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Abstract

I develop a dynamic general equilibrium overlapping generation model with endogenous fertility, human capital accumulation and intergenerational transfers to investigate the quantitative effect of one-child policy in China after 1980. The intergenerational transfers from mid-age worker to their old parents act as a channel through which parents have incentive to invest in quantity and quality of children. A calibrated version of the model implies that exogenous fertility restrictions imposed by one-child policy increase expected years of schooling by 2.91 years on average during policy periods, and the resulting faster human capital accumulation would generate a annual gain of moderate 5.01% in GDP per worker decades after the policy enacted. Quantitative results also suggest that the policy decreases population growth by 1.23% points on average in 1980 – 2020, and generates large shift of population age structure during demographic transition from high to low fertility. When dependency ratio reaches the peak around 2030, the share of old-age dependents rises by 17.8% points and the share of mid-age workers decreases by 5.92% points, compared to the no-policy scenario. The theory also predicts that for poor regions who generally face more binding fertility restriction, the policy offered greater incentive to invest in children’s quality, leading to a temporarily faster income growth.
Dedication

I dedicate my dissertation work to my family for their endless love.
I would like to express my sincere gratitude to my advisor, Dr. Robert Tamura for his generous help and continuous support of my research. I would like to thank the other members of my committee, Dr. Michal Jerzmanowski, Dr. Scott Baier, and Dr. Chungsang Lam for their engagement, insight comments, and practical advise.
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Chapter 1


1.1 Introduction

Over the last three decades, China experienced a spectacular economic transformation which included not only one of the biggest economic booms and widespread privatization on both agriculture and industry, but also a rapid human capital accumulation and radical change in demographic structure. Almost simultaneously with the Chinese economic reform from 1979, one-child policy, as the most influential birth control scheme in human history, was introduced to mitigate the fast population growth to adapt to the cultural mode of modernization. After its implementation, fertility rates are significantly falling over the last 30 years, which created an increased share of the working-age population and hence a boost to per capita income growth. During 1982-2010, total fertility rate has dropped about 54%, declining from 2.6 in 1980 to 1.2 in 2010. During the same time period, China
has undergone a sustained increase in human capital accumulation, with expected years of schooling rising about 56%, from 8.05 years in 1982 to 12.58 years in 2010. On October 29, 2015, China made a change from one-child policy to two-child policy. Chinese government claimed that the one-child policy is responsible for preventing 400 million excess births, which fueled China’s dramatic post-1978 economic boom. However the acclamation is questionable, since with economic growth and development, gradually established social security system and associated rising educational costs, fertility would decline because people tend to have fewer children and small families, which is widely observed in the history of developed countries.

Empirical evidences also show that since 1990, this policy is less effective in east and northeast provinces. In 2000, eight provinces in East and Northeast area already had total fertility rate below 1, indicating that this policy is not binding at all in these provinces. At the same time, a sizable decrease in fertility rate can be observed in Middle and West provinces, all of which are still under the influence of compulsory birth control. Due to its disparate impact across regions, the economic consequences of one-child policy would varied from East provinces to West provinces.

After its decades-long enforcement, one-child policy began to caught some attentions from academia in recent years. Choukhmane, Coeurdacier and Jin (2014) develop a life cycle model and argue that the fertility restrictions provide incentives for households to increase their children’s education and concludes that the policy significantly increased the human capital of the only child generation and accounted for large rise in aggregate savings. These effects on household saving was also examined in Ge, Yang and Zhang (2012) paper by cohort analysis. They focus on how the demographic structural change shaped by the policy has impact on saving decisions made by different cohorts of household.

However, one-child policy is still heavily under-studied. Several questions associated with it come up. How quantitatively the policy distorted people’s fertility choices and human capital investment during post-reform period. Whether the birth control policy contributed to China’s economic output although it is judged as the most cruel government policy in human history. And how it has shifted the demographic structure for the whole society. What are the consequences when it is totally abandoned. And how does it has impacted the economic disparity across regions.

To investigate its effects on these issues, in this paper, a general equilibrium overlapping generation model is developed and employed with endogenous human capital investment, fertility choice, and inter-generational transfer from mid-age children to their parents. Having children is assumed to
bring positive utility to parents since people are willing to have children. Furthermore, children can be treated as one kind of insurance for old-age people due to the potential transfer payments from them; also educating children well is helpful to generate future benefits when they grown up, which may become a potential economic gain for parents. According to the model, school efficiency are calibrated for 28 provinces and the whole country. It shows that the province facing a better school efficiency in time period $t$ associates with a higher level of GDP per worker in next time period. The reason is that a better school efficiency indicates a lower time cost of educating children, and it does trigger a higher human capital investment from parents on children. One facing better school efficiency at young age has higher human capital stock when she grows up, and the resulting higher human capital stock could contribute to a higher level of real GDP per worker. The model also predicts that with birth control, a obvious drop in fertility rate can successfully lead to faster human capital accumulation. The declines in fertility caused by the policy cause larger shift of population age structure during demographic transition. Quantitative results imply that the share of non-productive dependents rises from 55% to 72%, while the share of mid-age workers declines from 45% to 28% from 1990 to 2030. When dependency ratio reaches the peak around 2030, the share of old-age dependents would be increased by 17.8% points and the share of mid-age workers would be decreased by 5.92% points, compared to the no-policy scenario.

The theory yields additional predictions which is consistent with the evidence of Chinas economic inequality across regions. From establishment of the country, inequalities in output per worker gradually got deeper over time, and continued to proceed after 1980. But this pattern changed after 2000, an obvious catch-up was observed on most provinces in northeast, middle and west region. Figure 1.1 suggests that most of the east provinces face a relative loose constraint by the one-child policy, due to sizable lower fertility rates they already reached before the policy started. In 2006, for most east and northeast provinces, fertility constraint was even not binding, while all west and middle except two are still under its constraints. Due to the disparate influences, poor provinces under greater policy pressure has to decrease their fertility, inevitably inducing a higher investment in education according to the theory. This resulting rise in human capital accumulation helped West and Middle provinces to achieve a faster income growth for later generations, which is suggested by counterfactual simulation. At the same time, richer provinces under almost zero pressure from the policy would choose fertility and human capital investment as they like, as a result, the policy can
not induce the surge in income growth in these area. The theory in this paper successfully predicts a smaller gap in GDP per worker between rich and poor provinces occurs during policy period, compared to no-policy scenario.

This paper is also part of literature related to fertility, human capital accumulation and income growth. From the Barro and Becker (1989) paper, they provides a standard general equilibrium model where parents make choices of fertility along with decisions on consumption and intergenerational transfers to children. They assumed that parents can get utility from their own consumption and from their offspring. Becker, Murphy and Tamura (1990) then develop a growth model with endogenous fertility and investment in human capital, where agent is facing two kinds of budget constraints: time and resource. Under the key assumption that the rate of return to human capital rises as human capital stock increases, the paper generates multiple steady states: undeveloped scenario with large family size and little human capital investment, and the developed scenario with small family size and large amount of human capital investment. Their paper actually captures Malthusian and neoclassic elements. Later, in the Murphy, Tamura and Simon (2008), Tamura and Simon (2016), Murphy, Tamura and Simon (2016) papers school efficiency as key parameter is incorporated into the model to capture difference in time cost of human capital investment across space and time.

Ehrlich and Lui (1991) began to incorporated the role of children as old-age insurance for parents into their OLG model. Parents invest children’s education and receive future transfer committed by self-enforcing contract. They also put forward the companionship function, which allows parents to receive utility from quantity and quality of children. Following their work, Boldrin and Jones (2002) endogenized the material support old parents receive from children in their OLG model. They assumed that adult children cares about old parents’ consumption, and that they makes fertility choices along with decisions on their own consumptions, saving and transfer to their parents with either cooperative or non-cooperative mechanisms. In this paper, their model is extended to capture human capital investment made by adult children.

The rest of the paper is organized as follows. Section 1.3 describe empirical evidence. Section 1.4 present theoretical analysis. Section 1.5 show calibration and numerical results. And section 1.6 is conclusion.
1.2 Empirical Evidence

China’s one-child policy launched in 1980, and ended on October 29, 2015. Since then it was replaced by a two-child policy. It was not the first population control policy since the founding of P.R.China. During the time period of 1971-1979, there was a precedent one, known as later-longer-fewer campaign in a voluntary basis, which put forward three goals: later childbearing, greater spacing between first and subsequent children, and fewer children. During this period, fertility dropped about 30%. The progress did not satisfy central government when it realized that young age structure of population would produce economic growth. Then, one-child policy came into being. According to the policy as it was most commonly enforced, a couple was allowed to have one child. The policy is enforced at provincial level, and varies in accordance with local conditions. It allows many exceptions. In most rural areas, if the first child of a couple was a girl, they were allowed to have a second one. Some ethnic minorities can follow it on a voluntary basis. But for each couple of the dominant Han nationality in urban areas, only one child is allowed to be born. Without a birth permit, any pregnancy is considered “out-of-plan” and therefore illegal, and the pregnant mother would be forced to undergo abortion in many places during certain time period. Without a birth permit, fines were imposed on extra baby born in the family based on their income and other factors. Thirty-five years after it launched, China announced a new birth control policy which allows every family has two children, making an end of the one-child policy.

Figure 1.1 plots the fertility rates for 28 provinces (out of 31) during 1945-2010. The top left subfigure shows fertility rates for East provinces (Beijing, Tianjin, Hebei, Shanghai, Shandong, Fujian, Guangdong, Jiangsu, Zhejiang), the top right one presents the fertility rates for Northeast provinces (Heilongjiang, Liaoning, Jilin), the subfigures in left bottom and right bottom show the historical performance of Middle (Anhui, Hubei, Hunnan, Henan, Jiangxi, Shanxi) and West provinces (Sichuan, Guangxi, Yunan, Inner Mogolia, Guizhou, Shaanxi, Gansu, Qinghai, Ningxia, Xinjiang), respectively. Fertility rates for all provinces were actually falling down dramatically before the introduction of one-child policy. Several empirical facts are summarized as the following:

1. For provinces in East and Northeast, fertility rates were falling down in 1960s, and sharp decreases occurred before mid 1970s. (e.g. in 1963, fertility rate in Beijing is 6.9, while in 1974 it decreased to 1.3, and for Hebei province it decreased from 7.2 to 2.5.) For provinces
in Middle and West part of China, fertility rates were falling down from early 1970s, a decade later than in East.

2. After the one-child policy was introduced, no obvious decline in fertility for all provinces in early 1980s. At that time, family planning commission of China faced very strong public resistance and put low pressure on local birth planning cadres during the period. With the issue of Central Document 7 in 1984, Central Committee of the Communist Party reemphasized its stand on fertility control and urged adapting propaganda in accordance with local conditions (Bongaarts and Greenhalgh 1985).

3. For most provinces in East and Northeast, fertility rates were already around 2 before 1980, and did not decline a lot under the policy; for middle and west provinces, fertility rates experienced a relative bigger falling after one-child policy was enacted.

Another important observation is the patterns on expected years of schooling\(^1\) for 30 provinces in China. Figure 1.2 plots the expected years of schooling during the time period of 1949-2010. Almost all provinces experienced fast growth until late 1960s. During culture revolution (1967-1977) a sharp decrease in expected years of schooling occurred in every provinces. After that, for most of provinces, it continued to rise over time. some Middle and West provinces have already began to catch up most East and Northeast provinces since 1980s in the expected years of schooling.

During the same time the policy was enacted, Chinese economic reform was launched. A common argument is that regional incomes have been diverging across China during the post-reform period (Pedroni and Yao 2006). To observe the regional disparities more accurately, Figure 1.3 plots the ratios of real regional output per worker for 28 provinces to that of Shanghai. The curves in left top subfigure depicts the ratios for 9 provinces in East China. In right top subfigure, these curves denotes the ratios for 3 provinces in Northeast China. The subfigures in left bottom and right bottom, respectively, plot the historical performance for 6 provinces in Middle and that for 10 provinces in West. In the past 60 years, except for those provinces in East, all provinces account for lower and lower ratios over time. The gap between rich and poor provinces continues to widen after 1980 until 2000, since when obvious catching up can be pervasively observed in almost every province.

\(^1\) Expected years of schooling are the total number of years during which a child of age 7 can expect to spend full time schooling in their youth. See Appendix B for more details
So far, all provinces experienced sizable decline in fertility before and after the enforcement of one-child policy, and sustained increase in human capital accumulation during the policy period, which are measured by expected years of schooling. And income inequality is growing overtime until recent decade. One feature of the model described in next section is that the decrease in fertility associates with the increase in human capital accumulation, and the rate of human capital accumulation affects income growth. This make it possible to discuss the role of one-child policy in human capital accumulation and the income growth at national level and at provincial level.

1.3 Theoretical Analysis

Based on Boldrin and Jones (2002), Murphy, Tamura and Simon (2008), I develop a parsimonious 3-period overlapping generation model with endogenous fertility, human capital accumulation and intergenerational transfer. The set-up is meant to capture the feature of Chinese society: children are considered as a source of old-age support.

1.3.1 Set-up

Consider an overlapping generation model where people live up to three time periods: youth, middle-age and old-age. Let $N_t$ denote the population of mid-age people at time $t$. The total population, denoted as $N_{total,t}$, is the sum of the number of young, mid-age and old-age people. That is, $N_{total,t} = N_{t+1} + N_t + N_{t-1}$. Suppose that in period $t$, old-age agents depend on the support from mid-age agents and their savings to live. Mid-age agents work for one period and make decisions on quantity-quality of children, along with transfer to parents, siblings’ transfer are taken as given.

Preferences. The life-time utility for mid-age agent includes the consumption at middle-age $c_{m,t}$, the consumption at old-age $c_{o,t+1}$, parent’s consumption at old-age $c_{o,t}$, the benefits from having children:

$$U_t = ln c_{m,t} + \delta ln x_t + \eta \ln c_{o,t} + \beta \ln c_{o,t+1}$$

where $x_t = n_t(1 - \rho_t)$. $n_t$ is the number of children born by each individual, and $\rho_t$ is the young
adult mortality rate at time $t$, considered as exogenous.

**Budget constraints.** The mid-age individual faces two resource constraints and one time constraint at time $t$:

$$
c_{m,t} + \pi_t + a_t = m_t
$$

$$
c_{o,t+1} = \sum_{i=1}^{x_t} \pi_{i,t+1} + R_{t+1}a_t
$$

$$
1 = l_t + x_t(\tau_t\lambda_t + \nu)
$$

where $\nu$ is the fixed minimum time needed to rear each child, $\tau_i$ is the time spent on educating each child, $\lambda_t$ is the time efficiency of educating, $\pi_t$ is the amount of transfers to parents. And here I assume that every child take their siblings’ transfer as given. Agent in mid-age rears and educates children, make earnings by working $l_t$ hours, lends (or borrows) $a_t$ amount at market-determined interest rate $R_{t+1}$ and transfers part of earnings $\pi_t$, to parent. When come to old-age, agent receives transfer from children, collects (or pays) principal and interest if he lent (or borrowed). The intergenerational transfers from mid-age worker to their old parents act as a channel through which parents have incentive to invest in quantity and quality of children.\(^2\)

The mid-age agent’s earning at period $t$ is determined by the amount of time allocated on working, the human capital $h_t$ the agent has, and the wage rate $w_t$.

$$
m_t = w_th_t(1 - x_t(\tau_t\lambda_t + \nu))
$$

**Production.** The production side of the economy is given by an aggregate production function

$$
Y_t = F(K_t, z_t H_t)
$$

where $K_t$ is aggregate physical capital stock, $z_t$ is the labour augmented productivity, and $g_t = \frac{z_t+1}{z_t}.\(^3\)

Assuming capital depreciation rate is $\zeta$ at the end of every period, the dynamics of physical capital

\(^2\)The incorporation of endogenous transfer from children to parents not only put forward a fundamentally reason for rearing and educating children, but also drives a dynamic transition on fertility choices. See Appendix A for more details.

