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GAMMA-RAY LINES FROM YOUNG SUPERNOVA REMNANTS

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ABSTRACT

The gamma-ray luminosity of a typical type I supernova remnant has been calculated by assuming that the origin of the optical luminosity is due to the energy of the radioactive decay of Ni^{56}. It is expected that Ni^{56} is the most abundant nucleus resulting from silicon burning in the supernova shock conditions. The requisite mass of Ni^{56} (0.14 $M_\odot$) gives rise to gamma-ray lines with energies near 1 MeV that should be detectable in young supernova remnants at distances up to a few Mpc. Future detectors aboard satellites should be able to detect events at the rate of about two observable events per year. A few supernova remnants in the Galaxy should be observable at all times in lines following the decay of Ti^{44}.

The work of Bodansky, Clayton, and Fowler (1968*a*, *b*) has greatly clarified the earlier papers of Fowler and Hoyle (1964), Truran, Cameron, and Gilbert (1966) and Truran, Arnett, and Cameron (1967) on the abundances to be expected in the thermonuclear burning of silicon, and it now appears likely that silicon-burning shells in supernovae are responsible for most of the nucleosynthesis in the range of atomic weights $28 \leq A \leq 57$. A key feature of this work for the gamma-ray astronomy of supernova remnants is that the large natural abundances of some stable nuclei (particularly Fe^{56}) are to be attributed to the beta decay after expulsion of very abundant progenitors established during silicon burning in the shock-heated supernova shells. A few of these beta decays are followed by nuclear gamma radiation in intensities that may be expected to be observable in young supernova remnants. In particular the 6.10-day electron capture by Ni^{56} produces gamma rays and 77-day Co^{56}, whose decays to Fe^{56} are accompanied by a rich gamma-ray spectrum that may be observable for a year or so in supernova remnants to distances of several Mpc. Another interesting nucleus is Ti^{44}, which decays with a 48-year half-life to Sc^{44} and then to Ca^{44} with the emission of a 1.156 MeV gamma quantum. That gamma-ray line may be seen rather clearly in nebulae much too old to retain detectable concentrations of Co^{56}. It is our intention to assess the likelihood of detecting these gamma emissions. Such detection would be of outstanding importance for the theory of nucleosynthesis and of supernovae.

We cannot at this time make precise estimates of the silicon-burning yield in supernova shells, because that information depends upon a detailed model of the presupernova star and upon the fraction of the silicon that can survive the shock heating and expulsion. This material involves not only silicon shells in the presupernova star but oxygen shells as well, inasmuch as they will quickly collapse into a silicon-burning quasi-equilibrium during the shock heating of the oxygen and its resultant explosive burning. But we believe it cannot be far wrong to expect 0.1–1.0 $M_\odot$ of such material to be involved, depending upon the details of the presupernova star. Bodansky *et al.*

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(1968*b*) showed that the first 50 per cent of some of the Si^{28} is rapidly converted to a quasi-equilibrium with a peak at Ni^{56}, whereas the last 25 per cent or so is destroyed much more slowly. They also found that burning of about 65 per cent of the silicon closely matched natural abundances for a wide range of temperature and density, so that the assumption that supernovae are responsible for nucleosynthesis in the mass range $28 \leq A \leq 57$ is consistent with assuming that the average supernova burns about two-thirds of its silicon.

Further motivation for the expected radioactivity is to be found in the explanation of the supernova light curves. Colgate and McKee (1968), at the suggestion of J. Truran, have shown that the radioactive power delivered by the Ni^{56} to the expanding shell while it is still opaque does maintain a sufficiently high temperature to provide a high optical luminosity when the expanding nebula becomes optically thin. The important property of the decay of Ni^{56} is the relatively long half-life, 6.1 days, for the large energy difference, 2.11 MeV. The subsequent decay of Co^{56} to Fe^{56} has a still longer decay time, 77 days, and a still larger energy difference, 4.57 MeV, but is not as important for the peak of the light emission as the faster decaying Ni^{56}. The upshot is that the expected yield of Ni^{56} gives roughly 20 times the beta-decay energy in the critical period of initial optical transparency as would an arbitrary mixture of allowed beta decays. This is 10^3–10^4 times the optical luminosity expected from the shock-deposited energy alone. As a consequence of this unique feature as well as the fact that Ni^{56} is predicted to be the most abundant radioactive nucleus from the shock nucleosynthesis, we have investigated the expected X-ray and gamma-ray spectra.

