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PRACTICAL ROBUST GEOTECHNICAL DESIGN- METHODOLOGY AND APPLICATIONS

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ABSTRACT

This dissertation advances the robust geotechnical design methodology by offering improvements, which makes it more efficient and practical for the design of geotechnical systems. Robust geotechnical design (RGD) methodology seeks an optimal design, which is insensitive to, or robust against, the variation in the uncertain input parameters (called “noise factors”) by only adjusting the easy to control parameters (called “design parameters”). The main goal of robust design is to consider safety, cost and robustness simultaneously. Because the cost and the robustness are conflicting objectives, the multi-objective optimization that considers these two objectives while enforcing the safety constraint yields not a single best design but a set of non-dominated designs, which are neither superior nor inferior to one another. These non-dominated solutions form a Pareto front. All these non-dominated designs on the Pareto front are equally optimal in the sense that no improvement can be achieved in one objective without worsening in the other objective. To locate the best compromise between the objectives, knee point concept is often adopted.

In this dissertation, the existing RGD methodology, different robustness measures, and different methods for locating the knee point are examined, followed by the development of a new simplified procedure for determination of knee point. The reliability-based RGD approach is also improved in efficiency by coupling the reliability analysis of the system performance and design robustness evaluation. A simplified and efficient procedure is also proposed to implement the RGD optimization procedure in the Microsoft Excel spreadsheet.
With all improvements made in this research, the RGD approach can still be computationally challenging for the practicing engineer. In this regards, an efficient and practical RGD procedure using a Microsoft Excel spreadsheet is developed. Because the numerical software programs are often used to evaluate the system response, a response surface model is proposed to approximate the performance functions and integrated into the simplified RGD approach.

The significance and practicality of the proposed simplified RGD methodology is illustrated with multiple geotechnical applications, including the design of shallow foundations, rock slopes, drilled shafts, and supported excavations.
DEDICATION

I dedicate this dissertation to my beloved parents and my only brother, who have offered unwavering support and encouragement during all these years. I would also like to thank Shervin, who has supported me through this process and has never left my side.
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Chapter 1
INTRODUCTION

Purpose of the Research

Wide ranges of uncertainties that often exist in soil parameters, as well as the adopted models, can lead to uncertainty in the predicted response of the designed system. In traditional geotechnical designs, selected candidate designs are checked against safety requirements. Then, among the acceptable designs (in terms of safety requirements), the least cost design is adopted as the final design. In this approach, safety requirements are usually specified in terms of target factor of safety ($FS$) against shear failure or excessive deformation in a deterministic evaluation, or target probability of failure ($p_f$) or reliability index ($\beta$) in a reliability-based design (RBD) approach.

However, the deterministic methods do not explicitly consider the uncertainties; rather, a proper factor of safety is adopted to cope with these uncertainties. In the context of RBD approach, the performance of the system is analyzed using probabilistic methods that consider explicitly uncertainties (e.g., Harr 1987; Wu et al. 1989; Tang and Gilbert 1993; Christian et al. 1994; Lacasse and Nadim 1996; Griffiths et al. 2002; Phoon et al. 2003; Fenton and Griffiths 2008; Schuster et al. 2008; Juang et al. 2009; Juang et al. 2011; Wang et al. 2011; Zhang et al. 2011) but still the design is highly affected by the accuracy of statistical characterization of the uncertainties. The Load and Resistance Factor Design (LRFD) is becoming a design method of choice in geotechnical practice, in lieu of the factor of safety ($FS$)-based design and RBD approaches. The LRFD code employs partial factors to account for uncertainties. However, the standard LRFD
approach that involves fixed partial factors cannot cover all design scenarios involving different levels of uncertainties.

The traditional design, regardless of whether the FS-based approach, RBD approach, or the LRFD approach is employed, focuses mainly on safety and cost, but the design robustness is seldom considered. Thus, the predicted system response can vary widely, which is undesirable in a design situation. Of course, the variation of the system response may be reduced by reducing the variation in the uncertain input parameters. However, in many geotechnical projects the ability to reduce the uncertainties in soil variability is restricted by the nature of soil deposit (i.e., inherent soil variability) and/or the number of soil test data that is available. Assuming these uncertainties cannot be further reduced, robust design approach, originated from the field of Industrial Engineering (Taguchi 1986; Tsui 1992; Phadke 1989; Chen et al. 1996) can offer an alternative. The robust design seeks an optimal design that is insensitive to, or robust against, the input parameter uncertainties by only adjusting the “easy-to-control” parameters (termed design parameters in the concept of robust design). The main aim of this robust design approach is to consider robustness along with satisfying the safety and economic requirements.

Because of the high variations that often exist in geotechnical parameters and the adopted models in the geotechnical engineering, the robust design concept was introduced to the geotechnical field by Juang and his co-workers (Juang and Wang 2013; Juang et al. 2013a; Juang et al. 2013b; Wang et al. 2013; Gong et al. 2014a; Gong et al. 2014b; Gong et al. 2015; Juang et al. 2014; Wang et al. 2014).
In the existing reliability-based RGD methodology (Juang and Wang 2013), in order to increase the robustness of the design, the failure probability is taken as the system response of concern, and the standard deviation of the failure probability is adopted as a measure of design robustness. While mathematically sound, this reliability-based RGD approach is computationally demanding, as it involves computations using point estimation method (PEM), first order reliability method (FORM), and multi-objective optimization algorithm in three layers of computational loops. In that approach, FORM was used to compute the failure probability, PEM was used to evaluate the standard deviation of the failure probability (as a robustness measure), and the multi-objective optimization algorithm was used to locate the optimal design considering the failure probability (i.e., safety requirement), the standard deviation of the failure probability (i.e., design robustness), and the cost.

While a number of geotechnical applications of robust design concepts have been presented by Juang and his co-workers, each with different emphasis, there is no single and systematic introduction of the RGD methodology. Furthermore, the existing RGD methodologies are computationally challenging. The focus of this research is to present a complete RGD methodology, introduce new developments, and improve this methodology such that it becomes an efficient and practical geotechnical design tool.
Objectives and Scope

The objectives of this research are to (1) present a complete RGD methodology, propose an updated RGD methodology with a gradient-based robustness measure, and present a simplified procedure to locate the knee point, (2) provide a simplified procedure for reliability-based RGD using spreadsheet, and (3) propose a new response surface model to integrate the RGD methodology with numerical software. Various geotechnical problems including shallow foundations, rock slopes, drilled shafts, and braced excavations are adopted to demonstrate the efficiency and practicality of the improved RGD methodology.

Dissertation Organization

This dissertation consists of five chapters. The introduction is presented in current chapter, Chapter I, to set the stage for presentation of the entire dissertation.

In Chapter II, the RGD methodology is presented. Different robustness measures and different procedures to locate the most compromised design (knee point) with respect to conflicting objectives are provided, along with a new simplified procedure for determination of the knee point.

In Chapter III, an efficient reliability-based RGD approach is proposed for a design of a drilled shaft in clay. In the proposed framework, the evaluation of the design robustness and reliability analysis of the system performance is coupled, which results in computational efficiency. The entire reliability-based RGD approach is implemented in
the Microsoft Excel spreadsheets.

With all improvements, the RGD approach can be computationally challenging for the practicing engineer. In this regards, in Chapter IV, an efficient and practical RGD procedure using a single Microsoft Excel spreadsheet is developed. To make this practical procedure work with numerical software programs, which are often used to evaluate the system response, a new response surface model is proposed to approximate the performance functions.

Finally, in Chapter V, the last chapter, the main conclusions of this dissertation are presented, along with some recommendations for future study.
Chapter 2
ROBUST DESIGN IN GEOTECHNICAL ENGINEERING

Introduction

Wide ranges of uncertainties often exist in the geotechnical parameters, as well as in the adopted geotechnical models. These uncertainties can lead to uncertainty (or variation) in the “predicted response of the designed system under service loads,” which is termed “system response” in the context of robust design. In the engineering design, it is desirable and essential to reduce the variation of the computed system response so that the system will be robust against, or insensitive to, the uncertainties in the input parameters and the adopted models. In this regard, the design robustness is to be achieved by ensuring low variation in the system response, which is the focus of the newly developed design methodology, termed robust geotechnical design (RGD). In this chapter, the RGD methodology, along with the fundamental issues of how the design robustness is measured, how the optimization is conducted, and how the final design is selected, is presented and illustrated with multiple design examples.

To set the stage for discussing the robust design approach, the traditional geotechnical design approaches, both deterministic and probabilistic, are briefly reviewed in this chapter. In the deterministic design approach, the uncertainties in the input parameters and adopted models are not explicitly considered in the analysis. Rather, a factor of safety (FS) is adopted to cope with the acknowledged but unknown uncertainty in the system response.

However, without the quantitative knowledge of the uncertainty in the system response, the selection of a suitable $FS$ for a design is subjective, which may lead to an over-design or under-design relative to the target safety and cost level. Furthermore, without considering the uncertainties in the input parameters and the adopted models explicitly, it is virtually impossible to characterize and/or reduce the variation of the system response.

To evaluate the uncertainty in the system response so as to improve the design decision, the probabilistic or reliability-based approaches that explicitly consider all the uncertainties in the design analysis have been suggested (e.g., Harr 1987; Baecher and Christian 2003; Ang and Tang 2007; Phoon 2008; Fenton and Griffiths 2008; Bathurst et al. 2011; Zhang et al. 2011a; Wang 2013). In particular, the reliability-based design (RBD), along with one of its simpler variants, load and resistant factor design (LRFD), is becoming the design method of choice for many geotechnical engineers.

In a reliability-based design (RBD), multiple candidate designs are first checked against safety requirements (in terms of a target reliability or failure probability), and the acceptable designs are then screened based on cost, which yields the final design. The RBD will be straightforward if the system response obtained by the reliability analysis is certain and correct so that there will be no question whether a given design satisfies the safety requirement. However, the accuracy and precision of a reliability analysis depends upon how well the random parameters and models are characterized statistically. If the knowledge of the statistical characterization of the adopted model and its input parameters is “perfect,” the results of the reliability analysis will be accurate and certain, and the RBD can be easily implemented, for example, by selecting the least-cost design constrained with the target reliability requirement. In reality, our knowledge of the solution model and its geotechnical
parameters is not perfect, and the system response obtained by the reliability analysis cannot be evaluated with certainty. Thus, even with the RBD approach that considers explicitly the uncertainties in the input parameters and the adopted model, there is still a need to reduce the variation of the system response, as a high variation in the system response can lead to an over-design or under-design relative to the target safety level.

The variation of the system response may be reduced by focusing on reducing the variation of noise factors (i.e., uncertain input parameters and imperfect models); which is illustrated in Figure 2.1 as Approach 1. Alternatively, the same goal may be achieved by focusing on adjusting the design parameters (such as geometry and other parameters that can be controlled by the designer) without reducing the uncertainty of the noise factors. The latter, termed Approach 2 in Figure 2.1, is the essence of robust design and the focus of this chapter.

Figure 2.1 Two approaches to reduce the variation of the system response
Robust design concept was first proposed by Taguchi (1986) in the field of industry engineering. With this approach, a design that yields a minimal variation in the system response (thus achieving design robustness) is obtained by adjusting the design parameters without eliminating the uncertainties in noise factors. Early applications of robust design are closely related to product and mechanical design, primarily to avoid the effects of the uncertainty from environmental and operating conditions (Taguchi 1986; Phadke 1989). The more recent applications are found in various fields such as mechanical, structural and aeronautical design (e.g., Chen et al. 1996; Chen et al. 1999; Lee and Park 2001; Doltsinis et al. 2005; Park et al. 2006; Brik et al. 2007; Lagaros and Fragiadakis 2007; Kumar et al. 2008; Marano et al. 2008; Lee et al. 2010). Applications to geotechnical problems were introduced by Juang and his co-workers (Juang and Wang 2013; Juang et al. 2013a; Juang et al. 2013b; Wang et al. 2013; Gong et al. 2014a; Gong et al. 2015; Juang et al. 2014; Wang et al. 2014). Because of the distinct characteristic of geotechnical problems, which often involves high coefficients of variation (COVs) in the noise factors (i.e., uncertain input parameters and imperfect models), the term “Robust Geotechnical Design” (RGD) was coined by Juang et al. (2013a) for use in these geotechnical applications.

While a number of geotechnical applications of robust design concepts have been presented by Juang and his co-workers, each with different emphasis, there is no single and systematic introduction of the RGD methodology. In this chapter, I aim at presenting a complete RGD methodology with various design examples. While this necessitates a review of some previously published materials (e.g., Juang et al. 2013b), new developments are also presented, including a new robustness measure, a new algorithm to search for the knee point, and an efficient implementation of the multi-objective optimization. This chapter also
provides a systematic overview and discussions of all essential components of the RGD methodology.

Robust Geotechnical Design (RGD) Methodology

In a traditional geotechnical design, using either FS-based or RBD approach, the main considerations are safety and cost. Often, the engineer selects the least-cost design among a set of candidate designs that satisfy the safety requirements. In addition to satisfying the safety and cost requirements, robust geotechnical design (RGD) seeks an optimal design that is insensitive to, or robust against, the variation in noise factors (i.e., uncertain input parameters and imperfect models) by carefully adjusting design parameters (i.e., the parameters that can be easily adjusted by the designer). Thus, the objective of RGD in a given design is to achieve the design robustness, while satisfying the cost and safety requirements. Because the latter two requirements are the focus of both FS-based and reliability-based design approaches, the RGD approach is seen as complementary to these two traditional geotechnical design approaches.

The principle of the RGD methodology, as described in the previous paragraph, can be conveniently implemented as a multi-objective optimization problem. Figure 2.2 shows an example of such implementation.
Find \( d \) to optimize: \([C(d), R(d, \theta)]\) 
Subject to: Safety constraint as a function of \( g(d, \theta) \) 
where
- \( d \) – design parameters;
- \( \theta \) – noise factors;
- \( C \) – cost;
- \( R \) – robustness measure;
- \( g \) – system response.

Figure 2.2 Implementation of the RGD methodology in a multi-objective optimization

Here, RGD seeks an optimal design, represented by a set of design parameters \( (d) \), such that the design robustness \( R(d, \theta) \) and cost \( C(d) \) are optimized simultaneously, while the design (safety) constraint based on the system response \( g(d, \theta) \) is satisfied. Note that the design robustness and the safety constraint are a function of both design parameters \( d \) and noise factors \( \theta \), while the cost is oftentimes a function of only design parameters \( d \) [note: both \( d \) and \( \theta \) are a vector]. In the optimization setting shown in Figure 2.2, safety is a compulsory design constraint that must be satisfied, while the design robustness and the cost efficiency are the two objectives to be optimized. Of course, the robust design optimization concept described above is not the only way to achieve the desired optimal design, as will be discussed later.

The RGD methodology implemented in a multi-objective optimization setting shown in Figure 2.2 may be summarized in the following general steps:

**Step 1:** Describe the geotechnical problem of concern with mathematical models. Here, the system response of concern, noise factors, and design parameters are identified and a suitable mathematical model is established for computing the system response, which can
take a variety of forms as listed in Table 2.1.

Table 2.1 Various forms of system response and measures of design robustness for a number of robust geotechnical design (RGD) applications

<table>
<thead>
<tr>
<th>System</th>
<th>System Response</th>
<th>Robustness Measure</th>
<th>Optimization method</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow foundation</td>
<td>Failure probability</td>
<td>Variation in failure probability; feasibility robustness</td>
<td>RGD with NSGA-II*</td>
<td>Juang and Wang (2013)</td>
</tr>
<tr>
<td>Shallow foundation</td>
<td>Safety margin (difference between resistance and load)</td>
<td>Sensitivity index based on gradient of system response to noise factors</td>
<td>RGD with NSGA-II</td>
<td>Gong et al. (2014b)</td>
</tr>
<tr>
<td>Drilled shaft</td>
<td>Failure probability</td>
<td>Variation in failure probability; feasibility robustness</td>
<td>RGD with NSGA-II</td>
<td>Juang et al. (2013a)</td>
</tr>
<tr>
<td>Rock slope</td>
<td>Failure probability</td>
<td>Variation in failure probability; feasibility robustness</td>
<td>RGD with NSGA-II</td>
<td>Wang et al. (2013)</td>
</tr>
<tr>
<td>Shield tunnel</td>
<td>Factors of safety against tunnel segment safety and serviceability</td>
<td>Signal-to-noise ratio (SNR)</td>
<td>RGD with NSGA-II</td>
<td>Gong et al. (2014a)</td>
</tr>
<tr>
<td>Soil slope</td>
<td>Factor of safety against slope failure</td>
<td>Signal-to-noise ratio (SNR)</td>
<td>RGD with NSGA-II</td>
<td>Gong et al. (2015)</td>
</tr>
<tr>
<td>Diaphragm wall-strut supported excavation</td>
<td>Maximum wall deflection</td>
<td>Variation in maximum wall deflection</td>
<td>RGD with NSGA-II</td>
<td>Juang et al. (2014); Wang et al. (2014)</td>
</tr>
<tr>
<td>Supported excavation</td>
<td>Maximum wall deflection</td>
<td>Sensitivity index based on gradient of system response to noise factors</td>
<td>Simplified RGD without NSGA-II</td>
<td>Khoshnevisan et al. (2014)</td>
</tr>
</tbody>
</table>

*NSGA-II stands for “Non-dominated Sorting Genetic Algorithm” version II developed by Deb et al. (2002).
Step 2: Define the design space based on local experience but allowing for sufficiently wide possibility. Then, for each candidate design, compute the system response and evaluate the design (safety) constraint based on the system response, followed by the evaluation of the design robustness and the cost. Among these tasks, the definition and measure of the design robustness is the most critical, which is discussed later in this chapter.

Step 3: Perform the multi-objective optimization considering the design robustness, cost efficiency, and design (safety) constraint using the chosen optimization setting (Figure 2.2). Because the objectives of design robustness and cost efficiency are conflicting to each other, the optimization tends to yield a “Pareto front” (Cheng and Li 1997; Deb et al. 2002), which shows a tradeoff between design robustness and cost efficiency for all designs that satisfy the safety constraint. It should be noted that the optimization can be set up in many different ways (e.g., safety may be treated as a design objective instead of constraint; see Gong et al. 2015). The subject of the multi-objective optimization and Parent front is discussed later in this chapter.

Step 4: Select the final design based on the derived Pareto front. In general, either the least cost design that is above a specified level of design robustness or the most robust design that falls below a specified cost level can be selected as the final design based on the designer’s preference. In the case where a global knee point (Branke et al. 2004; Deb and Gupta 2011) exists on the Pareto front, it may be taken as the final design if no preference is specified by the designer or his/her client. The subject of the knee point is discussed later in this chapter.
Measures of Design Robustness

As illustrated in Figure 2.2, RGD can be implemented effectively as a multi-objective optimization problem that considers safety, cost, and robustness. The geotechnical engineer is more familiar with the subjects of cost and safety constraint, which are the basis for the traditional FS-based design or RBD. To this end, the focus of this section is placed on the subject of design robustness.