\(^3\)In this paper, $g_t$ specially refer to growth rate of productivity generated from institutional improvements and economic reforms in China.
accumulation is given by

\[ K_t = N_{t-1}a_{t-1} + (1 - \zeta)K_{t-1}. \]

And \( H_t \) is the aggregate stock of human capital used for producing goods, which is given by

\[ H_t = h_t l_t N_t = h_t (1 - x_t(\tau_t \lambda_t + \nu)) N_t \]

Therefore, the production function at time \( t \) is specified by

\[ F(K_t, z_t H_t) = K_t^\alpha \left( z_t h_t (1 - x_t(\tau_t \lambda_t + \nu)) N_t \right)^{1-\alpha}. \]

By solving firm’s profit maximization problem, \( F(K_t, z_t h_t l_t N_t) - w_t h_t l_t N_t - R_t K_t \) by choosing \( N_t \) and \( K_t \), get

\[ w_t = (1 - \alpha) k_t \alpha z_t \left( \frac{k_t}{z_t h_t l_t} \right)^{-\alpha}, \]

\[ R_t = \alpha \left( \frac{k_t}{z_t h_t l_t} \right)^{\alpha-1}. \]

where \( k_t = \frac{K_t}{N_t} \), per capita physical capital stock.

**Market clear condition.** The whole economy faces a market clear condition at every period.

\[ Y_t = (c_{m,t} + a_t) N_t + c_{o,t} N_{t-1}. \]

**Human capital accumulation.** The current per capita human capital in the economy is assumed to be determined by last generation’s per capita human capital, per capita investment made by last generation and the productivity to produce human capital, \( A \):

\[ h_{t+1} = Ah_t \tau_t \]

\[ h_t \] is the effective per capita human capital used in producing output. \( l_t = 1 - x_t(\tau_t \lambda_t + \nu) \) is how much time spent on working. Therefore, according to the definitions the marginal product of human capital is actually the wage rate of per unit of human capital, which is wage per unit of time per unit of human capital.
1.3.2 Optimal Decisions, balanced growth path and model dynamic

Mid-age agent maximizes life-time utility by choosing the number of children $n_t$, the investment on children’s education $\tau_t$, the transfer to parents $\pi_t$, the amount of capital to lend (or borrow) $a_t$. The first order conditions with respect to $n_t$, $\tau_t$, $a_t$ and $\pi_t$ yield

\begin{align}
- w_t h_t (1 - \rho_t) (\tau_t \lambda_t + \nu) + \beta \frac{\partial c_{o.t+1}}{\partial n_t} + \delta \frac{\partial c_{o.t+1}}{\partial n_t} &= 0 \quad (1.1) \\
- w_t h_t (1 - \rho_t) n_t \lambda_t + \beta \frac{\partial c_{o.t+1}}{\partial \tau_t} &= 0 \quad (1.2) \\
- \frac{1}{c_{m,t}} + \beta \frac{\partial c_{o.t+1}}{\partial a_t} &= 0 \quad (1.3) \\
- \frac{1}{c_{m,t}} + \eta c_{o,t} &= 0 \quad (1.4)
\end{align}

**Intergenerational transfer.** Given the optimal choices of fertility, education investment and saving, the decision on intergenerational transfer is the first order condition for $\pi_t$, Eq.(1.3), and resource constraint for old-age agent$^6$,

\begin{align}
\pi_t &= \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} (m_t - a_t) - \frac{R_t}{\eta + n_{t-1}(1 - \rho_{t-1})} a_{t-1}. \quad (1.6) \\
c_{o,t} &= \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} (m_t - a_t) + \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} R_t a_{t-1}. \quad (1.7)
\end{align}

$^6$The resource constraint for old-age agent is deduced from mid-age individual’s resource constraint:

\begin{equation}
 c_{o,t} = \sum_{i=1}^{s_{t-1}} \pi_{i,t} + R_t a_{t-1}. \quad (1.5)
\end{equation}

From Eq.(1.4), $c_{o,t} = \eta c_{m.t}$. And $c_{m.t} = m_t - a_t - \pi_t$. Combined the three equations, Eq.(1.6) is obtained. Then plug Eq.(1.6) into Eq.(1.5) with taking siblings' transfers as given, Eq.(1.7) is obtained.
Then, get its derivatives with respect to $\tau_{t-1}$, $n_{t-1}$, $a_{t-1}$,

$$\frac{\partial c_{o,t}}{\partial \tau_{t-1}} = \frac{\eta n_{t-1}(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \frac{m_t}{\tau_{t-1}}$$

$$\frac{\partial c_{o,t}}{\partial n_{t-1}} = \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} \pi_{t}$$

$$\frac{\partial c_{o,t}}{\partial a_{t-1}} = \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} R_t.$$

So for period $t+1$,

$$\frac{\partial c_{o,t+1}}{\partial \tau_{t}} = \frac{\eta n_{t}(1 - \rho_{t})}{\eta + n_{t}(1 - \rho_{t})} \frac{m_{t+1}}{\tau_{t}} \quad (1.8)$$

$$\frac{\partial c_{o,t+1}}{\partial n_{t}} = \frac{\eta(1 - \rho_{t})}{\eta + n_{t}(1 - \rho_{t})} \pi_{t+1} \quad (1.9)$$

$$\frac{\partial c_{o,t+1}}{\partial a_{t}} = \frac{\eta}{\eta + n_{t}(1 - \rho_{t})} R_{t+1} \quad (1.10)$$

The current decisions on the number of children and human capital investment on their descendants affect people’s future consumption through the transfers from children. Also the quantity and quality of child affect the total amount of transfer. Because each generation of mid-age agent cares about parents’ consumptions, by Eq.(1.6), the transfer is predictable, therefore future returns on investment on children’s quality are predictable as well. In this sense, parents consider children as an source of financial support in old-age.

---

$^6$ \[
\frac{\partial c_{o,t}}{\partial \tau_{t-1}} = \frac{\eta n_{t-1}(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \frac{\partial m_t}{\partial \tau_{t-1}}
= \frac{\eta n_{t-1}(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \eta_t A_{t-1} (1 - n_{t}(1 - \rho_{t-1})(\tau_{t} \lambda_t + \nu))
= \frac{\eta n_{t-1}(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \frac{m_t}{\tau_{t-1}}.
\]

$^7$ \[
\frac{\partial c_{o,t}}{\partial n_{t-1}} = \left( \frac{\eta(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \eta n_{t-1}(1 - \rho_{t-1})^2 \right) (m_t - a_t) - \frac{\eta(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \frac{R_0 a_{t-1}}{\eta + n_{t-1}(1 - \rho_{t-1})}.
\]

$$= \frac{\eta^2(1 - \rho_{t-1})}{(\eta + n_{t-1}(1 - \rho_{t-1}))^2} (m_t - a_t) - \frac{\eta R_0 a_{t-1}(1 - \rho_{t-1})}{(\eta + n_{t-1}(1 - \rho_{t-1}))^2}$$

$$= \frac{\eta(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \left( \frac{\eta}{\eta + n_{t-1}(1 - \rho_{t-1})} (m_t - a_t) - \frac{R_0 a_{t-1}}{\eta + n_{t-1}(1 - \rho_{t-1})} \right)$$

$$= \frac{\eta(1 - \rho_{t-1})}{\eta + n_{t-1}(1 - \rho_{t-1})} \pi_t$$
Saving. The first order condition for saving is

\[
\frac{1}{c_{m,t}} = \frac{\beta}{c_{o,t+1}} \frac{\partial c_{o,t+1}}{\partial a_t} = \frac{\beta}{c_{o,t+1}} \eta \frac{R_{t+1}}{\eta + n_t(1 - \rho_t)}
\]

That is,

\[
\frac{\beta}{c_{o,t+1}} = \frac{\eta + n_t(1 - \rho_t)}{\eta R_{t+1} c_{m,t}}
\]

(1.11)

Since first order condition for \(\pi_t\) indicates that \(c_{o,t+1} = \eta c_{m,t+1}\) by Eq.(1.4), plug it into Eq.(1.11), get

\[
\frac{c_{m,t+1}}{c_{m,t}} = \frac{\beta R_{t+1}}{\eta + n_t(1 - \rho_t)}
\]

(1.12)

The ratio of mid-age consumption for generation \(t + 1\) over mid-age consumption for generation \(t\) positively depends on gross return of physical capital at time \(t + 1\) and negatively depends on the fertility choice of generation \(t\).

Fertility and human capital investment. By Eq.(1.1), the first order condition with respect to \(n_t\) is

\[
- \frac{w_t h_t(1 - \rho_t)(\tau_t \lambda_t + \nu)}{c_{m,t}} + \frac{\beta}{c_{o,t+1}} \frac{\partial c_{o,t+1}}{\partial n_t} + \frac{\delta}{n_t} = 0
\]

Plug the Eqn.(8) and Eqn.(10) into it, get

\[
- \frac{w_t h_t(1 - \rho_t)(\tau_t \lambda_t + \nu)}{c_{m,t}} + \frac{\eta + n_t(1 - \rho_t)}{\eta R_{t+1} c_{m,t}} \frac{\eta(1 - \rho_t)}{\eta + n_t(1 - \theta_t)} \pi_{t+1} + \frac{\delta}{n_t} = 0
\]

That is,

\[
- \frac{w_t h_t(1 - \rho_t)(\tau_t \lambda_t + \nu)}{c_{m,t}} + \frac{\pi_{t+1}(1 - \rho_t)}{R_{t+1} c_{m,t}} + \frac{\delta}{n_t} = 0
\]
After rearrangement, it becomes,

\[
\frac{\delta}{n_t(1 - \rho_t)} = \frac{w_t h_t \lambda_t}{c_{m,t}} + \frac{\pi_{t+1}}{R_{t+1} c_{m,t}} - \frac{n_t(1 - \rho_t)}{\lambda_t} \frac{\partial c_{o,t+1}}{\partial \tau_t} \tag{1.13}
\]

The decision on number of children born depend on equating the utility of having an additional child to the net marginal cost of raising the child. LHS of Eq.(1.13) is the marginal benefit of having additional child. The first term in RHS is the wage loss of having one more child; the second term is actually the discount future transfer of the additional child.

The first order condition for \( \tau_t \) is

\[
-w_t h_t n_t(1 - \rho_t) \frac{\lambda_t}{c_{m,t}} + \frac{\beta}{c_{o,t+1}} \frac{\partial c_{o,t+1}}{\partial \tau_t} = 0
\]

Plug Eq.(1.11) and Eq.(1.9) into it, get

\[
-w_t h_t n_t(1 - \rho_t) \frac{\lambda_t}{c_{m,t}} + \frac{\eta + n_t(1 - \rho_t) \eta R_{t+1} c_{m,t}}{\eta + n_t(1 - \rho_t) \eta R_{t+1} c_{m,t}} \frac{m_{t+1}}{\tau_t} = 0
\]

That is,

\[
-w_t h_t n_t(1 - \rho_t) \frac{\lambda_t}{c_{m,t}} + \frac{n_t(1 - \rho_t) m_{t+1}}{R_{t+1} c_{m,t}} \frac{m_{t+1}}{\tau_t} = 0
\]

After rearrangement, it becomes

\[
0 = w_t h_t \lambda_t \tau_t - \frac{m_{t+1}}{R_{t+1}} \tag{1.14}
\]

Human capital investment hinges on equating the marginal cost of educating children, that is the first term in RHS, to the additional benefit from one more unit of human capital investment, that is the second term. Eq.(1.4), Eq.(1.11), Eq.(1.13) and Eq.(2.1) govern the behavior of the four endogenous variables in the whole system, and describe the transition process from initial period to balanced growth path.

**Balanced growth path.** For simplicity, I assume a constant school efficiency \( \lambda \), a constant productivity \( A \), and a zero young adult mortality rate. On balanced growth path \( n_t = n_{ss}, \tau_t = \tau_{ss}, \)
determined by the following equations:

\[ \tau_{ss} = \frac{\beta (1 - \nu n_{ss})}{\lambda (\eta + n_{ss} + \beta n_{ss})} \] (1.15)

\[ \left( \frac{\beta n_{ss}}{\eta + n_{ss}} - \frac{(1 - \beta)n_{ss} + \eta)\delta}{\beta \eta + \eta + n_{ss}} \right) s_{\pi,ss} = \frac{n_{ss}(\tau_{ss}\lambda + \nu)}{1 - n_{ss}(\tau_{ss}\lambda + \nu)} \frac{\delta(\eta + n_{ss})}{\beta \eta + \eta + n_{ss}} \] (1.16)

where

\[ s_{\pi,ss} = \frac{\eta}{\eta + n_{ss}} - \frac{\beta \eta + \eta + n_{ss}}{\eta + n_{ss}} \frac{\alpha}{1 - \alpha} \left( \frac{n_{ss}}{\eta + n_{ss}} - \frac{1 - \zeta}{\eta + n_{ss}Ag\tau_{ss}} \right) \]

And on balanced growth path, the growth rate of GDP per worker is determined by the following equation:

\[ \frac{y_{t+1}}{y_{t}} = Ag\tau_{ss}. \] (1.17)

Equation (1.15) is from the first order condition for \( \tau \), Eq.(1.2), and Eq.(1.16) is from the first order condition for \( n \), Eq.(1.1). The behavior of these two equations are depicted by the two curves in Figure 1.4. The red solid curve represents the combination of \( n \) and \( \tau \) that satisfies Eq.(1.15). It implies how educational investment response to more childbirth. More children born will reduce the resource available to educational investment on each of them. The black solid curve describes Eq.(1.16), which shows how fertility choice response to higher education investment. Higher education investment will cause higher human capital level, which will induce people to have fewer children.

1.3.3 Model Dynamics and implications

In the theory, the difference in schooling efficiency, \( \lambda \), would induce difference in human capital accumulation across regions. A higher value of \( \lambda \) means less time efficiency in investing education, indicating how much time cost parent pay out for one more year of schooling attained by children. Higher the time cost for parents, less investment parents would make in education. To reach the same level of schooling of children, parent has to spent less time on working, leading to a lower wage income in his/her mid-age.
Figure 1.5 depicts how state variables converges to balanced growth path over time with an illustrative calibration. As those figures show, \((n, \tau, a, \pi)\) converge quickly. From these figures, a better school efficiency cause a higher education investment, but slight change in fertility choices; a higher cost is compensated with higher return, that is, higher amount of transfer can be expected from each child; mid-age agents save fewer amount of income, since they have to reciprocate parents for their given at a such expensive price. However, a higher education cost doesn’t induce agent to make sizable change in fertility choice. The reason behind is that one more child means one more times of total wage cost (eg. educational cost and rearing cost) and one more year of schooling only means 0.05 more educational cost\(^8\). So, bigger \(\lambda\) would increases the wage foregone of having an additional child, but it won’t increase a lot since the educational cost spent on each child is relatively low. Although the decrease in marginal child’s human capital is significant (by the first subplot), the decrease in future transfers is not significant (by the forth subplot). According to Eq.(1.13), as \(\lambda\) increases, the first term in RHS increases a little and the second term decrease a little, which gives the total change in RHS is not huge. Therefore, the disincentive to having one more child only decrease slightly.

1.3.4 Basic model with one-child policy

One-child policy was introduced by Chinese government in 1979, and was enforced as a law in 1980. Suppose that under this policy the maximum number of children each individual can have is \(\bar{n}\), and \(\bar{n}\) varies at provincial level. If the most strict rule was undertaken, each individual have up to 0.5 child, so generally \(\bar{n} \in [0.5, 1]\). To capture the effect of birth control regulation, an additional constraint \(n_t \leq \bar{n}\) is added into the middle-age agent’s utility-maximization problem described already. However, population would shrink to zero when time goes to infinity, it is not realistic to implement the policy forever. Therefore, the one-child policy is unsustainable in long run. Also, considering this policy was abandoned on 2015, this model would become more realistic by making the following assumption:

**Assumption:** The one-child policy is carried on over a certain time period \([t', t' + T]\) and is public information for all agents.

