ELECTROMECHANICAL MODELING OF A HONEYCOMB CORE INTEGRATED VIBRATION ENERGY CONVERTER WITH INCREASED SPECIFIC POWER FOR ENERGY HARVESTING APPLICATIONS

Nataraj Chandrasekharan
Clemson University, cchandr@g.clemson.edu

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ELECTROMECHANICAL MODELING OF A HONEYCOMB CORE INTEGRATED VIBRATION ENERGY CONVERTER WITH INCREASED SPECIFIC POWER FOR ENERGY HARVESTING APPLICATIONS

A Dissertation
Presented to
The Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Mechanical Engineering

by
Nataraj Chandrasekharan
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Accepted by:
Dr. Lonny Thompson, Committee Chair
Dr. Gang Li
Dr. Mohammed Daqaq
Dr. Huijuan Zhao
Abstract

Innovation in integrated circuit technology along with improved manufacturing processes has resulted in considerable reduction in power consumption of electromechanical devices. Majority of these devices are currently powered by batteries. However, the issues posed by batteries, including the need for frequent battery recharge/replacement has resulted in a compelling need for alternate energy to achieve self-sufficient device operation or to supplement battery power. Vibration based energy harvesting methods through piezoelectric transduction provides with a promising potential towards replacing or supplementing battery power source. However, current piezoelectric energy harvesters generate low specific power (power-to-weight ratio) when compared to batteries that the harvesters seek to replace or supplement.

In this study, the potential of integrating lightweight cellular honeycomb structures with existing piezoelectric device configurations (bimorph) to achieve higher specific power is investigated. It is shown in this study that at low excitation frequency ranges, replacing the solid continuous substrate of a conventional piezoelectric bimorph with honeycomb structures of the same material results in a significant increase in power-to-weight ratio of the piezoelectric harvester. In order to maximize the electrical response of vibration based power harvesters, the natural frequency of these harvesters is designed to match the input driving frequency. The commonly used technique of adding a tip mass is employed to lower the natural frequency (to match driving frequency) of both, solid and honeycomb substrate bimorphs. At higher excitation frequency, the natural frequency of the traditional solid substrate bimorph can only be altered (to match driving frequency) through a change in global geometric design parameters, typically achieved by increasing the thickness of the harvester. As a result, the size of the harvester is increased and can be disadvantageous especially if the application imposes a space/size constraint. Moreover, the bimorph with increased thickness will now require a larger mechanical
force to deform the structure which can fall outside the input ambient excitation amplitude range. In contrast, the honeycomb core bimorph offers an advantage in terms of preserving the global geometric dimensions. The natural frequency of the honeycomb core bimorph can be altered by manipulating honeycomb cell design parameters, such as cell angle, cell wall thickness, vertical cell height and inclined cell length. This results in a change in the mass and stiffness properties of the substrate and hence the bimorph, thereby altering the natural frequency of the harvester.

Design flexibility of honeycomb core bimorphs is demonstrated by varying honeycomb cell parameters to alter mass and stiffness properties for power harvesting. The influence of honeycomb cell parameters on power generation is examined to evaluate optimum design to attain highest specific power. In addition, the more compliant nature of a honeycomb core bimorph decreases susceptibility towards fatigue and can increase the operating lifetime of the harvester.

The second component of this dissertation analyses an uncoupled equivalent circuit model for piezoelectric energy harvesting. Open circuit voltage developed on the piezoelectric materials can be easily computed either through analytical or finite element models. The efficacy of a method to determine power developed across a resistive load, by representing the coupled piezoelectric electromechanical problem with an external load as an open circuit voltage driven equivalent circuit, is evaluated. The lack of backward feedback at finite resistive loads resulting from such an equivalent representation is examined by comparing the equivalent circuit model to the governing equations of a fully coupled circuit model for the electromechanical problem. It is found that the backward feedback is insignificant for weakly coupled systems typically seen in micro electromechanical systems and other energy harvesting device configurations with low coupling. For moderate to high coupling systems, a correction factor based on a calibrated resistance is presented which can be used to evaluate power generation at a specific resistive load.
Dedication

This dissertation is dedicated to my parents Mr. and Mrs. Chandrasekharan and my sister Ms. Lekshmi Gomathy.
Acknowledgement

I would like to express my at most gratitude to my academic advisor Dr. Lonny Thompson for his continuous support he has offered over the years. His mentoring, criticism and guidance has played a significant part towards not only towards the completion of this dissertation, but also in acquiring valuable life skills.

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Chapter 1: Problem overview

1.1. Research motivation

The last couple of decades have witnessed a continuous decrease in power consumption, size miniaturization and cost reduction of electronic devices. For example, size and cost evolution of inertial sensors such as accelerometers is shown in Figure 1 where significant reduction in both size and cost is evident (Yole Developpment, 2014). Although the figure corresponds to accelerometers in specific, this trend is fairly strong across all low power electromechanical devices. Advancement in both micro-fabrication and process integration technology has contributed greatly to this decrease. The application of complementary metal-oxide semiconductor (CMOS) logic circuits in particular, which combine metal-oxide semiconductor (MOS) transistors in a symmetric complimentary circuit configuration, stands out as the major innovation leading to almost a zero power requirement during standby mode (low static power consumption), contributing to the overall low power requirement (Wanlass & Sah 1963). Along with CMOS logic, improvement in photolithography, wafer bonding, thin film technology and bulk micromachining has played a huge role in the volume production integrated circuits and hence low power electronic devices.
As a result, low power consuming electromechanical industry has flourished over the last decade transforming into a billion dollar market. The various low power electromechanical devices along with their contribution to market growth can be seen in Figure 2. It is interesting to note that energy scavengers in the market have been on the rise since 2012, whereas pressure sensors and other inertial sensors (accelerometers, gyroscopes etc.) have traditionally been the backbone of this industry. It is of importance to realize that the list of devices shown in Figure 2 encompasses a wide variety of target application including mechanical, biomedical and electrical disciplines. Moreover, products like MEMS level joysticks, speaker, Pico-projectors etc., are emerging and new additions to the market suggesting that these low power devices are on the rise in multiple domains.
In fact, research indicates continual growth in this industry over the coming years as many of these devices are yet to reach a market maturity (Applied Materials, 2013). Wireless sensors alone are estimated to grow to a $4 billion industry with an annual growth rate of 74% (IHS Technology 2013). Such steep growth requires more power for operation, and hence investing time and effort to explore means to power these low power electronics is rewarding.

Currently, majority of power electronic devices operate on battery power. Though the advances in transistor and manufacturing technology have led to the reduction in the mass and size of host devices, the mass and size of batteries powering them has not reduced proportionally. This size disparity between the host devices and batteries is evident, for instance in mobile phones where lithium ion batteries account for up to 36% of the entire weight (Cook-Chennault et al. 2008). The incapability of traditional batteries to provide higher power to weight ratio has prevented the mass and size reduction of batteries, and hence of the host device. A wide variety of batteries have been designed to power a multitude of different loads (applications). Power requirement for various electronic devices is obtained from (IDTechEx Report, 2009) ranging

![Figure 2: Growth in MEMS industry over the last decade (IHS Technology 2013)](image_url)
from nanowatts (nW) to hundreds of Watts, shown in Figure 3. For example, small non-rechargeable button batteries are able to power electronic devices like 32 KHz quartz oscillator (100 nW), electronic watch or calculator (1 μW), RFID tags (10 μW), hearing aids (100 μW) and miniature FM receivers (1 μW) all which require up to 1mW of power. AA and AAA batteries power slightly larger devices like transceiver Bluetooth (10 mW), while rechargeable batteries are required for cell phones and laptops that require well over 1W.

**Figure 3. Power requirement for various electronic devices along with battery source operating them. Data from (IDTechEx Report, 2009)**

One of the biggest issues batteries pose is the need for frequent recharging/replacement of the battery (due to finite lifetime) which might be physically difficult and/or financially steep, depending on the application. For example, replacing batteries in bio-implants is very inconvenient and expensive as it requires surgical procedures each time a replacement is needed. The inconvenience posed by batteries in battlefields and the need for an alternative source to operate battlefield related low power electronic devices are well documented (Ervin et al. 2013; Gilman et al. 2008). The continual need to re-charge/replace batteries can limit army surveillance, sensing, communications and survival which can prove detrimental to mobility, range and length
of operation. In addition, the environmental impacts due to battery production and more importantly battery disposal is well documented (McManus 2012).

Rechargeable lithium-ion batteries, with a power to weight ratio of around 340 mW/g (Panasonic 2010) and the more recent lithium-air and lithium-sulphur batteries however are major improvements over traditional battery electro-chemistries. However, their superior energy density does not meet the growing life cycle demands to power low energy consuming portable electronic devices. Taking into account Moore’s empirical law that predicts a doubling of transistors incorporated on a chip every two years and the fact that advancement in battery technology has been disproportionate (Paradiso & Starner 2005) as shown in Figure 4, the need to investigate alternative or a supplementing power sources is compelling.

Figure 4: Battery energy density evolution with respect to mobile computing evolution (Paradiso & Starner 2005)
However, the use of batteries in many applications is critical and currently unavoidable. While energy scavengers at present cannot replace batteries in applications that require larger power, for example a cell phone that requires a lithium ion battery, it can act as an energy supplement to battery power to reduce the frequency of recharging and/or replacing the battery. Button batteries and AA/AAA batteries are used at present to power devices that require up to 1mW as seen in Figure 3. This region of power requirement is recognized as the one where energy scavengers can contribute heavily, thereby negating the need for batteries to operate these low power electronic devices. Energy to weight ratio of disposable lithium button/coin battery is 198 mW-hr/g (Energizer Holdings Inc 2009) with a nominal voltage of 3V, while its power to weight ratio can be calculated depending on the current rating and amount of time it powers the application. Rechargeable lithium ion batteries that power slightly larger applications, for example mobile phones, is around 250-340 mW/g (Panasonic 2010). In order to replace these batteries, the power to weight ratio of the alternative power generation method should be comparable to that of the battery.

For applications that require higher power, energy harvesters can supplement battery power to reduce the frequency of battery recharge/replacement. However, it is then important to make sure that the mass and size of the harvester does not significantly increase the total mass and size of the device. Hence, these applications also require high power to weight ratio from energy harvesters in addition to attractive scalability and compactness. Consequently it is critical that the energy harvesters exhibit similar power to weight ratio when compared to the battery the harvesters seek to replace or supplement.
1.2. Review on current energy harvesting technologies

Energy to operate low power consuming devices can include a wide variety renewable and non-renewable sources like solar, vibration, fuel cells, heat engines, acoustic noise and thermoelectric power to name a few. Energy harvesting from each source has its own merits and demerits and a good discussion regarding power harvesting from various regenerative and non-regenerative sources can be found in (Cook-Chennault et al. 2008; Roundy et al. 2003). Non-regenerative sources though have a finite lifetime and pose many issues similar to battery use such as recharging and replacement. Hence subsequent discussions pertaining to power alternatives are limited to regenerative energy sources only. As a result renewable energy sources need to be investigated as means to obtain a suitable self-sustainable alternative to battery to power MEMS devices. Some of the suitable common renewable sources include solar, vibration and thermoelectric power.

1.2.1. Thermoelectric power

Thermoelectric power is obtained through the conversion of a temperature gradient into electric power through thermoelectric or Seebeck effect. A major roadblock in the form of lack of large temperature gradients in MEMS devices (typically devices less than 1cm³) has limited the adoption of thermoelectric power. Another issue it the requirement of thermal resistance match between the thermoelectric generator and the heat sink which is difficult especially with thin film thermoelectric devices as heat fluxes are drastically different from the heat sink (Bierschenk 2009) Moreover, It is reported in (Venkatasubramanian et al. 2006) that efficiency of traditional bulk and think film thermoelectric materials drops drastically when integrated to a device due to large amounts of electrical and thermal conduction losses. In fact, the efficiency of thermoelectric generators at low thermal gradients integrated in a device is between 0.5 - 0.8%. A
temperature gradient of <10 °C is commonly observed in MEMS applications (Cook-Chennault et al. 2008) for which a power density of around 15µW/cm³ can be harvested (Roundy et al. 2003). Note that with MEMS application in mind, power density (power/volume) is a better metric than power alone to quantify energy harvesting due to size constraints as MEMS devices is typically less than 1cm³. Hence it is concluded that thermoelectric power can only be viable source for energy harvesting at large temperature gradients which is difficult to obtain in MEMS devices.

1.2.2. Solar cells

In contrast, photovoltaic (PV) cells have readily available energy source in the form of sunlight which is converted to power through photovoltaic effect. A power density of 15,000µW/cm³ is obtained using PV cells under direct sunlight (outdoors) where the solar irradiance is 1000W/m². This value of power density is much higher than any other harvesting techniques. However, the power density value drops significantly (around 10µW/cm³) for indoor applications where artificial illumination is the major light source with light intensity of typically <10W/m². Silicon PV cells that has achieved a maximum efficiency of 25% (Green 2009; Zhao et al. 1999) in outdoor conditions is unfortunately less than 1-3% efficient under indoor light intensity (Randall & Jacot 2002). In addition, with an MEMS standpoint, PV cells are not suitable for applications that demand an embedded energy source due to lack of sufficient light.

Many studies investigating efficiency of solar cells report exciting improvements in efficiency. However some of these studies do not address the effect of series resistance on cell efficiency. 10-20% of the originally available power is lost with the additional series resistance of just one ohm (Prince 1955; Green 1977). Solar cells with small surface area (around 0.01cm²) will mostly be governed by shunt resistance resulting in good fill factor and high open circuit voltage. One of the findings in (Rajkanan & Shewchun 1979) requires that the active surface area of solar cell must be more than 0.25cm² in order to include series resistance effects (if any) which is necessary
for the correct evaluation of efficiency. Hence, it is recommended that studies and review papers report solar cell efficiency only from cells with an active area of above 0.25cm², which is quite often not the case.

1.2.3. Vibration energy harvesting

The concept of energy harvesting is to tap unused energy from ambient sources with conversion to usable electric power. Sources of ambient energy include vibrations from structures, machines and human activities such as walking, breathing etc. Energy harvesting from vibration can be realized through three basic mechanisms namely electromagnetic, electrostatic and piezoelectric conversion (Williams & Yates 1996). Electromagnetic power conversion involves generation of current on a conductor (usually shaped as a coil) in a magnetic field. Current is then generated by the relative movement of the magnet and conductor or by change in magnetic field. Hence, the current developed is proportional to the strength of the magnetic field, velocity of relative motion and the number of turns in the coil.

Some of the first studies on vibration to electricity conversion was developed by (Williams & Yates 1996; Shearwood & Yates 1997; Amirtharajah & Chandrakasan 1998) where electromagnetic conversion was the focus. A second order linear model was developed by (Williams & Yates 1996) and validated against experimental results obtaining a power density of 10-15 µW/cm³. There were issues associated with this model though as the vibration magnitude needed to drive the device was 380m/s² at 4.4 kHz. Such vibration amplitude and frequency of vibration are more or less impractical. Moreover, the voltage output was extremely low and amplification by a factor at least 100 would have been required. In spite of these practical limitations this model was the first illustrate power harvesting from vibration. A novel model was also developed by (Amirtharajah & Chandrakasan 1998) generating a maximum of 400 µW
without any mechanical damping albeit from a 4x4x10cm device rendering the power density to 2.5µW/cm³. The huge size of this device poses an issue with regard to power MEMS devices (<1cm³). Along with the model proposed by Williams, Yates and Shearwood, this model were among the first models to represent electromagnetic power conversion. Interested readers are encouraged to read (Arnold 2007) to gain further information on various electromagnetic power conversion models in the literature.

Electrostatic power generation utilizes the relative motion of two conductors separated by a dielectric medium to develop energy which is stored in a capacitor. One of the first models was proposed by (Meninger et al. 2001) through charge constrained and voltage constrained mechanisms with respect to varying capacitance. The device was 1.5cm x 0.5cm x 500µm which neatly fits the MEMS scale and was also integrated with CMOS process making it a good candidate to power MEMS devices. A model was also proposed by (Mitcheson et al. 2004) for electrostatic power generation under low frequency operations as models prior to this required high frequency vibrations to resonate the mechanical structure.

Piezoelectric energy conversion through direct piezoelectric effect causes a charge separation to be developed across the piezoelectric material thereby developing an electric field resulting in voltage generation when the material is under stress/strain. Reverse piezoelectric effect enables an applied electrical energy to cause deformation in the material. One of the earliest studies involving energy harvesting through piezoelectric conversion is investigated in (Häsler et al. 1984) converting the energy employed for respiration to electric power. A prototype converter was designed using polyvinylidene fluoride film (PVDF) that resulted in a low peak power generation of 17µW. In spite of the low power generation this work is significant as it is possibly the first reported work involving piezoelectric energy harvesting through a bio-
implantable device. One of the first works that focused on vibration to power conversion through piezoelectric mechanism involved a steel ball striking a piezoelectric plate (Umeda et al. 1996). An equivalent electrical model was used to represent and analyze the power conversion with the authors concluding that a large portion of the energy was lost as kinetic energy due to the ball bouncing off the plate. A simulated model preventing the steel ball from bouncing off is also proposed that resulted in a theoretical maximum efficiency of 52%.

Electromagnetic, electrostatic and piezoelectric energy conversions are all significant feasible harvesting techniques. However, as with any method certain limitations exist in each of them thereby promoting one method based on the application at hand. Many studies have analyzed these three conversion mechanisms in detail and have concluded that the power density obtained through piezoelectric energy harvesting is the highest among the three (Roundy & Wright 2004; Cook-Chennault et al. 2008). It was reported in (Cook-Chennault et al. 2008) that piezoelectric energy harvesting exhibit wide range of power densities and voltages. In fact, piezoelectric converter density is comparable to that of certain thin and thick film lithium and lithium ion battery.

Electromagnetic converters yield a relatively low voltage output (0.1 -0.2V) requiring the need of post processing voltage multipliers like transformers to amplify the voltage to charge a storage component. On the other hand, the voltage outputs from electrostatic converters are relatively high which could result in the requirement of implementing a potentially challenging custom circuit design due to the low current supplying capability (Beeby et al. 2006). In contrast, piezoelectric conversion yields voltage in a more useful range which can be used directly negating the need for further processing. A major drawback in electrostatic converters is the need for an external input voltage (bias voltage) to initiate a vibratory motion of the capacitor.
from which energy can then be harvested. On the contrary, electromagnetic and piezoelectric conversion mechanisms do not have this requirement as vibration can be induced through mechanical nature.

Nevertheless, the ease of integrating electrostatic energy harvesting techniques with MEMS fabrication is a significant advantage over both piezoelectric and electromagnetic schemes. In addition, miniaturization of electrostatic converter size only leads to increase in the capacitor energy density which is advantageous from a MEMS application standpoint. Thick and thin film piezoelectric layers have been implemented in MEMS applications (Lee & White 1996; Fang et al. 2006; Jeon et al. 2005; Choi et al. 2006) albeit with a drop in power density due to a resulting low coupling at MEMS scale (Verardi et al. 1997). In contrast, implementing and generating useful levels of power and voltage with an electromagnetic converter at micro scale remains a challenging task due to poor transduction properties of planar magnets, limitation on the number turns on the conducting planar coils and the issues with assembly and alignment associated with sub-millimeter electromagnetic configurations (Beeby et al. 2006). However, at larger scales these limitations are invalid and since at macro-scales larger and hence stronger magnets can be employed electromagnetic converters make a good candidate for energy harvesting. In summary, due the high energy density, lack of additional circuit elements for voltage amplification, direct generation of useful levels of voltage, moderate ease of integration with CMOS based MEMS devices and lack of external voltage input necessity piezoelectric energy harvesting provides a favorable platform for energy harvesting from vibration for MEMS application.

Despite the advantages in piezoelectric energy harvesting certain critical limitations do exist when compared with electrostatic and electromagnetic converters. Since a direct mechanical
strain need to be applied on the system and the fact that piezoelectric materials are brittle ceramics the operational lifetime and performance is reduced with time. In addition, piezoelectric converters are more expensive due the high cost of piezoelectric materials. Furthermore, the most commonly used piezoelectric material is lead zirconate titanate (PZT) due its favorable combination of moderate cost and coupling factor although lead free piezoelectric materials have been investigated by some which is discussed in a later section. The environmental impact of using lead based material is quite well documented in literature, for example read (Lanphear et al. 2005; Tong et al. 2000). These limitations could prove to be a roadblock in the widespread adoption and commercialization of piezoelectric converters if not addressed.

