Online Parameter Estimation and Adaptive Control of Magnetic Wire Actuators

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Online Parameter Estimation and Adaptive Control of Magnetic Wire Actuators

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Doctor of Philosophy
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Harshwardhan Karve
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Abstract

Cantilevered magnetic wires and fibers can be used as actuators in microfluidic applications. The actuator may be unstable in some range of displacements. Precise position control is required for actuation. The goal of this work is to develop position controllers for cantilevered magnetic wires.

A simple exact model knowledge (EMK) controller can be used for position control, but the actuator needs to be modeled accurately for the EMK controller to work. Continuum models have been proposed for magnetic wires in literature. Reduced order models have also been proposed. A one degree of freedom model sufficiently describes the dynamics of a cantilevered wire in the field of one magnet over small displacements. This reduced order model is used to develop the EMK controller here.

The EMK controller assumes that model parameters are known accurately. Some model parameters depend on the magnetic field. However, the effect of the magnetic field on the wire is difficult to measure in practice. Stability analysis shows that an inaccurate estimate of the magnetic field introduces parametric perturbations in the closed loop system. This makes the system less robust to disturbances. Therefore, the model parameters need to be estimated accurately for the EMK controller to work. An adaptive observer that can estimate system parameters on-line and reduce parametric perturbations is designed here. The adaptive observer only works if the
system is stable. The EMK controller is not guaranteed to stabilize the system under perturbations. Precise tuning of parameters is required to stabilize the system using the EMK controller. Therefore, a controller that stabilizes the system using imprecise model parameters is required for the observer to work as intended.

The adaptive observer estimates system states and parameters. These states and parameters are used here to implement an indirect adaptive controller. This indirect controller can stabilize the system using imprecise initial parameter estimates. The indirect adaptive controller overcomes the limitations of the EMK controller by stabilizing the closed loop system despite inaccurate initial parameter estimates.

The experiment setup used to test the controllers is also presented. Experiments were performed to test the adaptive controller using cantilevered cobalt and nickel wires. The closed loop system using the indirect controller is stable. The wire tracks continuous desired trajectories up to 30Hz. Experiments were also performed to test the robustness of the adaptive and EMK controllers when the wire is interacting with water.

The adaptive controller performs poorly when unmodeled disturbances are encountered, necessitating fall back to the EMK controller in some applications. The adaptive controller functions as an EMK controller if observer gain is set to 0. Thus, the indirect adaptive controller estimates model parameters, stabilizes the wire in the unstable region and can be switched into a non-adaptive mode for applications.
Dedication

Dedicated to my wife Apurva.

All journeys are easy if they lead me back to you.
Acknowledgments

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Chapter 1

Introduction

Flexural magnetic structures can be used as actuators in microfluidic applications such as droplet manipulation or fluid transport. Flexural actuators consist of a bending mechanical element made using ferromagnetic or paramagnetic material. When the actuator is placed in a magnetic field, the mechanical element bends towards the source of the field. This allows for actuation without direct physical contact with the bending element. Magnetic wires and fibers are particularly suited to microfluidic applications. Flexible polymer fibers can be fabricated with varying microfluidic properties. The polymer fibers can then be coated with paramagnetic particles. The surfaces of ferromagnetic wires can be treated to increase their affinity to fluids. In this way, an actuator can be fabricated for a desired application with desirable microfluidic properties. Nickel and Cobalt wires are used here as they are readily available at low cost. Nickel and Cobalt exhibit minimal magnetic hysteresis. Methods developed for paramagnetic fibers can be used for the wires as well.

Magnetic wires may be unstable in some range of operation. A position controller is required to stabilize the wires in a magnetic field and to ensure trajectory tracking before the wire can be used as an actuator. The literature on flexural mag-
Figure 1.1: A magnetic wire cantilevered between two electromagnets.

Magnetic structures is limited at present to fabrication and static modeling of such actuators. Feedforward control based on static models is the preferred method of control in literature. Some of the current literature on magnetic structures is reviewed below.

1.1 Literature review of magnetic actuators

Bending magnetic actuators with different geometries have been proposed for a variety of applications. Various magnetic microactuators have been fabricated [Khoo and Liu 2001; Hsieh et al. 2011; Liu and Yi 1999; Morega et al. 2013]. Shape memory actuators have been fabricated for microvalves and microscanners [Kohl et al. 2006]. Polymer magnets have been used to make magnetic actuators [Lagorce et al. 1999]. Magnetic flagella and cilia attached to microscopic swimming robots have been fabricated [Dreyfus et al. 2005; Gauger and Stark 2006]. Magnetic flaps of various shapes have been used to induce flow in a microfluidic channel [Belardi et al. 2011]. Magnetic actuators presented in literature for microfluidic applications consist of a bending or twisting magnetic element. The element bends when placed in a magnetic field. The amount of bending or twisting depends on the shape or the strength of the applied
magnetic field. The fibers and wires used in the current work follow the same principle of bending in a magnetic field. However, the literature has largely focused on fabrication and static modeling of magnetic actuators. The goal of this work is to increase the usability of magnetic actuators using feedback control.

The flexural magnetic structures presented in literature commonly use uniform or rotating uniform magnetic fields. Various bending structures made from ferromagnetic materials placed in uniform magnetic fields have been presented in literature [Moon and Pao 1968; Yang et al. 1999; Hsieh et al. 2011; Gerbal et al. 2015]. Bending filaments with paramagnetic and ferromagnetic materials placed in rotating or oscillating magnetic fields have been analyzed [Cebers and Cirulis 2007; Oukhaled et al. 2012]. Uniform or rotating magnetic fields are easier to analyze, but difficult to generate in practice compared to nonuniform fields. Nonuniform magnetic fields can be generated more easily. The disadvantage is that nonuniform fields are difficult to model. Nonuniform magnetic fields are used with ferromagnetic wires in the work presented here. The controllers can be used with paramagnetic fibers without modification.

The literature also presents various analytical and numerical methods for modeling flexural magnetic structures. The buckling of ferromagnetic magnetic plates and beams has been studied as early as the 60s and 70s [Moon and Pao 1968; Wallerstein and Peach 1972]. Buckling instability of ferromagnetic structures has been studied more recently as well [Yang et al. 1999; Gerbal et al. 2015]. Elastic magnetic elements have been modeled [Cebers 2005; Cebers and Cirulis 2007]. The bending of paramagnetic fibers in a nonuniform field has been studied [Groff et al. 2012; Karve 2012]. The literature primarily presents static models. A dynamic model is required for developing a controller. Dynamics of bending actuators can be modeled most accurately using continuum dynamics. However, continuum dynamic models are cumbersome to
use. Instead, an approximate model is used here.

For the flexural magnetic structure to be used as an actuator, the position of the bending element must be controlled accurately and robustly. In literature, feedforward control based on static data is commonly proposed for position control of magnetic actuators. Flexural magnetic actuators are unstable in some part of the operating range, as demonstrated by the analysis of buckling of magnetic elements in [Moon and Pao 1968] and [Gerbal et al. 2015]. Paramagnetic fibers and ferromagnetic wires exhibit the same instability, as shown in [Karve 2012] and [Groff et al. 2012]. This leaves a large range of motion where the actuator cannot be stabilized using feedforward control. A feedback controller is required to precisely control the motion of the actuator in this unstable region. This work presents an adaptive controller for magnetic wires placed in nonuniform fields using an approximate dynamic model. The controller is tested using electromagnets and a ferromagnetic wire arranged as shown in Figure 1.1. The following section covers preliminary modeling and sensing work that facilitates controller development.

1.2 Dynamic model and position sensing

There are two main prerequisites before position control can be designed and implemented. First, a dynamic model of the wire in the magnetic field is required. Also, a sensor that can sense the position of the wire accurately without interfering with the wire dynamics is required. A static continuum model of the wire and a one dimensional approximation were presented in [Groff et al. 2012] and [Karve 2012]. The one dimensional approximate model can be extended easily to the dynamic case. This model is used to develop the adaptive controller. Also, a high bandwidth optical sensor that can sense the position of the wire was presented in [Cheng 2013].
sensor is used to measure wire position for control.

A preliminary model based controller for magnetic fibers was presented in [Karve et al. 2013]. The model based or EMK controller assumes that the system parameters are known. Since this is not possible in practice, a parametric perturbation is introduced into the closed loop system. The challenge is to develop a controller that can control the position of the fiber or wire and estimate system parameters on-line. An adaptive observer can be used to estimate system parameters. The observer can also be extended to an indirect controller. The adaptive observer and controller developed here are presented in [Karve and Groff 2016].

1.3 Overview of dissertation

This thesis is organized as follows. Chapter 2 extends the one degree of freedom approximate model of the wire presented in [Groff et al. 2012] to the dynamic case. The preliminary feedforward-feedback controller from [Karve et al. 2013] is reviewed and analyzed in Chapter 3. The main limitation of the preliminary controller is that model parameters are not known accurately for the wires used in the experiments. An adaptive observer and controller that can estimate these system parameters on-line are presented in Chapter 4 and Chapter 5. The controller is validated experimentally in Chapter 6. Finally, Chapter 7 presents methods and techniques for using the preliminary and adaptive controllers in applications involving interaction with objects.
Chapter 2

Dynamic Model of the Wire

The wire is cantilevered at one end and placed in the field of an electromagnet. The electromagnetic field pulls the wire towards the magnet. The resulting bending generates strain in the opposite direction along the wire. In addition to these static forces, the wire experiences a small amount of damping when it is moving. The inertia of the wire also affects the motion of the wire. These are the important parts of the dynamics that affect the behavior of the wire in a magnetic field. All of these effects need to be modeled for controlling the position of the wire.

A static continuum model of a wire was presented in [Groff et al. 2012]. The model uses energy methods to predict static equilibria of the wire in a magnetic field. The equilibria predicted by the static model correspond well with measurements. However, extending the static model to the dynamic case and implementing it in real time is challenging. Measuring the shape of the wire at high sample rates is not possible. Also, modeling the interaction of the wire with other objects using the continuum framework is difficult. An approximate model of the wire as a rigid paramagnetic bar is presented in this chapter. The approximate model is validated by comparing the static behavior of the approximate model and the wire.
2.1 Approximate Model

A one degree of freedom (DoF) approximate model is used for developing a controller. The one DoF model is easier to adapt to new applications. The goal of the controller design is to control the position of the tip of the wire. The position of the tip of the wire can also be expressed as the angle of the line joining the base of the wire with the tip of the wire. Varying the strength of the applied magnetic field changes this angle. The wire is modeled as a rigid paramagnetic bar with a torsional spring at its base. The angle made by the bar is the same as the angle of the line joining the tip of the wire and its base. The wire and magnet system is shown in Figure 2.1(a) and the equivalent rigid bar arrangement is shown in Figure 2.1(b). The state of the system is the angle $\theta$ between the base of the wire and the tip of the wire. Thus, the displacement of the wire tip is the displacement of the tip of the bar. The bending along the wire is represented using a torsional spring at the base of the bar. Since the damping on the wire is relatively small, it is assumed to be linear. Finally, the magnetic torque on the bar is represented by a strictly positive function of $\theta$. Magnetics simulations show that the magnetic torque on the bar increases the closer the bar is to the magnets. Dynamics of the rigid bar in the magnetic field are
modeled as

$$\frac{mL^2}{3} \ddot{\theta} = -k_d \dot{\theta} - k_s \theta + \tau_{M_1}(\theta) I_1^2 - \tau_{M_2}(\theta) I_2^2 + \tau_{M_{12}}(\theta) I_1 I_2 \quad (2.1)$$

where \( k_d \) is the estimated damping coefficient, \( k_s \) is the spring constant of the torsional spring. Moment of inertia of the bar is given by \( \frac{mL^2}{3} \). Since the mass of the bar is very small, the gravitational force acting on the bar is negligible.

The magnetic torque on the bar due to magnet \( i \) for a 1A electromagnet coil current is given by \( \tau_{M_i}(\theta) \). Each term \( \tau_{M_i}(\theta) \) depends on the electromagnet and the position of the wire relative to the electromagnetic field. The third magnetic torque term, \( \tau_{M_{12}}(\theta) \) arises due to the interaction between the magnetic fields of the two magnets. This term complicates the design and implementation of a controller. However, the term vanishes when either coil current \( I_1 \) or \( I_2 \) is zero. Thus the controller is designed so that only one coil current is non zero at any given time. This means the interaction term can be ignored during the remaining analysis.

All model parameters of Equation 2.1 are required to implement a controller. The problem can be simplified by normalizing the system parameters by the moment of inertia as follows

$$\ddot{\theta} = -a \dot{\theta} - b \theta + c_1(\theta) I_1^2 - c_2(\theta) I_2^2 \quad (2.2)$$

where

$$a = \frac{3k_d}{mL^2}, \quad b = \frac{3k_s}{mL^2}, \quad c_1(\theta) = \frac{3\tau_{M_1}(\theta)}{mL^2}, \quad c_2(\theta) = \frac{3\tau_{M_2}(\theta)}{mL^2} \quad (2.3)$$

The magnetic field is computed using FEMM [Meeker] and then the magnetic torque on the bar \( c_1(\theta) \) and \( c_2(\theta) \) is computed as a function of \( \theta \) using the magnetic model described in [Groff et al. 2012]. The spring constant is determined by solving
Equation 2.2 for the steady state as follows

\[ b = \frac{c_1(\theta)I_1^2 - c_2(\theta)I_2^3}{\theta} \]  \hspace{1cm} (2.4)

Thus experimental data can be used to estimate \( b \). The current \( I_2 \) is set to zero while \( I_1 \) is held constant. The steady state value of \( \theta \) can be measured using a camera or an optical sensor. The spring constant \( b \) is then estimated using Equation 2.4.

Figure 2.2: Magnetic and strain torque on the rigid bar for \( u=0.39A \). Intersections of the two plots are the static equilibria.

The estimated parameters were used to generate steady state solutions of Equation 2.4 for constant currents. Figure 2.2 shows a plot of magnetic torque vs strain torque. The points at which the strain torque intersects the magnetic torque curve are the static equilibria. The system has two stable equilibria, where the bar is attracted towards the equilibrium. There is one unstable equilibrium between the two stable equilibria. The bar is pulled by magnetic and strain towards the stable equilibria and away from the unstable equilibrium. This shows the cause of the instability. The unstable region has no stable equilibria for any coil currents. Thus, if the wire is in the unstable region, but not at one of the unstable equilibrium points, it travels to the nearest stable equilibrium point.
Steady state solutions generated in simulations and the steady state equilibria observed experimentally are plotted in Figures 2.3(a) and 2.3(b) respectively. The stable solutions predicted by the model show a good correspondence with the ones observed in practice.

(a) Static equilibria of the rigid bar placed in a magnetic field. (b) Experimentally observed static equilibria of a cobalt wire.

Figure 2.3: Static equilibria of the wire in a magnetic field predicted by the model and experimentally observed equilibria.