\(^8\)Since one more year of schooling invested during a time period of 20 years is measured by \(\Delta \tau = \frac{1}{20}\).
Under the fertility constraint and the above assumption on the duration of birth control policy, two cases are discussed below.

**Case 1.** \( n^*_t = \bar{n} \). First order condition for \( n_t \) fails to hold for equality. So, optimal choices \( \tau^*_t, a_t^* \) and \( \pi_t^* \) is determined by first order conditions Eq.(1.2), Eq.(1.4), and Eq.(1.3) during the time period \([t', t' + T]\).

**Case 2.** \( 0 < n^*_t < \bar{n} \). Policy is not binding, the optimal solutions are exactly the same as the no-policy scenario described before.

### 1.4 Calibration and Simulation

In this section, I outline the procedure to calibrate all model parameters. The numerical values of those parameters for the whole country are contained in table 1.2 and table 1.3, together with brief explanations. Below follows some motivation for how those parameters are chosen.

1. I assume that each period corresponds to 20 years, and the initial cohort entering labour market is on 1950. Also assume that the new cohort entering labour market is ten years younger than the last one.\(^9\)

2. I set some parameters to commonly chosen values. The capital share of output, \( \alpha \), is set to 0.33. Time preference, \( \beta \), is set to 0.8179 for 20-year horizon, that is, 0.99 per annum. Depreciation rate of physical capital, \( \zeta \), is set to 0.558 for 20-year horizon, that is, 0.04 per annum. The time cost of rearing child, \( \nu \), is set to 0.125, which is consistent with Tamura and Simon (2008).

3. School efficiency in initial period of 1950-1970 for the whole country is set to 0.7057. It is calibrated according to Eq.(2.1). Firstly, rewrite Eq.(2.1) as,

\[
\lambda^\text{model}_t = \frac{k_t(1 - x_t\nu)}{(\alpha y_t + k_{t+1}x_t)\tau_t} \tag{1.18}
\]

Where \( y_t \) is real GDP per worker, \( x_t \) is from employment growth, \( k_t \) is real physical capital.

---

\(^9\)I have middle-aged cohorts in 1950-1970, 1960-1980, 1970-1990, and so on. And I assume that at initial periods, there is no old-age population. That means, mid-age agents at initial periods only care about number of childbirth and their life time consumption.
stock, and \( \tau_t \) is the ratio of expected years of schooling to 20 years. Then school efficiency in 1950-1970 can be obtained by the above equation using information from data.

To obtain school efficiency for later periods, assume that it converges linearly in 6th period from initial value to the value for US. The calibrated school efficiency in different time periods are contained in table 1.3. This assumption doesn’t affect comparisons of simulated results of between policy scenario and non-policy scenario, because it is the same in both economic scenarios and its effects can be canceled off under comparison.

School efficiency for US, \( \lambda_{us} \), is set to 0.21, which is calibrated according to Eq.(1.15) with \( n_{us} = 1, \tau_{us} = 0.7625, \nu = 0.125, \) and \( \beta = 0.8179 \) for 20-year horizon. The values for \( \tau_{us} \) and \( n_{us} \) are educational investment and fertility rate, respectively, on balanced growth path for US, and are consistent with Tamura and Simon (2008). More specifically, by Eq.(1.15), we can write,

\[
\lambda_{us} = \frac{\beta(1 - \nu n_{us})}{\tau_{us}(\eta_{us} + n_{us} + \beta n_{us})} \tag{1.19}
\]

Since from Eq.(1.17) and Eq.(1.31), we have,

\[
\eta_{us} + n_{us} = \frac{\beta R}{y_{t+1}/y_t}
\]

Then plug it into Eq.(1.19), we get

\[
\lambda_{us} = \frac{1 - \nu n_{us}}{\tau_{us}(\frac{R}{y_{t+1}/y_t} + n_{us})}
\]

Set \( y_{t+1}/y_t = 1.4859 \) for 20-year horizon, that is 1.02 per annum, and set \( R = 6.7274 \) for 20-year horizon, that is 1.1 on an annual basis. Note that \( R \) is constructed by cost share of physical capital times real physical capital stock, divided by real GDP for US during 1965-2014\(^{10}\). Then this setting produces \( \lambda_{us} = 0.21 \).

4. The productivity growth \( g_t \) in 1950-1970 for China is obtained according to production function with data on real GDP, real physical capital stock, expected years of schooling, and number

\(^{10}\)Data source: Federal Reserve Economic Data
of workers. According to production function,

$$g_t = \frac{zt+1}{zt} = \left(\frac{K_{t+1}^{\alpha} H_{t+1}^{1-\alpha}}{Y_t (K_{t}^{\alpha} H_{t}^{1-\alpha})}\right)^{1/\alpha}$$

(1.20)

(1.21)

$$= \left(\frac{K_{t+1}}{K_{t}^{1+1}}\right)^{\tau_{t+1}} \frac{h_{t+1} t_{t+1} N_{t+1}}{h_{t} t_{t} N_{t}}$$

With national data during 1950-1990, initial $g_t$ for China is obtained.

Then let productivity converge linearly to one in 2050 – 2070. And then assume that $z_t$ in initial period is normalized to one, therefore the national productivity can be obtained by $z_{t+1} = z_t g_t$. For each time period, the corresponding productivity and productivity growth used in simulation is contained in table 1.3. And figure 1.6 plot the productivity over time.


6. Preference for number of children, $\delta$, is set to 0.7529. Preference for old-age parents’ consumption, $\eta$, is set to 2.9057. Productivity to produce human capital, $A$, is set to 1.9. In previous discussion, I already assume that on balanced growth path annual gross growth rate of productivity due to economic reform and institutional improvement, is one. Further, I assume that on balanced growth path annual gross growth rate of GDP per worker is 1.02\% for China. That means $y_{t+1}/y_t = 1.4859$ for 20-year horizon. Then according to Eq.(1.17), we have $A = 1.9$.

11Young-adult mortality, $\rho$, defined as the probability of dying before age 35. It is specified as the unconditional probability of dying between the ages of 1 and 35, $\hat{p}_{1,35}$, plus the infant mortality rate, $m$, that is,

$$\rho = m + \hat{p}_{1,35}.$$  

This is very similar to the specification in Murphy, Tamura and Simon (2016) except that they use one third of infant mortality rate instead. They use one third of infant mortality because it explicitly distinguishes the cost of child death in infancy and a child death after the age of 1 with assumption that for infant deaths the woman has sufficient childbearing years remaining to replace the lost child and that the level of human capital investment in the child is much less than a child death after the age of 1 (Tamura and Simon (2016)). In this paper, the cost of rearing and investing in lost child doesn’t go into the budget constraint, therefore the unconditional probability of dying before age 35 is used to measure $\rho$ for simplicity and convenience. See Appendix.B for more details about how to obtain $\hat{p}_{1,35}$ and $m$.  

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Now with all these parameters already got, \((\delta, \eta)\) can be determined by Eq.(1.15), Eq.(1.16), and \((n_{ss}, \tau_{ss})\) on balanced growth path. I search for the two parameters \((n_{ss}, \tau_{ss})\) by “calibration exercise”. And \((n_{ss}, \tau_{ss})\) are chosen according to two criterion: fitting actual data on education investment as closely as possible and restricting \((n_{ss}, \tau_{ss})\) to reasonable ranges\(^{12}\). These exercise produces \(n_{ss}\) of 0.85 and \(\tau_{ss}\) of 0.7821 (15.6416 years of schooling over 20 years). And it also simultaneously generates \(\delta\) of 0.7529 and \(\eta\) of 2.9057.

I also calibrate model parameters and do simulation for provinces (Shanghai, Sichuan, Hunan, and Gansu). Compared with the procedure to produce parameters and do simulation for the whole country, there are several different criteria used to for provinces:

1. I use provincial level data to calibrate model parameters for corresponding provinces, while I use national level data to produce the ones for the whole country. As a result, simulated aggregate results are not the adding up of the provincial results. For instance, I use provincial school efficiency in 1960 – 1980 from table(1.1) for initial period.

2. Due to missing provincial level data in 1950, I assume that initial time period is 1960 – 1980, and new cohort is twenty years younger than the last one\(^{13}\).

3. Because fertility restriction is not binding for every province during policy periods, so the provinces where one-child policy is not binding are assumed to have no fertility restriction. For instances, I do not impose any fertility restriction in Shanghai in model simulation. For other three poor provinces (Sichuan, Hunan, and Gansu), they face fertility restriction in 1980 – 2020.

Except that, I follow the same procedure to produce model parameters and conduct simulations for provinces. In the following part of this section, I discuss the simulation results as well as more detail regarding to calibration and simulation.

**School efficiency.** The differences in time efficiency of schooling is an important cause of variations in human capital accumulation across provinces. To recovery the school efficiency parameter for each province, provincial data on employed labour, real GDP, real physical capital stock, expected years of schooling are used according to Eq.(1.18). Data on total number of employed people are from

\(^{12}\) The resulting \((n_{ss}, \tau_{ss})\) have to fall in reasonable ranges consistent with empirical evidences. For \(n_{ss}\), it has to be lower than 1, since developed Asian economy, such as Singapore, Japan, Hongkong, and Taiwan, has fertility rates below 2 for decades. And in recent years, those area have expected years of schooling above 14.

\(^{13}\) I have middle-aged cohorts in 1960-1980, 1980-2000, 2000-2020, and so on.
census data provided by NBSC, and data on real capital stock are from Yanrui Wu (2009), and data on real GDP are constructed with information on nominal GDP, real GDP growth, GDP deflator of base year (see Appendix B and Appendix C for details). The national school efficiency are calibrated by the same methodology with national level data\textsuperscript{14}.

Due to the missing data on Tibet, Chongqing, and Hainan, 28 provinces’ school efficiency in 1960-1980 and 1980-2000 are calculated and contained in table 1.1. School efficiency in different time period has similar pattern: East and Northeast province tends to have smaller school efficiency value, while for Middle or West province it tends to be larger. This pattern makes sense because East and Northeast are more economically developed. But as table 1.1 and figure 1.9 show, it was higher during 1980-2000 than that during 1960-1980 not only at provincial level, but also at national level. One possible explanation behind is that the cost of having children in theory is assumed to only incorporate educating cost and rearing cost, but other costs associated with the number of children arise after introducing birth control. Those other costs associated with having “out-of-plan” children, including huge amount of fines, risk of loosing job, run-away to avoid forced abortion. Moreover, many births of children without permit went unreported or were hidden from authorities. Those children, most of whom were undocumented, faced hardships in obtaining education. Unfortunately, data on this kind of “hidden” children was not collected, and financial penalties imposed on family who has “out-of-plan” children is varied by province and is not released by provincial governments. Because of those data issue, school efficiencies are overestimated during 1980-2000. But, those data issue are supposed to be common for all provinces, the cross sectional comparison between different provinces during two different periods are meaningful.

Figure 1.7 depicts the combinations of the real regional gross domestic output per labour at year 2010 and the school efficiency during 1960-1980 for 28 provinces. East provinces and northeast provinces are concentrated in left top area, middle provinces are clustered in the middle part, and west provinces are concentrated in the right bottom area except for Inner Mongolia. And it indicates that provinces with a better school efficiency associates with a higher real GDP per labour. The relationship between school efficiency during 1960-1980 and growth rate of regional real GDP per labour from 1980 to 2010 is described in figure 1.8. It’s hard to tell the true relationship between school efficiency and growth rate of real GDP per labour. After 1979, a series of dramatic economic

\textsuperscript{14}National level data on real GDP and real physical capital stock are from FRED. National level data on employed labour are from NBSC.
reforms and institutional changes varies by provinces. For example, the reform and opening-up policy favored coastal regions from the beginning of the reform. Probably because of this reason, the link between school efficiencies and GDP growth rates get weaken.

**Fertility, human capital accumulation, and GDP.** In this part, the effect of birth control policy is evaluated at national level. I simulate the behavior of two middle-aged cohorts which start at 1950 and 1960 respectively and their offspring. Also, suppose that for cohort of 1950, their offspring would face fertility constraint in 1970 – 1990 and 1990 – 2010, and for cohort of 1960, their offspring would face fertility constraint in 1980 – 2000 and 2000 – 2020. Table 1.2 and table 1.3 contains all parameters used in simulation.

In table 1.4 I use OLS and log-log regressions of data on simulation results. We run the following regressions:

\[
y_t = \alpha + \beta x_t
\]

\[
\ln(y_t) = \alpha + \beta \ln(x_t)
\]

In these two regressions, \( y_t \) is the data on fertility rate or expected years of schooling, and \( x_t \) is the simulation results on fertility rate or expected years of schooling. As table 1.4 shows, we have positive and 0.1% statistically significant \( \beta \)'s, and all \( \hat{R}^2 \)'s are more than 55%. I find that the evidence are broadly supportive.

As figure 1.10 shows, fertility declines for both policy scenario and non-policy scenario from the start, and it decreases more in the policy scenario. But after the policy ends, fertility for both scenarios gradually converge to 1.7 at balanced growth path. As figure 1.11 shows, an impressive impact of this policy on educational investment appears in 1980-2010, during which simulated expected years of schooling increase from 6.61 to 11.01 in policy scenario while those rise from 5.68 to 6.89 in no-policy scenario. And the simulated education investment in policy scenario are very close to actual data. But with the policy coming to an end, an undershoot of education investment occurs in transitional process because people would like to have more children and fertility will be higher than previous three decades. From figure 1.11, the policy encourages more educational investment and increase expected years of schooling by 2.91 years on average during policy periods, but induces lower investment after it ends, which is even slightly lower than that in no-policy scenario. Over the entire
periods, the policy causes a faster physical capital human capital accumulation, as table 1.5 shows\textsuperscript{15}. which leads to higher real GDP per worker after 2010, as figure 1.12 depicts. In 1980 – 2060, the model predicts an annual gain of moderate 5.01\% in real GDP per worker induced by the policy. That means each worker produces 5.01\% more real GDP on average in policy scenario during these periods.

**Population and Demographic transition.** Figure 1.13 depicts total population growths for policy scenario and non-policy scenario. Total population growths under the two scenario have the similar pattern since 1950: population sustainably grows until around 2010, and then continue to shrink since then. Compared with non-policy scenario, the annual population growth rate is 1.23\% points lower on average during policy period, and it is 0.85\% points lower on average in 2020 – 2080. Considered that the simulated annual growth rate of population for non-policy scenario during 1980 – 2020 is 1.28\%, I conclude that the policy largely decrease the population growth rate in those periods.

Figure 1.14 represents the simulated results in some aspects of demographic change generated in the two scenarios. As fertility rate declines, dependent children decreases and family size shrinks in both scenarios. The one-child policy reinforces those consequences. In policy scenario, the share of non-productive dependents rises from 55\% to above 72\%, and the share of mid-age workers declines from 45\% to 28\% from 1990 to 2030, while in non-policy scenario, the corresponding shares are from 63\% to 66\% and from 37\% to 34\%. From those subfigures in figure 1.14, the effective birth control creates three generations facing less non-productive dependents and subsequent several generations troubled with larger proportions of old-age dependents, compared with no-policy scenario. It imposes large shift of population age structure on demographic transitional process from high to low fertility society.