Piezoelectric energy conversion through direct piezoelectric effect causes a charge separation to be developed across the piezoelectric material thereby developing an electric field resulting in voltage generation when the material is under stress/strain. Typically, there are two different modes of operation to actuate a piezoelectric system namely a -31 and a -33 actuation mode. If the piezoelectric material is poled along the “3” direction then a -31 mode requires the strain to be developed perpendicular to the poling direction while a -33 mode allows for strain to be developed parallel to the poling direction. The former can be realized through bending while the latter can be achieved in a stack configuration (Goldfarb & Jones 1999). Comparison between these two configurations have been studied by many (Baker et al. 2005; Yu & Lan 2001). In an energy harvesting application typically a piezoelectric cantilever mounted to a vibrating support or excited by a driving load represented by a Fourier series (transform) of time harmonic force activates the efficient -31 mode. Moreover, the range of frequencies that can be achieved through a cantilever beam bending model can be more easily designed to match target ambient natural frequency of vibrations typically around 60-200 Hz (Roundy & Wright 2004) for maximum power generation through modal excitation. The stack configuration which is thicker would be
much stiffer with a large natural frequency (much higher than the target ambient frequency ranges) and would require a huge mass to lower the natural frequency of the structure to match excitation frequency for maximum power generation. Hence, it is advantageous and sometimes essential for energy harvesting from ambient sources that a bending configuration is preferred as it would be easier to drive the system at low excitation frequencies.

One of the most commonly employed in piezoelectric energy harvesting is that of a -31 mode beam where a piezoelectric layer is bonded to a substrate layer. If the piezoelectric harvester utilizes one layer of piezoelectric material it is called a unimorph configuration, while implementing two piezoelectric layers is called a bimorph. Typically a ductile structural layer is added to both unimorph and bimorph (center layer) to impart structural rigidity and improve mechanical reliability. For time harmonic excitation, the piezoelectric layers alternate between tension and compression. Since the piezoelectric bimorph contains two charge generating layers, they can either be connected in series or parallel. Further details regarding the connections and differences can be found in (Ng & Liao 2005; Chandrasekharan et al. 2013a).

The invention of piezoelectric bimorph with its full conceptual understanding was published in (Sawyer 1981), while (Steel et al. 1978) was among the first to analytically derive tip deflection of a unimorph excited by a voltage source. A comprehensive derivation of the static constitutive relationship of a unimorph with a passive substrate under various excitations was obtained by (Smits & Choi 1991) from fundamental principles by calculating the internal energy of thermodynamic equilibrium. (Smits & Dalke 1989; Smits et al. 1991) had previously similarly derived constitutive equations describing the electromechanical behavior of a two layer bimorph consisting of two bonded piezoelectric layers in series and parallel. Following this work, a similar approach involving estimation of internal energy in an infinitely small volume element in a
piezoelectric material under thermodynamic equilibrium is derived in (Wang & Cross 1999) for a three layer piezoelectric bimorph (two piezoelectric layers separated by a non-piezo center layer).

**Additional aspects in vibration based energy harvesting**

The significantly lower specific power or power to weight ratio of current energy harvester remains one of the major issues energy harvesting technologies. To attain the goal self-sustainable device operation or to supplement battery power, the power-to-weight ratio gap between piezoelectric energy harvesters and batteries need to be bridged and is the focus of this thesis. However, it is worthwhile to understand other current major issues in vibration based energy harvesting which is discussed subsequently.

As discussed before it is essential that under harmonic excitation the driving frequency and the fundamental frequency of vibration of the structure are matched to attain resonance to maximize power generation. This type of excitation technique is known as modal excitation. Such harvesters are linear harvesters in nature as linear resonance condition is exploited. The chief limitation that exists in linear harvesters is the significant drop in power generation when exact resonant frequency match is not achieved. One study, (Baker et al. 2005) reported one and two orders of magnitude drop in power when the system was driven at 25Hz and 50Hz away from the resonant frequency.

In order to attain resonance, the natural frequency of the system must to be known beforehand. Unfortunately, issues involved with precision in manufacturing, slight variations in material properties, influence of temperature variations and other design stage and first order uncertainties does not allow for exact natural frequency estimation with low tolerance/uncertainty. In addition, maintaining an exact driving frequency for long period is difficult as external factors such as wear and tear of host structure etc., lead to variation. The
excitation frequency is many cases is unknown or changes with operating conditions hindering the chances of efficient energy harvesting in those cases.

Consequently, methods to address resonant frequency match/mismatch are imperative for maximizing output power. One common method is to adopt a tuning mechanism to manipulate the natural frequency of vibration while methods to increase the frequency bandwidth of vibration so that mismatch in excitation and natural frequency does not cause a severe reduction in power, have also been investigated.

In principle an ideal tuning operation must be capable of the following: (a) detecting change in excitation frequency, (b) a mechanism to alter the natural frequency of vibration and (c) a closed loop control system to govern the change in frequency. Moreover, it is critical that the energy required to tune the harvester is significantly lower when compared to the energy harvested. It is also quite essential that the tuning mechanism achieves an attractive bandwidth with sufficient frequency resolution and high quality factor. The different means to alter and control harvester natural frequency can be categorized as manual and automatic tuning. Manual tuning mechanism typically involves manipulating the stiffness of the system to vary frequency as opposed to varying mass to shift the natural frequency (Leland & Wright 2006; Eichhorn 2008; Hu et al. 2007; Challa & Prasad 2008) while an auto tuning mechanism (Jo et al. 2011; Gu & Livermore 2010; Challa et al. 2011; Peters & Maurath 2009; Wu & Chen 2006; Roundy & Zhang 2005) should not require any manual input to change system parameters and such methods must detect change in excitation frequency and improvise appropriately to attain condition of resonance.

The fundamental limitation in linear resonant vibration harvesters as discussed previously can also be addressed by increasing the bandwidth thereby increasing the operational frequency
range of the harvester so that a slight shift away from resonance does not severely impact power generation. Many strategies to achieve an enlarged bandwidth have been proposed including, an array of multiple linear energy harvesters, targeting multiple modes of vibration from a structure and non-linear energy harvesters. Models containing multiple generators each with different natural frequency have been investigated as a method for improved bandwidth by (Shahruz 2006; Xue et al. 2008; Liu et al. 2008). Power harvesting from multiple bending modes of vibration resulting in an enhancement of the frequency bandwidth has also been studied. It involves power generation from frequencies associated with multiple modes (typically first two modes) and not just the fundamental frequency (Tadesse & Priya 2008; Ou et al. 2010; Arafa et al. 2011; Erturk et al. 2008; Berdy et al. 2011; Yang & Yang 2008; Kim et al. 2011).

Linear resonant energy harvesters operate under a very narrow frequency bandwidth and are only suitable under fixed harmonic excitation due to their sensitivity towards uncertainty in the system arising from imprecise characterization of host structure and/or manufacturing tolerance (Mann et al. 2012). Tuning mechanisms alleviate certain constraints at the expense of increased total size and/or mass (volume) and cost. As an alternative, non-linearity can be deliberately introduced into an energy harvesting system to conditionally increase the operational frequency bandwidth making it more suitable for ambient random and non-stationary excitation modes (Mann & Sims 2009; Gammaitoni et al. 2009; Wickenheiser & Garcia 2010; Masana & Daqaq 2011a). The most frequently implemented technique to induce non-linearity in the system is through the introduction of non-linear restoring force (Mann & Sims 2009; Gammaitoni et al. 2009; Wickenheiser & Garcia 2010; Masana & Daqaq 2011). Non-linearity imposed in this manner can be classified into two categories based on the number of stable equilibrium states. If the device has one equilibrium state it is called mono-stable while two equilibrium states correspond to bi-stable devices. The response of mono-stable and bi-stable non-linear energy harvesters can
be captured with the non-linear Duffing oscillator modeling that introduces cubic non-linearities in restoring force (the force that tends to push the system towards equilibrium). The shape of the potential energy function of a harvester can be generalized as,

$$U(x) = -\frac{1}{2}ax^2 + bx^3$$

(1)

where $U(x)$ is the potential energy of the system (Duffing potential), whose shape is dependent on the non-linearity present in the system while $a$ and $b$ correspond to stiffness coefficients. The restoring force is then given by the spatial derivative of potential energy,

$$F(x) = -ax + bx^3$$

(2)

It can be seen from the above equation that the restoring force and the displacement are governed by a cubic non-linear relationship. The system is said to be mono-stable if $a \leq 0$ while the system is bi-stable when $a > 0$ and $b > 0$ in the above equations. In a mono-stable case once $b > 0$, the system exhibits a hardening response i.e., the restoring force increases with an increase in displacement while a softening response i.e., restoring force decreases with an increase in displacement, is exhibited when $b < 0$. The bi-stable system exhibits two local minima points given by $\pm\sqrt{a/b}$ separated by a maximum at $x = 0$ whose height is given by $a^2/4b$. Moreover, it can be seen that the restoring force reduces to a linear case when $b = 0$. This is depicted in Figure 5 where the frequency response of a linear harvester is compared to a mono-stable non-linear harvester. Note that the bends in the frequency response is responsible for enhanced frequency bandwidth, a characteristic feature of non-linear harvesters. The bend towards the left and right is due to a softening and hardening type non-linearity respectively in mono-stable harvesters.
Unlike linear resonant energy harvesters where the associated particle response has a unique solution of physically realizable states at each frequency, non-uniqueness of solution exists due to the bends in the frequency response curves of non-linear energy harvesters. This leads to the creation of three branches of co-existing motion namely, the resonant ($B_r$), non-resonant ($B_n$) and unstable branches (dashed lines). Techniques to guarantee existence of solution on the resonant branch have been employed albeit at higher costs and reduction in net power harvesting.

![Bend in frequency response of a non-linear (mono-stable) energy harvester compared to a linear harvester (Daqaq et al. 2014).](image)

**Figure 5:** Bend in frequency response of a non-linear (mono-stable) energy harvester compared to a linear harvester (Daqaq et al. 2014).

A schematic of a potential energy function exhibited by a bi-stable harvester is shown in Figure 6. The region surrounding the local minimum of the potential energy is called a potential well. A bi-stable structure possesses two potential wells while a mono-table device will exhibit a single potential well. The schematic shows three sample dynamic trajectories each illustrating a different response. Depending on the level of external energy (excitation) dynamic behavior such as intra-well (low energy orbits), chaotic inter-well and inter-well oscillations (high energy orbits) is displayed.
If the excitation amplitude is too low, oscillations around only one potential well is exploited, known as intra-well oscillations shown by trajectory ‘a’ in Figure 6 while moderate to high excitation amplitudes enables dynamic trajectory transitions between the two potential wells, known as inter-well oscillations depicted by trajectories ‘b’ and ‘c’ in Figure 6. These inter-well oscillations could be transient or steady state in nature depending on the amplitude of excitation. Transient inter-well oscillations, followed by trajectory ‘b’, exhibit a chaotic response limiting the device to operate around one potential well. Sufficiently high amplitude is a requisite to maintain steady state periodic inter-well oscillations (trajectory ‘c’) to reap maximum benefits of a bi-stable harvester.

![Figure 6: Potential energy function of a bi-stable non-linear energy harvester with sample trajectories pertaining to a) intra-well b) chaotic-inter-well and c) inter-well oscillations (Harne & Wang 2013)](image)

In the next two subsections, some of the early works in the methods to induce mono and bi-stability along with their response towards harmonic and random excitation is outlined. A further subsequent subsection involves discussion pertaining to device configuration to achieve a piecewise liner response.
Mono-stable non-linear configurations

One of the earliest works pertaining to mono-stable non-linear modeling of piezoelectric power harvesting was presented in (Hu et al. 2006). Theoretical analysis of non-linear induced by large deformation near resonance is reported. The work demonstrated the multivalued nature and jumps in output power leading to ‘bends’ in frequency response typical of non-linear behavior resulting in increased bandwidth. Early studies such as (Beeby et al. 2007; Quinn et al. 2007) also demonstrated the advantages of non-linear modeling for energy harvesting.

While the above studies investigated intrinsic non-linearity in the system, a deliberate introduction of non-linearity in the system was probably first looked at by (Burrow & Clare 2007; Barton et al. 2010) using experimental methods involving electromagnetic transduction to convert vibration energy to power. Though these works does not pertain to piezoelectric energy harvesting, the author recognizes the need to acknowledge early studies in non-linear modeling.

One of the earliest work with regard to non-linear piezoelectric energy harvesting was investigated by (Stanton et al. 2009) that involved interaction between a magnetic end mass on a piezoelectric bimorph and external magnets to enforce non-linearity. Both, stiffness hardening and softening response were experimentally determined by manipulating the distance between the magnets and compared to a linear response demonstrating the improvement in frequency bandwidth. Figure 7 clearly demonstrates the enhancement in operational frequency bandwidth obtained.
Improved power output amplitude using a mono-stable non-linear mechanism under steady state harmonic excitation is not assured. The advantage in using such harvesters under harmonic excitations seems to be better tolerance towards shift in excitation frequency when compared to a linear resonant harvester (Quinn et al. 2011) which is more sensitive to the uncertainties due to manufacturing tolerances and uncertainty propagation (Mann et al. 2012). Moreover, it was demonstrated in (Sebald et al. 2011) that when the strength of non-linearity or the amplitude of excitation is small the response of a mono-stable non-linear harvester is comparable to that of a linear resonant harvester. In addition methods to increase the power amplitude by optimizing electric load are feasible but only with reduction in frequency bandwidth resulting in intolerance towards excitation frequency shifts. Optimization of electric load with respect to frequency poses the issue continuous alteration of load resistance which is not feasible for applications.

Figure 7: Frequency response comparison between a linear and non-linear piezoelectric harvester for a particular configuration (Stanton et al. 2009)
**Bi-stable non-linear configurations**

Perhaps, the earliest work in exploring the feasibility of energy harvesting from a bi-stable device was investigated by (Shahruz 2008; McInnes et al. 2008). A numerical investigation was conducted by (Shahruz 2008) focusing on large amplitude inter-well oscillations. It was reported that four times higher amplitude was obtained when compared to its linear counterpart under stochastic excitation. (McInnes et al. 2008) reported that periodic forcing can apparently be used to enhance the mechanical energy available for energy harvesting by attaining stochastic resonance.

The early practical descriptive models with respect to bi-stable energy harvesters were developed by (Cottone et al. 2009; Erturk et al. 2009). Implementation of bi-stable non-linear harvesters for piezoelectric energy harvesting was studied by (Cottone et al. 2009) where dynamical features based on stochastic non-linear oscillations were exploited through strategic arrangement of magnets and a piezoelectric inverted pendulum. (Erturk et al. 2009) also achieved power generation from a resistive load using the bi-stability concept through the addition of piezoelectric layers to the bi-stable magneto-elastic cantilever structure proposed by (Moon & Holmes 1979). The device consisted of a ferroelectric cantilever beam covered with piezoelectric layers with two external magnets located symmetrically near the free end of the beam.

Following their early work, other methods to induce bi-stability have been explored especially pure elastic beam buckling through axial loads (Masana & Daqaq 2011b; Sneller et al. 2011; Friswell et al. 2012) and buckling due to laminate asymmetry in composite plates (Arrieta et al. 2010). Bi-stable energy harvesters have also been developed and tested at MEMS scale using piezoelectric (Andò et al. 2010) and electrostatic (Nguyen et al. 2013) vibration energy transduction mechanisms, demonstrating enhanced operational frequency bandwidth.
The benefits of implementing a bi-stable energy harvester under harmonic excitation can be exploited only at certain excitation amplitudes. (Erturk et al. 2009) numerically and (Stanton et al. 2010) analytically demonstrated that when the excitation amplitude is low, the response is quite similar to that of a duffing type mono-stable harvester exploiting only a single potential well (intra well) in the potential energy-displacement diagram. Moderate excitation may trigger but not sustain inter-well oscillations leading to a chaotic response. Hence, large enough amplitude to trigger a steady-state periodic inter-well oscillation in the potential energy function is required to harness any benefit associated with bi-stable harvesters. This has been demonstrated by many researchers, for instance (Erturk et al. 2009; Masana & Daqaq 2011b; Stanton et al. 2010; Erturk & Inman 2011).

It can be then concluded that the performance of bi-stable energy harvesters is heavily dependent on prior knowledge of excitation level and the shape potential energy function of the harvester. However, different from linear and mono-stable harvesters, bi-stable harvesters can exhibit co-existence of aperiodic and chaotic behavior due to intra-well oscillations along with large-orbit high energy orbit inter-well oscillations thereby limiting the advantages of improved power harvesting due inter-well oscillations. It is worthwhile to note that in an investigation carried out in (Masana & Daqaq 2011b), power harvesting from a bi-stable and mono-stable axially loaded device was compared to each other under different excitation levels. It was reported that under low amplitudes of excitation the mono-stable configuration harvested more power, while even at large amplitudes of excitation a significant advantage over the mono-stable device was not observed.

In addition (Cottone et al. 2009) demonstrated that power enhancement using a bi-stable harvester is feasible only under white Gaussian noise provided the time constant of the harvesting
circuit is very large. It has been shown by many that when the time constant is not very large, the effect of non-linearity of power harvested diminishes significantly. Moreover, similar to harmonic excitation, prior knowledge of the excitation intensity is critical in exploiting benefits through appropriate design to increase power output under white noise excitation (Halvorsen 2013; Litak et al. 2010; Zhao & Erturk 2013). On comparison with mono-stable harvesters, bi-stable models generate higher output voltage under white noise excitation when the excitation variance is substantial. Bi-stable harvester’s response to colored noise has not been comprehensively analyzed due to inherent complexities. Interested readers can refer (Daqaq 2011; Masana & Daqaq 2013) to understand the limited research in this regard.

Non-linear configurations have been successfully analyzed numerically, analytically and experimentally to demonstrate conditional benefits through an enhanced frequency bandwidth. Dependence/prior knowledge on excitation amplitude and non-uniqueness of solution due to bends in frequency response seem to be issues of primary importance. As a result it is critical that the excitation source is analyzed correctly as model response varies considerably with respect to excitation source. This is essential to appropriate selection of techniques to model responses and analyze performance. This stresses the fact that only under certain design conditions a bi-stable outperforms a mono-stable or even a linear device. Further research in this regard is necessary to provide definitive design guidelines to guarantee superior performance of non-linear harvesters over linear resonant devices.
1.3. **Research Objective**

The need for an alternative self-sufficient power generator to replace or supplement battery power to operate low power consuming electromechanical devices was highlighted in Section 1.1. It is imperative that the energy source that powers these low power consuming electronics do not increase the overall mass of the device significantly. However, current piezoelectric energy harvesters exhibit significantly lower power-to-weight ratio when compared to a battery source the harvesters seek to replace. In lieu of these concerns, the primary objective of the dissertation is to increase the specific power or power-to-weight ratio of piezoelectric energy harvesters. The objective is achieved by integrating cellular honeycomb structures with existing piezoelectric energy harvesting configurations. One of the most common configurations, a piezoelectric bimorph is chosen to demonstrate the merits of this design. The improved design replaces the traditional solid homogenous continuous substrate with lightweight cellular honeycomb structures. The extremely low relative density and low effective stiffness properties of cellular honeycomb structures make them an attractive option to partner structural configurations in order to decrease the total mass of the structure, without severely compromising on strength. As a part of the objective, the influence of different honeycomb unit cell design parameters on power generation is studied to determine optimum design for highest specific power. Power generation from the piezoelectric bimorph with honeycomb substrate is compared to the conventional homogeneous solid core bimorph to highlight the advantages.

The second component of this dissertation focuses on the analysis of an uncoupled equivalent circuit model representation of piezoelectric generators. Vibration based energy harvesting from piezoelectric generators is an electromechanical problem that involves solving coupled governing equations. However, open circuit voltage can be determined quickly from either analytical or finite element models without the need for an external load resistor. Hence,
the second objective of the dissertation is to evaluate the efficacy of a method to calculate power generation from the open circuit voltage developed on the piezoelectric layer of the harvester. The equivalent model represents the coupled piezoelectric energy harvesting problem as an open circuit voltage driven equivalent circuit. Solving the uncoupled problem to evaluate power generation once the open circuit voltage has been determined can be used to rapidly conduct parametric and optimization studies for energy harvesting applications. The equivalent model representation is evaluated by comparing the power response obtained using the equivalent model against the electromechanical solution of a fully coupled piezoelectric energy harvesting problem. Potential applications where the equivalent model representation can be accurately used to evaluate power response is also analyzed as a part of evaluating the efficacy of the open circuit voltage driven equivalent model.