The simulations show that the instability is caused by the unstable static equilibria. To get the wire to one of the unstable equilibrium points, the coil current has to follow a very specific trajectory. This trajectory makes the wire follow stable and unstable equilibria. The slightest deviation from the trajectory causes the wire to go the nearest stable equilibrium. It is also difficult to hold the wire at an unstable equilibrium. Even if the coil current is kept constant, small disturbances can cause the wire to go to the nearest stable equilibrium. Another problem is that exact model knowledge for the unstable region is not available since static data cannot be obtained for this region from experiments. This can be seen by comparing the point at which the wire detaches from the magnet in Figure 2.3(a) vs in Figure 2.3(b). This problem can be alleviated in part by using feedback control. A simple feedback controller is presented in the next section.
Chapter 3

Exact Model Knowledge Controller

The goal of this chapter is to design a control law assuming the model is perfectly known [Karve et al. 2013]. The dynamics of Equation 2.2 can be used to implement a position controller for the fibers and wires. The model parameters may not be perfectly known in practice. These parametric perturbations can destabilize the system. However, implementing and analyzing an exact model knowledge controller is useful. The sensitivity of the closed loop system to perturbations in individual parameters can be analyzed. This analysis is presented here. Also, the preliminary controller can be modified to get adaptive controllers that can eliminate the perturbation.

3.1 EMK controller

The desired trajectory is denoted by $\theta_d$. The tracking error is defined as $\tilde{\theta} = \theta - \theta_d$. The first and second time derivatives of $\tilde{\theta}$ are denoted by $\dot{\tilde{\theta}}$ and $\ddot{\tilde{\theta}}$ respectively. Open loop error dynamics are derived from the dynamics of Equation
2.2 as follows

\[ \ddot{\theta} = -a \dot{\theta} - b \ddot{\theta} - a \dot{\theta}_d - b \dot{\theta}_d - \ddot{\theta}_d + c_1(\theta) I_1^2 - c_2(\theta) I_2^2 \]  

(3.1)

where the parameters \( a, b, c_1(\theta) \) and \( c_2(\theta) \) are as defined in Equation 2.3.

The magnetic torque is proportional to the square of the coil current. Therefore, only positive control inputs can be applied using magnet 1 and only negative control inputs can be applied using magnet 2. The two coil currents can be combined into one control input by switching them on and off as follows

\[ I_1 = \begin{cases} \sqrt{\frac{u}{c_1(\theta)}} & u \geq 0 \\ 0 & u < 0 \end{cases} \]

\[ I_2 = \begin{cases} 0 & u \geq 0 \\ \sqrt{-\frac{u}{c_2(\theta)}} & u < 0 \end{cases} \]  

(3.2)

where the control input \( u \) is designed as follows

\[ u = -\alpha \dot{\theta} - \beta \ddot{\theta} + \dot{\theta}_d + \dot{\theta}_d + \dot{\theta}_d \]  

(3.3)

where \( \alpha \) and \( \beta \) are control gains. The controller consists of potential and derivative feedback to stabilize the system, along with feedforward terms. The magnetic field is proportional to the square of the current. Therefore, the control input \( u \) is square rooted in Equation 3.2. The control input is then divided by the magnetic torque functions to cancel the magnetic torque terms. Finally, the switching action of Equation 3.2 combines the control inputs from the two currents into one input. Assuming
the parameter estimates are accurate, the closed loop error dynamics are given by

$$
\ddot{\theta} = \begin{cases} 
-a\dot{\theta} - b\dot{\theta} - a\dot{\theta}_d - b\theta_d - \dot{\theta}_d + u & u \geq 0 \\
-a\dot{\theta} - b\dot{\theta} - a\dot{\theta}_d - b\theta_d - \dot{\theta}_d - (-u) & u < 0 
\end{cases}
$$

(3.4)

The dynamics change to the following

$$
\ddot{\theta} = -a\dot{\theta} - b\dot{\theta} - a\dot{\theta}_d - b\theta_d - \dot{\theta}_d + u
$$

(3.5)

Substituting the control law of Equation 3.3 in Equation 3.5 gives the following closed loop error dynamics

$$
\ddot{\theta} = -(a + \alpha)\dot{\theta} - (b + \beta)\dot{\theta}
$$

(3.6)

This closed loop error system is exponentially stable. This can be demonstrated using the Lyapunov function defined as follows

$$
V = \frac{1}{2}(a + b + \alpha + \beta - 1)\dot{\theta}^2 + \frac{1}{2}(\ddot{\theta} + \dot{\theta})^2
$$

(3.7)

The derivative of $V$ along the trajectory of the closed loop error system is given by

$$
\dot{V} = -(b + \beta)\dot{\theta}^2 - (a + \alpha - 1)\dot{\theta}^2
$$

(3.8)

$\dot{V}$ is quadratic and negative definite since $(b + \beta) > 0$ and $(a + \alpha) > 1$. Therefore, the system is exponentially stable. Thus, the tracking error goes to zero exponentially fast. This controller is simple and effective when the parameters are known accurately. When the parameters are not known accurately, using the inaccurate parameter estimates causes perturbations in the closed loop system. The effects of these parametric perturbations are analyzed next. The adaptive controller presented later
has a similar structure as Equation 3.2 and Equation 3.3. Therefore, the following perturbation analysis can be used to analyze the adaptive controller as well.

## 3.2 Exact Model Knowledge controller

with perturbed equilibrium

The exact model knowledge controller uses parameter estimates for control. It is assumed that parameter estimates are accurate. This is not the case in practice since model parameters cannot be estimated accurately. This introduces parametric perturbations in the closed loop system. The simplest case of this perturbation is when the desired trajectory $\theta_d$ is constant. In this case, the wire may go to a perturbed equilibrium. The objective of this section is to determine this perturbed equilibrium as a function of $\theta_d$ and the perturbations in $b$ and $c(\theta)$. The second goal is to show that the perturbed equilibrium is also exponentially stable.

For the following analysis, the system with only one magnet is considered. The analysis holds for the case with two magnets as well. If $\theta_d$ is constant, the open loop error system with one magnet is given by

$$
\ddot{\theta} = -a\dot{\theta} - b\theta - b\theta_d + c(\theta)I^2 \tag{3.9}
$$

The corresponding exact model knowledge controller is given by

$$
I^2 = \frac{1}{\hat{c}(\theta)} \left\{-a\hat{\dot{\theta}} - \beta\hat{\theta} + \hat{b}\theta_d \right\} \tag{3.10}
$$

The parameters $\hat{b}$ and $\hat{c}(\theta)$ used in the controller are the estimated values of $b$ and $c(\theta)$ respectively. To ensure that the controller is realizable, $\hat{c}$ needs to be strictly
positive $\dot{c}(\theta) > 0$. Since the true values of the parameters are not known, the exact model knowledge assumption of Section 3.1 is not valid, i.e., $\hat{b} \neq b$ and $\dot{c}(\theta) \neq c(\theta)$.

The perturbed closed loop error dynamics are given by

$$
\ddot{\theta} = -(a + \alpha r_c(\theta)) \dot{\theta} - (b + \beta r_c(\theta)) \hat{\theta} - \epsilon_b(\theta) \theta_d \tag{3.11}
$$

where

$$
r_c(\theta) = \frac{c(\theta)}{\dot{c}(\theta)} \tag{3.12}
$$

$$
\epsilon_b(\theta) = b - \hat{b} r_c(\theta) \tag{3.13}
$$

The state vector is $x = \begin{bmatrix} \dot{\theta} & \ddot{\theta} \end{bmatrix}^T$. Solving for the equilibrium points of the perturbed system gives

$$
0 = -(b + \beta r_c(\theta_e)) \dot{\theta} - \epsilon_b(\theta_e) \theta_d \tag{3.14}
$$

The solution of this equation gives the perturbed steady state solution $\tilde{\theta}_e$ as

$$
\tilde{\theta}_e = -\frac{(b - \hat{b} r_c(\theta_e))}{(b + \beta r_c(\theta_e))} \theta_d \tag{3.15}
$$

Let $\theta_e$ be the position that the wire settles at when the set point is $\theta_d$. Then by the definition of $\tilde{\theta}$

$$
\tilde{\theta}_e = \theta_e - \theta_d \tag{3.16}
$$

Substituting the value of $\tilde{\theta}_e$ gives

$$
\theta_e = \left[1 - \frac{(b - \hat{b} r_c(\theta_e))}{(b + \beta r_c(\theta_e))}\right] \theta_d \tag{3.17}
$$
Thus $\theta_e$ is the solution of the following equation

$$\theta_e - g(r_c(\theta_e))\theta_d = 0 \quad (3.18)$$

where

$$g(r_c(\theta_e)) = \left[ 1 - \frac{(b - \hat{b}r_c(\theta_e))}{(b + \beta r_c(\theta_e))} \right] \quad (3.19)$$

If $g(r_c(\theta))$ is sufficiently close to 1, Equation 3.18 has at least one solution in $[\theta_{\text{min}}, \theta_{\text{max}}]$ for a given $\theta_d$. Additionally, if $r_c(\theta)$ is smooth enough, Equation 3.18 has a unique solution in $[\theta_{\text{min}}, \theta_{\text{max}}]$. Assuming that Equation 3.18 has a unique solution, the origin is shifted to the new equilibrium by defining $\hat{\theta}$ given by

$$\hat{\theta} = \theta - \theta_e \quad (3.20)$$

Then

$$\dot{\hat{\theta}} = \dot{\theta} \quad (3.21)$$

and

$$\ddot{\hat{\theta}} = \ddot{\theta} \quad (3.22)$$

Thus the closed loop error system changes to the following

$$\ddot{\hat{\theta}} = -(a + \alpha)\dot{\hat{\theta}} - (b + \beta)\hat{\theta}$$

**Nominal System**

$$-(\alpha(r_c(\theta) - 1))\dot{\hat{\theta}} - (\beta(r_c(\theta) - 1))\hat{\theta} + \frac{(b + \beta r_c(\theta))}{(b + \beta r_c(\theta_e))} \epsilon_b(\theta_e)\theta_d - \epsilon_b(\theta)\theta_d$$

**Vanishing Perturbation**

The nominal system is the same as the closed loop system of Equation 3.8 and it is exponentially stable as demonstrated in Section 3.1. Therefore, the system matrix of
the nominal system has two negative eigenvalues. The system can be written in state space form as follows

\[
\dot{\hat{x}} = \begin{bmatrix}
0 & 1 \\
-(b + \beta) & -(a + \alpha)
\end{bmatrix} \hat{x} + \begin{bmatrix}
0 & 0 \\
-\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1)
\end{bmatrix} \hat{x} \\
\text{Nominal System}
\]

\[
+ \begin{bmatrix}
0 \\
(b + \beta r_c(\theta)) (b + \beta r_c(\theta)) \epsilon_b(\theta_e) \hat{\theta}_d - \epsilon_b(\theta) \hat{\theta}_d
\end{bmatrix} \\
\text{Vanishing Perturbation}
\]

\[
\hat{z} = \begin{bmatrix}
\hat{\theta} \\
\hat{\theta}_d
\end{bmatrix}^T
\]

(3.24)

where \( \hat{x} = \begin{bmatrix}
\hat{\theta} \\
\hat{\theta}_d
\end{bmatrix} \). The perturbation terms are called vanishing perturbations because they go to 0 as \( \hat{x} \to 0 \). The nominal system is given by

\[
\dot{\hat{x}} = A \hat{x}
\]

(3.25)

where

\[
A = \begin{bmatrix}
0 & 1 \\
-(b + \beta) & -(a + \alpha)
\end{bmatrix}
\]

(3.26)

The nominal system can be diagonalized using eigenvalues and eigenvectors with the change of basis

\[
A = C \Lambda C^{-1}
\]

and \( \hat{z} = C^{-1} \hat{x} \)

(3.27)

The eigenvalues of \( A \) are strictly negative. Therefore, \( \Lambda \) is a diagonal matrix with
negative terms. The diagonalized system is given by

\[
\dot{\hat{z}} = \Lambda \hat{z} + C^{-1} \left[ \begin{array}{cc}
0 & 0 \\
-(\beta(r_c(\theta) - 1)) & -(\alpha(r_c(\theta) - 1))
\end{array} \right] C \hat{z} \\
+ C^{-1} \left[ \begin{array}{c}
0 \\
\frac{(b+\beta r_c(\theta))}{(b+\beta r_c(\theta_e))} \epsilon_b(\theta_e) \theta_d - \epsilon_b(\theta) \theta_d
\end{array} \right]
\] (3.28)

When \( \hat{\theta} = 0, \theta = \theta_e \) and \( \hat{x} = \hat{z} = 0 \). Substituting \( \hat{z} = 0 \), or equivalently \( \theta = \theta_e \), in the second perturbation term gives

\[
C^{-1} \left[ \begin{array}{c}
0 \\
\frac{(b+\beta r_c(\theta_e))}{(b+\beta r_c(\theta_e))} \epsilon_b(\theta_e) \theta_d - \epsilon_b(\theta_e) \theta_d
\end{array} \right] = C^{-1} \left[ \begin{array}{c}
0 \\
\epsilon_b(\theta_e) \theta_d - \epsilon_b(\theta_e) \theta_d
\end{array} \right] = 0
\] (3.29)

Therefore, the perturbation is bounded and vanishes with \( \hat{x} \) and also with \( \hat{z} \). Let \( \gamma \) be the bound on the vanishing perturbation

\[
\left| \left| C^{-1} \left[ \begin{array}{cc}
0 & 0 \\
-\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1)
\end{array} \right] C \hat{z} + C^{-1} \left[ \begin{array}{c}
0 \\
\frac{(b+\beta r_c(\theta_e))}{(b+\beta r_c(\theta_e))} \epsilon_b(\theta_e) \theta_d - \epsilon_b(\theta) \theta_d
\end{array} \right] \right| \right|
\leq \gamma \left| \left| \hat{z} \right| \right|
\] (3.30)

Define the Lyapunov candidate function as

\[
V(\hat{z}) = \hat{z}^T \hat{z}
\] (3.31)

Let \( -\lambda_1 \) and \( -\lambda_2 \) be the eigenvalues of \( \Lambda \), with \( \lambda_1 \leq \lambda_2 \). Then the Lyapunov function
of Equation 3.31 satisfies the following three properties

\[ \|\hat{\delta}\|^2 \leq V(\hat{\delta}) \leq \|\hat{\delta}\|^2 \]
\[ \dot{V}(\hat{\delta}) = 2\hat{\delta}^T \Lambda \hat{\delta} \leq -2\lambda_1 \|\hat{\delta}\|^2 \]  
(3.32)
\[ \|\partial V/\partial \hat{\delta}\| = \left(4\hat{\delta}^T \Lambda \hat{\delta}\right)^{1/2} \leq 2\|\hat{\delta}\| \]

Therefore, \( \dot{V} \) is negative definite for the nominal system. A well known theorem on vanishing perturbations can be used to prove the exponential stability of the perturbed system [Khalil 1996]. The theorem is stated below.

