PREDICTIVE ENERGY MANAGEMENT IN SMART VEHICLES: EXPLOITING TRAFFIC AND TRAFFIC SIGNAL PREVIEW FOR FUEL SAVING

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PREDICTIVE ENERGY MANAGEMENT IN SMART VEHICLES: EXPLOITING TRAFFIC AND TRAFFIC SIGNAL PREVIEW FOR FUEL SAVING

A Thesis
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the Graduate School of
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Master of Science
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by
Behrang Asadi
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Accepted by:
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Abstract

This master thesis proposes methods for improving fuel economy and emissions of vehicles via use of future information of state of traffic lights, traffic flow, and deterministic traffic flow models.

The first part of this thesis proposes use of upcoming traffic signal information within the vehicle’s adaptive cruise control system to reduce idle time at stop lights and lower fuel use. To achieve this goal an optimization-based control algorithm is formulated for each equipped vehicle that uses short range radar and traffic signal information predictively to schedule an optimum velocity trajectory for the vehicle. The objectives are timely arrival at green light with minimal use of braking, maintaining safe distance between vehicles, and cruising at or near set speed. Three example simulation case studies are presented to demonstrate potential impact on fuel economy, emission levels, and trip time.

The second part of this thesis addresses the use of traffic flow information to derive the fuel- or time-optimal velocity trajectory. A vehicle’s untimely arrival at a local traffic wave with lots of stops and goes increases its fuel use. This paper proposes predictive planning of the vehicle velocity for reducing the velocity transients in upcoming traffic waves. In this part of the thesis macroscopic evolution of traffic pattern along the vehicle route is first estimated by combining a traffic flow model and real-time traffic data streams. The fuel optimal velocity trajectory is calculated by solving an optimal control problem with the spatiotemporally varying constraint imposed by the traffic. Simulation results indicate
the potential for considerable improvements in fuel economy with a little compromise on travel time.
Dedication

I sincerely dedicate this work to my parents, my sister and all of my friends over the world.
Acknowledgments

Many people deserve my gratitude for making this thesis possible. First of all, I would like to thank my advisor Dr. Ardalan Vahidi for his encouragement, trust, support, and knowledge. I would also like to thank my committee members, Dr. Nader Jalili and Dr. Thomas R. Kurfess, for their input and advice regarding this study. My master thesis would have been incomplete without the assistance and collaboration of my fellow lab-mates, Chen Zhang and Ali Borhan, and former graduate student Dr. Saeid Bashash; I benefited and learned so much from all of them.

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Last, but not least, I would like to thank Ms. Valerie Holmes of the Traffic Engineering Department of the city of Greenville, South Carolina for providing the in-city traffic signal phase and timing information which was used in this study.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE PAGE</td>
<td>i</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iv</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Motivation</td>
<td>1</td>
</tr>
<tr>
<td>Approach</td>
<td>2</td>
</tr>
<tr>
<td>Overview of Thesis</td>
<td>3</td>
</tr>
<tr>
<td>2. EXISTING INTELLIGENT HIGHWAY AND INTELLIGENT VEHICLE INITIATIVES</td>
<td>5</td>
</tr>
<tr>
<td>Introduction</td>
<td>5</td>
</tr>
<tr>
<td>Intelligent Transportation Systems</td>
<td>5</td>
</tr>
<tr>
<td>3. PREDICTIVE CRUISE CONTROL: UTILIZING UPCOMING TRAFFIC SIGNAL INFORMATION FOR IMPROVING FUEL ECONOMY AND REDUCING TRIP TIME</td>
<td>13</td>
</tr>
<tr>
<td>Introduction</td>
<td>13</td>
</tr>
<tr>
<td>Methodology</td>
<td>16</td>
</tr>
<tr>
<td>Simulation Case Studies</td>
<td>26</td>
</tr>
<tr>
<td>Conclusion</td>
<td>37</td>
</tr>
<tr>
<td>4. TRAFFIC FLOW PREVIEW FOR PLANNING FUEL OPTIMAL VEHICLE VELOCITY</td>
<td>39</td>
</tr>
</tbody>
</table>
Table of Contents (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>39</td>
</tr>
<tr>
<td>Methodology</td>
<td>43</td>
</tr>
<tr>
<td>Simulation Results</td>
<td>52</td>
</tr>
<tr>
<td>Conclusion</td>
<td>64</td>
</tr>
<tr>
<td>5. CONCLUSION</td>
<td>66</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>70</td>
</tr>
<tr>
<td>A. Model Predictive Control</td>
<td>72</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>77</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>MPC Parameters</td>
<td>27</td>
</tr>
<tr>
<td>3.2</td>
<td>Drive-cycle statistics for PCC and baseline vehicles</td>
<td>31</td>
</tr>
<tr>
<td>3.3</td>
<td>PSAT simulation results for an economy-size vehicle</td>
<td>31</td>
</tr>
<tr>
<td>3.4</td>
<td>Initial speed and position for the fleet vehicles</td>
<td>34</td>
</tr>
<tr>
<td>3.5</td>
<td>Total traveled distance for PCC and baseline fleets</td>
<td>35</td>
</tr>
<tr>
<td>3.6</td>
<td>Fuel economy comparison for PCC and baseline fleets</td>
<td>36</td>
</tr>
<tr>
<td>4.1</td>
<td>Fuel economy results in mile per gallon for conventional and PCC equipped vehicles</td>
<td>56</td>
</tr>
<tr>
<td>4.2</td>
<td>Macroscopic traffic model parameters</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>Initial and boundary condition parameters value for case A</td>
<td>59</td>
</tr>
<tr>
<td>4.4</td>
<td>Drive cycle statistic for Case A: Conventional and PCC-equipped vehicles</td>
<td>60</td>
</tr>
<tr>
<td>4.5</td>
<td>Fuel economy results in mile per gallon for passenger and heavy vehicle - Case A</td>
<td>61</td>
</tr>
<tr>
<td>4.6</td>
<td>Initial and boundary condition parameters value for case B</td>
<td>62</td>
</tr>
<tr>
<td>4.7</td>
<td>Drive cycle statistic for Case B: Conventional and PCC-equipped vehicles</td>
<td>63</td>
</tr>
<tr>
<td>4.8</td>
<td>Fuel economy results in mile per gallon for passenger and heavy vehicle - Case B</td>
<td>63</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Schematic of telematics-based predictive cruise control</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>Schematic map of red lights distributed over space-time</td>
<td>19</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic of PSAT powertrain</td>
<td>25</td>
</tr>
<tr>
<td>3.4</td>
<td>Trajectory of PCC and baseline vehicle with respect to red-light map</td>
<td>28</td>
</tr>
<tr>
<td>3.5</td>
<td>Velocity, control inputs and the position for a vehicle without advanced signal information</td>
<td>29</td>
</tr>
<tr>
<td>3.6</td>
<td>Velocity, control inputs and the position for a vehicle with advanced signal information</td>
<td>30</td>
</tr>
<tr>
<td>3.7</td>
<td>Google map of a part of Pleasantburg Drive in Greenville</td>
<td>32</td>
</tr>
<tr>
<td>3.8</td>
<td>First set of PCC and baseline trajectories simulated with Pleasantburg Drive signal timing information</td>
<td>33</td>
</tr>
<tr>
<td>3.9</td>
<td>Second set of PCC and baseline trajectories simulated with Pleasantburg Drive signal timing information</td>
<td>34</td>
</tr>
<tr>
<td>3.10</td>
<td>Illustration of $\gamma$ as the gap activation area</td>
<td>35</td>
</tr>
<tr>
<td>3.11</td>
<td>Trajectories of fleet of PCC vehicle</td>
<td>35</td>
</tr>
<tr>
<td>3.12</td>
<td>Trajectories of fleet of baseline vehicle</td>
<td>36</td>
</tr>
<tr>
<td>3.13</td>
<td>Gap and gap constraint between each two vehicles in the PCC fleet</td>
<td>37</td>
</tr>
<tr>
<td>4.1</td>
<td>Real-time traffic information shown by green and red circles on traffic layer of Google Earth</td>
<td>40</td>
</tr>
<tr>
<td>4.2</td>
<td>Schematic of DP grid and value function iteration</td>
<td>50</td>
</tr>
<tr>
<td>4.3</td>
<td>Schematic of PSAT powertrain model</td>
<td>52</td>
</tr>
</tbody>
</table>
4.4 Trajectories delivered for conventional vehicle versus PCC vehicle for forward congestion wave ...........................................55
4.5 Trajectories delivered for conventional vehicle versus PCC vehicle for backward forward congestion wave ...........................56
4.6 Spatiotemporal traffic flow surface generated for Case A .......................59
4.7 Trajectories delivered for conventional vehicle versus PCC vehicle for Case A.................................................................60
4.8 Spatiotemporal traffic flow surface generated for Case B ......................62
4.9 Trajectories delivered for conventional vehicle versus PCC vehicle for Case B.................................................................63
A.1 Schematic of a Model Predictive Controller............................................74
Chapter 1

Introduction

1.1 Motivation

Consider the following two examples in which lack of information of upcoming events has a negative influence on fuel economy and possibly emissions of a vehicle:

1. Vehicles speeding up toward a green light and having to come to a sudden stop when the light turns red (amber) losing fuel and wearing their brakes and engines. (Unknown future status of traffic lights)

2. A vehicle’s untimely arrival at a local traffic wave with lots of stops and goes. (Unknown future traffic flow)

Interestingly preview information of traffic light status and traffic flow is not far-fetched today with GPS-enabled vehicle navigation systems. Google currently streams real-time traffic information of major U.S. cities and includes accurate average speed of vehicles in each road segment (see the latest edition of Google Earth and its traffic layer.). Traffic light information is not currently broadcast to vehicles, however serious research is underway in making this information available under Cooperative Intersection Collision
Avoidance Systems (CICAS) initiative of the U.S. Department of Transportation [58]. In fact the protocol for communicating signal phase and timing via Dedicated Short Range Communications (DSRC) is now being finalized by a team of government, industry, and university researchers [57].

Most uses of such information have been for navigation and routing purposes using mostly ad-hoc or proprietary routines [7, 21, 27, 66]. An untapped opportunity lies in utilizing this vast source of dynamic information for better energy management of conventional and hybrid vehicles. Preview can help plan an eco-friendly speed profile which saves fuel and reduces emissions without increasing trip time. This eco-friendly speed can be suggested to the driver or directly incorporated in vehicle’s adaptive cruise control module.

The objective of this Masters thesis is to, i) propose an effective velocity trajectory planning for improving the fuel economy and reducing trip time on a particular driving route when the traffic light information is available to the vehicle in advance, and ii) utilize the spatiotemporal traffic information and dynamic behavior of traffic flow along the road to find the fuel optimal velocity trajectory without compromising trip time.

1.2 Approach

In the first section of this thesis, the desired velocity trajectory is obtained through a two-level optimization based control strategy: A supervisory level controller which is based on logical rules generates a velocity trajectory that ensures timely arrival of the vehicle to the green phase of the intersection, and a model predictive controller (MPC) [23, 51] to track the generated trajectory. The second section of this thesis is allocated to the use of traffic flow information to generate a near fuel optimal velocity trajectory via use of dynamic programming approach. In this method spatiotemporal traffic flow pattern is first predicted using a partial differential equation macroscopic traffic model which can be ini-
initialized using real-time traffic information. The predicted traffic surface serves as an upper constraint on vehicle’s velocity in solving the fuel optimal problem.

1.3 Overview of Thesis

In Chapter 2 an overview is given on existing intelligent transportation system initiatives and methods. A brief introduction of existing traffic management through intelligent highways and intelligent vehicle is given in this chapter with an overview of available communication technologies used in the intelligent transportation systems. Also in this chapter some of the existing algorithms in traffic management are reviewed.

Chapter 3 is on utilization of upcoming traffic signal information for improving fuel economy and reducing emissions. In this chapter a two-level optimization-based control strategy is introduced to derive a near-optimal trajectory which improves fuel economy and emissions and may also reduce trip time. Taking the advantage of future state of traffic light information, the resulting near-optimal velocity trajectory is shown to considerably reduce fuel consumption and trip time. A same investigation on implementation of such control strategy is done for fuel economy and trip time evaluation of a multi-vehicle scenario and the reported results demonstrate improvement in fuel economy, emissions, and trip time.

