SUPPLY CHAIN NETWORK DESIGN: RISK-averse vs. Risk-Neutral Decision Making

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SUPPLY CHAIN NETWORK DESIGN:
RISK-APERSE VS. RISK-NEUTRAL DECISION MAKING

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Industrial Engineering

by
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ABSTRACT

Recent events, such as the Heparin tragedy, highlight the necessity for designers and planners of supply chain networks to consider the risk of disruptions in spite of their low probability of occurrence. One effective way to hedge against supply chain network disruptions is to have a robustly designed supply chain network. This involves strategic decisions, such as choosing which markets to serve, which suppliers to source from, the location of plants, the types of facilities to use, and tactical decisions, such as production and capacity allocation. In this dissertation, we focus on models for designing supply chain networks that are resilient to disruptions.

We consider two types of decision making policies. A risk-neutral decision making policy is based on the cost minimization approach, and the decision-maker defines the set of decisions that minimize expected cost. We also consider a risk-averse policy wherein rather than selecting facilities that minimize expected cost, the decision-maker uses a Conditional Value-at-Risk approach to measure and quantify risk. However, such network design problems belong to class of NP hard problems. Accordingly, we develop efficient heuristic algorithms and metaheuristic approaches to obtain acceptable solutions to these types of problems in reasonable runtimes so that the decision making process is facilitated with at most a moderate reduction in solution quality. Finally, we perform statistical analyses (e.g., logistic regression) to assess the likelihood of selection for each facility. These models allow us to identify the factors that impact facility selection in both the risk-neutral and risk-averse policies.
DEDICATION

I dedicate this dissertation to my family without whom I would not be here.
ACKNOWLEDGMENTS

Over the past four years I have received support and encouragement from a great number of individuals. In the first place I would like to express my genuine gratitude to my advisor, Dr. Mary Beth Kurz, for her supervision, advice, constructive criticisms, and guidance from the very early stage of this research as well as giving me extraordinary experiences throughout the work. Thank you for your invaluable support and for being patient with me. My sincerest gratitude is also extended to the other members of my dissertation committee. I would like to gratefully acknowledge Dr. Julia Sharp for her guidance and crucial contribution, which made her a backbone of this research. I truly enjoyed working with you Dr. Sharp, and benefited from your vast knowledge of statistic. I gratefully thank Dr. Scott Mason and Dr. Kevin Taaffe for their constructive and insightful comments and valuable collaborations on this work over the last few years. Dr. Maria Mayorga has been a great inspiration on doing innovative and insightful research. I would like to extend my appreciation to other faculty members and staff at Department of Industrial Engineering.

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CHAPTER ONE

1. INTRODUCTION

1.1. Motivation

Over the last several decades, considerable effort has been expended in order to make supply chains leaner. Recent studies indicate that, while this effort has effectively reduced operational costs, it has also increased the vulnerability of supply chains to disruption risks [1,2]. Disruptions in supply chains can be defined as “unplanned and unanticipated events that disrupt the normal flow of goods and materials within a supply chain [3].” As an example, in 1999, an earthquake in Taiwan had a dramatic impact on the global semiconductor market. This caused a temporary global shortage of semiconductor components with production down times that ranged from two to four weeks [4]. This disruption resulted in significant physical costs (e.g., damage to facilities, inventory, electronic networks, and infrastructure) as well as lost revenue due to decreased sales and production, and missed deliveries.

Some supply chain disruptions are not only costly, but may have catastrophic consequences in spite of their low probability of occurrence. For instance, in the healthcare supply chain, it is not acceptable to experience a late delivery or product shortage if patients’ lives are in danger. Nevertheless, several examples of disruptions in healthcare supply chains exist. For example, the disruption of a flu vaccine manufacturer in Bristol, U.K. in 2004 resulted in disastrous consequences. The U.K. government stopped production when U.S. regulators inspected a manufacturing plant and found evidence of bacterial contamination problems. This reduced the U.S.’s
supply of the vaccine by nearly 50% during the 2004-2005 flu season [5]. While many may feel influenza is a minor disease, influenza causes 250,000 to 500,000 deaths per year and is the sixth most common cause of death in the U.S. when combined with pneumonia [6]. Pharmaceutical and healthcare supply chains are also susceptible to disruptions caused by contamination. Heparin, a widely-used blood-thinning medicine that is made from pig intestines, was contaminated by an undetected outbreak of blue ear pig disease in China in 2008. This contamination led to 81 patient deaths and to hundreds of allergic reactions in the US [7]. The investigation of the event involved several government agencies, university researchers, and a biotech company that had a generic heparin under FDA review. Although no one at the time knew what was causing the reactions, members of Congress concluded that the issue was the result of “regulatory failure” [7]. In another supply chain disruption, a baby food producer who purchased vitamin supplements from a Chinese supplier found out that the supplements were contaminated by cement [8]. This incident involved twenty-two Chinese and ten global manufacturers and led to kidney problems and kidney stones in Chinese babies and illustrates the result of poor or failed inspection by FDA or production facilities [9]. A 2008 US Government Accountability Office (GAO) report indicated that in 2007, the FDA inspected approximately 8% of foreign facilities and declared that, at that rate, it would take 13 years to inspect all such facilities [10]. On the other hand, inspection of raw materials is usually a substantial portion of manufacturing costs and manufacturing facilities search for cost-reduction opportunities [11]. As a result, while consumers may assume all goods are inspected, some goods are either not inspected or are inadequately inspected.
All of these examples with the exception of the influenza example could have been detected by inspecting the upstream suppliers’ (suppliers of raw materials) and/or producers’ production line and manufacturing plants; however, the case of influenza vaccine was undetectable and was only a consequence of facilities’ failure.

These types of incidents and the reality that not all goods entering the pharmaceutical supply chain can be inspected by the FDA or other equivalent regulatory agencies accentuate the need to consider supply chain risks and (supply) disruptions in the design and planning stages. However, managers typically tend to underestimate the impact of supply chain disruptions, deceived by their small probability of occurrence, and design supply chain networks to minimize the operational cost without considering any disruption scenarios. Unfortunately, once a disruption occurs, there are few opportunities to change existing supply chain infrastructure. Therefore, to hedge against supply chain disruptions, it is critical to consider potential disruptions during the design of the supply chain networks so that the supply chain can perform acceptably in the event of an unplanned disruption.

The risk-neutral policy may arise due to forces of globalization, which encourage firms to aggressively design their supply network base around the world in order to find opportunities for reducing supply chain costs. However, emphasizing supply chain costs may make that chain fragile and more susceptible to the risk of disruption.

This research focuses on risks that impact strategic and tactical decisions. We discuss models for designing supply chain networks that are resilient to disruptions. We focus on tainted materials, inspired by the contamination cases described above. We reduce (but do not completely eliminate) tainted materials by introducing
producer-implemented inspections. In all cases, we assume that some tainted and untainted materials are shipped and model the risk of shipping tainted materials with a penalty cost; when we inspect facilities; we also incur costs related to inspection and the disposal of discovered tainted materials.

The objective is to design the supply chain infrastructure under the risk of disruption, so that it operates with highest possible efficiency (i.e., at low cost) both normally and when a disruption occurs.

Supply network design problems are usually modeled as a 0–1 mixed integer programming problem and many have been shown to be NP-hard [12],[13]. While substantial efforts have been devoted to develop exact methods for this type of problem (see, e.g., [14,15]), (meta) heuristics must be resorted to when dealing with large sized instances. Accordingly, in this dissertation, we develop efficient heuristic algorithms and various metaheuristics approaches to obtain acceptable solutions to these types of problems in reasonable runtimes.
1.2. Main Contribution

In this dissertation, three models will be considered to address the problem described above. The techniques utilized in our study encompass mathematical models, computational algorithm design and solution procedures, and statistical analysis.

The first model considers a single-period, single-product supply chain with capacitated facilities, modeled as a two-stage mixed integer stochastic program to minimize the expected cost. The first-stage decisions represent the strategic decisions of facility selection. In the second-stage, tactical decisions, namely inspection policy and capacity allocation, are considered in response to disruptions at facility sites.

The consideration of facility inspection is a key parameter in our model. Tragedies, such as the Heparin and Chinese baby food manufacturing incidents, inspired this aspect of the work. If the risk of shipping tainted materials can be minimized prior to such tragedies, producers can decrease liability and improve consumer safety. Designers of several types of supply chains, such as healthcare, pharmaceutical, cosmetic and beauty, and food and dairy products should be interested in the network design insights provided by our model related to reducing the risk of tainted products reaching consumers.

To deal with uncertainty, a scenario-based model is proposed. Researchers have used this type of model to deal with uncertainties of supply or demand. The decision-maker identifies the set of potential scenarios and estimates the likelihood of each scenario occurring. Increasing the number of consumers, facilities and consequently, the number of scenarios to describe the underlying distributions
provides a better description of the actual problem, but it also increases the size of the problem. Experience from solving similar problems using commercial software in this research shows that the number of scenarios used has a significant impact on the solution time. Our contribution is the development of several heuristics and metaheuristics to solve the complicated models presented in our work effectively and to obtain acceptable solutions to the models in a reasonable time, demonstrated through extensive computational tests.

In the second model, we consider a risk-averse policy wherein rather than selecting facilities that minimize the expected cost; the decision-maker uses a method from financial engineering – Conditional Value-at-Risk (CVaR) – to measure and quantify risk and to define what qualifies as a worst-case scenario. This methodology allows the user to specify the extent to which these worst-case scenarios should be avoided. The CVaR approach also allows a decision-maker to control the amount of supply to procure based on a desired risk level to avoid the worst-case operational scenarios. To the best of our knowledge, no authors have addressed the problem of the CVaR and facility/supplier selection concurrently. Our contribution here is two-fold. First, we will develop various heuristic and metaheuristic methods to solve the resultant large models. Second, we will provide valuable managerial insights by comparing the results of the CVaR model with the results of the cost minimization model.

Finally, we will perform a statistical analysis to consider a logistic regression and multinomial logistic regression models to identify the factors that impact our strategic decisions in both the CVaR and cost minimization models. To the best of
our knowledge, the problem we address and this type of analysis have not been tackled previously in the academic literature.

1.3. Significance of the Dissertation

The outcome of our models and the statistical analyses will enable managers to select the most appropriate suppliers for their pharmaceutical supply chain and to make capacity allocation and inspection implementation decisions under both risk-neutral and risk-averse policies. The proposed models also determine when and where inspections and monitoring should be performed to reduce the risk of tainted material reaching consumers. This study will aid practitioners designing supply chains and policy makers devising various disruption mitigation strategies related to the costs and risks in supply chain. The significance of this research is summarized as following:

- Our goal will be to solve the above-stated problem using heuristics or metaheuristics to effectively solve large sized problems.
- The objective of most of the papers we reviewed is to minimize the cost when supply is random. However, our approach will be to use CVaR and to define what qualifies as a worst-case scenario. Instead of selecting facilities and allocating production across them to minimize expected costs, the CVaR approach allows a decision-maker to select the facilities and control the amount of supply based on avoidance of the worst-case scenario.
- In this research, we perform logistic and multinomial logistic regression statistical analysis to identify the factors that impact our strategic decisions in both the CVaR and cost minimization models. To the best of our knowledge,
the problem we address and the regression model we use have not been previously used in this type of research.

- This research will explore the trade-off between expected cost/profit and risk. The outcome of the model will allow managers and decision-makers to decide whether they should order from low cost, risky facilities or more reliable but more expensive facilities.

1.4. Structure of Dissertation

The dissertation consists of six chapters. Chapter 1 presents an overview of the research issue, goal and objectives of the research.

Chapter 2 reviews literature concerning supply network design under disruptions, risk minimization in supply chain network design, and finally, multi-objective approaches in supply chain network design problems.

Chapter 3 considers the modeling approach and the solutions procedures that can be applied to Model 1.

Chapter 4 presents CVaR to minimize risk, resulting in Model 2. Heuristic and metaheuristic approaches are developed to solve Model 2.

Chapter 5 presents the logistic regression model and the multinomial logistic regression model, which are then used to identify the factors that impact strategic decisions in both the CVaR and cost minimization models. This dissertation is concluded in Chapter 6 with a discussion of future research directions.
CHAPTER TWO

2. Literature Review

In this section, we provide a brief overview of the current literature state on the following research streams:

- Supply chain design under risk and uncertainty
- Risk minimization in supply chain network design

2.1. Supply chain design under risk and uncertainty

2.1.1. Introduction

In the literature, supply chain disruptions can be classified as either small-scale random disruptions or large-scale major disruptions. Small-scale random disruptions are those that are often caused by the usual random variations in, for example, consumer demand or delivery lead time. This category has received a fairly large amount of attention in the literature. Normally, the effects of such disruptions can be reduced by standard approaches such as maintaining increased safety stock inventories [16]. The concentration of this research is on large-scale disruptions which have received less attention in the literature. These kinds of disruptions tend to occur infrequently but have major effects on the operation of the supply chain. Recent events such as the Heparin tragedy, hurricane Katrina, the September 11 disaster, and the 2011 Japanese earthquake and tsunami belong to this category. Such events highlight the necessity for designers and planners to consider the risk of disruptions in supply chain networks.
The design decisions of a supply chain network are made on two levels: the strategic level and the tactical level. Strategic level design involves deciding the configuration of the network, i.e., the number, selection, location, and technology of the facilities. Tactical level design involves deciding the operation of the network, i.e., controlling material flows for purchasing, processing, distribution of products, etc.

2.1.2. Related Literature

Supply chain network design under the risk of disruption has recently received an increasing amount of attention from the research community. Table 2-1 summarizes a number of the most recent research efforts in this area. In each column, we identify the author(s) (in alphabetical order), year, description of their research, each study's problem type (Problem Type: Deterministic or Stochastic), the source of uncertainty (Random Source: Supply or Demand), decision type (Strategic, Tactical), capacitated or uncapacitated, approach details, and if a metaheuristic approach is used or not.

We note that the vast majority of the supply chain disruption literature focuses on demand uncertainty. In terms of the classification scheme in Table 2-1, our model is stochastic and focuses on the supply-side, the decision type is strategic and tactical, the capacity is limited (capacitated), the type of model is mixed integer stochastic programming, and the solution approach is heuristic and metaheuristic. The literature summary indicates that this has not been considered in the current body of literature.
Table 2-1 Literature review summary on supply chain design under uncertainty

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Problem type</th>
<th>Source of uncertainty</th>
<th>Decision type</th>
<th>Capacitated?</th>
<th>Approach details</th>
<th>Metaheuristic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aghezzaf [17]</td>
<td>2004</td>
<td>S</td>
<td>Demand</td>
<td>Strategic</td>
<td>Yes</td>
<td>Robust optimization</td>
<td>No</td>
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<tr>
<td>Agrawal et al. [18]</td>
<td>2000</td>
<td>S</td>
<td>Demand</td>
<td>Tactical</td>
<td>No</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
<tr>
<td>Alliparmak et al.  [19]</td>
<td>2006</td>
<td>D</td>
<td>-</td>
<td>Strategic</td>
<td>Yes</td>
<td>MIP</td>
<td>Yes</td>
</tr>
<tr>
<td>Applequist et al.  [20]</td>
<td>2000</td>
<td>S</td>
<td>Demand</td>
<td>Strategic</td>
<td>No</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
<tr>
<td>Arora, Arora [21]</td>
<td>2010</td>
<td>D</td>
<td>-</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP</td>
<td>No</td>
</tr>
<tr>
<td>Blanchini et al.   [22]</td>
<td>1997</td>
<td>S</td>
<td>Demand</td>
<td>Tactical</td>
<td>No</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
<tr>
<td>Caserta and Rico   [23]</td>
<td>2008</td>
<td>D</td>
<td>-</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP</td>
<td>Yes</td>
</tr>
<tr>
<td>Cui et al. [25]</td>
<td>2010</td>
<td>S</td>
<td>Supply</td>
<td>Strategic,</td>
<td>No</td>
<td>Stochastic Modeling</td>
<td>No</td>
</tr>
<tr>
<td>Hoff et al. [26]</td>
<td>2007</td>
<td>S</td>
<td>Demand</td>
<td>Tactical</td>
<td>No</td>
<td>Stochastic Modeling</td>
<td>Yes</td>
</tr>
<tr>
<td>Jayaraman and Ross [27]</td>
<td>2003</td>
<td>D</td>
<td>-</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP</td>
<td>Yes</td>
</tr>
<tr>
<td>Kasilingam and Lee [28]</td>
<td>1996</td>
<td>S</td>
<td>Demand</td>
<td>Strategic,</td>
<td>No</td>
<td>MIP-Commercial Software</td>
<td>No</td>
</tr>
<tr>
<td>Keshkin and Uster [29]</td>
<td>2007</td>
<td>D</td>
<td>-</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP</td>
<td>Yes</td>
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<tr>
<td>Lin [30]</td>
<td>2009</td>
<td>S</td>
<td>Demand</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP/Lagrangean relaxation</td>
<td>No</td>
</tr>
<tr>
<td>Miranda and Garrido [31]</td>
<td>2004</td>
<td>S</td>
<td>Demand</td>
<td>Strategic,</td>
<td>No</td>
<td>MIP/Lagrangean relaxation</td>
<td>No</td>
</tr>
<tr>
<td>Qi and Shen [32]</td>
<td>2007</td>
<td>S</td>
<td>Supply</td>
<td>Strategic,</td>
<td>No</td>
<td>MIP/stochastic programming</td>
<td>No</td>
</tr>
<tr>
<td>Qi et al. [33]</td>
<td>2010</td>
<td>S</td>
<td>Supply</td>
<td>Strategic,</td>
<td>No</td>
<td>MIP/stochastic programming</td>
<td>No</td>
</tr>
<tr>
<td>Shen et al. [34]</td>
<td>2008</td>
<td>S</td>
<td>Supply</td>
<td>Strategic,</td>
<td>No</td>
<td>MIP/stochastic programming</td>
<td>No</td>
</tr>
<tr>
<td>Xu and Nozick [35]</td>
<td>2009</td>
<td>S</td>
<td>Supply</td>
<td>Strategic,</td>
<td>Yes</td>
<td>MIP/Lagrangean relaxation</td>
<td>No</td>
</tr>
</tbody>
</table>
2.2. Risk minimization in supply chain network design

2.2.1. Introduction

As noted earlier, some supply chain disruptions are not only costly, but may have catastrophic consequences in spite of their low probability of occurrence. For instance, we discussed the healthcare supply chain, where it is not acceptable to experience a late delivery or product shortage if patients’ lives are in danger. Hence, catastrophic healthcare delivery problems must be avoided earnestly. Nevertheless, to the best of our knowledge, the objective of the majority of the papers reviewed is based on a cost minimization approach and the decision-makers are risk-neutral. That might arise due to globalization where firms are aggressively designing their supply network base around the world to find opportunities for reducing supply chain costs. However, emphasizing supply chain cost might make the supply chain fragile and more susceptible to the risk of disruptions.

Our plan is to manage the cost associated with the risk of supply disruptions. Therefore, we consider a risk-averse policy where the decision-maker, rather than selecting facilities and identifying the assignment base set that minimizes expected cost, uses a Conditional Value-at-Risk (CVaR) approach to measure and quantify the risk and also define what qualifies as a worst-case scenario. This methodology allows the user to specify to what extent these worst-case scenarios should be avoided. In the next section, we discuss the articles on risk management in supply chain network design.
2.2.2. Related Literature

A small body of literature has addressed the risk-averse approach to decision-making in the supply chain network design. With the objective of either minimizing the expected opportunity loss or minimizing the maximum opportunity loss, Current et al. [36] studied problems in which the total number of facilities to be located is uncertain over a planning horizon. Gaonkar and Vsiwanadham [37] developed a model for selecting suppliers to minimize the expected shortfall under disruption. The general idea was to match demand and supply using cost as the single criterion. Other researchers also developed risk-based analytical approaches to supplier selection and evaluation [38-40]).

Researchers have also applied the concept of mean-variance optimization (see [41]) in the supply chain network design problem [42-48]. The general idea was that the firms consider both costs and risks in their model by using a mean-variance approach to minimize the expected total cost and valuation of the risk. The objective function is of the form \( Z = E(\tilde{s}) - \lambda \text{Var}(\tilde{s}) \) where \( \tilde{s} \) denotes the random payoff and \( \lambda \) is a measure of risk aversion [47]. However, several limitations are associated with this mean-variance formulation. For instance, the estimate of risk by mean-variance is only suitable when returns are normally distributed (see Pardalos et al. [49]).

Other researchers have used Value at Risk (VaR) to make strategic/tactical decisions in the supply chain network design [50-54]. VaR is a risk measure that mostly focuses on rare events and provides the value that can be expected to be lost during severe, adverse market fluctuations [55]. However, there are some problems associated with VaR, which will be discussed in Section 4.2. Therefore, these issues led some researchers to use an alternative measure CVaR, in a few areas such as
portfolio optimization [56], transportation and fleet allocation [57], market/demand selection problem [58], electricity procurement problem [59], and facility location problem (see [60]). By utilizing the CVaR concept, Chen et al. [61] addressed an uncapacitated stochastic $p$-median problem in which the objective was to minimize the expected regret associated with a subset of worst-case scenarios whose collective probability of occurrence is not more than $1 - \alpha$. In their model, the demand and the distance between the demand nodes and the facilities were stochastic.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Problem type</th>
<th>Source of uncertainty</th>
<th>Decision type</th>
<th>Risk measure</th>
<th>Approach details</th>
<th>Metaheuristic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Chen et al. [61]</td>
<td>2006</td>
<td>D</td>
<td>-</td>
<td>Strategic</td>
<td>$\alpha$-reliable</td>
<td>MIP</td>
<td>No</td>
</tr>
<tr>
<td>2 Noyan [62]</td>
<td>2011</td>
<td>S</td>
<td>Demand</td>
<td>Strategic, Tactical</td>
<td>CVaR</td>
<td>MIP/Decomposition Method</td>
<td>No</td>
</tr>
<tr>
<td>3 Ravindaran [39]</td>
<td>2010</td>
<td>S</td>
<td>Demand</td>
<td>Strategic, Tactical</td>
<td>VaR</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
<tr>
<td>4 Sawik [63]</td>
<td>2010</td>
<td>S</td>
<td>Supply</td>
<td>Strategic, Tactical</td>
<td>CVaR</td>
<td>MIP</td>
<td>No</td>
</tr>
<tr>
<td>5 Shen et al. [64]</td>
<td>2008</td>
<td>S</td>
<td>Demand</td>
<td>Strategic</td>
<td>CVaR</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
<tr>
<td>6 Wagner et al. [53]</td>
<td>2009</td>
<td>S</td>
<td>Demand</td>
<td>Strategic</td>
<td>VaR</td>
<td>Stochastic modeling</td>
<td>No</td>
</tr>
</tbody>
</table>

2.3. Summary

This chapter has reviewed the literature that pertains to the modeling of the effects of disruptions on supply chain operations. We initially considered the literature that pertains to the supply chain design under risk and uncertainty in Section 2.1. The literature summary indicates the research goal and some modeling assumptions have not been addressed in the current body of literature. Next we also employ a risk-averse approach in Chapter 4, as discussed in Section 2.2. The material in this chapter contributes important understanding of supply chain risk management particularly, within the context of the pharmaceutical supply chain.
CHAPTER THREE


3.1. Introduction

In this chapter, we consider a single-period, single-product supply chain with capacitated facilities, which is modeled as a two-stage stochastic 0-1 problem. The goal of the model consists of selecting facilities in the first-stage, and defining capacity (product) allocation among the consumers and the inspection policy in the second stage. The objective is to minimize the expected cost composed of the fixed cost of selecting facilities, the cost of shipping untainted products, the cost of shipping tainted products, the cost of inspecting facilities, and the cost of discarding tainted products.

