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Protocols for dynamic spectrum access in cognitive radio networks

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PROTOCOLS FOR DYNAMIC SPECTRUM ACCESS IN COGNITIVE RADIO NETWORKS

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Electrical Engineering

by
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Accepted by:
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Spectrum access protocols permit secondary users to transmit on frequency bands that are not being utilized by the primary owners. A cognitive radio that wishes to transmit in a band must first decide if the band is available (i.e., not being used by the owner) and then it must periodically re-evaluate the band’s availability once it begins transmitting in the band to ensure that a signal from a primary owner has not emerged. To accomplish these tasks, spectrum access protocols employ periodic sensing of the channel. Frequent sensing intervals are required to ensure that cognitive radios wishing to access the band are not disrupting transmissions by the owners of the band. Because spectrum sensing requires that radios cease transmission to observe the channel, the potential for throughput by the secondary users is reduced.

A proposed enhancement to standard spectrum access protocols is presented that permits secondary users to monitor the frequency bands while communicating. This capability increases the amount of time that radios can transmit on the band and it decreases the amount of time required to detect the emergence of transmissions by a primary owner. Both improvements are obtained via a protocol that observes statistics obtained in the receiver of the cognitive radio during packet reception. The statistics are obtained with little or no additional hardware and do not require complicated channel measurements or pilot symbols. The proposed protocol for spectrum access is applicable to both single-link networks and multi-link cooperative networks.
ACKNOWLEDGMENTS

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CHAPTER 1
INTRODUCTION

Dynamic spectrum access protocols improve spectrum utilization by allowing radios to transmit on spectrum bands when they are not in use by the primary owners. The network of radios with the primary rights to transmit in a particular band is the primary network and a radio in this network is a primary radio. The radios that do not have primary rights to transmit in the band are referred to as secondary radios. The standard approaches for dynamic spectrum access involve a procedure referred to as spectrum sensing, in which secondary radios examine a frequency band for primary radio activity prior to initiating communications on the band. If the frequency band is deemed vacant by the spectrum sensing protocol, then secondary radios are permitted to access the band for a short period of time. After this time, however, the secondary radios must vacate the channel and repeat the spectrum sensing procedure to ensure that a primary radio has not begun transmitting on the band. Frequent examination of the band is required to ensure that a transmission from the primary network has not emerged.

A number of techniques can be used for spectrum sensing in dynamic spectrum access networks [1]–[15]. Protocols of varying complexity exist, depending on the underlying relationship between the primary and secondary networks. For example, if little or no information about the primary signal is available to secondary radios, then common techniques such as energy detection [13], [14] are required for spectrum sensing. More advanced techniques use known features of the primary signal (e.g., cyclostationary signatures) to aid in identifying the presence of the primary signal [12], [15]. In multiuser secondary networks, collaboration amongst secondary
cognitive radios enables improved detection capabilities [3]–[5].

An enhancement to traditional spectrum sensing, referred to as spectrum monitoring is presented in this thesis. The goal of spectrum monitoring is to detect the emergence of a primary radio’s transmission while the secondary radios are communicating in the band. The proposed enhancement permits less frequent examination of the band by means of traditional spectrum sensing, which allows the secondary network to achieve greater throughput. Additionally, spectrum monitoring reduces the time between the emergence of a primary radio’s signal and its detection by the secondary radios.

Initial results for a protocol that achieves sensing while communicating are presented in [16], where it is shown that simple receiver statistics from the demodulator or decoder can be used to detect the presence of a primary signal in the band. Secondary radios monitor statistics obtained over the course of several packet receptions to identify an abrupt change in the measured statistics, which is indicative of a primary radio utilizing the band. Receiver statistics are used in a variety of applications such as adaptive transmission [17]–[19], signal-to-noise ratio (SNR) estimation [20]–[22], and soft-decision decoding [23], [24]. Receiver statistics provide information about the quality of the channel and prevent the need for pilot symbols or complicated channel measurements. In subsequent chapters, various receiver statistics are evaluated with regard to their applicability to spectrum monitoring.

In this thesis, I propose and evaluate a spectrum monitoring protocol. A large portion of the study is geared towards the use of receiver statistics for primary signal detection, which is the main aspect of the protocol. The detection decision is a binary hypothesis test for declaring the presence or absence of a primary signal. Performance of detection via monitoring is quantified in terms of the relation between
the primary and secondary signals (e.g., signal strength, phase offset, time offset). The spectrum monitoring protocol incorporates some features of standard spectrum sensing protocols with the additional capability to detect the emergence of a primary signal while communicating. The proposed monitoring techniques can augment many spectrum sensing protocols, but results are provided in this thesis for energy detection, which is the most common type of sensing.
CHAPTER 2
SPECTRUM MONITORING AND SENSING

2.1 Protocol model

The general operation of spectrum sensing protocols is described to illustrate the standard approach for dynamic spectrum access. While the underlying mechanisms of various sensing protocols are specific to the network and the cognitive radio capabilities, most protocols adhere to the same general model. The proposed spectrum monitoring enhancements can be applied to any spectrum sensing protocol. For both the sensing and monitoring protocols, the same procedure is used to identify available frequency bands and gain initial access to the spectrum. The difference between the protocols occurs after a secondary user has identified and begun utilizing the frequency band. The focus of this investigation is on the performance of the protocols once a secondary radio has gained access to a frequency band.

The spectrum sensing protocol periodically checks the band for the presence of a primary signal in short intervals called *sensing intervals*. Following each sensing interval is a *transmission interval* during which the secondary radio transmits one or more packets. During a transmission interval, a system that uses only spectrum sensing makes no attempt to detect the presence of a primary signal. The spectrum monitoring protocol alternates between sensing intervals and *monitoring intervals*. During monitoring intervals and transmission intervals, the secondary radio is transmitting packets, but a critical difference is that the secondary radios have several opportunities to detect the emergence of a primary signal during each monitoring interval. Example operation of the spectrum sensing and spectrum monitoring protocols is illustrated in Figures 2.1 and 2.2, respectively. Note that both protocols revert
to continuous sensing intervals once a primary signal has been detected. One key difference is that the spectrum monitoring protocol is able to detect the primary signal prior to the end of the monitoring interval, which allows the secondary cognitive radio to vacate the band immediately. If a primary signal enters the band during a transmission, then a spectrum sensing protocol is not able to detect the primary signal until the next sensing interval. Prior to the sensing interval, the secondary radio is interfering with the operation of the primary network. Minimizing this disruption is a major priority in the design of dynamic spectrum access protocols.

The spectrum sensing protocol has one type of detection opportunity and the spectrum monitoring protocol has two types of detection opportunities. For each detection opportunity, the protocol must declare one of two hypotheses. Let $H_0$ be the hypothesis that a primary signal is not present in the band and let $H_1$ be the hypothesis that a primary signal is present in the band. Associated with each detection opportunity is a false-alarm probability (i.e., probability of choosing $H_1$ when $H_0$ is correct) and a detection probability (i.e., probability of choosing $H_1$ when
Figure 2.2: Example operation of a spectrum monitoring protocol when no primary signal emerges (top) and when a primary signal emerges (bottom).

$H_1$ is correct). Let $\hat{P}_f$ and $\hat{P}_d$ denote the false-alarm and detection probabilities achieved by each spectrum sensing decision and let $P_f$ and $P_d$ be the corresponding probabilities for each spectrum monitoring decision.

2.2 Detection rules for monitoring

When there is no primary signal in the band, the channel is said to be in its quiescent state. The receiver estimates the characteristics (i.e., the distribution) of the receiver statistics in the quiescent state so that a change to the characteristics can be detected. The general idea behind monitoring is that the emergence of a primary signal will cause the receiver statistic to take on a value that is unlikely to occur in the quiescent state. In practice, the quiescent state of the channel must be estimated at the receiver and estimation techniques for this purpose are provided in Section 5.4.

The decision statistic, which is the receiver statistic from the most recent packet, is compared with a detection threshold to yield a hypothesis decision (i.e., decide $H_0$...
or $H_1$). Performance of this decision mechanism is governed by the receiver’s ability to select a proper threshold. If the quiescent distribution of the receiver statistic used for detection is a well known distribution (e.g., binomial) or if the distribution is well approximated by a known distribution, then one approach is to employ a Neyman-Pearson (NP) decision rule [25], which either maximizes the detection probability subject to a constraint on the probability of false-alarm or minimizes the false-alarm probability subject to a constraint on the detection probability. If the quiescent distribution of the receiver statistic is not a known distribution, then heuristic decision rules must be established based on empirical distributions.

2.3 Cognitive capabilities

A network of cognitive radios can combine information to improve the detection performance over that which is achievable by individual links. In Chapter 6, protocols for cooperative detection are presented that employ statistics collected at multiple radios. Detection performance can vary greatly from link to link; thus, the cognitive radios should exploit the best performing links if they can be identified. If multiple cognitive radios conduct spectrum sensing or spectrum monitoring, each radio employs some form of detection algorithm. The result of the algorithm might be a hypothesis decision, or it might be a collection of statistics and estimates that must be sent elsewhere for further processing. One possible procedure for operation of cooperative detection by two cognitive radios is illustrated in Figure 2.3.

The outcome of the detection algorithm at each radio is passed to a central location (e.g., a designated radio in the secondary network). For a multicast transmission, this location is the transmitting radio and the outcome of the detection algorithm at each receiver can be fed back to the transmitting radio as a few bits in the acknowledgment.
packet. Because the detection protocol collects and processes data from multiple cognitive radios, the protocol can be thought of as a data fusion process. The primary function of the process is to use the data from all secondary radios to decide on an appropriate network response. The response could be for all radios to vacate the band (e.g., if a primary user was detected by one or more radios) or the response could be for certain radios to tweak the detection mechanism to produce fewer false alarms. The method and rationale for this adjustment are described further in Chapter 5. Additionally, if poor or highly variable channel conditions arise, which is indicated by frequent false alarms, then traditional spectrum sensing might be preferable over spectrum monitoring. Thus, the network response can inform all radios to switch to traditional spectrum sensing until channel conditions improve or it could inform all radios to vacate the band altogether while alternative frequency bands are sought.
Figure 2.3: Example of cooperative detection by cognitive radios in a secondary network.
CHAPTER 3
DETECTION STATISTICS

Statistics that are easily derived in the receiver of the cognitive radio are ideal candidates for detection statistics. Examples include decoder statistics, such as the error count and iteration count, or demodulator statistics, such as the distance statistic and ratio statistic [17]. The error count is of interest in our system since it is very simple to calculate and has proven to be an effective statistic for adaptive transmission. Furthermore, its known distribution (i.e., binomial) lends itself well to analysis.

