{v_{13}^p}\right)^2} = \frac{(R_{11})^2}{1 + \left(1 + \frac{R_{11}}{R_{33}}\right)^2 + \left(1 + \frac{R_{33}}{R_{11}}\right)^2}. \]

Examination of this expression for \( f(\sigma, R_{11}, R_{33}, R_{23}) \) reveals that in the case of an isotropic material, when \( R_{11} = R_{33} = 1.0 \), \( f(\sigma, R_{11}, R_{33}, R_{23}) = 2/3 \) as expected.

Before the parameterization procedure could be employed, the following was established:

(i) \( R_{11} \) was set to 1.0;

(ii) since, as shown above, the relationship between the equivalent creep rate and uniaxial creep rate does not involve \( R_{33} \), \( R_{33} \) could not be obtained using the uniaxial-creep data and will have to be determined separately. The procedure used for evaluation of \( R_{33} \) will be presented later;

(iii) since the creep data given in Figure 4-4 do not involve any shear component of the creep strain, strength ratio \( R_{23} \) also cannot be directly evaluated using the results presented in this figure. The procedure used for evaluation of \( R_{23} \) will be presented later.

Determination of the remaining four parameters \( (A, Q, n \text{ and } m) \) pertaining to the primary-creep-deformation model for SiC/SiC CMCs is carried out as a constrained single-objective-function optimization problem in which the material parameters act as design variables. The constraints imposed on the parameters required that all the parameters except \( m \) be positive (to yield a positive value of the equivalent creep strain) while \( m \) was required to remain negative (in order to reflect a decrease in the primary creep-deformation rate with time).

The general form of the objective function to be minimized can be written as:
\[ \text{obj\_func} = \sum_{I} \sum_{i} \left[ \varepsilon_{I,i}^{cr} - A e^{-Q/RT_I} \left( \frac{q_I(\sigma, R_{11}, R_{33}, R_{23})}{\sigma_0} \right)^n \frac{t_{I}^{m+1}}{m+1} \right]^2 \]  

(4.10)

where summation index \( I = 1−4 \) and corresponds to the four temperature/axial-stress combinations given in Figure 4-4, while the summation index \( i = 1−9 \) corresponds to the data points in the primary-creep section of each of the four curves in the same figure.

The optimization procedure described above yielded the following optimal values of the four primary-creep SiC/SiC CMC material-model parameters: \( A = 1.169 \cdot 10^{-10} \) \( 1/\text{s}^{0.4672} \), \( Q = 99.492 \) kJ/mol, \( n = 3.8147 \) and \( m = -0.5328 \). The remaining primary-creep material-model parameters appearing in Eq. (4.10) are set as follows: (a) \( R_{11} = 1.0 \); (b) \( R_{33} = R_{11} \cdot \left( \frac{\sigma_{33}}{\sigma_{11}} \right) = 1.0 \cdot 0.54 = 0.54 \), where a typical value for the through-the-thickness to in-plane strength ratio of 0.54 is assumed; (c) \( R_{23} = R_{12} = \sqrt{ \frac{3}{R_{22}^2 + \frac{1}{R_{11}^2} - \frac{1}{R_{33}^2}} } = 2.29 \); and (d) \( \sigma_0 = 1.0 \) MPa. It should be noted that the assigned values for these four parameters will not affect the outcome of the material-model validation presented in the next subsection. However, \( R_{33} \) and \( R_{23} \) may have some effect on the results obtained in the last portion of the present work dealing with the creep-deformation response of a gas-turbine engine blade.
Creep-Deformation Material-Model Validation: Validation of the primary-creep-deformation model for SiC/SiC CMCs formulated and parameterized in the previous two subsections has been carried out in two steps:

(a) Within the first step of validation, the “goodness of fit” was assessed by comparing the “experimental” equivalent creep-strain data ($\varepsilon_{i,i}^{cr}$ in Eq. (4.10)) and their computed counterparts ($A e^{-Q/RT_i} \left( \frac{q_t}{\sigma_0} \right)^n \frac{t_i^{m+1}}{m+1}$, also in Eq. (4.10)) corresponding to identical conditions of applied temperature, axial stress and time. The outcome of this procedure is depicted using the correlation plot shown in Figure 4-5(a). Application of a linear regression analysis to the scatter-plot results yields $y = 0.995x - 3.565 \cdot 10^{-5}$, depicted in this figure as a solid line, suggesting that the parameter-identification procedure employed in the previous section was quite successful in accounting for the measured primary-creep behavior of the SiC/SiC CMCs.

To further quantify the goodness of fit, a bar graph of the distribution of the relative error, defined as the difference between the calculated and experimental equivalent creep strain divided by the half-sum of these strains for the non-zero equivalent creep strain data shown in Figure 4-5(a), is shown in Figure 4-5(b). Since examination of this figure reveals that for 28 out of 32 (nonzero creep-strain) data points, the magnitude of the relative error is less than 3.5%, the parameter-identification procedure can be considered as reasonably successful; and

(b) In this portion of the creep-model validation, the model predictions for the steady-state creep rate are compared with their experimental counterparts (the data not used in the
Figure 4-5(a). Correlation plot of experimental [9] vs. computed primary-creep-strain data, corresponding to identical conditions of applied temperature, axial stress and time. The linear regression line $y = 0.995x - 3.565 \cdot 10^{-5}$ is also shown; and (b) distribution of the relative error in primary equivalent creep-strain.
parameter-identification analysis) as reported in Refs. [9,1]. As mentioned earlier, the steady-state creep-rate model predictions are obtained by evaluating the creep rate at $t^* = 20\,\text{h}$ using Eq. (4.2) and the values of the time-dependent creep-deformation model parameters given earlier. As far as the experimental values of the equivalent creep-strain rates are concerned, they are evaluated differently using the data provided in Refs. [9,1]. In Ref. [9], as shown in Figure 4-4, uniaxial creep strain vs. time data are given in the steady state creep-deformation regime, $t^* > 20\,\text{h}$. Consequently, a linear regression analysis was applied to each set of steady state creep-deformation data shown in Figure 4-4 and the resulting slope used as a measure of the corresponding average uniaxial steady-state creep rate. In the case of Ref. [1], Table 4-1, uniaxial creep rates were directly provided for the four test conditions corresponding to the full factorial design of experiments, associated with two levels of temperatures, $T = 1093\,^\circ\text{C}$ and $T = 1204\,^\circ\text{C}$, and two levels of uniaxial stress, $\sigma = 125\,\text{MPa}$ and $\sigma = 140\,\text{MPa}$, and a creep run-out time of 1000 hours. The uniaxial steady-state creep-rate data obtained from both Refs. [9] and [1] are converted to their corresponding equivalent steady-state creep rates using the procedure explained above.

The success of the time-dependent creep-deformation model in accounting for the observed steady-state creep behavior of the SiC/SiC CMCs is assessed in Figure 4-6(a), in which the measured equivalent steady-state creep rates, $\dot{\varepsilon}_{\text{stead, exper}}^{\text{cr}}$, as evaluated using the creep-strain data reported in Ref. [9] and reproduced in Figure 4-4, are plotted along the $x$-axis and the corresponding model predictions, $\dot{\varepsilon}_{\text{stead, model}}^{\text{cr}}$, along the $y$-axis. The experimental creep-rate data reported in Ref. [1] are not included in Figure 4-6(a) since their magnitudes are one to two orders of magnitude lower than those obtained in Ref. [9] and, hence, in a linear-linear plot like the one
Figure 4-6(a). Correlation plot of experimental [9] vs. computed steady-state creep-rate data. The linear regression line $y = 1.29x + 6.28 \cdot 10^{-9}$ is also shown; and (b) correlation plot redrawn as a log-log plot, with the steady-state equivalent creep-strain results as reported in Ref. [1] superposed.
shown in this figure, the latter data are clustered at the plot origin. Application of a linear regression analysis to the scatter-plot results in Figure 4-6(a) yields $y = 1.29x + 6.28 \cdot 10^{-9}$, depicted in this figure as a solid line, suggesting that the use of the primary-creep equivalent rate at $t^* = 20\, h$ as a measure of the steady-state creep rate yields reasonable predictions when compared to their measured counterparts as reported in Ref. [9].

To overcome the aforementioned problem related to the substantially lower values of the steady-state creep rate as reported in Ref. [1], the correlation plot along with the correlation line originally shown in Figure 4-6(a) is redrawn as a log-log plot in Figure 4-6(b) and the steady-state creep-rate results as reported in Ref. [1] are superposed. Examination of the results displayed in Figure 4-6(b) reveals that the present creep-deformation material model is less successful in accounting for the steady-state creep-rate data reported in Ref. [1]. There may be several reasons for this discrepancy, the primary of which could be identified as:

(a) In the case of the SiC/SiC CMCs creep tested in Ref. [9], the SiC-matrix was processed via CVI, resulting in a finer grain- and, thus, more creep-prone microstructure. In Ref. [1], on the other hand, the SiC-matrix was fabricated using the molten-silicon infiltration process [e.g. 16, 17], yielding a coarser grain- and, thus, more creep-resistant matrix microstructure;

(b) While CMC used in both studies contained Hi-Nicalon fibers, in the case of Ref. [9], the fibers were coated with an unspecified thin layer of pyrolytic carbon, while in the case of Ref. [1], fibers were coated with a BN/C/Si$_3$N$_4$/C four-layer coating with carefully controlled thicknesses of each coating layer;

(c) The volume fraction of the fibers was not specified in Ref. [9], while in Ref. [1] it was specified as 32 vol. %;
(d) In Ref. [9], CMCs possessed a 2.5-D ply architecture, while in Ref. [1] the CMC had a cross-ply architecture; and

(e) The geometries and sizes of the creep-test specimens, as well as the experimental setups used to carry out the creep experiments, were substantially different in the two studies.

