Modeling the Galactic 511 keV Positron Annihilation Emission, Production, and Propagation

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Modeling the Galactic 511 keV Positron Annihilation Emission, Production and Propagation

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Science
Physics

by
Bethany R. Johns
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Accepted by:
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Abstract

The Galactic 511 keV positron annihilation emission has a strong bulge component to the emission and a weak disk component. The bulge emission is about 1.4–6 times as the disk emission. The bulge emission is defined to be a diffuse emission centered at the Galactic center and extends to about a FWHM of 8° in Galactic longitude and latitude. An asymmetry has also been observed in the disk emission, where the negative longitudes are brighter than the positive longitudes by a factor of 1.8.

This research examines the morphology of the Galactic 511 keV positron annihilation emission by modeling each stage of the positron from production, propagation, to finally annihilation. The production of positrons is modeled using the most probable positron production sources, radioactive isotopes, $^{26}$Al, $^{44}$Ti, and $^{56}$Ni. After the positron is produced the model accounts for the energy loss and annihilation mechanisms a positron from a radioactive isotope may undergo as it propagates through a warm neutral and ionized Galactic medium. The propagation of positrons is modeled in three different scenarios: positrons are trapped in the turbulent Galactic magnetic field and annihilate where they are produce, positrons escape the turbulent field and travel along the Galactic poloidal field, and positrons travel along the poloidal field but slow and annihilate in an extra density of gas placed at the Galactic center.

Results show the model of a turbulent field and propagation along a poloidal field can explain the flux from positron annihilation emission and the observed asymmetry using positrons from radioactive isotopes. However, the bulge emission cannot be explained by positron propagation from the disk to the bulge. In the transport model along the Galactic
poloidal field there is no emission from the bulge region because there is no gas for positrons to slow and annihilate with. The model that includes an extra density of gas in the Galactic center shows that even with gas to slow and annihilate with the positrons are produced at an energy such that they may slow in the extra gas, but do not necessarily annihilate there.

Therefore, positron propagation is not sufficient to explain the Galactic 511 keV positron annihilation emission morphology. The conclusion is that there must be a central positron production source or population of sources that produces positrons at low energy, such that they will be confined to and annihilate in the bulge region.
Dedication

This work is dedicated to my husband, D. Adam Johns.

He was always there for me through all the ups, downs, twists and turns. He has made an untold number of sacrifices in order to help me during my research. The first five years of our marriage has been accompanied by me being in graduate school. Without his love and support I could never have finished. I have only hope to give back to him all that he has given me.

Thank you. I have always, do always, and will always love you.
I would like to acknowledge my advisor Dr. Mark Leising for his encouragement and support. I would also like to thank those of my dissertation committee Dr. Dieter H. Hartmann, Dr. Jeremy R. King, Dr. Fivos Drymiotis for their patience and support.

My family also played an integral role in helping me keep my sanity through this research. My mother and father have given generous amounts of love and support. Whenever I doubted myself and my ability to continue they were always there to encourage me and give advice. They were also there for me on the good days to tell me how proud they are of me. I also thank my sister for her support and encouragement. Conversations with my nieces via the web relieved a bit of the mothering instinct since I chose to pursue my PhD rather than start a family. I also thank my grandparents for their support in my education ever since I was a child. They have always encouraged me to learn and have always been supportive. I am thankful every day to have wonderful, close, and loving family, who may not fully understand the physics, but understands how important this experience was for me and was with me every stop along the way.

I would also like to thank my other cohorts in the Physics and Astronomy Department, Adria Updike, Eric Bubar, and Ginger Bryngelson. We have helped each other numerous times on various projects, problems, and programming techniques. I wish them all the best in their future careers in astronomy and astrophysics.
# Table of Contents

Title Page ................................................................. i
Abstract ................................................................. ii
Dedication ............................................................... iv
Acknowledgments ....................................................... v
List of Tables ............................................................ viii
List of Figures ........................................................... ix

1 Introduction ........................................................... 1
   1.1 Positron Annihilation Emission Morphology .................... 2
   1.2 Positronium Annihilation Fraction .............................. 7
   1.3 Annihilation in Gaseous Phases of Interstellar Medium ........ 9
   1.4 History ................................................................ 11
   1.5 Summary .......................................................... 17

2 Positron Production ..................................................... 20
   2.1 Positron Production Sources .................................... 20
   2.2 Assessment of Positron Production Sources .................... 42
   2.3 Modeling Positron Production Sources .......................... 46
   2.4 Summary .......................................................... 53

3 Positron Propagation .................................................... 54
   3.1 Propagation of Positrons ........................................ 54
   3.2 Galactic Magnetic Field ......................................... 57
   3.3 Modeling the Galactic Magnetic Field and Positron Propagation 64
   3.4 Summary .......................................................... 68

4 Positron Annihilation ................................................... 69
   4.1 Positron Annihilation Process .................................. 69
   4.2 ISM Gas Phases .................................................. 82
   4.3 Modeling Gas Phases ............................................. 86
   4.4 Summary .......................................................... 88
5 Results & Discussion ......................................................... 93
  5.1 Turbulent Model ....................................................... 93
  5.2 Transport Model ....................................................... 95
  5.3 Extra Gas Model ..................................................... 98
  5.4 Flux Measurements .................................................. 99
  5.5 The Positronium Fraction ........................................ 101
  5.6 Asymmetry ............................................................ 102

6 Conclusion ................................................................. 103

Bibliography ................................................................. 109
List of Tables

1.1 Two models that describe the Galactic positron annihilation emission data, a bulge-thick disk model and halo-thin disk model. ......................... 7
1.2 Timeline of history of positron annihilation research. ..................... 16

2.1 Variables for the calculations of positron annihilation rate and references. . 28
2.2 Results of positron annihilation rate calculations. .......................... 29
2.3 The values of the shared energy between the positron and neutrino, $\epsilon_0 m_e c^2$, and the most probable energy value for each isotope. ......................... 34
2.4 Assessment of the Parameters of Positron Production Sources. ............ 43

4.1 Charge exchange energy thresholds for certain atoms. ..................... 79
4.2 Parameters for the Radial Density Function ................................. 88

5.1 Results: Flux in units of ph cm$^{-2}$ s$^{-1}$ with a distance to the Galactic center of 8.5 kpc. ................................................................. 99
5.2 Positronium fraction for each model: turbulent, transport, and extra gas. . 101
5.3 Asymmetry ................................................................. 102
## List of Figures

1.1 Image of 511 kev positron annihilation emission morphology from Knödlseder et al. (2005) .......................................................... 3
1.2 Model image reconstruction of the Galactic 511 keV positron annihilation. Aitoff projection, Galactic center is in the middle, and contours are intensity levels of $10^{-4}, 10^{-3}, 10^{-2}$ ph cm$^{-2}$ s$^{-1}$ sr$^{-1}$. ........................................... 6
1.3 The different ways for a positron to annihilate, either directly or through Ps formation. Each process produces a specific spectrum of photons. .............. 8
1.4 Modeling the spectrum of the positron annihilation emission from Jean et al. (2006) .................................................................. 10
1.5 The profile line shapes of positron annihilation in each phase of the ISM. a) cold, molecular hydrogen medium; b) cold, atomic hydrogen medium; c) warm, neutral medium; d) warm, ionized medium; e) hot medium. (Gues-soum et al. 2005) ............................................................. 12
1.6 The global spectrum constructed from the positron annihilation in each phase of the ISM. ................................................................. 13
1.7 The map of the flux of the Galactic 511 keV positron annihilation emission from Purcell et al. [1997]. Contours are exponentially spaced. .............. 18
1.8 The map of the Galactic 511 keV positron annihilation emission from Milne [2006]. ............................................................. 19

2.1 Schematic of the relevant decays of $^{26}$Al ........................................... 21
2.2 The COMPTEL 1.809 MeV photon map from Plüschke [2001]. Note that the emission is from the disk of the Galaxy. ........................................... 22
2.3 Schematic of the relevant decays of $^{44}$Ti ........................................... 23
2.4 Simplified decay scheme of $^{56}$Ni from Nadyozhin [1994]. ..................... 25
2.5 Simplified decay scheme of $^{56}$Co from Nadyozhin [1994]. ..................... 26
2.6 default .......................................................................................... 31
2.7 The positron $\beta$ decay energy spectrum for $^{26}$Al (red), $^{56}$Ni (i.e. $^{56}$Co, yellow), $^{44}$Ti (i.e. $^{56}$Sc, green), and $^{22}$Na (blue). .................................................. 35
2.8 The positron energy spectrum from energetic $p - p$ collisions Murphy et al. (1987) ................................................................. 36
2.9 All-sky map of cosmic rays above 100 MeV from the decay of neutral pions $\pi^0$. The bright section along the Galactic plane indicates that the positrons from cosmic ray interactions are not solely responsible for the bulge positrons. Credit: EGRET Team .................................................. 37
2.10 The distribution of LMXB in the Galaxy. The color is scaled with X-ray flux, the bright white yellow color are the brightest LMXB. This figure was created by using data from Liu et al. (2001).

2.11 The energy spectrum of the inner Galaxy. Data points come from ISGRI, SPI, COMPTEL and EGRET. The red dashed and black line is the modeled flux from positrons with energies of 100, 30, 10, and 3 MeV (from top). (Sizun et al. 2006)

2.12 Location of the spiral arms used in determining the free electron density. The sun is located at (0,8.5 kpc).

2.13 The distribution of $^{26}$Al in the xy plane.

2.14 The all sky map of the intensity of $^{26}$Al using model from Taylor and Cordes 1993. Intensity scale is not zero but maximum at $2.89\times10^{-3}$ photons/cm$^2$/s.st.

3.1 The hydrogen fine and hyperfine splitting that creates the 21 cm line.

3.2 The 2 dimensional model of the Galactic magnetic field on the axis of x and z.

3.3 The 3D model of the Galactic magnetic field.

3.4 Components of velocity vector of particle with velocity not parallel with magnetic field. In this example the magnetic field is along the x-axis. The equations assume the magnetic field is aligned along the z-axis.

3.5 The effect of the pitch angle on the distance traveled for a positron starting at an initial energy of 1 MeV.

4.1 Diagram for the process and mechanisms of positron annihilation.

4.2 Ionization energy loss per column depth for a positron going through a uniform gas density of $n_e=1$ cm$^{-3}$ and $n_H=1$ cm$^{-3}$. The black line is IE energy loss, the blue line is Coulomb, and the red line is the total.

4.3 Ionization energy loss per column depth for a positron through a uniform gas density of 2 particles of hydrogen per cm$^3$. The black line is without a magnetic field and the magenta line is when the positron gyrates around the field line at a pitch angle of $\theta = \pi/3$ as it travels.

4.4 Diagram of (a) direct annihilation in center of mass frame and (b) in-flight which produces shifted photons greater than 511 keV.

4.5 The energy spectrum of photons from the annihilation of ortho-positronium.

4.6 The cross sections in units of $10^{-16}$ cm$^2$ for each annihilation mechanism and the discrete energy loss of ionization and excitation. The green line is the Maxwell Boltzman distribution at T=8000 K in a warm medium to show at what energy a positron will start to thermalize. After thermalization the important annihilation mechanisms are CE, RR, DAH, and DA.

4.7 Radial density distribution for HI (red) and H$_2$ (green).

4.8 Neutral hydrogen in the xy plane and in galactic coordinates.

4.9 Molecular hydrogen in the xy plane and in galactic coordinates.

4.10 Neutral and molecular hydrogen combination in the xy plane and in galactic coordinates.
5.1 The map of the 511 keV annihilation emission, where positrons are assumed to be trapped due to the turbulent Galactic magnetic field and annihilate where produced. Intensity$_{\text{max}}$ = 1.2 × 10$^{-3}$ ph cm$^{-2}$ s$^{-1}$ sr$^{-1}$.

5.2 The map of positron annihilation emission, where positrons are assumed to be transported along the Galactic poloidal field going through the Galactic center and annihilate with the gas. Intensity$_{\text{max}}$ = 1.0 × 10$^{-3}$ ph cm$^{-2}$ s$^{-1}$ sr$^{-1}$.

5.3 The map of the 511 keV annihilation emission with an artificial density of gas in the Galactic center. Intensity$_{\text{max}}$ = 2.5 × 10$^{-3}$ ph cm$^{-2}$ s$^{-1}$ sr$^{-1}$.
Chapter 1

Introduction

The Galactic 511 keV positron annihilation emission has a unique morphology. There are two basic components, a bulge and disk component. The bulge component of the emission is at least three times as bright as the disk component. The morphology that is observed is counterintuitive to what is known about how and where positrons are produced.

Many of the sources that have the largest rates of positron production exist in the Galactic disk and the gas that positrons annihilate with is mostly in the disk. If most of the positrons are produced in the disk and assuming that positrons annihilate close to where they are produced, then we would expect most of the positron annihilation emission to be in the disk.

In order to explain the bright emission from the bulge, positrons either have to be produced, propagate to, or annihilate in the bulge at a greater rate than in the disk. Removing the assumption that positrons annihilate where produced, another explanation, is that positrons could propagate from the disk to the bulge along Galactic magnetic field lines.

This research explores the production, propagation and annihilation of positrons in the Galaxy to explain the observed Galactic 511 keV positron annihilation emission morphology.
1.1 Positron Annihilation Emission Morphology

The International Gamma-Ray Astrophysical Laboratory (INTEGRAL) instrument Spectrometer on-board INTEGRAL (SPI) has mapped the Galactic positron annihilation emission. SPI has the capability for imaging and high resolution spectroscopy with a spatial resolution of 3° (FWHM) and spectral energy resolution of ~2.1 keV (FWHM, at 0.5 MeV).

Both Knödlseder et al. (2005) and Weidenspointner et al. (2008b) have modeled the morphology of the positron annihilation emission using SPI data. Knödlseder et al. (2005) created a model independent image map of the 511 keV line using the Richardson-Lucy (RL) algorithm, which deconvolves the observed data from the instrument response over time to obtain an estimate of the true image of the emission. The observed image is the true image convolved with the instrument response.

\[ O = T \times I \] (1.1)

where, \( O \) is the observed intensity, \( T \) is the true intensity, and \( I \) is the response of the instrument. The instrument response includes the instrument background. The count rate of SPI is the sum of the instrument response and the signal from the Galactic sources. The instrument background consists of a flat continuum and a broad intrument 511 keV feature from positrons annihilating within the telescope. To invert this equation to solve for the true image you must use Fourier transforms. The convolution of two functions corresponds to the multiplication of their Fourier transforms.

\[ F(O) = F(T \times I) = F(T) \times F(I) \] (1.2)

Solve for the true image by doing the inverse Fourier transform.

\[ T = F^{-1} \left[ \frac{F(O)}{F(I)} \right] \] (1.3)

The RL iterative algorithm is used to reduce the instrumental broadening. The \((n + 1)^{th}\)
approximation is related to the instrument response and the $n^{th}$ approximation.

$$T_{n+1} = T_n \frac{O}{T_n \times I}$$  \hfill (1.4)

The observed data is usually the first approximation. Image deconvolution is easily affected by image noise and exposure biases. Figure 1.1 is the all sky image of the 511 keV line emission using the RL deconvolution algorithm. The contours from the center outwards are intensity levels of $10^{-2}$, $10^{-3}$, $10^{-4}$ photons cm$^{-2}$ s$^{-1}$ sr$^{-1}$. The emission appears to be centered at the Galactic center and symmetric. Latitude and longitude profiles indicate the extent of the emission to be $\sim 10^\circ$ in radius from the Galactic center.

Weidenspointner et al. (2008b) fit bulge and disk models to the most recent four years of SPI data to obtain a positron annihilation rate for the spheroidal and disk components of the emission. The flux in units of photons cm$^{-2}$ s$^{-1}$ in Galactic coordinates $l, b$ is found by integrating the volume emissivity $\epsilon(r, l, b)$ in units of photons cm$^{-3}$ s$^{-1}$ over a volume element $dV$ divided by $4\pi r^2$ assuming the photons are emitted isotropically.

$$F(l, b) = \int \frac{\epsilon(r, l, b)}{4\pi r^2} dV$$  \hfill (1.5)
In spherical coordinates \( dV = r^2 \cos(b) dldbd\Omega = r^2 drd\Omega \). Intensity is measured in units of photons cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\). The all sky 511 keV intensity is proportional to the volume emissivity integrated along the line of sight \( dr \).

\[
I(l, b) = \frac{dF(l, b)}{d\Omega} = \frac{1}{d\Omega} \int \epsilon(r, l, b) \frac{r^2}{4\pi r^2} drd\Omega
\]

The 511 keV photon luminosity \( L_{\text{photon}} \) is calculated by integrating \( \epsilon(r, l, b) \) over the Galactic volume.

\[
L_{\text{photon}} = \int \epsilon(r, l, b) r^2 drd\Omega
\]

The 511 keV photon luminosity does not equal the positron luminosity. Positron annihilation does not always produce two photons of equal energy. If the positron annihilates from the ground state of positronium with the statistical ratio of total spin states it will produce two 511 keV photons only a quarter of the time in the parapositronium state.

\[
L_{\text{photon}} = (2(1 - f_{Ps}) + \frac{1}{4} 2f_{Ps})L_{e^+} = (2 - 1.5f_{Ps})L_{e^+}
\]

where, \( f_{Ps} \) is the fraction of positrons that annihilate via positronium and \( L_{e^+} \) is the rate of positron annihilation luminosity.

Two descriptions of spatial distribution fit the data equally well. There is a bulge-thick disk model and a halo-thin disk model. No unique description exists because the spheroid and disk can have faint extensions contributing to their total gamma-ray emissivities. The thick disk model represents an old disk population and the thin disk represents a young disk population.

The halo model component explains the bright peaked emission from the Galactic central region and an extended emission which is roughly spherically symmetric about
the Galactic center. The halo model is used to help determine the scale of the emission. The bulge model component describes the bulge region proper using a narrow and wide Gaussian distribution with FWHM of $3^\circ$ and $11^\circ$, respectively. The halo model has a larger flux, luminosity and annihilation rate than the bulge model because of the presence of a flat extended tail in the halo distribution. INTEGRAL data does not detect this faint emission tail and therefore both halo and bulge models are viable solutions to the emission morphology.

Models for the disk and the halo are from Robin et al. (2003). Each model (disk, bulge, halo) describes a population by a star formation rate history, initial mass function, age or age range, set of evolutionary tracks, kinematics, and metallicity characteristics. The halo model is distinct from the bulge model in construction. Their stellar formation scenario may be similar, but the halo spheroid population is older and more metal poor. The bulge is old, but the metallicity distribution is closer to the disk metallicity. Model values come from Weidenspointner et al. (2008a).

In the bulge-thick disk model the sky distribution is approximated by a bulge and single disk component. The bulge is described as a superposition of two Gaussians centered at the Galactic center parameterized by their FWHM. The disk was modeled using a young stellar population distribution parameterized by the scale lengths of the disk and central hole, $R_d$ and $R_h$, and the axis ratio, $\epsilon$. Parameters are adjusted to fit the data. Best fit values are FWHM$_n$=3.4 and FWHM$_w$=11. For the disk, $R_d$=3.8 kpc, $R_h$=3.2 kpc and $\epsilon$=0.09. The disk density is model by,

$$\rho_D(x, y, z) = \rho_0 \left( \exp\left(-\frac{a}{R_0}\right)^2 - \exp\left(-\frac{a}{R_i}\right)^2 - \exp\left(\frac{z}{z_0}\right) \right)$$

$$a_c^2 = x^2 + y^2 + \left(\frac{z}{\epsilon}\right)^2$$

where the vertical exponential scale height is $z_0$=70 pc, the disk scale radius is $R_0=5$ kpc, the inner disk truncation radius is $R_i=3$ kpc, and $\epsilon=0.014$ is the fixed disk axis ratio. In the latitude direction the best best fitting disk model has a FWHM$_b$ of $7^\circ$ which
agrees with models of old stellar populations older than 3 Gy with FWHM_b of about 8° (Weidenspointner et al. 2008a).