3.1 System model

One secondary radio, the source, transmits a session of packets to another secondary radio, the destination. In our model, binary error control codes are applied to the information bits at the source and modulated using QPSK. At the input to the destination’s receiver is the signal from the source’s transmission, which is referred to as the desired signal, and potentially a signal from a radio in the primary network. The secondary signal waveform $s_2(t)$, is given by

$$s_2(t) = A_2[u_2 \cos(\omega_c t + \phi_2) - \nu_2 \sin(\omega_c t + \phi_2)]p_t(t),$$  \hspace{1cm} (3.1)$$

where $A_2$ is the signal amplitude of the secondary signal at the secondary receiver, the pair of binary elements $(u_2, \nu_2)$ represents the in-phase and quadrature polarities of the QPSK symbol, and the function $p_t(t) = 1$ for $0 \leq t \leq \tau$ and $p_t(t) = 0$ otherwise. The matched filters for the in-phase and quadrature components are $h_M(t) = \cos[\omega_c(\tau - t) + \phi_2]p_t(t)$, and $h_M(t) = -\sin[\omega_c(\tau - t) + \phi_2]p_t(t)$, respectively. The energy per
QPSK symbol in the secondary signal is $E_2 = A_2^2 \tau$.

The primary signal has the same waveform as (3.1) except that it may be offset in phase and time. Let $\phi = \phi_1 - \phi_2$ denote the phase offset between the signals and let $\hat{\tau}$ denote the time offset. Similarly, let $A_1$ be the amplitude of the primary signal at the secondary receiver and let $E_1 = A_1^2 \tau$ be the corresponding energy per QPSK symbol. Let $u_1$ and $v_1$ be the in-phase and quadrature bit polarities for the primary signal for the interval $[0, \hat{\tau})$ and let $x_1$ and $y_1$ be the in-phase and quadrature bit polarities for the primary signal for the interval $(\hat{\tau}, \tau]$. The primary signal waveform is given by

$$s_1(t) = A_1[u_1 \cos(\omega_c t + \phi_1) - v_1 \sin(\omega_c t + \phi_1)]p_{\hat{\tau}}(t) + A_1[x_1 \cos(\omega_c t + \phi_1) - y_1 \sin(\omega_c t + \phi_1)]p_{\tau-\hat{\tau}}(t - \hat{\tau}).$$

(3.2)

Let $\rho = E_2/N_0$ be the ratio of the energy per QPSK symbol in the secondary signal to the one-sided spectral density of the thermal noise $N_0$. When expressed in decibels (dB), this quantity is denoted $QSENR = 10 \log_{10}(E_2/N_0)$. The secondary-to-primary power ratio is given by $SPR = 10 \log_{10}(A_2^2/A_1^2) = 10 \log_{10}(E_2/E_1)$.

3.2 Error count

The binary symbols at the output of the source’s encoder form the source’s data sequence and the binary hard-decisions at the output of the destination’s demodulator form the destination’s data sequence. The number of positions for which the two sequences disagree is referred to as the error count. The error count (EC), which is known to have a binomial distribution for a binary memoryless channel, is useful for deriving analytical bounds on the performance of optimal protocols. If a packet does not decode correctly, then the receiver typically cannot determine the destination’s
data sequence, so it cannot determine the EC. The receiver’s error count (REC) for a correctly decoded packet is the number of differences between the source’s data sequence and the destination’s data sequence. Some decoders, such as sum-product algorithm (SPA) decoders for LDPC codes, can provide the REC directly. Alternatively, for a correctly decoded packet, the destination can encode the information bits at the output of the decoder and compare the encoded sequence with the destination’s data sequence to obtain the REC, as illustrated in Figure 3.1 [26].

The error count is used in this thesis for all simulated and analytical results. The EC and REC have indistinguishable distributions if the packet error probability is negligible [26]. However, packet errors will cause the two statistics to have different distributions. Namely, the mean of the REC will be lower than the EC, but it’s density still resembles that of a binomial distribution. For the study of detection, we are interested in high values of SPR for which the effect of the primary signal is difficult to detect. If the primary signal is strong enough such that packet errors results from its emergence, then monitoring becomes trivial. In practical implementations of the spectrum monitoring protocol, packet errors will automatically results in the monitoring protocol declaring the emergence of the primary signal.

3.3 Demodulator statistics

Many of the results provided in this thesis are for detection using the error count statistic. However, the same techniques can be applied to other statistics such as the distance statistic, which is derived from the distance metric originally defined in [27]. If a packet decodes correctly (e.g., as determined by a CRC code), then the source’s data sequence is known at the receiver. For each received symbol, the squared distance from the received point to the correct symbol in the constellation
is calculated. The average of the squared distances for all symbols in the set is the distance statistic for the set. Performance of spectrum monitoring using the distance statistic is evaluated in Section 8.
CHAPTER 4
ERROR COUNT

A significant increase in the error count from one packet to the next is indicative of a primary signal emerging on the band. Statistical distributions of the error count are determined for a single primary signal that may be offset in both time and phase from the secondary signal. The error count for a packet follows a binomial distribution with parameters \( n \) and \( p \), where \( n \) is the number of binary symbols in the packet and \( p \) is the binary symbol error probability. All results provided in this thesis are for packets containing \( n = 4096 \) binary symbols. Derivations are provided for primary and secondary signals employing QPSK modulation. Similar (albeit more complicated) derivations can be obtained for M-QAM modulation.

4.1 Binary symbol error probability

The average binary symbol error probability for the secondary receiver is

\[
P_e = \frac{1}{16} \sum_{\text{w}_i} Q \left( \sqrt{\frac{E_2}{N_0}} + \left( \frac{u_1^\hat{\tau}}{\tau} + \frac{x_1^\tau}{\tau} \right) \cos(\phi) \right.
- \left. \left( \frac{v_1^\hat{\tau}}{\tau} + \frac{y_1^\tau}{\tau} \right) \sin(\phi) \right) \sqrt{\frac{E_1}{N_0}} \right),
\]

(4.1)

where \( \text{w}_i \) is the \( i \)th possible binary vector \( \text{w}_i = (u_1^i, v_1^i, x_1^i, y_1^i) \) out of 16 such vectors. Without loss of generality, let \( 0 \leq \hat{\tau} \leq \tau/2 \) and \( 0 \leq \phi < \pi/4 \). For any phase offset, the minimum error probability occurs for \( \hat{\tau} = \tau/2 \) and the largest error probability occurs for \( \hat{\tau} = 0 \). A proof of this is provided in Appendix A. We cannot conclude that a given value of \( \phi \) will always result in the minimum error probability for the system under consideration. Consider the following example. Let \( \hat{\tau} = 0 \) and consider
$\phi = 0$ and $\phi = \pi/4$. From equation 4.1, the error probability for $\phi = 0$ and $\hat{\tau} = 0$ is

$$P_e = \frac{1}{2} Q\left(\sqrt{\frac{E_2}{N_0}} + \sqrt{\frac{E_1}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{E_2}{N_0}} - \sqrt{\frac{E_1}{N_0}}\right). \quad (4.2)$$

and the error probability for $\phi = \pi/4$ and $\hat{\tau} = 0$ is

$$P_e = \frac{1}{4} Q\left(\sqrt{\frac{E_2}{N_0}} + \sqrt{2}\sqrt{\frac{E_1}{N_0}}\right) + \frac{1}{4} Q\left(\sqrt{\frac{E_2}{N_0}} - \sqrt{2}\sqrt{\frac{E_1}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{E_2}{N_0}}\right). \quad (4.3)$$

The decision boundaries for a QPSK symbol in the first quadrant are shown in Figure 4.1. The solid black circle is the mean of the decision statistic and the other circles are the means of this statistic when conditioned on the in-phase and quadrature polarities of the primary signal. For large values of $E_1/N_0$, $\phi = 0$ results in the largest error probability. However, for small values of $E_1/N_0$, it is possible for $\phi = 0$ to yield a slightly lower binary symbol error probability.

Binary symbol error probabilities are provided in Table 4.1 for QSENR = 6 dB and $\hat{\tau} = 0$ for several values of SPR. As is clear from the table, neither value of $\phi$ achieves the minimum binary symbol error probability for all values of SPR. For large values of SPR, which make the primary signal the most difficult to detect,
the phase has little effect on the binary symbol error probability. The binary symbol error probabilities for QSENR = 6 dB and \( \hat{\tau} = \tau/2 \) are provided in Table 4.2.

\[ \begin{array}{c|cc}
\text{SPR} & \phi = 0 & \phi = \pi/4 \\
\hline
12 & 0.036925 & 0.037144 \\
9 & 0.051213 & 0.051868 \\
6 & 0.080588 & 0.081736 \\
3 & 0.140183 & 0.136748 \\
\end{array} \]

Table 4.1: Binary symbol error probabilities for \( \hat{\tau} = 0 \) and QSENR = 6 dB.

\[ \begin{array}{c|cc}
\text{SPR} & \phi = 0 & \phi = \pi/4 \\
\hline
12 & 0.029966 & 0.029990 \\
9 & 0.037110 & 0.037165 \\
6 & 0.051797 & 0.051758 \\
3 & 0.081595 & 0.080162 \\
\end{array} \]

Table 4.2: Binary symbol error probabilities for \( \hat{\tau} = \tau/2 \) and QSENR = 6 dB.

\[ \begin{array}{c|cc}
\text{SPR} & \phi = 0 & \phi = \pi/4 \\
\hline
12 & 0.029966 & 0.029990 \\
9 & 0.037110 & 0.037165 \\
6 & 0.051797 & 0.051758 \\
3 & 0.081595 & 0.080162 \\
\end{array} \]

Table 4.2: Binary symbol error probabilities for \( \hat{\tau} = \tau/2 \) and QSENR = 6 dB.

The distribution of the error count is shown in Figure 4.2 for \( \phi = 0 \) and \( \phi = \pi/4 \) for QSENR = 6 dB, \( \hat{\tau} = 0 \) and several values of SPR. The corresponding distributions are shown in Figure 4.3 for \( \hat{\tau} = \tau/2 \). The binary symbol error probability is evaluated for each \( \phi \in \{0, \frac{\pi}{64}, \frac{2\pi}{64}, \ldots, \frac{\pi}{4}\} \). For each value of SPR, the values of \( \phi \) and \( \tau \) from among the values in the set that results in the highest and lowest error probabilities are determined. The analytical distribution of the error count corresponding to the lowest binary symbol error probability, referred to as the lower extreme distribution, is shown in Figure 4.4 for \( n = 4096 \) and QSENR = 6 dB, which occurs for when \( \hat{\tau} = \tau/2 \) and some value of \( \phi \). The lower extreme distribution is of interest since it exhibits
Figure 4.2: Error count distributions for QSENR = 6 dB and $\hat{\tau} = 0$ for $\phi = 0$ (solid lines) and $\phi = \pi/4$ (dotted lines).

the least distinction from the quiescent distribution. The *upper extreme distribution* is the distribution that results from parameters causing the highest binary symbol error probability. The extreme distributions are illustrated in Figure 4.5 for several values of SPR. Also shown in Figures 4.4 and 4.5 is the conditional distribution of the error count given that the primary signal is absent (i.e., $E_1 = 0$).

