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MODELING OF THE EFFECT OF TOOL WEAR ON CUTTING FORCES IN TURNING

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MODELING OF THE EFFECT OF TOOL WEAR ON CUTTING FORCES IN TURNING

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Mechanical Engineering

by
Yu Long
May 2008

Accepted by:
Dr. Yong Huang, Committee Chair
Dr. Georges M. Fadel
Dr. Thomas R. Kurfess
Dr. John C. Ziegert
ABSTRACT

Machining is the most widely used and efficient material removal process, and tool wear in machining is usually of great interest in order to improve machining efficiency and effectiveness. Cutting tool wear patterns such as the flank and crater wear and the dead metal zone (DMZ) may induce the cutting tool geometry variation in machining. As a result, cutting forces may increase or decrease due to the change of tool effective cutting geometry during machining operations. A quantitative understanding of and the ability to predict cutting forces in relation to tool wear are important to the tool life estimation, chatter prediction, and tool condition monitoring. Most available force models are limited to fresh tool conditions or worn tool conditions only considering the flank wear. Furthermore, the effect of DMZ on cutting forces, especially when a chamfered tool is used, is frequently ignored. The objective of this dissertation is to analytically model the combined effect of the crater and flank wear as well as the effect of DMZ on cutting forces in 2D and 3D turning.

Using turning as the most common machining example, analytical 2D and 3D force models are first proposed to model cutting forces under worn tool conditions with both crater and flank wear presented using the slip-line based plasticity theory. An uncertainty study is further proposed to validate the proposed 2D force model using the noninformative Bayesian linear regression approach in cutting CK45 steels. The validated 2D force model is further extended for 3D oblique cutting by using the geometric and coordinate transformations based on the cutting chip discretization, and the
developed 3D force model is validated in hard turning of hardened 52100 bearing steels.

The effect of stagnant DMZ on cutting forces is studied based on a three-zone model. The total energy consumption under the DMZ zone is modeled due to excess, extrusion, and friction using a slip-line approach. Satisfactory match between the predictions and the measurements has been achieved in turning of P20 mold steels. An improved modeling accuracy is also observed when compared with a previous modeling effort.

This study leads to a new turning force models under the combined effect of flank and crater wear in 2D and 3D cutting, a new Bayesian analysis-based methodology to evaluate force measurements and predictions, and a new approach to model the effect of DMZ on cutting forces in using chamfered or worn tools.
DEDICATION

To my father (1940-2006)
ACKNOWLEDGEMENTS

I will miss the days and nights at Clemson University.

Thank a lot for my advisor, Dr. Yong Huang, to teach me about manufacturing, also help me to revise my papers, and my dissertation. Also, I really appreciate helps from my committee members, Professor Thomas R. Kurfess, Professor John C. Ziegert, and Professor Georges M. Fadel. Without any of the helps, I cannot finish my work.

I have had a good time in Clemson Advanced Manufacturing and System Integration Laboratory. There are lots of fun among me and our group members: Xiaoyu Wang, Lei Tang, Wei Wang, Yafu Lin, Jun Yin, Mason D. Morehead, and Kevin Foy. Thank all of you very much!

At last, I would like to thank supports from my parent, my brothers, and my wife Bin Yang.
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\(= M_{vG,L} + M_{vLN} + M_{vNP} \)  

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| $$n$$, $$m$$ | Number of experimental sceneries, number of inputs |
|---|

| $$N(\cdot, \cdot), t(\cdot; \cdot)$$ | Normal distribution, the student’s t distribution |
|---|

| $$p(\cdot | \cdot), p(\cdot)$$ | Conditional and joint probability densities with the arguments determined by the context |
|---|

| $$P_{H}, P_{G_1}, P_{G_2}, P_{L}$$ | Hydrostatic stress of point $$H$$, $$G_1$$, $$G_2$$ and $$L$$, respectively |
|---|

| $$r_1, r_2, r_3$$ | Radius of slip line $$LG_1$$, $$OG_2$$, $$G_1G_2$$, respectively |
|---|

| $$R, R'$$ | Velocity jump across the slip line $$HIGG_1O$$ and $$B_CC_D$$ |
|---|

| $$R_1, R_o, R_{G_1}, R_{H}, R_{L}$$ | Velocity of inner and outer radiiuses of chip, velocities at points $$G_1$$, $$H$$, and $$L$$ |
|---|

| $$t_1$$, $$t_c$$ | Undeformed chip thickness and chip thickness |
|---|

| $$t$$ | Time |
|---|

| \(s^2\) | Scale factor: \(s^2 = \frac{1}{n-m} \left( y - X\hat{\beta} \right) \left( y - X\hat{\beta} \right) \) |
|---|

| \(t_{\alpha/2, n-m}\) | \(\alpha/2\)-upper percentile of student distribution with n-m degree of freedom |
|---|

| $$V$$, $$V_c$$, $$V_f$$ | Cutting speed, chip velocity, velocity of workpiece under the flank wear land |
|---|

| $$V_\beta$$ | Scale factor: \(V_\beta = (X'X)^{-1} \) |
|---|

<p>| $$VB$$ | Flank wear length |</p>
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<td>$\bar{\alpha}$</td>
<td>Significance level</td>
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<tr>
<td>$\bar{\beta}$</td>
<td>Parameter vector (a vector $1 \times m$): $\bar{\beta} = (\bar{\beta}_0, \bar{\beta}_1, \ldots, \bar{\beta}_m)$</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
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**Chapter 3**