The findings obtained in the present subsection suggest that:

(i) the present creep-deformation material model can be utilized with relatively high fidelity for the material the data of which are used to parameterize the model, to predict the aspects of the creep behavior of the same material which were not used in the material-model parameterization; and

(ii) the model is less successful when applied to the similar grades of CMCs, but with differing fiber- and ply-level composite-material architectures, processing routes and testing conditions. This finding is fully justified since it is well-established [1] that the composite-material architectures and processing routes have a major effect on the microstructure, and thus, properties (including the creep-deformation behavior) of the CMCs.

4.3.1 Creep-Rupture Material-Model

As in the case of the creep-deformation model, development of the creep-rupture model will involve three separate steps: (i) formulation; (ii) parameterization; and (iii) validation.
Creep-Rupture Material-Model Formulation: A creep-rupture material model is typically formulated using a functional form which relates the time to rupture to the applied stress and temperature. This approach is adopted in the present work. Furthermore, the following functional form for the creep-rupture time, related to the steady-state creep-rate functional form, is adopted.

\[
\tau_r = A_r e^{q R T / \sigma_0} \left( \frac{q}{\sigma_0} \right)^{n_r}
\]  

(4.11)

where subscript \( r \) is used to denote creep-rupture material-model parameters, and \( q \) again denotes the Hill’s potential as given by Eq. (4.6).

Examination of Eq. (4.11) and the recognition that the strength ratios \( R_{11}, R_{33} \) and \( R_{23} \) were already determined, and that \( \sigma^0 \) was set to 1 MPa, suggests that the complete definition of the creep-rupture model involves specification of the following three additional parameters: \( A_r, Q_r \) and \( n_r \). In the next subsection, a procedure will be described for evaluating these parameters using the experimental results (pertaining to the effect of the applied uniaxial-stress and temperature on the rupture-time) reported in Ref. [9] and replicated in Figure 4-4. This will be followed by a procedure used to validate the formulated and parameterized model against the experimental results reported in Ref. [1].

Creep-Rupture Material-Model Parameterization: To parameterize the creep-rupture material model formulated in the previous subsection, the results depicted in Figure 4-4 pertaining to the creep-rupture time are utilized. Determination of the remaining three parameters (\( A_r, Q_r \) and \( n_r \)) pertaining to the creep-rupture model for SiC/SiC CMCs is carried out as a constrained single-objective-function optimization problem in which the material parameters act as design
variables. The constraints imposed on the parameters required that $A_r$ and $Q_r$ be positive (to yield a positive value of the rupture time) while $n_r$ was required to be negative (in order to reflect a decrease in the rupture time with an increase in the applied stress).

The general form of the objective function to be minimized can be written as:

$$\text{obj_func} = \sum_{I} \left[ \tau_{r,I} - A_r e^{Q_r/RT_I} \left( \frac{q_I(\sigma,R_{11},R_{33},R_{23})}{\sigma_0} \right)^{n_r} \right]^2$$

(4.12)

where summation index $I = 1-4$ and corresponds to the four temperature/axial-stress combinations given in Figure 4-4.

The optimization procedure described above yielded the following optimal values of the three creep-rupture SiC/SiC CMC material-model parameters: $A_r = 7.57 \cdot 10^5$ h, $Q_r = 95.007$ kJ/mol, and $n_r = -3.26$. It should be noted that the activation energy obtained for the creep-rupture process (95.007 kJ/mol) is quite comparable to its counterpart for the creep deformation process ($Q = 99.492$ kJ/mol).

Creep-Rupture Material-Model Validation: As in the case of the primary-creep-deformation model, validation of the creep-rupture model will be carried out in two steps:

(a) Within the first step of validation, the “goodness of fit” was assessed by comparing the “experimental” creep-rupture times taken from Figure 4-4 (the data used in the material-model parameterization) against the model predictions. The outcome of this procedure is depicted using the correlation plot shown in Figure 4-7(a). Application of a linear regression analysis to the scatter-plot results shown in this figure yields $y = 1.0012x - 2.43$ (h), depicted in this figure as a solid line, suggesting that the parameter-identification procedure employed in the
previous section was quite successful in accounting for the measured creep-rupture time of the SiC/SiC CMCs; and

(b) In this portion of the creep-rupture-model validation, the model predictions for the rupture time are compared with their experimental counterparts from Ref. [1], as replicated in Table 4-1 (the data not used in the parameter-identification analysis). It should be noted that in this portion of the creep-rupture material-model validation, only eight data points from Table 4-1 pertaining to the rupture-times shorter than the creep-runout time of 1000 hours were used. It should also be noted that these eight data points correspond to four different temperature-stress conditions. The results of this comparison are depicted as an eight-point scatter plot in Figure 4-7(b), in which the experimental creep-rupture data taken from Table 4-1 are plotted along the x-axis, while the corresponding model predictions, as given by Eq. (4.11), are plotted along the y-axis. Also shown in Figure 4-7(b) is a solid line corresponding to the perfect experiment/model correlation (i.e. a line with a slope of 1.0 and an intercept of 0.0). Examination of the results depicted in Figure 4-7(b) reveals that the present creep-rupture material model is not very successful in accounting for the measured creep-rupture times, as reported in Ref. [1]. This finding again is not that unexpected, considering the fact that: (i) the experimental creep-rupture results reported in Ref. [1] under identical temperature-stress conditions could differ by as much as one to two orders of magnitude; and (ii) as in the case of the steady-state creep rate, creep-rupture time is expected to be a sensitive function of the CMC architecture, material microstructure and processing route, and these are substantially different for the CMCs used in Refs. [9] and [1].
Figure 4-7 (a) Correlation plot of “experimental” vs. computed creep-rupture times. The linear regression line $y = 1.0012x - 2.43$ (h) is also shown; and (b) scatter plot of the experimental creep-rupture data taken from Table 4-1 against the corresponding model predictions, as given by Eq. (4.11). A solid line $y = x$ corresponding to perfect experiment/model correlation is also shown.
4.3 Application Of The Creep Models To A Turbine Blade

The creep-deformation and creep-rupture models for SiC/SiC CMCs developed in the previous section are subsequently implemented in the appropriate user-material subroutine and coupled with a finite element solver so that the problem of the engineering performance and durability of a gas-turbine engine component can be investigated computationally. In this section, an example is given for the use of these models within a finite-element-based computational analysis of the performance and durability of a prototypical blade located within the low-pressure section of a gas-turbine (aviation) engine.

4.3.1 Problem Formulation

The problem analyzed here involves a prototypical blade located within the low-pressure section of a gas-turbine (aviation) engine. The blade is attached to a high-speed (10,000 rpm) rotating shaft and has a small (ca. 0.25 mm) clearance between its tip and the shroud. A schematic containing a single blade attached to the hub and a single shroud-segment is depicted in Figure 4-8. In the course of normal operation, the combination of high temperatures and high centrifugal forces will cause the blade to creep outward (in the radial direction of the gas-turbine engine) and the blade-tip/shroud clearance to decrease and ultimately become zero. To prevent the resulting undesirable blade-tip rub of the shroud, the gas-turbine engine operation will have to be temporarily halted and the blade-tip trimmed (as needed). This procedure could be repeated a few times to extend the life of the blade. However, eventually the extent of creep-deformation within the blade will become excessive particularly in the blade-root region, i.e. in the region where the blade is attached to the hub, and the blade may rupture. Since blade rupture is not permitted, the rupture time of the blade material adjacent to the hub controls the useful life of the blade. The creep-deformation material model developed in the previous section enables the prediction of the
maintenance schedule of the gas-turbine engine, while the creep-rupture model can be used to predict the useful life of the blade. These two aspects of the blade performance and durability are addressed in the remainder of this section. However, trimming of the blade-tip will not be modeled explicitly. Instead, once the condition for the blade-tip rub has been attained for the first time, the contact between the blade-tip and the shroud-segment will be suppressed so that the blade could continue to creep (while “penetrating” the shroud). Once the blade has experienced sufficient amount of creep, the conditions for the creep-rupture will be attained at its root, enabling blade-rupture process to be modeled explicitly.

Figure 4-8 Geometrical model employed in the present work, consisting of three distinct components: (a) hub; (b) blade; and (c) a single shroud-segment. Key dimensions of the three components are indicated.
4.3.2 Finite Element Analysis

A prototypical finite element analysis of the performance and durability of a structural component like a gas-turbine engine blade, in general, requires specification of the following: (a) geometrical model; (b) meshed model; (c) computational algorithm; (d) initial conditions; (e) boundary conditions; (f) contact interactions; (g) material models; and (h) computational tool. These items are briefly discussed in the remainder of this subsection.

Geometrical Model: The geometrical model employed in the present work is shown in Figure 4-8. It consists of three distinct components: (a) hub; (b) blade; and (c) a single shroud-segment. Key dimensions of the three components are indicated in Figure 4-8.

Meshed Model: Each of the components listed above is meshed using first-order four-node tetrahedron continuum elements. The total number of elements in each of the three components was as follows: (a) hub – ca. 350000; (b) blade – ca. 74000; and (c) shroud-segment – ca. 3500, with an average element-edge length of ca. 2 mm. The mesh size used was selected in such a way that a tradeoff was achieved between computational accuracy and computational cost. The blade is attached to the hub by having the two components share nodes at the joint.

Computational Algorithm: All the calculations carried out in this portion of the work involved two distinct static loading steps:

(a) an elastic step within which the hub and blade were subjected to the centrifugal distributed loading (with respect to the hub axis as the axis of rotation) associated with the hub rotation about its axis at an angular velocity of 10,000 rpm; and

(b) a visco-type loading step, within which allowance was made for the creep-deformation within the hub and blade, as well as for the creep-rupture in the root region of the blade.
**Initial Conditions:** Initially, that is, at the beginning of the first loading step, all the components of the model were assumed to be stress-free. As far as the thermal state of the model was concerned, it was assumed that initially (as well as throughout the analysis), the temperature was uniform and constant (1473 K) throughout the entire computational model.

**Boundary Conditions:** Within the first loading step, the hub and blade were subjected to distributed centrifugal loading associated with the hub rotation about its axis at an angular velocity of 10,000 rpm (while the translation degree of freedom of the rotation axis are fixed). Within the second step, the same loading was retained while the hub and blade were enabled to undergo time-dependent deformation (creep) and ultimate rupture. As mentioned above, the temperature within the model was assumed to be uniform and constant throughout the entire analysis.