Based on observation of the error count, the secondary radio decides whether the primary signal is present. A primary signal with $\hat{\tau} = \tau/2$ yields the lowest error probability of any time offset and is therefore the most difficult to detect. For this reason, many of the analytical results are provided for $\hat{\tau} = \tau/2$. For the simulation results presented in Section 5.4, performance is evaluated using random time and phase offsets for the primary signal.
Figure 4.3: Error count distributions for QSENR = 6 dB and $\hat{\tau} = \tau/2$ for $\phi = 0$ (solid lines) and $\phi = \pi/4$ (dotted lines).

Figure 4.4: The lower extreme distributions of the error count for QSENR = 6 dB.
Figure 4.5: The upper extreme distributions (dashed lines) and lower extreme distributions (solid lines) of the error count for QSENR = 6 dB.
CHAPTER 5
DETECTION RULES BASED ON ERROR COUNT

5.1 Neyman-Pearson decision rule

The detection protocol observes the error count $X$ and chooses one of two hypotheses after each packet reception. Under hypothesis $H_0$, $X$ is binomial with parameters $n$ and $p_0$, where $p_0 = Q(\sqrt{E_2/N_0})$. Under hypothesis $H_1$, $X$ is binomial with parameters $n$ and $p_1$, where $p_1$ is the binary symbol error probability given by (4.1). Let $f_i(x)$ be the density of $X$ under hypothesis $H_i$. The NP decision rule is to choose $H_1$ if $L(x) = f_1(x)/f_0(x) > \eta$, where $\eta$ is selected to achieve a target false alarm probability or detection probability. The likelihood ratio $L(x)$ is expressed as

$$L(x) = \frac{\binom{n}{x} p_1^n (1-p_1)^{n-x}}{\binom{n}{x} p_0^n (1-p_0)^{n-x}} = \left(\frac{1-p_1}{1-p_0}\right)^n \left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]^x, \; x \in \{0, 1, \ldots, n\}. \quad (5.1)$$

The NP decision rule simplifies to

Choose \begin{align*}
H_1 \quad \text{if} \quad x > \frac{\ln\left(\eta/(1-p_0)/(1-p_1)\right)}{\ln\left(p_1(1-p_0)/p_0(1-p_1)\right)}, \\
H_0 \quad \text{otherwise}.
\end{align*} \quad (5.2)

Since $\eta$ represents an arbitrary threshold, let $\eta' = \left[\frac{\ln(\eta/(1-p_0)/(1-p_1))}{\ln(p_1(1-p_0)/p_0(1-p_1))}\right]$ and rewrite the decision rule as

Choose \begin{align*}
H_1 \quad \text{if} \quad x \geq \eta', \\
H_0 \quad \text{otherwise}.
\end{align*} \quad (5.3)
The probability of false alarm and the probability of detection are

\[ P_f = P(X \geq \eta'|H_0) = \sum_{i=\eta'}^{n} \binom{n}{i} p_0^i (1-p_0)^{n-i} \]  

(5.4)

and

\[ P_d = P(X \geq \eta'|H_1) = \sum_{i=\eta'}^{n} \binom{n}{i} p_1^i (1-p_1)^{n-i} \]  

(5.5)

5.2 Analytical detection performance

Equations (5.4) and (5.5) show that \( P_f \) and \( P_d \) are nonincreasing functions of \( \eta' \). Similarly, the miss probability \( P_m = 1 - P_d \) is an increasing function of \( \eta' \). If the probabilities \( p_0 \) and \( p_1 \) are known, then the false-alarm, detection, and miss probabilities can be calculated for a given value of \( \eta' \). Given a constraint on the false-alarm probability (e.g., \( P_f \leq \alpha \)), the optimal (i.e., smallest) value of \( \eta' \) that satisfies the constraint can be determined. Similarly, given a constraint on the miss probability (e.g., \( P_m \leq \beta \)), the optimal (i.e., largest) value of \( \eta' \) that satisfies the constraint can be determined.

The proposed protocol’s performance is evaluated for a secondary receiver with QSENR = 6 dB and \( \hat{\tau} = \tau/2 \). In Figure 5.1, a curve for the false-alarm probability is given for the constraint \( P_m \leq \beta \) for four values of \( \beta \). Even if the secondary signal is 8 dB weaker than the primary signal (i.e., SPR = 8 dB), the false-alarm probability is negligible for each value of the detection probability. A false alarm probability less than 0.05 and a miss probability of less than 0.05 are achieved for primary signals that are 11 dB weaker than the secondary signals. Corresponding curves for the detection probability for a given constraint on the false-alarm probability are shown in Figure 5.2.
5.3 Detection space

Let the detection space be the two-dimensional space of all values of $QSENR = 10 \log_{10}(E_2/N_0)$ and $PQSENR = 10 \log_{10}(E_1/N_0)$. For a given point in the space, the detection probability is evaluated subject to a constraint on the false-alarm probability. Results are given in Figure 5.3 for a receiver with $\hat{\tau} = \tau/2$ and $P_f \leq 0.05$. Each curve corresponds to a constraint $P_d \geq \gamma$, and it divides the detection space into two regions: the detectable region and the undetectable region. It is claimed that all points in the detectable region satisfy $P_d \geq \gamma$ and $P_f \leq 0.05$ if the optimal value of $\eta'$ is selected. It can be shown that values of PQSENR less than 0 dB correspond to primary signals that are below the secondary receiver’s noise floor. Detection capability is reduced for large values of QSENR if PQSENR is held constant. As the value of QSENR is increased, the number of symbol errors becomes very low and the error count becomes less useful as a statistic for detection. However, certain demodulator
statistics, such as the distance statistic, have shown promise as detection statistics for large values of QSENR.

5.4 Simulated detection performance

In practice, cognitive radios in the secondary network must establish detection thresholds based on the perceived quiescent state of the channel. To achieve this task, the radios estimate the distribution of the receiver statistics under hypothesis $H_0$. Performance is evaluated for the proposed protocol by simulating a secondary user’s session during which a primary signal appears in the band. To estimate the quiescent distribution of the error count, it is sufficient to estimate the parameter $p_0$. No knowledge of the channel parameters such as fading levels, transmit power levels, or radio locations are assumed by the estimator employed by the cognitive radio of the secondary user. Rather, an estimate of the parameter $p_0$ is formed using statistics
Results are provided for both static channels (e.g., AWGN) and channels with slow fading modeled by a finite-state Markov chain of the type shown in Figure 5.4. Finite-state Markov chains have been used to model Rayleigh fading channels [28], [29] and channels with other forms of slow fading [30], [31]. To model time-varying propagation loss, each of the $K$ states in the Markov chain represents a fade level beyond some nominal value. This model is employed to evaluate the performance of the protocol for two fading channels. For both fading channels, the Markov chain contains $K = 5$ states where state $i$, $0 \leq i \leq 4$, represents a fading level of $i\Delta$ dB relative to some nominal value. Thus, the fading channel is characterized by the received signal strength in the nominal state (i.e., state 0 of the chain) and by the parameters $p$ and $\Delta$. For the first fading channel (FC1), the strength of the secondary signal in the nominal state is $\text{QSENR} = 7$ dB, with $\Delta = 0.5$ dB and $p = 0.05$. The second
Figure 5.4: Markov chain used to model channels with slow fading.

The destination establishes a detection threshold based on the perceived quiescent state of the channel. An estimate of $p_0$ is formed from the previous packet for which hypothesis $H_0$ was declared. The destination first estimates the signal-to-noise ratio $\rho = E_2/N_0$ for the secondary signal and then uses $\hat{p}_0 = Q(\sqrt{\hat{\rho}})$ to estimate $p_0$, where $\hat{\rho}$ denotes the estimate of $\rho$. The estimate $\hat{\rho}$ is obtained from the signal-to-variance (SNV) estimator [32]–[34]. The threshold is established by substituting the estimate of $p_0$ in the expression for (5.4). Inaccuracies in the estimate of $p_0$ and small variations in the channel from packet to packet (e.g., as a result of slow fading) can result in false-alarm probabilities larger than the target probability. To counter this issue, the protocol includes a margin on $\hat{\rho}$ to increase the detection threshold and offset increases in the false-alarm probability.

Detection performance of the proposed monitoring scheme is shown in Figure 5.5 for the AWGN channel and for both fading channels. For these results, the time
offset is uniformly distributed on the interval \([0, \tau]\) and the phase offset is uniformly distributed on \([0, 2\pi]\). Both the time offset and phase are fixed for the duration of a packet, but can differ from packet to packet. For the AWGN channel, the received power of the secondary signal is fixed such that \(\text{QSENR} = 6\, \text{dB}\); so the value of SPR is determined by the received power of the primary signal. For the fading channels, the link from the primary and secondary radios experience independent fading modeled by FC1 or FC2. The received power of the secondary signal can take on one of five values corresponding to the five fade levels of the Markov chain; the SPR for these results is defined as the ratio of the reference power level for the secondary signal, which is fixed at 6 dB, to the reference power level of the primary signal. The estimate of \(p_0\) is used to set the detection threshold using a nominal false-alarm probability of \(\alpha = 0.025\).

For the curves in Figure 5.5, no margin is applied when setting the threshold. The resulting false-alarm probabilities are 0.04 for the AWGN channel, 0.07 for FC1, and 0.11 for FC2. Also shown in the figure is the performance of a monitoring protocol that is provided with perfect knowledge of the SNR so that an ideal threshold can be selected to achieve a target false-alarm constraint (i.e., \(\alpha = 0.025\)). This curve represents the performance that would be achieved if the estimator provided a perfect estimate of \(p_0\).

Due to uncertainties in the channel that might arise from slow-varying conditions such as fading, a margin \(m\) is introduced to reduce the probability of false alarms. The margin reduces the estimate of \(\rho\), which results in a larger detection threshold. Although the false-alarm probability is reduced, increasing the detection threshold also reduces the detection probability. This tradeoff is illustrated in Figure 5.6 for the AWGN channel and for fading channel FC2. The corresponding false-alarm probabilities are provided in Table 5.1. The simulated false-alarm probabilities in the
Figure 5.5: Simulated detection performance of the spectrum monitoring protocol for the AWGN channel and for channels with slow fading.

<table>
<thead>
<tr>
<th>Margin, $m$</th>
<th>AWGN</th>
<th>FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>0.04552</td>
<td>0.11684</td>
</tr>
<tr>
<td>0.5 dB</td>
<td>0.00003</td>
<td>0.05684</td>
</tr>
<tr>
<td>1.0 dB</td>
<td>0</td>
<td>0.00034</td>
</tr>
<tr>
<td>1.5 dB</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1: Simulated false-alarm probabilities for 500,000 detection opportunities. Note that for the AWGN channel, no false alarms occurred for $m \geq 1$ dB and for channel FC2, no false alarms occurred for $m \geq 1.5$ dB. While a moderate value for the margin provides some protection against false alarms, large values degrade the detection capability of the protocol.