The original robust design procedure recommended by Taguchi (1986) involved two steps: bringing the mean of the “product performance” (referred to herein as system response) to the target, and minimizing the variation of the system response (e.g., Chen et al. 1996; Park et al. 2006). Bringing the mean of the system response to the target can be easily implemented by satisfying the design (safety) requirements that are specified based on the system response. Thus, the main focus of a robust design is to minimize the variation of the system response. To this end, the variation of the system response can be an effective measure of the design robustness (Juang et al. 2013a; Wang et al. 2014). A system is deemed robust (i.e., having a high degree of design robustness) if the system response has low variation. For a given geotechnical problem, the system response of concern may be the deformation, factor of safety, or probability of failure. Listed in Table 2.1 are various combinations of the forms of system response and the measures of design robustness for a number of RGD applications.

The signal-to-noise ratio (SNR) has also been used as a measure of design robustness. In fact, numerical experiments are often conducted in the quality engineering to identify the most robust design that yields the most favorable SNR. Although SNR has been defined in various forms, the following definition by Phadke (1989) has been shown effective in the RGD application in geotechnical engineering (Gong et al. 2015):
\[ SNR = 10 \log_{10} \left( \frac{E[g(d, \theta)]}{\sigma[g(d, \theta)]} \right)^2 \]  

(2.1)

where \( E[g(d, \theta)] \) and \( \sigma[g(d, \theta)] \) are the mean and standard deviation of the computed system response \( g(d, \theta) \) of a given design \( d \). Higher robustness can be achieved by securing a higher value of \( SNR \), which signals a lower variability of the system response.

Another robustness measure listed in Table 2.1 is “feasibility robustness” (Parkinson et al. 1993; Du and Chen 2000), which is the probability that the system remains feasible (i.e., safe) even when the inputs undergo variation. For example, if the failure probability of the system is taken as the system response of concern, the feasibility robustness concept may be expressed as follows (Juang and Wang 2013):

\[ P[p_f - p_T \leq 0] > P_c \]  

(2.2)

where \( p_f \) is the computed failure probability of the system, which is a random variable given the uncertainty in the estimated statistics (i.e., COV) of noise factors; \( p_T \) is a pre-specified target failure probability; \( P[p_f - p_T \leq 0] \) is the probability that the target failure probability can be satisfied; and \( P_c \) is a pre-specified confidence level of the feasibility robustness.

Computation of \( P[p_f - p_T \leq 0] \) requires knowledge of the distribution of \( p_f \), which is difficult to ascertain. In many cases, the histogram of the reliability index \( \beta \) (corresponding to \( p_f \)) may be approximated with a lognormal distribution. Thus, an equivalent counterpart in the form of \( P[(\beta - \beta_T) \geq 0] \), where \( \beta_T \) is the target reliability index, may be used for assessing the feasibility robustness. If the mean and standard deviation
of $\beta$, denoted as $\mu_\beta$ and $\sigma_\beta$, can be determined, then Eq. (2.2) becomes:

$$P[(\beta - \beta_r) \geq 0] = \Phi(\beta_\beta) \geq P_c$$

(2.3)

where $\Phi$ is the cumulative standard normal distribution function, and $\beta_\beta$ is defined as:

$$\beta_\beta = \frac{\ln \left[ \frac{\mu_\beta}{\sqrt{1+\left(\frac{\sigma_\beta}{\mu_\beta}\right)^2}} \right] - \ln(\beta_T)}{\ln \left[ 1+\left(\frac{\sigma_\beta}{\mu_\beta}\right)^2 \right]}$$

(2.4)

Thus, $\beta_\beta$ may also be used as an index for feasibility robustness. The approach of measuring robustness with a feasibility robustness index ($\beta_\beta$) allows for an informed and effective design decision.

If the factor of safety (FS), rather than the failure probability, is taken as the system response of concern, Eq. (2.2) may be modified as follows:

$$P[FS - FS_T \leq 0] > P_c$$

(2.5)

where $FS_T$ is the target factor of safety, and $FS$ is the computed factor of safety for the system, which is a random variable given the uncertainty in the noise factors. The interpretation and evaluation of Eq. (2.5) is similar to that of Eq. (2.2), although the assessment of the uncertainty of the parameters in Eq. (2.5) is much simpler.

The robustness measures discussed so far are those commonly seen in the literature. In a given geotechnical problem, some measures may be more applicable than others. However, these robustness measures generally require an evaluation of the variation of the system response. Monte Carlo Simulation (MCS) method or reliability methods may be used to evaluate the variation of the system response. Evaluation of the variation of system response
is a time-consuming task, especially for the robust design that takes the failure probability as the system response, as the evaluation of the failure probability itself requires use of MCS or reliability methods.

To reduce the computational effort, Gong et al. (2014b) proposed a gradient-based robustness measure. For a system with design parameters $d$ and noise factors $\theta$ as inputs, its system response can be denoted as $g(d, \theta)$. The gradient of the system response ($\nabla g$), which represents the relative change of the system response caused by the relative change in noise factors, is used as a measure for the sensitivity of the system response to the noise factors. Figure 2.3 shows that the variation of the system response (and thus, the design robustness) is related to the gradient of the system response to noise factors ($\theta$).

![Diagram](image)

Figure 2.3 Variation of system response as revealed by the gradient of the system response
The design with a lower gradient of the system response to noise factors yields a lower variation in the system response. The gradient of the system response to the noise factors, \( \nabla g \), at a check point of noise factors, \( \theta' \), can be expressed as follows:

\[
\nabla g_{\theta' \rightarrow \theta} = \left\{ \frac{\partial g(d, \theta)}{\partial \theta_1_{\theta' \rightarrow \theta}}, \frac{\partial g(d, \theta)}{\partial \theta_2_{\theta' \rightarrow \theta}}, \ldots, \frac{\partial g(d, \theta)}{\partial \theta_n_{\theta' \rightarrow \theta}} \right\}
\]

(2.6)

where \( n \) represents the number of noise factors and \( \theta' \) represents the check point.

As the gradient is an \( n \)-dimensional vector and the noise factors have different units, it is desirable to normalize the gradient vector into a dimensionless vector, \( J \), as follows:

\[
J = \left\{ \frac{\partial g(d, \theta)}{\partial \theta_1_{\theta' \rightarrow \theta} \times 100\%}, \frac{\partial g(d, \theta)}{\partial \theta_2_{\theta' \rightarrow \theta} \times 100\%}, \ldots, \frac{\partial g(d, \theta)}{\partial \theta_n_{\theta' \rightarrow \theta} \times 100\%} \right\}
\]

(2.7)

Ideally, the scaling factors for normalization should reflect the levels of variation (i.e., \( COV \)) of noise factors. However, if the system response is evaluated with a deterministic model that does not consider the variation of noise actors explicitly in the analysis, an approximation of \( J \) could be obtained using the scaling factors, \((\theta_1' \times 100\%), (\theta_2' \times 100\%), \ldots, (\theta_n' \times 100\%)\), as shown in Eq. (2.7). Finally, a sensitivity index (\( SI \)) is defined by taking the Euclidean norm of the normalized gradient vector as follows:

\[
SI = \| J \| = \sqrt{JJ^T}
\]

(2.8)

The sensitivity index \( SI \) is a single value that is effective in measuring the design robustness. A lower \( SI \) value means less sensitivity of the system response to the noise factors, and thus implies higher design robustness.
Multi-Objective Optimization and Pareto Front

The goal of RGD as illustrated in Figure 2.2 is to seek an optimal design with respect to design robustness and cost, while satisfying the safety constraint. Once the system response of concern is chosen, and the safety, design robustness, and cost are evaluated, the optimal design may be obtained through multi-objective optimization. However, a single best optimal design is generally unattainable in this case since the two objectives, robustness and cost, are conflicting. The multi-objective optimization in this scenario tends to yield a set of “non-dominated” designs (e.g., Deb et al. 2002). The collection of all these non-dominated designs is known as Pareto front (Cheng and Li 1997; Deb et al. 2002). Among all the designs on the Pareto front, none is superior or inferior to others on the Pareto front with respect to both objectives, but they are all superior to the dominated designs in the feasible domain. Figure 2.4 shows a conceptual sketch of Pareto front.

![Figure 2.4 Conceptual sketch of Pareto front and knee point](image-url)
Notice that the utopia point is an unattainable design, while the concept of knee point is discussed later.

One of the more popular multi-objective optimization algorithms is “Non-dominated Sorting Genetic Algorithm” version II (NSGA-II) developed by Deb et al. (2002).

With reference to Figure 2.5, the NSGA-II algorithm is summarized in the following (Juang et al. 2012a; Juang and Wang 2013). First, a random “parent population” \( P_0 \) from the design space is created with a size of \( N \). The term “parent population” is widely used in Genetic Algorithm (GA); here, it can be thought of as the first trial set of “optimal” designs. A series of genetic algorithm (GA) operations such as mutation and crossover are performed on “parent population” \( P_0 \) to generate the “offspring population” \( Q_0 \) with the same size of \( N \). Then, an iterative process is adopted to refine the parent population. In the GA, each step in the iteration is termed as a “generation.”
In the $t^{th}$ generation, the parent population $P_t$ and the offspring population $Q_t$ are combined to form an intermediate population $R_t = P_t \cup Q_t$ with a size of $2N$. Non-dominated sorting is next performed on $R_t$, which groups the points in $R_t$ into different levels of non-dominated fronts. For example, the best class is labeled $F_1$, and the second best class is labeled $F_2$, and so on. The best $N$ points are selected into parent population of the next generation, $P_{t+1}$. Using the scenario illustrated in Figure 2.5 as an example, if the number of points in $F_1$ and $F_2$ is less than $N$, they will all be selected into $P_{t+1}$. Then, if the number of points in $F_1$ and $F_2$ and $F_3$ exceeds the population size $N$, the points in $F_3$ are sorted using the “crowding distance” sorting technique (Deb et al. 2002), which aims to maintain the diversity in the selected points. Thus, the best points in $F_3$ are selected to fill all remaining slots in the next population $P_{t+1}$. After obtaining $P_{t+1}$ in the $t^{th}$ generation, $P_{t+1}$ is then treated as the parent population in the next generation and the process is repeated until $P_{t+1}$ is converged. The final, converged $P_{t+1}$ is the Pareto front.

The derived Pareto front is problem-specific, and for a given problem, it may be used as a design aid to assist in making an informed design decision. For example, at a preferred (pre-specified) cost level, the design with the highest robustness among all points on the Pareto front can be taken as the final design. On the other hand, at a pre-specified robustness level, the design with the least cost among all points on the Pareto front can be taken as the final design. For other situations, a tradeoff between design robustness and cost can be made, and a suitable optimal (or non-dominated) design can be selected as the final design.
Simplified Methods for Determining Knee Point

The Pareto front reveals a tradeoff relation between the two conflicting objectives (cost and robustness). To help with the tradeoff decision, a knee point may be identified, which represents the best compromise solution among all non-dominated designs on the Pareto front. Three existing methods (Reflex Angle approach, Normal Boundary Intersection approach and Marginal Utility Function approach) and a new procedure developed in an ongoing research for locating the knee point on the Pareto front are summarized in this chapter.

To begin with, a transformation, which normalizes the objective functions into a value ranging from 0.0 to 1.0, is usually taken:

\[
f'_i(d) = \frac{f_i(d) - [f_i(d)]_{\text{min}}}{[f_i(d)]_{\text{max}} - [f_i(d)]_{\text{min}}} \tag{2.9}
\]

where \([f_i(d)]_{\text{max}}\) and \([f_i(d)]_{\text{min}}\) are the maximum and minimum values of the 1th objective function \(f_i(d)\).

In the reflex angle (RA) approach (see Figure 2.6), the reflex angle at each point on the Pareto front is an indication of the bend of the front from its left to right (Branke et al. 2004; Deb and Gupta 2011). The point with the maximum reflex angle on the Pareto front is taken as the knee point. The problem with this method is that the identified knee point is not always a global knee point.
In order to eliminate the possibility of getting a local knee point, the normal boundary intersection (NBI) approach may be applied. In this method, a straight line is constructed by connecting the lowest point on the Pareto front to the highest point (see Figure 2.7). The knee point is a design point on the Pareto front, which has the maximum distance from this constructed line (Deb and Gupta 2011).
The NBI approach and the RA approach are only valid for bi-objective optimization problems. For a problem with more than two objectives, the marginal utility function (MUF) approach can be used (Branke et al. 2004). The marginal utility function, denoted as \( U'(d, \lambda) \), is formulated as:

\[
U'(d, \lambda) = \min \left[ U(d, \lambda) - U(d, \lambda) \right], \quad (i \neq j)
\]

(2.10)

where

\[
U(d, \lambda) \text{ is a linear utility function defined as:}
\]

\[
U(d, \lambda) = \sum \lambda_i f_i(d)
\]

(2.11)

\( \lambda_i \) = weighting parameter with a value ranging from 0.0 to 1.0 (\( \sum \lambda_i = 1.0 \))

\( f_i(d) \) = \( i \)th normalized objective function of the \( j \)th design \((d_j)\).

With an assumption of uniform distribution, random values of \( \lambda_i \) are generated using Monte Carlo simulations (MCS). For each design on the Pareto front, the expected marginal utility function can be computed. The knee point is identified as the design with the maximum expected marginal utility.

Unlike the aforementioned three methods, in which the knee point is to be identified after the Pareto front has been established, the new procedure, referred to herein as the minimum distance (MD) approach, identifies the knee point through a series of single-objective optimization. The MD approach is summarized in the following steps:

**Step 1**: Perform a single-objective optimization with respect to each objective function of concern, \( f_i(d) \), using the following setting:
Figure 2.8 Single objective optimization setting

Find: \( d_i^* \) (design parameters)  
Subject to: \( d \in S \) (design pool)  
Safety requirements  
Objective: \( \min \{[f_i(d)]_{\min} = f_i(d_i^*)\} \)

where \( d_i^* \) represents the optimal design based on the \( i^{th} \) objective, which meets the safety requirements and yields a minimum in the \( i^{th} \) objective function, \( [f_i(d)]_{\min} \). By repeating the single-objective optimization in Figure 2.8 for each and every design objective, a utopia point of \( \{[f_1(d)]_{\min},[f_2(d)]_{\min},\ldots,[f_m(d)]_{\min}\} \) can be identified in the design pool, where \( m \) represents the number of objectives to be optimized. While the utopia point defined this way is in reality not attainable, the location of this utopia point is required in the MD approach.

**Step 2**: Determine the corresponding maximum value of each objective function among all designs \( \{d_1^*,d_2^*,\ldots,d_m^*\} \), which is expressed as follows:

\[
[f_i(d)]_{\max} = \max\left([f_i(d_j^*)]\right) \quad (j \in 1, 2, \ldots, m) \tag{2.12}
\]

**Step 3**: Normalize the objective functions into values ranging from 0.0 to 1.0 using the transformation described in Eq. (2.9). As a result, the coordinates of the normalized utopia point are all equal to 0.

**Step 4**: Compute the distance from the normalized utopia point to the normalized objective functions for each candidate design in the design pool. The design that meets the safety requirements and yields the minimum distance is the knee point, as it is an acceptable design that is closest to the utopia point, as illustrated in Figure 2.9.
The minimum distance (MD) approach is a global knee point search algorithm. This approach does not require a Pareto front to begin with, and it is applicable to optimization with more than two objectives, although Figure 2.9 is illustrated with only two objectives. Although many outstanding studies on knee points have been published (e.g., Bechikh et al. 2011; Deb et al. 2006), the MD approach is easy to use and very efficient computationally. When adopted within the RGD framework, it offers a practical geotechnical design tool.

**Example 2.1 - RGD Application in Shallow Foundations**

*Problem description, design parameters, and design space*

The first example concerns the robust design of a spread foundation subjected to axial (compressive) and moment loads. This design example is similar to the one solved by Juang and Wang (2013) with the exception of an additional moment load, which necessitates a slight modification to the adopted deterministic model for the system response.
In reference to Figure 2.10, the axial load is applied at the center of foundation, which consists of two components; a permanent load component of 900 kN \((G)\) and a random variable component \((Q)\) with a mean of 458.7 kN and a coefficient of variation \((COV)\) of 0.15. The moment load \((M)\) is also applied at the center of the foundation with a fixed value of 500 kN-m. The spread foundation is to be installed in a stiff till with a fixed total unit weight of 22 kN/m\(^3\), a mean effective friction angle \(\phi'\) of 36.4° \((c' = 0)\), a mean undrained shear strength \(c_u\) of 235.3 kPa, and a mean coefficient of volume compressibility \(m_v\) of 0.01875 m\(^2\)/MN. The last three soil parameters \(\phi'\), \(c_u\) and \(m_v\) are uncertain (random) variables, and the variations of these random variables are described later. The unit weight of concrete is 24 kN/m\(^3\). The
foundation is founded at just above the groundwater table at a depth of \( D = 0.8 \) m.

For this design example, the footing width \( (B) \) and footing length \( (L) \) are considered as design parameters. For illustration purpose, a discrete design space is considered, in which both footing width \( (B) \) and footing length \( (L) \) will be selected from the range of 1.0 m to 4.0 m with an increment of 0.1 m (note: in practice the design space where the final design is selected from is usually a choice of the designer). A length-to-width ratio \( (L/B) \) of between 1 and 10 is maintained. Given these ranges, there are 496 possible designs in this discrete design space. The goal of robust design is to select the design parameters \( (B, L) \) so that the design robustness is maximized, the cost is minimized, and the safety requirements are satisfied.

**Noise factors**

Three soil parameters \( \phi' \), \( c_u \) and \( m_v \) are treated as random variables or noise factors in this design example. According to Orr and Breysse (2008), the \( COV \) of \( \phi' \), denoted as \( COV[\phi'] \), typically ranges from 4\% to 11\% ; the \( COV \) of \( c_u \), denoted as \( COV[c_u] \), typically ranges from 20\% to 40\%; and the \( COV \) of \( m_v \), denoted as \( COV[m_v] \), typically ranges from 20\% to 40\%. If these \( COVs \) can be accurately and precisely determined, the system response of concern (in terms of the failure probability; discussed later) will be a fixed value for each design in the discrete design space, and the least-cost design among all designs that satisfy the target failure probability (or the corresponding reliability level) can be taken as the final design. However, these \( COVs \) are difficult to characterize precisely, and can vary significantly; thus, the system response will not be a fixed value. The objective of RGD is to minimize the
variation of the system response, given the variation in the estimated $COV$s of these soil parameters. For illustration purpose, $COV[\phi']$ is assumed to have a mean of 0.08 and a $COV$ of 25% (roughly to cover the typical range of $COV[\phi']$). Similarly, $COV[m_v]$ is assumed to have a mean of 0.30 and a $COV$ of 17%; $COV[m_v]$ is assumed to have a mean of 0.30 and a $COV$ of 17%.