It seems that the situation future generations will meet is not so terrible as many people think. The surge in the proportion of old-age dependent accompanies with low proportion of young dependent approaching toward balanced growth path gradually from below. The patterns on old-age dependent and young dependent can be observed in bottom subplots in figure 1.14. The peak in the proportion of non-productive dependents comes in around 2030, during which fertility constraint induce about

\textsuperscript{15}Table 1.5 shows the per worker physical capital and human capital accumulation for both economic scenarios. The human capital accumulation rates are less than one in the early periods for both economic scenarios. And the growth rates of physical capital and human capital are counterfactual. They are much too fast for physical capital and too small for human capital. This is because of large growth rates of TFP during these times.
17.8% points more share of old-dependent and about 5.92% points less share of mid-age worker, compared to situation without fertility constraint. Fortunately, the peak is alleviated by smaller ratio of children born. However, the model may underestimate the proportion of old-age dependents. Since 1970, the life expectancy has largely increased as population health get improved. This feature is not incorporated into model in which I assume that any generation can only live up to 60 years. Therefore, in a realistic sense the influence of larger proportion of old-age dependents would be longer and larger than the prediction of the theory.

**Welfare Analysis.** In this part, I turn to the effect of one-child policy on the welfare of the whole society. Given the calibrated model parameters \((\beta, \eta, \delta, \zeta, \alpha)\) and optimal choices on fertility, \(\{n_t\}\), educational investment, \(\{\tau_t\}\), transfer, \(\{\pi_t\}\), and saving, \(\{a_t\}\), at any point in time I can construct maximal utility and optimal mid-age consumption of an agent living in policy regime at each time, that is, \(\hat{V}_{\text{policy}}\) and \(C_m(\hat{V}_{\text{policy}}, \hat{n}_{\text{policy}}, \hat{\alpha}_{\text{policy}}, \hat{\pi}_{\text{policy}})\). As well, for an agent living in non-policy regime, I can construct her maximal utility and optimal mid-age consumption at each time, denoted as \(V_{\text{nonpolicy}}^*\) and \(C_m(V_{\text{nonpolicy}}^*, \hat{n}_{\text{nonpolicy}}^*, \hat{\alpha}_{\text{nonpolicy}}^*, \hat{\pi}_{\text{nonpolicy}}^*)\).

To measure the change in welfare due to one-child policy, we construct two kinds of indexes. The first one is the ratio of utility of those living in policy regime to the utility of those living in non-policy regime at each time, that is, \(\widehat{V}_{\text{policy}}/V_{\text{nonpolicy}}^*\). The second one is the ratio of mid-age consumption that is required to make those living in policy regime as well as those living in non-policy regime to mid-age consumption of those living in policy regime at each time. That is,

\[
C_m(V_{\text{nonpolicy}}^*, \hat{n}_{\text{policy}}^*, \hat{\alpha}_{\text{policy}}^*, \hat{\pi}_{\text{policy}}^*)/C_m(\hat{V}_{\text{policy}}, \hat{n}_{\text{policy}}, \hat{\alpha}_{\text{policy}}, \hat{\pi}_{\text{policy}})\]

Table (1.6) records welfare loss and welfare gains measured by different indexes. The first column is the weighted average utility of those (including mid-age and old-age agents) living in policy regime.

\(16^\text{V}_{\text{nonpolicy}}^*\) denotes the maximal utility for non-policy scenario, \(\hat{V}_{\text{policy}}\) denotes the maximal utility for policy scenario, and \(\hat{n}_{\text{policy}}, \hat{\alpha}_{\text{policy}}, \hat{\pi}_{\text{policy}}\) are optimal choices on fertility, saving and transfer of those living in policy regime.
divided by the weighted average utility of those living in non-policy regime at each time\textsuperscript{17}. The second column is the ratio of mid-age consumption required to make the weighted average utility of those living in policy regime at the same level of the weighted average utility of those living in non-policy regime at each time. The third column is the utility of a mid-age agent living in policy regime divided by the utility of a mid-age agent living in non-policy regime at each time. The forth column is the ratio of mid-age consumption required to make the mid-age agent in policy scenario as well as the one in non-policy scenario over the mid-age consumption in policy scenario. The fifth column is the ratio of utility of an old-age agent in policy scenario over that in non-policy scenario for each time. In this paper, I assume that there is no old-age population at initial periods. That is to say, there is no old-age consumption in 1950 – 1970 and 1960 – 1980.

As the first and second column in the table shows, the whole society experiences welfare loss at early periods since the beginning of the policy. At each time the average utility the whole society can achieve in policy scenario is sustainably lower than that in non-policy scenario until decades after the policy ended. That is mainly because the policy reduces the mid-age population, increases the shares of old-age dependents. The utility of old-age agent become more important because they account for higher share of population. At the same time, in non-policy regime, society is more weighted towards mid-age agent. Since the policy enacted, welfare costs of society are rising until around 2040, and the costs range from 1.03 to 113,965 times of mid-age consumption. After that, welfare costs are declining. As previously discussed, the policy promotes creation of human capital so that faster human capital accumulation help accelerate GDP per workers in long run. Therefore, in later periods when society in policy scenario accumulates higher enough human capital per worker and physical capital per worker, welfare of society eventually surpass that in non-policy scenario. Also, from the rest three columns in the table, I found that an agent living in policy regime has lower utility in mid-age but higher utility in old-age during 1980 – 2000. At these time, the welfare loss for a mid-age agent range from 1.04 to 1.11 times of mid-age consumption. But after 2000, the utility of an agent in mid-age in policy regime begins to reach and surpass that in non-policy regime because GDP per worker and human capital stock per worker get higher enough. While the utility of an old-age agent in policy regime is sustainable higher because they can collect higher return from

\textsuperscript{17}The weighted average utility of society at each time is in policy regime calculated by the following:

\[ \hat{U}_{\text{policy},t} = \frac{\text{mid-age population}_t}{\text{mid-age and old-age population}_t} \hat{u}_{\text{mid-age},t} + \frac{\text{old-age population}_t}{\text{mid-age and old-age population}_t} \hat{u}_{\text{old-age},t} \]

Then weighted average utilities of society in non-policy regime at different times are calculated in the same way.
saving and higher return from investing in children. Conclusively, the first generations facing the fertility restriction get worse in mid-age, but get better in old-age; one-child policy improve welfare of society in the later periods at the cost of welfare loss of society in the early periods\textsuperscript{18}.

**Economic disparity.** Rapid fertility decline is found to make a quantitatively relevant contribution to reducing the incidence and severity of poverty. Empirical evidence implies that fertility in most east and northeast provinces starts to decline from early 1960s and is close to two in 1980. For example, Shanghai has fertility rate of below one even on 1980. This suggests that the policy is not binding in Shanghai since the beginning. Fertility in East and Northeast Provinces decline since early 1960s, while in middle and west provinces it began to decrease one decade later, as figure 1.1 shows. And after 2000, the policy is not binding for most east and northeast provinces. Table 1.8 records the fertility targets of the policy and actual fertility rates for 28 provinces in 2006. It indicates that in 2006, fertility targets of the policy are almost irrelevant in east and northeast provinces except for Hebei, Jiangsu, Shandong and Guangdong, and are still binding in all middle and west provinces except for Shaanxi and Inner Mongolia.

Now I turn to the comparison in simulated GDP per worker of between Shanghai (the richest province in China) and three poor provinces (Sichuan, Hunan, Gansu) from the West and Middle. These provinces are selected because they are bounded by the policy before and now, and they have relatively less population of minorities, which indicates that the policy in these provinces is more strict than in minority dominant provinces. In this part, I simulate the behavior of cohort of 1960 and their offspring for selected provinces. Preference parameters ($\eta, \beta$) for Shanghai are calibrated to fit actual fertility rates and expected years of schooling. But those parameters for the provinces from West and Middle are calibrated to fit actual expected years of schooling and fertility rate of around 1.7 on balanced growth path\textsuperscript{19}. This is because according to fertility data and policy target information from table 1.8 and figure 1.1, the policy is not binding for Shanghai\textsuperscript{20}. However, the three provinces from West and Middle are still under fertility restrictions during policy periods.

\textsuperscript{18} And for the first generations facing the fertility restriction, they get worse in mid-age, but get better in old-age, and their life time utility still get improved by the policy. This is reasonable because agent who makes decisions in mid-age and receive transfer in old-age can not choose the exact amount of transfer to maximize their utility. And also because the competitive equilibrium solutions of this general equilibrium overlapping generation model is not Pareto optimal.

\textsuperscript{19} Fertility rate of 1.7 is the fertility rate on balanced growth path for the whole China in previous calibration and simulation part.

\textsuperscript{20} In 1980, Shanghai's total fertility rate was already below than 1. For some years in 1980s, total fertility rate in Shanghai is slightly higher than 1. Considered the exceptions allowed by the policy, we can assume that the policy is not binding in Shanghai.
Therefore, fertility constraints are enforced in 1980 – 2020 for the three poor provinces. The preference parameter values are given in table 1.7. And the rest other parameter values are chosen by exactly the same methodology described in the beginning of the section.

Figure 1.15 plots the ratios of simulated GDP per worker for selected poor provinces to that of Shanghai. These solid lines represent simulated results in policy scenario, while the dash lines represent simulated results in non-policy scenario. The simulated results predict an alleviation in economic inequality between the richest province, Shanghai, and those poor provinces after 1990. And its biggest influence is predicted on the next generation after the policy expires, that is, after 2010. This prediction suggests that this policy delivers a smaller gap in income between rich and poor provinces.

1.5 Conclusion

In this paper, I have studied a tractable dynamical general equilibrium model with overlapping generations. This model is capable of capturing the Chinese social norms that adult children offer financial support to their parents. This feature provides incentive for mid-age workers to invest in their children’s quality and quantity. This simple model yields semi-closed solutions that reveal the quantity-quality trade-off in children: parents are encouraged to increase their offspring’s education to compensate for the reduction in the number of children. Therefore, when fertility constraint is enforced, a distinct rise in educational investment emerges. And more strictly the constraint is imposed, more educational investment is increased. Our counterfactual work presents that the exogenous fertility restrictions imposed by one-child policy have lead to a surge in human capital investment, further lead to a higher growth rate of GDP per worker when the more-educated generations grow up and enter labor market. The model predicts an annual gain of moderate 5.01% in real GDP per worker during 2020 – 2080.

This paper also demonstrates that the policy obviously decrease population growth by 1.23% points on average in 1980 – 2020, and by 0.85% points on average for later periods. It also generates large shifts in demographic transitional process. Predictions imply that 19.47% points more share of old-dependents and 6.47% points less share of mid-age workers caused by the policy when the peak of non-productive dependent proportion arrives in 2030. Moreover, the model probably underestimates
the severity of this social problem, because the increased life expectancy is not involved in the theory. I expect a greater impact of the policy on heavy social burdens. Beyond that, this paper also predict welfare changes due to one-child policy. It improves welfare of society at the later periods at the cost of welfare loss at the early periods.

In this paper, educational cost acts as a exogenous sources of income imbalance across regions in the theoretical framework. Large educational cost suppress the increase in educational investment, leading to a slow human capital accumulation and then a slow income growth. Our calibration work shows that the difference in educational cost between rich and poor provinces doesn’t narrow down over time, even after economic reforms launched. But the one-child policy provided a natural experiment linking fertility, educational investment, income growth across regions. It delivered larger incentive for poor provinces make more educational investment to attain a temporal higher income growth under its influence.
1.6 References


Wu Yanrui, “Chinas capital stock series by region and sector”, The University of Western Australia Discussion Paper, 2009.


1.7 Appendix A: Theory

A.1 Model Comparison Consider a standard overlapping generation model where mid-age agent do not give transfer to their old-age parents, but they still make decisions on quality and quantity of their children. The utility function is rewrote as:

$$U_t = ln c_{m,t} + \delta ln x_t + \eta ln h_{t+1} + \beta ln c_{o,t+1}.$$ 

Where $h_{t+1}$ denotes per capita human capital of next generation, and $x_t = n_t(1 - \rho_t)$. And the budget constraints are rewrote as:

$$c_{m,t} + a_t = m_t,$$
$$c_{o,t+1} = R_{t+1} a_t,$$
$$1 = l_t + x_t(\tau_t \lambda_t + \nu).$$

Suppose other assumptions do not change. Mid-age agent maximizes life-time utility by choosing the number of children $n_t$, the investment on children’s education $\tau_t$, the amount of capital to lend (or borrow) $a_t$. The first order conditions with respect to $n_t$, $\tau_t$ and $a_t$ yield

$$\frac{\delta}{n_t} - \frac{w_t h_t (\tau_t \lambda_t + \nu)(1 - \rho_t)}{m_t - a_t} = 0, \quad (1.23)$$
$$- \frac{w_t h_t n_t (1 - \rho_t) \lambda_t}{m_t - a_t} + \frac{\eta}{\tau_t} = 0, \quad (1.24)$$
$$- \frac{1}{m_t - a_t} + \frac{\beta}{a_t} = 0. \quad (1.25)$$

Combine Eq.(1.23) and Eq.(1.24), we get,

$$1 = \frac{\delta (m_t - a_t)}{\eta (m_t - a_t) + w_t h_t n_t (1 - \rho_t)}.$$ 

That is,

$$n_t = \frac{(\delta - \eta)(m_t - a_t)}{w_t h_t \nu (1 - \rho_t)}. \quad (1.26)$$
From Eq.(1.25),

\[ m_t - a_t = \frac{1}{1 + \beta} m_t. \]

Plug it into Eq.(1.26), we get

\[ n_t = \frac{\delta - \eta}{\nu(1 - \rho_t)(1 + \beta)} l_t. \] (1.27)

Combine Eq.(1.23) and Eq.(1.25), we have

\[ 1 - l_t = \frac{\delta}{1 + \beta} l_t. \] (1.28)

From Eq.(1.27) and Eq.(1.28),

\[ n_t = \frac{\delta - \eta}{(1 + \beta + \delta) \nu}. \]

So the model result shows that fertility \( n_t \) is constant over time. This simple set-up without endogenous transfer can not drive dynamic transition of \( n_t \).
A.2 Balanced growth path. Define $s_{π,t} = \frac{n_t}{m_t}$, $s_{a,t} = \frac{a_t}{m_t}$. As time $t$ goes to infinity, the optimal choices of $n^*, τ^*, s^*, s_{π,t}$ converges respectively to $n_{ss}$, $τ_{ss}$, $s_{a,ss}$ and $s_{π,ss}$ for constant $λ$, zero young adult mortality rate, and as well $R_t = R$ for all $t$.