In the Halo-thin disk model the sky distribution is approximated by a halo component and a single disk component. The halo component peaks in the Galactic center and has fainter emission extending beyond the bulge region. The disk component is modeled like the disk in the bulge-thick disk model with slightly different best fit parameters. The halo density is modeled as,

\[ \rho_H(R, z) = \frac{\rho_0}{d_0} \left( \frac{a_c}{R_{\odot}} \right)^n \]

\[ a_c^2 = x^2 + y^2 + (z/\epsilon)^2 \]

where the best fit values for the halo core radius, \( a_c \), slope of density profile \( n \), and axis ratio \( \epsilon \) are \( a_c=0.21 \) kpc, \( n=2.7 \), and \( \epsilon=0.8 \). For the disk the best fit values are \( R_d=4.0 \) kpc, \( R_h=3.5 \) kpc, and \( \epsilon=0.06 \).

![Figure 1.2: Model image reconstruction of the Galactic 511 keV positron annihilation. Aitoff projection, Galactic center is in the middle, and contours are intensity levels of \( 10^{-4}, 10^{-3}, 10^{-2} \) ph cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\).](image)

The flux of 511 keV photons \( f_{511} \), the luminosity of the 511 keV photons from positron annihilation \( L_{511} \), and the rate of the positron annihilation \( L_p \) is found in Table 1.1. The positron annihilation rate in the spheroid and disk components is in the range of 1.2–3.1 s\(^{-1}\) and 8.1–5.2 s\(^{-1}\). The bulge to disk ratio \( B/D \) is from 1.4–6. The bulge is at least 1.4 times as bright as the disk and at most 6 times as bright.
Table 1.1: Two models that describe the Galactic positron annihilation emission data, a bulge-thick disk model and halo-thin disk model.

<table>
<thead>
<tr>
<th></th>
<th>Flux (10^{-4} \text{ cm}^{-2} \text{ s}^{-1})</th>
<th>(L_{511}) (10^{42} \text{ s}^{-1})</th>
<th>Rate (10^{42} \text{ s}^{-1})</th>
</tr>
</thead>
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<tr>
<td><strong>Bulge-Thick Disk</strong></td>
<td></td>
<td></td>
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<tr>
<td>Bulge</td>
<td>7.5</td>
<td>6.4</td>
<td>11.5</td>
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<tr>
<td>Thick Disk</td>
<td>9.4</td>
<td>4.5</td>
<td>8.1</td>
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<tr>
<td>B/D</td>
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<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Halo-Thin Disk</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>17.4</td>
<td>31.3</td>
</tr>
<tr>
<td>Thin Disk</td>
<td>7.3</td>
<td>2.9</td>
<td>5.2</td>
</tr>
<tr>
<td>H/D</td>
<td>2.9</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

1.2 Positronium Annihilation Fraction

Positrons annihilate one of two ways. One way is by direct annihilation where the positron annihilates with an electron creating two photons in opposite directions of equal energy, in the center of mass frame, totaling the mass of the system before annihilation, which is 1022 keV. A positron and electron have equal mass of 511 keV, so the two photons will each have energy of 511 keV.

The other way for positrons to annihilate is through the formation of positronium (Ps). Positronium is an atom-like state of an electron and positron. They have captured each other before they annihilate. The energy and direction of the resultant photons from positronium annihilation depends on the state of positronium. There are two states: para- and ortho-positronium. The para-positronium state has a short lifetime of \(\tau = 0.125\, \text{ns}\) and the spins of the electron and positron are anti-parallel. This is called the singlet state and forms a quarter of the time. In this state two photons are created at equal energy but opposite direction similar to direct annihilation. The ortho-positronium state has a longer lifetime of \(\tau = 140\, \text{ns}\), which forms three quarters of the time called the triplet state, where the electron and positron spins are parallel. When they annihilate three photons are emitted. The sum of the energies from the resultant photons is 1.022 Mev, but each
photon is no greater than 511 keV. The photon energy spectrum from the annihilation of ortho-positronium is a continuum zero to 511 keV. Figure 1.3 shows a schematic of the annihilation of Ps and the resulting energy spectrum of photons.

![Figure 1.3: The different ways for a positron to annihilate, either directly or through Ps formation. Each process produces a specific spectrum of photons.](image)

The fraction of positrons that annihilate via Ps can be calculated from the intensity of the 511 keV line and the continuum. The 511 keV photons come from direct annihilation and the annihilation via para-positronium and the intensity is related to the positronium fraction, $f_{Ps}$.

$$I_{2\gamma} = 2(1 - f_{Ps}) + \frac{1}{4}2f_{Ps}$$

(1.15)

The intensity from the annihilation of ortho-positronium is proportional to the positronium fraction and the three quarters of the time Ps is formed in the ortho-positronium state.
Ortho-positronium produces three photons.

\[ I_{3\gamma} = \frac{3}{4} 3 f_{Ps} \]  

(1.16)

Solve for \( f_{Ps} \) by,

\[ \frac{I_{2\gamma}}{I_{3\gamma}} = \frac{2(1 - f_{Ps}) + \frac{1}{2} f_{Ps}}{9/4 f_{Ps}} \]  

(1.17)

\[ f_{Ps} = \frac{2}{3/2 + 9/4 \frac{I_{2\gamma}}{I_{3\gamma}}} \]  

(1.18)

Figure 1.4 shows the 511 keV line profile. The spectrum was modeled using a narrow and broad Gaussian. When fitted with a narrow line there is a significant excess of counts in the wings of the line. This broad line is due to annihilation of the para-positronium state formed in-flight and is about a third of the line flux. The continuum is also modeled and labeled OrthoPs. The narrow and broad flux is \( I_{2\gamma} \) and the continuum is \( I_{3\gamma} \). Jean et al. (2006) (and references therein) found the ratio of the intensities to be \( \frac{I_{3\gamma}}{I_{2\gamma}} = 3.95 \pm 0.32 \). This corresponds to a \( f_{Ps} = 0.967 \pm 0.022 \). Of all the positrons annihilated in the Galaxy \( \sim 97\% \) annihilate through the formation of positronium.

There are two positron annihilation mechanisms that form Ps, charge exchange and radiative recombination. Therefore, these annihilation mechanisms will be important in the modeling. The annihilation mechanisms will be discussed in Chapter 4.

### 1.3 Annihilation in Gaseous Phases of Interstellar Medium

Jean et al. (2006) also modeled the line spectrum from the shape of the annihilation in different phases of the interstellar medium, called the ISM Model. In the model the different phases of the ISM are characterized by their temperature and composition. A molecular medium in the model has a temperature of 10 K. A cold medium has a temperature of 80 K and is composed only of neutral hydrogen. A warm medium can have neutral hydrogen and free electrons at a temperature of 8000 K. A hot medium in the model has only free
Figure 1.4: Modeling the spectrum of the positron annihilation emission from Jean et al. (2006)
electrons and ions at a temperature of 10^6 K.

Positron annihilation in each gas phase of the ISM has a distinct profile shape. Figure 1.5 shows the line profile shapes in different phases of the ISM (Guessoum et al. 2005).

Jean et al. (2006) combined these profiles into a global spectrum, $S_{ISM}$. $S_{ISM}$ is constructed from the relative contributions of each phase, $f_i$.

$$S_{ISM} = I_{e^+e^-} \sum_{i=1}^{5} f_i S_i + A_c \left( \frac{E}{511 \text{ keV}} \right)^s$$

where $S_i$ is the normalized spectral distribution in units of keV$^{-1}$ of annihilation photons in phase $i$=[molecular, cold, warm neutral, warm ionized, hot]. $I_{e^+e^-}$ is the annihilation flux in units of photons cm$^{-2}$ s$^{-1}$. Figure 1.6 shows the best fit of the global spectrum to the data, which has contributions from only the warm neutral and warm ionized medium. The contribution from the warm neutral medium $f_{WNM} = 0.49$ and warm ionized medium $f_{WIM} = 0.51$. All other contributions are negligible. Therefore, neutral hydrogen and free electrons which comprise the warm medium are the most important gas phases to model.

1.4 History

Johnson and Haymes from the Rice University Gamma Ray Astronomy Group are credited with the discovery of the 511 keV positron annihilation emission from the galactic center (Johnson & Haymes 1973). Their balloon borne gamma ray detector was launched in 1970 and measured a spectral feature at 476 ± 24 keV. They speculated that the line feature was the 511 keV emission from electron-positron annihilation even though their 1σ uncertainty in the measurement did not include 511 keV. Later measurements by Leventhal et al. (1978) established the positron annihilation hypothesis by confirming that the feature was centered on 511 keV.

Motivation for detecting cosmic gamma-rays came from the prediction of cosmic processes that produce gamma radiation. Feenberg & Primakoff (1948) concluded that
Figure 1.5: The profile line shapes of positron annihilation in each phase of the ISM. a) cold, molecular hydrogen medium; b) cold, atomic hydrogen medium; c) warm, neutral medium; d) warm, ionized medium; e) hot medium. (Guessoum et al. 2005)
gamma rays are produced from collisions between thermal photons and primary cosmic ray particles. Hayakawa (1952) predicted diffuse gamma-ray emission from the decay of neutral pions, $\pi^0 \rightarrow 2\gamma$, which are produced by interactions of cosmic rays with interstellar matter. Hutchinson (1952) predicted gamma-rays from cosmic ray bremsstrahlung. Gamma rays were also predicted to come from astrophysical objects by means of nuclear processes (Burbidge et al. 1957; Morrison 1958).

The first detection of the 511 keV line emission by Johnson and Haymes was succeeded by the launch of other detectors designed to measure the gamma ray universe. A few notable observatories are summarized and Table 1.2 gives the timeline of events.

The High Energy Astrophysical Observatory (HEAO-3), a satellite with germanium detectors, launched in 1979. It made two measurements of the 511 keV flux in the Galactic center region. In 1979 it measured a narrow line flux of $1.85 \times 10^{-3}$ photons cm$^{-2}$ s$^{-1}$ centered at 510.9 keV with width 3.13 keV. Then in 1980 it measured a flux of $0.65 \times 10^{-3}$ photons cm$^{-2}$ s$^{-1}$. Riegler et al. (1981) noted that the flux was variable suggesting...
that the emission was from a point source.

The Gamma Ray Spectrometer (GRS) on the Solar Maximum Mission (SMM) consisted of an array of seven NaI detectors launched in 1980. Share et al. (1988a) measured a flux from the central Galaxy of $2 \times 10^{-3}$ photons cm$^{-2}$ s$^{-1}$. They concluded that the annihilation emission was a diffuse distribution and that a variable source at the Galactic center was unlikely.

The Gamma Ray Imaging Spectrometer (GRIS) was a high-resolution balloon borne germanium detector flown twice in 1988. Gehrels et al. (1991) measured a flux of $1.2 \times 10^{-3}$ photons cm$^{-2}$ s$^{-1}$ narrowly concentrated toward the Galactic center. The positron annihilation line was spectroscopically resolved with a width of 3 keV with the centroid at 511 keV.

The Transient Gamma-Ray Spectrometer (TGRS) was launched in 1994. It measured a narrow line flux with centroid at 510.8 keV and width of 2.6 keV and flux from the Galactic center region of $1.36 \times 10^{-3}$ photons cm$^{-2}$ s$^{-1}$. The data suggested a point source but could not rule out a diffuse distributed source. There was no evidence of variability (Teegarden et al. 1996; Harris et al. 1998).

The Oriented Scintillation Spectrometer Experiment (OSSE) was the first to map both the morphology of the 511 keV emission and the positronium continuum fluxes (Purcell et al. 1993). The satellite was launched in 1991 and measured a symmetric non-variable distribution sharply peaked at the Galactic center. The data were consistent with a two component spatial distribution model of a Galactic bulge and symmetric Galactic disk producing emission up to $l = \pm 20^\circ$.

Purcell et al. (1997) modeled the 511 keV distribution by separate components. The Galactic plane was modeled by a flat top ridge distribution with a Gaussian profile in latitude. The Galactic bulge was modeled by a two dimensional circular Gaussian. Using only OSSE data the flux from the bulge was $3.5 \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$ and from the disk $8.9 \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$. The combination of OSSE, TGRS, and SMM data was also modeled with results of the flux from the bulge $3.3 \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$ and from
the disk $11.5 \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$. See Figure 1.7 for the map of the 511 keV emission from Purcell et al. (1997).

Milne et al. (2000) also used two separate components when modeling the 511 keV positron annihilation emission using combined OSSE, TGRS, and SMM data, but used more OSSE data since the publication of Purcell et al. (1997). The Galactic bulge component was modeled by either a Galactic center point source, a Gaussian, or a projection of a truncated $R^{1/4}$ radial function (Milne et al. 2000). The 28 disk models are in order of increasing disk thickness of Gaussian latitude profile FWHM ranging from $2^\circ - 22^\circ$. Each bulge and disk component model combination were tested. The Galactic center source model did not fit the data, thus showing that an extended bulge model is optimal. The $R^{1/4}$ bulge model fits the data better when paired with a thin disk and the Gaussian with a thick disk. The $R^{1/4}$ favors the thin disk because the function possesses “wings” that extend beyond the Gaussian model and beyond the Galactic center. The Gaussian model requires a thick disk to explain the data because the Gaussian model is not as extended as $R^{1/4}$. Therefore, since the $R^{1/4}$ model is bulge-dominated the bulge to disk flux ratio will be larger than ratio of the disk-dominated Gaussian bulge model. The bulge to disk ratio of the flux ranged from 0.2 – 3.3. One can see that the B/D ratio is dependent on how the Galactic component emission is modeled (Milne et al. 2000). The Galactic 511 keV emission modeled by Milne (2006) can be seen in Figure 1.8

SPI, the spectrometer aboard INTEGRAL (launched in 2002) was designed to study the 511 keV line emission and conducted a more detailed survey that indicated significant emission toward the Galactic bulge and a weak disk. The INTEGRAL data was also modeled by Knödlseder et al. (2005) using separate Galactic components. The bulge component was tested using triaxial stellar bar models where the parameters of bar radius and scale height are allowed to vary. The disk emission was too faint to conclude anything about its morphology (Figure 1.2). For the disk component two models were tested, a young and old stellar population model with scale heights of 70 pc and 200 pc, respectively.
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948</td>
<td>Feenburg and Primakoff calculated collisions between thermal photons and primary cosmic ray particles.</td>
</tr>
<tr>
<td>1952</td>
<td>Hayakawa researched the production of pions from cosmic ray spallation. Neutral pions decay into two 6.5MeV gamma rays.</td>
</tr>
<tr>
<td>1957</td>
<td>Burbidge, Burbidge, Fowler, and Hoyle predicted gamma-rays from nuclear processes.</td>
</tr>
<tr>
<td>1958</td>
<td>Morrison identified gamma-ray emission processes and possible sources for gamma-rays.</td>
</tr>
<tr>
<td>1970</td>
<td>Johnson and Haymes discovered the 511 keV line from the Galactic center.</td>
</tr>
<tr>
<td>1978</td>
<td>Leventhal et al. Confirmed the 511 keV is from positron annihilation.</td>
</tr>
<tr>
<td>1981-1990</td>
<td>GRS [SMM], Gamma Ray Spectrometer on the Solar Maximum Mission Share et al. (1988b)</td>
</tr>
<tr>
<td>1991</td>
<td>GRIS, Gehrels et al. (1991)</td>
</tr>
<tr>
<td>1994</td>
<td>TGRS [WIND], Teegarden et al. (1996)</td>
</tr>
<tr>
<td>1991-2000</td>
<td>OSSE [CGRO], Oriented Scintillation Spectrometer Experiment on the Compton Gamma Ray Observatory Purcell et al. (1993)</td>
</tr>
<tr>
<td>2002</td>
<td>SPI [INTEGRAL], Spectrometer on INTEGRAL, the International Gamma Ray Astrophysical Laboratory Knödlseder et al. (2005)</td>
</tr>
</tbody>
</table>

Table 1.2: Timeline of history of positron annihilation research.
1.5 Summary

The Galactic 511 keV positron annihilation emission has been well measured by INTEGRAL. From this data we can model the morphology of the emission and the line shape. Modeling the morphology gives the intensity, flux, luminosity of the emission, as well as the positron annihilation rate. The two models that equally fit the data, the bulge-thick and halo-thin disk, give a bulge to disk ratio $B/D = 1.4 - 6$. The bulge is $\sim 3$ times brighter on average than the disk.

The fraction of positrons that annihilate through the formation of positronium and the gas phase in which the positrons annihilate can be modeled from the positron annihilation spectrum. The positronium fraction is $f_{Ps} = 97\%$. Positrons annihilate in an ISM that is 49\% neutral hydrogen and 51\% ionized. Positronium is formed by the annihilation mechanisms charge exchange and radiative recombination. Charge exchange is when a positron takes an electron from a neutral hydrogen atom and forms positronium. Radiative recombination is the formation of positronium from capturing a free electron. Therefore, the neutral hydrogen and free electron gas phases and the positronium annihilation mechanisms of charge exchange and radiative recombination are the most important aspects to model.

This research will produce physically based maps of the Galactic 511 keV positron annihilation emission by modeling what is known about positron production and positron annihilation in neutral and ionized medium using the most probable annihilation mechanisms charge exchange and radiative recombination. The model also tracks the annihilation of para- and ortho-positronium, thus the positronium fraction can also be calculated. The mechanism of positron propagation is unknown. Different mechanisms of propagation are modeled.
Figure 1.7: The map of the flux of the Galactic 511 keV positron annihilation emission from Purcell et al. [1997]. Contours are exponentially spaced.
Figure 1.8: The map of the Galactic 511 keV positron annihilation emission from Milne [2006].
Chapter 2

Positron Production

2.1 Positron Production Sources

2.1.1 Radioactive Isotopes

Radioactive isotopes produce positrons through $\beta^+$-decay. $\beta^+$-decay is when a proton decays into a neutron emitting a positron, neutrino, and a photon. Prominate radioactive isotopes in the Galaxy are $^{26}$Al, $^{44}$Ti, $^{56}$Ni, and $^{22}$Na. The observations of the $^{26}$Al $\gamma$-ray photon show that $^{26}$Al clusters along spiral arms, which indicate that $^{26}$Al comes from massive stars. $^{26}$Al is produced hydrostatically during H-burning and ejected in Wolf-Rayet stellar winds or produced explosively and ejected when the star explodes via the mechanism of core collapse supernovae (CCSN). $^{44}$Ti is also created in CCSN and thought to be exclusively formed in supernovae events. However, the Cas A supernovae remnant is the only place where $^{44}$Ti has been detected by its $\gamma$-ray lines. Therefore, $^{44}$Ti production is inferred from the solar abundance of its decay product $^{44}$Ca. The $^{44}$Ti decay product $^{44}$Sc is what $\beta^+$-decays into $^{44}$Ca. $^{56}$Ni is created in a thermonuclear type Ia supernovae (SNIa). A SNIa is a binary system of a white dwarf that accretes matter from a main sequence or red giant companion star. The accreted mass from the companion causes the carbon-oxygen white dwarf to exceed the gravitational pressure, previously balance by the electron degeneracy pressure, igniting fusion and proceeding until the star explodes. The
$^{56}$Ni decay product $^{56}$Co is what $\beta^+$-decays. $^{22}$Na is created in oxygen-neon novae. Novae result from explosive hydrogen burning on the surfaces of white dwarves from accreting mass from the companion star. The ejected mass from the explosion is enriched with the material of the white dwarf.

### 2.1.1.1 $^{26}$Al

![Diagram of $^{26}$Al decay](image)

Figure 2.1: Schematic of the relevant decays of $^{26}$Al

The decay of the radioactive isotope $^{26}$Al can partially explain the 511 keV emission from the disk. $^{26}$Al has a lifetime of $\tau_{Al} = 1.03 \times 10^6$ years. The ground state of $^{26}$Al will decay by 82\% positron emission (15\% electron capture) to the first excited state of $^{26}$Mg. After 0.49 ps de-excitation will produce a gamma ray photon of 1.8 MeV.

$$^{26}Al \rightarrow ^{26}Mg + \beta^+ + \gamma(1.8\,MeV) \quad (2.1)$$

The rate of 1.8 MeV photon production $\dot{N}_{1809}$ can be calculated using the total Galactic mass of $^{26}$Al, $M_{Al} = 1.7\,M_\odot = 3.38 \times 10^{33}$ g (Knödlseder 1997), the atomic mass of $^{26}$Al is $A = 26$ g mol$^{-1}$, and the rate of radioactive decay $\lambda = 9.67 \times 10^{-7}$ per year, 82\% by positron emission. The rate is,

$$\dot{N}_{1809} = 0.82 \frac{M_{Al}}{A} N_A \lambda = 2 \times 10^{42} \, e^+ \, s^{-1} \quad (2.2)$$
where \( N_A \) is Avogadro’s number, \( 6.022 \times 10^{23} \) particles per mol and \( \lambda = \ln(2)/\tau_{\text{Al}} \). The positron rate from the decay of \(^{26}\text{Al}\) account for 25–40\% of the \( 0.5 - 0.8 \times 10^{43} \text{ e}^+ \text{ s}^{-1} \) from the disk.