It is noted that in practice, the engineer usually tries to estimate the $COV$ of an uncertain soil parameter based on limited data, guided by the $COV$ values reported in the literature and engineering judgment. In most situations, the estimate may be expressed in a range $[l, u]$ and by taking 2-sigma rule, the mean and standard deviation of the $COV$ of a soil parameter may be estimated. The uncertainty in the estimated $COV$ is handled in the robust design, which is aimed at reducing the effect of such uncertainty.

**System response and design robustness**

For this example, the “effective area” method (Meyerhoff 1953) is adopted as the deterministic model for computing the ultimate limit state (ULS) bearing capacity of the spread foundation subjected to the applied loads, and Eurocode 7 is used for evaluating the foundation settlement and the serviceability limit state (SLS) requirement (Orr and Farrell 1999; Orr and Breysse 2008). The ULS failure occurs when the computed bearing capacity is less than the applied load and the SLS failure occurs when the computed total settlement exceeds the maximum allowable settlement. Because of the uncertainty in the soil parameters, reliability methods are generally preferred for the design analysis. For this example, the target failure probability for ULS is set as $7.2 \times 10^{-5}$ (corresponding to a reliability index of 3.8) and
the target failure probability for SLS is set as $6.7 \times 10^{-2}$ (corresponding to a reliability index of 1.5), which are typically recommended (e.g., Wang 2011). However, previous studies indicated that the ULS controlled the design of the shallow foundation in this case (Wang and Kulhawy 2008; Wang 2011). Therefore, in this illustrative design example, the probability of failure with respect to the ULS requirement is treated as the system response of concern.

The variation of the system response is often taken to gauge the design robustness. In this example, the standard deviation ($\sigma_p$) of the ULS failure probability is used to measure the design robustness. Thus, for each candidate design in the design space, the mean ($\mu_p$) and standard deviation ($\sigma_p$) of the ULS failure probability must be determined. As an example, a procedure that integrates point estimate method (PEM) with first order reliability method (FORM), proposed by Juang and Wang (2013), is taken to compute $\mu_p$ and $\sigma_p$. This procedure is a main part of the reliability-based RGD methodology illustrated in Figure 2.11.
Figure 2.11 Reliability-based robust geotechnical design flowchart (adapted from Juang and Wang 2013)
Robust design of shallow foundation

The third aspect in a robust design is cost. In this example, the cost for each candidate design is computed using the procedure proposed by Wang and Kulhawy (2008). The total cost is the summation of the costs of excavation, concrete, formwork, reinforcement and backfill, which is approximately a function of the design parameters ($B$, $L$). Thus, the safety, robustness, and cost are all function of the design parameters, which enables the design optimization.

Once the system response (for safety), the variation of the system response (for robustness), and the cost of each of the designs in the discrete design space are evaluated, a multi-objective optimization can be performed using NSGA-II, considering safety, robustness, and cost simultaneously, as shown in Figure 2.12.

<table>
<thead>
<tr>
<th>Find:</th>
<th>$d = [B, L]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject to:</td>
<td>$B \in {1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \ldots, 4.0\text{m}}$</td>
</tr>
<tr>
<td></td>
<td>$L \in {1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \ldots, 4.0\text{m}}$</td>
</tr>
<tr>
<td></td>
<td>$\mu_r &lt; p_T = 0.000072$</td>
</tr>
<tr>
<td></td>
<td>$1 \leq L/B \leq 10$</td>
</tr>
<tr>
<td>Objectives:</td>
<td>Minimizing the std. dev. of ULS failure probability</td>
</tr>
<tr>
<td></td>
<td>Minimizing the cost for shallow foundation</td>
</tr>
</tbody>
</table>

Figure 2.12 Optimization setting for RGD of shallow foundation
The results are typically shown as a Pareto front (Figure 2.13), which consists of 80 non-dominated designs for this example. The Pareto front shows a tradeoff relationship between robustness (in terms of \( \sigma_p \)) and cost. Based on the target level of cost or robustness, the final design (i.e., most preferred design) can be chosen from the Pareto front.

![Figure 2.13 Pareto front derived for design of shallow foundation](image)

**Example 2.2- RGD Application in Rock Slopes**

*Problem description, design parameters, and design space*

For the second example of RGD application, a hypothetical rock slope composed of two blocks separated by a vertical tension crack is studied. As shown in Figure 2.14 (Jimenez-Rodriguez et al. 2006), the tension crack is randomly located, either at the slope top or the slope face.
Figure 2.14 Rock slope consisting of two blocks separated by a tension crack: (a) tensile crack on slope top, (b) tensile crack on slope face (modified after Jimenez-Rodriguez et al. 2006)

The original slope geometry is defined by a slope height of $H = 25$ m and a slope angle of $\psi_f = 50^\circ$. The location of the slip surface is assumed to be certain with a dip angle $\psi = 32^\circ$, and the unit weight of rock ($\gamma$) is considered a fixed value of 25 kN/m$^3$. The
The purpose of this example is to demonstrate how the RGD can be applied to design of this rock slope to prevent slope failure. For demonstration purpose, the slope height \((H)\) and the slope angle \((\psi_f)\) are selected as the design parameters; no anchored tiebacks are used. This design example is similar to the one reported by Xu et al. (2014); however, the variation of the system response (in terms of system reliability or failure probability) is analyzed using the same procedure as described in Example No. 1 (i.e., PEM integrated with FORM; see Figure 2.11), instead of the fuzzy set-based method as reported by Xu et al. (2014).

For illustration purpose, a discrete design space is considered, in which the slope height will be selected from the range of 20 m to 25 m with an increment of 0.2 m, and the slope angle will be selected from the range of 40° to 50° with an increment of 0.2°. Given these ranges and selected increments for the design parameters, there are 1326 possible designs in the chosen discrete design space.

For the hypothetical two-block rock slope, block B can be either stable (no interaction between block A and B) or unstable with a tendency to slide and imposing an interaction force on block A. Detailed formulations for evaluating the factor of safety (i.e., the stability of this rock slope system) can be obtained from Jimenez-Rodriguez et al. (2006).

**Noise factors**

As noted by Jimenez-Rodriguez et al. (2006), the parameters describing rock properties along the slip surface, as well as the position of tension crack and water depth should be considered as random variables in the reliability analysis of rock slope with two removable blocks. These random variables include the cohesion along slip surface of block A.
and block B ($c_A$ and $c_B$), friction angle along slip surface of block A and block B ($\phi_A$ and $\phi_B$), as well as friction angle along the contact surface between two blocks ($\phi_{AB}$), the ratio (or proportion) of the tension crack depth filled with water ($\xi_{z_w}$), and the location of tension crack $\xi_{X_s}$ (Jimenez-Rodriguez et al. 2006). The variables $c_A$, $c_B$, $\phi_A$, $\phi_B$, and $\phi_{AB}$ are assumed to follow truncated normal distribution (Hoek 2006). The variable $\xi_{z_w}$ is assumed to follow the exponential distribution with a mean of 0.25 and truncated within the interval [0, 0.5]. The variable $\xi_{X_s}$ is assumed to follow a non-symmetric beta distribution with model parameters $q = 3$, $r = 4$, $a = 0$, $b = 1$ (Low 2007b; Ang and Tang 2007). These statistical parameters are listed in Table 2.2.

Furthermore, the cohesion and friction angle are assumed to be negatively correlated with correlation coefficient $\rho_{c_A, \phi_A} = \rho_{c_A, \phi_B} = -0.5$ (Low 2007b). On the other hand, shear strength parameters between the two blocks are assumed positively correlated with the correlation coefficients $\rho_{c_A, c_B} = \rho_{\phi_A, \phi_{AB}} = \rho_{\phi_B, \phi_{AB}} = \rho_{\phi_A, \phi_B} = 0.3$ (Jimenez-Rodriguez et al. 2006). All other random variables are assumed independent with each other.

It is generally more difficult to estimate the COVs of the parameters $c_A$, $c_B$, $\phi_A$, $\phi_B$, and $\phi_{AB}$, denoted as $COV[c_A]$, $COV[c_B]$, $COV[\phi_A]$, $COV[\phi_B]$, and $COV[\phi_{AB}]$, respectively. As such, these COVs may be uncertain and cannot be treated as fixed values. The failure probability of the slope determined with reliability analysis using uncertain COVs will be uncertain, which tends to complicate the design of rock slope. In this situation, the reliability-based RGD method is readily applicable.
Table 2.2 Statistics of random variables for rock slope example with multiple failure modes (after Jimenez-Rodriguez et al. 2006; Low 2007b; Hoek 2006)

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Probability distribution</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion along slip surface of block A $c_A$ (kPa)</td>
<td>Normal</td>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>Cohesion along slip surface of block B $c_B$ (kPa)</td>
<td>Normal</td>
<td>18</td>
<td>0.2</td>
</tr>
<tr>
<td>Friction angle along slip surface of block A $\phi_A$ (°)</td>
<td>Normal</td>
<td>36</td>
<td>0.075</td>
</tr>
<tr>
<td>Friction angle along slip surface of block B $\phi_B$ (°)</td>
<td>Normal</td>
<td>32</td>
<td>0.075</td>
</tr>
<tr>
<td>Friction angle along the contact surface between two blocks $\phi_{AB}$ (°)</td>
<td>Normal</td>
<td>30</td>
<td>0.075</td>
</tr>
<tr>
<td>Location of tension crack $\xi_{x_a}$</td>
<td>Beta distribution with model parameters, $q = 3, r = 4, a = 0, b = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio (or proportion) of the tension crack depth filled with water $\xi_{z_a}$</td>
<td>Exponential with mean 0.25, truncated to [0, 0.5]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the context of RGD, these parameter COVs are treated as noise factors (random variables). For illustration purpose in this example, the mean of these random variables are assumed to be the same as those listed in Table 2.2. Furthermore, the coefficients of variation of $COV[c_A]$, $COV[c_B]$, $COV[\phi_A]$, $COV[\phi_B]$, and $COV[\phi_{AB}]$ are assumed to be 0.17, and the coefficients of variation of $COV[c_A]$, $COV[c_B]$, $COV[\phi_A]$, $COV[\phi_B]$, and $COV[\phi_{AB}]$ are assumed to be 0.12. The two coefficients of variation, 0.17 and 0.12, are assumed so that the ranges of COVs of the parameters $c_A, c_B$, $\phi_A$, $\phi_B$, and $\phi_{AB}$ are consistent with those reported previously (e.g., Low 2007b; Hoek 2006).
System response and design robustness

For the reliability analysis of rock slope system with multiple failure modes, a disjoint cut-set formulation (Jimenez-Rodriguez et al. 2006; Jimenez-Rodriguez and Sitar 2007) may be employed. To calculate the failure probability of the rock slope, the slope is modeled as a series of disjointed cut-sets that define the failure modes. The summation of individual failure probabilities of the cut-sets (failure modes) represents the failure probability of the complete rock slope. The complete procedure to calculate the probability of failure of the rock slope system can be obtained from Jimenez-Rodriguez and Sitar (2007).

Thus, for a given design of rock slope, the point estimate method (PEM) integrated with first order reliability method (FORM), described in Example 2.1, can be used to compute the mean ($\mu_p$) and standard deviation ($\sigma_p$) of the failure probability of the slope, given the estimated statistics of the described noise factors described. The standard deviation ($\sigma_p$) of the failure probability of the slope is used to measure the design robustness in this example.

Robust design of rock slope

The described RGD methodology, which is illustrated with Figure 2.2 is applied to design of rock slope. The system response of concern is the failure probability of the slope, and for this example, the robustness is measured with the feasibility robustness defined in Eq. (2.2) or (2.4), which required the knowledge of the mean and standard deviation of the failure probability (or equivalently, the mean and standard deviation of the reliability index). For each design in the design space, consisting of two design parameters, slope height ($H$) and the slope angle ($\psi_f$), the cost is measured with the volume of the rock mass that needs to be excavated (Wang et al. 2013).
Based on the above discussions, the robust design optimization can be set up as illustrated in Figure 2.15.

\[
\begin{align*}
\text{Find} & \quad d = [H, \ \psi_f] \\
\text{Subjected to:} & \quad H \in \{20\text{m}, 20.2\text{m}, 20.4\text{m}, \ldots, 25\text{m}\} \\
& \quad \psi_f \in \{40^\circ, 40.2^\circ, 40.6^\circ, \ldots, 50^\circ\} \\
& \quad \mu_H < p_T = 0.0062 \\
\text{Objectives:} & \quad \text{Minimizing } \sigma_f \text{ (i.e., maximizing robustness)} \\
& \quad \text{Minimizing the cost for the given design}
\end{align*}
\]

Figure 2.15 Robust design of rock slope ( \(p_T = 0.0062\) corresponding to \(\beta_T = 2.6\) )

In this example, the multi-objective optimization is performed using NSGA-II, which yields 48 non-dominant designs that form a Pareto front. For all designs on the Pareto front, a plot of cost versus feasibility robustness index (\(\beta_P\)) is shown in Figure 2.16.

Figure 2.16 Pareto front showing a tradeoff between cost and feasibility robustness level
Each point on this plot is a unique design that satisfies the target failure probability (or reliability) requirement. The Pareto front, showing a tradeoff between cost and robustness, can aid in making an informed decision. If a target feasibility robustness level ($\beta_T^T$) is specified by the designer, the least cost design can be easily identified from the Pareto front. For example, when the target feasibility level is set at $\beta_T^T = 1$, which corresponds to a confidence probability of $P_c = 0.84$, the least cost design is slope height $H = 24.6$ m and slope angle $\psi_f = 40.4^\circ$.

**Summary**

Geotechnical design is almost always performed in the face of uncertainties. Rather than reducing the sources of uncertainties (termed noise factors in the context of robust design), the RGD seeks a safe and cost efficient design that has a low variation in the system response (or a high degree of design robustness) by adjusting the design parameters. Thus, the goal of RGD is to seek an optimal design with respect to both cost and robustness, while satisfying the safety requirements. The original RGD methodology and its simplified version are presented and demonstrated as an effective design tool that meets this goal.

This chapter provides an update of the recently developed robust geotechnical design (RGD) methodology with two illustrative examples. A comprehensive review of all main components of the RGD methodology is presented in this chapter. First, the system response of concern may be chosen in the form of deformation, factor of safety, or probability of failure. Depending on the given problem and the chosen system response, an appropriate measure of design robustness is selected. In terms of design robustness, the existing measures
such as variation of the system response, signal-to-noise ratio, and feasibility robustness, along with the new gradient-based robustness measure are introduced and demonstrated. To seek an optimal design with respect to cost and robustness simultaneously, while satisfying the safety requirements, multi-objective optimization is required. In this chapter, the NSGA-II algorithm for such optimization is introduced, which generally yields a set of non-dominant designs, collectively form a Pareto front, when the objectives are conflicting with each other, as in the examples studied in this chapter. To aid in making an informed design decision based on the derived Pareto front, the knee point concept is introduced. The existing methods for knee point identification such as normal boundary intersection, reflex angle and marginal utility function methods are reviewed along with a new simplified procedure, called minimum distance (MD) approach. These methods are demonstrated with the illustrated design examples, including design of shallow foundation, rock slope, and supported excavations.
Chapter 3

EFFICIENT ROBUST GEOTECHNICAL DESIGN OF DRILLED SHAFTS IN CLAY
USING SPREADSHEET

Introduction

Performance or response of a geotechnical system under loading can be over- or under-estimated due to the uncertainty in soil parameters, loading conditions, analysis model errors, and construction variation. To cope with the uncertainty in the predicted system response, the engineer often selects a conservative design that may be cost inefficient. While the reliability-based methods are capable of considering explicitly the uncertainty in input parameters and solution models (e.g., Wu et al. 1989; Christian et al. 1994; Baecher and Christian 2003; Phoon et al. 2003; Fenton and Griffiths 2008), reliability analysis requires an accurate statistical characterization of such uncertainty, which is often challenging in practice. If the variability of soil parameters is under-estimated, the results of the least-cost reliability-based design (RBD) may not meet the minimum reliability index requirement and thus, the safety constraint may be violated (Juang et al. 2013a).

To deal with the uncertainty in the estimated statistics of soil parameters (called noise factors herein), Juang et al. (2013a; 2013b; 2014) proposed a reliability-based robust geotechnical design (RGD) approach. The goal of RGD is to reduce the effect of uncertainty in the noise factors on the predicted performance or response of the geotechnical system. Rather than reducing the uncertainty in the noise factors, RGD reduces the variation of the

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response of the system by increasing its design robustness, which is a measure of how sensitive the response of a given system is to the variation of noise factors. A design is considered robust if the system response is insensitive to the variation of the noise factors. RGD is achieved by adjusting easy-to-control design parameters (e.g., geometry parameters) so that the variation of the system response is reduced to an extent at which the system is considered insensitive to the variation in the hard-to-control noise factors.

RGD is not a new concept in engineering; rather, it is adapted from the well-established robust design methodology, initiated by Taguchi (1986), in the field of industrial engineering. In fact, numerous applications of this concept have been reported in various engineering fields (e.g., Tsui 1992; Chen et al. 1996; Doltsinis et al. 2005; Zhang et al. 2005; Park et al. 2006; Beyer and Sendhoff 2007; Marano et al. 2008; Lee et al. 2010). Adapting the concept of robust design to the field of geotechnical engineering, however, requires some refinements as the levels of uncertainty in input parameters, solution model, and construction variation are generally much higher than that in other fields of engineering. Therefore, how to select the most effective and efficient approach is a critical issue that must be resolved for a successful implementation of robust design principles in geotechnical engineering.

In the previous study, Juang et al. (2013a) presented a reliability-based RGD approach, in which the performance of drilled shafts is analyzed using reliability methods, and the variation of the system performance, in terms of failure probability, is utilized as a measure of design robustness. The goal of RGD is to optimize the design robustness and the cost simultaneously, while satisfying the safety or performance requirements. To this end, RGD is usually implemented as a multi-objective optimization problem. The reliability-based
RGD approach by Juang et al. (2013a) involves three loops, one for computing failure probability using reliability methods, one for computing the variation of the failure probability considering the uncertainty in the estimated statistics of input parameters, and the last one for conducting the multi-objective optimization. Although this reliability-based RGD approach is shown as an effective design tool in geotechnical engineering (i.e., Juang et al. 2013a; Juang and Wang 2013; Wang et al. 2013), its multi-objective optimization-based implementation is computationally demanding. A more efficient RGD approach is desirable.

The efficiency of a robust design is affected by how the system response is measured and computed, and how the design robustness is defined. For example, Juang et al. (2014) described an RGD approach using the deformation, in lieu of failure probability, as the system response. In this approach, the evaluation of the variation of the system response involves only one loop of iterative procedure rather than two, which yields significant savings in computation time. Furthermore, the type of chosen system response also dictates the choice and efficiency of a particular robustness measure. In the literature, the commonly adopted robust design approaches include Taguchi’s method (e.g., Taguchi 1986; Phadke 1989), worst-case approach (e.g., Nagy and Braatz 2004; Zhu and Fukushima 2009), mean-variation compromising approach (e.g., Chen et al. 1996; Park et al. 2006), and feasibility robustness approach (e.g., Parkinson 1993; Du and Chen 2000).