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<th>Constants used to model the chip contact length</th>
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<tr>
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</tr>
<tr>
<td>( \psi_0^j, \psi_r^j )</td>
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</tr>
<tr>
<td>( \eta_c^j, \eta_{cr}^j, \eta_{cr}^0 )</td>
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Chapter 4

A, B, C, D, E, m, n | Constants of the Johnson-Cook material model |
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<td>Shear strain rate constant of primary shear zone</td>
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<td>( F_2 )</td>
<td>Tangential force on the bottom edge ON of DMZ</td>
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<td>( F_{c1} )</td>
<td>Cutting force of the primary shear zone</td>
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<td>( F_{c2} )</td>
<td>Cutting force from the bottom edge of DMZ</td>
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</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<td>Thrust force of the primary shear zone</td>
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<td>Thrust force from the bottom edge of DMZ</td>
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<td>Shear flow stress of chip at the tool-chip interface</td>
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<td>Shear flow stress of workpiece in primary shear zone</td>
</tr>
<tr>
<td>$k_{work}$</td>
<td>Shear flow stress of work piece along edge ON</td>
</tr>
<tr>
<td>$K$</td>
<td>Thermal conductivity</td>
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<tr>
<td>$N_1$</td>
<td>Normal force on the rake face</td>
</tr>
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<td>$t$</td>
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<td>$t_0$</td>
<td>Undeformed chip thickness</td>
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<td>$\Delta t$</td>
<td>Time of material passing the bottom edge of DMZ</td>
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<td>Effective up flow undeformed material thickness</td>
</tr>
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<td>$t_{e2}$</td>
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</tr>
<tr>
<td>$t_{chip}$</td>
<td>Chip thickness</td>
</tr>
<tr>
<td>Symbol</td>
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</tr>
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</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Room temperature</td>
</tr>
<tr>
<td>$T_{\text{chip}}$</td>
<td>Temperature of chip along the tool-chip interface</td>
</tr>
<tr>
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<td>Melt temperature of the workpiece</td>
</tr>
<tr>
<td>$T_{\text{work}}$</td>
<td>Temperature along the tool – work piece interface</td>
</tr>
<tr>
<td>$T_{MN}$</td>
<td>Temperature of primary shear zone</td>
</tr>
<tr>
<td>$\Delta T_{\text{chip}}$</td>
<td>Average temperature rise in the chip</td>
</tr>
<tr>
<td>$\Delta T_{\text{max}}$</td>
<td>Maximum temperature rise in the chip</td>
</tr>
<tr>
<td>$\Delta T_{MN}$</td>
<td>Temperature rise of primary shear zone</td>
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<tr>
<td>$\Delta T_{\text{work}}$</td>
<td>Average temperature rise of the zone under the DMZ</td>
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<td>$\Delta T_{\text{wMax}}$</td>
<td>Maximum temperature rise along the interface of DMZ and work piece</td>
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<tr>
<td>$u_{\text{total}}$</td>
<td>Total specific energy of the deformation under the DMZ</td>
</tr>
<tr>
<td>$u_{p1}$</td>
<td>Specific energy of extrusion deformation</td>
</tr>
<tr>
<td>$u_{p2}, u_{p2\text{entrance}}$</td>
<td>Specific energy of excess deformation, the specific energy of entrance deformation</td>
</tr>
<tr>
<td>$u_{p3}$</td>
<td>Specific energy of friction deformation</td>
</tr>
<tr>
<td>$V$</td>
<td>Cutting speed</td>
</tr>
<tr>
<td>$V_{\text{chip}}$</td>
<td>Velocity of chip</td>
</tr>
</tbody>
</table>
\( V_s \) | Workpiece velocity along the primary shear plane
---|---
\( V_{under} \) | Workpiece velocity under the DMZ
\( w \) | Cutting width
\( \alpha \) | Rake angle
\( \beta \) | Chamfer angle
\( \gamma \) | Angle between the bottom of DMZ and cutting direction
\( \gamma_{MN} \) | Shear strain along the primary shear plane
\( \dot{\gamma}_{MN} \) | Shear strain rate along the primary shear plane
\( \delta_1 \) | Ratio of the thickness of thin layer along the tool-chip interface to the chip thickness
\( \delta_2 \) | Ratio of the thickness of the tool-work piece interface plastics layer to the influenced layer thickness
\( \varepsilon, \dot{\varepsilon} \) | Strain, strain rate
\( \dot{\varepsilon}_0 \) | Reference strain rate, usually assumed as 1.0
\( \varepsilon_{int} \) | Strain on the interface of tool-chip
\( \dot{\varepsilon}_{int} \) | Strain rate on the interface of tool-chip
\( \varepsilon_{MN} \) | Strain along the primary shear zone
\( \dot{\varepsilon}_{MN} \) | Strain rate along the primary shear zone
\( \varepsilon_{p1} \) | Strain of extrusion deformation
\( \dot{\varepsilon}_{p1} \) | Strain rate of extrusion deformation
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\varepsilon_{p2}$</td>
<td>Strain of excess deformation</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{p2}$</td>
<td>Strain rate of excess deformation</td>
</tr>
<tr>
<td>$\varepsilon_{p3}$</td>
<td>Strain of friction deformation</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{p3}$</td>
<td>Strain rate of friction deformation</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Friction angle on the bottom edge of DMZ</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of inclination of the resultant cutting force $R$ to the primary shear zone $MN$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Friction angle of rake face</td>
</tr>
<tr>
<td>$\lambda_{\text{chamfer}}$</td>
<td>Length of the chamfer edge</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Proportion that deformation energy converted into heat</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of workpiece</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\sigma_{N1}, \sigma'_{N1}$</td>
<td>Normal stress on tool-chip interface</td>
</tr>
<tr>
<td>$\sigma_{N2}, \sigma'_{N2}$</td>
<td>Normal stress on the bottom edge ON of DMZ interface</td>
</tr>
<tr>
<td>$\tau_{\text{int}}$</td>
<td>Shear stress on the interface of tool-chip</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Shear angle of primary shear zone</td>
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</table>
CHAPTER ONE
INTRODUCTION AND LITERATURE REVIEW

