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Viscoelastic Damping of Hexagonal Honeycomb Sandwich Panels

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VISCOELASTIC DAMPING OF HEXAGONAL HONEYCOMB SANDWICH PANELS

A Thesis
Presented to
The Graduate School of
Clemson University

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science
Mechanical Engineering

By
Nikhil Kumar Seera
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Accepted by:
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ABSTRACT

Composite sandwich structures with a viscoelastic core material constrained between thin stiff face sheets are an effective means of damping in engineering applications. Damping introduces energy dissipation which helps control vibration amplitudes. Conventional Hexagonal honeycombs are often referred to as regular honeycombs and are defined by cellular geometry with effective positive Poisson’s ratio. Regular honeycombs are commonly used for the cores of sandwich plates because of their low density and high stiffness properties. Honeycombs with negative in-plane Poisson’s ratio are known as Auxetic honeycombs and offer enhancement of mechanical properties such as impact absorption, damage resistance, when compared to regular honeycombs.

In this study the modal vibration and damping capabilities of honeycomb sandwich plates with viscoelastic core are analyzed using a finite element model developed in ABAQUS. The viscoelastic material used for the base material of the honeycomb core is defined using a Prony series corresponding to the generalized Maxwell model. Damping loss factors are calculated from the ratio of energy dissipation over elastic strain energy for both a quasi-static analysis with a sinusoidal pressure load, and an implicit dynamic analysis with instantaneous pressure load. Additionally loss factors are calculated using a direct steady-state frequency response analysis, using half-power bandwidth method.
Comparisons are made between regular and two configurations of auxetic honeycomb. The first auxetic honeycomb (Auxetic-I) considered has the same extensional in-plane effective moduli as regular honeycomb. In the other auxetic honeycomb, (Auxetic-II), the mass is the same as that of regular honeycombs. In addition, comparisons are made between in-plane and out-of-plane loading.

Results showed that in the frequency domain, for both in-plane and out-of-plane loading honeycomb sandwich plates with both the Auxetic configurations show higher damping than the regular counterpart, and also shifts the natural frequencies to lower values. Results also show that for both regular and auxetic with in-plane loading display higher loss factors compared to out-of-plane loading. In the time domain, when a stiffer viscoelastic material is assigned to the core, Auxetic honeycomb showed higher loss factors compared to Regular. Whereas when softer viscoelastic moduli is defined, regular showed higher loss factors.
DEDICATION

This thesis is dedicated to my parents, Mr. S. Appala Naidu and Mrs. S. Aruna Kumari.
ACKNOWLEDGMENTS

I would like to convey my heartfelt regards to my advisor Dr. Lonny L. Thompson for putting me on this project and guiding me throughout my graduate career. I attribute my Master degree to his encouragement and effort without which this thesis would not have been completed. I would like to thank Dr. Paul F. Joseph, and Dr Jaehyung Ju for being on my advisory committee and for their valuable suggestions for improving my thesis.
# TABLE OF CONTENTS

ABSTRACT ................................................................................................................................. ii
DEDICATION .............................................................................................................................. iv
ACKNOWLEDGMENTS .............................................................................................................. v
TABLE OF CONTENTS .............................................................................................................. vi
LIST OF FIGURES ................................................................................................................ xii
Chapter 1 : INTRODUCTION ................................................................................................. 1
  1.1 Damping .......................................................................................................................... 1
  1.2 Damping in Sandwich Panels ....................................................................................... 3
  1.3 Honeycomb Sandwich Panels ...................................................................................... 5
  1.4 Research Motivation ..................................................................................................... 8
  1.5 Thesis Objective .......................................................................................................... 10
  1.6 Thesis Outline .............................................................................................................. 11
Chapter 2 : THEORY OF VISCOELASTICITY ...................................................................... 13
  2.1 Characteristics of Viscoelastic Materials ..................................................................... 13
  2.2 Boltzmann Superposition Integrals for Creep and Relaxation ................................... 16
  2.3 Maxwell Model ............................................................................................................. 18
  2.4 Time and Frequency Dependent Viscoelastic Properties ............................................ 20
Chapter 3 : GEOMETRIC AND EFFECTIVE MATERIALS PROPERTIES ......................... 22
  3.1 Base Materials .............................................................................................................. 22
  3.2 Effective Properties of Honeycomb cores .................................................................. 23
  3.3 Viscoelastic Properties of the Core Material ............................................................... 28
  3.4 Modeling of the Sandwich Plate .................................................................................. 31
    3.4.1 Estimation of Sandwich Plate geometry from the Unit Cell geometry ............... 32
    3.4.2 In-plane Loading model ......................................................................................... 34
    3.4.3 Out-of-Plane Loading model ................................................................................ 36
Chapter 4: FINITE ELEMENT MODELS

4.1 In-Plane Loading model ................................................................. 40
4.2 Out-of-Plane Loading model ............................................................ 41
4.3 Constraints ....................................................................................... 43
4.4 Boundary Conditions and Loads ......................................................... 45
4.5 Analysis Procedures .......................................................................... 47

Chapter 5: MODAL VIBRATION AND FREQUENCY RESPONSE OF THE SANDWICH PANELS ................................................................................. 48

5.1 Modal Vibration of Sandwich Panels ................................................... 48
  5.1.1 Results of In-Plane Loading model ................................................ 48
  5.1.2 Results of Out-of-Plane model ...................................................... 50
5.2 Frequency Response ........................................................................... 51
  5.2.1 Direct Steady State Analysis .......................................................... 52
5.3 Results of In-plane loading model ....................................................... 52
  5.3.1 Soft Polycarbonate material assigned to the core ............................. 52
  5.3.2 Stiff Polycarbonate material assigned to the core ............................ 55
5.4 Results of Out-of-plane loading model ............................................... 55
  5.4.1 Soft Polycarbonate ......................................................................... 55
  5.4.2 Stiff Polycarbonate material .......................................................... 58
5.5 Mesh Convergence ............................................................................ 58
5.6 Calculation of loss factor using Half–Power Bandwidth method ............. 60

Chapter 6: CALCULATION OF LOSS FACTORS OF SANDWICH PLATES USING QUASI STATIC ANALYSIS .............................................................................. 62

6.1 Quasi Static Analysis .......................................................................... 62
6.2 Loss Factors ....................................................................................... 63
  6.2.1 Hysteretic Energy Loss factor ......................................................... 64
  6.2.2 Quasi-static Loss factor ................................................................. 64
6.3 Loss factors calculated for the in-plane loading model .......................... 65
  6.3.1 Stiff Polycarbonate core ................................................................. 65
  6.3.1.2 NLGEOM ON ........................................................................... 69
  6.3.2 Soft Polycarbonate core ................................................................. 73
6.4 Loss factors calculated for the Out-of-plane loading model ..................... 77
  6.4.1 Stiff Polycarbonate core ................................................................. 77
  6.4.1.2 NLGEOM ON ........................................................................... 81
  6.4.2 Soft Polycarbonate core ................................................................. 85
Chapter 7: DYNAMIC BEHAVIOR OF SANDWICH PLATES

7.1 Implicit Dynamic Analysis ................................................................. 90
7.2 Dynamic Loss Factor .......................................................................... 91
  7.2.1 Results of in-plane loading model .................................................. 91
  7.2.2 Results of Out-of-plane loading model .......................................... 98

Chapter 8: CONCLUSION AND FUTURE WORK ................................. 105

8.1 Conclusive remarks ........................................................................... 105
8.2 Suggestions for Future work .............................................................. 108
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.1 Definition and Examples of the Hierarchical Building Blocks</td>
<td>7</td>
</tr>
<tr>
<td>Table 3.1 Elastic Material Properties of face sheets and core</td>
<td>22</td>
</tr>
<tr>
<td>Table 3.2 Material definition of Polycarbonate based on time dependent properties</td>
<td>23</td>
</tr>
<tr>
<td>Table 3.3 Effective properties of Honeycomb cores calculated using CMT formulas</td>
<td>27</td>
</tr>
<tr>
<td>Table 3.4 Three term Prony series coefficients obtained from curve fit</td>
<td>29</td>
</tr>
<tr>
<td>Table 3.5 Mass of the three configurations of in-plane loading sandwich plates</td>
<td>36</td>
</tr>
<tr>
<td>Table 3.6 Mass of the three configurations of three configurations of out-of-plane loading models</td>
<td>39</td>
</tr>
<tr>
<td>Table 5.1 First 10 modal frequencies of the in-plane models</td>
<td>49</td>
</tr>
<tr>
<td>Table 5.2 First 10 modal frequencies of the out-of-plane models</td>
<td>50</td>
</tr>
<tr>
<td>Table 5.3 Frequency Loss factor ($\eta_f$) of in-plane loading models</td>
<td>61</td>
</tr>
<tr>
<td>Table 5.4 Frequency Loss factor ($\eta_f$) of out–of–plane loading models</td>
<td>61</td>
</tr>
<tr>
<td>Table 6.1 Hysteretic loss factor ($\eta_{hys}$) of in-plane models</td>
<td>66</td>
</tr>
<tr>
<td>Table 6.2 Energy dissipated $D_q$</td>
<td>68</td>
</tr>
<tr>
<td>Table 6.3 Strain Energy generated $U_q$</td>
<td>69</td>
</tr>
<tr>
<td>Table 6.4 Qusai-static loss factor ($\eta_q$) of in-plane models</td>
<td>69</td>
</tr>
</tbody>
</table>
Table 6.5 Hysteretic loss factor ($\eta_{hys}$) of in-plane models ............................................. 70
Table 6.6 Energy dissipated $D_q$ ........................................................................................................... 72
Table 6.7 Strain Energy generated $U_q$ .................................................................................................. 73
Table 6.8 Quasi-static loss factor ($\eta_q$) of in-plane models ................................................................. 73
Table 6.9 Hysteretic loss factor ($\eta_{hys}$) of in-plane models ................................................................. 74
Table 6.10 Energy dissipated $D_q$ ........................................................................................................... 76
Table 6.11 Strain Energy generated $U_q$ .................................................................................................. 76
Table 6.12 Quasi-static loss factor ($\eta_q$) of in-plane models ................................................................. 77
Table 6.13 Hysteretic loss factor ($\eta_{hys}$) of out-of-plane models .......................................................... 78
Table 6.14 Dissipated energy $D_q$ ........................................................................................................... 80
Table 6.15 Strain energy generated $U_q$ .................................................................................................. 81
Table 6.16 Quasi-static loss factor ($\eta_q$) out-of-plane models .............................................................. 81
Table 6.17 Hysteretic loss factor ($\eta_{hys}$) out-of-plane models .............................................................. 82
Table 6.18 Dissipated Energy $D_q$ ........................................................................................................... 84
Table 6.19 Strain Energy generated $U_q$ .................................................................................................. 84
Table 6.20 Quasi-static loss factor ($\eta_q$) out-of-plane models .............................................................. 85
Table 6.21 Hysteretic loss factor ($\eta_{hys}$) out-of-plane models .............................................................. 86
Table 6.22 Dissipated Energy $D_q$ ........................................................................................................... 88
Table 6.23 Strain Energy generated $U_q$ .................................................................................................. 88
Table 6.24 Quasi-static loss factor ($\eta_q$) out-of-plane models .............................................................. 88
Table 7.1 Dissipated Energy $D_d$ ................................................................. 94
Table 7.2 Strain energy generated $U_d$ .......................................................... 94
Table 7.3 Dynamic Loss factor $\eta_d$ .............................................................. 94
Table 7.4 Dissipated Energy $D_d$ ................................................................. 97
Table 7.5 Strain energy generated $U_d$ .......................................................... 97
Table 7.6 Dynamic Loss factor $\eta_d$ .............................................................. 97
Table 7.7 Dissipated energy $D_d$ ................................................................. 100
Table 7.8 Strain energy generated $U_d$ .......................................................... 100
Table 7.9 Dynamic Loss factor $\eta_d$ .............................................................. 100
Table 7.10 Dissipated Energy $D_d$ ................................................................. 103
Table 7.11 Strain energy generated $U_d$ .......................................................... 103
Table 7.12 Dynamic Loss factor $\eta_d$ .............................................................. 103

Table 8.1 Loss factors for three configurations of honeycombs with soft polycarbonate core ................................................................. 106
Table 8.2 Loss factors for three configurations of honeycombs with stiff polycarbonate core ................................................................. 107
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1 Hierarchical Building blocks of Macro structure of composite sandwich plate</td>
<td>6</td>
</tr>
<tr>
<td>Figure 1.2 Regular and Auxetic Unit cells</td>
<td>9</td>
</tr>
<tr>
<td>Figure 2.1 Creep strain at various constant stresses [31]</td>
<td>14</td>
</tr>
<tr>
<td>Figure 2.2 Relaxation response for constant strains [31]</td>
<td>14</td>
</tr>
<tr>
<td>Figure 2.3 Maxwell unit showing spring and dashpot [31]</td>
<td>18</td>
</tr>
<tr>
<td>Figure 2.4 Generalized Maxwell model [32]</td>
<td>19</td>
</tr>
<tr>
<td>Figure 3.1 Unit cells of Regular and Auxetic Honeycombs showing geometric parameters</td>
<td>24</td>
</tr>
<tr>
<td>Figure 3.2 Normalized Relaxation modulus $G_R/G_0$ for polycarbonate</td>
<td>29</td>
</tr>
<tr>
<td>Figure 3.3 Normalized Loss modulus $G_L/G_0$ as a function of frequency</td>
<td>30</td>
</tr>
<tr>
<td>Figure 3.4 Normalized Storage modulus $G_S/G_0$ as a function of frequency</td>
<td>30</td>
</tr>
<tr>
<td>Figure 3.5 Damping loss factor $G_L/G_S$ as a function of frequency</td>
<td>31</td>
</tr>
<tr>
<td>Figure 3.6 Hexagonal honeycomb core showing directions of loading [19]</td>
<td>32</td>
</tr>
<tr>
<td>Figure 3.7 Horizontal and Vertical dimensions of Regular and Auxetic Unit cells</td>
<td>33</td>
</tr>
<tr>
<td>Figure 3.8 Overall dimensions of Regular &amp; Auxetic In-plane loading models</td>
<td>35</td>
</tr>
</tbody>
</table>
Figure 3.9 Completely assembled Regular & Auxetic in-plane loading models .......................................................................................................................... 36