**Theorem 3.1.** The perturbed system is given by

\[ \dot{x} = f(x, t) + g(x, t) \]  
(3.33)

Consider the nominal system,

\[ \dot{x} = f(x, t) \]  
(3.34)

Suppose \( x = 0 \) is an exponentially stable equilibrium point of the nominal system. Let \( V(x, t) \) be a Lyapunov function that satisfies

\[ c_1 \|x\|^2 \leq V(x, t) \leq c_2 \|x\|^2 \]
\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x, t) \leq -c_3 \|x\|^2 \]  
(3.35)
\[ \left\| \frac{\partial V}{\partial x} \right\| \leq c_4 \|x\| \]

for all \((t, x) \in [0, \infty) \times D\) for some positive constants \(c_1, c_2, c_3\) and \(c_4\). Suppose the perturbation term \(g(x, t)\) satisfies the linear growth bound \(\forall t \geq 0, \forall x \in D\)

\[ \|g(x, t)\| \leq \gamma \|x\| \]  
(3.36)
where \( \gamma \) is some nonnegative constant. If \( c_3, c_4 \) and \( \gamma \) satisfy the following condition

\[
\gamma \leq \frac{c_3}{c_4}
\]

(3.37)

then the origin of the perturbed system is exponentially stable.

The equilibrium point of the nominal system \( \dot{z} = 0 \) is exponentially stable. The perturbation bound \( \gamma \) defined in Equation 3.30 satisfies Equation 3.36 and the Lyapunov function of Equation 3.31 satisfies the three conditions of Equation 3.35. Therefore, by Theorem 3.1, the perturbed system is stable if the following condition is satisfied

\[
\gamma < |\lambda_1|
\]

(3.38)

If this condition is satisfied, \( \theta \to \theta_e \), i.e., the fiber goes to a perturbed equilibrium. Thus, if the perturbation term is smaller than the smallest eigenvalue of the nominal system, then the perturbed system is exponentially stable.

This result holds if \( \theta_d \) is a constant. If \( \theta_d \) is not constant, the fiber may not go to an equilibrium. Therefore, a more general result is needed to demonstrate the robustness of the controller.

### 3.3 Non-vanishing perturbations

The result of Section 3.2 holds if \( \theta_d \) is constant. If \( \theta_d \) is not constant, the perturbation terms do not vanish with the tracking error. These perturbations are called non-vanishing perturbations. In this case, the system states go to a ball around the origin of the error system. This ball is the ultimate bound on the state trajectory. Khalil [Khalil 1996] presents a way to determine the ultimate bound for systems with non-vanishing perturbations. However, the ultimate bound predicted by this
 theorem is too conservative. To get a more reasonable bound, the nominal system
is first diagonalized. Then the theorem on non-vanishing perturbations is applied to
the diagonalized system. This gives a tighter ultimate bound in terms of a matrix
norm.

Assuming the parameter estimates \( \hat{a}, \hat{b} \) and \( \hat{c} \) are not accurate, the dynamics
of Equation 3.6 changes to the following perturbed dynamics

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-(b+\beta) & -(a+\alpha)
\end{bmatrix} \begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
-\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1)
\end{bmatrix} \begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix}
\]

Nominal System

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 \\
\epsilon_b(\theta)\dot{\theta}_d + \epsilon_a(\theta)\dot{\theta}_d + \epsilon_c(\theta)\ddot{\theta}_d
\end{bmatrix}
\]

Vanishing Perturbation

Non-vanishing Perturbation

\[ (3.39) \]

There is a vanishing perturbation due to the difference between \( c(\theta) \) and \( \hat{c}(\theta) \) and a
nonvanishing perturbation due to parameter errors in the feedforward terms that in-
clude \( \theta_d, \dot{\theta}_d \) and \( \ddot{\theta}_d \). Unlike the vanishing perturbation, the nonvanishing perturbation
does not vanish as system states go to zero.

The nominal system is linear

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-(b+\beta) & -(a+\alpha)
\end{bmatrix} \begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix}
\]

By the stability result of Section 3.1 the nominal system is exponentially stable.

Denoting \( y = \begin{bmatrix} \dot{\theta} & \ddot{\theta} \end{bmatrix}^T \), the nominal system is given by

\[ \dot{y} = Ay \]

(3.41)
where

\[ A = \begin{bmatrix} 0 & 1 \\ -(b + \beta) & -(a + \alpha) \end{bmatrix} \]  \hspace{1cm} (3.42)

The matrix \( A \) can be diagonalized using eigenvalues and eigenvectors using the following transformation

\[ A = C\Lambda C^{-1} \]  \hspace{1cm} (3.43)

and \( z = C^{-1}y \)

where \( \Lambda \) is the matrix of eigenvalues of \( A \). Since the nominal system is exponentially stable, both eigenvalues of \( A \) are negative. The diagonalized form of the perturbed system is

\[
C^{-1}\dot{y} = \Lambda C^{-1}y + C^{-1}\begin{bmatrix} 0 & 0 \\ -\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1) \end{bmatrix}y \\
+ C^{-1}\begin{bmatrix} 0 \\ \epsilon_b(\theta)\theta_d + \epsilon_a(\theta)\dot{\theta}_d + \epsilon_c(\theta)\ddot{\theta}_d \end{bmatrix}
\]  \hspace{1cm} (3.44)

Changing the co-ordinates from \( y \) to \( z \) gives

\[
\dot{z} = \Lambda z + C^{-1}\begin{bmatrix} 0 & 0 \\ -\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1) \end{bmatrix}Cz \\
+ C^{-1}\begin{bmatrix} 0 \\ \epsilon_b(\theta)\theta_d + \epsilon_a(\theta)\dot{\theta}_d + \epsilon_c(\theta)\ddot{\theta}_d \end{bmatrix}
\]  \hspace{1cm} (3.45)

Pick a Lyapunov candidate function for the nominal system as follows

\[ V(z) = z^Tz \]  \hspace{1cm} (3.46)
Let the eigenvalues of $A$ be $-\lambda_1$ and $-\lambda_2$, with $\lambda_1 < \lambda_2$. Then the Lyapunov function satisfies the following equations

\[
||z||^2 \leq V(z) \leq ||z||^2
\]

\[
||\partial V/\partial z|| = (4z^Tz)^{1/2} \leq 2||z||
\]

\[
\dot{V}(z) = 2z^T\Lambda z \leq -2\lambda_1||z||^2
\]  

(3.47)

The two perturbation terms can be bounded as follows

\[
\left\| C^{-1} \begin{bmatrix} 0 & 0 \\ -\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1) \end{bmatrix} \right\| Cz \leq \gamma ||z||
\]

and

\[
\left\| C^{-1} \begin{bmatrix} 0 \\ \epsilon_b(\theta)\dot{\theta}_d + \epsilon_a(\theta)\ddot{\theta}_d + \epsilon_c(\theta)\dddot{\theta}_d \end{bmatrix} \right\| \leq \Delta
\]

(3.48)

where $\Delta$ is some strictly positive constant and $\gamma$ is defined as

\[
\gamma = \sqrt{2} \left| \lambda_1 \begin{bmatrix} 0 & 0 \\ -\beta(r_c(\theta) - 1) & -\alpha(r_c(\theta) - 1) \end{bmatrix} C \right|
\]

(3.49)

The vanishing perturbation imposes an additional condition for the system to be exponentially stable. This condition must be satisfied first before the theorem on nonvanishing perturbations can be applied.

Applying Theorem 3.1 gives the first condition under which system trajectories go to a ball around the origin as follows,

\[
\gamma < |\lambda_1|
\]

\[
0 < |\lambda_1| - \gamma
\]

(3.50)
This means \( \|z\| \) decreases exponentially as long as \( \dot{V} \) is negative. This is true if \( \gamma \) is less than \( |\lambda_1| \), i.e., the vanishing perturbation is smaller than the smallest magnitude eigenvalue of \( A \). The theorem on nonvanishing perturbations can now be applied. The theorem is stated below.

Theorem 3.2. Consider the system defined in Theorem 3.1 in Equation 3.33 and Equation 3.34. Let \( x = 0 \) be an exponentially stable equilibrium point of the nominal system. Let \( V(t, x) \) be a Lyapunov function of the nominal system that satisfies Equation 3.35 in \([0, \infty) \times D\), where \( D = \{x \in \mathbb{R}^n \mid \|x\| < r\} \). Suppose the perturbation term \( g(x, t) \) satisfies

\[
\|g(x, t)\| < \delta < c_3 c_4 \sqrt{c_1 \theta r} \tag{3.51}
\]

for all \( t \geq 0 \), all \( x \in D \), and some positive constant \( \theta < 1 \). Then, for all \( \|x(t_0)\| < \sqrt{c_1/c_2} r \), the solution \( x(t) \) of the perturbed system of Equation 3.33 satisfies \( \forall t_0 \leq t < t_0 + T \),

\[
\|x(t)\| \leq k \exp[-\gamma(t - t_0)]\|x(t_0)\| \tag{3.52}
\]

and \( \forall t \geq t_0 + T \)

\[
\|x(t)\| < b \tag{3.53}
\]

for some finite \( T > 0 \), where,

\[
k = \sqrt{c_2/c_1}, \quad \gamma = \frac{(1 - \theta)c_3}{2c_2}, \quad b = \frac{c_4}{c_3} \sqrt{c_2 \delta/\theta} \tag{3.54}
\]

Suppose the system dynamics are defined for all states within a ball of size \( r \) around the origin.

\[
\|z\| \leq r \tag{3.55}
\]

Applying Theorem 3.2 and Equation 3.50 gives the following condition for the states
to remain bounded

\[ \Delta < (|\lambda_1| - \gamma)r \]  

(3.56)

If this condition is met, then for all initial conditions \( z_0 \) such that \(|z_0| < r\), the system states go to a ball \( \rho(\Delta) \), defined as

\[ \rho(\Delta) = \Delta \frac{|\lambda_1| - \gamma}{\rho(\Delta)} \]  

(3.57)

This equation shows that the size of the ultimate bound is directly proportional to the size of the nonvanishing perturbation. Also, the ultimate bound decreases if the eigenvalues of the nominal system are larger. Finally, the ultimate bound increases if the vanishing perturbation is larger. This means perturbation in \( \hat{c} \) decreases the stability margin of the system.

The stability analysis is performed using \(|z|\). The balls of radius \( r \) and \( \rho(\Delta) \) in \(|z|\) are ellipses in \(|y|\). These ellipses can be obtained by applying the inverse transformation as

\[ z^T z = y^T (C^{-1})^T C^{-1} y = r \]

\[ z^T z = y^T (C^{-1})^T C^{-1} y = \rho(\Delta) \]  

(3.58)

This theoretical result can be summarized visually with an example. The system parameters are selected as follows

\[ a = 180 \quad b = 35000 \]
\[ \Delta = 700 \quad \theta_d = 0.2 \]
\[ \alpha = 100 \quad \beta = 100 \]  

(3.59)

The given value of \( \Delta \) corresponds to 10% error in the estimated value of \( b \) and \( \gamma = 0 \).

Applying Equation 3.57 gives the ultimate bound. Figure 3.1 shows what the ellipses
Figure 3.1: Level sets of Initial Conditions and Ultimate Bound for the parameters of Equation 3.59. The initial conditions are selected so that the ellipse contains all possible initial conditions such that $\theta_0 \in [0, \theta_{\text{max}}]$ and $\dot{\theta}_0 = 0$.

Look like in state space for fixed gains. For a perturbation bounded by $\Delta = 700$, for all initial conditions $y_0$ bounded by the red ellipse ($r$), system states go to the region bounded by the blue ellipse ($\rho$).

The smallest eigenvalue of the nominal system $A$ is a good indicator of the total perturbation the system can tolerate. Also, for a given perturbation size $\Delta$, this

Figure 3.2: Effect of controller gains $\alpha$ and $\beta$ on the Ultimate Bound and the size of the perturbation the system can tolerate. Increasing the gains makes the system more robust to perturbations. Also, the size of the ultimate bound is smaller for larger gains.
eigenvalue can be used to calculate the resulting ultimate bound. The ultimate bound and the tolerable perturbation are both affected by the control gains $\alpha$ and $\beta$. The effect of the gains on the size of the tolerable perturbation and the ultimate bound are plotted in Figure 3.2(a) and Figure 3.2(b) respectively. The vanishing perturbation bound $\gamma$ was assumed to be 0 for convenience. Figure 3.2(a) shows that larger gains make the system robust to larger perturbations. Figure 3.2(b) shows that the size of the ultimate bound decreases for larger gains. These numerical studies verify the result of the stability analysis.

![Figure 3.3: Tracking error response when $\hat{b}$ perturbed by 10%.

![Figure 3.4: Tracking error response when $\hat{a}$ is perturbed.](image)

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Figure 3.5: Tracking error response when \( \hat{c} \) is perturbed by a constant factor.

### 3.4 Simulations

The following simulations illustrate how perturbation in each parameter affect the closed loop system. The perturbed parameters are selected so that the total perturbation in the system corresponds to \( \Delta = 700 \). First, \( \hat{b} \) is perturbed by 10%. If the desired trajectory is constant, as shown in Figure 3.3(a) the wire goes to a perturbed equilibrium resulting in a constant steady state error as expected from the result of Section 3.2. If the desired trajectory is not constant, the steady state error is not constant, as shown in Figure 3.3(b). Next, \( \hat{a} \) is perturbed so that \( \Delta = 700 \). A perturbation in \( \hat{a} \) does not cause a steady state error, as shown in Figure 3.4(a). However, there is some bounded error in the response to the sine wave, as shown in Figure 3.4(b). Finally, \( \hat{c} \) is perturbed. Again, this leads to bounded steady state error in the closed loop system, as shown in Figure 3.4. In each case, the trajectories of the system were ultimately bounded by the predicted bound, as shown in Figure 3.6.

The simulations show that the system states go to the ultimate bound. However, larger perturbations lead to larger steady state errors. The trajectory the system states follow inside the ultimate bound cannot be predicted since the true system parameters are not known. Also, if the perturbation is large enough, the closed loop system may not be stable. This means to get trajectory following control that can be
used in an application, system parameters still need to be estimated accurately. The next chapter addresses this by proposing an adaptive observer to estimate system parameters and states.

Figure 3.6: Tracking error response for the perturbed closed loop system.
Chapter 4

Adaptive Observer

Chapter 3 presented an exact model knowledge controller and analyzed the stability of the closed loop system. The analysis shows that asymptotic stability of the origin cannot be guaranteed in the presence of parametric perturbations. Also, if the perturbations are large, the system may become unstable. The system is especially sensitive to perturbations in \( \hat{c} \). Thus, it is important to get accurate estimates of the system parameters.

![Diagram](image)

Figure 4.1: Basic observer structure that consists of model or observer dynamics and a parameter estimator.

An adaptive observer, like the one shown in Figure 4.1 can estimate system states and system parameters. The observer consists of a model and a parameter estimator. The model generates an estimate of the system states from control inputs.
The parameter estimator adapts model parameters to minimize the error between the measured state and estimate of the measured state.

In Chapter 2, the wire-magnet system is modeled as a linear system with a nonlinear input. Adaptive observers for linear systems have been proposed in the 70s [Kreisselmeier 1977; Narendra and Kudva 1974b,a; Kudva and Narendra 1973; Luders and Narendra 1974]. Adaptive observers for linear systems can be extended to nonlinear systems that are linear in parameters [Bastin and Gevers 1988; Marino 1990; Marino and Tomei 1995; Rajamani and Hedrick 1995; Cho and Rajamani 1997]. If the magnetic torque function can be represented as a linear in parameters function of the states and inputs, an adaptive observer similar to the one in [Bastin and Gevers 1988] can be used to estimate system states and parameters. An indirect adaptive controller can also be constructed using the state and parameter estimates. This adaptive observer-controller overcomes the limitations of the model based controller presented in [Karve et al. 2013] and Chapter 3.