In this chapter, the RGD approach proposed by Juang et al. (2013a) is further refined. The main objective is to enhance the RGD approach as an effective and efficient design tool. Several new features are introduced into the refined RGD framework. First, a fundamentally sound and intuitive measure of the design robustness based on the variation of the performance function is presented. Second, greater computational efficiency is achieved by
“coupling” the evaluation of design robustness with the evaluation of performance requirement using reliability methods (note: the word “coupling” here means that the two evaluation procedures share common computational steps). Third, an effective procedure to transform the multi-objective optimization into a series of single-objective optimizations is developed so that the entire RGD process can be easily implemented in the user friendly Excel spreadsheet environment. The improved RGD approach presented in this chapter is not only efficient but also user friendly and practical.

**Variation of the Performance Function as a Robustness Measure**

Robust design is aimed at deriving an optimal design, represented by a set of design parameters \( \mathbf{d} \), the system response of which is insensitive to, or robust against, the variation of noise factors \( \mathbf{\theta} \) (e.g., Taguchi 1986; Tsui 1992; Chen et al. 1996; Beyer and Sendhoff 2007; Juang et al. 2013a; Juang et al. 2013b; Juang et al. 2014; Gong et al. 2014a). In principle, the design robustness can be increased by reducing the variation of the system response that comes from the variation of noise factors. In this chapter, the variation of the response or performance of a geotechnical system, which may be obtained through a reliability analysis, is used as a robustness measure. Use of such robustness measure enables a “coupling” of the evaluation of design robustness with the evaluation of system performance requirement.
Formulation of the performance function in the context of robust design

Within the context of robust design, the input parameters are classified into two categories, one is the *easy-to-control* design parameters (*d*) and the other is the *hard-to-control* noise factors (*θ*). For a geotechnical system with design parameters of *d* and noise factors of *θ* as inputs, the performance function, denoted as *g(d, θ)*, can be set up as:

\[
g(d, θ) = R(d, θ) - S(d, θ)
\]

(3.1)

where \( R(d, θ) \) and \( S(d, θ) \) are the resistance term and the load term, respectively. The performance of a design (\( d \)) is considered unsatisfactory if \( g(d, θ) < 0 \). Noise factors such as uncertain soil parameters, loading conditions, model errors, and construction variation are treated as random variables in a reliability analysis. To meet the performance requirement, the evaluated failure probability (\( P_f \)) of a design, referred to herein as the *probability of unsatisfactory performance*, must be less than a specified target value (\( P_T \)).

Variation of the performance function as a robustness measure

With the performance function \( g(d, θ) \) defined in Eq. (3.1), the design robustness can be intuitively measured by its variation (standard deviation). Fundamentally, a greater variation of the performance function indicates a higher level of sensitivity of the system response to the variation of noise factors and thus a lower degree of robustness. Using Taylor series expansions, the design robustness of a design (\( d \)), in the form of the standard deviation of the performance function, can be approximated as (e.g., Ang and Tang 2004; Zhang et al. 2011a):

\[
σ[g] ≈ \sqrt{GC_θG^T}
\]

(3.2)
where $C_\theta$ is the covariance matrix among noise factors ($\theta$), and $G$ is the gradient vector of the performance function to noise factors ($\theta$), computed as:

$$G = \begin{bmatrix} \frac{\partial g(d, \theta)}{\partial \theta_1}_{\theta=\mu_\theta} & \frac{\partial g(d, \theta)}{\partial \theta_2}_{\theta=\mu_\theta} & \cdots & \frac{\partial g(d, \theta)}{\partial \theta_n}_{\theta=\mu_\theta} \end{bmatrix}$$  \hspace{1cm} (3.3)

where $\mu_\theta$ presents the mean of noise factors ($\theta$). Generally, a lower standard deviation of the performance function ($\sigma[g]$) signals a higher degree of design robustness.

Two possible approaches to improve design robustness

The measure of design robustness expressed in Eq. (3.2) is affected by two terms: the gradient vector ($G$) and the covariance ($C_\theta$). Hence, two approaches are possible to improve the design robustness, either by minimizing the gradient (referred to as Approach 1 in Figure 3.1) or by minimizing the variation of noise factors (referred to as Approach 2 in Figure 3.1).

The first approach is to find the design parameters ($d$) such that the gradient ($G$) is reduced for a given variation of noise factors ($\theta$). This is precisely the approach taken in many existing robust design methods (e.g., Taguchi 1986; Chen et al. 1996; Nagy and Braatz 2004; Juang et al. 2013a; Juang et al. 2013b; Juang et al. 2014; Liu et al. 2013; Gong et al. 2014a). The second approach seeks to increase the design robustness by reducing the variation of noise factors ($\theta$).
MFOSM-Based Procedure for Reliability Analysis of System Performance

Given noise factors, the question of whether the performance requirement of a system is satisfied is not possible to evaluate with certainty using deterministic methods. Thus, it may be more appropriate to evaluate the system performance using reliability methods. Although such evaluation can be done using rigorous methods such as Monte Carlo simulation (MCS) and first order reliability method (FORM), the computational demand of the RGD optimization that involves MCS or FORM is often prohibitive. Therefore, the mean first order second moment method (MFOSM), a much simpler reliability method, is employed herein to evaluate the system performance within the proposed RGD framework.
Mean first order second moment method (MFOSM)

Within the context of MFOSM, the mean \((E[g])\) of the performance function \(g(d, \theta)\) can be approximated as follows (e.g., Ang and Tang 2004; Zhang et al. 2011a):

\[
E[g] \approx g(\mu_\theta, d)
\]  

(3.4)

The standard deviation \((\sigma[g])\) of the performance function \(g(d, \theta)\) is readily available from the formulation of design robustness (see Eq. 3.2), the reliability index \((\beta)\) of the system performance, \(g(d, \theta) > 0\), can be computed with an assumption that \(g(d, \theta)\) follows a normal distribution (e.g., Zhao and Ono 2001; Ang and Tang 2004; Zhang et al. 2011a):

\[
\beta = \frac{E[g]}{\sigma[g]}
\]  

(3.5)

The failure probability \((P_f)\) or the probability of unsatisfactory performance is related to the reliability index \((\beta)\):

\[
P_f = \Phi(-\beta)
\]  

(3.6)

It is noted that through the common formulation of the variation of the performance function, the evaluation of design robustness and the evaluation of system performance requirement, two main procedures in the RGD, share common computational steps or are “coupled” from the perspective of computational effort. This “coupling” for computational efficiency is a significant feature of the efficient RGD approach proposed herein, as it greatly reduces the required computation time in the entire RGD process.
Evaluation of system performance requirement using MFOSM-based procedure

Though MFOSM is often favored for its simplicity over more rigorous methods such as MCS and FORM, the accuracy of MFOSM might be questionable in many cases. To overcome the dilemma between accuracy and computation efficiency, the procedure presented by Zhang et al. (2011a) is employed here. Zhang et al. (2011a) pointed out that the reliability index \( \beta^M \) that is computed using MFOSM is usually highly correlated with the reliability index \( \beta^T \) that is computed using more sophisticated and accurate reliability methods such as MCS and FORM. Thus, a mapping function can be established for a given problem:

\[
\beta^M = f(\beta^T)
\]  

(3.7)

Note that the reliability index obtained using FORM that has been calibrated by MCS is considered a close approximation of the “true” reliability index, and thus the superscript “T” (for “true”) is used for the reliability index computed with FORM. Similarly, the superscript “M” is for the reliability index computed using MFOSM. The mapping function defined in Eq. (3.7) can be constructed through the following procedures. First, compute the values of \( \beta^M \) using MFOSM and \( \beta^T \) using FORM (that has already been calibrated with MCS) for a series of designs in a given design space of a given problem. Second, carry out the least-square regression analysis to develop a mapping function of \( \beta^M = f(\beta^T) \). Third, validate the obtained mapping function.

Once the mapping function shown in Eq. (3.7) is successfully established in a given design space of a given problem, the target reliability index \( \beta^M_t \) for system performance requirement, within the context of MFOSM analysis, can be set up as \( \beta^M_t = f(\beta^T) \), where
\( \beta^T \) is the specified target reliability index (Zhang et al. 2011a). Thus, the MFOSM analysis along with the calculated \( \beta^M_T \) can readily be used to analyze the system performance requirement without the need of performing FORM or MCS analysis.

**Framework for Efficient Robust Geotechnical Design**

Figure 3.2(a) illustrates the robust design implemented as a multi-objective optimization problem that considers *design robustness* and *cost* as objectives while the system performance requirement and the design space are treated as constraints. Here, the robust design seeks an optimal design in the design space that the design robustness is maximized and the cost \( (C) \) is minimized simultaneously, while the system performance requirements are satisfied.

**A simplified optimization procedure**

Typically, the design robustness and the cost efficiency are two conflicting objectives. Thus, a single best solution that is optimal with respect to both objectives simultaneously is not attainable; rather, a set of non-dominated optimal solutions often exist that are superior to all others in the design space, but within which, none of them are superior or inferior to others. These non-dominated solutions form a Pareto front. As can be seen later, all these non-dominated designs on the Pareto front are equally optimal in the sense that no improvement can be achieved in one objective without worsening the other objective. Figure 3.3 shows a conceptual illustration of the non-dominated solution and Pareto front obtained from a robust design optimization.
Find: \( \mathbf{d} \) (Design parameters)

Subjected to: \( \mathbf{d} \in \mathbf{DS} \) (Design space)
\[
\beta_i^M \geq \beta_i^M \quad \text{(Requirements of system performances)}
\]

Objectives: \( \min \sigma[g] \) (Maximizing the design robustness)
\( \min C \) (Minimizing the cost)

(a) Robust design through multi-objective optimization

Find: \( \mathbf{d} \) (Design parameters)

Subjected to: \( \mathbf{d} \in \mathbf{DS} \) (Design space)
\[
\beta_i^M \geq \beta_i^M \quad \text{(Requirements of system performances)}
\]
\[
C < C_{Tj} \quad \text{(Cost level is dealt as an additional constraint)}
\]

Objectives: \( \min \sigma[g] \) (Maximizing the design robustness)

(b) Robust design through proposed optimization procedure

Figure 3.2 Optimization setting of robust design

The multi-objective optimization problem shown in Figure 3.2(a) may be solved using genetic algorithms such as Non-dominated Sorting Genetic Algorithm version II, NSGA-II (Deb et al. 2002). Although NSGA-II is a fast algorithm, such multi-objective optimization requires the knowledge of genetic algorithms and considerable programming skills, and is usually computationally demanding.

In this chapter, the multi-objective optimization problem shown in Figure 3.2(a) is solved through a series of single-objective optimizations, as shown in Figure 3.2(b).
Unlike the weighted-sum-of-objectives approach (e.g., Kim and De Weck 2005; Marler and Arora 2010), the cost is modeled in the proposed single-objective optimization approach as a constraint, instead of an objective, along with the constraints of design space and system performance requirements. Using the single-objective optimization setting in Figure 3.2(b), the most robust design within a given acceptable cost level \((C_{Tj})\) can readily be screened out using an Excel spreadsheet. As depicted in Figure 3.3, the resulting optimal design, denoted as \((C_{Tj}, \sigma_j[g])\), is a non-dominated solution on the Pareto front that would have been obtained using multi-objective optimization in Figure 3.2(a). Thus, the Pareto front can be collectively formed by a series of optimal designs obtained through single-objective optimizations.

The approach of transforming multi-objective optimization into a series of single-objective optimizations is inspired by previous studies (e.g., Haimes et al. 1971; Hämäläinen and Mäntysaari 2002). While algorithms such as NSGA-II may be more suitable...
for multi-objective optimization, it would be more efficient to implement it through a series of single-objective optimizations when the optimization involves only two objectives. Since the geotechnical problems such as the design of drilled shafts can generally be modeled as a bi-objective optimization problem, the proposed approach is effective. The proposed approach, however, has a tremendous advantage as it can be easily implemented in an Excel spreadsheet without the knowledge of genetic algorithms and considerable programming skills. As will be shown later, the proposed approach is efficient and able to yield practically the same results as those obtained with NSGA-II multi-objective optimization algorithm. Thus, the proposed RGD approach has a potential as a practical design tool for engineering applications.

Procedures to implement the proposed RGD approach

The procedures to implement the proposed efficient robust geotechnical design (RGD) approach are summarized in the following four main steps:

**Step 1:** Characterize the problem of concern. Here, the system performance of concern, noise factors, and design parameters are identified; meanwhile, the design space, requirements of system performance, design robustness, and cost are formulated.

**Step 2:** Construct the mapping function that relates $\beta^T$ (computed using MCS or FORM) to $\beta^M$ (computed using MFOSM) for the domain problem characterized in Step 1. This step enables an accurate evaluation of the system performance using MFOSM analysis.

**Step 3:** Carry out the robust design optimization considering the design robustness, cost efficiency, and design constraints using the optimization procedure shown in Figure
3.2(b). Here, the most robust design within a given cost level is identified using Excel “Solver”.

**Step 4:** Construct the Pareto front to reveal the tradeoff between design robustness and cost efficiency in the given design space. The Pareto front may be constructed through following steps. First, determine the least cost design and the most robust design among all feasible designs (i.e., those that satisfy the performance requirements) in the design space and denote the costs for these designs as \( C_L \) and \( C_R \), respectively. Second, divide the resulting cost interval of \([C_L, C_R]\) into a series of cost levels, denoted as \( C_T = \{C_{T1}, C_{T2}, C_{T3}, \ldots, C_{Tn}\} \). Third, determine the most robust design within each cost level. Fourth and lastly, plot all the resulting designs, in terms of design robustness versus cost, which yields a Pareto front.

**Application of RGD in Drilled Shaft in Clay**

To demonstrate the proposed efficient robust geotechnical design (RGD) approach, a design example published by ETC10 (2009) for the design of a drilled shaft in clay, shown in Figure 3.4, is adopted here.

![Figure 3.4 Schematic diagram of a drilled shaft in clay (after ETC 10)](image-url)
The RGD of this example of drilled shaft is accomplished with the following 4 steps:

**Step 1: Model characterization and parameters setting**

Two fundamental requirements of system performance in the design of drilled shafts in clay are bearing capacity (safety requirement) and settlement (serviceability requirement). These performance requirements are generally specified in terms of ultimate limit state (ULS) and serviceability limit state (SLS). The performance functions with respect to ULS and SLS, in terms of $g_1(d, \theta)$ and $g_2(d, \theta)$, are formulated respectively as follows:

$$g_1(d, \theta) = Q_{ULS} - F_d - F_l$$

$$g_2(d, \theta) = Q_{SLS} - F_d - F_l$$

(3.8)

(3.9)

where $Q_{ULS}$ represents the ULS bearing capacity; $Q_{SLS}$ is the SLS bearing capacity that is derived based on a specified maximum allowable settlement of $s_a$ (e.g., taken as 20 mm in this example); and, $F_d$ and $F_l$ represent the dead load and live load on the drilled shaft, respectively. For drilled shafts in clay, the ULS bearing capacity, $Q_{ULS}$, can be estimated using the following equation:

$$Q_{ULS} = Q_s + Q_b - W$$

(3.10)

where $Q_s$ and $Q_b$ are the side resistance and tip resistance, respectively, and $W$ is the weight of drilled shaft. The side resistance, $Q_s$, in Eq. (3.10) is estimated as (Phoon and Kulhawy 2005):

$$Q_s = \left[0.33 + 0.17/(c_{ul}/p_a) + \varepsilon\right](\pi BDc_{ul})$$

(3.11)

where $c_{ul}$ is the average undrained shear strength over the shaft length; $p_a$ is the atmosphere pressure (taken as 100 kPa); $B$ and $D$ are the diameter and length of drilled shaft, respectively; and, $\varepsilon$ is a normal random variable with a mean of 0 and a standard deviation of 0.12. The tip resistance, $Q_b$, in Eq. (3.10) is computed as (Phoon and Kulhawy 2005):
\[ Q_b = M_{tc} \left[ \left( 6.17 c_{u2} \xi_{cd} \xi_{cr} + q \right) \left( \pi B^2 / 4 \right) \right]^{-1.12} \]  

(3.12)

where \( c_{u2} \) is the average undrained shear strength within a depth of \( B \) below the tip; \( \xi_{cd} \) and \( \xi_{cr} \) are modifiers for the shaft depth and soil rigidity, respectively; \( q \) is the total vertical earth stress at shaft tip, taken as \( \gamma D \) (\( \gamma \) is the total unit weight of clay); and, \( M_{tc} \) is a lognormal random variable with a mean of 0.45 and a standard deviation of 0.13. The depth modifier in Eq. (3.12), \( \xi_{cd} \), is evaluated as (Phoon and Kulhawy 2005):

\[ \xi_{cd} = 1 + 0.33 \tan^{-1} \left( D / B \right) \]  

(3.13)

The rigidity modifier in Eq. (3.12), \( \xi_{cr} \), is generally taken as 1.0. Finally, the SLS bearing capacity of drilled shafts in clay, \( Q_{SLS} \), can be estimated as (Dithinde et al. 2010):

\[ \frac{Q_{SLS}}{Q_{ULS}} = \frac{s_a}{a + bs_a} \]  

(3.14)

where \( a \) and \( b \) are two hyperbolic curve-fitting parameters for the normalized load-settlement curve, which can be statistically represented with lognormal variables; and, \( s_a \) is the maximum allowable settlement of drilled shafts. The mean of \( a \) and \( b \) are 2.79 mm and 0.82, respectively, while the standard deviations are 2.04 mm and 0.09, respectively; and the correlation between these two fitting parameters is -0.801. It is noted that the model errors with respect to both ULS bearing capacity and SLS bearing capacity are explicitly considered through the model parameters of \( \varepsilon, M_{tc}, a, \) and \( b \).

Use of the aforementioned empirical models to estimate the ULS and SLS bearing capacity of drilled shafts in clay is not a limitation of the proposed RGD approach. Indeed, the more sophisticated finite element method (FEM) that considers explicitly the non-linear and time-dependent behaviors of soil and the non-linear soil-structure interaction may also be
used, in lieu of these empirical models, for evaluating the bearing capacity. Alternatively, the response surface method (e.g., Faravelli 1989; Li et al. 2011) may also be adopted to emulate the results of FEM solutions, and the well-developed response surface can then be treated as an empirical model to be used within the RGD framework.

For this drilled shaft example, a well-documented and well-studied example (ETC10, 2009), the water table is at the ground surface and the unit weight of clay ($\gamma$) is 20 kN/m$^3$. Hence, the uncertainty of geotechnical parameters mainly comes from the undrained shear strength ($c_u$). In reference to Figure 3.5, the undrained shear strength of clay, obtained from undrained triaxial tests, increases linearly with depth; the normalized undrained shear strength, $c^u = c_u / z$, is sufficiently represented by a lognormal variable with a mean of 13.04 and a standard deviation of 4.15. The adopted lognormal distribution of $c^u$ is validated, with 95% confidence level, using Chi-Square test ($\chi^2$) and the statistics are estimated using the maximum likelihood principle.