**Contact Interactions:** Normal interactions between the blade-tip and shroud-segment were all modeled using the so-called “Hard Contact Pair” type of contact algorithm [14, 15]. Within this algorithm, contact pressures between two bodies were not transmitted unless the nodes on the *slave surface* contact the *master surface*. No penetration/overclosure between the slave and master surfaces was allowed, and there was no limit to the magnitude of the contact pressure that could be transmitted when the surfaces were in contact. Transmission of shear stresses across the contact interfaces was assumed to be controlled by a modified Coulomb friction law. This law utilizes a static, $\mu_{st}$, and a kinetic, $\mu_{kin}$, friction coefficient and an upper-bound shear stress limit, $\tau_{slip}$ (a maximum value of shear stress which can be transmitted before shearing within the softer material, rather than interfacial sliding, begins to take place).

As mentioned earlier, contact between the blade-tip and shroud-segment was enabled from the onset of the finite element analysis until the condition of the blade-tip rub was attained
for the first time. Past this point, the blade-tip/shroud-segment contact was removed, enabling the blade to continue to creep and to ultimately reach the condition for creep-rupture in the region where the blade is attached to the hub.

**Material Models:** The three components present in the computational model are assumed to be made of the following materials: (a) hub – GTD-111 (equiaxed); (b) blade – SiC/SiC CMC; and (c) shroud-segment – Inconel 718 (equiaxed). In the preliminary finite-element analysis for all the participating materials, the constitutive model included elastic and creep components. However, the results obtained revealed that except for the SiC/SiC CMC present in the blade, the other two materials/components experience substantially less amount of creep. Consequently, in all the subsequent calculations, only the elastic portion of the material constitutive response was prescribed for GTD-111 and Inconel 718. Specifically, due to their equiaxed grain-size character, GTD-111 and Inconel 718 are treated as being isotropic linear-elastic materials and, hence, their associated material constitutive models are fully defined by specifying the Young’s modulus $E$ and Poisson’s ratio $\nu$. The following values of $E$ and $\nu$ are used: (a) GTD-111 – 170 GPa, 0.28; and (b) Inconel 718 – 203 GPa, 0.30. As far as the SiC/SiC CMC material is concerned, its constitutive response under high-temperature sustained-loading condition is assumed to be appropriately accounted for by the creep-deformation and creep-rupture models developed in the previous section of this manuscript.

**Computational Tool:** All the calculations carried out in the present work were done using ABAQUS/Standard, a general purpose finite-element program [18]. The problem at hand is solved numerically using an implicit nonlinear static finite-element algorithm within which the mechanical equilibrium within the system is solved repeatedly as a function of time, since the mechanical equilibrium established at the end of the previous time increment is perturbed, by the
operation of the creep-deformation processes, within the current time increment. A typical simulation run pertaining to the creep-deformation and subsequent creep-rupture within the blade required 12 hours of (wall-clock) time on a 12-core, 3.0 GHz machine with 12 GB of memory.

4.3.3 Results and Discussion

In this section representative results pertaining to the creep-deformation and creep-rupture of the SiC/SiC CMC gas turbine blade are presented and discussed. These results are divided in two groups: (i) those pertaining to the creep-deformation up to the point of blade-tip rub; and (ii) those pertaining to the times past the first incident of blade-tip rub, the times which result in excessive creep-deformation within the blade and its ultimate creep-induced rupture at its root.

Attainment of the First Blade-tip Rub Condition: Figure 4-9(a) shows the spatial distribution of the Hill’s potential in the blade just prior to the establishment of the shroud/blade-tip contact. For clarity, in this figure, as well as in the subsequent figures the hub is not shown. Examination of the results depicted in Figure 4-9(a) shows that: (a) the Hill’s potential values are highest in the blade region adjacent to the blade/hub joint and they taper off towards the blade-tip; (b) due to the fact that the blade is rigidly connected to the hub and the hub in this region shares the blade-borne centrifugal load, the blade regions in the joint exhibit lower values of Hill’s potential (i.e. axial stress due to centrifugal force); and (c) due to the fact that the shroud segment is stationary and has no contact with the blade, it exhibits a zero value of the Hill’s potential.

Figure 4-9(b) shows the spatial distribution of the Hill’s potential in the blade just after the establishment of the shroud/blade-tip contact. Examination of the results depicted in this figure and their comparison with the results displayed in Figure 4-9(a) shows: (a) variation of the Hill’s potential through the blade is effectively identical in the two cases. If the contour levels are
refined in the blade-tip region, relative to the contour levels used in Figures 4-9(a)-(b) some small differences in the Hill’s potential are observed in the two cases. However, further away from the blade-tip these differences vanish; and (b) due to the fact that the blade-tip has established contact with the shroud segment, the shroud segment now exhibits non-zero values of the Hill’s (or more precisely, due to the isotropic nature of the shroud material, the von Mises) potential. It should be noted that since the contact stresses observed are quite small in comparison to the centrifugal stresses experienced by the blade, different Hill’s/von Mises potential ranges and contour levels are used for the blade and the shroud.
Figure 4-9 Spatial distribution of Hill’s potential on the SiC/SiC CMC blade: (a) just prior to the blade-tip/inner shroud contact event; and (b) right after the blade-tip/inner shroud contact event
Figure 4-10 shows the spatial distribution of equivalent creep strain in the blade at the moment when the blade-tip establishes contact with the shroud for the first time (9763 hours of engine operation). Examination of the results depicted in this figure shows that the highest values of the equivalent creep strain are seen in the region of the blade adjacent to the blade/hub joint, gradually reducing towards the blade-tip. This distribution of the equivalent creep strain is expected given a similar distribution of Hill’s equivalent potential (depicted in Figures 4-9(a)-(b)). Since the creep phenomenon was not modeled for the hub and the inner shroud material, these components are not shown in Figure 4-10.

Figure 4-10 Spatial distribution of equivalent creep strain over the SiC/SiC CMC blade at the moment of first blade-tip/inner-shroud contact event (9763 hours of engine operation).
As mentioned above, the computational analysis carried out in this portion of the work established that it took 9763 hours for the blade tip to make a contact with the shroud for the first time under the assumed design and operating conditions: (a) blade-tip/shroud clearance of 0.25 mm; (b) operating temperature of 1473K; and (c) hub rotational speed of 10000 rpm. This finding suggests that, if the rotor overhaul is to be carried out every 5000 hours, then the blade-tip/shroud clearance can be reduced to 0.128 mm. This would improve gas-turbine engine efficiency by reducing the amount of combustion gases escaping around the blade-tip.

**Creep-Rupture of the Blade at its Root:** As mentioned earlier, trimming of the blade tip to prevent blade-tip rub is not modelled explicitly in the present work. Instead, once the blade-tip/shroud contact is established for the first time, the shroud/blade-tip contact algorithm is deactivated allowing the blade to “penetrate” the shroud. Since the tip portion of the blade which should have been trimmed is of a sub-millimeter thickness and, hence, makes only a minor contribution to the overall mass of the blade, this simplification is not expected to give rise to significant numerical errors in the analysis being carried out.

Figure 4-11 shows spatial distribution of equivalent creep strain over the blade (5237 hours after the first blade-tip/shroud contact event or, equivalently, after 15000 hours of engine operation time). Examination of this figure and its comparison with Figure 4-10 shows that: (a) the blade-tip in Figure 4-11 “protrudes” through the shroud since, as mentioned earlier, blade-tip/shroud contact algorithm was deactivated after the first blade-tip/shroud contact event; (b) higher values of the equivalent creep strains are seen over the entire surface of the blade in Figure 4-11 than in Figure 4-10; and (c) in the region of the blade adjacent to the hub, the equivalent creep strain has acquired quite large values (> 0.45 %) suggesting that the SiC/SiC CMC may be approaching its creep-ductility limit (i.e. the onset of creep-rupture).
Figure 4-11. Spatial distribution of equivalent creep strain over the SiC/SiC CMC blade (5237 hours after the first blade-tip/shroud contact event or, equivalently, after 15000 hours of engine operation time). Due to the fact that the blade-tip/shroud contact was disabled after the first contact event the blade is allowed “penetrate” the shroud.

Spatial distribution of the Hill’s potential over the blade just prior to its creep-rupture is depicted in Figure 4-12(a). The corresponding spatial distribution of the Hill’s potential right after “creep-rupture” has taken place (16,781 hours of engine operation) is shown in Figure 4-12(b). It should be noted that creep rupture in the present analysis is characterized by the structural failure and the removal of a relatively large number of finite elements within a section of blade adjacent to its joint with the hub. However, the rupture does not result in a complete detachment of the blade from the hub. Examination and the comparison of the results displayed in Figures 4-12(a)-(b) reveals:
(a) creep-rupture causes a substantial increase in the Hill’s potential in the region of the blade which is adjacent to the hub and still attached to it; and

(b) increased values of the Hill’s potential extend along the blade axis from the blade root to about half of its length. That is, the Hill’s potential in the tip portion of the blade remains fairly unaffected by the blade rupture.

As mentioned above, the present analysis has predicted that the onset of creep-rupture within the blade takes place after 16,781 hours of the engine operation. This time period, or more appropriately a fraction of it (to address the safety concerns) can be considered as the expected service life of the subject blade under the aforementioned design and operating conditions.
Figure 4-12 Spatial distribution of Hill's potential over the SiC/SiC CMC blade: (a) just prior to the blade “creep-rupture” event; and (b) right after the blade “creep-rupture” event (16,781 hours of engine-operation time).
4.4 Summary and Conclusions

Based on the results obtained in the present work, the following main summary remarks and conclusions can be drawn:

1. Creep-deformation and creep-rupture models were developed for SiC/SiC ceramic-matrix composites (CMCs). The models account for the transversely isotropic character of the CMCs and for the general three-dimensional nature of the loading.

2. One set of experimental data available in the open literature was subsequently used to parameterize the models, while another set of experimental data also available in the open literature was used to validate the models.