When \(^{26}\text{Al}\) decays it produces a positron, neutrino, and a photon at 1.809 MeV. The 1.809 MeV emission has been observed in the Galaxy and is well measured (Figure 2.2. Figure 2.2 for the map of the 1.809 MeV survey Plüschke et al. (2001). The COMPTEL data

![Figure 2.2: The COMPTEL 1.809 MeV photon map from Plüschke [2001]. Note that the emission is from the disk of the Galaxy.](image)

show the sites of positron production for \(^{26}\text{Al}\). Notice that the emission is mainly in the disk with no bulge component. Therefore, if positrons annihilate where they are produced then the positron annihilation emission should look like the 1.809 MeV emission. However, the positron annihilation emission has a strong bulge and almost no disk component. This makes a strong case for positron propagation which is discussed in Chapter 3.

\(^{26}\text{Al}\) is produced by massive stars. If massive stars are a major source of ionizing photons, then \(^{26}\text{Al}\) can be traced by free-free radio emission. Free-free emission is produced by free electrons scattering off of ions without being captured. The ionizing photons from
massive stars ionize the region creating free electrons and ions. To create a 3D model of $^{26}\text{Al}$, I used the model of free electrons from Taylor & Cordes (1993) as a basis.

### 2.1.1.2 $^{44}\text{Ti}$

![Figure 2.3: Schematic of the relevant decays of $^{44}\text{Ti}$](image)

Another percentage of the disk positron rate can be explained by the decay of $^{44}\text{Ti}$. The ground state of $^{44}\text{Ti}$ will decay into an excited state of $^{44}\text{Sc}$. Once it reaches the ground state it will decay 94% by positron emission.

\[
^{44}\text{Ti} \rightarrow ^{44}\text{Sc}^* \\
^{44}\text{Sc} \rightarrow ^{44}\text{Ca}^* + e^+ + \gamma + \nu_e \quad 95\% \, \beta^+ \quad (2.3)
\]
The Galactic $^{44}$Ti production is inferred from the solar ratio $(^{44}\text{Ca}/^{56}\text{Fe})_{\odot} = 1.2 \times 10^{-3}$ Leising & Share (1994). $^{44}$Ti production rate $\dot{M}_{Ti}$,

$$\dot{M}_{Ti} = \left(\frac{^{44}\text{Ca}}{^{56}\text{Fe}}\right)_{\odot} \left(R_{SNIa}Y_{56}^{SNI} + R_{CC}Y_{56}^{CC}\right)$$  \hspace{1cm} (2.4) \\
= 6.4 \times 10^{-6} \text{M}_{\odot} \text{y}^{-1} \hspace{1cm} (2.5) \\
= 1.3 \times 10^{28} \text{g y}^{-1} \hspace{1cm} (2.6)$$

Total Galactic supernovae rates are $R_{SNIa} = 0.5 - 0.6$ and $R_{CC} = 2.13 - 2.33$ SN per century (Mannucci et al. 2005). The $^{56}\text{Fe}$ yield in SNIa $Y_{56}^{SNIa} = 0.5 \text{M}_{\odot}$ and in CCSN $Y_{56}^{CC} = 0.1 \text{M}_{\odot}$ (Knödlseder 2000). The $^{44}$Ti positron production rate $\dot{N}_{44}$ is

$$\dot{N}_{44} = 0.95 \left(\frac{\dot{M}_{Ti}}{A}\right) N_{A} \hspace{1cm} (2.7)$$

$A = 44 \text{ g/mol}$, $N_{A}$ is Avogadro's number, $6.022 \times 10^{23}$ particles per mole. Together, both $^{26}\text{Al}$ and $^{44}\text{Ti}$ can account for the positron annihilation rate of the disk if the positrons produced in the disk annihilate in the disk. The distribution is similar to $^{26}\text{Al}$ because $^{44}\text{Ti}$ also is made in massive stars.

2.1.1.3 $^{56}\text{Ni}$

SNIa create large numbers of positrons from the decay of $^{56}\text{Ni}$, which is the most abundant radioisotope produced in SNIa. SNIa synthesize about 0.1 to 1.0 $\text{M}_{\odot}$ of $^{56}\text{Ni}$ Hoyle & Fowler (1960). $^{56}\text{Ni}$ has a half life of $t_{1/2} = 6.1$ days.

$$^{56}\text{Ni} \rightarrow ^{56}\text{Co}^* + \nu_e$$  \hspace{1cm} (2.8) \\
$$^{56}\text{Co}^* \rightarrow ^{56}\text{Co} + \gamma$$
Figure 2.4: Simplified decay scheme of $^{56}\text{Ni}$ from Nadyozhin [1994].
Figure 2.5: Simplified decay scheme of $^{56}$Co from Nadyozhin [1994].
The daughter isotope $^{56}$Co will decay into $^{56}$Fe via electron capture or positron emission, i.e. $\beta$-decay. $^{56}$Co has a half life of $t = 77.2$ days and a life time of $\tau_{Co} = 111.3$ days. $^{56}$Co decays 19% of the time by $\beta$ decay Nadyozhin (1994). See Figure 2.4 and 2.5 for a simplified decay scheme.

$$
\begin{align*}
^{56}Co & \rightarrow ^{56}Fe + e^+ + \gamma + \nu_e & 19\% \beta^+ \\
^{56}Co & \rightarrow ^{56}Fe + \gamma + \nu_e & 81\% \text{ EC}
\end{align*}
$$

If a SNIa produces an average amount of $\sim 0.5 \, M_\odot$ of $^{56}$Ni then a yield of $2 \times 10^{54}$ positrons will be produced per SNIa event. Where $M$ is the mass of the radioisotope in grams, $A$ is the atomic mass of 56 grams per mole, $N_A$ is Avogadro's number, $6.022 \times 10^{23}$ particles per mole, and $f$ is the percent of $\beta$ decay which is 19%.

$$
\text{Yield} = f \left( \frac{M_{Ni}}{A} \right) N_A \\
= (0.19) \left( \frac{0.5 M_\odot}{56 \, \text{g/mol}} \right) \left( \frac{2 \times 10^{33} \, \text{g}}{1M_\odot} \right) 6.022 \times 10^{23} \, \text{particles/mol} \\
= 2 \times 10^{54} \, \text{ particles}
$$

Of the $2 \times 10^{54}$ positrons that are created from the expected yield of $\sim 0.5 M_\odot$ of $^{56}$Co only the fraction of $8 \times 10^{52}$ positrons possibly escape the ejecta. (Milne et al. 1999).

The general equation for the positron production rate $\dot{N}_{Ni}$ is the product of the observed supernovae rate, the luminosity of the particular component of the galaxy times, and the number of positrons that escape a supernova. For example, to determine the positron annihilation rate for the bulge $\dot{N}_{Ni,Bulge}$, the supernovae rate per unit K luminosity (inferred from elliptical galaxies) must be multiplied by the luminosity of the bulge in the corresponding observational band (the K-band), and the escape yield. The positron annihilation rate for the bulge $\dot{N}_{Ni,Bulge}$ is

$$
\dot{N}_{Ni,Bulge}^K = (SNR_{E0}^K) \left( \frac{L_{Bulge}^K}{SN} \right) \left( \frac{e^+}{S^N} \right)
$$

27
Where \( SNR_{E0}^B \) is the supernovae rate of old stellar populations per unit B luminosity inferred from ellipticals in the B-band, \( L_{Bulge}^B \) is the luminosity of the bulge in the B-band, and \( \frac{e^+}{SN} \) is the number of positrons that escape from supernovae.

Calculate the \(^{56}\text{Ni}\) positron production rate using the values from Table 2.1. Table 2.2 has the results.

\[
\dot{N}_{Ni,Bulge} = \left( \frac{0.035SN}{10^2y10^{10}L_K^\odot} \right) \left( 1.2 \times 10^{10}L_K^\odot \right) \left( \frac{8 \times 10^{52}e^+}{SN} \right) \left( \frac{1y}{31556926s} \right)
\]

\[= 1.1 \times 10^{42} \frac{e^+}{s} \] (2.12)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Component</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bulge</td>
<td></td>
</tr>
<tr>
<td>( L_K^{Bulge} )</td>
<td>1.2 \times 10^{10}L_K^\odot</td>
<td>Reshetnikov (2000)</td>
</tr>
<tr>
<td>( SNR_{E0}^K )</td>
<td>0.035 \pm 0.013 SNuK</td>
<td>Mannucci et al. (2005)</td>
</tr>
<tr>
<td></td>
<td>Disk</td>
<td></td>
</tr>
<tr>
<td>( L_K^{Disk} )</td>
<td>5.5 \times 10^{10}L_K^\odot</td>
<td>Reshetnikov (2000)</td>
</tr>
<tr>
<td>( SNR_S^K )</td>
<td>0.088 \pm 0.035 SNuK</td>
<td>Mannucci et al. (2005)</td>
</tr>
</tbody>
</table>

Table 2.1: Variables for the calculations of positron annihilation rate and references.

The SNIa rate for each component of the Galaxy, bulge and disk, is needed to calculate the rate positrons annihilate to produce the 511 keV emission. The supernovae rate of the Milky Way is poorly known. What has commonly been done is to infer the supernovae rate of the Galaxy from observations of supernovae in other nearby galaxies similar to the Galaxy’s Hubble type. However, for this research, the bulge and disk supernovae rates must be differentiated. The assumption is made that the bulge of the Galaxy is similar to elliptical galaxies and the disk can be interpreted as a spiral galaxy. Supernovae rates can
be found for both elliptical and spiral galaxies to calculate the positron annihilation rates in our Galaxy.

Supernovae rates for a galaxy should depend on its number of stars or its mass. The supernovae rate has been observed to depend on the luminosity of the parent galaxy (Tammann 1970). Luminosity is related to the amount of stellar mass, so the luminosity of a galaxy is a measure of its mass. An abundance of stars means more mass and more luminosity. The more stars the higher the probability to have systems that have SNIa progenitors. So, the assumption is

\[ \text{Luminosity} \propto \text{Mass of stars} \propto \text{Number of stars} \propto \text{Number of SNIa progenitors} \propto \text{SNR} \quad (2.13) \]

<table>
<thead>
<tr>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{N}_{Ni,Bulge}$</td>
</tr>
<tr>
<td>$1.1 \pm 0.4 \times 10^{42} \text{e}^+ s^{-1}$</td>
</tr>
<tr>
<td>$\dot{N}_{Ni,Disk}$</td>
</tr>
<tr>
<td>$1.2 \pm 0.5 \times 10^{43} \text{e}^+ s^{-1}$</td>
</tr>
<tr>
<td>$\dot{N}_{Ni,Total}$</td>
</tr>
<tr>
<td>$1.2 \pm 0.5 \times 10^{43} \text{e}^+ s^{-1}$</td>
</tr>
<tr>
<td>$B/D$</td>
</tr>
<tr>
<td>$0.17 \pm 0.09$</td>
</tr>
</tbody>
</table>

Table 2.2: Results of positron annihilation rate calculations.

The supernovae rates are usually normalized to the galaxy luminosity in certain bands such as the B and K bands (centered at .44 and 2.2 micrometers, respectively) and in units per century. A common unit for supernovae rates is supernovae per century per $10^{10} L_\odot$ in the B-band, $SNu = \left( \frac{SN}{10^{10}L_\odot 10^2 y} \right)$. Rates can also be normalized to the K-band, the number of supernovae per century per $10^{10}$ K-band solar luminosity ($SNuK$). For reference the solar luminosity in the B-band is $L_{\odot}^B = 5.2 \times 10^{32} \text{erg}/s$ (O’Sullivan et al. 2001) and in the K-band $L_{\odot}^K = 5.67 \times 10^{31} \text{erg}/s$ (Mannucci et al. 2005).
Historically, the B-band luminosity was used to measure the mass of galaxies. The B flux is a combination of emission from old stars and emission from young stellar populations. However, the B emission is absorbed by dust (Mannucci et al. 2005). Emission whether from a young or old stellar population will contribute a different amount of B flux, either more or less, along the Hubble Sequence. The presence of young stellar populations in late Hubble type galaxies will supply a significant amount of B luminosity. Therefore, B luminosity is a poor tracer of mass along the Hubble sequence and is unspecific as to stellar age.

As a stellar population evolves, the young massive stars will die leaving behind an older, less massive, i.e. cooler, stellar population. Star formation will continue to make massive and less massive stars. However, since less massive stars have much longer lifetimes than massive stars, the less massive stars will accumulate while the number of massive stars will stay more constant. Therefore, to get a general measure of a galaxy’s mass one uses the K-band whose near infrared wavelength measures the light of less massive stars. The less massive stars are where the mass of the stellar population has been adding up over time.

Thus, the K-band luminosity of a galaxy of a particular Hubble type is a good measure of its old population. The number of low mass stars is proportional to the number of low mass binary star systems in which SNIa occur. The number of SNIa progenitors is proportional to the rate of SNIa (Tammann et al. 1994). Therefore, the supernovae rate normalized to the luminosity in the K-band should be a better measure of the supernovae rate along the Hubble sequence than normalized to the B-band luminosity. The luminosity in the near IR of a galaxy is proportional to the number of low mass stars and the low mass binaries which is proportional to the supernova rate since low mass binaries are supernovae progenitors.

\[
\text{Luminosity}_{NIR} \propto N(\text{low mass stars}) \propto N(\text{low mass binaries}) \propto \text{SNR} \quad (2.14)
\]
White dwarfs are associated with old stellar populations. The life cycle of a star depends on its mass. A low or medium mass star of \( \leq 8 - 10 \, M_\odot \), the star will become a white dwarf. Stars that become white dwarfs evolve more slowly than more massive stars since their low mass star progenitors stimulate nucleosynthesis at a much slower rate. SNIa white dwarf progenitor stars have predicted lifetimes of \( 10^9 \) to \( 10^{10} \) years Langer et al. (2000). The bulge of our Galaxy is composed of old stars of ages \( 8 \times 10^9 \) years Wyse et al. (1997). The stars in the bulge appear to be K and M stars of high metallicity. K and M stars live long lives on the main sequence and the bulge stars must have been formed from the remains of previous generation of stars to achieve high metallicity.

2.1.1.4 22Na

\[
^{22}_{\text{n}}\text{Na} \rightarrow ^{22}_{\text{10}}\text{Ne} + e^+ + \gamma
\]  

(2.15)

The decay of \(^{22}Na\) produces positrons and a photon of 1.275MeV.

Novae cannot easily be seen in the Milky Way because of interstellar extinction toward the galactic bulge. Therefore it is difficult to study their spatial distribution in the galaxy. However, novae are seen in other galaxies in particular M31, our closest neighbor.
galaxy. Novae have been observed to reside primarily in the bulge of M31. Novae produce positrons via $\beta$ decay of the radioactive isotopes $^{13}\text{N}$, $^{18}\text{Fe}$, and $^{22}\text{Na}$, (lifetimes = 14min, 2.6h, and 3.75y). $^{22}\text{Na}$ yields in novae are $6 \times 10^{-9} \text{M}_\odot$ (Knödlseder et al. 2005). To fuel the bulge emission from the decay of $^{22}\text{Na}$ alone, the novae rate would be 1600 per year. This rate is unreasonable considering that the estimate for the novae rate is $35 \pm 11$ per year for novae in the Galaxy.

Leising et al. (1988) observed the Galaxy using the SMM observatory searching for the positron tracer of the 1.275MeV gamma-ray. There was no evidence of the $^{22}\text{Na}$ line in the spectrum in the region of the Galactic center. $^{13}\text{N}$ yields $2 \times 10^{-7} \text{M}_\odot$ per novae (Knödlseder et al. 2005), a higher rate than $^{22}\text{Na}$ requiring 26 novae per year. However, $^{13}\text{N}$ has a much shorter lifetime to escape the ejecta to annihilate in the interstellar medium.

Novae fit the criteria of a bulge source of positrons. However, the fraction of positrons that escape novae remains uncertain albeit insignificant compared to other positron producing sources (Leising & Clayton 1987). Even if the positrons escape to annihilate, the 1.275 Mev line tracer of the positrons from novae has not been detected. Novae are not significant positron production sources.

### 2.1.1.5 Radioactive Isotope Positron Initial Energy

$\beta^+$-decay is when a proton decays into a neutron. Since the mass of a neutron is greater than a proton it takes energy to produce the reaction. The energy comes from the difference in the binding energy from parent and daughter nucleus. The parent binding energy must be less than the daughter to provide energy for the reaction.

\[
\text{energy} + p \rightarrow n + e^+ + \nu_e
\] (2.16)

For $\beta^+$-decay to occur there must be a threshold of $2m_e c^2 \sim 1 \text{ MeV}$ between parent and daughter isotopes, where $m_e = 0.511 \text{ MeV}$. The parent must be at least $2m_e c^2$ as massive
than the daughter.

\[ [M(A, Z) - M(A, Z - 1) - 2m_e] c^2 > 0 \]  \hspace{1cm} (2.17)

In this reaction a proton leaves (changes to neutron), the atomic number \( Z \) goes down one, and thus an electron leaves. The product and reactant atomic masses are defined in their neutral atomic state by their number of protons, \( Z \), and equal number of electrons. The mass of the electron and positron are lost in this reaction.

An example using \( ^{26}\text{Al} \):

\[ ^{26}\text{Al} \rightarrow ^{26}\text{Mg} + e^+ + \nu_e + \gamma (1.8\text{MeV}) \]  \hspace{1cm} (2.18)

The binding energy of \( ^{26}\text{Al}, E_{Al} = 8149.771 \text{ keV} \) which is less than the binding energy of \( ^{26}\text{Mg}, E_{Mg} = 8333.872 \text{ keV} \). The difference, \( E_{Al} - E_{Mg} \), is -184.101 keV. Thus the reaction can occur if the difference in the mass is greater than \( 2m_e c^2 \). The mass of \( ^{26}\text{Al} \), \( M_{Al} = 12210.31 \text{ keV} \). The mass of \( ^{26}\text{Mg}, M_{Mg} = 16214.58 \text{ keV} \). The difference in mass is, \( M_{Mg} - M_{Al} = 4004.52 \text{ keV} \). We know that it takes at least \( 2m_e c^2 \) for the reaction to occur and that a photon is created at 1809 keV. This leaves a left over energy of \( \sim 1176 \text{ keV} \) to share between the positron and neutrino that was created from the reaction.

Initially, the continuous spectrum of the energy of the positron (or electron) from \( \beta^- \)-decay presented a problem in conserving energy because the decay is a transition between two definite energy states. To conserved energy it is necessary to account for the energy that does not appear as kinetic energy of the positron. This lead to the discovery of the neutrino. In \( \beta^- \)-decay, as a radioactive isotope decays it emits a positron and a neutrino. The left over energy from the difference in mass from the parent and daughter isotope is shared by the both the positron and neutrino. The upper limit of the shared energy is the difference in the mass of the reaction. The spectrum of energies for the positron and neutrino is based on the transition probability and the density of the final state. See Segre (1977) for complete details.

The spectrum of positron and neutrino initial energies are calculated from the for-
mula by Segre (1977). The distribution of energies for positrons is,

$$\omega(\epsilon) = (\epsilon_0 - \epsilon)^2 \epsilon \sqrt{\epsilon^2 - 1}$$  \hspace{1cm} (2.19)

Where $\epsilon$ is the energy of the positron in units of $m_e c^2$, $E = \epsilon m_e c^2$. When the kinetic energy of the positron is zero $\epsilon = 1$, it can go no lower because of the rest energy of the positron.