Motivation and Objective

Machining is a common material removal process, and the United States alone
spends some $300 billion per year in machining (Komanduri, et al., 2001). To increase
production flexibility and agility of machine tools, manufacturers tried to improve cutting
performance of these tools in high-speed machining and hard turning. However, these
high-speed machining and hard turning techniques typically have a high tool wear rate,
primarily in the form of crater and flank wear on the tool rake and flank faces
respectively. Another wear pattern, the dead metal zone (DMZ) before the cutting edge,
often happens in cutting with negative rake angle tools such as chamfered tools. The
effects of tool wear on cutting forces is often of great interest because they affect the
accuracy and efficiency of the machining process and, hence, the product quality and
profit.

Up to now, little has been done to quantitatively understand and predict cutting
performance under the conditions of tool wear, which is important for thermal modeling,
tool life estimation, chatter prediction, and tool condition monitoring. Current analytical
methods for modeling cutting force only consider tools that are nearly new or tools only
with flank wear or the DMZ. When the flank and crater wear, and/or the DMZ are
present, there is no systematic work to predict cutting forces. This dissertation aims to
provide analytical methods to model the effect of the aforementioned tool wear patterns on cutting forces in turning.

Machining process is a process full of uncertainties. As a result, force measurement under the same cutting conditions can be different from time to time depending on the degree of machining uncertainties. Conventional validation of analytical force model is to simply compare the model predictions with some experimental measurements and draw conclusions based on the visual comparisons. Due to machining uncertainties, the limited measurements taken cannot reliably and completely represent force information in cutting. Therefore, to better evaluate a force model, it is necessary to find an efficient and effective methodology. Fortunately, the Bayesian approach-based validation methodology provides a promising alternative to the conventional model validation methodology.

The objective of this dissertation is to develop a scientific, systematic, and reliable methodology to predict cutting force under worn tool conditions for given material information. This study leads to new turning force models under the combined effect of flank and crater wear in 2D and 3D cutting, a new Bayesian analysis-based methodology to evaluate force measurements and predictions, and a new approach to model the effect of DMZ on cutting forces when using chamfered or worn tools.

Considering the extensive existence of tool wear in machining, this research provides a scientific step to guide the design of cutting tools and the optimization of cutting parameters in metal cutting.
Background

Tool Wear and Cutting Force

The time period between 1940 and 1960 is considered as the Golden age of machining research when the foundation of basic chip mechanics of metal cutting was developed. Recently, machining research has boomed again due to the emergence of advanced cutting tools and workpiece materials along with highly rigid, computer-controlled machining centers. Typically, metal cutting research studies how to carry out machining economically, and investigates what happens at the cutting edge during cutting. Several factors that affect the performance of metal cutting are mechanical properties and microstructures of workpieces, strength and geometry of tools, and accuracy and requirements of machine. In order to evaluate the cutting performance, tool wear and cutting forces are usually investigated.

Tool wear occurs on every kind of metal cutting tools. Some researches have been done on the formation of tool wear in high speed steel (HSS) tool such as (Okushima, et al., 1957; Hitomi, 1961; Marques, et al., 1991; and Stern, et al., 1993). Also tool wear happens in carbide tool (including uncoated carbide tool and coated carbide tool) (Hitomi, 1961; Karag, et al., 1996; Yang, 2001; Gekonde, et al., 2002; Molinari, et al., 2002; and Subramanian, et al., 2004), and polycrystalline cubic boron nitride (PCBN) tools (Barry, et al., 2001; Huang, 2002; Scheffer, et al., 2003; and Poulachon, 2004).