3. The model validation step clearly indicated that model parameterization is highly sensitive to the details of the CMC-material microstructure and processing route. For example, parameterization established for a CMC in which the matrix was fabricated via the chemical vapor infiltration route may not be very reliable if applied to a CMC in which the matrix was fabricated using a melt-infiltration route (even if the other aspects of the CMC microstructure and architecture are quite comparable in the two grades of this material).

4. To demonstrate the utility of the creep-deformation and creep-rupture models developed in the present work, the problem of the performance and durability of a CMC-blade located in the low-pressure stage of a gas-turbine engine is investigated computationally using a finite-element analysis. The results obtained are used to help establish the needed gas-turbine engine-maintenance schedule (needed in order to prevent blade-tip rub condition) and to assess the useful life of the blade (as controlled by the possibility of creep-induced blade rupture at its root).
4.4 References


5.1 Abstract

This paper addresses the problem of materials selection for springs used to clamp an inner shroud segment to the outer shroud block in utility and industrial gas turbine engines. Clamping is achieved through the application of an initial compressive load to the spring. However, since the spring is subjected to high-temperature and oxidizing conditions, it experiences creep and surface oxidation. Both of these processes result in the loss of the compressive load within the spring with time. A material selection procedure is developed, which identifies optimum materials (design variables), with respect to the minimum loss in the clamping-spring load (objective function) for a given set of geometrical constraints (i.e. maximum size of the spring is constrained by the outer-shroud cavity which houses the spring) and functional constraints (force retention should persist over the expected life of the inner-shroud segment). Two material selection procedures are devised: (a) one, fairly rigorous and computationally intensive, based on the use of a finite element analysis; and (b) the other, less rigorous but computationally less expensive, based on the use of a simplified analytical/numerical procedure. In the absence of oxidation, the two approaches yielded different, but mutually consistent, results with identical ranking of the clamping-force candidate materials. The inclusion of the oxidation effects showed that oxidation-induced loss in the spring material increases the extent of clamping-force relaxation and may affect the ranking of the candidate materials.
5.2 Introduction

The present work addresses the problem of material selection for the springs used to attach/clamp ceramic-matrix composite (CMC)-based inner-shroud segments to the metallic-based outer-shroud blocks in utility and industrial gas-turbine engines. One of the key factors influencing material selection in this case is the creep-induced reduction in the clamping force provided by the spring, the problem which can ultimately result in undesirable mobility of the inner-shroud segments, their cyclic loading, and hot-gas leakage into the interior cavity of the outer shroud blocks. Based on the description of the present problem, the key aspects which will be briefly reviewed in the remainder of this section include: (a) the basics of operation and construction of gas-turbine engines; (b) the use of CMCs in the construction of gas-turbine engine hot-section components; and (c) prior work on the problem of material selection in the designs in which the component performance is controlled by the creep behavior of the material of which the component is made.

5.2.1 Basics of Operation and Construction of Gas-Turbine Engines

While gas-turbine engines as a system are not the subject of the present work, a brief description of the principles of their operation and construction is warranted from the standpoint of fully conceptualizing the problem of clamping-spring material selection. A gas-turbine engine is a type of internal combustion engine which utilizes air as the working fluid, and its operation involves the four basic steps of the internal-combustion-engine cycle (described below) [1]. This enables the conversion of the chemical energy of the combusted fuel to the mechanical energy of the working fluid (the energy which drives the turbine shaft).

The four steps of the gas-turbine engine cycle include:

(i) air intake;
(ii) air compression;

(iii) fuel injection and combustion, followed by expansion of the combustion gases and working fluid, and their interaction with the turbine blades, which produces the shaft work output (driving a compressor and other devices like an electric generator); and

(iv) exhaust of the combustion gases and working fluid.

In contrast to a reciprocating-piston/cylinder internal combustion engine, the four steps occur not sequentially within the same general location (i.e. cylinder), but rather simultaneously at different locations within the engine.

A block-diagram-type schematic of a gas-turbine engine is provided in Figure 5-1(a), and the main sections of the engine are labeled, along with the flow paths for the fuel and working fluid. A solid model of a prototypical aviation gas-turbine engine is shown in Figure 5-1(b), with the compressor, combustor and turbine sections labeled. The component (i.e. the clamping spring) for which material selection is conducted, as well as the components which are clamped using the spring, are all located within the turbine section of the engine.

Among the key factors governing the new designs of gas-turbine engines are the maximization of the engine efficiency and the minimization of the undesirable environmental effects. Engine efficiency is often quantified by specific fuel consumption (i.e. the fuel consumption per unit shaft power/thrust output). Many factors, such as the mass flow rate of air, pressure ratio of the compressor, turbine-inlet temperature, and individual component efficiencies, influence engine efficiency. As far as the minimization of environmental effects is concerned, operation of the gas-turbine engines at higher temperatures is generally preferred since release of the products of incomplete combustion to the environment is minimized. As will be discussed below, operation of the gas-turbine engines at higher temperatures requires the
availability of structural heat- and oxidation-resistant materials (e.g. CMCs) in place of the traditionally-used metallic superalloys.

Figure 5-1 (a) Block-diagram-type schematic of a gas-turbine engine. The main sections of the engine are labeled, along with the flow paths for the fuel and working fluid; and (b) solid model of a prototypical aviation gas-turbine engine.
5.2.2 Use of CMCs in the Construction of Gas-Turbine Engine Hot-Section Components

High-temperature metallic materials such as nickel-, cobalt- or iron-based superalloys used in gas-turbine engines have been pushed to their thermal-stability limit since they are often made to operate at temperatures which are within 50 degrees of their melting point. To increase power density and energy efficiency of the gas-turbine engines, new materials are needed which can operate at temperatures as high as 1400 K [2]. The main candidate materials currently identified for use in the next generation of gas-turbine engines are (monolithic) ceramics and CMCs. Since these materials can withstand extremely high temperatures, their use in hot sections of gas-turbine engines can yield a number of benefits such as: (i) improvements in thrust and fuel efficiency; (ii) lower pollutant emissions; (iii) reduced cooling requirements; (iv) simplification of the engine-component design; and (v) reduced requirements for the strength/weight of the supporting structure.

However, due to their relatively low fracture toughness, tensile strength and damage tolerance, monolithic ceramics are not being perceived as respectable candidate materials for use in critical turbine-engine structural applications (e.g. turbine blades). On the other hand, CMCs consisting of a ceramic matrix and ceramic fibers possess superior structural properties relative to their monolithic-ceramic counterparts, while retaining their high-temperature stability and integrity. This is the reason that the CMCs are being aggressively researched and developed for use in future gas-turbine engines. The potential of the CMCs in revolutionizing the performance of the gas-turbine engines is shown schematically in Figure 5-2 [2]. In this figure, the x-axis represents the approximate period of dominance in usage of the particular class of high-temperature materials (and associated high-temperature technologies), while the y-axis denotes the temperature capability of the material class in question. It is seen that the temperature
capability of CMCs lies above the fitting line for the temperature capabilities of the past and present gas-turbine engine materials. Presently, CMCs are being perceived as candidate materials in gas-turbine engine components which experience low in-service loads such as (turbine) inner shroud, nozzles, combustion liners, airfoils and exhaust components.

The component (i.e. clamping spring) for which material selection is carried out in the present work is used to more rigidly clamp a CMC internal shroud to the metallic superalloy outer-shroud block (the latter two are already connected using a hole-and-pin-attachment mechanism). However, the connecting pins are only allowed to loosely attach the inner and outer shroud components (forming a so-called floating assembly). The reason for this is a quite large thermal-expansion mismatch between the CMCs (2E-6/K to 3E-6/ K) and superalloys (13E-6/K
to 15E-6/K). The use of tight pin connections would create excessive thermal stresses and, in turn, inner-shroud damage/failure in the regions adjacent to the pins. The primary function of the internal shroud is to provide an internal cylindrical surface so that the blade-tip clearance leakage is minimized, resulting in higher engine efficiency. Additionally, this internal shroud insulates the turbine-casing from the hot gases, and so minimizes the cooling requirement for the casing, contributing to still higher engine efficiency. A close-up of the inner turbine section of a prototypical aviation gas-turbine engine is depicted in Figure 5-3, in which the blades and inner shroud are labeled (the outer shroud, located between the inner shroud and the casing, is not visible).

Figure 5-3 Close-up of the inner turbine section of a prototypical aviation gas-turbine engine, with the blades and inner shroud labeled (the outer shroud, located between the inner shroud and the casing, is not visible).
When made of CMCs, an internal shroud is not a single-piece component, but rather composed of individual segments running circumferentially and connected via a tongue and groove mechanism. For maximum safety and efficiency of operation, each inner-shroud segment should be firmly clamped/attached to the corresponding outer-shroud block. As mentioned above, to ensure good inner-shroud to outer-shroud clamping (despite the fact that the two only form a floating assembly), mechanisms (like the one analyzed in the present work) involving pre-loaded helical springs are often used. Due to clamping-spring exposure to in-service high temperatures, creep relaxation within the spring and the associated loss of the clamping force within it may become a life-limiting factor for the CMC internal-shroud segments. Consequently, when selecting materials for the clamping-spring a detailed consideration should be given to the creep-induced stress-relaxation phenomenon, the subject addressed in the present work.

5.2.3 Creep-Controlled Material Selection

Material Selection for Ambient-Temperature Structural Applications: Since creep-controlled material selection is generally carried out using a method which represents an extension of Ashby’s material-selection approach for mechanical components operating at ambient or near-ambient temperatures [3], a brief description of Ashby’s approach is first given in this subsection. The two key concepts utilized within Ashby’s approach include: (a) performance indices – a functional relationship involving material properties which control the performance of a component; and (b) material-selection charts – charts in which the axes are defined in terms of the material properties appearing in the performance index. Furthermore, Ashby’s approach can be characterized as a constrained-optimization problem, the solution of which yields the performance index (and, in turn, the associated material-selection chart).

Here a case of material selection for a light (optimization objective), strong (constraint),
constant-cross-section shape (constraint) beam (function, i.e. a component that is capable of supporting bending stresses/moments) is considered.