While mixed integer stochastic programming models for this problem may allow for an exact solution in some situations, it can be very challenging to draw concrete analytical insights from such models and to obtain good solutions for large instances within a limited time frame since the problems are NP-hard [13]. As a result, we develop several heuristics and metaheuristics to efficiently solve and handle large sized problems. Finally, some experimental results are reported to obtain more insight from the model.

The remainder of this chapter is organized as follows. In Section 3.2 the problem description is discussed. The mathematical formulation is introduced in Section 3.3. In Sections 3.4 and 3.5, the data generation method and solution procedure are presented, respectively. Computational results are discussed in Section
3.6. Finally, conclusions and recommendations for future research comprise Section 3.8.

3.2. Problem Description

The earliest work in supply chain network design was developed by Geoffrion and Graves [65]. They introduced a multi-commodity logistics network design model for optimizing finished product flows from plants to distribution centers to final consumers. Beginning with the work of Geoffrion and Graves, a large number of optimization-based approaches have been proposed for the design of supply chain networks. These works have resulted in significant improvements in the modeling of these problems as well as in algorithmic and computational efficiency. However, generally this research assumes that the design parameters for the supply chain are deterministic [23,66-70]. Unfortunately, critical parameters such as the consumers’ demand, resource supply, and the price of the material are generally uncertain. Therefore, traditional deterministic optimization is not suitable for truly capturing the behavior of the real-world problem.

The significance of uncertainty has encouraged a number of researchers to address stochastic parameters in their research. However, most of the stochastic approaches for supply chain network design only consider tactical level decisions usually related to demand uncertainty [26,71-73], while supply uncertainty is often ignored and supply capacity assumed to be unlimited (see [74] for more details). For instance, Santoso et al. [75] proposed a stochastic programming approach for addressing demand uncertainty in supply chain network design. Alonso-Ayuso et al.
developed a two-stage stochastic model for strategic production planning under
demand uncertainty. Tsiakis et al. [76] and Alonso-Ayuso et al. [72] also considered a
two-stage stochastic programming model for supply chain network design under
demand uncertainty.

In contrast with most prior research, we focus on the supply (capacity)
management required to mitigate the impact of facility capacity disruptions.
Moreover, we assume that supply quantities can be influenced by inspection, which
might be conducted at facility locations. The decision to inspect products before they
are shipped is made in some cases (e.g., by the FDA in pharmaceutical and food
supply chains).

We consider a mixed integer stochastic programming model that is formulated
as a two-stage optimization problem. We consider the selection of the facilities as the
first-stage variables, which is modeled as either selected or unselected. The binary
character of the strategic decision variables is one of the most fundamental
characteristics of the first-stage problem. The second-stage decision variables include
tactical decisions which are made after the realization of the random events (supply
disruption) is known. The second-stage decisions indicate the production/capacity
allocation policies as well as the decision to inspect each facility. Therefore, this stage
is referred to as a product allocation problem in which cost is minimized by allocating
the capacity and determining whether or not inspection should be implemented in
each selected facility. The inspection decision is modeled as a binary variable for each
facility. Therefore, the model enables us to determine when and where inspections
should be performed with the intent of minimizing the amount of tainted products
shipped to consumers by utilizing appropriate penalty costs. Insights on how networks should be configured to avoid the risk of tainted products reaching consumers are of interest to several types of supply chains such as healthcare, pharmaceutical, cosmetic and beauty, food or automotive industries. Consequences and real-world examples of shipped tainted product are addressed in Section 1.1.

In Figure 3-1, we provide a hypothetical supply network with an initial assignment of consumers to facilities at a point in time before any disruptions have occurred. We consider a set of facilities and consumers. In some cases disruptions can be a consequence of tainted raw material (received from suppliers). Hence, for the sake of clarity and in order to show the flow of raw materials from the suppliers to the facility, we have illustrated the set of suppliers as well. Three facilities were selected and the capacity was sufficient to fulfill all the demand of all the consumers.

Figure 3-1 Initial demand allocation (before disruption)

The scenario where a disruption occurred at two of the facilities is illustrated in Figure 3-2. We assume that the disruption was the consequence of a disruption in the suppliers of raw materials. This disruption caused the facilities to produce tainted items and ship them to consumers.
Once inspection is implemented in a facility, a portion of tainted items is discarded and fewer tainted items are delivered to the consumers. This idea is illustrated in Figure 3-3. However, discarding the tainted items might result in consumer demand being unsatisfied. Then, the unmet demand can be fulfilled by adding another facility as illustrated in Figure 3-3.
3.3. **Mathematical Model (Cost Minimization Model)**

Consider a supply chain network $\mathcal{N} = (L, C)$ where $L$ is the set of facilities and $C$ is the set of consumers. In the first stage, $x_i$ is 1 if facility $i$ is selected and is 0 otherwise (where $i \in L$ is an index for facilities). Let $Q(x, \bar{s})$ represent the optimal solution of the second-stage problem corresponding to the first-stage decision variable $x$ and the random scenario $\bar{s}$. Thus, the stochastic formulation of the problem can be written as

$$\min \sum_{i \in L} x_i f_i + E\left[ Q(x, \bar{s}) \right]$$  \hspace{1cm} (1)

Subject to \hspace{1cm} $x_i \in \{0,1\} \ \forall i \in L,$  \hspace{1cm} (2)

where $E\left[ Q(x, \bar{s}) \right]$ is the expected cost taken with respect to random scenario $\bar{s}$. The objective (1) in the first-stage problem is the sum of the cost of selecting facilities. The first-stage constraint (2) restricts the decision variables $x_i$ to be binary. Given a feasible first-stage solution vector $\bar{x}$, the objective of the second-stage problem for random scenario $\bar{s}$ minimizes the sum of the allocation (shipping) cost of the untainted products, the cost of shipping tainted product, the cost of discarding tainted product after inspection, and the cost of inspection. In this model, we discard tainted products. An alternative is to repair (or rework) the tainted product, an option which we may consider in future research.

To deal with the uncertainty in the second stage, a scenario-based modeling approach is proposed that has been used in stochastic programing problems [13,72].
In the second stage, let us consider random scenario $s$ with $\Pr(\tilde{s} = s) = \rho_s$ where $\rho_s$ is the probability of occurrence for scenario $s$. Given a finite set of scenarios, with associated probabilities $\rho_s$, $E\left[Q(x, \tilde{s})\right]$ can be evaluated as $E\left[Q(x, \tilde{s})\right] = \sum_{s \in S} \rho_s Q(x, s)$. Hence, we can present the deterministic equivalent of the formulation (1). To simplify, we denote this as the Supply Chain Design (SCD) model. We first summarize the complete notation for the SCD as sets and parameters:

Sets

$C$ the set of consumers, indexed by $c$
$L$ the set of candidate facilities, indexed by $l$
$S$ the set of realized scenarios, indexed by $s$

Parameters

$f_l$ the fixed cost of opening facility $l$
$k_l$ the capacity of facility $l$
$n_l$ the fixed cost of implementing an inspection at candidate facility $l$
$b_c$ the total demand of consumer $c$
$\lambda_l$ the cost of shipping an untainted product from facility $l$ to consumer $c$
$\omega_l$ the penalty cost for shipping a tainted product from facility $l$ to consumer $c$
$\gamma_l$ the cost of discarding a tainted product at facility $l$ after inspection originally destined for consumer $c$
$\rho_s$ the probability of occurrence for scenario $s$
the fraction of tainted products produced at facility \( l \) in scenario \( s \)

\( r_{ls} \)

the fraction of tainted products produced at facility \( l \) after inspection in scenario \( s \) (we assume \( q_{ls} > r_{ls} \))

To make the definitions of \( q_{ls} \) and \( r_{ls} \) clearer, suppose that, under scenario \( s \), the extent of failures at the unreliable facility \( l \) is given by \( q_{ls} = 0.20 \) and \( r_{ls} = 0.05 \).

This means that for every 100 units of production at facility \( l \), 100\( q_{ls} = 20 \) of them will be tainted. If no inspection is implemented, these 20 tainted units will be shipped to consumers. If inspection is implemented, 15 of these 20 tainted units will be detected and discarded while 100\( r_{ls} = 5 \) units will be undetected and shipped to consumers.

**Decision Variables**

\[ x_l = \begin{cases} 
1, & \text{if facility } l \text{ is selected,} \\
0, & \text{else}
\end{cases} \]

\[ z_l = \begin{cases} 
1, & \text{if inspection is used at facility } l, \\
0, & \text{else}
\end{cases} \]

\( p_{ls} \)

the number of products shipped from facility \( l \) to consumer \( c \) in scenario \( s \)

\( k_{ls} \)

the number of tainted products produced at facility \( l \) intended to be shipped to consumer \( c \) in scenario \( s \)

\( d_{ls} \)

the number of tainted products produced at facility \( l \) intended to be shipped to consumer \( c \) but discarded after inspection in scenario \( s \)

The deterministic equivalent of the formulation is proposed in [77]. However, for the convenience of the reader, we also present the formulation. Note that the
second-stage decision variables are indexed by a scenario index. The SCD model follows:

\[
\begin{align*}
\text{[SCD]} \quad & \min \sum_{l \in L} x_l f_l + \sum_{s \in S} \rho_s \left( \sum_{l \in L, s \in C} \sum_{l_c \in C} \left[ (1 - q_{ls}) p_{lcs} \right] + \sum_{l \in L, c \in C} \sum_{l_c \in C} \gamma_{l,k} k_{lcs} + \sum_{l \in L, c \in C} \gamma_{l,d} d_{lcs} + \sum_{l \in L} n_{l} z_{ls} \right) \\
\text{subject to} \quad & \sum_{c \in C} \left[ (1 - q_{ls}) p_{lcs} + k_{lcs} + d_{lcs} \right] \leq \kappa_{l} x_{l} \quad \forall l \in L, s \in S \quad (3) \\
& k_{lcs} + d_{lcs} = q_{ls} p_{lcs} \quad \forall c \in C, l \in L, s \in S \quad (4) \\
& k_{lcs} - (r_{ls}) p_{lcs} \leq M (1 - z_{ls}) \quad \forall c \in C, l \in L, s \in S \quad (5) \\
& d_{lcs} - (q_{ls} - r_{ls}) p_{lcs} \leq M (1 - z_{ls}) \quad \forall c \in C, l \in L, s \in S \quad (6) \\
& d_{lcs} \leq M (z_{ls}) \quad \forall c \in C, l \in L, s \in S \quad (7) \\
& \sum_{l \in L} \left[ (1 - q_{ls}) p_{lcs} + k_{lcs} \right] = b_{c} \quad \forall c \in C, s \in S \quad (8) \\
& z_{ls} \leq x_{l} \quad \forall l \in L, s \in S \quad (9) \\
& k_{lcs}, d_{lcs}, p_{lcs} \geq 0 \quad \forall c \in C, l \in L, s \in S \quad (10) \\
& z_{ls} \in \{0,1\} \quad \forall l \in L, s \in S \quad (11) \\
& x_{l} \in \{0,1\} \quad \forall l \in L. \quad (12)
\end{align*}
\]

The objective function (3) in the first stage problem is the sum of fixed cost of selecting facilities. The second stage consists of four distinct terms. The first term \( \left( \sum_{l \in L, c \in C} \sum_{l_c \in C} \left[ (1 - q_{ls}) p_{lcs} \right] \right) \) represents the expected transportation cost of shipping untainted products. The second term \( \sum_{l \in L, c \in C} \sum_{l_c \in C} \gamma_{l,k} k_{lcs} \) and the third term \( \sum_{l \in L, c \in C} \gamma_{l,d} d_{lcs} \) represent the penalty cost of supplying tainted products for the consumers and the cost
of discarding tainted products, respectively. Finally, the last term \( \sum_{l \in L} n_l z_{ls} \) is the cost of inspection, which is implemented at a facility site.

Constraint set (4) requires a facility to be open if any portion of the consumer demand is served from the facility. In addition, constraint set (4) ensures that the total consumer demand assigned to any facility does not exceed the facility's capacity. Constraint sets (5)-(8) together represent the amount of tainted product that is shipped to the consumer. Hence, without inspection, when \( z_{ls} = 0 \), constraint set (8) implies that \( d_{ks} = 0 \). Given constraint set (5), all of the tainted products will reach the consumer. However if inspection is implemented, constraint sets (6) and (7) imply that only products passing inspection (which may include some tainted products) will be shipped to the consumer. Constraint set (9) requires that the demand of every consumer is met. Constraint set (10) implies that inspection is applied only to the selected set of facilities. Constraint set (11) requires that \( k_{ks}, d_{ks}, \) and \( p_{ks} \) are positive values. Finally, constraint sets (12) and (13) place binary restrictions on variables \( z_{ls} \) and \( x_i \).

3.4. Generation of Test Data

We model a facility as either being in a pristine condition, producing up to the full capacity with no tainted goods, or as being in a condition of producing some tainted material. Let 0 indicate that a facility is capable of operating at full capacity with no tainted material produced and let 1 indicate that a facility is producing some tainted materials. Let \( \Theta_l \in [0.50, 0.95] \) be the probability of facility \( l \) being in State 0,
which we call the reliability of facility \( l \), drawn from a continuous uniform distribution. Therefore, the assumption that all facilities have an identical probability of working or failing is relaxed [78]. If a facility is in State 1, the proportion of tainted product is randomly selected from a continuous uniform distribution in the range \([0.10,0.30]\). The proportion of tainted product that is detected after inspection is randomly drawn from a continuous uniform distribution in the range \([0.01,0.09]\).

To determine the probability of scenario \( s \left( \rho_s \right) \), we need to define a scenario. A scenario is defined as an event where a subset of facilities (say \( L' \)) are in State 0 and where facilities in the set \( L \setminus L' \) are in State 1. Given the number of facilities \( |L| \), the total number of scenarios in which at least one facility is in State 1 is given by

\[
\sum_{i=1}^{\frac{|L|}{i}} \binom{|L|}{i} = 2^{|L|} - 1.
\]

Including the scenario in which all facilities are in State 0, the total number of scenarios is \( 2^{|L|} \). Hence, the probability of realizing a scenario \( s \in S \) is defined as \( \rho_s = \prod_{i \in L} \Theta_i \prod_{i \in L \setminus L'} (1 - \Theta_i) \). We list other assumptions as follows:

- The fixed cost of opening a facility is drawn from a discrete uniform distribution between $1,000,000 and $2,000,000.
- The demand for each consumer is drawn from a discrete uniform distribution between 100 and 300 units.
- The cost of inspection at each facility is drawn from a discrete uniform distribution between $50,000 and $100,000.
• The cost of shipping untainted products is drawn from a discrete uniform distribution between $100 and $1000.

• The penalty cost of shipping tainted products is drawn from a discrete uniform distribution between $10,000 and $20,000.

• The cost to discard products is equal to 25% of the penalty cost of shipping untainted products.

• The fraction of tainted products produced at facility \( l \) is correlated with the probability of facility \( l \) being in State 0. Hence, more reliable facilities produce less tainted products.

• The cost of selecting a facility is correlated with the capacity so that the highest capacity has the highest selecting cost.

• The cost of inspection is correlated to the percentage of improvement, which is the difference between \( q_l \) and \( r_l \).

• The total capacity is 35% higher than the total demand before implementing inspection and discarding tainted items.

3.5. Solution Procedure

3.5.1. Heuristic

We present a few constructive (greedy) heuristics in this section. In constructive algorithms, we start from scratch (empty solution) and construct a solution by assigning values to one decision variable at a time, until a complete solution is generated [79]. Constructive algorithms are popular techniques as they are simple to design. Moreover, their complexity compared to other algorithms such as iterative algorithms is low. However, in most optimization problems, the performance
of constructive algorithms may be low as well. Therefore, we also develop improvement algorithms to improve the quality of the solution achieved by constructive algorithms. In our improvement algorithm, we start with a complete solution (i.e., a constructive algorithm solution) and transform it at each iteration using some search operators to hopefully find a better solution.

In our solution procedure, we first determine the set of selected facilities, \( x \). Given the set of selected facilities, we determine the values for inspection i.e., \( z_{ls} \). Having \( x \) and \( z_{ls} \) determined and fixed to their binary values, equation (3) reduces to a capacitated transportation problem, which is relatively easier to solve. We call this model SCD-Sub, and its formulation is stated as follows:

\[
[\text{SCD-Sub}] \quad \min \sum_{s \in S} \sum_{l \in L, c \in C} \left( \lambda_c \left(1 - q_{ls}\right) p_{lcs} \right) + o_{lc} k_{lcs} + \gamma_c d_{lcs}
\]

Notice that \( k_{lcs} \) and \( d_{lcs} \) are auxiliary decision variables which depend solely on \( p_{lcs} \) and \( z_{ls} \). However, given the fact that \( z_{ls} \) is already determined and fixed, we can rewrite the SCD-Sub as follows:

\[
[\text{SCD-Sub}] \quad \min \sum_{s \in S} \left( \sum_{l \in L, c \in C} \lambda_c \left(1 - q_{ls}\right) + o_{lc} q_{ls} \left(1 - \bar{z}_{ls}\right) + o_{lc} r_{ls} \bar{z}_{ls} + \gamma_c \left(1 - \bar{z}_{ls}\right) \left(q_{ls} - r_{ls}\right) \right) p_{lcs}
\]

subject to

\[
k_{lcs} + d_{lcs} = q_{ls} p_{lcs}, \quad \forall c \in C, l \in L, s \in S \tag{14}
\]

\[
k_{lcs} - (r_{ls}) p_{lcs} \leq M \left(1 - \bar{z}_{ls}\right), \quad \forall c \in C, l \in L, s \in S \tag{15}
\]

\[
d_{lcs} - (q_{ls} - r_{ls}) p_{lcs} \leq M \left(1 - \bar{z}_{ls}\right), \quad \forall c \in C, l \in L, s \in S \tag{16}
\]

\[
d_{lcs} \leq M \left(\bar{z}_{ls}\right), \quad \forall c \in C, l \in L, s \in S \tag{17}
\]
\[
\sum_{i=1}^{c} \left( (1 - q_{i,s}) p_{i,s} + k_{i,s} \right) = b_c, \quad \forall c \in C, s \in S
\]  
\[
k_{i,s}, d_{i,s}, p_{i,s} \geq 0, \quad \forall c \in C, l \in L, s \in S
\]

where \( z_{i,s} \) is the fixed and known value of \( z_{i,s} \). We will refer to this as SCD-Sub herein.

### 3.5.1.1. Constructive Heuristics

Our constructive heuristics operate in three stages. In stage one we determine the set of selected facilities. In stage two we determine the inspection values, and finally in the last stage, we solve SCD-Sub in two phases. In the following, all three stages are presented.

**Stage one methods**

We develop three constructive heuristics to determine the set of selected facilities as follows:

- **Basic Greedy Heuristic (BGH):** One way to determine vector \( x \) is to simply open all the facilities. Therefore, we have \( x_l = 1, \forall l \in L \).

- **Selective Greedy Heuristic (SGH):** In this method, we first start with an empty set for selected facilities. Steps are illustrated in Figure 3-4. We estimate the total costs of selecting a facility includes the fixed cost, and mean costs of shipping untainted products, shipping tainted products, and discarding tainted products and then we select the facilities with the minimum cost estimated total costs.
Figure 3-4 Pseudocode of the Selective Greedy Heuristic (SGH)

- Capacity-Based Greedy Heuristic (CBGH): In this method, we first start with an empty set for selected facilities. Then we choose a facility from the set of remaining candidates that reduces the total demand of the consumer the most. The corresponding steps are illustrated in Figure 3-5.
Stage two methods

Once the selected facilities have been set, we determine the inspection values for each scenario. We develop the following construction heuristics:

- Failed Scenario Inspection Heuristic (FSIH): Consider \( S'_l, \forall l \in L \) as the set of scenarios where facility \( l \) works with full capacity and \( S''_l, \forall l \in L \) as the set of scenarios where facility \( l \) will produce tainted products where \( S'_l \cup S''_l = S \). We define \( z_{sl} \) as follows:

\[
    z_{sl} = \begin{cases} 
        1, & s \in S'_l \land x_l = 1, \forall l \in L, \forall s \in S \\
        0, & s \in S''_l \land x_l = 0, \forall l \in L, \forall s \in S 
    \end{cases}
\]

This heuristic performs inspection for only those facilities that are selected in stage one and belong to set of scenarios where facility \( l \) produces tainted products.

- Greedy Inspection Heuristic (GIH): In this method, we define a desired shipping untainted level \( \Delta \) \((\Delta \in (0,1])\), where \((\Delta)100\%\) of the shipping products to consumers must be untainted. Let’s start with an empty set for \( z \).
Given a scenario \( s \in S^*_s \), the following relation should be satisfied

\[
\sum_{l \in L} \left[ q_{ls} x_l \kappa_l (1 - z_{ls}) + r_{ls} x_l \kappa_l z_{ls} \right] \leq (1 - \Delta) \sum_{c \in C} b_c .
\]

Otherwise, we perform inspection until we reach the desired level of untainted products. Note that we consider facilities in decreasing order of maximum reduction in the fraction tainted products or \( \max \left\{ (q_l - r_l) \right\} \forall l \in L \). We consider \( \Delta = 0.90 \) in our computations.