Truran *et al.* (1967) showed that the supernova shock conditions are appropriate for silicon-burning nucleosynthesis. The shock temperature (5×10^9 ° K), density (1.3×10^7 g cm^{-3}), and expansion time (0.1 sec) were taken from calculations of Colgate, Grasberger, and White (1951) which were later reproduced with the neutron-star, neutrino-transport calculations of Colgate and White (1966). These temperature and density conditions correspond to ejecting 0.5 $M_\odot$ at a mean final velocity of 1.7×10^9 cm sec^{-1}. The thermal energy at the shock is hydrodynamically transformed into the kinetic energy of the rapidly outflowing fluid behind the shock. This fluid velocity expands a 0.5 $M_\odot$ shell by a Δr equal to its thickness in a time of about 0.1 sec, which is roughly the duration of the silicon-burning nucleosynthesis. These conditions are independent of the supernova mechanism. They depend only on having 0.5 $M_\odot$ shock ejected from a highly evolved star at a velocity that is close to that observed for the most recent supernova remnant, Tycho's—as analyzed in detail by Minkowski (1967). Bodansky *et al.* (1968*b*) showed, furthermore, that the product abundances are not very sensitive to the temperature and density of the silicon quasi-equilibrium as long as they are such that Ni^{56} is the major nucleus synthesized, and as long as the same mass fraction of silicon remains in each case. For definiteness, therefore, we take the conditions of burning as $T_9 = 4.0$, $\rho = 10^7$, and $X(Si^{28}) = 0.25$ (which corresponds exactly to Fig. 21 of Bodansky *et al.* 1968*b*). These relative concentrations hardly differ, for example, from those corresponding to $T_9 = 4.8$, $\rho = 10^9$, and $X(Si^{28}) = 0.25$. Table 1 lists the number of nuclei per gram for several of the most interesting species in such a burning state, as well as their half-lives and associated gamma-ray lines. The branching ratios shown for the pair-annihilation lines (0.511 MeV) are calculated as being twice the branching ratio for positron decay. The Ni^{56} mass fraction corresponding to Table 1 is about 32 per cent, so that if we conservatively assume about 0.5 $M_\odot$ of silicon-burning debris, we have about 0.16 $M_\odot$ of Ni^{56} in the nebula. Approximately the same mass of Ni^{56} is required for the radiant energy of the supernova, which has been estimated by Minkowski as 10^{49} ergs released in roughly 10^6 seconds. The decay of 0.14 $M_\odot$ of Ni^{56} releases 10^{49} ergs of energy, so for the purpose of estimating the gamma-ray line fluxes we will normalize our calculations to that yield. A more detailed calculation must include the less efficient radioactive heating due to the decay of other nuclear species (especially Co^{56} and Fe^{52}) and the partial losses due to adiabatic expansion (Colgate and McKee 1968). A yield of 0.14 $M_\odot$ of Ni^{56} is nonetheless a convenient normalization. It is consistent with a total of about 0.5 $M_\odot$ of ejected matter, and any change in supernova yield obtained from a more detailed theory can, in any case, be scaled almost linearly except during the initial rise of the light curve. We have also assumed that the hydrogen and helium mass fractions are small; a larger supernova (type II) should have an external envelope of hydrogen and helium which would somewhat modify the gamma-ray transparency at early times.