This section presents an adaptive observer that can update the parameter estimates and eliminate the parametric perturbations. The magnetic torque functions $c_1(\theta)$ and $c_2(\theta)$ are approximated using tent functions. This makes the system equations linear in parameters. An adaptive observer proposed for linear systems [Kudva and Narendra 1973] and [Narendra and Annaswamy 1989] is modified for systems that are linear in parameter functions of measured variables and inputs. This gives an adaptive observer similar to the one presented in [Bastin and Gevers 1988].

Section 4.1 presents the linear-in-parameters box spline approximation of the torque functions. Section 4.2 presents the construction of an adaptive observer for magnetic wires based on the one presented by [Kudva and Narendra 1973]. Convergence properties of the adaptive observer are analyzed in Section 4.3.
4.1 B-spline approximation of magnetic torque function

The magnetic torque function is represented as a linear combination of basis function. Tent functions, i.e., uniform linear box splines are used here as a basis. First, the operating range of the wire is divided into \( n-1 \) segments using \( n \) knots, \([t_1, \ldots, t_n]\). A linear tent function \( i_j(\theta) \) located at the \( j^{th} \) knot is defined as

\[
i_j(\theta) = \begin{cases} 
0 & \theta < t_j - w/2 \\
1 - |\theta - t_j| & t_j - w/2 \leq \theta \leq t_j + w/2 \\
0 & t_j + w/2 < \theta
\end{cases} \tag{4.1}
\]

where \( w \) is the width of each spline, defined by

\[
w = 2(t_j - t_{j-1}) \tag{4.2}
\]

The approximation of a torque function using linear and quadratic splines is shown in Figure 4.2. The approximation of \( c_1(\theta) \) constructed using box splines on \( \theta \) is given...
by
\[ c_1(\theta) = \sum_{j=1}^{n} k_j i_j(\theta) \quad (4.3) \]

Define the vector of tent functions as
\[ i(\theta)^T = \begin{bmatrix} i_1(\theta) & i_2(\theta) & \ldots & i_n(\theta) \end{bmatrix} \quad (4.4) \]

And parameter vector
\[ k^T = \begin{bmatrix} k_1 & k_2 & \ldots & k_n \end{bmatrix} \quad (4.5) \]

Thus the approximation can be written in vector form as
\[ c_1(\theta) = k^T i(\theta) \quad (4.6) \]

Similarly, \( c_2(\theta) \) is represented using the parameter vector \( l \) as follows
\[ c_2(\theta) = l^T i(\theta) \quad (4.7) \]

The open loop system with the magnetic torque functions represented by a piecewise linear approximation is given by
\[ \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \odot_{1\times n} \quad \odot_{1\times n} \end{bmatrix} \begin{bmatrix} i(\theta) \end{bmatrix} \begin{bmatrix} I_1^2 \\ l^T \end{bmatrix} (i(\theta))(-I_2^2) \quad (4.8) \]

The precision of the approximation can be improved by using more basis functions. It should be noted that Equation 4.6 is an approximation of the torque function of Equation 2.2. Therefore, even with the best possible set of parameters \( k \), there may
still be a small perturbation in the system. In practice, it was observed that with 6 to 8 equally spaced basis functions, the perturbation was smaller than the resolution of the sensor. Thus, this approximation is sufficiently accurate for control.

4.2 Observer Design

The adaptive observer presented here has the same structure as the observer presented in [Kudva and Narendra 1973]. Before the adaptive observer can be designed, the system of Equation 4.8 has to be transformed to observer canonical form. This is achieved through the similarity transform $C$ defined as follows

$$C = \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$$

The state vector of the system in this form is given by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \theta \\ a\theta + \dot{\theta} \end{bmatrix}$$

The open loop dynamics in observer canonical form are given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a & 1 \\ -b & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \sum_{j=1}^{n} k_j i_j(x_1) \end{bmatrix} I_1^2 + \begin{bmatrix} 0 \\ \sum_{j=1}^{n} l_j i_j(x_1) \end{bmatrix} (-I_2^2)$$
The dynamics are rewritten as the sum of known and unknown parts as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-a_0 & 1 \\
-b_0 & 0
\end{bmatrix} \begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} + \begin{bmatrix}
a_0 - \hat{a} \\
b_0 - \hat{b}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \sum_{j=1}^{n} \begin{bmatrix}
k_ji_j(x_1) \\
l_ji_j(x_1)
\end{bmatrix} \begin{bmatrix}
x_1 \\
-x_2
\end{bmatrix} + \sum_{j=1}^{n} \begin{bmatrix}
k_ji_j(x_1) \\
l_ji_j(x_1)
\end{bmatrix} \begin{bmatrix}
x_1 \\
-x_2
\end{bmatrix} (-I_2^2)
\]

where \(a_0\) and \(b_0\) can be selected freely. The true system parameters \(a, b, k_j\) and \(l_j\) are not known. Denote the estimates of these parameters as \(\hat{a}, \hat{b}, \hat{k}_j\) and \(\hat{l}_j\) respectively.

Parameter estimation errors are defined as follows

\[
\phi^T = \begin{bmatrix}
a - \hat{a} & b - \hat{b}
\end{bmatrix}
\]

\[
\psi_{k_j}^T = \begin{bmatrix}
0 & \hat{k}_j - k_j
\end{bmatrix}
\]

\[
\psi_{l_j}^T = \begin{bmatrix}
0 & \hat{l}_j - l_j
\end{bmatrix}
\]

Observer equations for this system are obtained by rewriting the canonical form dynamics using observer states \(\hat{x}_1\) and \(\hat{x}_2\) and the parameter estimates. Auxiliary inputs \(v_A, v_k, v_l\) (defined below) are also added. This gives observer dynamics given by

\[
\begin{bmatrix}
\dot{\hat{x}}_1 \\
\dot{\hat{x}}_2
\end{bmatrix} = \begin{bmatrix}
-a_0 & 1 \\
-b_0 & 0
\end{bmatrix} \begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} + \begin{bmatrix}
a_0 - \hat{a} \\
b_0 - \hat{b}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \sum_{j=1}^{n} \begin{bmatrix}
k_ji_j(x_1) \\
l_ji_j(x_1)
\end{bmatrix} \begin{bmatrix}
x_1 \\
-x_2
\end{bmatrix} + \sum_{j=1}^{n} \begin{bmatrix}
k_ji_j(x_1) \\
l_ji_j(x_1)
\end{bmatrix} \begin{bmatrix}
x_1 \\
-x_2
\end{bmatrix} \begin{bmatrix}
x_1 \\
-x_2
\end{bmatrix} (-I_2^2) + v_A + \sum_{j=1}^{n} v_k + \sum_{j=1}^{n} v_l
\]

\[(4.15)\]
Define the observer error as $e = \dot{x}_1 - x_1$. The error system for the observer is obtained by subtracting Equation 4.13 from Equation 4.15.

$$
\dot{e} = \begin{bmatrix} -a_0 & 1 \\ -b_0 & 0 \end{bmatrix} e + \phi x_1 + \sum_{j=1}^n \psi_{kj} i_j(x_1) I_1^2 + \sum_{j=1}^n \psi_{lj} i_j(x_1)(-I_2^2) \\
+ v_A + \sum_{j=1}^n v_{kj} + \sum_{j=1}^n v_{lj}
$$

(4.16)

The inputs to the observer $y$ and $i_j(x_1) I_1^2$, $i_j(x_1)(-I_2^2)$ are filtered using matrix transfer functions

$$(w^A)^T = \begin{bmatrix} -\frac{s}{s+d_2} & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} x_1$$

$$(w^{kj})^T = \begin{bmatrix} 0 & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1) I_1^2$$

$$(w^{lj})^T = \begin{bmatrix} 0 & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1)(-I_2^2)$$

(4.17)

The auxiliary inputs $v_A$, $v_{kj}$ and $v_{lj}$ are defined as

$$
v_A^T = \begin{bmatrix} \dot{\hat{a}} & \dot{\hat{b}} \end{bmatrix} \begin{bmatrix} 0 & -\frac{d_2}{s+d_2} & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} x_1 = \begin{bmatrix} 0 & \frac{\hat{a}}{s+d_2} x_1 - \hat{b} \frac{1}{s+d_2} x_1 \end{bmatrix}
$$

$$
v_{kj}^T = \begin{bmatrix} \dot{\hat{k}}_j \end{bmatrix} \begin{bmatrix} 0 & -\frac{d_2}{s+d_2} & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1) I_1^2 = \begin{bmatrix} 0 & \frac{\hat{k}_j}{s+d_2} i_j(x_1) I_1^2 \end{bmatrix}
$$

(4.18)

$$
v_{lj}^T = \begin{bmatrix} \dot{\hat{l}}_j \end{bmatrix} \begin{bmatrix} 0 & -\frac{d_2}{s+d_2} & \frac{1}{s+d_2} \\ 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1)(-I_2^2) = \begin{bmatrix} 0 & \frac{\hat{l}_j}{s+d_2} i_j(x_1)(-I_2^2) \end{bmatrix}
$$
Finally, the parameter estimates are updated using the equations

\[
\dot{\phi}_k = \begin{bmatrix} -\dot{\hat{a}} & -\dot{\hat{b}} \end{bmatrix} = -\Gamma e_1 \begin{bmatrix} \frac{1}{s+d_2} & \frac{1}{s+d_2} \end{bmatrix} x_1
\]

\[
\dot{\psi}_{k,j} = \begin{bmatrix} 0 & \dot{\hat{k}}_j \end{bmatrix} = -\gamma e_1 \begin{bmatrix} 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1) I_1^2
\]

\[
\dot{\psi}_{l,j} = \begin{bmatrix} 0 & \dot{\hat{l}}_j \end{bmatrix} = -\eta e_1 \begin{bmatrix} 0 & \frac{1}{s+d_2} \end{bmatrix} i_j(x_1)(-I_2^2)
\]

(4.19)

where \(\gamma\) and \(\eta\) are constant gains and \(\Gamma\) is a \(2 \times 2\) diagonal positive definite gain matrix. Larger gains lead to faster observer convergence. The parameter \(d_2\) is a constant tuning parameter that is selected as a trade off between speed of parameter convergence and speed of observer error convergence. If \(e_1\) is very small, the parameter estimates do not vary quickly. If \(e_1\) is large, parameters vary more quickly. It was observed during simulations that large \(d_2\) leads to slower observer convergence.

The adaptive observer consists of Equations 4.15, 4.18 and 4.19. The following section examines the convergence of the observer states to the system states. The conditions under which the parameter estimates converge to their true values are also examined.

## 4.3 Proof of convergence of observer

The stability analysis presented here follows the method presented in [Kudva and Narendra 1973] and [Narendra and Annaswamy 1989]. The convergence of the observer is demonstrated in two steps. First, the error system of Equation 4.16 is shown to be equivalent to the system of Equation 4.21 defined below. Then the system of Equation 4.21 is shown to be asymptotically stable. Thus, the stability of the error system 4.21 implies the stability of the observer error dynamics. The
observer error dynamics are given by

$$
\dot{e} = Ke + \phi x_1 + \sum_{j=1}^{n} \psi_{k_j} i_j(x_1) I_1^2 + \sum_{j=1}^{n} \psi_{l_j} i_j(x_1)(-I_2^2) + v_A + \sum_{j=1}^{n} v_{k_j} + \sum_{j=1}^{n} v_{l_j}
$$

$$
e_1 = h^T e
$$

where \( h^T = \begin{bmatrix} 1 & 0 \end{bmatrix} \).

**Claim 4.1.** Define the vector \( d^T = \begin{bmatrix} 1 & d_2 \end{bmatrix} \). Define the dynamics of \( \epsilon \) as

$$
\dot{\epsilon} = K\epsilon + d\phi^T w^A + \sum_{j=1}^{n} d\psi_{k_j}^T w^{k_j} + \sum_{j=1}^{n} d\psi_{l_j}^T w^{l_j}
$$

$$
\epsilon_1 = h^T \epsilon
$$

If the auxiliary inputs \( v_A \) and \( v_{k_j}, v_{l_j} \) are selected as per Equation 4.18, then, \(|e - \epsilon| \to 0 \) as \( t \to \infty \).

**Proof.** The proof assumes \( I_2 = 0 \) for brevity. The proof for the case with nonzero \( I_2 \) follows the same method. The goal is to show that the input-output response of the systems of Equation 4.20 and Equation 4.21 is the same.

The input-output response of the two systems is compared by subtracting the transfer function of the system of Equation 4.21 from the transfer function of the system of Equation 4.20. If the difference between the two transfer functions is 0, then \(|e - \epsilon| \to 0 \) as \( t \to \infty \). The \((h, K)\) transfer function is given by

$$
h^T (sI - K)^{-1} = \frac{1}{k(s)} \begin{bmatrix} s & 1 \end{bmatrix}
$$

where \( k(s) = s^2 + a_0s + b_0 \) and \( s \) is the time derivative operator. The difference
between the input-output response of the two systems is

\begin{equation}
\begin{aligned}
 h^T(sI - K)^{-1} \left[ \phi x_1 + v_A - d\phi^T w^A + \sum_{j=1}^{n} \psi_{kj} i_j(x_1) I_1^2 + \sum_{j=1}^{n} v_{kj} - \sum_{j=1}^{n} d\psi_{kj}^T w^{kj} \right] \\
= \frac{1}{k(s)} \left[ \begin{array}{c}
 \phi x_1 + v_A - d_1 \phi^T w^A \\
 \phi_2 x_1 + v_{A_2} - d_2 \phi^T w^A \\
 \end{array} \right] \\
+ \frac{1}{k(s)} \left[ \begin{array}{c}
 \sum_{j=1}^{n} \psi_{kj_1} i_j(x_1) I_1^2 + \sum_{j=1}^{n} v_{kj_1} - \sum_{j=1}^{n} d_1 \psi_{kj_1}^T w^{kj} \\
 \sum_{j=1}^{n} \psi_{kj_2} i_j(\theta) I_1^2 + \sum_{j=1}^{n} v_{kj_2} - \sum_{j=1}^{n} d_2 \psi_{kj_2}^T w^{kj} \\
 \end{array} \right]
\end{aligned}
\end{equation}

Multiplying the vectors gives

\begin{equation}
\begin{aligned}
 h^T(sI - K)^{-1} \left[ \phi x_1 + v_A - d\phi^T w^A + \sum_{j=1}^{n} \psi_{kj} i_j(x_1) I_1^2 + \sum_{j=1}^{n} v_{kj} - \sum_{j=1}^{n} d\psi_{kj}^T w^{kj} \right] \\
= \frac{1}{k(s)} \left\{ s[\phi_1 x_1 + v_1 - d_1 \phi^T w^A] + \phi_2 x_1 + v_2 - d_2 \phi^T w^A \right\} \\
+ \frac{1}{k(s)} \sum_{j=1}^{n} \left\{ s \left[ \psi_{kj_1} i_j(\theta) I_1^2 + v_{kj_1} - d_1 \psi_{kj_1}^T w^{kj} \right] + \psi_{kj_2} i_j(\theta) I_1^2 + v_{kj_2} - d_2 \psi_{kj_2}^T w^{kj} \right\}
\end{aligned}
\end{equation}

The right hand side of the above equation is 0. This is demonstrated in Appendix A. Thus, the input to output transfer functions of the systems of Equation 4.20 and Equation 4.21 is the same. Therefore, \(|e - \epsilon| \to 0\) as \(t \to \infty\). Since the output of the system of Equation 4.21 is the same as the output of the observer error system, the remaining stability analysis uses Equation 4.21 as observer error dynamics. \(\square\)

The following well known lemmas are required for the second part of the proof.