- Random Greedy Inspection Heuristic (RGIH): The basic idea of this method is to estimate how much we can save by implementing inspection in a facility. If this saving is substantial enough, then the inspection for a facility will be implemented. The steps are defined in Figure 3-6.

\[
\text{Figure 3-6 Pseudocode of the Random Greedy Inspection Heuristic (RGIH)}
\]

```
for \( s = 1 \) to \( |S| \)
1. Determine the amount of saving: calculate the saving for each facility by following equation. \( \phi_{ls} = \frac{(q_{ls} - r_{ls}) \sum_{c=1}^{K_l} o_{lc}}{n_l + (q_{ls} - r_{ls}) \sum_{c=1}^{K_l} y_{lc}} \), \forall l \in L \), where \( \phi_{ls} \) is the amount of saving for facility \( l \) in scenario \( s \). Let the vector \( \Phi \) be the set of calculated savings where \( |\Phi| = |x| \).
2. Normalize vector \( \Phi : \hat{\Phi} = \frac{\Phi}{|\Phi|} \)
3. Generate a set of random numbers, \( \hat{R} \in (0,1) \), where \( |\hat{R}| = |\Phi| = |x| \).
4. Compare each element of vector \( \hat{R} \) with the corresponding element of vector \( \Phi \). If that is greater than the corresponding normalized saving value, then \( z_{ls} = 0 \), and 1 otherwise.
end for
```
Stage three methods

In this stage we solve SCD-Sub in two phases. In the first phase, we set $p_{lc}$ in a greedy fashion, based on the unit transportation cost to the consumers ($\lambda_c$) and capacity of the selected facilities. Note that in Figure 3-7, $a_c$ represents the demand of consumer $c$, and $g_l$ represents the capacity for facility $l$. In the second phase, given the obtained value for $p_{lc}$, we simply compute the values of auxiliary variables $k_{lc}$ and $d_{lc}$. 
**Input:** \( c \in C \) set of consumers, \( l \in L \) set of facilities, \( s \in S \) set of scenarios, \( f_i, \kappa_i, n_i, b_c, \rho \), \( \lambda_c \), \( \alpha_c \), \( q_{is} \) and \( r_{is} \).

**Output:** \( x_i, z_{is}, p_{is}, k_{is}, d_{is} \) and the total cost.

**Stage One:** Determine \( x \)
1. Use \( BGH, SGH \) or \( CBGH \).

**Stage Two:** Determine \( z_{is} \)
2. Use \( FSIH, GIH \), or \( RGIH \).

**Stage Three:** Solve SCD-Sub

### Phase 1

3. \( \forall s \in S \)
4. Sort \( \lambda_c \) in increasing order. \( a_c \leftarrow \lambda_c ; g_i \leftarrow \kappa_i \).
5. \( \forall c \in C \)
6. \( \text{while } a_c > 0 \text{ do} \)
7. \[ l = \arg \min \{ \lambda_c \} \forall l \in L, c \in C \]
8. \( \text{if } g_l > a_c \text{ then} \)
9. \[ p_{is} \leftarrow a_c, g_l \leftarrow g_l - a_c, a_c \leftarrow 0 \]
10. \( \text{else if } g_l > 0 \text{ then} \)
11. \[ p_{is} \leftarrow g_l, a_c \leftarrow a_c - g_l, g_l \leftarrow 0 \]
12. \( \text{end if} \)
13. \( \text{end while} \)
14. \( \text{end for} \)
15. \( \text{end for} \)

### Phase 2

16. \( \forall s \in S, \forall l \in L \)
17. \( \text{if } z_{is} = 0 \text{ then} \)
18. \( \forall c \in C \)
19. \[ d_{is} \leftarrow 0 \]
20. \[ k_{is} \leftarrow q_{is} p_{is} \]
21. \( \text{end for} \)
22. \( \text{else if } Z_{is} = 1 \text{ then} \)
23. \( \forall c \in C \)
24. \[ d_{is} \leftarrow (q_{is} - r_{is}) p_{is} \]
25. \[ k_{is} \leftarrow r_{is} p_{is} \]
26. \( \text{end for} \)
27. \( \text{end if} \)
28. \( \text{end for} \)
29. \( \text{end} \)

---

Figure 3-7 Pseudocode to solve SCD-Sub
3.5.1.2. Improvement Heuristic

In this section we develop improvement heuristics to improve the solution obtained from one of the heuristic methods presented above (note that improvement heuristics operate on a solution found by a constructive heuristic). First, we present a heuristic that begins with a feasible solution and seeks improvement to the original. The improvement heuristic iteratively closes one facility if the facility is already selected and opens a facility if a facility is not selected. This iteration enables us to generate a new neighborhood and explore if the new set of selected facilities provides a cheaper solution or not. In order to maintain feasibility, only moves are allowed which provide enough capacity to satisfy the total demand of the consumers. The details of this heuristic are shown in Figure 3-8.

```
1. for each facility \( l \in X \), if \( x_l = 1 \) then \( x_l = 0 \) otherwise if \( x_l = 0 \) then \( x_l \leftarrow 1 \). Let \( \psi \) be the new set of selected facilities \( \{|\psi| = |x|\} \).
2. Compute saving as:
   \[ \sigma_l = SCD(x) - SCD(\psi) \]
3. If \( \sigma_l < 0 \) then \( x_l \leftarrow \psi_l \). Go to 1.
4. end for
```

Figure 3-8 Pseudocode of the local_X

In the second improvement algorithm, we apply a Variable Neighborhood Search (VNS). The basic idea of VNS is to find a set of predefined neighborhoods to achieve a better solution. This algorithm is used to explore either at random or deterministically a set of neighborhoods to get different local optima and to escape from local optima (for general pseudocode of VNS see [80]). The purpose of the second improvement is to minimize transportation cost for each individual scenario, i.e., minimize \( \sum_{l \in L, c \in C} (\lambda_{hc} \left( (1 - q_{hc}) P_{hsc} \right)) \).

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VNS Algorithm

1. **Initialization**: Select the set of neighborhood structures \( N_k \), for \( k = 1, \ldots, k_{\text{max}} \), that will be used in the search; find an initial solution \( v \); choose a stopping condition;

2. **Repeat** the following steps until the stopping condition is met:
   a. Set \( k \leftarrow 1 \);
   b. Repeat the following steps until \( k = k_{\text{max}} \):
      i. **Shaking**: Generate a point \( v' \) at random from the \( k^{th} \) neighborhood of \( v \) \( (v' \in N_k(v)) \).
      ii. **Improve or not**: If \( v' \) is better than \( x \), do \( v \leftarrow v' \) and continue the search with \( N_1(k \leftarrow 1) \); otherwise, set \( k \leftarrow k + 1 \)

We implement the VNS for our problem as illustrated in Figure 3-10.

1. Set \( k \leftarrow 1 \)
2. for all \( s \in S \)
3. while \( k < K_{\text{max}} \)
   Shaking:
   a. Set \( u \leftarrow p \)
   b. Define a neighborhood strategy in \( u \)
   c. Apply a mechanism to generate a new solution for \( u \)
   Improve or not:
   7. Calculate the cost for \( u \) from equation (21).
   8. If \( \text{cost}(u) < \text{cost}(p) \) then \( p \leftarrow u \) else \( k \leftarrow k + 1 \)
   9. end while
   10. end for

The neighborhood strategy that we apply is structured by randomly choosing a point in the matrix of transportation. Subsequently, we identify the closed path leading to that point which consists of horizontal and vertical lines as illustrated. In order to generate a new solution, we move \( \hat{R} \) unit(s) from the chosen point and another point at a corner of the closed path and modify the remaining points at the other corners of the closed path to reflect this move. Note that \( \hat{R} \) is a random variable.
over the range of zero and the minimum value of the four selected points. This scheme is demonstrated in Figure 3-11. The selected point is shown by *.

<table>
<thead>
<tr>
<th>Consumer facility</th>
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<th>3</th>
<th>4</th>
</tr>
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<td>85</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>45</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
<td>36</td>
<td>73</td>
<td>73</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
<td>78</td>
<td>91</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 3-11 Neighborhood strategy in VNS

3.5.2. Simulated Annealing

Simulated Annealing is a metaheuristic approach inspired by nature. In this case, the process of a heated metal being cooled at a controlled rate (annealed) to improve its physical properties is simulated. The method was popularized by the work of Kirkpatrick et al. [81] which continued the earlier work of Metropolis et al. [82]. The fundamental idea is to allow moves resulting in solutions of worse quality than the current solution in order to escape from local optima [79]. The probability of doing such a move is decreased during the search.

An important consideration in SA is to set the initial value of the temperature $T_0$. If the initial temperature is set very high, the search may be relatively close to a random local search. Otherwise, if the initial temperature is very low, the search might be an improving local search algorithm [79]. The choice of a suitable cooling schedule is also crucial for the performance of the algorithm. The cooling
schedule defines the value of temperature $T$ at every iteration. The pseudo code of the simulated annealing algorithm is illustrated Figure 3-12.

3.5.2.1. **Defining Initial Temperature and Cooling Schedule**

The temperature $T$ is decreased during the search process, thus at the beginning of the search the probability of accepting uphill moves is high and it gradually decreases. As stated, the choice of an appropriate cooling schedule and initial value of temperature are crucial for the performance of the algorithm. The cooling schedule defines the value of $T$ at each iteration $k$, $T_{k+1} = R(T_k, k)$, where $R(T_k, k)$ is a function of the temperature at the previous step and of the iteration number. In this research, we use one of the most common cooling schedule which follows a geometric law as $T_{k+1} = \theta T_k$, where $\theta \in (0, 1)$, which corresponds to an exponential decay of the temperature [83]. Furthermore, experience has shown that $\theta$ should be between 0.5 and 0.99 (see [79]). Hence, we considered four values, $\theta = 0.95, 0.90, 0.80$ and $0.75$; and we obtained the best minimum regret in less computational time at $\theta = 0.75$ (see Section 3.6.1 for more detail).

Another important factor in SA is to define the initial value of the temperature $T_0$ properly. There is a tradeoff between a very high initial temperature and a lower one. The high temperature explores more of the solution space at the cost of increased running time. For this research, we use acceptance deviation methods proposed by Huang et al. [84]. The starting temperature is computed by $t\sigma$ using preliminary experimentations on each data set, where $\sigma$ represents the standard deviation of difference between values of objective functions and $t = -\frac{3}{\ln(\theta)}$ with the acceptance
probability of $\tilde{\omega}$, which is greater than $3\sigma$. Finally, a sufficient number of iterations at each temperature should be performed. If too few iterations are performed at each temperature, the algorithm may not be able to reach the global optimum. Given the presented formula and after several experiments, we set the value for the initial temperature, $T_0 = 8000$.

![Diagram](image)

**Figure 3-12 General scheme of Simulated Annealing (SA) (cf. Yang, [85]).**

### 3.5.2.2. Neighborhood Selection

The manner in which a metaheuristic technique moves from one solution to its neighbor is a critical component. In our SA algorithm, we define a neighborhood which combines four neighborhood structures: (1) swapping one randomly selected facility with another randomly selected facility (SA-	extit{swap}), (2) selecting one more facility (SA-	extit{add}), (3) closing one selected facility (SA-	extit{remove}), and finally (4) closing two facilities while selecting another two (SA-2	extit{swap}). Note that we apply the same neighborhood strategy to determine the inspection values and afterward.
compute the values of $p_{lcs}^{iter}$, $k_{lcs}^{iter}$, and $d_{lcs}^{iter}$ by using algorithm presented in Figure 3-7.

3.5.2.3. **Stopping criterion**

Various stopping criteria have been developed in the literature. A popular stopping criteria and the temperature reaches a set value (such as 0.01). Another criterion can be completing a predetermined number of iterations. In this chapter, a combination of these two criteria is considered in which we stop at the earlier of the temperature reaching 0.01 or the completion of 100 (350) iterations for small (large) size problems. We discuss this condition and the convergence of SA algorithm in more detail in Section 3.6.1.

3.5.2.4. **SA Algorithm**

According to above explanation, the SA algorithm is outlined in the Figure 3-13.
1. Initialize the parameters of the annealing schedule (Initial temperature, final temperature and total number of iterations)
2. Generate an initial solution by determining vector $x^0, z^0, p^0_{t,s}, k^0_t, d^0_{t,s}$ by the represented constructive or improvement heuristics and define relevant total cost $f(x^0, z^0, p^0_{t,s}, k^0_t, d^0_{t,s})$
3. $iter \leftarrow 1$; Temperature $\leftarrow$ Initial Temperature
4. while Temperature $>$ Final Temperature or $iter <$ total number of iterations do
5. 
6. aZeroElem $\leftarrow$ Number of zero elements in vector $x^{new}$ and $z^{new}$
7. aOneElem $\leftarrow$ Number of one elements in vector $x^{new}$ and $z^{new}$
8. aRand $\leftarrow$ Generate a Random Number
9. if $0 \leq aRand < \frac{1}{4}$
10. create a new solution using SA-swap method and return $x^{new}$ and $z^{new}$
11. done $\leftarrow$ true
12. else if $\frac{1}{4} \leq aRand < \frac{1}{2}$ and $aZeroElem \geq 1$
13. create a new solution using SA-add method and return $x^{new}$ and $z^{new}$
14. done $\leftarrow$ true
15. else if $\frac{1}{2} \leq aRand < \frac{3}{4}$ and $aOneElem > 1$
16. create a new solution using SA-remove method and return $x^{new}$ and $z^{new}$
17. done $\leftarrow$ true
18. else if $\frac{3}{4} \leq aRand < 1$ and $aOneElem \geq 2$
19. create a new solution using SA-2swap method and return $x^{new}$ and $z^{new}$
20. done $\leftarrow$ true
21. end while
22. Obtain the values of $p^0_{t,s}, k^0_t, d^0_{t,s}$ by using SCD-Sub.
23. if $f(x^{new}) - f(x^0) \leq 0$ then $f(x^0) = f(x^{new})$, $x^0 = x^{new}$
24. Update Temperature
25. end while
26. return the final solution

Figure 3-13 Pseudo code of the SA algorithm

3.5.3. **Commercial Software**

The optimization problem is modeled by using the AMPL mathematical programming language and solved with Gurobi 4.5.6. Each problem instance is solved on four cores (threads=4) of a Dell Optiplex 980 with an Intel Core i7 860 Quad @ 2.80GHz and 16GB RAM. The operating system is Windows 7 Enterprise 64-bit. In
our computational analysis, we terminate Gurobi when the CPU time limit of 14,400 seconds is reached. Table 3-1 summarizes the results from the solution, and the discussion is presented in Section 3.6.

3.6. Computational Result

In this section, we perform computational experiments to assess the effectiveness of the algorithms. In Section 3.5.1, we presented three heuristics (BGH, SGH, and CBGH) to determine set of selected facilities, $x$, and also three heuristics (FSIH, GIH, and RGIH) to determine the set of inspections to conduct, $z$. By combining these six heuristics, we construct nine different heuristics for determining $x$ and $z$. For instance, our first heuristic can be denoted as BGH&FSIH. Finally, we employ the greedy heuristic presented in Figure 3-7 to solve SCD-Sub. All the algorithms were implemented and executed in MATLAB 7.9 (2009b) and tested on a single core of a Dell OptiPlex 980 computer running the Windows 7 Enterprise 64 bit operating system with an Intel(R) Core(TM) i7 CPU860@ 2.80GHz, and 8GB RAM.

We consider twelve sets of problems with ten data instances in each. Hence, we solve 120 instances of varying sizes as illustrated in Table 3-1. The second, third and the fourth columns represent the size of the problems under consideration. We also report the average of the optimal value and average solution time for each set. Finally, the last column represents the total number of optimal solutions obtained from ten data instances.
<table>
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<tr>
<th>Set no.</th>
<th>No. of consumers</th>
<th>No. of facilities</th>
<th>No. of scenarios</th>
<th>Avg. optimal value/best values found</th>
<th>Avg. solution time (s)</th>
<th>No. of optimal solutions in ten data instances</th>
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</table>

*: Average of best objective values found
**: The CPU time exceeded the prescribed time limit of 14400 seconds.

As observed from Table 3-1, increasing the number of facilities implies an increase in the number of scenarios and the size of the problem has a large impact on the solution time. For instance, for the case of ten consumers or twenty consumers and ten facilities, Gurobi did not return any optimal solutions within the prescribed time limit of 14400 seconds. This is also illustrated in Figure 3-14. In order to calculate the relative optimality gap, we use the objective function value that is provided by Gurobi when the prescribed time limit is reached. To assess each heuristic, we consider solution quality and solution (computational) time. For the solution quality, we consider a quality criterion which is the gap between the result of heuristic/SA and the optimal/best solution obtained from Gurobi. This gap is defined according to the following equation:

\[
% \text{gap} = \frac{(SA\text{ or Heuristics Solution} - \text{Best Found (or Optimal Solution)})}{\text{Best Found (or Optimal Solution)}}.
\]

42
Furthermore, given the random characteristic of GIH, RGIH, Local\_x, VNS\_p, and SA, the corresponding objective values and solution times are the average across thirty independent replications. Table 3-2 reports the result for 2 facilities and 2, 5, 10, and 20 consumers. Note that bold-faced values indicate achievement of the best objective value among constructive heuristics and improvement heuristics/SA, respectively.

The results in Table 3-2 show that, regardless of the number of consumers, the heuristic algorithms always provide solutions within 3\% of the solution found by Gurobi. Heuristic algorithms are fast and their solution time generally does not vary with the number of consumers. We see that SGH&FSIH and CBGH&FSIH algorithms provide better solution quality and Local\_x does not provide any
improvement in the solution of the constructive heuristics. The VNS\(_p\) procedure provides better quality solutions than the Local\(_x\), however, this improvement comes with an increase in the solution time. Another observation from Table 3-2 is that even though the solution time for SA algorithm is notably higher than the other algorithms, its performance are not as good as VNS\(_p\) when we limit the problem instances to 2 facilities.
## Table 3-2: Comparison of algorithms results for 2 facilities

<table>
<thead>
<tr>
<th>Constructive Heuristics</th>
<th>Improv. Heuristic and Metaheuristic</th>
<th>BGH&amp;FSIH</th>
<th>BGH&amp;GIH</th>
<th>BGH&amp;RGIH</th>
<th>SGH&amp;FSIH</th>
<th>SGH&amp;GIH</th>
<th>SGH&amp;RGIH</th>
<th>CBGH&amp;FSIH</th>
<th>CBGH&amp;GIH</th>
<th>CBGH&amp;RGIH</th>
<th>Local$_L$</th>
<th>VNS$_P$</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min gap</td>
<td></td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.08%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Avg. gap</td>
<td></td>
<td>0.40%</td>
<td>0.47%</td>
<td>0.22%</td>
<td>0.21%</td>
<td>0.64%</td>
<td>0.36%</td>
<td>0.36%</td>
<td>0.56%</td>
<td>0.25%</td>
<td>0.21%</td>
<td>0.06%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Max gap</td>
<td></td>
<td>0.89%</td>
<td>0.77%</td>
<td>1.59%</td>
<td>0.76%</td>
<td>2.49%</td>
<td>0.76%</td>
<td>0.56%</td>
<td>1.59%</td>
<td>0.59%</td>
<td>0.56%</td>
<td>0.09%</td>
<td>0.89%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
<td></td>
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<td>Min gap</td>
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<tr>
<td>Avg. gap</td>
<td></td>
<td>1.34%</td>
<td>1.59%</td>
<td>1.86%</td>
<td>0.97%</td>
<td>1.36%</td>
<td>1.33%</td>
<td>0.84%</td>
<td>1.08%</td>
<td>1.23%</td>
<td>0.84%</td>
<td>0.25%</td>
<td>0.55%</td>
</tr>
<tr>
<td>Max gap</td>
<td></td>
<td>3.75%</td>
<td>3.62%</td>
<td>3.83%</td>
<td>3.83%</td>
<td>3.83%</td>
<td>3.83%</td>
<td>1.47%</td>
<td>2.53%</td>
<td>2.33%</td>
<td>1.47%</td>
<td>1.37%</td>
<td>1.47%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
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<td>0.002</td>
<td>0.002</td>
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</tr>
<tr>
<td>Min gap</td>
<td></td>
<td>0.00%</td>
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<td>0.00%</td>
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<td>0.00%</td>
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<td>0.00%</td>
<td>0.00%</td>
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<td>0.00%</td>
</tr>
<tr>
<td>Avg. gap</td>
<td></td>
<td>0.96%</td>
<td>2.12%</td>
<td>0.68%</td>
<td>0.48%</td>
<td>2.19%</td>
<td>1.08%</td>
<td>0.48%</td>
<td>0.81%</td>
<td>1.08%</td>
<td>0.48%</td>
<td>0.05%</td>
<td>0.50%</td>
</tr>
<tr>
<td>Max gap</td>
<td></td>
<td>4.77%</td>
<td>7.02%</td>
<td>1.84%</td>
<td>1.84%</td>
<td>7.02%</td>
<td>4.55%</td>
<td>1.84%</td>
<td>2.16%</td>
<td>2.84%</td>
<td>1.84%</td>
<td>0.40%</td>
<td>1.84%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
<td></td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.006</td>
<td>0.03</td>
</tr>
<tr>
<td>Min gap</td>
<td></td>
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<td>0.00%</td>
<td>0.43%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
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<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Avg. gap</td>
<td></td>
<td>1.58%</td>
<td>1.58%</td>
<td>2.31%</td>
<td>1.58%</td>
<td>1.58%</td>
<td>1.97%</td>
<td>1.58%</td>
<td>1.90%</td>
<td>1.90%</td>
<td>1.58%</td>
<td>1.29%</td>
<td>1.58%</td>
</tr>
<tr>
<td>Max gap</td>
<td></td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>5.37%</td>
<td>4.49%</td>
<td>5.37%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
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<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.005</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Bold-faced values indicate achievement of the average of the best solution gap.
We now evaluate the effectiveness of our algorithms for five facilities. The results are presented in Table 3-3. We observe that BGH&FSIH, BGH&GIH, and BGH&RGIH do not perform well compared to other constructive heuristics. The reason is that in these three heuristics we use BGH to select all the facilities while the total demand can be satisfied by selecting fewer facilities. SGH&FSIH, SGH&GIH, and SGH&RGIH provide solutions on average within 8% of the best found solution with a remarkably fast solution time in comparison to the optimal solution time. Both Local_x and VNS_p are capable of improving the solution quality even for a larger number of consumers, with an average solution gap within 5% of the optimal solution. In particular, SA clearly provides the best overall solution cost for the range of problems tested and requires only a moderate extra computational time than other algorithms. SA achieves solutions which are, on average, within 3% of the optimality gap. For 5 facilities and \(|c| \in [10]\), as shown in Table 3-1, we found 9 optimal solutions in ten data instances. Therefore, in Table 3-3 we show the gap with the optimal and best solutions individually.
Table 3-3 Comparison of algorithms results for 5 facilities

<table>
<thead>
<tr>
<th></th>
<th>Constructive Heuristics</th>
<th>Improv. Heuristic and Metaheuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BGH&amp;FSIH</td>
<td>BGH&amp;GSH</td>
</tr>
<tr>
<td>2 Consumers</td>
<td>Min gap*</td>
<td>7.09%</td>
</tr>
<tr>
<td></td>
<td>Avg. gap*</td>
<td>16.05%</td>
</tr>
<tr>
<td></td>
<td>Max gap*</td>
<td>30.08%</td>
</tr>
<tr>
<td></td>
<td>Avg. time (s)</td>
<td>0.002</td>
</tr>
<tr>
<td>5 Consumers</td>
<td>Min gap*</td>
<td>13.10%</td>
</tr>
<tr>
<td></td>
<td>Avg. gap*</td>
<td>18.37%</td>
</tr>
<tr>
<td></td>
<td>Max gap*</td>
<td>25.37%</td>
</tr>
<tr>
<td></td>
<td>Avg. time (s)</td>
<td>0.002</td>
</tr>
<tr>
<td>10 Consumers</td>
<td>Min gap*</td>
<td>8.37%</td>
</tr>
<tr>
<td></td>
<td>Avg. gap*</td>
<td>14.24%</td>
</tr>
<tr>
<td></td>
<td>Max gap*</td>
<td>19.20%</td>
</tr>
<tr>
<td></td>
<td>Avg. gap w/ non-opt sol. **</td>
<td>9.63%</td>
</tr>
<tr>
<td></td>
<td>Avg. time (s)</td>
<td>0.002</td>
</tr>
<tr>
<td>20 Consumers</td>
<td>Min gap*</td>
<td>2.01%</td>
</tr>
<tr>
<td></td>
<td>Avg. gap*</td>
<td>9.70%</td>
</tr>
<tr>
<td></td>
<td>Max gap*</td>
<td>15.06%</td>
</tr>
<tr>
<td></td>
<td>Avg. time (s)</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Bold-faced values indicate achievement of the best solution gap.