![Figure 3.5 Characterization of undrained shear strength from test data](image-url)
The unit weight of concrete, an essential input parameter for evaluating $Q_{ULS}$ and $Q_{SLS}$, is treated as a constant at 24 kN/m$^3$. To account for the uncertainty in loading conditions, both the dead load ($F_d$) and live load ($F_l$) are represented as lognormal variables. Given the characteristic value of load $F$ (from Eurocode 7: $F_{dc} = 300$ kN and $F_{lc} = 150$ kN), the mean of load $F$, denoted as $E[F]$, can be approximated as follows (Wang et al. 2011):

$$E[F] = \frac{F_c}{1 + 1.645 \delta_F}$$

(3.15)

where $F_c$ represents the characteristic value of load $F$, and $\delta_F$ is the coefficient of variation of load $F$. With an assumption that the coefficients of variation of the dead load ($F_d$) and live load ($F_l$) are 0.1 and 0.18, respectively (Zhang et al. 2011a), the mean of the dead load ($F_d$) and live load ($F_l$) are calculated as 258 kN and 116 kN, respectively.

Due to construction imperfection and unforeseen geological conditions, differences between the designed geometry and the constructed geometry of a drilled shaft should be expected. The effect of such discrepancy on the pile capacity can be significant (O’Neill 2001; Iskander et al. 2003; Poulos 2005). Yet, a quantitative index measuring such differences is rarely reported in literature. In the context of robust design, the as-built geometry parameters of a drilled shaft (i.e., $B_T$ and $D_T$) are herein modeled as lognormal variables. The mean of these parameters (i.e., $B_T$ and $D_T$) are assumed to be the same as the parameters of the designed geometry (i.e., $B$ and $D$), while the standard deviations of these parameters are both set to 0.05 m; further, the correlation coefficient between the two geometry parameters is assumed as $\rho_{B_T,D_T} = 0.5$. The standard deviations and the correlation coefficient assumed above are just an example for the purpose of illustrating the effect of construction noise. In
practice, other distribution type and statistics can be adopted based on local experience. Additional discussion on the effect of the level of construction variation is presented later.

In summary, the geotechnical parameters, loading parameters, construction variations, and model errors are explicitly included in the robust design with noise factors of $c^b$, $F_d$, $F_l$, $B_T$, $D_T$, $\varepsilon$, $M_{tc}$, $a$, and $b$. The designed geometry parameters (i.e., $B$ and $D$) are treated as design parameters. For illustration purpose, the design space is set up as a continuous space, $DS = \{(B, D) \mid B \in [0.3 \text{ m}, 0.6 \text{ m}], \ D \in [10.0 \text{ m}, 25.0 \text{ m}]\}$. Within the framework of robust design, the target reliability indexes (i.e., $\beta_{T1}$ and $\beta_{T2}$) with respect to ULS performance and SLS performance are set as 3.2 and 2.6, respectively, to ensure an adequate system performance. These target reliability indexes correspond to the target failure probabilities (i.e., $P_{T1}$ and $P_{T2}$) of $6.9 \times 10^{-4}$ and $4.7 \times 10^{-3}$, respectively. For this design example, the study performed by Orr et al. (2011) found that the critical factor that controls the design is the ULS performance requirement. Thus, the variation of the performance function with respect to ULS performance, $\sigma[g_1(d, \Theta)]$, is adopted herein to measure the design robustness. In this chapter, the cost of the drilled shaft ($C$) is simply represented with the volume of concrete ($\pi B^2 D/4$), although other, more sophisticated cost estimate models may be adopted. Figure 3.6 illustrates all the elements considered in the robust design of the drilled shaft in clay.
Step 2: Mapping from FORM to MFOSM

To establish the mapping function that relates $\beta^T$ (computed using FORM that has been calibrated with MCS) to $\beta^M$ (computed using MFOSM), 18 designs, equally distributed in the design space ($DS$), are analyzed. The analysis is carried out with respect to both ULS and SLS performance requirements, in terms of $\beta_1$ and $\beta_2$, respectively. FORM is adopted herein to evaluate the true reliability indexes of drilled shaft performance, as it can be implemented in the Excel spreadsheet environment and yields practically the same results as that obtained using MCS. Shown in Figure 3.7 is an example of FORM analysis of a drilled shaft in clay, with respect to ULS performance, in a spreadsheet environment that follows the procedures advanced by Low and Tang (2007a).

Note that for simplicity, the correlation matrix is assumed to retain the original unmodified correlation matrix. Shown in Figure 3.8 is the reliability analysis of drilled shafts...
in clay, with respect to both ULS performance and SLS performance, using MFOSM in the spreadsheet environment.

As reflected in Figure 3.9(a) & 3.9(b), the results are fitted well with a third-order polynomial function, yielding the following mapping functions:

\[
\beta_1^M = 0.0163(\beta_1^T)^3 - 0.1453(\beta_1^T)^2 + 0.8862(\beta_1^T) + 0.1438 \\
\beta_2^M = 0.0099(\beta_2^T)^3 - 0.1024(\beta_2^T)^2 + 0.8164(\beta_2^T) + 0.0890
\]

(16)

(17)

where \( \beta_1^M \) and \( \beta_2^M \) are the reliability indexes with respect to ULS and SLS, respectively, computed using MFOSM; and, \( \beta_1^T \) and \( \beta_2^T \) are the reliability indexes with respect to ULS and SLS, respectively, computed using FORM.
Initially, enter the mean \( \mu \), standard deviation \( \sigma \), and correlation matrix \( [R] \) of noise factors \( \theta \) and the geometry of drilled shaft (in terms of \( B \) and \( D \)), followed by invoking Excel Solver to automatically minimize reliability index \( \beta_{T_1} \) by changing column \( n \), subjected to \( g_1(d, \theta) = 0 \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( \mu_n )</th>
<th>( \sigma_n )</th>
<th>( n_i )</th>
<th>( \theta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_n ) (kPa/m)</td>
<td>Lognormal</td>
<td>13.04</td>
<td>4.15</td>
<td>2.520</td>
<td>0.311</td>
<td>-1.008</td>
<td>9.09</td>
</tr>
<tr>
<td>( B_T ) (m)</td>
<td>Lognormal</td>
<td>0.45</td>
<td>0.05</td>
<td>-0.805</td>
<td>0.111</td>
<td>-0.893</td>
<td>0.41</td>
</tr>
<tr>
<td>( D_T ) (m)</td>
<td>Lognormal</td>
<td>10.00</td>
<td>0.05</td>
<td>2.303</td>
<td>0.005</td>
<td>-0.478</td>
<td>9.98</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Normal</td>
<td>0.00</td>
<td>0.12</td>
<td>0.000</td>
<td>0.120</td>
<td>-1.141</td>
<td>-0.14</td>
</tr>
<tr>
<td>( M_{sc} )</td>
<td>Lognormal</td>
<td>0.45</td>
<td>0.13</td>
<td>-0.839</td>
<td>0.283</td>
<td>-0.439</td>
<td>0.38</td>
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<tr>
<td>( a ) (mm)</td>
<td>Lognormal</td>
<td>2.79</td>
<td>2.04</td>
<td>0.812</td>
<td>0.654</td>
<td>0.000</td>
<td>2.25</td>
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<tr>
<td>( b )</td>
<td>Lognormal</td>
<td>0.82</td>
<td>0.09</td>
<td>-0.204</td>
<td>0.109</td>
<td>0.000</td>
<td>0.82</td>
</tr>
<tr>
<td>( F_d ) (kN)</td>
<td>Lognormal</td>
<td>258.00</td>
<td>25.80</td>
<td>5.548</td>
<td>0.100</td>
<td>0.442</td>
<td>268.29</td>
</tr>
<tr>
<td>( F_{f_1} ) (kN)</td>
<td>Lognormal</td>
<td>116.00</td>
<td>20.88</td>
<td>4.738</td>
<td>0.179</td>
<td>0.359</td>
<td>121.72</td>
</tr>
</tbody>
</table>

**Correlation matrix \([R]\)**

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -0.8 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.8 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

**Notes:**
For this case, in solver’s option, use **automatic scaling**
**Quadratic** for Estimates, **Central** for Derivatives and **Newton** for Search
Others as default options.

Figure 3.7 Layout of a spreadsheet for FORM analysis of a drilled shaft based on ULS
Initially, enter the mean $\mu$, standard deviation $\sigma$, and covariance matrix $[C_{\theta}]$ of noise factors $\theta$ and the geometry of drilled shaft (in terms of $B$ and $D$), followed by automatic computation of the reliability indexes (in terms of $\beta_M^1$ and $\beta_M^2$).

<table>
<thead>
<tr>
<th>Statistics characterization of noise factors $\theta$</th>
<th>Partial derivatives of performance functions $G$</th>
<th>Statistics of performance functions $g_1$ and $g_2$</th>
<th>See Eqs. (2) &amp; (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^n$ (kPa/m)</td>
<td>$\frac{\partial g_1 (d, \theta_i)}{\partial \theta_i}$, $\frac{\partial g_2 (d, \theta_i)}{\partial \theta_i}$</td>
<td>$E[g_1 (d, \theta)]$</td>
<td>$B$ (m) 0.45</td>
</tr>
<tr>
<td>$B_T$ (m)</td>
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<tr>
<td>$D_T$ (m)</td>
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</tr>
<tr>
<td>$c$</td>
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<tr>
<td>$M_{nc}$</td>
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<td>0.13</td>
<td>438.53</td>
</tr>
<tr>
<td>$a$ (mm)</td>
<td>2.79</td>
<td>2.04</td>
<td>0.00</td>
</tr>
<tr>
<td>$b$</td>
<td>0.82</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>$F_1$ (kN)</td>
<td>258.00</td>
<td>25.80</td>
<td>-1.00</td>
</tr>
<tr>
<td>$F_1$ (kN)</td>
<td>116.00</td>
<td>20.88</td>
<td>-1.00</td>
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</table>

### Covariance Matrix $[C_{\theta}]$

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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>435.974</td>
</tr>
</tbody>
</table>

Notes:
The partial derivative of performance function is approximated as $\frac{\partial g}{\partial \theta_i} = \Delta g / \Delta \theta_i$, where $\Delta \theta_i = 0.1 \sigma \theta_i$.

Figure 3.8 Layout of a spreadsheet for MFOSM analysis of a drilled shaft based on ULS and SLS
(a) Mapping function of reliability index based on ULS

(b) Mapping function of reliability index based on SLS

Figure 3.9 Mapping from FORM to MFOSM
As shown in Figure 3.9, the computed coefficients of determination ($R^2$), with respect to ULS and SLS, are 0.9909 and 0.9839, respectively, indicating the true reliability indexes of the drilled shaft performance can be accurately captured through MFOSM analysis and corresponding mapping functions. It is noted that the error in the computed reliability index becomes noticeable at the very high end of reliability index, as some scatter is observed in this range. Nevertheless, this discrepancy is considered acceptable for two reasons: (1) the scatter is quite modest even in this range, and (2) the typical design generally involves a smaller reliability index well below that range.

Based upon the obtained mapping functions (see Eqs. 3.16 & 3.17), the target reliability indexes (i.e., $\beta_{T_1}^{M}$ and $\beta_{T_2}^{M}$) with respect to ULS and SLS, within the context of MFOSM analysis, are set as 2.03 and 1.69, respectively. The resulting target reliability indexes (i.e., $\beta_{T_1}^{M}$ and $\beta_{T_2}^{M}$) are readily applicable to the MFOSM-based procedure for evaluating the system performance requirements. Thus, no FORM or MCS is required in the subsequent robust design process, which is another feature of the proposed RGD approach that reduces the computational effort.

**Step 3: Efficient multi-objective optimization**

As formulated previously, the proposed efficient robust geotechnical design (RGD) can be implemented through a series of single-objective optimization problems, in which the cost is treated as an additional constraint (see Figure 3.2b). The single objective optimization-based RGD can be efficiently implemented in the spreadsheet environment using Excel’s built-in optimization routine “Solver”, as shown in Figure 3.10.
Initially, enter the mean $\mu$, standard deviation $\sigma$, and covariance matrix $[C]_\Theta$ of noise factors $\Theta$, the geometry of drilled shaft (in terms of $B$ and $D$), and the cost level $C_T$, followed by invoking Excel Solver to automatically minimize the standard deviation of performance functions $\sigma[g_1(d, \Theta)]$ by changing parameters $B$ and $D$, subjected to $B \in [0.3m, 0.6m], D \in [10.0m, 25.0m], \beta^M_1 > \beta^M_1, \beta^M_2 > \beta^M_2$, and $C < C_T$.

<table>
<thead>
<tr>
<th>Statistics characterization of noise factors $\Theta$</th>
<th>Partial derivatives of performance functions $G$</th>
<th>Statistics of performance functions $g_1$ and $g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\partial g_1(d, \Theta)/\partial \Theta_i$</td>
</tr>
<tr>
<td>$c_n$ (kPa/m)</td>
<td>13.04</td>
<td>4.15</td>
</tr>
<tr>
<td>$B_1$ (m)</td>
<td>0.42</td>
<td>0.05</td>
</tr>
<tr>
<td>$D_1$ (m)</td>
<td>16.19</td>
<td>0.05</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.00</td>
<td>0.12</td>
</tr>
<tr>
<td>$M_{nc}$</td>
<td>0.45</td>
<td>0.13</td>
</tr>
<tr>
<td>$a$ (mm)</td>
<td>2.79</td>
<td>2.04</td>
</tr>
<tr>
<td>$b$</td>
<td>0.82</td>
<td>0.09</td>
</tr>
<tr>
<td>$F_d$ (kN)</td>
<td>258.00</td>
<td>25.80</td>
</tr>
<tr>
<td>$F_l$ (kN)</td>
<td>116.00</td>
<td>20.88</td>
</tr>
</tbody>
</table>

Covariance Matrix $[C]_\Theta$

$$
egin{bmatrix}
17.223 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.003 & 0.001 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.001 & 0.003 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.014 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.017 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4.162 & -0.147 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.147 & 0.008 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 665.640 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 435.974 & 0
\end{bmatrix}
$$

Notes:

The partial derivative of performance function is approximated as $\partial g/\partial \Theta = \Delta g / \Delta \Theta_i$, where $\Delta \Theta_i = 0.1 \sigma_i$.

For this case, in solver’s option, use automatic scaling

**Quadratic** for Estimates, **Central** for Derivatives and **Newton** for Search

Others as default options.

Figure 3.10 Layout of a spreadsheet for robust design of a drilled shaft
It is noted that the evaluation of design robustness, in terms of the standard deviation of the performance function with respect to the ULS performance, and the evaluation of system performance requirements, in terms of the reliability indexes with respect to both ULS and SLS, share common computational steps (i.e., they are “coupled” computationally). With the spreadsheet shown in Figure 3.10, the most robust design within a given cost level ($C_T$) can readily be identified. For example, the most robust designs for three arbitrarily specified cost levels are identified and listed in Table 3.1.

Table 3.1 Examples of most robust design at various cost levels

<table>
<thead>
<tr>
<th>Specified cost level, $C_T$ (m$^3$)</th>
<th>Most robust design within the cost level</th>
<th>Design parameters</th>
<th>Actual cost, $C$ (m$^3$)</th>
<th>Resulting robustness, $\sigma_{[g_1(d, \theta)]}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.04</td>
<td>Design 1</td>
<td>Diameter, $B$ (m)</td>
<td>0.32</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Depth, $D$ (m)</td>
<td>25.00</td>
<td></td>
</tr>
<tr>
<td>2.24</td>
<td>Design 2</td>
<td></td>
<td>0.42</td>
<td>2.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16.19</td>
<td></td>
</tr>
<tr>
<td>2.91</td>
<td>Design 3</td>
<td></td>
<td>0.60</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.29</td>
<td></td>
</tr>
</tbody>
</table>

**Step 4:** Pareto front – tradeoff between design robustness and cost efficiency

Although the most robust design within a given cost level ($C_T$) can readily be identified using the spreadsheet shown in Figure 3.10, it should be noted that the optimal cost level is generally unknown in prior, and thus the resulting robust design may be a biased design. To guide the design decision, it is desirable to construct a complete Pareto front that shows the tradeoff between design robustness and cost efficiency.

For the given drilled shaft problem, the costs of the least cost and the most robust designs are 2.04 m$^3$ and 2.91 m$^3$, respectively. The resulting cost interval of [2.04 m$^3$, 2.91 m$^3$] is then divided into 10 cost levels, denoted as $C_T = \{2.04$ m$^3$, 2.14 m$^3$, 2.24 m$^3$, 2.34 m$^3$, ...]
2.44 m$^3$, 2.54 m$^3$, 2.64 m$^3$, 2.74 m$^3$, 2.84 m$^3$, 2.91 m$^3$}. For each cost level in $C_T$, the most robust design is identified using the spreadsheet shown in Figure 3.10. Thereafter, all the resulting optimal designs are plotted as shown in Figure 3.11.

The resulting Pareto front shows a global tradeoff between design robustness and cost efficiency in the design space. The Pareto front may be used as a decision aid in selecting the most preferred design at a desired cost level.

Also shown in Figure 3.11 is the Pareto front obtained using the genetic algorithm of NSGA-II (Deb et al. 2002; Juang et al. 2013b; Juang et al. 2014). For solutions using NSGA-II, the population size was set at 200 and the generation number was set at 500; these values were selected using a trial-and-error procedure. At convergence, the multi-objective optimization algorithm yields a Pareto front, as shown in Figure 3.11. The graph in Figure 3.11 demonstrates that the proposed efficient RGD approach (Figure 3.2b) can derive

![Figure 3.11 Pareto front showing tradeoff between design robustness and cost efficiency](image-url)
practically the same Pareto front as the existing RGD approach (Figure 3.2a) that relies on the multi-objective optimization algorithms such as NSGA-II.

As mentioned previously, all the designs on the Pareto front are equally optimal in the sense that no improvement can be achieved in one objective without worsening in the other objective. Nevertheless, if one wishes to select the most preferred design (i.e., the best compromise solution) from a Pareto front, the “knee point” concept (Branke et al. 2004; Deb and Gupta 2011) may be used. The knee point on the Pareto front conceptually yields the best compromise among conflicting objectives in the design space. According to Branke et al. (2004), the knee point may be determined using the following marginal utility function:

$$U'(d_i, \lambda) = \min \left(U(d_j, \lambda) - U(d_i, \lambda) \right) \quad (i \neq j) \quad (3.18)$$

where $$U(d, \lambda)$$ is a linear utility function defined as (Branke et al. 2004):

$$U(d, \lambda) = \sum \lambda_i f_i(d) \quad (3.19)$$

where $$d$$ represents design parameters, in this example: $$d = (B, D)$$; $$\lambda_i$$ is random parameter with a value ranging from 0.0 to 1.0, and $$\sum \lambda_i = 1.0$$; and, $$f_i(d)$$ = objective functions to be minimized, in this case: $$f_1(d) = (\sigma[g_1] - \sigma_{\min}[g_1]) / (\sigma_{\max}[g_1] - \sigma_{\min}[g_1])$$, and $$f_2(d) = (C - C_{\min}) / (C_{\max} - C_{\min})$$, where $$\sigma_{\max}[g_1]$$ and $$\sigma_{\min}[g_1]$$ represent the maximum value and minimum value of $$\sigma[g_1]$$ on the Pareto front, respectively; and, $$C_{\max}$$ and $$C_{\min}$$ represent the maximum value and minimum value of $$C$$ on the Pareto front, respectively.