Figure 3.10 Overall dimensions of Regular & Auxetic Out-of-plane loading models .................................................................................................................. 37

Figure 3.11 Section view of completely assembled Regular & Auxetic out-of-plane loading models ...................................................................................................... 38

Figure 4.1 Meshed Face sheet .................................................................................................................................................................................................... 42

Figure 4.2 Meshed Regular and Auxetic Honeycomb cores ................................................................................................................................................. 42

Figure 4.3 Figure showing tie constraints of in-plane loading model ................................................................................................................................................. 44

Figure 4.4 Figure showing tie constraints of out-of-plane loading model ................................................................................................................................................. 44

Figure 4.5 Regions on which boundary conditions are applied in in-plane loading model .............................................................................................................. 45

Figure 4.6 Regions on which boundary conditions are applied in out-of-plane loading model ..................................................................................................... 46

Figure 4.7 Loading scenario in in-plane loading model ............................................................................................................................................................................. 46

Figure 4.8 Loading scenario in out-of-plane loading model ............................................................................................................................................................... 47

Figure 5.1 First 10 mode shapes of in-plane model with regular honeycomb core ................................................................................................................................................. 50

Figure 5.2 First 10 mode shapes of in-plane model with regular honeycomb core ................................................................................................................................................. 51

Figure 5.3 Frequency response of in-plane model with regular honeycomb core ................................................................................................................................................. 53

Figure 5.4 Frequency response of in-plane model with Auxetic -I honeycomb core ................................................................................................................................................. 53

Figure 5.5 Frequency response of in-plane model with Auxetic -II honeycomb core ................................................................................................................................................. 54
Figure 5.6 Comparison of frequency response of Regular, Auxetic –I, Auxetic –II for the first mode ................................................................. 54

Figure 5.7 Comparison of frequency response of Regular, Auxetic –I, Auxetic –II for the first mode ................................................................. 55

Figure 5.8 Frequency response of out-of-plane model with regular honeycomb core ...................................................................................... 56

Figure 5.9 Frequency response of out-of-plane model with Auxetic -I honeycomb core ............................................................................. 56

Figure 5.10 Frequency response of out-of-plane model with Auxetic -II honeycomb core ............................................................................. 57

Figure 5.11 Comparison of frequency response of Regular, Auxetic –I, Auxetic –II .................................................................................. 57

Figure 5.12 Comparison of frequency response of Regular, Auxetic –I, Auxetic –II for the first mode ................................................................. 58

Figure 5.13 Comparison of frequency response for a finer mesh, in-plane model ......................................................................................... 59

Figure 5.14 Comparison of frequency response for a finer mesh, out-of-plane model ............................................................................... 59

Figure 5.15 Half Power Bandwidth method .............................................................................................................................. 60

Figure 6.1 Sinusoidal amplitude of load applied during Quasi-static analysis ................................................................................................. 63

Figure 6.2 Hysteresis curves of three configurations of in-plane loading models .......................................................................................... 66

Figure 6.3 ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................................................. 67

Figure 6.4 ALLSE plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................................................. 68

Figure 6.5 Hysteresis curves of three configurations of in-plane loading models .......................................................................................... 70
Figure 6.6 ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................. 71

Figure 6.7 ALLSE plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................. 72

Figure 6.8 Hysteresis curves of three configurations of in-plane loading models ............................................................ 74

Figure 6.9 ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................. 75

Figure 6.10 ALLSE plots of Regular, Auxetic-I, Auxetic-II in-plane models ................................................................. 76

Figure 6.11 Hysteresis curves of three configurations of out-of-plane loading models ....................................................... 78

Figure 6.12 ALLCD plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 79

Figure 6.13 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 80

Figure 6.14 Hysteresis curves of three configurations of out-of-plane loading models ....................................................... 82

Figure 6.15 ALLCD plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 83

Figure 6.16 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 83

Figure 6.17 Hysteresis curves of three configurations of out-of-plane loading models ....................................................... 86

Figure 6.18 ALLCD plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 87

Figure 6.19 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models ............................................................. 87
Figure 7.1 Comparison of displacement of a node at the center of face sheet for the three configurations of in-plane model with long term viscoelastic core ................................................................. 92

Figure 7.2 ALLCD comparison of the three configurations of sandwich plates .................................................................................................................. 93

Figure 7.3 ALLSE comparison of the three configurations of sandwich plates .................................................................................................................. 93

Figure 7.4 Comparison of displacement of a node at the center of face sheet for the three configurations of in-plane models with instantaneous viscoelastic core .................................................................................................................. 95

Figure 7.5 ALLCD comparison of the three configurations of sandwich plates .................................................................................................................. 96

Figure 7.6 ALLSE comparison of the three configurations of sandwich plates .................................................................................................................. 96

Figure 7.7 Comparison of displacement of a node at the center of face sheet for the three configurations of sandwich plates ........................................ 98

Figure 7.8 ALLCD comparison of the three configurations of sandwich plates .................................................................................................................. 99

Figure 7.9 ALLSE comparison of the three configurations of sandwich plates .................................................................................................................. 99

Figure 7.10 Comparison of displacement of a node at the center of face sheet for the three configurations of sandwich plates ............................ 101

Figure 7.11 ALLCD comparison of the three configurations of sandwich plates .................................................................................................................. 102

Figure 7.12 ALLSE comparison of the three configurations of sandwich plates .................................................................................................................. 102
CHAPTER 1 : INTRODUCTION

1.1 Damping

Flexural Vibration control or damping of composite laminates and sandwich panels is essential in engineering applications. The information on structure-sensitive material property damping is required to solve several problems in structural mechanics. Damping not only helps in noise or vibration control but also plays a major role in reducing structural fatigue and equipment malfunction [1]. Higher damping reduces the amplitude of vibrations at resonance of the structure thereby improving the performance of a structure subjected to dynamic loading. It also results in faster decay of free vibrations, reduced dynamic stress levels [2].

Damping can be defined in various ways. It is often associated with the amount of energy dissipated. Lazan [3] referred the term mechanical damping to be the energy dissipated within a material or a structure subjected to cyclic stress. Rao [4] referred damping to be the process of extracting mechanical energy from a vibrating system usually by conversion into heat or other form of energy. Kareem [5], defined damping capacity of a system to be the ratio of the amount of energy dissipated in one cycle of oscillation to the maximum amount of energy gained by the system during the cycle.

There are two types of damping, material damping and structural damping [3, 4]. Material damping is the inherent damping property within a material and it does not depend on the shape, stress distribution or volume of the system or structure [3]. Structural damping includes energy dissipation at supports, boundaries, joints in addition
to the material damping. The damping energies dissipated in the case of material and structural damping can be defined as

For a material

\[ D = \int \sigma d \varepsilon \]  

(1.1)

For a structure

\[ D_s = \int P dX \]  

(1.2)

where \( D \) is the unit damping energy dissipated by a macroscopically uniform material per unit volume of material per cycle of loading (in-lb/in\(^3\)-cycle), \( D_s \) is the total damping energy dissipated per cycle by an entire member or structural assembly. \( \sigma \) and \( \varepsilon \) are the unit stress and strain in the material. \( P \) is the load with which the structure is subjected to a deflection of \( X \) [3]. The material and structure damping can be related to each other as

\[ D_s = \int_{V_s}^{V} D dV \]  

(1.3)

where \( V \) is the volume of the structural assembly subjected to stress less than \( \sigma \) and \( V_s \) is the total effective volume of specimen or part contributing to the damping energy dissipation.

The current study focuses on determining the amount of structural damping of sandwich plates with same base material but different geometric configurations. Damping of a material can be measured using a loss factor parameter defined by [2,3,6]

\[ \eta = \frac{D}{2\pi U} = \frac{E''}{E'} \]  

(1.4).

\( E' \) and \( E'' \) are the real and imaginary parts of a complex modulus respectively. Also \( \eta = 2\xi \), where \( \xi \) is viscous damping ratio. The \( D \) and \( U \) can be replaced with \( D_s \) and \( U_s \) to obtain loss factor \( \eta_s \) of the structural member. In the current study loss factor \( \eta_s \) and
1.2 Damping in Sandwich Panels

The term sandwich panel refers to a structure with two thin stiff face sheets bonded to a thick low density core. This kind of arrangement enables the sandwich panels to have increased moment of inertia at lower weights. With this attribute sandwich panels found numerous applications in automotive and aircraft structures where high flexural stiffness to weight ratio is required. Another advantage of sandwich panels is that different configurations of sandwich panels can be generated for different applications just by changing the material or geometric properties of the face sheets and the core.

Damping can be achieved from sandwich panels by incorporating a viscoelastic material as the core material. This is referred to as constrained layer damping [1]. Viscoelastic materials have high energy dissipation property and their usage in constrained layer damping results in suppressing noise and vibrations [7]. The properties of viscoelastic materials will be discussed in detail in Chapter 2.

The use of viscoelastic materials for damping applications dates back to 1950s. Oberst [8] first proposed applying thin viscoelastic material to flexural members for vibration control. Kerwin [1] introduced the usage of constrained layer viscoelastic damping. He proposed that the shear motion produced by constraining the outer surface of viscoelastic layer with a stiff material results in greater dissipation of energy, in other words greater amount of damping. The major limitation of his study is the two face sheets sandwiching the viscoelastic core cannot have the same thickness. Since the introduction
of this basic concept many developments were made by others to improve the damping performance. The ability of constraining layers to induce shear into the viscoelastic core without themselves experiencing much of the shear is important for damping.

Di Taranto [9] derived a sixth order differential equation of motion in terms of longitudinal displacement based on complex shear modulus of viscoelastic material to study the viscoelastic behavior of the sandwich panels. Ungar [10] suggested a multiple constrained layer approach of alternate viscoelastic and elastic layers for an improved damping. He also derived expressions for loss factors of sandwich beams. Based on his work two conclusions were drawn, for a higher system damping the constraining layers should be thinner than the viscoelastic layer and the loss factor will have a maximum value when three layered sandwich panel is symmetric about neutral axis. Ungar and Kerwin [11] reexamined the definition of loss factor, ratio of energy dissipated per cycle to the total energy associated with vibration for viscoelastic systems. Based on this work Johnson and Kienholz [12] develop a method using finite element analysis to predict the amount of damping in structures with constrained viscoelastic layers.

Mead and Markus done similar work [13] of deriving loss factors of encastre sandwich beams for different boundary conditions other than simply supported boundary condition. They showed that the maximum values of damping are not sensitive to boundary conditions, only the frequency at which they occur shifts based on the boundary condition used. Plunkett and Lee [14] proposed that the amount of damping also depends on the effective length of constrained layer apart from its stiffness. They have also determined the optimal length of constrained layer to produce higher damping.
In the present days Birman [6] developed methodologies for prediction of damping in sandwich structures using two approaches. One approach is based on the analyses of free vibrations and the other is based on mechanics of materials. Wang and Werely [15] developed a spectral finite element analysis model in frequency domain to study the dynamic behavior of sandwich beams with viscoelastic cores. Amichi and Atalla [16] proposed a new finite element for sandwich beams with viscoelastic core which allows for symmetrical and unsymmetrical configurations.

1.3 Honeycomb Sandwich Panels

Sandwich panels with honeycomb cores are being used widely in aerospace and building industry where weight sensitive structures with high flexural rigidity are required. Honeycomb sandwich panels offer good compromise between stiffness and lightness. Conventional hexagonal honeycombs are commonly used for the cores of the sandwich plates in structural application. Various other honeycomb core geometries, square, rectangular, triangle diamond, circular are suggested for multifunctional applications where just stiffness or strength are not enough but other mechanical properties like thermal conductivity, heat convection etc are also considered to be essential. They can be even manufactured from a wide range of materials such as aluminum, titanium, fiber reinforced plastics or even resin impregnated paper [17, 18].

The construction of honeycomb from the micro level of the base materials to the macro structure of the composite sandwich plate can be described by the hierarchy shown in Figure 1.1. The core of the sandwich structure can be considered the Meta structure. Therefore by modifying the intermediate Meso properties the behavior of the Meta
structure can be controlled to give different configurations which can be used for a wide range of applications. The Meso middle level scale is defined by the unit cell geometric properties of the cellular honeycomb core structure.