From the solution to firm’s profit maximization problem,

$$R_t K_t = αY_t.$$  

That is,

$$R_t \frac{K_t}{N_t} = α \frac{Y_t}{N_t}.$$  

Combined it with market clear condition, we get

$$R_t \frac{a_{t-1}N_{t-1} + (1 - ζ)K_{t-1}}{N_t} = α(c_{m,t} + a_t + π_t + \frac{R_t a_{t-1}}{n_{t-1}}).$$  

That is,

$$R_t \left((1 - α) \frac{a_{t-1}}{n_{t-1}} + (1 - ζ) \frac{k_{t-1}}{n_{t-1}}\right) = αm_t.$$  

After arrangement, it becomes

$$R_t = \frac{αm_t n_{t-1}}{(1 - α)a_{t-1} + (1 - ζ)k_{t-1}}.$$  

Since

$$\frac{m_{t+1}}{m_t} = \frac{w_{t+1} h_{t+1} \left(1 - n_{t+1} (λτ_{t+1} + ν)\right)}{w_t h_t \left(1 - n_t (λτ_t + ν)\right)} = \frac{w_{t+1} h_{t+1} \left(1 - n_{t+1} (λτ_{t+1} + ν)\right)}{w_t h_t \left(1 - n_t (λτ_t + ν)\right)}.$$  

On balanced growth path, with the assumption that $z_t$ grows at a constant rate $g$,

$$\frac{m_{t+1}}{m_t} = Agτ_{ss}.$$  

(1.29)
So,

\[ R = \frac{\alpha A \tau_{ss} n_{ss}}{(1 - \alpha) s_{a,ss} + (1 - \zeta) s_{k,ss}}. \]  

(1.30)

From physical capital accumulation technology, get

\[
\begin{align*}
k_{t+1} &= \frac{a_t}{n_t} + (1 - \zeta) \frac{k_t}{n_t}, \\
\frac{k_{t+1}}{m_{t+1}} &= \frac{a_t}{n_t m_t} \frac{m_t}{m_{t+1}} + (1 - \zeta) \frac{k_t}{n_t m_t} \frac{m_t}{m_{t+1}}, \\
\frac{s_{k,t+1}}{m_t} &= \frac{s_{a,t}}{n_t} + (1 - \zeta) \frac{s_{k,t}}{n_t}.
\end{align*}
\]

On balanced growth path,

\[ s_{k,ss} A \tau_{ss} = \frac{s_{a,ss}}{n_{ss}} + (1 - \zeta) \frac{s_{k,ss}}{n_{ss}}. \]

Since

\[
\begin{align*}
s_{k,ss} &= \frac{k_t}{m_t} \\
&= \frac{k_t}{(1 - \alpha) k_t^\alpha (z_t h_t l_{ss})^{\alpha - 1}} \\
&= \frac{1}{(1 - \alpha) k_t^{\alpha - 1} (z_t h_t l_{ss})^{1 - \alpha}} \\
&= \frac{\alpha}{1 - \alpha R}.
\end{align*}
\]

So, on balanced growth path,

\[
\begin{align*}
\frac{k_{t+1}}{z_{t+1} h_{t+1} l_{ss}} &= \frac{k_t}{z_t h_t l_{ss}}, \\
h_{t+1} &= Ah_t \tau_{ss}.
\end{align*}
\]

So,

\[
\begin{align*}
k_{t+1} &= \frac{z_{t+1} h_{t+1} k_t}{z_t h_t} \\
&= A \tau_{ss} k_t.
\end{align*}
\]
From Eq.(1.29),
\[
\frac{(1 - s_{a,t+1} - s_{\pi,t+1})m_{t+1}}{(1 - s_{a,t} - s_{\pi,t})m_t} = \frac{\beta R_{t+1}}{\eta + n_t}.
\]

So, on balanced growth path, it is
\[
Ag_{\tau_{ss}} = \frac{\beta R}{\eta + n_{ss}}. \tag{1.31}
\]

So,
\[
s_{a,ss} = \left(Agn_{ss}\tau_{ss} - (1 - \zeta)\right)\frac{\alpha}{1 - \alpha R} = \left(Agn_{ss}\tau_{ss} - (1 - \zeta)\right)\frac{\alpha}{1 - \alpha (\eta + n_{ss})Ar_{ss}}.
\]

Arrange Eq.(2.1) and get,
\[
0 = \frac{m_{t}h_{t}\tau_{t}\lambda_{t}}{R_{t+1}m_{t}} - \frac{m_{t+1}}{R_{t+1}}
\]

On balanced growth path,
\[
0 = \frac{\tau_{ss}\lambda_{t}}{(1 - n_{ss}(\tau_{ss}\lambda + \nu))} - \frac{Ag_{\tau_{ss}}}{R}
\]
\[
= \frac{\tau_{ss}\lambda_{t}}{(1 - n_{ss}(\tau_{ss}\lambda + \nu))} - \frac{\beta}{\eta + n_{ss}}.
\]

That is,
\[
\tau_{ss} = \frac{\beta (1 - n_{ss}\nu)}{\lambda(\eta + n_{ss} + \beta n_{ss})}. \tag{1.32}
\]
Arrange Eq.(1.13) and get,

\[
\delta = \frac{w_t h_t}{c_{m,t}} (\tau_t \lambda_t + \nu) - \frac{\pi_{t+1} n_t}{R_{t+1} c_{m,t}}
\]

\[
= n_t (\tau_t \lambda_{p,t} + \nu) \frac{w_t h_t}{c_{m,t}} - \frac{n_t \pi_{t+1}}{R_{t+1} c_{m,t}}
\]

\[
= n_t (\tau_t \lambda_t + \nu) \frac{w_t h_t}{(1 - s_{a,t} - s_{\pi,t}) n_t} - \frac{n_t s_{\pi,t+1} m_{t+1}}{R_{t+1} (1 - s_{a,t} - s_{\pi,t}) n_t}
\]

\[
= n_t (\tau_t \lambda_t + \nu) \frac{(1 - s_{a,t} - s_{\pi,t}) n_t}{(1 - s_{a,t} - s_{\pi,t}) (1 - n_t (\tau_t \lambda_t + \nu))} - \frac{n_t s_{\pi,t+1} m_{t+1}}{R_{t+1} (1 - s_{a,t} - s_{\pi,t}) n_t}.
\]

On balanced growth path,,

\[
\delta = \frac{n_{ss} (\tau_{ss} \lambda + \nu)}{(1 - s_{a,ss} - s_{\pi,ss}) (1 - n_{ss} (\tau_{ss} \lambda + \nu))} - \frac{Ag n_{ss} \tau_{ss} s_{\pi,ss}}{R (1 - s_{a,ss} - s_{\pi,ss})}.
\]

That is,

\[
1 - s_{a,ss} - s_{\pi,ss} = \frac{1}{\delta} \left( \frac{n_{ss} (\tau_{ss} \lambda + \nu)}{1 - n_{ss} (\tau_{ss} \lambda + \nu)} - \frac{\beta n_{ss}}{\eta + n_{ss}} s_{\pi,ss} \right).
\] (1.33)

Arrange Eq.(1.6), get

\[
\frac{\pi_t}{m_t} = \frac{\eta}{\eta + n_{t-1}} (1 - a_t) \frac{R_t}{\eta + n_t m_t} - \frac{a_{t-1}}{m_t} \frac{R_t}{\eta + n_{t-1} m_{t-1}}.
\]

On balance growth path,, that is,

\[
s_{\pi,ss} = \frac{\eta}{\eta + n_{ss}} (1 - s_{a,ss}) - \frac{R}{\eta + n_{ss} \nu} \frac{1}{\tau_{ss}} s_{a,ss}
\]

\[
= \frac{\eta}{\eta + n_{ss}} - \frac{\eta}{\eta + n_{ss}} s_{a,ss} - \frac{1}{\eta + n_{ss}} \frac{\beta}{\eta + n_{ss}} s_{a,ss}
\]

\[
= \frac{\eta}{\eta + n_{ss}} - \left( \frac{\eta}{\eta + n_{ss}} + \frac{1}{\beta} \right) s_{a,ss}.
\]

So,

\[
s_{a,ss} = \frac{\beta \eta - \beta (\eta + n_{ss}) s_{\pi,ss}}{\beta \eta + \eta + n_{ss}}.
\]
So,

\[
(Ag_{ss}\tau_{ss} - (1 - \zeta)) \frac{\alpha}{1 - \alpha \eta + n_{ss}} = \frac{\beta}{\eta + n_{ss}} = \frac{\beta \eta - \beta(n + n_{ss})s_{\pi,ss}}{\beta \eta + \eta + n_{ss}}.
\]

\[
\frac{\alpha \beta n_{ss}}{\eta + n_{ss}} - \frac{\alpha \beta (1 - \zeta)}{1 - \alpha \eta + n_{ss}} = \frac{\alpha \beta n_{ss}}{\eta + n_{ss}} - \frac{\alpha \beta (1 - \zeta)}{1 - \alpha \eta + n_{ss}} = \frac{\beta \eta - \beta(n + n_{ss})s_{\pi,ss}}{\beta \eta + \eta + n_{ss}}.
\]

\[
s_{\pi,ss} = \frac{n}{\eta + n_{ss}} - \frac{\beta \eta + \eta + n_{ss}}{\beta(\eta + n_{ss})} \frac{\alpha \beta}{1 - \alpha \eta + n_{ss}} \left( \frac{n_{ss}}{\eta + n_{ss}} - \frac{1 - \zeta}{(\eta + n_{ss})Ag_{\tau_{ss}}} \right).
\]  

(1.34)

\[
s_{a,ss} + s_{\pi,ss} = \frac{\beta \eta + n_{ss} + \eta}{\beta \eta + \eta + n_{ss}} + \frac{(1 - \beta)n_{ss} + \eta}{\beta \eta + \eta + n_{ss}} s_{\pi,ss}.
\]

\[
1 - s_{\pi,ss} - s_{a,ss} = \frac{n + n_{ss}}{\beta \eta + \eta + n_{ss}} - \frac{(1 - \beta)n_{ss} + \eta}{\beta \eta + \eta + n_{ss}} s_{\pi,ss}.
\]

(1.35)

From Eq.(1.33) and Eq.(1.35),

\[
\left( \frac{\beta n_{ss}}{\eta + n_{ss}} - \frac{(1 - \beta)n_{ss} + \eta}{\beta \eta + \eta + n_{ss}} \right) s_{\pi,ss} = \frac{n_{ss}(\tau_{ss} \lambda + \nu)}{1 - n_{ss}(\tau_{ss} \lambda + \nu)} - \frac{\delta (\eta + n_{ss})}{\beta \eta + \eta + n_{ss}}.
\]

Where

\[
s_{\pi,ss} = \frac{n}{\eta + n_{ss}} - \frac{\beta \eta + \eta + n_{ss}}{\beta(\eta + n_{ss})} \frac{\alpha \beta}{1 - \alpha \eta + n_{ss}} \left( \frac{n_{ss}}{\eta + n_{ss}} - \frac{1 - \zeta}{(\eta + n_{ss})Ag_{\tau_{ss}}} \right).
\]

Conclusively, \( n_{ss} \) and \( \tau_{ss} \) are determined by Eq.(1.36) and Eq.(1.32).
1.8 Appendix B: Data Construction

The literature has mainly constructed those data sets: the expected years of schooling, the real regional GDP, and the mortality rates.

**Expected years of schooling.** Expected years of schooling data consists of provincial annual observations covering the period from 1949 to 2010. Those data are constructed from primary school enrollment rates, secondary school enrollment rates and higher education enrollment rates, according to this formula: for a child of age 7 at year $t$, expected years of schooling he will attain is the sum of primary school enrollment rates from year $t$ to $t + 5$, secondary school enrollment rates from year $t + 6$ to $t + 11$ and higher education enrollment rates from year $t + 12$ to $t + 16$. Since the enrollment rates data only spans the time period of 1949-2010, for children born after year 1985 the forward higher education enrollment rates are not available. Following the procedure in Murphy, Simon and Tamura (2008), I use the linear projections of three enrollment rates on time trend, of which primary school enrollment rates are truncated by 1 if projected values are above 1.

The annual primary school enrollment rates, used to constructed expected years of schooling, start in 1949 and end in 2010, covering 30 provinces, obtained from published book "China's Provincial Statistics: 1949-1982" and from China Statistic Yearbooks. The primary school ages in China are from 7 to 12. For annual enrollment rates for secondary school and higher education, the school ages are from 13 to 18 and from 19 to 23, respectively. The data are constructed for each province $i$, by those formula:

$$
enrolment\ rates\ for\ secondary\ school_{i,t} = \frac{\text{the\ number\ of\ students\ in\ secondary\ school}_{i,t}}{\text{population\ of\ ages\ 13-18}_{i,t}}.$$

$$
enrolment\ rates\ for\ higher\ education_{i,t} = \frac{\text{the\ number\ of\ students\ in\ higher\ education\ institutional}_{i,t}}{\text{population\ of\ ages\ 19-23}_{i,t}}.$$

Where the population distribution since 1982 are calculated from the raw data obtained from each provinces' statistic yearbooks, the population of ages 13-18 for year $t$ before 1982 are estimated by the population of ages 7-12 for year $t - 1$, and the population of ages 19-23 for year $t$ before 1982 are
estimated by the sum of newborns 19-23 years ago from that year with the assumption that these groups of people do not move across provinces since born.

**Real regional GDP.** Since National Bureau of Statistics of China only reports nominal GDP levels and real GDP growth, but not real GDP levels, the real GDP for each provinces are constructed with information on nominal GDP, real GDP growth. GDP deflator of the base year (1953) is normalized to one. Year-over-year real growth rates for each province are used to construct the growth rate of each provinces deflator.


**Young mortality.** National young mortality, deaths before age 35, are obtained from CHLR during 1988-1990, 1994-2007, 2010 on annual basis. Let \( p_{1,35} \) denoted the probability of dying before 35 conditional on surviving the infant period, and \( m \) denotes the probability of dying in infant period. Table 1.9 presents the results of regressions of log of young survival rates on log of infant survival rates, with/out time effect. The first column shows that without time effect, the variations of infant survival rates can explain 79% of young survival rates, the second column indicates that controlled by year, the result get improved with \( \bar{R}^2 \) of 90%, and the final column indicates that controlled by time and time-squared, 89% of variations of young survival rates can be explained by infant survival rates. Therefore, young mortality rate prior to 1988, \( p_{1,35} \), is projected by regression function of year and infant mortality, \( m \). And the unconditional probabilities of dying between the ages of 1
and 35, \( \hat{p}_{1,35} \), are constructed by,

\[
\hat{p}_{1,35} = p_{1,35} \times (1 - m).
\]

And the conditional and unconditional probabilities of dying between the ages of 1 and 35 for each provinces are constructed by the similar procedure. Specifically, \( p_{1,35} \) are constructed by the same regression function with same coefficients from national data, and \( \hat{p}_{1,35} \) are still obtained by the equation mentioned above.
1.9 Appendix C: Data Sources

The data are constructed from a variety of sources, including:


Nominal output


Number of workers


Real physical capital stock.

1953-2005, real physical capital stock in provincial level from Yanrui Wu (2009), which is in 1953 price.

Fertility rates

1945-1981, annual total fertility rates in provincial level from “Basic data on fertility in the provinces of China, 1940-82”.


Mortality rates


1950-1990, infant mortality at national level are from United Nations Population Division, and observations are taken every five year.


Primary school enrollment rates


Secondary school enrollment rates and higher education enrollment rates


2009-2010, number of students in school for each provinces from China Statistical Yearbooks (2010-2011), and population in school ages for each provinces from China Human Capital Measurement and Human Capital Index Project.
1.10 Appendix D: Figures

Figure 1.1: Fertility rates for 28 provinces from 1945-2010.
Figure 1.2: Expected years of schooling for 30 provinces from 1949-2010.
Figure 1.3: Ratios of real GDP per Worker for 28 provinces over that of Shanghai from 1952-2010.
Figure 1.4: Human capital and fertility choices on balanced growth path using illustrative calibration.

Note: $\alpha = 0.333$, $\beta = 0.998$ per annum, $\zeta = 0.04$, $\delta = 1.05$, $\eta = 0.85$, $\lambda = 0.46$, $Ag = 3.5$, $\nu = 0.125$. 
Figure 1.5: State variables on balanced growth path using illustrative calibration.

Note: $\alpha = 0.333$, $\beta = 0.998$ per annum, $\zeta = 0.04$, $\delta = 1.05$, $\eta = 0.85$, $Ag = 3.5$, $\nu = 0.125$. 
Figure 1.6: National productivity used in simulation.
Figure 1.7: Real GDP of region per worker and school efficiency across 28 provinces in 2010.

Note: black ones represent east and northeast provinces, while red ones represent middle and west provinces.
Figure 1.8: Growth rate of real GDP and school efficiency across 28 provinces during time period of 1980-2010.

Note: black ones represent east provinces, red ones represent west provinces, blue ones represent middle provinces, and purple ones represent northeast provinces.
Figure 1.9: School efficiency for 28 Provinces.

Note: black line is a 45 degree line.
Figure 1.10: Number of childbirths per household.

Note: blue line represents simulated results in non-policy scenario, red line represents simulated results in policy scenario, black line represents actual data.
Figure 1.11: Educational investment, unit: year.

Note: blue line represents simulated results in non-policy scenario, red line represents simulated results in policy scenario, black line represents actual data.
Figure 1.12: Log of GDP per worker.

Note: blue line represents simulated results in non-policy scenario, red line represents simulated results in policy scenario.
Figure 1.13: Log of total population.