The credibility of this yield is attested to by the following interesting but inconclusive argument. If the average composition of the Galaxy is taken to be the same as that of the Sun, it contains about 3×10^7 $M_\odot$ of Fe^{56} or about 3×10^8 supernovae at a yield of 0.1 $M_\odot$ of Ni^{56} per supernova. An active galactic age of 10^{10} years could produce such a result if supernovae occur at a rate of about 1 per 30 years per galaxy. This rate is quite reasonable according to arguments presented by Katgert and Oort (1967) and Kukarkin (1965).

TABLE 1

CONCENTRATION OF KEY NUCLEI*

Nucleus	Number per Gram	$\tau_{1/2}$	E_γ (MeV)†
Si^{28}	5.38×10^{21}	Stable	....
Ti^{44}	$5\ 43 \times 10^{18}$	48 years	(Sc^{44}) 1 156 (100), 0 511 (188)
Cr^{48}	2.71×10^{19}	23 hours	0.31 (100)
V^{48}	Cr^{48} decay	16 days	0.983 (100), 0.511 (100), 1.312 (97), 0.945 (10), 2.241 (3)
Ni^{56}	$3\ 40 \times 10^{21}$	6.10 days	0 812 (85), 0.748 (51), 0 472 (34), 1.56 (15)
Co^{56}	Ni^{56} decay	77 days	0.847 (100), 1.24 (67), 0 511 (40), 2.60 (17), 1.03 (16), 1 76 (14), 3.26 (13), 2.02 (11)

* Nuclear data from Lederer, Hollander, and Perlman (1967).

† Numbers in parentheses are per cent per disintegration.

We calculate the emergence of the gamma rays from the expanding nebula as follows. The effective mass fraction visible at any given wavelength as a function of time corresponds to the surface condition

$$\int_{r_s}^{\infty} \rho dr = 1/K\,, \tag{1}$$

where K is the mass absorption coefficient for the gamma ray or X-ray in question. The mass fraction F_s corresponds to the fraction of the ejected mass external to r_s:

$$F_s = \frac{1}{M} \int_{r_s}^{\infty} 4\pi r^2 \rho dr\,, \tag{2}$$

where M is the total ejected mass. The observable fraction for constant surface brightness is then

$$F_{\text{obs}} = \tfrac{1}{4} F_s \quad \text{for} \quad F_s \ll 1\,, \qquad F_{\text{obs}} \simeq \tfrac{1}{2} F_s \quad \text{for} \quad F_s = 1\,,$$

$$F_{\text{obs}} = 1 \quad \text{if} \quad \int_0^{\infty} K \rho dr \ll 1\,.$$

The velocity distribution of the ejected matter has been discussed by Colgate and White (1966) and Colgate (1967) and can be approximated by the analytical expression

$$U = U_0 F^{-1/4}\,, \qquad F \leq 0.1\,, \tag{3}$$

$$U = 10^{1/4} U_0 (1.1 - F)^{1/3}\,, \qquad F \geq 0.1\,. \tag{4}$$

Expression (3) corresponds to the shock wave speeding up in the near-plane-parallel density gradient of the envelope and expression (4) corresponds to a near-uniform expansion of the bulk (90 per cent) of the ejected mass. When the matter has expanded to distances large compared with its initial radius in the presupernova star, then the matter distribution in terms of the F-coordinate is surprisingly model-independent, with the exception of U_0 and the total expected mass. For the smaller-mass supernova (type I), $U_0 \simeq 1.7 \times 10^9$ cm sec^{-1} which gives a mass average for the 90 per cent fraction of 2×10^9 cm sec^{-1} as observed for Tycho's nebula.