Chapter 4 is dedicated to the use of spatiotemporal traffic flow preview in calculating the fuel optimal velocity trajectory enroute to a destination. In this chapter a partial differential equation model of traffic flow is assumed and the optimal control strategy is implemented with this traffic constraint to to find the fuel minimal velocity trajectory. In this chapter we also describe the gas dynamic macroscopic model of traffic flow which is used to predict spatiotemporal evolution of traffic down the rout. We report preliminary results on fuel economy gains for both a passenger and a heavy commercial vehicle which are positive and encourage further work in this area.
Finally, in chapter 5 the contributions of this thesis are summarized. The areas in which future potential research can be conducted have also been proposed in this chapter.
Chapter 2

Existing Intelligent Highway and Intelligent Vehicle Initiatives

2.1 Introduction

Various definitions, methodologies, and tools are largely in use in the areas of intelligent highways and intelligent vehicles. The purpose of this chapter is to introduce technologies and tools used in these areas with a brief overview of research trends in the field.

2.2 Intelligent Transportation Systems

Intelligent transportation systems (ITS) refers to the systems that benefit from information transport between the infrastructure and vehicles in an effort to improve safety, traffic flow, and fuel economy. From the control perspective, technologies in ITS could be categorized into two main topics: Intelligent Highways that deals with centralized intelligent control of highways and Intelligent Vehicles which is focused on decentralized control
of intelligent vehicles.

2.2.1 Intelligent Highways Initiatives

The Federal Highway Administration (FHWA) in cooperation with private industry, and academia, established a program called "Intelligent Vehicle Highway Systems (IVHS)" in the 1990s. The general goal of IVHS was to use advanced communication technologies to increase the capacity of current roadways, to improve the safety on highways, and to improve fuel efficiency of moving vehicles.

One example of projects pursued under IVHS was vehicle platooning which would allow partial automation of a large fleet of vehicles, increasing their fuel efficiency and also with the target of increasing the highway throughput. However, USDOT canceled implementation of this program in 1998, citing budget difficulties and the importance of close driving safety systems[72]. During its active days, IVHS attracted many researchers from different disciplines. Part of the research focused on modeling individual vehicles in a fleet [38] and developing control systems for them, string stability of a fleet has also been extensively studied [48, 48]. There has been also major research focus and findings on communication technologies[68].

Although the idea of IVHS is partially shelved due to many institutional and organizational issues, the research findings of this period is valuable in the current trend in the area of intelligent vehicles and networked vehicles. Many of the communication schemes developed and control strategies are applicable today.

2.2.2 Intelligent Vehicles Initiative

More recently the focus has moved from centralized intelligent highway ideas in the 1990s, to decentralized intelligence within each vehicle. Under this initiative the goal
is to equip vehicles with communication devices that enable them to communicate with surrounding vehicles and the infrastructure, via ad-hoc networks [11, 63] or cellular networks. Such technology will enable 1) improve the safety of the future vehicles by collision prevention, 2) improve fuel economy by allowing smart vehicles to choose better routes and avoid congestion and traffic stops, and 3) reduce traffic congestion by allowing the infrastructure control the flow of smart vehicles. Research is being actively done on communication schemes and hardware by electrical engineers, on routing schemes by computer science and operation research experts, and on traffic optimization by traffic engineers. Of course many human factor issues need to be addressed as well.

In this chapter we provide an overview of the existing technologies, the ongoing research which leads to the motivation for the current thesis: i.e. to use the vast amount of information available today and the communication capability to help improve the fuel economy of such vehicles; those that are equipped with communication capabilities and have access to real-time information about their surrounding conditions.

### 2.2.2.1 Communication Technology

Two different forms of wireless communication technologies are suggested for intelligent vehicles: 1) Vehicle-to-Vehicle Communication and 2) Vehicle-Infrastructure. In 1999, the Federal Communications Commission has dedicated the 5.9GHz frequency exclusively for vehicular communications that can transmit data over distances of up to 1km (3,280 feet) between vehicles and with roadside infrastructures [69]. This communication protocol is referred to as Dedicated Short Range Communication (DSRC) protocol.

1. **Vehicle-to-Vehicle Communication**: Vehicle to Vehicle (V2V) communication empowers automobiles to exchange information with each other. Vehicle to vehicle communication is expected to be more widely deployed in the near future because
they have the potential to improve convenience and safety of vehicles in traffic[75]. For instance, equipped cars that can communicate via wireless links and hence create ad hoc networks can be used to reduce traffic accidents and facilitate traffic flow. An international standard, IEEE 802.11p, also known as Wireless Access in Vehicular Environments (WAVE), is recently published [54].

2. Vehicle-Infrastructure: Vehicle-Infrastructure (VI) communication is a technology used to provide a communications link between vehicles and the roadside infrastructure (via Roadside Equipments), in order to increase the safety, fuel efficiency, and convenience of the transportation system. It is based on deployment of dedicated short-range communication (DSRC)[16] protocols. Recently, Vehicle-Infrastructure communication technology is also envisioned through use of cell phones and GPS navigators. GPS navigators are becoming standard in many new vehicles and are an option on most new mid- and high-range vehicles. In addition, many users have cellular phones which continuously transmit trackable signals (and may also be GPS-enabled). Automatic registration plate recognition can also provide high levels of data in many geographical regions. Ford recently showcased its own Smart Intersection technology, which relies on GPS and wireless communication technologies to enable traffic lights and street signs to send warnings to approaching vehicles, and both Nissan and Volvo have confirmed development plans for similar systems in the past. In the near future, it is likely to have a standardized system where every vehicle on the road will recognize the presence of other vehicles. The technology also has the potential for creating a world where such communications may enable automated driving [31].

end


2.2.2.2/ Algorithms

i. Traffic Signal Management: As a part of ITS, extensive research has been conducted in traffic signal management. These works would be classified into several topics according to their main goals.

Collision avoidance and intersection safety[65, 13] has been discussed in the literature as one of the major goals of traffic signal management. Traffic management at intersections has also been widely discussed in the literature with the hope of improving traffic throughput optimization[10, 12]. The benefits of traffic signal synchronization are also investigated. It has been shown that in many cases the green-light wave solutions can be derived by taking advantage of this implementation[39]. Another method of improving traffic flow for any density has been brought into literature as ”self-organizing traffic light” method[26]. In self-organizing traffic light method, each traffic signal, i.e. intersection keeps a counter which is set to zero when the light turns red and then increases with the number of cars approaching the red light. When the counter (representative of number of cars by the red light) reaches a specified threshold, the green light at crossing way turns yellow.

Along with the widespread use of adaptive cruise control and installation of communication equipments in luxury cars, many papers have focused on traffic system traveling time reduction and safety improvement via utilization of data interchange between signal and equipped vehicle. The idea is that the autonomous vehicles call the upcoming intersection to reserve a time-space gap to pass; which among other things can help improve the fuel economy as well[15, 73].
ii. Traffic Routing and Optimization: The origin of traffic routing optimization problems goes back to graph theory emerged in 1736\cite{17} in which the goal is to find a route with particular specifications. For instance, if a graph represents a road network, the weights assigned on its edges may represent the length, grade, and traffic load of each road. This way, many traffic routing optimization problem could be morphed to a graph.

Using graph theory, many papers in the literature have introduced algorithms to find the optimum route based on various criteria such as less fuel, less time, and less distance\cite{18, 47, 64}.

Older models and algorithms for routing problems were commonly developed based on default constant travel times between all locations in a traffic network. This assumption is in contradiction with real dynamic travel time between locations, particularly in urban area. However, consideration of travel time that varies with the time of traveling is recently taken into account and more reliable solutions for routing optimization problem have been derived\cite{19}. With the increasing availability of real-time traffic information and communication systems, the need for more effective routing algorithms arises. More recently, a dynamic routing system has been developed for shipping purchases quicker. This algorithm dispatches a fleet of vehicles according to customer orders. Each customer order requires a transport from a pickup location to a shipping address in a given time horizon\cite{20}.

With the advent of global position system (GPS), developing new routing technology is increasing in such a pace that online routing motors with the hourly updated traffic information are available on public networks\cite{1} and even on equipped handy devices\cite{2}. The exponential growth of online
routing technology makes this hope brighter that in near future all transportation means can generate their optimum path online.

iii. Traffic Flow Modeling and Control: Modeling of traffic flow has been the topic of many research projects since 1950s. The original motivation to model the traffic flow was to investigate the situations in which traffic jam appears, moves, and damps[24]. The initial idea was to derive mathematical equations based on either behavior of drivers in the road or analogy between fluid dynamics and vehicles in traffic flow. Considerable research has been done to modify vehicular traffic flow models to match realistic traffic stream.

In general, traffic flow modeling could be categorized into three main branches of study based on level of details: Microscopic, Mesoscopic, and Macroscopic.

In microscopic traffic models individual vehicles are represented as moving particle and their procession is modeled by simple rules or using a cellular automata approach. Traffic scenarios are created by simulated a large number of point vehicle models.

In mesoscopic traffic model, the mathematical representation of traffic system focuses on individual vehicle action but with a more aggregate representation of traffic dynamics. However, they are still restricted in their ability to represent details of traffic operations, especially as related to ITS systems. Typical applications of mesoscopic models are evaluations of traveler information systems.

Macroscopic modeling of traffic flow refers to the representation of the traffic with high level of aggregation as a flow without distinguishing its constituent parts [37]. Typically, a macroscopic model defines a relation
between the traffic density (the number of vehicles per kilometer), the average velocity, and the traffic flow (the number of vehicles passing a certain point per hour) [8]. The attractive feature of this approach is that it does not involve a detailed microscopic investigation of dynamical behavior of each car in the system. This model was first introduced based on the assumption that at any point of the road, the traffic flow is a function of traffic density (the number of vehicles per unit distance) at the corresponding point [67]. However, it should be mentioned that the only accurate physical law in traffic flow is "conservation of mass". It means that other introduced equations for macroscopic flow of traffic are based on coarse approximations and has been inspired by equations govern on fluid mechanics.

More recently, with the benefit of traffic flow modeling, control strategies have been developed to avoid congestion along traffic stream either via enforcing dynamic speed limit in segments of a road or by ramp metering. Using traffic model information, a control strategy is also proposed to avoid traffic shock waves [32].
Chapter 3

Predictive Cruise Control:
Utilizing Upcoming Traffic Signal Information for Improving Fuel Economy and Reducing Trip Time

3.1 Introduction

American drivers spend a total of 40 hours per year idling in traffic. The fuel used during this idle time adds to 78 billion dollars per year [4]. A big portion of our idle time is the time spent behind traffic lights. Poor traffic signal timing is believed to account for an estimated 10 percent of all traffic delay (about 300 million vehicle-hours) on major roadways alone [6]. Effective advanced traffic signal control methods such as traffic-actuated signals and signal synchronization have been widely deployed across many of our traffic intersections which
help save us precious time and expensive fuel every day. Such measures however are very costly to implement and maintain; just the annual cost of signal timing updates is estimated at 217 million dollars a year according to [5]. Even with these measures in place, we often cruise at full speed toward a green and have to come to a sudden halt whenever the light turns red. This lack of information about “future” state of the traffic signal increases fuel use, trip time, and engine and brake wear. In an ideal situation if the future of a light timing and phasing are known, the speed could be adjusted for a timely arrival at green.

While maybe unrealistic a few years ago, communicating traffic signal state to the vehicles in advance is not far-fetched today. In fact researchers are now experimenting with broadcasting red light warnings to vehicles to improve traffic intersection safety [52, 59]. As demonstrated in [59], the required information broadcast technology is available today and is expected to be more widely deployed in near future.