1.4. Thesis Organization

The remainder of the document is organized as follows. Chapter 2 provides a brief overview on cellular structures with emphasis on elastic behavior and the deformation mechanisms in hexagonal honeycomb structures. The chapter provides information on the effective elastic properties and design of honeycomb structures. Electromechanical modeling of piezoelectric harvesters in general from fundamental principles is presented in Chapter 3. The model is then specialized to a traditional piezoelectric bimorph with solid substrate before developing the honeycomb substrate piezoelectric bimorph utilizing the effective elastic properties detailed in Chapter 2. Power generation, frequency profile, honeycomb cell parameter influence on power harvesting and comparison to the traditional solid core bimorph under different excitation is also discussed in Chapter 3. In Chapter 4, a computationally expedited method to evaluate power generation by representing the fully coupled piezoelectric energy harvesting problem as an open
circuit voltage driven equivalent circuit, is investigated. Correction factors and the
efficacy/accuracy of the equivalent model is studied by comparing it to a fully coupled distributed
parameter analytical solution. A broader impact and future directions to extend this study are
discussed in Chapter 5.
Chapter 2: Review of the elastic behavior of cellular structures

Cellular solids are clusters of cells, derived from the Latin word ‘cella’ meaning a small compartment or enclosed space. They are formed through the interconnections of individual cells or unit cells filling space when their edges or faces are joined and packed together. These interconnections may be in an orderly periodic fashion or could be in a random chaotic arrangement. The unique properties resulting from the cellular structure have been widely exploited leading to the use of cellular solids in a wide variety of engineering applications. Some of the naturally existing examples of such structure include wood and corals.

Periodic cellular structures can be represented in two and three dimensional form. An example of a two-dimensional periodic structure is the hexagonal honeycomb made by bees. It is for this reason that 2D periodic structures are typically termed honeycombs. Three dimensional cellular structures are referred to as foams. If the cells are connected through open faces, they are known as open-celled while if each cell is enclosed completely they are known as closed-cells, with partially open/closed cells also exist. Discussion pertaining to foams is beyond the scope of this chapter while relevant information regarding the mechanics of deformation that makes periodic honeycomb structures an attractive and effective option to partner vibrating piezoelectric structures for energy harvesting applications is provided.

2.2. Characteristic features of the unit cell.

The most important unique feature of cellular structures is the relative density $\rho' = \rho'/\rho_s$, where $\rho'$ is the density of the cellular structure and $\rho_s$ is the density of the corresponding solid, the cellular structure is obtained from. The discussion in this section is limited to hexagon shaped
honeycomb cellular structures, while triangle, square, diamond shaped honeycombs have also been manufactured (Wang & McDowell 2004). A schematic of the unit cell of hexagonal honeycombs in displayed in Figure 8 (a) below, where $t_{\text{wall}}$, $h$, $l$ and $\theta$ refer to the cell wall thickness, vertical cell height, inclined cell length and cell angle respectively. If $h=l$ the hexagon is termed as regular while irregular hexagons have $h/l \neq 1$. Recently, an enhanced core design was suggested by changing honeycomb geometry with in-plane negative Poisson’s ratio, called auxetic honeycombs which requires a negative cell angle, which showed improved static and dynamic properties (Scarpa & Tomilson 2000; Ruzzene & Scarpa 2003). In order to induce the auxetic effect where a tensile force along $x_1$ direction increases the dimension along $x_2$ axis, the cell angle $\theta$ is designed to be negative. The unit cell of a conventional regular hexagon honeycomb and the more recent auxetic honeycomb is shown in Figure 8 (a) and (b) respectively. The unit cell length and height are defined by $L_u$ and $H_u$. A unit cell can be defined as the smallest component by volume which upon repetition/translation makes the final structure.
Firstly, the relative density can be derived from geometric and trigonometric principles. From the hexagonal honeycomb cell below, the ratio of the honeycomb density to its solid counterpart can be evaluated by,

\[
\frac{\rho}{\rho_s} = \frac{A_{\text{wall}}}{A_{\text{cell}}} \tag{3}
\]

where \( A_{\text{wall}} \) refers to the area of the cell wall while \( A_{\text{cell}} \) corresponds to area of the cell. The area of the cell wall can be determined by,

\[
A_{\text{wall}} = \frac{(2h + 4t) t_{\text{wall}}}{2} \tag{4}
\]

where the division by 2 is due to the fact the each cell wall is shared by two cells. On the other hand, the area of the hexagonal cell is,
\[ A_{\text{eff}} = (h \ 2l \cos \theta) + 2(l \cos \theta \ l \sin \theta) = 2l \cos \theta(h + l \sin \theta) \] \hspace{1cm} (5)

The relative density can then be determined by,

\[ \frac{\rho'}{\rho_s} = \frac{(\alpha + 2)\beta}{2 \cos \theta(\alpha + \sin \theta)} \] \hspace{1cm} (6)

where \( \alpha = h/l \) and \( \beta = t_{\text{wall}}/l \) respectively. It is assumed that \( t<<l \) which essentially means low relative density in addition to small deformation assumption neglecting small changes in geometry. The relations need to be modified and refined by including shear and axial deformation in addition to cell wall bending, if \( t/l >> 1/4 \) or if strains are greater than 20%. The deformation mechanism in a honeycomb structure depends on the type of loading exerted on the structure. The honeycombs can be deformed in-plane (\( X_1-X_2 \) plane) by applying in-plane forces or it can be deformed out-of-plane when loaded along the \( X_3 \) (through the thickness) direction.

### 2.3. Deformation mechanism of hexagonal honeycomb structures

The deformation mechanism in a honeycomb structure depends on the type of loading exerted on the structure. The honeycombs can be deformed in-plane by applying an in-plane force or it can be deformed out-of-plane when loaded out-of-plane. The in-plane corresponds to the \( X_1-X_2 \) plane while the out-of-plane corresponds to the \( X_3 \) plane in Figure 12, while the properties exhibited in each type of loading are different and named in-plane and out-of-plane properties. The in-plane stiffness and strength is much lower as the cell walls undergo bending unlike in an out-of-plane loading where the cell walls require axial extension or compression.
As the honeycombs are compressed in-plane the cell walls initially bend resulting in a linear-elastic behavior (provided the material is linear elastic). Above a critical strain the cells collapse till the opposing cell walls begin to touch each other. The cell collapse depend on the material of the cell with elastomers undergoing buckling, ductile metals yielding, and brittle materials resulting in fracture. Once the cell collapse ends, the structure undergoes densification leading to rapid increase in stiffness. Under in-plane tensile loading, the deformation mechanism is quite similar to compression. As a tensile load is applied, the cell walls initially bend and beyond a critical strain the cells yield plastically or fails (no buckling) that is exhibited by the entire honeycomb structure as well. Note that under tensile and compressive in-plane loading, an increase in relative density of the honeycomb increases the relative cell wall thickness resulting in a larger resistance to cell wall bending and collapse leading to higher stiffness. The deformation
mechanism under out-of-plane compression or tension is a relatively simpler one. The honeycombs suffer compression or extension in the $X_3$ direction under compressive and tensile loads respectively resulting in elastic or plastic buckling before failure with much higher stiffness than in-plane stiffness due to the lack of cell wall bending.

The stress-strain relationship under the linear elastic deformation can be represented by,

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{23} \\
\gamma_{13} \\
\gamma_{12}
\end{bmatrix} = \begin{bmatrix}
1 / E_1^* & -\nu_{12} / E_1^* & -\nu_{13} / E_2^* & 0 & 0 & 0 \\
-\nu_{12} / E_1^* & 1 / E_1^* & -\nu_{23} / E_1^* & 0 & 0 & 0 \\
-\nu_{13} / E_1^* & -\nu_{23} / E_1^* & -1 / E_3^* & 0 & 0 & 0 \\
0 & 0 & 0 & 1 / G_{23}^* & 0 & 0 \\
0 & 0 & 0 & 0 & 1 / G_{13}^* & 0 \\
0 & 0 & 0 & 0 & 0 & 1 / G_{12}^*
\end{bmatrix} \begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix}
\] (7)

where $E$, $G$, and $\nu$ denote the Young’s modulus, shear modulus and Poisson’s ratio respectively. The orthotropic honeycomb properties are defined through 9 independent elastic constants with 4 in-plane ($E_1^*, E_2^*, G_{12}^*, \nu_{12}^*$) and 5 out-of-plane ($E_3^*, G_{13}^*, G_{23}^*, \nu_{13}^*, \nu_{23}^*$) independent constants.

2.4. In-plane elastic behavior of hexagonal honeycombs

The in-plane properties exhibited by the honeycombs when loaded in the $X_1$-$X_2$ plane, in the $X_1$ or $X_2$ direction as shown in Figure 11. Properties pertaining to only linear-elastic deformation are discussed, as other deformation mechanisms involving elastic buckling, plastic collapse, brittle failure, viscoelastic deformation, creep and creep buckling is beyond the scope of this document but can be found in (Gibson & Ashby 1997). Note that only hexagonal honeycombs are discussed as they are of primary interest due to reasons discussed previously, while properties of triangular and square honeycombs can also be found in (Gibson & Ashby
1997). Only uniaxial loading case is discussed here, but the same procedure can be extended to bi-axial loading as well.

The elastic properties include Young’s modulus $E^*$, shear modulus $G^*$ and Poisson’s ratio $\nu^*$. The elastic moduli may be isotropic or anisotropic partially depending on whether the hexagon is regular or irregular. If $h=l$ the hexagon is termed as regular while irregular hexagons have $h/l \neq 1$. In the case of an irregular honeycomb or varying cell wall thickness four independent elastic moduli has to be defined namely $E_1^*, E_2^*, G_{12}^*, \nu_{12}^*$. In contrast, if the hexagon is regular and cell wall thickness is constant across all cells, only two independent elastic moduli ($E^*$ and $G^*$) need to be defined.

The cell wall bending due to in-plane tensile or compressive loading can be described through $E_1^*, E_2^*, G_{12}^*, \nu_{12}^*$ and $\nu_{21}^*$. They are not completely independent and are related by,

$$E_1^* \nu_{21}^* = E_2^* \nu_{12}^*$$

Figure 11: Representation of a uniaxial in-plane tensile loading.

rendering the number of independent variables to four. If the load is in the $X_1$, the Young’s modulus in the $X_i$ direction is calculated using Hooke’s law, $E_i^* = \sigma_i / \epsilon_i$. Similarly,
Young’s modulus in the $X_2$ direction when the force is along $X_2$ is evaluated by $E'_2 = \sigma_2 / \epsilon_2$. The strain in $X_1$ and $X_2$ direction can then be calculated in terms of the cell wall deflection from standard beam theory while the stresses can be calculated in terms of the moment tending to bend the cell. The resulting effective Young’s modulus along $X_1$ is calculated as,

$$E'_1 = E_s \beta^3 \frac{\cos \theta}{\sin^2 \theta (\alpha + \sin \theta)}$$

where, $E_{s}$ is the Young’s modulus of the solid the honeycomb is made from. Through similar derivation,

$$E'_2 = E_s \beta^3 \frac{(\alpha + \sin \theta)}{\cos^3 \theta}$$

Note that the effect of axial and shear loads in addition to bending components is neglected in the derivation of the modulus of elasticity due to the assumption of small $t_{wall}/l$ and small deflections. With the help of strains evaluated in the $X_1$ and $X_2$ directions Poisson’s ratio for the loading $X_1$ is evaluated by,

$$\nu'_{12} = -\frac{\epsilon'_2}{\epsilon'_1} = \frac{\cos^2 \theta}{\sin \theta (\alpha + \sin \theta)}$$

while, Poisson’s ratio when the loading is along $X_2$ is given by the reciprocal of $\nu'_{12}$.
As a special case, if the hexagon is regular and cell wall thickness is uniform across all cell walls, then the Poisson ratios conveniently reduces to $\nu_{12}^* = \nu_{21}^* = 1$. Note that in structures where the cell angle is $\theta < 0$, the Poisson’s ratio becomes negative implying that unlike in the normal case where tension in one direction, say $X_2$ direction, causes contraction in $X_1$ direction, due to negative Poisson’s ratio, an expansion occurs along $X_1$. Such structures are termed Auxetic structures and have also been manufactured with honeycomb profiles called Auxetic honeycombs. Along with the two Young’s moduli and two Poisson’s ratios, a fifth elastic constant namely the shear modulus ($G_{12}^*$) completely describes the in-plane elastic properties of honeycomb structures.

As the honeycomb is sheared, the shear modulus can be evaluated by first determining the shear deflection which can then be used to solve for shear strain angle $\gamma$. This along with the readily available shear stress, $\tau = F / (2b\cos\theta)$ can then be made use of to compute shear modulus, $G_{12}^* = \tau / \gamma$ resulting in,

$$G_{12}^* = E_{1}r^3 \frac{(\alpha + \sin \theta)}{\alpha^2(1 + 2\alpha)\cos\theta}$$

Note that the relations provided for the five in-plane elastic constants generally hold good although they are sensitive to cell wall thickness and cell angle. These constants agree to a good degree with experiments on elastomeric and metal honeycombs (L J Gibson et al. 1982; Gibson 1981) for $t_{wall}/l < 0.4$. 

The linear elastic response of honeycombs under a biaxial loading (loading in both $X_1$ and $X_2$ directions) can also be described in a similar fashion. Unlike in uniaxial loading, axial deformation is not neglected and adds to bending displacements. Interested readers are encouraged to read (Gibson & Ashby 1997) to gain information regarding the description of elastic constants for biaxial loads. In addition, their response to elastic buckling, plastic collapse, brittle failure, viscoelastic deformation, creep and creep buckling can also be found.

2.5. Out-of-plane elastic properties of hexagonal honeycombs

Out-of-plane elastic response arises when the honeycombs are loaded in the $X_3$ direction. As discussed previously unlike in-plane loadings, out-of-plane loadings deform honeycombs through extension or compression of the cells and not through cell wall bending. This results in much higher stiffness exhibited when loaded out-of-plane. Elastic constants under out-of-plane loading along with the previously discussed in-plane elastic constants completely describe the elastic properties of honeycomb structures. As with in-plane loading it is assumed the $t_{wall} \ll l$ and that the cell wall thickness is uniform.

Similar to in-plane elastic constants, Young’s modulus, Poisson’s ratio and shear modulus due to load along $X_3$ define the linear elastic out-plane honeycomb properties. The Young’s modulus along $X_3$ is simply the Young’s modulus of the solid scaled by the load bearing section and can be estimated by,

$$E_3' = E_s \frac{(\alpha + 2)\beta}{2 \cos \theta (\alpha + \sin \theta)}$$ (14)
It is interesting to note that the ratio of the Young’s modulus of the honeycomb and the solid is equivalent to the ratio of their densities as seen in Eq. (6). In addition four out-of-plane Poisson’s ratios can be defined that conveniently conform to the following simple relations,

\[ \nu_{31}^* = \nu_{32}^* = \nu_s \]

\[ \nu_{13}^* = \frac{E_s^*}{E_s} \nu_s \approx 0; \nu_{23}^* = \frac{E_s^*}{E_s} \nu_s \approx 0 \]

It is seen that the two Poisson ratios \( \nu_{31}^* \) and \( \nu_{32}^* \) are just equivalent to the Poisson ratio of the solid while \( \nu_{13}^* \) and \( \nu_{23}^* \) are approximately zero, derived from the reciprocal theorem discussed previously in Eq. (8). The exact estimation of out-of-plane shear modulus (\( G_{13}^* \) and \( G_{23}^* \)) is not as straightforward the Young’s modulus and Poisson ratios, as the stress distribution in a honeycomb under shearing is not simple. Initially plane honeycomb may not remain plane as each cell may suffer non-uniform deformation. However, an upper and lower bound within which the exact shear modulus lies can be determined using the theorems of minimum potential energy and minimum complimentary energy respectively. Both theorems involve strain energy calculations strain and stress distribution estimation associated with strain distribution allowing deformation and stress distribution satisfying equilibrium conditions. More information regarding the theorem of minimum potential energy and minimum complimentary energy can be found in (Sololnikoff 1956; McClintock & Argon 1966), while detailed derivation pertaining to the application of these theorems to derive the shear modulus terms can be found in (Kelsey et al. 1958; Gibson & Ashby 1997). Note that the upper and lower bound to estimate \( G_{13}^* \) conveniently
reduces to a single value and hence can be calculated exactly. However, only a range can be approximated to estimate $G'_{23}$. The final expressions to estimate both shear moduli is given by,

$$G_{i3} = G_i \frac{\beta \cos \theta}{\alpha + \sin \theta}$$ (17)

$$G_i \frac{1}{2} \frac{\beta(\alpha + 2 \sin^2 \theta)}{\cos \theta(\alpha + \sin \theta)} \geq G_{23} \geq G_i \frac{\beta(\alpha + \sin \theta)}{\cos \theta(1 + 2\alpha)}$$ (18)

where $G_i$ is the shear modulus of the solid. Along with the four independent in-plane elastic properties five more independent out-plane properties totaling nine independent elastic constants ($E_1', G_{13}', G_{23}', v_{13}', v_{23}'$) completely describe the linear elastic deformation of honeycomb structures. The elastic properties of a general irregular hexagonal honeycomb structure are summarized in Table 1. Note that the elastic constants presented for both in-plane and out-of-plane loading have been validated and are well supported by experiments (Patel & Finnie 1970; Masters & Evans 1996; Abd El-Sayed et al. 1979; L. J. Gibson et al. 1982; Gibson & Ashby 1997)

The elastic properties of a general (regular or irregular, conventional or auxetic) hexagonal honeycomb structure are summarized in table 1 obtained using cellular materials theory (see (Gibson & Ashby 1997) for details). Note that the elastic constants presented for both in-plane and out-of-plane loading have been validated and are well supported by experiments (Patel & Finnie 1970; Masters & Evans 1996; Abd El-Sayed et al. 1979; L. J. Gibson et al. 1982; Gibson & Ashby 1997). These effective elastic properties generally hold strong for low relative density, for $t_{wall}/l < 0.4$, while refinements to the model can be made by including shear and axial deformation in addition to bending deformation.
Table 1: Effective elastic constants of hexagonal honeycomb structures

<table>
<thead>
<tr>
<th>Effective tensile modulus</th>
<th>Effective shear modulus</th>
<th>Effective Poisson’s ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1^* = E_1 \beta^1 \frac{\cos \theta}{\sin^2 \theta (\alpha + \sin \theta)}$</td>
<td>$G_{12}^* = E_1 \beta^1 \frac{(\alpha + \sin \theta)}{\alpha^2 (1 + 2\alpha) \cos \theta}$</td>
<td>$\nu_{12}^* = \frac{\nu_{12}}{E_1} = \frac{\cos^2 \theta}{\sin \theta (\alpha + \sin \theta)}$</td>
</tr>
<tr>
<td>$E_2^* = E_2 \beta^1 \frac{(\alpha + \sin \theta)}{\cos^3 \theta}$</td>
<td>$G_{13}^* = G_1 \frac{\beta \cos \theta}{\alpha + \sin \theta}$</td>
<td>$\nu_{23}^* = \frac{\nu_{23}}{E_2} = \frac{\sin \theta (\alpha + \sin \theta)}{\cos^3 \theta}$</td>
</tr>
<tr>
<td>$E_3^* = E_3 \frac{(\alpha + 2)\beta}{2 \cos \theta (\alpha + \sin \theta)}$</td>
<td>$G_2 \frac{1}{2 \cos \theta (\alpha + \sin \theta)} \geq G_{13} \geq G_3 \frac{\beta (\alpha + \sin \theta)}{\cos \theta (1 + 2\alpha)}$</td>
<td>$\nu_{31}^* = \nu_{32}^* = \nu_s$</td>
</tr>
</tbody>
</table>

$\nu_{12}^* = \frac{E_1^*}{E_s} \nu_s \approx 0$ $\nu_{23}^* = \frac{E_2^*}{E_s} \nu_s \approx 0$

2.6. Design of honeycomb structures

Hexagonal unit cell design parameters namely inclined cell length ($l$), vertical cell height ($h$), cell angle ($\theta$) and cell wall thickness need to be estimated to design honeycomb structures. In this regard, depending on the application a global length and width need to be prescribed besides the number of cells intended for use. For example, under a rectangular configuration, under a fixed global length, $L$ and width, $W$, a honeycomb’s unit cell length, $L_u$ and unit cell width, $H_u$ are derived once the number of unit cells are defined which are given by $L_u = L/N_L$ and $H_u = W/N_W$ (see Figure 8), where $N_L$ and $N_W$ are the number of unit cells along the length and width, respectively. Once the unit cell length, $L_u$ and unit cell width, $H_u$ are obtained, the inclined cell length, $l$, and the vertical cell length, $h$, can then be derived from geometric and trigonometric principles given by,

$$l = \frac{L_u}{4 \cos \theta} = \frac{L/N_L}{4 \cos \theta}$$

(19)

$$h = \frac{H_u}{2} - l \sin \theta = \frac{H_u}{2} - \frac{L_u \sin \theta}{4 \cos \theta} = \frac{W/N_W}{2} - \frac{(L/N_L) \sin \theta}{4 \cos \theta}$$

(20)
Based on the above equations, example configurations for a rectangular honeycomb structure under fixed global length and width is shown in the table below. Note that the cell wall thickness does not play a role in determining the inclined cell length and vertical cell height.