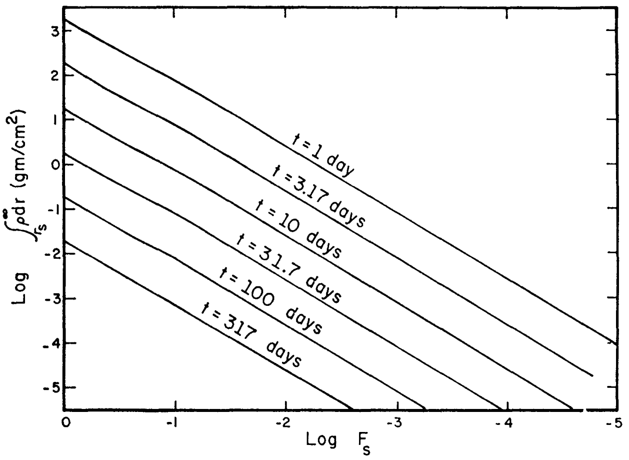

FIG. 1.—Thickness in grams per unit column above the mass fraction F_s is shown as a function of F_s for several different values of the time. A given thickness corresponding to a $1/e$ length for a specific gamma-ray energy penetrates to increasingly greater values of F_s as time progresses.

The surface condition then becomes

$$\begin{aligned} \int_{r_s}^{\infty} \rho dr &= \frac{M}{6\pi U_0^2 t^2} F_s^{3/2}\,, \qquad F \leq 0.1 \\ \int_{r_s}^{\infty} \rho dr &= \frac{0.9M[1-(1.1-F_s)^{1/3}]}{\frac{4}{3}\pi(10^{1/2})U_0^2 t^2} + \frac{10^{-3/2}M}{6\pi U_0^2 t^2}\,, \qquad F \geq 0.1\,. \end{aligned} \tag{5}$$

Figure 1 shows the matter thickness as a function of the corresponding external mass fraction for several values of time by using expression (5).

We are interested in just what mass fraction $F_{\text{obs}}(E)$ is actually observable from a fixed direction with photons of energy E. The mass fraction from which photons may emerge radially is such that the external thickness per unit column

$$\int_{r_s}^{\infty} \rho dr \simeq 1/K(E)\,,$$

where $K(E)$ is the mass absorption coefficient at energy E. We have found that numerically that mass fraction is given approximately by $F_s(E,t) \simeq 0.038\,[E(\text{MeV})]^{0.39}\,[t(\text{days})]^{1.39}$.

But the observable mass fraction from a given direction is derivable only by rather involved geometrical arguments which take into account the self-shielding of the nebula. We find that detailed calculations are well reproduced by the following simple treatment of Figure 1. The column thickness $1/K(E)$ determines the ordinate value of Figure 1. The point where a given time line intersects that value yields the radially transparent fraction $F_s(E)$, and the fraction of that value observable from a selected direction is well approximated by

$$\begin{aligned} F_{\text{obs}} &\simeq \tfrac{1}{4}F_s\,, & F_s &< \tfrac{1}{2}\,; \\ F_{\text{obs}} &\simeq \tfrac{3}{4}F_s - \tfrac{1}{4}\,, & \tfrac{1}{2} < F_s &< 1\,; \\ F_{\text{obs}} &\simeq \tfrac{1}{2}F_s^*\,, & 1 \leq F_s^* &< 2\,; \\ F_{\text{obs}} &\simeq 1\,, & 2 &\leq F_s^*\,. \end{aligned}$$

The quantity F_s^* is not the real mass fraction, which has an upper limit of unity, but is an extrapolation of the curves of Figure 1 that correctly characterizes the results of a more detailed calculation. The problem of emergence of the gamma rays is one of absorption only rather than, as for the problem of optical luminosity, one of thermal diffusion.

The problem of the continuum at all wavelengths is a more difficult one than the gamma-ray spectrum. Colgate and McKee (1968) have provided further discussion of the optical continuum. Near 1 MeV where the gamma lines are, the continuum should be quite small, however, so the lines will be easily extractable from the continuum. Bremsstrahlung by the positrons emitted produces a negligible continuum near the energies in question.

The possibility exists that the Kα X-rays can be observed as well as the characteristic gamma-ray lines. The X-ray transitions are excited by the electron-capture process in the primary nucleus and to a certain extent by the photoelectric absorption of the primary and cascade gamma rays. The number of Kα X-rays is comparable to the number of gamma rays, but because of their much lower photon energy and because of the large X-ray continuum, the line X-rays will probably be more difficult to detect.