Tool wear progression is a function of tool material, tool geometry, and physical, mechanical, and chemical properties of workpiece. Cutting speed has been found to have
the greatest effect on tool wear. Although advances in cutting materials over past decades have tremendously improved tool life, predicting and understanding tool wear have found to be complex. Tool wear generally occurs because of several wear mechanisms. These mechanisms, or any combination of them, include adhesion, abrasion, diffusion, fracture, thermal shock, surface fatigue, and chemical corrosion (Huang, 2002). Adhesion wear is caused when welded asperities junctions between workpiece and cutting tool fracture, which develop during high pressures and temperatures cutting process. Abrasion wear occurs when hard particles located either on the chip or the workpiece rub over the face of the tool causing microcutting or microcracking. Diffusion can happen when atoms from the cutting tool disperse into the chip or workpiece due to solubility of the two materials under high contact. Surface fatigue typically takes place under cyclic deformations caused by sliding asperities in the workpiece or tool. Depending on specific cutting conditions, workpiece, and cutting tool materials, these mechanisms produce marks or scars on the cutting tool in the form of different wear patterns. Typical wear patterns include, but are not limited to, flank wear, crater wear, built-up edge, DMZ, thermal shock crack, chipping, and notch or groove wear. Numerous models have been proposed to describe general wear volume loss and/or wear rate for different wear mechanisms, including some applications in metal cutting (Huang, 2002).
Tool wear usually influences the cutting forces because it may change the geometry of tool, alter the cutting edge condition of tool and the surface finish of workpiece. Figure 1.1 shows how force modeling works with typical wear modeling. The force model also works for wear model by providing stress and velocity, as well as the temperature field. Also knowledge of cutting forces is required when design a machining tool/insert and workpiece holding fixtures, making the workpiece and tool be able to withstand the generated cutting forces to prevent distortion and chatter. In general, cutting forces is the most effective variable to evaluate the cutting performance. For the models shown in Figure 1.1, force modeling under worn tool conditions is very critic. Figure 1.2(a) shows a turning application. Typically three cutting forces are measured: the tangential cutting force, the radial thrust force, and the axial feed force, as seen in Figure 1.2(b).
Figure 1.2: (a) Typical turning application, (b) illustration of three dimensional cutting forces in a turning process
Figure 1.3 shows cutting process using a fresh tool and a worn tool. The objective of this dissertation is mainly to answer how the tool wear patterns which include the crater and flank wear and/or DMZ have effect on the cutting forces as the cutting configuration is shown in Figure 1.3(b).
Plasticity-based Machining Modeling

Analysis of the stresses in metal cutting processes has been an active area of applied plasticity for several decades. In another word, the plasticity theory of metal is the fundamental of metal cutting modeling.

(1) The theory of plasticity

The theory of plasticity deals with the behavior of materials at strains where Hooke’s law is no longer valid. A number of aspects of plastic deformation make the mathematical formulation of the theory of plasticity more difficult than the description of the behavior of elastic solids. Plastic deformation is not a reversible process like elastic deformation, and plastic strain is dependent on the loading path by which the final state is achieved. Nevertheless, the theory of plasticity has been one of the most popular areas of continuum mechanics, and considerable progress has been made in developing theories that can solve important engineering problems.

In general, a complete solution for plasticity must satisfy the equilibrium equations of motion, the boundary conditions, a basic relationship of flow law, and a yielding criterion.

(1.1) Flow law and flow potential

The stress-strain curve obtained by uniaxial loading is of fundamental interest in plasticity. A true stress-strain curve is frequently called a flow curve because it allows for the stress required to cause the metal to flow plastically to any given strain.
Many attempts have been made to find mathematical equations for this law. The most commonly used equations for metal cutting are the power law model and the Johnson-Cook model.

(1.2) Yielding criteria

The problem of deducing mathematical relationships for predicting the conditions at which plastic yielding begins when a material is subjected to any possible combination of stresses is an important issue in the field of plasticity. It is expected that yielding under a situation of combined stresses can be related to some particular combination of principal stresses. Currently, there is no theoretical way of representing the relationship between the stress components to correlate yielding for a three-dimensional state of stress with yielding in the uniaxial tension test.

The yielding criteria are essentially empirical relationships. Currently, the yield criterion is believed to be some function of the invariants of the stress deviator, which is independent on hydrostatic pressure. At present, there are two main criteria: Von Mises’ or distortion-energy criterion as well as maximum-shear-stress or Tresca criterion.

(2) Slip-line field theory

Since forces and deformations in reality are generally quite complex, it is usually necessary to use simplification assumptions to obtain a tractable solution. Because the strains involved in deformation processes are large, it is usually possible to neglect elastic strains and only consider the plastic strains (plastic-rigid solid). As a first approximation, strain hardening is often neglected due to the high temperature during metal cutting process. The principal objective of analytical studies of metal cutting processes is to
determine the forces required to produce a given deformation for a certain geometry prescribed by the cutting process. Such studies are useful for selecting or designing the equipment to perform a particular metal cutting job. A mechanics’ analysis of a process may also be used to develop information about the frictional conditions in the process. Particularly, all analyses assume the material to be isotropic and homogeneous.

In general cases, there are nine independent equations containing nine unknowns, six stress components and three velocity (strain-rate) components. While an analytical solution is attainable if a sufficient number of boundary conditions are specified, the complicated mathematics in a general solution is formidable. Thus, most analyses of actual metal cutting processes are limited to two-dimensional problems. Applying the assumption of perfectly rigid plasticity to metal under plane strain condition, the theory, named as slip-line field theory, can be derived. The details can be referred to in Appendix A: Slip-line Field Theory.

In the slip-line field analysis, flow stress is assumed to vary only due to strain-hardening. However, in the predictive machining theory and by comparing flow stress results obtained from machining tests with those obtained from high speed compression and tension tests, strain-rate and temperature are also found having effects on flow stress. Plane strain and uniaxial conditions are still related using an effective stress and strain, but it is assumed that for a given material, the effective stress would only be a unique function of the effective strain for given values of effective strain-rate and temperature. If $\sigma$, $\varepsilon$, and $\dot{\varepsilon}$ are taken as the uniaxial stress, strain, and strain-rate values and $k$, $\gamma$, and
\( \dot{\gamma} \), as the plane strain maximum shear stress, shear strain, and shear strain-rate values, then the following relationships hold:

\[
k = \frac{\sigma}{\sqrt{3}} \quad \gamma = \sqrt{3} \varepsilon \quad \dot{\gamma} = \sqrt{3} \dot{\varepsilon}
\]

(3) Finite element method

Practical metal cutting is complex, and a computational method to investigate the metal cutting process is needed. The finite element method is one of the choices at the current stage. The FEM method begins from the governing equations, which are usually partial differential equations, and from the material constitutive model, including flow law and yield criteria. Using the variational method or the weight residual method, the governing equations are transformed to the weak form of the equations. Then, it is convenient to assume the physical domain as consisting of an assemblage of a finite number of subdomains called finite elements that are connected to one another along their interfaces. The distribution of a governing physical parameter within each element is approximated by a suitable continuous function called the shape function, which is uniquely defined in terms of its values at a specified number of nodal points usually located along the boundary of the element. Therefore, the solution to the initial or boundary value problem is often reduced to that of a problem involving the nodal point values of the unknown variable.