This chapter focuses on employing upcoming light time and phase information within the vehicle’s adaptive cruise control system to reduce i) wait time at stop lights and ii) fuel use which in turn may also reduce total trip time and CO₂ emissions. To achieve this goal an optimization-based control algorithm will be formulated for each equipped vehicle that uses short range radar and traffic signal timing information to schedule an optimum velocity trajectory for the vehicle. The objectives are timely arrival at green light with minimal use of braking, maintaining safe distance between vehicles, and cruising at or near set speed. Figure 3.1 shows a schematic of this proposed concept.

Adaptive cruise control is now in production and a well-matured technology. Many ideas on intelligent transportation system (ITS) have been explored ex-
tensively during the 1990s within intelligent highway initiatives in the US, Japan, and Europe [72]. Voluntary use of future signal and traffic information has only recently attracted attention under CICAS (Cooperative Intersection Collision Avoidance Systems) initiative mainly for improving intersection safety [65, 13]. Optimal traffic management at intersections has been mainly studied from a signal-timing optimization perspective e.g. signal synchronization [10, 39, 26]. More recently and for futuristic autonomous vehicles, Dresner et al. [15, 73] have proposed replacing traffic lights and stop signs by intelligent lights: via a two way communication protocol, the autonomous vehicles call the intersection ahead to reserve a time-space slot to pass; which among other things can help improve the fuel economy.

To the best knowledge of the authors, the Predictive Cruise Control (PCC) concept that we propose in this chapter is first in its kind that utilizes the adaptive cruise control function in a predictive manner to simultaneously improve fuel economy and reduce signal wait time. The proposed predictive speed control mode differs from current adaptive cruise control systems in that i) besides maintaining a safe gap between vehicles, it decreases use of brakes, thus reducing brake wear and kinetic energy loss, ii) is applicable in stop and go traffic,
and more importantly iii) receives a timing signal from an upcoming traffic light in advance to safely and smoothly speed up or down to a timely arrival at green light whenever possible, therefore reducing idling at red.

These sometimes conflicting objectives are unified under an optimization-based model predictive control (MPC) framework. The proposed MPC formulation allows tracking a target speed, calculated based on traffic signal information, while reducing brake use. At the same time it enforces several physical constraints including a safe distance to the front vehicle. Simulation of complex stop and go situations is facilitated relying on MPC as the “driving brain” of each vehicle. Because model predictive control is an optimization-based approach it may handle the traffic imposed constraints more systematically than the existing microscopic and macroscopic models for traffic simulation [56, 55, 37]. Many underlying functions or rules required to determine procession of vehicles in the existing methods limit embedding systematic optimization routines in them.

Section II formulates the methodology for planning a desired velocity profile around red lights and the tracking of this target velocity under motion constraints using model predictive control. Section II-C describes a detailed powertrain model used for evaluating the fuel economy and CO$_2$ emissions of the vehicle. Three simulation case studies are presented in Section III to illustrate application of the proposed methodology in single- and multi-vehicle scenarios. Conclusions are presented in Section IV.
3.2 Methodology

One of the analytical challenges unique to this optimal control problem is dynamic switching of lights to red and green. These types of motion constraints render the feasible solution space non-convex. Solution of a non-convex optimization problem is computationally intensive and may not converge to the global optimum. In order to find a near-optimal solution with reasonable level of computations, we handle the problem at two levels: i) a set of logical rules that calculates a reference velocity for timely arrival at green lights combined with ii) a model predictive controller that tracks this target velocity. The resulting solution may be sub-optimal but is real-time implementable. A simple model of the vehicle will be used at the supervisory level for velocity planning; but the fuel economy, CO$_2$ emissions, and drivability will be later evaluated using a detailed model of the powertrain.

3.2.1 Reference Velocity Planning

A reference velocity $v_{target}$ is determined based on the driver’s set cruise speed, and also the signal received from the upcoming traffic light. The basic idea is to safely i) increase $v_{target}$, up to a maximum allowable, when there is enough green time to pass, or otherwise ii) decrease $v_{target}$, down to a minimum allowable, to arrive at the next green. All will be done considering driver’s set cruise control speed. The objective is to avoid stopping at a red light if feasible.

It is assumed that the approximate distance to the next traffic light(s) is known at each time and shown by $d_i$ where the subscript $i$ denotes the light number in a sequence of traffic lights, i.e. $d_1$ is the approximate distance to the first upcoming light and $d_2$ to the second light at each time. The light(s) update and
broadcast an expected sequence of their green and red times regularly. Suppose \( g_{ij} \) is start of the \( j^{th} \) green of the \( i^{th} \) traffic light and \( r_{ij} \) is start of the \( j^{th} \) red of the \( i^{th} \) light. For example, light number 1 broadcasts, at regular intervals, a sequence

\[
[g_{11}, r_{11}, g_{12}, r_{12}, g_{13}, \ldots] = [40, 100, 150, 200, 240, \ldots]
\]

which implies the first traffic light is currently red, it will turn green in 40 seconds, red in 100 seconds, green again in 150 seconds, and so forth. Figure 3.2 shows a schematic of the map formed at each time step based on the information received from the lights. Equipped vehicles can use the remaining distance to the next light(s) and the green and red sequence to set their target speed. This target speed (slope of each path) should be in the feasible range \([v_{min}, v_{max}]\) where \( v_{min} \) is the road’s minimum speed limit and \( v_{max} \) is the smaller of two quantities: the velocity set by the driver and the road’s maximum speed limit. Other constraints such as acceleration constraints, maintaining safe distance to the front vehicle, and reducing use of brakes are handled separately by a dynamic optimization scheme (details in Section 3.2.2).

The following steps determine the target speed at each step \( k \):

i. For a vehicle to pass during the first green of the first light, its velocity should be in the interval \([\frac{d_1}{r_{11}}, \frac{d_1}{g_{11}}]\). This is only feasible if this interval has a set intersection with the feasible speed interval of \([v_{min}, v_{max}]\). If this set intersection is empty, passing through the first green without stopping at red is deemed infeasible. In that event, feasibility of passing during the next green interval is checked and the process is repeated until for some \( i^{th} \) interval \([\frac{d_1}{r_{ii}}, \frac{d_1}{g_{ii}}]\) has a set intersection with \([v_{min}, v_{max}]\). This set intersection
is mathematically characterized by:

\[
\left[ \frac{d_1}{r_{1i}}, \frac{d_1}{g_{1i}} \right] \cap [v_{\text{min}}, v_{\text{max}}] \quad (3.1)
\]

and determines the range of speed that ensures passing the first light without having to stop at a red.

For example assume the speed limits are \([v_{\text{min}}, v_{\text{max}}] = [5, 20] \text{m/s}\) and the distance to the first traffic light is 1000\(m\). The first light broadcasts,

\[g_{11} = 5s, \quad r_{11} = 25s, \quad g_{12} = 40s, \quad r_{12} = 100s\]

then

\[\left[ \frac{d_1}{r_{11}}, \frac{d_1}{g_{11}} \right] = [40, 200] \text{m/s}\]
does not meet the speed limit. The second interval

\[
\left[ \frac{d_1}{r_{12}}, \frac{d_1}{g_{12}} \right] = [10, 25] \, \text{m/s}
\]

intersects with the feasible speed at [10, 20] m/s. Therefore, if the velocity of the vehicle is chosen between 10 m/s and 20 m/s, the vehicle passes the first light without having to stop. If no feasible set intersection is found, stopping at the light will be unavoidable and no further check is necessary.

ii. If passing without stop at the first light is determined to be feasible, the process in step 1 is repeated for the second traffic light by checking the set intersections

\[
\left[ \frac{d_2}{r_{2i}}, \frac{d_2}{g_{2i}} \right] \cap [v_{\text{min}}, v_{\text{max}}]
\]

and picking the first non-empty one.

iii. Next, the set intersection of the feasible range of speeds determined in step 1 and that of step 2 is calculated. A non-empty solution \([v_{\text{low}}, v_{\text{high}}]\) indicates feasibility of passing the two lights without having to stop at a red. However an empty solution does not imply that stopping at red is necessarily required. It only means that passing the two consecutive lights with the same speed is not feasible. In that event, the vehicle can re-adjust its target speed after passing the first light to pass the green of the second light.

iv. The process is continued by checking the next lights until a stop at red becomes unavoidable. The last feasible range \([v_{\text{low}}, v_{\text{high}}]\) is an appropriate target velocity. In this chapter we set \(v_{\text{target}} = v_{\text{high}}\) for reducing trip time.\(^1\)

\(^1\)One can argue that in some scenarios a decreasing target velocity profile may require less fuel than a constant target velocity with same travel time. For example consider the scenario where the vehicle needs
Note that the target velocity is updated at each sampling time and therefore may change at each instant based on vehicle’s position and the most recent information from the lights. This set of rules is not necessarily “optimal”, but helps break down a fundamentally non-convex optimization problem to a simpler real-time implementable one. Tracking this target velocity, maintaining a safe distance to the front vehicle, and reducing use of brakes are handled by this optimization scheme described next.

### 3.2.2 Optimal Tracking of the Reference Velocity

A simple model of the vehicle is used at the supervisory level for calculating the vehicle acceleration based on effective traction force of the engine $f_{\text{engine}}$ or braking force $f_{\text{brake}}$ and the road forces $f_d$. For the $i^{th}$ vehicle with mass $m_i$, the longitudinal dynamics is [28]:

$$m_i \frac{d^2 x_i}{dt^2} = f_{\text{engine}}^i - f_{\text{brake}}^i + f_d^i$$  \hspace{1cm} (3.2)

where $f_d^i$ lumps the road forces including aerodynamic drag, rolling resistance, and road grade forces:

$$f_d^i = -c_D v_i^2 - m_i g (\sin(\theta) + \mu \cos(\theta))$$  \hspace{1cm} (3.3)

to slow down after passing the first light to go through the second light without stopping. In this scenario a decreasing target speed before the first light may be more fuel economical than a constant speed. In particular, one can check feasibility of a target velocity decreasing linearly where the constant deceleration rate $a$ before the first light can be found from the following kinematic equation,

$$d_1 = \frac{1}{2} a g_{11}^2 + v_0 g_{11}$$

where $v_0$ is the initial speed. Because searching for variable speed profiles increases the search space and the computational time, such profiles are not considered in this chapter.
where $\theta$ is the road grade, $c_D$ is a “lumped” drag coefficient, $\mu$ is the coefficient of rolling resistance, and $g$ is gravitational acceleration. The $f_d^i$ term is treated as a measured disturbance and updated at each sample time. Equation (3.2) can be written in the following state-space discretized form:

$$z_i(k+1) = Az_i(k) + B_u u_i(k) + B_w w_i(k)$$

$$y_i(k) = Cz_i(k)$$

(3.4)

where $z_i = [x_i \ v_i]^T$ is the state vector, $u_i = [f_{engine}^i \ f_{brake}^i]^T$ is the control input, and $w_i = [f_d^i]$ is the measured disturbance. The main outputs of interest are $y_i = [x_i \ v_i]^T$; however other outputs are introduced in the simulation code to handle the gap inequality constraint described later. The matrices $A \in \mathbb{R}^{2 \times 2}$, $B_u \in \mathbb{R}^{2 \times 2}$, $B_w \in \mathbb{R}^{2 \times 1}$, and $C \in \mathbb{R}^{2 \times 2}$ are the discretized system matrices. The engine and brake forces are manipulated for tracking the target speed as closely as possible while maintaining a safe distance to the front vehicle. These objectives along with the desire to reduce use of service brakes can be unified in a Model Predictive Control (MPC) framework. The control performance index at each step $k$ for the $i^{th}$ vehicle is defined as:

$$J_i(k) = \sum_{j=k}^{k+P-1} \left[ w_1(v_i(j) - v_{target})^2 + w_2(f_{brake}^i(j))^2 \right]$$

(3.5)

The trip time is reduced by setting $v_{target}$ equal to maximum feasible speed as explained in the previous section. This constant-velocity solution may be suboptimal; the truly optimal solution requires explicit optimization of trip time over space of all functionals $v_i$. Here $w_1$ and $w_2$ are simply penalty weights for each term. The above index penalizes deviations of vehicle speed $v_i$ from the target speed $v_{target}$ and also
reduces use of brake force over a future prediction window of \( P \) steps. Reduced use of service brakes in the cost function indirectly contributes to fuel savings. Fuel use is not explicitly penalized; this allows use of the simpler vehicle model for control design. Fuel savings will be later evaluated using a detailed model of the vehicle’s powertrain.