**Objective function:** Mass, \( m \), of a beam of length \( L \), and a constant rectangular cross-sectional shape, \( \alpha = b/h \), \( b \) – breadth and \( h \) – height, is to be minimized.

\[
m = A \cdot L \cdot \rho = \alpha h^2 \cdot L \cdot \rho
\]  

(5.1)

where \( A = bh = \alpha h^2 \) is the beam cross-sectional area and \( \rho \) is the material mass-density.

**Functional constraint:** When subjected to transverse load \( F \), the beam should not collapse plastically due to the formation of a “plastic hinge.”

\[
F \leq F_{\text{collapse}} = C_1 \sigma_y \frac{bh^2}{L} = C_1 \sigma_y \frac{\alpha h^3}{L}
\]  

(5.2)

where \( C_1 \) is a (known) load-distribution-dependent constant and \( \sigma_y \) is the yield strength of the material.

If inequality, Eq. (5.2) is solved for the free variable \( h \), the resulting relationship substituted in Eq. (5.1) and rearranged, one obtains

\[
m \geq \left(L^{5/3} \alpha^{1/3}\right) \left(F/C_1 \right)^{2/3} \left(\rho/\sigma_y^{2/3}\right) = f_1(\text{geom}) \cdot f_2(\text{func}) \cdot f_3(\text{mat})
\]  

(5.3)

where symbols \( f_1(\text{geom}) \), \( f_2(\text{func}) \) and \( f_3(\text{mat}) \) are used to denote (decoupled) functional relationships related to the geometrical and functional requirements and material contributions to the component performance, respectively. Since the component performance is maximized when its mass is minimized, the contribution of the material to the component performance is defined by a performance index \( M = \sigma_y^{2/3} / \rho \) so that the larger the value of \( M \), the larger the contribution of the associated material to the component performance. It should be noted that \( M \)
only depends on the material properties, \( \sigma_y \) and \( \rho \), i.e. the material contribution to the component performance is unaffected by the details of geometrical and functional requirements. As will be shown below, in the case of creep-based material selection, this will not generally be the case.

As far as the material-selection chart is concerned here, based on the definition of the performance index, the material-selection chart involves only two material properties, \( \sigma_y \) and \( \rho \). This chart is depicted in Figure 5-4 as a log-log plot along with the bubbles showing the ranges of the two material properties for various grades of materials (e.g. PP – polypropylene) and envelopes for different material classes (e.g. polymers and elastomers). In this chart, lines with slope 3/2 are associated with the same value of the performance index defined above and the larger the \( y \)-axis intercept of such a line, the larger the value of the performance index. This finding can be used to graphically identify the best materials for use in light, strong, constant-cross-sectional-shape beam applications. An example of the “guideline” with a slope of 3/2 is depicted in Figure 5-4. The guideline is positioned in such a way that it identifies a small subset of best materials, including composites and ceramics.
Figure 5-4 Yield strength vs. density material-selection chart used for graphical material selection in applications demanding light, strong, materials. A guideline having a slope of 3/2 is depicted and used to identify a subset of materials ideal for light, strong, constant-cross-section-shape beams.

Material Selection for Creep-Controlled-Relaxation Applications: Examination of the open-domain literature revealed that, to a first-order approximation, selection of the materials for use in creep-controlled relaxation applications can follow the same material-selection procedure as that described above for the ambient-temperature applications. While a number of reports were identified dealing with the creep-controlled material-selection problems, the basis for material selection employed is essentially the same as that presented in the seminal work of Ashby and Abel [4]. Consequently, in the remainder of this subsection, only a brief overview of the approach developed by Ashby and Abel[4] for creep-controlled material selection will be presented. It
should be noted that, while one can generally identify four subtypes of the creep-controlled material-selection problems, i.e.: (i) excessive creep deformation; (ii) creep rupture; (iii) creep relaxation; and (iv) creep-induced buckling, only the case of creep relaxation will be analyzed here. Furthermore, the approach of Ashby and Abel[4] is strictly applicable only to the case of the components subjected to a uniform (but time-varying) stress-state, e.g. the case of a bolt pre-tensioned to provide the necessary clamping force at high operating temperatures at which creep relaxation is active. The problem of material selection in this case can be formulated as follows: the mass of the bolt should be minimized subject to the constraint that the pre-tension force, \( F(t_0) \), within the bolt not drop below a minimum acceptable value \( F_{\text{min}} \) for a specified duration \( t_{\text{life}} \). Thus, the problem can be mathematically cast as follows:

**Objective function:** Mass, \( m \), of the bolt having a non-specified (uniform) cross-sectional area \( A \) and a fixed length \( L \) should be minimized.

\[
m = A \cdot L \cdot \rho \tag{5.4}
\]

**Constraining Relation:** Clamping force, \( F \), after specified time \( t = t_{\text{life}} \) should not be lower than a minimum acceptable pre-tension force \( F_{\text{min}} \).

\[
F(t_{\text{life}}) \geq F_{\text{min}} \tag{5.5}
\]

**Associated Kinematic Constraint:** Total axial strain \( \varepsilon_{\text{total}} \) within the bolt, which contains an elastic component \( \varepsilon_{\text{elastic}} \) and a creep component \( \varepsilon_{\text{creep}} \), remains constant.

\[
\varepsilon_{\text{total}} = \varepsilon_{\text{elastic}} + \varepsilon_{\text{creep}} = \text{constant} \tag{5.6}
\]
\[ \dot{\epsilon}_{total} = \dot{\epsilon}_{elastic} + \dot{\epsilon}_{creep} = 0 \quad (5.7) \]

Associated Material Constitutive Relation: Hooke’s Law,

\[ \varepsilon_{elastic} = \frac{\sigma}{E}, \quad \dot{\varepsilon}_{elastic} = \frac{\dot{\sigma}}{E} \quad (5.8) \]

where \( \sigma \) denotes the axial stress, \( E \) the Young’s modulus and the raised dot a time derivative.

Associated Material Constitutive Relation: Steady-state creep,

\[ \dot{\varepsilon}_{creep} = A_1 \exp \left( -\frac{Q}{RT} \right) \left( \frac{\sigma}{\sigma_0} \right)^n = A'_1 (T) \sigma^n \quad (5.9) \]

where \( A_1, A'_1, Q, \sigma_0 \) and \( n \) are material parameters, \( R \) the universal gas constant, and \( T \) the prescribed temperature.

Free-variable Functional Relationship:

\[ F(t_{life}) = A \sigma(t_{life}) \quad (5.10) \]

To derive the material selection index for the light (objective), creep-relaxation-resistant (constraint) bolt (function), Eqs. (5.4)–(5.10) are used as follows:

(a) Eqs. (5.8) and (5.9) are substituted into Eq. (5.7) and the resulting first-order ordinary differential equation integrated between \( t = 0 \) and \( t = t_{life} \), yielding \( \sigma(t_{life}) \). Generally, \( F(t_{life}) \) is defined at this point only implicitly via the resulting single nonlinear algebraic equation. Under the conditions that \( n > 3.0 \) and \( \sigma(t_{life})/\sigma(t_0) \leq 0.5 \), the algebraic equation can be simplified to yield an explicit expression for \( \sigma(t_{life}) \);

(b) The resulting equation for \( \sigma(t_{life}) \) is substituted in Eq. (5.10), and in turn in Eq. (5.5). Since
the resulting inequality contains $A$ (on its left-hand side), it is solved for $A$ and plugged into Eq. (5.4) to yield:

$$m \geq L \cdot F_{\text{min}} \cdot \frac{\rho}{\left( n-1 \right) E \cdot A'_{1}(T) \cdot t_{\text{life}}} \quad (5.11)$$

Examination of Eq. (5.11) reveals that the functional requirement, $t_{\text{life}}$, and the material parameter $n$ could not be separated. Thus, one cannot express Eq. (5.11) as a product of a geometry-dependent term (containing $L$), a functionalRequirement term ($F_{\text{min}}$ and $t_{\text{life}}$) and a material-property-dependent term (containing $\rho$, $E$, $A_{1}$, $A'_{1}(T)$ and $n$). In other words, a material performance index which combined only material properties cannot be defined, in the present case. Instead, for each case of the geometrical and functional requirements ($t_{\text{life}}$), one has to evaluate the entire right side of Eq. (5.11) and use these results as the basis for materials selection (that is, the lower is the value of the bolt mass, the more suitable is the candidate material for bolt construction). Alternatively, one can define an “operational” performance index which includes contributions of not only the material properties but also of the functional requirement $t_{\text{life}}$ and the thermal operating condition $T$. In other words, the denominator in the last term of Eq. (5.11), which has stress units and is a measure of the stress-relaxation resistance of a material (and is labeled $\sigma_{D}$), can be evaluated for a given material, $t_{\text{life}}$ and $T$. Then the operational material index is defined as $M = \sigma_{D}/\rho$ and a conventional log-log material selection chart $\sigma_{D}$ vs. $\rho$, like the one depicted in Figure 5-4, can be constructed defining a “bubble” for each candidate material. In the present case, guidelines used for the graphical
selection of the light, creep-induced relaxation-resistant materials have a slope of 1 (and not 3/2 as in the case of the light, strong, constant-cross-section-shape beam). However, material selection is graphically carried out in the same way by increasing the y-intercept of the guideline in order to isolate an optimum subset of materials suitable for use in light, creep-induced-relaxation-resistant components.

It should be recalled that the procedure for creep-relaxation-controlled material selection described above is strictly valid for components experiencing a uniform-stress state. As will be discussed later, the component of interest here, a helical spring, is not characterized by a uniform-stress state.

5.2.4 Main Objective

There are two main objectives of the present work: (a) development of a fairly rigorous finite-element-based procedure for selection of the materials for use in creep-induced-relaxation-resistance applications; and (b) examination of the suitability of the approach of Ashby and Abel [4] for material selection in the same types of mechanical designs.