In the early 1970s, FEM was first introduced to simulate the orthogonal machining process. In the early stage, the Eulerian formulation technique is used, which is currently still being used by some active researchers. In this technique, the finite
element grid is spatially fixed, and the material flows through it. Such an approach is usually used to study the steady state cutting process without the need to simulate the lengthy transition from incipient to steady state cutting conditions or the use of chip separation criteria. The major benefit of using Eulerian formulation is that fewer elements are required to specify the chip and workpiece, thereby reducing the computation time. The disadvantage of using such an approach is that experimental work must be carried out in order to ascertain the chip geometry and shear angle. Furthermore, only continuous chip formation can be modeled.

With the development of faster processors with large memory in the late 1980s, model limitations and computational difficulties have been solved to some extent; therefore, the use of a Lagrangian formulation has increased. The major advantages of this approach are that the tool motion can be simulated from some initial conditions to steady state cutting and that the chip geometry together with workpiece residual stress can be predicted. Here, the elements are attached to the workpiece material, and chip separation criteria can be found in order to allow the chip to break freely from the workpiece. Various researchers have proposed different chip separation criteria for FEM simulation in machining, which are either classified as physical or geometric. The former includes strain energy density, effective plastic strain, and stress, while geometric criteria relate to the distance between the overlapping nodes and the tool tip. Neither criterion has a substantial effect on chip geometry, distribution of shear stress, effective stress, or effective plastic strain in the chip and in the machined surface. However, the magnitude
designated for these criteria do have a major effect on mesh distortion, together with the value of maximum shear stress and the effective stress in the machined surface.

Before the mid-1990s, most researchers used finite element code written by themselves. But recently, the use of commercial packages has increased dramatically. These software mainly include ADVANCEDGE, DEFORM, ABAQUS/Standard and Explicit. More details about how FEM software is applied to metal cutting simulation can be referred to in Appendix B: Guide for Using AdvantEdge to Simulate Turning Process.

Problem related to tool wear has seldom been investigated by FEM due to the complicated nature of tool wear mechanisms during the cutting process, although FEA has proven promising in estimating the process variables, such as stress and temperature on the tool face which can approximately determine how cutting parameters affect tool life and tool performance. Recent research shows that FEA simulation cannot exactly predict tool wear or tool wear rate by far (Yen, et al., 2002; and Xie, 2005).

Current State of Modeling Approaches in Metal Cutting

2D Force Modeling

Tool wear, primarily in the form of crater and flank wear present on rake and flank faces of machine tools, has been studied using a variety of previously developed force models.

When a tool is fresh and sharp, cutting force modeling is conducted using three types of modeling approaches: the theoretical approach, the mechanistic approach, and
the computational approach. The theoretical approach uses minimum energy (Merchant, 1945; and Usui, 1978a) and plasticity theory based methods for modeling (Lee, et al., 1951; Hill, 1954). Theoretical approaches include the minimum energy method, also known as the upper bound (kinematic) method since it uses a kinematically admissible hodograph to obtain unknown process information through energy minimization with respect to the geometry of the deformation zone. However, minimum energy method often overestimates the actual power required (Seethaler, et al., 1997; Adibi-Sedeh, et al., 2002). Other theoretical approaches include plasticity theory based modeling methods such as the dislocation theory based approach (Shaw, 1950; von Turkovich, et al., 1970), slip line field model, and slip line field theory based modeling approach (Johnson, 1962; Kudo, 1965; Dewhurst, 1978; Oxley, 1989; Fang, 2003a, b). The mechanistic approach uses a minimal number of sharp tool cutting tests to predict the cutting forces under a range of conditions by assuming that the cutting forces are proportional to the chip load (Kapoor, et al., 1998; Armarego, et al., 1999). Finally, the computational approach primarily includes finite element modeling (Strenkowski, et al, 1985; Marusich, et al., 1995; Guo, et al., 2002; Ng, et al., 2002) and molecular dynamics simulation (Komanduri, et al., 2001). Although these three approaches are primarily classified based on flat face cutting tools in orthogonal cutting, they have been extended to other diverse tool types such as restricted contact, grooved, large negative rake angle, honed, or chamfered tools in three-dimensional oblique cutting.

When a tool becomes worn, the effects of both flank and crater wear must be considered when modeling the total cutting force of that tool. While much research has
been done on either the cutting forces or the subsequent tool wear, very few force modeling studies have been conducted to address both the effects of the flank and crater wear, which will be the major contribution of this dissertation.