Table 2: Example configurations of possible honeycomb structure design

<table>
<thead>
<tr>
<th>Example configurations</th>
<th>Number of horizontal cells, $N_L$</th>
<th>Number of vertical cells, $N_W$</th>
<th>Inclined cell length, $l$ (mm)</th>
<th>Vertical cell height, $h$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
<td>12.5</td>
<td>3</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>Configuration 2</td>
<td>18.5</td>
<td>4.5</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Configuration 3</td>
<td>25</td>
<td>6</td>
<td>0.76</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Based on different factors such as manufacturability of a particular dimension, cost and application, a particular configuration can be selected. A 2D representation of Configuration 1 is shown in the figure below. The effective elastic properties of honeycomb structures discussed in this chapter depend more strongly on the cell shape and not the cell size. This can be seen by observing the equations summarized in table 1. Hence, the number of cells along the length and width does not play a significant role when evaluating the effective properties of the honeycomb structure.
Once, the design parameters are determined, the effective elastic properties can then be calculated using the expressions for the elastic constants in table 1. The cell wall thickness, $t_{\text{wall}}$, of a honeycomb is the most difficult part to manufacture due to the smallest dimension. Therefore a reasonable dimension realizable through conventional honeycomb manufacturing methods such as conventional forming processes and the wire electrical discharge machining (EDM) is determined beforehand. Moreover, the dimension of $t_{\text{wall}}$ should be carefully designed to conform to the condition $\rho^*/\rho_s \not> 1$ while typically the ratio $t_{\text{wall}}/l < 0.3$ is maintained to qualify the material as cellular (Gibson & Ashby 1997).
Chapter 3: Electromechanical modeling of piezoelectric harvesters with honeycomb substrate

In this chapter, an electromechanical model of piezoelectric structures is developed from fundamental principles. Governing equations, strong from, and weak form formulations are detailed through an energy variation approach (Hamiltonian’s principle) as well discussed from force equilibrium stand point. The derived governing equations are then specialized to one of the most common piezoelectric energy harvesting configuration, a piezoelectric bimorph with solid homogenous continuous substrate. Design parameters to be modified, to replace the solid homogenous substrate with the effective properties of honeycomb structures derived earlier is then analyzed. Power generation and frequency profile when using the honeycomb substrate piezoelectric bimorph is discussed. The results are then compared with the power response of the traditional solid core piezoelectric bimorph. The advantages in using the honeycomb substrate bimorph under different excitation frequencies are analyzed. In addition, the influence of unit cell parameters on power generation and frequency is also examined to determine optimum design for highest specific power.

3.1. Weak form formulation of the piezoelectric problem

The governing equations for the piezoelectric boundary value problem can obtained through an energy variation (Hamiltonians principle) or through a force equilibrium analysis.

The coupled governing equation for piezoelectric elements for using an energy variation method neglecting the magnetic effects is represented in terms of kinetic energy ($T_k$), internal potential energy ($U$), electrical energy ($W_e$) and external work ($W$) as,
\[
\delta W = \int_\Omega \left[ \delta (T - U + W_e) + \delta W \right] dt = 0
\] (21)

where \( \delta \) represents the variation function. The individual energy terms are defined as,

\[
T = \frac{1}{2} \int_\Omega \rho \dot{\mathbf{u}}^T \dot{\mathbf{u}} d\Omega
\] (22)

\[
U = \frac{1}{2} \int_\Omega S^T T d\Omega
\] (23)

\[
W_e = \frac{1}{2} \int_{\Gamma_e} E^T D d\Omega
\] (24)

\[
\delta W = \sum_{k=1}^{\Omega} \delta \mathbf{u}_k f_k + \sum_{j=1}^{\Omega} \delta \phi_j q_j
\] (25)

In the above set of energy equations, \( S \) and \( T \) represent the strain and stress matrices, while \( E \) and \( D \) are electric field and electric displacement matrices. The displacement is denoted by \( \mathbf{u}(x,t) \), while the scalar potential is denoted by \( \phi(x,t) \) and density denoted by \( \rho \). The discrete forces and charges are represented by \( f \) and \( q \) respectively. The external work term can may also consist of surface traction \( t \) acting on the boundary,

\[
\delta W = \sum_{k=1}^{\Omega} \delta \mathbf{u}_k f_k + \sum_{j=1}^{\Omega} \delta \phi_j q_j + \int_{\Gamma} \mathbf{u} \cdot t d\Gamma
\] (26)

Piezoelectric energy conversion can be better understood with the help of coupled piezoelectric constitutive relations shown below in tensorial notation as,

\[
\begin{bmatrix}
S \\
D
\end{bmatrix} =
\begin{bmatrix}
\mathbf{s}^T & \mathbf{d}^T \\
\mathbf{d} & \mathbf{\varepsilon}^T
\end{bmatrix}
\begin{bmatrix}
T \\
E
\end{bmatrix}
\] (27)

where, the parameters \( \mathbf{d} \) and \( \mathbf{\varepsilon} \) denote piezoelectric (coupling) strain coefficient and dielectric constant matrices respectively with \( \mathbf{s} \) denoting the compliance matrix. Superscripts \( E \) and \( T \) stand indicate that corresponding quantities are measured at constant electric field and
stress respectively. The above vectorial notation is expanded to a full matrix form below where
the symmetries of the transversely isotropic material is applied.

\[
\begin{pmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{pmatrix} =
\begin{pmatrix}
\frac{E}{\rho} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{E}{\rho} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{E}{\rho} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{E}{\rho} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{E}{\rho} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{E}{\rho}
\end{pmatrix}
\begin{pmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{pmatrix}
+ \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
E_1 \\
E_2 \\
E_3
\end{pmatrix}
\]

(28)

Voight notation is used to conveniently represent the components $11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow
3$, $23 \rightarrow 4$, $13 \rightarrow 5$, $12 \rightarrow 6$. The constitutive relations can also be expressed in stress-displacement form as,

\[
\begin{pmatrix}
T \\
D
\end{pmatrix} =
\begin{pmatrix}
e^T & e^T \\
e & e^T
\end{pmatrix}
\begin{pmatrix}
S \\
E
\end{pmatrix}
\]

(30)

where the coupling is represented through piezoelectric stress constant $e$.

Now, by first substituting the constitutive relations into the individual energy equations, the first variation of (21) results in,

\[
VI = \int_{\Omega} \left( \frac{1}{\rho} \delta \dot{u} \dot{u} d\Omega - \int_{\Omega} \delta S' (eS - eE) d\Omega + \int_{\Omega} \delta E' (e' S + SE) d\Omega \right) + \sum_{i=1}^{n} \delta \dot{u}_i f_i + \sum_{j=1}^{m} \delta p_j q_j dt = 0
\]

(31)
The essential (Dirchlet) boundary condition that must be explicitly stated in the weak
form requires that,

$$\delta u_i = 0 \quad \text{on } \Gamma_a$$  \hspace{1cm} (32)

$$\delta \phi = 0 \quad \text{on } \Gamma_e$$  \hspace{1cm} (33)

Note that the natural boundary condition is embedded in the external work term presented
above. Using the relation,

$$\int_0^h \frac{d}{dt}(\delta u \dot{u}) dt = \int_0^h (\delta u \ddot{u}) dt + \int_0^h (\delta \dot{u} \ddot{u}) dt$$  \hspace{1cm} (34)

The variational indicator equation, Eq (31) can be rewritten as,

$$VI = \int_{\Omega} \left[ -\rho \delta u \ddot{u} \right] d\Omega - \int_{\Omega} \delta S' (cS - eE) d\Omega + \int_{\Omega_p} \delta S' (e' S + SE) d\Omega_p + \sum_{k=1}^{nf} \delta u^f + \sum_{j=1}^m \delta \phi_j q_j \right] dt = 0$$  \hspace{1cm} (35)

as the variation must disappear at the boundary. Representing the above equation
separately in terms mechanical and electrical domains yield,

$$\int_{\Omega} \left[ -\rho \delta u \ddot{u} \right] d\Omega - \int_{\Omega} \delta S' (cS - eE) d\Omega + \sum_{k=1}^{nf} \delta u^f \right] dt = 0$$  \hspace{1cm} (36)

$$\int_{\Omega_p} \left[ \int_{\Omega} \delta S' (e' S + SE) d\Omega_p + \sum_{j=1}^m \delta \phi_j q_j \right] dt = 0$$  \hspace{1cm} (37)

In order for the above equations to be satisfied by any weighting function, the integrand
must be equal to the right hand side (zero) resulting in,

$$\int_{\Omega} \rho u \ddot{u} \ddot{u} d\Omega + \int_{\Omega} \delta S' (cS - eE) d\Omega = \sum_{k=1}^{nf} \delta u^f$$  \hspace{1cm} (38)
The forcing functions namely the forces and charge on the right hand side of the above equations can be represented as summation of body and surfaces components. Consequently, the final weak form of the piezoelectric problem is represented as,

\[
\int_{\Omega_p} \delta E' (e' S + \text{SE}) d\Omega_p = -\sum_{j=1}^{n} \delta \phi_j q_j
\]

(39)

The weak form can also be obtained through a force equilibrium approach. For a piezoelectric body with volume \(\Omega\) the field equations comprises of the governing equation for elastodynamics and electrostatics given by,

\[
\int_{\Omega} \rho \delta \ddot{u} \ddot{u} d\Omega + \int_{\Omega} \delta S' (cS - \text{eE}) d\Omega = \int_{\Gamma} \delta u' f \Gamma d\Gamma + \int_{\Omega} \delta u' b d\Omega
\]

(40)

\[
\int_{\Omega_p} \delta E' (e' S + \text{SE}) d\Omega_p = -\int_{\Gamma} \delta \phi_j Q_j d\Gamma - \int_{\Omega} \delta \phi_j q_j d\Omega
\]

(41)

In the above equations, \(T_{ij, j}\) denotes the divergence of the symmetric stress tensor, while \(D_{ij}\) denotes the divergence of the electric displacement vector. The set of body force and charge is represented by \(b\) and \(q\) respectively. Note that absence of body forces and charge under equilibrium reduces the above equations to

\[
T_{ij, j} = 0 \quad \text{in} \quad \Omega
\]

(42)

\[
D_{ij} = 0 \quad \text{in} \quad \Omega
\]

(43)

(44)

(45)

For convenience, the coupled constitutive relations discussed earlier are presented below in index notation form as
\[ T_i = C_{\sigma i j} S_{\sigma j} - \varepsilon_{i\sigma} E_{\sigma} \quad (46) \]
\[ D_i = \varepsilon_{\sigma i j} S_{\sigma j} + \varepsilon_{\sigma} E_{\sigma} \quad (47) \]

Note that the electric field and the symmetric strain tensor are related to the primary variables \( \phi \) and \( u \) through,
\[ E_i = -\phi_i \quad (48) \]
\[ S_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) \quad (49) \]

Although not presented here, the weak form derived from the energy variational approach (Hamiltonian’s principle) can be formulated through a weighted residual method.

### 3.2. Constitutive relations in reduced form

If the coupled piezoelectric behavior of thin beams based on Euler-Bernoulli is assumed then the only non-zero stress component is the 1D bending stress \( T_j \) rendering other stress components \( T_2 = T_3 = T_4 = T_5 = T_6 = 0 \). In addition, under the assumption that the electrode pair covers the piezoelectric faces along the length which is perpendicular to the 3-direction, the above constitutive equations can be reduced to,
\[
\begin{bmatrix}
S_i \\
D_j
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{ii}^E & d_{31} \\
d_{31} & \varepsilon_{33}^T
\end{bmatrix}
\begin{bmatrix}
T_i \\
E_3
\end{bmatrix}
\quad (50)
\]

The above constitutive relation that is expressed in strain-electric displacement form can also be written in the form as stress-electric displacement form as,
\[
\begin{bmatrix}
T_i \\
D_j
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{ii}^E & -\varepsilon_{31} \\
\varepsilon_{31} & \varepsilon_{33}^T
\end{bmatrix}
\begin{bmatrix}
S_i \\
E_3
\end{bmatrix}
\quad (51)
\]
where super script ‘$S$’ denotes measurement of the quantity at constant strain, $c_{11}$ is the piezoelectric bending stiffness and $e$ is the piezoelectric stress constant. The relation between the stiffness and compliance, piezoelectric strain and stress constants, and the dielectric term measured at constant stress and strain is given by,

$$
\epsilon_{11}^E = \frac{1}{s_{11}^E}, \quad e_{31} = \frac{d_{31}}{s_{11}^E}, \quad \varepsilon_{33}^S = \varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}
$$  \tag{52}

For a moderately thick beam, in addition to the bending stress $T_1$ the shear stress component $T_5$ would also be a non-zero entity rendering $T_2 = T_3 = T_4 = T_6 = 0$ and the constitutive relations in strain-electric displacement form can be written as,

$$
\begin{bmatrix}
S_1 \\
S_5 \\
D_3
\end{bmatrix} =
\begin{bmatrix}
\epsilon_{11}^E & 0 & d_{31} \\
0 & \epsilon_{55}^E & 0 \\
d_{31} & 0 & \varepsilon_{33}^T
\end{bmatrix} \begin{bmatrix} T_1 \\ T_5 \\ E_3 \end{bmatrix}
$$  \tag{53}

The relation between the stiffness and compliance terms, piezoelectric stress and strain coefficients and the dielectric terms at constant stress and strain from the Euler-Bernoulli assumptions is valid in this case as well while in addition $c_{55}^E = \frac{1}{s_{55}^E}$ and $T_5 = \kappa c_{55}^E S_s$ which is the shear stress included by Timoshenko (Timoshenko 1922; Timoshenko 1921), where $\kappa$ is the shear correction factor (Mindlin 1951; Mindlin 1952; Cowper 1966; Kaneko 1975; Timoshenko 1922; Timoshenko 1921).

If the piezoelectric structure is modeled as thin plate using classical Kirchoff-Love plate theory the non-zero stress components would now be $T_1$, $T_2$, and $T_5$ rendering $T_3 = T_4 = T_6 = 0$ and the constitutive relations in strain displacement form can be given by,
\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_6 \\
D_1
\end{bmatrix} =
\begin{bmatrix}
S_{11}^E & S_{12}^E & 0 & d_{31} \\
S_{12}^E & S_{11}^E & 0 & d_{31} \\
0 & 0 & S_{66}^E & 0 \\
d_{31} & d_{31} & 0 & E_{33}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_6
\end{bmatrix}
\]

(54)

The relation between the reduced compliance and stiffness components are given by,

\[
\begin{align*}
\varepsilon_{11}^E &= \frac{S_{11}^E}{(S_{11}^E + S_{12}^E)(S_{11}^E - S_{12}^E)}, \\
\varepsilon_{12}^E &= \frac{-S_{12}^E}{(S_{11}^E + S_{12}^E)(S_{11}^E - S_{12}^E)}, \\
\varepsilon_{66}^E &= \frac{1}{S_{66}^E}
\end{align*}
\]

(55)

while the piezoelectric stress and strain coefficients are related through

\[
\varepsilon_{33} = \frac{d_{31}}{S_{11}^E + S_{12}^E}
\]

(56)

and the dielectric terms measured under constant stress and strain is linked by,

\[
\varepsilon_{33}^2 = \varepsilon_{33}^T = \frac{2d_{31}^2}{S_{11}^E + S_{12}^E}
\]

(57)

3.3. Electromechanical modeling of honeycomb core piezoelectric bimorph

As discussed previously, to increase the power to weight ratio of piezoelectric energy harvesters, the solid continuous core is replaced with honeycomb core. To illustrate this idea, the bimorph piezoelectric cantilever beam, one of the most common physical configurations in piezoelectric energy harvesting is used as an example and the idea is represented in Figure 13. However, the ability of honeycomb substrates to replace the solid substrate is not limited to bimorph configuration only but any configuration that utilizes an elastic solid substrate.
Figure 13. Schematic of piezoelectric cantilever bimorph with the piezoelectric facesheets sandwiched between a) solid continuous brass substrate and b) periodic regular hexagonal honeycomb structures.

The electromechanical model of a piezoelectric energy harvester has been developed by many through global distributed analytical and finite element approximation of the weak form derived earlier. For this study, the electromechanical modeling of the piezoelectric bimorph with honeycomb substrate under base excitation is developed through a distributed parameter approach using Rayleigh-Ritz discretization technique. In addition, a 3D finite element model of the same problem is developed.

The Rayleigh-Ritz approach approximates the unknown variables namely relative displacement and electric potential. The displacement is expressed the sum of $nr$ individual mode shapes $\psi_{ri}(x)$ multiplied by a generalized mechanical coordinate $r_i(t)$, while the electric potential for each of the $nq$ electrode pairs is expressed as a function of potential distribution $\psi_{vj}(x)$ multiplied by a generalized electrical coordinate $v_j(t)$ given by,

$$u(x,t) = \sum_{i=1}^{nr} \psi_{ri}(x)r_i(t) = \psi_r(x)r(t)$$  \hspace{1cm} (58)

$$\phi(x,t) = \sum_{j=1}^{nq} \psi_{vj}(x)v_j(t) = \psi_v(x)v(t)$$  \hspace{1cm} (59)
The piezoelectric bimorph is assumed to be a thin beam. Euler-Bernoulli beam theory allows for the axial strain to be expressed in terms of the beam neutral axis displacement and distance from the neutral axis.

\[ S(x, t) = -\gamma \frac{\partial^2 u(x, t)}{\partial x^2} = -y \psi_r \dot{r}(t) \]  

(60)

where, the primes denote the derivative with respect to the axial position \( x \). Incorporating these approximations into the weak form yields the electromechanical governing equation in a compact form as,

\[ M \ddot{r} + Cr + Kr - \Theta v = B_f \]  

(61)

\[ \Theta^T r + C_p v + q = 0 \]  

(62)

where \( M, K, C_p \), and \( \Theta \) are the mass, stiffness, damping, and coupling matrices, while \( v \) and \( q \) are the voltage and charge developed. Subscripts ‘\( s \)’ and ‘\( p \)’ throughout the remainder of the paper refer to the substrate and piezoelectric layer respectively. These matrices can be expanded as,

\[ M = \int_{V_s} \psi_s \rho_s \psi_s \, dV_s + \int_{V_p} \psi_p \rho_p \psi_p \, dV_p \]  

(63)

\[ K = \int_{V_s} (\gamma \psi_s \vec{y}) \eps_s (\gamma \psi_s \vec{y}) \, dV_s + \int_{V_p} (\gamma \psi_p \vec{y}) \eps^E (\gamma \psi_p \vec{y}) \, dV_p \]  

(64)

\[ \Theta = \int_{V_s} (\nabla \psi_s \vec{y}) \vec{e} \, (\nabla \psi_s \vec{y}) \, dV_p \]  

(65)

\[ C_p = \int_{V_p} (\nabla \psi_p \vec{y}) \eps^s (\nabla \psi_p \vec{y}) \, dV_p \]  

(66)
The problem domain $\Omega$ is divided into piezoelectric and substrate volume $V_p$ and $V_s$ respectively. The term on the right hand side of Eq. (61) represents the mechanical forcing function here is represented as set of finite discrete forces assumed to be in the form of a distributed force proportional to the base acceleration equivalent to the product of total mass and the acceleration term given by,

$$B_f = \sum_{i=1}^{n_f} \phi(x_i) f_i$$  \hspace{1cm} (67)

Under the assumption of uniform cross-section in the axial direction, the forcing function for a base excitation can also be defined through a forcing vector, $B_f$ where

$$B_f = \int_0^L m(x_u) \psi, dx_u = \int_0^L \psi, dx_u$$  \hspace{1cm} (68)

where the beam under base excitation consists of infinitely many discrete forces due to local inertias of the infinitesimal elements yielding an integral for the mechanical forcing term as seen above instead of a summation term as seen in Eq. (67) that represents a set of discrete forces. Then the mechanical structural actuation governing equation can be written as,

$$M \ddot{\ddot{w}} + C \dot{w} + K w - \Theta \dot{v} = -B_f \ddot{w}_B$$  \hspace{1cm} (69)

where $\ddot{w}_B$ is the base acceleration term.