**Lemma 4.1. Lefschetz-Kalman-Yakubovich (LKY) Lemma [Narendra and Annaswamy 1989]:** If the \((h, K, d)\) transfer function is strictly positive real, then
there exist positive definite matrices $P$ and $Q$, such that

$$K^TP + PK = -Q$$

$$Pd = h$$ (4.25)

\[\square\]

**Theorem 4.1. Application of Barbalat’s Lemma [Khalid 1996]:** Let $D \subset \mathbb{R}^n$ be a domain containing $x = 0$ and suppose $f(t,x)$ is piecewise continuous in $t$ and locally Lipschütz in $x$, uniformly in $t$, on $[0,\infty) \times D$. Furthermore, suppose $f(t,0)$ is uniformly bounded for all $t \geq 0$. Let $V : [0,\infty) \times D \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$W_1(x) \leq V(t,x) \leq W_2(x)$$

$$\dot{V}(t,x) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) \leq -W(x)$$ (4.26)

$\forall t \geq 0, \forall x \in D$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions and $W(x)$ is a continuous positive semidefinite function on $D$. Choose $r > 0$ such that $B_r \subset D$ and let $\rho < \min_{\|x\|=r} W_1(x)$. Then, all solutions of $\dot{x} = f(t,x)$ with $x(t_0) \in \{x \in B_r | W_2(x) \leq \rho\}$ are bounded and satisfy

$$W(x(t)) \to 0 \text{ as } t \to \infty$$ (4.27)

Moreover, if all the assumptions hold globally and $W_1(x)$ is radially unbounded, the statement is true for all $x(t_0) \in \mathbb{R}^n$.

\[\square\]

**Claim 4.2.** Suppose the roots of the $(h,K,d)$ transfer function are strictly positive real. Then the system of Equation 4.21 is asymptotically stable if the parameter
estimates are updated by

\[ \dot{\phi} = -\Gamma \epsilon_1 w^A \]
\[ \dot{\psi}_{k_j} = -\gamma \epsilon_1 w^{k_j} \]
\[ \dot{\psi}_{l_j} = -\eta \epsilon_1 w^{l_j} \] (4.28)

Proof. The \((h, K, d)\) transfer function must be positive real. Writing out the transfer function explicitly gives

\[ h^T(sI - K)^{-1}d = \frac{s + d_2}{s^2 + a_0 s + b_0} \] (4.29)

where \(d_2, a_0\) and \(b_0\) can be selected freely to ensure that the transfer function \(h^T(sI - K)^{-1}d\) is strictly positive real.

Define the Lyapunov candidate function as follows

\[ V(\epsilon, \phi, \psi_{k_j}, \psi_{l_j}) = \epsilon^T P \epsilon + \phi^T \Gamma^{-1} \phi + \frac{1}{\gamma} \sum_{j=1}^{n} \psi_{k_j}^T \psi_{k_j} + \frac{1}{\eta} \sum_{j=1}^{n} \psi_{l_j}^T \psi_{l_j} \] (4.30)

where \(P\) is a positive definite symmetric matrix to be defined later. Time derivative of \(V\) is given by

\[ \dot{V} = 2\epsilon^T P \dot{\epsilon} + 2\phi^T \Gamma^{-1} \dot{\phi} + \frac{2}{\gamma} \sum_{j=1}^{n} \psi_{k_j}^T \dot{\psi}_{k_j} + \frac{2}{\eta} \sum_{j=1}^{n} \psi_{l_j}^T \dot{\psi}_{l_j} \] (4.31)

Substituting the observer error dynamics and the estimate update equations

\[ \dot{V} = \epsilon^T (K^T P + PK) \epsilon + 2(\epsilon^T P d \phi^T w^A - \phi^T \epsilon_1 w^A) \]
\[ + 2 \sum_{j=1}^{n} (\epsilon^T P d \psi_{k_j}^T w^{k_j} - \psi_{k_j}^T \epsilon_1 w^{k_j}) \]
\[ + 2 \sum_{j=1}^{n} (\epsilon^T P d \psi_{l_j}^T w^{l_j} - \psi_{l_j}^T \epsilon_1 w^{l_j}) \] (4.32)
Since the \((h, K, d)\) transfer function is strictly positive real, by the LKY Lemma 4.1, there exist positive definite matrices \(P\) and \(Q\) such that

\[
K^T P + PK = -Q
\]

\[
Pd = h
\]

Substituting in \(\dot{V}\) gives

\[
\dot{V} = -\epsilon^T Q \epsilon + 2(\epsilon^T h \phi^T w^A - \phi^T \epsilon_1 w^A)
\]

\[
+ 2 \sum_{j=1}^{n} (\epsilon^T h \psi_{kj}^T w^{kj} - \psi_{kj}^T \epsilon_1 w^{kj})
\]

\[
+ 2 \sum_{j=1}^{n} (\epsilon^T h \psi_{lj}^T w^{lj} - \psi_{lj}^T \epsilon_1 w^{lj})
\]

By Equation 4.21, \(\epsilon^T h = h^T \epsilon = \epsilon_1\), which is a scalar. Therefore

\[
\dot{V} = -\epsilon^T Q \epsilon \leq 0
\]

\(\dot{V}\) is negative semidefinite. This means \(V\) satisfies the conditions for Theorem 4.1. Therefore, by Theorem 4.1, \(\epsilon^T Q \epsilon \to 0\) as \(t \to \infty\). Since \(Q\) is positive definite, \(\epsilon^T Q \epsilon \to 0\) means \(\epsilon \to 0\) as \(t \to \infty\). Also, if \(\epsilon = 0\), then the observer error system reaches steady state. This means \(e \to 0\) as \(t \to \infty\) regardless of parameter convergence. That is, the estimated state approaches the true state asymptotically.

\[\Box\]

### 4.3.1 Condition for Persistent Excitation

The stability analysis above only proves that \(e \to 0\) as \(t \to \infty\). Parameter estimation errors also go to zero asymptotically if the inputs to the system \(I_1\) and \(I_2\) are sufficiently exciting. If the desired trajectory is not sufficiently exciting, parameter
estimates may go to steady state values that are different from true parameter values.

For an observer for a linear system, a sufficiently exciting input for estimating 2n parameters of a linear system consists of n different frequencies. This condition cannot be applied for estimation of the parameters of a nonlinear system directly. It is necessary to derive conditions on the inputs I₁ and I₂ that guarantee sufficient excitation. Substituting ε = 0 in Equation 4.21 gives

\[ \phi^T w^A + \sum_{j=1}^{n} \psi_{kj}^T w^{kj} + \sum_{j=1}^{n} \psi_{lj}^T w^{lj} = 0 \] (4.36)

Thus the parameter estimation errors are the solution to the linear equation in the filtered signals \( w^A, w^{kj} \) and \( w^{lj} \). If the filtered signals are all linearly independent, the only solution to this equation is given by

\[ \phi = \psi_{kj} = \psi_{lj} = 0 \quad \forall j \in [1, n] \] (4.37)

The signals \( w^{kj} \) are all linearly independent of each other. Similarly, \( w^{lj} \) are all linearly independent of each other. Thus, the four terms of Equation 4.36 can be treated as a linear combination of four signals and four parameters.

If the open loop system is stable, a control input with 2 frequencies may be sufficient to estimate system parameters. However, the open loop wire-magnet system is not stable over the entire range of wire displacements. Thus the system parameters cannot be estimated directly from open loop response to a control input. The observer generates estimates of system parameters \( \hat{a}, \hat{b}, \hat{k} \) and \( \hat{l} \) and system states \( \hat{x} \). An indirect controller can be implemented using these parameter estimates and state estimates. However, if a feedback controller is implemented using the coil currents, the coil currents cannot be selected freely. Thus, the sufficient excitation
condition cannot be guaranteed for the coil currents. The following chapter proposes an indirect adaptive controller using the parameter and state estimates generated by the observer. The persistent excitation condition is updated for the closed loop system at the end of the stability analysis.
Chapter 5

Controller Design

The exact model knowledge controller presented in Chapter 3 is susceptible to parametric perturbations caused by inaccurate parameter estimates. The adaptive observer designed in Chapter 4 can estimate system parameters if the input is sufficiently exciting. However, the adaptive observer cannot be used directly since the open loop system is not stable. An indirect controller is presented here based on the adaptive observer. Also, the persistent excitation condition for the adaptive observer is updated for the closed loop system at the end of the analysis.

5.1 Indirect Adaptive Control

The closed loop system with the indirect controller is represented in Figure 5.1. The observer generates system state estimates and parameter estimates. Thus the adaptive observer is used to design a tracking controller that uses just the angular position measurement. This is achieved by driving the observer states \( \hat{x}_1 \) and \( \hat{x}_2 \) to a desired trajectory \( x_{1d} \) and \( x_{2d} \). Since \( \hat{x}_1 \to x_1 \) and \( \hat{x}_1 \to x_{1d} \), it follows that \( x_1 \to x_{1d} \).
Figure 5.1: Closed loop system with the indirect controller. The observer drives the state estimate \( \hat{x}_1 \) to \( x_1 \) and the controller ensures that the state estimate \( \hat{x}_1 \) tracks \( x_{1d} \).

The observer dynamics are given by

\[
\begin{bmatrix}
\dot{\hat{x}}_1 \\
\dot{\hat{x}}_2
\end{bmatrix} = \begin{bmatrix}
-a_0 & 1 \\
-b_0 & 0
\end{bmatrix} \begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} + \begin{bmatrix}
a_0 - \hat{a} \\
b_0 - \hat{b}
\end{bmatrix} x_1 + \begin{bmatrix} 0 \\
\sum_{j=1}^{n} \hat{k}_{ij}(x_1)
\end{bmatrix} I_1^2 + \begin{bmatrix} 0 \\
\sum_{j=1}^{n} \hat{l}_{ij}(x_1)
\end{bmatrix} I_2^2 \\
+ v_A + \sum_{j=1}^{n} v_{kj} + \sum_{j=1}^{n} v_{lj}
\]

\[
\hat{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} \tag{5.1}
\]

Define the tracking error state as

\[
\tilde{x}^T = \begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2
\end{bmatrix} = \begin{bmatrix}
\hat{x}_1 - x_{1d} \\
\hat{x}_2 - x_{2d}
\end{bmatrix} \tag{5.2}
\]

Open loop error dynamics are given by

\[
\begin{bmatrix}
\dot{\tilde{x}}_1 \\
\dot{\tilde{x}}_2
\end{bmatrix} = \begin{bmatrix}
-a_0 & 1 \\
-b_0 & 0
\end{bmatrix} \begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2
\end{bmatrix} + \begin{bmatrix}
-x_{1d} - a_0 x_{1d} + x_{2d} \\
-x_{2d} - b_0 x_{1d}
\end{bmatrix} + \begin{bmatrix}
0 \\
\sum_{j=1}^{n} \hat{k}_{ij}(x_1)
\end{bmatrix} I_1^2 + \begin{bmatrix} 0 \\
\sum_{j=1}^{n} \hat{l}_{ij}(x_1)
\end{bmatrix} I_2^2 + v_A + \sum_{j=1}^{n} v_{kj} + \sum_{j=1}^{n} v_{lj} \tag{5.3}
\]
The controller is designed with the same structure as the EMK controller of Chapter 3. First, the coil currents $I_1$ and $I_2$ are selected to give a continuous control law $u$. Then $u$ is designed with feedforward terms that cancel the desired trajectory terms and the auxiliary inputs. Feedback terms are added so that the closed loop eigenvalues can be selected freely. The undesired terms in the $\tilde{x}_1$ dynamics are canceled by using $x_{2d}$ as a backstepping variable.

The coil currents are selected as per Equation 3.2. The equations are restated here:

$$ I_1 = \begin{cases} \sqrt{\sum_{j=1}^{n} k_{jj}(x_1)} & u \geq 0 \\ 0 & u < 0 \end{cases} \quad (5.4) $$

$$ I_2 = \begin{cases} \sqrt{-u} & u \geq 0 \\ 0 & u < 0 \end{cases} \quad u < 0, $$

This combines the two control input terms. The dynamics of the system with the switched currents are given by

$$ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_0 & 1 \\ -b_0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} -\dot{x}_{1d} - a_0 x_{1d} + x_{2d} \\ -\dot{x}_{2d} - b_0 x_{1d} \end{bmatrix} + \begin{bmatrix} a_0 - \hat{a} \\ b_0 - \hat{b} \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ u \end{bmatrix} + v_A + \sum_{j=1}^{n} v_{k_j} + \sum_{j=1}^{n} v_{l_j} \quad (5.5) $$

The control law is selected as follows:

$$ u = \underbrace{-\beta \tilde{x}_1 - \alpha \tilde{x}_2 + \dot{x}_{2d} + b_0 x_{1d} - (b_0 - \hat{b}) \tilde{x}_1 - v_{A_1}}_{\text{Feedback}} - \underbrace{\sum_{j=1}^{n} v_{k_j} - \sum_{j=1}^{n} v_{l_j}}_{\text{Feedforward}} \quad (5.6) $$

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The feedback terms in Equation 5.6 are added so that the eigenvalues of the closed loop system can be selected as desired. The feedforward terms are added to cancel unwanted dynamics. The variable $x_{2d}$ is used as a backstepping variable to help achieve $\hat{x} \rightarrow x_{1d}$

$$x_{2d} = \dot{x}_{1d} + a_0 x_{1d} + (\hat{a} - a_0) \hat{x}_1$$  \hfill (5.7)

Finally, the first derivative of $x_{2d}$ is implemented by substituting the $\dot{x}_1$ dynamics as follows

$$\dot{x}_{2d} = \ddot{x}_{1d} + a_0 \dot{x}_{1d} + \dot{\hat{a}} \dot{x}_1 + (\hat{a} - a_0) [ -a_0 \dot{x}_1 + \dot{x}_2 + (a_0 - \dot{a}) x_1 ]$$  \hfill (5.8)

Substituting the control law in the error dynamics Equation 5.3 gives

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -a_0 & 1 \\ -(b_0 + \beta) & -\alpha \end{bmatrix} & A_c \\ \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} a_0 - \dot{a} \\ b_0 - \dot{b} \end{bmatrix} \\ b_c \end{bmatrix} (x_1 - \hat{x}_1)$$  \hfill (5.9)

The nominal linear part $A_c$ is stable and its eigenvalues can be selected using $\alpha$ and $\beta$. If the eigenvalues of $A_c$ are negative, then there exist a pair of positive semidefinite matrices $P_c$ and $Q_c$ such that the following is true

$$A_c^T P_c + P_c A_c = -Q_c$$  \hfill (5.10)