Italicized indicate a better solution than the best solution found by Gurobi within the time limit.

*: Values indicate the average gap with optimal solutions found

**: Values indicate the average gap with the best solution found
Realistic sized problems are commonly larger than those tested above. Hence, we examine a larger size problem for ten facilities. In Table 3-1, we show that for set 9 and set 12 we were not able to find the optimal solution for any of the ten instances in 14400 seconds. In addition, in set 3 and 5 only 50% and 10% of the data instances were solved to optimality, respectively. This indicates how increasing the number of facilities and correspondingly the number of scenarios has a significant impact on the solution time. We presented the result of algorithms for the tested problem in Table 3-4.

Negative values in Table 3-4 indicate that the heuristics or SA achieved a better solution than the best solution found by Gurobi. For ten facilities and $|C| \in \{10, 20\}$, SGH&FSIH, SGH&GIH, and SGH&RGIH perform well based on the average solution gap. For the case of ten facilities and $|C| \in \{2, 5\}$ consumers CBGH&FSIH, CBGH&GIH, and CBGH&RGIH achieved a better performance. However, the SA solutions outperform those found by all the other algorithms, even though they require less computational time than SA. Also, the minimum and maximum gap is usually somewhat better for the SA. Hence, for large sized problems we recommend using the SA algorithm, although reasonable results can still be achieved by some of the algorithms. For ten facilities and $|C| \in \{5\}$, we found only 1 optimal solution in ten data instances. Hence, we separate the result for this data instance from the others and display the gap between the optimal solution and the algorithms in the corresponding row of Table 3-4.

It is observable that the number of facilities and consequently the number of scenarios has a significant impact on the computational time in our model. However, the results indicate the effectiveness of the SA algorithm we proposed, particularly for
larger sized problems. For problems in practice (that can have even larger sizes), our SA heuristic shows promising results.
Table 3-4 Comparison of algorithms results for ten facilities

<table>
<thead>
<tr>
<th></th>
<th>BHG&amp;FSIH</th>
<th>BHG&amp;GIH</th>
<th>BGH&amp;GIH</th>
<th>SGH&amp;FSIH</th>
<th>SGH&amp;GIH</th>
<th>CBGH&amp;FSIH</th>
<th>CBGH&amp;GIH</th>
<th>CBGH&amp;RGIH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min gap</strong></td>
<td>20.01%</td>
<td>21.25%</td>
<td>20.52%</td>
<td>4.24%</td>
<td>4.17%</td>
<td>2.52%</td>
<td>0.46%</td>
<td>0.46%</td>
</tr>
<tr>
<td><strong>Avg. gap</strong></td>
<td>24.10%</td>
<td>24.02%</td>
<td>22.75%</td>
<td>7.04%</td>
<td>6.86%</td>
<td>6.57%</td>
<td>3.93%</td>
<td>3.89%</td>
</tr>
<tr>
<td><strong>Max gap</strong></td>
<td>29.06%</td>
<td>28.11%</td>
<td>28.00%</td>
<td>10.63%</td>
<td>8.72%</td>
<td>8.78%</td>
<td>9.65%</td>
<td>9.98%</td>
</tr>
<tr>
<td>Avg. gap w/ non-opt sol. **</td>
<td>21.17%</td>
<td>21.26%</td>
<td>21.26%</td>
<td>6.72%</td>
<td>6.69%</td>
<td>6.44%</td>
<td>3.21%</td>
<td>3.45%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
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<td>0.005</td>
<td>0.005</td>
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<table>
<thead>
<tr>
<th></th>
<th>BHG&amp;FSIH</th>
<th>BHG&amp;GIH</th>
<th>BGH&amp;GIH</th>
<th>SGH&amp;FSIH</th>
<th>SGH&amp;GIH</th>
<th>CBGH&amp;FSIH</th>
<th>CBGH&amp;GIH</th>
<th>CBGH&amp;RGIH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min gap</strong></td>
<td>20.00%</td>
<td>21.25%</td>
<td>20.52%</td>
<td>4.24%</td>
<td>4.17%</td>
<td>2.52%</td>
<td>0.46%</td>
<td>0.46%</td>
</tr>
<tr>
<td><strong>Avg. gap</strong></td>
<td>24.10%</td>
<td>24.02%</td>
<td>22.75%</td>
<td>7.04%</td>
<td>6.86%</td>
<td>6.57%</td>
<td>3.93%</td>
<td>3.89%</td>
</tr>
<tr>
<td><strong>Max gap</strong></td>
<td>29.06%</td>
<td>28.11%</td>
<td>28.00%</td>
<td>10.63%</td>
<td>8.72%</td>
<td>8.78%</td>
<td>9.65%</td>
<td>9.98%</td>
</tr>
<tr>
<td>Avg. gap w/ non-opt sol. **</td>
<td>21.17%</td>
<td>21.26%</td>
<td>21.26%</td>
<td>6.72%</td>
<td>6.69%</td>
<td>6.44%</td>
<td>3.21%</td>
<td>3.45%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>BHG&amp;FSIH</th>
<th>BHG&amp;GIH</th>
<th>BGH&amp;GIH</th>
<th>SGH&amp;FSIH</th>
<th>SGH&amp;GIH</th>
<th>CBGH&amp;FSIH</th>
<th>CBGH&amp;GIH</th>
<th>CBGH&amp;RGIH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min gap</strong></td>
<td>20.00%</td>
<td>21.25%</td>
<td>20.52%</td>
<td>4.24%</td>
<td>4.17%</td>
<td>2.52%</td>
<td>0.46%</td>
<td>0.46%</td>
</tr>
<tr>
<td><strong>Avg. gap</strong></td>
<td>24.10%</td>
<td>24.02%</td>
<td>22.75%</td>
<td>7.04%</td>
<td>6.86%</td>
<td>6.57%</td>
<td>3.93%</td>
<td>3.89%</td>
</tr>
<tr>
<td><strong>Max gap</strong></td>
<td>29.06%</td>
<td>28.11%</td>
<td>28.00%</td>
<td>10.63%</td>
<td>8.72%</td>
<td>8.78%</td>
<td>9.65%</td>
<td>9.98%</td>
</tr>
<tr>
<td>Avg. gap w/ non-opt sol. **</td>
<td>21.17%</td>
<td>21.26%</td>
<td>21.26%</td>
<td>6.72%</td>
<td>6.69%</td>
<td>6.44%</td>
<td>3.21%</td>
<td>3.45%</td>
</tr>
<tr>
<td>Avg. time (s)</td>
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<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Bold-faced values indicate achievement of the best solution gap. Italicized indicate a better solution than the best solution found by Gurobi within the time limit.

*: Values indicate the average gap with optimal solutions found

**: Values indicate the average gap with the best solution found
Figure 3-15 Gap of the algorithms under different number of facilities
Figure 3-16 Solution time of the algorithms under different number of facilities
3.6.1. The Convergence of the SA

We discussed the settings of SA parameters in Section 3.5.2. In this section, the convergence of the SA algorithm is verified for the specific cases of $\theta = 0.95$ and 0.75. We also limit the number of iterations to 350 in order to focus on the early convergence. The result is illustrated in Figure 3-17. Note that the presented results in Figure 3-17 are the average of ten independent runs. We observe that the SA with $\theta = 0.75$ converges to a better solution faster than the SA with $\theta = 0.95$. Figure 3-17 also indicates that setting the number of iterations on 350 iterations is appropriate as the algorithm converged prior to this value.

![Figure 3-17 Convergence curves of SA for ten facilities and five customers](image-url)
3.7. **Results of Algorithms Comparison**

The performance of the 9 different heuristics with improvement heuristics and SA is compared in this section. The test problems attempted are classified into two sizes (problem structures):

i. Medium: 5 facilities 5 consumers.

ii. Large: ten facilities ten consumers.

Note that the data used in the analyses are ten data instances we defined in Section 3.6.

The normal probability plots of the residuals indicate that the normality assumption for conducting an analysis of variance (ANOVA) is not satisfied. Thus, a nonparametric method (Kruskal-Wallis) to compare the algorithms is used. The Kruskal-Wallis procedure is carried out to test the following hypothesis:

\[ H_0: \quad \text{"There is not a difference in the total cost obtained for the problem instances using the 9 different heuristics with improvement heuristics and SA."} \]

\[ H_1: \quad \text{"There is a difference in the total cost obtained for the problem instances using the 9 different heuristics with improvement heuristics and SA."} \]

A significance level if 0.05 is used for each test. The results of Kruskal-Wallis test for both the medium sized problem and the large sized problem indicate that there is enough statistical evidence that at least one of the algorithms tends to yield a different total cost than the others for both the medium sized problem and the large sized problem.

A Pairwise Wilcoxon rank sum test (also known as the Mann-Whitney U test) is conducted to compare the performance of each of a pair of presented algorithms.
The results are presented in Table (a) and Table (b) in Appendix B. The results for the medium sized problem indicate that, SA is outperformed by all constructive heuristics except SGH&FSIH, SGH&GIH, and SGH&RGIH. However, the results show that there is no significant difference among the SA, VNS_p, and Local_x. Among the constructive heuristics SGH&FSIH, SGH&GIH, and SGH&RGIH is recommended for the medium size problems. For large size problems SA outperforms the constructive heuristics and the Local_x at the 5% level. However, SA and VNS_p are marginally insignificant (p-value=0.056). Therefore, SGH&FSIH and SGH&GIH are also recommended for larger size problems.

3.8. Conclusions and Future Research

In response to some catastrophic events, particularly in healthcare/pharmaceutical supply chains, Chapter 3 addresses a supply chain network design to hedge against the risk of supply disruptions and sending tainted materials to consumers. We considered a mixed-integer stochastic programming model with capacitated facilities. The model was formulated as a two-stage optimization problem. The aim of the model consists of the facility selection, actual capacity allocation among the consumers, and determination of inspection policy with the objective of minimizing the total cost. The impact of supply/capacity uncertainty is explicitly modeled in all our models in order to design a reliable supply chain network. To capture the uncertainty, a scenario-based approach was presented.

Experience from solving the problem using commercial software indicated that the number of facilities, and consequently the number of scenarios, has a significant impact on the computational time. As a result, we developed several
heuristic methods and a metaheuristic approach to effectively solve the presented model.

Based on our computational studies, the SA approach is not efficient in terms of solution quality and solution time for the small sized problems or small number of scenarios. However, some of the heuristics, in particular SGH&FSIH, SGH&GIH, SGH&RGIH and CBGH&FSIH, achieved good solution qualities in a more reasonable time when compared to the optimal or best found solution. Local_\text{x} and VNS_\text{p} were able to improve the solutions obtained from constructive heuristics. Therefore, constructive and improvement heuristics are preferable on small sized problems. However, for practical sized problems, i.e. ten facilities and more, SA outperforms constructive and improvement heuristics, even though it requires higher computational time.

There are several interesting future research directions. We assumed a deterministic demand in our model whereas in real world this may not be a valid assumption. Moreover, we assumed an inspection and discard policy but in some industries like automotive and electronics industry this can be considered as inspection and fixed policy where items defected after detecting can be repaired. Another extension is to develop other metaheuristic techniques such as Genetic Algorithm or Tabu Search to compare their effectiveness with SA algorithm.
CHAPTER FOUR


4.1. Introduction

In the previous chapter, we solely identified the set of decisions made by a risk-neutral decision-maker in order to minimize the expected cost. In the literature, however, it has been indicated that minimizing expected cost is not always satisfactory from a practical point of view, and managers in the real world are also concerned with the other objectives such as downside risk minimization.

In this chapter, a risk-averse policy wherein, rather than selecting facilities and identifying the pertinent supplier-consumer assignments that minimize the expected cost, the decision-maker uses a Conditional Value-at-Risk (CVaR) approach to measure and quantify risk and to define what comprises a worst-case scenario. The CVaR methodology allows the decision-maker to specify to what extent worst-case scenarios should be avoided and the corresponding costs associated with such a policy. We first reformulate the problem SCD as a mean-risk model. After introducing the underlying optimization models, we present computational analysis to compare the results of the risk-averse (SCD-CVaR) and risk-neutral (SCD) policies. In addition, we provide several managerial insights.

In this chapter, the CVaR description and the mathematical formulations of the risk-averse policy are introduced in Section 2 and Section 3, respectively. In Section 3 the data generation method is presented. Solution procedures, computational
experiments and sensitivity analysis are discussed in Section 4. Finally, Section 5 includes our conclusions and our recommendations for future work.

4.2. The Conditional Value at Risk (CVaR) Concept

The CVaR builds upon the measure called Value-at-Risk (VaR). VaR is a popular method to measure risk in a portfolio. VaR focuses on all outcomes below a specific level. Therefore, given a probability \( \alpha \), VaR answers the question: “What is the maximum loss associated with \( 100\% \alpha \) probability over a target horizon?” Despite the popularity of VaR in finance and risk management, this technique has a few important undesirable properties. Artzner et al. [86] pointed out that VaR is not a coherent measure of risk since it fails to hold the sub-additivity property. Therefore, the VaR of a portfolio can be higher than the sum of VaRs of the individual assets in the portfolio (i.e., \( \text{VaR}(x + y) \geq \text{VaR}(x) + \text{VaR}(y) \)) where \( f(\cdot) \) is the risk measure). Moreover, VaR is difficult to optimize when it is calculated using the scenario-based approach [87]. These reasons have led us to use an alternative measure, CVaR.

The CVaR measure leads to a minimization of VaR because CVaR is greater than or equal to VaR (see Figure 4-1 and equation (25)). The CVaR measure considers those outcomes in which losses over a specific period of time exceed VaR. In other words, we allow \( (1-\alpha)100\% \) of the outcomes to exceed VaR, and the average value of these outcomes is represented by CVaR. Generally, \( \alpha \) indicates the level of conservatism that a decision-maker is willing to adopt. As \( \alpha \) approaches one, the range of acceptable worst-cases becomes narrower in the corresponding optimization problem. Figure 4-1 illustrates the relationship between CVaR and VaR:
CVaR is always greater than or equal to VaR. Moreover, the distribution is skewed to the right and therefore, the number of worst-case outcomes is reduced when $\alpha$ is increased.

![Illustration of relation between CVaR and VaR](image.png)

Figure 4-1 Illustration of relation between CVaR and VaR

We provide the formal definition of VaR and CVaR in the following equation.

Consider, for example, a random variable $\tilde{x}$ that represents loss from an outcome. Given a risk level $\alpha$, the VaR of the random variable $\tilde{x}$ is given by

$$\text{VaR}_\alpha [\tilde{x}] := \min \{ \eta : \Pr(\tilde{x} \geq \eta) \leq 1 - \alpha \}. \tag{22}$$

Given the equation (22), the CVaR at risk level $\alpha$, is defined by Rockafellar and Uryasev [87] as

$$\text{CVaR}_\alpha (\tilde{x}) = \mathbb{E} \{ \tilde{x} | \tilde{x} \geq \text{VaR}_\alpha (\tilde{x}) \}. \tag{23}$$
Rockafellar and Uryasev [87] proved that for a minimization problem, the CVaR can be computed as

$$\text{CVaR}_\alpha (\tilde{x}) = \min \left\{ \eta + \frac{1}{1-\alpha} \max \left( \tilde{x} - \eta, 0 \right) \right\}. \quad (24)$$

In [87], Rockafellar and Uryasev also proved that for a set of pre-defined scenarios with corresponding probabilities, equation (24) can be transformed into a linear programming model by introducing the auxiliary variables $\tau_i (i = 1, ..., N)$ as

$$\min \eta + \frac{1}{1-\alpha} \sum_{i=1}^{N} \rho_i \tau_i \quad (25)$$

subject to:

$$\tau_i \geq L_i - \eta \quad \forall i, \quad (26)$$

$$\tau_i \geq 0 \quad \forall i, \quad (27)$$

where $L_i$ is the realization of the expected loss related to scenario $i$. 
Figure 4.2 Representation of CVaR and VaR for a finite set of scenarios (discrete case)

4.3. The CVaR Model

In expanding the formulation of the SCD model for a risk-averse objective, we define $\eta$ as a decision variable denoting the optimal value for VaR. The CVaR is a weighted measure of $\eta$ and the costs greater than $\eta$. We define $\tau_s$ as the tail loss for scenario $s$, where tail loss is defined as the amount by which the loss in scenario $s$ exceeds $\eta$. Given Equation (25) and the SCD model, a risk-averse supply chain network model with unreliable supply sources is defined as

$$[\text{SCD-CVaR}] \quad \min \eta + \frac{1}{1-\alpha} \sum_{i \in S} p_i \tau_s$$

subject to:

$$\tau_s \geq \sum_{i \in I} x_i f_i + \sum_{i \in I, c \in C} \left\{ \lambda_{ic} \left[ (1-q_{is}) p_{isc} \right] + o_{ic} k_{ic} + r_{ic} d_{ic} \right\} + \sum_{i \in I} p_i z_{is} - \eta, \quad \forall s \in S$$

(29)
\[
\sum_{c \in C}[(1-q_{ls}) p_{lc} + k_{lc} + d_{lc}] \leq \kappa_l x_l, \quad \forall l \in L, s \in S
\] (30)

\[
k_{lc} + d_{lc} = q_{ls} p_{lc}, \quad \forall c \in C, l \in L, s \in S
\] (31)

\[
k_{lc} - (r_{ls}) p_{lc} \leq M (1-z_{ls}), \quad \forall c \in C, l \in L, s \in S
\] (32)

\[
d_{lc} - (q_{ls} - r_{ls}) p_{lc} \leq M (1-z_{ls}), \quad \forall c \in C, l \in L, s \in S
\] (33)

\[
d_{lc} \leq M (z_{ls}), \quad \forall c \in C, l \in L, s \in S
\] (34)

\[
\sum_{l \in L}[(1-q_{ls}) p_{lc} + k_{lc}] = b_c, \quad \forall c \in C, s \in S
\] (35)

\[
z_{ls} \leq x_l, \quad \forall l \in L, s \in S
\] (36)

\[
k_{lc} , d_{lc} , p_{lc} \geq 0, \quad \forall c \in C, l \in L, s \in S
\] (37)

\[
z_{ls} \in \{0,1\}, \quad \forall l \in L, s \in S
\] (38)

\[
x_l \in \{0,1\}, \quad \forall l \in L
\] (39)

\[
\tau_s \geq 0 \quad \forall s \in S
\] (40)

In the above formulation, constraint set (29) computes the tail cost for scenario \(s\). Constraint set (40) indicates that we only consider the scenarios in which the loss exceeds \(\eta\).

4.4. Solution Procedures

4.4.1. Exact Solution by Using Commercial Software

The optimization problem is modeled in the AMPL mathematical programming language and solved with Gurobi 4.5.6. Each problem instance is solved on four cores (threads=4) of a Dell Optiplex 980 with an Intel Core i7 860 Quad @
2.80GHz and 16GB RAM. The operating system is Windows 7 Enterprise 64-bit. In our computational analysis, we terminate Gurobi when the CPU time limit of 14,400 seconds is reached.

All of our computational experiments are based on the data that was generated from the procedure presented in Section 3.4. We considered ten data instances for a supply chain network consisting of five facilities and five consumers. We selected five levels of $\alpha$: 0.50, 0.65, 0.75, 0.85 and 0.95. Results comparing the derived solutions from the SCD model with those from the SCD-CVaR model, under various risk-level values, are summarized in Table 4-1.

4.4.2. Computational Experiments for Exact Models

This section presents numerical studies on both the SCD and the SCD-CVaR models, as outlined above, in order to highlight the differences between the risk-neutral and risk-averse policies. Our observations indicate that higher values of $\alpha$ imply a higher level of risk-aversion and a narrower range of worst-case scenarios. From Table 4-1, it can be observed that the average expected cost, VaR, and CVaR increase with associated increases to $\alpha$ values. This is because, as a decision-maker or a supply chain designer becomes more risk-averse, he or she is willing to accept a higher total cost in order to avoid more worst-case scenarios. Hence, our derived SCD-CVaR model restricts the number of scenarios that exceed VaR, and the right-hand tail cost will be minimized at the price of increasing the total expected cost (see Figure 4-1).

In Table 4-1, we have divided the expected total cost into the fixed cost, expected untainted delivered cost, the expected tainted penalty cost, the expected
discard cost, and the expected inspection cost. As per the results obtained, the fixed cost increases with respect to increasing risk-level \( \alpha \). The reason for this increase is that, for higher values of risk-level, the average number of selected facilities gradually increases. However, in SCD-CVaR\( _{\alpha=0.85,0.95} \), even though the average number of selected facilities is equal, the corresponding average fixed costs are different. The difference is because increasing the risk-level \( \alpha \) also leads to the selection of different types of facilities. In Figure 4-3, we show the output for one data instance in order to illustrate this observation. We notice that the number of selected facilities and/or the type of the facility changes with respect to the value of \( \alpha \). For instance, in the SCD-CVaR\( _{\alpha=0.95} \) model, Facility 1 is not selected. In contrast, in the SCD-CVaR\( _{\alpha=0.85} \) model, Facility 5 is not selected, which is a facility with a higher fixed cost.

Another key observation from Table 4-1 (also illustrated in Figure 4-4 and Figure 4-5) is that becoming more risk-averse results in remarkable increases in the cost of shipping untainted products to consumers. This implies that capacity allocation decisions change by varying \( \alpha \). We observe growth in the cost of inspection with respect to level of risk aversion level particularly for \( \alpha \in \{0.85,0.95\} \). This increase noticeably indicates that increasing \( \alpha \) leads to conducting more inspections in the facilities. Additionally, a remarkable reduction in the expected penalty cost of shipping tainted products is noticeable. Given Equations (5)–(8), we notice that \( k_{lx} \) and \( d_{lx} \) are auxiliary decision variables that depend solely on variables \( p_{lx} \) and \( z_{lx} \). Therefore, implementation of more inspections at the facilities and larger values of
shipping untainted products results in the discarding of more tainted products and, subsequently, a reduction in the number of tainted products shipped. Furthermore, inspection decisions at the facilities under different scenarios change as the value of $\alpha$ changes.