These equations represent the electromechanical system when an electric load is not connected for power generation. Once an external load resistor ($R_l$) is connected, Eq. (62) can be modified by first taking a single time derivative throughout the equation and then using the relation $i=\dot{q}$ (where the dot represents time derivative) and $v=iR_l$. The equations can be then recast in the form,
\[
\begin{align*}
(M_s + M_p)\ddot{r}(t) + C\dot{r}(t) + (K_s + K_p) r(t) + qR\dot{q}(t) = \sum_{i=1}^{nf} \phi(x_i) f_i(t) \\
R\dot{q}(t) - \frac{\theta^T}{C_p} r(t) + \frac{q}{C_p} q(t) = 0
\end{align*}
\]

It is known from literature that to maximize the electrical response of the piezoelectric energy harvester it is desirable to drive the harvester near the fundamental resonance frequency structure (modal excitation). As a result, using a closed loop reduced scalar electromechanical model that assumes that beam motion is dominated by a single mode due to the resonant frequency of the bimorph designed to be around the input driving frequency is a fair one. Hence the electromechanical model implemented in (Liao & Sodano 2008) is adopted in this paper to capture the power response of the piezoelectric bimorph at a single mode, typically the fundamental mode, hence the selection of the geometric and material properties provided in (Liao & Sodano 2008). Assuming constant electric potential through the thickness of the piezoelectric layer and that the potential developed across the surface the piezoelectric layers is uniform, using structural modes the coupled vector quantities in Eqs. (70) and (71) can be reduced to coupled modal equations in terms of scalar variables of voltage and displacement. The mode shape integrals of the effective system parameters namely the mass \((M_s + M_p)\), stiffness \((K_s + K_p)\), coupling \(\theta\), capacitance \(C_p\), and effective input mass \(D\) which is the inertial load from a base excitation are then evaluated for a piezoelectric bimorph at a single mode and their closed form scalar expressions for a bimorph in a parallel connection are computed in (Liao & Sodano 2008) given by,
\[ M = \left( \rho_s t_s + 2\rho_p t_p \right)b \]  

(72)

\[ K = \left[ 1.0302c_i \left( \frac{t_s^3}{L^2} \right) + 2.0604c_p \left( \frac{3t_s^3 t_p + 6t_s t_p^2 + 4t_p^3}{L^2} \right) \right]b \]  

(73)

\[ \theta = -2.753d_{31} c_p b \left( \frac{t_s + t_p}{\sqrt{L}} \right) \]  

(74)

\[ C_p = 2K_c \epsilon_i b \left( \frac{L}{t_p} \right) \]  

(75)

\[ D = -0.783M \sqrt{\epsilon} \]  

(76)

Note that these reduced expressions were computed for a traditional piezoelectric bimorph with a solid continuous substrate. The global geometric parameters such as the overall length \( L \) and width \( b \) are 66.62 mm and 13.68 mm respectively while the piezoelectric (PZT-5H) and the solid substrate (brass) layer thicknesses are 0.26 mm \( (t_p) \) and 0.76 mm \( (t_s) \) respectively.

PZT-5H has a density \( \rho_p \) 7200 kg/m\(^3\), piezoelectric strain constant \( d_{31} = -320 \) pm/V, relative dielectric constant \( K_f \) of 3800 and an elastic modulus \( c_p \) of 62 GPa, while brass density \( \rho_s \) is 8700 kg/m\(^3\) with a Young’s modulus \( c_s \) of 97 GPa. The piezoelectric layers cover the entire length of the substrate while it is assumed that the entire top and bottom surfaces of the piezoelectric layers are covered by perfectly conductive continuous electrodes with negligible thickness when compared to the thickness of the piezoelectric \( (t_p) \) and substrate \( (t_s) \) layer.
Damping is imparted to the system in the form of Rayleigh’s proportional damping which is frequently employed in modal analysis due to the ease of diagonalization of the damping matrix, as it is a linear combination of mass and stiffness matrices. Proportional damping is expressed as
\[ C = \alpha_d M + \beta_d K \]
where \( \alpha_d \) and \( \beta_d \) are damping coefficients evaluated by solving the equation
\[ \zeta_i = 0.5 (\alpha_d + \omega_i \beta_d) \]
usually at the first two vibration modes \((i=1,2)\) to obtain two equations and two unknowns for the same damping ratio \((\zeta)\). For the current study a proportional damping with \( \zeta = 0.019 \) is introduced to electromechanical system for the first mode to be consistent with the reference model in (Liao & Sodano 2008).

Steady stage current (or charge) is initially evaluated by solving the scalar form of the coupled governing Eqs. (70) and (71) and peak power response is then conveniently expressed as a function of current developed across the resistor through,
\[ P = i(t)^2 R_l = \dot{q}(t)^2 R_l \]. Resistive loads have been commonly used to extract power from a vibrating energy harvesting system, due to its simplicity although it is not an optimal electronic circuit to harvest energy. In practice however, the piezoelectric harvester would typically be combined with a full wave rectifier to convert AC (alternating current) signals generated to DC (direct current) as most applications required DC signals which then charges an energy storage element like a capacitor. Further, a DC-DC convertor is then utilized to regulate voltage transferred to the load. However, despite the inefficient mechanism, a simple resistive load is frequently used as it provides with a quick technique to analyze power harvesting and calculating power generated with sufficient certainty. The closed form expression to determine power generated across the resistor as given by (Liao & Sodano 2008) is,
\[
P = \left\{ D^2 \omega^2 R \right\} \left[ \left( \frac{K}{\theta} - \frac{M}{\theta} \omega^2 \right) - \left( \frac{CC}{\omega^2} \right) R \right]^{-1} + \left[ \left( \frac{C}{\theta} \omega \right) + \left( \frac{KC}{\omega^2} \theta \omega - \frac{MC}{\omega^2} \right) R \right]^{-1}
\] (77)
The optimal load resistor for maximum power in the circuit depends on the electro-mechanical coupling and effective capacitance which could be obtained by solving an optimization problem. A common technique is to approximate the optimal resistance with a stand-alone purely electrical (mechanically uncoupled) RC circuit modeled resulting in an optimal load of \( R_l = \frac{1}{\omega_c C_p} \) using resistive impedance matching technique for maximum power transfer.

Before analyzing the honeycomb substrate bimorph, a 3D finite element model of the piezoelectric bimorph with solid substrate was modeled in Abaqus/CAE to validate the reduced single mode solution. The short and open circuit natural frequency of the bimorph from the finite element model was found to be 135 Hz and 139.4 Hz which closely matches (less than 1% difference) the corresponding values obtained through the analytical model. The validity of the reduced scalar model evaluated at a single model was also verified in (Liao & Sodano 2008) by comparing it to the full modal solution (Sodano et al. 2004) it was reduced from, demonstrated using a numerical example.

Now, as the honeycomb structure replaces the solid continuous substrate the mass and stiffness properties of the central layer is altered. The effective mass and stiffness of the honeycomb core are evaluated through the effective properties determined through cellular material theory summarized in Table 1. The number of cells required along the length \((N_L)\) is chosen as 12.5 and 3 along the width \((N_H)\) which are both reasonable values with respect to the global geometry of the piezoelectric bimorph. Once the number of cells required is determined, then the unit cell length, height, cell wall thickness and cell angle are evaluated. For this study, conventional regular hexagonal honeycombs are utilized that renders the cell angle 30°. Note that in a regular hexagon the incline cell length equals the vertical cell height as outlined in the
previous section. The cell wall thickness, $t_{\text{wall}}$, of a honeycomb is the most difficult part to manufacture due to the smallest dimension. Therefore, in this study, $t_{\text{wall}}$ is initially chosen to be 0.15 mm, which is a reasonable dimension considering conventional metallic honeycomb manufacturing methods such as conventional forming processes and the wire electrical discharge machining (EDM). Inclined cell length (1.52 mm), vertical cell height (1.52 mm) are then obtained using Eqs. (19) and (20). The complete effective elastic properties and relative density of the brass honeycombs structure is summarized and compared to solid continuous brass in Table 3 below.

Table 3: Complete elastic behavior and relative density of brass honeycombs compared to solid continuous brass.

<table>
<thead>
<tr>
<th>Property of Bimorph substrate</th>
<th>Homogenous continuous solid substrate</th>
<th>Regular hexagonal honeycomb substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1^*$ (GPa)</td>
<td>97</td>
<td>0.22</td>
</tr>
<tr>
<td>$E_2^*$ (GPa)</td>
<td>97</td>
<td>0.22</td>
</tr>
<tr>
<td>$E_3^*$ (GPa)</td>
<td>97</td>
<td>11.05</td>
</tr>
<tr>
<td>$G_{12}^*$ (GPa)</td>
<td>36.2</td>
<td>0.05</td>
</tr>
<tr>
<td>$G_{13}^*$ (GPa)</td>
<td>36.20</td>
<td>2.06</td>
</tr>
<tr>
<td>$G_{23}^*$ (GPa)</td>
<td>36.2</td>
<td>2.06</td>
</tr>
<tr>
<td>$v_{12}^* = v_{21}^*$</td>
<td>0.34</td>
<td>1</td>
</tr>
<tr>
<td>$\rho^+ = \rho^*/\rho_s$</td>
<td>1</td>
<td>0.114</td>
</tr>
</tbody>
</table>

The effective in-plane elastic modulus in the $x_1$ and $x_2$ directions are defined by $E_1$ and $E_2$ while the effective out-of-plane elastic modulus in $x_3$ direction is defined by $E_3$ while the in-plane shear modulus is represented by $G_{12}$ and the out-of-plane shear modulus denoted by $G_{13}$ and $G_{23}$. 
The bimorph cantilever beam under bending with Bernoulli beam assumptions is primarily and largely dictated by $E_i$. It can be seen from Table 3 that effective $E_i$ for the honeycomb core is 2 orders of magnitude lower than its corresponding solid. In addition a potential advantage honeycomb structures exhibit is its reduced mass. The mass of the regular honeycomb structures is only around 10% of the mass of the solid homogeneous substrate.

The effective elastic properties obtained however are applicable only to the honeycomb core and not the sandwich piezoelectric bimorph. The flexural rigidity or the equivalent bending stiffness of the sandwich structure can be evaluated as the sum of the flexural rigidity of the individual layers. In particular, the equivalent bending stiffness of the piezoelectric bimorph is approximated as the sum of the bending stiffness contributed by the piezoelectric face sheets and the core. The equivalent bending stiffness can be evaluated by (Allen 1969),

$$
(El)_w = E_f \frac{b t_f^3}{6} + E_c \frac{b t_c d^2}{2} + E_c \frac{b t_c^3}{12}
$$

where, $E$ denotes the Young’s modulus, $I$ is the second moment of area, $b$ the width of the structure, $t$ the thickness and the subscripts ‘$f$’ and ‘$c$’ represent the face sheet and core respectively. The variable $d=t_f+t_c$ is the distance between the centroids of the two face sheets. The first term in Eq. (78) denotes the flexural rigidity contributed by the bending of the face sheets about the neutral axis while the third term captures the bending stiffness contribution of the core. The second term is the stiffness of the face sheets associated with bending about the centroid axis of the sandwich structure. The equivalent bending stiffness or flexural rigidity of the bimorph can be then evaluated by substituting appropriate geometric and elastic properties into Eq. (78).
The contribution of first and third term in Eq. (78) towards the equivalent bending stiffness/flexural rigidity of the sandwich panel with regular hexagonal honeycomb is negligible compared to the second term. This is expected as the first term represents bending stiffness contribution of the face sheets about its own neutral axis with \( t_f \) being 3 times lower than \( t_c \) while the third term denotes flexural rigidity contribution from the core alone with the elastic constants of the core being significantly lower than the face sheet. Hence the dominant contribution towards the effective bending stiffness of the sandwich is through the second term in Eq. (78) which is associated with the bending of the face sheets about the centroid of the sandwich. In fact, the second term accounts for 98% of the equivalent bending stiffness computed through Eq. (78) and summarized in Table 4 below where it can be clearly seen that any change in the honeycomb core modulus \( E_c \) has a negligible impact on the flexural rigidity of the sandwich bimorph. Moreover, a closer look at the second term in Eq. (78) reveals that the equivalent bending stiffness is more sensitive to the face sheet thickness than the core thickness along the face sheet stiffness.

<table>
<thead>
<tr>
<th>Flexural rigidity of sandwich bimorph</th>
<th>Solid Substrate bimorph</th>
<th>Honeycomb substrate bimorph</th>
</tr>
</thead>
<tbody>
<tr>
<td>((EI)_{eq}); Flexural rigidity of sandwich structure ((Nm^2))</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td>Term 1; Face sheet bending about neutral axis (%)</td>
<td>1.5</td>
<td>2.1</td>
</tr>
<tr>
<td>Term 2; Face sheet bending about the centroid (%)</td>
<td>69.2</td>
<td>97.8</td>
</tr>
<tr>
<td>Term 3; Bending stiffness of core (%)</td>
<td>29.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Note that as stated earlier the dominant elastic modulus in bending, \( E_f \) of honeycomb core is almost 3 orders of magnitude lower than its solid counterpart (see Table 2). In fact the elastic modulus of slid brass is around 440 times higher than brass honeycomb. However, the
ratio of equivalent bending stiffness of the bimorph with solid brass substrate to brass honeycomb substrate is 1.4, and not as drastic. This allows the piezoelectric bimorph with honeycomb core exhibit moderate strength even when the elastic modulus of the core is 3 orders of magnitude lower. Hence, since the equivalent bending stiffness is relatively comparable, similar average stress/strain distribution across the surface area of the PZT facesheet is expected.

Substituting the evaluated effective bending stiffness \( E_i \) and honeycomb density \( \rho^* \) summarized in Table 3 as \( c \) and \( \rho \), in equations (72) and (73) now reflect mass and stiffness of the sandwich bimorph with honeycomb substrate. By replacing the solid substrate with honeycomb structures, the total mass and effective stiffness of the bimorph device is altered resulting in a shift in the natural frequency of the structure. The short circuit natural frequency of the honeycomb core piezoelectric bimorph using the reduced analytical model was found to be 172.16 Hz, while the 3D finite element model in Abaqus/Standard reported a value of 173 Hz, a less than 1% difference between the two models. The finite element mesh of the unit cell used to model the honeycomb structure is shown in Figure 14 below. The unit cell is meshed with 4 quadratic elements along the inclined cell length and vertical cell height along with 2 quadratic elements through the thickness.

![Figure 14. Finite element mesh of the unit cell of the conventional regular honeycomb](image-url)
The power generated from the honeycomb core bimorph can be evaluated by using the same equation (77), used to compute power harvested from the bimorph with solid core. Note that substrate density \( \rho_s \) and core stiffness \( c_s \) are the parameters that change when the honeycomb replaces the solid brass as substrate. Power response of both, the solid and honeycomb core bimorph, excited through a base acceleration of 1g (constant for the remainder of the paper) is shown in Figure 15 below. As expected it can be seen that due to the significantly light honeycomb substrate, the short circuit natural frequency of the piezoelectric bimorph has shifted from 137.5 Hz when solid core was used, to 172.16 Hz when it is replaced with honeycomb substrate. In addition, as expected, the power generated from the solid core bimorph is higher due to the larger (2 times) mass of the sandwich bimorph when compared to the honeycomb core. Moreover, the stiffer solid substrate resists bending more effectively in the case conventional bimorph when compared to the more compliant honeycomb core leading to more stress/strain on the PZT layers. This results in the solid substrate bimorph having 1.4 times higher bimorph flexural rigidity compared to the honeycomb core bimorph as shown in Table 1.

![Figure 15. Increase in the natural frequency of the bimorph when the solid substate is replace by honeycomb substrate.](image-url)
Although the higher flexural rigidity is beneficial in terms of power generation, under cyclic loading conditions the operating lifetime of the harvester is lowered due to larger strains developed on the brittle piezoelectric ceramic contributing to the degradation of the piezoelectric material reducing the durability of the harvester. The honeycomb core bimorph, being more compliant increases the operating lifetime of the harvester thus reducing the need for replacement due to fatigue.

3.4. Cell parameter influence on mechanical and power harvesting characteristics

From the discussion in the previous section (as summarized in Table 4) it is clear that a variation in honeycomb core stiffness has negligible impact on the equivalent sandwich stiffness. However, reduction in core mass decreases the total mass of the bimorph greatly. The impact of variation in honeycomb core stiffness and mass, on power generation is shown in Figure 16 below.

![Figure 16](image)

**Figure 16. Influence of a) honeycomb core stiffness with 0.05 Pa < \( E_{\text{honeycomb}} \) < 2 GPa and b) core mass with \( \rho^* < 1 \), on peak power generation**

It can be seen that change in honeycomb stiffness has an insignificant effect on the sandwich stiffness and hence power generated, while variation in honeycomb core mass alters
the sandwich mass and hence has a substantial impact on power generation. As a result, cell parameters that influence honeycomb core mass is studied and its effect on power harvesting is analyzed. It can be seen from Eq.(6) that the honeycomb core density is dependent on the cell angle \((\theta)\), ratio of wall thickness \((t_{\text{wall}})\) to inclined cell length \((l)\) and the ratio of vertical cell height \((h)\) to inclined cell length \((l)\). In particular the effect of \(\theta\), \(t_{\text{wall}}\) and \(h\) on the mass of the bimorph and ultimately power generation is studied, maintaining constant inclined cell length.

It can be seen from Eq.(6), that the density of the honeycombs is directly proportional to cell wall thickness. Increase in cell wall thickness increases the mass of the core and hence the bimorph, lowering the natural frequency of the honeycomb substrate piezoelectric bimorph, as shown in Figure 17 (a) below. The honeycomb considered for this study is a regular hexagonal honeycomb with \(\theta=30^\circ\) and \(h=l\). Studies with auxetic honeycomb core with \(\theta=-30^\circ\) was also carried out but not presented as results are very similar. As expected, the bimorph with the largest cell wall thickness and hence largest mass generates the highest power. Maximum power generated increases linearly with cell wall thickness (due to its linear relation with mass) as shown in Figure 17(b). Note that the cell wall thickness can only be increased until the relative density of the core is 1 at which point the honeycomb becomes solid. This method of varying the natural frequencies of the bimorph provides with alternative approach to vary harvester resonant frequency.
Figure 17. a) Variation in natural frequency and power generation with varying cell wall thickneeses and b) Influence of cell wall thickness on peak power.

The effect of cell angle variation on honeycomb density can also be predicted from Eq. (6). A closer look at the denominator \( \alpha + \sin \theta \) indicates that the more negative the cell angle is, smaller is the denominator thereby increasing the honeycomb density. This relation is captured in Figure 18 (a) and (b) where it can be seen that power generation is the highest when the cell angle is \(-30^\circ\), as the honeycomb core and hence sandwich bimorph is the heaviest. Moreover, peak power generated decreases with an increase in cell angle. It is worthwhile to recall that the honeycombs with a negative cell angle as discussed previously (see Figure 8 (b)) are known as auxetic honeycombs, where an increase in width along \( x_2 \) axis is observed when stretched along the length in \( x_1 \) direction known as the auxetic effect. The ratio \( t_{wall}/l \) of cell wall thickness to inclined cell length was kept constant at 0.1 with \( h=l \) for this study.
Figure 18. a) Variation in natural frequency and power generation with varying cell angles $\theta$ and b) Influence of cell angle $\theta$ on peak power.