The $b_c(x_1 - \hat{x}_1)$ term is a vanishing perturbation when the full state $\begin{bmatrix} \hat{x}^T & e^T \end{bmatrix}^T$ is considered. The perturbation vanishes when the observer error $e_1$ goes to 0. Therefore, observer error dynamics must be included in the stability analysis of the controller. The stability of the combined closed loop system is analyzed next.
5.2 Stability Analysis of the Combined Observer-Controller System

Using the equivalent dynamics for the observer, the combined dynamics of the closed loop system are as follows

\[
\begin{bmatrix}
\dot{\tilde{x}} \\
\dot{e}
\end{bmatrix} =
\begin{bmatrix}
A_c & \otimes_{2 \times 2} \\
\otimes_{2 \times 2} & K
\end{bmatrix}
\begin{bmatrix}
\tilde{x} \\
e
\end{bmatrix}
+ \begin{bmatrix}
b_c e_1 \\
\otimes_{2 \times 1}
\end{bmatrix}
+ \begin{bmatrix}
\otimes_{2 \times 1} \\
d\phi^T w^A + \sum_{j=1}^{n} d\psi_{kj}^T w_{kj} + n \sum_{j=1}^{n} d\psi_{lj}^T w_{lj}
\end{bmatrix}
\]

\[e_1 = h^T e\]  

(5.11)

Define the combined Lyapunov function as

\[V = \frac{1}{2} \tilde{x}^T P_c \tilde{x} + \frac{1}{2} e^T P_o e + \frac{1}{4} \phi^T \Gamma^{-1} \phi + \frac{1}{2} \gamma \sum_{j=1}^{n} \psi_{kj}^T \psi_{kj} + \frac{1}{2} \eta \sum_{j=1}^{n} \psi_{lj}^T \psi_{lj}\]

(5.12)

where \(P_c\) and \(P_o\) are positive definite \(2 \times 2\) matrices to be defined later. The time derivative of \(V\) along the trajectories of Equation 5.9 is given by

\[\dot{V} = \dot{x}^T P_c A_c \dot{x} + e^T P_o K e + \dot{x}^T P_o b_c e_1 \]

\[+ e^T P_o d\phi^T w^A + e^T P_o \sum_{j=1}^{n} d\psi_{kj}^T w_{kj} + e^T P_o \sum_{j=1}^{n} d\psi_{lj}^T w_{lj}\]

(5.13)
Grouping the parameter estimation error terms, $\dot{V}$ reduces to the following scalar

$$
\dot{V} = \ddot{x}^TP_cA_c\ddot{x} + e^TP_oKe + \ddot{x}^TP_c b_c e_1 \\
+ (e^TP_o d\phi^T w^A - \phi^T e_1 w^A) + \sum_{j=1}^{n} (e^TP_o d\psi_{kj}^T w^{kJ} - \psi_{kj}^T e_1 w^{kJ}) \\
+ \sum_{j=1}^{n} (e^TP_o \psi_{kj}^T w^{kJ} - \psi_{kj}^T e_1 w^{kJ})
$$

(5.14)

From the observer stability analysis, $a_0$, $b_0$ and $d_2$ are selected such that $(h, K, d)$ is a positive real transfer function. Therefore, by the Lefschetz-Kalman-Yakubovich Lemma Equation 4.25, there exist positive definite matrices $P_o$ and $Q_o$ such that

$$
K^T P_o + P_o K = -Q_o
$$

(5.15)

$$
P_o d = h
$$

Substituting in $\dot{V}$ gives

$$
\dot{V} = \ddot{x}^TP_cA_c\ddot{x} + e^TP_oKe + \ddot{x}^TP_c b_c e_1 \\
+ (e^T h\phi^T w^A - \phi^T e_1 w^A) + \sum_{j=1}^{n} (e^T h\psi_{kj}^T w^{kJ} - \psi_{kj}^T e_1 w^{kJ}) \\
+ \sum_{j=1}^{n} (e^T h\psi_{kj}^T w^{kJ} - \psi_{kj}^T e_1 w^{kJ})
$$

(5.16)

where $e^T h = \epsilon_1$, which is a scalar. Substituting in $\dot{V}$ gives

$$
\dot{V} = -\ddot{x}^T Q_c \ddot{x} - e^T Q_o e + \ddot{x}^T P_c b_c e_1
$$

(5.17)

where

$$
A_c^T P_c + P_c A_c = -Q_c
$$

(5.18)
Since the observer eventually converges, \( \dot{a} \to 0 \) and \( \dot{b} \to 0 \), i.e., \( b_c \) is bounded. Thus, the quadratic term \( \tilde{x}^T P_x b_c e_1 \) is a bounded vanishing perturbation. Therefore, \( \dot{V} \) can be made negative semidefinite by selecting the initial parameter estimates \( a_0, b_0 \) and feedback gains \( \alpha \) and \( \beta \) appropriately. Thus, by Theorem 4.1, the closed loop error \( \tilde{x} \) and observer error \( e \) go to zero asymptotically.

The observer error and tracking error go to zero asymptotically regardless of parameter estimation errors, i.e., \( \tilde{x} \to 0 \) and \( e \to 0 \) as \( t \to \infty \). Therefore, \( \|\hat{x}_1 - x_{1d}\| \to 0 \) and \( \|x_1 - \hat{x}_1\| \to 0 \) as \( t \to \infty \). The triangle inequality gives

\[
\|x_1 - x_{1d}\| \leq \|x_1 - \hat{x}_1\| + \|\hat{x}_1 - x_{1d}\| \quad (5.19)
\]

Therefore, \( \|x_1 - x_{1d}\| \to 0 \) as \( t \to \infty \). This means the adaptive controller causes system states to track the desired trajectory asymptotically.

5.2.1 Sufficient Excitation Condition for Closed Loop System

Substituting \( \ddot{x} = 0 \) and \( e = 0 \) in Equation 5.11 gives the same linear combination of the filtered signals \( w^A, w^j \) and \( w^k_j \) as Equation 4.36

\[
\phi^T w^A + \sum_{j=1}^{n} \psi^T_{kj} w^{kj} + \sum_{j=1}^{n} \psi^T_{lj} w^{lj} = 0 \quad (5.20)
\]

If the filtered signals are all linearly independent, the parameter estimation errors all go to zero asymptotically. For the adaptive observer, this persistent excitation condition can be guaranteed by ensuring that the input has a sufficient number of frequencies. This is not possible for the closed loop system since the control input is generated by the controller. The only way to ensure persistent excitation in this case is to select the desired trajectory \( x_{1d} \) appropriately.
For all frequencies less than the smallest closed loop eigenvalue that the physical system can achieve, $x_1$ tracks $x_{1d}$. The maximum frequency the system can track is referred to here as the limiting frequency. Thus, $x_1$ contains all the frequencies in $x_{1d}$ that are less than the limiting frequency of the system. The control signals also contain the same frequencies as those in $x_1$ and $x_{1d}$. Thus, any input that contains frequencies that are strictly less than the limiting frequency of the wire is not guaranteed to generate accurate parameter estimates. On the other hand, if $x_{1d}$ contains frequencies greater than the limiting frequency of the system, $x_1$ does not contain these higher frequencies. This ensures that the signals $w^A$, $w^{b_j}$ and $w^{k_j}$ are linearly independent. Thus, for parameter estimation, it is necessary to use a desired trajectory that contains frequencies both greater and smaller than the limiting frequency of the system.

The adaptive observer and controller solve the parameter estimation and control problem. The controller is tested using simulations and experiments in the next chapter. The response of the system is also tested with various desired trajectories to check which trajectories generate accurate parameter estimates.

5.2.2 Desired Trajectories Constraints

The stability analysis assumes that the desired trajectory is one the system can follow. Suppose $\mathcal{U}$ is the set of all realizable control inputs. Suppose the set of all possible $x(t)$ is denoted as $\mathcal{F}$ and defined as $\mathcal{F} = \{x(t)|\dot{x} = f(x, u), \ u \in \mathcal{U}\}$. If $x(t) - x_d(t) = 0$, then $x_d(t) \in \mathcal{F}$. This means there exists some $u(t) \in \mathcal{U}$ that causes $x(t)$ to follow $x_d(t)$. Alternatively, if $x_d(t) \notin \mathcal{F}$, then $x(t) - x_d(t) \neq 0$ for any $u \in \mathcal{U}$. Therefore, possible desired trajectories have to be constrained to the trajectories the closed loop system can follow using some realizable $u(t)$. If this is not
true, asymptotic tracking is not possible.

The constraint on $x_{1d}$ for the electromagnet-wire system can be obtained from the response of the wire. The electromagnet coil currents $I_1$ and $I_2$ are limited to maximum 2A. The speed of response of the wire is limited to the rise time for a 2A step input. This limits the possible desired trajectories to frequencies smaller than this frequency. For example, Figure 5.2 shows the response of the simulated system with the current limit at 2A and 1A respectively. The system cannot track the same frequency sine wave for a lower current limit since the current saturates.

![Figure 5.2: Desired trajectory frequency is limited by coil current limit.](image)

(a) Current limit at 2A
(b) Current limit at 1A
Chapter 6

Controller Validation

The two magnet model of Equation 2.2, the observer of Equations 4.15, 4.18 and 4.19 and the controller of Equations 5.4, 5.6, 5.7 and 5.8 were all implemented in Simulink. The controller was also tested on a physical system consisting of a fiber and two electromagnets. The model and the interface to the physical system were implemented as a switched subsystem; thus the same controller and observer can be tested in simulation and experiments.

6.1 Simulation Results

The frequency-richness of the control input is not guaranteed in the stability analysis of the closed loop system of Section 5.1. However, the response of the parameter estimator to various desired trajectories can be tested in simulation. The trajectories that cause parameter estimates to converge to true values can then be applied to the physical system to estimate system parameters. Also, the effect of varying the observer and controller gains on the speed of convergence of the closed loop system can be analyzed in simulation.
6.1.1 System Parameter Estimation

To get accurate parameter estimates and reliable gains, the model parameters used in the simulation should be as close as possible to system parameters. Thus, system parameters are estimated for use as true values in simulation. System parameters can be estimated as follows.

A 2A current is applied to one of the two magnets. This causes the wire to travel to the magnet, i.e., $\theta = \theta_{\text{max}}$. Then both the coil currents are set to 0A. This results in underdamped oscillations as shown in Figure 6.1. The characteristic equation of the system is given by

$$s^2 + as + b = 0 \quad (6.1)$$

The frequency of the oscillations is a good estimate of the natural frequency ($\omega_n$) of the wire. The stiffness coefficient $b$ is given by

$$b = w_n^2 \quad (6.2)$$

The response in Figure 6.1 shows that the natural frequency of the wire is around 37Hz. Thus $b$ is set as $5.7 \times 10^4$ in the simulations. The damping ratio of the wire
is between 0.25 and 0.5. Thus, $a$ is set as 50. The true magnetic torque function is difficult to estimate since the wire cannot be stabilized directly in the unstable region. It is easier to generate a torque curve for the rigid bar from the geometry using FEMM. The curves for $c_1$ and $c_2$ are represented using look up tables in the simulations.

The perturbed initial parameter estimates $\hat{a}$ and $\hat{b}$ are obtained by multiplying the true values of $a$ and $b$ by a perturbation factor. The initial torque curve estimates $\hat{c}_1$ and $\hat{c}_2$ are set as constant over $\theta$. Each curve has 9 knots, with one knot at the origin and one at each magnet and 6 knots between.

### 6.1.2 Simulations

The adaptive observer and controller were implemented in Simulink. The model and the hardware interface were implemented as switched subsystems. This allows for the same controller to be applied to the model and the physical system without modification. The parameters estimated from the physical system were used as simulation model parameters. The perturbed estimates $\hat{a}$, $\hat{b}$ and $\hat{c}_1$, $\hat{c}_2$ were used to initialize the observer.

Figure 6.2 shows the response of the simulated closed loop system to a step input. The simulated state $x_1$ goes to the set point $x_{1d}$. The step input is not frequency rich, hence, the parameter estimates do not go to their true values. This is illustrated in Figure 6.4 by comparing the true values of $c_1(\theta)$ and $c_2(\theta)$ with their estimates. Figure 6.3 shows the response of the simulated system to a 13Hz sine wave and a 28Hz sine wave. Again, the closed loop system converges quickly, but the parameter estimates do not converge to their true values.

A chirp input sweeps through a range of frequencies and contains an infinite
number of frequencies. Thus a chirp signal that sweeps frequencies between 18Hz and 28Hz was also tested as an input for parameter identification. The observer error and tracking error go to zero asymptotically. Figure 6.5 shows the evolution of $\hat{a}$ and $\hat{b}$ over time. The parameter estimates also go to true values asymptotically. Also, as

Figure 6.3: Tracking behavior for $f = 13$Hz and $f = 28$Hz triangle waves. The closed loop system can track both sine inputs well.
Figure 6.4: Comparison of $c_1(\theta)$ and $c_2(\theta)$ with their estimates $\hat{c}_1(\theta)$ and $\hat{c}_2(\theta)$ at $t = 60s$ with a 13Hz sine input. The estimates do not go to their true values as the sine input only contains one frequency.

seen in Figure 6.6, estimates of $c_1(\theta)$ and $c_2(\theta)$ go to true values asymptotically.

Figure 6.7 shows the response of the closed loop system to a 1Hz triangle wave and a series of small steps. Both of these inputs contain multiple frequencies. Thus

Figure 6.5: Evolution of parameter estimates $\hat{a}$ and $\hat{b}$ in response to the chirp input. The parameter estimates go to the true values of the parameters asymptotically.
both inputs generate more accurate parameter estimates than step or sine inputs. Also, these inputs are more interesting from an application point of view. The controller converges to the desired trajectory with the triangle input. The step input contains very high frequencies at the stepping points. Thus, the response degrades at these points. The controller tracks the inputs well otherwise. The simulation results are summarized as follows. For all desired trajectories containing frequencies less

Figure 6.6: Comparison of $c_1(\theta)$ and $c_2(\theta)$ with their estimates $\hat{c}_1(\theta)$ and $\hat{c}_2(\theta)$ at $t = 60s$ with a chirp input. The estimates are significantly better than the estimates obtained using the sine input.

Figure 6.7: Response of the simulated system to a 1Hz triangle wave and a series of small steps. The controller tracks both inputs well.
Figure 6.8: Comparison of $c_1(\theta)$ and $c_2(\theta)$ with their estimates $\hat{c}_1(\theta)$ and $\hat{c}_2(\theta)$ at $t = 60$ s obtained using a series of small steps. The estimates are fairly accurate.

than or equal to the natural frequency of the wire, the observed state $\hat{x}_1$ converges to the true state $x_1$ and the tracking error $x_1 - x_{1d}$ also goes to zero asymptotically. The observer also converges for desired trajectories containing frequencies greater than the natural frequency of the wire. However, the tracking error does not go to zero for high frequencies.

It was observed that parameter estimates go to their true values if the desired trajectory contains frequencies both lower and greater than the limiting frequency of the system. However, the high frequencies in the desired trajectory cannot be tracked by the controller. Thus, these trajectories can only be used for parameter identification. Desired trajectories with frequency less than the limiting frequency of the system can be used for applications in which the wire is used as an actuator.