Table 4-1 A comparison between optimal solutions to the SCD and SCD-CVaR models with various risk-levels.

<table>
<thead>
<tr>
<th></th>
<th>SCD</th>
<th>SCD-CVaR $\alpha=0.50$</th>
<th>SCD-CVaR $\alpha=0.65$</th>
<th>SCD-CVaR $\alpha=0.75$</th>
<th>SCD-CVaR $\alpha=0.85$</th>
<th>SCD-CVaR $\alpha=0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVaR</td>
<td>-</td>
<td>7,080,578</td>
<td>7,226,298</td>
<td>7,382,727</td>
<td>7,540,617</td>
<td>7,797,082</td>
</tr>
<tr>
<td>VaR</td>
<td>-</td>
<td>6,673,971</td>
<td>6,850,485</td>
<td>6,992,332</td>
<td>7,335,176</td>
<td>7,587,006</td>
</tr>
<tr>
<td>Avg. expected total cost</td>
<td>6,587,944</td>
<td>6,790,520</td>
<td>6,910,247</td>
<td>6,972,212</td>
<td>7,214,093</td>
<td>7,250,803</td>
</tr>
<tr>
<td>Avg. fixed cost</td>
<td>5,210,752</td>
<td>5,694,548</td>
<td>5,861,480</td>
<td>5,943,178</td>
<td>6,247,912</td>
<td>6,257,492</td>
</tr>
<tr>
<td>Avg. expected untainted delivered cost</td>
<td>592,174</td>
<td>632,791</td>
<td>643,899</td>
<td>652,651</td>
<td>681,041</td>
<td>687,738</td>
</tr>
<tr>
<td>Avg. expected tainted penalty cost</td>
<td>755,792</td>
<td>403,878</td>
<td>339,150</td>
<td>304,849</td>
<td>204,321</td>
<td>228,408</td>
</tr>
<tr>
<td>Avg. expected inspection cost</td>
<td>26,185</td>
<td>52,314</td>
<td>58,365</td>
<td>63,418</td>
<td>68,202</td>
<td>68,566</td>
</tr>
<tr>
<td>Avg. expected discard cost</td>
<td>3,041</td>
<td>6,988</td>
<td>7,353</td>
<td>8,117</td>
<td>8,685</td>
<td>8,599</td>
</tr>
<tr>
<td>Avg. no. of selected facilities</td>
<td>3.4</td>
<td>3.8</td>
<td>3.9</td>
<td>3.9</td>
<td>4.1</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Figure 4-3 Fixed cost vs. reliability on Facility Selection at various risk levels
As an example, let us consider Scenario 21, where all facilities are in State 1, except for Facility 3 and Facility 5, and Scenario 32 where all facilities are in State 1. As illustrated in Figure 4-6, in the SCD model for scenario 21, inspection is implemented in Facility 3 and Facility 5, whereas in the SCD-CVaR$_{\alpha=0.95}$, inspection is implemented only in Facility 5. Furthermore, in Scenario 32, inspection is performed in all the facilities for SCD-CVaR$_{\alpha=0.95}$, while only three facilities are inspected in the SCD model.

Finally, recall that increasing $\alpha$ leads to a higher average number of selected facilities. This can be justified because a risk-averse decision-maker provides more capacity (by selection of more facilities) in order to be able both to perform more inspections and to discard more tainted products and still satisfy the total demand. The most salient conclusion that can be drawn from these results is that the risk-averse policy results in different strategic and tactical decisions compared to the risk-neutral policy, which is probably a more suitable design for a pharmaceutical supply network. However, we should note that the magnitude of change is also highly dependent upon the value of the risk level. In the next section, we utilize a sensitivity analysis for the further investigation of this subject.
Figure 4-4 Average cost of selection facilities for various risk-level values

Figure 4-5 Expected costs for various risk-level values
4.4.3. **Sensitivity Analysis of the Exact Models**

Our computational experiments display the sensitivity of the solution relative to the various values of the risk level (i.e., $\alpha$). In this section, we analyze the SCD and the SCD-CVaR outcomes for various settings of some of the parameters in order to provide insights that can assist decision-makers. Note that we compare the results of our sensitivity analysis to the results obtained in our computational analysis, which we refer to as the “base case.” We also perform the single-factor experiment in order to observe the effect of each cost factor. To make the results more interpretable and for the sake of simplicity, we consider the SCD model along with $\text{SCD-CVaR}_{\alpha=0.50,0.85}$ in our sensitivity analysis. Table 4-2 presents the settings for each parameter used in the sensitivity analysis.
Table 4-2 Parameter setting for sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range (s) of the Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed cost of selecting a facility ( f_i )</td>
<td>([300k, 500k], U[2M, 3M])</td>
</tr>
<tr>
<td>Fixed cost of implementing an inspection ( a_i )</td>
<td>([25k, 50k], U[75k, 150k])</td>
</tr>
<tr>
<td>Cost shipping tainted products ( c_{lk} )</td>
<td>([5k, 10k], U[15k, 30k])</td>
</tr>
</tbody>
</table>

4.4.3.1. Varying fixed cost of selecting a facility \( f_i \)

In this section, we examine the sensitivity of the fixed cost. For this purpose, let us first assume that our facilities are small-size facilities where the fixed cost of selecting a facility is drawn from a discrete uniform distribution between $300,000 and $500,000. We also consider larger size facilities where the fixed cost of selecting a facility is drawn from a discrete uniform distribution between $2,000,000 and $3,000,000. Note that we keep all other parameters constant. We use the same ten data instances that were previously described, changing only the fixed cost of selecting a facility. The result is summarized in Table 4-3. Note that the numbers in parentheses denote the reduction or growth in the costs compared to values obtained in the base case.

For the case where \( f_i \in U[300k, 500k] \), the average number of selected facilities increases as the fixed cost decreases in both the SCD model and SCD-CVaR model. We do not observe a notable change in the average expected cost of shipping untainted products. We observe a remarkable reduction in both the average expected penalty cost of shipping tainted products and the average expected cost of discarding tainted products, particularly in the SCD and SCD-CVaR\(_{\alpha=0.50}\) models. This is a
consequence of an increase in the average number of selected facilities as well as the implementation of more inspections at these facilities. For $SCD-CVaR_{\alpha=0.85}$, we observe an increase in the average expected cost of inspection and a slight change in the average expected cost of discarding tainted products. We observe that the ability to reduce the selecting cost of facilities can result (i.e., considering smaller size facilities) in increasing the average number of selected facilities and subsequently implementation of more inspection at the facilities particularly in the risk-neutral policy.

We notice that the average number of selected facilities drastically decreases when $f_i \in U[2m,3m]$. Furthermore, we observe a considerable increase in the average expected penalty cost of shipping tainted products and a decrease in the average expected cost of discarding tainted products, which are the consequence of considerable reduction in the implementation of inspection at the facilities. The reason for these changes is because of the reduction in the average number of selected facilities, which causes the decrease of the available capacity. Hence, the inspection at the facilities is refused and the tainted products are not discarded in order to provide enough capacity to be able to satisfy the total demand of consumers (i.e., to satisfy constraint set (9)). As a result, we observe that higher fixed costs of selecting a facility results in a reduction in implementation of inspection at the facilities and subsequently, increased quantities of tainted products reaching consumers.
Table 4-3 Comparison between optimal solutions to the SCD and SCD-CVaR models at varying fixed cost.

<table>
<thead>
<tr>
<th></th>
<th>SCD</th>
<th>SCD-CVaR_{α=0.50}</th>
<th>SCD-CVaR_{α=0.85}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_i \in [300k, 500k]$</td>
<td>$f_i \in [2M, 3M]$</td>
<td>$f_i \in [300k, 500k]$</td>
</tr>
<tr>
<td>CVaR</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VaR</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Avg. expected total cost</td>
<td>2,538,062</td>
<td>10,514,286</td>
<td>2,699,462</td>
</tr>
<tr>
<td>Avg. fixed cost</td>
<td>1,642,266</td>
<td>8,994,122</td>
<td>1,675,595</td>
</tr>
<tr>
<td>Avg. expected untainted delivered cost</td>
<td>587,447</td>
<td>597,569</td>
<td>627,086</td>
</tr>
<tr>
<td>Avg. expected tainted penalty cost</td>
<td>232,563(-70%)</td>
<td>907,643(20%)</td>
<td>191,559(-53%)</td>
</tr>
<tr>
<td>Avg. expected inspection cost</td>
<td>68,523(162%)</td>
<td>13,203(-50%)</td>
<td>73,607(41%)</td>
</tr>
<tr>
<td>Avg. expected discard cost</td>
<td>7,262(138%)</td>
<td>1,748(-42%)</td>
<td>7,587(9%)</td>
</tr>
<tr>
<td>Avg. no. of selected facilities</td>
<td>4.1</td>
<td>3.5</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.2</td>
</tr>
</tbody>
</table>
4.4.3.2. Varying fixed cost of implementing an inspection \( (n_i) \)

In Section 3.4, the cost of inspection at each facility was uniformly distributed over the range of $50,000 and $100,000. We consider two different ranges for our sensitivity analysis. In the first range, we will have a 50% reduction in the cost of inspection (i.e., inspection cost drawn from \( U[25k,50k] \)), and in the second range, we consider a 50% increase in the cost of inspection (i.e., inspection cost drawn from \( U[75k,150k] \)). Table 4-4 presents our findings for various levels of \( \alpha \). The numbers in parentheses denote the reduction or growth in the costs compared to values obtained in the “base case.”

For \( n_i \in U[25k,50k] \), despite some slight variations, we observe that average fixed cost, average expected penalty cost of shipping tainted products, and average number of selected facilities are all insensitive to the change. In the SCD and SCD-CVaR \( \alpha=0.50 \) models, we note that while the inspection cost decreases considerably, the average discard cost increases and the average expected penalty cost of shipping tainted products decreases, which is more considerable in the SCD-CVaR \( \alpha=0.50 \) model. We also note an increase in the average expected cost of discarding tainted products which implies that the reduction in cost of implementing inspection results in more inspections being performed, as would be expected.

For \( n_i \in U[75k,150k] \), we observe a reduction in the number of selected facilities as \( \alpha \) increases. This reduction is also valid when compared to the base case and the case where \( n_i \in U[25k,50k] \). We observe that, for high inspection cost cases and for higher values of \( \alpha \), the SCD-CVaR model tends to select more reliable
facilities, whereas for lower inspection costs, the tendency is toward selecting facilities with larger capacities. Hence, we observe lower fixed costs for $n_i \in [75k,150k]$ compared to the base case and also $n_i \in [25k,50k]$. The corresponding analysis is presented in more detail in Chapter 5. The results also show fewer inspections implemented in both the SCD and SCD-CVaR$_{\alpha=0.50}$ models and 26 percent and 11 percent decreases in the average expected costs of discarding tainted products for $\alpha = 0.50$ and $\alpha = 0.85$, respectively. This reduction results in a nearly six percent increase in the average expected penalty cost of shipping tainted products, which is still 56 percent less than what we observe for $\alpha = 0.50$. The obtained result indicates that managers and decision-makers should either maintain inspection cost at the lowest possible value or become more risk-averse when the cost of inspection implementation is high.
### Table 4-4 Comparison between optimal solutions to the SCD and SCD-CVaR models at varying inspection cost.

<table>
<thead>
<tr>
<th></th>
<th>SCD</th>
<th>SCD-CVaR_{\alpha=0.50}</th>
<th>SCD-CVaR_{\alpha=0.85}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_i \in [25k, 50k]$</td>
<td>$n_i \in [75k, 150k]$</td>
<td>$n_i \in [25k, 50k]$</td>
</tr>
<tr>
<td>CVaR</td>
<td>-</td>
<td>6,374,835</td>
<td>6,716,224</td>
</tr>
<tr>
<td>VaR</td>
<td>-</td>
<td>5,980,693</td>
<td>6,472,782</td>
</tr>
<tr>
<td>Avg. expected total cost</td>
<td>6,569,842</td>
<td>6,620,171</td>
<td>6,778,385</td>
</tr>
<tr>
<td>Avg. fixed cost</td>
<td>5,210,752</td>
<td>5,210,752</td>
<td>5,728,242</td>
</tr>
<tr>
<td>Avg. expected untainted delivered cost</td>
<td>592,205</td>
<td>592,152</td>
<td>627,905</td>
</tr>
<tr>
<td>Avg. expected tainted penalty cost</td>
<td>751,566(-0.6%)</td>
<td>775,485(3%)</td>
<td>389,182(-3%)</td>
</tr>
<tr>
<td>Avg. expected inspection cost</td>
<td>12,265(-52%)</td>
<td>38,865(48%)</td>
<td>26,052(-50%)</td>
</tr>
<tr>
<td>Avg. expected discard cost</td>
<td>3054(0.4%)</td>
<td>2,942(-4%)</td>
<td>7003(0.4%)</td>
</tr>
<tr>
<td>Avg. no. of selected facilities</td>
<td>3.4</td>
<td>3.4</td>
<td>3.8</td>
</tr>
</tbody>
</table>
4.4.3.3. Varying penalty cost of shipping tainted products ($o_{hc}$)

Table 4-5 reports the relative differences in the optimal expected costs and the average number of selected facilities with respect to changes in the penalty cost of shipping tainted products. For $o_{hc} \in U[5k,10k]$, there is a considerable reduction in the average number of selected facilities for both SCD-CVaR$_{\alpha=0.50}$ and SCD-CVaR$_{\alpha=0.95}$. However, for the SCD model, the average number of selected facilities is insensitive to the change. We also notice a remarkable reduction in the inspection cost and the average expected discard cost, particularly in the SCD and SCD-CVaR$_{\alpha=0.50}$ models. This reduction implies the implementation of fewer inspections at the facilities. These observations indicate that the use of a low penalty cost for shipping tainted products results in decisions that do not support the detection of tainted materials nor the selection of enough facilities to protect against requiring shipping tainted products.

For $o_{hc} \in U[15k,30k]$, we observe a notable increase in the number of selected facilities in the SCD and SCD-CVaR$_{\alpha=0.50}$ models. We also notice increase in the inspection cost and the average expected discard cost in the SCD and SCD-CVaR$_{\alpha=0.50}$ models. However, in the SCD-CVaR$_{\alpha=0.95}$ model, the average number of selected facilities and the average expected penalty cost of shipping tainted products are both insensitive to the change. The results decidedly indicate that decision-makers should consider the high penalty cost of shipping tainted products when dealing with both the risk-neutral policy and the risk-averse policy. Our result also supports the position that managers and decision-makers should avoid
considering a low penalty cost for shipping tainted products when they are willing to be risk-averse.
Table 4-5 Comparison between optimal solutions to the SCD and SCD-CVaR models at varying shipping tainted cost.

<table>
<thead>
<tr>
<th></th>
<th>SCD ( \alpha_e \in [5k,10k] )</th>
<th>SCD ( \alpha_e \in [15k,30k] )</th>
<th>SCD-CVaR ( \alpha = 0.50 )</th>
<th>SCD-CVaR ( \alpha = 0.85 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVaR</td>
<td>-</td>
<td>-</td>
<td>5,595,021</td>
<td>6,990,944</td>
</tr>
<tr>
<td>VaR</td>
<td>-</td>
<td>-</td>
<td>5,340,650</td>
<td>6,604,175</td>
</tr>
<tr>
<td>Avg. expected total cost</td>
<td>6,000,543</td>
<td>6,605,522</td>
<td>6,044,112</td>
<td>6,737,116</td>
</tr>
<tr>
<td>Avg. fixed cost</td>
<td>5,068,814</td>
<td>5,373,692</td>
<td>5,090,760</td>
<td>5,887,011</td>
</tr>
<tr>
<td>Avg. expected untainted delivered cost</td>
<td>582,557</td>
<td>485,849</td>
<td>614,505</td>
<td>516,871</td>
</tr>
<tr>
<td>Avg. expected tainted penalty cost</td>
<td>340,375 (-55%)</td>
<td>698,080 (-8%)</td>
<td>326,791 (-19%)</td>
<td>258,296 (-36%)</td>
</tr>
<tr>
<td>Avg. expected inspection cost</td>
<td>7,971 (-70%)</td>
<td>43,276 (65%)</td>
<td>10,916 (-79%)</td>
<td>67,946 (30%)</td>
</tr>
<tr>
<td>Avg. expected discard cost</td>
<td>826 (-72%)</td>
<td>4,625 (52%)</td>
<td>1,140 (83%)</td>
<td>6,991 (0.05%)</td>
</tr>
<tr>
<td>Avg. no. of selected facilities</td>
<td>3.3</td>
<td>3.7</td>
<td>3.3</td>
<td>4.0</td>
</tr>
</tbody>
</table>


4.4.3.4. Experiments

This section presents a few experiments on both the SCD and the SCD-CVaR models.

Experiment 1. Fixed capacity and varying reliability of the facilities

The demand for each consumer is drawn from a discrete uniform distribution between 100 and 300 units. We consider an identical constant value for the capacity of the facilities in order to maintain the ratio of the total capacity 35% higher than the total demand before implementing inspection and discarding tainted items. Furthermore, since the cost of selecting a facility is correlated with the capacity, hence this cost is constant and identical for all the facilities as well. As we observe in Figure 4-7, for higher values of risk-level $\alpha$, facilities with higher reliability have higher likelihood of selection. Note that the reliability of the facilities is drawn from a discrete uniform distribution between 0.50 and 0.95.
Experiment 2. Varying capacity and fixed reliability of the facilities

In our second observation, we maintain the reliability of the facilities at a constant rate while varying the capacity of the facilities. The result is illustrated in Figure 4-8. We observe that, the number of selected facilities changes with respect to the value of $\alpha$ and for higher values of risk-level, the number of selected facilities increases. We also observe the tendency toward selecting facilities with larger capacities and also providing more available capacity for high values of $\alpha$. 
4.4.4. Heuristic

4.4.4.1. A Metaheuristic Approach

From solving the SCD-CVaR problem using commercial software we observed that, the number of facilities, and consequently the number of scenarios, has a significant impact on the computational time (see Table 4-7 as an example). As a result, we develop a metaheuristic-based solution approaches for analyzing this challenging, practically-motivated problem.

In this research, a simulated annealing (SA) is proposed to solve the SND-CVaR problem. Our SA operates in five phases as follows:
• Phase 1: determine set of selected facilities \( (x_i) \)
• Phase 2: determine the inspection decisions \( (z_{it}) \)
• Phase 3: calculate \( p_{lts}, k_{lts} \) and \( d_{lts} \) or capacity allocation
• Phase 4: calculate VaR(\( \eta \))
• Phase 5: calculate SND-CVaR and evaluate the solution

For phase 1 and phase 2, we utilize the neighborhood strategy which was discussed in Section 3.5.2. We apply the same neighborhood strategy to determine \( z_{it} \).

After the set of selected facilities and inspection decisions are fixed and known, we need to allocate the available capacity to customers. We conduct the capacity allocation in a greedy fashion and in three steps as applied for SCD-Sub in Chapter 3. Recall that we followed three steps. In step 1, the demand vector and transportation cost are sorted. In step 2, the biggest demand is assigned to the smallest cost. And finally in step 3, decision variables \( k_{lts} \) and \( d_{lts} \) are computed. In phase 4, we use the following definition to calculate VaR (see [88]).

**Definition 1.** Consider a set of scenarios \( S \) where the likelihood of each scenario is \( \rho \)

\( I(k) \) denote the \( k \)th order statistic of the scenario sample \( (I_{(1)} \leq I_{(2)} \leq \cdots \leq I_{(k)}) \).

The empirical cdf can be defined as follows:

\[
\hat{F}(y) = \begin{cases} 
0 & \text{if } y < I_{(1)} \\
\frac{k}{s} & \text{if } I_{(k)} \leq y < I_{(k+1)} \\
1 & \text{if } y \geq I_{(s)}
\end{cases}
\]
One popular estimator of \( \text{VaR}(\alpha) \) is the inverse of the above empirical cdf, i.e.,

\[
\text{VaR}(\alpha) = \hat{F}^{-1}(\alpha) = I(k) \quad \text{where} \quad \frac{k-1}{S} < \alpha \leq \frac{k}{S}.
\]

The structure of the SA algorithm is presented in Figure 4-9. In the next section we conduct some computational analysis to evaluate the performance of the proposed SA.

A computational experiment is conducted to test the performance of the SA procedure against the exact solution. For that experiment we employ two datasets where the first consists of 5 facilities and 5 consumers and the second consists of ten facilities and 5 consumers. We selected three levels of \( \alpha \): 0.50, 0.75, and 0.99.
The main observation from Table 4-6 is that the solution time decreases with increases in the value of $\alpha$ in the exact solution. However, we observe a steady trend for the solution time in SA. In other word, in SA, the solution time is not a function of the value of $\alpha$ while the opposite is true when solving the SCD-CVaR model exactly.

Figure 4-10 illustrates that the average fixed cost of the exact solution and SA solution increases by increasing $\alpha$. This indicates that the number of selected suppliers increases with the risk-level $\alpha$ in order to increase the available capacity. From Figure 4-11 we also observe that the average cost of shipped untaited product in both exact solution and SA solution increases when the decision-maker is more risk-averse.