Similar parametric study was conducted with the ratio $h/l$ and the result is shown below in Figure 19 (a)-(d). It can be seen from Figure 19 (a)-(d) that when the cell angle $\theta = 30^\circ$ the ratio $h/l$ does not have a significant impact on the natural frequency and power generation. The core mass when using regular honeycomb is not affected significantly when the $h/l$ ratio is varied. In contrast when the cell angle $\theta = -30^\circ$ it can be seen that $h/l$ ratio has considerable effect on the mass of honeycomb core bimorph with auxetic honeycombs. Low ratios of $h/l$ increases the overall mass while higher ratio $h/l$ reduces the mass resulting in a corresponding shift in the natural frequency of the harvester. This behavior can also be predicted by analyzing Eq.(6) where the ratio $(\alpha+2) / (\alpha+\sin\theta)$ tends to vary significantly with variation in $\alpha$, only when $\theta = -30^\circ$. 
Figure 19. Variation in natural frequency and power generation for different ratios of $h/l$ when (a) $\theta = 30^\circ$ and (b) $\theta = -30^\circ$ while peak power variation with the ratio $h/l$ when (c) $\theta = 30^\circ$ and (d) $\theta = -30^\circ$.

In summary, any unit cell parameter that induces a change in honeycomb core mass can shift the natural frequency of the structure as core stiffness has negligible effect of sandwich stiffness. Increased cell wall thickness, decreased cell angle and small ratio of $h/l$ result in heavier core and bimorph mass and hence lowers natural frequency of the structure. Note that while changing these parameters the relative density of honeycomb to solid was not allowed to exceed a ratio of 1 preventing the honeycomb core from getting heavier than its solid counterpart. It is evident from the above discussion that the honeycomb core bimorph provides with more design parameters to alter the natural frequencies of the bimorph impacting power generation when
compared to the traditional solid core bimorph. The greater design flexibility when using honeycomb substrate is hence advantageous.

3.5. **Comparison between honeycomb and solid core piezoelectric bimorph**

It is essential that under harmonic excitation, the input driving frequency and the fundamental frequency of vibration of the structure are matched to attain resonance to maximize power generation. Such harvesters are linear harvesters in nature as linear resonance condition is exploited. The chief limitation that exists in linear harvesters is the significant drop in power generation when exact resonant frequency match is not achieved. One study, (Baker et al. 2005) reported one and two orders of magnitude drop in power when the system was driven at 25 Hz and 50 Hz away from the resonant frequency. As a result methods to address resonant frequency match/mismatch are imperative for maximizing output power. In the simplest case, a mass is added to the end/tip of the bimorph and is adopted in this study.

In order to attain resonance, prior knowledge of the driving frequency is required which is difficult in many cases. In addition issues involved with precision in manufacturing, slight variations in material properties, influence of temperature variations and other design stage and first order uncertainties does not allow for exact natural frequency of the harvester estimation with low tolerance/uncertainty. However, for the sake of investigating the potential advantageous of using a honeycomb core over solid core bimorph it is assumed that the input driving frequency ranges are known beforehand and uncertainty in prior determination of natural frequency is neglected. The method of adding a tip mass to alter the natural frequency of the harvester however requires prior knowledge of driving frequency and exact amount of tip mass required can then be determined. The advantage then in using the honeycomb core bimorph can be
analyzed under two conditions: (a) lower input driving frequency of around 100 Hz, and (b) higher input driving frequency of around 200 Hz.

Under low input driving frequency conditions, the reference piezoelectric bimorph with solid substrate with a short circuit natural frequency of around 140 Hz requires a tip mass to lower the natural frequency to match the excitation frequency. The honeycomb core bimorph presented earlier that has a natural frequency of 172.16 Hz needs a heavier tip mass to lower the natural frequency to around 100 Hz. Exact analytical modeling of the tip mass is not developed as the study primarily focuses on comparison of honeycomb substrate with solid substrate. The effective mass of the piezoelectric bimorph with a tip mass (which is placed completely on the piezoelectric layer) is modeled as, $M_{\text{eff}} = M_{\text{bimorph}} + 3M_{\text{tip}}$, to be consistent with the distributed parameter modeling approach. The addition of the tip mass is also reflected in the effective input mass or inertial load $D$, through $M$ in Eq. (76). This is analogous to the lumped mass approach where the equivalent mass is approximated as one-third mass of the beam lumped with the tip mass which is frequently adopted in many studies.

To evaluate mass per unit length ($L'$) of the bimorph with tip mass, effective bimorph length $L'$ is defined as $L_{\text{beam}} - L_{\text{tip}}/2$. With the modeling of tip mass and effective mass calculation, electromechanical response of piezoelectric bimorph with tip mass can now be modeled with solid and honeycomb bimorph and is studied next. In the following study, honeycomb cores with 6 different cell wall thicknesses were modeled ranging from 0.076 mm (lowest) to 4.52 mm (highest) which is then compared to the reference solid core bimorph with tip mass. Finite element models of each of honeycomb core piezoelectric bimorphs and solid core bimorph were developed and the tip mass to lower the short circuit natural frequency of the bimorph to 105 Hz.
was determined. Once the tip mass was determined, effective mass was calculated by $M_{\text{eff}} = M_{\text{bimorph}} + 3M_{\text{tip}}$ and effective sandwich short circuit stiffness can determined as $K_{\text{eff}} = M_{\text{eff}} \alpha^2$.

Power generated from the bimorph with tip mass can be then estimated with the help of Eq. (77). Power generated across the resistor (6.4 kΩ) from the traditional solid core bimorph is compared to six different honeycomb core models with varying cell wall thicknesses and the result is shown in Figure 20 below. Each curve corresponds to the power response of the bimorph where the curve indicated by $\rho^* = \rho^* / \rho_s = 1$ indicates the solid core bimorph while the remaining curves represents honeycomb core bimorph. A relative density $\rho^* = 0.03$ indicates the lightest honeycomb core (smallest cell wall thickness) while $\rho^* = 0.35$ signifies the heaviest honeycomb core (largest cell wall thickness). It can be seen from Figure 20 that power generated from the solid core bimorph is higher than any of the honeycomb core harvesters. This is expected as it is known from literature that at the same natural frequency the harvester with larger mass and higher stiffness generates larger power. This can be observed even in the different honeycomb core bimorphs where the heaviest honeycomb core bimorph generates maximum power with the power decreasing with honeycomb core mass. Note that the effective mass and stiffness of the solid core bimorph is 1.55 times higher than the honeycomb core with lowest cell wall thickness.
Figure 20. Power amplitude response comparison between solid core bimorph and bimorphs with honeycomb core of different cell wall thickness. Legend indicates ratio of solid to honeycomb core density, $\rho^+$. However, the total mass of the harvester is different from the effective mass which is simply the mass of the bimorph plus mass of the tip, $M_{\text{total}} = M_{\text{bimorph}} + M_{\text{tip}}$. Power to weight ratio or specific power is computed by dividing the power amplitude as seen in the figure above by the total mass of the harvester as shown in Figure 21 below.

![Figure 20](image1.png)

Figure 21. Specific power response comparison between solid core bimorph and bimorphs with honeycomb core of different cell wall thickness. Legend indicates ratio of solid to honeycomb core density, $\rho^+$. It can be seen that power to weight ratio is the highest in the case of the lightest honeycomb core bimorph ($\rho^+ = 0.03$) and is lowest for the solid core bimorph ($\rho^+ = 1$). A 25% increase in power to weight ratio is observed in the lightest honeycomb core bimorph over the traditional
bimorph with solid core. The increase in specific is due to the honeycomb structure with smallest cell wall thickness (hence lightest) requiring heaviest tip mass to lower the natural frequency to around 105 Hz when compared to the solid core bimorph. The tip mass is the dominant factor in the vibration of a cantilever beam with end mass and not the distributed beam mass along the length. The effective input mass distribution can be represented by $M_{\text{eff}} = M_{\text{bimorph}} + 3M_{\text{tip}}$, where the greater impact of the tip mass can be seen. Hence, this addition of a larger tip mass to the light honeycomb core bimorph allows for higher power generation at a lower total mass $M_{\text{total}} = M_{\text{bimorph}} + M_{\text{tip}}$. However, as the total mass of the honeycomb bimorph is reduced considerably, the rate of decrease in power amplitude is not linear with total mass. The rate of decrease in power amplitude is lower than the rate of decrease in total mass when using the honeycomb core bimorph. This results in a higher power/mass when using the honeycomb core bimorph for power generation. This can be better explained with the help of a parametric study to observe the power amplitude relation to total mass of the harvester.

Each time the mass of the bimorph is altered, the tip mass needs to be varied accordingly to keep the natural frequencies of the harvester constant. The above results presented considered 6 honeycomb core models with varying cell wall thicknesses and a solid core bimorph. To obtain more data points a quadratic curve fit between total and effective mass was utilized. Two goodness of fit characteristics were evaluated with norm of the residuals being 7.14e-5 while the $R^2$ value was 0.9998 both suggesting a good fit.
Once the relation between total and effective mass is established, their impact on power generation can be studied. It can be observed from Figure 23 (a) that decrease in total mass of the harvester decreases power amplitude as expected, but not drastically. On the other hand Figure 23 (b) shows that peak specific power is the highest when the total mass of the harvester is the lowest (honeycomb core with smallest $t_{wall}$). It can be seen that the specific power decreases with increase in total mass as the honeycomb core tends to become solid continuous.
To observe the rate of decrease in power amplitude and total mass with respect to effective mass the power amplitude generated and the total mass of honeycomb core harvesters is scaled with the one generated from the solid core harvester. This trend to highlight the disproportionate reduction in the two scaled parameters (power amplitude and total mass) is captured in Figure 24 below.

![Graph showing scaled power amplitude and total mass variation with respect to effective mass.](image)

**Figure 24. Scaled power amplitude and total mass (with respect to solid core bimorph) variation with respect to effective mass.**

It can be clearly seen that the lightest honeycomb core bimorph when compared to the solid core bimorph generates around 65% of the power amplitude at around 50% of the total mass. This disproportionality in reduction between power amplitude and total mass results in the increased power to weight ratio of honeycomb core harvesters. Power amplitude and specific power generated from the honeycomb core harvester and its percentage difference (increase/decrease) is compared to solid core bimorph. The percentage increase in power to weight ratio of honeycomb core bimorphs depends on the cell wall thickness (hence core mass resulting in total mass) of honeycomb cores. As the cell wall thickness of honeycomb increases, the percentage difference between the honeycomb and solid core harvester reduces as the honeycomb core increasingly tends to become a solid. The variation of percentage difference in power amplitude and specific
Power with respect to total harvester mass is studied, represented in Figure 25 below. The negative percentage for the power amplitude curve indicates the amount of decreased power the honeycomb core harvester generates with respect to its solid core counterpart.

![Figure 25](image.png)

**Figure 25. Percent change in power amplitude and specific power generated by honeycomb core bimorph when compared to solid core bimorph.**

As the total mass of the harvester decreases, it can be seen that the trend in power amplitude and specific power is increasingly diverging in nature. With decrease in total mass, the honeycomb core harvester generates lower power amplitude but higher power to weight ratio. The lightest honeycomb core harvester (6g) on comparison with the solid core harvester (11.5g) generates around 35% less power amplitude but with a 25% increase in specific power. This improved power to weight ratio of honeycomb core piezoelectric energy harvesters is a step forward in bridging the gap between the specific power of energy harvesters and batteries which is critical to achieve battery-less electronic device operation. Moreover, the increased specific power has the potential to help supplement battery power without significantly increasing the mass of the host device in low power electronic applications where rechargeable batteries are currently used. In applications where battery use is necessary, the honeycomb core bimorph can work in synergy with the batteries without significantly increasing the overall mass of device due
to higher power to weight ratio. In addition, the higher power to weight ratio could benefit human powered wearable energy harvesters due to its lightness proving to be a convenient factor. As stated earlier, the flexural rigidity of the solid core bimorph is 1.55 times the honeycomb core bimorph with lowest cell wall thickness. This increased resistance to bending enables more stress/strain development on the PZT layers which under cyclic loading conditions decreases the harvester device longevity/lifetime whereas the honeycomb core bimorphs are more compliant increasing the harvester device lifetime as larger load cycles can be sustained. Note that in order to replace or supplement batteries which is the objective, it is necessary to counter one of the biggest limitations posed by batteries that pertains to their periodic replacement. Honeycomb core bimorphs can help increase the operating lifetime of the harvester reducing the frequency of replacement due to fatigue.

Under higher input excitation levels the honeycomb core bimorphs exhibit a different type of advantage. Assuming that the driving frequency range is around 185 Hz, the natural frequency of reference case traditional reference solid substrate bimorph without tip mass studied earlier, with a short circuit natural frequency of around 140 Hz has to be increased this time to match the higher excitation frequency. To attain higher natural frequencies, the reference solid core bimorph need to be lighter and/or stiffer. A decrease in mass is not favored as it will result in decrease in power generated, so the commonly adopted method is to thicken the harvester making it stiffer. This can be achieved by increasing the thickness of the piezoelectric layer or the substrate. Increasing the thickness of the piezoelectric layer, although allows of more energy harvesting due to more piezoelectric material, becomes quite expensive and is not possible/not effective when piezoelectric patches half beam length coverage situations. Hence, increasing the thickness of the solid substrate layer is the most viable option. It can be seen upon inspecting Eqs (72) and (73) that the substrate thickness has a much larger impact on the sandwich stiffness than
the mass. This can also be seen through Eqs (6), (9) and (78). Hence, increasing substrate thickness even though increases both mass and stiffness, cubic relationship between substrate thickness and stiffness dominates, resulting in an increased natural frequency. In addition a closer look at Eq. (74) suggests that increasing the thickness of the bimorph harvester increases the electromechanical coupling leading to higher power generation.

Regardless of which layer needs to be thickened, the total depth of the traditional solid core harvester \((2t_p + t_s)\) needs to be increased to increase the natural frequency. This results in a change in the global dimension increasing the overall size of the harvester which could be an issue in applications that imposes space/size constraints. Note that in many cases the global dimensions and especially the piezoelectric and substrate layer thickness of the harvester is designed at an optimum value. The influence of the substrate layer to piezoelectric layer thickness ratio \((t_s/2t_p)\) was studied in (Wang et al. 1999) concluding that if the piezoelectric layer thickness is constant, voltage sensitivity decreases with increasing thickness ratio \(t_s/2t_p\) for a piezoelectric bimorph. Any increase in the substrate layer thickness induces more strain in the PZT layer in bending and thus more charge (Kim et al. 2005a; Kim et al. 2005b; Yoon et al. 2005). However, there is a maximum substrate thickness within where this increase can be observed, and any further increase to the optimum value could be non-beneficial. These studies among many others (Chandrasekharan et al. 2013a) stress the significance of substrate thickness on piezoelectric bimorph performance and is hence designed at an optimum value. It must also be noted that increased substrate thickness will require a larger mechanical force to deform the structure which may fall outside the input ambient excitation amplitude. Consequently, any change to this optimum thickness (increase/decrease) might prove a hindrance.
In contrast, the honeycomb core bimorphs have the unique capabilities to preserve the global dimensions (size) of the harvester while increasing the natural frequencies of the harvester. An increase in the natural frequency of the honeycomb bimorph is possible through cell parameter manipulations. As shown earlier, decreasing cell wall thickness, increasing the cell angle or increasing cell height to cell length ratio, results in decreasing the mass of the honeycomb core bimorph without affecting the bimorph sandwich stiffness significant. As a result, it enables an increase in the natural frequency of the structure.

To demonstrate this potential, power response using a solid core bimorph with increased solid core thickness (to increase sandwich stiffness) is compared to a honeycomb core bimorph with reduced cell wall thickness (to reduce sandwich mass), both leading to an increase in natural frequencies. It is assumed that the input driving frequency is around 185 Hz and hence to match this excitation frequency for maximum power transfer, the harvester needs to be designed to vibrate with a short circuit – open circuit frequency range near the excitation frequency.

It was determined that with the piezoelectric layer thickness kept constant, the solid substrate thickness need to be increased by 50% to shift the short circuit natural frequency from 137.46 Hz to 183 Hz, while the honeycomb core thickness was designed to be 0.04 mm with $\theta = 30^\circ$ and $l = h$, to attain the same short circuit natural frequency of 183 Hz. A finite element model of both the solid substrate with increased thickness (1.145 mm) and honeycomb core with reduced cell wall thickness (0.042 mm) was developed as a measure of validating the mechanical properties. The short circuit natural frequency (172 Hz) predicted by the FE model differed from the analytical scalar model by less than 1%.

The power generated across the resistor (3.7 kΩ) from the solid and honeycomb core bimorphs are compared and is shown in Figure 26 (a). As expected the power amplitude
generated from the solid substrate bimorph is significantly higher due to significant difference (around 3.3 times) in the mass and sandwich stiffness of the solid and honeycomb core bimorphs.

Figure 26. (a) Power amplitude and (b) specific power (right) response of honeycomb and solid core bimorph at a higher excitation frequency of around 185 Hz.

Reduction in the mass of the honeycomb bimorph is accompanied by reduction in power amplitude of the harvester and they decrease linearly in this case (as there is no tip mass) resulting in the same power to weight ratio as seen in Figure 26 (b). However, the unique unit cell parameter manipulation of the honeycomb substrate allowed for increase in the natural frequency without any increase in the harvester size thereby preserving all optimal global dimensions, especially the piezoelectric and substrate layer thickness.

The increase in the overall thickness of the solid substrate bimorph resulted in a 62mm\(^2\) envelope area of the harvester. If the application is dictated primarily by the harvester size, the honeycomb bimorph would prove beneficial. The increased total thickness of the traditional solid core bimorph could limit is applicability while the honeycomb bimorph is relatively more scalable. The 50\% increase in the substrate thickness also would now require higher mechanical force to deform the structure (as the solid substrate bimorph has 3.3 times higher flexural rigidity)
which may not be feasible while the honeycomb core bimorph due to much lower flexural rigidity will undergo bending deformation at lower mechanical force as well.

The honeycomb core bimorph can be particularly advantageous when the application imposes a constraint of the total thickness of the harvester as power/total depth of the honeycomb core bimorph is significantly higher when compared to the traditional solid core bimorph. Many applications demand dimensions and tolerances which cannot accommodate larger sizes/global dimensions and or larger/mass and in such situations honeycomb bimorphs would be beneficial. In addition, due to the increased bimorph thickness to achieve higher natural frequency the solid core bimorph results in 3.3 times higher equivalent bending stiffness compared to the honeycomb core bimorph. As previously discussed under cyclic loading conditions the operating lifetime of the harvester is shortened due to higher stresses on the brittle piezoelectric ceramics. The honeycomb core bimorph on the other hand being significantly more compliant increases the device lifetime (able to sustain larger load cycles) reducing the frequency of fatigue induced replacement which is critical towards addressing a major limitation posed by batteries pertaining to its frequent periodic replacement.

3.6. Conclusion

The decrease in power consumption due to advancement in transistor technology and improved microfabrication processes has resulted in many low power consuming electromechanical devices currently being operated through batteries. The disproportionate advancement in computing technology and battery technology, issues when using battery such as frequent recharge/replacement, limitation to miniaturization of the host device etc., are compelling reasons to replace or supplement battery power. Current specific power of vibration based piezoelectric energy harvesters are significantly lower than the specific power of most
battery types. Hence, techniques to improve existing power to weight ratio from piezoelectric harvesters is rewarding. The current paper demonstrates the potential of integrating lightweight honeycomb structures with existing piezoelectric energy harvester configuration to increase power to weight ratio of the harvester. A piezoelectric bimorph is chosen to demonstrate the potential advantages of using honeycomb core. The traditional solid continuous substrate in a bimorph is replaced with cellular honeycomb structures, reducing the total mass of the harvester significantly without compromising on flexural strength.