5.2.5 Chapter Organization

A formulation of the problem being addressed in the present work is given in Section 5.3. Details regarding the finite-element analysis employed and the results obtained for the clamping-spring material selection are presented in Section 5.4. The problem of suitability of the Ashby and Abel [4] approach for clamping-spring material selection is addressed in Section 5.5, and the main conclusions resulting from the present work are summarized in Section 5.6.
5.3 Problem Formulation

To help with the formulation of the clamping-spring material-selection problem at hand, a labeled solid-model type of schematic is depicted in Figure 5-5. To help clarify the relationship between the schematic shown in Figure 5-5 and the attendant cross-section of the gas-turbine engine, the engine axis, engine-interior side and engine-casing side are labeled. The main components identified in this figure include the inner-shroud segment and the outer-shroud block to which the inner-shroud segment is loosely connected via pin-and-hole/-groove mechanisms. This type of joining mechanism allows for differential thermal expansion between the CMC inner-shroud segment and the metallic outer-shroud wall to be accommodated without the development of undesirable excessive thermal stresses. The outer-shroud block has a cavity which houses a helical spring (the component for which the material selection procedure is being developed) and a piston (the component through which the compression/clamping force of the spring is transmitted to the inner-shroud segment). To ensure positive seating of the inner-shroud segment against the fixing pins (critical from the standpoint of obtaining limiting inner-shroud component mobility), the clamping force provided by the spring should be of sufficiently high magnitude. This can initially be ensured by sufficient preloading of the spring. However, due to the relatively high operating temperatures experienced by the spring (resulting from the combustion-gas leakage from the engine interior into the outer-shroud cavity and the heat transfer through the faces of the inner-shroud segment), creep relaxation within the spring becomes active, leading to a loss in its clamping force. Thus, a material selection procedure is needed which could be used to ensure that over the expected life of the inner-shroud segment, the relaxation of the clamping force is minimum (or, alternatively, that the force within the spring does not fall below a minimum acceptable value). Due to the fact that the stress-field within the
spring is not uniform, a rigorous finite-element-based material-selection procedure will be
developed and applied first. Then the suitability of a simplified Ashby-Abel-type procedure
(based on the use of the maximum shear stress within the spring) will be addressed. Furthermore,
initially it will be assumed that clamping force relaxation is completely controlled by the creep
processes within the spring. In the very last portion of the work, it will also be recognized that
oxidation of the spring and the potential spalling of the oxide acts as a material recession
mechanism which reduces the wire diameter of the spring and, in turn, may provide additional
contributions to the spring clamping force relaxation.

Figure 5-5  A solid-model of a gas-turbine engine, hot-section shroud assembly involving the
inner-shroud segment and the associated outer-shroud block, attaching pins and the
clamping-spring and piston assembly.

It should be noted that, as stated earlier, clamping-spring material selection is carried out
for the case of the utility and industrial gas turbine engines for which the mass of the spring is not
generally of a primary concern. However, due to the fact that the spring has to be housed within
the internal cavity of the outer shroud block, the maximum size/diameter of the spring has to be
constrained. Consequently, the materials selection problem will be cast as follows: materials
(design variables) are needed which, for the prescribed size of the clamping spring (geometrical constraint), minimizes the loss of the force within the spring (objective) over the expected life (24,000 hrs) of the spring/inner-shroud (functional constraint).
5.4 Finite Element Analysis, Results And Discussion

In this section, details are presented regarding the finite-element-based material selection procedure for creep-induced stress-relaxation-resistant clamping helical springs, as well as the main results obtained.

5.4.1 Finite-Element Analysis

The finite-element analysis (FEA) employed in the present work requires specification of the following: (a) geometrical model; (b) meshed model; (c) computational algorithm; (d) initial conditions; (e) boundary conditions; (f) contact interactions; (g) material models; and (h) computational tool. These aspects are briefly overviewed in the remainder of this sub-section.

**Geometrical Model:** The geometrical model employed in the present work was already shown in Figure 5-5. It should be noted that, in Figure 5-5, a portion of the outer shroud was cut out in order to reveal the interior cavity within the outer shroud, as well as the placement of the clamping-spring/piston assembly. The finite-element analysis employed utilized the complete outer-shroud block as well as the other components of the geometrical model. It should be noted that the geometrical model includes the following components: (a) inner-shroud segment; (b) outer-shroud block; (c) four inner-shroud-to-outer-shroud connecting pins; (d) one piston; and (e) one clamping helical spring.

**Meshed Model:** Each of the components listed above is meshed using first-order four-node tetrahedron continuum elements. The total number of elements in the model was ca. 480,000 with 55,000 elements residing within the spring (the component the mechanical response of which is of primary concern) while the remaining elements reside in the other components of the model. A close-up of the meshed model showing the contact region between the helical spring and the piston is depicted in Figure 5-6. A standard mesh-sensitivity analysis was conducted in order to
ensure that further refinement in the mesh size does not significantly alter the accuracy of the key computational results.

Figure 5-6 A close-up of the meshed model showing the contact region between the helical spring and the piston.

**Computational Algorithm:** All the calculations carried out in this portion of the work involved two distinct loading steps:

(a) an elastic step within which the helical spring is preloaded in compression and placed into the model in order to provide a force, via the piston, to the inner-shroud segment. The equilibrium state of the system is then solved for using the standard static-equilibrium finite-element analysis; and

(b) a visco-type loading step, within which allowance is made for the creep deformation within the spring (as well as within the remaining components in the model). Since the attendant stresses were found to be substantially larger in the spring than in the remaining components, significant creep was detected only within the spring while the remaining components experience little to no creep deformation.
**Initial Conditions:** Initially, that is, at the beginning of the first loading step, all the components of the model were assumed to be stress-free. The computational domain under investigation was assumed to be in a steady state relative to its temperature distribution. The steady-state temperature distribution within the computational domain was obtained in a separate conjugate thermal finite-element analysis (details and the results of this analysis will be presented in a future communication). Within this analysis, the interaction of the computational domain used here with the turbine-engine interior combustion gases and the outer-shroud block cooling air is analyzed. The results obtained show some, but not significant (19 K maximum) variation in the temperature along the length of the clamping spring. For this reason, and for the fact that the results of the present finite-element analysis are to be compared with their analytical/numerical counterparts, temperature within the spring was assumed to be uniform and equal to 750 K. As far as the temperatures within the remaining components of the computational domain analyzed here are concerned, it was set equal to the ones predicted by the conjugate thermal analysis. This (constant but non-uniform) thermal condition was applied initially and throughout the finite-element analysis of the creep-relaxation within the spring.

**Boundary Conditions:** Within the first loading step, the spring is subjected to compression in order to attain the desired level of reaction force within it. Within the second step, the portion of the outer-shroud block facing the turbine-casing is fixed, the compressed spring inserted into the model and allowed to interact with the outer-shroud block at one end, and with the piston on the other. As mentioned above, constant and non-uniform (except for the spring) temperature boundary conditions are prescribed.

**Contact Interactions:** Normal interactions between the components are all modeled using the so-called “Hard Contact Pair” type of contact algorithm [5, 6]. Within this algorithm, contact
pressures between two bodies are not transmitted unless the nodes on the slave surface contact the master surface. No penetration/overclosure between the slave and master surfaces is allowed, and there is no limit to the magnitude of the contact pressure that could be transmitted when the surfaces are in contact. Transmission of shear stresses across the contact interfaces is assumed to be controlled by a modified Coulomb friction law. This law utilizes a static, \( \mu_{\text{st}} \), and a kinetic, \( \mu_{\text{kin}} \), friction coefficient and an upper-bound shear stress limit, \( \tau_{\text{slip}} \) (a maximum value of shear stress which can be transmitted before shearing within the softer material, rather than interfacial sliding, begins to take place).

Material Models: As mentioned earlier, the main components of the computational model and the materials of which the components are made include: (a) an inner-shroud segment – CMC; (b) an outer-shroud block – Inconel 718; (c) inner-shroud-to-outer-shroud attaching pins – Carpenter L605, a cobalt-based superalloy; (d) a piston – Haynes 230, a Ni-Cr superalloy; and (e) a clamping spring – one of the five candidate materials discussed below. In the preliminary finite-element analysis for all the participating materials, the constitutive model included elastic and creep component. However, the results obtained revealed that except for the clamping spring, all other components of the model experience very little creep. Consequently, in all the subsequent calculations, only the elastic portion of the material constitutive response was prescribed for the non-spring materials. Specifically, Inconel 718, Carpenter L605 and Haynes 230 are treated as being isotropic linear-elastic materials and, hence, the associated material constitutive model is fully defined by specifying the Young’s modulus \( E \) and Poisson’s ratio \( \nu \). The following values of \( E \) and \( \nu \) are used: (a) Inconel 718 – 203 GPa, 0.30; (b) Carpenter L605 – 231 GPa, 0.30; and (c) Haynes 230 – 211 GPa, 0.33. As far as CMC is concerned, it is treated as being a transversely isotropic, linear-elastic material with the unique axis being aligned in the laminate through-the-
thickness direction \((x_3)\). The material constitutive model in this case is defined in terms of five elastic moduli as \(E_{11}=E_{22}=245\) GPa, \(E_{33}=195\) GPa, \(\nu_{13}=\nu_{23}=0.25\), \(\nu_{12}=0.22\) and \(G_{13}=G_{23}=83\) GPa [7-10]. As far as the spring materials are concerned, five proprietary candidates labeled \(M_1, M_2, M_3, M_4\) and \(M_5\) are considered. In addition to the isotropic linear-elastic portion, the constitutive model for these materials also involves the steady-state creep law, Eq. (5.14), and the associated material parameters, \(A_i, \sigma_0, n\) and \(Q\). Table 5-1 provides numerical values for the constitutive-model parameters for materials \(M_1-M_5\).