Some research on force modeling have been done by only considering the effect of flank wear (Kobayashi, et al., 1960; Thomsen, et al., 1962; Waldorf, 1996) believed that the size of the plastic flow region on the tool flank face increases with the progression of wear land length. Contact theory (Elanayar, et al., 1996) and slip line field theory (Shi, et al, 1991; Waldorf, et al, 1998) are frequently used to model the ploughing effect in metal cutting. While some researchers have argued that the ploughing forces due to flank wear and the chip formation forces are coupled (Wang, et al., 1999), the worn tool ploughing forces are typically modeled as uncoupled (Marques, et al., 1991; Elanayar, et al., 1996; Smithey, et al., 2000; Huang, et al., 2005b). Generally, the total cutting forces will increase due to the ploughing effect as the effect of flank wear is dominant.

When only crater wear is considered, the cutting forces may decrease (Marques, et al., 1991; Armarego, et al., 1999) as the effective rake angle turns smaller and the tool turns sharper. This effect of crater wear has been modeled using an average rake angle based empirical method (Marques, et al., 1991) and a finite element approach (Komvopoulos, et al., 1991; Li, et al., 2002). Unfortunately, neither of these works has completely elucidated the underlying physics involved.

When considering both flank and crater wear, the total cutting forces due to tool wear may increase or decrease depending on the collective effect. Finite element method
is most frequently used to model two-dimensional cutting forces based on updated crater wear and flank wear geometries (Yen, et al., 2002; Xie, 2005). Unfortunately, the FEM based approach is still too computationally intensive for practical use. The total cutting forces have also been predicted with encouraging results by an empirical approach when the forces are caused by both crater and flank wear (Marques, 1991). However, the limited ability of the empirical approach in identifying and understanding the individual contributions of both crater and flank wear also prevents its extension into three-dimensional force modeling. For machining applications where the effect of both types of wear is significant, there is a need to develop an analytical modeling approach that discovers the underlying machining physics, which can then capture the effects of both crater and flank wear on the total cutting forces.

3D Force Modeling

While modeling of orthogonal cutting has been studied intensively, three-dimensional oblique cutting is not well understood. A typical cutting operation normally involves a main (primary) cutting edge, a minor cutting edge, and a rounded tool nose, which is generally used to connect these two cutting edges to improve workpiece surface finish and tool toughness. Unlike orthogonal cutting, chip flow direction and undeformed chip cross sections must be carefully captured in 3D oblique cutting no matter the tool is worn or not.

The first modeling attempts to understand 3D oblique cutting processes occurred mainly in the 1950s (Merchant, 1944; Stabler, 1951; Shaw, et al., 1952; Colwell, 1954).
More recent efforts were undertaken in the 1980s and 1990s (Morcos, 1980; Oxley, 1989; Armarego, et al., 1999; Adibi-Sedeh, et al., 2002; Molinari, et al., 2005), and efforts in this decade included computational modeling efforts by Guo (Guo, et al., 2002). While investigations into 3D oblique cutting using tool geometries such as one or two straight cutting edges (Usui, et al., 1978b; Venuvinod, 1996), chamfered tools (Fuh, et al., 1995; Huang, et al., 2005b), and grooved tools (Parakkal, 2002) have been conducted, most research has focused on flat-faced nose radius tools (Oxley, 1989; Armarego, et al., 1999; Adibi-Sedeh, et al., 2002; Molinari, et al., 2005).

The most recent representative modeling efforts in this area can be classified into two categories based on the methods for considering the cutting edges: the equivalent cutting geometry approach and the edge discretization approach. The equivalent approach, first proposed by Colwell (Colwell, 1954) assumes that the chip flows in the direction perpendicular to the line connecting the two end points of contact area between the tool and the workpiece, which is known as the single equivalent cutting edge. This equivalent approach has been improved through modeling in more generalized cases (Oxley, 1989; Arsecularatne, et al., 1995). There are two different types of discretization approaches: one considers the major, minor, and tool nose cutting edges individually without further discretization (Usui, et al., 1978b), and the other discretizes the cutting edges into a number of (infinitesimal) elements, modeling the collective process information from each element (Seethaler, et al., 1997; Armarego, et al., 1999; Adibi-Sedeh, et al., 2002; Molinari, et al., 2005). The key to this modeling approach is to transform 3D cutting configurations into a set of single cutting edge scenarios. The
cutting process information can then be modeled using the available orthogonal modeling approaches as summarized above, and the 3D information can be further calculated based on this two-dimensional information using a geometry transformation.

The edge discretization approach, which provides a straightforward and powerful approach for investigating 3D oblique cutting process, uses three methods: (1) dividing the engaged cutting edges into concentric trapezoids (Oxley, 1989; Armarego, et al., 1999), (2) dividing the engaged cutting edges parallel to the feed velocity direction (Molinari, et al., 2005), and (3) dividing the cutting edges parallel to the chip flow direction (Adibi-Sedeh, et al., 2002). Since the third approach (Adibi-Sedeh, et al., 2002) provides the most natural understanding of the chip formation process and of the chip flow motion of the discretized elementary chip segments, it will be the one used in this study.

When modeling fresh tools with complex geometries, most current modeling approaches are based on modifications or extensions of previous approaches for flat-faced nose radius tools. For example, the equivalent cutting geometry approach has frequently been used to model 3D oblique cutting using chamfered (Huang, et al., 2005b) and grooved tools (Parakkal, et al., 2002).