6.2 Experiment Setup

The experimental setup shown in Figure 6.9 consists of a horizontally cantilevered wire, placed between two electromagnets. The electromagnets (E-09-150) were purchased from Solenoid City. The coil resistance is 6.5$\Omega$ and coil inductance is approximately 13mH. The coil consists of approximately 1270 turns of 26 AWG
copper wire.

The wire is 35mm long. It is a 100\(\mu\)m diameter Cobalt wire. The magnets are placed co-axially such that the axis of the magnets is close to the tip of the wire. Each magnet is approximately 10mm away from the wire. This distance can be varied using micrometer positioning stages.

A two channel voltage controlled current source circuit is used to drive the magnets. This circuit is described in Appendix B. A high bandwidth optical sensor is used to sense the position of the fiber. The controller is implemented on a Simulink Real Time system with Quanser Q8 I/O hardware. A modified PS3 Eye camera with a C-mount macro lens is used for sensor calibration.

The switched subsystem block in the Simulink model has two modes. In simulation mode, the controller output is connected to the Simulink implementation of Equation 2.2. The output of the model used as the measurement. The control inputs and measurements can be quantized and discretized to make the simulation realistic. Band limited white noise is also added to the measurement. In hardware mode, the current signals are fed to the Q8 D/A block. The five sensor inputs measured using the Q8 A/D block. Singular value decomposition is used to obtain
fiber angle from the raw measurements.

The switched model implementation means the same controller and observer can be used with the model and the physical system. This setup makes debugging convenient. Also, the simulation and experimental results presented in the following chapters are obtained with identical observer and controller structures.

6.2.1 Current Control

The inductance of each electromagnet coil is 13mH and the resistance of each coil is 6.5Ω. Thus the magnet acts as a low pass filter with a cutoff frequency of 80Hz. This is very close to the natural frequency of the fibers and wires used (25–30Hz). Thus controlling coil voltage introduces a phase shift in the controller output. It is necessary to control the electromagnet coil current directly. This can be achieved by closing the coil current loop with an op-amp and a current sensing resistor. The op-amp circuit has a bandwidth on the order of 1MHz. Also, a voltage divider can be used on the input of the voltage to current converter to increase the input voltage range. This circuit gives better resolution at the controller output. The circuit specifications are presented in Appendix B.

6.2.2 High Bandwidth Sensor

The fibers and wires used in the experiments have a natural frequency of around 20 to 40Hz. A high bandwidth sensor is required to sense the position of the wire for control. Cameras with image processing algorithms can achieve speeds of around 100Hz. Thus, cameras are not a viable sensor for control. Therefore, a high bandwidth fiber optic sensor was designed and implemented to sense the wire position [Cheng 2013].
An LED is placed on one side of the wire. Five optical fibers are placed on the other side of the wire. The light from the LED is modulated at 50kHz. The light from the LED is transported to phototransistors by the optical fibers. Phototransistor current is converted to voltage and demodulated in hardware. An offset stage and a gain stage are added to give a higher voltage range to increase sensor output resolution. The sensor output voltage is then measured using the Analogue Input channels on the Q8 board.

Thus, the wire angle maps to 5 voltage levels. The wire angle needs to be reconstructed using the voltage levels. A k-nearest neighbor was originally used to reconstruct the wire angle from sensor measurements [Cheng 2013]. The k-nearest neighbor algorithm is easy to implement, but it is computationally expensive. A more efficient algorithm to reconstruct the wire angle using singular value decompositions is presented here. The calibration procedure and the new angle reconstruction method is described below.

6.2.2.1 Sensor Calibration

The sensor is calibrated by collecting data with the wire moving all the way from one magnet to the other magnet and back.

The sensor is calibrated by performing a feedforward experiment. First, a 2A current is applied to magnet 2. This causes the wire to move to $-\theta_{\text{max}}$. Then a 2A current is applied to magnet 1 while magnet 2 is turned off. This causes the wire to
move to $\theta_{\text{max}}$ as shown in Figure 6.10. The wire is moved from $-\theta_{\text{max}}$ to $\theta_{\text{max}}$ and back using this technique a number of times. The sensor measurements over these runs are averaged to get an average set of data. The average dataset is shown in Figure 6.11(a). Since there are 5 channels sensing one variable, some of the data plotted in Figure 6.11(a) is redundant. Therefore, singular value decomposition is performed on the data, which gives the two principle components plotted in Figure 6.11(b). The remaining three components are redundant. Plotting the first significant component vs the second component shows that the wire angle maps to a section of a circle on a 5 dimensional plane, as shown in Figure 6.11(c). Thus performing an arctan operation gives the mapping between angle of a point on the circle vs the true angle, as shown
To reconstruct the angle from the raw sensor measurements, the raw data is projected onto the first two left singular vectors to get a point on the circle of Figure 6.11(c). The angle is obtained using the \( \arctan \) function on the projections. The reconstructed angle is mapped to the true angle using the line shown in Figure 6.11(d).

### 6.3 Experiment Results

The system parameters estimated in Section 6.1.1 are used to initialize the controller tested experimentally. First, the response of the system to exact model knowledge controller of Chapter 3 was tested first as a baseline. The adaptive observer and controller presented in Chapter 5 was tested next. The estimated parameters were used with the exact model knowledge controller. The amount of steady state error observed with the EMK controller is an indicator of the accuracy of the parameters.

#### 6.3.1 Exact Model Knowledge Controller

The exact model knowledge controller linearizes the system by canceling out the magnetic torque function. Thus, if the torque function is not modeled accurately, there is a significant perturbation in the closed loop system. This perturbation may cause the closed loop system to be unstable in some region of operation.

Since accurate system parameter estimates cannot be obtained in practice, some perturbation is expected. Inaccurate estimates of the spring constant \( b \) and the magnetic torque functions \( c_1 \) and \( c_2 \) lead to steady state errors when \( \theta_d \) is a constant. Inaccurate estimates of the damping \( a \) and the magnetic torque functions \( c_1 \) and \( c_2 \) lead to errors when tracking trajectories.
Figure 6.12: Tracking behavior for a Cobalt Wire using the EMK controller with parameters obtained from static data.

An open loop experiment was performed and \( \hat{c}_1(\theta) \) and \( \hat{c}_2(\theta) \) were estimated from static data. The response of the system with the EMK controller to a 1Hz triangle wave and a 13Hz sine wave is plotted in Figure 6.12. The measured angle \( \theta \) does not follow the desired trajectory well. This is the effect of parametric perturbations. Specifically, \( \hat{c}_1(\theta) \) and \( \hat{c}_2(\theta) \) cannot be estimated accurately for the unstable region.

Figure 6.13: Coil currents for the EMK following the desired trajectory of Figure 6.12. The current is very noisy due to measurement noise.
from the steady state data. This causes the wire to stick to the magnets. The wire touching the magnets destabilizes the closed loop system. The adaptive controller overcomes this limitation by estimating $\hat{c}_1(\theta)$ and $\hat{c}_2(\theta)$ on-line. The EMK controller is also susceptible to measurement noise. As shown in Figure 6.13, measurement noise leads to noisy coil currents. The tracking results for the adaptive controller are presented next.

### 6.3.2 Adaptive Controller

The parameter estimates used in the exact model knowledge controller were used to initialize the adaptive controller. The adaptive observer updates the parameter estimates online. The adaptive controller causes the states to converge to the desired trajectory. The system parameters go to values that minimize the parametric perturbation. These parameters may not be close to their true values as the stability analysis of Chapter 4 and Chapter 5 does not guarantee parameter convergence.

It was observed during simulations that the chirp input is useful for parameter estimation. However, the EMK controller does not perform well with the parameter estimates obtained using the adaptive controller with a chirp input. This is possibly because a lower frequency chirp was used than in the simulations. Using higher frequency inputs with the physical system causes problems as this can excite an additional vibration mode. Since these modes are not modeled, this causes closed loop system to become unstable. Therefore, a high frequency, high amplitude chirp signal cannot be used to identify parameters of the physical system.

Figure 6.14 shows the response of the system to a 13Hz sine wave and a 1Hz triangle wave. The controller tracks both inputs well. Another advantage of the adaptive controller is that the observer filters measurements. This leads to cleaner
control inputs, as shown in Figure 6.15.

The triangle wave input is more interesting from the point of view of applications. Also, the triangle wave input is at a low frequency, but it leads to good parameter estimates. It was observed that the parameter estimates obtained from the triangle wave input of Figure 6.14 result in significant improvement in the tracking performance of the EMK controller. The updated parameters from the response to the triangle wave input were used to initialize the adaptive controller. Figure 6.16 shows the response of the system to a positive step input and a negative step in-
Figure 6.16: Adaptive controller response to set points at 0.19 and $-0.19$ respectively. The wire goes to the set point within 15ms. The wire goes to the set point within 30ms. Also, these set points are inside the unstable region, indicating that the adaptive controller stabilizes the wire in the unstable region.

As desired trajectory frequency approaches the limiting frequency of the wire, large displacements excite a second mode in the system. Since this mode is not modeled, it causes the observer and the controller to fail. The results from these experiments are not plotted as the current frequently switches up to 2A. This can damage the magnets if the controller is left running. It may be possible to alleviate this problem by modeling extra modes. The adaptive controller as it is performs very well at frequencies up to 25Hz. This should be sufficient for most applications.

Overall, the adaptive controller tracks the desired trajectory asymptotically. It performs better than the EMK controller, without accurate initial parameter estimates. The adaptive controller stabilizes the wire in the unstable region, increasing the range of displacements over which the wire can be used as an actuator. The accuracy of the parameter estimates is tested next by using the estimates in the EMK controller.
6.3.3 EMK Controller with updated parameters

The adaptive controller works as long as the model approximates system dynamics closely. If the system dynamics change, the adaptive controller may cause the system to become unstable. On the other hand, the exact model knowledge controller is more robust to changes in dynamics. The main drawback of the exact model knowledge controller is that parametric perturbations may lead to instability. However, the adaptive observer-controller gives parameter estimates that lead to minimum perturbation. Using these parameters in the exact model knowledge controller minimizes the parametric perturbations. Thus it is useful to test the EMK controller with the parameter estimates obtained from the adaptive controller. Figure 6.17(a) shows the response of the system with the adaptive controller using parameter estimates obtained from the triangle wave response of Figure 6.14. The parameter adaptation gain is set to 0 for this test. Figure 6.17(b) shows the response of the system with the EMK controller using the same parameter estimates. As shown in Figure 6.18 the control input was significantly noisier for the response of Figure 6.17(b) than the response of Figure 6.17(a). This is because of measurement noise. The EMK controller relies on low pass filters to compensate for measurement noise. The filter frequency
Figure 6.18: Coil currents for the response of Figure 6.17. The adaptive observer filters measurement noise, giving cleaner control inputs.

needs to be high due to the high natural frequency of the wires. On the other hand, the adaptive controller uses the state estimates from the observer for feedback. This reduces the effect of the measurement noise.

The adaptive controller can be used to estimate the model parameters. The updated parameter estimates can be used in the EMK controller. Experiments show that the adaptive and EMK controllers can be used to control the position of the wire in a magnetic field. These controllers can now be used with the wire in microfluidic applications. The following chapter covers preliminary testing done on the wire interacting with water and presents extension of the controls work for applications.
Chapter 7

Controller Extensions for Applications

The controller work presented before is extended for applications in this chapter. The effect of different desired trajectories on parameter convergence is studied in simulations. Response of the controller to changing desired trajectories is tested. Variable observer gain algorithms are studied to improve speed of convergence. Ways of detecting interactions with a fluid surface are also proposed. A combination of these techniques is most useful for applications.

7.1 Parameter Convergence in Experiments

Estimated parameters can be verified by using the controller with updated parameters and estimation gain set to zero. Accurate parameters give good tracking. However, there may be different sets of parameters that give equally good tracking behavior. A small amplitude chirp superimposed on a triangle wave was used to identify parameters of the physical system. For comparison, a combination of 5, 15
and 25Hz sine waves were used to identify parameters. Figure 7.1 shows the response of the system to these two signals. The controller tracks both signals well.

Evolution of $\hat{a}$ and $\hat{b}$ for the triangle-chirp and for the sine wave is plotted in Figure 7.2. At around 7.5s and 9.5s, the wire touches the magnet. This causes the parameter estimates to change suddenly. This does not affect convergence of parameters to true values. For example, $\hat{a}$ returns to the previous value at 20s after the jump at 9.5s.

Figure 7.3 shows the estimated value of $\hat{c}_1$ and $\hat{c}_2$ obtained from the triangle-chirp input. The estimates are monotonically increasing, except near $\theta = 0$. This may be because of small amount of magnetic hysteresis in the wire. The increasing shape is expected since the magnetic field is stronger nearer to the magnets. Figure 7.4 shows the estimated value of $\hat{c}_1$ and $\hat{c}_2$ obtained from the sinusoid combination. The estimate of $\hat{c}_1$ is close to the estimate obtained using the triangle-chirp. However, the estimate of $\hat{c}_2$ is not monotonically increasing.

The resulting parameter estimates were used with the adaptive controller with
the adaptation gain set to 0. If the parameter estimates are good, they give good tracking for all control inputs. The response of the closed loop system to various inputs is plotted in Figures 7.5, 7.6 and 7.7. The response shows that the parameters identified using the triangle-chirp combination result in excellent tracking performance. The parameters identified from the combination of sine waves are not as

Figure 7.3: Final values of $\hat{c}_2$ and $\hat{c}_1$ in response to the triangle-chirp.

Figure 7.2: Evolution of $\hat{a}$ and $\hat{b}$ in response to the input of Figure 7.1
Figure 7.4: Final values of $\hat{c}_1$ and $\hat{c}_2$ in response to the sinusoid input.

Specifically, the wire tends to stick to the magnet when using the parameters from the sinusoid response. This does not occur when using the parameters from the triangle-chirp. This is because the magnetic field is stronger nearer the magnet. The controller should compensate for this by decreasing the current when the wire is near the magnet. The estimate of $\hat{c}_2$ from the triangle-chirp reflects this as it is monotonically increasing closer to the magnet. The estimate from the sinusoid

Figure 7.5: Response of the closed loop system to a sine wave using parameters estimated from triangle-chirp and sinusoids.
combination decreases nearer the magnet. This causes the wire to stick to the magnet, as seen on Figure 7.5 at 0.15s and every 0.2s after that.

The response to the triangle wave and the steps, plotted in Figure 7.6 and Figure 7.7 also shows larger tracking errors for the parameter estimates obtained from the sinusoid input than for the parameter estimates obtained from the triangle-chirp.

Figure 7.7: Response of the closed loop system to a series of small steps using parameters estimated from triangle-chirp and sinusoids.
7.2 Switching Between Trajectories

![Graph showing switching between trajectories](image)

Figure 7.8: Switching from an identifying chirp signal to a "holding" position at $\theta = 0.15\text{rad}$ followed by a triangle wave.