The results presented in Figures 5.5 and 5.6 correspond to the probability of detection that occurs during the first opportunity for detection (i.e., the first packet received at the secondary radio after the primary signal has emerged). If the primary
Figure 5.6: Simulated detection performance as a function of the margin for the AWGN channel (solid lines) and for fading channel FC2 (dashed lines).

signal emerges on the band, then the ideal network response is for all secondary transmitters within range of the primary receivers to cease immediately; thus, the primary performance criterion considered is the detection probability corresponding to this first opportunity. We also evaluate the conditional probability that detection occurs during the \( j \)th packet after the primary signal emerges given that misses occurred during the previous \( j - 1 \) packets. Let \( P_m(j) \) denote the probability that the destination does not detect the primary signal during the first \( j \) transmission attempts. Simulated values for \( P_m(j) \) are provided in Table 5.2 for fading channel FC2. For a margin of \( m = 0.5 \) dB, there is a noticeable reduction in the miss probability as \( j \) increases; however, little additional reduction is achieved for larger values of the margin.
<table>
<thead>
<tr>
<th>SPR Margin</th>
<th>$P_m(1)$</th>
<th>$P_m(2)$</th>
<th>$P_m(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 dB 0.5 dB</td>
<td>0.00038</td>
<td>0.00032</td>
<td>0.00030</td>
</tr>
<tr>
<td>4 dB 0.5 dB</td>
<td>0.00424</td>
<td>0.00382</td>
<td>0.00350</td>
</tr>
<tr>
<td>5 dB 0.5 dB</td>
<td>0.01366</td>
<td>0.01252</td>
<td>0.01168</td>
</tr>
<tr>
<td>6 dB 0.5 dB</td>
<td>0.03098</td>
<td>0.02848</td>
<td>0.02640</td>
</tr>
<tr>
<td>3 dB 1.0 dB</td>
<td>0.00574</td>
<td>0.00562</td>
<td>0.00556</td>
</tr>
<tr>
<td>4 dB 1.0 dB</td>
<td>0.01774</td>
<td>0.01752</td>
<td>0.01734</td>
</tr>
<tr>
<td>5 dB 1.0 dB</td>
<td>0.04942</td>
<td>0.04868</td>
<td>0.04812</td>
</tr>
<tr>
<td>6 dB 1.0 dB</td>
<td>0.13072</td>
<td>0.12910</td>
<td>0.12802</td>
</tr>
</tbody>
</table>

Table 5.2: Simulated miss probabilities for detection over multiple packets.
CHAPTER 6
COOPERATIVE DETECTION

The protocol described in the previous sections, which operates at a single secondary radio, can be extended to allow cooperative detection among multiple radios in a secondary network. Suppose two secondary radios are destinations for a single session whose packets are from one source (i.e., multicast transmission) or two different sessions with packets from two different sources. In either case, the emergence of the primary signal affects both sessions. If the two secondary radios are able to communicate with each other (e.g., via a separate control channel), then one radio can notify the other radio if a primary signal emerges. In this chapter several cooperative detection protocols are proposed and evaluated. Analytical results are derived and presented in Sections 6.1 and 6.2. For the analytical results, the time and phase offsets that result in the lower extreme distribution are employed. For the simulated results provided in Section 6.3, random time and phase offsets are employed.

6.1 Techniques for cooperative detection

Detection rules are derived for a network in which two secondary radios have active sessions; however, the derivation can be generalized to accommodate a larger number of secondary radios. In the following development, it is assumed that the error counts for the two links are independent, which allows the NP decision rule to be derived. Two protocols for cooperative detection are evaluated. The decentralized cooperative detection (DCD) protocol requires each secondary radio to make an individual decision based on its own receiver statistics. The decision is then relayed to the other secondary radio. If one or both of the secondary radios detects a primary
signal, then both radios will decide that the primary signal is present and terminate their sessions. Let $P_{f_i}$ be the false-alarm probability that results from the individual hypothesis decision of secondary radio $i$ when $\eta_i'$ is the detection threshold and let $P_{d_i}$ be the detection probability. Let $P_{i,k}$ denote the binary symbol error probability for secondary radio $i$ under hypothesis $H_k$. The false alarm probability for secondary radio $i$ is

$$P_{f_i} = \sum_{j=\eta_i'}^{n} \binom{n}{j} P_{i,0}^j (1 - P_{i,0})^{n-j},$$  

(6.1)

where $n$ is the number of binary symbols per packet and the detection probability is

$$P_{d_i} = \sum_{j=\eta_i'}^{n} \binom{n}{j} P_{i,1}^j (1 - P_{i,1})^{n-j}.$$  

(6.2)

The false alarm probability for the DCD protocol is

$$P_f = P_{f_1} + P_{f_2} - P_{f_1} P_{f_2}$$  

(6.3)

and the detection probability for the DCD protocol is

$$P_d = P_{d_1} + P_{d_2} - P_{d_1} P_{d_2}.$$  

(6.4)

Note that (6.3) and (6.4) require that the error count at secondary radio 1 is independent of the error count at secondary radio 2. Secondary radio $i$ selects the maximum threshold $\eta_i'$ that satisfies $P_{f_i} \leq \alpha_i$, where $\alpha_i = 1 - \sqrt{1 - \alpha}$. It follows that $P_f = P_{f_1} + P_{f_2} - P_{f_1} P_{f_2} \leq \alpha_1 + \alpha_2 - \alpha_1 \alpha_2 = \alpha$.

The centralized cooperative detection (CCD) protocol makes one detection decision after receiving information from all secondary radios. The decision could take place
at one of the secondary radios or at some centralized network location. In either case, this protocol involves a more complicated decision mechanism than the DCD protocol. For the CCD protocol, the decision rule is derived in a way that is similar to the derivation in Section 5. Let the binomial random variable \( Y_i \) correspond to the error count for secondary radio \( i \). The likelihood ratio for the joint error count statistic \((Y_1, Y_2)\) is \( L(y_1, y_2) = f_1(y_1, y_2)/f_0(y_1, y_2) \), which is expressed as

\[
L(y_1, y_2) = \left( \frac{1 - P_{1,1}}{1 - P_{1,0}} \right)^n \left[ \frac{P_{1,1}(1 - P_{1,0})}{P_{1,0}(1 - P_{1,1})} \right]^{y_1} \left( \frac{1 - P_{2,1}}{1 - P_{2,0}} \right)^n \left[ \frac{P_{2,1}(1 - P_{2,0})}{P_{2,0}(1 - P_{2,1})} \right]^{y_2}. \tag{6.5}
\]

The decision rule can be written as

\[
\text{Choose } \begin{cases} H_1 & \text{if } y_1 \ln \left[ \frac{P_{1,1}(1 - P_{1,0})}{P_{1,0}(1 - P_{1,1})} \right] + y_2 \ln \left[ \frac{P_{2,1}(1 - P_{2,0})}{P_{2,0}(1 - P_{2,1})} \right] \geq \eta', \\ H_0 & \text{otherwise}, \end{cases} \tag{6.6}
\]

where

\[
\eta' = \ln \left[ \frac{(1 - P_{2,0})(1 - P_{1,0})}{(1 - P_{2,1})(1 - P_{1,1})} \right] n + \ln(\eta). \tag{6.7}
\]

The false-alarm probability is

\[
P_f = P \left( Y_1 \ln \left[ P_{1,1}(1 - P_{1,0}) \right]/P_{1,0}(1 - P_{1,1}) \right) + Y_2 \ln \left[ P_{2,1}(1 - P_{2,0}) \right]/P_{2,0}(1 - P_{2,1}) \geq \eta' \mid H_0 \right), \tag{6.8}
\]

where \( Y_1 \) and \( Y_2 \) are independent binomial random variables with parameters \( n \) and \( P_{i,0}, i = 1, 2 \). Solving equation (6.8) gives

\[
P_f = \sum_{i=0}^{n} \left[ \binom{n}{i} P_{2,0}^{i}(1 - P_{2,0})^{n-i} \sum_{j=\eta_i}^{n-i} \binom{n}{j} P_{1,0}^{j}(1 - P_{1,0})^{n-j} \right] \tag{6.9}
\]
where
\[ \eta_i = \max \left\{ 0, \left[ \eta' - i \ln \left( \frac{P_{2,1}(1-P_{2,0})}{P_{2,0}(1-P_{2,1})} \right) \right] \right\}. \] (6.10)

The detection probability is
\[ P_d = \sum_{i=0}^{n} \left( \binom{n}{i} P_{2,1}^i (1-P_{2,1})^{n-i} \sum_{j=\eta_i}^{n} \binom{n}{j} P_{1,1}^j (1-P_{1,1})^{n-j} \right). \] (6.11)

Given \( P_{1,0}, P_{2,0}, P_{1,1}, \) and \( P_{2,1} \), equation (6.9) can be solved numerically in terms of \( \eta' \) to satisfy a constraint (e.g., \( P_f \leq \alpha \)). Similarly, given the decision threshold \( \eta' \), the detection probability can be evaluated analytically. The CCD protocol requires that the parameters \( y_1, y_2, P_{1,0}, P_{2,0}, P_{1,1}, \) and \( P_{2,1} \) are known at a central location. The parameter \( P_{1,1} \) is the binary symbol error probability of secondary radio \( i \) when the primary signal is present. Knowing this parameter requires that the secondary radio knows the SPR, which is often not a practical assumption. The CCD protocol is able to weigh the error counts from each secondary radio based on the respective SPRs. The DCD protocol, however, will weight each link equally since no information about the SPR is assumed to be known.

6.2 Analytical detection performance

The detection performance is evaluated for the cooperative DCD and CCD protocols as well as for the standard single-link detection (SLD) protocol described in Section 5. For secondary radio \( J \) (\( J \) is either 1 or 2 in the results), single-link detection is denoted by SLDJ and the secondary-to-primary power ratio is SPRJ. All numerical results are for QSENR = 6 dB with half-symbol time offset and phase alignment between the primary and secondary signals for both secondary receivers. The ideal
threshold is selected to achieve $P_f \leq 0.025$. Results on the detection probability are shown in Figure 6.1 for $SPR_1 = SPR_2$. The DCD and CCD protocols achieve better detection performance than the SLD protocol as a result of the additional information from secondary radio 2.