The speed limit, engine and brake force limits, and the minimum safe following distance are imposed as pointwise-in-time inequality constraints. The constraints should be satisfied over the future prediction horizon \( \forall j \in \{k, k + 1, \cdots, k + P - 1\} \). The speed limit constraint is,

\[
v_{\min} \leq v_i(j) \leq v_{\max}
\]  

(3.6)

where \( v_{\min} \) and \( v_{\max} \) are speed limits and should also be smaller than the driver set speed. Bounds on the traction force are represented by,

\[
0 \leq f_{i\text{engine}}^j \leq f_{\text{acceleration}}^{\text{max}}
\]

\[
0 \leq f_{i\text{brake}}^j \leq f_{\text{deceleration}}^{\text{max}}
\]  

(3.7)

where \( f_{\text{acceleration}}^{\text{max}} \) and \( f_{\text{deceleration}}^{\text{max}} \) depend on tire and road condition and also maximum engine and braking torque capability. The minimum safe distance between the vehicle \( i \) and the front vehicle (target) should be a function of the vehicle speed and is chosen as [72]:

\[
x_{\text{target}}^j - x_i^j \geq \alpha v_i(j) + \beta
\]  

(3.8)

where \( \beta \) is a “static gap” parameter and determines the minimum gap needed when the vehicles are stopped and \( \alpha \) is a “dynamic gap” parameter providing
extra gap with increased speed. Note that when the vehicle is approaching a red
light, the light is considered similar to a stopped vehicle and the position $x_{target}$
is fixed to the position of the light. This ensures that the vehicle comes to a stop
with distance $\beta$ from the light ($x_{target} \geq x_i + \beta$).

The cost function (3.5) subject to the model equation (3.4) and inequality con-
straints (3.6), (3.7), and (3.8) is minimized at each sample time to determine the
sequence of next $N \leq P$ control inputs $U_i(k) = [u_i(k) \ u_i(k+1) \ \cdots \ u_i(k+N-1)]$ over the future horizon $P$. When $N < P$ the remaining control moves
$[u_i(k+N) \ u_i(k+N+1) \ \cdots \ u_i(k+P-1)]$ are assumed to be zero. Ac-
cording to the standard MPC design, only the first entry of the control sequence
$U_i(k)$, is applied to the vehicle; the optimization horizon is moved one step
forward, the model and constraints are updated if necessary, and the optimiza-
tion process is repeated to obtain the next optimal control sequence $U_i(k+1)$
[51, 22, 9].

### 3.2.3 Evaluation of Fuel Savings Potential with a Detailed
Powertrain Model

The MPC solution generates a constraint-admissible velocity profile that fol-
low the set target speed as closely as possible. In order to estimate the fuel
economy of the vehicle when following this optimal velocity trajectory, a pro-
duction vehicle is selected and its powertrain model is assembled from the ex-
tensive database of Powertrain System Analysis Toolkit (PSAT). PSAT, devel-
oped by Argonne National Laboratory [46], is a powerful simulation tool for
evaluating the fuel economy of conventional and hybrid vehicles when follow-
ing a prescribed velocity cycle. Its physics-based component models combined
with empirical maps obtained from production vehicles allow high-fidelity evaluation of fuel economy. Figure 4.3 shows schematics of a PSAT powertrain model. This is a conventional (non-hybrid) powertrain with an automatic transmission. The models for torque converter, transmission, and vehicle dynamics are all very detailed and include several dynamic states and switching modes. Details such as electrical accessory loads, the starter, generator, etc. are not overlooked, and are modeled for simulation accuracy.

PSAT is a “forward-looking” causal simulation tool in which the vehicle speed is determined by the combined influence of road loads and engine (or brake) torque at the wheels. The resulting velocity is compared to the prescribed desired velocity; the difference is fed to a driver model (a PI controller) which in turn determines a torque demand. The torque demand is met by the engine (or brake) torques and the above simulation loop is repeated. The engine fuel rate is determined using an empirical engine map and as a function of engine speed and engine torque. The fuel rate is integrated over the whole cycle time to determine the amount of fuel used.
3.3 Simulation Case Studies

This section presents the results of a few simulations performed to demonstrate validity of the proposed PCC methodology and to observe the fuel economy, emissions, and travel time gains in these example simulations. To understand the impact of PCC on the average and under different traffic lights and vehicle parameters, an extensive simulation study is needed which is outside the scope of the current thesis. We hope the following simulation results motivate a detailed statistical analysis in the future.

The simulations are run first with the Predictive Cruise Control (PCC) off which serves as a baseline for comparison and then with PCC on during which advanced information of the lights phasing and timing is available. The comparison baseline is a vehicle without advanced access to signal phasing and timing information. For a fair comparison, the baseline vehicle is assumed to operate in adaptive cruise control mode as well\(^2\). The baseline vehicle tracks a target velocity using the MPC strategy explained in section 3.2.2. Its controller minimizes (3.5) subject to the same model equation (3.4) and inequality constraints (3.6), (3.7), and (3.8). However the target velocity \(v_{\text{target}}\) is always equal to the driver set speed for the baseline vehicle. The need for a timely stop at red light is enforced through the constraint (3.8) and by fixing \(x_{i+1}\) to the position of the light as soon as the light turns amber or if an upcoming light is found to be red (thus no advanced phase and time information).

The parameters of the supervisory level controller are summarized in Table 3.1. In all simulations, the vehicle mass is assumed to be \(m = 1000\) kg. Maximum acceleration is assumed to be \(a_{\text{max}} = 3\) m/s\(^2\) which is a conservative estimate\(^2\).

---

\(^2\)Adaptive cruise control assumption can be thought of as a systematic mean to model a driver behavior in flowing traffic. In other words the comparison is not limited only to ACC equipped vehicles.
of the maximum acceleration capability of a midsize vehicle. From there we calculate $f_{\text{acceleration}}^{\text{max}} = ma_{\text{max}} = 3000N$. Assuming braking on dry asphalt, the coefficient of braking is chosen to be $\mu_b = 0.69$ [28]. The maximum possible braking force is then calculated as $f_{\text{deceleration}}^{\text{max}} = \mu_b mg \approx 6800N$. The sampling time of 0.2 seconds allowed capturing the relevant dynamics. After several trials prediction and control horizons of 8 and 2 seconds respectively, were found to be adequate beyond which the performance did not change considerably. The penalty weights $W_1$ and $W_2$ were tuned to track the target velocity with reasonable braking effort. The gap parameters are most relevant in multi-vehicle simulations and are chosen to ensure sufficient distance between vehicles.

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>units (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>sample time</td>
<td>0.2</td>
<td>s</td>
</tr>
<tr>
<td>$P$</td>
<td>prediction horizon</td>
<td>8</td>
<td>s</td>
</tr>
<tr>
<td>$N$</td>
<td>control horizon</td>
<td>2</td>
<td>s</td>
</tr>
<tr>
<td>$W_1$</td>
<td>penalty weight 1</td>
<td>3000</td>
<td>(m/s)$^{-2}$</td>
</tr>
<tr>
<td>$W_2$</td>
<td>penalty weight 2</td>
<td>150</td>
<td>N$^{-2}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>dynamic gap parameter</td>
<td>0.2</td>
<td>s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>static gap parameter</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>$f_{\text{acceleration}}^{\text{max}}$</td>
<td>max positive traction</td>
<td>3000</td>
<td>N</td>
</tr>
<tr>
<td>$f_{\text{deceleration}}^{\text{max}}$</td>
<td>max negative traction</td>
<td>6800</td>
<td>N</td>
</tr>
</tbody>
</table>

### 3.3.1 Single Vehicle Scenario

**Case I-Suburban Driving**

The first simulation case study is created to approximate suburban driving conditions: the driver set speed is 30 m/s, the maximum speed is $v_{\text{max}} = 30$ m/s, and the minimum speed $v_{\text{min}}$ is zero. A sequence of 10 traffic lights spaced at 1km intervals is assumed for this simulation study. The light timing and phasing is
Figure 3.4: Trajectory of PCC and baseline vehicles with respect to the red-light map. Horizontal solid lines represent red intervals.

assumed to be fixed and independent of the incoming traffic. Future work can consider situations of synchronized or traffic-actuated lights. Figure 3.4 summarizes the light timing information. Also on this graph we show the trajectory of PCC and baseline vehicles.

Figure 3.5 shows the velocity profile, control inputs, and the distance traveled by the baseline vehicle. Zero portions of the velocity profile show that the baseline vehicle stops at multiple red signals. In a period of 400 seconds, the vehicle travels the distance of 7.66 km and passes 7 lights. The average velocity is therefore 19.15 m/s. During the same time and with the same initial conditions the PCC-equipped vehicle was able to travel 8.92 km as shown in Figure 3.6. By predictive use of signal information, the PCC vehicle schedules
its velocity to a timely arrival at a green light whenever possible. As a result the average velocity is 22.32 m/s which is a 16.5 percent improvement over the baseline vehicle. During the simulation the minimum and maximum speed constraints as well as all other constraints are met.

To evaluate the resulting fuel economy and emissions, an economy-sized passenger vehicle with the mass of 1000 kg and 5-speed automatic transmission was selected in PSAT. The vehicle has a 1.7 L 4-cylinder gasoline engine with the maximum power of 115 hp. The detailed vehicle model is assembled in PSAT v6.2. The velocity profiles shown in the first subplot of Figures 3.5 and 3.6 are fed as inputs to the PSAT simulation environment. A driver-model follows this input velocities very closely. Table 3.2 summarizes the statistics of the resulting velocity and acceleration. The maximum acceleration and decel-
Figure 3.6: Velocity, control inputs and the position for a vehicle with advanced signal information.

Operation for both PCC and baseline vehicles are within physical constraints and comparable to maximum acceleration and deceleration levels in many standard city cycles\(^3\). The calculated fuel economy and CO\(_2\) emissions are shown in Table 3.3. In this particular simulation, the PCC-equipped vehicle uses 47 percent less fuel with 56 percent less CO\(_2\) emissions than the vehicle with the conventional ACC for the same travel time. This is while the PCC vehicle travels a longer distance.

To determine if real-time implementation of the proposed optimization-based strategy is computationally viable, we also recorded the total computational time for solving the MPC optimization problem. The simulations were run in

\(^3\)For example US06 Supplemental Federal Test Procedure (SFTP) which has been developed to address the shortcomings with the FTP-75 test cycle in the representation of aggressive, high speed and/or high acceleration driving behavior, rapid speed fluctuations, and driving behavior following startup has a maximum acceleration of 3.75 m/s\(^2\) and maximum deceleration of -3.08 m/s\(^2\).
SIMULINK on a dual-core Intel\(^4\) Pentium IV processor with 1GHz processing speed per core, 4MB of cache, and 2GB of RAM. An estimate of CPU time was obtained using the CPU command in MATLAB\(^5\). For a simulation interval of 400 seconds the CPU time for running the MPC optimization was 19.1 seconds.

Table 3.2: Drive-cycle statistics for PCC and baseline vehicles.

<table>
<thead>
<tr>
<th>PCC vehicle</th>
<th>Max</th>
<th>Average</th>
<th>Standard Dev.</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>30.00</td>
<td>22.32</td>
<td>7.81</td>
<td>m/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>2.04</td>
<td>0.28</td>
<td>0.58</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>Deceleration</td>
<td>-3.23</td>
<td>-0.21</td>
<td>0.57</td>
<td>m/s(^2)</td>
</tr>
</tbody>
</table>

Baseline vehicle

<table>
<thead>
<tr>
<th>Max</th>
<th>Average</th>
<th>Standard Dev.</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>30.00</td>
<td>19.15</td>
<td>11.79</td>
</tr>
<tr>
<td>Acceleration</td>
<td>2.04</td>
<td>0.97</td>
<td>0.87</td>
</tr>
<tr>
<td>Deceleration</td>
<td>-3.10</td>
<td>-0.79</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 3.3: PSAT simulation results for an economy-size vehicle.