Note: blue line represents simulated results in non-policy scenario, red line represents simulated results in policy scenario.
Figure 1.14: Demographic transition.

Note: blue line represents simulated results in non-policy scenario, red line represents simulated results in policy scenario. Clockwise from top left subfigure: non-productive dependent ratio; mid-age population ratio; young-age population ratio; old-age population ratio.
Figure 1.15: Ratios of simulated GDP per worker of poor provinces over that of Shanghai.

Preference parameters for each provinces (Shanghai, Sichuan, Hunan, Gansu) are contained in Table 1.7. Solid lines represent simulated results in policy scenario, and dash lines represents simulated results in non-policy scenario.
Table 1.1: School efficiencies for 28 provinces.

<table>
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<tr>
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<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td>National</td>
<td>0.5276</td>
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<td>Beijing</td>
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<td>0.5071</td>
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<td>0.6007</td>
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<td>Anhui</td>
<td>0.4075</td>
<td>0.5836</td>
</tr>
<tr>
<td>Fujian</td>
<td>0.4753</td>
<td>0.5211</td>
</tr>
<tr>
<td>Jiangxi</td>
<td>0.5235</td>
<td>0.6211</td>
</tr>
<tr>
<td>Shandong</td>
<td>0.4885</td>
<td>0.5664</td>
</tr>
<tr>
<td>Henan</td>
<td>0.5537</td>
<td>0.6215</td>
</tr>
<tr>
<td>Hubei</td>
<td>0.4571</td>
<td>0.6134</td>
</tr>
<tr>
<td>Hunan</td>
<td>0.4596</td>
<td>0.6416</td>
</tr>
<tr>
<td>Guangdong</td>
<td>0.3777</td>
<td>0.4542</td>
</tr>
<tr>
<td>Guangxi</td>
<td>0.6925</td>
<td>0.8331</td>
</tr>
<tr>
<td>Sichuan</td>
<td>0.8166</td>
<td>0.8972</td>
</tr>
<tr>
<td>Guizhou</td>
<td>0.8287</td>
<td>0.9397</td>
</tr>
<tr>
<td>Yunnan</td>
<td>0.7511</td>
<td>0.9083</td>
</tr>
<tr>
<td>Shaanxi</td>
<td>0.5284</td>
<td>0.6131</td>
</tr>
<tr>
<td>Gansu</td>
<td>0.6185</td>
<td>0.832</td>
</tr>
<tr>
<td>Qinghai</td>
<td>0.7773</td>
<td>0.9502</td>
</tr>
<tr>
<td>Ningxia</td>
<td>0.6968</td>
<td>0.7415</td>
</tr>
<tr>
<td>Xinjiang</td>
<td>0.7351</td>
<td>0.7499</td>
</tr>
</tbody>
</table>
Table 1.2: Calibration of parameter values and initial conditions for the whole country.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>description</th>
<th>detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.333</td>
<td>cost share of capital</td>
<td>commonly used</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.8179</td>
<td>time preference</td>
<td>commonly used</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.9057</td>
<td>preference for old parent’s consumption</td>
<td>by Eq.(1.15) and Eq.(1.16)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.7529</td>
<td>preference for number of children</td>
<td>by Eq.(1.15) and Eq.(1.16)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.125</td>
<td>rearing cost of children</td>
<td>Tamura and Simon(2008)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.558</td>
<td>respectively</td>
<td>commonly used</td>
</tr>
<tr>
<td>$A$</td>
<td>1.9</td>
<td>productivity to produce human capital</td>
<td>Eqn.(1.17) using $\tau_{ss} = 0.7821$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.21</td>
<td>school efficiency on BGP</td>
<td>by Eqn.(1.15)</td>
</tr>
<tr>
<td>$n_{ss}$</td>
<td>0.85</td>
<td>fertility rate on BGP</td>
<td></td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>0.7821</td>
<td>educational investment on BGP</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{50,0}$</td>
<td>0.7057</td>
<td>initial school efficiency for cohort 1950</td>
<td>by Eqn.(1.18), data in 1950-1970</td>
</tr>
<tr>
<td>$n_{50,0}$</td>
<td>3</td>
<td>number of children per person during 1950-1970</td>
<td>fertility rate in 1950-1970</td>
</tr>
<tr>
<td>$n_{60,0}$</td>
<td>2.42</td>
<td>number of children per person in 1960-1980</td>
<td>fertility rate in 1960-1980</td>
</tr>
<tr>
<td>$\bar{n}_{50,1}$</td>
<td>2</td>
<td>fertility constraint in 1970-1990</td>
<td>fertility rate in 1990-2010</td>
</tr>
<tr>
<td>$\bar{n}_{60,1}$</td>
<td>0.8</td>
<td>fertility constraint in 1980-2000</td>
<td>fertility rate in 1980-2000</td>
</tr>
<tr>
<td>$\bar{n}_{50,2}$</td>
<td>0.6</td>
<td>fertility constraint in 1990-2010</td>
<td>fertility rate in 1990-2010</td>
</tr>
<tr>
<td>$\bar{n}_{60,2}$</td>
<td>0.54</td>
<td>fertility constraint in 2000-2020</td>
<td>fertility rate in 2000-2010</td>
</tr>
<tr>
<td>$k_{50,0}$</td>
<td>1725</td>
<td>initial physical capital per worker for cohort of 1950</td>
<td>from FRED, unit:2005 US dollar</td>
</tr>
<tr>
<td>$k_{60,0}$</td>
<td>2418</td>
<td>initial physical capital per worker for cohort of 1960</td>
<td>from FRED, unit:2005 US dollar</td>
</tr>
<tr>
<td>$y_{50,0}$</td>
<td>20306</td>
<td>initial GDP per worker in 1950-1970</td>
<td>from FRED, unit:2005 US dollar</td>
</tr>
<tr>
<td>$y_{60,0}$</td>
<td>26057</td>
<td>initial GDP per worker in 1960-1980</td>
<td>from FRED, unit:2005 US dollar</td>
</tr>
</tbody>
</table>

Note: Table reports the values of model parameters and initial conditions for 20-period horizon. We set $\beta$ to 0.8179 on a 20-year basis, that is, 0.99 on an annual basis. $\zeta$ is 0.558 on a 20-year basis, that is, 0.04 on an annual basis. Initial values for physical capital per worker and GDP per worker are sum of corresponding data in 20 years.
### Table 1.3: School efficiency and TFP for the whole country.

<table>
<thead>
<tr>
<th>Time</th>
<th>School Efficiency</th>
<th>Productivity</th>
<th>Productivity Growth (20-year horizon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950-1970</td>
<td>0.7057</td>
<td>1</td>
<td>4.2695</td>
</tr>
<tr>
<td>1960-1980</td>
<td>0.6561</td>
<td>2.0707</td>
<td>4.0667</td>
</tr>
<tr>
<td>1970-1990</td>
<td>0.6066</td>
<td>4.2695</td>
<td>3.705</td>
</tr>
<tr>
<td>1980-2000</td>
<td>0.5570</td>
<td>8.4208</td>
<td>3.3433</td>
</tr>
<tr>
<td>1990-2010</td>
<td>0.5974</td>
<td>15.8186</td>
<td>2.9814</td>
</tr>
<tr>
<td>2000-2020</td>
<td>0.4579</td>
<td>28.1533</td>
<td>2.6194</td>
</tr>
<tr>
<td>2010-2030</td>
<td>0.4083</td>
<td>47.1622</td>
<td>2.2571</td>
</tr>
<tr>
<td>2020-2040</td>
<td>0.3587</td>
<td>73.7455</td>
<td>1.8945</td>
</tr>
<tr>
<td>2030-2050</td>
<td>0.3091</td>
<td>106.4535</td>
<td>1.5313</td>
</tr>
<tr>
<td>2040-2060</td>
<td>0.2596</td>
<td>139.7144</td>
<td>1.1667</td>
</tr>
<tr>
<td>After 2050</td>
<td>0.2100</td>
<td>163.0088</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1.4: Pooled regression of data on model solutions: fertility and expected years of schooling.

<table>
<thead>
<tr>
<th></th>
<th>$\tau^\text{data}$</th>
<th>$n^\text{data}$</th>
<th>$\ln\tau^\text{data}$</th>
<th>$\ln n^\text{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^\text{model}$</td>
<td>0.807 ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0650)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n^\text{model}$</td>
<td>-</td>
<td>2.220 ***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.2206)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln\tau^\text{model}$</td>
<td>-</td>
<td>-</td>
<td>0.691 ***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0558)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln n^\text{model}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.428 ***</td>
</tr>
<tr>
<td></td>
<td>(0.1129)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>3.150 ***</td>
<td>-1.139 **</td>
<td>0.826 ***</td>
<td>0.154 **</td>
</tr>
<tr>
<td></td>
<td>(0.596)</td>
<td>(.3387)</td>
<td>(0.1223)</td>
<td>(0.0499)</td>
</tr>
<tr>
<td>$N$</td>
<td>61</td>
<td>77</td>
<td>61</td>
<td>77</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.7185</td>
<td>0.5690</td>
<td>0.7173</td>
<td>0.6765</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: The row marked by $p$ reports the results of the test that constant term is zero and coefficient of explanatory variable is one.
Table 1.5: Per worker physical capital and human capital accumulation.

<table>
<thead>
<tr>
<th>Time</th>
<th>$k_{t, non-policy}$</th>
<th>$k_{t, policy}$</th>
<th>$h_{t, non-policy}$</th>
<th>$h_{t, policy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950-1970</td>
<td>0.3284</td>
<td>0.3284</td>
<td>0.5193</td>
<td>0.5193</td>
</tr>
<tr>
<td>1960-1980</td>
<td>0.3467</td>
<td>0.3467</td>
<td>0.7410</td>
<td>0.7410</td>
</tr>
<tr>
<td>1970-1990</td>
<td>8.8925</td>
<td>10.2823</td>
<td>0.5395</td>
<td>0.6287</td>
</tr>
<tr>
<td>1980-2000</td>
<td>13.3534</td>
<td>18.8926</td>
<td>0.5723</td>
<td>0.8228</td>
</tr>
<tr>
<td>1990-2010</td>
<td>3.0309</td>
<td>4.8279</td>
<td>0.5977</td>
<td>0.9719</td>
</tr>
<tr>
<td>2000-2020</td>
<td>3.4510</td>
<td>5.3752</td>
<td>0.6548</td>
<td>1.0452</td>
</tr>
<tr>
<td>2010-2030</td>
<td>1.5425</td>
<td>1.4458</td>
<td>0.7431</td>
<td>0.7028</td>
</tr>
<tr>
<td>2020-2040</td>
<td>2.1412</td>
<td>1.9704</td>
<td>0.8031</td>
<td>0.7430</td>
</tr>
<tr>
<td>2030-2050</td>
<td>1.7294</td>
<td>1.6921</td>
<td>0.9079</td>
<td>0.8789</td>
</tr>
<tr>
<td>2040-2060</td>
<td>1.6952</td>
<td>1.6551</td>
<td>1.0427</td>
<td>1.0041</td>
</tr>
<tr>
<td>2050-2070</td>
<td>1.7036</td>
<td>1.6934</td>
<td>1.2177</td>
<td>1.1977</td>
</tr>
<tr>
<td>2060-2080</td>
<td>1.3799</td>
<td>1.3736</td>
<td>1.5167</td>
<td>1.4903</td>
</tr>
<tr>
<td>after 2070</td>
<td>1.486</td>
<td>1.486</td>
<td>1.486</td>
<td>1.486</td>
</tr>
</tbody>
</table>

Note: The column marked by $k_{t, non-policy}$ reports the growth rates of physical capital per worker, that is $g_k^{k_{t, non-policy}} = k_{t+1}/k_t$ in non-policy regime. The column marked by $k_{t, policy}$ reports the growth rates of physical capital per worker, that is $g_k^{k_{t, policy}} = k_{t+1}/k_t$ in policy regime. The column marked by $h_{t, non-policy}$ reports the growth rates of human capital per worker, that is $g_h^{h_{t, non-policy}} = A\tau_t$ in non-policy regime. The column marked by $h_{t, policy}$ reports the growth rates of human capital per worker, that is $g_h^{h_{t, policy}} = A\tau_t$ in policy regime. The last row reports the values all variables converge to.
Table 1.6: Welfare analysis.

<table>
<thead>
<tr>
<th>Time</th>
<th>$\hat{U}<em>{\text{policy}}/U^{*}</em>{\text{nonpolicy}}$</th>
<th>$\hat{C}/C_{\text{policy}}$</th>
<th>$\hat{u}<em>{m,\text{policy}}/u^{*}</em>{m,\text{nonpolicy}}$</th>
<th>$\hat{c}<em>{m}/c</em>{m,\text{policy}}$</th>
<th>$\hat{u}<em>{o,\text{policy}}/u^{*}</em>{o,\text{nonpolicy}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950-1970</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>1960-1980</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>1970-1990</td>
<td>0.9993</td>
<td>1.0288</td>
<td>0.9992</td>
<td>1.0353</td>
<td>1.0015</td>
</tr>
<tr>
<td>1980-2000</td>
<td>0.9981</td>
<td>1.09384</td>
<td>0.9977</td>
<td>1.1099</td>
<td>1.0014</td>
</tr>
<tr>
<td>1990-2010</td>
<td>0.9601</td>
<td>8.4391</td>
<td>1.0013</td>
<td>0.9446</td>
<td>1.0028</td>
</tr>
<tr>
<td>2000-2020</td>
<td>0.9045</td>
<td>462.81</td>
<td>1.0067</td>
<td>0.7202</td>
<td>1.0092</td>
</tr>
<tr>
<td>2010-2030</td>
<td>0.8540</td>
<td>27048</td>
<td>1.0213</td>
<td>0.3704</td>
<td>1.0153</td>
</tr>
<tr>
<td>2020-2040</td>
<td>0.8305</td>
<td>113965</td>
<td>1.0274</td>
<td>0.2429</td>
<td>1.0241</td>
</tr>
<tr>
<td>2030-2050</td>
<td>0.9554</td>
<td>13.481</td>
<td>1.0304</td>
<td>0.2287</td>
<td>1.0305</td>
</tr>
<tr>
<td>2040-2060</td>
<td>0.9452</td>
<td>37.6535</td>
<td>1.0374</td>
<td>0.1352</td>
<td>1.0313</td>
</tr>
<tr>
<td>2050-2070</td>
<td>0.9948</td>
<td>1.0892</td>
<td>1.0345</td>
<td>0.1749</td>
<td>1.0384</td>
</tr>
<tr>
<td>2060-2080</td>
<td>0.9962</td>
<td>0.9441</td>
<td>1.0426</td>
<td>0.0984</td>
<td>1.0345</td>
</tr>
<tr>
<td>2070-2090</td>
<td>1.1063</td>
<td>0.2792</td>
<td>1.0362</td>
<td>0.1519</td>
<td>1.0425</td>
</tr>
<tr>
<td>2080-2100</td>
<td>1.0214</td>
<td>0.1705</td>
<td>1.0445</td>
<td>0.0835</td>
<td>1.0358</td>
</tr>
</tbody>
</table>

Note: the first column records the ratio of the weighted average utility of society under non-policy scenario to that under policy scenario at each time; the second column records the ratio of the mid-age consumption that is required to make the weighted average utility of society in policy regime at the same level of that in non-policy regime to the mid-age consumption in the policy regime at each time; the third column records the ratio of utility of an agent in mid-age in policy scenario to that in non-policy scenario; the forth column records the mid-age consumption that is required to make the mid-age agent in policy regime at the same level of utility of the one in non-policy regime, divided by the mid-age consumption of the agent in policy regime; the fifth column records the ratio of utility of an old-age agent in policy regime to that in non-policy regime.
Table 1.7: Preference parameters for provinces.