By means of Monte Carlo simulation (MCS), different random values of $$\lambda_1$$ and $$\lambda_2$$ are generated with an assumption of uniform distribution, and the marginal utility function is computed for each of these 10 non-dominated optimal designs on the Pareto front. The design with the maximum expected marginal utility is then taken as the knee point on the Pareto front.
front. Based on the calculated *expected* marginal utility, Design 2 (see Table 3.1) is identified as the most preferred solution in the design space (or knee point on the Pareto front).

As can be observed in Figure 3.11, on the left side of the knee point, a slight reduction in the cost can lead to a large increase in the variation of the ULS performance function (indicating a drastic reduction in the design robustness); on the right side of the knee point, a slight reduction in the variation of the ULS performance function (indicating a slight improvement of design robustness) requires a large increase in cost. Therefore, the knee point on the Pareto front represents the best compromise between the design robustness and the cost efficiency and can be treated as the most preferred design in the design space.

**Further Discussions on the Proposed RGD Approach**

*Effectiveness of the variation of the performance function as a robustness measure*

Figure 3.12 shows the variation or distribution of the system performance with respect to ULS performance for the three designs listed in Table 3.1.

![Figure 3.12 Distribution of ULS performance of three drilled shaft designs](image)

*Figure 3.12 Distribution of ULS performance of three drilled shaft designs (standard deviation of each and every noise factor is a constant)*
Through 1,000,000 MCS runs, the distribution of the system performance (defined in Eq. 3.8) caused by the variation of noise factors can be obtained for each design. The results show that a design with a higher variation of the performance function (indicating lower design robustness) has a wider distribution of system performance, which implies a higher degree of uncertainty as to whether the system performance can satisfy the pre-defined performance requirement.

Figure 3.13(a) & 3.13(b) show the gradient of the system performance to noise factors and the variation of the system performance that arises from noise factors, respectively, for the three designs listed in Table 3.1. Figure 3.13(a) depicts that the system performance is most sensitive to the variation of the as-built diameter of drilled shafts ($B_T$) and model parameters of $Q_{ULS}$ (i.e., $\varepsilon$ and $M_{tc}$). Figure 3.3 (b) illustrates that the variation of the system performance is mainly caused by the variation of the normalized undrained shear strength ($c^0$), as-built diameter of drilled shafts ($B_T$), and model parameters of $Q_{ULS}$ (i.e., $\varepsilon$ and $M_{tc}$).
Figure 3.13 Sensitivity of performance function with respect to noise factors

Compared with the existing robust measures, the presented robustness measure in this chapter shows a significant advantage as it is an integration of the gradient (or sensitivity) and the variation of noise factors (see Eq. 3.2). Thus, the knowledge of how the variation of the system performance is affected by the variation of each noise factor is reflected in the formulation of the proposed robustness measure.
Figure 3.14 shows the distribution of the system performance for the three designs listed in Table 3.1, which is similar to Figure 3.11, but in Figure 3.14, the uncertainty in the estimated statistics of noise factors is included.

![Distribution of ULS performance of three drilled shaft designs](image)

Figure 3.14 Distribution of ULS performance of three drilled shaft designs (standard deviation of every noise factor is a random variable with 20% COV)

As in most geotechnical problems, it is often challenging to determine the statistics of key geotechnical parameters with certainty; such uncertainty in the estimated statistics can lead to another layer of uncertainty in the system performance. For illustration purposes, the standard deviations of noise factors are treated as independent lognormal variables with a coefficient of variation (COV) of 20%. Through 1,000,000 MCS runs, the distribution of the system performance can be achieved as shown in Figure 3.14. The results indicate that the design with a lower variation of the performance function (indicating higher design robustness) can tolerate a greater uncertainty in the estimated statistics of noise factors, as it yields a shallower distribution of the system performance and a lower failure probability, defined as probability of unsatisfactory performance (see Table 3.2).
Table 3.2 Probability of unsatisfactory performance of each of the three designs at each of the three levels of variation of noise factors

<table>
<thead>
<tr>
<th>Design</th>
<th>Level of variation of noise factors (in terms of COV of the standard deviation of each of noise factors)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COV = 10%</td>
</tr>
<tr>
<td>Design 1</td>
<td>1.169 × 10^{-3}</td>
</tr>
<tr>
<td>Design 2</td>
<td>0.949 × 10^{-3}</td>
</tr>
<tr>
<td>Design 3</td>
<td>0.530 × 10^{-3}</td>
</tr>
</tbody>
</table>

Therefore, the design with higher design robustness has an additional benefit of being able to guard against the uncertainty in the estimated statistics of noise factors.

Comparison between the proposed RGD and the traditional RBD

The Pareto front shown in Figure 3.11 is employed herein to illustrate the difference between the robust geotechnical design (RGD) and the reliability-based design (RBD). Since all designs on the Pareto front meet the system performance requirements, the design that yields the least cost (in this case, Design 1) can be selected as the final design. This least cost design using the RBD approach, however, is associated with the highest variation of the system performance (see Figure 3.11), highest sensitivity of the system performance to the variation of noise factors (see Figure 3.12), and least tolerance of the system performance to the variation in the estimated statistics of noise factors (see Figure 3.13). Thus, the least-cost RBD design is not necessarily the most preferred design since a modest variation in the noise factors could lead to a dramatic variation of the system performance. On the other hand, the RGD design approach that considers explicitly the safety (performance requirements), cost efficiency, and design robustness produces a Pareto front that offers a tradeoff between design
robustness and cost efficiency. Thus, the effect of the uncertain noise factors is realistically managed in the design decision using the proposed RGD approach.

**Improving design robustness by reducing the variation of noise factors**

So far the discussion focuses on improving the design robustness by adjusting only the design parameters, assuming that the uncertainty in noise factors cannot be reduced or it is not economical to do so. An alternative is to improve the design robustness by reducing the variation of noise factors, referred to as Approach 2 shown in Figure 3.1. In this section, the construction variation as a noise factor is used as an example to illustrate the robust design using Approach 2.

The three designs listed in Table 3.1 are analyzed for their design robustness, in terms of the variation of the performance function, at three levels of construction variation. Here, the level of construction variation is represented with the standard deviation of as-built geometry (applied to both $B_T$ and $D_T$). Three levels of standard deviation, 0.05 m, 0.10 m, and 0.15 m, are adopted for illustration purposes. The results of the analysis are plotted in Figure 3.14.
As expected, the design robustness can be increased (meaning that the variation of the system performance can be lowered) by reducing the variation of the as-built geometry of drilled shafts caused by the construction variation. Thus, Approach 2 shown in Figure 3.1 is demonstrated as an effective alternative for improving the design robustness, although an additional cost that is associated with noise variation reduction may be incurred. On the other hand, cost savings may be achieved from the resulting optimal design at a lower level of construction noise. Although further study to determine the net effect on the overall cost is needed, the results of the analysis demonstrate that reducing the variation of noise factors may be a cost-effective step in the robust design of a geotechnical system.
Summary

This chapter presents an efficient robust geotechnical design (RGD) approach, in which the design robustness is represented with the variation of the performance function that is readily available in a reliability analysis. Through the coupling of the evaluation of design robustness with the reliability analysis of system performance, additional savings in the computational effort are achieved. Finally, the goal of multi-objective optimization, typically required in a robust design framework, can be achieved with a series of single objective optimizations using Excel Solver. In short, an efficient and practical RGD approach is developed.
Chapter 4
PRACTICAL ROBUST GEOTECHNICAL DESIGN OF SUPPORTED EXCAVATION USING RESPONSE SURFACE

Introduction

Uncertainties are considered implicitly or explicitly in a geotechnical design, regardless of whether the factor of safety (FS)-based approach, the reliability-based design (RBD) method (Harr 1987, Baecher and Christian 2003; Ang and Tang 1984) or the load and resistance factor design (LRFD) method is used. However, in these traditional design methods, the focus is placed on cost and safety, and the robustness is not considered in the design. Juang et al. (2013) has shown that even with the more rigorous RBD method, over-design or under-design can still happen if the uncertainties in the parameters, solution model and/or construction variation are not precisely characterized. To this end, they introduced the concept of robust design into geotechnical problems. The robust design (Taguchi 1986) seeks an optimal design that simultaneously and explicitly considers safety, cost efficiency, and design robustness. A design is considered robust if the system response (i.e., the performance or response of a designed system under loading) is insensitive to the variation in the uncertain, “hard-to-control” input parameters, known as noise factors in the literature of robust design (Taguchi 1986). The robust design seeks a design that is insensitive to the variation in the noise factors by carefully adjusting the “design parameters” (i.e., the parameters that can be controlled by the engineer, such as the geometry, dimension, and other design settings).

In an effort to promote the robust design concept in the design of geotechnical structures, a term “Robust Geotechnical Design” (RGD) has been coined by Juang and his
co-workers (Juang et al. 2013a; Wang et al. 2013; Khoshnevisan et al. 2014). Both the reliability-based RGD (Juang et al. 2013a), in which the safety requirement is evaluated using the reliability analysis, and the $FS$-based RGD (Gong et al. 2015), in which the safety requirement is evaluated using the deterministic $FS$-based approach, have been proposed, and the effectiveness of these RGD methods has been demonstrated. However, even for simple problems such as drilled shaft design and rock slope design, the RGD approach can be computationally challenging, as the reliability analysis for the safety requirement and the evaluation of the variation in the system response for assessing the design robustness must be repeated within the multi-objective robust optimization framework. Another potential barrier for the practicing engineer to adopt the RGD approach is the programming skills required for performing the multi-objective robust optimization.

In this chapter, the RGD approach is simplified and new features such as the modified gradient-based robustness measure, the response surface surrogate model for the evaluation of the system response, and the minimum distance (MD) algorithm that eliminates the need for multi-objective optimizations in the search for the optimal design, are introduced. Detailed formulations and procedures for the robust geotechnical design are presented using a real-world supported excavation as an example. In fact, a soldier-piles-tieback-anchors supported excavation in sandy and gravelly site is selected for this study to demonstrate the applicability of the RGD approach in a sufficiently complex geotechnical problem where the numerical solution using a sufficiently complex computer code is a necessity. With the new features, the simplified RGD approach becomes a practical design tool for the design of a sufficiently complex geotechnical system.
Robustness Measures for Robust Geotechnical Design

The goal of the robust geotechnical design (RGD) is to seek an optimal design in the design pool such that the cost is minimized and the design robustness (to be defined later) is maximized, while the safety or performance requirements with respect to both stability and serviceability are satisfied. In addition to cost and safety, the two objectives in a traditional design, the design robustness is equally emphasized in RGD. Robust design seeks an optimal design that is insensitive to the variation of noise factors by carefully adjusting design parameters. Using the supported excavation as an example, the noise factors are referring to uncertain soil parameters and the surcharge behind the wall, and the design parameters are referring to the dimensions of the soldier pile, and layouts of the tieback anchors (see Figure 4.3 presented later). The RGD process requires a workable definition of the design robustness. In other words, the key here is how the robustness is measured. While many robustness measures have been reported (Khoshnevisan et al. 2014), the focus here is the gradient-based robustness measure discussed next.

Weighted Sensitivity Index ($SI_w$)

Let’s define $d$ as a vector of design parameters, $\theta$ as a vector of noise factors, and $g(d,\theta)$ is the system response of a system with design parameters $d$ and noise factors $\theta$ as inputs. The sensitivity of the system response $g(d,\theta)$ to the noise factors, $\theta$, can be measured by its gradient $G$ evaluated at the nominal values or means of noise factors, $\mu_\theta$. (Gong et al. 2014b):
where $m$ represents the number of noise factors.

In general, a design (or system) with a lower gradient exhibits a lower variation in the predicted system response, thus, by definition, is a more robust design. However, different noise factors may have different levels of variation, and their contribution to the gradient-based robustness could be different and should be considered. Thus, the gradient-based sensitivity index ($SI$) by Gong et al. (2014b) is modified as follows:

$$SI_w = \|J_w\| = \sqrt{J_w^T J_w}$$

(4.2)

where $J_w$ is a normalized gradient vector defined below:

$$J_w = \left\{ \left( w_1 \right) \frac{\mu_{\theta_1}}{g(d, \mu_\theta)} \frac{\partial g(d, \theta)}{\partial \theta_1}_{\theta = \mu_\theta}, \left( w_2 \right) \frac{\mu_{\theta_2}}{g(d, \mu_\theta)} \frac{\partial g(d, \theta)}{\partial \theta_2}_{\theta = \mu_\theta}, ..., \left( w_m \right) \frac{\mu_{\theta_m}}{g(d, \mu_\theta)} \frac{\partial g(d, \theta)}{\partial \theta_m}_{\theta = \mu_\theta} \right\}$$

(4.3)

The modification is implemented with the weighting factors, $(w_j, j = 1, m)$ that depend on the relative levels of variation (or variability) of the $m$ noise factors.

One way to determine the weighting factors based on different variability of the noise factors is to conduct a pairwise comparison (Saaty 2004). Pairwise comparisons of the variability of all pairs of noise factors may be performed using engineering judgment or based on coefficients of variation (COVs), estimated based on available data and/or those reported in the literature. By pairwise comparison, a comparison matrix is formed:

$$A = \left[ a_{ij} \right]_{m \times m}$$

(4.4)
where $a_{ij} = \text{COV}_i / \text{COV}_j$, for $i= 1, m$ and $j = 1, m$.

Then, the matrix $A$ is normalized:

$$A^N = \begin{bmatrix} a^{N}_{ij} \end{bmatrix}_{m \times m}$$

(4.5)

$$a^{N}_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}$$

where $a^{N}_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}$, for $i= 1, m$ and $j = 1, m$.

Finally, the weight of each noise factor is calculated as:

$$w_i = \left( \frac{\sum_{j=1}^{m} a^{N}_{ij}}{m} \right) / m \text{ for } i = 1, m$$

(4.6)

While both $G$ and $J_w$ are vectors, the weighted sensitivity index $SI_w$ is a scalar value and thus is suitable as a robustness measure for robust design optimization. A lower $SI_w$ value means less sensitivity of the system response to the noise factors, and thus implies higher design robustness.

It is noted that the pairwise comparison procedure presented previously allows for the weighting factors to be determined based on COVs of the noise factors, when abundant data are available for full statistical characterization. However, in the absence of sufficient data, as in most cases in the geotechnical practice, the weighting factors may be determined reasonably based on pairwise comparisons of the perceived variability of the noise factors by engineering judgment and local experience.

**Signal to Noise Ratio (SNR)**

The signal-to-noise-ratio (SNR), defined below (Phadke 1989), is a well-accepted robustness measure in many fields of engineering:
\[ SNR = 10 \log_{10} \left( \frac{E[g(d, \theta)]}{\sigma[g(d, \theta)]} \right)^2 \]  

(4.7)

where \( E[g(d, \theta)] \) and \( \sigma[g(d, \theta)] \) are the mean and standard deviation of the predicted system response \( g(d, \theta) \) of a given \( d \). A higher \( SNR \) indicates higher robustness.

Previous studies (Juang et al. 2013a; Wang et al. 2013) revealed that whether a given robustness measure, such as \( SNR \), is suitable for robust design optimization depends on the system response of concern. No single robustness measure is most suitable for all types of designs with various system responses of concern. In this chapter, \( SI_w \) is focused in the robust design of the supported excavation and \( SNR \) is used as a reference for comparison.

Elements of Robust Design Focusing on Supported Excavation

Design of supported excavation – a case history

For illustration purposes, a real-world supported excavation project in Taipei, Taiwan is studied herein. This excavation project was for the construction of a student dormitory and underground parking of Wesley Girls High School. The layout of the excavation was roughly a rectangular shape with a length of 140 m and a width of 45 m (Figure 4.1). The excavation depths ranged from 6.9 m to 11.9 m.

The deposits at the site consisted of three cohesionless layers (in reference to Table 4.1 and Figure 4.2): the first layer (0 to 3.35 m) is a silty sand (SM), the second layer (3.35 to 11.15 m) is a poorly graded gravel (GM), the third layer (11.15 to 15.50 m) is another GM layer, which is underlain by a rock formation (15.50 to 20.00 m).
<table>
<thead>
<tr>
<th>Section</th>
<th>Excavation depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section A</td>
<td>6.9 m</td>
</tr>
<tr>
<td>Section B</td>
<td>9.15 m</td>
</tr>
<tr>
<td>Section C</td>
<td>11.4 m</td>
</tr>
<tr>
<td>Section D</td>
<td>11.9 m</td>
</tr>
<tr>
<td>Section E</td>
<td>8.5 m</td>
</tr>
</tbody>
</table>

Figure 4.1 Plan layout of the excavation site in the case study
Figure 4.2 Three possible tieback anchor layouts of the shoring system
Table 4.1 lists the geotechnical parameters for each of these layers, including the unit weight, effective cohesion ($c'$), effective friction angle ($\phi'$), standard penetration blow count (N), and modulus of horizontal subgrade reaction ($k_h$).

At this site, the groundwater table was well below the excavation depth. Based on local experience, a soldier pile wall that consists of reinforced concrete piles was selected by the designer as the retaining structure, which was supported by tieback anchors.