The performance of the protocols is given in Figure 6.2 for $SPR_2 = SPR_1 - 3$ dB. In this case, the single-link performance of secondary radio 2 is expected to be better than that of secondary radio 1 since lowering the SPR makes primary signal detection easier. The CCD protocol is guaranteed to achieve better performance than either of the individual secondary radios since it uses the optimal decision rule for the joint error count statistic. The DCD protocol, however, does not know which link is likely to have better performance (i.e., as a result of lower SPR). Thus, the DCD protocol might have worse performance than that achieved by single-link detection at secondary radio.
2. The detection probability as a function of SPR1 for SPR2 = SPR1 – 6 dB is given in Figure 6.3. As the difference between SPR1 and SPR2 increases, the CCD protocol places less emphasis on the error count from secondary radio 1. However, even if SPR2 = SPR1 – 6 dB, then useful information is obtained from secondary radio 1, which is evident from the gain achieved by the CCD protocol over the single-link detection protocol for secondary radio 2. The performance of the DCD protocol is comparable to the best performing individual secondary radio. If the DCD protocol had knowledge of the SPR for each link (or if learned such knowledge over time) then it could place more weight on the link with lower SPR and achieve performance closer to the CCD protocol. The analytical results in Figures 6.1–6.3 establish that significant performance gains can be achieved by cooperative detection protocols.
6.3 Simulated detection performance

Simulated performance of the DCD protocol is evaluated for a multicast transmission with two destination radios. The time offset for each link is uniformly distributed on the interval \([0, \tau]\) and the phase offset is uniformly distributed on \([0, 2\pi]\). The offsets for link 1 are assumed to be independent from the offsets for link 2 and are drawn according to the random distribution for each packet. For the AWGN results, each link has \(\text{QSENR} = 6\, \text{dB}\) and for the fading results, the reference power level of the secondary signal at each destination is 6 dB. Independent Markov chains are used to model the fading for all channels. In order to provide a fair comparison between single-link and multiple-link detection protocols, the maximum threshold margin (to the nearest hundredth of a decibel) that satisfies \(P_f \leq 0.025\) is employed. In the case of multiple-link detection, the same margin is employed at both destinations; thus,
the resulting false-alarm probability is the same for both links. Detection results are provided for all channels in Figure 6.4 with solid lines corresponding to single-link detection and dashed lines corresponding to the DCD protocol. Recall that the DCD protocol requires each destination to make an individual hypothesis decision. If either destination decides that the primary signal is present, then the network responds by ceasing transmission at all radios. The cooperative-detection capabilities yield approximately 1 dB improvement in SPR detection capabilities with an equivalent false-alarm probability. The margins used for both the single-link and two-link networks are provided in Table 6.1 with the resulting false-alarm probabilities.

The primary benefit of cooperative monitoring results from the notion that at least one link in the secondary network can achieve a high detection probability. To illustrate this, the multicast network of two links is altered such that two links have different reference power levels for the primary signal. For the previous results
Table 6.1: Threshold margins and simulated false-alarm probabilities for single-link and two-link detection.

<table>
<thead>
<tr>
<th>Channel, Links</th>
<th>Margin, $m$</th>
<th>$P_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWGN, 1</td>
<td>0.07 dB</td>
<td>0.023</td>
</tr>
<tr>
<td>AWGN, 2</td>
<td>0.07 dB</td>
<td>0.024</td>
</tr>
<tr>
<td>FC 1, 1</td>
<td>0.16 dB</td>
<td>0.024</td>
</tr>
<tr>
<td>FC 1, 2</td>
<td>0.22 dB</td>
<td>0.023</td>
</tr>
<tr>
<td>FC 2, 1</td>
<td>0.74 dB</td>
<td>0.024</td>
</tr>
<tr>
<td>FC 2, 2</td>
<td>0.81 dB</td>
<td>0.023</td>
</tr>
</tbody>
</table>

in Figure 6.4, $\text{SPR}_1 = \text{SPR}_2$, where $\text{SPR}_i$ is the secondary-to-primary power ratio corresponding to middle state of the Markov chain that is used to model the fade level for the primary link. In the following results, the secondary-to-primary power ratio for link 2 is reduced by either 3 dB or 6 dB, which yields a better detection performance for link 2 relative to link 1. Performance is shown in Figure 6.5 for the single-link detection protocol for links 1 and 2, which are shown as dashed lines, and for the DCD protocol, which is shown as solid lines. The corresponding performance for a 6 dB offset is shown in Figure 6.6. The same threshold margins used to obtain the results in Figure 6.4 are used to obtain the results shown in Figures 6.5 and 6.6; thus, the false-alarm probabilities provided in Table 6.1 apply to all results. Performance of the cooperative detection protocol is comparable to the best performing single-link detection protocol (i.e., SLD2). This is despite the fact that the DCD protocol weighs the decisions for each link equally. Performance results for the CCD protocol are not included since such a protocol requires knowledge of the SPR, which is not known by the receivers.
Figure 6.5: Simulated performance of cooperative detection protocols vs. single-link detection ($P_f < 0.025$, $\text{SPR2} = \text{SPR1} - 3 \text{dB}$).

Figure 6.6: Simulated performance of cooperative detection protocols vs. single-link detection ($P_f < 0.025$, $\text{SPR2} = \text{SPR1} - 6 \text{dB}$).
CHAPTER 7
PROTOCOL IMPLEMENTATION

The primary focus of the previous sections is on the performance of detection via monitoring. In practice, the spectrum monitoring protocol must still schedule periodic examination of the band via traditional spectrum sensing techniques. In this section, a protocol that integrates spectrum monitoring with spectrum sensing is proposed.

7.1 Analytical performance evaluation

Let $T_p$ be the duration of a data packet in the secondary network. Two simplifying assumptions are made with regard to time granularity to simplify the protocol model. First, it is assumed that the length of the sensing interval is $K_s T_p$ for some positive integer $K_s$. Second, it is assumed that the primary signal will emerge at some integer multiple of $T_p$ according to a specified probability distribution. Let $P_a(j)$ be the probability that the primary signal arrives at time $t = jT_p$, for $0 \leq j < \infty$. Once the primary signal arrives in the band, it is assumed to remain there indefinitely.

The spectrum sensing protocol alternates between sensing intervals and transmission intervals, whereas the spectrum monitoring protocol alternates between sensing intervals and monitoring intervals. If either protocol declares $H_1$ (i.e., that the primary signal is present) at the end of the sensing interval, then the radio continues sensing the band and does not initiate a transmission or start a monitoring interval. Let $K_t$ denote the number of packets contained in a transmission interval and let $K_m$ denote the maximum number of packets contained in a monitoring interval. For the monitoring techniques presented, there is one detection opportunity for each packet.
in the monitoring interval. If the monitoring protocol declares $H_1$ at the conclusion of a packet within the monitoring interval, then the secondary radio vacates the band and reverts back to full-time spectrum sensing until it deems that a frequency band is available for use. If the spectrum monitoring protocol declares $H_0$ for a packet, then the monitoring interval continues until $K_m$ packets have been transmitted. At that time, a mandatory sensing interval is required.

The performance metrics of interest for the system are the channel utilization and the detection delay. The secondary network is utilizing the channel if a secondary radio is transmitting and the primary signal is not present in the band. Channel utilization is defined as the percentage of time that the secondary network is utilizing the band under $H_0$. Note that increasing the frequency of sensing intervals will decrease the potential channel utilization since the secondary network cannot transmit packets during sensing intervals. The channel utilization defined in this thesis is a measure of the long-term average use of the channel by the secondary network when the primary signal is absent. If the primary signal emerges and vacates the band frequently, then an alternative measure of utilization or throughput should also be considered. The detection delay is the time from the emergence of the primary signal to its detection by the secondary network. Spectrum etiquette might dictate a constraint on the average detection delay or a constraint on the probability that the detection delay exceeds some specified value. The goal of either protocol is to achieve a high channel utilization while adhering to the constraints on the detection delay imposed by the spectrum etiquette protocol.

The detection and false-alarm probabilities depend on the location of the primary signal arrival within the sensing-transmitting or sensing-monitoring cycles. The probability distribution for the detection delay specifies the probability that the primary
signal is first detected at time $iT_p$ given that the primary signal arrived at time $jT_p$.

For the spectrum sensing protocol, nonzero values of this probability occur only at values of $iT_p$ that correspond to the end of a spectrum sensing interval. Time $t = 0$ is defined to be the end of the initial sensing period by the secondary radio, which is also the beginning of the first transmission interval. Opportunities for detection occur at times $t = n(K_t + K_s)T_p$, for integer values of $n \geq 1$ as is illustrated in Figure 7.1. Given that the primary signal arrives at time $jT_p$, the first opportunity for detection occurs at time $m_j(K_t + K_s)T_p$ where $m_j = \lceil j/(K_t + K_s) \rceil$. Let $t = D_1T_p$ be the time that the spectrum sensing protocol detects the primary signal. We use the notation $P(D_1 = i|j)$ to denote the probability that detection occurs at time $t = iT_p$ given that the primary signal arrived at time $t = jT_p$. Recall that $\hat{P}_d$ is the detection probability achieved by the sensing interval detection decision. Note that $P(D_1 = m_j(K_t + K_s)|j) = \hat{P}_d$, since $t = m_j(K_t + K_s)T_p$ is the first detection opportunity following an arrival at time $t = jT_p$. If this first detection opportunity fails, the next opportunity is at time $t = (m_j + 1)(K_t + K_s)T_p$; the detection probability associated for this next opportunity is $P(D_1 = (m_j + 1)(K_t + K_s)|j) = (1 - \hat{P}_d)\hat{P}_d$.

For all values of $i$ and $j$ we can express the conditional delay probability as

$$P(D_1 = i|j) = \begin{cases} (1 - \hat{P}_d)^{n-m_j}\hat{P}_d, & i = n(K_t + K_s), \\ 0, & \text{otherwise}, \end{cases}$$

for integers $n \geq m_j$.

A similar equation is derived for spectrum monitoring with the exception that additional detection opportunities exist after each packet in the monitoring interval. Detection opportunities for the spectrum monitoring protocol are illustrated in Figure 7.2 for the first two cycles of monitoring and sensing intervals. As can
be seen in the figure, opportunities for detection via monitoring occur at times $t = [a(K_m + K_s) + b]T_p$, for integers $a \geq 0$ and integers $1 \leq b \leq K_m$. The spectrum monitoring protocol still has sensing intervals that result in detection opportunities at times $t = c(K_m + K_s)T_p$, for $c > 0$. Let $t = D_2 T_p$ be the time that the spectrum monitoring protocol detects the primary signal. The probability that detection occurs at time $t = iT_p$ given that the primary signal arrived at time $t = jT_p$ has the general form given by

$$P(D_2 = i|j) = (1 - P_d)^{\theta_1}(1 - \hat{P}_d)^{\theta_2}(P_d)^{\theta_3}(\hat{P}_d)^{1 - \theta_3},$$  \hspace{1cm} (7.2)$$

where $\theta_1$ is the number of missed detection opportunities via monitoring and $\theta_2$ is the number of missed detection opportunities via sensing. The value of $\theta_3$ depends on the type of detection opportunity at time $t = iT_p$ (i.e., $\theta_3 = 1$ for monitoring and $\theta_3 = 0$ for sensing). Values for $\theta_1, \theta_2$, and $\theta_3$ depend on the location of the primary signal arrival within the monitoring or sensing intervals. The equation for $P(D_2 = i|j)$ is
Figure 7.2: Detection opportunities for the spectrum monitoring protocol.

best expressed recursively as

$$P(D_2 = i|j) = \begin{cases} 
1 - \sum_{k=j}^{i-1} P(D_2 = k|j) P_d, & i = a(K_m + K_s) + b, \\
1 - \sum_{k=j}^{i-1} P(D_2 = k|j) \hat{P}_d, & i = c(K_m + K_s), \\
0, & \text{otherwise},
\end{cases} \quad (7.3)$$

for $i > j$, where $P(D_2 = k|k) = 0$ for all $k$.