To switch to a plot of the energy spectrum

$$\epsilon = \frac{E}{m_e c^2} = \frac{E_0}{m_e c^2} + \frac{K_{e^+}}{m_e c^2} = 1 + \frac{K_{e^+}}{m_e c^2}$$ \hspace{1cm} (2.20)

$$\epsilon_0 = \frac{E_{\nu} + K_{e^+}}{m_e c^2} + 1$$ \hspace{1cm} (2.21)

The energy the positron and neutrino share is $E_{\nu} + K_{e^+}$. For the example of $^{26}$Al, $\epsilon_0 = 1176/511$.

$$\omega(K_{e^+}) = N \left( \epsilon_0 - \left( 1 + \frac{K_{e^+}}{m_e c^2} \right)^2 \right) \left( 1 + \frac{K_{e^+}}{m_e c^2} \right) \sqrt{\left( 1 + \frac{K_{e^+}}{m_e c^2} \right)^2}$$ \hspace{1cm} (2.22)

$N$ is the normalization constant. Figure 2.7 shows the spectrum of energies a positron can have from different isotopes.

Table 2.3: The values of the shared energy between the positron and neutrino, $\epsilon_0 m_e c^2$, and the most probable energy value for each isotope.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\epsilon_0 m_e c^2$ [keV]</th>
<th>Max of function [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$Na</td>
<td>545</td>
<td>156</td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>1176</td>
<td>415</td>
</tr>
<tr>
<td>$^{56}$Ni</td>
<td>1459</td>
<td>544</td>
</tr>
<tr>
<td>$^{44}$Ti</td>
<td>1475</td>
<td>551</td>
</tr>
</tbody>
</table>
2.1.2 High-Energy Processes

2.1.2.1 Cosmic Rays

Cosmic rays interact with the interstellar medium producing pions. The positively charged pion decays to a muon which decays producing a positron.

\[
\pi^+ \rightarrow \mu^+ + \nu \\
\mu^+ \rightarrow e^+ + \nu + \bar{\nu}
\]  

Positrons are secondary particles from cosmic-ray interactions. The energy spectrum of these positrons is determined from the kinematics of the collision and the energy spectrum
of the cosmic-rays. The production rate of positrons depends on the assumed propagation model of cosmic rays. Propagation of cosmic rays is typically modeled by diffusion. If the diffusion is fast then a small amount of secondary particles are produced. If the diffusion is slow then a large amount of secondary particles are produced. Porter et al. (2008) calculated a positron rate from cosmic-rays as \((1 - 2) \times 10^{42}\) positrons \(s^{-1}\).

Figure 2.8 shows the spectrum of energies a positron can have from an energetic \(p - p\) collision. In the \(p - p\) collision one proton at rest the other is very energetic with kinetic energy ranging from 0.3 to 100 GeV. The positron has a spectrum of energies with a typical maximum at 30–40 MeV.

![Figure 2.8: The positron energy spectrum from energetic \(p - p\) collisions Murphy et al. (1987)](image-url)
The positively charged pion is accompanied by the creation of the neutral pion, $\pi^0$. Thus the neutral pion can be a tracer for the positrons from charged pions. The high energy emission $>100$ MeV that is seen along the Galactic plane is the decay of neutral pions. The Energetic Gamma Ray Experiment Telescope (EGRET) on-board CGRO has mapped the Galaxy in high energy gamma rays (Figure 2.9). The evidence of emission along the Galactic plane indicates where the cosmic rays are and where they interact with the ISM to produce positrons. There is no indication that there is a bulge component to the EGRET map. Therefore, cosmic rays cannot be the dominant source of bulge emission locally.

Figure 2.9: All-sky map of cosmic rays above 100 MeV from the decay of neutral pions $\pi^0$. The bright section along the Galactic plane indicates that the positrons from cosmic ray interactions are not solely responsible for the bulge positrons. Credit: EGRET Team
2.1.2.2 Pulsars

Pulsars produce positrons by pair creation. The rotation of a pulsar leads to an induced electric field that extracts electrons from the surface of the star. The electrons then lose energy by curvature radiation when propagating along the star’s magnetic field. The photons from the radiation are so energetic that positron-electron pairs form in the intense neutron star magnetic field. In principle, this process can repeat producing many pairs.

\[ \gamma + \gamma \rightarrow e^+ + e^- \quad (2.24) \]

One photon can produce pairs a magnetic field \( \geq 10^{12} \) G.

The rate \( \dot{n}_{e^+} \) positrons are produced in pulsars depends on the strength and topology of the magnetid field and the pulsar spin period.

\[ \dot{n}_{e^+} \approx 2.8 \times 10^{37} B^{10/7} P^{-8/21} \text{ s}^{-1} \quad (2.25) \]

where \( B \) is on the order of magnitude of \( 10^{12} \) G and \( P \) is the period of the pulsar (\( \tau \)). Only a fraction of positrons escape in the pulsar winds to be injected into the Galaxy.

\[ \dot{N}_{e^+} = \dot{n}_{e^+} N = 5 \times 10^{42} \text{ s}^{-1} \quad (2.26) \]

where \( N \) is the number of sources in the Galaxy calculated but the birthrate \( R \) and lifetime \( \tau \) of pulsars, \( N = R \tau \).

The initial energy of positrons from pulsars is \( > 30 \) MeV Lemiere et al. (2008). Since pulsars are from young stars their distribution follows the star formation rate distribution, which has a small scale height (\( \sim 100 \) pc) and an insignificant bulge population.
2.1.2.3 Low Mass X-ray Binaries

An X-ray binary is a system of two stars orbiting around each other. One star is a compact object and the other companion star can be on the main sequence, a white dwarf or an evolved star. X-ray binaries might produce positrons by pair production in the area around the compact object or through nuclear interactions that form unstable nuclei emitting positrons as they decay. An X-ray binary system that ejects particles via jets is known as a microquasar. These sources eject $10^{41} - 10^{42}e^+s^{-1}$ in relativistic jets into the interstellar medium (Dermer & Murphy 2001).

There are two classes of X-ray binaries, high and low mass, named such for the mass of the donor star. Low mass X-ray binary (LMXB) companion stars are of late spectral type with a mass less than or equal to $2M_\odot$. High mass X-ray binaries (HMXB) have a giant or supergiant O or B donor companion star. HMXB are associated with young stellar populations and have a distribution spread along the disk Grimm et al. (2002). Therefore, HMXB are excluded from the list of sources. The X-ray binaries mentioned in the rest of the section refer to LMXB.

The distribution of LMXB was studied by Grimm et al. (2002). From this survey, LMXB were found to be centrally located in the Galactic longitudinal and latitudinal directions. A third of the LMXB were within the $\sim 2$ kpc extent of the 511 keV emission from the galactic bulge. However, of the 12 most luminous LMXB known, only 30% are actually located in the bulge. If the luminosity of LMXB in X-rays is proportional to the positron annihilation luminosity then the bulge LMXB are not luminous enough in X-rays to create the B/D ratio comparable to the SPI measurements. Grimm’s value for the B/D ratio is $\sim 0.9$. Both Grimm and Guessoum considered the scale height of 410 pc for LMXB from the galactic plane. Bright LMXB in the disk could have positrons escape the inner gas layer of the disk at $\sim 100$ pc. Positrons would escape to the halo and diffuse before annihilation. The positrons would diffuse to large scale heights and be hard to detect. After subtraction of the disk emission from $^{26}$Al about 80% of positrons from disk LMXB would have to escape to meet the criteria of a B/D ratio of 3-9. Whether such a large fraction can
escape is uncertain.

The positron production by X-ray binaries and the distribution of X-ray binaries cannot explain the B/D ratio measured by SPI using current data on X-ray binaries. Modeling the LMXB in the Galaxy and their luminosity may provide more insight as to how much LMXB contribute to the 511 keV emission. However, preliminary research indicates that the LMXB emission morphology will not correspond to the 511 keV emission morphology that has been observed. If the luminosity of the the X-ray binary scales with the positron luminosity then the morphologies will not match (Guessoum et al. 2006). There is a slight asymmetry to the LMXB distribution (Figure 2.10). Recently, Weidenspointner et al. (2008a) calculated the source number ratio of LMXB to be \( N(l < 20^\circ) / N(l > 20^\circ) = 1.7 \) and concluded that the asymmetry that was measured in the 511 keV emission could be from

![Liu LMXB Distribution Scaled to Flux](image)

Figure 2.10: The distribution of LMXB in the Galaxy. The color is scaled with X-ray flux, the bright white yellow color are the brightest LMXB. This figure was created by using data from Liu et al. (2001).
these sources. However, Bouchet et al. (2008), using different methods of data analysis did not detect an asymmetry.

### 2.1.2.4 Galactic Center Black Hole

The black hole (BH) in the Galactic center seems an obvious suggestion for a central source of positron production. However, the 511 keV emission is diffuse over a region of $8^\circ$ and not completely from a point source. If one central source produced positrons they would have to travel $\sim 2$ kpc before they annihilate to explain a diffuse emission.

The rate may be proportional to the x-ray luminosity. However, the x-ray luminosity from the BH is $10^4$ times weaker than the combined luminosity of Galactic LMXB. If $L_{e^+} \propto L_{x-rays}$ then the BH can not be an important source of positrons.

However, it may be the case that the BH does not produce positrons in steady state. Positrons are produced by $p - p$ collisions with initial energy similar to cosmic-rays $\sim 30$ MeV or by $\gamma - \gamma$ pair production (Cheng et al. 2007). The large rate of positrons could be from a rare active event in the past which may have made the BH brighter in x-rays and produce more positrons. The event in the past would also give positrons time to travel away from the Galactic center region to annihilate elsewhere. However, it is uncertain wether positrons could survive the trip from center to the outer bulge. Positrons can slow and annihilate with molecular hydrogen, which is dense in the Galactic center.

### 2.1.2.5 Dark Matter

Boehm & Silk (2008) have proposed light dark matter as a candidate source of galactic positrons, where the term light describes a particle, "a thousand to a hundred times lighter than a proton, i.e., with a mass comparable to the electron mass." Positron-electron pairs can come from decays or annihilation of light dark matter particles. There are two mechanisms, the exchange of a charged heavy fermion or neutral light boson. The existence of both particles have been speculated theoretically, but current particle physics experiments have not confirmed their existence.
The heavy fermion could explain the 511 keV morphology while the light boson may explain why 80% of the universe is dark matter (Boehm & Silk 2008). Only a cuspy galactic density profile can indicate whether light dark matter responsible is for the 511 keV morphology (Ascasibar et al. 2006). Light dark matter particles are required to have a mass < 100 MeV to explain the 511 keV observed morphology by INTEGRAL.

The flux from light dark matter has been compared to the observed 511 keV flux from SPI by (Hooper et al. 2004). If the 511 keV emission is due to light dark matter particles, then we should see a positron annihilation flux from galaxies known to have high densities of dark matter. Dwarf spheroidal galaxies have environments devoid of gas and dust compared to the Galactic center and are known to have a high density of dark matter. This is due to the large mass to light ratios for dwarf spheroidals, which indicates that a large fraction of the mass is dark matter Gilmore et al. (2007). Therefore, observations of 511 keV positron annihilation from dwarf spheroidals could provide strong evidence for light dark matter particles. The nearby Sagittarius dwarf spheroidal was predicted to produce a 511 keV flux in the range of $1 - 7 \times 10^{-4}$ ph cm$^{-2}$ s$^{-1}$. However, the Sagittarius dwarf galaxy was observed by SPI (Cordier et al. 2004) and no source was detected. The flux 2σ upper limit reported was $4.8 \times 10^{-4}$ ph cm$^{-2}$ s$^{-1}$. Knödlseder et al. (2005) reported a lower upper limit of $1.7 \times 10^{-4}$ ph cm$^{-2}$ s$^{-1}$ for the Sgr dwarf spheroidal and did not confirm the central cusp model the light dark matter for this galaxy.

More research needs to be done before light dark matter is excluded as a plausible source for the Galactic annihilation emission. The cuspy density profile of the light dark matter is still be to be constrained by observations.

2.2 Assessment of Positron Production Sources

Radioactive isotopes are the most probable sources of Galactic positrons. They produce positrons at a rate comparable to the observed Galactic 511 keV positron annihilation emission, are produced at an initial energy within the constraint, and have an
observed Galactic distribution. The other proposed positron production sources either have no reasonable value for a positron production rate, an initial energy to large to satisfy the constraint, no reasonable value for initial energy, or no value for production rate and positron initial energy. See Table 2.4 for values.

Table 2.4: Assessment of the Parameters of Positron Production Sources.

<table>
<thead>
<tr>
<th>Sources</th>
<th>Production Method</th>
<th>Production Rate $[e^+/s]$</th>
<th>Initial Energy [keV]</th>
<th>B/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{26}\text{Al}$</td>
<td>$\beta$-Decay</td>
<td>$10^{42}$</td>
<td>1176</td>
<td>Disk</td>
</tr>
<tr>
<td>$^{44}\text{Ti}$</td>
<td>$\beta$-Decay</td>
<td>$10^{42}$</td>
<td>1475</td>
<td>Disk</td>
</tr>
<tr>
<td>$^{56}\text{Ni}$</td>
<td>$\beta$-Decay</td>
<td>$10^{43}$</td>
<td>1149</td>
<td>Disk</td>
</tr>
<tr>
<td>$^{22}\text{Na}$</td>
<td>$\beta$-Decay</td>
<td>$10^{41}$</td>
<td>545</td>
<td>Disk</td>
</tr>
<tr>
<td>Cosmic-rays p-p</td>
<td></td>
<td>$10^{42}$</td>
<td>$3 \times 10^4$</td>
<td>Disk</td>
</tr>
<tr>
<td>Pulsars $\gamma - \gamma$</td>
<td></td>
<td>$10^{42}$</td>
<td>$&gt; 3 \times 10^4$</td>
<td>Disk</td>
</tr>
<tr>
<td>LMXB $\gamma - \gamma$</td>
<td></td>
<td>?</td>
<td>?</td>
<td>Disk</td>
</tr>
<tr>
<td>Black Hole p-p</td>
<td></td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Dark Matter Annihilation</td>
<td></td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

### 2.2.1 Positron Production Rate

The observed Galactic positron annihilation rate is about 2–6 times larger in the Galactic central bulge region than in the disk. The positron annihilation rate for the bulge region and the disk are $(11.5–31.3) \times 10^{42} \text{ s}^{-1}$ and $(8.1–5.2) \times 10^{42} \text{ s}^{-1}$, respectively. The average total Galactic positron annihilation rate is $28.1 \times 10^{42} \text{ s}^{-1}$. The positron production source must produce positrons close to this rate in order for it to be considered a viable production source to explain the observed annihilation emission.

The radioactive isotopes $^{26}\text{Al}$, $^{44}\text{Ti}$, $^{56}\text{Ni}$ and cosmic rays produce a significant portion. These isotopes produce positrons at a rate of $10^{42} – 10^{43} \text{ e}^+/s$, which is on the order of the observed annihilation rate of $10^{43} \text{ e}^+/s$. The rate of positron production of pulsars, LMXB, black hole, dark matter are still speculative.
2.2.2 Positron Initial Energy

The energy at which a positron can annihilate ‘in-flight’ is constrained to below 3 MeV. The energy spectrum of the inner Galaxy within $|l| = 10^\circ$ and $|b| = 10^\circ$ as measured by various instruments (Sizun et al. 2006) shows that the flux above the 511 keV line follows a power law. The extra flux above the power law is a model of relativistic positrons annihilating ‘in-flight’, internal Bremsstrahlung, Bremsstrahlung, and electron Bremsstrahlung. The photons produced from relativistic direct annihilation in-flight will be Doppler shifted. The range of energies for the photons from this process is $m_e c^2 / 2 \leq E_{\gamma} \leq E + m_e c^2 / 2$. See 4.1.2 for information about in-flight annihilation.

The extra flux above the continuum is calculated for positrons with energies of 100, 30, 10, and 3 MeV injected into the inner region of the Galaxy. For positrons with energies above 3 MeV the extra flux is not observed. Therefore, only positrons with injection energy below 3 MeV annihilate within the inner Galaxy. Positrons could be created at higher energies and loose most of the energy before they reach the inner Galaxy or positrons could be created at energies below 3 MeV. See Figure 2.11. Significant positron production sources are the radioactive isotopes $^{26}\text{Al}$, $^{44}\text{Ti}$, $^{56}\text{Ni}$, $^{22}\text{Na}$ which all fulfill the requirement by being below 1 MeV. Pair production by $\gamma - \gamma$ collisions in LMXB and black hole may contribute, but the mechanism of positron production is uncertain. The high energy $\gamma - \gamma$ pair production in pulsars and the energetic $p - p$ in cosmic rays produce positrons at energies above the threshold of 3MeV. The initial energy of positrons from dark matter is unknown.

2.2.3 Production Source Morphology

The morphology of the Galactic positron annihilation emission is contrary to the distribution of many of the positron production sources. The B/D of the emission is $> 2$, but the B/D of the production sources is $< 1$. Many of the sources—such as radioactive isotopes, cosmic-rays, and pulsars—are considered disk population sources. These sources mainly exist in the Galactic disk and show no sign of producing positrons in a distribution
that will increase their B/D. If positrons from these sources follow the assumption that they annihilate where they are produced, then the observed B/D ratio would be less than one. Therefore, positrons from these sources may be required to propagate in order to produce the observed positron annihilation emission. Bulge population sources have a central distribution—such as LMXB, dark matter and the black hole at the Galactic center—and were conceived under the assumption that positrons annihilated where they are produced because the Galactic magnetic field was sufficiently tangled to keep positrons close to the source. However, much is uncertain about these sources, such as their positron production rate and the energies at which the positron is produced.

In this dissertation I will focus on the source that is most probable, radioactive isotopes from massive stars. This source has the largest positron production rate that has actually been observed and measured. Also, studying a disk population source, such as radioactive isotopes, allows us to study how positrons may propagate through the Galaxy along magnetic field lines. The other positron production sources suffer the uncertainty
of how they produce positrons, the production rate, and what energies the positrons are produced, which makes it difficult to obtain quantitative results about their contribution to the positron annihilation emission.

### 2.3 Modeling Positron Production Sources

I model the three dimensional distribution of $^{26}$Al and $^{44}$Ti. As discussed in the previous section radioisotopes are the source of positrons that produce positrons at a rate comparable to the observed annihilation rate and produce positrons under the threshold of 3 MeV. There is also a well known distribution of $^{26}$Al from observations of the 1.809 MeV line which is from the decay that produces positrons.

Knödlseder (1997) using multi wavelength correlation study found correlation between free-free emission and the 1.809 MeV gamma-ray line emission. The correlation suggests that massive stars are the origin of $^{26}$Al. The correlation between free-free emission and $^{26}$Al is to be expected if massive stars are sources of ionizing radiation. The method for determining the density of free electrons in the Galaxy is from pulsar dispersion measure (DM). A pulsar emits periodically a spectrum of photons spanning the range in radio frequency. The electromagnetic waves traveling through ionized medium interact with the free electrons causing them to oscillate and retarding the progress of the wave. The interaction is such that waves’ group velocity decreases with increasing wavelength. Therefore, radio waves with larger wavelength are more slowed. In one pulse of light from a pulsar the high frequency photons arrive first and the low frequency waves arrive late. The delay in the arrival time is proportional to the density of free electrons along the line of sight. The spread is called the DM.

$$DM = \int_0^L n_e ds$$

(2.27)

where $n_e$ is the density of free electrons and $ds$ is the line of sight from 0 – $L$. 

46
Taylor & Cordes (1993) modeled the free electron density. This research employs their model for the free electron density and for the density of radioisotopes, $^{26}\text{Al}$ and $^{44}\text{Ti}$, normalized to the observed flux from $^{26}\text{Al}$.