Table 4-6 Summary of SA result for SCD-CVaR problem

<table>
<thead>
<tr>
<th></th>
<th>5 facilities and 5 consumers</th>
<th>ten facilities and 5 consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha=0.50$</td>
<td>$\alpha=0.75$</td>
</tr>
<tr>
<td><strong>Exact Solution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CVaR</td>
<td>7556517</td>
<td>7722731</td>
</tr>
<tr>
<td>Average Solution Time (s)</td>
<td>62.8</td>
<td>63.6</td>
</tr>
<tr>
<td>No. of optimal solutions in 5 instances</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td><strong>SA Solution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CVaR</td>
<td>7,856,700</td>
<td>8,277,300</td>
</tr>
<tr>
<td>Average Solution Time (s)</td>
<td>1.43</td>
<td>1.48</td>
</tr>
<tr>
<td>Gap-CVaR (%)</td>
<td>4.0</td>
<td>7.2</td>
</tr>
</tbody>
</table>
4.4.4.2. Sample Average Approximation

Sample Average Approximation (SAA) is a solution method for stochastic optimization problems with large numbers of scenarios which was first proposed by Kleywegt et al. [89]. They analyzed the behavior of the SAA method when applied to stochastic discrete optimization problems. The basic idea of this method is to
randomly generate samples, then the expected objective function of the stochastic problem is approximated by these samples. The resulting sample average approximation problem is then solved by deterministic optimization techniques [90]. Consider equation (41):

\[
\text{[SAA]} \quad w = \min \sum_{i \in L} x_i f_i + E\left[ Q(x, \tilde{s}) \right]
\]

(41)

subject to \( x_i \in \{0, 1\}, \forall l \in L \) \hspace{1cm} \text{(42)}

The main idea of the SAA method is that by generating \( N \) samples \( s^1, s^2, ..., s^N \) of scenarios from \( S \), the expected value function \( E\left[ Q(x, \tilde{s}) \right] \) (where

\[
E\left[ Q(x, \tilde{s}) \right] = \sum_{s=1}^{N} \rho_s Q(x, s)
\]

(41)) is approximated by the sample average function

\[
\frac{1}{N} \sum_{n=1}^{N} Q(x, s^n)
\]

Then we obtain the following problem:

\[
\min \sum_{i \in L} x_i f_i + \frac{1}{N} \sum_{n=1}^{N} Q(x, s^n)
\]

(43)

Let \( w^* \) and \( x^* \) denote the optimal objective function value and the first-stage vector solution of the original problem (equation (41)). In the SAA method, \( M \) independent batches, each of which consists of \( N \) scenarios, are generated and the SAA problem is solved \( M \) times repeatedly. Denote the objective values which are obtained at each time by \( w^*_N, w^*_N, ..., w^*_N \), then let
signify the average of the $M$ optimal values of the SAA problem. It is proven that $E\left[w_N^*\right] \leq w^*$ [89,91,92]. Therefore, $w_N^*$ provides a statistical estimate of a lower bound of the optimal value of the true problem (see [75]). The variance of the estimator $w_N$ can be estimated by

$$\hat{\sigma}_{w_N}^2 = \frac{1}{(M-1)} \sum_{m=1}^{M} (w_{N_m}^* - w_N^*)^2$$

(45)

The above procedure results in $M$ different candidate solutions. It is natural to consider $x^*$ as one of the optimal solutions $\hat{x}^1, \hat{x}^2, ..., \hat{x}^M$ of the $M$ of the SSA problem which has the smallest estimated objective value. Hence, we have

$$x^* \in \arg\min \{\hat{x}^1, \hat{x}^2, ..., \hat{x}^M\}$$

(46)

We can also evaluate the quality of the solution by computing the following gap

$$w_N^* - w(x^*)$$

(47)

Furthermore, for any feasible point $x$, the objective value of $\sum_{i \in L} x_i f_j + E[Q(x, \bar{s})]$ is an upper bound for $w^*$. This upper bound can be estimated by equation (48)

$$\bar{w}_{N'} = \min \left\{ \sum_{i \in L} x_i f_j + \frac{\sum_{n=1}^{N'} Q(x, s^n)}{N'} \right\}$$

s.t (2)
where \( N' \) is the sample size of scenarios and it is chosen to be significantly larger than \( N (N' \gg N) \). Given the above explanation, the SAA method applied in this research is presented in the following.

We stated that increasing the number of facilities, and consequently the number of scenarios significantly increase the computational time of the SCD-CVaR model. Therefore, we apply the SAA method in addressing the large sized problems for identifying a good lower bound to the original problem.

In this research the general SAA algorithm is implemented with some modifications to fit the problem studied. We denote this algorithm as modified-SAA. However, for the sake of comparison with the results of modified-SAA, we also present the result of the general SAA algorithm in Section 4.4.4.2.1. The modifications are summarized in the following.

First, in Section 4.4 we mentioned that the optimization problem, SCD-CVaR, is modeled by using the AMPL mathematical programming language and solved with Gurobi 4.5.6. We terminate Gurobi when the CPU time limit of 14,400 seconds is reached. In this research, an estimate of the upper bound to equation (41), \( \bar{\pi}_{w'} \), can be derived by using the objective value of the solution to the SCD-CVaR which is reported by Gurobi.

Second, in the general SAA algorithm a random scenario \( N = \{ s^1, s^2, ..., s^N \} \) of realizations of the random vector \( \bar{s} \) is generated and an identical weight of \( \frac{1}{N} \) is assigned to each single scenario (see [89]). In this research, we calculate the
probability of each scenario following the procedure presented in Section 3.4 where the probability (weight) is equal $\rho_j$.

![Modified-SAA Algorithm](image)

4.4.4.2.1. Computational Result of modified-SAA Algorithm

To illustrate the applicability of the proposed modified-SAA method, a numerical experiment is conducted in this section. The modified-SAA algorithm is tested by using different combinations of sample sizes $|N| \in \{8, 16, 32, 64\}$ and $|M| = \{128, 64, 32, 16\}$. The general SAA algorithm is tested by using the sample size $|N| \in \{20, 30, 50, 100, 300\}$ and $|M| = 10$.

The SAA algorithm presented in Figure 4-12 implemented and executed in MATLAB 7.9 (2012) and tested on a single core of a Dell OptiPlex 980 computer running the Windows 7 Enterprise 64 bit operating system with an Intel(R) Core(TM) i7 CPU860@ 2.80GHz, and 8GB RAM.
All of our computational experiments are based on the data that was generated from the procedure presented in Section 3.4. We considered ten data instances for a supply chain network consisting of ten facilities and ten consumers and we select 0.75 for the risk-level $\alpha$. Table 4-7 reports the average of the optimal value/ best solution found from Gurobi for each data instances. As per the results obtained, none of the ten data instances resulted in an optimal solution within the prescribed time limit of 14400 seconds. We consider the average of ten data instances as the upper bound and we consider 14210174 as $\bar{w}_N$ in our computation.

<table>
<thead>
<tr>
<th>Data Instance no.</th>
<th>best solution found (upper bound)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16709100*</td>
</tr>
<tr>
<td>2</td>
<td>14934928*</td>
</tr>
<tr>
<td>3</td>
<td>12993917*</td>
</tr>
<tr>
<td>4</td>
<td>15981710*</td>
</tr>
<tr>
<td>5</td>
<td>13799333*</td>
</tr>
<tr>
<td>6</td>
<td>12847419*</td>
</tr>
<tr>
<td>7</td>
<td>14348449*</td>
</tr>
<tr>
<td>8</td>
<td>15010500*</td>
</tr>
<tr>
<td>9</td>
<td>11525948*</td>
</tr>
<tr>
<td>10</td>
<td>13950441*</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>14210174</strong></td>
</tr>
</tbody>
</table>

*: Best objective values found

Table 4-8 summarizes corresponding computational results of the modified-SAA algorithm in detail. Note that the presented results are the average of ten data instances. The first and second columns present the number of replications and the sample size, respectively. The third column provides the estimated objective value of the corresponding problem. The fourth and fifth columns represent the gap between the best solution found by modified-SAA algorithm and the estimated objective value,
and the standard deviations of the estimator \( w_N \), respectively. The CPU time, the average optimality gap (average gap between the lower bound and the upper bound) and finally the 95% confidence interval for the optimality gap display in the sixth, seventh and eighth columns, correspondingly. It is clear from Table 4-8 that the solution quality can be improved by increasing the sample size however, we observe that average optimality gap remains above 0.50 even if we increase the sample size \( N \). That is because in the set of scenarios we have some scenarios with very small probability values. Hence, these values result in small objective value. As a result, in this research we also implemented the general SAA algorithm to compare the results of these two algorithms. The results are presented in Table 4-9.

| Table 4-8 Computational results of modified-SAA algorithm |
|---|---|---|---|---|---|
| \( |M| \) | \( |N| \) | \( w_N \) | \( \% w_N - w(x^*) \) | \( \sigma_{\hat{\Delta}_w} \) | Time (s) | \% Avg. optimality gap | Avg. CI of LB at 95% |
| 128 | 8 | 395388 | 0.93 | 36173 | 58 | 0.96 | (390056, 400718) |
| 64 | 16 | 795,204 | 0.83 | 48225 | 190 | 0.92 | (794000, 796409) |
| 32 | 32 | 2485926 | 0.71 | 121609 | 832 | 0.82 | (2479102, 2492750) |
| 16 | 64 | 3309112 | 0.49 | 296420 | 1301 | 0.76 | (3022096, 3340950) |

The computational results in Table 4-9 indicate that the general SAA algorithm leads to a considerably lower average optimality gap compared to the modified-SAA algorithm. We also observe that, in the general SAA algorithm the quality gap \( (w_N - w(x^*)) \) and the computational time are notably smaller than the values achieved in Table 4-8 for modified-SAA algorithm.

To summarize the results of this section, the general SAA leads to a better gap for the SCD-CVaR model with a shorter computational time compared to the modified-SAA algorithm. However, these solutions differed from the exact solutions obtained by Gurobi for the first-stage decision variable. Based on our computational
experiences, the modified-SAA algorithm resulted in solutions that better matched the first-stage decision variables from the exact solution in spite of the worse lower bound. For larger instances however, the solution times exceeded 2 hours of computations i.e., solution times are remarkably longer as problem size increases.

| | | | | | | |
|---|---|---|---|---|---|
| | | | | | |

Table 4-9 Computational results of general SAA algorithm

| | | | | | | |
|---|---|---|---|---|
| | | | | | |

4.5. Conclusions and Future Research

In this chapter, we presented a supply network design problem with application in the pharmaceutical industry to hedge against unreliability of capacity and prevent shipping of tainted materials to the consumers. We studied a risk-neutral decision-making policy and a risk-averse decision-making policy. We characterized the trade-off between the risk and cost, which provides several insights on the impact of risk-aversion on the facilities’ optimal decisions in a pharmaceutical supply chain. Our studies demonstrated how strategic and tactical decisions change with respect to the risk level. We found that an increase in the risk level $\alpha$ leads to the selection of not only more reliable facilities but also a different number of facilities. The risk-averse policy also resulted in fewer worst-case scenarios as compared to the risk-neutral policy. Our computations also revealed that becoming more risk-averse resulted in remarkable increases in the cost of shipping untainted products to consumers.
Our experience from solving the problem using AMPL-Gurobi indicated that the number of facilities considerably increases the computational time. As a result, we developed a metaheuristic approach to effectively solve the presented model. We observed that the SA algorithm provides solutions within 5% of the solutions found by Gurobi for both small-size and large-size problems. We also applied the SAA algorithm to obtain a good lower bound for SCD-CVaR problem and also solutions to the first-stage problem. We considered two approaches for our SAA algorithm. In the first approach, we modified SAA algorithm in order to be able to use the set of scenarios based on our procedure we developed in this research. For the second approach, we considered the general SAA algorithm. Our results indicated that the general SAA led to a lower average optimality gap with a shorter computational time compared to the modified-SAA algorithm. However, based on our computational experiences, the modified-SAA algorithm resulted in better solutions for the first-stage decision variables. For larger instances however, the solution times exceeded 2 hours of computations, i.e., solution times are remarkably longer as problem size increases. As a future extension, we wish to investigate some techniques such as Bender Decomposition or L-shaped methods to efficiently solve the problems.

The significance of this chapter is two-fold. First of all, to the best of our knowledge there is no currently available research to evaluate pharmaceutical (or healthcare) supply chain network design with respect to sending tainted materials to the consumer. Secondly, there is also little prior research to date that investigates supply chain risk within the context of the pharmaceutical supply chain. As pharmaceutical availability and drug safety clearly are key components to effective
patient quality of care, our models can assist supply chain designers enhance patient safety and quality of patient care.

There are some interesting future research extensions. An extension of the presented work is to include demand uncertainty and/or seasonal demand. Moreover, we assumed an inspection and discard approach, which is not a valid assumption in some supply chains like the automotive and electronics industries. This assumption can be shifted to an inspection and fix (rework) approach where defective products can be repaired after detection. Ultimately, we have considered instances that included five facilities. However, experience from solving the models using commercial software indicated that the number of facilities can dramatically increase the computational time. We also think it is important to design and develop heuristic techniques to obtain acceptable solutions to these larger size problems in reasonable runtimes and with good solution quality.
CHAPTER FIVE

5. An Statistical Analysis of Facility Selection Process

5.1. Statistical Analyses Motivation

In this chapter, we will consider statistical analyses for the SCD and SCD-CVaR models. In review of our computational analysis, we found that the selection of some of facilities were obvious based on parameter values such as capacity or fixed cost. In other cases, the facilities which appeared to be desirable were not selected. Hence, it may be difficult to predict or determine which facilities are selected or unselected; it may also be difficult to interpret the output of the SCD or SCD-CVaR models. We perform a regression analysis in order to identify factors for predicting the selection of a facility in the SCD and SCD-CVaR frameworks at various risk levels and to analyze relationships among variables. The second purpose for conducting a regression analysis in this context is to assess the probability of selection for each individual facility at various risk levels ($\alpha$). We will also determine the probability of selection for a set of selected facilities or alternatives.

Let us consider a supply chain network consisting of five facilities and five consumers where the total demand of the consumers is 908 units. The values of the parameters for this example are presented in Table 5-1.
Table 5-1 Values for an example of a supply chain network

<table>
<thead>
<tr>
<th>Facility #</th>
<th>Fixed cost ((f_i))</th>
<th>Inspection cost ((n_i))</th>
<th>Fraction of tainted items before inspection ((q_i))</th>
<th>Fraction of remained tainted items after inspection ((r_i))</th>
<th>Reliability ((\Theta_i))</th>
<th>Capacity ((\kappa_i))</th>
<th>Ratio capacity/total demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1068072</td>
<td>90025</td>
<td>0.28</td>
<td>0.08</td>
<td>90%</td>
<td>136</td>
<td>15%</td>
</tr>
<tr>
<td>2</td>
<td>1764329</td>
<td>77067</td>
<td>0.25</td>
<td>0.07</td>
<td>52%</td>
<td>359</td>
<td>40%</td>
</tr>
<tr>
<td>3</td>
<td>1565588</td>
<td>53977</td>
<td>0.17</td>
<td>0.08</td>
<td>89%</td>
<td>317</td>
<td>35%</td>
</tr>
<tr>
<td>4</td>
<td>1358214</td>
<td>95728</td>
<td>0.26</td>
<td>0.03</td>
<td>73%</td>
<td>247</td>
<td>27%</td>
</tr>
<tr>
<td>5</td>
<td>1302468</td>
<td>61519</td>
<td>0.12</td>
<td>0.02</td>
<td>85%</td>
<td>176</td>
<td>19%</td>
</tr>
</tbody>
</table>

From Table 5-1, we observe the following relations

\[ f_2 > f_3 > f_4 > f_5 > f_1 \]

\[ n_4 > n_1 > n_2 > n_5 > n_3. \]

Figure 5-1 illustrates this tradeoff between \(\kappa_i\) and \(q_i\) for various values of \(\alpha\) which is the result of an optimal solution. For the SCD and SCD-CVaR_{\alpha=0.50} models, we expect either a smaller number of selected facilities or facilities with lower fixed cost. As we observe in Figure 5-1, for SCD and SCD-CVaR_{\alpha=0.50}, facility 1 and facility 5 are not selected which are facilities with the lowest fixed cost, respectively. However, as we observe in Table 5-1, facility 1 and facility 5 are the facilities with the highest and lowest fraction of tainted products before inspection, respectively. Facility 1 and Facility 5 also have the lowest capacity among all facilities. That indicates that in SCD and SCD-CVaR_{\alpha=0.50} model, the selection of facilities was a trade-off between fixed cost or capacity and fraction of tainted products.
In SCD-CVaR_{0.65}, SCD-CVaR_{0.75} and SCD-CVaR_{0.85}, facility 4 is not selected which is the most expensive facility for performing inspections. In SCD-CVaR_{0.95} and SCD-CVaR_{0.99}, decision making is based on a highly risk-averse policy and almost no risk is acceptable. Therefore, the model mainly focuses on minimizing 5% and 1% of the worst outcomes and shipping the least amount of tainted items to consumers, respectively. However, it is unclear why Facility 5 (which is the facility with highest fraction of untainted products before inspection) is not selected. Consequently, another method should be considered to predict the selection of facilities based on reliability or other factors. Moreover, when moving from a risk-neutral to a risk-averse policy, fixed costs, capacity, or perhaps other factors may not be sufficient in predicting which facilities will be selected or unselected. In the next section, a regression analysis is conducted.
5.2. Logistic Regression Analysis

The response variable, selecting a facility, is dichotomous or binary. A linear regression model is inappropriate to predict a binary response variable for the following reasons:

- A linear regression analysis assumes a continuous response which is unconstrained and can vary over the range of $-\infty$ to $+\infty$ whereas for
dichotomous variables, the observed outcome is bounded and binary, with only two as possible values.

- The error terms produced from such a model will not be normally distributed.

A logistic regression models the relationship between a two-level categorical response variable (binary response) and a set of explanatory (independent/predictor) variables which can be quantitative or qualitative. Assume we have observed independent variables $X_1, X_2, \ldots, X_K$ on a group of subjects. We wish to use this information to describe the probability that a facility will be selected given the values of the $k$ independent variables $X_1, X_2, \ldots, X_k$. For notational convenience, we denote the probability statement $P(y = 1 | X_1, X_2, \ldots, X_k)$ as simply $\pi(X)$. The status of a facility, $y_1$, is a binary response variable coded as $y = 1$ or $y = 0$ with respective probability $\pi(X) = P(y_1 = 1 | X_1, X_2, \ldots, X_k)$ and $1 - \pi(X) = 1 - P(y_1 = 1 | X_1, X_2, \ldots, X_k)$. The probability of selecting a facility is described by following relationship

$$P(y_1 = 1 | X_1, X_2, \ldots, X_k) = \frac{e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k}}{1 + e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k}}. \quad (49)$$

The terms $\beta_0, \beta_1, \ldots, \beta_k$ in this model represent unknown parameters and must be estimated based on data obtained on the values of the predictor and response variables for a group of subjects. By rearranging equation (49) the odds of selecting a facility can be expressed as:

$$\frac{\pi(X)}{1 - \pi(X)} = e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k}. \quad (50)$$

Therefore,
\[ Y = \ln \left( \frac{\pi(X)}{1 - \pi(X)} \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon. \]  

(51)

This transformation \( Y \), in equation (51), is called the logit transformation. This also turns equation (49) into a linear regression model. Note that \( \ln \left( \frac{\pi(X)}{1 - \pi(X)} \right) \) is the log odds.

The odds are another way to express the likelihood that an event will occur. The odds is a ratio of the probability that some event will occur over the probability that it will not occur. For instance, if \( p = 0.8 \) is the probability of selecting a facility (e.g., \( y = 1 \)), then \( 1 - p = 0.2 \) which is the probability of that facility not being selected. Therefore, the odds \( 0.8/0.2 = 4 \) which indicates that the likelihood that this facility is selected is four times the likelihood of this facility is not being selected. The advantage with using the odds is that, unlike a probability value, odds have no upper bound. Therefore, a small change in a probability value results in large change in odds. A value of one for odds indicates that the probability of selection and probability of not selecting a facility are the same. Our interest is in the change of the dependent variable for a one unit change in the independent variables, or the “Odds Ratio (OR).” The OR is the ratio of the odds of the event for one value of one independent variable (e.g., capacity) divided by the odds for a different value of that independent variable (for instance, a value one unit lower for capacity when other variables are constant in the model). Hence, the OR indicates the amount of change in the odds and the direction of the relationship between an independent variable and dependent variable.
In general, we can state that an OR of one indicates that the odds of selecting a facility are the same; an OR greater than one indicates that the odds of selecting a facility increases (a positive relationship); and an OR less than one indicates that the odds of selecting a facility decreases (a negative relationship). Table 5-2 gives a comparison of the standard linear regression model with the logistic regression model.

<table>
<thead>
<tr>
<th>Table 5-2 Comparison of linear and logistic regression models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Response</td>
</tr>
<tr>
<td>Covariates</td>
</tr>
<tr>
<td>Meaning of coefficient</td>
</tr>
</tbody>
</table>

Let $\hat{\pi}_l$ and $\hat{\beta}$ denote the fitted value for facility $l$ and the corresponding estimated coefficients, respectively. We can rewrite the estimated odds equation as:

$$\frac{\pi(X)}{(1-\pi(X))} = e^{\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \cdots + \hat{\beta}_K X_K}$$

or,

$$\hat{Y} = \ln \left( \frac{\hat{\pi}(X)}{1-\hat{\pi}(X)} \right) = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \cdots + \hat{\beta}_K X_K + \epsilon.$$  \hspace{1cm} (52)

Thus, given the above equation for each one unit increase in $X_k$ while keeping other predictors constant, the predicted odds is increased by a factor of $exp(\hat{\beta}_k)$. In other words, $exp(\hat{\beta}_k)$ is an odds ratio, which denotes the odds at $X_k + 1$ divided by the odds $X_k$. Similarly, if all predictors are set equal to 0, the predicted odds is $exp(\hat{\beta}_0)$. 
We will also consider the interaction between the variables. If two independent variables interact, then the effect of one of them on the dependent variable varies depending on the value of the other independent variable. The presence of interactions can have important effects for the interpretation of regression model. Interaction is assessed by adding the cross-product term (e.g., $X_1 \times X_2$) to the model. For instance, by assuming an interaction between $X_{k-1}$ and $X_k$ we can rewrite equation (52) as following:

$$\hat{Y} = \ln \left( \frac{\hat{\pi}(X)}{1 - \hat{\pi}(X)} \right) = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \ldots + \hat{\beta}_k X_k + \hat{\beta}_{k+1} X_{k-1} X_k + \epsilon. \quad (53)$$

Now, for each one unit increase in $X_k$, the increase in the predicted odds is a factor of $\exp(\hat{\beta}_k + \hat{\beta}_{k+1} X_{k-1})$. That is, an independent variable whose effect on the dependent variable is considered to vary as a function of the $X_{k-1}$. Note that an increase in the logit is in favor of the probability of selecting a facility since by increasing $\frac{\hat{\pi}(X)}{1 - \hat{\pi}(X)}$, $\hat{\pi}(X)$ would also increase.

### 5.2.1. Description of the Data Source

In this section, we define a set of continuous predictor variables that are hypothesized to be associated with the dependent variable, which is the status of a facility being selected or unselected. The predictors for facility $l$ include the following: the reliability of the facility ($\Theta_l$), the fixed cost of opening the facility ($f_l$), the capacity of the facility ($\kappa_l$), the fixed cost of implementing an inspection at the facility ($n_l$), the total demand of the consumers ($b = \sum_{c \in C} b_c$), the fraction of tainted
products at the facility $l$ ($q_{ls}$); the fraction of tainted products at facility $l$ after inspection ($r_l$); and risk level ($\alpha$). We will investigate the impact of these factors on the status of a facility. The status of a facility is represented by an indicator variable defined as follows:

$$y_l = \begin{cases} 1, & \text{if facility } l \text{ is selected}, l \in L \\ 0, & \text{else} \end{cases}$$

We use 50 data files for a specific case containing five facilities and five consumers. We consider seven levels for $\alpha$: 0, 0.50, 0.65, 0.75, 0.85, 0.95 and 0.99. Therefore, in total we will have 1750 ($50 \times 5 \times 7$) observations. To ensure that the data generated are uniformly spread out over the defined ranges (defined in 3.4); we consider the histogram for each single factor in Figure 5-2.