The spectrum etiquette protocol might impose certain constraints on the distribution of detection delay or on the average detection delay that is achieved by a spectrum access protocol. Let the random variable $Z$ denote the detection delay. The mean delay is given by

$$\overline{Z} = \sum_{j=0}^{\infty} P_a(j) \sum_{i=j+1}^{\infty} (i - j) T_p P(D_\ell = i|j). \quad (7.4)$$

where $\ell = 1$ for the spectrum sensing protocol and $\ell = 2$ for the spectrum monitoring protocol.

While detection delay is used to measure a protocol’s adherence to the spectrum etiquette, the channel utilization measure is employed to determine the efficiency with
which the spectrum access protocol permits secondary access to the band. Channel utilization is evaluated by first determining the percentage of time that the protocol is sensing the band (i.e., not transmitting packets). Under ideal operation of the spectrum sensing protocol each transmission interval of duration $K_t T_p$ is followed by a mandatory sensing interval of duration $K_s T_p$; thus the maximum possible channel utilization for the spectrum sensing protocol is $K_t / (K_t + K_s)$. At a minimum, the spectrum sensing protocol will be sensing the band $K_s / (K_t + K_s)$ percent of the time. However, if a sensing interval detection opportunity yields a false alarm, then the protocol is required to initiate another sensing interval. Thus, it is possible to have two or more consecutive sensing intervals if one or more false alarms occur, which increases the percentage of time that the channel is sensing. Let $\alpha_s$ be the average number of sensing intervals that occur between transmission intervals. Note that $\alpha_s \geq 1$ with equality only if $\hat{P}_f = 0$. The channel utilization for the spectrum sensing protocol is

$$U_1 = \frac{K_t}{K_t + \alpha_s K_s}. \quad (7.5)$$

In a similar manner, the channel utilization achieved by the spectrum monitoring protocol is evaluated by first determining the average length of the monitoring interval. In addition to false alarms that might occur at the end of a sensing interval, a false-alarm can occur at the end of each packet within the monitoring interval, which will result in a monitoring interval with less than the maximum $K_m$ packets. Let $\alpha_m$ be the average number of packets transmitted in a monitoring interval. The channel utilization for spectrum monitoring is given by

$$U_2 = \frac{\alpha_m T_p}{\alpha_m T_p + \alpha_s K_s}. \quad (7.6)$$
To calculate $\alpha_s$, we first determine $P(V_s = v)$ for $v \geq 1$, where $V_s$ is the number of consecutive sensing intervals that occur between transmission intervals. This probability is expressed as

$$P(V_s = v) = (1 - \hat{P}_f)(\hat{P}_f)^{v-1}, \quad (7.7)$$

for $v \geq 1$, and $P(V_s = v) = 0$, otherwise. It follows that

$$\alpha_s = \sum_{v=1}^{\infty} vP(V_s = v). \quad (7.8)$$

In a similar manner we determine $P(V_m = v)$ for $1 \leq v \leq K_m$, where $V_m$ is the number of packets in a monitoring interval. Note that $P(V_m > K_m) = 0$ since the maximum size of the monitoring interval is $K_m$. A monitoring interval will only contain $K_m$ packets if no false alarms occur for the first $K_m - 1$ packets in the interval. Thus, $P(V_m = K_m) = (1 - P_f)^{K_m - 1}$. At the other extreme, a monitoring interval will contain only one packet if a false alarm results from the first monitoring opportunity, which occurs with probability $P_f$. It follows that

$$P(V_m = v) = \begin{cases} 
P_f(1 - P_f)^{v-1}, & 1 \leq v < K_m, \\
(1 - P_f)^{K_m - 1}, & v = K_m,
\end{cases} \quad (7.9)$$

and the average length of the monitoring interval is given by

$$\alpha_m = \sum_{v=1}^{K_m} vP(V_m = v). \quad (7.10)$$
7.2 Protocol implementation

Energy detection, which is the most common spectrum sensing technique, is employed during the sensing intervals of both the spectrum sensing and spectrum monitoring protocols. Both the primary and secondary signals employ QPSK modulation with the same symbol duration, which is consistent with the primary and secondary signals having the same bandwidth. Expressions for the detection and false-alarm probabilities are provided in [14]. During a sensing interval, the band is sampled with frequency $f_s$ symbols per second to produce $N = f_sKST_p$ samples. Under hypothesis $H_0$, samples of the received signal at the secondary radio are represented as $y(n) = u(n)$, for $1 \leq n \leq N$, where $u(n)$ is an independent and identically distributed Gaussian process with mean zero and variance $\sigma_u^2$. Under hypothesis $H_1$, samples are represented by $y(n) = s(n) + u(n)$, where $s(n)$ is an independent and identically distributed random process with mean zero and variance $\sigma_s^2$. The detection decision compares the test statistic

$$T(y) = \sum_{n=1}^{N} |y(n)|^2$$

(7.11)

to a threshold to yield the hypothesis decision. The threshold can be determined to achieve a target $\hat{P}_d$ or a target $\hat{P}_f$. It is assumed that the threshold is set to achieve a target detection probability $\hat{P}_d^*$. An approximation for the false alarm probability is given in terms of the target detection probability as

$$\hat{P}_f = Q \left( \sqrt{2\gamma + 1}Q^{-1}(\hat{P}_d^*) + \gamma \sqrt{N} \right),$$

(7.12)

[14], where $\gamma = \frac{\sigma_s^2}{\sigma_u^2}$. For a given channel (i.e., a given value of $\gamma$), the performance of the energy detector is governed by the target detection probability and the number
of samples.

The monitoring technique proposed in Chapter 5 is employed to detect the presence of the primary signal. A sharp increase in the error count from one packet to the next suggests that the channel has degraded significantly, possibly as a result of primary signal emergence. If such an increase is detected via the spectrum monitoring detection opportunity, the secondary radio will cease transmission and revert to traditional spectrum sensing techniques (e.g., energy detection in this case) to further investigate the channel for the presence of primary network activity.

7.3 Performance results

The channel utilization and detection delay achieved by a radio link in the secondary network are evaluated. Each packet contains 2048 QPSK symbols, QSENR = 6 dB, and $\gamma = -12$ dB. Thus the primary signal is 18 dB weaker than the secondary signal. During the sensing interval the channel is sampled once per symbol to yield $N = 2048K_s$ samples. The probability that the primary signal emerges at time $jT_p$ is

$$P_a(j) = \begin{cases} 
\rho, & j = 0, \\
(1-\rho)^j\rho, & j > 0,
\end{cases} \quad (7.13)$$

where $\rho$ is the arrival probability.

The performance of the energy detector depends on the selection of the appropriate threshold. Using (7.12), the false-alarm probability is evaluated as a function of the target detection probability, $\hat{P}_d^*$. The average detection delay is evaluated for several values of $\hat{P}_d^*$ to determine which value achieves the best performance as a function of the number of packets contained in a transmission interval, $K_t$. The average detection delay as a function of the target detection probability is shown in Fig. 7.3
Figure 7.3: Average detection delay achieved by the spectrum sensing protocol ($K_s = 1$). For the spectrum sensing protocol with $K_s = 1$, arrival probability $\rho = 0.01$, and several values of $K_t$. The units on the ordinate axis of Fig. 7.3 are multiples of the data packet duration $T_p$. Lower values of the transmission interval size $K_t$ provide more frequent opportunities for detection and thus result in a lower detection delay than higher values of $K_t$ for the same target detection probability. Suppose the target detection probability is selected to give an average detection delay less than $4T_p$ for the system, which might correspond to the specifications set forth by the spectrum etiquette protocol. From Fig. 7.3 it is clear that if the spectrum sensing interval size is $K_s = 1$, then values of $K_t \geq 6$ do not satisfy the detection delay constraint.

We next compare the performance of the spectrum sensing protocol with that of the spectrum monitoring protocol. For the spectrum sensing protocol, values of the false-alarm probability are evaluated for several values of the target detection probability. Given the false-alarm and detection probabilities, the resulting detection delay
and channel utilization are determined. A low target detection probability yields few false alarms, which results in high utilization and a large detection delay. A high target detection probability results in a lower detection delay and low channel utilization. The channel utilization achieved as a function of the detection delay is shown in Figure 7.4 for the spectrum sensing protocol (solid lines) and the spectrum monitoring protocol (dashed lines). For the spectrum monitoring results, the threshold for the monitoring detection opportunity is selected to achieve a false alarm probability of \( P_f < 0.05 \) and the threshold for the spectrum sensing protocol is fixed to achieve a target detection probability of \( \hat{P}_d^* = 0.9 \). For a given detection delay, significant gains in channel utilization are achieved if spectrum monitoring is employed. For example, for an average detection delay of \( 3T_p \), the spectrum monitoring protocol achieves a channel utilization of 92.4 whereas the spectrum sensing protocol achieves only 73.8. Because the spectrum monitoring protocol permits detection within the monitoring interval, more packets can be transmitted consecutively prior to scheduling a sensing interval in comparison to the spectrum sensing protocol.
Figure 7.4: Channel utilization achieved by the spectrum sensing and spectrum monitoring protocols ($\gamma = -12$ dB).
CHAPTER 8
DEMODULATOR STATISTICS FOR DETECTION

8.1 Distance statistic

Monitoring methods similar to those presented for the error count statistic can also be applied to demodulator statistics such as the distance statistic. To establish the NP decision rule for the distance statistic, the conditional density must be determined for each hypothesis. Let the random variable $A_i = B_i^2 + C_i^2$ be the squared distance between the $i$th received point and the $i$th transmitted symbol, where $B_i$ is the Euclidean distance between the in-phase components of the received point and the correct symbol location and $C_i$ is the corresponding quadrature Euclidean distance. Under hypothesis $H_0$, $B_i$ and $C_i$ are independent, zero-mean Gaussian random variables with variance $\sigma^2$. The distance statistic $D$ for a packet with $m$ QPSK symbols is

$$D = \frac{1}{m} \sum_{i=1}^{m} (B_i^2 + C_i^2), \quad (8.1)$$

which is equivalent to

$$D = \frac{1}{m} \sum_{i=1}^{2m} (\sigma F_i)^2 = \frac{\sigma^2}{m} \sum_{i=1}^{2m} F_i^2, \quad (8.2)$$

where $F_{2j} = C_j/\sigma$ and $F_{2j-1} = B_j/\sigma$ for $1 \leq j \leq m$. The random variable

$$G = \sum_{i=1}^{2m} F_i^2 \quad (8.3)$$
is a chi-squared random variable with $2m$ degrees of freedom. The density for the $D$ under $H_0$ is

$$f_0(x) = \frac{m}{\sigma^2} f_G \left( \frac{mx}{\sigma^2} \right) = \frac{m}{2\sigma^2(m-1)!} \left( \frac{mx}{2\sigma^2} \right)^{m-1} \exp \left( -\frac{mx}{2\sigma^2} \right), \quad (8.4)$$

for $x > 0$, and $f_0(x) = 0$, otherwise. The mean and variance of $D$ under $H_0$ are given by

$$E[D|H_0] = \frac{\sigma^2}{m} E[G] = 2 \sigma^2 \quad (8.5)$$

and

$$\text{Var}(D|H_0) = \frac{\sigma^4}{m^2} \text{Var}(G) = \frac{4 \sigma^4}{m}. \quad (8.6)$$