![Figure 1.1 Hierarchical Building blocks of Macro structure of composite sandwich plate](image)

Table 1.1 Definition and Examples of the Hierarchical Building Blocksshowss the definitions and examples of the terms mentioned in Figure 1.1 Hierarchical Building blocks of Macro structure of composite sandwich plate in the perspective of honeycomb cellular meso structures.
The mechanical behavior of the honeycomb cellular meso structures has received significant attention due to the variety of benefits it offers. The effective properties of honeycomb cores are studied for the analysis and design of honeycomb sandwich panels. The effective properties of hexagonal honeycombs were derived by Gibson and Ashby, considering the honeycomb cells as linear elastic beams [19]. This study is named as Cellular Material Theory (CMT). Grediac [20] used finite element methods to calculate the transverse shear of honeycomb cores. Noor [18] predicted the free vibration response of infinitely long and rectangular honeycomb sandwich panels using finite element models. Nilsson and Nilsson [21] derived dynamic properties of honeycomb sandwich panels using Hamilton’s principle. Wang and Yang [22] carried out an experimental investigation to determine damping in honeycomb sandwich panels. Jung and Aref [7] combined honeycomb and solid viscoelastic material citing honeycomb structure will

<table>
<thead>
<tr>
<th>Hierarchical Level</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro</td>
<td>Material properties of base material assigned to honeycombs</td>
<td>Base material’s density, elastic modulus, Poisson’s ratio etc</td>
</tr>
<tr>
<td>Meso</td>
<td>Intermediate level parameters that define the honeycomb unit cell geometry</td>
<td>Cell wall height, length, thickness, cell angle etc</td>
</tr>
<tr>
<td>Meta</td>
<td>Effective properties corresponding the honeycomb core</td>
<td>Effective moduli, effective density, effective Poisson’s ratio etc</td>
</tr>
<tr>
<td>Macro</td>
<td>Properties of the sandwich panel</td>
<td>Effective in-plane, out-of-plane behavior of the panel</td>
</tr>
</tbody>
</table>

Table 1.1 Definition and Examples of the Hierarchical Building Blocks
enhance the stiffness of the entire structure and the viscoelastic material will provide energy dissipation when subjected to in-plane shear loading.

In general, honeycombs are stiffer or stronger in the out-of-plane direction considering that the cell walls take only axial loads. They are prone to bending failures or buckling failures when loaded in the in-plane direction [17, 19]. For this reason the in-plane applications of honeycomb sandwich panels are very limited.

1.4 Research Motivation

Conventional Hexagonal honeycombs referred to as Regular honeycombs hereafter in this study are most commonly used as the core for honeycomb sandwich panels. Regular Honeycombs have positive in-plane Poisson’s ratio. A new configuration of honeycomb meso structure with negative in-plane Poisson’s ratio, known as Auxetic honeycomb showed to have better mechanical properties than the regular counterparts.

The term Auxetic is first introduced by Evans [23], pointing out that regular honeycomb when bent in out-of-plane produces an anticlastic or saddle-shaped curvature due to their positive effective in-plane Poisson’s ratio, whereas the auxetic ones with negative Poisson’s ratio produces synclastic or domed curvatures. This behavior is extremely useful in manufacturing curved sandwich shells.
Auxetic honeycombs when pulled in one-direction expand in the transverse direction leading to a stiffening geometric effect. The negative Poisson’s ratio also leads to the enhancement of several properties such as impact absorption, damage resistance and tolerance, shear modulus and indentation resistance. They are also being used in structural acoustics due to their low-cut off frequency. [24, 25].

Figure 1.2 shows the difference in unit cells of regular and auxetic honeycombs.

![Figure 1.2 Regular and Auxetic Unit cells](image)

Scarpa, Tomilson [24] studied the vibration characteristics of sandwich plates with auxetic honeycomb core. Ruzzene, Scarpa [25] studied the wave propagation characteristics of sandwich plates with honeycomb core. Both the works suggested that proper selection of auxetic honeycomb’s cell angle helps in developing a sandwich panel with better static and dynamic characteristics.

Ju has done a significant study on auxetic honeycombs. In [26] he stated that auxetic honeycombs have lower geometric non linearity and higher shear flexibility compared to regular counter parts. In [27] he implied that auxetic honeycombs are candidates for shear flexure design considering their low effective shear moduli and higher maximum effective strain compared to the regular honeycombs. In [28] he equated
the shear modulus of auxetic and regular honeycombs using cellular material theory (CMT) and found that auxetic honeycombs showed a low cyclic energy loss under shear loading. In this study he also developed a viscoelastic honeycomb model by combining Prony series of the generalized Maxwell model with CMT.

The damping capability of auxetic honeycombs is never investigated. Taking into account the benefits auxetic honeycombs offer compared to the regular counterparts, if they can even provide better damping capability, they can take over the place of regular honeycombs in the multifunctional applications where high shear flexibility is needed along with vibration control.

1.5 Thesis Objective

The main goal of this study is to study the vibration and damping capabilities of honeycomb sandwich plates with viscoelastic cores. The effect of cellular geometries of hexagonal honeycomb meso structures on the damping properties of sandwich panels is investigated.

Sandwich panels made of polycarbonate regular honeycomb core and aluminum face sheets are compared against two different configurations of auxetic honeycomb sandwich panels made of same base materials. The first auxetic configuration considered has same extensional in-plane effective moduli as regular honeycomb. The second auxetic honeycomb has same mass and out-of-plane elastic modulus as that of regular honeycomb. In other words for the same base material, the structural damping of the honeycomb sandwich panels for three different honeycomb meso structures is being investigated.
Also, it was mentioned that honeycomb behaves differently in in-plane and out-of-plane loadings. Therefore the effect of direction of loading on the damping capability of the sandwich panels is also studied.

1.6 Thesis Outline

Chapter 1 discusses about the importance of damping in structural mechanics, how previous researchers achieved damping or vibration control by assigning viscoelastic material and honeycomb cellular meso structures to the cores of sandwich panels, the benefits of hexagonal honeycomb cells with negative in-plane Poisson’s ratio over the conventional hexagonal honeycomb cells with positive in-plane Poisson’s ratio which motivated this study and the objective of the current study.

Chapter 2 gives a brief overview on viscoelastic materials, their characteristics, how their behavior can be related to simple mechanical spring-dash pot models like the Maxwell model, the equivalence of Maxwell model with Prony series and the time and frequency dependent viscoelastic properties.

Chapter 3 discusses how the effective material and geometric properties used in this study are calculated using CMT, how to define a Maxwell generalized prony series model in ABAQUS using uni–axial shear stress data. It also talks about the steps followed in developing the two-dimensional in-plane model and three-dimensional out-of-plane model used in this study.

Chapter 4 discusses the finite element details of the models used in this study. The details include mesh, constraints, analysis procedures, boundary conditions and loads.
Chapter 5 discusses about the undamped modal frequencies, damped and undamped frequency response of the three configurations of honeycomb sandwich panels and the procedure applied to calculate loss factors from the frequency response data.

Chapter 6 tells about how the quasi static analysis step set up in ABAQUS for this study, the procedure followed to obtain the loss factors based on the results from the quasi static analysis.

Chapter 7 tells about how the implicit dynamic analysis step is set up in ABAQUS for this study, the procedure followed to obtain the dynamic loss factor based on the results from the implicit dynamic analysis.

The results are summarized and conclusions of the study were made in the Chapter 8. Suggestions for future work are also mentioned in this chapter.
CHAPTER 2 : THEORY OF VISCOELASTICITY

Viscoelastic materials are the materials which exhibit characteristics of both viscous fluids and elastic solids. Ideal Hookean elastic solids can only store energy under loading, whereas ideal Newtonian fluids can only dissipate energy under non hydrostatic stresses. Viscoelastic materials on the other hand are capable of both storage and dissipation of energy when subjected to loads. The linear relationship between stress and strain of a linear elastic solid is independent of time, but for a viscoelastic material the linearity depends on the time history of the input [29]. Many polymeric materials like rubbers, plastics, acrylics, silicones, vinyl’s, adhesives having long-chain molecules exhibit viscoelastic behavior. The material properties of viscoelastic materials depend on several factors like environmental temperature, vibration frequency, pre-load, dynamic load, environment humidity and so on [4].

2.1 Characteristics of Viscoelastic Materials

A viscoelastic member when subjected to oscillatory loading, the resulting stress strain curve is called hysteresis loop. The area enclosed by the hysteresis loop is a measure of the damping or dissipation of energy in the member [29]. A viscoelastic member under a constant stress undergoes an increased deformation until an asymptotic level of strain is reached. This phenomenon is called creep [30]. The creep behavior at various stress level is as shown in Figure 2.1
When a viscoelastic material is subjected to constant strains, the resulting stress will exhibit time-dependent stress relaxation as shown in Figure 2.2.
A viscoelastic structure when subjected to a sinusoidally varying stress the resulting strain is also sinusoidal of same frequency, but the phase lags by an angle of $\delta$ [31]. The stress and strain functions can be written as

$$\varepsilon = \varepsilon_0 \cos \omega t$$
$$\sigma = \sigma_0 \cos(\omega t + \delta)$$  \hspace{1cm} (2.1)

The stress function can be written in a complex form whose real part is in phase and imaginary part is 90° out of phase with the strain as shown

$$\sigma^* = \sigma'_0 \cos \omega t + i \sigma''_0 \sin \omega t$$  \hspace{1cm} (2.2)

From the above equation the following can be written

$$|\sigma^*| = \sigma_0 = \sqrt{(\sigma'_0)^2 + (\sigma''_0)^2}$$
$$\tan \delta = \frac{\sigma''_0}{\sigma'_0}$$  \hspace{1cm} (2.3)

Two different elastic moduli can be defined using the above complex stress function. One of the moduli is called the “real” or “storage” modulus ($E'$) which is the ratio of the in-phase stress to the strain

$$E' = \frac{\sigma'_0}{\varepsilon_0}$$  \hspace{1cm} (2.4)

The second one is called “imaginary” or “loss” modulus ($E''$)

$$E'' = \frac{\sigma''_0}{\varepsilon_0}$$  \hspace{1cm} (2.5)
2.2 Boltzmann Superposition Integrals for Creep and Relaxation

According to Boltzmann Superposition principle the response of a non aging linear viscoelastic material at a constant temperature at any time \( t \) due to an input at time \( t=\tau \) is a function of the input and the elapsed time \( (t-\tau) \) only [29]. If the material is subjected to strains \( \Delta \epsilon_1, \Delta \epsilon_2, \) and \( \Delta \epsilon_3 \) at times \( \tau_1, \tau_2, \) and \( \tau_3 \) then according to Boltzmann superposition principle the stress function can be written as

\[
\sigma(t) = \Delta \epsilon_1 C(t - \tau_1) + \Delta \epsilon_2 C(t - \tau_2) + \Delta \epsilon_3 C(t - \tau_3) \tag{2.6}
\]

where \( C(t) \) is the relaxation modulus, which is zero for \( t<0 \). For infinite strains applied the stress response can be generalized using Boltzmann Superposition integral as

\[
\sigma(t) = \int_{-\infty}^{t} C(t - \tau) \frac{d\epsilon(t)}{d\tau} d\tau \tag{2.7}
\]

A similar expression can be developed for the strain response for infinite amount of stresses applied,

\[
\epsilon(t) = \int_{-\infty}^{t} S(t - \tau) \frac{d\sigma(t)}{d\tau} d\tau \tag{2.8}
\]

where \( S(t) \) is the creep compliance, which is zero at \( t < 0 \).

The Boltzmann superposition integral can be converted into an ordinary differential equation using Laplace transforms. These ODE’s can be used to interpret physical models for viscoelastic behavior. The Laplace transform of both sides of equation (2.7) gives,

\[
\mathcal{L}\left[\sigma(t)\right] = \bar{\sigma}(s) = \mathcal{L}\left[ \int_{-\infty}^{t} C(t - \tau) \frac{d\epsilon(t)}{d\tau} d\tau \right] \tag{2.9}
\]
Using convolution integral the right hand side of the equation (2.9) can be written as

\[ \mathcal{L} \left[ \int_{-\infty}^{t} C(t-\tau) \frac{d\epsilon(t)}{d\tau} d\tau \right] = \bar{C}(s) \frac{d\bar{\epsilon}(s)}{d\tau} \]  \hspace{1cm} (2.10)

Inverse Laplace transform of above equation gives

\[ \left[ \int_{-\infty}^{t} C(t-\tau) \frac{d\epsilon(t)}{d\tau} d\tau \right] = \mathcal{L}^{-1} \left[ \bar{C}(s) \frac{d\bar{\epsilon}(s)}{d\tau} \right] \]  \hspace{1cm} (2.11)

Using the above equation, equation (2.9) can be written as

\[ \bar{\sigma}(s) = \mathcal{L} \left[ \mathcal{L}^{-1} \left( \bar{C}(s) \frac{d\bar{\epsilon}(s)}{d\tau} \right) \right] = \left( \bar{C}(s) \frac{d\bar{\epsilon}(s)}{d\tau} \right) \]  \hspace{1cm} (2.12)

Applying Laplace transforms of derivatives and neglecting initial conditions, the above equation can be written as

\[ \bar{\sigma}(s) = s\bar{C}(s)\bar{\epsilon}(s) \]  \hspace{1cm} (2.13)

Similarly, \( \bar{\epsilon}(s) = s\bar{S}(s)\bar{\sigma}(s) \)

The Relaxation modulus and creep compliance can be related to each other as,

\[ \bar{S}(s)\bar{C}(s) = \frac{1}{s^2} \]  \hspace{1cm} (2.14)

Equation (2.13) can be written as a ratio of two polynomials using the Laplace parameter \( s \) as:

\[ \bar{\sigma}(s) = \frac{s\bar{C}(s)}{\bar{S}(s)}\bar{\epsilon}(s) = \frac{N(s)}{M(s)}\bar{\epsilon}(s) \]

where \( M(s) = a_0 + a_1s + a_2s^2 + \ldots + a_ns^n \)

\( N(s) = b_0 + b_1s + b_2s^2 + \ldots + b_ns^n \)

\[ \therefore M(s)\bar{\sigma}(s) = N(s)\bar{\epsilon}(s) \]  \hspace{1cm} (2.15)
Using inverse Laplace transform of the \( n \)th derivative of a function and neglecting initial conditions above Laplace equation can be written as an ordinary differential equation as,

\[
a_n \frac{d^n \sigma}{dt^n} + a_{n-2} \frac{d^{n-2} \sigma}{dt^{n-2}} + \ldots + a_1 \frac{d \sigma}{dt} + a_0 \sigma = b_n \frac{d^n \varepsilon}{dt^n} + \ldots + b_2 \frac{d^2 \varepsilon}{dt^2} + b_1 \frac{d \varepsilon}{dt} + b_0 \varepsilon \quad (2.16)
\]

Using this ODE several physical models of linear viscoelastic behavior can be explained [29].