Response of the system to various periodic and aperiodic trajectories is useful for testing tracking behavior. For applications, it may be required that the controller switch between a variety of trajectories. For example, it may be required that the controller follow a chirp signal for parameter identification, then switch seamlessly to tracking a planned path in a series of small steps. The performance of the controller was tested for inputs that switch between identifying trajectories, such as the chirp and more practical trajectories, such as triangle waves, steps and set points. Figure 7.8 shows the response of the system to one such trajectory. The trajectory changes from a chirp to a constant to a triangle wave. The controller tracks the input well over each switching of the trajectories.

7.3 Variable Gain Algorithms

One extension of the observer is to use time varying gains to speed up convergence. The varying gain algorithm presented in [Narendra and Annaswamy 1989]
Figure 7.9: Evolution of $\Gamma$ over time. The gain $\Gamma_{11}$ associated with $\dot{a}$ decreases much faster than $\Gamma_{22}$.

was implemented and tested. One advantage of the variable gain algorithm is that the adaptation gain goes to zero asymptotically. This means the adaptive observer begins to behave more like a filter after a sufficient amount of time has passed. Thus, the controller switches automatically from parameter estimation to tracking using estimated parameters.

The gains $\Gamma$, $\gamma$ and $\eta$ are updated as follows

\[
\dot{\Gamma} = -\Gamma w^T w^TA\Gamma \\
\dot{\gamma} = -\gamma^2 w^2 k_j^2 \\
\dot{\eta} = -\eta^2 w^2 l_j^2
\] (7.1)

The evolution of $\Gamma_{11}$ and $\Gamma_{22}$ are plotted in Figure 7.9. The velocity of the wire is much higher than the displacement. In other words, the signal that detunes $\Gamma_{11}$ is much larger than the one that detunes $\Gamma_{22}$. Therefore, $\Gamma_{11}$ decreases by a few orders of magnitude within 0.2s, while $\Gamma_{22}$ does not. The adaptive gain algorithm adjusts parameter estimation gains in this way so that all parameter estimators are weighed evenly. This ensures faster observer convergence.
The convergence of the observer using this algorithm is difficult to prove since the adaptation gains tend to 0 asymptotically. However, this means the observer stops adapting over time. Therefore, the adaptive observer with variable gains of Equation 7.1 transitions over time into a robust nonadaptive mode.

![Comparison of the response of the constant gain observer with the variable gain observer for tapping on water.](image)

**Figure 7.10:** Comparison of the response of the constant gain observer with the variable gain observer for tapping on water.

The adaptive gain algorithm was tested by comparing the response of the system with and without adaptive gains, while the wire is tapping on water. A triangle wave with increasing amplitude is used as the desired trajectory. The two responses are plotted in Figure 7.10. The variable gain observer response plotted in Figure 7.10(b) is significantly better than the constant gain observer response plotted in Figure 7.10(a). In Figure 7.10(a), the wire sticks to the water. Detaching from the water excites the second mode, which destabilizes the controller. This does not happen with the variable gains because the adaptive observer gains decay asymptotically. The observer is more robust to the unmodeled mode due to the reduced gains. This algorithm is more useful for applications compared to constant gains.
The wire tapping the surface of water Figure 7.11: Example of the wire interacting with unmodeled objects. The controller was modified to improve performance while tapping on water.

7.4 Identifying Interactions with Objects

The adaptive controller ensures tracking control. This means for a sufficiently smooth desired trajectory, tracking error is very small $< 0.03\text{rad}$. If the wire then interacts with an unmodeled object, such as a fluid surface, the tracking error increases suddenly. This behavior can be used to identify the location of the unmodeled object. Figure 7.12 shows the response of the system to an increasing amplitude triangle wave. The amplitude stops increasing and the parameter adaptation gain is set to zero at the point at which the error deviates from zero. This gives a good practical combination of adaptive and model based control for applications.

This leads to an extension of the observer for applications. First, the object the wire interacts with is modeled as a linear in parameters function of measurements and inputs. These additional terms are included in the observer, but set to zero initially. The associated adaptation gains are also set to zero. When the tracking error deviates from zero, the adaptation gains for the wire parameters are set to zero, while the adaptation gains for the object parameters are set to nonzero values. The interaction terms are added to the observer and the controller. This causes the
Figure 7.12: A triangle wave with increasing amplitude. The amplitude stops increasing when the measurement deviates from desired trajectory. The adaptive observer gain is set to 0 when this deviation occurs.

adaptive controller to compensate for these additional interactions without disturbing the previously identified observer parameters.

Figure 7.13: The basis function used to model contact force at the surface of the water.

Using this technique the adaptive controller can be modified for applications where the wire is crossing boundaries into domains with varying dynamics. For example, the contact force from the water surface is modeled as a one sided function of $\theta$, as shown in Figure 7.13. A scalar parameter is used to adapt to the contact
The parameter estimate $\hat{g}$ is updated as follows

$$\dot{\hat{g}} = -\delta e_1 w_g$$  \hspace{1cm} (7.3)$$

where $\delta$ is a positive gain and

$$w_g = \frac{1}{s + d_2} f(x_1)$$  \hspace{1cm} (7.4)$$

Figure 7.14: Comparison of the response obtained using a varying gain observer vs varying gain observer with a model of the contact force at the water surface.
The modified controller applies more current on magnet 1 to try to pull the wire out of the water. The control input switches due to measurement noise.

The auxiliary input $v_g$ is defined as

$$
    v_g^T = \begin{bmatrix}
        0 \\
        \dot{\gamma}_{\pi+d_2} f(x_1)
    \end{bmatrix}
$$

The controller is also modified to compensate for the interaction terms. As shown in Figure 7.14, this method gives slightly better tracking at the point at which the wire touches the fluid. As shown in Figure 7.15, the modified controller applies more current when the wire is touching the water. This behavior is as expected. The current switches to 2A and back due to measurement noise. It is possible that the magnet is not strong enough to pull the wire out of the water. The modified controller may work better for a sensor with less measurement noise and stronger magnets.
Chapter 8

Conclusion

Problems with position control of magnetic wires and fibers using electromagnets were identified and addressed in this dissertation. An approximate model of the magnet-wire system was derived from the static model of [Groff et al. 2012]. A preliminary model based controller was implemented first. The effect of parameter uncertainty on system stability was analyzed for the model based controller. Analysis of the model based controller highlighted the need for accurate parameter and state estimates to achieve tracking control.

An adaptive observer was implemented to estimate system states and parameters. Stability analysis demonstrates that the adaptive observer estimates system states accurately. The control input must contain a sufficient number of frequencies to get accurate parameter estimates. Also, the adaptive observer cannot be used directly since the open loop system is unstable. Thus, a controller that can stabilize the system using observer generated state and parameter estimates is required.

An indirect adaptive controller was used to control the wire position using the parameter estimates and state estimates. The controller stabilizes the wire. The desired trajectory must contain sufficient number of frequencies for the parameter
estimates to converge to true values. Trajectories that ensure this were identified using simulations. Trajectories that give parameter estimates that ensure good tracking were also identified experimentally.

The combined adaptive observer-controller system was tested in simulations and experiments. The indirect adaptive controller ensures tracking control of desired trajectories without the need for accurate parameter estimates. For certain frequency rich inputs the parameter estimates converge to true parameter values. Accurate parameter estimates can be used with a non-adaptive controller in applications.

Alternatively, a variable adaptation gain algorithm like the one presented in Section 7.3 can be used to combine identification and tracking into one controller. Because of the linear in parameters structure of the adaptive observer, the adaptive controller can be also modified easily for applications where the wire moves across domains with varying dynamic properties.

In summary, an EMK controller was presented and analyzed. The analysis shows that the EMK controller is susceptible to parametric perturbations. An adaptive observer was designed and implemented to estimate system parameters and states. The state and parameter estimates were used to implement an indirect adaptive controller that ensures tracking control. The indirect controller stabilizes the wire in the unstable region using rough estimates of the initial parameters. The magnetic function is estimated from very little initial knowledge. The adaptive controller is more robust to measurement noise than the EMK controller. The controller can also be extended easily for applications by varying adaptation gains and desired trajectories.
8.1 Future Work

Ideally, the eigenvalues of the closed loop system can be as large as possible. Thus the speed of the response should only be limited by the coil current saturation. As mentioned in Section 6.3.2, second mode vibrations occurred in the wire for desired trajectories with frequencies greater than 30Hz and amplitudes of 0.2 rad or greater. Since this mode is not included in the model, it destabilizes the controller. If the model, the observer and the controller are modified to include this second mode, it may improve high frequency performance.

The adaptive controller is not robust to unmodeled interactions. However, the linear in parameter structure makes it easy to include interactions with objects. Basis functions can be used to approximate effects that vary over $\theta$. These terms can then be added to the adaptive observer and controller as presented in Section 7.4. The modified observer and controller should perform better than the EMK controller in this case.
Appendices
Appendix A  Proof of Claim 4.1

The right hand side of Equation 4.24 is demonstrated to be zero by comparing the terms separately. First the $\phi$ terms are tested

$$\{s[\phi_1 x_1 + v_1 - d_1 \phi^T w^A] + \phi_2 x_1 + v_2 - d_2 \phi^T w^A\} \Rightarrow 0 \quad (1)$$

These six terms are expanded and grouped by coefficients of $\phi_1$, $\phi_2$, $\dot{\phi}_1$ and $\dot{\phi}_2$ as follows

$$\phi_1: \quad \mathcal{L}^{-1}(sX_1(s)) - \mathcal{L}^{-1}\left(\frac{s^2X_1(s)}{s + d_2}\right) - \mathcal{L}^{-1}\left(\frac{d_2sX_1(s)}{s + d_2}\right)$$

$$= \mathcal{L}^{-1}\left\{\left(1 - \frac{s}{s + d_2} - \frac{d_2}{s + d_2}\right)X_1(s)\right\} = 0 \quad (2)$$

$$\phi_2: \quad - \mathcal{L}^{-1}\left(\frac{sX_1(s)}{s + d_2}\right) + \mathcal{L}^{-1}(X_1(s)) - \mathcal{L}^{-1}\left(\frac{d_2X_1(s)}{s + d_2}\right)$$

$$= \mathcal{L}^{-1}\left\{\left(1 - \frac{s}{s + d_2} + 1 - \frac{d_2}{s + d_2}\right)X_1(s)\right\} = 0 \quad (2)$$

$$\dot{\phi}_1: \quad \mathcal{L}^{-1}(X_1(s)) - \mathcal{L}^{-1}\left(\frac{sX_1(s)}{s + d_2}\right) - \mathcal{L}^{-1}\left(\frac{d_2X_1(s)}{s + d_2}\right)$$

$$= \mathcal{L}^{-1}\left\{\left(1 - \frac{s}{s + d_2} - \frac{d_2}{s + d_2}\right)X_1(s)\right\} = 0 \quad (2)$$

$$\dot{\phi}_2: \quad - \mathcal{L}^{-1}\left(\frac{X_1(s)}{s + d_2}\right) + \mathcal{L}^{-1}\left(\frac{X_1(s)}{s + d_2}\right)$$

$$= \mathcal{L}^{-1}\left\{\left(1 - \frac{1}{s + d_2} + \frac{1}{s + d_2}\right)X_1(s)\right\} = 0 \quad (2)$$

Let $z(t) = i_j(\theta)I_1^2$. The $\psi_{kj}$ terms from Equation 4.24 are tested next

$$\{s[\psi_{kj_1} z + v_{kj_1} - d_1 \psi^T_{kj} w^{kj}] + \psi_{kj_2} z + v_{kj_2} - d_2 \psi^T_{kj} w^{kj}\} \Rightarrow 0 \quad (3)$$
The six terms are expanded and grouped by coefficients of \( \psi_{kj1} \), \( \psi_{kj2} \), \( \dot{\psi}_{kj1} \) and \( \dot{\psi}_{kj2} \) as follows

\[
\psi_{kj1} : 0
\]

\[
\psi_{kj2} : -\mathcal{L}^{-1} \left( \frac{sZ(s)}{s + d_2} \right) + \mathcal{L}^{-1}(Z(s)) - \mathcal{L}^{-1} \left( \frac{d_2Z(s)}{s + d_2} \right) \\
= \mathcal{L}^{-1} \left\{ \left( -\frac{s}{s + d_2} + 1 - \frac{d_2}{s + d_2} \right) Z(s) \right\} = 0
\]

\[
\dot{\psi}_{kj1} : 0
\]

\[
\dot{\psi}_{kj2} : -\mathcal{L}^{-1} \left( \frac{Z(s)}{s + d_2} \right) + \mathcal{L}^{-1} \left( \frac{Z(s)}{s + d_2} \right) = \mathcal{L}^{-1} \left\{ \left( -\frac{1}{s + d_2} + \frac{1}{s + d_2} \right) Z(s) \right\} = 0
\]

All terms on the right hand side of Equation 4.24 are 0.
Appendix B  Current Control Circuit

The current control circuit is specified here. The spice schematic of the circuit is shown in Figure B.1. The magnet is modeled as an inductance (L1) and a resistance (R1). An OP-AMP (X1) provides high gain feedback. The MOSFET (X2) controls the current through the magnet. A freewheeling diode D1 is used to protect the MOSFET from current surges due to switching of the magnet. If V1 increases, the OP-AMP increases the gate voltage to allow more current through the magnet until the voltage drop across current sense resistor R2 matches V1.

<table>
<thead>
<tr>
<th>Component</th>
<th>Type</th>
<th>Serial Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 and R1</td>
<td>Magnet</td>
<td>Solenoid City E-09-150</td>
</tr>
<tr>
<td>D1</td>
<td>Diode</td>
<td>1N5822</td>
</tr>
<tr>
<td>X1</td>
<td>OP-AMP</td>
<td>LM358P</td>
</tr>
<tr>
<td>X2</td>
<td>MOSFET</td>
<td>IRFZ44VPBF</td>
</tr>
<tr>
<td>R2</td>
<td>Current Sense Resistor</td>
<td>MCPRW0AWJW10JB00</td>
</tr>
</tbody>
</table>
Appendix C  Modifications to Sensor Signal Conditioning Circuit

The signal conditioning circuit used to process signals from the optical sensor in [Cheng 2013] was modified. The modifications are as follows

C.1 Power Source

![Power rail connections for the sensor circuit](image)

Figure C.1: Power rail connections for the sensor circuit

The DC adapter for the LED provides 12V input. The MAX274 ICs are rated for ±5V. The on-board power regulator (UrA) and virtual ground OP-AMP (U1) were removed to avoid damaging the filter ICs. Power to the LED is provided using the DC adapter. Power to the signal conditioning circuit is provided through a bench top DC power supply. Figure C.1 shows the power leads soldered onto the power rails. The connections are listed in the table below

<table>
<thead>
<tr>
<th>Orange</th>
<th>White</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3V</td>
<td>Common</td>
<td>-3V</td>
</tr>
</tbody>
</table>

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C.2 Unused OP-AMPs

The quad OP-AMP ICs used have 4 OP-AMPs each. Only 3 of the OP-AMPs are used in each channel. The unused OP-AMP can cause problems if the inputs are left floating. The inverting input of each unused OP-AMP was shorted to the output. A lead was soldered to the non-inverting terminal as shown in Figure C.2. This lead needs to be connected to the common terminal of the supply.

Figure C.2: Unused OP-AMP termination for the sensor circuit
Bibliography


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