Power generation from different honeycomb core bimorph designs is compared to the traditional solid core bimorph at low (~100 Hz) and high (~200 Hz) excitation frequency. Under lower excitation frequencies, a tip mass is utilized to lower the natural frequency of the harvester in order to match the input excitation frequency for maximum power generation. Different honeycomb core configurations were modeled by varying the cell wall thickness, cell angle, and ratio of cell thickness to cell length ratio of the honeycomb cell, leading to a change in the total mass of the harvester. It was shown that the honeycomb core bimorph with the smallest cell wall thickness and hence lightest honeycomb core, generated around 25% more specific power than the traditional solid core bimorph. The improved specific power of honeycomb core piezoelectric bimorph is a step forward in bridging the power to weight ratio gap between existing piezoelectric harvesters and batteries. Moreover, due to higher power to weight ratio with significantly reduced mass, it can also serve as a viable energy source to work in synergy with batteries to alleviate some of the battery issues. Increased power to weight ratio of the harvester can particularly benefit human powered wearable harvesters due to significantly reduced mass. Moreover, the significantly more compliant honeycomb core bimorph harvester increases the operating lifetime of the harvester under cyclic loading conditions.
Under higher input excitation frequencies, the natural frequency of the traditional solid core bimorph is typically increased to match the driving frequency by increasing the thickness of the solid substrate, thus increasing the stiffness of the sandwich structure. This requires a change in the global dimension (typically substrate thickness) and may prove to be non-beneficial in applications that are dictated by space/size constraints. The substrate thickness in a bimorph typically has an optimum value beyond which any increase negatively impacts harvester performance. Increased substrate thickness leads to stiffer structure/central layer, which requires larger forces to induce bending deformation, which may not be practical. The honeycomb core bimorph on the other hand, can preserve the designed optimum global dimensions as the natural frequency of the bimorph is increased to match the driving frequency by manipulating the cell parameters like the cell wall thickness, cell angle, and ratio of cell thickness to cell length. Low cell wall thickness, large cell angle or higher cell thickness to cell length ratio, all result in a lighter honeycomb core without significantly affecting sandwich stiffness, thereby able to increase the natural frequency of the harvester without the need to alter global dimensions. The honeycomb core bimorph under high excitation frequency is particularly beneficial when a constraint on size/space is imposed. Moreover, since the honeycomb core has considerably lower effective stiffness properties when compared to the solid core, the significantly more compliant honeycomb core bimorph increases the operating lifetime of the harvester by decreasing susceptibility towards fatigue, under cyclic loading conditions.

Design flexibility of the honeycomb substrate piezoelectric bimorph is demonstrated as unit cell parameters of honeycomb structures can be manipulated to alter mass and stiffness properties of the substrate, resulting in unit cell parameter significantly influencing power generation. The improved power to weight ratio of honeycomb core bimorphs can be exploited by the many facets of energy harvesting technologies including the promising non-linear energy
harvesting. Although not explored in this paper, other vibration to power conversion mechanisms like electromagnetic and electrostatic transduction configuration that currently utilize an elastic continuous solid substrate can benefit from replacing the solid substrate with honeycomb structures to obtain higher power to weight ratio from their harvester.
Chapter 4: Analysis of an open circuit voltage driven equivalent circuit representation of piezoelectric energy harvesters

Several efforts have been made to model the electromechanical response of piezoelectric energy harvesters. The electromechanical response of a cantilever beam with piezoelectric elements was first published in (Hagood et al. 1990) by applying energy variation principles (Hamiltonian’s principle) for active structural control applications. The same distributed parameter method along with piezoelectric actuation equations from (Crawley & Anderson 1990) and constitutive relations detailed in (Smits et al. 1991) was implemented in (Sodano et al. 2004; du Toit et al. 2005) to predict the electromechanical response of cantilever beams for energy harvesting applications. Exact analytical solution for the energy harvesting problem was developed in (Chen et al. 2006; Erturk & Inman 2008) where the vibratory motion of the beam is expressed as a convergent series of eigenfunctions consisting of circular and hyperbolic sine and cosine functions. Other piezoelectric generator modeling techniques include equivalent circuit modeling (Elvin & Elvin 2008) where coupled electromechanical equations are developed through a distributed parameter method. These equations are then decoupled through standard eigenvalue analysis and the decoupled mass, stiffness, damping terms are replaced with equivalent inductive, capacitive and resistive elements respectively. Furthermore, a large number of studies have focused on the finite element (FE) formulations for piezoelectric actuation and sensing problems, for instance (Chen et al. 1997; HWANG & PARK 1993; Tzou & Tseng 1990). More recently, FE modeling of piezoelectric structures for energy harvesting applications have been developed, for example (Zhu et al. 2009; De Marqui Junior et al. 2009a). In addition to
distributed parameter and finite element models, many researchers (Stephen 2006; Ajitsaria et al. 2007; Roundy et al. 2003) have also developed a lumped parameter or single degree of freedom (SDOF) models for piezoelectric energy harvesting. In a SDOF model, the mechanical domain of the piezoelectric problem is represented by a spring-mass-damper system.

The majority of studies modeling the electromechanical response of piezoelectric generators utilize non-complicated geometries. Simple beam models account for the majority of the piezoelectric configurations, while few researchers have also investigated piezoelectric energy harvesting from plates using classical Kirchoff-Love plate theory (Kim 2005; De Marqui Junior et al. 2009b; Lee 1990). Developing exact coupled analytical electromechanical models for complex geometries (including thick beams and plates) is tedious; in such cases finite element (FE) models can be beneficial. Open circuit voltage from either analytical or finite elements models can be determined without the need for an external load resistor. The current paper evaluates a method to calculate power generation from the open circuit voltage developed on the piezoelectric material of the harvester by representing the energy harvesting problem as an open circuit voltage driven equivalent circuit. This simple method to evaluate power generation once the open circuit voltage has been determined can be used to rapidly conduct parametric and optimization studies for energy harvesting applications. One of the most common piezoelectric energy harvesting configurations, the piezoelectric cantilever bimorph is chosen to evaluate the method to determine power generation. The approximations involved in representing the coupled piezoelectric problem as an open circuit voltage driven equivalent circuit is examined by comparing the governing equations of the equivalent and fully coupled circuit models. The degree of approximation is examined by varying the strength of coupling which provides insight into the effect of electromechanical coupling on power generation. The strength of device coupling is measured through the difference in short circuit to open circuit natural frequencies. A correction
factor is introduced for strongly coupled systems to calibrate the equivalent open circuit voltage driven model by varying the optimal load resistance.

4.1. Equivalent circuit model representation of the piezoelectric problem

A bimorph cantilever beam is modeled with a substrate sandwiched between two piezoelectric strips, as shown in Figure 27. Both substrate and piezoelectric surfaces are clamped on one side to ensure full transfer of strain energy to the piezoelectric material. The piezoelectric layers cover the entire length \( L \) and width \( b \) of the substrate. It is assumed that the entire top and bottom surfaces of the piezoelectric layers are covered by perfectly conductive continuous electrodes with negligible thickness when compared to the thickness of the piezoelectric \( t_p \) and substrate \( t_s \) layers. The piezoelectric and substrate layer thickness is uniform over the length of the beam.

![Figure 27: A schematic of the cantilever piezoelectric bimorph used in the study](image)

Depending on the polarization direction of the piezoelectric layers, the bimorph can be electrically series or parallel. The system is electrically in parallel (adopted in the current paper).
when the polarization in both piezoelectric layers are in the same direction. In contrast, the system is in series if the polarization is in opposing directions. Further details and differences between series and parallel configurations can be found in (Chandrasekharan et al. 2013b).

The coupled governing equations for a vibrating piezoelectric energy harvester is given by,

\[
M \ddot{r} + C \dot{r} + K r - \Theta v = F \quad (79)
\]

\[
\Theta^T r + C_p v + q = 0 \quad (80)
\]

where \( M, K, C, C_p \) and \( \Theta \) are the mass, stiffness, damping, capacitance and coupling matrices. Under the constraint that the voltage \( v \) is the same across the surface area of the electrode, charge \( q \) is developed when the forcing term \( F \) is applied resulting in a displacement that can be evaluated through \( r \). As mentioned before, the above electromechanical representation can been obtained using a global distributed parameter, lumped parameter or finite element methods.

Note that the actuation and sensing equations in Eqs. (79) and (80) respectively represent the electromechanical system when an electric load is not connected for power generation. Resistive loads have been commonly used to extract power from a vibrating energy harvesting system due to its simplicity, although it is not an optimal electronic circuit to harvest energy. However, despite the simplistic nature of the load, a resistor is frequently used as it provides with a quick technique to calculate power generated. Once an external load resistor \( (R_l) \) is connected, Eq. (80) can be modified by first taking a single time derivative throughout the equation and then using the relation \( i=\dot{q} \) (where the dot represents time derivative) and \( v=iR_l \). The equation can be then recast in the form,
\[ \Theta^T \ddot{v} + C_p \dot{v} + \frac{1}{R_i} v = 0 \]  

(81)

Due to parallel connection and symmetrical nature of the bimorph (top and bottom layers have same thickness and equally distant form neutral axis) the voltage developed on the electrodes across the top and bottom surfaces of the PZT layers are equal, \( v = v_1 = v_2 \) where subscripts ‘1’ and ‘2’ refer to the top and bottom piezoelectric layers respectively. The total charge however is the sum of individual charges developed on the surfaces of the PZT layers, \( q = q_1 + q_2 \).

The electromechanical response of the device is amplified when the driving or excitation frequency is near the natural frequency of the harvester. As a result it is beneficial to evaluate power generation at a single mode, typically the fundamental mode. Under the assumption that the voltage developed across the surface of the piezoelectric layer is uniform, by applying mode superposition with structural modes the coupled vector governing equations for a bimorph in Eqs. (79) and (81) can be reduced and expressed in terms of coupled electromechanical equations with scalar parameters under an assumed harmonic excitation given by,

\[ \left( K + j \omega C - \omega^2 M \right) \ddot{\theta} + \omega^2 \theta \dot{v} = F \]  

(82)

\[ j \omega \theta \dot{\theta} + j \omega C_{pe} \frac{\dot{v}}{R_i} \]  

(83)

where the hat indicates time harmonic representation of the variable, \( j \) the imaginary number and \( \omega \) the frequency. In the above representation, \( M, C, \) and \( K \), are scalar modal mass, damping, and stiffness coefficients, while \( \theta_e = \theta_1 + \theta_2 \), the effective coupling, and \( C_{pe} = C_{p1} + C_{p2} \), the effective capacitance, are the sum of the corresponding individual piezoelectric layer contributions for a parallel connection. Moreover, since the bimorph is symmetric and due to the parallel connection
of the piezoelectric layers, $\theta = \theta_1 = \theta_2$ and $C_p = C_p1 = C_p2$ resulting in $\theta_e = 2\theta$ and $C_{pe} = 2C_p$. The effective capacitance is given by,

$$C_{pe} = 2 \frac{\varepsilon_{33} b L}{t_p} \quad (84)$$

where, $\varepsilon_{33}$ denotes the ‘33’ component of the permittivity matrix (equals product of relative dielectric constant, $K_i^*$ and permittivity of free space, $\varepsilon_0$). The coupling term $\theta_e$ in Eq. (82) is responsible for the electromechanical coupled behavior which when absent ($\theta_e = 0$), reduces the equation to a purely mechanical structural form. The coupling is directly proportional to the piezoelectric strain ($d$) or stress constant ($e$), thickness of piezoelectric ($t_p$) and substrate layers ($t_s$).

Once, the resistive load is included in the governing equation, electric power can be harvested at a finite resistance between zero and infinity. The two limiting cases however are of importance and is normally termed the short circuit ($R_l = 0$) and open circuit ($R_l = \infty$) conditions with the corresponding frequencies, short circuit ($\omega_{sc}$) and open circuit ($\omega_{oc}$) frequency. Note that no voltage is developed in a short circuit condition while no current is developed in an open circuit condition, hence resulting in no power generation at these two conditions. The open circuit condition is simulated by setting $R_l = \infty$ in Eq. (83) and in the limit when the load resistor tends to infinity ($R_l = \infty$) the governing equations Eqs. (82) and (83) can be then rewritten as,

$$\left( K + j\omega C - \omega^2 M \right) \hat{r}_{oc} - \theta_e \hat{v}_{oc} = F \quad (85)$$

$$j\omega r_p \theta_e + j\omega C_{pe} \hat{v}_{oc} = 0 \quad (86)$$
The open circuit voltage $v_{oc}$ can be evaluated in terms of $r_{oc}$ from Eq. (86) and is substituted into Eq. (85) resulting in,

$$\left( \{ K + \theta_e^2 C^{-1} \} + j \omega C - \omega^2 M \right) \hat{r}_{oc} = F \quad (87)$$

Again, it can be seen that the above governing equation reduces to a purely structural form if the electromechanical coupling is absent ($\theta_e = 0$). In addition, it can be observed that the stiffness term is altered due to electromechanical coupling resulting in an increased stiffness termed the open circuit stiffness, while the open circuit natural frequency ($\omega_{oc}$) can be obtained by solving the corresponding eigenvalue problem. The short circuit condition is simulated by first representing governing equations Eqs. (82) and (83) in terms of current (using $v = iR_l$) and then setting $R_l = 0$ in both the actuation and sensing equations resulting in,

$$\left( K + j \omega C - \omega^2 M \right) \hat{r}_{sc} = F \quad (88)$$

$$j \omega \hat{r}_{oc} \theta_e + i_{sc} = 0 \quad (89)$$

It can be seen from Eq. (88) that the actuation equation represents a purely structural problem with the same structural stiffness (short circuit stiffness) as electromechanical coupling is absent. The short circuit natural frequency ($\omega_{sc}$) is obtained by solving the corresponding eigenvalue problem. The short circuit current can be evaluated by solving Eqs. (88) and (89) through substitution method.

Open circuit voltage and short circuit current can also be discussed in terms of the constitutive relations governing the coupled piezoelectric problem. In the case of beam models, with nonzero normal stress and normal strain components in the longitudinal $x_1$ direction, and electric field in the $x_3$-direction through the thickness of the piezoelectric layers of the bimorph beam structure, the constitutive equations can be represented in the strain-displacement form as,
\[ S_i = s_{i1} T_i + d_{i1} E_i \quad \text{and} \quad D_i = d_{i3} T_i + \varepsilon_{i3}^T E_i \]

where the variables \( T, S, E, \) and \( D \) represent the stress, strain, electric field and electric displacement respectively while material properties \( s_{i1} \) indicates the compliance component along \(-11\) direction, \( d_{i1} \) represents the piezoelectric strain constant and \( \varepsilon_{i3}^T \) the dielectric constant. Superscripts ‘\( E \)’ and ‘\( T \)’ denote measurement under constant electric field and stress respectively. Under open circuit conditions, the electric displacement term is assumed to zero (\( D = 0 \)) while on the other hand under short circuit condition the electric field term is zero (\( E = 0 \)), resulting in decoupling of the constitutive relation in both conditions.

In many cases, for example a beam model, it is easy to evaluate the open circuit voltage generated as an external electric load is not required for its evaluation. Regardless of whether the open circuit voltage is determined numerically or analytically a quick method to evaluate power generation across a resistive load once the open circuit voltage is determined, is advantageous. The equivalent open circuit voltage driven circuit shown in Figure 28 is adopted to evaluate power generated across a resistive load, where the piezoelectric generator is modeled as a voltage source in series with a capacitor connected to a load resistor across which power generation can be evaluated as a function of load voltage. The open circuit voltage is the source voltage that drives the resistor-capacitor (\( RC \)) circuit. The electromechanical representation of the piezoelectric generator has been equivalently modeled as a current source in series or a voltage source in parallel with a capacitor by many, for instance (Roundy et al. 2003; Ng & Liao 2005; Priya 2005; Roundy et al. 2004; Ottman et al. 2002). The consequences of such a representation and the degree of approximation when representing the coupled electromechanical model as an equivalent circuit model is investigated subsequently.

Power generated across the resistor can be evaluated as a function the voltage drop across a finite resistive load. For a fully coupled problem, the optimal resistance is dependent on electromechanical coupling determined through an optimization problem. However, for the open
circuit voltage driven equivalent circuit model representation adopted, the optimal resistive load is based on the stand-alone $RC$ circuit. Using resistive impedance matching technique, the optimal load of the $RC$ circuit is obtained as $R_{opt} = 1/\omega_s C_p$.

![Piezoelectric generator](image)

**Figure 28: Open circuit voltage driven equivalent circuit representation of piezoelectric harvester connected to a resistive load**

To evaluate the voltage drop across the resistive load, first the relation between the source open circuit voltage and the load voltage is obtained by applying Kirchoff’s voltage law. The voltage law states the algebraic sum of the voltages around a closed circuit must be zero, $V_{oc} = V_c + V_l$ where $V_c$ and $V_l = iR_l$ are the voltages developed across the capacitor and load resistor respectively. Then using the fundamental relations $q = V_c C_{pe}$ and $i = dq/dt$ in addition to the time harmonic representation, $d/dt \rightarrow j\omega$, an equation for the current $i$ flowing through the circuit can be obtained as,

$$\hat{i} = \frac{V_{oc}}{\left(1 + j\omega C_p R_l\right)}$$  \hspace{1cm} (90)

Since the current flowing through the circuit is constant, the load voltage across the resistor can be determined by,
Peak power across the load resistor can be then evaluated as \( P = \frac{v_i^2}{R_i} \). The open circuit voltage driven equivalent electrical circuit is analyzed further by comparing it to the coupled electrical governing equation in Eq. (83). Re-arranging terms in Eq. (91) and substituting the value of \( v_{oc} \) in terms of \( r_{oc} \) obtained from Eq. (86) yields,

\[
\hat{v}_j = iR_i = v_{oc} \left( \frac{R_i}{R_i + \frac{1}{j\omega C_{pe}}} \right)
\]

The governing equation of the equivalent model in Eq. (92) on comparison with the sensing equation in Eq. (83) mathematically reveals the approximation. The first term in Eq. (83) \( j\omega \hat{r} \theta_c \) represents the driving current source while the same driving current source in the equivalent model seen in Eq. (92) is represented by \( j\omega r_{oc} \theta_c \), where \( r_{oc} \) is the displacement at open circuit conditions. Other terms in Eq. (92) correctly represent the current passing through the capacitor and resistor. Representing the current source as \( j\omega r_{oc} \theta_c \) instead of \( j\omega \hat{r} \theta_c \) results in the beam deflection and source voltage being independent of load resistance. As a result, by using the equivalent circuit at finite resistive load, the backward voltage feedback into the mechanical domain due to electromechanical coupling is not captured as the mechanical domain is not coupled in the circuit. The inability to address this backward feedback due to electromechanical coupling reduces the accuracy of the open circuit voltage driven equivalent circuit representation. Hence this equivalent circuit model to evaluate power generation represented by Eq. (92), serves as an approximate model to the fully coupled model represented by Eqs. (82) and (83).
the lack of backward voltage feedback effect due to electromechanical coupling, the open circuit voltage driven equivalent circuit model is referred to as the uncoupled model in the remainder of the paper. However, note that the determination of open circuit voltage discussed earlier is through solving a coupled electromechanical problem at infinite load.

4.2. Investigation into degree of approximation of the uncoupled equivalent model

From the discussion in the previous section it is clear that the estimation of power generation using the equivalent model is an approximation. To understand the degree of approximation involved it is beneficial to compare power generated from the open circuit voltage driven equivalent uncoupled equivalent circuit model to a fully coupled electromechanical model.

As discussed before, to maximize the electrical response of the piezoelectric energy harvester it is desirable to drive the harvester near the resonant frequency of the structure. As a result, using a reduced scalar electromechanical model that assumes beam motion is dominated by a single mode is valid. Hence, the electromechanical model presented in (Liao & Sodano 2008) where a global distributed parameter approach combining Rayleigh-Ritz discretization method and Bernoulli beam assumptions is reduced and evaluated at a single mode (fundamental mode) is adopted as a reference model to which the equivalent model is compared. The mode shape integrals of the reduced single mode scalar effective system parameters namely the mass ($M$), stiffness ($K$), coupling ($\theta$), capacitance ($C_{pe}$) and effective input mass ($D$) resulting from the inertial load due to base excitation are evaluated for the piezoelectric cantilever bimorph in parallel connection and is given by,
\[ M = \left( \rho t_s + 2 \rho_p t_p \right) b \]  
(93)

\[ K = \left[ 1.0302 c_t \left( \frac{t_s^2}{L^2} \right) + 2.0604 c_p \left( \frac{3t_s^2 t_p + 6t_t^2 t_p^2 + 4t_p^3}{L^2} \right) \right] b \]  
(94)

\[ \theta_e = -2.753 d_{31} c_p b \left( \frac{t_s + t_p}{\sqrt{L}} \right) \]  
(95)

\[ D = -0.783 b \left( 2 \rho_p t_p + \rho_p t_s \right) \sqrt{L} \]  
(96)

where, \( \rho \) denotes the density of the material. Subscripts ‘s’ and ‘p’ denote the substrate and piezoelectric layer in the remainder of the paper. Note that in Eq. (95) the coupling term \( \theta_e \) is linearly proportional to PZT strain coefficient \( d_{31} \), piezoelectric layer thickness \( t_p \) and substrate thickness \( t_s \). Damping is imparted to the system in the form of Rayleigh proportional damping which is frequently employed in modal analysis due to the ease in diagonalization of the damping matrix as it is a linear combination of mass and stiffness matrices. Damping is given by \( C = \alpha_d M + \beta_d K \) where \( \alpha_d \) and \( \beta_d \) are damping coefficients evaluated by solving the equation \( \zeta_i = 0.5 \) \((\alpha_d + \omega \beta_d)\) typically at the first two vibration modes \((i=1,2)\) to obtain two equations and two unknowns for the same damping ratio \((\zeta)\). For the current study a proportional damping with \( \zeta = 0.019 \) is introduced to electromechanical system to the fundamental mode to be consistent with the reference model. The closed form expression to determine power generated across the resistor at the first mode as given by is,

\[ P = \left\{ D^2 A^2 \omega^2 R \right\} \left[ \left( \frac{K}{\theta_e} - \frac{M}{\theta_e \omega^2} \right) - \left( \frac{CC_{pe} \omega}{\theta_e \omega^3} \right) R \right]^2 + \left[ \left( \frac{C}{\theta_e \omega} \right) + \left( \frac{KC_{pe} \omega + \theta_e \omega - MC_{pe} \omega^3}{\theta_e \omega^3} \right) R \right]^2 \right\}^{-1} \]  
(97)
The geometric and material properties of the reference model is adopted in the current paper for power generation comparison purposes. The global geometric parameters, overall length $L$ and width $b$ are 66.62 mm and 9.74 mm respectively while the piezoelectric (PZT-5H) and the solid substrate (brass) layer thicknesses are 0.26 mm ($t_p$) and 0.76 mm ($t_s$) respectively. PZT-5H has a density $\rho_p$ 7200 kg/m$^3$, piezoelectric strain constant $d_{31}=-320$ pm/V, relative dielectric constant $K^T$ of 3800 and an elastic modulus $c_p$ of 62 GPa, while brass density $\rho_s$ is 8700 kg/m$^3$ with elastic modulus $c_s$ of 97 GPa.