The relation for the free electron density is

$$n_e(x, y, z) = n_1g_1(r)sech(z/h_1)^2 + n_2g_2(r)sech(z/h_2)^2$$

$$+n_a sech(z/h_a)^2\sum_{j=1}^{4} f_j g_a(r, s_j) + n_G g_G(u) \quad (2.28)$$

where $r = \sqrt{x^2 + y^2}$. The summation is over the spiral arms. The model parameters are evaluated by the position of the spiral arms (Figure ??). The coordinates of the spiral arms are from seven fiducial markers in Taylor & Cordes (1993) and are interpolated to produce the spiral arms. In the z-direction the free electron density falls off by the squared hyperbolic secant, $sech(z)$ characterized by a scale height $h$.

The scale factors $f_j$ are,

$$f_1 = f_4 = 1 \quad (2.30)$$

$$f_2 = \begin{cases} 
1, & \theta < 215^\circ \\
1 + (\theta - 215^\circ)/20^\circ, & 215^\circ < \theta < 235^\circ \\
2, & \theta > 235^\circ 
\end{cases} \quad (2.31)$$

$$f_3 = \begin{cases} 
(3 + \cos[2\pi(\theta - 120^\circ)/40^\circ])/4, & 120^\circ < \theta < 160^\circ \\
1, & \text{elsewhere} 
\end{cases} \quad (2.32)$$

where $\theta$ is the angle with vertex in the Galactic center, counterclockwise from negative y-direction.
Figure 2.12: Location of the spiral arms used in determining the free electron density. The sun is located at (0, 8.5 kpc).
The remaining dependencies are,

\[ g_1(r) = sech^2(r/A_1)/sech^2(8.5/A_1) \] (2.33)

\[ g_2(r) = \exp -[(r - A_2)/1.8]^2 \] (2.34)

\[ g_a(r,s_j) = \begin{cases} 
\exp -(x_j/w_a)^2 sech^2[(r - A_a)/2], & r > A_a \\
\exp -(x_j/w_a)^2, & r \leq A_a 
\end{cases} \] (2.35)

\[ g_G(u) = \begin{cases} 
\exp -[(u - 0.13)/0.05]^2, & u > 0.13 \text{ kpc} \\
1.0, & U \leq 0.13 \text{ kpc} 
\end{cases} \] (2.36)

where \( s_j = \sqrt{(x - x_0)^2 + (y - y_0)^2} \) is the distance from the point \((x_0, y_0, 0)\) on spiral arm \(j\) to the point \((x, y, 0)\), \( u = \sqrt{(x + 0.492)^2(y - 8.587)^2 + z^2} \), and the distance to the Galactic center is 8.5 kpc. The units for the distances are all in kpc. Constants and other values can be found in Taylor & Cordes (1993).

Figure 2.13 is the model density of free electrons at the same orientation as Figure 2.12. There are four distinct spiral arms where it is assumed star formation, massive stars, and free electrons are concentrated. There is also a background free electron density which can be seen as a ring at about 4 kpc from the Galactic center. The largest densities \( n_e = 0.18 \text{ cm}^{-3} \) and the density in the solar region is \( n_e = 0.019 \text{ cm}^{-3} \).

What is noticeable is the dearth of density in the Galactic center region. This distribution is used to place the positron sources \(^{26}\text{Al}\) and \(^{44}\text{Ti}\). Therefore, chances of positrons produced in the bulge region are slim. After positrons are produced they slow in the ionized gas and sometimes annihilate directly with electrons or after the an interaction that forms positronium then annihilates. The small amount of gas in the bulge region for the positrons to interact with means that emission from this region is minimal.

The distribution of the density of \(^{26}\text{Al}\) and \(^{44}\text{Ti}\) is derived from the free electron
density but normalized to the flux of Galactic $^{26}\text{Al}$ as seen in Figure 2.2. Wang et al. (2009) measured the $^{26}\text{Al}$ gamma-ray flux from the inner Galaxy region $-30^\circ < l < 30^\circ$ and $-10^\circ < b < 10^\circ$ as $(2.93 \pm 0.15) \times 10^{-4} \text{ph cm}^{-2}\text{s}^{-1}\text{rad}^{-1}$. The density of $^{26}\text{Al} n_{\text{Al}}$ is proportional to the density of free electrons $n_e$. The volume emissivity of 1.809 MeV photons $\epsilon_{\text{Al}}$ from $^{26}\text{Al}$ is in units of [photons/cm$^3$/s],

$$\epsilon_{1809} = (0.82)n_{\text{Al}}\lambda_{\text{Al}}$$ (2.37)

where the radioactive decay of $^{26}\text{Al}$ is $\lambda = 9.67 \times 10^{-7}$ y$^{-1}$ and the fraction of $^{26}\text{Al}$ that produce a 1.809 MeV photon is 0.82.

To calculate the flux, first calculate the intensity of $^{26}\text{Al}$ photons.

$$I_{1809} = \frac{1}{4\pi} \int \epsilon_{\text{Al}} dr$$ (2.38)
In the code the distance along the line of sight is 30 kpc with step sizes of $0.05kpc = 1.545 \times 10^{20}$ cm. The integral is calculated by a sum at every step along the line of sight. The flux is related to the intensity by the solid angle $d\Omega$ as seen in Chapter 1. The flux of the inner Galaxy is calculated using the limits,

$$F(l, b) = \int_{l=-30^\circ}^{30^\circ} \int_{b=-10^\circ}^{10^\circ} I(l, b)_{1809} \cos b \, db \, dl$$ (2.39)

The unnormalized flux is $F(l,b)=3.019\text{ photons/cm}^2\text{/s}$, compared to the observed flux the normalization factor is ,

$$n_{Al} \propto n_e$$ (2.40)

$$= \frac{2.93 \times 10^{-4}}{3.02 \times 10^{13}} n_e$$ (2.41)

$$= 9.7 \times 10^{-18}$$ (2.42)

Testing the flux after incorporating this normalizing factor in the $^{26}\text{Al}$ density shows agreement with observed flux.

Knödlseder et al. (2005) compare a value of the flux of $^{26}\text{Al}$ 1.809 MeV from Knödlseder (1997), $F_{Al} = 9 \times 10^{-4}$ [photons/cm$^2$/s] to the flux of 511 keV $F_{511}$ photons using a positronium fraction of $f_{Ps} = 0.93$.

$$F_{511} = 0.82(2 - 1.5f_{Ps})F_{Al} = 5 \times 10^{-4} \text{ [photons/cm}^3\text{/s]},$$ (2.43)

Compared to the observed disk $F_{511}$ this flux accounts for about 60% of the emission. The more recent value for the flux of $^{26}\text{Al}$ compared to observed 511 keV emission accounts for only 25% of the 511 keV emission. Figure 2.14 is the normalized 1.809 MeV intensity. The intensity is used to predict the distribution of the rate of positron production.

The distribution of $^{44}\text{Ti}$ is found also from the tracer of free electrons. The normal-
Figure 2.14: The all sky map of the intensity of 26Al using model from Taylor and Cordes 1993. Intensity scale is not zero but maximum at $2.89 \times 10^{-3}$ photons/cm$^2$/s/st.
2.4 Summary

The $^{26}\text{Al}$ and $^{44}\text{Ti}$ positron production sources meet the criteria of producing a rate of positrons comparable to the observed rate of positron annihilation, producing positrons with initial energy under the threshold of 3 MeV, and have a known Galactic distribution. This research tests whether $^{26}\text{Al}$ and $^{44}\text{Ti}$ can be the source of positrons that annihilate with the observed 511 keV line emission. Knödlseder et al. (2005) has suggested that $^{26}\text{Al}$ and $^{44}\text{Ti}$ can already explain the observed disk annihilation emission.

Approximately $10^{42} \text{ e}^+ \text{ s}^{-1}$ come from each source. This research uses a Monte Carlo simulation of positrons with initial conditions based on the location and energy from each source. The location is based on the density of $^{26}\text{Al}$ and $^{44}\text{Ti}$ in the inferred from the distribution of free electrons. The initial energy is based on the spectrum of energy a positron can have after the decay of $^{26}\text{Al}$ or $^{44}\text{Ti}$. 

\[
\frac{n_{\text{Ti}} = \text{norm} \, n_{\text{Al}}}{n_e} = \frac{\dot{M}_{\text{Ti}}}{\dot{M}_{\text{Al}}} \frac{2.93 \times 10^{-4}}{3.02 \times 10^{13}} \times n_c 
\]

where $\dot{M}_{\text{Ti}} = 6.4 \times 10^{-6}$ and $\dot{M}_{\text{Al}} = 2.7 \times 10^{-6} \, \text{M}_{\odot}$ per year. Therefore the normalization for $^{44}\text{Ti}$ is $2.3 \times 10^{-17}$.
Chapter 3

Positron Propagation

3.1 Propagation of Positrons

The observed 511 keV positron annihilation emission indicates that positrons annihilate in the bulge region of the Galaxy. The sources that produce positrons are mainly in the disk. How is the dichotomy between the production of positrons and annihilation positrons reconciled? In previous studies (Knödlseder et al. 2005) the assumption was the positrons annihilated where they are produced. This lead to many of the suggestions for positron production sources, including centralized Galactic source populations.

The argument for positrons annihilating where produced without much propagation is according to Bohm diffusion (Jean et al. 2009), which approximates that the mean free path for propagation is related to the the gyroradius of the positron around a Galactic field line. The gyroradius \( r_g \) is defined as,

\[
  r_g = \frac{\gamma m_0 v}{qB}
\]

(3.1)

where, \( \gamma \) is the Lorentz factor, \( m_0 \) is the rest mass of the charged particle, \( v \) is the velocity, \( q \) is the charge, and \( B \) is the strength of the magnetic field. The velocity and \( \gamma \) can be
written as,

\[ v = c \sqrt{1 - \frac{1}{\gamma^2}} \quad (3.2) \]

\[ \gamma = \frac{E}{mc^2} + 1 \quad (3.3) \]

If a positron is created at energy 1 MeV in a magnetic field of average strength of 1 \( \mu \)G, then the radius the positron gyrates is,

\[ r_g = \frac{\gamma m_0 c \sqrt{1 - \frac{1}{\gamma^2}}}{qB} \quad (3.4) \]

\[ = 1.6 \times 10^9 \text{ cm} = 1.6 \times 10^{-12} \text{ kpc} \quad (3.5) \]

Given that the Galaxy is about 40 kpc in diameter and has a scale height of about 1 kpc, the gyroradius for an average field strength is very small. A small gyroradius is not enough to assume particles are confined to their production sites. If a particle gyrates around a field line it can also travel along the field line. Therefore, the Galactic magnetic field must be sufficiently tangled for no propagation out of the disk.

The complexities of the ISM may argue against the assumption that positrons annihilate where they are produced. Jean et al. (2006) modeled the distance positrons travel using a diffusion theory at high energies and collisions with the ISM atoms and molecules at low energies. Using the gas content of the bulge they concluded that positrons injected into this type of gas at 1 MeV by radioactive isotopes would not escape. They calculated a distance positrons can travel as a function of their kinetic energy. For a positron at 1 MeV, it could travel a distance of 40–160 pc in a warm ionized medium.

Prantzos (2006) used positrons from SNIa in an old disk stellar population transported along a Galactic magnetic field. In the less dense regions on the ISM with scale height about >300 pc positrons have a lifetime of \( \tau \sim 10^5 \) years. Positrons from SNIa and 1 MeV wander the low density before annihilating. During these timescales positrons interact and are transported along field lines. The turbulent fields at this scale height are unknown
making it possible for positrons to propagate along the poloidal field. To enter the bulge positrons may need to avoid the magnetic mirroring effect from a strong gradient of the poloidal field in the Galactic center. The effect is small if the components of the velocity are parallel with the magnetic field. The effect of $v_\parallel \gg v_\perp$ may be natural. Diffusion motion studied by Casse et al. (2002) shows the diffusion coefficient $D_\parallel$ is related to the magnetic field $B_0$ and $D_\perp$ by,

$$\frac{D_\perp}{D_\parallel} = \left( \frac{B^2}{B_0^2 + B^2} \right)^{2.3}$$  \hspace{1cm} (3.6)

where $B$ is the mean intensity of the inhomogeneous turbulent component. Near the border of the diffusion zone where $B < B_0$, $D_\perp \ll D_\parallel$. Where the toroidal field transitions to the poloidal field, positrons have diffused such that they are parallel with the field. So along the poloidal field positrons have velocity components parallel to the field and do enter the bulge.

Prantzos (2006) concluded that SNIa may suffice to explain the observed positron annihilation rate and bulge to disk ratio if 50% of the positrons from SNIa escape the disk and transport to the bulge. This percentage depends on the escape fraction of positrons from SNIa, which is still debatable.

Higdon et al. (2009) model Galactic positron transport using a mean free path from magnetohydrodynamics with positron energies of 1 MeV in the interplanetary turbulence. However, this method for transport is valid for the interplanetary medium and the ISM is likely to be different. They use positrons from radioactive isotopes mainly $^{56}\text{Ni}$ from SNIa, which is uncertain.

To resolve the dichotomy, one may remove the assumption of positrons annihilating where they are produced and allow positrons to propagate. The poloidal Galactic magnetic field has the ability to move positrons from the disk into the bulge region. However, properties of the Galactic magnetic field is not well known except for the knowledge of a regular field in the plane of the Galaxy and poloidal field outside the disk. In the disk there is also
a turbulent field.

### 3.2 Galactic Magnetic Field

#### 3.2.1 Strength

The strength of the Galactic magnetic field can be measured using the Zeeman effect. The Zeeman splitting of the hydrogen 21cm line is used as a tracer for magnetic fields in radio astronomy. The hydrogen 21 cm line originates from the $^2S_{1/2}$ state. The magnetic field that causes the 21cm line splitting comes from the atom. To measure the strength of the Galactic magnetic field one must measure the strength of splitting the 21 cm line. The unsplit line is typically labeled $\pi$ and the left and right splitting are labeled $\sigma^-$ and $\sigma^+$. The quantum mechanical derivation of the Zeeman effect show that the split frequency is proportional to the unsplit frequency $\nu_L$, the Larmor precession frequency, by a factor $g$.

$$\Delta \nu = g \nu_L$$  \hspace{1cm} (3.7)
The Larmor precession is $\omega_L = \frac{e}{2mc}B$. The relation between frequency and precession is $\omega_L = 2\pi\nu$. Therefore,

$$\Delta \nu = g \frac{eB}{4\pi mc}$$

(3.8)

The quantity $\frac{e}{4\pi mc}$ is known as the “Zeeman displacement” and is equivalent to the Bohr magneton $\mu_B$ divided by Planck’s constant $h$.

$$\Delta \nu = g \frac{\mu_B}{\hbar} B$$

(3.9)

The Zeeman displacement is the shift of the unsplit line in either $\sigma^-$ and $\sigma^+$ direction. The splitting can be measured as the total splitting between $\sigma^-$ and $\sigma^+$ called the splitting coefficient $b$ defined as twice the displacement times $g$.

$$\Delta \nu = \frac{b}{2} B$$

(3.10)

The factor $g$ is the Lande $g$-factor. The value is given by,

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

(3.11)

where $S$ is the total spin of the atom, $L$ is the total orbital angular momentum, and $J = L + S$ is the total angular momentum. Since the 21 cm line originates from the $^2S_{1/2}$ state, $J = 1/2$, $S = 1/2$ and $L = 0$. Giving a Laude factor of $g = 2$. This must be multiplied by the nuclear $g$-factor,

$$g_N = \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$

(3.12)

where $J = 1/2$ as before, $F = 1$ is the total angular momentum quantum number, and $I = 1/2$ the nuclear spin quantum number. The nuclear g-factor $g_N = 1/2$. This yields a
total g-factor of \( g = 1 \). Therefore, the 21 cm line hyperfine splitting will be,

\[
\Delta \nu = \frac{\mu_B B}{\hbar} = 2.80 \text{ Hz } \mu \text{G}^{-1}
\]  

(3.13)

where, \( b = 2.80 \text{ Hz } \mu \text{G}^{-1} \). The profile of the 21cm is broad and the field in the ISM is weak, so that it is not typical to actually see the lines from \( \sigma^- \) and \( \sigma^+ \) but observe the Zeeman effect as HI broadening.

### 3.2.2 Direction

The direction of the Galactic magnetic field is determined by the polarization of light from an electron bound to an atom in the presence of the magnetic field. For a non-relativistic electron bound to a nucleus, the electric field of the accelerating charge is

\[
\vec{E}(\vec{x}, t) = \frac{e}{4\pi \epsilon_0 c^2} \frac{\hat{n} \times (\hat{n} \times \hat{v})}{R}
\]

(3.14)

The unit vector \( \hat{n} \) points from the electron to the observer, \( \vec{v} \) is the velocity of the electron, and \( R \) is very large. The velocity in cartesian coordinates

\[
\hat{v} = \hat{x} \hat{x} + \hat{y} \hat{y} + \hat{z} \hat{z}
\]

(3.15)

Where the external magnetic field is aligned along \( \hat{z} \). The triple cross-product can be rewritten as

\[
\hat{n} \times (\hat{n} \times \hat{v}) = \hat{n}(\hat{n} \cdot \hat{v}) - \hat{v}
\]

(3.16)

However, due to the large distance from observer to the atom \( \hat{n} = \hat{r} \)

\[
\hat{n} \times (\hat{n} \times \hat{v}) = \hat{r}(\hat{r} \cdot \hat{v}) - \hat{v}
\]

(3.17)
Solving this expression for spherical coordinate system

\[\begin{align*}
\hat{x} &= \sin \theta \cos \phi \hat{r} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi} \\
\hat{y} &= \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} - \cos \phi \hat{\phi} \\
\hat{z} &= \cos \theta \hat{r} - \sin \theta \hat{\theta}
\end{align*}\]

(3.18)

(3.19)

(3.20)

Substituting into above equation

\[
\hat{r}(\hat{r} \cdot \vec{v}) - \vec{v} = -\dot{\theta}(\vec{x} \cos \theta \cos \phi + \vec{y} \cos \theta \sin \phi) - \ddot{\vec{z}} \sin \theta + \dot{\phi}(-\vec{x} \sin \phi + \vec{y} \cos \phi)
\]

(3.21)

This expression is symmetric in \(\phi\), there is no preferred azimuthal direction in the \(xy\) plane.