The governing equations associated with the spring-materials include [11, 12]:

(i) **Generalized Hooke’s Law** –

\[
\{\sigma\} = [C]\{\varepsilon_{\text{elastic}}\} = [C]\{\varepsilon_{\text{total}}\} - \{\varepsilon_{\text{creep}}\} = [C]\{\varepsilon_{\text{total}}\} - \{\varepsilon^0_{\text{creep}}\} - \{\Delta\varepsilon_{\text{creep}}\}
\]  

(5.12)

where symbols \{\ldots\} and […] are used to denote a 6x1 vector and 6x6 matrix, respectively, \{\sigma\} and \{\varepsilon\} are respectively the stress and strain vectors, \([C]\) is the elastic stiffness matrix, \(\Delta\) designates an increment in a dependent variable over a time increment of duration \(\Delta t\), and superscript \(0\) defines the state of a quantity at the end of the previous time increment. Since \{\varepsilon_{\text{total}}\}, \{\varepsilon^0_{\text{creep}}\}, and \([C]\) are all known, Eq. (5.12) represents a set of six equations with twelve (six \{\sigma\} and six \{\Delta\varepsilon_{\text{creep}}\}) unknowns.

(ii) **Normality Flow Rule** –

\[
\{\Delta\varepsilon_{\text{creep}}\} = \Delta t\left(\hat{\varepsilon}_{\text{creep}}\right)\frac{\partial \sigma_{eq}}{\partial [\varepsilon]}
\]  

(5.13)
where $\dot{\varepsilon}_{\text{creep}}$ is the equivalent creep-rate, and $\sigma_{eq}$ is the von Mises equivalent stress, defined as

$$\sigma_{eq} = \frac{3}{2}\sqrt{\langle S \rangle : \langle S \rangle},$$

and the deviatoric stress is defined as $\{S\} = \langle \sigma \rangle - \frac{1}{3} \text{tr}(\{\sigma\}) \{I\}$.

(iii) Steady-State Equivalent Creep Rate Relation –

$$\dot{\varepsilon}_{\text{creep}} = A_i \exp \left( \frac{-Q}{RT} \right) \left( \frac{\sigma_{eq}}{\sigma_0} \right)^n$$  \hspace{1cm} (5.14)

Eqs. (5.12)–(5.14) represent a system of 13 algebraic equations with 13 unknowns: 6 $\{\sigma\}$, 6 $\{\Delta \varepsilon_{\text{creep}}\}$ and $\dot{\varepsilon}_{\text{creep}}$.

Table 5-1 Constitutive model parameters for the five clamping spring materials.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value for alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>GPa</td>
<td>M₁ M₂ M₃ M₄ M₅</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>N/A</td>
<td>0.3 0.32 0.33 0.3 0.29</td>
</tr>
<tr>
<td>Creep-rate parameter</td>
<td>$A_1$</td>
<td>s⁻¹</td>
<td>1.0E-7 1.8E-7 1.6E-7 2.4E-7 2.1E-7</td>
</tr>
<tr>
<td>Stress parameter</td>
<td>$\sigma_0$</td>
<td>MPa</td>
<td>1.0 1.0 1.0 1.0 1.0</td>
</tr>
<tr>
<td>Stress exponent</td>
<td>$n$</td>
<td>N/A</td>
<td>8.4 8.45 8.7 8.1 8.22</td>
</tr>
<tr>
<td>Activation energy</td>
<td>$Q$</td>
<td>J/mol</td>
<td>500E3 505E3 515E3 490E3 495E3</td>
</tr>
</tbody>
</table>
Computational Tool: All the calculations carried out in the present work were done using ABAQUS/Standard, a general purpose finite-element program [13]. The problem at hand is solved numerically using the aforementioned finite-element algorithm within which the mechanical equilibrium within the system is solved repeatedly as a function of time, since the mechanical equilibrium established at the end of the previous time increment is perturbed, by the operation of the creep-deformation processes, within the current time increment. A typical simulation run pertaining to the creep-induced stress-relaxation within the spring, over its expected life of 24000 hours required 18 hours of (wall-clock) time on a 12-core, 3.0 GHz machine with 12 GB of memory.

5.4.2 Results and Discussion

A typical evolution of the von Mises stress within the helical spring as a function of duration of its exposure at 750 K is depicted in Figures 5-7(a)–(d). It is seen that as the time advances, the stress within the spring relaxes (at a progressively lower rate). This finding is consistent with the fact that the creep rate is a function of the attendant stress and as the stress decreases within the spring, so does the rate of creep relaxation.
Figure 5-7 A typical evolution of the von Mises stress within the helical spring as a function of duration of its exposure at 750 K: (a) 0 hr; (b) 8,000 hr; (c) 16,000 hr; and (d) 24,000 hr.
Temporal evolution of the clamping force within the spring for the five spring-material candidates, \( M_1 - M_5 \), is depicted in Figure 5-8. It is seen that:

(a) since the objective of the material selection here is to minimize the degree of clamping-force loss within the spring over the expected life of the inner-shroud segment (24,000 hours), material \( M_1 \) appears to be the best candidate;

(b) since some of the curves in Figure 5-8 cross, one can conclude that, in principle, the identity of the optimum material, or at least the ranking of the materials, is affected by the prescribed expected life of the inner-shroud segment; and

(c) the fact that some curves intersect in Figure 5-8 also suggests that creep-relaxation-resistant-material selection based on the initial creep-rate value (a simplified approach [14, 15], frequently used to circumvent a more rigorous and correct material-selection procedure like the one presented here) may be incorrect if the expected life of the inner-shroud component is sufficiently long (i.e. longer than the time at which the force vs. time curves intersect).
Figure 5-8 Temporal evolution of the clamping force within the spring for the five spring-material candidates, M₁–M₅, as yielded by the rigorous and computationally expensive finite element procedure.

A bar graph is depicted in Figure 5-9 showing the relative retention of the clamping-spring force after exposure to 750 K for 24,000 hours. This relative retention of the clamping force is defined as the ratio of the initial force and the one after 24,000 hours 750 K exposure. Thus, the higher is the value of this quantity, the more suitable candidate material it is in the inner-shroud-to-outer-shroud clamping applications. It should be noted that Figure 5-9 contains three sets of bars. The ones labeled “FEM” were yielded by the present FEM analysis while the ones labeled “Analytical” and “Analytical + Oxidation” were yielded by the analyses presented in the next section. Examination of the results labeled “FEM” in Figure 5-9 reveals the following ranking of the candidate alloys: M₁, M₅, M₂, M₃, M₄.
Figure 5-9 Relative retention of the clamping-spring force after exposure to 750 K for 24,000 hours for the five spring materials, M₁–M₅. The results labeled “FEM” and “Analytical” include the effect of creep relaxation alone, while the ones labeled “Analytical + Oxidation” include both the effects of creep relaxation and oxidation.
5.5 Simplified Analytical/Numerical Approach

The procedure developed and employed in the previous section can be readily used for the selection of materials in applications requiring materials with high creep-induced-stress-relaxation resistance. Unfortunately, the procedure entails at least an intermediate level of working knowledge in the area of the thermomechanical finite element analyses. In this section, a simplified, non-finite-element-based procedure is developed and employed for the selection of the materials for use in the same applications, i.e. the applications requiring high-resistance to creep-induced stress relaxation.

5.5.1 Analytical/Numerical Procedure

As mentioned earlier, the stress state within a helical spring is non-uniform. It was also discussed in Section 5.2.3 that the Ashby-Abel approach is limited to the components subjected to uniform stress fields. To make the Ashby-Abel approach applicable to a helical-spring materials selection problem, the non-uniform stress-state within the spring will be substituted in the analysis, by its maximum (shear) stress component. It is recognized that this substitution will make the analysis less accurate but the extent of this inaccuracy is not clear up front and, as will be shown below, material ranking established through the use of the finite-element analysis may not be altered.

**Governing Equation:** Since the force within the spring depends only on the spring elastic displacement, the governing equation for the force within the compressed spring at the expected life of the component, \( F(t_{life}) \), can be defined as:

\[
F(t_{life}) = k \left[ \delta_{total} - \delta_{\text{creep}}(t_{life}) \right] = k \left[ \delta_{total} - L \varepsilon_{\text{creep}}(t_{life}) \right]
\]  

(5.15)
where \( k \) is the spring stiffness, defined as:

\[
k = \frac{Gd}{8C^3n_{\text{turns}}} \left( \frac{C^2}{C^2 + 0.5} \right)
\] (5.16)

and \( G \) is the material shear modulus, \( d \) the spring-wire diameter, \( C = D/d \), \( D \) the spring diameter, \( n_{\text{turns}} \) the number of turns in the spring, \( \delta_{\text{total}} \) and \( \delta_{\text{creep}} \) are respectively the total axial and creep axial displacements, and \( L \) is the uncompressed length of the spring.

**Creep Constitutive Law:** The creep strain after \( t = t_{\text{life}} \) is defined as:

\[
\varepsilon_{\text{creep}}(t_{\text{life}}) = \int_{0}^{t_{\text{life}}} \dot{\varepsilon}_{\text{creep}}(t') dt'
\] (5.17)

where \( \dot{\varepsilon}_{\text{creep}}(t') \) is given by Eq. (5.9), in which \( \sigma \) is replaced with \( \sigma(t') = \sqrt{3} \tau_{\text{max}}(t') \) and denotes the equivalent stress at the location of the maximum shear stress \( \tau_{\text{max}} \).

**Kinetic Relationship:** \( \tau_{\text{max}}(t') \) is related to \( F(t') \) through the following kinetic equation:

\[
\tau_{\text{max}} = \frac{8DK}{\pi d^3} F(t)
\] (5.18)

where \( K = \left( \frac{4C-1}{4C+4} \right) + 0.615C \). By substituting Eqs. (5.9) and (5.16)–(5.18) into Eq. (5.15) one obtains an integral equation in the form:

\[
F(t_{\text{life}}) = k \left[ \delta_{\text{total}} - A_1 \exp \left( \frac{-Q}{RT} \right) \left( \sqrt{3} \cdot \frac{8DK}{\pi d^3} \cdot \frac{1}{\sigma_0} \right)^n L \int_{0}^{t_{\text{life}}} [F(t')]^n dt' \right]
\] (5.19)
This equation can be solved numerically by: (a) discretizing the time scale between 0 and $t_{life}$; and (b) for each $0 < t \leq t_{life}$, employing recursively a numerical integration scheme to evaluate the right hand side of Eq. (5.19) and a non-linear algebraic equation solver to solve for $F(t)$. 