### 2.3 Maxwell Model

A convenient way of describing the time dependent viscoelastic response is to use the spring-dashpot models. These mechanical analogues are constructed from simple elements such as “Hookean” springs and “Newtonian” dashpots. The spring of modulus \( k \) is assumed to follow Hooke’s Law \( \sigma = k \varepsilon \) and the dashpot is assumed to be filled with a Newtonian fluid of viscosity \( \eta \) and the corresponding equation that governs the dashpot is \( \sigma = \eta \frac{d \varepsilon}{dt} \) [29].

The Maxwell model is a simple spring-dashpot model consisting of a spring and a dashpot in series, as shown in Figure 2.3

![Figure 2.3 Maxwell unit showing spring and dashpot](image)

The total strain across the element is a summation of strains in the spring and the dashpot,

\[
\varepsilon = \varepsilon_s + \varepsilon_d \quad (2.17)
\]

The strain rate across the model is then
\[
\dot{\varepsilon} = \dot{\varepsilon}_s + \dot{\varepsilon}_d = \frac{\dot{\sigma}}{k} + \frac{\sigma}{\eta} \quad (2.18)
\]

Maxwell model serves as a building block for more complex linear viscoelastic models. Consider a generalized Maxwell model (Wiechert Model) with a spring of stiffness \( k_\infty \) and \( n \) Maxwell units connected in parallel as shown in Figure 2.4.

![Generalized Maxwell model](image)

**Figure 2.4 Generalized Maxwell model [32]**

In each Maxwell unit the stress in the spring and the stress in the dashpot are the same.

\[
\sigma_i = \eta_i \dot{\varepsilon}_i = k_i (\varepsilon - \varepsilon_i) \quad (2.19)
\]

The above equation can be rearranged as

\[
\dot{\varepsilon}_i + \frac{k_i}{\eta_i} \varepsilon_i = \frac{k_i}{\eta_i} \varepsilon \quad (2.20)
\]

In relaxation case, when a constant strain \( \varepsilon = \varepsilon_0 \) is applied

\[
\dot{\varepsilon}_i + \frac{k_i}{\eta_i} \varepsilon_i = \frac{k_i}{\eta_i} \varepsilon_0 \quad (2.21)
\]

The solution of the above first order differential equation is
\[ \epsilon_i = \epsilon_0 (1 - e^{-\frac{t}{\tau_i}}) \quad (2.22) \]

where \( \tau_i = \frac{\eta_i}{k_i} \)

Equation (2.19) can be written using equation (2.22) as,

\[ \sigma_i = k_i (\epsilon_0 - \epsilon_0 (1 - e^{-\frac{t}{\tau_i}})) = k_i \epsilon_0 e^{-\frac{t}{\tau_i}} \quad (2.23) \]

The stress in the whole model is the summation of stress in the spring and the \( n \) Maxwell units.

\[ \sigma = k_{\infty} \epsilon_0 + \sum_{i=1}^{n} k_i \epsilon_0 e^{-\frac{t}{\tau_i}} \quad (2.24) \]

Normalizing the above equation with \( \epsilon_0 \) gives the relaxation modulus of the system

\[ C(t) = \frac{\sigma}{\epsilon_0} = k_{\infty} + \sum_{i=1}^{n} k_i e^{-\frac{t}{\tau_i}} \quad (2.25) \]

Equation (2.25) is often referred to as Prony or Dirichlet series [32].

2.4 Time and Frequency Dependent Viscoelastic Properties

The prony series expansion of time domain shear relaxation modulus can be written using equation (2.25) as

\[ G_R(t) = G_0 \left( 1 - \sum_{k=1}^{N} g_k (1 - e^{-\frac{t}{\tau_k}}) \right) \quad (2.26) \]
where $G_0$ is the instantaneous relaxation modulus, and $g_k$ and $\tau_k$ are the prony series material constants obtained from curve fitting the shear relaxation modulus $G_R$. When $t \to 0$, $G_R \to G_0$, and when $t \to \infty$, $G_R \to G_\infty$, where

$$G_\infty = G_0 \left(1 - \sum_{k=1}^{N} g_k\right) \quad (2.27)$$

is the long term shear relaxation modulus. Using equation (2.27) the time dependent shear relaxation modulus can also be expressed as

$$G_R(t) = G_\infty + G_0 \sum_{k=1}^{N} g_k e^{-t/\tau_k} \quad (2.28)$$

Using Fourier transforms the time dependent shear relaxation modulus can be expressed in frequency domain as $G^s(\omega) = G_s(\omega) + iG_l(\omega)$ where,

**Storage modulus** $G_s = G_\infty + G_0 \sum_{k=1}^{N} g_k \frac{(\tau_k \omega)^2}{1 + (\tau_k \omega)^2} \quad (2.29)$

**Loss modulus** $G_l = G_0 \sum_{k=1}^{N} \frac{g_k (\tau_k \omega)}{1 + (\tau_k \omega)^2} \quad (2.30)$

When $\omega \to 0$, $G_s(0) = G_\infty$, the storage modulus approaches the long-term relaxation modulus. For larger frequencies $\omega \to \infty$, $G_s(0) = G_0$, the storage modulus approaches the instantaneous relaxation modulus. The loss modulus approaches zero for both high and low frequencies [33].
3.1 Base Materials

As mentioned in the earlier chapter, this study is to investigate the member damping capability of sandwich plates with different configurations of honeycomb cores. Therefore, instead of choosing a metal which has negligible viscoelastic material damping effect as a base material for the honeycomb core a polymer polycarbonate is selected. There are other materials like elastomers which appear to have higher viscoelastic dissipation energy, but they may not have the ability to withstand the general structural loading when used for the base material of honeycombs.

Polycarbonate is chosen to be the base material of the honeycomb core in this study due its moderate stiffness, impact resistance and viscoelastic behavior. Also polycarbonate is impact resistant at room temperature unlike other amorphous polymers which becomes impact resistant only above their glass transition temperature [34]. An aluminum alloy, Al-5052-H39 is considered to be material for the face sheet since it is stiff and has a negligible viscoelastic material damping effect. Table 3.1 shows the elastic properties of Al-5052-H39 and polycarbonate.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus, $E$ (GPa)</th>
<th>Poisson’s Ratio, $\nu$</th>
<th>Density, $\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-5052-H39</td>
<td>68.97</td>
<td>0.34</td>
<td>2700</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>2.075</td>
<td>0.37</td>
<td>1200</td>
</tr>
</tbody>
</table>
Taking into account the time dependent behavior of elastic moduli of polycarbonate as shown in equation 2.28 two different approaches are used to define the material. In one case the elastic modulus of polycarbonate given in Table 3.1 is considered to be the instantaneous elastic modulus and in the other case the elastic modulus is considered to be the instantaneous modulus. The details of the material and their corresponding time dependent modulus calculated using equation 2.27 is reported in the table below.

**Table 3.2 Material definition of Polycarbonate based on time dependent properties**

<table>
<thead>
<tr>
<th>Material</th>
<th>Instantaneous Modulus, $E_s$ (GPa)</th>
<th>Long-term Modulus, $E_\infty$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Polycarbonate</td>
<td>2.075</td>
<td>0.00753</td>
</tr>
<tr>
<td>Stiff Polycarbonate</td>
<td>571.546</td>
<td>2.075</td>
</tr>
</tbody>
</table>

Though the definition of elastic modulus differs for the soft and stiff polycarbonate the viscoelastic relaxation behavior remains same for both the materials.

### 3.2 Effective Properties of Honeycomb cores

The unit cell of the regular and auxetic honeycombs are as shown in the Figure 3.1, where $\theta$ is the internal cell angle, $h$ is the vertical cell length, $l$ is the inclined cell length and $t$ is the cell wall thickness.
Gibson and Ashby developed Cellular Materials Theory (CMT) [19] to predict the linear elastic behavior of honeycomb structures by considering the unit cell walls as Euler-Bernoulli beams. The effective in-plane and out-of-plane properties of honeycomb based on CMT are as follows:

\[
\frac{\rho^*}{\rho_s} = \frac{\beta(\alpha + 2)}{2 \cos \theta (\alpha + \sin \theta)}
\]

\[
\frac{E_{1}^*}{E_s} = \frac{\beta^3 \cos \theta}{\sin^3 \theta (\alpha + \sin \theta)}
\]

\[
\frac{E_{2}^*}{E_s} = \frac{\beta^3 (\alpha + \sin \theta)}{\cos^3 \theta}
\]

\[
\frac{E_{3}^*}{E_s} = \frac{\beta(\alpha + 2)}{2(\alpha + \sin \theta) \cos \theta}
\]
\[
\frac{G_{12}^*}{E_s} = \frac{\beta^3 (\alpha + \sin \theta)}{\alpha^3 (1 + 2\alpha) \cos \theta}
\]

\[
\frac{G_{13}^*}{G_s} = \frac{\beta \cos \theta}{(\alpha + \sin \theta)}
\]

\[
\left(\frac{G_{23}^*}{G_s}\right)_{\text{upper}} = \frac{\beta (\alpha + 2\sin^2 \theta)}{2(\alpha + \sin \theta) \cos \theta}
\]

\[
\left(\frac{G_{23}^*}{G_s}\right)_{\text{lower}} = \frac{\beta (\alpha + \sin \theta)}{(1 + 2\alpha) \cos \theta}
\]

\[
v_{12}^* = \frac{\cos^2 \theta}{(\alpha + \sin \theta) \sin \theta}
\]

\[
v_{21}^* = \frac{(\alpha + \sin \theta) \sin \theta}{\cos^2 \theta}
\]

(3.1)

where \( \alpha = \frac{h}{l} \) is the aspect ratio of the cell sides, \( \beta = \frac{t}{l} \) is the relative thickness, \( E_s \) is the Young’s modulus, \( G_s \) is the shear modulus and \( \rho_s \) is the mass density of the material with which the honeycomb core is made of. \( E_1^* \) and \( E_2^* \) are the in-plane effective moduli and \( E_3^* \) is the out-of-plane effective modulus. \( G_{12}^* \) is the in-plane effective shear modulus, \( G_{13}^* \) and \( G_{23}^* \) are the out-of-plane effective shear moduli. The lower and upper bounds are calculated using the minimum potential energy and minimum complementary energy theorems. \( v_{12}^* \) and \( v_{21}^* \) are the in-plane effective Poisson’s ratios of the honeycomb.

Based on the above mentioned equations unit cells of three different configurations are developed for this study. They are

1. Regular Honeycomb
2. Auxetic-I Honeycomb

3. Auxetic-II Honeycomb

Regular honeycomb is defined with \( \theta = 30^\circ \) and \( h = l \). Both the auxetic models are defined with \( \theta < 0 \). Auxetic-I is defined with \( \theta = -30^\circ \) and \( h = 2l \). Auxetic-I has same effective cell size and in-plane Young’s moduli (\( E_{11}^* \) and \( E_{22}^* \)). The cell wall thickness (\( t \)) remains the same in both cases. Auxetic-II is defined with \( \theta = -30^\circ \) and \( h = 2l \) and a cell wall thickness (\( t \)) which is three fourth of the cell wall thickness of regular honeycomb. Auxetic-II has same mass and the same effective out-of-plane modulus (\( E_{33}^* \)). The effective properties of the three honeycombs corresponding to polycarbonate core material in table are given in the table below.
Table 3.3 Effective properties of Honeycomb cores calculated using CMT formulas

<table>
<thead>
<tr>
<th>Core Geometry</th>
<th>Regular Honeycomb</th>
<th>Auxetic-I Honeycomb</th>
<th>Auxeti-II Honeycomb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (mm)</td>
<td>0.423</td>
<td>0.423</td>
<td>0.31725</td>
</tr>
<tr>
<td>$l$ (mm)</td>
<td>4.23</td>
<td>4.23</td>
<td>4.23</td>
</tr>
<tr>
<td>$h$ (mm)</td>
<td>4.23</td>
<td>8.46</td>
<td>8.46</td>
</tr>
<tr>
<td>$E_1^*$ (MPa)</td>
<td>4.79</td>
<td>4.79</td>
<td>2</td>
</tr>
<tr>
<td>$E_2^*$ (MPa)</td>
<td>4.79</td>
<td>4.79</td>
<td>2</td>
</tr>
<tr>
<td>$E_3^*$ (MPa)</td>
<td>238.69</td>
<td>319.47</td>
<td>238.69</td>
</tr>
<tr>
<td>$G_{12}^*$ (MPa)</td>
<td>1.20</td>
<td>0.18</td>
<td>0.075</td>
</tr>
<tr>
<td>$G_{13}^*$ (MPa)</td>
<td>43.72</td>
<td>43.72</td>
<td>32.67</td>
</tr>
<tr>
<td>$G_{23}^*$ (MPa) upper bound</td>
<td>43.72</td>
<td>72.87</td>
<td>54.45</td>
</tr>
<tr>
<td>$G_{23}^*$ (MPa) upper bound</td>
<td>43.72</td>
<td>26.23</td>
<td>19.06</td>
</tr>
<tr>
<td>relative density, $\frac{\rho_0^*}{\rho_a}$</td>
<td>0.12</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>$v_{12}^*$</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$v_{21}^*$</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

From the Table 3.3 we can see that regular honeycomb has the same effective in-plane moduli ($E_1^*$ and $E_2^*$), same effective out-of-plane shear moduli ($G_{13}^*$ and $G_{23}^*$) and
same effective Poisson’s ratio \( \nu_{12}^* = \nu_{21}^* = 1 \). Auxetic-I honeycomb with the same relative thickness \( \beta \) has the same effective in-plane moduli \( (E_1^* = E_2^*) \) and effective out-of-plane shear modulus \( (G_{13}^*) \) as that of the regular honeycomb. It also has the same effective Poisson’s ratio \( \nu_{12}^* = \nu_{21}^* = -1 \). Auxetic-II honeycomb has same mass density and effective out-of-plane modulus \( (E_3^*) \) as that of regular honeycomb. It can be seen that the effective in-plane moduli \( (E_1^* = E_2^*) \) of Auxetic-II has been reduced by more than half. The effective in-plane shear modulus \( (G_{12}^*) \) has reduced by a large magnitude for both the auxetic cases.