Given the histograms in Figure 5-2, we observe that data is distributed nearly uniformly over the variable ranges. We also consider a similar histogram for each
single facility to verify that each individual facility also has data over the predefined ranges. As an example, histograms for facility 1 are presented in Figure 5-3.

![Histograms for Facility 1](image)

**Figure 5-3 Histogram of the data for Facility 1**

Strong linear relationships among predictor variables might imply multicollinearity and cause imprecise regression coefficients that are difficult to interpret. The relationship between two quantitative variables may be displayed graphically by means of a scatterplot. The scatter plots between pairs of predictor variables are illustrated in Figure 5-4.
A positive linear relationship between the two quantitative predictor variables is indicated on a scatterplot by an upward linear trend whereas a negative linear relationship is indicated by the opposite effect. Otherwise, if the points do not show a linear trend, multicollinearity should not be an issue. When we inspect the plots in Figure 5-4, it becomes apparent that a positive association between capacity and fixed cost (correlation = 0.70) and $q$ and inspection cost (correlation = 0.73) exist. This association may be due to the assumption that we considered during generating the data. One way to overcome this issue is to exclude one of the variables from the model. Combining two or multiple variables into a single variable may be another alternative. Hence, we exclude fixed cost ($f_i$) and we define a function for the relationship between capacity and total demand of consumers ($\sum_{c \in C} b_c$) which is
define as \( \Gamma_{i,1} = \sum_{c \in c} b_c \). This ratio implies the fraction of the total demand that can be covered by a facility. In addition, we exclude inspection cost and consider \( \Gamma_{i,2} = 1 - (q_i - r_i) \). This fraction indicates the maximum percent of untainted products that we can expect from facility \( l \) after performing an inspection. Therefore, we consider risk-level (\( \alpha \)), reliability of facility \( l \) (\( \Theta_i \)), the fraction of the total demand that can be covered by facility \( l \) (\( \Gamma_{i,1} \)), and the maximum percent of untainted products that we can expect from facility \( l \) after performing an inspection (\( \Gamma_{i,2} \)) as our independent variables in the statistical analyses. In the next section, we discuss our regression model.

5.2.2. Model Description

The main analysis of this section uses a logistic regression model to consider the relationship of the independent variables with the dependent variable. The first order logistic regression model is presented as follows:

\[
P(y_i = 1 | \alpha, \Theta_i, \Gamma_{i,1}, \Gamma_{i,2}) = \frac{e^{\beta_0 + \beta_1 \alpha + \beta_2 \Theta_i + \beta_3 \Gamma_{i,1} + \beta_4 \Gamma_{i,2}}}{1 + e^{\beta_0 + \beta_1 \alpha + \beta_2 \Theta_i + \beta_3 \Gamma_{i,1} + \beta_4 \Gamma_{i,2}}}
\] (54)

and given (51)

\[
Y_i = \ln(\text{odds}) = \beta_0 + \beta_1 \alpha + \beta_2 \Theta_i + \beta_3 \Gamma_{i,1} + \beta_4 \Gamma_{i,2} + \varepsilon.
\] (55)

And given the equation (55), the mean response function for the log odds is written as

\[
E\{Y_i\} = \beta_0 + \beta_1 \alpha + \beta_2 \Theta + \beta_3 \Gamma_1 + \beta_4 \Gamma_2.
\] (56)

The regression model with interaction is as follows:
\[ Y_i = \beta_0 + \beta_1 \alpha + \beta_2 \Theta_i + \beta_3 \Gamma_{i,1} + \beta_4 \Gamma_{i,2} + \beta_5 \alpha \Theta_i + \beta_6 \alpha \Gamma_{i,1} + \beta_7 \alpha \Gamma_{i,2} + \beta_8 \Theta_i \Gamma_{i,1} + \beta_9 \Theta_i \Gamma_{i,2} + \beta_{10} \Gamma_{i,1} \Gamma_{i,2} + \epsilon. \] (57)

The mean response function for equation (57) can be written as follows:

\[ E\{Y_{ij}\} = \beta_0 + \beta_1 \alpha + \beta_2 \Theta + \beta_3 \Gamma_1 + \beta_4 \Gamma_2 + \beta_5 \alpha \Theta + \beta_6 \alpha \Gamma_1 + \beta_7 \alpha \Gamma_2 + \beta_8 \Theta \Gamma_1 + \beta_9 \Theta \Gamma_2 + \beta_{10} \Gamma_1 \Gamma_2. \] (58)

We consider two approaches for our statistical analysis. In the first approach, as we assumed in Section 3.4, we assume the cost of selecting a facility is correlated with the capacity such that the facility with the highest capacity has the highest fixed cost of selection. In the second approach, however, we assume that cost of selecting a facility is correlated with the reliability of a facility or the probability of facility being in state 0 (\( \Theta_i \)). The statistical analysis in this research was executed in the software package R (64-bit) version 2.11.1.

5.2.2.1. Approach 1

The coefficients from the estimated regression model for the first approach, when we assume the cost of selecting a facility is correlated with the capacity are given in Table 5-3. In Table 5-3, the coefficients, their standard errors, Wald test statistic, and the associated p-values are given. In logistic regression, the significance of a predictor variable is assessed via the likelihood ratio Chi-square test or Wald test [86]. At any step in the procedure, the variable with the smallest p-value is the one that produces the greatest change in the log-likelihood relative to a model not containing the variable. The Wald test statistic is computed by dividing the estimated coefficient of interest by its standard error \( (SE(\beta)) = var(\hat{\beta})^{1/2} \) where \( \hat{\beta} \) is the estimated coefficient). For instance, from Table 5-3 we observe that the regression
coefficient for $\alpha$ is significant at 0.05 level and $\Gamma_1$ is significant at 0.01 level whereas $\Gamma_2$ is not significant in this approach. Table 5-3 also indicates that interaction between $\alpha$ and $\Gamma_1$, and the interaction between $\Gamma_1$ and $\Gamma_2$ are highly significant at 0.001 level.

Given Table 5-3, we have

$$\hat{Y}_i = \ln(\text{odds}) = 5.53 - 8.41\alpha + 6.10\Theta - 60.20\Gamma_1 - 11.81\Gamma_2 + 1.53\alpha\Theta + 20.22\alpha\Gamma_1$$
$$+ 2.46\alpha\Gamma_2 - 6.77\Theta\Gamma_1 - 5.15\Theta\Gamma_2 + 99.26\Gamma_1\Gamma_2$$

(59)

| Research Model Predictors | Logistic Coefficients | Std. Error | z value (significance level) | Pr(>|z|) |
|---------------------------|-----------------------|------------|-----------------------------|---------|
| (Intercept)               | 5.35                  | 8.06       | 0.66                        | 0.51    |
| $\alpha$                  | -8.41                 | 4.01       | -2.09*                      | 0.03    |
| $\Theta$                  | 6.10                  | 9.09       | 0.67                        | 0.50    |
| $\Gamma_1$                | -60.41                | 19.23      | -3.14**                     | 0.00    |
| $\Gamma_2$                | -11.81                | 9.97       | -1.19                       | 0.24    |
| $a\Theta$                 | 1.53                  | 2.36       | 0.65                        | 0.52    |
| $a\Gamma_1$               | 20.22                 | 4.10       | 4.93***                     | 0.00    |
| $a\Gamma_2$               | 2.46                  | 4.40       | 0.56                        | 0.58    |
| $\Theta\Gamma_1$          | -6.77                 | 14.18      | -0.48                       | 0.63    |
| $\Theta\Gamma_2$          | -5.15                 | 9.77       | -0.53                       | 0.60    |
| $\Gamma_1\Gamma_2$        | 99.26                 | 27.81      | 3.57***                     | 0.00    |

Significant codes: ‘***’ $p < 0.001$; ‘**’ $p < 0.01$; ‘*’ $p < 0.05$; ‘.’ $p < 0.1$; ‘’ $p < 1$, AIC = 925.0

5.2.2.1.1. **Approach 1 Interpretation**

As stated, when a model has interaction between variables, it attempts to describe how the outcome for an independent variable is different depending on the level of another independent variable. We present an example of how to interpret the fitted model here to clarify the interaction interpretation in our regression model. From Equation (59), it can be shown that the change in the logit (or log odds) with 0.25 unit increase in $\alpha$ when other variables are held constant is
Equation (60) indicates that when a model has interaction term(s) of two independent variables, the effect of one independent variable on the dependent variable varies depending on the value of the other independent variable. Let’s assume the reliability of a facility is 50 percent ($\Theta = 0.50$), the facility can cover 15 percent of the demand or $\Gamma_1 = 0.15$ and we expect a maximum of 70 percent untainted products from facility (i.e., $\Gamma_2 = 0.70$) when $\alpha = 0.50$. From equation (60) we observe that increasing $\alpha$ to 0.75 will result in changing the odds of the selecting the facility by approximately a factor of 0.53. On the other hand, by considering a higher value for $\Gamma_2$, say 0.75 (which means reducing the fraction of expected tainted product), and keeping $\Gamma_1$ at the same level, we will observe approximately 3 percent increase in odds of selecting the facility. By holding $\Gamma_2$ at 0.70 and increasing $\Gamma_1$ to 0.16, a 5% increase is observed which indicates an increase in the likelihood of selecting larger facilities for higher values of confidence level.

5.2.2.2. **Approach 2**

The coefficients from the estimated regression model for approach 2 are given in Table 5-4. From Table 5-4 we observe that, the regression coefficient for $\alpha$ is highly significant at 0.001 level while in approach 1 this value is significant at 0.05 level. Furthermore, $\Gamma_2$ is significant at 0.01 level whereas this ratio itself was not significant in approach 1. From Table 5-4 we also observe that unlike approach 1, $\Theta$ and $\alpha \Theta$ are significant at 0.05 and 0.01 level, respectively. Interaction between $\alpha$ and $\Gamma_2$ is significant at 0.1 level. Given Table 5-4,
\[
\hat{Y}_i = \ln(\text{odds}) = 48.73 - 20.12\alpha - 28.39\Theta - 77.950\Gamma_1 - 50.62\Gamma_2 + 8.96\alpha\Theta + 12.03\alpha\Gamma_1 \\
+ 12.24\alpha\Gamma_2 + 14.52\Theta\Gamma_1 + 15.32\Theta\Gamma_2 + 111.15\Gamma_1\Gamma_2.
\]

(61)

| Research Model Predictors | Logistic Coefficients | Std. Error | \(z\) value (significance level) | \(Pr(>|z|)\) |
|---------------------------|-----------------------|------------|---------------------------------|-------------|
| (Intercept)               | 48.73                 | 12.38      | 3.94***                         | 0.00008     |
| \(\alpha\)               | -20.12                | 5.25       | -3.84***                        | 0.00013     |
| \(\Theta\)               | -28.39                | 14.30      | -1.99*                          | 0.04704     |
| \(\Gamma_1\)             | -77.95                | 42.70      | -1.83                           | 0.06790     |
| \(\Gamma_2\)             | -50.62                | 17.85      | -2.84**                         | 0.00458     |
| \(\alpha\Theta\)         | 8.96                  | 3.14       | 2.86**                          | 0.00430     |
| \(\alpha\Gamma_1\)       | 12.03                 | 8.53       | 1.41                            | 0.15843     |
| \(\alpha\Gamma_2\)       | 12.24                 | 6.85       | 1.79                            | 0.07384     |
| \(\Theta\Gamma_1\)       | 14.52                 | 27.23      | 0.53                            | 0.59370     |
| \(\Theta\Gamma_2\)       | 15.32                 | 13.62      | 1.13                            | 0.26045     |
| \(\Gamma_1\Gamma_2\)     | 111.15                | 63.26      | 1.76                            | 0.07890     |

Significant codes: ‘***’ \(p < 0.001\); ‘**’ \(p < 0.01\); ‘*’ \(p < 0.05\); ‘.’ \(p < 0.1\); ‘’ \(p > 0.1\), AIC = 999.15

5.2.2.2.1. **Approach 2 Interpretation**

From equation (61), the change in the log odds with 0.25 unit increase in \(\alpha\) when other variables are held constant is

\[
\hat{Y}_{i,\Delta\alpha} = \hat{Y}_{i}^{\alpha+0.25} - \hat{Y}_{i}^{\alpha} = -5.03 + 2.24\Theta + 3.01\Gamma_1 + 3.06\Gamma_2.
\]

(62)

Given \(\Theta = 0.50\), \(\Gamma_1 = 0.15\) and \(\Gamma_2 = 0.70\), the change in the odds of the selecting the facility will be a factor of 0.26. If we increase \(\Theta\) to \(\Theta = 0.60\) the increase in odds of the selecting the facility of will be 0.23 percent which indicates increasing the reliability of a facility will result in increasing the likelihood of selecting the facility.
5.2.3. **Stepwise Regression**

From Table 5-3 and Table 5-4, it is observable that not all coefficients are significant. For instance, the coefficient for the interaction between $\alpha$ and $\Gamma_2$ is not a significant predictor in approach 1. We therefore apply a stepwise selection procedure to obtain only the variables are significant predictors for status. The stepwise logistic regression (SLR) method allows the model to be assessed as it is being built. In the SLR, predictor variables are selected for inclusion or exclusion from the regression model based on a stepwise selected method.

Stepwise regression method is a combination of the forward selection and backward elimination, testing at each iteration for variables to be included or excluded. Forward selection starts with no variables and selects variables if they are statistically significant. Backward elimination starts with all candidate variables, examining them one by one for statistical significance, and finally, eliminating the ones that are not significant. The R software package has an option that allows the user to perform the stepwise procedure.

The Akaike Information Criterion (AIC) approach is used to obtain the log-likelihood value. At any step in the procedure, a statistically significant variable is the one that produces the greatest change in the log-likelihood relative to a model not containing the variable [93]. The AIC is based on a likelihood theory and is a function of the number of observations $n$, the sum of squared errors $SSE$ and the number of parameters estimated in the model $p$, as shown in the following:
The first term in the above equation is a measure of the model lack of fit while the second term is a penalty term for additional parameters in the model. Therefore, as the number of parameters $p$ included in the model increases, the lack of fit term should decrease due to the decrease of the error while the penalty term increases. On the other hand, as variables are dropped from the model, the lack of fit term should increase while the penalty term decreases. The model with the smallest AIC is considered the best model since it minimizes the difference from the given model to the true model. The AIC after implementing the stepwise selection method for approach 1 is 918.9 while the calculated AIC for equation (59) was 925.0. Therefore, we select the model resulting from the combination model. The coefficients from the estimated regression model for approach 1 after performing the stepwise selection method are given in Table 5-5.

| Research Model Predictors | Logistic Coefficients | Std. Error | z value (significance level) | Pr(>|z|) |
|---------------------------|-----------------------|------------|-----------------------------|---------|
| (Intercept)               | 5.81                  | 3.48       | 1.67**                      | 0.10    |
| $\alpha$                 | -2.59                 | 0.79       | -3.25**                     | 0.00    |
| $\theta$                 | 1.57                  | 0.75       | 2.09*                       | 0.04    |
| $\Gamma_1$               | -55.99                | 18.08      | -3.10**                     | 0.00    |
| $\Gamma_2$               | -12.30                | 4.23       | -2.91**                     | 0.00    |
| $\alpha\Gamma_3$        | 19.02                 | 3.99       | 4.76***                     | 0.00    |
| $\Gamma_1\Gamma_2$      | 88.79                 | 22.22      | 4.00***                     | 0.00    |

Significant codes: ‘***’ $p < 0.001$; ‘**’ $p < 0.01$; ‘*’ $p < 0.05$; ‘.’ $p < 0.1$; ‘ ’ 1, AIC = 918.87

From Table 5-5, performing a stepwise procedure resulted in excluding a few of the interactions and reducing the AIC by almost seven units to 918.9. Given Table 5-5, let us rewrite equation (59) as follows:
\[ \hat{Y}_i = 5.81 - 2.59\alpha + 1.57\Theta - 55.99\Gamma_1 - 12.30\Gamma_2 + 19.02\alpha\Theta + 88.796\Gamma_1\Gamma_2. \] (63)

In the logistic regression model (63), some of the interaction term still exists. From equation (63), the coefficient for \( \alpha \) is -2.59 and by increasing \( \alpha \) by 0.25 unit, the odds ratio will be \( e^{(-0.65+4.75\Gamma_1)} \). For instance, when \( \Gamma_1 \) is held at 0.15, we will have 0.06 for the odds increase and the odds for \( \Gamma_1 \) equal 0.16 will 0.31 which indicates more than 60 percent increase in odds of selecting the facility.

The coefficients from the estimated regression model for approach 2 after performing the stepwise method are given in Table 5-6. From Table 5-6 we observe that performing a stepwise procedure resulted in excluding a few of the interactions and reducing the AIC by nearly 3 units to 996. Given Table 5-6, the regression model for approach 2 with interaction is as follows:

\[ \hat{Y}_i = 44.61 - 17.45\alpha - 12.39\Theta - 85.80\Gamma_1 - 47.12\Gamma_2 + 10.04\alpha\Theta + 142.16\alpha\Gamma_2 - 12.39\Gamma_1\Gamma_2. \] (64)

| Research Model Predictors | Logistic Coefficients | Std. Error | \( z \) value (significance level) | Pr(>|z|) |
|---------------------------|-----------------------|------------|----------------------------------|----------|
| (Intercept)               | 41.66                 | 9.25       | 4.50***                          | 0.0000   |
| \( \alpha \)              | -17.45                | 4.99       | -3.49***                         | 0.0005   |
| \( \Theta \)              | -12.39                | 2.35       | -5.28**                          | 0.0000   |
| \( \Gamma_1 \)            | -85.80                | 32.92      | -2.61**                          | 0.0092   |
| \( \Gamma_2 \)            | -47.12                | 11.39      | -4.14***                         | 0.0000   |
| \( \alpha\Theta \)        | 10.04                 | 3.06       | 3.29*                            | 0.0010   |
| \( \alpha\Gamma_2 \)      | 142.16                | 39.44      | 3.61                             | 0.0003   |
| \( \Gamma_1\Gamma_2 \)    | -12.39                | 2.35       | -5.28**                          | 0.0000   |

Significant codes: ‘***’ \( p < 0.001 \); ‘**’ \( p < 0.01 \); ‘*’ \( p < 0.05 \); ‘.’ \( p < 0.1 \); ‘’ \( p > 0.1 \); AIC = 996.62
5.2.4. Prediction

To evaluate the predictive ability of the model based on the estimated logistic coefficients in Table 5-5 and Table 5-6, we perform prediction for both approach 1 and approach 2 for a set of 5 facilities. We consider two risk-level values for all the facilities, i.e., $\alpha = 0.50, 0.95$. The data and the result of prediction are presented in Table 5-7 and Table 5-8. Note that in approach 1 facilities are sorted based on their capacity and in approach 2 facilities are sorted based on their reliability.

The results in Table 5-7 indicate that $\Gamma_1$ is a very significant attribute when selecting a supplier in approach 1. For instance, the likelihood of selecting the facility with the highest capacity (i.e., Facility 1) is 0.607 for $\alpha=0.50$ whereas this facility has the worst reliability among the others and only 71 percent of its capacity can be used after inspection. We also observe 9% growth in likelihood of selection for Facility 1 for $\alpha=0.95$. As another example, Facility 4 is the most reliable facility and provides a large portion of untainted products even after inspection. However, we observe a small likelihood of selection since its capacity is small. The likelihood of selection reduces even more when $\alpha$ increases to 0.95.

<table>
<thead>
<tr>
<th>Facility #</th>
<th>reliability</th>
<th>$\Gamma_1$</th>
<th>$\Gamma_2$</th>
<th>$\alpha=0.50$</th>
<th>$\alpha=0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Probability of selection</td>
<td>logit</td>
<td>Probability of selection</td>
<td>logit</td>
</tr>
<tr>
<td>2</td>
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<td>0.2782</td>
<td>0.7587</td>
<td>0.780</td>
<td>1.265</td>
</tr>
<tr>
<td>4</td>
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<td>0.1264</td>
<td>0.8992</td>
<td>0.425</td>
<td>-0.304</td>
</tr>
<tr>
<td>3</td>
<td>0.8392</td>
<td>0.1916</td>
<td>0.9074</td>
<td>0.711</td>
<td>0.901</td>
</tr>
<tr>
<td>5</td>
<td>0.8326</td>
<td>0.1068</td>
<td>0.8324</td>
<td>0.276</td>
<td>-0.965</td>
</tr>
<tr>
<td>1</td>
<td>0.5024</td>
<td>0.3006</td>
<td>0.7146</td>
<td>0.607</td>
<td>0.435</td>
</tr>
</tbody>
</table>
A notable observation from Table 5-7 is that, for higher risk-level the likelihood of selecting a facility decreases for facilities with smaller capacities. This indicates that in approach 1, a risk-averse decision-maker tends to provide more capacity to hedge against risk.

The results in Table 5-8, however, indicate that higher weight assigned to reliability of a facility in approach 2 as compared to approach 1. A notable observation from Table 5-8 is that Facility 5 has the lowest reliability but its capacity is the largest. The probability of selecting facility 5 for $\alpha=0.50$ is 0.995. However, after increasing risk-level $\alpha$ to 0.95, the probability of selecting facility 5 decreases to 0.965. This decrease indicates that more reliable facilities are selected in risk-averse decision making policy. These results can be useful for facilities as well since it enables them to analyze their situation and compare it with other facilities (competitors) and attempt to change their behaviors in order to increase their likelihood of selection. We believe that other important implications in practice can be achieved through this result.