5.5.2 Results and Discussion

Temporal evolution of the clamping force within the spring for the five spring-material candidates, \( M_1 \)–\( M_5 \), obtained using the analytical/numerical procedure in this section, is depicted in Figure 5-10. Using the same results, a second set of bars labeled “Analytical” for the relative retention of the clamping-spring force is depicted in Figure 5-9. Examination of the results depicted in Figure 5-10 and their comparison with the ones shown in Figure 5-8, as well as comparison of the two sets of bars in Figure 5-9, reveals that:

(a) the degree of clamping force retention predicted by the analytical/numerical approach, for all five candidate alloys, is somewhat smaller than that predicted by the more rigorous finite element based approach. For example, for alloy \( M_1 \) the force retention ratios predicted by the analytical and FEM approaches are 0.77 and 0.79, respectively. This finding was expected considering the fact that, in the simplified analysis, the maximum value of the stress was used; and

(b) the ranking of the five candidate alloys \( M_1 \)-\( M_5 \), as predicted by the two approaches has not changed.
Figure 5-10 Temporal evolution of the clamping force within the spring for the five spring-material candidates, M₁–M₅, as yielded by the approximate and computationally less expensive analytical/numerical procedure.

5.5.3 The Effect of Spring-Material Oxidation

As mentioned earlier, the clamping spring is not only expected to be subjected to creep but also to oxidation. Since the latter phenomenon reduces the effective wire diameter of the spring, it is expected to cause a reduction in the spring stiffness and, thus, also contribute to the
clamping force relaxation within the spring. Before one can include the effect of oxidation into the clamping force relaxation analysis, the question of diffusion-limited vs. chemical reaction limited oxidation regimes has to be resolved (since the oxidation-reaction rate laws are different for the two regimes). Typically, this question is resolved by evaluating the so-called Pilling-Bedworth parameter, \( R_{PB} \). This parameter is defined as the ratio of the oxide unit-cell volume and the associated metallic unit-cell volume. When \( R_{PB} \) is less than 1, the oxide layer (being of a smaller volume) is subjected to tension and cracks, enabling oxygen a direct access to the unoxidizing metal. In this case, oxidation is a chemical reaction controlled process. Furthermore, if \( R_{PB} \) is greater than 2, oxide (having a larger volume) is subjected to compressive stresses by the adjoining metal, which causes it to chip off from the unoxidized metal and leave the unoxidized metal fully exposed to oxygen. In this case, oxidation is again a chemical-reaction controlled process. Lastly, when \( 1 < R_{PB} < 2 \), oxide tends to adhere to (typically loosely), and act as a protective layer for, the underlying metal. In this case, oxidation is a diffusion-controlled process, since oxygen has to diffuse through the oxide layer in order to reach the unoxidized metal.

All five clamping-spring candidate materials, \( M_1-M_5 \), have \( 1 < R_{PB} < 2 \) and, thus, are associated with a diffusion-controlled oxidation-reaction regime.

In the case of the diffusion-controlled oxidation reaction, oxide thickness, \( h_{oxide} \), increases parabolically with oxidation time, \( t \), as \( h_{oxide} = k_{oxide} t^{1/2} \) where \( k_{oxide} \) is a material, temperature and oxygen partial pressure dependent reaction-rate constant. To include the effect of oxidation in the analytical/numerical analyses presented in Section 5.5.1, wire diameter, \( d \), in Eq. (5.19) should be replaced with \( d - 2k_{oxide} t^{1/2} \). For the candidate spring materials \( M_1-M_5 \), the parabolic rate constant
$k_{oxide}$ at 750 K and partial pressure of oxygen of 0.2 are obtained in the ongoing work as: 1.1E-08, 1.8E-08, 1.5E-08, 2.2E-08 and 7.5E-09 m/s$^{0.5}$, respectively.

The results obtained in this portion of the work are shown in Figure 5-9, the bars labeled “Analytical + Oxidation”. A comparison of these results with the ones labeled “Analytical” in the same figure reveals: (a) oxidation indeed decreases the degree of clamping force retention within the spring; and (b) the ranking of the candidate materials has changed to M$_1$, M$_5$, M$_3$, M$_2$, M$_4$. However, alloy M$_1$ remains the best candidate material for the clamping-spring under the thermomechanical and oxidizing conditions investigated in the present work.
5.6 Summary And Conclusions

Based on the results obtained in the present work, the following main summary remarks and conclusions can be drawn:

1. The problem of materials selection for springs used to clamp an inner shroud segment to the outer shroud block in utility and industrial gas turbine engines is addressed by developing two alternative material-selection procedures. The first of these procedures is fairly rigorous, computationally intensive and based on the use of a finite element analysis. The other procedure is less rigorous, computationally less expensive and based on the use of a simplified analytical/numerical procedure.

2. The spring is initially pre-compressed and placed into the outer-shroud cavity in order to provide a clamping force between the inner shroud segment and the outer shroud block. However, since the spring is subjected to high-temperature and oxidizing conditions, it experiences creep and surface oxidation, and in turn a reduction in its clamping force. Consequently, the material selection problem is stated as: materials are needed, which minimize the reduction in the spring clamping force over the expected life of the inner-shroud segment.

3. In the absence of oxidation, the two approaches yielded different, but mutually consistent, results with identical ranking of the clamping-force candidate materials.

4. The inclusion of the oxidation effects, on the other hand, showed that oxidation-induced loss in the spring material increases the extent of clamping-force reduction and affects the ranking of the candidate materials.
5.7 References


8. M. Grujicic, J. S. Snipes, R. Galgalikar, R. Yavari, V. Avuthu and S. Ramaswami, “Multi-length-scale derivation of the room-temperature material constitutive model for SiC/SiC


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6.1 Conclusions

The present work was focused on material model development for SiC/SiC CMCs. This material is intended to replace the existing metallic materials in the industrial and aviation gas turbine engines. The advantages of using CMCs in the gas-turbine engines are manifold: increase in the combustion temperatures, increased operating efficiency, weight reduction and lower emissions. Initially only stationary, low stress experiencing components are being manufactured using CMC material. As the development of the CMC material continues, in future it is expected that more complicated components will be made from CMCs. Thus the multi-length scale computational model developed for predicting microstructure-property relationships for CMCs in the present work will complement future experimental design efforts by providing predictive capabilities to reduce development time and cost. The work presented in the previous chapters can be summarized as follows:

1. A multi-length scale room temperature material constitutive model has been developed for SiC/SiC CMCs. The component length scale material model is developed by conducting homogenization at two characteristic microstructural length scales: fiber/tow length scale and ply/lamina length scale. The brittle and stochastic failure of the constituents has been included in the material response by using Weibull distribution. The resulting component length scale homogenized material model shows the characteristic elastic and inelastic behavior (resulting from material cracking) demonstrated by CMCs. This model has been implemented as a user defined subroutine for a commercial finite element program ABAQUS. Suitability of
this material model is demonstrated by conducting FOD impact testing on an inner shroud, a prototypical CMC stationary component used in gas-turbine hot-section.

2. The room temperature component length scale material model is extended to include the in-service material degrading environmental effects experienced by CMC components in the hot-section of gas-turbine. Four such effects have been identified: prolonged high temperature exposure in the absence of mechanical loading, differential creep, dry oxidation and wet-oxidation. The component length CMC material properties are made a function of the nature, duration and extent of exposure to a given environmental condition. Finally, an inner shroud that is exposed to aforementioned degrading environments is tested for FOD impact resistance. The obtained results clearly show that the wet-oxidation has the highest material degrading effects for a given exposure duration and temperature.

3. The CMC material model is next extended to include the creep effects. Although the creep is not expected to cause a very high damage in the low-stress experience stationary component like the inner shroud, for rotating components like blade, creep deformation and creep rupture is a primary life limiting phenomenon. The CMC material behavior is anisotropic (transversely isotropic) and based on the attendant microstructure it is expected that the creep response would also be anisotropic. An anisotropic creep material model is developed and parameterized using the experimental results for CVI based CMCs. The primary and secondary regimes of the CMC creep response are predicted quite accurately by the developed model. A separate model is developed and parameterized for predicting creep rupture time. This material model is again implemented as user defined subroutine for a
commercial FEA software ABAQUS. The usability of this creep model is demonstrated by applying it to a CMC blade used in a low pressure turbine of an aviation engine.

4. The CMC components face a unique challenge when used in hot-sections of gas turbine engines due to significantly different (approximately 3-5 times lower) thermal expansion coefficients compared to the surrounding metallic components. Thus the conventional attachment techniques cannot be used for CMC components. To allow for differential thermal expansion a “floating” type of assembly is used, which uses a metallic spring to clamp the CMC component with the surrounding metallic components. Since this assembly is located in the hot-section of the gas turbine, the metallic spring experiences creep and oxidation leading to a loss in its clamping force, potentially dictating life of the CMC component. A material selection procedure is presented for the clamping spring based on FEA and an analytical method. Both methods yield consistent results when only creep problem is considered, however the presence of oxidation shows to affect the material selection.

6.2 Suggestions for future work

1. The multi-length scale material model developed for SiC/SiC CMCs should be expanded to include the high-strain rate effects. The material is subjected to impact loading conditions in the hot-sections of the gas turbine, under FOD. Inclusion of high strain rate effects will enhance the material failure prediction under such conditions.

2. Since in future, it is expected that SiC/SiC CMCs will be used for components experiencing cyclic loads such as blades; the effect of fatigue should be included in the multi-length scale CMC material model.
3. The thermal expansion effects should be included in the multi-length scale CMC material model by making it a coupled temperature-displacement type of model.

4. The four in-service environmental effects on the CMC material behavior have been considered in isolation. Their combined effect on CMC material behavior should be considered since it will contribute to more realistic life prediction for CMC components.

5. The generalized 3D anisotropic creep model presented in chapter 4, should be enriched by additional experimental data to determine values of the shear ratios. This will lead to fully defined 3D creep model for CMCs.