3.3 **Viscoelastic Properties of the Core Material**

The viscoelastic properties of the polycarbonate material assigned to the honeycomb core are defined in time domain by a prony series expansion (generalized Maxwell model) of shear relaxation modulus in the form shown in equation (2.26).

Normalized shear relaxation modulus data \( \frac{G_k}{G_0} \) for polycarbonate is taken from Mercier. The material constants \( g_k \) and \( \tau_k \) are obtained from a nonlinear least squares fit done on this data using a N=3 term Prony series in Abaqus. Figure 3.2 shows the normalized shear relaxation modulus \( \frac{G_k}{G_0} \) for polycarbonate taken from Mercier [34] and the nonlinear least square curve fit using a N=3 prony series. The prony series curve fits the test data for most of the time of significant relaxation.
The frequency dependent shear modulus is defined by a prony series in the form

\[ G'(\omega) = G_s(\omega) + G_l(\omega) \]. \( G_s(\omega) \) and \( G_l(\omega) \) are defined in the equations (2.29, 2.30)

![Graph showing normalized relaxation modulus](image)

**Figure 3.2 Normalized Relaxation modulus \( G_R/G_0 \) for polycarbonate**

The prony coefficients \( g_k \) and \( \tau_k \) obtained from the curve fit are as shown in table below.

**Table 3.4 Three term Prony series coefficients obtained from curve fit**

<table>
<thead>
<tr>
<th>( k )</th>
<th>( g_k )</th>
<th>( \tau_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0601089</td>
<td>0.0015332</td>
</tr>
<tr>
<td>2</td>
<td>0.84558</td>
<td>2.1425</td>
</tr>
<tr>
<td>3</td>
<td>0.0906806</td>
<td>19.791</td>
</tr>
</tbody>
</table>
The corresponding frequency dependent loss and storage modulus for the Polycarbonate are shown in Figure 3.3 and Figure 3.4 and the material damping loss factor $\eta = \frac{G_i}{G} \,$ is shown in Figure 3.5.

Figure 3.3 Normalized Loss modulus $G_i/G_0$ as a function of frequency

Figure 3.4 Normalized Storage modulus $G_s/G_0$ as a function of frequency
It can be seen from the above figure that the material damping loss factor of polycarbonate is maximum at lower frequencies of the order 0-500 Hz, and decreases as the frequency approaches infinity.

3.4 Modeling of the Sandwich Plate

Modeling of the sandwich plate is carried out in Abaqus 6.9.1. A polycarbonate honeycomb core is sandwiched between two aluminum face sheets. Two different sandwich plate models are created for the three configurations of honeycomb cores. The models are

1. In-Plane Loading model
2. Out-of-Plane Loading model
The in-plane loading model corresponds to a model in which load is applied along either $X_1$ or $X_2$ directions as shown in Figure 3.6. When loaded in the in-plane honeycomb cell walls bend. They may even buckle in the case of compressive in-plane load. The out-of-plane loading model corresponds to a model in which load is applied along the $X_3$ direction as shown in Figure 3.6. In the case of out-of-plane loading cell walls experience either compression or extension. For a honeycomb the elastic moduli are higher in the out-of-plane direction ($X_3$) compared to the in-plane direction ($X_1$, $X_2$).

### 3.4.1 Estimation of Sandwich Plate geometry from the Unit Cell geometry

The dimensions of the sandwich plate are dependent on the number of unit cells present in the core. A closer look at the horizontal and vertical dimensions of the unit cell is given in the figure below,
Figure 3.7 Horizontal and Vertical dimensions of Regular and Auxetic Unit cells

Let the horizontal length of the unit cell be $L_h$ and the vertical length be $L_v$. From the above figures,

For a regular honeycomb (3.3)

$$L_h = 2l \cos \theta$$

$$L_v = 2(h + l \sin \theta)$$

For an auxetic honeycomb (3.4)

$$L_h = 2l \cos \theta$$

$$L_v = 2(h - l \sin \theta)$$

Let $H$ be the horizontal length of the sandwich plate and $N_h$ be the desired number of unit cells along the horizontal length. Then

$$H = N_h L_h = 2lN_h \cos \theta \text{ for both the honeycombs (3.5)}$$

Let $V$ be the vertical length of the sandwich plate and $N_v$ be the desired number of unit cells along the vertical length. Then

$$V = N_v L_v = 2N_v(h + l \sin \theta) \text{ for a regular honeycomb (3.6)}$$
\( V = N_v L_v = 2N_v (h - l \sin \theta) \) for an auxetic honeycomb (3.7)

Therefore by deciding the desired number of unit cells along each direction and the critical dimensions of the unit cells \((l, h)\), the overall dimensions of the sandwich plate can be derived using the above equations.

3.4.2 In-plane Loading model

In this case the loading is only along the in-plane directions \((X_1 \text{ or } X_2)\). Therefore a two dimensional model is developed in Abaqus 6.9.1 for this case which helps in the significant reduction of computational time.

The number of unit cells along the horizontal direction \((X_1)\) are chosen to be 11 and along the vertical direction \((X_2)\) to be 2. Using table for the dimensions of the critical parameters of the unit cell and the equations (3.5, 3.6, 3.7) the overall dimensions of the honeycomb core are 80.59227 mm along the \(X_1\) direction and 25.38 mm along the \(X_2\) direction. The cell wall thickness (t), material polycarbonate and a beam section is assigned during the section assignment phase. Figure 3.8 shows the regular, auxetic honeycomb cores. Auxetic-I and II are not separately shown since the difference in cell wall thickness cannot be seen in a two dimensional model.
In the next step, the face sheet of length 80.59227 mm is generated. The thickness of the face sheet is 0.2 mm. The thickness, material aluminum and beam section are assigned to the face sheet in the section assignment phase. The face sheets are assembled to the top and bottom of the honeycomb core. The completely assembled sandwich plate looks as shown in the figure below,
Figure 3.9 Completely assembled Regular & Auxetic in-plane loading models

Mass properties of the sandwich plates are as shown in the table below

**Table 3.5 Mass of the three configurations of in-plane loading sandwich plates**

<table>
<thead>
<tr>
<th>Core geometry</th>
<th>Mass of the honeycomb core (in kg)</th>
<th>Mass of the face sheets (in kg)</th>
<th>Total mass of the sandwich plate (in kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.287534</td>
<td>0.087039</td>
<td>0.374574</td>
</tr>
<tr>
<td>Auxetic- I</td>
<td>0.386486</td>
<td>0.087039</td>
<td>0.473526</td>
</tr>
<tr>
<td>Auxetic- II</td>
<td>0.289865</td>
<td>0.087039</td>
<td>0.376904</td>
</tr>
</tbody>
</table>

3.4.3 **Out-of-Plane Loading model**

In this case the loading is along the out-of-plane direction ($X_3$). Therefore a three dimensional model is developed in Abaqus 6.9.1. The base feature is selected to be a deformable shell instead of a solid. This is also done in order to reduce computational time.
The number of unit cells along the horizontal direction ($X_1$) are chosen to be 11 and along the vertical direction ($X_2$) to be 6. Using table for the dimensions of the critical parameters of the unit cell and the equations (3.1) the overall dimensions of the honeycomb core are 80.59227 mm along the $X_1$ direction and 76.14 mm along the $X_2$ direction. The core is extruded along the $X_3$ direction to a depth (D) of 4.23mm. The cell wall thickness (t), material polycarbonate and shell section are assigned during the section assignment phase. Figure 3.10 shows the regular, auxetic honeycomb cores. Auxetic-I and II are not separately shown since the difference in cell wall thickness cannot be seen in a two dimensional model.

![Diagram of overall dimensions](image)

*Figure 3.10 Overall dimensions of Regular & Auxetic Out-of-plane loading models*
In the next step, the face sheet of length 80.59227 mm, width 76.14mm is generated. The thickness of the face sheet is 0.2 mm. The thickness, the material aluminum and shell section are assigned to the face sheet in the section assignment phase. The face sheets are assembled to the top and bottom of the honeycomb core. The cut section of completely assembled sandwich plate looks as shown in the figure.

![Figure 3.11 Section view of completely assembled Regular & Auxetic out-of-plane loading models](image)

Figure 3.11 Section view of completely assembled Regular & Auxetic out-of-plane loading models
Mass properties of the sandwich plates are as shown in the table below,

Table 3.6 Mass of the three configurations of three configurations of out-of-plane loading models

<table>
<thead>
<tr>
<th>Core geometry</th>
<th>Mass of the honeycomb core (in kg)</th>
<th>Mass of the face sheets (in kg)</th>
<th>Total mass of the sandwich plate (in kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.003651</td>
<td>0.006627</td>
<td>0.010278</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.004904</td>
<td>0.006627</td>
<td>0.011531</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.003678</td>
<td>0.006627</td>
<td>0.010305</td>
</tr>
</tbody>
</table>
CHAPTER 4 : FINITE ELEMENT MODELS

The in-plane loading and out-of-plane loading models generated in the previous chapter are meshed and set up for various analyses using Abaqus 6.9.1. Previous works [26, 27, 28, 35] used ABAQUS for the finite element study of honeycomb structures. Beam elements are used for the two dimensional models and shell elements are used for the three dimensional model. These elements are preferred over conventional solid 3D elements to reduce the computational time. The details of the final finite element models and various analyses procedures adopted in this study after several iterations and troubleshooting are discussed in the following sections.

4.1 In-Plane Loading model

In the in-plane loading model B-22 beam elements are assigned to the honeycomb core and face sheets. B22 corresponds to a planar beam that uses quadratic interpolation. It has 3 nodes per element, one internal middle node and two external nodes. Each node has three degrees of freedom, displacement along X_1 and X_2 direction and rotation about X_3 direction. They follow Timoshenko beam theory thereby allows for transverse shear deformation [33].

The honeycomb core is meshed with 4 elements per edge and the face sheet is meshed with 44 elements per edge. This gives a total of 632 elements for the honeycomb core and a total of 720 elements for the sandwich plate.
4.2 Out-of-Plane Loading model

In the out-of-plane loading model S4R shell elements are assigned to the honeycomb core and face sheets. S4R corresponds to a 4-node, quadrilateral, stress/displacement shell element with reduced integration and a large-strain formulation. Each node has six degrees of freedom, three displacement and three rotational freedoms. These elements allow transverse shear deformation. They use thick shell theory if the shell thickness is high or else uses Kirchhoff thin shell formulation if the shell thickness is small. Reduced integration elements uses when compared to the fully integrated elements. As a result the run time is significantly reduced while maintaining the accuracy. Eight noded S8R elements are not preferred because they capture only small strains unlike S4R elements which capture finite-strains and also they increase the computational time significantly [33].

The honeycomb core is meshed with 4 elements per edge and the face sheet is meshed with 44 elements per edge. This gives a total of 7488 elements for the honeycomb core, 1936 elements for each of the face sheets and a total of 11360 elements for the sandwich plate. Figure 4.1 and Figure 4.2 shows the individual meshed face sheet and honeycomb core respectively.
Figure 4.1 Meshed Face sheet

Figure 4.2 Meshed Regular and Auxetic Honeycomb cores
4.3 Constraints

The assembly shown in figure is just a visual representation that face sheets are attached to the honeycomb core. When the analysis is performed on the sandwich plate these face sheets are prone to lose contact. Therefore the face sheets are to be constrained to the honeycomb core such that both of them move or deform together as a single entity. Abaqus provides a TIE constraint which enables to tie the face sheet to the honeycomb core.

A surface based tie constraint allows the user to tie two surfaces together for the duration of the simulation. Of the two surfaces the one which is stiffer is called the master surface and the other surface is called the slave surface. The nodes on the slave surface are constrained to have same motion, same temperature or electric potential as that of a point of on the master surface closest to it. In other words the degrees of freedom of the slave surface nodes are eliminated. There are two types of surface based tie constraint formulations, surface to surface and node to surface [33]. In this study the honeycomb core is the slave surface and the aluminum face sheets is the master surface.

In the in-plane loading model the node to surface tie formulation is used. As shown in the Figure 4.3 the slave nodes of the honeycomb core are tied to the master aluminum face sheet surface.
In the out-of-plane loading case a surface to surface tie formulation is used. The slave surface of the honeycomb core is tied to the bottom surface of master aluminum face sheet. The tie constraint can be clearly seen in the three dimensional model shown in the Figure 4.4. The yellow circles correspond to the tie constraint.
4.4 **Boundary Conditions and Loads**

In the in-plane-loading model the three degrees of freedom \(U_1, U_2\) and \(UR_3\) of the B22 element are constrained along the edges shown in the figure below.

![Figure 4.5 Regions on which boundary conditions are applied in in-plane loading model](image)

In the out-of-plane model the six degrees of freedom \(U_1, U_2, U_3, UR_1, UR_2\) and \(UR_3\) of the S4R element are constrained along the edges shown in the figure below,
Figure 4.6 Regions on which boundary conditions are applied in out-of-plane loading model

A uniform pressure load is applied on one of the face sheets in both the models as shown in Figure 4.7 and Figure 4.8. The arrows correspond to the direction of application of pressure load.