For worn tools, only the effect of flank wear has been investigated in modeling 3D oblique cutting (Huang, et al., 2005b; Smithey, et al., 2000). However, for high-speed machining and hard turning where crater wear is pronounced, the effect of crater wear must be considered.
Force Modeling of DMZ in Cutting with Chamfered Tool

To improve the cutting process efficiency and effectiveness, the tool geometry should be carefully designed in addition to the optimization of cutting conditions. As a result of such efforts, chamfered tools were first introduced in Japan (Hitomi, 1961). Such chamfered tools can offer good cutting toughness and edge chipping/breakage resistance, and they also reduce the energy consumption and prolong the tool life (Shaw, 1984). Due to their attractive properties, recently chamfered tools are broadly applied in interrupted cutting (Hirao, et al., 1982) as well as cutting hard materials (Huang, 2002).

Typical chamfered tools have a negative chamfer angle from -10 to -25 degrees. Since the chamfer edge is mainly responsible for material removal, the dead metal zone (DMZ) has been frequently reported in cutting (Hirao, et al., 1982; Jacobson, et al., 1988; and Zhang, et al., 1991) as in cutting with large negative rake angle tools (Lortz, 1979; Abebe, et al., 1981; and Kita, et al., 1982). The DMZ is a zone filled with a trapped volume of stagnant dead metal before the chamfered edge. It differs from the built-up edge (BUE) formed before the tool tip since BUE is an unstable structure and breaks up eventually. As a special tool wear pattern, this DMZ induces the tool geometry change. It should be pointed that DMZ effectively deposits mass on tool, instead of reducing mass from tool as the crater/flank wear does. And the DMZ induced effect on cutting forces should be carefully investigated in addition to those of the flank and crater wear. This dissertation devotes to studying this DMZ induced effect on turning.
Although Hitomi (Hitomi, 1961) and Chang et al. (Change et al., 1998) reported that the workpiece materials formed under such a chamfer edge are removed as the secondary chip, it was more frequently observed that they form a stagnant dead metal zone and this zone acts as an additional cutting edge (Hirao et al., 1982; Jacobson, et al., 1988; and Zhang, et al., 1991). Besides the experimental efforts for better design of chamfered tools, some notable analytical work has been made to provide a better understanding on their cutting mechanism. Zhang et al. (Zhang, et al., 1991) proposed a three-zone (primary, tool-chip interface/secondary, and dead metal deformation zones) cutting model and solved the shear angle based on the minimum energy principle. Ren et al., (Ren, et al., 2000) further developed this three-zone model (Zhang, et al., 1991) by using Oxley’s predictive machining theory (Oxley, 1989) to model the primary and secondary deformation zones. A power-law material model was used to predict the material behavior in the modeling approach (Ren, et al., 2000). Recently, Movahhedy et al., (Movahhedy et al., 2002) proposed a finite element based numerical analysis to model the continuous chip formation process in cutting with chamfered tools. They found that a stagnant material zone under the chamfer edge is evident in simulation results and chamfer angle does not affect the chip removal process significantly.

For better application of chamfered tools in interrupted cutting and cutting advanced materials, there is an increasing need to further the understanding of the chamfered tool cutting mechanism. Since theoretical modeling provides more physical insights on process fundamentals and is much less time consuming than numerical approaches, it is favored in this dissertation.
Organization of This Dissertation

This dissertation is divided into five chapters. The flow chart (Figure 1.4) is the organization of the main chapters in this dissertation.

![Flow Chart]

Figure 1.4: Organization of the dissertation

The first chapter is the introduction and research review, where the motivation is first introduced. The principal goal and background are presented. Overall knowledge about metal cutting, such as tool wear and cutting force, is briefly reviewed. Force models are the main parts of this dissertation. For each of them, the research progress is presented. The discussions include what is available in the current literature on the topic and what are the research opportunities in this dissertation. Finally, the organization of this dissertation is provided.

In Chapter 2, a new analytical force modeling approach for orthogonal cutting
under worn tool conditions is presented that considers both crater and flank wear effects. The starting point is the basic assumption of plane strain for a perfect rigid plasticity. Then a slip line field model is proposed for the 2D condition. The proposed model is verified based on the published experimental data of the high-speed cutting. And it shows satisfactory accuracy and an improved match. The developed approach is ready to be used for force modeling in oblique cutting modeling, which will be the topic of the next chapter. Considering the uncertainties in the real process, Chapter 2 also introduces a model validation method using a noninformative Bayesian linear regression method to account for the machining process uncertainties.

A further 3D force model is constructed in Chapter 3. The complex geometry of worn tools brings challenges when considering a 3D force model. In this chapter, the worn tool profile is approximated using some assumption about the wear area. Based on the simplified geometry of the worn tool, the chip is separated into many segments, and each segment can be treated equally as orthogonal cutting conditions, so the 2D force model can be applied to predict the cutting forces for each segment. Bringing in the concept of inter-element interaction and considering the overall force normal to the cutting direction is zero, the 3D force can be assembled and solved. The prediction forces match the experimental data very well.

Then in the Chapter 4, another tool wear pattern, DMZ, is modeled to predict cutting forces. This chapter begins with a review of DMZ in metal cutting. After observing the experimental flow around the zone, one analytical force model based on slip-line field theory is proposed. Combining thermal effect and strain hardening, each
variable has been derived analytically. Then the proposed model is verified with experimental measurements and is compared to another developed model. The results show that improved accuracy and more reasonable force tendency have been revealed. Hence the new model can be used for force modeling under the DMZ effect.