<table>
<thead>
<tr>
<th>Value</th>
<th>PCC</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Economy (miles/gallon)</td>
<td>28.72</td>
<td>19.22</td>
</tr>
<tr>
<td>CO(_2) Emissions(g/mile)</td>
<td>290</td>
<td>453</td>
</tr>
</tbody>
</table>

Case II-City Driving

The second single vehicle simulation case study represents inner-city driving. For this we were able to acquire traffic signal phasing and timing data from a stretch of Pleasantburg Drive\(^6\) inside the city of Greenville, South Carolina. Figure 3.7 shows a Google Map\(^7\) of this street and 10 of its consecutive intersections selected for this study. The distances between these intersections have been measured using the map. Observing the posted speed limit of 45

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\(^4\)Intel is a registered trademark of Intel Corporation, Santa Clara, CA.

\(^5\)MATLAB and SIMULINK are registered trademarks of The MathWorks Inc. of Natick, MA.

\(^6\)Pleasantburg Drive, south bound, starting from Century Drive and ending at Cleveland Street. The lights phasing and timing are those in place in April of 2009 and obtained from the Traffic Engineering Department of the City of Greenville.

\(^7\)Google Map is a registered trademark of Google Inc. Mountainview, CA.
Figure 3.7: Google map (Google Inc.) of a part of Pleasantburg Drive in Greenville, South Carolina used in the second simulation case study.

mph along Pleasantburg Drive, we set $v_{\text{max}} = 20 \text{ m/s}$. The driver set speed is also selected at 20 m/s, and the minimum speed $v_{\text{min}}$ is set to zero. The other simulation parameters are those of Case I.

The simulations were run with two sets of initial conditions. Figures 3.8 and Figure 3.9 show the trajectories of PCC and baseline vehicles in these two scenarios. With the first set of initial conditions, the PCC vehicle saves 65 seconds of trip time with 29% less fuel (25.97 mpg versus 20.07 mpg for the baseline). If the start time is delayed by 20 seconds, the PCC’s trip time advantage will only be 2 seconds and the fuel economy gain will reduce to 24 % (25.11 mpg versus 20.26 mpg for the baseline).
3.3.2 Multi-Vehicle Scenario

In this section we investigate the trip time and fuel economy for a fleet of PCC-equipped vehicles in a multi-vehicle simulation. Each vehicle runs a copy of the control strategy presented in Section 4.2 in a decentralized fashion. All vehicle and signal parameters are those chosen in Case I of single vehicle scenario. The former set of constraints in equations (3.6) and (3.7) remains unchanged. When a vehicle is detected at a distance $\gamma$ in front, the constraint in (3.8) is also activated with $x_{\text{target}}$ set as the position of the lead vehicle. Otherwise $x_{\text{target}}$ will be the position of the next targeted red light. The parameter $\gamma$ can be chosen based on the vehicle’s maximum braking distance (See Fig. 3.10).

Simulations are performed for two fleets of vehicles: A PCC-equipped fleet and
Figure 3.9: Second set of PCC and baseline trajectories simulated with Pleasantburg Drive signal timings. Horizontal solid lines represent red intervals.

A fleet of the same vehicles without PCC. Each fleet has six vehicles aligned initially with the set of initial conditions shown in table 3.4.

Table 3.4: Initial speed and position for the fleet vehicles.

<table>
<thead>
<tr>
<th>heightVehicle</th>
<th>Initial Position(m)</th>
<th>Initial Speed(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9000</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>8950</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>8900</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>8800</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>8750</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>8720</td>
<td>15</td>
</tr>
</tbody>
</table>

Figures 3.11 and 3.12 show the trajectories of PCC and baseline fleets for a simulation period of 400 seconds. The distance traveled by each vehicle as well as the total distance traveled by the vehicles of each fleet during this period are tabulated in table 3.5 for this simulation case study. The average velocity of the PCC fleet is 19.31 m/s as compared to the 18.53 m/s of the baseline
Figure 3.10: Illustration of $\gamma$ as the gap constraint activation area. Top views of Honda Accord and S2000, Honda Motor Co..

Figure 3.11: Trajectories of fleet of PCC vehicles.

flock; in other words the PCC fleet is 4.2 percent faster than the baseline fleet. Figure 3.13 shows the gap and gap constraint activation area between each two vehicles in the PCC fleet. It can be seen that the gap always remains above the velocity-dependent gap constraint line.

The vehicle velocity trajectories are fed to PSAT to determine the fuel used by each fleet. The vehicle configuration and parameters are those described in
Figure 3.12: Trajectories of fleet of Baseline vehicles.

Table 3.5: Total traveled distance for PCC and Baseline fleets.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCC(km)</td>
<td>8.9</td>
<td>7.6</td>
<td>7.5</td>
<td>7.5</td>
<td>7.4</td>
<td>7.4</td>
<td>46.3</td>
</tr>
<tr>
<td>Baseline(km)</td>
<td>7.6</td>
<td>7.5</td>
<td>7.5</td>
<td>7.4</td>
<td>7.3</td>
<td>7.0</td>
<td>44.4</td>
</tr>
</tbody>
</table>

Section 3.3.1. Table 3.6 summarizes the results. The average fuel economy of the PCC fleet is 41.8 percent better than that of the baseline fleet in this case study.

Table 3.6: Fuel economy comparison for PCC and Baseline fleets.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCC(mpg)</td>
<td>28.7</td>
<td>27.7</td>
<td>27.9</td>
<td>28.1</td>
<td>28.1</td>
<td>27.5</td>
<td>28.0</td>
</tr>
<tr>
<td>Baseline(mpg)</td>
<td>19.2</td>
<td>19.7</td>
<td>20.2</td>
<td>21.3</td>
<td>20.3</td>
<td>17.3</td>
<td>19.7</td>
</tr>
</tbody>
</table>
3.4 Conclusion

Communicating the signal state to vehicles has been recently proposed for improving traffic intersection safety. The positive simulation results of this chapter promise that signal-to-vehicle communication technology may also enable reduction of fuel consumption, greenhouse gas emissions, and trip time of future vehicles by predictive velocity planning. By this, we hope to encourage further research and innovation towards more intelligent traffic intersection control systems. Of course, any gain from the proposed PCC methodology depends on timing and phasing of traffic lights and the distance between them and the vehicle parameters. A detailed statistical analysis using Monte Carlo simulations
is one possible way of determining average gains with PCC which is a good direction for future simulation analysis.

From an analytical perspective, formulation of the trip optimization in this chapter in a model predictive control framework is novel and lends itself well to many traffic-imposed hard constraints. In an ongoing work we hope to evaluate the impact of traffic on the PCC strategy and vice versa by combining a macroscopic traffic model and the microscopic MPC methodology.
Chapter 4

Traffic Flow Preview For Planning

Fuel Optimal Vehicle Velocity

4.1 Introduction

Frequent stops and goes in traffic waves increase the fuel usage and emissions of passenger and commercial vehicles. Many of such stop and go conditions occur due to lack of information about the upcoming traffic pattern down the road. Many drivers choose to aggressively speed up to near the speed limit, only to be forced to abruptly decelerate their vehicles when faced with the slower traffic ahead of them and then perhaps idle or crawl in slow-moving traffic. If the upcoming traffic pattern is somehow “revealed” to the drivers in advance, the opportunity exits to adjust the speed more predictively to reduce harsh deceleration and idling or crawling intervals. Such planning of velocity could lower fuel use and emissions, improve the ride, and reduce brake and engine wear.

In the connected vehicles of today, the vehicle navigation system or handheld
wireless devices put real-time traffic information at our fingertips. Via these devices it is possible to now retrieve coarse traffic flow information en route to our destination via several online traffic information services. For example Google Maps provides real-time traffic information around 30 major U.S. cities now []; the traffic layer of Google Earth also shows the average velocity of vehicles at numerous nodes on a road (see Figure 4.1). In the near future it may be technically plausible to get higher resolution information on the immediate state of traffic; i.e. the velocity of individual nearby vehicles. In the Mobile Millennium project [3], a collaboration between Nokia, UC Berkeley’s California Center for Innovative Transportation (CCIT), California Department of Transportation (CALTRANS) and NAVTEQ, in-vehicle cellular phones are used as traffic sensors and form traffic velocity fields that can then be transmitted back to participating vehicles. One can envision access to a complete map of surrounding traffic, once such technologies are more widely deployed.

Figure 4.1: Real-time traffic information shown by green and red circles on traffic layer of Google Earth (Google Inc.). Average velocity at each node is also shown. This is a portion of Interstate 385 inside Washington DC, time is 3:00 pm on February 4th, 2009.
This level of information can potentially revolutionize the way we drive our vehicles. In fact many navigation systems today suggest alternate shortest-time routes based on traffic conditions. Several research papers have addressed the routing problem based on traffic information [43, 70]. The authors believe that there is more that can be done: By calculating a speed profile that reduces time spent behind a local traffic jam, fuel can be saved with little influence on trip time. To the best knowledge of the authors such an investigation is missing in the literature; its results if positive can cut the cost of running fleets of commercial heavy trucks and provide an eco-friendly option for passenger cars. The proposed approach relies mostly on software and information and needs minimal hardware investments. This impacts not only the high-tech vehicle of the future but the current fleet when equipped with add-on accessories such as a smart phone.

This chapter formulates such predictive planning of velocity and demonstrates its impact on fuel economy and emissions of passenger and commercial vehicles via several simulation case studies. We cast the problem as an optimal control problem with the goal of reducing velocity transients while also penalizing trip time. This optimal problem will be solved numerically by a two dimensional dynamic program. A key to this work is realizing the traffic-imposed constraints on the velocity which requires a spatiotemporal estimation of traffic velocity. In this we differentiate between inner-city driving and inter-city highway driving:

- In inner-city or urban driving, sensing/predicting the immediate state of traffic is probably critical and short preview horizons will be most effective.

At the same time short preview horizons call for more accurate traffic in-
formation than the averaged information provided by services like Google traffic. Therefore in an urban driving scenario we assume such information is available to us via vehicle-to-vehicle [71, 74] or infrastructure-to-vehicle [40] communication. The required communication technology is expected to be more widely deployed in the future.

- In Inter-city highway trips a longer traffic preview horizon along with coarser traffic information can be effective. For this type of planning knowledge of current state of traffic is not sufficient and a predictive traffic model may be used to estimate the evolving pattern of traffic down the road. The averaged information from Google, projected forward in time and space using a PDE model, could be utilized and will be explored in this work. An alternative is using historic traffic data to estimate the speed of traffic at each segment of a route at different times of a day.

Because a feedforward traffic estimator that predicts evolution of traffic along the vehicle route is a key part of this work, it is discussed in detail next in Section 4.2.1.1. This is followed by formulation of the optimal velocity planning problem in Section 4.2.2. The numerical dynamic programming solution process is described in Section 4.2.3. Section 4.2.4 describes the modeling process used for evaluation of fuel economy. Several simulation case studies are presented in Section 4.3 followed by Conclusions.
4.2 Methodology

4.2.1 Traffic Models

There is a vast body of work by traffic engineers, physicists, and computer scientists on traffic modeling. Excellent and thorough review of such models can be found in [37, 55, 56]. Microscopic traffic models use simple car-following rules to model procession and interaction of individual vehicles. These models are described by a system of ordinary differential equations or in the cellular automata approach by rules for advancing individual vehicle in a fine grid in discrete time steps. The drawback of microscopic models is the high computational load as the number of vehicles increases. Macroscopic models use the analogy of traffic flow to fluid flow and formulate spatiotemporal evolution of speed and traffic density using coupled PDEs. Aggregating a large number of vehicles into a continuum macroscopic model has the advantage of being much faster computationally. At the same time via macroscopic models it is possible to capture complex traffic phenomena such as a congestion wave, instabilities, and phase transitions of traffic flow [34]. Macroscopic models have also been used for calculating average travel times, fuel consumption and emission levels [61], for short-term forecasts of traffic flow for rerouting [35, 45], and for design of traffic flow control systems [33, 41]. More recently in [29], a gas-kinetic traffic model is used to simulate the influence of ramps on future velocity of a plug-in hybrid vehicle. Missing from the literature are methods that help plan the velocity of an individual vehicle to reduce the chance of its untimely arrival at a local traffic wave.