Figure 4.7 Loading scenario in in-plane loading model
4.5 Analysis Procedures

Three analyses are employed on the sandwich plates, the results which will be used to determine the damping characteristics. They are

- Natural Frequency extraction
- Direct Steady State analysis
- Quasi Static analysis
- Implicit Dynamic analysis

In all the analyses results were generated with both instantaneous and long-term elastic modulus assigned to the polycarbonate viscoelastic material, to study the effect of time dependent modulus. These analyses will be explained in detail in the coming sections.
CHAPTER 5: MODAL VIBRATION AND FREQUENCY RESPONSE OF THE SANDWICH PANELS

5.1 Modal Vibration of Sandwich Panels

Undamped modal frequencies and the corresponding mode shapes of the sandwich plates are investigated using the natural frequency extraction analysis in ABAQUS to verify the stability of the sandwich plates developed. The analysis is setup in ABAQUS as follows:

- Step 0 – Initial: The boundary conditions are specified.
- Step 1 – Linear Perturbation: Frequency, first 10 natural frequencies of the system are requested.

Loads cannot be applied during this analysis.

5.1.1 Results of In-Plane Loading model

Table 5.1 shows the first 10 undamped natural frequencies of the three sandwich plates. The first modal frequency of regular honeycomb sandwich panel occurs at 571.36 Hz. The modal frequencies of auxetic honeycomb are found to be lower than those of regular honeycomb. The first modal frequency of Auxetic-I occurs at 233.18 Hz which is 59.1% lesser than the regular case. Even though the Auxetic-I core is designed to have same $E_{11}$, $E_{22}$, $G_{13}$ and higher $G_{23}$, the relatively large increase in mass by 20.9% resulted in reduction of the natural frequencies. The first mode of Auxetic-II occurs at 179.55 Hz which is 68.57% lower than regular sandwich plate. Auxetic-II is designed to have same mass as that of regular but the lower effective $E_{11}$, $E_{22}$ and $G_{13}$ resulted in reduction of the natural frequencies.
Table 5.1 First 10 modal frequencies of the in-plane models

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Regular Frequency (Hz)</th>
<th>Auxetic-I Frequency (Hz)</th>
<th>Auxetic-II Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>571.36</td>
<td>233.18</td>
<td>179.55</td>
</tr>
<tr>
<td>2</td>
<td>1158.1</td>
<td>484.43</td>
<td>378.85</td>
</tr>
<tr>
<td>3</td>
<td>1769.4</td>
<td>776.19</td>
<td>633.91</td>
</tr>
<tr>
<td>4</td>
<td>2034.5</td>
<td>1096.3</td>
<td>908.63</td>
</tr>
<tr>
<td>5</td>
<td>2134.3</td>
<td>1231.5</td>
<td>934.63</td>
</tr>
<tr>
<td>6</td>
<td>2314.7</td>
<td>1460.7</td>
<td>1237.3</td>
</tr>
<tr>
<td>7</td>
<td>2408.8</td>
<td>1682</td>
<td>1271.7</td>
</tr>
<tr>
<td>8</td>
<td>2689.7</td>
<td>1827.6</td>
<td>1398.3</td>
</tr>
<tr>
<td>9</td>
<td>3076.3</td>
<td>1849.4</td>
<td>1521.8</td>
</tr>
<tr>
<td>10</td>
<td>3233.4</td>
<td>2191.1</td>
<td>1683.4</td>
</tr>
</tbody>
</table>

Figure 5.1 shows the corresponding mode shapes of regular honeycomb sandwich panel. The auxetic ones showed similar modes due to similarity in overall macro structure.
5.1.2 Results of Out-of-Plane model

Table 5.2 shows the first 10 undamped natural frequencies of the three sandwich plates. The first modal frequency of regular honeycomb sandwich panel occurs at 2925.7 Hz. The modal frequencies of auxetic honeycomb are found to be lower than those of regular honeycomb. The first modal frequency of Auxetic-I occurs at 2716 Hz and Auxetic-II occurs at 2533.5 Hz which are 7.17% and 13.41% lower than regular sandwich plate.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Frequency (Hz)</th>
<th>Regular</th>
<th>Auxetic-I</th>
<th>Auxetic-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2925.7</td>
<td>2716</td>
<td>2533.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4581.5</td>
<td>4321.4</td>
<td>3996.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4755.1</td>
<td>4362.1</td>
<td>4084.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6012.3</td>
<td>5600.1</td>
<td>5198.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6590.3</td>
<td>6255.1</td>
<td>5736.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6916.8</td>
<td>6331.5</td>
<td>5924.5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7670.5</td>
<td>7222.6</td>
<td>6650.3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7846.4</td>
<td>7276.8</td>
<td>6769.3</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8685.5</td>
<td>8256.1</td>
<td>7571.4</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9127.1</td>
<td>8442.3</td>
<td>7899.6</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.2 shows the corresponding mode shapes of regular honeycomb sandwich panel.

The auxetic ones showed similar modes due to similarity in overall macro structure.
Figure 5.2 First 10 mode shapes of in-plane model with regular honeycomb core

5.2 Frequency Response

To determine the frequency response of the sandwich panels a direct steady state analysis is done in ABAQUS.
5.2.1 **Direct Steady State Analysis**

This analysis helps in determining the steady-state amplitude and phase of the response of the system due to harmonic excitation at given frequency. The analysis is setup in abaqus as follows.

- **Step 0 – Initial:** The boundary conditions are specified.
- **Step 1- Linear Perturbation, Direct Steady State:** A uniform harmonic pressure load is applied on the face sheet.

The displacement of a node at the center of the face sheet is recorded. Displacement in the direction $X_2$ is recorded for in-plane model and displacement in the direction of $X_3$ is recorded for out-of-plane model. This displacement is normalized with the static displacement which corresponds to displacement at zero frequency. In the step 1, a pressure load of 0.1Mpa is applied on the in-plane loading model and a load of 1MPa is applied on the out-of-plane loading model. The viscoelastic material is defined in frequency-domain for this analysis. The frequencies calculated in natural frequency extraction step are provided as frequency sweep input for this analysis. More number of data points is used for analysis around the modal frequency range compared to the entire frequency sweep to obtain resonant peaks and reduce computational time.

5.3 **Results of In-plane loading model**

5.3.1 **Soft Polycarbonate material assigned to the core**

Figure 5.3 Figure 5.4 Figure 5.5 shows the normalized logarithmic displacement vs. frequency of three configurations of in-plane loading models. Both undamped and damped responses are reported till the third modal frequency is reached. The soft
polycarbonate material mentioned in Table 3.1 is assigned to the honeycomb cores in this case. Both the undamped and damped responses exactly match with the frequencies of modes 1, 3 shown in Table 5.1.

Figure 5.3 Frequency response of in-plane model with regular honeycomb core

Figure 5.4 Frequency response of in-plane model with Auxetic-I honeycomb core
Figure 5.5 Frequency response of in-plane model with Auxetic-II honeycomb core

Figure 5.6 shows the comparison of damped frequency response of three configurations of honeycomb sandwich plates till the first modal frequency is reached.

Figure 5.6 Comparison of frequency response of Regular, Auxetic-I, Auxetic-II for the first mode
5.3.2 Stiff Polycarbonate material assigned to the core

Figure 5.7 shows the damped frequency response of three configurations of in-plane loading models defined with the stiff polycarbonate material mentioned in Table 3.2. The storage and loss moduli of the complex elastic modulus in frequency domain increases making the model very stiff. As a result the first mode is shifted to higher frequency compared to undamped first modal frequency.

![Figure 5.7 Comparison of frequency response of Regular, Auxetic-I, Auxetic-II for the first mode](image)

5.4 Results of Out-of-plane loading model

5.4.1 Soft Polycarbonate

Figure 5.8 Figure 5.9 Figure 5.10 shows the normalized logarithmic displacement vs. frequency of three configurations of out–of–plane loading models. Both undamped and damped responses are reported till the sixth modal frequency is reached. The
undamped responses exactly match with the frequencies of modes 1, 5, 6 shown in Table 5.2. The damped responses are shifted slightly to a lesser magnitude.

Figure 5.8 Frequency response of out-of-plane model with regular honeycomb core

Figure 5.9 Frequency response of out-of-plane model with Auxetic-I honeycomb core
Figure 5.10 Frequency response of out-of-plane model with Auxetic-II honeycomb core

Figure 5.11 shows the comparison of damped frequency response of three configurations of honeycomb sandwich plates

Figure 5.11 Comparison of frequency response of Regular, Auxetic-I, Auxetic-II
5.4.2 **Stiff Polycarbonate material**

Figure 5.12 shows the damped frequency response of three configurations of out-of-plane loading models defined with a stiff polycarbonate material. Even in this case the damped frequencies are shifted to a higher magnitude.

![Graph](image)

**Figure 5.12** Comparison of frequency response of Regular, Auxetic-I, Auxetic-II for the first mode

### 5.5 Mesh Convergence

A mesh convergence study is performed to see the effects of mesh on the output. A finer mesh is chosen for both the in-plane and out-of-plane models. In the case of in-plane model each edge of the honeycomb core is meshed with 8 B22 elements, whereas in the case of out-of-plane model 4 S8R (quadratic elements) are assigned along each edge. Figure 5.13 and Figure 5.14 shows the frequency response of both models with regular honeycomb core.
Figure 5.13 Comparison of frequency response for a finer mesh, in–plane model

Figure 5.14 Comparison of frequency response for a finer mesh, out-of–plane model

From both the figures it can be seen that the mesh used in the current study almost matches the finer mesh frequency response. Therefore the rest of the analyses are conducted with the mesh mentioned in Chapter 4.
5.6 Calculation of loss factor using Half–Power Bandwidth method

Damping ratio can be estimated from the frequency response using half-power bandwidth method. According to this method the loss factor \( \eta_f \) is defined as the ratio of the frequency range between the two half power points to the natural frequency at a mode.

\[
\eta_f = \frac{\omega_2 - \omega_1}{\omega_n}
\]

where \( \omega_n \) is the frequency corresponding to the amplitude \( X \) of the resonant peak in the frequency response. \( \omega_1 \) and \( \omega_2 \) are the frequencies corresponding to the half power points. Half power points are the points on the frequency response corresponding to \( \frac{X}{\sqrt{2}} \) amplitude [36].

![Figure 5.15 Half Power Bandwidth method](image)
In the current study loss factors are calculated only for the resonant peaks corresponding to the first mode. Table below summarizes the loss factors of in-plane models calculated using above method.

**Table 5.3 Frequency Loss factor ($\eta_f$) of in-plane loading models**

<table>
<thead>
<tr>
<th>Honeycomb Configuration</th>
<th>Soft Polycarbonate</th>
<th>Stiff Polycarbonate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.009188</td>
<td>0.000415</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.018303</td>
<td>0.001291</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.021449</td>
<td>0.001775</td>
</tr>
</tbody>
</table>

Results show that Auxetic-II configuration has the highest loss factor compared to the other configurations in the case of in-plane loading model.

Table below summarizes the loss factors of out-of-plane models

**Table 5.4 Frequency Loss factor ($\eta_f$) of out – of-plane loading models**

<table>
<thead>
<tr>
<th>Honeycomb Configuration</th>
<th>Soft Polycarbonate</th>
<th>Stiff Polycarbonate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.001963</td>
<td>0.000123</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.001936</td>
<td>0.000230</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.002050</td>
<td>0.000170</td>
</tr>
</tbody>
</table>

Results show that Auxetic-II configuration has higher loss factor in the case of out –of-plane model defined with soft polycarbonate and Auxetic-I has higher loss factor in the case of model with stiff polycarbonate core.
6.1 Quasi Static Analysis

A static stress analysis ignores the effects of a time dependent material in solving a problem. Since the honeycomb core is assigned with a rate dependent viscoelastic polycarbonate material, a quasi static stress analysis is performed on the sandwich plates to determine the damping characteristics from their hysteresis behavior. The Quasi static stress analysis includes the time dependent material response (creep, swelling, viscoelasticity and two layer viscoplasticity) [33]. This analysis is set up in Abaqus as follows

- Step 0: Initial – The boundary conditions are specified
- Step 1: Visco step – A uniform pressure load with sinusoidal amplitude as shown in Figure 6.1 is applied for a step time of 0.25 secs.

The displacement of a node at the center of the face sheet and the energies of the whole model are found out with this analysis. In the step 1, a pressure load of 0.1Mpa is applied on the in-plane loading model and a load of 1MPa is applied on the out-of-plane loading model. The viscoelastic material is defined in time-domain for this analysis. Two different quasi-static analyses are done, in one case with non-linear geometry (NLGEOM) turned off and in the other case with NLGEOM turned on.
6.2 Loss Factors

The displacement of a node at the center of the face sheet, ALLCD and ALLSE are the outputs generated from the analysis which are used in the calculation of loss factors of the sandwich plates. ALLCD is an abaqus history output which refers to the amount of energy dissipated by viscoelasticity. ALLSE is an abaqus history output refers to the strain energy of the system. Two types of energy dissipation measures are calculated in this chapter using the above data, a) Hysteretic Energy Loss factor ($\eta_{\text{hys}}$) and b) Quasi static Loss factor ($\eta_{\text{q}}$). The higher the loss factor the higher is the damping capability of the system.
6.2.1 **Hysteretic Energy Loss factor**

Hysteretic energy loss factor ($\eta_{hys}$) is calculated from a hysteresis curve generated between the force applied on the sandwich plates and the displacement of a node at the center of the face sheet. Maximum displacement of the sandwich plate occurs at the node at the center of the face sheet, hence it is considered to generate the hysteresis curve. The amount of area enclosed by the hysteresis curve generated from the sinusoidal loading of the sandwich plate is proportional to the amount of damping energy dissipated, which can be used to comment on the damping capability of the structure [3,7]. This area is considered to be the hysteretic energy loss factor ($\eta_{hys}$) in this study.