The presence of a primary signal in the band will shift the location of the received point (i.e., the demodulator outputs corresponding to the in-phase and quadrature components) in the secondary receiver. The magnitude of the shift is proportional to the strength of the primary signal and the direction of the shift depends on the polarity of the transmitted primary symbols, the time offset, and the phase offset. Thus, the emergence of a primary signal will increase the distance statistic, in general. Under $H_1$, the distance statistic is given by

$$D = \frac{1}{m} \sum_{i=1}^{2m} P_i^2, \quad (8.7)$$

where $P_i$ is a Gaussian random variable with mean $A_1 \tau / 2$ and variance $\sigma^2$. We can also express $D$ as

$$D = \frac{\sigma^2}{m} \sum_{i=1}^{2m} \left( \frac{P_i}{\sigma} \right)^2 = \frac{\sigma^2}{m} J, \quad (8.8)$$

where $J$ is a noncentral chi-squared random variable with $2m$ degrees of freedom and
 parameter $\lambda$, where
\[
\lambda = \frac{m A^2 \tau^2}{2 \sigma^2}
\] (8.9)

The density for the distance statistic under $H_1$ is
\[
f_1(x) = \frac{m}{\sigma^2} f_G \left( \frac{x m}{\sigma^2} \right) \exp \left( \frac{-x m}{2 \sigma^2} \right) \exp \left( \frac{-x m}{2 \sigma^2} \right) I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right),
\] (8.10)

for $x > 0$, and $f_1(x) = 0$, otherwise. The mean and variance of $D$ under $H_1$ are given by
\[
E[D|H_1] = \frac{\sigma^2}{m} E[J] = \frac{\sigma^2}{m} (2m + \lambda) = 2\sigma^2 + \frac{A^2 \tau^2}{2m}
\] (8.11)
and
\[
\text{Var}(D|H_1) = \frac{\sigma^4}{m^2} \text{Var}(J) = \frac{4\sigma^4}{m^2} + \frac{2\sigma^2 A^2 \tau^2}{m}.
\] (8.12)

As is shown in Appendix B, the phase offset has no effect on the distance statistic and the time offset $\hat{\tau} = \tau/2$ results in the smallest average distance statistic, which is generally the most difficult situation to detect. Distributions of the distance statistic are shown in Figure 8.1 for QSENR = 12 dB. The upper and lower extreme distributions occur for $\hat{\tau} = 0$ and $\hat{\tau} = \tau/2$, respectively. The distribution for SPR = 12 dB with $\hat{\tau} = 0$ is indistinguishable from the distribution for SPR = 9 dB with $\hat{\tau} = \tau/2$. The separation between the quiescent distribution and the distributions corresponding to a primary signal in the band indicates that the distance statistic could be useful for higher values of QSENR, which is the region in which the protocols that use the error count perform poorly.
8.2 Decision rules based on distance statistic

The NP decision rule is to choose $H_1$ if $L(x) = f_1(x)/f_0(x) > \eta$, where $\eta$ is selected to achieve a target $P_f$, $P_d$, or $P_m$. Using the conditional densities provided in 8.4 and 8.10, the likelihood function can be expressed as

$$L(x) = \frac{m}{2\sigma^2} \exp\left(\frac{-xm}{2\sigma^2}\right) \exp\left(\frac{-\lambda}{2}\right) \left(\frac{xm}{\lambda\sigma^2}\right)^{(m-1)/2} I_{m-1}\left(\sqrt{\frac{\lambda xm}{\sigma^2}}\right),$$

which reduces to

$$L(x) = (m - 1)! \exp\left(\frac{-\lambda}{2}\right) \left(\frac{x\lambda m}{4\sigma^2}\right)^{-(m-1)/2} I_{m-1}\left(\sqrt{\frac{\lambda xm}{\sigma^2}}\right),$$

Figure 8.1: Empirical distributions of the distance statistic for $\tau = \tau/2$ (solid lines) and $\tau = 0$ (solid lines).
for $x > 0$, and $L(x) = 0$, otherwise. If $L(x)$ is monotonically increasing for all $x > 0$, then $L(x) > \eta$ is equivalent to $x > \eta' = L^{-1}(\eta)$. It is clear from (8.14) that for $x > 0$, $L(x)$ is a product of continuous functions. It is now shown that $L(x)$ is a strictly increasing function for $x > 0$. Let

$$
\tilde{L}(x) = x^{-(m-1)/2} I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right).
$$

(8.15)

The modified Bessel function of the first kind can be expressed as

$$
I_\alpha(x) = \sum_{k=0}^\infty \frac{1}{k! \Gamma(k + \alpha + 1)} \left( \frac{x}{2} \right)^{2k+\alpha},
$$

(8.16)

so $I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right)$ can be expressed as

$$
I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right) = \sum_{k=0}^\infty \frac{1}{k!(k+m-1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-1}.
$$

(8.17)

The derivative of (8.17) with respect to $x$ is

$$
\frac{d}{dx} I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right) = \frac{\lambda m}{8\sigma^2} \sum_{k=0}^\infty \frac{2k + m - 1}{k!(k+m-1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-3}.
$$

(8.18)
The derivative of (8.15) with respect to $x$ is

$$
\frac{d}{dx} \tilde{L}(x) = x^{-(m-1)/2} \left[ \frac{d}{dx} I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right) \right] + I_{m-1} \left( \sqrt{\frac{\lambda x m}{\sigma^2}} \right) \left[ \frac{-(m-1)}{2} x^{-(m+1)/2} \right]
$$

(8.19)

$$
= x^{-(m-1)/2} \left[ \frac{\lambda m}{8\sigma^2} \sum_{k=0}^{\infty} \frac{2k + m - 1}{k!(k + m - 1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-3} \right]
$$

$$
+ \sum_{k=0}^{\infty} \frac{1}{k!(k + m - 1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-1} \left[ \frac{-(m-1)}{2} x^{-(m+1)/2} \right]
$$

(8.20)

$$
= \frac{1}{2} x^{-(m+1)/2} \sum_{k=0}^{\infty} \frac{2k + m - 1}{k!(k + m - 1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-1}
$$

$$
- \frac{(m - 1)}{2} x^{-(m+1)/2} \sum_{k=0}^{\infty} \frac{1}{k!(k + m - 1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-1}
$$

(8.21)

$$
= \sum_{k=0}^{\infty} (2k + m - 1) \omega_k - \sum_{k=0}^{\infty} (m - 1) \omega_k
$$

(8.22)

$$
= \sum_{k=0}^{\infty} 2k \omega_k,
$$

(8.23)

where

$$
\omega_k = \frac{1}{2} x^{-(m-1)/2} \sum_{k=0}^{\infty} \frac{1}{k!(k + m - 1)!} \left( \sqrt{\frac{\lambda x m}{4\sigma^2}} \right)^{2k+m-1}.
$$

(8.24)

Because $\omega_k > 0$ for all $k \geq 0$, it follows that the derivative of $\tilde{L}(x)$ is positive for all $x > 0$. Thus, $\tilde{L}(x)$ is increasing for $x > 0$ which implies that $L(x)$ is increasing for
Let $\eta' > 0$. Expressions for $P_f$, $P_d$, and $P_m$ are given by

$$P_f = P(D > \eta'|H_0) = \int_{\eta'}^{\infty} \frac{m}{2\sigma^2(m-1)!} \left(\frac{xm}{2\sigma^2}\right)^{m-1} \exp\left(-\frac{xm}{2\sigma^2}\right) dx,$$  \hspace{1cm} (8.26)\\

$$P_d = P(D > \eta'|H_1) = \int_{\eta'}^{\infty} \frac{m}{2\sigma^2} \exp\left(-\frac{xm}{2\sigma^2}\right) \exp\left(-\frac{\lambda}{2}\right) \left(\frac{xm}{\lambda\sigma^2}\right)^{(m-1)/2} I_{m-1} \left(\sqrt{\frac{\lambda xm}{\sigma^2}}\right) dx,$$  \hspace{1cm} (8.27)\\

and

$$P_m = P(D < \eta'|H_1) = \int_{0}^{\eta} \frac{m}{2\sigma^2} \exp\left(-\frac{xm}{2\sigma^2}\right) \exp\left(-\frac{\lambda}{2}\right) \left(\frac{xm}{\lambda\sigma^2}\right)^{(m-1)/2} I_{m-1} \left(\sqrt{\frac{\lambda xm}{\sigma^2}}\right) dx.$$  \hspace{1cm} (8.28)

The threshold $\eta'$ can be selected to satisfy a given false alarm probability using equation (8.26). However, this requires that $\sigma^2$ must be known or estimated at the receiver. One potential estimate for $\theta = \sigma^2$ is the ML estimate derived from the statistics $F_i$ defined in (8.2). The distance statistic for $m$ QPSK symbols is the sum of $2m$ independent Gaussian random variables, each with mean 0 and variance $\sigma^2$. The joint density for the variables is given by

$$f(y) = \frac{1}{(2\pi\theta)^m} \prod_{i=1}^{2m} \exp\left(\frac{-y_i^2}{2\theta}\right).$$  \hspace{1cm} (8.29)
The value of $\theta$ that maximizes (8.29) is desired, which is also the maximum of $\ln[f(y)]$. Setting the derivative equal to zero gives

$$
\frac{d}{d\theta} \ln f(y) = \frac{-m}{\theta} + \frac{1}{2\theta^2} \sum_{i=1}^{2m} y_i^2 = 0
$$

(8.30)

and solving for $\theta$ yields

$$
\theta = \frac{1}{2m} \sum_{i=1}^{2m} y_i^2 = \frac{1}{2} D.
$$

(8.31)

Taking the second derivative yields

$$
\frac{d^2}{d\theta^2} \ln f(y) = \frac{m}{\theta^2} - \frac{1}{\theta^3} \sum_{i=1}^{2m} y_i^2 = \frac{2m}{\theta^3} \left[ \frac{\theta}{2} - \frac{1}{2m} \sum_{i=1}^{2m} y_i^2 \right]
$$

$$
= \frac{2m}{\theta^3} \left[ \frac{\theta}{2} - \theta \right] = -\frac{m}{\theta^2},
$$