<table>
<thead>
<tr>
<th>Province</th>
<th>$\delta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai</td>
<td>0.7772</td>
<td>3.7549</td>
</tr>
<tr>
<td>Sichuan</td>
<td>0.7904</td>
<td>2.3103</td>
</tr>
<tr>
<td>Gansu</td>
<td>0.7715</td>
<td>2.4307</td>
</tr>
<tr>
<td>Hunan</td>
<td>0.7543</td>
<td>2.5511</td>
</tr>
</tbody>
</table>

Note: $\delta$ is the preference for number of childbirth, and $\eta$ is the preference for old parent’s consumption.
Table 1.8: Policy targets and actual fertility rates for 28 provinces in 2006.

<table>
<thead>
<tr>
<th>Province</th>
<th>Policy target</th>
<th>Actual rate</th>
<th>Binding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>1.01</td>
<td>0.795</td>
<td>No</td>
</tr>
<tr>
<td>Tianjin</td>
<td>1.23</td>
<td>1.073</td>
<td>No</td>
</tr>
<tr>
<td>Heibei</td>
<td>1.391</td>
<td>1.702</td>
<td>Yes</td>
</tr>
<tr>
<td>Shanxi</td>
<td>1.36</td>
<td>1.408</td>
<td>Yes</td>
</tr>
<tr>
<td>Inner Mongolia</td>
<td>1.309</td>
<td>1.149</td>
<td>No</td>
</tr>
<tr>
<td>Liaoning</td>
<td>1.419</td>
<td>1.028</td>
<td>No</td>
</tr>
<tr>
<td>Jilin</td>
<td>1.426</td>
<td>1.1</td>
<td>No</td>
</tr>
<tr>
<td>Heilongjiang</td>
<td>1.08</td>
<td>1.045</td>
<td>No</td>
</tr>
<tr>
<td>Shanghai</td>
<td>1.173</td>
<td>0.801</td>
<td>No</td>
</tr>
<tr>
<td>Jiangsu</td>
<td>1.21</td>
<td>1.351</td>
<td>Yes</td>
</tr>
<tr>
<td>Zhejiang</td>
<td>1.423</td>
<td>1.345</td>
<td>No</td>
</tr>
<tr>
<td>Anhui</td>
<td>1.416</td>
<td>1.642</td>
<td>Yes</td>
</tr>
<tr>
<td>Fujian</td>
<td>1.438</td>
<td>1.330</td>
<td>No</td>
</tr>
<tr>
<td>Jiangxi</td>
<td>1.393</td>
<td>1.802</td>
<td>Yes</td>
</tr>
<tr>
<td>Shandong</td>
<td>1.389</td>
<td>1.532</td>
<td>Yes</td>
</tr>
<tr>
<td>Henan</td>
<td>1.402</td>
<td>1.934</td>
<td>Yes</td>
</tr>
<tr>
<td>Hubei</td>
<td>1.427</td>
<td>1.226</td>
<td>Yes</td>
</tr>
<tr>
<td>Hunan</td>
<td>1.437</td>
<td>1.53</td>
<td>Yes</td>
</tr>
<tr>
<td>Guangdong</td>
<td>1.369</td>
<td>1.451</td>
<td>Yes</td>
</tr>
<tr>
<td>Guangxi</td>
<td>1.432</td>
<td>1.786</td>
<td>Yes</td>
</tr>
<tr>
<td>Sichuan</td>
<td>1.117</td>
<td>1.561</td>
<td>Yes</td>
</tr>
<tr>
<td>Guizhou</td>
<td>1.59</td>
<td>1.962</td>
<td>Yes</td>
</tr>
<tr>
<td>Yunnan</td>
<td>1.323</td>
<td>1.701</td>
<td>Yes</td>
</tr>
<tr>
<td>Shaanxi</td>
<td>1.338</td>
<td>1.28</td>
<td>No</td>
</tr>
<tr>
<td>Gansu</td>
<td>1.454</td>
<td>1.757</td>
<td>Yes</td>
</tr>
<tr>
<td>Qinghai</td>
<td>1.384</td>
<td>1.877</td>
<td>Yes</td>
</tr>
<tr>
<td>Ningxia</td>
<td>1.3</td>
<td>1.838</td>
<td>Yes</td>
</tr>
<tr>
<td>Xinjiang</td>
<td>1.461</td>
<td>1.63</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Source: China Economic Yearbook 2013.
### Table 1.9: Log of survival rates between the ages of 1 and 35

<table>
<thead>
<tr>
<th>Variable</th>
<th>(\ln(1-p_{1,35}))</th>
<th>(\ln(1-p_{1,35}))</th>
<th>(\ln(1-p_{1,35}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(1-m))</td>
<td>0.018***</td>
<td>0.011***</td>
<td>0.012***</td>
</tr>
<tr>
<td>time</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>timesq</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.79</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>Observations</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: *** p<0.01, ** p<0.05, * p<0.1.
Table 1.10: Regression of log of survival rates by age group.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ln(1-p_{1,5})</th>
<th>ln(1-p_{5,10})</th>
<th>ln(1-p_{10,15})</th>
<th>ln(1-p_{15,20})</th>
<th>ln(1-p_{20,25})</th>
<th>ln(1-p_{25,30})</th>
<th>ln(1-p_{30,35})</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1-m)</td>
<td>0.050***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{1,5})</td>
<td>-</td>
<td>0.204***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{5,10})</td>
<td>-</td>
<td>-</td>
<td>0.652***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{10,15})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.086**</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{15,20})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.923***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{20,25})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.794***</td>
<td>-</td>
</tr>
<tr>
<td>ln(1-p_{25,30})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.870***</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.62</td>
<td>0.77</td>
<td>0.59</td>
<td>0.37</td>
<td>0.60</td>
<td>0.68</td>
<td>0.86</td>
</tr>
<tr>
<td>Observations</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: *** p<0.01, ** p<0.05, * p<0.1.
Table 1.11: Pooled regression of infant survival rates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ln(1-m\text{provincial})</th>
<th>0.925***</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(1-m\text{national})</td>
<td></td>
<td>0.925***</td>
</tr>
<tr>
<td>Std. Err.</td>
<td></td>
<td>0.122</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>172</td>
</tr>
</tbody>
</table>

Note: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. 
Chapter 2

Empirical Evidence from School Efficiency

2.1 Introduction

The negative correlation between quality and quantity of children per family has been widely discussed by economic literature. Becker and Lewis (1973) put forward a theory that sheds a deep insight on understanding on the trade-off relationship between number of children born per family and quality of these children. In their study, increase in marginal cost of additional child lead to higher outcome for their quality, held those children’s quantity constant; increase in marginal cost of children’s quality lead to higher outcome for their quantity, held their quality constant. Becker, Murphy, and Tamura (1990) modeled this negative correlation into a endogenous growth model, which generates two steady states for society: when human capital stock is scare, society chooses large family size and few investment in children’s quality; when human capital stock is abundant, society chooses small family size and huge investment in children’s quality. Later in Tamura, Simon and Murphy (2016), school efficiency was introduced as a parameter into marginal costs of quantity and quality to capture the difference in transforming schooling years spent on children into human capital obtained by children. And a slight decrease in school efficiency can produce a large decrease in years of schooling but a small increase in number of children born. Incorporating this parameter,
school efficiency, makes the theoretical model capable of explaining the phenomenon that relatively large variations in human capital investment in children is even associated with small variations in family sizes across regions.

In the first chapter, I develop a dynamic general equilibrium model with endogenous fertility, human capital investment, intergenerational transfer from child to parents, based on Tamura, Simon and Murphy (2016) and Boldrin and Jones (2002). The theory provides an understanding on the relationship between quantity and quality of children: mid-age workers invest on their children’s quantity and quality, not only to receive positive utility from number of children, but also to receive more financial support from children in later period; but with the budget constraint, mid-age workers have to tradeoff investment in quantity (quality) against quality (quantity). The model is developed to discuss the effect of one-child policy on economic growth in China after 1980. This policy imposed a strict fertility restriction on family. That is, it allow most families to have only one child. After the policy enacted, a relatively large variation in human capital investment that measured by expected years of schooling, is still observed across provinces in China. To absorb the distinct differences across provinces, schooling efficiency, is used as a key parameter to measure the time cost spent on each child for an additional year of schooling the child obtains. It plays an important role to produce the differences in educational outcomes for provinces with strict restrictions on family size.

According to the theoretical model, data on real GDP, number of workers, real physical stock and expected years of schooling are used to produce the theoretical school efficiency, but data on expenditure on education are not used. This paper compares model school efficiency with data school efficiency constructed from educational expenditure data and other data, by the means of OLS and fixed effect regressions. The statistical results prove that theoretical data is consistent with real data. This is in the same line with the findings of Tamura, Simon and Murphy (2016), and it provides an evidence that the theoretical model is reasonable and capable of describing economic behavior.

The paper is organized as the following: Section 2.2 present the data used for calibration and regression analysis; Section 2.3 describe the theoretical model and key equations used for calibration; Section 2.4 present regression analysis; Section 2.5 is the conclusion.
2.2 Data Description

In this section, I discuss how I construct provincial level data, and describe the Dataset I use to test implications from the theoretical model.

**Expected years of schooling.** Expected years of schooling data consists of provincial annual observations covering the period from 1949 to 2010. Those data are constructed from primary school enrollment rates, secondary school enrollment rates and higher education enrollment rates, according to this formula: for a child of age 7 at year $t$, expected years of schooling he will attain is the sum of primary school enrollment rates from year $t$ to $t + 5$, secondary school enrollment rates from year $t + 6$ to $t + 11$ and higher education enrollment rates from year $t + 12$ to $t + 16$. Since the enrollment rates data only spans the time period of 1949-2010, for children born after year 1985 the forward higher education enrollment rates are not available. Following the procedure in Murphy, Simon and Tamura (2008), I use the linear projections of three enrollment rates on time trend, of which primary school enrollment rates are truncated by 1 if projected values are above 1.

The annual primary school enrollment rates, used to constructed expected years of schooling, start in 1949 and end in 2010, covering 30 provinces, obtained from published book *China’s Provincial Statistics: 1949-1982* and from *China Statistic Yearbooks*. The primary school ages in China are from 7 to 12. For annual enrollment rates for secondary school and higher education, the school ages are from 13 to 18 and from 19 to 23, respectively. The data are constructed for each province $i$, by the formulae:

$$\text{enrolment rates for secondary school}_{i,t} = \frac{\text{the number of students in secondary school}_{i,t}}{\text{population of ages 13-18}_{i,t}}.$$

$$\text{enrolment rates for higher education}_{i,t} = \frac{\text{the number of students in higher education institution}_{i,t}}{\text{population of ages 19-23}_{i,t}}.$$

Where the population distribution since 1982 are calculated from the raw data obtained from each provinces’ statistical yearbooks, the population of ages 13-18 for year $t$ before 1982 are estimated by the population of ages 7-12 for year $t - 1$, and the population of ages 19-23 for year $t$ before 1982 are estimated by the sum of newborns 19-23 years ago from that year with the assumption that these groups of people do not move across provinces since birth.
**Real physical capital stock.** Annual data during 1953-2005 on real physical capital stock are in 1953 price, obtained from Yanrui Wu (2009).

**Real regional GDP.** Annual real regional GDP for each province are constructed by nominal regional GDP, real GDP growth, and GDP deflator of the base year (1953). Since National Bureau of Statistics of China do not report real GDP at provincial level, but nominal GDP levels and real GDP growth for each province are provided. So nominal regional GDP and real regional GDP growth from 1949-2008 are obtained from *China Compendium of Statistics: 1949-2008*, and the data from 2009-2010 are obtained from China Statistical Yearbooks (2010 and 2011). The Year-over-year real GDP growth rates for each province are used to construct the growth rate of each provinces deflator. And GDP deflator of the base year (1953) is normalized to one.


**Real school efficiency.** School efficiency is the time efficiency of education time. It is defined as how much time parents have to spend for an additional year of schooling their child attains. In this paper, based on Tamura, Simon, and Murphy (2016), the time cost for total expected years of schooling is measured by the share of output per worker spent on education per student according to the following:

\[ S_t^{\text{data}} = \frac{[\text{nominal government educational expenditure}]_t}{[\text{nominal GDP}]_t} \frac{[\text{number of employed workers}]_t}{[\text{number of students}]_t} \]

To construct the time cost for each province, I collected annual provincial data on nominal educational expenditure, nominal GDP, number of employed workers, number of students, and expected years of schooling from 1960-1988 and from 1998-2000.

Then actual data for school efficiency is constructed by:

\[ \lambda_t^{\text{data}} = \frac{S_t^{\text{data}}}{\tau_t^{\text{data}}} \]

where \( \tau_t^{\text{data}} \) is the estimated expected years of schooling divided by 20 years.
2.3 Model and Theoretical School Efficiency

Based on Boldrin and Jones (2002), Murphy, Tamura and Simon (2008), we develop a parsimonious 3-period overlapping generation model with endogenous fertility, human capital accumulation and intergenerational transfer. The set-up is meant to capture the feature of Chinese society: children are considered as a source of old-age support.

2.3.1 Model

Consider an overlapping generation model where people live up to three time periods: youth, middle-age and old-age. Let \( N_t \) denote the population of mid-age people at time \( t \). The total population, denoted as \( N_{\text{total},t} \), is the sum of the number of young, mid-age and old-age people. That is, \( N_{\text{total},t} = N_{t+1} + N_t + N_{t-1} \). Suppose that in period \( t \), old-age agents depend on the support from mid-age agents and their savings to live. Mid-age agents work for one period and make decisions on quantity-quality of children, along with transfer to parents, siblings’ transfer are taken as given.

Preferences. The life-time utility for mid-age agent includes the consumption at middle-age \( c_{m,t} \), the consumption at old-age \( c_{o,t+1} \), parent’s consumption at old-age \( c_{o,t} \), the benefits from having children:

\[
U_t = \ln c_{m,t} + \delta \ln x_t + \eta \ln c_{o,t} + \beta \ln c_{o,t+1}
\]

where \( x_t = n_t(1 - \rho_t) \). \( n_t \) is the number of children born by each individual, and \( \rho_t \) is the young adult mortality rate at time \( t \), considered as exogenous.

Budget constraints. The mid-age individual faces two resource constraints and one time constraint at time \( t \):

\[
\begin{align*}
   c_{m,t} + \pi_t + a_t &= m_t \\
   c_{o,t+1} &= \sum_{i=1}^{x_t} \pi_{i,t+1} + R_{t+1}a_t \\
   1 &= \ell_t + x_t(\tau_t \lambda_t + \nu)
\end{align*}
\]
where $\nu$ is the fixed minimum time needed to rear each child, $\tau_t$ is the time spent on educating each child, $\lambda_t$ is the time efficiency of educating, $\pi_t$ is the amount of transfers to parents. And here I assume that every child take their siblings’ transfer as given. Agent in mid-age rears and educates children, make earnings by working $l_t$ hours, lends (or borrows) $a_t$ amount at market-determined interest rate $R_{t+1}$ and transfers part of earnings $\pi_t$ to parent. When come to old-age, agent receives transfer from children, collects (or pays) principal and interest if he lent (or borrowed). The intergenerational transfers from mid-age worker to their old parents act as a channel through which parents have incentive to invest in quantity and quality of children.\footnote{The incorporation of endogenous transfer from children to parents not only put forward a fundamentally reason for rearing and educating children, but also drives a dynamic transition on fertility choices. See Appendix A for more details.}

The mid-age agent’s earning at period $t$ is determined by the amount of time allocated on working, the human capital $h_t$ the agent has, and the wage rate $w_t$.

$$m_t = w_t h_t (1 - x_t (\tau_t \lambda_t + \nu))$$

**Production.** The production side of the economy is given by an aggregate production function

$$Y_t = F(K_t, z_t H_t)$$

where $K_t$ is aggregate physical capital stock, $z_t$ is the labour augmented productivity, and $g_t = \frac{z_{t+1} - z_t}{z_t}$.\footnote{In this paper, $g_t$ specially refer to growth rate of productivity generated from institutional improvements and economic reforms in China.}

Assuming capital depreciation rate is $\zeta$ at the end of every period, the dynamics of physical capital accumulation is given by

$$K_t = N_{t-1} a_{t-1} + (1 - \zeta) K_{t-1}.$$

And $H_t$ is the aggregate stock of human capital used for producing goods, which is given by

$$H_t = h_t l_t N_t = h_t (1 - x_t (\tau_t \lambda_t + \nu)) N_t$$
Therefore, the production function at time $t$ is specified by

$$F(K_t, z_t H_t) = K_t^\alpha \left( z_t h_t \left( 1 - x_t (\tau_t \lambda_t + \nu) \right) N_t \right)^{1-\alpha}.$$ 

By solving firm's profit maximization problem, $F(K_t, z_t h_t l_t N_t) - w_t h_t l_t N_t - R_t K_t$ by choosing $N_t$ and $K_t$, get

$$w_t = (1 - \alpha) K_t^\alpha z_t \left( z_t h_t l_t N_t \right)^{-\alpha},$$

$$= (1 - \alpha) z_t \left( \frac{k_t}{z_t h_t l_t} \right)^\alpha.$$ 

$$R_t = \alpha \left( \frac{k_t}{z_t h_t l_t} \right)^{\alpha-1}.$$ 

where $k_t = \frac{K_t}{N_t}$, per capita physical capital stock.