6.2.2 **Quasi-static Loss factor**

Quasi-static loss factor ($\eta_q$) is calculated from the ALLCD and ALLSE plots generated during the quasi static analysis. It is the measure of the energy dissipated or lost by the system relative to the energy stored within the system. The quasi-static loss factor ($\eta_q$) can be written as

\[ \eta_q = \frac{D_q}{2\pi U_q} \]  

(5.1)

where $D_q$ is the amount of damping energy dissipated associated with one cycle of loading; $U_q$ is the average amount of strain energy generated.

$D_q$ is calculated from ALLCD plots by subtracting the ALLCD value at the end of the cycle (0.25 secs) from the ALLCD value at the start of the cycle (0.05 secs). The period between 0.05 seconds and 0.25 seconds corresponds to the fully enclosed
hysteresis curve, whose area is proportional to the amount of damping energy dissipated.

From [3] average strain energy of the system \( U_q \) is proportional to the area under the hysteresis curve during the initial loading phase. The initial loading phase corresponds to the loading from 0 to 0.05 secs in the Figure 6.1. Therefore \( U_q \) is taken to be the value of ALLSE at 0.05 seconds which corresponds to the area under the hysteresis curve during the initial loading phase.

6.3 Loss factors calculated for the in-plane loading model

6.3.1 Stiff Polycarbonate core

6.3.1.1 NLGEOM OFF

6.3.1.1.1 Calculation of Hysteretic energy loss factor

The hysteresis curves of the sandwich plates with the three configurations of honeycomb core Regular, Auxetic-I and Auxetic-II are as shown in the Figure 6.2. The results show that during loadings and unloading during the sinusoidal cycle, very little hysteresis is found, indicating a small hysteretic energy loss factor.
Figure 6.2 Hysteresis curves of three configurations of in-plane loading models

\( \eta_{hys} \), the area enclosed by the hysteresis curves shown in Figure 6.2, is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_{hys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.003202</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.016472</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.035881</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Auxetic-II configuration is 91% greater Regular configuration and 54% greater than Auxetic-I configuration. Auxetic-I shows an 80.5% higher loss factor than Regular configuration. Regular sandwich plate has a significantly higher effective in-plane shear modulus \( G_{12}^* \)
making it stiffer than both the auxetic cases in the in-plane direction which resulted in a lesser displacement which in turn resulted in lesser area under force vs. displacement curve.

6.3.1.1.2 Calculation of Quasi-static loss factor

The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores Regular, Auxetic-I and Auxetic-II as a function of time are as shown in the Figure 6.3 and Figure 6.4.

![ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models](image)

Figure 6.3 ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models

Figure 6.3 shows that Auxetic-II has higher dissipation energy and Regular has lower dissipation energy, which is similar to the trend shown in section 6.3.1.1.1
Auxetic-II being the least stiff of the three configurations shows higher strain energy generated as shown in the ALLSE plot above.

$D_q$, the amount of damping energy dissipated per cycle is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.002068</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.010295</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.022160</td>
</tr>
</tbody>
</table>

$U_q$, the average amount of strain energy is reported in the table below
Table 6.3 Strain Energy generated $U_q$

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.7809</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>3.3126</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>6.7152</td>
</tr>
</tbody>
</table>

$\eta_q$, the quasi-static loss factor is reported in the table below

Table 6.4 Quasi-static loss factor ($\eta_q$) of in-plane models

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.000421</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.000494</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.000525</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the quasi-static loss factor of Auxetic-II configuration is 19.8% greater than Regular configuration and 5.9% greater than Auxetic-I configuration. Auxetic-I has 14.8% higher loss factor compared to Regular configuration.

6.3.1.2 NLGEOM ON

To account for the effects of non-linear geometry on the results NLGEOM is turned on and the results are discussed in the sections below.
6.3.1.2.1 Calculation of Hysteretic energy loss factor

The hysteresis curves of the sandwich plates with the three configurations of honeycomb cores are as shown in the figure below.

![Hysteresis curves of three configurations of in-plane loading models](image)

Figure 6.5 Hysteresis curves of three configurations of in-plane loading models

\( \eta_{hys} \), the area enclosed by the hysteresis curves shown in Figure 6.5, is shown in the table below.

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_{hys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.003189</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.013267</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.017091</td>
</tr>
</tbody>
</table>
From the above table it can be seen that the hysteretic energy loss factor of Auxetic-II configuration is 81.33% greater Regular configuration and 22.37% greater than Auxetic-I configuration. Auxetic-I has 76% higher loss factor than the Regular configuration.

6.3.1.2.2 Calculation of Quasi-static loss factor

The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores Regular, Auxetic-I and Auxetic-II are as shown in the Figure 6.6 and Figure 6.7

![ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models](image-url)
Figure 6.7 ALLSE plots of Regular, Auxetic-I, Auxetic-II in-plane models

$D_q$, the amount of damping energy dissipated per cycle is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.00205769</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.00830871</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.01092636</td>
</tr>
</tbody>
</table>
$U_q$, the average amount of strain energy is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.78752</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>2.5898</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>3.3681</td>
</tr>
</tbody>
</table>

$\eta_q$, the quasi-static loss factor is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.000415</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.000510</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.000516</td>
</tr>
</tbody>
</table>

From the table it can be seen that the quasi-static loss factor of Auxetic-II configuration is 19.4 % greater than Regular configuration and 1.1% greater than Auxetic-I configuration. Auxetic-I has 18.6% higher loss factor than the Regular configuration.

6.3.2 Soft Polycarbonate core

6.3.2.1 NLGEOM ON

6.3.2.1.1 Calculation of Hysteretic loss factor

Figure 6.8 shows the hysteresis curves of the three configurations of in-plane model.
$\eta_{hys}$, the area enclosed by the hysteresis curves shown in Figure 6.8, is shown in the table below.

**Table 6.9 Hysteretic loss factor ($\eta_{hys}$) of in-plane models**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_{hys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.410152</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.242799</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.152919</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Regular configuration is 40.8% and 62.72% greater than Auxetic-I, Auxetic-II configuration respectively. This trend is not similar to the previous case where viscoelastic material is
defined with a long-term elastic modulus. Also the magnitude of $\eta_{hys}$ are higher in this case.

6.3.2.1.2 Calculation of Quasi-static loss factor

The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores Regular, Auxetic-I and Auxetic-II are as shown in the Figure 6.9

Figure 6.10

![Figure 6.9 ALLCD plots of Regular, Auxetic-I, Auxetic-II in-plane models](image-url)
Figure 6.10 ALLSE plots of Regular, Auxetic-I, Auxetic-II in-plane models

\( D_q \), the amount of damping energy dissipated per cycle is reported in the table below

**Table 6.10 Energy dissipated \( D_q \)**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( D_q ) (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.271172</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.176528</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.118172</td>
</tr>
</tbody>
</table>

\( U_q \), the average amount of strain energy is reported in the table below

**Table 6.11 Strain Energy generated \( U_q \)**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( U_q ) (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>3.29554</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>3.2938</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>3.34209</td>
</tr>
</tbody>
</table>
\( \eta_q \), the quasi-static loss factor is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.013109</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.008529</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.005627</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Regular configuration is 34.94\% and 57\% greater than Auxetic-I, Auxetic-II configuration respectively.

6.4 Loss factors calculated for the Out-of-plane loading model

6.4.1 Stiff Polycarbonate core

6.4.1.1 NLGEOM OFF

6.4.1.1.1 Calculation of Hysteretic energy loss factor

The hysteresis curves of the sandwich plates with the three configurations of honeycomb core are as shown in the figure below
Figure 6.11 Hysteresis curves of three configurations of out-of-plane loading models

$\eta_{hys}$, the area enclosed by the hysteresis curves shown in Figure 6.11, is shown in the Table 6.13. Since the honeycomb is significantly stiffer in $X_3$ direction compared to the in-plane directions, lesser displacement is obtained at the center node resulting in lower $\eta_{hys}$ values compared to that of in-plane loading models.

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_{hys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.006712</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.007696</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.008037</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Auxetic-II configuration is 16.48% greater than Regular configuration and 4.24% greater than
Auxetic-I configuration. Auxetic-I has 12.79% higher loss factor compared to Regular configuration. Even though Auxetic-I has higher $E_{33}^s$ when compared to Regular, the high shear flexibility of Auxetic configuration [26] might have resulted in a higher displacement, there by resulting in higher area under the hysteresis curve.

6.4.1.1.2 Calculation of Quasi-static loss factor

The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores are as shown in the Figure 6.12 and Figure 6.13.
Figure 6.13 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models

It can be seen in the above figures that, Auxetic-II configuration has higher ALLCD, ALLSE compared to the other configurations. Auxetic-I has a higher ALLCD values but lower ALLSE values than regular configuration indicating that it might have higher loss factor.

$D_q$, the amount of damping energy dissipated per cycle is reported in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.003030</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.003328</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.003610</td>
</tr>
</tbody>
</table>

$U_q$, the average amount of strain energy is reported in the table below
Table 6.15 Strain energy generated $U_q$

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.331345</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.325659</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.343797</td>
</tr>
</tbody>
</table>

$\eta_q$, the quasi-static loss factor is reported in the table below

Table 6.16 Quasi-static loss factor ($\eta_q$) out-of-plane models

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.0014553</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.001626</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.001671</td>
</tr>
</tbody>
</table>

From the table it can be seen that the quasi-static loss factor of Auxetic-II configuration is 12.9% greater than Regular configuration and 2.69% greater than Auxetic-I configuration. Auxetic-I has a 10.5% higher loss factor compared to regular configuration.

6.4.1.2 NLGEOM ON

6.4.1.2.1 Calculation of Hysteretic energy loss factor

The hysteresis curves of the sandwich plates with the three configurations of honeycomb core are as shown in the figure below
Figure 6.14 Hysteresis curves of three configurations of out-of-plane loading models

\( \eta_{hys} \), the area enclosed by the hysteresis curves shown in Figure 6.14, is shown in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_{hys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.006714</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.007693</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.008042</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Auxetic-II configuration is 16.51% greater than Regular configuration and 4.34% greater than Auxetic-I configuration. Auxetic-I has a 12.7% higher loss factor compared to regular configuration.
6.4.1.2.2 Calculation of Quasi-static loss factor

The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores Regular, Auxetic-I and Auxetic-II are as shown in the Figure 6.15 and Figure 6.16.

![ALLCD plots](image1.png)

Figure 6.15 ALLCD plots of Regular, Auxetic-I, Auxetic-II out-of-plane models

![ALLSE plots](image2.png)

Figure 6.16 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models
ALLCD plot shows that Auxetic-II has higher dissipation energy and Regular has lower dissipation energy, which is similar to the trend shown in section 6.4.2.1

$D_q$, the amount of damping energy dissipated per cycle is shown in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.003031</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.003327</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.003608</td>
</tr>
</tbody>
</table>

$U_q$, the average amount of strain energy is shown in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.331817</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.324359</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.342445</td>
</tr>
</tbody>
</table>
\( \eta_q \), the quasi-static loss factor is given in the table below

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.001454</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.001632</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.001677</td>
</tr>
</tbody>
</table>

From the table it can be seen that the quasi-static loss factor of Auxetic-II configuration is 13.3% greater than Regular configuration and 2.68% greater than Auxetic-I configuration. Auxetic-I has a 10.9% higher loss factor than Regular configuration.

6.4.2 Soft Polycarbonate core

6.4.2.1 NLGEOM ON

6.4.2.1.1 Calculation of Hysteretic loss factor

The hysteresis curves of the sandwich plates with the three configurations of honeycomb core are as shown in the figure below.
Figure 6.17 Hysteresis curves of three configurations of out-of-plane loading models

\( \eta_{hys} \), the area enclosed by the hysteresis curves shown in Figure 6.17, is shown in the table below.

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_{hys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.448840</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.401990</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.415792</td>
</tr>
</tbody>
</table>

From the above table it can be seen that the hysteretic energy loss factor of Regular configuration is 10.43%, 7.36% greater than Auxetic-I and Auxetic-II configurations respectively.

6.4.2.1.2 Calculation of Quasi-static loss factor
The ALLCD and ALLSE curves of the sandwich plates with three configurations of honeycomb cores Regular, Auxetic-I and Auxetic-II are as shown in the

Figure 6.18 ALLCD plots of Regular, Auxetic-I, Auxetic-II out-of-plane models

Figure 6.19 ALLSE plots of Regular, Auxetic-I, Auxetic-II out-of-plane models
$D_q$, the amount of damping energy dissipated per cycle is reported in the table below

**Table 6.22 Dissipated Energy $D_q$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.210476</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.188156</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.19609</td>
</tr>
</tbody>
</table>

$U_q$, the average amount of strain energy is reported in the table below

**Table 6.23 Strain Energy generated $U_q$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_q$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>2.36739</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>2.32494</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>2.47118</td>
</tr>
</tbody>
</table>

$\eta_q$, the quasi-static loss factor is reported in the table below

**Table 6.24 Quasi-static loss factor ($\eta_q$) out-of-plane models**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.014149</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.01288</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.012629</td>
</tr>
</tbody>
</table>
From the above table it can be seen that the hysteretic energy loss factor of Regular configuration is 10.43%, 7.36% greater than Auxetic-I and Auxetic-II configurations respectively.
CHAPTER 7: DYNAMIC BEHAVIOR OF SANDWICH PLATES

7.1 Implicit Dynamic Analysis

Transient dynamic behavior of the sandwich plates is studied using this analysis. The analysis is setup in abaqus as follows.

- Step 0- Initial: The initial boundary conditions are specified
- Step 1- Implicit dynamic analysis: A uniform pressure load of linear ramp amplitude is applied for the step time of 0.001 secs.
- Step 2- Implicit dynamic analysis: A uniform pressure load of constant amplitude is applied for the step time of 0.099 secs for the out-of-plane model and 0.299 secs in the case of in-plane model.