One way of forming a spatiotemporal traffic map is through these existing gas-
kinetic PDE models of traffic to predict its evolution. Specifically in inter-city highway driving use of a predictive model is important due to the long planning horizon.

In this section a progressive development of macroscopic traffic model description is provided based on the fundamental equations of fluid dynamics.

4.2.1.1 Derivation of Macroscopic traffic flow

A good starting point for derivation of macroscopic traffic flow would be the mathematical interconnection of three dependent traffic flow variables: $V(x,t)$ (velocity), $\rho(x,t)$ (density), and $q(x,t)$ (flow). For a flow of a fluid, the continuity equation could be expressed as:

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} q = 0 \quad (4.1)$$

where, $x$ (position) and $t$ (time) are independent variables and $q = \rho V$. This implies that the rate of traffic density is a function of traffic flow gradient and net input flow of vehicles to the considered road segment (right hand side of the equation 4.1). Equation (4.1) has two unknown variables. This, highlights the need of at least one more equation to relate two unknown variables $V$ and $\rho$. One suggestion is to assume that the velocity is a function of traffic density only. Based on this assumption, Lighthill and Whitham proposed a modified expression for continuity equation for traffic flow as:[49]:

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial \rho} q + \frac{\partial}{\partial x} \rho = 0 \quad (4.2)$$
This equation can be solved by using a linear change of variable method[30]. However, due to difference between amplitude of input and output waves in certain shock areas, a traffic density discontinuity appears in particular locations of the stream. Therefore, in practice, a second order viscosity term is added to equation 4.2 to play as a damping factor:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial \rho} q + \frac{\partial}{\partial x}\rho = \lambda \frac{\partial^2}{\partial x^2}\rho$$

(4.3)

Further development of the traffic flow equations deals with inclusion of conservation of momentum into the traffic flow characteristic equation. To this end, Payne [60] suggested the first continuum traffic flow model based on conservation of momentum in the flow of traffic as:

$$\frac{\partial}{\partial t}V + V \frac{\partial}{\partial x}V = \frac{[V^*(\rho) - V]}{\tau} - \left( \frac{c_0^2}{\rho} \right) \frac{\partial}{\partial x}\rho$$

(4.4)

Two major terms are introduced in the Payne model: Anticipation and Relaxation. Anticipation represents the deriver’s readiness in adjusting the speed on forthcoming spatial changes in traffic density. It is usually shown by:

$$\left( \frac{c_0^2}{\rho} \right) \frac{\partial}{\partial x}\rho$$

where $c_0$ is a constant and is typically chosen equal to 4.16 $m/s$[44]. Relaxation describes the tendency of the traffic flow to reach the desired speed. Commonly it is presented as:

$$\frac{[V^*(\rho) - V]}{\tau}$$

where $V^*(\rho)$ is a predefined function which is representative of desired velocity.
and $\tau$ is a constant and is on the order of $10^{-3}\text{sec}$[14]. A good equilibrium velocity function could be suggested as:

$$V^*(\rho) = V_f(1 - \frac{\rho}{\rho_{jam}}) \quad (4.5)$$

where, $\rho_{jam}$ represents the minimum density required to expect a traffic jam and $V_f$ represents the maximum vehicle’s speed when only one vehicle is on the road[25].

It is worth mentioning that the model proposed by Payne fails to explain a few issues:

i. Vehicles can only react to downstream traffic flow conditions.

ii. Personality of the vehicles remains unaffected by prevailing traffic conditions.

iii. In interaction between two vehicles the slow vehicle remains virtually unaffected by the faster vehicle.[36]

Since traffic is a compressible flow, a more general form of equation 4.4 has been suggested to modify the behavior of the model to a more realistic traffic flow[50, 42]:

$$\frac{D}{Dt}V = \frac{\partial}{\partial t}V + V \frac{\partial}{\partial x}V = \frac{[V^*(\rho) - V]}{\tau} - \left(\frac{C_0^2}{\rho}\right) \frac{\partial}{\partial x}\rho + \alpha \frac{\partial^2}{\partial x^2}V \quad (4.6)$$

where, $\frac{D}{Dt}$ is material derivative operator and $\alpha$ represent the viscosity of the flow. Increasing viscosity leads to a smoother velocity profile.

Summarizing the aforesaid modified continuity and modified momentum equation, a system of partial differential equations may render the solution for the
discussed flow of traffic:

\[
\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial \rho} q + \frac{\partial}{\partial x} \rho = \lambda \frac{\partial^2}{\partial x^2} \rho \\
\frac{\partial}{\partial t} V + V \frac{\partial}{\partial x} V = \left[ V^*(\rho) - V \right] - \left( \frac{c_0^2}{\rho} \right) \frac{\partial}{\partial x} \rho + \alpha \frac{\partial^2}{\partial x^2} V
\]  

(4.7)

Approximate boundary and initial conditions and the ramp inputs \( q(x,t) \) can be retrieved from real-time traffic information services such as Google traffic (Fig. 4.1) or streamed from local traffic information channels. The set of coupled PDEs will be solved using a finite-difference approach in real-time to determine traffic-imposed constraints in the future path of a vehicle. For inner-city driving, the \textit{immediate} traffic-imposed bounds on speed can be obtained via infrastructure-to-vehicle communication [40] or via ad-hoc [71, 74] vehicle-to-vehicle communication networks.

### 4.2.2 Optimal Control Problem Setup

The average traffic velocity \( v_t(x,t) \) estimated above will be an upper limit to the velocity each vehicle can assume at position \( x \) at time \( t \). The goal is to find a velocity profile that i) meets this traffic-imposed speed limit (and the speed limits of the road) and ii) lowers fuel use without compromising trip time. In other words, the slope of each feasible path is now upper-bounded by the spatiotemporally varying limit \( v_t(x,t) \) imposed by traffic. The problem of finding the optimal speed trajectory \( v(x,t) \) can be formalized as an optimal control problem which will be solved numerically. The cost function is:

\[
\min_{v(x,t)} J = \int_{x_i}^{x_f} \| L(v(x,t)) \|^2 dx \quad \frac{dx}{Q(v(x,t))}
\]  

(4.8)
subject to road speed limits \([v_{\min}, v_{\max}]\), traffic-imposed bound on speed \(v_t(x,t)\), and driver set speed \(v_{\text{set}}\):

\[
 v_{\min} \leq v(x,t) \leq \min(v_{\max}, v_t(x,t), v_{\text{set}})
\]  

(4.9)

and with acceleration and deceleration constraints imposed on \(\dot{v}(x,t)\). In (4.8), \(x_i\) and \(x_f\) are the origin and destination and \(\| \cdot \|_Q\) denotes the weighted 2-norm with the diagonal penalty weighting matrix \(Q\). Appropriate choice of the the cost functional \(L(v(x,t))\) is an open problem. For example the choice \(L(v) = \begin{bmatrix} \dot{m}_f & NO_x \end{bmatrix} (v(x,t) - v_{\text{set}}) \) penalizes the fuel rate \(\dot{m}_f\) and \(NO_x\) emissions, while also penalizing deviations from the driver set speed. The latter ensures travel time is not compromised. Another choice is to explicitly penalize trip time by selecting \(L(v) = \begin{bmatrix} \dot{m}_f & NO_x & 1 \end{bmatrix}^T\) which will result in trip time \(t_f - t_i\), appearing in the cost function. However inclusion of fuel rate and emissions in the cost function add to the complexity of this optimal control problem, because it requires inclusion of a detailed model of the vehicle powertrain. Therefore in this first work on the topic we use a simpler cost functional \(L(v) = \begin{bmatrix} v^2 & 1 \end{bmatrix}^T\) to penalize trip time and velocity transients \(\ddot{v}\) (accelerations and decelerations) which indirectly contribute to increase in fuel use. The other factor increasing the fuel use is idling at zero speed; penalizing the total trip time should cut unnecessary idling. The solution \(v(x,t)\) can then be issued as a reference to the low-level vehicle controller. Alternatively the velocity \(v(x,t)\) can be suggested to the driver as the eco-friendly speed.
4.2.3 Numerical Solution Via Dynamic Programming

The optimal control problem posed above cannot be solved analytically due to the spatiotemporally varying constraints imposed on its optimization variables along with several other pointwise-in-time constraints. In this work we solve this problem numerically using a dynamic program.

The vehicle kinematics is represented by the following two-state dynamic equations:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= u
\end{align*}
\]  

(4.10)

where \( x \) and \( v \) are position and velocity of the vehicle respectively and \( u \) is its acceleration which is selected as an input. Therefore \( L = \begin{bmatrix} u^2 & 1 \end{bmatrix}^T \) is set in the cost function (4.8). In addition to the velocity constraint (4.9), we impose the acceleration constraint on the input \( u \):

\[
a_{\text{min}} \leq u(x,t) \leq a_{\text{max}}
\]

(4.11)

where \( a_{\text{max}} \) is positive maximum allowable acceleration and \( a_{\text{min}} \) is negative maximum allowable deceleration.

The cost function can be written as follows:

\[
J = \int_{x_i}^{x_f} u^2 \frac{dx}{v(x,t)} + \phi(t_f,t_i)
\]

(4.12)

where \( \phi(t_f,t_i) \) is a terminal cost on trip time and proportional to \( t_f - t_i \) by a penalty weight.

The cost function in (4.12) is rewritten in discretized space calculated back-
Figure 4.2: Schematic of the DP grid and value function iteration.

ward:

\[ J = \sum_{n=0}^{N_{\text{max}}} u^2(x_n, t_{x_n}) \Delta x + \phi(t_f, t_i) \]  

(4.13)

We also define the cost function \( J_{X,N}(v, t) \) as the cost-to-go from position \( x_N \) to the final position which is a function of variables \( v \) and \( t \):

\[ J_{X,N}(v, t) = \sum_{n=N}^{N_{\text{max}}} \frac{u^2(x_n, t_{x_n})}{v(x_n, t_{x_n})} \Delta x + \phi(t_f, t_i) \]  

(4.14)

The optimal cost-to-go from position \( x_N \) to the final position will then be:

\[ J_{X,N}^*(v, t) = \min_u \sum_{n=N}^{N_{\text{max}}} \frac{u^2(x_n, t_{x_n})}{v(x_n, t_{x_n})} \Delta x + \phi(t_f, t_i) \]  

(4.15)

The optimal acceleration \( u \) can be found relying on Bellman’s optimality principle and by value function iterations backward-in-position as shown in Figure 4.2.
4.2. Given the optimal cost-to-go $J^*_X$ iterations over each node on the planar grid at $x_{N-1}$ will yield the optimal cost-to-go $J^*_X_{N-1}$:

$$J^*_X_{N-1}(v,t) = \min_u (J^*_X(v,t) + \frac{u^2(x_{N-1})}{v(x_{N-1})}) \tag{4.16}$$

and also determines the optimal control $u(x_{N-1})$. The process is continued backward-in-position until the sequence of optimal control inputs over the entire trip is determined.

4.2.4 Evaluation of Fuel Savings Potential with a Detailed Powertrain Model

In order to estimate the fuel economy of the vehicle when following the optimal velocity trajectory, a production vehicle is selected and its powertrain model is assembled from the extensive database of Powertrain System Analysis Toolkit (PSAT). PSAT developed by Argonne National Laboratory [46] is a powerful simulation tool for evaluating the fuel economy of conventional and hybrid vehicles when following a prescribed velocity cycle. Its physics-based component models combined with empirical maps obtained from production vehicles allow high-fidelity evaluation of fuel economy. Figure 4.3 shows schematics of a PSAT powertrain. This is a conventional (non-hybrid) powertrain with an automatic transmission. The models for torque converter, transmission, and vehicle dynamics are all very detailed and include several dynamic states and switching modes. Details such as electrical accessory loads, the starter, generator, etc. are not overlooked and modeled for simulation accuracy.