A finite element model was developed in Abaqus/CAE to determine the open circuit voltage generated on the electrodes covering the piezoelectric layers due to base excitation of 1g. From the FE analysis the short circuit and open circuit natural frequencies of the vibrating piezoelectric bimorph was determined to be 135.5 Hz and 139.4 Hz respectively which matches very closely (less than 1% variation) with the reference model. Peak power response at the optimal resistance of the stand-alone $RC$ circuit ($R_{opt}=7.07$ k$\Omega$) model is compared to that obtained using the fully coupled analytical single mode solution in Eq. (97) and presented in Figure 29 below.

![Figure 29: Power generated from the uncoupled equivalent circuit model compared to that from the fully coupled analytical single mode solution](image-url)
As expected, the equivalent circuit model overestimates the power generated from the system as seen in Figure 29, as the backward voltage feedback due to electromechanical coupling is overlooked. It can also be observed that the power generated from the equivalent circuit model is exactly at the open circuit natural frequency of 139.4 Hz when it is known from literature (Liao & Sodano 2008) that peak power frequency is slightly lower than the open circuit natural frequency. As discussed earlier the equivalent model does not capture the dependence of beam deflection, voltage, and natural frequency on resistive load. Hence, it is beneficial to compare power response from the equivalent circuit model to the behavior exhibited by the coupled analytical single mode solution across a range of resistive loads. The variation in power generated with resistance using the equivalent and fully coupled model is shown in Figure 30 where it can be seen that a disagreement between the two models exists near the optimal resistance of the stand-alone RC circuit \( R_l = 7.07 \, k\Omega \). In contrast, away from optimal resistance the difference is relatively small and matches exactly with the single mode solution at 2 resistance values (2 intersections).

![Figure 30: Variation in power obtained from the uncoupled equivalent circuit model and the analytical single mode solution with resistance](image-url)
The two exact resistance values where peak power amplitude agreement between the two models were determined to be at around 3 times and 1/3 the $R_{opt}$=$1/\omega_{sc}C_p$ value. Subsequently, power generated at $(1/3)R_{opt}$ and $3R_{opt}$ is computed and compared to the analytical single mode solution, presented in Figure 31(a) and (b) respectively.

![Figure 31](image)

**Figure 31:** A comparison of power response evaluated by the analytical single mode solution and the uncoupled equivalent circuit model at (a) $(1/3)R_{opt}$ and (b) $3R_{opt}$

It can be seen from the above figure that peak power amplitude and peak power frequency evaluated through the equivalent circuit model closely matches the analytical solution at $3R_{opt}$ as seen in Figure 31(b). On the other hand, at $(1/3)R_{opt}$ a peak power amplitude match with a 3.5% variation in peak power frequency is observed from Figure 31(a). This variation in peak power frequency (between short and open circuit natural frequency) with change in resistance that is observed when using the fully coupled electromechanical model as shown in Figure 32(a) is not captured through the equivalent circuit model. As the backward voltage feedback due to electromechanical coupling is overlooked by equivalent model, only a power amplitude variation and not a frequency variation (frequency is constant at open circuit natural
frequency) with resistance is observed, as seen in Figure 32(b). It can be seen from Figure 32 (a) and (b) that peak power generated by the equivalent circuit model at $3R_{opt}$ and $(1/3)R_{opt}$ is the same, a trend also exhibited by the coupled analytical model. However, unlike the fully coupled model the peak power frequency does not vary and remains at the open circuit natural frequency.

![Figure 32: Power response at three resistive loads obtained from (a) the single mode solution and (b) the uncoupled equivalent circuit model](image)

Note that from Figure 30 and Figure 32(a), the peak power generated when using the fully coupled single mode solution is fairly constant across a wide range of resistance (between $1/3R_{opt}$ and $3R_{opt}$). In fact, variation in power generated at $3R_{opt}$ and $R_{opt}$ is less than 5% with only a 2% change peak power frequency. In addition Figure 31(b) suggests that the equivalent circuit model is more accurate at $3R_{opt}$. Hence, it is concluded that the equivalent circuit model can be used at a calibrated resistance $3R_{opt}$, capturing a more accurate peak power amplitude and peak power frequency along with good match in the frequency bandwidth as seen in Figure 31(b).

The calibration factor however is dependent on the short circuit and open circuit natural frequency range ($\omega_{oc} - \omega_{sc}$) which in turn is dependent on the electromechanical coupling. A strong coupling will have a pronounced effect on $\omega_{oc} - \omega_{sc}$. A larger short circuit – open circuit range will require larger calibration factor while moderate coupling will result in smaller $\omega_{oc} - \omega_{sc}$.
needing a small calibration factor. A weakly coupled system will have an insignificant shift from short circuit to open circuit natural frequency requiring no calibration. The electromechanical coupling coefficient \( k_{31} \) is a piezoelectric material property that measures the ability of the material to convert mechanical energy into electrical energy or vice versa and as given by (IEEE Standard on Piezoelectricity 1987) is,

\[
k_{31} = d_{31} \sqrt{\frac{c_{11}^e}{K_{11}^e \varepsilon_0}}
\]  

(98)

where \( c_{11} \) is the short circuit modulus which is the elastic modulus along the \(-11\) direction. Note that the piezoelectric strain constant \( d_{31} \) has a stronger influence on the coupling coefficient when compared to the other parameters, dielectric constant and elastic modulus. The impact of \( d_{31} \) on power generation is also studied in (Chandrasekharan et al. 2013b) through a parameter sensitivity analysis and is found to have the strongest influence on power generation among many design geometric and material parameters. The linear relation between \( d_{31} \) and coupling seen in Eq. (98) is also seen earlier in Eq. (95). The short circuit – open circuit range dependence on electromechanical coupling which in turn is strongly influenced by \( d_{31} \) is shown in Figure 33. It can be seen that a piezoelectric bimorph with larger piezoelectric strain constant \( (d_{31}) \) has a stronger electromechanical coupling coefficient and hence larger \( \omega_{oc} - \omega_{sc} \) while there is reduction in short circuit to open circuit frequency range with decrease in \( d_{31} \). At low values of \( d_{31} \) it can be seen that there is an insignificant shift in the short circuit to open circuit frequency.
Figure 33: The variation in the shift from short circuit to open circuit natural frequency with piezoelectric strain constant $d_{31}$

However, $k_{31}$ is a material property and not a property of the device itself. The effective system coupling of the device is typically lower than that of the piezoelectric material coupling itself due to addition of the structural layer and can be computed through (Lesieutre 1998; Roundy & Wright 2004),

$$k_{31} = \frac{\omega_{oc}^2 - \omega_{sc}^2}{\omega_{sc}^2}$$  \hspace{1cm} (99)

The relation between effective system coupling and the amount of shift between short and open circuit frequency is also highlighted in Eq. (99). Lower system coupling is indicative of a small range between short and open circuit natural frequency and vice-versa. Consequently, at lower coupling the resonant frequency variation with resistance is not prominent and the short circuit and open circuit frequency is approximately the same, rendering $\omega_{sc} = \omega_{oc} = \omega_r$. Hence it can be concluded that, lower the effective system coupling smaller the calibration factor required as the system is weakly coupled and the uncoupled equivalent circuit shown in Figure 28 tends to
become more accurate representation of the coupled piezoelectric problem. To confirm this behavior, power generated from the approximate model with a piezoelectric strain constant with one order of magnitude lower \( d_{31} = -25 \) pm/V was compared to the fully coupled single mode analytical solution shown in Figure 34.

![Figure 34: Power response comparison between the uncoupled equivalent circuit model and the fully coupled analytical model at low coupling, \( d_{31} = -25 \) pm/V](image)

The relation between effective system coupling and the amount of shift between short and open circuit frequency is also highlighted in Eq. (99). Lower system coupling is indicative of a small range between short and open circuit natural frequency and vice-versa. Consequently, at lower coupling the resonant frequency variation with resistance is not prominent and the short circuit and open circuit frequency is approximately the same, rendering \( \omega_0 = \omega_{oc} = \omega_r \). Hence it can be concluded that, lower the effective system coupling smaller the calibration factor required as the system is weakly coupled and the uncoupled equivalent circuit shown in Figure 28 tends to become more accurate representation of the coupled piezoelectric problem. To confirm this behavior, power generated from the approximate model with a piezoelectric strain constant with
one order of magnitude lower test $d_{31} = -25 \text{ pm/V}$ was compared to the fully coupled single mode analytical solution shown in Figure 34.

Subsequently a parameter study on $d_{31}$ ranging from -1 pm/V to -320 pm/V was then studied to observe the effect of system coupling on the degree of approximation, shown in Figure 35. As expected, at low values of $d_{31}$ that in turn results in low/weak system coupling ($k_{sys} < 0.1$) a good agreement exists between power generated through the equivalent circuit model matches and the fully coupled single mode analytical solution. Ignoring the backward voltage feedback for weakly coupled system does not drastically reduce the accuracy of the power generation estimation using equivalent circuit model. However, at higher/stronger coupling ($k_{sys} > 0.1$) an over prediction of power generation from the equivalent model exists, as considerable backward voltage feedback effect is introduced and not accounted for in the mechanical governing equation resulting in its divergence from the fully coupled solution.

![Graph](image_url)

**Figure 35:** A peak power response comparison between the uncoupled equivalent circuit and the fully coupled analytical model at different system coupling conditions

At higher/stronger coupling conditions however power generation can be estimated with the help of the corrected uncoupled model where power generation is evaluated at a calibrated resistance
of $3R_{opt}$. Figure 36 below compares power response of the calibrated open circuit voltage driven uncoupled equivalent circuit model at different system coupling values and it can be seen that it closely matches the fully coupled analytical solution closely even at higher coupling conditions ($k_{sys} > 0.1$).

![Power Response Comparison](image)

**Figure 36: A peak power response comparison between the calibrated uncoupled equivalent circuit and the fully coupled analytical model at different system coupling conditions**

From the above discussion it is concluded that the open circuit voltage driven uncoupled equivalent circuit model can be utilized to determine power generation in energy harvesting applications where the effective system coupling is low. In order to use the model in applications where the system coupling is higher, a corrected calibrated uncoupled model based on resistance can be adopted with good accuracy at a single resistive load. However, the calibration factor for resistance is case specific and dependent on the shift in short circuit to open circuit frequency.

A particular application where the open circuit voltage driven uncoupled equivalent circuit model can be used is in MEMS energy harvesters. The overall dimension of the harvester for such an application is typically less 1cm$^3$ that requires thin piezoelectric film deposition with dimensions usually in micrometers. The electromechanical coupling as seen in Eq. (95) is directly proportional to the piezoelectric stress/strain constant, the piezoelectric and substrate layer thickness along with the overall width and length of the harvester. At MEMS scale, all geometric
dimensions are very small, especially the piezoelectric film thickness leading to the very low electromechanical coupling. In fact the effective system coupling $k_{sys}$ for PZT based MEMS energy harvester ($k_{sys} = 0.02$) is an order of magnitude lower than the PZT ceramic ($k_{sys} = 0.3$) (Kamel et al. 2010; Renaud et al. 2008). A look at Figure 35 reveals that at $k_{sys} = 0.02$, a very good agreement between the equivalent circuit model and the analytical single mode solution exists suggesting that the equivalent model can be an accurate method to predict the power response for micro-machined piezoelectric energy harvesters for MEMS applications.

The accuracy of the equivalent model for energy harvesters at MEMS scale is also investigated with a different piezoelectric material. The feasibility of using aluminum nitride (AlN) as the piezoelectric material for micro-machined piezoelectric harvesters has been investigated by many, for instance see (Elfrink et al. 2009). It is understood from their efforts that AlN proves to be a superior choice to the more common PZT in terms of ease of integration with CMOS based electronics and deposition technology (Elfrink et al. 2009; Kim et al. 2012). However the piezoelectric strain constant $d_{31}$ for AlN is -1.5 pm/V compared to -320 pm/V for PZT-5H, two orders of magnitude smaller. Despite the low piezoelectric strains constant the effective system coupling of the harvester when using AlN is comparable to that when using thin film PZT due superior sputter film deposition techniques for AlN and lower dielectric constant for AlN that increases the material coupling $k_{31}$, as seen in Eq. (98). In fact, a higher system coupling of around 0.06 is reported in (Kamel et al. 2010) when compared to 0.02 for the PZT thin film. A closer look at Figure 35 shows that even at $k_{sys} = 0.06$ a good agreement between the open circuit voltage driven equivalent circuit approximate model and the analytical single mode solution is obtained.
The above results and discussion indicates the validity of the open circuit voltage driven equivalent circuit approximate model to predict power generated from weakly coupled systems with good accuracy. In weakly coupled systems, the effective system coupling is significantly reduced when compared to bulk piezoelectric ceramics. As a result, representing the electromechanically coupled piezoelectric generator connected a resistive load as an open circuit voltage driven equivalent circuit tends to become a more accurate representation at low coupling conditions. It is worthwhile to recall that power generated from vibration based piezoelectric energy harvesters find their use majorly in MEMS applications where electromechanical coupling is typically low/weak.

4.3. Conclusion

The accuracy of an uncoupled open circuit voltage driven equivalent model representation of the fully coupled piezoelectric energy harvester is investigated by comparing it to a fully coupled piezoelectric energy harvester model. The degree of equivalent model approximation and its consequences along with applications where the equivalent circuit model can be used is also examined. It was found that when the system coupling is low/weak such as in MEMS energy harvesters, the backward feedback effect (voltage induced force) due to electromechanical coupling is insignificant and hence the equivalent circuit model is a good approximation of the piezoelectric problem. This is due to the effective system coupling of weakly coupled systems being reduced significantly (by an order of magnitude in MEMS). At higher coupling, the backward feedback effect due to coupling becomes more pronounced and the inability of the equivalent circuit model to capture the backward feedback effect results in an overestimation of power generation across the resistive load. This overestimation of power at higher coupling is corrected by calibrating the equivalent circuit model based on resistance to significantly improve
accuracy of the equivalent model at the calibrated load when compared to a fully coupled electromechanical model. Hence, the equivalent circuit representation is best suited to model weakly coupled systems such as micro machined piezoelectric energy harvesters while at higher coupling a resistance based calibration can be used to more accurately predict power response at a specific resistive load.
Chapter 5: Broader impact and future direction

The proposed design method involving integration of honeycomb structures with energy harvesting devices could be adopted as the prototype energy generator for conducting future research in the field of energy harvesting. The proposed model has the potential to replace the traditional bimorph as the most commonly used physical configuration as it can be conveniently adopted in a multitude of different areas including non-linear energy harvesting, study of tuning mechanisms, research in piezoelectric materials among many others.

The improved power to weight ratio of honeycomb core bimorphs can be exploited by the many facets of energy harvesting technologies including the promising non-linear energy harvesting. In fact although not explored in this dissertation, other vibration to power conversion mechanisms like electromagnetic and electrostatic transduction configurations especially a cantilever beam based configuration (Boisseau et al. 2011) can benefit from partnering the existing energy harvesting configurations with cellular substrates to increase power to weight ratio of the harvester.
Appendix: Historical perspective on piezoelectricity

Energy harvesting through piezoelectric conversion is a relatively new field when compared to the discovery of piezoelectric effect. The author understands that a historical perspective on piezoelectricity is not essential for energy harvesting through piezoelectric effect but is of the opinion that it is only appropriate to acknowledge and pay tribute to the discovery and early works pertaining to piezoelectricity that is fundamental to the field of piezoelectric energy harvesting. The discovery, key early contributions, and modelling of piezoelectric phenomenon are outlined in the subsequent discussion.

Piezoelectric effect was experimentally discovered by Jacques and Pierre Curie in 1880. It was reported that under compression/decompression of an asymmetrical crystal along their hemihedral axes, an electric polarization was observed. This phenomenon was unique to certain type of crystals and absent in amorphous materials (Curie & Curie 1880). The word piezoelectricity was however coined by Hankel in 1881 from the Greek word, ‘piezein’ meaning, ‘to press’.

It must be stressed that the discovery by the Curie brothers was the generation of an electric charge in a substance by a mechanical stress, which is known as direct piezoelectric effect. A reciprocal effect that involved deformation of material under an applied electric field had not been discovered yet. Reverse or inverse piezoelectric effect was theoretically proposed and mathematically deduced by Lippmann in 1881 (Lippmann 1881), based on thermodynamic principles. This was later experimentally validated by the Curie brothers later that year (Curie & Curie 1881).

Furthermore, on comparison of the crystal axes that were excited by pressure to those which were not, an empirical rule for the generation of electric effects was formulated in (Curie
that states that the crystal should not have a center, i.e., no plane of symmetry perpendicular to the polar axis and that no axis of symmetry of an even order perpendicular to the direction should exist. These conditions are necessary and sufficient for crystals to exhibit piezoelectric effect. It was concluded that the variation in electrical effect with pressure on crystals is linear and that it should be represented through a coefficient. Their further work was focused on experimentally measuring these coefficients.

A second wave of research pertaining to piezoelectricity came in the form of explaining the source of piezoelectricity. Rongten, J. and P. Curie, Kundt and Friedel concluded that the cause of piezoelectricity lies in the internal tension of the asymmetrical crystallographic axes of the crystal caused by the changes in molecular distances that are permanently polarized. These theories were mostly qualitative and a mathematical account was limited to charge development on just one axis. No mathematical account was put forward to include variation in electric effects with directions of pressure. Though the molecular explanation of the Curies agreed with many observations in early 1880s, later observation revealed inconsistencies that were made evident in a landmark torsion test conducted by Rongten that exposed the weaknesses in the molecular explanatory theory. Details pertaining to the experiment can be found in (Katzir 2006). This led to the construction of a phenomenological theory put forward by Voigt which replaced the molecular theory as the reference on which future work would be built on. However, note that it was due to a hypothetical molecular explanatory theory that led to the discovery of piezoelectric effect in 1880 by the Currie brothers, hence its importance.

Voigt’s theory was based on the piezoelectric phenomenon arising due to linear strain effects (experimentally determined) and general principles of crystal physics, expressed through continuous differential equations (Wiedemann 1894; Pockels 1895). It was a general theory as it was applicable to all crystals under any stress. It could be argued that Voigt’s general theory
represented a paradigm shift in the modelling approach towards explaining the piezoelectric effects.

Further elaboration of the phenomenological theory included thermodynamic formulation of the theory between 1892 and 1894. To this end, work by Duhem, Pockel, Riecke, Voigt and Lord Kelvin are all significant. Interested readers can refer to (Katzir 2006) to understand the evolution of the continuum model that culminated in Voigt’s theory of secondary phenomena with full thermodynamic formulations in 1894, the basis of the current piezoelectric theory. By 1895 piezoelectric research had firmly embodied both empirical and theoretical knowledge and it can be concluded that molecular, continuum and experimental models all played important roles.
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