The displacement of a node at the center of the face sheet and the energies of the whole model are recorded for this analysis. The analysis is divided into two steps to prevent the sandwich panels from experiencing a high magnitude instantaneous load. In the step 1, 2 a pressure load of 0.1Mpa is applied on the in-plane loading model and a load of 1MPa is applied on the out-of-plane loading model. The viscoelastic material is defined in time-domain for this analysis. Abaqus provides a numerical damping control parameter $\alpha$ for this analysis to introduce artificial damping effects. Artificial damping grows with the ratio of the time increment to the period of vibration of a mode. In order to give more importance to the damping effects due to material in this study the $\alpha$ value is changed from the default value of -0.05 to -0.01 to introduce minimal artificial damping.
7.2 Dynamic Loss Factor

In this study the damping capability of the honeycomb sandwich panels from the implicit dynamic analysis is predicted using a dynamic loss factor ($\eta_d$), which is defined as

$$\eta_d = \frac{D_d}{U_d} \quad (7.1)$$

In the above equation $D_d$ corresponds to the amount of energy dissipated by the system, which is given by ALLCD output at the end of the step time. $U_d$ corresponds to the strain energy of the system, given by ALLSE output at the end of the step time. The higher the value of $\eta_d$, the greater is the amount of damping.

7.2.1 Results of in-plane loading model

7.2.1.1 Stiff Polycarbonate core

Figure 7.1 shows the displacement of a node at the center of the face sheet over the period of step time. From the figure it can be seen that the regular honeycomb is stiffer in the in-plane direction compared to the auxetic honeycombs. Auxetic-II shows a higher displacement for the same pressure load applied. This behavior is as expected from the effective properties mentioned in Table 3.3.
Figure 7.1 Comparison of displacement of a node at the center of face sheet for the three configurations of in-plane model with long term viscoelastic core.

Figure 7.2 and Figure 7.3 shows the ALLCD and ALLSE plots for the whole model. The numerator and denominator of the dynamic loss factor ($\eta_d$) are taken from these plots.
Figure 7.2 ALLCD comparison of the three configurations of sandwich plates

Figure 7.3 ALLSE comparison of the three configurations of sandwich plates
$D_d$, the energy dissipated by the system is listed in the table below

**Table 7.1 Dissipated Energy $D_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.007743</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.069545</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.089546</td>
</tr>
</tbody>
</table>

$U_d$, the strain energy of the system is listed in the table below

**Table 7.2 Strain energy generated $U_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.78716</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>2.79734</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>2.83826</td>
</tr>
</tbody>
</table>

$\eta_d$, the dynamic loss factor calculated from the ratio $\frac{D_d}{U_d}$ is given in the table

**Table 7.3 Dynamic Loss factor $\eta_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.0098366</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.0248611</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.0315496</td>
</tr>
</tbody>
</table>
The dynamic loss factor of Auxetic-II configuration is 68.82% greater than Regular configuration and 21.2% greater than Auxetic-I configuration. Auxetic-I has 60.4% higher loss factor than Regular configuration.

7.2.1.2 Soft Polycarbonate core

Figure 7.4 shows the displacement of a node at the center of the face sheet over the period of step time.

![Graph showing displacement over time for different configurations](image)

**Figure 7.4 Comparison of displacement of a node at the center of face sheet for the three configurations of in-plane models with Instantaneous viscoelastic core**

Figure 7.5 and Figure 7.6 shows the ALLCD and ALLSE plots for the whole model.
Figure 7.5 ALLCD comparison of the three configurations of sandwich plates

Figure 7.6 ALLSE comparison of the three configurations of sandwich plates
$D_d$, the energy dissipated by the system is listed in the table below

**Table 7.4 Dissipated Energy $D_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>1.58556</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>2.9768</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>2.5044</td>
</tr>
</tbody>
</table>

$U_d$, the strain energy of the system is listed in the table below

**Table 7.5 Strain energy generated $U_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>3.17803</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>3.43406</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>3.80818</td>
</tr>
</tbody>
</table>

$\eta_d$, the dynamic loss factor calculated from the ratio $\frac{D_d}{U_d}$ is given in the table

**Table 7.6 Dynamic Loss factor $\eta_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.498912</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.866684</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.6576367</td>
</tr>
</tbody>
</table>

97
The dynamic loss factor of Auxetic-I configuration is 42.43% greater than Regular configuration and 24.12% greater than Auxetic-II configuration.

7.2.2 Results of Out-of-plane loading model

7.2.2.1 Stiff Polycarbonate core

Figure 7.7 shows the displacement of a node at the center of the face sheet over the period of step time.

**Figure 7.7 Comparison of displacement of a node at the center of face sheet for the three configurations of sandwich plates**

From the Figure 7.7 it can be seen that the Regular and Auxetic-I has approximately the same amount of displacement, even though Auxetic-I has higher $E_{33}^c$. As mentioned earlier, this might be due to the high shear flexibility of Auxetic configuration

Figure 7.8 and Figure 7.9 shows the ALLCD and ALLSE plots for the whole model.
Figure 7.8 ALLCD comparison of the three configurations of sandwich plates

Figure 7.9 ALLSE comparison of the three configurations of sandwich plates
$D_d$, the energy dissipated by the system is listed in the table below

**Table 7.7 Dissipated energy $D_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$D_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.005625</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.006077</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.006574</td>
</tr>
</tbody>
</table>

$U_d$, the strain energy of the system is listed in the table below

**Table 7.8 Strain energy generated $U_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$U_d$ (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.331863</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.322726</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.339762</td>
</tr>
</tbody>
</table>

$\eta_d$, the dynamic loss factor calculated from the ratio $\frac{D_d}{U_d}$ is given in the table

**Table 7.9 Dynamic Loss factor $\eta_d$**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>$\eta_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.016950</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.018831</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.019351</td>
</tr>
</tbody>
</table>
The dynamic loss factor of Auxetic-II configuration is 12.4% greater than Regular configuration and 2.68% greater than Auxetic-I configuration. Auxetic-I has 10% higher loss factor than Regular configuration.

7.2.2.2 Soft Polycarbonate core

Figure 7.10 shows the displacement of a node at the center of the face sheet over the period of step time

Figure 7.10 Comparison of displacement of a node at the center of face sheet for the three configurations of sandwich plates

Figure 7.11 and Figure 7.12 shows the ALLCD and ALLSE plots for the whole model.
Figure 7.11 ALLCD comparison of the three configurations of sandwich plates

Figure 7.12 ALLSE comparison of the three configurations of sandwich plates
\( D_d \), the energy dissipated by the system is listed in the table below

**Table 7.10 Dissipated Energy \( D_d \)**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( D_d ) (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.636999</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.581556</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.753345</td>
</tr>
</tbody>
</table>

\( U_d \), the strain energy of the system is listed in the table below

**Table 7.11 Strain energy generated \( U_d \)**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( U_d ) (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>3.50369</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>3.68605</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>4.53673</td>
</tr>
</tbody>
</table>

\( \eta_d \), the dynamic loss factor calculated from the ratio \( \frac{D_d}{U_d} \) is given in the table

**Table 7.12 Dynamic Loss factor \( \eta_d \)**

<table>
<thead>
<tr>
<th>Honeycomb configuration</th>
<th>( \eta_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>0.181808</td>
</tr>
<tr>
<td>Auxetic-I</td>
<td>0.157772</td>
</tr>
<tr>
<td>Auxetic-II</td>
<td>0.155054</td>
</tr>
</tbody>
</table>
The dynamic loss factor of Regular configuration is 13.22% greater than Regular configuration and 14.72% greater than Auxetic-I configuration.
CHAPTER 8 : CONCLUSION AND FUTURE WORK

8.1 Conclusive remarks

The main goal of this thesis is to investigate the damping capability of honeycomb sandwich plates with viscoelastic cores and the effect of cellular geometry of honeycomb unit cells on the structural damping of the sandwich plates. Honeycomb cores made of conventional hexagons (Regular) and un-conventional hexagons with effective negative in-plane Poisson’s ratio (Auxetic) are considered to be of interest. Therefore Finite element models of Sandwich plates with three different core configurations a) Regular b) Auxetic–I c) Auxetic -II are developed in ABAQUS. Viscoelastic Prony series coefficients derived by curve fitting uni-axial shear stress data are used to define polycarbonate viscoelastic material for the honeycomb cores. Aluminum is assigned to the face sheets of the sandwich plates. As a secondary objective the effect of in-plane and out-of-plane loading of honeycombs is also studied. Natural frequency extraction, direct –steady state, quasi-static and implicit dynamic analyses are conducted on the sandwich plates using ABAQUS.

Natural frequency extraction showed that sandwich plates with Regular honeycomb core displayed higher undamped modal frequencies when compared to both of the Auxetic configurations for any type of loading. Loss factors calculated using half –power band width method showed that in-plane Auxetic-II had higher damping capacity compared to the other configurations irrespective of the polycarbonate material definition. In the case of out- of-plane model Auxetic-II showed higher damping when
soft polycarbonate material is used for the core and Auxetic-I displayed higher damping when stiff polycarbonate is used. The models with soft polycarbonate core shifted the damped frequencies to a slightly lesser value than the undamped natural frequencies, where as the models with stiff polycarbonate core shifted the damped frequencies to a higher magnitude. As a result following the trend of material damping factor of polycarbonate shown in Figure 3.5 the loss factors of models with soft polycarbonate core are higher compared to the ones with stiff polycarbonate core.

Loss factors calculated from quasi-static analysis and implicit dynamic analysis with non-linear geometry effects included and soft polycarbonate core are reported in the table below.

**Table 8.1 Loss factors for three configurations of honeycombs with soft polycarbonate core**

<table>
<thead>
<tr>
<th>Loading direction</th>
<th>Honeycomb configuration</th>
<th>Hysteric loss factor ($\eta_{hys}$)</th>
<th>Quasi-static loss factor ($\eta_q$)</th>
<th>Dynamic loss factor ($\eta_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-plane Loading</td>
<td>Regular</td>
<td>0.410152</td>
<td>0.013109</td>
<td>0.498912</td>
</tr>
<tr>
<td></td>
<td>Auxetic-I</td>
<td>0.242799</td>
<td>0.008529</td>
<td>0.866684</td>
</tr>
<tr>
<td></td>
<td>Auxetic-II</td>
<td>0.152919</td>
<td>0.005627</td>
<td>0.6576367</td>
</tr>
<tr>
<td>Out-of plane Loading</td>
<td>Regular</td>
<td>0.448840</td>
<td>0.014149</td>
<td>0.181808</td>
</tr>
<tr>
<td></td>
<td>Auxetic-I</td>
<td>0.401990</td>
<td>0.01288</td>
<td>0.157772</td>
</tr>
<tr>
<td></td>
<td>Auxetic-II</td>
<td>0.415792</td>
<td>0.012629</td>
<td>0.155054</td>
</tr>
</tbody>
</table>
Loss factors calculated from quasi-static analysis and implicit dynamic analysis with non-linear geometry effects included and stiff polycarbonate core are reported in the table below.

**Table 8.2 Loss factors for three configurations of honeycombs with stiff polycarbonate core**

<table>
<thead>
<tr>
<th>Loading direction</th>
<th>Honeycomb configuration</th>
<th>Hysteretic loss factor ($\eta_{hys}$)</th>
<th>Quasi-static loss factor ($\eta_q$)</th>
<th>Dynamic loss factor ($\eta_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-plane Loading</td>
<td>Regular</td>
<td>0.003189</td>
<td>0.000415</td>
<td>0.0098366</td>
</tr>
<tr>
<td></td>
<td>Auxetic-I</td>
<td>0.013267</td>
<td>0.000510</td>
<td>0.0248611</td>
</tr>
<tr>
<td></td>
<td>Auxetic-II</td>
<td>0.017091</td>
<td>0.000516</td>
<td>0.0315496</td>
</tr>
<tr>
<td>Out-of plane Loading</td>
<td>Regular</td>
<td>0.006714</td>
<td>0.001454</td>
<td>0.016950</td>
</tr>
<tr>
<td></td>
<td>Auxetic-I</td>
<td>0.007693</td>
<td>0.001632</td>
<td>0.018831</td>
</tr>
<tr>
<td></td>
<td>Auxetic-II</td>
<td>0.008042</td>
<td>0.001677</td>
<td>0.019351</td>
</tr>
</tbody>
</table>

From this study and Table 8.1 Table 8.2 following things can be concluded,

- Auxetic honeycombs can be designed to have same effective extensional in-plane modulus and mass as that of regular honeycombs.
- Frequency and time dependent viscoelastic material properties can be directly applied using Prony series coefficients in ABAQUS simulations.
- Regular honeycomb sandwich plates have higher natural frequencies than the Auxetic configurations considered in this study.
- Viscoelastic damping offers shift in the resonant frequencies and also controls the resonant peak amplitudes.
• Loss factors are higher for the models with a soft polycarbonate core compared to the models with stiff polycarbonate core.

• Regular honeycomb sandwich plates showed higher loss factors compared to auxetic honeycomb sandwich plates when a soft polycarbonate material is assigned to the core.

• Auxetic-II honeycomb sandwich plates showed higher loss factors compared to the other two configurations for any type of loading when a stiff polycarbonate material is assigned to the core.

8.2 Suggestions for Future work

1. In the present work only two configurations of Auxetic honeycombs were studied. This work can be extended by deriving other configurations of Auxetic honeycombs by modifying unit cell parameters which may offer better damping capabilities than the ones used in this study.

2. Develop an analytical model and perform physical experiments to validate the FEA results for sandwich structures under quasi-static and dynamic loads.

3. Foam filled honeycomb foam cores with in-plane negative Poisson’s can also be tried as the core material for sandwich plates to see their advantages over honeycomb structures in terms of damping.
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