Market clear condition. The whole economy faces a market clear condition at every period.

$$Y_t = (c_{m.t} + a_t) N_t + c_{o.t} N_{t-1}.$$ 

Human capital accumulation. The current per capita human capital in the economy is assumed to be determined by last generation’s per capita human capital, per capita investment made by last generation and the productivity to produce human capital, $A$:

$$h_{t+1} = A h_t \tau_t.$$ 

\textbf{2.3.2 Optimal Decision on Educational Investment}

Mid-age agent maximizes life-time utility by choosing the number of children $n_t$, the investment on children’s education $\tau_t$, the transfer to parents $\pi_t$, the amount of capital to lend (or borrow) $a_t$. According to first order conditions with respect to $n_t$, $\tau_t$, $a_t$ and $\pi_t$, we can obtain the following

$h_t$ is the effective per capita human capital used in producing output. $l_t = 1 - x_t (\tau_t \lambda_t + \nu)$ is how much time spent on working. Therefore, according to the definitions the marginal product of human capital is actually the wage rate of per unit of human capital, which is wage per unit of time per unit of human capital.
equation to calibrate theoretical school efficiency:

\[ 0 = w_t h_t \lambda_t \tau_t - \frac{m_{t+1}}{R_{t+1}} \]  

(2.1)

The equation implies that the optimal human capital investment can be determined by equate the marginal cost of educating children, that is the first term in RHS, to the additional benefit from one more unit of human capital investment, that is the second term.

**Theoretical school efficiency.** To recovery the theoretical school efficiency for each province, provincial data on employed labour, real GDP, real capital stock, expected years of schooling are collected and used. According to the eq.(2.1), we have

\[ 0 = (1 - \alpha) y_t \lambda_t \tau_t \frac{1}{1 - x_t (\lambda_t \tau_t + \nu)} - \frac{1}{\alpha} k_{t+1} \]

After arrangement, we get

\[ \lambda_{t, \text{model}} = \frac{k_t (1 - x_t \nu)}{(\alpha y_t + k_{t+1} x_t) \tau_t} \]

where \( y_t \) is real GDP per worker, \( x_t \) is employment growth, \( k_t \) is real capital stock, and \( \tau_t \) is the ratio of expected years of schooling to 20 years. Due to the missing data on Tibet, Chongqing, and Hainan, 28 provinces’ school efficiency during different time periods are calibrated and presented in Table 1.1.

### 2.4 Regression Analysis

To parametric the model and calibrate the school efficiency, I use data on expected years of schooling, real GDP, real capital stock, employment, but don’t use any data on educational expenditure. In the section, we use provincial data on school efficiency constructed from public educational expenditure and other information, and use empirical regression to produce measures of goodness of fit of the

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4The procedure to get the equations is a little bit complicated, and more detail can be found in my job market paper
models solutions with the data. Now consider the two following regression equations:

\[ \lambda_i^{data} = \alpha + \beta \lambda_i^{model} \]
\[ \ln \lambda_i^{data} = \alpha + \beta \ln \lambda_i^{model} \]

Table 2.1 records the results for regression of average \( \lambda_i^{data} \) on \( \lambda_i^{model} \). Since any observation of \( \lambda_i^{model} \) are for time period of 1960-1980 or of 1980-2000, we calculated the average \( \lambda_i^{data} \) for 1960-1980 and for 1980-2000 in provincial level. And because theoretical model captures the enforcement of one-child policy, \( \lambda_i^{model} \) contains information about the effect of one-child policy. Therefore in this paper we only focus on statistical results generated by regressions without control for one-child policy, although we still present result of the case with one-child policy as a reference.

In Table 2.1, slope coefficients with and without control are positive and statistically significant, and introducing an control variable for one-child policy hurts the explanatory power. In table 2.2, we present the results for log-log regressions. Without any control variable, the slope coefficient is positive and significant in 0.1% level, and the \( \bar{R}^2 \) are 0.28. Although we reject the hypothesis that \( \alpha = 0 \) and \( \beta = 1 \), we cannot reject \( \beta = 1 \) in 5% significant level. It is clear that theoretical model can produce the time series on school efficiency quite a bit of consistent with reality.

Table 2.3-2.6 show the results for regression relating annual \( \lambda_i^{data} \) on \( \lambda_i^{model} \). Here, I introduce a dummy variable to control for 9-year compulsory education policy period (after 1994). That is,

\[
D_{school}^{i,t} = \begin{cases} 
1, & \text{if in compulsory education policy period.} \\
0, & \text{if not.} 
\end{cases}
\] (2.2)

In table 2.3, slope coefficients are positive and significant in 5% level with and without controls, but \( \bar{R}^2 \) is very small. Table 2.4 produces the results for log-log regressions. The slope coefficient are all positive, very close to one, and significant in 0.1% level with and without controls. The first column in table 2.4 records the regression results for the case without any control, that shows \( \beta \) is 1.015 and we cannot reject the hull hypothesis that \( \beta \) is one in 5% significant level. The third column records the regression results for the case with a control on compulsory education policy. It suggests that \( \beta \) is 1.075, and we still cannot reject the null hypothesis that \( \beta \) is one in 5% significant level.
Compared with the case without any control variable, introducing the dummy variable to control for compulsory education policy period can slightly improve $R^2$, from 19.58% to 20.37%. Despite that we reject the null hypothesis that $\alpha = 0$ and $\beta = 1$, the closeness of $\beta$ to one and relatively high $R^2$ provide evidence that model data fit real data with quite a bit of success.

Table 2.5 and 2.6 show the results for fixed effect regression relating annual $\lambda^\text{data}_{i,t}$ on $\lambda^\text{model}_{i,t}$. The regression equations are the following,

$$\lambda^\text{data}_{i,t} = \alpha + \beta\lambda^\text{model}_{i,t} + \nu_i + u_{i,t}$$

$$ln\lambda^\text{data}_{i,t} = \alpha + \beta ln\lambda^\text{model}_{i,t} + \nu_i + u_{i,t}$$

We introduce fixed effect coefficient to control for specific effect of each province. More specifically, there are 28 provinces in dataset. In table 2.5, the third column records the regression results for the case with a control on compulsory education policy period, which has $p$ value of 0.0878. That means the significant level of the test is 8.78%. Since the null hypothesis is that $\alpha$ is zero and $\beta$ is one, we cannot reject the null hypothesis at any significant level of below than 8.78%. In table 2.6, although we reject the null hypothesis that $\alpha = 0$ and $\beta = 1$, we still cannot reject that $\beta = 1$ and $R^2$'s are improved to about 37%.

To further examine the relationship between $\lambda^\text{data}$ and $\lambda^\text{model}$, we consider the following regression equation:

$$\Delta\lambda^\text{data}_{i,t} = \alpha + \beta\Delta\lambda^\text{model}_{i,t} + u_{i,t}$$

Where $\Delta\lambda^\text{data}_{i,t} = \lambda^\text{data}_{i,t+20} - \lambda^\text{data}_{i,t}$, and $\Delta\lambda^\text{model}_{i,t} = \lambda^\text{model}_{i,t+20} - \lambda^\text{model}_{i,t}$. And the regression results are presented in table 2.7. These regressions have very small $R^2$’s, and statistically insignificant coefficients. However, as the last row of the table shows, the regression without any control has $p$ value of 0.426, which is the significant level of the test that $\alpha$ is zero and $\beta$ is one. That means we cannot reject the hypothesis at any significant level below 42.6%. Also, for the regression with a control on compulsory education policy, we cannot reject the hypothesis at any significant level below 54.03%. This provide a solid evidence supporting the consistency between model data and
real data.

2.5 Conclusion

The correlation between theoretical school efficiency and real cost of schooling are examined in this paper. In five of seven tables, we cannot reject the hypothesis that $\beta$ is one. In two tables we cannot reject the hypothesis that $\alpha$ is zero and $\beta$ is one. Among these, we find a very strong evidence that relationship between the change in real data and the change in model data is one-to-one. Beside that, introducing the control variable for compulsory education policy and log specification can improve explanatory power of regressions. And introducing the control variable for one-child cannot obviously improve statistical results, because model data already contain information about the effect of this policy. These statistical analysis suggest that theoretical data is consistent with real data, and imply that theoretical model is quite reasonable and is capable of describing economic activities.
2.6 References


Wu Yanrui, “Chinas capital stock series by region and sector”, The University of Western Australia Discussion Paper, 2009.


2.7 Appendix A: Tables

Table 2.1: Pooled regressions of data on model solutions.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{data}$</th>
<th>$\lambda_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.350**</td>
<td>0.395*</td>
</tr>
<tr>
<td></td>
<td>(0.168)</td>
<td>(0.200)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.128</td>
<td>-0.141</td>
</tr>
<tr>
<td></td>
<td>(0.106)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>one-child</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0650</td>
<td>0.0484</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The final row is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. 
Table 2.2: Pooled regressions of data on model solutions.

<table>
<thead>
<tr>
<th></th>
<th>$\ln \lambda_{data}$</th>
<th>$\ln \lambda_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.362***</td>
<td>1.624***</td>
</tr>
<tr>
<td></td>
<td>(0.308)</td>
<td>(0.366)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-2.154***</td>
<td>-1.881***</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.273)</td>
</tr>
</tbody>
</table>

one-child | No | Yes
---|---|---
$N$ | 49 | 49
$\bar{R}^2$ | 0.2795 | 0.2900
$p$ | 0.000 | 0.000

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The final row is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. 
Table 2.3: Pooled regressions of data on model solutions.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{data}$</th>
<th>$\lambda_{data}$</th>
<th>$\lambda_{data}$</th>
<th>$\lambda_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.403*</td>
<td>0.441*</td>
<td>0.440*</td>
<td>0.441*</td>
</tr>
<tr>
<td></td>
<td>(0.174)</td>
<td>(0.196)</td>
<td>(0.177)</td>
<td>(0.196)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.152</td>
<td>-0.163</td>
<td>-0.163</td>
<td>-0.163</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.109)</td>
<td>(0.106)</td>
<td>(0.109)</td>
</tr>
<tr>
<td>One-child</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>compulsory edu</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>579</td>
<td>579</td>
<td>579</td>
<td>579</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0076</td>
<td>0.0061</td>
<td>0.0077</td>
<td>0.006</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The final row is the $p$-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. 

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Table 2.4: Pooled regressions of data on model solutions.

<table>
<thead>
<tr>
<th></th>
<th>$\ln \lambda^{data}$</th>
<th>$\ln \lambda^{data}$</th>
<th>$\ln \lambda^{data}$</th>
<th>$\ln \lambda^{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.015***</td>
<td>1.136***</td>
<td>1.075***</td>
<td>1.146***</td>
</tr>
<tr>
<td></td>
<td>(0.0855)</td>
<td>(0.1000)</td>
<td>(0.0882)</td>
<td>(0.0999)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-2.415***</td>
<td>-2.299***</td>
<td>-2.361***</td>
<td>-2.293***</td>
</tr>
<tr>
<td></td>
<td>(0.0503)</td>
<td>(0.0706)</td>
<td>(0.0542)</td>
<td>(0.0705)</td>
</tr>
</tbody>
</table>

One-child: No, Yes

compulsory edu: No, Yes

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>$\bar{R}^2$</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-child</td>
<td>576</td>
<td>0.1958</td>
<td>0.000</td>
</tr>
<tr>
<td>compulsory edu</td>
<td>576</td>
<td>0.2019</td>
<td>0.000</td>
</tr>
<tr>
<td>Yes</td>
<td>576</td>
<td>0.2037</td>
<td>0.000</td>
</tr>
<tr>
<td>Yes</td>
<td>576</td>
<td>0.2054</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The final row is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. 

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Table 2.5: Fixed effect regressions of data on model solutions

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{data}$</th>
<th>$\lambda_{data}$</th>
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<th>$\lambda_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.510</td>
<td>1.178</td>
<td>0.629</td>
<td>1.175</td>
</tr>
<tr>
<td></td>
<td>(0.324)</td>
<td>(0.742)</td>
<td>(0.343)</td>
<td>(0.742)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.215</td>
<td>-0.552</td>
<td>-0.273</td>
<td>-0.548</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.388)</td>
<td>(0.201)</td>
<td>(0.388)</td>
</tr>
<tr>
<td>One-child</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>compulsory edu</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>579</td>
<td>579</td>
<td>579</td>
<td>579</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.0093</td>
<td>0.0092</td>
<td>0.0111</td>
<td>0.0101</td>
</tr>
<tr>
<td>$p$</td>
<td>0.0065</td>
<td>0.0882</td>
<td>0.0878</td>
<td>0.0878</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

The final row is the p-value on the null hypothesis that $\beta = 1$ and $= 0$. 

84
<table>
<thead>
<tr>
<th></th>
<th>$\ln\lambda^{data}$</th>
<th>$\ln\lambda^{data}$</th>
<th>$\ln\lambda^{data}$</th>
<th>$\ln\lambda^{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.835***</td>
<td>0.989***</td>
<td>0.941***</td>
<td>0.941***</td>
</tr>
<tr>
<td></td>
<td>(0.129)</td>
<td>(0.289)</td>
<td>(0.136)</td>
<td>(0.136)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-2.415***</td>
<td>-2.263***</td>
<td>-2.310***</td>
<td>-2.310***</td>
</tr>
<tr>
<td></td>
<td>(0.142)</td>
<td>(0.292)</td>
<td>(0.148)</td>
<td>(0.148)</td>
</tr>
<tr>
<td>One-child</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>compulsory edu</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>576</td>
<td>576</td>
<td>576</td>
<td>576</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.3704</td>
<td>0.3697</td>
<td>0.3757</td>
<td>0.3757</td>
</tr>
<tr>
<td>$p$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The final row is the p-value on the null hypothesis that $\beta = 1$ and $= 0$. 

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Table 2.7: First difference regressions of data on model solutions

<table>
<thead>
<tr>
<th></th>
<th>$\lambda^{data}$</th>
<th>$\lambda^{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.719</td>
<td>1.712</td>
</tr>
<tr>
<td></td>
<td>(1.217)</td>
<td>(1.221)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.207</td>
<td>-0.198</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.205)</td>
</tr>
<tr>
<td>compulsory edu</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>164</td>
<td>164</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0061</td>
<td>0.0001</td>
</tr>
<tr>
<td>$p$</td>
<td>0.4160</td>
<td>0.5403</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.  * $p < 0.1$,  ** $p < 0.05$,  *** $p < 0.01$

The final row is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. 