The electric field can be viewed from any angle around the magnetic field.

\[
\begin{align*}
E_\theta &= \frac{e}{4\pi\varepsilon_0 c^2} (\ddot{x} \cos \theta \cos \phi - \ddot{z} \sin \theta) \\
E_\phi &= \frac{e}{4\pi\varepsilon_0 c^2} (\vec{y})
\end{align*}
\]

(3.22)

(3.23)

The components of the electric field can be used to described the polarization from this system of an electron in an external magnetic field. The Stokes parameters are four parameters that completely quantify the propagation of polarized radiation. The Stokes parameters in spherical coordinates are written

\[
\begin{align*}
I &= \langle E_\phi E_\phi^* \rangle + \langle E_\theta E_\theta^* \rangle \\
Q &= \langle E_\phi E_\phi^* \rangle - \langle E_\theta E_\theta^* \rangle \\
U &= \langle E_\phi E_\theta^* \rangle + \langle E_\theta E_\phi^* \rangle \\
V &= i \left( \langle E_\phi E_\phi^* \rangle + \langle E_\theta E_\theta^* \rangle \right)
\end{align*}
\]

(3.24)

(3.25)

(3.26)

(3.27)

The parameters each describe a quality of the polarization. The parameter \(I\) is the total intensity of the radiation it does not contain any polarization info. The parameter \(Q\) is the tendency to align horizontal, \(Q > 0\) means horizontal and \(Q < 0\) means vertical. \(U\) is
the tendency to align at 45°. $V$ is a measure of circularity. The parameters are written in matrix form

$$S = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

As the electron precesses around the magnetic field the emitted radiation will be shifted from the original Zeeman frequency. The shifting of the line and the polarization can all be written in terms of the Stokes parameters. To quantify the shift consider the equation of motion

$$m_e \ddot{r} + k\vec{r} = -e(\vec{v} \times \vec{B})$$ (3.28)

where, $m_e$ is the mass of the electron, $k\vec{r}$ is the restoring force, and the right hand term, $-e(\vec{v} \times \vec{B})$ is the Lorentz force. Remember the $\vec{B}$ is along $\hat{z}$. The equations in each direction are

$$\ddot{x} + \omega_0^2 x = \left( \frac{e\vec{B}}{m_e} \right) \dot{y}$$ (3.29)

$$\ddot{y} + \omega_0^2 y = \left( \frac{e\vec{B}}{m_e} \right) \dot{x}$$ (3.30)

$$\ddot{z} + \omega_0^2 z = 0$$ (3.31)

Where $\omega_0$ is the frequency of oscillation, $\omega_0 = (k/m_e)^{1/2}$. Solving the second order differential equations

$$x(t) = A \cos \omega_L t \cos \omega_0 t$$ (3.32)

$$y(t) = A \sin \omega_L t \cos \omega_0 t$$ (3.33)

$$z(t) = A \cos \omega_0 t$$ (3.34)
Where $\omega_L$ is the Lamor precession frequency, $\omega_L = \frac{eB}{2m_e}$ and $A$ is the amplitude of oscillation. Expressing the equations of motion in complex form, with $\omega_\pm = \omega_0 \pm \omega_L$, and differentiating twice,

\[
\dddot{x} = -\frac{A}{2} \left( \omega_+^2 \exp i\omega_+ t + \omega_-^2 \exp i\omega_- t \right) \tag{3.35}
\]

\[
\dddot{y} = \frac{iA}{2} \left( \omega_+^2 \exp i\omega_+ t - \omega_-^2 \exp i\omega_- t \right) \tag{3.36}
\]

\[
\dddot{z} = -A\omega_0^2 \exp i\omega_0 t \tag{3.37}
\]

Now the electric field components can be written as

\[
E_\theta = \frac{eA}{8\pi\epsilon_0 c^2 R} \left[ \cos \theta \left( \omega_+^2 \exp i\omega_+ t + \omega_-^2 \exp i\omega_- t \right) + 2\omega_0^2 \exp i\omega_0 t \right] \tag{3.38}
\]

\[
E_\phi = \frac{e}{4\pi\epsilon_0 c^2 R} \left( \omega_+^2 \exp i\omega_+ t - \omega_-^2 \exp i\omega_- t \right) \tag{3.39}
\]

Note that the $\theta$ angle is between the observer and the magnetic field $\vec{B}$. Using the formula 3.2.2 to solve for the Stokes parameters,

\[
S = \frac{8}{3} \left( \frac{eA}{8\pi\epsilon_0 c^2 R} \right)^2 \begin{pmatrix}
1/2(\omega_+^4 + \omega_-^4)(1 + \cos^2 \theta) + 1/2\omega_0^4 \sin^2 \theta \\
-1/4(\omega_+^4 + \omega_-^4) \sin^2 \theta + 1/2\omega_0^4 \sin^2 \theta \\
0 \\
1/2(\omega_+^4 + \omega_-^4) \cos \theta
\end{pmatrix}
\]

Absorbing constants and rewriting in terms of the line frequencies

\[
S = \frac{1}{4} \begin{pmatrix}
I(\nu - \nu_-) \\
-\sin^2 \theta \\
0 \\
-2\cos \theta
\end{pmatrix}
\begin{pmatrix}
(1 + \cos^2 \theta) \\
-\sin^2 \theta \\
0 \\
-2\cos \theta
\end{pmatrix}
+ \begin{pmatrix}
2\sin^2 \theta \\
0 \\
0 \\
2\cos \theta
\end{pmatrix}
+ \begin{pmatrix}
(1 + \cos^2 \theta) \\
-\sin^2 \theta \\
0 \\
2\cos \theta
\end{pmatrix}
\]

62
If the magnetic field is orientated parallel along the line of sight of the observer so that \( \theta = 0^\circ \), then

\[
S = \frac{1}{4} \begin{bmatrix}
I(\nu - \nu_-) \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + I(\nu - \nu_+) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
\end{bmatrix}
\]

Only circularly polarized light will be seen when the line of sight is along the magnetic field since the only non-zero parameter that measures polarization is the \( V \) parameter which measures the circularity of the light. The left-handed circularly polarized light will be shifted to the lower frequency \( \nu_- \) and the right-handed circularly polarized component will be shifted to the higher frequency \( \nu_+ \). This is known as the longitudinal Zeeman effect.

If the line of sight is perpendicular to the magnetic field so that \( \theta = 90^\circ \), then

\[
S = \frac{1}{4} \begin{bmatrix}
I(\nu - \nu_-) \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + 2I(\nu - \nu_0) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + I(\nu - \nu_+) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}
\end{bmatrix}
\]

In this case, only linearly polarized light will be seen because the only non-zero parameter is \( Q \) which is the measure of linear polarization. Note that the value for the shifted lines is \( Q < 0 \), which means that the polarization is purely vertical and the unshifted line is aligned horizontal, \( Q > 0 \).

Observing the polarization of the light from an electron oscillating at a frequency \( \nu_0 \) and at the shifted frequencies \( \nu_- \) and \( \nu_+ \) from Lamor precession will tell you the direction of the magnetic field along the line of sight.
3.3 Modeling the Galactic Magnetic Field and Positron Propagation

The model for a poloidal magnetic field was taken from Prouza & Šmída (2003). The components of the Galactic magnetic field are like a dipole field,

\[ B_x = -\frac{3K}{2R^3} \sin 2\theta \cos \phi \] \hspace{1cm} (3.40)
\[ B_y = -\frac{3K}{2R^3} \sin 2\theta \sin \phi \] \hspace{1cm} (3.41)
\[ B_z = -\frac{K}{R^3} (3 \cos^2 \theta - 1) \] \hspace{1cm} (3.42)

To avoid singularities in the center a cylinder of height 300 pc and diameter 100 pc with a constant field strength of 2 mG in the negative z direction is put into the Galactic center. The constant \( K \) is set to correspond to the observed features of the Galactic field and the field at the Sun is equal to 0.2 \( \mu \)G.

\[ K = \begin{cases} 
200 \text{ G pc}^3 & \text{R} < 2 \text{ kpc} \\
10^{-6} \text{ G} & 2 \text{ kpc} < \text{R} < 5 \text{ kpc} \\
10^5 \text{ G pc}^3 & \text{R} > 5 \text{ kpc}
\end{cases} \] \hspace{1cm} (3.43)

The distance a positron propagates along a magnetic field is related to the pitch angle between the direction of the magnetic field and the direction of the positron. The distance along the field is called \( dl \) and the arc-length of the gyration is called \( ds \). The distance along the field line \( dl \) is calculated using the component of the velocity in the \( z \)-direction. The arc-distance \( ds \) is calculated using the magnitude of the velocity.

\[ dl = v \| dt = v \cos \alpha dt \] \hspace{1cm} (3.44)
\[ ds = v dt \] \hspace{1cm} (3.45)
\[ \frac{dl}{ds} = \frac{v \cos \alpha dt}{v dt} = \cos \alpha \] \hspace{1cm} (3.46)
Figure 3.2: The 2 dimensional model of the Galactic magnetic field on the axis of x and z.

Figure 3.3: The 3D model of the Galactic magnetic field.
Figure 3.4: Components of velocity vector of particle with velocity not parallel with magnetic field. In this example the magnetic field is along the x-axis. The equations assume the magnetic field is aligned along the z-axis.

where $\alpha$ is the pitch angle, i.e. the angle between the magnetic field line and the velocity vector. If the positron is traveling along the magnetic field line then the pitch angle is zero and the positron travels as if in a straight line. The distance the positron travels should be the same without the term $\frac{dz}{ds}$ that corrects for the presence of the magnetic field. If the pitch angle is 90$^\circ$ then, $\cos \alpha = 0$ and the positron circles around going nowhere since all of the velocity is along the perpendicular direction, $v = v_\perp$.

The distance the positrons travels is affected by the column density of the gas as it travels along the arclength of the helix. A positron will not travel as far along a magnetic field as it would traveling freely, because of the increase of column depth. The column depth is $d\xi$ and is related to the density if the gas $\rho$ and the arc-distance by,

$$d\xi = \rho ds$$

(3.47)

Therefore, the relation between the distance traveled along the field and the arc-distance
is,

\[ dl = ds \cos \alpha \]  (3.48)

where, \( dl \) is the distance traveled along the field line, \( ds \) is the arc-distance along the helix the positron travels, and \( \alpha \) is the pitch angle.

**Distance Positron Travels Along Magnetic Field as a Function of Pitch Angle**

![Figure 3.5: The effect of the pitch angle on the distance traveled for a positron starting at an initial energy of 1 MeV.](image)

In a uniform medium of average density of one particle per cubic centimeter a positron can travel up to 20 kpc. Figure 3.5 shows how far a positron can travel in the average medium at different pitch angles. Only once the pitch angle becomes very close to 90° does the affect of pitch angle considerably change the distance the positron can travel. As positrons are created from the radioactive isotopes their pitch angles with the magnetic field are isotropic. There is no preferred direction of propagation upon production.

67
Therefore, in the model the initial conditions of the positrons are set such that the direction (i.e. pitch angle) is random.

At the scale of the model the change in propagation is assumed to follow the direction of the magnetic field. In the model the strength of the field is about a $\mu$G and the gyroradius is $\sim 10^{-13}$ kpc. From the values in the code of the magnetic field and velocity of the positron the pitch angle can be solved.

\[
\vec{B} \cdot \vec{v} = \cos \alpha \left| \vec{B} \right| \left| \vec{v} \right|
\] (3.49)

\[
\vec{B} \cdot \vec{v} = (v_x B_x) + (v_y B_y) + (v_z B_z)
\] (3.50)

\[
\alpha = \arccos \left( \frac{\vec{B} \cdot \vec{v}}{\left| \vec{B} \right| \left| \vec{v} \right|} \right)
\] (3.51)

\[
\alpha = \arccos \left( \frac{(v_x B_x) + (v_y B_y) + (v_z B_z)}{\left| \vec{B} \right| \left| \vec{v} \right|} \right)
\] (3.52)

This pitch angle determines the distance of positron propagation.

### 3.4 Summary

In order to explain the observed emission of the Galactic positron annihilation emission in the bulge from positron production sources in the disk, positrons are assumed to transport along the Galactic magnetic field. There is a basis for this transport if positrons can escape the turbulent field of the disk. The turbulent disk field is unknown. In fact, Prouza & Šmída (2003) models the turbulence as “field cells” of random magnetic direction displaced in the disk. This effect can trap positrons in the disk. This research models the two scenarios of the turbulent field trapping positrons in the disk and of positrons escaping the disk to transport along the regular poloidal field.
Chapter 4

Positron Annihilation

Positrons are created at energies above the energy of the ISM. They must slow down before annihilating. Some positrons may annihilate before thermalizing with the medium, which is called in-flight annihilation. After thermalization a positron can annihilate directly with an electron or through the formation of positronium. Positrons interact differently with different phases of the interstellar medium. Positrons have different cross sections for different media. The cross section is proportional to the probability that a positron will interact with a particle of a particular media. There are five ways a positron can annihilate: charge exchange with neutral atom through positronium, radiative combination with free electrons through positronium, direct annihilation with bound electrons, direct annihilation with free electrons, and annihilation on dust. Figure 4.1 diagrams the process of positron annihilation from positron production to the mechanisms of annihilation.

4.1 Positron Annihilation Process

4.1.1 Energy Loss

Positrons produced under the energy of 1 MeV lose energy by the energy loss mechanisms Coulomb collisions and ionization/excitation collisions with atoms. Energy loss due to Coulomb collisions is a continuous process, whatever the energy of the positron. Ion-
Figure 4.1: Diagram for the process and mechanisms of positron annihilation.

The Coulomb energy loss of a positron per column depth in the ionized components of the medium can be approximated by the formula from Axelrod (1980).

\[
\left( \frac{dE}{d\xi} \right)_{\text{plasma}} = -\frac{4\pi r_0^2 m_e c^2}{\beta^2 A m_n} \left( \ln \frac{\gamma \beta^2}{\hbar \omega_P / m_e c^2} + \ln 2 - \frac{\beta^2}{2} \right)
\]

(4.1)

where \( \beta = v/c \), gamma is the Lorentz factor, \( m_e c^2 \) is the rest mass, \( r_0 = 2.8 \times 10^{-13} \text{ cm} \) is the classical electron radius, \( \chi_e \) is the electron fraction, \( A \) is the atomic mass, \( m_n \) is the atomic mass unit, \( \hbar \omega_P = 5.6 \times 10^4 \sqrt{n_e} \text{ eV} \), and \( n_e \) is the electron number density.

The ionization and excitation collision energy loss for a positron with any medium...
is from (Berger & Seltzer 1964).

\[
\left( \frac{dE}{d\xi} \right)_{IE} = -\frac{4\pi r_0^2 mc^2 Z}{\beta^2 Amw} \left[ \ln \frac{\sqrt{\gamma - 1} \gamma \beta}{(I/mc^2)} + 2 \log 2 - \frac{\beta^2}{12} \left( \frac{23}{2} + \frac{7}{\gamma + 1} + \frac{5}{(\gamma + 1)^2} + \frac{2}{(\gamma + 1)^3} \right) \right]
\]  

(4.2)

(4.3)

where \( Z \) is the atomic number and essentially the number of electrons bound to the atom, and \( I \) is the mean excitation/ionization energy. The ionization of hydrogen is 13.6 eV and excitation is 10.2 eV. For molecular hydrogen the ionization is 15.4 eV and the excitation is 12.0 eV.

The total continuous energy loss per column depth of the positron is the sum of Coulomb and ionization/excitation in the densities of ionized gas, neutral hydrogen, and molecular hydrogen.

\[
\left( \frac{dE}{d\xi} \right)_{\text{Total}} = \left( \frac{dE}{d\xi} \right)_{\text{plasma}} + \left( \frac{dE}{d\xi} \right)_{IE,H} + \left( \frac{dE}{d\xi} \right)_{IE,H_2}
\]

(4.4)

Figure 4.2 shows the energy loss per column depth for Coulomb, ionization/excitation in hydrogen and the total.

Below 1 keV the interaction of positrons with the medium should be modeled by Monte Carlo simulation since at these energies a positron looses a significant portion of its energy in one interaction. After a positron reaches 1 keV the energy loss is either from ionization or excitation with discrete energy loss of either 13.6 eV or 10.2 eV.

The amount of energy a positron looses is measured per column depth. If a positron travels freely along a straight line then the distance it travels will be,

\[
R = \int dx = \int dx \frac{dE}{dE} = \int_{E_i}^{E_f} \frac{dx}{dE} dE
\]

(4.5)

where \( \frac{dx}{dE} \) is the energy loss formula Equation 4.3 converting the column depth to length by \( d\xi = \rho dx \). If the positron travels along a magnetic field line, then the positron will go through more gas per distance because the positron gyrates around the magnetic field line.
Figure 4.2: Ionization energy loss per column depth for a positron going through a uniform gas density of $n_e=1\text{ cm}^{-3}$ and $n_H=1\text{ cm}^{-3}$. The black line is IE energy loss, the blue line is Coulomb, and the red line is the total.
The more gas the positron travels through the more energy it will lose per distance (Figure 4.2). The gyroradius will decrease as the energy of the positron decreases. The distance the positron goes along the z-axis is \( dl \). The arclength is \( ds \).

\[
R = \int dl = \int dl \frac{ds}{dE} dE = \int_{E_i}^{E_f} \frac{dl}{ds} ds dE
\]  

(4.6)

Where \( \frac{ds}{dE} \) is the energy loss formula, Equation 4.3. The relationship between \( dl \) and \( ds \) is \( dl = ds \cos \alpha \), where \( \alpha \) is the pitch angle between the direction of the positron and the direction of the magnetic field. This relationship was discussed in Chapter 3. After each interaction that causes energy loss the pitch angle of the positron’s velocity with the magnetic field will change. For a positron starting at an initial energy of 1 MeV, undergoing ionization and excitation collisional energy loss (Equation 4.3) without a magnetic field present, will travel a distance \( R \approx 21 \) kpc. When a magnetic field is present, 70% of positrons will travel a significant distance of \( R \approx 5 - 21 \) kpc. A positron travels less than 1 kpc for a pitch angle above 88°. Figure 3.5 shows how far a positron travels at a certain pitch angle. Figure 4.3 compares the energy loss per column depth of a positron freely traveling (black line) and the energy loss of a positron confined to gyrate around a magnetic field line (magenta line). Therefore, if a magnetic field is present to propagate positrons, the positron will go through more gas per step than if there was no magnetic field, and the result is to lose energy faster per step and thermalize with the medium faster. A positron confined to propagate along a magnetic field will lose more energy per column depth and thus not travel as far before annihilation.

4.1.2 In-flight Annihilation

A positron can annihilate in-flight before it thermalizes with the medium of annihilation. In-flight annihilation produces shifted photons above 511 keV. The energy of the photons, \( E_\gamma \), that are produced from in-flight annihilation are in the range of \( mc^2/2 \leq E_\gamma \leq E + mc^2/2 \), where \( E \) is the total energy of the positron. To show how to calculate the
Figure 4.3: Ionization energy loss per column depth for a positron through a uniform gas density of 2 particles of hydrogen per cm$^3$. The black line is without a magnetic field and the magenta line is when the positron gyrates around the field line at a pitch angle of $\theta = \pi/3$ as it travels.
minimum and maximum energy of the photons from in-flight annihilation start with momentum and energy conservation. Before annihilation the momentum is \( \vec{p} \) and the energy is \( E + mc^2 \), \( E^2 = m^2c^4 + p^2c^2 \). After the annihilation the momentum and energy of the photons is \( \vec{p}_1, \vec{p}_2 \) and \( E_1 = p_1c, E_2 = p_2c \).

Conservation of momentum:

\[
\vec{p} = \vec{p}_1 + \vec{p}_2
\]  

Conservation of energy:

\[
E + mc^2 = E_1 + E_2
\]  

Figure 4.4: Diagram of (a) direct annihilation in center of mass frame and (b) in-flight which produces shifted photons greater than 511 keV
We will solve for the energy of the first photon given $\theta = \theta_1 + \theta_2$. To solve for $E_1$ use Equation 4.1.2 to solve for $E_2$. The cosine comes from the cosine rule.

\[
\begin{align*}
\vec{p}_2 &= \vec{p} - \vec{p}_1 \quad (4.9) \\
p_2^2 &= p^2 + p_1^2 + 2pp_1 \cos \theta_1 \quad (4.10) \\
p_2^2 &= (p^2 + p_1^2 + 2pp_1 \cos \theta_1)c^2 \quad (4.12) \\
E_2^2 &= E^2 - m^2c^4 + E_1^2 + 2pcE_1 \cos \theta_1 \quad (4.13)
\end{align*}
\]

Using conservation of energy equation

\[
\begin{align*}
E_2 &= E + mc^2 - E_1 \quad (4.14) \\
E_2^2 &= E^2 + m^2c^4 + E_1^2 + 2Emc^2 - 2EE_1 - 2E_1mc^2 \quad (4.15)
\end{align*}
\]

Equate 4.13 and 4.15.

\[
E^2 - m^2c^4 + E_1^2 + 2pcE_1 \cos \theta_1 = E^2 + m^2c^4 + E_1^2 + 2Emc^2 - 2EE_1 - 2E_1mc^2 \quad (4.16)
\]

Solve for $E_1$

\[
E_1 = \frac{mc^2(m^2 + E)}{E + mc^2 - pc \cos \theta_1} \quad (4.17)
\]

Write in terms of the particle kinetic energy $K$, $E = K + mc^2$.

\[
E_1 = \frac{mc^2(K + 2mc^2)}{K + 2mc^2 - pc \cos \theta_1} \quad (4.18)
\]
Replace \( pc \) with kinetic energy term

\[
E^2 = p^2c^2 + m^2c^4 = K^2 + m^2c^4 + 2Kmc^2 \tag{4.20}
\]

\[
p^2c^2 + m^2c^4 = K^2 + m^2c^4 + 2Kmc^2 \tag{4.21}
\]

\[
p^2c^2 = K(K + 2mc^2) \tag{4.22}
\]

\[
pc = \sqrt{K(K + 2mc^2)} \tag{4.23}
\]

Therefore,

\[
E_1 = \frac{mc^2(K + 2mc^2)}{K + 2mc^2 - \sqrt{K(K + 2mc^2)} \cos \theta_1} \tag{4.24}
\]

This can be simplified by dividing the top and bottom of the fraction by \((K + 2mc^2)\)

\[
E_1 = \frac{mc^2}{1 - \frac{K\sqrt{1+2mc^2/K}}{K(1+2mc^2/K)}}
\]

\[
= \frac{mc^2}{1 - \cos \theta_1(1 + 2mc^2/K)^{-1/2}} \tag{4.25}
\]

Use the binomial theorem \((1 + x)^{-1/2} \approx 1 - x/2\) for \(K \gg mc^2\) to solve for the energy range \(E_{min}\) to \(E_{max}\)

\[
E_1 \approx \frac{mc^2}{1 - \cos \theta_1(1 - mc^2/K)} \tag{4.26}
\]

\[
E_{min}(\theta_1 = 180^\circ) = \frac{mc^2}{1 + 1 - mc^2/K} = \frac{mc^2}{2} \tag{4.27}
\]

\[
E_{max}(\theta_1 = 0^\circ) = \frac{mc^2}{1 - 1 + mc^2/K} = K \tag{4.28}
\]

The in-flight annihilation will shift the energy of the 511 keV photon which can be seen in the line profile of the annihilation. Jean et al. (2006) fit the line with a narrow and broad Gaussian and concluded that the broad fit is from the annihilation of the para-
positronium state formed in-flight. See the introduction for more.