PSAT is a “forward-looking” causal simulation tool in which the vehicle speed
is determined by the combined influence of road loads and engine (or brake) torque at the wheels. The resulting velocity is compared to the prescribed desired velocity; the difference is fed to a driver model which in turn determines a torque demand. The torque demand is met by the engine (or brake) torques and the above simulation loop is repeated. The engine fuel rate is determined using an empirical engine map and as a function of engine speed and engine torque. The fuel rate is integrated over the whole cycle time to determine the amount of fuel used.

4.3 Simulation Results

In this section, we present the outcome of implementation of the discussed optimal control strategy to an arbitrary traffic pattern. Simulations are performed to determine the potential impact on fuel economy and trip time of a vehicle when future state of traffic is available either via predictive databases or through the model formulated in section 4.2.1. For the fuel economy evaluation, two different size of vehicles have been considered: a passenger vehicle and a mid-size truck. The passenger vehicle is an economy-sized car with 5-speed automatic
transmission, 1000 kg mass and 115 hp maximum power, and the midsize truck has 6-speed automatic transmission, 8500 kg mass and 500 hp maximum power. The fuel economy evaluation process is done in PSAT v6.2 where the detailed vehicle models are assembled. For the first part of this section, two pre-planned spatiotemporal traffic velocity profiles are us and are fed to the optimal control problem as the velocity constraint. These velocity profiles are obtained based on the expected distribution of traffic along a mid-load traffic road. Then the optimal control problem is run and the results demonstrate the level of fidelity of the proposed optimal controller. The second part of this section corresponds to a preliminary fuel economy evaluation and optimal control simulation results for the velocity planning based on macroscopic traffic flow models.

According to the shape of the traffic flow surface, four different set of cases have been investigated in this study. Then, in each case, simulations are run for a conventional vehicle and for a vehicle with a predictive cruise control system. Simulations are repeated for two different penalty weights on trip times to investigate the sensitivity of the results. In all simulations the maximum acceleration is assumed to be $2 \text{m/s}^2$ which is a conservative estimate of maximum acceleration capability of a midsize vehicle. Assuming braking on dry asphalt, the friction coefficient of $\mu_b = 0.69$ yields the maximum possible deceleration of $6.7 \text{m/s}^2$. However, to avoid aggressive driving, maximum braking deceleration of $3 \text{m/s}^2$ is taken into consideration.
4.3.1 Velocity Planning Based on Pre-defined Traffic Velocity Distribution

Case I: The first set of simulations belongs to a case in which a pre-defined traffic flow velocity distribution is suggested based on estimated forward wave of traffic congestion. In this case the appeared congestion in the road section moves forward along the road as a function of time. In order to allocate a specific average velocity to each location of the vehicle’s path, a section of a road is assumed with the length of 9km and for the duration of 1000 seconds. We discretized the length of the road to 18 elements in space and 9 time intervals. All simulations start with the initial velocity of 25km/h. The road speed limit is $v_{max} = 60km/h$ and the minimum allowable velocity in all of elements is $v_{min} = 0$. In order to clarify the role of normalized penalty weight of terminal time (Trip Time) for vehicles equipped with future traffic conditions, two simulations are run; the first and second simulations are run with normalized terminal time penalty of 0.5 and 50 respectively. Simulations are performed in the space and time limits of 9km and 1000 seconds.

Figure 4.4 shows the velocity trajectory of a conventional vehicle compared to the trajectories of PCC-equipped vehicles. As shown, the conventional vehicle reaches the target in 795 seconds while the PCC vehicles with normalized terminal time penalty of 0.5 and 50 pass the target in 845 and 855 seconds respectively. This observation indicates trip time loss of 7 and 8 percents compared to the velocity trajectory of a same conventional vehicle. All the speed and acceleration constraint are met during the above-mentioned simulations.

Case II: Another set of simulations is arranged with a different spatiotemporal distribution of velocity over the same time and space limit on the defined road.
Figure 4.4: Trajectories delivered for conventional vehicle (solid blue) versus PCC vehicles for forward congestion wave. Solid red and dashed blue line corresponds to the normalized time penalty of 0.5 and 50 respectively. In this case the appeared congestion in the road moves backward along the road as a function of time. Output and state constraints are same as the above simulations.

Figure 4.5 shows the velocity trajectory of a conventional vehicle compared to the trajectories of PCC-equipped vehicles. As shown, the conventional vehicle passes the target in 910 seconds while the PCC vehicles with normalized terminal time penalty of 0.5 and 50 pass the target in 966 and 970 seconds respectively. This observation indicates trip time loss of 6 and 6.5 percents compared to the velocity trajectory of a same conventional vehicle. All the speed and acceleration constraint are met during the above-mentioned simulations.
Figure 4.5: Trajectories delivered for conventional vehicle (solid blue) versus PCC vehicles for backward congestion wave. Solid red and dashed blue line corresponds to the normalized time penalty of 0.5 and 50 respectively.

Next, a fuel economy evaluation is done for each case in PSAT. A driver model follows these trajectories very closely. Table 4.1 summarizes the PSAT evaluation of fuel economy for each case. For the case I, the PCC-equipped vehicle can save up to 21 percent fuel over the conventional vehicle (56 mpg versus 46 mpg). In case II, the PCC-equipped vehicle can save up to 8 percent fuel as compared to the conventional vehicle (54 mpg versus 50 mpg).

Table 4.1: Fuel economy results in mile per gallon for conventional and PCC-equipped vehicles

<table>
<thead>
<tr>
<th></th>
<th>Conventional</th>
<th>PCC (Wt = 0.5)</th>
<th>PCC (Wt = 50)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>46</td>
<td>56</td>
<td>54</td>
</tr>
<tr>
<td>Case II</td>
<td>50</td>
<td>54</td>
<td>52</td>
</tr>
</tbody>
</table>
4.3.2 Velocity Planning Based on Macroscopic Traffic Flow Model

In this section two different traffic models are assumed based on the possible practical boundary conditions seen in the daily commutes. It should be noted that due to lack of information about the parameters of the models, surfaces generated as the traffic flow velocity distribution do not necessarily represent the actual traffic flow behavior. Hence, we consider this section as an ongoing research area and present our preliminary results. The main goal is to present the idea behind using a traffic flow model for better velocity planning and more research is needed to investigate the merits in realistic traffic scenarios.

In all traffic flow models in this section, the resolution of the velocity distribution surface is 900 by 450 which corresponds to 20 meters and 1 second along position and time vectors. Other parameters of the system of partial differential equations 4.7 are summarized in Table 4.2.

Table 4.2: Macroscopic traffic model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>110</td>
<td>veh/m.s</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.01</td>
<td>s</td>
</tr>
<tr>
<td>$c_0$</td>
<td>4.16</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_f$</td>
<td>20</td>
<td>m/s</td>
</tr>
<tr>
<td>$\rho_{jam}$</td>
<td>0.2</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.1</td>
<td>1/m</td>
</tr>
</tbody>
</table>

In order to generate a three dimensional spatiotemporal traffic flow surface, the macroscopic traffic flow governing equations (system of PDEs in 4.7) need to be provided with a set of boundary and initial conditions. Two different sets of initial and boundary conditions form two case studies $A$ and $B$. 

57
### 4.3.2.1 Traffic Model-Case A

The first traffic model was created based on semi-sinusoidal initial and boundary conditions. In this case the general form of the boundary conditions were assumed to be a first-order constant value functions as:

\[
\begin{align*}
V(x_{\text{left}}, t) &= A \\
\rho(x_{\text{left}}, t) &= \tilde{A} \\
V(x_{\text{right}}, t) &= A' \\
\rho(x_{\text{right}}, t) &= \tilde{A}'
\end{align*}
\]  

(4.17)

The constant functions of 4.17 imply that the flow of vehicles entering to and exiting from the boundaries of the simulated road segment has a constant velocity of \(Am/s\) and \(Bm/s\) respectively. It is worth mentioning that the density \(\rho\) and velocity \(V\) are assumed to counter balance each other. It means that the higher values of density corresponds to the lower values of velocity in all locations of the flow. For the initial condition, a sinusoidal function of position is assumed to present the state of the traffic over the road at the beginning of the simulation as:

\[
\begin{align*}
V(x, t_0) &= B \sin(\omega x) + B' \\
\rho(x, t_0) &= \tilde{B} \cos(\tilde{\omega} x) + \tilde{B}'
\end{align*}
\]  

(4.18)

The form of equations 4.18 is arbitrary; however it shows a reasonable arrangement of traffic through the road based on real life driving experience. The simulation parameters appeared in equations 4.17 and 4.18 are tabulated in Table 4.3. Figure 4.6 shows the surface indicating traffic velocity distribution for case study A.
Table 4.3: Initial and boundary conditions parameters value for Case A

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>18</td>
<td>m/s</td>
<td>$A$</td>
<td>0.074</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$A'$</td>
<td>12</td>
<td>m/s</td>
<td>$A'$</td>
<td>0.087</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$B$</td>
<td>6</td>
<td>m/s</td>
<td>$B$</td>
<td>0.053</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$B'$</td>
<td>12</td>
<td>m/s</td>
<td>$B'$</td>
<td>0.084</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$16\pi/9000$</td>
<td>rad/s</td>
<td>$\omega$</td>
<td>$16\pi/9000$</td>
<td>rad/m</td>
</tr>
</tbody>
</table>

Figure 4.6: Spatiotemporal traffic flow surface generated for case A.

Given the traffic flow information derived above, a velocity planning analysis is done via dynamic programming and is then compared to the case in which a conventional vehicle follows the traffic. The conventional vehicle is expected to move with the traffic stream. In other words, a conventional vehicle trajectory moves on the solution surface of Figures 4.6 and 4.6. In order to gain a better understanding of sensitivity of the cost function and the shape of the trajectories on the penalty factors, two different values of penalty factors have been
Figure 4.7: Trajectories delivered for conventional vehicle (in red) versus PCC vehicle for Case A. Dashed and solid blue curves represents time penalties of 0.5 and 50 respectively.

applied to the cost function and the results are shown in Figure 4.7. Table 4.4 summarizes the statistics of the resulting velocity profiles of conventional and PCC-equipped vehicles with the two different penalties on trip time.

Table 4.4: Drive-Cycle statistics for Case A: Conventional and PCC-equipped vehicles.

<table>
<thead>
<tr>
<th></th>
<th>Conventional</th>
<th>PCC ($W_t = 0.1$)</th>
<th>PCC ($W_t = 10$)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Max$ velocity</td>
<td>12.76</td>
<td>11.4</td>
<td>12.59</td>
<td>m/s</td>
</tr>
<tr>
<td>$Min$ Velocity</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>m/s</td>
</tr>
<tr>
<td>Trip time</td>
<td>702</td>
<td>748</td>
<td>732</td>
<td>s</td>
</tr>
</tbody>
</table>

The fuel economy analysis were run for a passenger and a midsize heavy vehicle. The results are reflected in the Table 4.5. For case A, the fuel economy evaluation results in up to 12 percent fuel saving for a passenger vehicle and 8 percent fuel saving for the heavy vehicle when the traffic information is predic-
tively utilized.

Table 4.5: Fuel economy results in miles per gallon for passenger and heavy vehicle - Case A.

<table>
<thead>
<tr>
<th>Fuel economy</th>
<th>Conventional</th>
<th>PCC ($W_i = 0.5$)</th>
<th>PCC ($W_i = 50$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger Vehicle</td>
<td>49.26</td>
<td>53.05</td>
<td>55.00</td>
</tr>
<tr>
<td>Heavy Vehicle</td>
<td>7.38</td>
<td>8.05</td>
<td>8.00</td>
</tr>
</tbody>
</table>

4.3.2.2 Traffic Model-Case B

The second traffic model is created based on fully-sinusoidal initial and boundary conditions. In this case the general form of the boundary conditions is assumed to be a sinusoidal function of time as:

\[
V(x_{\text{left}}, t) = C \sin(\epsilon t) + D \\
\rho(x_{\text{left}}, t) = \dot{C} \cos(\epsilon t) + \dot{D}
\]

\[
V(x_{\text{right}}, t) = C' \sin(\epsilon' t) + D' \\
\rho(x_{\text{right}}, t) = \dot{C}' \cos(\epsilon' t) + \dot{D}'
\]

The sinusoidal boundary conditions imply that the flow of vehicles on the boundaries of the simulation road section has a peak. Constants and coefficients brought into equations 4.19 constitute the shape of the input and output flow of the road section. For the initial condition, similar to case A a sinusoidal function of position with the same values of parameters indicated in 4.3 is assumed to present the state of the traffic over the road at the beginning of the simulation time. The parameters appeared in equations 4.19 are tabulated in Table 4.6. Figure 4.8 shows the surface indicating traffic velocity distribution for case study B.