(8.32)

which shows that (8.31) is a maximum, and thus $\hat{\theta} = D/2$ is the ML estimate of $\theta = \sigma^2$. This estimator is unbiased and the variance is given by

$$
\text{Var}(\hat{\theta}) = \frac{1}{4} \text{Var}(D) = \frac{\sigma^4}{m}.
$$

(8.33)

8.3 Approximations for detection performance

For a large value of $m$, a chi-squared random variable is well-approximated by a Gaussian random variable. The approximate detection capability using the distance statistic is evaluated using the distributions corresponding to hypotheses $H_0$ and $H_1$. Let $D_0$ be a Gaussian random variable with mean and variance given by (8.5) and (8.6), respectively. Similarly, let $D_1$ be a Gaussian random variable with mean and variance given by (8.11) and (8.12), respectively. Let $f_{D_i}(x)$ be the density function for $D_i$. The approximate probability of false alarm, detection, and miss for
a given threshold $\eta'$ are given by

\[ P_f \approx P(D_0 > \eta') = \int_{\eta'}^{\infty} f_{D_0}(x)dx = Q \left( \frac{(\eta_d - 2\sigma^2)\sqrt{m}}{2\sigma^2} \right), \quad (8.34) \]

\[ P_d \approx P(D_1 > \eta') = \int_{\eta'}^{\infty} f_{D_1}(x)dx = Q \left( \frac{\left[ \eta_d - 2\sigma^2(1 + \frac{\mu^2}{\sigma^2}) \right] \sqrt{m}}{2\sigma^2(1 + 2\frac{\mu^2}{\sigma^2})} \right), \quad (8.35) \]

and

\[ P_m \approx P(D_1 < \eta') = \int_{-\infty}^{\eta'} f_{D_1}(x)dx = Q \left( \frac{\left[ 2\sigma^2(1 + \frac{\mu^2}{\sigma^2}) - \eta_d \right] \sqrt{m}}{2\sigma^2(1 + 2\frac{\mu^2}{\sigma^2})} \right). \quad (8.36) \]
Enhanced techniques for dynamic spectrum access are proposed and evaluated. The proposed methods increase the percentage of time that secondary radios are permitted to transmit on the frequency band compared with traditional methods for spectrum access. At the same time, the proposed techniques are shown to reduce the time it takes for the secondary network to detect the emergence of a primary signal. This is accomplished through a technique known as spectrum monitoring, which permits secondary radios to detect the presence of the primary signal during a session of packets as opposed to traditional methods that require all secondary radios to vacate the band in order for the radios to conduct channel sensing.

Spectrum monitoring does not require additional hardware or overhead; instead, it employs simple statistics obtained during packet reception to enhance traditional spectrum sensing methods. The error count, which is obtained in the decoder, and the distance statistic, which is obtained in the demodulator, are both shown to be useful statistics for the purpose of primary signal detection. Simple threshold tests are used to make a binary hypothesis decision to decide if the primary signal is present or not present. The threshold for the decision is selected to achieve a target detection or false-alarm probability based on the perceived state of the channel. Techniques for selecting the threshold are evaluated for static channels and for time-varying channels such as those that experience slow fading. The proposed techniques are also demonstrated to work well in multi-link cooperative networks. Cooperative protocols that use information gathered from several cognitive radios are shown to outperform equivalent single-link protocols. Analytical and simulated results are provided to
justify the utility of spectrum monitoring as an enhancement to traditional spectrum sensing protocols.
APPENDIX A
PROOF OF WORST CASE TIME OFFSET

For any value of \( \phi \), the average binary symbol error probability is minimized for \( \hat{\tau} = \tau/2 \). This is shown by making use of the following inequality. For \( 0 < \epsilon_1 < \epsilon_2 \) and \( x > 0 \),

\[
2Q(x) < Q(x + \epsilon_1) + Q(x - \epsilon_1) < Q(x + \epsilon_2) + Q(x - \epsilon_2), \quad (A.1)
\]
which follows from strict convexity of $Q(x)$ over $x \geq 0$. Let $\beta_i = \sqrt{\frac{2}{N_0}}$ and expand equation (4.1) to yield

$$P_e =$$

\[
\frac{1}{16} \left\{ Q \left( \beta_2 + \beta_1 \cos \phi + \beta_1 \sin \phi \right) + Q \left( \beta_2 + \beta_1 \cos \phi - \beta_1 \sin \phi \right) \right. \tag{A.2} \\
+ Q \left( \beta_2 - \beta_1 \cos \phi + \beta_1 \sin \phi \right) + Q \left( \beta_2 - \beta_1 \cos \phi - \beta_1 \sin \phi \right) \right. \tag{A.3} \\
+ Q \left( \beta_2 + \beta_1 \cos \phi + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \sin \phi \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \sin \phi \right] \right) \right. \tag{A.4} \\
+ Q \left( \beta_2 + \beta_1 \sin \phi + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \right. \tag{A.5} \\
+ Q \left( \beta_2 - \beta_1 \cos \phi + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \sin \phi \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \sin \phi \right] \right) \right. \tag{A.6} \\
+ Q \left( \beta_2 + \beta_1 \sin \phi + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \right. \tag{A.7} \\
+ Q \left( \beta_2 - \beta_1 \sin \phi + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \cos \phi \right] \right) \right. \tag{A.8} \\
+ Q \left( \beta_2 + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \left( \cos \phi + \sin \phi \right) \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \left( \cos \phi + \sin \phi \right) \right] \right) \right. \tag{A.9} \\
+ Q \left( \beta_2 + \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \left( \cos \phi - \sin \phi \right) \right] \right) \\
\left. \left[ \left( 1 - \frac{2\bar{\tau}}{\tau} \right) \beta_1 \left( \cos \phi - \sin \phi \right) \right] \right) \right. \tag{A.10} \\
\left\} . \right. \]
The inequality of (A.1) is only true for $x > 0$. Note that $\beta_2 > \beta_1$ is a sufficient condition to achieve $x > 0$ (where $x$ is the term not in square brackets for each $Q$ function). For $\beta_1 \geq \beta_2$, the primary signal is stronger than the secondary signal and detection is trivial. For the purposes of this analysis it is assumed that $\beta_2 > \beta_1$.

We find the value of $\hat{\tau}$ that minimizes each individual line in the above equation and show that each line (i.e., sum of two terms) is minimized for $\hat{\tau} = \tau/2$. It immediately follows that $P_e$ is minimized for $\hat{\tau} = \tau/2$. We need only consider the lines that are a function of $\hat{\tau}$. For these lines, we write the terms in the form of $Q(x + \epsilon) + Q(x - \epsilon)$. For each line, the terms in square brackets represent the value $\epsilon$. From (A.1) we know that a value of $\epsilon = 0$ minimizes the sum of $Q(x + \epsilon) + Q(x - \epsilon)$, which is achieved for $\hat{\tau} = \tau/2$. The inequality of (A.1) requires $\epsilon > 0$. This requirement is satisfied for $0 \leq \phi \leq \pi/4$. 
Figure B.1 shows the signal constellation that results from a primary signal that is time aligned with the secondary signal. The effect of the thermal noise is ignored initially to evaluate the effect of the primary signal on the mean of decision statistic. The point \((x_1, y_1)\) is the mean of the decision statistic if the primary signal is absent. The four blue points represent the possible means of the decision statistic if the primary signal is aligned in phase with the secondary signal and the red points \((A, B, C, \text{ and } D)\) represent the means if the primary signal has a phase offset of \(\phi_1\). If the primary signal is not aligned in time then the mean of the decision statistic will be a convex combination of two points in the space.

Consider the reception of the \(i\)th QPSK symbol in the presence of a primary signal with a phase offset of \(\phi_1\) and a time offset \(\hat{\tau}\). The polarities of two QPSK symbols in the primary signal will affect the mean of the \(i\)th decision statistic at the secondary receiver. Let \(S_1\) denote the QPSK symbol being transmitted for the first \(\hat{\tau}/\tau\) seconds of the \(i\)th QPSK symbol and let \(S_2\) denote the QPSK symbol being transmitted for the last \((\tau - \hat{\tau})/\tau\) seconds of the \(i\)th QPSK symbol. The points \(S_1\) and \(S_2\) are from the set \(\{A, B, C, D\}\) with equal probability. By symmetry we need only consider \(S_1 = A\) and evaluate the four possibilities for \(S_2\). Let \(D(s_1, s_2, \alpha, \phi_1)\) be the square of the Euclidean distance given that \(S_1 = s_1\) and \(S_2 = s_2\), where \(\alpha = \hat{\tau}/\tau\) and \(\phi_1\) is the phase offset. The range of interest for \(\alpha\) is [0,1]. For our system \(|x_1| = |y_1| = A_2\tau/2\) and \(u = A_1\tau/2\). The remaining distances shown in Figure B.1 are \(v = \sqrt{2}u\), \(z = \sqrt{2}u\sin(45 - \phi_1)\), and \(s = \sqrt{2}u\cos(45 - \phi_1)\). We can express the
desired distances as

\[ D(A, A, \alpha, \phi_1) = z^2 + s^2 = 2u^2, \quad (B.1) \]

\[ D(A, B, \alpha, \phi_1) = [\alpha z + (1 - \alpha)s]^2 + [\alpha s - (1 - \alpha)z]^2 = 2u^2(2\alpha^2 - 2\alpha + 1), \quad (B.2) \]

\[ D(A, C, \alpha, \phi_1) = [\alpha z - (1 - \alpha)z]^2 + [\alpha s - (1 - \alpha)s]^2 = 2u^2(2\alpha - 1)^2, \quad (B.3) \]

\[ D(A, D, \alpha, \phi_1) = [\alpha z - (1 - \alpha)s]^2 + [\alpha s + (1 - \alpha)z]^2 = 2u^2(2\alpha^2 - 2\alpha + 1). \quad (B.4) \]

It follows immediately from (B.1)–(B.4) that the distance statistic is independent of \( \phi_1 \) and that a value of \( \alpha = 0.5 \) (or \( \hat{\tau} = \tau/2 \)) results in the smallest distance for each case. To obtain the distribution for the distance statistic for asynchronous signals, we assume that each of the four phase offsets is equally likely. The only parameter that is affected is the value of \( \lambda \) defined in (8.9). For asynchronous signals, \( \lambda \) is a function of \( \hat{\tau} \) and is given by

\[ \lambda_{\hat{\tau}} = \frac{mA_1^2\tau^2}{2\sigma^2} - \frac{mA_1^2}{\sigma^2}\hat{\tau}(\tau - \hat{\tau}). \quad (B.5) \]
Figure B.1: QPSK constellation points corrupted by a primary signal with phase offset $\phi_1$. 
REFERENCES