The shoring system described previously is re-designed in this chapter using the RGD approach. For illustration purposes, only section D (see Figure 4.1; this is the section with the largest excavation depth) is designed, although other sections can be designed using the same approach. Figure 4.2 shows the excavation depth in each excavation stage and the possible layouts of the tieback anchors, which depend on the vertical spacing of the tieback anchors and the installed angle of tieback anchors.
Table 4.1 Basic soil properties adopted in the shoring system for the deep excavation project

<table>
<thead>
<tr>
<th>Layer</th>
<th>Soil type</th>
<th>Depth (m)</th>
<th>Unit weight (t/m³)</th>
<th>Effective Cohesion c (t/m²)</th>
<th>Effective Friction angle $\phi$ (°)</th>
<th>Modulus of horizontal subgrade reaction $k_h$ (t/m²)</th>
<th>$q_{skin,u}$ (t/m²) = N/5</th>
<th>N</th>
<th>Elasticity Modulus</th>
<th>$E_{load}$ (t/m²)</th>
<th>$E_{reload}$ (t/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SM</td>
<td>0 - 3.35</td>
<td>1.80</td>
<td>0.0</td>
<td>28.0</td>
<td>500</td>
<td>1</td>
<td>5</td>
<td></td>
<td>1274</td>
<td>3823</td>
</tr>
<tr>
<td>2</td>
<td>GP₁</td>
<td>3.35-11.15</td>
<td>1.90</td>
<td>0.0</td>
<td>30.0</td>
<td>4000</td>
<td>8.2</td>
<td>41</td>
<td></td>
<td>10197</td>
<td>30591</td>
</tr>
<tr>
<td>3</td>
<td>GP₂</td>
<td>11.15-15.5</td>
<td>2.30</td>
<td>0.0</td>
<td>36.0</td>
<td>5000</td>
<td>10</td>
<td>&gt;50</td>
<td></td>
<td>12746</td>
<td>38239</td>
</tr>
<tr>
<td>4</td>
<td>Rock</td>
<td>15.5-20.0</td>
<td>2.40</td>
<td>2.0</td>
<td>34.0</td>
<td>5000</td>
<td>10</td>
<td>&gt;50</td>
<td></td>
<td>12746</td>
<td>38239</td>
</tr>
</tbody>
</table>
Note that Trinity Foundation Engineering Consultants (TFEC) using a special-purpose computer code TORSA (Sino-Geotechnics 2010) performed the original design. The geotechnical parameters listed in Table 4.1, including the unit weight ($\gamma$), effective cohesion ($c'$), effective friction angle ($\phi'$), standard penetration blow count (N), and modulus of horizontal subgrade reaction ($k_h$) were derived by the designer (Hsii-Sheng Hsieh, personal communication 2014) based on sampling and testing performed by TFEC. These data are assumed in the present study. Further, additional data are needed as a different software DeepEx (Deep Excavation LLC 2015) is used in this study. These data are also listed in Table 4.1. Here, the elastic modulus, $E_{load}$ is estimated based on an empirical relation, $E_{load} = 2.55 k_h$; and the modulus during the phase of reloading, $E_{reload}$, is assumed to be three times of $E_{load}$ (Hsii-Sheng Hsieh, personal communication 2014). The parameter, $q_{skin,u}$ (in the unit of t/m$^2$) is the ultimate bond resistance for the tieback anchors, which is estimated as N/5 (Hsii-Sheng Hsieh, personal communication 2014). The data listed in Table 4.1 are assumed for the deterministic analysis of the supported excavation investigated in this paper.

**Deterministic model for responses of the supported excavation**

In traditional deterministic methods, the safety requirement is defined through the use of limiting factors of safety and the serviceability requirement is defined through limiting maximum wall and/or ground deformation (JSA 1988; TGS 2001; PSCG 2000; Ou 2006). As noted previously, DeepEx2015 (short-handed as “DeepEx” hereinafter) is adopted for the analysis and design of the shoring system in this paper. DeepEx is a commercially available special-purpose computer program for analysis and design of supported excavation. It should
be noted that other computer codes such as TORSA (Sino-Geotechnics 2010) or PLAXIS (Plaxis by 2015) may be used for the analysis and design of the shoring system for excavation. DeepEx is based on the beam on elastic foundation theory, in which the soldier pile wall is modeled as an elastic beam. The pressure acting on the back of the soldier pile wall is assumed to be the active earth pressure and the resistance of soil inside the excavation is modeled as soil springs (Ou 2006; Sino-Geotechnics 2010; Deep Excavation LLC 2015). Using DeepEx, the system responses of the supported excavation system, such as the factor of safety against push-in failure ($FS_{\text{push-in}}$), the factor of safety against basal stability ($FS_{\text{basal}}$), and the maximum wall deflection ($\delta_{\text{max}}$), can be obtained.

In the robust design of a shoring system for a supported excavation project, the maximum wall deflection ($\delta_{\text{max}}$) is most suitable to be chosen as the system response of concern, since the ground movement and the damage potential of the excavation support and the adjacent structures are strongly related to the maximum wall deflection (Juang et al. 2012b; Clough and O’Rourke 1990). Furthermore, the maximum wall deflection is relatively easy to measure and convenient to be used as a control for field monitoring of the performance of the excavation (Kung et al. 2007). Finally, the safety requirements, such as the $FS$ against push-in failure ($FS_{\text{push-in}}$) and $FS$ against basal heave ($FS_{\text{basal}}$), must be considered along with the cost for each candidate design.

In the proposed robust design optimization framework, presented later, the system responses predicted by DeepEx will be substituted with their counterparts computed based on a response surface surrogate model. Development and use of a surrogate model allows for a more efficient and practical implementation of the RGD approach in an Excel spreadsheet, a practical engineering design tool.
Characterization of noise factors for robust design

Inherent variability, measuring error, and transformation error (Phoon and Kulhawy 1999) may lead to variation in soil parameters. For the supported excavation shown in Figure 4.2, the main uncertainties in the geotechnical parameters are the effective cohesion ($c'$) and the effective friction angle ($\phi'$). Another uncertain parameter considered herein is the surcharge loading behind the wall ($q_0$). These uncertainties are known as noise factors in the context of robust design optimization. Limited test data and high variability in the soil parameters often make it impossible to obtain an accurate statistical characterization of these noise factors. In this study, the values given for these noise factors listed in Table 4.1 are assumed as the mean values. For illustration purposes, the effective cohesion is assumed to have a COV of 0.3, the COV of the effective friction angle is assumed to be 0.07 (Orr and Breysse 2008), and the surcharge loading is assumed to have a COV of 0.2 (Wang et al. 2013). It is noted that the top three layers at the site are cohesionless deposits with $c' = 0$ (see Table 4.1), thus, a total of 6 noise factors are identified for the robust design optimization. They are: $\phi_{SM}$, $\phi_{GP_1}$, $\phi_{GP_2}$, $\phi_{Rock}$, $c'_{Rock}$, and $q_0$. These noise factors are summarized in Table 4.2. The correlation coefficient between the effective cohesion ($c'$) and the effective friction angle ($\phi'$) of the rock is assumed to have a fixed value of -0.5.

<table>
<thead>
<tr>
<th>Noise factor</th>
<th>Coefficient of variation, COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Cohesion, $c$</td>
<td>0.3</td>
<td>Normal</td>
</tr>
<tr>
<td>Effective Friction angle, $\phi$</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td>Surcharge Loading, $l$</td>
<td>0.2</td>
<td>Normal</td>
</tr>
</tbody>
</table>
Design parameters and design pool for robust design

In a robust design, the design parameters are those input parameters that are easy-to-control by the designer. For the supported excavation studied, the diameter of the soldier pile ($D$), the length of the soldier pile ($L$), the vertical spacing of tieback anchors ($V$), the horizontal spacing of tieback anchors ($H$), and the installed angle of tieback anchors with respect to the horizontal direction ($\alpha$) are treated as the design parameters. The soldier piles are spaced at set intervals, typically 6 ft. to 10 ft. (1.8 m to 3 m). Here, the spacing of the soldier piles ($I$) is taken as a fixed value of 2 m (GDP 2015). For the supported excavation system studied (Figure 4.2), the preload of tieback anchors are all set up as 20 ton/tieback; and the free length (the first part) and fixed length (the grouted part) of tieback anchors are both designed as 8.0 m. The location of each tieback anchor is set at approximately 0.8 m above the excavation depth at each excavation stage, except for the last excavation stage.

Based on local practice, the feasible values of each of the five design parameters ($D$, $L$, $V$, $H$, $\alpha$) for the supported excavation shown in Figure 4.2 are determined, and listed in Table 4.3.

**Table 4.3 Design parameter ranges in the design pool**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of pile, $D$ (m)</td>
<td>0.6m, 0.7m, 0.8m</td>
</tr>
<tr>
<td>Length of pile, $L$ (m)</td>
<td>16m, 17m, 18m, 19m, 20m</td>
</tr>
<tr>
<td>Vertical spacing of tieback anchors, $V$ (m)</td>
<td>2.5m, 3m, 3.5m</td>
</tr>
<tr>
<td>Horizontal spacing of tieback anchors, $H$ (m)</td>
<td>1.5m, 2m, 2.5m</td>
</tr>
<tr>
<td>Installed angle of tieback anchors, $\alpha$ (°)</td>
<td>15°, 20°, 25°</td>
</tr>
</tbody>
</table>

Note: Horizontal interval of soldier pile, $I$ (m), is fixed at 2m.

Thus, a discrete design pool is selected for the robust design of the supported excavation, which consists of 270 combinations of the five design parameters.
Cost estimation of the supported excavation

As stated previously, the goal of robust design is to obtain an optimal design, which is safe, robust and cost efficient. For the given excavation project, the dimensions and depth of the excavation site are fixed according to either the structural or architectural requirements. In other words, the cost of excavation is fixed in the robust design optimization for a given excavation project. Thus, the cost of concern for performing the robust design optimization may be taken as the sum of the cost for the soldier pile wall ($C_w$) and the cost for the tieback anchor systems ($C_t$), expressed as:

$$ C = C_w + C_t $$

(4.8)

Using the geotechnical practice in Taiwan as an example, the cost for the soldier pile wall ($C_w$) may be estimated as (Hsii-Sheng Hsieh, personal communication 2014):

$$ C_w = \left( \frac{SL}{T} \right) (L) \left( \frac{D}{0.6} \right)^2 (t_1) $$

(4.9)

where $SL$ is the perimeter of the excavation site (m). In this paper, $SL$ is equal to 54.08 m. The parameter $t_1$ is the unit price (US dollar) for a 0.6 m diameter pile, and in this study, $t_1= 66$ US dollars.

Similarly, the cost for the tieback systems ($C_t$) may be estimated as (Hsii-Sheng Hsieh, personal communication 2014):

$$ C_t = \left( \frac{SL}{H} \right) (VL) (t_2 \times l_2 + t_3) $$

(4.10)

where $VL$ is the number of vertical levels of the tieback anchors, which are determined based on the configuration (see Figure 4.2). The parameter $t_2$ is the unit price (US dollar) for the unit length of the anchor, $l_2$ is the length of the anchor, and $t_3$ is the unit price (US dollar) for the
anchor head. In this study, these unit prices are assumed: \( t_2 = t_3 = 66 \text{ US dollars} \) based on local practice in Taipei (note: it is a coincidence that \( t_1 = t_2 = t_3 \) in this design example).

It should be noted that the cost estimation models adopted herein are only used for illustration purposes, although they are quite realistic for similar excavations in Taipei. Any other reasonable cost estimation model may be adopted.

**Response Surface for Robust Design Focusing on Supported Excavation**

As stated previously, use of software package such as DeepEx for computing the responses of a supported excavation, such as one supported by a solider pile wall with tieback anchors shown in Figure 4.2, within the robust design optimization framework that has to consider different scenarios for noise factors and a large number of designs in a design pool is computationally challenging. Multiple features are introduced in this paper into the proposed RGD approach to make it efficient and practical; the construction of a response surface is a focus towards this goal. Here, the response surface (Xu and Low 2006; Zhang et al. 2011b) is a problem-specific surrogate model that replaces the numerical model (i.e., software package) for computing the responses (safety factors against instability and maximum wall deflection) in a supported excavation.

**Procedure for constructing a response surface**

For the supported excavation shown in Figure 3, the noise factors \( \theta = \{ \theta_i, i = 1, 6 \} = \{ \phi_{SM}, \phi_{GP1}, \phi_{GP2}, \phi_{Rock}, c_{Rock}', q_0 \} \) and the design parameters \( d = \{ d_j, j = 1, 5 \} = \)
\{D, L, V, H, \alpha\}. For a design \(d\) (i.e., a combination of the five design parameters), the system response \(g(d, \theta)\) is computed in this paper using DeepEx. The responses of particular interest for the robust design of the supported excavation are the factor of safety against push-in (\(FS_{\text{push-in}}\)), factor of safety against basal heave (\(FS_{\text{basal}}\)), and maximum wall deflection (\(\delta_{\text{max}}\)). Symbolically, the response for a given design is \(y = g(d, \theta) = \{y_j, j = 1, 3\}\) where \(y_1 = FS_{\text{push-in}}, y_2 = FS_{\text{basal}}, \) and \(y_3 = \delta_{\text{max}}\). Note that each given \(y_j\) is a function of noise factors, \(y_j = f(\theta)\). A popular model for the response surface is a second-order polynomial (Bucher and Bourgund 1990; Xu and Low 2006):

\[
y_j = a_0 + \sum_{i=1}^{m} a_i \theta_i + \sum_{i=1}^{m} a_{mi} \theta_i^2
\]  

(4.11)

where \(m\) is the number of noise factors. In the example supported excavation studied, \(m = 6\), and thus the number of the unknown coefficients is \(2m+1 = 13\).

To determine the 13 coefficients \(\{a_0, a_1, a_2, \ldots, a_{12}\}\), a set of 13 \(y_j\) values is required. These \(y_j\) values may be obtained by 13 repeated analyses using DeepEx, each with a sampling point strategically placed. A commonly used scheme (for example, Xu and Low 2006) for strategic combinations of noise factor values is shown in Table 4.4.
Table 4.4 Different noise factor scenarios adopted for developing the response surface

<table>
<thead>
<tr>
<th>Noise factors scenario</th>
<th>$\phi'_{SM}$</th>
<th>$\phi'_{GR_1}$</th>
<th>$\phi'_{GR_2}$</th>
<th>$\phi'_{Rock}$</th>
<th>$c'_{Rock}$</th>
<th>$q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>2</td>
<td>$\mu+\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>3</td>
<td>$\mu-\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>4</td>
<td>$\mu$</td>
<td>$\mu+\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>5</td>
<td>$\mu$</td>
<td>$\mu-\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>6</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu+\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>7</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu-\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>8</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu+\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>9</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu-\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
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<tr>
<td>10</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu+\sigma$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>11</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu-\sigma$</td>
<td>$\mu$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>12</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu+\sigma$</td>
</tr>
<tr>
<td>13</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$\mu-\sigma$</td>
</tr>
</tbody>
</table>

Note: $\mu$ = mean of a noise factor
$\mu+\sigma$ = mean plus one standard deviation of a noise factor
$\mu-\sigma$ = mean minus one standard deviation of a noise factor

The first scenario is to assign the mean values for all 6 noise factors in DeepEx for a given design. Then, for each noise factor, two sampling points are taken at the mean ± one standard deviation of this noise factor while assigning the mean to each of the other 5 noise factors. The latter step yields 12 sampling points (scenarios) and thus a total of 13 scenarios are obtained. By repeating the DeepEx analysis for each of these noise factor scenarios, 13 sets of responses $\{y_j, j = 1, 3\}$ are obtained. Thus, for each of the three types of responses ($y_1$, $y_2$, or $y_3$), 13 output values are secured, which are then used to solve for the 13 coefficients $(a_0, a_1, a_2, \ldots, a_{12})$.

The process described previously yields a response surface for each $y_j$ for a given design $d$. As noted previously, there are 270 designs in the design pool that represent 270
combinations of the five design parameters (D, L, V, H, \( \alpha \)). To construct a general response surface that is effective for the robust design optimization, each of the 13 coefficients \( (a_0, a_1, a_2, \ldots, a_{12}) \) must be expressed as a function of the five design parameters (D, L, V, H, \( \alpha \)). To represent the whole design pool without going through each and every design, a quasi-response surface approach is taken herein, in which each design parameter in Table 4.3 is treated as a discrete random variable.

As shown in Table 4.5, 11 scenarios (combinations) of design parameters are strategically selected (sampled) to cover the extent of the whole design pool. One design scenario involves taking the middle value of the range of each of the five design parameters. Next, for each design parameter, two sampling points are taken at the lower and upper bounds of the range while the middle value is taken for each of the other 4 design parameters. The latter step yields \( 2n = 10 \) sampling points or scenarios (\( n \) is the number of design parameters; \( n = 5 \)). Thus, a total of \( 2n + 1 = 11 \) scenarios are obtained.

For each of the 11 scenarios of design parameters, the response surface approach described previously is used to determine the 13 coefficients \( (a_0, a_1, a_2, \ldots, a_{12}) \). This process is repeated for each of the 11 scenarios. Thus, for each type of response \( (y_1, y_2, \text{or } y_3) \), a matrix of coefficients \( [a_{k, l-1}]_{k=1,11; l=1,13} \) is obtained. The \( l^{th} \) column of this matrix is a vector; for example, the first column consists of \( 2n + 1 = 11 \) values of \( a_0 \), the second column consists of 11 values of \( a_1 \), and so on. The quasi-response surface is then established with the following second-order polynomial function:
To determine the $2n+1 = 11$ coefficients, $(b_0, b_1, b_2, ..., b_{11})$ in Eq. (4.12), 11 values of $a_{l-1}$ are required. Thus, for each of the 13 sets ($l = 1, 13$) of 11 values of $a_{l-1}$, the coefficients $(b_0, b_1, b_2, ..., b_{11})$ can be determined.

Table 4.5 Different scenarios for design parameters in the response surface

<table>
<thead>
<tr>
<th>Design parameters scenarios</th>
<th>Diameter of pile $(D)$</th>
<th>Length of pile $(L)$</th>
<th>Vertical spacing of tieback anchors $(V)$</th>
<th>Horizontal spacing of tieback anchors $(H)$</th>
<th>Installed angle of tieback anchors $(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>2</td>
<td>U</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
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<tr>
<td>3</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>4</td>
<td>M</td>
<td>U</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>5</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>6</td>
<td>M</td>
<td>M</td>
<td>U</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>7</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>U</td>
<td>M</td>
</tr>
<tr>
<td>9</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>10</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>U</td>
</tr>
<tr>
<td>11</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>L</td>
</tr>
</tbody>
</table>

Note: M, U, L = middle point, upper point, and lower point of the chosen range, respectively.

Table 4.6 lists the 13 sets of coefficients $(b_0, b_1, b_2, ..., b_{11})$, which is the building block of the general response surface.

The first set of the 11 coefficients $(b_0, b_1, b_2, ..., b_{11})$ in Table 4.6 can be used, along with the values of the five design parameters of a given design $d$, to determine the coefficient
$a_0$ based on Eq. (4.12); the second set of the 11 coefficients can be used to determine the coefficient $a_1$; and so on. Thus, the 13 sets of coefficients listed in Table 4.6 allow for determination of the 13 coefficients $(a_0, a_1, a_2, \ldots, a_{12})$, which in turn, can be used, along with the values of the six noise factors, to determine the response $y_j$ based on Eq. (4.11). The process to determine the response $y_j$ based on Eqs. (4.11) and (4.12) is referred to herein as the response surface model. Note that when applying the developed response model, all input parameters are entered with their nominal values, and no variation is considered. The variation in the noise factors, however, is explicitly considered in the robust design optimization process presented later.