In Figure 2 we show the gamma-ray fluxes as a function of time for several key lines emitted with the decay of the species labeling the curves. We have calculated these fluxes for a supernova distance of 10^6 pc to emphasize their observability in extragalactic objects. The flux at an arbitrary distance may, of course, be obtained by dividing the values in Figure 2 by $[d(\text{Mpc})]^2$. We have shown only one line from each decay because the remainder can be inferred adequately from Table 1. The 0.511 MeV line results from the annihilation of positrons emitted by Co^{56} and Sc^{44}.

The growth rate of the fluxes at short times reflects primarily the thinning of the nebula during its early hydrodynamic expansion. This growth rate reflects the assumptions of spherical hydrodynamics and of a negligibly thin envelope of hydrogen and helium. The massive envelope of a type II event would provide an additional opacity to the gamma rays, with the result that their flux would emerge less rapidly. Measurement of the rise time of these fluxes would provide a datum of unique importance for the understanding of supernova hydrodynamics, namely a combination of U_0 and the total ejected mass. An effective supernova-watch program will be important for such a study.

After a period of about 1 month the fluxes are almost independent of the hydrodynamics because the nebula is by that time sufficiently thin near 1 MeV to see all of the ejecta with high efficiency. The flux levels at a specified distance depend primarily upon the argument that 0.14 $M_\odot$ of Ni^{56} is produced in the explosion and upon the various half-lives involved.

The question of the detectability of these gamma-ray lines is a very interesting one. Balloon-borne experiments have been performed (Haymes *et al.* 1968) to search for the gamma-ray line emission below 0.5 MeV from the Crab Nebula that would be expected on the basis of the californium hypothesis (Clayton and Craddock 1965). Although no line gammas were detected, the detector was sufficiently sensitive that a strong signal (five standard deviations above the background) would have resulted from a line gamma-ray flux of 2×10^{-3} photon cm^{-2} sec^{-1} at an energy of 0.5 MeV. (The strongest expected lines were at the 10^{-4} flux level.) Calculations show that a detector of the type used by Haymes *et al.* (1968) has a sensitivity to line gamma rays that is practically energy-independent from 0.5 to 3.0 MeV; the decreasing detector efficiency at increased energies is offset by the decreasing background radiation. So we take 2×10^{-3} to be the flux level detectable from balloons for the particular lines of this investigation. Below 0.5 MeV the gamma-ray background flux increases more rapidly than the detector efficiency, decreasing the photon sensitivity of present detectors. Thus the K X-ray photons near 0.01 MeV as well as the lower-energy line gamma rays from extragalactic supernovae would most likely be lost in the background.