Given the traffic flow information derived for case B, a velocity planning anal-
Table 4.6: Initial and boundary conditions parameters value for Case B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>6</td>
<td>m/s</td>
<td>$C$</td>
<td>0.06</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$D$</td>
<td>12</td>
<td>m/s</td>
<td>$D$</td>
<td>0.08</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$C'$</td>
<td>0.5</td>
<td>m/s</td>
<td>$C'$</td>
<td>0.01</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$D'$</td>
<td>12</td>
<td>m/s</td>
<td>$D'$</td>
<td>0.07</td>
<td>vehicle/m</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>1/300</td>
<td>rad/s</td>
<td>$\varepsilon'$</td>
<td>1/300</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$16\pi/9000$</td>
<td>rad/s</td>
<td>$\omega'$</td>
<td>$16\pi/9000$</td>
<td>rad/m</td>
</tr>
</tbody>
</table>

Figure 4.8: Spatiotemporal traffic flow surface generated for case $B$.

ysis is done via dynamic programming. Similar comparisons of case $A$ are performed for case $B$. The conventional vehicle’s trajectory is supposed to move on the solution surface of Figure 4.8. Two different values of normalized penalty factors mentioned in case $A$ have been applied to the cost function and the results are shown in Figure 4.9. Table 4.7 summarizes the statistics of the resulting velocity profiles of conventional and PCC-equipped vehicles with
two different penalties on trip time.

Figure 4.9: Trajectories delivered for conventional vehicle (in red) versus PCC vehicle for Case B. Dashed and solid blue curves represents time penalties of 0.5 and 50 respectively.

Table 4.7: Drive-Cycle statistics for Case B: Conventional and PCC-equipped vehicles.

<table>
<thead>
<tr>
<th></th>
<th>Conventional</th>
<th>PCC ($W_t = 0.5$)</th>
<th>PCC ($W_t = 50$)</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MaxVelocity</td>
<td>13.56</td>
<td>11.4</td>
<td>13.50</td>
<td>m/s</td>
</tr>
<tr>
<td>MinVelocity</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>m/s</td>
</tr>
<tr>
<td>Triptime</td>
<td>635</td>
<td>675</td>
<td>655</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 4.8: Fuel economy results in miles per gallon for passenger and heavy vehicle - Case B.

<table>
<thead>
<tr>
<th>Fuel economy</th>
<th>Conventional</th>
<th>PCC ($W_t = 0.5$)</th>
<th>PCC ($W_t = 50$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger Vehicle</td>
<td>43.55</td>
<td>54.7</td>
<td>44.17</td>
</tr>
<tr>
<td>Heavy Vehicle</td>
<td>7.12</td>
<td>8.05</td>
<td>7.77</td>
</tr>
</tbody>
</table>

The fuel economy analysis were run for two passenger and typical heavy vehicles studied in case B. The results are reflected in the Table 4.8. The fuel
economy evaluation shows that a passenger vehicle driving in traffic flow modeled in case B saves 25 percent fuel with predictive velocity planning. This improvement is 13 percent for the heavy vehicle.

4.4 Conclusion

In this chapter we proposed the idea of predictively planning a vehicle’s speed for reducing the velocity transients in upcoming traffic waves in order to reduce its fuel consumption. It was assumed that future state of traffic in space and time can be estimated to be used as an spatiotemporal upper bound to how fast individual vehicles can travel. One possible method for estimation of velocity that was proposed in this paper is using real-time traffic information as initial conditions to a macroscopic traffic model represented by a set of coupled nonlinear partial differential equations.

An optimal control problem was cast with the estimated traffic flow surface as the constraint for the velocity and with the target of improving fuel economy. The validity of the approach was investigated in four different simulation case studies; in two cases spatiotemporal distribution of traffic speed was assumed and in two other cases the PDE traffic model was solved to generate the traffic surface. The fuel evaluation simulations showed up to 25 percent improvement of fuel economy was possible in some scenarios when the future state of traffic was known. This improvement was achieved at the cost of 6 to 8 percent increase in trip time.

It should be noted that the ideas presented in this chapter are still preliminary and more work is needed to understand the validity of the claims here in more realistic traffic scenarios. As the future work, we believe that application of
macroscopic traffic flow as a reference for an optimal control problem is a good tool to determine the best departure time and in some particular cases to generate a fuel/trip-time optimal trajectory. Since the computational load of the proposed algorithm is high, investigating running part of the computations off-vehicle and on a backend cluster than communicates with the vehicle via cellular network is another direction envisioned for future work.
Chapter 5

Conclusion

This manuscript focused on use of traffic signal and traffic flow preview for predictively planning a fuel minimal velocity trajectory for a “connected” vehicle that has access to realtime traffic information services. To the best of our knowledge predictive use of traffic signal and traffic flow for reducing fuel use and emissions has not be addressed in the past (at least systematically). Therefore a main contribution of this thesis may be showing the opportunities that exist for fuel saving by mere use of information and software and with minimal hardware investments. This is also a timely topic because traffic information is now available through various providers and can be integrated into the vehicle navigation system or into its add-on accessories. In this work we also present novel solutions based on a mix of advanced optimal control and optimization techniques with predictive macroscopic traffic models and logical rules that can be considered analytical contributions of this work.

Positive simulation results in the first part of this work promise that signal-to-vehicle communication technology may enable reduction of fuel consumption, greenhouse gas emissions, and trip time of future vehicles by predictive veloc-
ity planning. By this, we hope to encourage further research and innovation towards more intelligent traffic intersection control systems. Of course, any gain from the proposed PCC methodology depends on timing and phasing of traffic lights and the distance between them and the vehicle parameters. A detailed statistical analysis using Monte Carlo simulations is one possible way of determining average gains with PCC which is a good direction for future simulation analysis. Also business models for getting access to real-time signal information and effective communication protocols for broadcasting them is an open area for future work.

From an analytical perspective, formulation of the trip optimization in this approach in a model predictive control framework is novel and lends itself well to many traffic-imposed hard constraints.

In the second portion of this thesis, we proposed the idea of predictively planning a vehicle’s speed for reducing the velocity transients in upcoming traffic waves in order to reduce its fuel consumption. It was assumed that future state of traffic in space and time can be estimated to be used as an spatiotemporal upper bound to how fast individual vehicles can travel. One possible method for estimation of velocity that was proposed in this thesis is using real-time traffic information as initial conditions to a macroscopic traffic model represented by a set of coupled nonlinear partial differential equations.

An optimal control problem was cast with the estimated traffic flow surface as the constraint for the velocity and with the target of improving fuel economy. The validity of the approach was investigated in four different simulation case studies; in two cases spatiotemporal distribution of traffic speed was assumed and in two other cases the PDE traffic model was solved to generate the traffic
surface. The fuel evaluation simulations showed up to 25 percent improvement of fuel economy was possible in some scenarios when the future state of traffic was known. This improvement was achieved at the cost of 6 to 8 percent increase in trip time.

It should be noted that the ideas presented in the second part of this thesis are still preliminary and more work is needed to understand the validity of the claims here in more realistic traffic scenarios. As the future work, we believe that application of macroscopic traffic flow as a reference for an optimal control problem is a good tool to determine the best departure time and in some particular cases to generate a fuel/trip-time optimal trajectory. Since the computational load of the proposed algorithm is high, investigating running part of the computations off-vehicle and on a backend cluster than communicates with the vehicle via cellular network is another direction envisioned for future work.
Appendices
Appendix A

Model Predictive Control

A.1 Introduction

In this section we introduce the control approach that is used in chapter 3 to solve lower level optimal controller module. We will start with definition of model predictive control framework and its application. Then, a general formulation of such control problems will be explained briefly.

A.1.1 Definition

Model predictive control (MPC) or model-based predictive control or receding horizon control is a powerful control framework which is receiving continues interest from academia and industry.

In general, model predictive control refers to a strategy in control engineering in which current control action is derived by solving a finite horizon open-loop optimal control problem on-line and at each sampling step, using the current state of the plant as the initial state [53]. Given the current states as initial states, a
model predictive controller has an internal model to predict the response of the plant over a future prediction horizon. Then the controller is able to calculate the appropriate control move for optimizing the pre-defined desired criteria in the overlooked prediction horizon. Dealing with a tracking problem rather than a regulation problem and a set of constraints to be satisfied through determination of control moves, makes MPC a strong control approach candidate to apply to this problem.

A.1.2 Basic Structure

Figure A.1 shows a schematic structure of a model predictive controller. Taking all constraints applied to the dynamics and outputs of the system, the first control input is calculated in the optimizer to minimize the cost function over the prediction horizon. Then, the following control input is applied to the system and the optimization process is repeated in a receding horizon manner.

A.1.3 Formulation

In order to reduce the computational time, the system model and constraints in this thesis are considered as a discrete-time linear time invariant system. Several notations have been suggested in literature for MPC formulation. In this section we introduce a popular notation for a simple model predictive control problem. Consider a discrete-time linear time invariant system regulation problem to the origin as:

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k + Gw_k \\
y_k &= Cx_k + \xi_k
\end{align*}
\]  

(A.1)
Figure A.1: Schematic of a Model Predictive Controller

Where, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, and $u \in \mathbb{R}^l$ are the process states to be controlled, measured process output, and process inputs (manipulated variables) vectors respectively. $w_k$ and $\xi_k$ represent the state measured disturbance and measurement noise respectively. The initial state $x_0$ is assumed to be Gaussian with non-zero mean [62].

The state-space system of equations A.1 is supposed to fulfill the constraints:

$$y_{\text{min}} \leq y(k) \leq y_{\text{max}}$$
$$\Delta y \leq M$$
$$u_{\text{min}} \leq u(k) \leq u_{\text{max}}$$
$$\Delta u \leq N$$

(A.2)

Where, $\Delta$ represents the variation of a particular variable over one step.
Suppose that a full measurement of states of A.1 is available at the current step $k$. Then objective is to minimize the cost function $J$ which penalizes the squared deviation of input and state from a reference trajectory. A basic formulation of the cost function would be defined as:

$$J(k) = \sum_{i=H_w}^{H_p} \|z(k+i|k) - r(k+i|k)\|^2_{W(i)} + \sum_{i=0}^{H_u-1} \|\Delta u(k+i|k)\|^2_{R(i)}$$  \hspace{1cm} (A.3)$$

where, $H_p$ is the prediction horizon, $H_u$ is the control horizon, $r$ is the reference trajectory, $W$ and $R$ are the weighting vectors for states and control moves and "$|$" denotes the step to which the state or control output is allocated. Selecting appropriate weighting vectors, The cost function A.3 penalizes deviation of the controlled outputs from the reference desired trajectory $r(k+i|k)$. The main aim of the controller is to select the control signal $u(k)$ that can minimize the performance index A.3 in the region confined by the set of state, output, and input constraints. The control input set is the optimum solution only for the prediction horizon $H_p$. Then, the first element of calculated control move will be applied to the plant among the elements included in derived control move set. Tuning the parameters of the cost function such as prediction horizon, control horizon and wights will adjust the behavior of the systems. However, there is a tradeoff between the computation time and the performance of the controller.

### A.1.4 Conclusion

In this chapter we have presented a formal description of model predictive control. It could be stated that the model predictive control would be a proper ap-
proach in dealing with linear and nonlinear systems with complex constraints with the goal of tracking fed time varying trajectories.
Bibliography


[29] Q. Gong, Y. Li, and Z.-R. Peng. Trip based near globally optimal power management of plug-in hybrid electric vehicles using gas-kinetic traffic