4.1.3 Annihilation Mechanisms

Once the positron has thermalized with the medium the positron can annihilate directly with an electron or through the formation of positronium. Positronium can form in two states, the para- and ortho-positronium states. In the para-positronium state the positron with produce two photons of equal energy 511 keV. The ortho-positronium state produces a spectrum of photon energies. The distribution of energies was solved by Ore & Powell (1949). Figure 4.5 shows the energy spectrum of the photons from the annihilation via ortho-positronium.

\[
\omega(\epsilon) = \frac{2}{\pi} - \frac{9}{2} \left( \frac{\epsilon (1 - \epsilon)}{(2 - \epsilon)^2} + \frac{2(1 - \epsilon)}{\epsilon} \ln 1 - \epsilon + \frac{2 - \epsilon}{\epsilon} - \frac{2(1 - \epsilon)^2}{(2 - \epsilon)^3} \ln 1 - \epsilon \right) \tag{4.30}
\]

Charge Exchange is when a positron has sufficient energy to take the electron from a bound atom \( X \), such as neutral or molecular hydrogen, and form positronium.

\[
e^+ + X \rightarrow Ps + X^+ \tag{4.31}
\]

The positron has to be above an energy threshold to form positronium via charge exchange. First, to get the electron away from the atom, there has to be enough energy to overcome the binding energy, or the ionization potential \( E_i \), of the electron to the atom \( X \) minus the binding energy of positronium, which has a binding energy of -6.8 eV.

\[
E \geq E_i - 6.8 \text{ eV} \tag{4.32}
\]

For example, the binding energy of hydrogen is -13.6 eV, the ionization potential \( E_i = 13.6 \text{ eV} \). Therefore, the energy threshold will be 6.8 eV.
Figure 4.5: The energy spectrum of photons from the annihilation of ortho-positronium.

Table 4.1: Charge exchange energy thresholds for certain atoms.

<table>
<thead>
<tr>
<th>Process</th>
<th>Ionization Potential [eV]</th>
<th>Threshold [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ + H \rightarrow Ps + H^+$</td>
<td>13.6</td>
<td>6.8</td>
</tr>
<tr>
<td>$e^+ + H_2 \rightarrow Ps + H_2^+$</td>
<td>15.4</td>
<td>8.6</td>
</tr>
<tr>
<td>$e^+ + He \rightarrow Ps + He^+$</td>
<td>24.6</td>
<td>17.8</td>
</tr>
</tbody>
</table>
**Radiative Recombination** is the formation of positronium when a positron captures a free electron and emits a photon.

\[ e^+ + e^- \rightarrow Ps + \gamma \]  \hspace{1cm} (4.33)

**Direct Annihilation** is when a positron annihilates with a free electron and produces two photons of equal energy.

\[ e^+ + e^- \rightarrow \gamma + \gamma \]  \hspace{1cm} (4.34)

**Direct Annihilation with Bound Electron** produces two photons of equal energy. The electron is bound to an atom, X, such as neutral or molecular hydrogen. After the interaction the atom is ionized, X⁺.

\[ e^+ + X \rightarrow \gamma + \gamma + X^+ \]  \hspace{1cm} (4.35)

The cross sections for charge exchange, ionization/excitation with hydrogen, and radiative recombination were taken from Guessoum et al. (2005). The cross sections for direct annihilation and direct annihilation with hydrogen are from Milne (1998) using the formulas,

\[ \sigma_{DA} = \frac{\pi r_0^2}{\gamma + 1} \left( \frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} - \ln \left( \gamma + \sqrt{\gamma^2 - 1} - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right) \right) \]  \hspace{1cm} (4.36)

At low energies the cross section must be corrected for Coulomb interactions. At about 1 keV the cross section is,

\[ \sigma_{DA} = \frac{\pi r_0^2}{1 - \exp \left( -2\pi\alpha/\beta \right)} \]  \hspace{1cm} (4.37)
Figure 4.6: The cross sections in units of $10^{-16}$ cm$^2$ for each annihilation mechanism and the discrete energy loss of ionization and excitation. The green line is the Maxwell Boltzman distribution at $T=8000$ K in a warm medium to show at what energy a positron will start to thermalize. After thermalization the important annihilation mechanisms are CE, RR, DAH, and DA.
where, $\alpha = 1/137.036$ is the fine structure constant. The cross section for direct annihilation with hydrogen is,

$$
\sigma_{DAH} = \sqrt{\frac{13.6}{E}} \pi a_0^2 \alpha^3 \left(13 - 8 \exp \left(-\left(\sqrt{\frac{E}{13.6}} - 0.45\right)^2/0.36\right)\right) \tag{4.38}
$$

4.2 ISM Gas Phases

The gas of the ISM is what slows the positrons and what the positrons annihilate with. For energy loss the positron goes through an ionized gas, neutral hydrogen gas and the molecular hydrogen gas. For annihilation, positrons annihilate in a warm ionized and neutral medium (i.e. a partially ionized medium). According to Guessoum et al. (2005) and Jean et al. (2006) the gas that is responsible for the 511 keV is composed of neutral hydrogen and ionized gas at a temperature considered warm $T = 8000K$ compared to the temperature of the cold phase of $T = 10 - 80K$ and the hot phase $T = 10^6$. In this research the positrons annihilate in the warm medium.

Each type of gas has its own Galactic distribution. The ionized gas distribution was discussed in Chapter 2 along with the distribution of massive stars that create the ionized gas. The 3D model of the ionized gas is from Taylor & Cordes (1993). HI is measured with the hydrogen 21 cm line. This line is a "forbidden" transition in the ground state of hydrogen, called the spin-flip transition when the spins of the proton and electron change from parallel to antiparallel. The structure and dynamics of the Milky Way are found using the technique of the Doppler shifted neutral hydrogen 21 cm radio line from the ground state of hydrogen Kalberla & Kerp (2009). The molecular hydrogen density is inferred from the map of CO molecules.

4.2.1 The Hyperfine Splitting of the Ground State of Hydrogen

Hyperfine splitting of the hydrogen ground state energy level is a result of the electron dipole moment interacting with the magnetic field of the proton magnetic dipole
moment. The magnetic moment of the electron and proton are proportional to their spin. The magnetic field set up by the proton is proportional to the proton’s magnetic dipole moment. The Hamiltonian of an electron is a magnetic field created by the proton is written as

\[ H = -\mu_e \cdot B_p \] (4.39)

Therefore, the energy of the now split ground level is proportional to the dot product of the spins.

\[ E \propto S_e \cdot S_p \] (4.40)

The alignment of the spins could be parallel or anti-parallel, which creates two states, the triplet state and the singlet state, respectively. The difference in energy between these two states in the hyperfine structure of the ground state of hydrogen is 5.88 \times 10^{-6} \text{ eV}, equivalent to a wavelength of 21 cm (1420 MHz).

\[ \Delta E = 5.88 \times 10^{-6} \text{ eV} \] (4.41)
\[ \nu = \Delta E/h = 1420 \text{ MHz} \] (4.42)
\[ \lambda = c/\nu = 21 \text{ cm} \] (4.43)

A more detailed analysis of hyperfine splitting can be found in Griffiths (1995). The spontaneous transition probability, the Einstein A coefficient, \( A = 2.869 \times 10^{-15} \text{ s}^{-1} \) is very small and considered forbidden. However, in the interstellar medium the volume of hydrogen is 90\% hydrogen. Due to the large volume of hydrogen, the emission transitions can be seen.
4.2.2 Mapping Neutral Hydrogen Densities

The relationship between an body emitting at a particular wavelength or frequency at rest and emitting while at a certain velocity is called the Doppler effect.

\[
\frac{v}{c} = \frac{\nu - \nu_0}{\nu_0} = \frac{\Delta \nu}{\nu_0}
\] (4.44)

A cloud of HI emitting at 21 cm moving with some speed along the line of sight relative to Earth, will be shifted to some other wavelength. The shift in the wavelength corresponds to the velocity at which the cloud travels. The velocity of the HI cloud is related to its distance from the Galactic center. The velocity is converted into distance using the Galactic rotation curve. Therefore, a distance to a cloud of HI can be found just by using the Doppler shift of the line. The amount of HI at a particular distance depends on the brightness of the shifted line profile, which is directly proportional to the amount of HI, i.e. the optical depth \( \tau \). One way to characterize brightness (specific intensity \( I_\nu \)) at a frequency \( \nu \) is to give the temperature of the blackbody having some brightness at the frequency.

\[
I_\nu = B_\nu(T_B)
\] (4.45)

In the radio wavelengths, \( h\nu \ll kT \), which is in the Rayleigh-Jeans limit.

\[
B_\nu(T_B) = \frac{2\nu^2k}{c^2}T_B
\] (4.46)

Thus, the brightness can be written in terms of brightness temperature. \( T \) is the temperature of the gas.

\[
T_B = T(1 - e^{-\tau})
\] (4.47)

The optical depth, \( \tau \), is a function of the number of emitters in some volume along a line of sight (written here as column depth, \( N_H \)), the absorption cross section \( s \), and the Maxwell
line shape function $\phi$.

$$\tau = N_H \cdot S \cdot \phi \quad (4.48)$$

$$= N_H \left( 0.02654 \frac{h\nu}{kT} f \right) \left( \frac{\lambda_0}{\sqrt{\pi} b} e^{-v^2/b^2} \right) \quad (4.49)$$

$$= 5.49 \times 10^{-14} \frac{N_H e^{-v^2/b^2}}{\sqrt{\pi} b T} \quad (4.50)$$

where $f = 5.75 \times 10^{-12}$ is proportional to the Einstein A probability transition coefficient and $b$ is $\sqrt{2kT/m}$. To get the column depth integrate over all velocities.

$$N_H = 1.823 \times 10^{13} \int T_B(v) \left[ \frac{\tau}{1 - e^{-\tau}} \right] dv \quad (4.51)$$

For a more detailed description see Chapter 3 of Spitzer (1978).

### 4.2.3 Mapping Molecular Hydrogen

Molecules have rotational and vibrational energy levels. Celestial molecules are observed from the lines created by their transitions between rotational and vibrational energy levels. The transition lines from molecular hydrogen are in the ultra violet regime, which is difficult to measure in the Galaxy because of extinction from dust. However, molecular hydrogen is found with other molecules in a molecular cloud. Molecules are destroyed, i.e. dissociate, by UV radiation from hot stars. In a molecular cloud the outer layers contain molecules that have been destroyed protecting those molecules in the center. The outer layer absorbs the UV radiation. Therefore, molecular hydrogen can be traced by emission from other molecules, most notable the CO molecule. The CO $J = 1 \to 0$ rotational transition can be seen at 2.6 mm in radio wavelengths. CO is a tracer for molecular hydrogen.
4.3 Modeling Gas Phases

The ionized gas phase was modeled using Taylor & Cordes (1993) 3D model of free electrons. The model was already discussed in Chapter 2. The dispersion measure of radio emission from pulsars is directly proportional to the column density of free electrons along the line of sight. The measurement of large scale distribution of interstellar free electrons is done by applying the dispersion measure from pulsars with independent distance estimates. Cordes et al. (1991) used pulsar distances and their dispersion measures to construct an axisymmetric model of the free electron space average density. Taylor & Cordes (1993) added spiral arm structure based on the spiral arm patterns observed in optical and radio wavelengths of the ionized gas regions. The location of the spiral arms are fixed and the dependence of the dispersion measure of pulsars is used to determine the features of the arms. A least squares grid search procedure is used to evaluate the remaining free parameters of the three dimensional ionized gas model.

I used Amôres & Lépine (2005) distribution to model the neutral and molecular hydrogen distribution. Amôres & Lépine (2005) were using the distribution of neutral and molecular hydrogen to model interstellar extinction in the Galaxy. The hypothesis is that dust is well mixed with the gas and extinction can be measured by the column density of the gas. Data for HI came from the Berkeley survey, the Parkes survey, and the NRAO survey. H$_2$ data is from the Columbia survey (Dame et al. 2001) and the ratio of integrated intensity of CO to H$_2$ using a constant of $1.8 \times 10^{20}$ cm$^{-2}$ $(K\text{kms}^{-1})^{-1}$.

The radial density function for HI and H$_2$ is of the form

$$n_{HI,H_2} = c \exp \left( -\frac{r}{a} - \left( \frac{b}{r} \right)^2 \right)$$ (4.52)

with parameters, $a$, $b$ and $c$ in Table 4.2. The large distribution of H$_2$ in the center region is modeled separately using,

$$n_{H_2} = \beta \exp \left( -\left( \frac{r}{\alpha_{\beta}} \right)^2 \right)$$ (4.53)

86
where \( \alpha = 0.1 \) kpc and \( \beta = 240.0 \) cm\(^{-3}\). Figure 4.7 shows the radial density distribution for both media.

The scale height density depends on the Galactic radius,

\[
n_{HI, H_2}(r, z) = n_{HI, H_2}(r, z_c) \exp \left( - \left( \frac{z - z_c}{1.2z_{1/2}} \right)^2 \right) \tag{4.54}
\]

where \( z_c = 20 \) pc since the sun is not located exactly in the middle of the disk and \( z_{1/2} \), the scale height half-width at half-maximum, is,

\[
z_{1/2} = 45 \exp(0.1r) \tag{4.55}
\]
where $z_{1/2}$ is in parsecs and $r$ is in kiloparsecs. This expression is fine for $H_2$. In the case of HI the $z_{1/2}$ is multiplied by a factor of 1.8.

<table>
<thead>
<tr>
<th>Medium</th>
<th>$a$ [kpc]</th>
<th>$b$ [kpc]</th>
<th>$c$ [cm$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HI</td>
<td>7.0</td>
<td>1.9</td>
<td>0.7</td>
</tr>
<tr>
<td>$H_2$</td>
<td>1.2</td>
<td>3.5</td>
<td>58.0</td>
</tr>
</tbody>
</table>

Amôres & Lépine (2005) also include spiral arm structure along with the axisymmetric model of the gas distribution to test how spiral arms affect interstellar extinction. The discussion on the construction of the spiral arms was not in their paper. It is clear that the arms do not necessarily reflect the usual four major arms of the Milky Way. However, this research includes the location of arms because these areas of high density are important for the location of annihilation of positrons.

Figures 4.8, 4.9, and 4.10 show the neutral hydrogen, molecular hydrogen and the combination in the xy plane showing the arm structure and an all-sky view in Galactic coordinates.

The color scale may be misleading. The most dense areas are in the arms not outside the arms. In the map of neutral hydrogen there is actually a dearth of density in the Galactic center region. This region is about 2 kpc in diameter. In the map of molecular hydrogen the dense areas are the arms and in the Galactic center not the surrounding area around the arms. These areas appear white because of such low density and the scaling of the colors. In the maps of the xy plane the peak density for neutral hydrogen is 4 cm$^{-3}$ and for molecular hydrogen the peak density occurs in the center at 240 cm$^{-3}$.

### 4.4 Summary

There is a dearth of neutral hydrogen and ionized gas in the Galactic bulge for positrons to annihilate in that region. The molecular hydrogen is in the very center region for positrons to slow and possibly annihilate. The annihilation emission from propagating
Figure 4.8: Neutral hydrogen in the xy plane and in galactic coordinates.
Figure 4.9: Molecular hydrogen in the xy plane and in galactic coordinates.
Figure 4.10: Neutral and molecular hydrogen combination in the xy plane and in galactic coordinates.
positrons is expected to trace the density of the partially ionized medium.

The diameter of the hole in neutral hydrogen is 2 kpc. The extent of the 511 keV positron annihilation emission is measured to be $\sim 8^\circ$ (FWHM) (Knödlseder et al. 2005). If the sun is located at 8.5 kpc from the Galactic center then the radius of the emission is 600 pc. The observed diffuse bulge 511 keV annihilation emission is within the radius where there the hydrogen density is practically zero. If annihilating with hydrogen is the most probable mechanism for the observed emission then from this model of hydrogen density, there should be no indication of annihilation in the Galactic center. However, the positrons are propagated to the center by the poloidal magnetic field and are slowed by the high density of molecular hydrogen in this region. But, even if they are propagated and slowed in the Galactic center the likelihood of annihilating is slim compared to the other areas of the Galaxy that have a high density of neutral hydrogen.
Chapter 5

Results & Discussion

Three models of positron transport were investigated: the turbulent, transport, and extra gas models. The turbulent model uses the assumption that the turbulent magnetic field confines positrons to the disk and positrons annihilate where they are produced. The transport model assumes that positrons will leave their place of origin and propagate along the Galactic poloidal magnetic field, and go through the gas of Galaxy to slow and then annihilate in the dense regions of gas. The extra gas model is where an extra amount of gas is placed in the Galactic bulge region in order to see how propagated positrons would annihilate with the gas to explain the observed emission from the bulge region.

5.1 Turbulent Model

The turbulent gas model makes the assumption that positrons annihilate where they are produced because the turbulent Galactic magnetic field confines them to not travel far from their production source. The 511 keV emission from this map is showing where the production sources are located. The positrons in models are created from the radioactive isotopes $^{26}$Al and $^{44}$Ti, whose distribution does not include a central distribution. Therefore, we would not see the observed morphology of the emission in the turbulent model.

What is notable about the emission of the turbulent model is the asymmetry in the
Figure 5.1: The map of the 511 keV annihilation emission, where positrons are assumed to be trapped due to the turbulent Galactic magnetic field and annihilate where produced. \[ \text{Intensity}_{\text{max}} = 1.2 \times 10^{-3} \, \text{ph cm}^{-2} \, \text{s}^{-1} \, \text{sr}^{-1} \]
emission in negative longitudes versus positive longitudes. The ratio of right to left shows that the right is 1.2 times brighter than the left. An asymmetry in the 511 keV positron annihilation emission has been observed by Weidenspointner et al. (2008a) with a ratio of 1.8. See Section 5.6 for more about the emission asymmetry.

5.2 Transport Model

Figure 5.2: The map of positron annihilation emission, where positrons are assumed to be transported along the Galactic poloidal field going through the Galactic center and annihilate with the gas. \( \text{Intensity}_{\text{max}} = 1.0 \times 10^{-3} \, \text{ph cm}^{-2} \, \text{s}^{-1} \, \text{sr}^{-1} \)

Figure 5.2 shows the model where positrons are transported from the disk along the Galactic magnetic field. The transport model makes the assumption that positrons will propagated along the Galactic poloidal magnetic field lines. Positrons will leave the disk where they are produced to travel along the field lines and then slow and annihilate with the high densities of gas in the disk region. Since positrons are allowed to propagate, this
model shows the location of the gas and where it is most dense to slow and annihilate the positrons.