<table>
<thead>
<tr>
<th>Facility #</th>
<th>reliability</th>
<th>$\Gamma_1$</th>
<th>$\Gamma_2$</th>
<th>Probability of selection</th>
<th>logit</th>
<th>$\alpha=0.50$</th>
<th>$\alpha=0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.6027</td>
<td>0.1098</td>
<td>0.8443</td>
<td>0.006</td>
<td>-5.141</td>
<td>0.005</td>
<td>-5.279</td>
</tr>
<tr>
<td>4</td>
<td>0.5624</td>
<td>0.2045</td>
<td>0.7535</td>
<td>0.408</td>
<td>-0.371</td>
<td>0.341</td>
<td>-0.643</td>
</tr>
<tr>
<td>1</td>
<td>0.8701</td>
<td>0.1277</td>
<td>0.8068</td>
<td>0.044</td>
<td>-3.068</td>
<td>0.102</td>
<td>-2.725</td>
</tr>
<tr>
<td>2</td>
<td>0.8385</td>
<td>0.2915</td>
<td>0.7989</td>
<td>0.809</td>
<td>1.895</td>
<td>0.866</td>
<td>2.053</td>
</tr>
<tr>
<td>5</td>
<td>0.5563</td>
<td>0.3501</td>
<td>0.7167</td>
<td>0.995</td>
<td>5.254</td>
<td>0.965</td>
<td>3.915</td>
</tr>
</tbody>
</table>

5.2.5. **Key Observations**

Recall that in approach 1 we assumed that the cost of selecting a facility is correlated with the capacity such that the facility with the highest capacity has the
highest fixed cost of selection. In approach 2, however, we assume that cost of selecting a facility is correlated with the reliability of a facility or the probability of facility being in state 0. We summarize our observations from approach 1 and approach 2 and also risk-neutral and risk-averse decision making policies as following:

- Facilities with larger capacities have higher likelihood of selection when capacity drives facility cost.
- When capacity drives facility cost, for higher value of $\alpha$ (risk-averse policy) the likelihood of selection goes down if capacity of facility goes down.
- When reliability drives facility cost, increasing the reliability of a facility will result in increasing the likelihood of selecting the facility.
- When reliability drives facility cost, for higher value of risk-level $\alpha$ the likelihood of selection goes down if reliability and/or the fraction untainted products that we can expect from facility $l$ after performing an inspection go down.
- Provided that the problem is capacitated (i.e., the capacity is limited), “size” of the facility is perceived to be the most important factor for facility selection in the risk-neutral and risk-averse policies and also when capacity drives facility cost and when reliability drives facility cost.
- Under the risk-neutral policy, “reliability” of the facility is perceived to be the least important factor.
5.3. Multinomial Logistic Regression (MLR)

In Section 5.2, we conducted a logistic regression analysis to assess the likelihood of selection for each individual facility at various risk levels. In this section, we assess the likelihood of selection for a set of selected facilities at various risk levels. For instance, we assess the likelihood of selecting all the facilities together or selecting all the facilities except Facility 1.

Suppose the set of selected facilities, $Y$, is a categorical response variable with $J$ categories. Let $\{\pi_1, \pi_2, \ldots, \pi_J\}$ be the response probabilities satisfying $\sum \pi_j = 1$. When one takes $n$ independent observations, the probability distribution for the number of outcomes that occur within each of the $J$ types is a multinomial distribution [94]. One value (typically the first, the last, or the value with the highest frequency) of the dependent variable is designated as the reference or baseline category. The probability of the other categories is compared to the probability of the reference category. For instance, if there are four categories (alternatives), A, B, C, and D, and the reference category is A, MLR will calculate the log odds (or estimates the set of coefficients) for being in category B versus A, category C versus A, and category D versus A. Each set of coefficients then represents the effect of a unit change in the independent variables on the log odds of each category relative to the reference category.

For a dependent variable $Y$ with $J$ categories, we require the calculation of $J-1$ log odds, one for each category relative to the reference category, to describe the relationship between the dependent variable and the independent variables. As a
result, if the last category is the reference, then, for \( j = 1, \ldots, J-1 \), the odds ratio of category \( j \) with predictor \( x \) is

\[
\log \left( \frac{\pi_j}{\pi_J} \right) = \alpha_j + \beta_j x, \quad j = 1, \ldots, J - 1.
\] (65)

The choice probability for the reference category \( J \) is

\[
\pi_J = \frac{1}{1 + \sum_{j=1}^{J-1} e^{\alpha_j + \beta_j x}}.
\] (66)

The response probabilities for \( j = 1, \ldots, J - 1 \) can be expressed as

\[
\pi_j = \frac{e^\alpha e^{\beta_j x}}{1 + \sum_{j=1}^{J-1} e^{\alpha_j + \beta_j x}}.
\] (67)

5.3.1. Description of the Data Source

We consider risk-level (\( \alpha \)), the fraction of the total demand that can be covered by facility \( l \) (\( \Gamma_{l,1} \)) and the maximum percent of untainted products that we can expect from facility \( l \) after performing inspection (\( \Gamma_{l,2} \)) as our independent variables. In the next section, we discuss our regression model. We use 120 data instances for a specific case containing five facilities and five consumers. Hence, we have \( 2^{10} - 1 \) possible sets of alternatives of selected facilities as illustrated within Table (c) in Appendix C. For instance, in Table (c) alternative “eee” indicates the alternative that all facilities are not selected except Facility 1 and alternative “a” is the alternative that all facilities are selected. We consider seven levels for the risk level \( \alpha \): 0, 0.50, 0.65, 0.75, 0.85, 0.95 and 0.99. As a result, in total we will have 840 (120*7)
observations. Finally, for the sake of interpretability, we sort the reliability and consequently $\Gamma_{i,2}$ in a descending manner so that Facility 1 is the most reliable facility and Facility 5 is the least reliable facility. Moreover, as mentioned, strong linear relationships among predictor variables might imply multicollinearity and cause imprecise regression coefficients that are difficult to interpret. The relationship between the independent variables is illustrated in Figure 5-5. By inspecting Figure 5-5, it becomes apparent that a positive linear association between some of the independent variables exists. For instance, a positive linear association between $\Gamma_{3,2}$ and $\Gamma_{4,2}$ (correlation = 0.74), $\Gamma_{1,2}$ and $\Gamma_{2,2}$ (correlation = 0.62), and $\Gamma_{1,1}$ and $\Gamma_{3,1}$ (correlation = -0.43) exists. To overcome this issue, we convert $\Gamma_{i,2}$ from a quantitative variable to a qualitative variable with three levels e.g., “1 = High expectation” where $\Gamma_{i,2} \in [0.86, 0.94)$, “2 = Medium expectation” where $\Gamma_{i,2} \in [0.78, 0.86)$, and finally, “3 = Low expectation” where $\Gamma_{i,2} \in [0.70, 0.78)$. We also exclude $\Gamma_{3,1}$ from the model.
5.3.2. Model Description

In our MLR model, the two-way interaction effects of the independent variables are also considered. As an example, the coefficients from the estimated results of multinomial logistic regression model (\( \alpha = 0.75 \)) are given in Table 5-9. Note that in Table 5-9, alternative “b” (which is the alternative that all facilities are selected except the most reliable, i.e., Facility 1) is the reference category and the probability of categories is compared to the probability of the reference category. Hence, given Table 5-9 and equation (65) the log odds of the estimated MLR model for alternative “c” (the alternative that all facilities are selected except the most reliable i.e., Facility 2) is

\[
\log\left( \frac{\pi_c}{\pi_b} \right) = -39.917 + 33.024 \Gamma_{1,1} - 52.932 \Gamma_{2,1} - 33.941 \Gamma_{4,1} - 30.185 \Gamma_{5,1} - 2.9763 \Gamma_{1,2} - 0.39135 \Gamma_{2,2} \\
+ 0.77611 \Gamma_{3,2} + 22.184 \Gamma_{5,2}.
\]

(68)
### Table 5-9 Multinomial regression model result for $\alpha = 0.75$

| Research Model Predictors | Multinomial Coefficients | Std. Error | t-value (significance level) | Pr(>|t|) |
|----------------------------|---------------------------|------------|-----------------------------|---------|
| altc                       | -39.917                   | 91761      | -0.0004                     | 1.000   |
| alte                       | -108.37                   | 97011      | -0.0011                     | 0.999   |
| alti                       | 12.194                    | 13.27      | 0.9189                      | 0.358   |
| altk                       | 112.24                    | 68579      | 0.0016                      | 0.999   |
| altq                       | 12.708                    | 12.075     | 1.0525                      | 0.293   |
| alty                       | -29.662                   | 93273      | -0.0003                     | 1.000   |
| altc:ratio1.1              | 33.024                    | 18.848     | 1.7521                      | 0.080   |
| alteratio1.1               | 83.312                    | 21.476     | 3.8793                      | 0.000***|
| alti:ratio1.1              | 41.676                    | 18.887     | 2.2065                      | 0.027*  |
| altk:ratio1.1              | 196.82                    | 41190      | 0.0048                      | 0.996   |
| altratio1.1                | 38.122                    | 18.188     | 2.0966                      | 0.036   |
| altk:ratio2.1              | 248.92                    | 158690     | 0.0016                      | 0.999   |
| altratio2.1                | -52.932                   | 19.467     | -2.719                      | 0.007** |
| alternatio2.1              | 40.735                    | 18.386     | 2.2156                      | 0.027*  |
| alti:ratio2.1              | -9.789                    | 16.319     | -0.5999                     | 0.549   |
| altk:ratio2.1              | -496.77                   | 129650     | -0.0038                     | 0.997   |
| altratio2.1                | 0.72686                   | 15.197     | 0.0478                      | 0.962   |
| altk:ratio1.2              | 160.25                    | 91802      | 0.0017                      | 0.999   |
| alti:ratio1.4              | -33.941                   | 18.119     | -1.8732                     | 0.061   |
| alteratio1.4               | 18.031                    | 19.3       | 0.9343                      | 0.350   |
| alti:ratio1.4              | -32.748                   | 17.018     | -1.9243                     | 0.054   |
| altk:ratio1.4              | -344.04                   | 76278      | -0.0045                     | 0.996   |
| altratio1.4                | -37.84                    | 16.147     | -2.3435                     | 0.019*  |
| alti:ratio1.4              | -221.36                   | 76698      | -0.0029                     | 0.998   |
| alti:ratio1.5              | -30.185                   | 18.861     | -1.6004                     | 0.110   |
| altratio1.5                | 35.907                    | 22.438     | 1.6003                      | 0.110   |
| alti:ratio1.5              | -17.671                   | 18.982     | -0.9309                     | 0.352   |
| altratio1.5                | -311.28                   | 85893      | -0.0036                     | 0.997   |
| altratio1.5                | -39.092                   | 18.02      | -2.1694                     | 0.030*  |
| altratio1.5                | -263.6                    | 140030     | -0.0019                     | 0.998   |
| altratio2.1                | -2.9762                   | 2.6725     | -1.0774                     | 0.281   |
| altratio2.1                | -1.7366                   | 2.2707     | -0.7648                     | 0.464   |
| altratio2.1                | -2.7198                   | 2.1427     | -1.2694                     | 0.204   |
| altratio2.1                | 29.213                    | 25202      | 0.0012                      | 0.999   |
| altratio2.1                | -2.7911                   | 2.0057     | -1.3916                     | 0.164   |
| altratio1.1                | 14.456                    | 27381      | 0.0005                      | 1.000   |
| altratio1.1                | -0.39135                  | 1.9627     | -0.1994                     | 0.842   |
| altratio2.2                | -0.83265                  | 1.657      | -0.5025                     | 0.615   |
| altratio2.2                | -0.38954                  | 1.5294     | -0.2547                     | 0.799   |
| altratio2.2                | 44.571                    | 14191      | 0.0031                      | 0.997   |
| altratio2.2                | -1.5184                   | 1.4893     | -1.0195                     | 0.308   |
| altratio2.2                | 27.466                    | 11428      | 0.0024                      | 0.998   |
| altratio2.3                | 0.77611                   | 2.1987     | 0.353                       | 0.724   |
| altratio2.3                | -0.8342                   | 1.9577     | -0.4261                     | 0.670   |
| altratio2.3                | -1.5974                   | 1.9132     | -0.8349                     | 0.404   |
| altratio2.3                | -33.976                   | 17309      | -0.002                      | 0.998   |
| altratio2.3                | -0.80602                  | 1.8586     | -0.4337                     | 0.665   |
| altratio2.3                | -8.3163                   | 13081      | -0.0006                     | 0.999   |
| altratio2.5                | 22.184                    | 30587      | 0.0007                      | 0.999   |
| altratio2.5                | 22.296                    | 32337      | 0.0007                      | 0.999   |
| altratio2.5                | 1.1626                    | 2.0254     | 0.574                       | 0.566   |
| altratio2.5                | 13.27                     | 15474      | 0.0009                      | 0.999   |
| altratio2.5                | 3.2877                    | 2.01       | 1.6357                      | 0.102   |
| altratio2.5                | -13.914                   | 24695      | -0.0006                     | 1.000   |

Significant codes: ***p < 0.001; **p < 0.01; *p < 0.05; .p < 0.1; "p > 1

Log-Likelihood: -77.582
McFadden R^2: 0.53146
Likelihood ratio test : chisq = 176 (p.value=< 2.22e-16)
5.3.3. **Prediction**

This section is concerned with the predictive ability of our resultant MLR model. The data and the result of prediction are presented in Table 5-10 and Table 5-11, respectively. Note that we use identical data for all values of risk-level $\alpha$.

<table>
<thead>
<tr>
<th>$\Gamma_{ij}$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{11}$</td>
<td>0.250</td>
</tr>
<tr>
<td>$\Gamma_{21}$</td>
<td>0.223</td>
</tr>
<tr>
<td>$\Gamma_{41}$</td>
<td>0.278</td>
</tr>
<tr>
<td>$\Gamma_{51}$</td>
<td>0.258</td>
</tr>
<tr>
<td>$\Gamma_{12}$</td>
<td>1</td>
</tr>
<tr>
<td>$\Gamma_{22}$</td>
<td>2</td>
</tr>
<tr>
<td>$\Gamma_{32}$</td>
<td>3</td>
</tr>
<tr>
<td>$\Gamma_{52}$</td>
<td>3</td>
</tr>
</tbody>
</table>

The results of prediction are summarized in Table 5-11 (and illustrated in Figure (e)). Several observations can be made from this table. For instance, we observe how the probability of alternative “$a$” increases with increases to $\alpha$. This observation confirms our results in Chapter 4 where we observed that for higher values of risk-level, the number of selected facilities gradually increases. This increase can be justified because a risk-averse decision-maker provides more capacity (by selection of more facilities) in order to be able both to perform more inspections and to discard more tainted products and still satisfy the total demand. Furthermore, for $\alpha = 0.05$, there exists a (small) likelihood of selecting alternatives with a lower number of selected facilities (i.e., the alternatives “$u$” and “$y$”).

Another result that we obtained in Chapter 4 was that, for higher values of $\alpha$, a risk-averse decision-maker tends to select more reliable facilities and vice versa. For instance, alternative “$b$” is the alternative where the most reliable facility (Facility 1) is not selected. From Table 5-11, we observe that increasing $\alpha$ leads to a lower
likelihood of selecting this alternative which indicates that the tendency to close the most reliable facility decreases with respect to increasing the risk-level $\alpha$. Moreover, Facility 5 and Facility 4 are the least reliable facility and we observe the probability of the alternatives “i” and “q” is larger when becoming more risk-averse. The result of MLR verified the result of our computational analysis in Chapter 4 where we showed that the risk-averse policy results in different strategic compared to the risk-neutral policy.

<table>
<thead>
<tr>
<th>alpha</th>
<th>Reference (baseline) category</th>
<th>Alternatives</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>e</th>
<th>i</th>
<th>q</th>
<th>u</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>b</td>
<td>0.000</td>
<td>0.272</td>
<td>0.270</td>
<td>0.013</td>
<td>0.090</td>
<td>0.339</td>
<td>0.005</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>b</td>
<td>0.000</td>
<td>0.222</td>
<td>0.315</td>
<td>0.022</td>
<td>0.096</td>
<td>0.343</td>
<td>0.000</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>0.65</td>
<td>b</td>
<td>0.000</td>
<td>0.038</td>
<td>0.481</td>
<td>0.002</td>
<td>0.076</td>
<td>0.403</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>b</td>
<td>0.000</td>
<td>0.008</td>
<td>0.690</td>
<td>0.000</td>
<td>0.058</td>
<td>0.244</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>a</td>
<td>0.000</td>
<td>0.000</td>
<td>0.551</td>
<td>0.002</td>
<td>0.070</td>
<td>0.376</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>a</td>
<td>0.000</td>
<td>0.245</td>
<td>0.000</td>
<td>0.172</td>
<td>0.000</td>
<td>0.043</td>
<td>0.540</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.99</td>
<td>a</td>
<td>0.000</td>
<td>0.434</td>
<td>0.000</td>
<td>0.048</td>
<td>0.000</td>
<td>0.074</td>
<td>0.444</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

5.3.4. **Conclusion**

In conclusion, this chapter firstly identified the factors for predicting the selection of a facility in the SCD and SCD-CVaR frameworks at various risk levels and to analyze relationships among variables. Secondly, by conducting a logistic regression analysis and multinomial regression analysis we assessed the probability of selection for each individual facility at various risk levels along with probability of selection for a set of selected facilities or alternatives.

The obtained results enable managers to identify the criteria that impact our strategic decisions in both the risk-neutral and risk-averse decision-making policies.
The results also enable the facilities to analyze their situations and compare them with other facilities (competitors) and to change their behaviors in order to increase their likelihood of selection.
CHAPTER SIX

6. Conclusions and Future Research

6.1. Research Conclusions

In response to certain catastrophic events, particularly in the healthcare and pharmaceutical supply chains, this research focused on risks that impact strategic and tactical decisions in these types of supply chains. In contrast with most prior research, in this dissertation we focused on the supply (capacity) management required to hedge against the unreliability of capacity and to prevent the shipping of tainted materials to the consumers.

We discussed models for designing supply chain networks resilient to capacity disruptions. The goal was to design a supply chain infrastructure under the risk of disruption, so that it operates with the highest possible efficiency (i.e., at low cost) both normally and when a disruption occurs. However, such network design problems belong to the class of NP-hard problems. Accordingly, heuristic algorithms and metaheuristic approaches were developed to obtain the solutions to these types of problems.

In Chapter 3, we considered a single-period, single-product supply chain with capacitated facilities. We considered a risk-neutral decision-making policy based on a cost-minimization approach and utilized a mixed integer stochastic programming model formulated as a two-stage optimization problem. The goal of the model consisted of the facility selection in the first stage, defining capacity (product) allocation among the consumers, and inspection decisions. The objective was to minimize the expected cost composed of the fixed cost of selecting the facility, the
cost of shipping untainted products, the cost of shipping tainted products, the cost of inspection the facility, and the cost of discarding tainted products.

From solving the problem in AMPL, we observed that the number of facilities, and consequently the number of scenarios, had a significant impact on the computational time of our problem. Therefore, we developed nine constructive heuristic methods, two improvement heuristic methods, and a Simulated Annealing (SA) approach to solve the presented model effectively. Based on our computational studies and statistical analyses, the SA approach is not efficient in terms of solution quality and solution time for small-sized problems or a small number of scenarios. However, some developed heuristics achieved good solution qualities in a more reasonable time compared with the optimal or best found solution. Moreover, our improvement heuristic algorithms were able to improve the solutions obtained from constructive heuristics. Therefore, constructive and improvement heuristics were preferable in small sized problems. However, for problems with ten or more facilities, the SA approach outperformed constructive and improvement heuristics even though it required higher computational time.

In Chapter 4, we addressed a risk-averse decision-making policy wherein rather than selecting facilities and identifying the pertinent supplier-consumer assignments that minimize the expected cost, the decision-maker uses a CVaR approach to measure and quantify risk and to define what comprises a worst-case scenario.

We characterized the trade-off between the risk and the cost and this provided several insights on the impact of risk-aversion on the optimal decisions. Our studies demonstrated how strategic and tactical decisions changed with respect to the risk
level. We found that an increase in the risk level $\alpha$ leads to the selection of more reliable facilities and a different number of facilities. The risk-averse policy also resulted in fewer worst-case scenarios compared with the risk-neutral policy. Our computations also revealed that becoming more risk-averse resulted in remarkable increases in the cost of shipping untainted products to consumers.

Lastly, we considered statistical analyses for the risk-neutral and risk-averse decision-making policies in Chapter 5. We found that the selection of certain facilities was obvious based on parameter values, such as capacity or fixed cost. In other cases, facilities that appeared to be desirable were not selected. Hence, it may be difficult to predict or determine which facilities are selected or unselected; it may also be difficult to interpret the output of the SCD or SCD-CVaR models. Hence, we conducted a regression analysis to identify factors for predicting the selection of a facility at various risk levels and to identify relationships among independent variables.

The result from the logistic regression analysis revealed that in the risk-averse policy when sufficient capacity is available, reliability ($\Theta$) and the maximum percent of untainted products that we can expect from a facility ($\Gamma_2$) are perceived to be the most important factors for facility selection. Moreover, if the capacity is limited, the “size” of the facility, that is, the fraction of the total demand that can be covered by a facility ($\Gamma_{1,1}$), is perceived to be the most important factor for facility selection in both the risk-neutral and risk-averse policies. Under the risk-neutral decision-making policy, “reliability” of the facility is perceived to be the least important factor, and cost of selection is a significant factor.

From the multinomial regression analysis, we observed that for higher values of risk-level, the number of selected facilities increased. The reason is that a risk-
averse decision-maker provides more capacity (by selection of more facilities) to be able both to perform more inspections and to discard more tainted products and still satisfy the total demand. Another result was that the tendency toward not selecting the most reliable facility decreases as a decision-maker becomes more conservative. The outcome of our models and the statistical analysis could enable managers to select the most qualified suppliers for their supply chain and to make capacity allocation and inspection implementation decisions under both risk-neutral and risk-averse policies. Furthermore, results enable decision-makers to change their behaviors to increase their likelihood of selection.

6.2. Future Research Directions

The models and solution methods addressed in this dissertation can be extended for future research directions.

In Chapter 3, we assumed an inspection and discard approach, which is not a valid assumption in certain supply chains, such as the automotive and electronics industries. This assumption can be shifted to an inspection and fix (rework) approach, where defective products can be repaired after detection. Another good extension is to develop other metaheuristic techniques, such as a Genetic Algorithm or a Tabu Search, for comparison with the SA algorithm.

In Chapter 4, we considered instances that included five facilities. However, experience from solving the models using commercial software indicated that the number of facilities could increase the computational time dramatically. As a future extension, we can consider other methodologies, such as Bender’s Decomposition, Lagrangian relaxation, or L-shaped methods, to solve larger-sized problems. We also
think it is important to design and develop heuristic techniques to obtain acceptable solutions to these larger size problems in reasonable runtimes and with good solution quality.

In our research, we considered deterministic or estimated demands and that might not always be a valid assumption. Hence, an interesting extension of the presented work is to include demand uncertainty and/or seasonal demand as they exist in real-world supply chains.

The model itself can be extended by considering a multi-commodity model instead of a single-product model. Adding more tier levels is another possible future direction. For example, we could extend the model to include multiple raw material suppliers or multiple distribution centers.
APPENDIX A

Normal probability plots of the residuals

Figure (a) Normal Probability Plot of Residuals set 0-4 (5 facility 5 customers)

Figure (b) Normal Probability Plot of Residuals set 5-9 (5 facility 5 customers)
Figure (c) Normal Probability Plot of Residuals set 0-4 (ten facility ten customers)

Figure (d) Normal Probability Plot of Residuals set 5-10 (ten facility ten customers)
## APPENDIX B

### Pairwise Wilcoxon test

#### Table (a) R package output for Pairwise Wilcoxon Test (5 facility and 5 customers)

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#### Table (b) R package output for Pairwise Wilcoxon Test (ten facility and ten customers)

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**APPENDIX C**

**Possible set of alternatives of selected facilities**

Table (c) Set of alternatives of selected facilities for 5 facilities

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APPENDIX D

Prediction performance of MLR model

Figure (e) Likelihood of various alternatives
BIBLIOGRAPHY