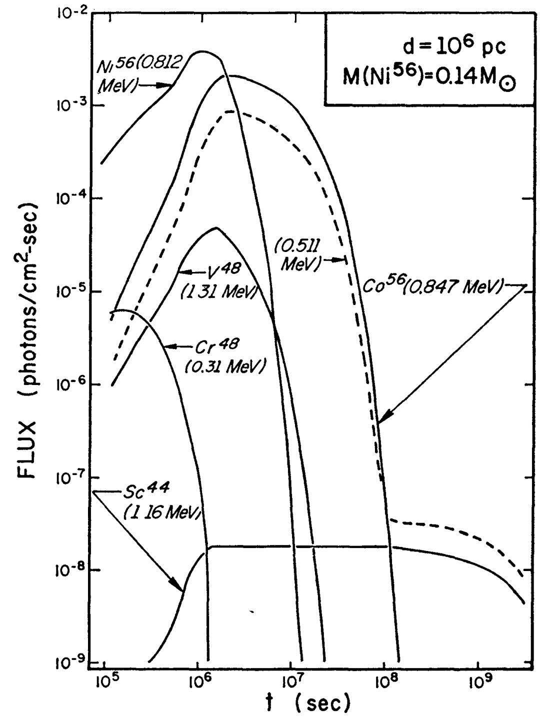

FIG. 2.—Prominent gamma-ray line fluxes as a function of time. The calculation assumes that about 0.5 $M_\odot$ of silicon-burning shells containing 0.14 $M_\odot$ of Ni^{56} are ejected from a supernova at a distance of 10^6 pc. Fluxes in other circumstances vary linearly with the yield and inversely with the square of the distance. Lines are identified by the chemical symbol of the decaying nucleus and the energy of the associated gamma ray. Relative strengths of other lines from a thin nebula are listed as branching ratios in Table 1. Dashed line labeled 0.511 MeV is calculated by taking the rate of annihilation of positrons to be equal to their rate of emission from Co^{56} plus Sc^{44}.

Haymes (private communication) has made a design study of the effectiveness of such a telescope in a satellite. Due to the decreased background (assuming an optimum low-altitude, low-inclination orbit) and longer observation times, the detectability threshold could be reduced for that same instrument by a factor of about 50, or to a line flux of 4×10^{-5} cm^{-2} sec^{-1}. The ORNL group has designed a Ge(Li) diode telescope (cf. Chapman *et al.* 1968) in an attempt to take advantage of the improved energy resolution of such detectors, but it is not known what threshold in flux they can ultimately expect to detect. But for the purposes of this paper we are content to note that a satellite threshold of 4 × 10^{-5} cm^{-2} sec^{-1} should make supernovae detectable to distances up to about 10 Mpc.

It is quite apparent from Figure 2 that the best opportunity for detection is provided by the 0.85 and 1.24 MeV Co^{56} lines during the first few months following the explosion. These line fluxes are above the balloon threshold, for example, for the period 15–40 days following a supernova at 10^6 pc, and for considerably longer periods of time at the flux level corresponding to satellite threshold. For less distant supernovae of course, the chances are correspondingly improved. For older supernovae (>5 years) within our Galaxy, the 1.15 MeV line of Sc^{44} is the most easily detectable line. It should be detectable by satellite for ages of 100 years or so at distances within 10^4 pc.

For the foregoing arguments it will be clear that the chance of observing these events depends upon the frequency with which supernovae may be expected to occur within a given distance and upon the likelihood that they will be detected early enough. The frequency is an elusive quantity that is very much in doubt. Only three supernovae within the last 900 years are attributed to our Galaxy. All three are estimated to be within about 3 kpc of the Earth or about one-fourth of the galactic radius. It is thus probable that the galactic-supernova frequency is greater than 0.003 by a factor of about 10–20. A different observational problem exists in determining the frequency of supernovae from external galaxies. There is a certain element of "chance" that the supernova will be found near its maximum brightness and that it will not be lost in the luminous inner regions of a galaxy. Recently, Katgert and Oort (1967) have examined in detail the list of supernovae discovered since 1885 as prepared by Zwicky (1964). They find significant deficiencies in the number of supernovae discovered in the southern hemisphere, in fainter galaxies, and during certain intervals of less intensive supernova searching. They derive a supernova frequency of 0.03 per year per galaxy, in agreement with the galactic estimate given above.

TABLE 2

DETECTABILITY SUMMARY

GAMMA-RAY LINES	TIME AFTER OUTBURST	BALLOON-BORNE DETECTOR (2×10^{-3} cm^{-2} sec^{-1})		SATELLITE-BORNE DETECTOR (4×10^{-5} cm^{-2} sec^{-1})	
		Supernova Distance (Mpc)	Approx. Number Detectable per Year	Supernova Distance (Mpc)	Approx. Number Detectable per Year
Ni^{56} (0 812, 0.748 MeV)..	10–20 days	1.3	0.1	9.0	2–5
Co^{56} (0.84, 1.24 Mev) ...	15–40 days	1.0	.1	7.0	2
Co^{56} (weaker lines); V^{48} (0.983, 1.31 MeV)..	15–30 days	0.1	.03	0.7	0.1
Sc^{44} (1.156 MeV)........	15 days–50 years	0.003	0 003?	0.02	Continuous

We have made rough estimates of the frequency of observable events based on a supernova frequency of 0.03 per year per mean galaxy (10^{11} stars). Due to inhomogeneities in the distribution of the local galaxies, the estimated number of galaxies within a radius d is not a smooth function but was obtained from a list of the brightest galaxies (Allen 1963) up to $d = 4$ Mpc. For distances greater than 4 Mpc, it is expected that the frequency will be proportional to the volume of space (d^3).