What is notable about this model is that positrons will slow and annihilate differently in the different gas phases. The neutral gas phase is modeled to have a minimal scale height in the Galactic center and flare out as the radius increases. The ionized gas is modeled with a larger scale height than the neutral gas at radii close to the center. Therefore, as positrons travel out of the disk and propagate into the Galactic center the positrons will slow and annihilate via direct annihilation in the ionized gas region rather than the neutral gas. One can see the direct in-flight annihilation emission between the longitude $45^\circ < l < -45^\circ$ and $15^\circ < b < -15^\circ$.

The direct in-flight annihilation leads to a lower positronium fraction. Positronium if formed in-flight by the mechanism of charge exchange with neutral hydrogen. However, in the region of the Galactic center as positrons are transported by the field, the positrons lose energy and annihilate in the region where the density of ionized gas is much greater than the density of neutral hydrogen.

There is also an asymmetry in the 511 keV emission in this model. As the positrons are transported along the magnetic field lines the positrons slow and annihilate in the gas of high density. The transport of positrons shows the same asymmetry as the turbulent model.

This model does not explain the observed 511 keV emission morphology. There is no significant emission coming from the bulge region where 511 keV emission is observed. Positrons are not created in the bulge region, but are allowed to be transported there by the Galactic magnetic field. However, the positrons travel through the bulge region without much slowing or annihilation because there is no gas in the bulge region for slowing positrons or to annihilate them.
Figure 5.3: The map of the 511 keV annihilation emission with an artificial density of gas in the Galactic center. Intensity$_{max} = 2.5 \times 10^{-3}$ ph cm$^{-2}$ s$^{-1}$ sr$^{-1}$
5.3 Extra Gas Model

Figure 5.3 shows the 511 keV emission in the case where an extra density of gas was placed in the bulge region. In the extra gas model a density of gas was placed in the Galactic bulge region to explore how the observed 511 keV positron annihilation may be explained, since positrons annihilate where the gas density is high enough to slow and annihilate them. The extra density of gas is a spherical Gaussian shape that was added to the model of the gas from Amôres & Lépine (2005) with a width to fill the lack of density within the inner 1 kpc and a height of about twice the maximum density of the gas in the arms of model gas.

Amôres & Lépine (2005) derived their radial distribution of neutral hydrogen from surveys of the neutral hydrogen column depth. At intervals of 1° for $-180° < l < 180°$ they compared the observed column density with a theoretical column density obtained by integration along the line of sight of a tentative radial density function varying the parameters to minimize the rms difference. This resulted in a radial density function with a dearth of neutral hydrogen within a radius of 1 kpc of the Galactic center.

Ferrière et al. (2007) notes that within the innermost 3 kpc of the Galaxy neutral hydrogen is confined to a noticeably tilted layer extended out to a radius of 1.5 kpc. The density of neutral hydrogen within a radius of 1.5 kpc (which defines the entire central molecular zone) is about 1 cm$^{-3}$ in their axisymmetric model. The density of hydrogen gas within the Galactic center is constrained to be about 1 cm$^{-3}$ using observations from Burton & Liszt (1978). Therefore, the observed density of the neutral hydrogen within 1 kpc may not be enough to explain how positrons will slow or annihilate within this region.

This model shows that positron slowing and annihilation can happen in the bulge region if there is a significant amount of gas for positrons to interact with. However, positrons are created at a spectrum of energies. The low energy positrons will slow and annihilate in the extra gas. The positrons created at higher energies will slow in the extra gas, but do not annihilate there. This model shows 511 keV in the positive and negative
latitudes from direct in-flight annihilation where positrons are slowed by going through the extra bulge gas, but annihilate in the region where the ionized gas density is much larger than the neutral gas density. The extra bulge gas is efficient at slowing the positrons so that they can annihilate outside of the region. This creates a reduction in the fraction of positrons that annihilate in-flight through charge exchange. Therefore, even if positrons could propagate to the bulge region and have gas to annihilate with, that does not guarantee that positrons will slow enough to annihilate in the Galactic bulge region.

5.4 Flux Measurements

Table 5.1: Results: Flux in units of ph cm$^{-2}$ s$^{-1}$ with a distance to the Galactic center of 8.5 kpc.

<table>
<thead>
<tr>
<th></th>
<th>Turbulent</th>
<th>Transport</th>
<th>Extra Gas</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>$2.3 \times 10^{-6}$</td>
<td>$2.3 \times 10^{-6}$</td>
<td>$8.7 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$^{44}$Ti</td>
<td>$3.5 \times 10^{-6}$</td>
<td>$3.5 \times 10^{-6}$</td>
<td>$1.3 \times 10^{-5}$</td>
<td>$7.3–21.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{56}$Ni</td>
<td>$1.2 \times 10^{-5}$</td>
<td>$1.2 \times 10^{-5}$</td>
<td>$4.3 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$1.8 \times 10^{-5}$</td>
<td>$1.8 \times 10^{-5}$</td>
<td>$6.5 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Disk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$8.2 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$^{44}$Ti</td>
<td>$1.9 \times 10^{-4}$</td>
<td>$1.9 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$7.3–9.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{56}$Ni</td>
<td>$6.2 \times 10^{-4}$</td>
<td>$6.3 \times 10^{-4}$</td>
<td>$4.1 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$9.3 \times 10^{-4}$</td>
<td>$9.5 \times 10^{-4}$</td>
<td>$6.12 \times 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 shows the results of the fluxes from each model (turbulent, transport, and extra gas) compared to observed fluxes from Weidenspointner et al. (2008b). The fluxes are shown for the bulge and disk. The bulge is defined by being between $4^\circ$ and $-4^\circ$ in both Galactic longitude and latitude. This is the FWHM of the observed bulge emission. For the bulge and disk the fraction of the flux that comes from each radioactive isotope is shown.

In the turbulent and transport models the fraction of the disk flux that comes from
$^{26}$Al and $^{44}$Ti is about 30-40% compared to the observed flux. The positrons from these isotopes are not enough to explain the disk emission in whether in the limit positrons annihilate where they are produce or if they are allowed to propagated along the Galactic field lines. However, we have calculated that $^{56}$Ni can produce $1 \times 10^{43} \text{ e}^+ \text{ s}^{-1}$ in the Galaxy. If the contribution of positrons from $^{56}$Ni is accounted for then the positron annihilation emission from the disk in the turbulent and transport models can explain the observed positron annihilation emission.

In the extra gas model the total flux in the disk from $^{26}$Al, $^{44}$Ti, and $^{56}$Ni decreases. This is because the positrons are propagating from the disk into the extra gas in the bulge region and annihilating. The flux in the bulge from the extra gas increases. However, the flux does not increase enough to explain the observed emission.

In the turbulent and transport models the disk flux can be explained. In the extra gas model the positrons are transported to the bulge region and annihilate there, but the flux does not increase significantly to explain the observed bulge flux and the disk flux decrease. Therefore, positron propagation from the production sites of positron production sources cannot explain neither the observed disk flux nor the observed bulge flux.

The observed flux measurements and the model flux measurements are based on a system where the sun in at a distance of 8.5 kpc from the Galactic center. However, the exact distance to the Galactic center is still disputed and can range from 7–8.5 kpc. If the distance to the Galactic center is actually 7 kpc then the flux measurements should be scaled and can present some uncertainty.

$$F \propto \frac{1}{r^2}$$

(5.1)

The flux at a distance of 7 kpc, $F_7$, can be scaled using the ratio of the flux at a distance of 8.5 kpc, $F_{8.5}$.

$$F_7 = \left(\frac{8.5}{7}\right)^2 F_{8.5}$$

(5.2)
If the distance to the Galactic center is 7 kpc then the flux measurements of the turbulent, transport and extra gas models will increase by a factor of $\sim 1.5$. The sum of the scaled $^{26}\text{Al}$ and $^{44}\text{Ti}$ flux will be $4.6 \text{ ph cm}^{-2} \text{ s}^{-1}$. The scaled flux of $^{26}\text{Al}$ and $^{44}\text{Ti}$ can explain $50 - 63\%$ of the observed flux. It doesn’t take as many positrons per second from each source to explain the flux from the disk. However, the scaled flux still does not explain the observed emission from the bulge.

5.5 The Positronium Fraction

Table 5.2: Positronium fraction for each model: turbulent, transport, and extra gas.

<table>
<thead>
<tr>
<th>$f_{Ps}$</th>
<th>Turbulent</th>
<th>Transport</th>
<th>Extra Gas</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.94</td>
<td>0.89</td>
<td>0.73</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Table 5.2 shows the fraction of positrons that annihilate through the formation of positronium for each model of positron propagation: turbulent, transport, and extra gas. Positronium forms either in-flight via charge exchange or after thermalization with the gas via charge exchange or radiative recombination.

As positrons propagate through the gas of the Galaxy they will slow and annihilate in the gas with high density. In the turbulent model where positrons are assumed to annihilated where they are produced, the positrons annihilate in a region where the neutral gas density is greater than the ionized gas density. Therefore, the positrons are more likely to annihilate in-flight via charge exchange and form positronium. In the turbulent model the positronium fraction is comparable to the observed positronium fraction.

In the transport and extra gas models where positrons can be transported out of the disk and out of regions of high neutral hydrogen densities the positronium fraction decreases. In the transport model the fraction is lower because the positrons escape the disk to slow and annihilate in the region above and below the disk where the ionized gas has a higher density than the neutral gas. In the extra gas model the positrons slow in the
extra gas and annihilate in the region above and below the center in the region where the
ionized gas density is greater than the neutral gas density.

Therefore, positron propagation from the disk to the bulge cannot explain the ob-
served positronium fraction. The propagation of positrons allows positrons to lose energy
and annihilate in the regions above and below the Galactic bulge region where the ionized
gas density is greater than the neutral gas density.

5.6 Asymmetry

<table>
<thead>
<tr>
<th>Results</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ &lt; l &lt; -50^\circ$</td>
<td>1.2</td>
</tr>
<tr>
<td>$50^\circ &lt; l &lt; 0^\circ$</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Weidenspointner et al. (2008a) detected an asymmetry in the disk emission. The
flux from the inner disk at $-50^\circ < l < 0^\circ$ exceeds the flux from the positive longitudes by
a factor of 1.8. An asymmetry in the 511 keV positron annihilation emission can be seen
in the turbulent and transport models of positron annihilation. The ratio of the flux from
negative longitudes versus the positive longitudes for both models is 1.2.

The asymmetry may be seen in both models, but may not be significant enough to
explain the observed asymmetry. The annihilation of positrons from radioactive isotopes
can still explain the disk flux.
Chapter 6

Conclusion

This research explored how the observed Galactic 511 keV positron annihilation emission morphology could be explained by investigating the life of positron through three stages: production, propagation and annihilation.

Many Galactic positron production sources have been suggested based on the distribution of the source population and the rate of positron production from the source. However, the positrons from the radioactive isotopes are the most probable source of Galactic positrons. The positron production rate from the radioactive isotope $^{26}$Al is derived from the observations of the photon of 1.8 MeV that accompanies the decay that produces the positron. In the case of $^{26}$Al we have a definitive rate and location of the positrons based on the observations of the 1.8 MeV photon. $^{44}$Ti comes from the same stars that produce $^{26}$Al. The photon from the decay of $^{44}$Ti has only been observed in the Cas A supernovae remnant. The mass of $^{44}$Ti is derived from the solar ratio of daughter isotope $(^{44}$Ca/$^{56}$Fe)$_{⊙}$ and the production rate of $^{56}$Fe from the supernovae rate. $^{26}$Al produces positrons at a rate of $2 \times 10^{42}$ e$^+$ s$^{-1}$. $^{44}$Ti produces positrons at a rate of $3 \times 10^{42}$ e$^+$ s$^{-1}$.

The distribution of $^{26}$Al can be traced by the distribution of free elections. There exists a three dimensional model for the density of free electrons. The initial positions of the positrons created from $^{26}$Al and $^{44}$Ti are based on the density of free electrons.

Radioactive isotopes also create positrons under 1 MeV, which falls under the con-
straint that to explain the observed shape of the line emission the positron must be created under 3 MeV. A positron created at higher energies has the possibility to annihilate in-flight before thermalizing with the medium. The in-flight annihilation broadens the shape of the line emission. Therefore, positrons with energies above 3 MeV create a line shape that is much broader than the observed line shape.

The initial energy of a positron created from the $\beta$-decay of radioactive isotopes can have a spectrum of energies. When an isotope $\beta$-decays it produces a positron and a neutrino. Both of these particles share the energy left over from the decay. The spectrum of energies is determined by the density of final states from the decay. The maximum energy the positron or neutrino can have is the difference in energy of the decay.

The radioactive isotopes are the most probable positrons producers to that are modeled in this research because they have a particular three dimensional distribution, the produce positrons at a rate comparable to the observed positron annihilation rate, and produce positrons with initial energies under the constraint of 3 MeV.

The distribution of positron production sources had a bulge to disk ratio of less than one. The propagation of positrons from the disk to the bulge was suggested in order to explain the observed 511 keV positron annihilation emission morphology. A positron at 1 MeV can travel $\sim 20$ kpc in the average density of the ISM of 2 particles cm$^{-3}$. However, since positrons are charged particles the propagation and direction of the positron is dictated by the Galactic magnetic field. Positrons do not just fly out of the disk once they are created, but are constrained by the Galactic magnetic field.

This research explores the two cases where the positron is trapped in the turbulent Galactic magnetic field and where positrons can escape the turbulent field and travel along the Galactic poloidal field to funnel toward the bulge region. In the turbulent field the positron is assumed to annihilate where it is produced. This is based on the assumption that the positron propagates with a mean free path equal to its gyroradius of $\sim 10^9$ cm in the turbulent field. This corresponds to a distance of $3 \times 10^{-13}$ kpc. The propagation along the poloidal field can happen if a positron escapes the turbulent field.
As the positron propagates it loses energy and interacts with the gas to annihilate. The positron loses energy by ionization or excitation in a neutral medium or by Coulomb interactions in an ionized gas. Both processes are considered continuous energy processes until the positron reaches an energy of about 1 keV. At this energy the positron can still lose energy continuously in a ionized gas, but in a neutral gas the positron loses a significant fraction of its energy in each ionization or excitation interaction. Under 1 keV the positron loses energy as a discrete process in a neutral medium. In this research a positron can lose energy continuously in a density of molecular hydrogen, neutral hydrogen and ionized gas.

A positron traveling along a magnetic field will gyrate around the field line. The distance a positron travels along the field line is related to the distance the positron travels through the gas by the pitch angle of direction of the velocity vector of the positron and the direction of the magnetic field. A positron at a pitch angle of 90° with gyrate in place and not travel along the magnetic field because there is no component of the velocity parallel with the magnetic field. A positron with a pitch angle of 0° with travel right along the field line since the entire velocity vector is in the direction of the magnetic field.

The energy a positron loses is related to the column depth of the gas it goes through. If a positron gyrates around a field line the positron will lose more energy than a positron that travels straight along the field line because it goes through more column depth of gas per step along the field line. Including a model of the Galactic magnetic field does not guarantee all positrons will leave the disk to annihilate elsewhere, since some can gyrate around a field line without propagating because of the pitch angle.

Before a positron can thermalize with the medium it can annihilate in-flight by charge exchange or by direct annihilation. Annihilation via charge exchange depends on the density of neutral hydrogen, because the mechanism of charge exchange takes the electron from a hydrogen atom and forms positronium with the positron and electron. Direct annihilation depends on the density of the ionized gas, since the mechanism is the interaction of positron with an electron.

After the positron has thermalized with the medium the positron can annihilate
through the formation of positronium via charge exchange or radiative recombination. Radiative recombination is when a positron and an electron come together to form positronium but have a combined energy greater than the binding energy of positronium and the left over energy is radiated away. The positron can also directly annihilate with an electron or directly annihilate with an electron that is bound to an atom, such as hydrogen.

This research uses the cross sections of positron interactions with the medium and the density of the medium to determine the energy loss and the annihilation process.

The results of the research are based on the most probable positrons producers radioactive isotopes $^{26}$Al and $^{44}$Ti were modeled as the source of Galactic positrons. The positron production rate of both isotopes is $\sim 5 \times 10^{42}$. The annihilation of the positrons from these isotopes were modeled in the case where the positrons annihilated where they were produced and in the case where positrons could be transported along the Galactic field lines. In both cases, the most likely positron producers explain about $30 - 40\%$ of the positron annihilation emission from the disk and cannot explain the bulge emission morphology. $^{56}$Ni is also produced in the Galaxy and can account for the rest of the disk flux. However, whether positrons are transported to the bulge or not, the bulge positron annihilation emission cannot be explained.

The morphology of the 511 keV emission is tied to the density of the gas since the gas is what slows the positrons and is what the positrons annihilate with. In the turbulent model where positrons annihilate where they are produced and in the transport model where positrons propagate along the Galactic field lines the 511 keV emission is in the disk where the gas density is high enough to slow them and cause them to annihilate. An asymmetry can be seen in the 511 keV annihilation emission whether in the case positrons are transported or not. The annihilation flux of positrons from radioactive isotopes is comparable to the observed annihilation flux. Therefore, the positrons from radioactive isotopes can explain the observed disk emission, but not the bulge emission.

Many of the proposed positron production sources are considered disk population sources, including the mostly likely positrons production sources radioactive isotopes $^{26}$Al
and \(^{44}\text{Ti}\) which were modeled in this research. Positron propagation from the disk production sources to the bulge was suggested in order to explain the observed bulge to disk ratio, \(B/D = 1.4 - 6\), of the positron annihilation emission. However, positron propagation is not enough to explain the observed 511 keV emission morphology.

In the transport model where positrons propagate along the Galactic magnetic field the 511 keV emission from the disk is comparable to the observed emission. However, there is no bulge emission. This is because there is no gas in the Galactic bulge region for positrons to slow or annihilate with. Positrons that propagate out of the disk and along the Galactic magnetic field will propagate through the Galactic center and slow and annihilate with the high densities of gas in the disk as they come back to the disk after going through the center region.

The extra gas model shows that positrons can annihilate in the bulge region if there is a high density of gas for positrons to slow and annihilate with. However, since positrons are created with a spectrum of energies the high energy positrons only slow in the high density bulge region to annihilate via direct in-flight annihilation in the region outside of the bulge where the ionized gas density is higher than the neutral gas density.

To create the observed 511 keV positron annihilation emission morphology, positrons that propagate out of the disk and into the bulge region have to slow at just the right rate for them to preferentially annihilate in the bulge. Also, if positrons do slow at just the right rate there must be a significant neutral hydrogen gas density in the bulge region to measure the 93% of positrons that annihilate through the formation of positronium, i.e. the mechanism of annihilation via charge exchange with neutral hydrogen.

The models presented in this research show that the positron annihilation emission from the disk can be explained by the annihilation of positrons from radioactive isotopes. However, the bulge emission could not be explained by these sources even if the positrons could propagate from the disk to the bulge. The positrons from the other suggested positron production sources that have a bulge to disk ratio less than one, would also have the same problem of not being able to explain the bulge emission since the positrons from these
sources are not created in the bulge. Therefore, since the propagation of positrons from the disk to the bulge cannot explain the observed emission morphology, the observed bulge positron annihilation emission must come from a central positron production source.

However, a centralized source of positrons cannot fully explain the observed emission morphology. Even if created in the bulge, the positrons must still be able to annihilate with gas in the bulge. The constraints on the observed density of gas in the central radius of 3 kpc is $\sim 1 \text{ cm}^{-3}$. The density of the gas in the bulge region in the extra gas model has a density of $\sim 17 \text{ cm}^{-3}$ within the radius of 3 kpc. This extra density was not enough to slow the high energy positrons that were transported from the disk. The extra density did slow the positrons, but the positrons went beyond the bulge region to annihilate in the extended region of ionized gas.

To summarize, positrons from radioactive isotopes can explain the disk flux if they annihilate where they are produced. The observed 511 keV bulge emission must then be created from positrons that are produced by a central source. To annihilate in the bulge the positrons created by the source must be at low energies, otherwise the positrons will propagate out of the bulge before they have a chance to annihilate with the small density of gas that exists in the bulge region. Propagation of positrons from the disk to the bulge cannot explain the observed 511 keV positron annihilation emission morphology. In order to understand the observed 511 keV positron annihilation emission morphology we must know more about how positrons are created in the bulge region and how positrons can slow and annihilate in the low density of gas that is observed in the bulge region.
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