In summary, Tables 4.6, 4.7, and 4.8 list the coefficients that are required for the determination of the maximum wall deflection ($\delta_{\text{max}}$), the factor of safety against push-in failure ($FS_{\text{push-in}}$), and the factor of safety against basal heave ($FS_{\text{basal}}$), respectively, using the response surface model described previously that involves Eqs. (4.11) and (4.12). It should be noted that for excavation in sandy and gravelly deposits, basal heave is generally not a concern.
Table 4.6 The coefficients required for the determination of the maximum wall deflection using the response surface model (Eqs. 4.11 and 4.12)

<table>
<thead>
<tr>
<th></th>
<th>b₀</th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>b₅</th>
<th>b₆</th>
<th>b₇</th>
<th>b₈</th>
<th>b₉</th>
<th>b₁₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>1262.908</td>
<td>-800.489</td>
<td>-115.192</td>
<td>-16.825</td>
<td>-2.011</td>
<td>111.185</td>
<td>512.107</td>
<td>3.055</td>
<td>6.903</td>
<td>0.067</td>
<td>-17.453</td>
</tr>
<tr>
<td>a₁</td>
<td>-10.356</td>
<td>-4.149</td>
<td>0.991</td>
<td>-0.056</td>
<td>-0.005</td>
<td>1.678</td>
<td>3.772</td>
<td>-0.027</td>
<td>0.010</td>
<td>0.000</td>
<td>-0.281</td>
</tr>
<tr>
<td>a₂</td>
<td>-21.212</td>
<td>26.815</td>
<td>1.911</td>
<td>0.236</td>
<td>-0.012</td>
<td>-4.916</td>
<td>-17.296</td>
<td>-0.051</td>
<td>-0.103</td>
<td>0.000</td>
<td>0.818</td>
</tr>
<tr>
<td>a₃</td>
<td>-22.374</td>
<td>11.182</td>
<td>2.177</td>
<td>0.637</td>
<td>0.040</td>
<td>-2.545</td>
<td>-7.455</td>
<td>-0.056</td>
<td>-0.227</td>
<td>-0.001</td>
<td>0.401</td>
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Table 4.7 The coefficients required for the determination of the factor of safety against push-in failure using the response surface model
(Eqs. 4.11 and 4.12)

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Table 4.8 The coefficients required for the determination of the factor of safety against basal failure using the response surface model
(Eqs. 4.11 and 4.12)

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</table>
Finally, it might be of interest to note that for the design of supported excavation at this site where the deposits mainly consist of sandy and gravelly layers (Figure 4.2), both the factor of safety against basal heave ($FS_{\text{basal}}$) and the factor of safety against push-in failure ($FS_{\text{push-in}}$) are much greater than the required minimum factors of safety (1.5 as per design codes such as JSA 1988, and PSCG 2000). In general, if the serviceability requirement (e.g., the maximum wall deflection is less than a limiting value, say, 0.7% of the final excavation depth) is satisfied, the safety requirements (in terms of $FS_{\text{basal}}$ and $FS_{\text{push-in}}$) will generally be satisfied.

**Validation of the response surface model**

It is important to check for the accuracy of the obtained response surface model. To validate this response surface model, 20 designs are randomly selected from the design pool, in which those designs that were used for the development of the response surface models are excluded. For each design, the values of the six noise factors are randomly selected from the assumed probability distributions. For each of these designs, the $FS$ against basal stability, the $FS$ against push-in failure, and the maximum wall deflection are computed with DeepEx. The results are then compared with those obtained from the developed response surface model. As shown in Figure 4.3, the accuracy of the response surface model is found satisfactory.

It should be noted that the response surface model is problem-specific, and a new response surface model may have to be developed for a given problem.
Figure 4.3 Validation of the proposed response surface models for: (a) factors of safety; (b) maximum wall deflection
Robust Geotechnical Design (RGD) of Supported Excavation

Simplified robust design optimization setting

The goal of RGD is to seek an optimal design that simultaneously considers safety, cost efficiency, and design robustness (Juang et al. 2013a). Specifically, RGD seeks an optimal design \( (d) \) in the design pool \( (S) \) such that the cost \( (C) \) is minimized and the design robustness is maximized (which is achieved herein by minimizing the weighted sensitivity index \( SI_w \)), while the safety requirements with respect to both stability (in terms of factor of safety against failures) and serviceability (in terms of maximum wall deflection) are satisfied. Thus, RGD is typically modeled as a multi-objective optimization problem. Previous studies (e.g., Juang et al. 2013a; Khoshnevisan et al. 2014) on robust designs in geotechnical engineering relied on multi-objective optimization algorithms such as NSGA-II ((Deb et al. 2002).

Due to the conflict between the objectives (to minimize cost and to maximize robustness), the multi-objective optimization yields a set of non-dominated designs that form a Pareto front (Deb 2001). Thus, an additional step is required to find the best compromised design by locating the knee point (Branke et al. 2004; Deb and Gupta 2011) on the Pareto front. To eliminate the need for NSGA-II, the multi-objective optimization may be solved through a series of single objective optimizations (Khoshnevisan et al. 2015). However, this approach still depends on an approximate Pareto front to locate the knee point (the best compromised design). To further simplify the process so that the RGD can be implemented in a single spreadsheet, the optimization setting shown in Figure 4.4 is adopted, in which the minimum distance (MD) approach is adopted for
optimization. The MD approach is based on the concept of “utopia” design (Chen et al. 1999).

$$\begin{align*}
\text{Find:} & \quad \mathbf{d} \text{ (design parameters)} \\
\text{Subject to:} & \quad \mathbf{d} \in S \text{ (design pool)} \\
& \quad \text{FS}_{\text{push-in}} > 1.5 \\
& \quad \text{FS}_{\text{basal}} > 1.5 \\
& \quad \delta_{\max} = f(\mathbf{d}, \mathbf{\theta}) < 0.7\% H_j \quad (H_j \text{ is the final excavation depth}) \\
\text{Objectives:} & \quad \text{min (distance from utopia)}
\end{align*}$$

Figure 4.4 New RGD optimization setting

Among all he designs that meet the safety requirements, the lowest and highest values of cost, denoted as $[C(d)]_{\text{min}}$ and $[C(d)]_{\text{max}}$, respectively, and the lowest and highest values of weighted sensitivity index, denoted as $[SL_w(d)]_{\text{min}}$ and $[SL_w(d)]_{\text{max}}$, respectively, can easily be obtained. As the cost and the design robustness (in terms of weighted sensitivity index) are conflicting objectives, it is impossible to obtain a design that simultaneously yields $[C(d)]_{\text{min}}$ and $[SL_w(d)]_{\text{min}}$. However, on a the 2-D plot of $C(d)$ versus $SL_w(d)$, a point $\{[C(d)]_{\text{min}}, [SL_w(d)]_{\text{min}}\}$ exists, which is termed utopia design (see Figure 4.5).
The main idea of the MD approach is that among all the designs that meet the safety requirements, the design $d_k$ that gives a minimum distance between itself, represented by a point $\{C(d_k), SI_w(d_k)\}$, and the utopia design, represented by $\{[C(d)]_{\text{min}}, [SI_w(d)]_{\text{min}}\}$, is deemed the best compromised design between the robustness and the cost. Theoretically, the computed distance from the utopia design to a candidate design may be interpreted as the additional “price”, in terms of a combination of design robustness and cost efficiency, the designer (or owner) has to pay to select this candidate design as the final design. The idea may be traced back to the marginal utility concept (Branke et al. 2004) or the compromise-programming concept (Chen et al. 1999).

Note that when computing the distance, it is necessary to normalize the objective functions $C(d_k)$ and $SI_w(d_k)$. Symbolically, the normalization is carried out as:

$$f_i^n(d) = \frac{f_i(d) - [f_i(d)]_{\text{min}}}{[f_i(d)]_{\text{max}} - [f_i(d)]_{\text{min}}} \quad \text{(4.16)}$$
where \([f_i(d)]_{\text{max}}\) and \([f_i(d)]_{\text{min}}\) are the maximum and minimum values, respectively, of the \(i^{th}\) objective function, \(f_i(d)\). In the design example investigated herein, \(i = 2\).

The MD-based optimization setting (Figure 4.4), along with the response surface model, greatly simplifies the RGD procedure and enables a practical implementation of the RGD procedure in a single Excel spreadsheet. The robust design process can be easily performed using the \textit{Solver} feature of Microsoft Excel that returns the minimum of a constrained nonlinear multivariable function.

\textit{Robust geotechnical design of supported excavation – the spreadsheet solution}

For each of the candidate designs in the selected design pool (see Table 4.3), the safety requirements, cost, and design robustness can be readily evaluated. A design that satisfies the safety requirements and has the minimum distance from the utopia point is selected as the most optimal design. For the soldier pile tieback anchors supported excavation system shown in Figure 4.2, the RGD process is implemented in a spreadsheet, as shown in Figure 4.6.
### Figure 4.6 Spreadsheet for the RGD optimization of the shoring system of the deep excavation project (\(SI_w\) adopted as the robustness measure)
In Figure 4.6, the input data and the final outcome are color-coded and shown in bold face. The final robust design (represented by design parameters: D, L, H, α, and V in Cells H5:H9) is searched (i.e., optimized) using the Solver feature of Excel by minimizing the distance between a given design (represented by its cost and $SI_w$) and the utopia design (represented by two objective functional values, minimum cost and minimum $SI_w$, each searched separately from the design space). Cells L14:M14 show the utopia point. The minimum distance is shown in Cell Q14. For a given design in the design space, the statistics of noise factors are entered in Cells C5:D10. The safety constraints are listed in Cells N4:N6. For the calculation of the weighted sensitivity index ($SI_w$), the gradient with respect to noise factors is computed first, as shown in Cells J5:J10. For the optimal design obtained using Solver, the cost and $SI_w$ are shown in Cell Q11 and Q12, respectively. The factors of safety and the maximum wall deflection of this optimal design are shown in Cells Q8:Q10. The final design is represented by the design parameters ($D = 0.5 \text{ m}, L = 19 \text{ m}, V = 3 \text{ m}, H = 2.5 \text{ m}, \alpha = 25^\circ$) shown in Cells H5:H9.

Further Discussion

The results of the optimal design obtained by the simplified RGD procedure are shown in Table 4.9. For comparison, three other designs, the first representing the least-cost design from the design pool, the second representing the most robust design from the design pool, and the third representing the original project design by TFEC (Hsii-Sheng Hsieh, personal communication 2014), are also shown in Table 4.9.
Table 4.9 Results of the three designs of the shoring system for the deep excavation project

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<th>Maximum wall deflection, $\delta_{\text{max}}$ (cm)</th>
<th>Cost (US$ \times 10^3$)</th>
<th>$SI_w$</th>
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<td>$D$ (m) $L$ (m) $V$ (m) $H$ (m) $\alpha$ (°)</td>
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<td>Most preferred design</td>
<td>0.5 16 3.0 2.5 25</td>
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<td>(optimal)</td>
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</tr>
<tr>
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</tbody>
</table>

Note: Horizontal interval of soldier pile (I) is fixed at 0.6 m for the original design; for the other three designs, $I = 2.0$ m. The original design does not consider the uncertainty and has no $SI_w$ value (N/A).
While all four designs listed in Table 4.9 meet the safety requirements ($F_{S_{basal}} > 1.5$, $F_{S_{push-in}} > 1.5$, and $\delta_{\max} < 0.7\%H_f = 8.33$ cm, as per PSCG 2000), each design has a different level of design robustness and cost efficiency. In this case, the least-cost design is also the least robust design, in which a slight variation in noise factors can result in highest variation in the system response (the maximum wall deflection). As the nominal value of the maximum wall deflection for the least cost design (7.82 cm) is already close to the maximum allowable wall deflection (8.33 cm), high variation in the predicted system response could lead to an unsafe design (meaning that the maximum wall deflection requirement is violated), which is not desirable. The most robust design from the selected design pool is also most costly and appears to be unnecessary. The most preferred or optimal design in the design pool is the best compromise design. It is noted that the original design by the project engineer is used as a reference for information only, as a direct comparison is difficult. Nevertheless, the original design, which is based on the design’s experience and not obtained through optimization, is the most costly of the four designs. Note that in the first three designs, the horizontal interval of the solider pile ($I$) is set to 2.0 m, while the value is set to 0.6 m in the original design. It is interesting to see the original design is roughly as conservative as the most robust design in the selected design tool, and the computed maximum wall deflection are approximately the same even two different software are used (DeepEx versus TORSA).

Finally, it should be of interest to compare the results of the RGD procedure using the $SI_w$ versus $SNR$ as a robustness measure. The RGD process is repeated using $SNR$ as a robustness measure, and for the same excavation problem, a spreadsheet is created, as
shown in Figure 4.7.

The best compromise (or optimal) design using $SNR$ yields $D = 0.5\, \text{m}$, $L = 19\, \text{m}$, $V = 3\, \text{m}$, $H = 2.5\, \text{m}$, $\alpha = 25^\circ$. The cost is $991,000$ and the maximum wall deflection is $6.6\, \text{cm}$. These results are practically the same as those shown in Table 4.9 for the most preferred design (the optimal design). This again confirms the validity of the weighted sensitivity index as a robustness measure, as $SNR$ has been widely used as a robustness measure in many engineering fields. The advantage of $SI_w$ as the robustness measure is its greater tolerance for imprecise statistical characterization of noise factors, while the advantage of $SNR$ as the robustness measure is its well-known, well-accepted status.
Figure 4.7 Spreadsheet for the RGD optimization of the shoring system of the deep excavation project (SNR adopted as the robustness measure)
Summary

In this chapter, a simplified robust geotechnical design (RGD) approach is proposed with multiple new features, such as the modified gradient-based robustness measure, the response surface surrogate model for evaluation of the system response, and the minimum distance (MD) algorithm that eliminates the need for multi-objective optimizations in the search for the optimal design. These new features allow for a more efficient and practical implementation of the RGD approach in a single spreadsheet, a practical engineering tool. The simplified RGD approach is demonstrated effective and efficient for the robust design of a soldier pile tieback anchors supported excavation, a sufficiently complex geotechnical engineering problem. In such robust design, the safety, cost efficiency and design robustness are explicitly and simultaneously considered.
Chapter 5
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The following conclusions are drawn from the results of the study on the robust design in geotechnical engineering presented in Chapter II:

1. Through the illustrated examples, the RGD methodology and its simplified version (i.e., simplified RGD method) are shown as an effective tool for the design of various geotechnical systems that considers safety, robustness, and cost simultaneously.

2. The weighted gradient-based robustness measure ($SI_w$) is shown to be effective, which reflects the relative variation in system response caused by the relative change in noise factors. Use of this gradient-based robustness measure enables an efficient implementation of the RGD methodology.

3. Knee point concept is shown as an effective tool to aid in making an informed design decision based on the Pareto front that has been established using multi-objective optimization algorithms such as NSGA-II. The newly developed minimum distance (MD) procedure is found to be effective for determining the knee point without performing multi-objective optimization. This is significant, as it greatly reduces the computational effort, enabling the simplified RGD method that employs the MD procedure as a practical geotechnical design tool.
The following conclusions are drawn from the results of the study on the efficient robust geotechnical design of drilled shafts in clay using spreadsheet presented in Chapter III:

1. The proposed efficient RGD approach is shown to be effective and efficient in producing the best compromise design with respect to both objectives of design robustness and cost efficiency, while satisfying all performance requirements.

2. The developed robustness measure, in terms of the variation of the performance function, is demonstrated to be effective, intuitive, and fundamentally sound. Higher variation of the performance function signals lower design robustness, which implies a higher degree of uncertainty as to whether the system can satisfy the pre-defined performance requirement.

3. Within the context of the proposed reliability-based RGD approach, the evaluation of design robustness and the evaluation of system performance requirement, two essential procedures in the robust design, share common computational steps, as both can be analyzed using MFOSM analysis. Thus, the computational efficiency is greatly improved over other existing reliability-based robust design approaches.

4. Although the proposed RGD is presented as a multi-objective optimization problem, it can be efficiently solved using a series of single-objective optimizations that does not require the more sophisticated genetic algorithms and programming skills. The Pareto front obtained using the proposed procedure implemented with Excel Solver is shown to be identical to the one obtained using the more sophisticated multi-objective optimization algorithm.
5. While the robust design using Approach 2 as shown in Figure 3.1 has been shown effective, it is possible that Approach 1, which allows for consideration of some reduction in the variation of noise factors within the framework of the robust design, can yield more cost-efficient designs while improving the design robustness.

6. As shown in Figure 3.6, the proposed RGD approach considers the noises from construction variation, loading conditions, model errors, and geotechnical parameters in the design of drilled shafts in clay. The proposed RGD approach is flexible in allowing use of either Approach 1 or Approach 2 or both for the most preferred design with respect to cost efficiency, design robustness, and performance requirements. In light of the recognition that the bearing capacity and settlement of a drilled shaft are strongly affected by the construction quality, the proposed RGD approach offers a practical and effective tool to examine possible alternatives.

The following conclusions are drawn from the results of the study on the practical robust geotechnical design of supported excavation using response surface presented in Chapter IV:

1. A new procedure for developing a response surface surrogate model to replace the outcome of a computer program is proposed. This new procedure, which considers the effects of noise factors and design parameters in two steps, is found effective. The surrogate model is found to be able to duplicate the response of the soldier pile tieback anchors supported excavation computed using DeepEx. This procedure is formulated in such a way that it can be implemented with different computer programs for the analysis of the system response.
2. The gradient-based robustness measure, in terms of weighted sensitivity index, is shown to be intuitive and effective, as it reflects the variation of the system response due to a relative variation in noise factors. The design with a lower sensitivity index yields a lower variation of the system response, and thus is more robust. The outcome of the RGD using the weighted sensitivity index, as the robustness measure is found practically identical to that obtained using the signal-to-noise ratio (SNR) as the robustness measure.

Recommendations for future research

1. Use of the response surface, as a surrogate model for complex numerical procedures (such as PLAXIS or DeepEx) within the RGD framework is shown effective in the supported excavation problem investigated. However, this approach should be further studied, by exploring different ways of constructing and validating the response surface and by assessing its applicability in other geotechnical problems.

2. The RGD approaches formulated in this dissertation seek designs that are optimized with respect to cost and robustness, while satisfying the safety requirements. The design robustness focuses on making the performance or response of the designed system insensitive to the variation in the noise factors. It should be worth exploring the optimization setting in which the robustness is defined based on insensitivity of cost with respect to the variation in the noise factor. Regardless of how the design robustness is measured, however, safety, cost, and robustness should be explicitly considered in a RGD approach.
3. The RGD approach may be applied to calibration of resistance factors in the Load and Resistance Factor Design (LRFD) method. In this problem, the objective would be to reduce the effect of the generally unknown or imprecisely characterized geotechnical parameters on the calibrated resistance factors. The existing LRFD codes often specify single resistance factors regardless of the levels of variation in the uncertain geotechnical parameters, and thus the design based on LRFD codes can be over-design or under-design, just like the traditional FS-based approaches. Considering the robustness in the calibration may offer resistance factors that can yield a robust design, albeit at higher costs. This issue appears to be worth investigating.
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