In Table 2 we have summarized our expectations regarding the observational possibilities. We have listed for the relevant decays the approximate number of supernovae per year that should be detectable, at the flux thresholds of both balloon and satellite detectors, out to the indicated distances. The second column shows the period of time during which the supernova should be detectable if it is at the distances indicated. With the use of balloon-borne telescopes we must await a suitable discreet opportunity which may present itself roughly every 10 years or so. With the use of a satellite-borne telescope the situation changes remarkably, for several detectable events per year are expected in the lines of Ni^{56} and Co^{56}. The operational need here is not only that of a satellite-borne telescope, but also of an efficient supernova watch capable of early detection of all supernovae out to a distance of 10^7 pc. The Sc^{44} lines present another fascinating possibility for the satellite-borne telescope, moreover, for if the supernova frequency in the Galaxy is 0.03 per year, two or three galactic remnants should always be detectable inasmuch as the supernova frequency is greater than the Ti^{44} decay rate.

Thus, the observation of gamma-ray line emission from a young supernova seems very promising in the near future. This observation, or even a null observation at a low threshold, will have great significance in the fields of nuclear astrophysics and supernova theory. The scientific importance of a positive measurement would be analogous with and comparable to the importance of the successful detection of neutrinos from the Sun.

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REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (London: Athlone Press).
Bodansky, D., Clayton, D. D., and Fowler, W. A. 1968*a*, *Phys. Rev. Letters*, **20**, 161.
———. 1968*b*, *Ap. J. Suppl.*, **16**, 299.
Chapman, G. T., Macklin, R. L., Gibbons, J. H., and Nutt, R. 1968, *Bull. Am. Phys. Soc.*, **13**, 695.
Clayton, D. D., and Craddock, W. L. 1965, *Ap. J.*, **142**, 189.
Colgate, S. A. 1967, *Ap. J.*, **150**, 163.
Colgate, S. A., Grasberger, W. H., and White, R. H. 1961, University of California Radiation Laboratory Rept. No. 6741.
Colgate, S. A., and McKee, C. 1968, 126th meeting, A.A.S., Charlottesville, Virginia, April 1968 (to be published).
Colgate, S. A., and White, R. H. 1966, *Ap. J.*, **143**, 626.
Fowler, W. A., and Hoyle, F. 1964, *Ap. J. Suppl.*, **9**, 201.
Haymes, R. C., Ellis, D. V., Fishman, G. J., Kurfess, J. D., and Tucker, W. H. 1968, *Ap. J.* (*Letters*), **151**, L9.
Katgert, P., and Oort, J. H. 1967, *B.A.N.*, **19**, 239.
Kukarkin, B. V. 1965, *Astrofizika* (Russian ed.), **1**, 465.
Lederer, C. M., Hollander, J. M., and Perlman, I. 1967, *Table of Isotopes* (6th ed.; New York: John Wiley & Sons).
Minkowski, R. L. 1967, in *Stars and Stellar Systems*, ed. L. H. Aller and B. M. Middlehurst (Chicago: University of Chicago Press), Vol. **7**, chap. xi.
Truran, J. W., Arnett, W. D., and Cameron, A. G. W. 1967, *Canadian J. Phys.*, **45**, 2315.
Truran, J. W., Cameron, A. G. W., and Gilbert, A. 1966, *Canadian J. Phys.*, **44**, 563.
Zwicky, F. 1964, Interim Report to the Committee for Research on Supernovae within Commission 28 of the I.A.U.