At last, as the conclusion of the dissertation, Chapter 5 first summarizes the work and achievements of this dissertation. Further, a critical comment is given to this dissertation to provide recommendations for future work.
CHAPTER TWO

MODELING OF THE COMBINED EFFECT OF THE CRATER AND FLANK WEAR ON CUTTING FORCES IN ORTHOGONAL CUTTING

Introduction

Progressive tool wear mainly in the forms of flank and crater wear, presents on the tool rake and flank faces respectively. Cutting forces may increase or decrease depending on the respective contribution from the flank and crater wear. Quantitative understanding and prediction of cutting forces under both tool flank and crater wear conditions is important to cutting process thermal modeling, tool life estimation, chatter prediction, and tool condition monitoring purposes. However, there is no documented study to capture the effect of both flank and crater wear on the total cutting forces and discover the underlying machining physics. So a slip line based force modeling approach is proposed to capture the cutting mechanism under the effect of both flank and crater wear.

The chapter is organized as follows. After reviewing the previous slip line analysis in worn tool force modeling, a slip line field and the associated hodograph are introduced to capture the contributions of both flank and crater wear. The slip line models for the primary shear zone, the flank wear land and each part of the secondary shear zone are discussed in detail. Further, the model implementation procedure is proposed in terms of the hodograph geometric relationship, the equilibrium of force and moment in order to
predict cutting forces of interest. And, the simulation procedure of the force model is presented. Then, the force model is verified using the experimental data. Not satisfied with the graphical comparison for the force model, one model validation method based on Bayesian method is presented. The result shows that most predictions of the proposed slip-line field model locate in the 75% confidence interval. After the model has been validated, furthermore, the effects of crater wear depth, flank wear land, the ratio of crater sticking and sliding regions, and friction coefficients of interest on intermediate slip line angles and/or cutting forces are studied based on the proposed model. Finally, some conclusions are drawn for the proposed worn tool force model.

Proposed Analytical Model

Slip-Line Analysis for Worn Tool Force Modeling

Since the boundary conditions at the tool-chip interface and the tool flank-work material interface are not known at the outset, an analytic solution to the machining problem requires experimental observations to support the assumed boundary conditions and verify the obtained results.

There are many literatures about the formation of crater wear in high speed steel (HSS) tool such as (Okushima, et al., 1957; Hitomi, 1961; Manuel, et al., 1991; Stern, et al., 1993). Similar condition is suitable for carbide tool (including uncoated carbide tool and coated carbide tool) (Hitomi, 1961; Karag, et al., 1996; Yang, 2001; Molinari, et al., 2002; Subramanian, et al., 2004). Figures 2.1(a) and (b) are typical crater profiles of
carbide tool. Literatures (Barry, et al., 2001; Huang, 2002; Scheffer, et al., 2003; and Poulachon, et al., 2004) have verified good crater wear profile for PCBN tool as shown in Figure 2.1(c). Therefore, it can be approximated that the crater wear land of carbide tool, TiN coated tool and hard turning tool such as CBN tool is a circular arc. The highest temperature point of tool-chip interface is believed nearly in the middle of tool-chip interface (Chao, et al., 1955; Trent, 1977; Dearnley, 1986). It is the best explanation for the circular arc profile for the crater wear.

Figure 2.1: Representative crater wear pictures (a) uncoated carbide tool (Subramanian, et al., 2004), (b) coated carbide tool (Karag, et al., 1996), (c) PCBN (Barry, et al., 2001)

Near the tool tip, there is usually a transient edge where BUE forms as in Figure 2.1(b). Some researchers (Devillez, et al., 2004; Poulachon, 2004) used White Light Interferometer to measure the profile of crater wear of CBN tool. Their measurements verified the existence of the short transient edge at the worn tool tip. As further discussion on stress distribution along the crater wear (Hsu, 1966; Bailey, 1975; and
Chungchoo, et al., 2002), the contact condition along the circular arc was divided into two parts: near the tip was sticking state, the far part was the sliding contact.

In order to analytically model the orthogonal metal cutting process, the plane-strain rigid-plastic slip line theory based approach is highly favored by many machining researchers, and there are various types of slip-line models developed. The most significant reason for using the slip-line theory is that slip line analysis can show the material flow pattern much more clearly over the whole shear deformation zone (Fang, 2001), and save the simulation time compared to other methods like FEM. The slip line analysis was pioneered by Merchant (Merchant, 1944) to model the metal cutting process and followed by many important works (Lee, et al., 1951; Kudo, 1965; Dewhurst, 1978; Abebe, et al., 1981; Oxley, 1989). Recently, Fang (Fang, 2003b) further proposed a generalized slip line model for restricted and/or groove tools with a honed tool tip and other eight representative slip line models (Merchant, 1944; Lee, et al., 1951; Johnson, 1962; Kudo, 1965; Dewhurst, 1978; Shi, et al., 1993; Fang, 2001; Fang, 2002) can be derived from Fang’s model (Fang, 2003b).

![Figure 2.2](image.png)

Figure 2.2: A typical worn tool (Kountanya, et al., 2004) and its representative geometry
This paper also applies the slip line theory to capture the worn tool cutting mechanism. A typical worn tool is shown in Figure 2.2.

From point of the slip line modeling approach, cutting with flank wear can be similarly modeled as that of cutting using honed or chamfered tools. Generally, the cutting edge geometry defined by edge radius, chamfer angle, or flank wear significantly influences the material flow pattern in metal cutting, and such a geometry helps form a stagnant point before the tool tip (Palmer, et al., 1963; Shi, et al., 1991; Waldorf, 1996; Waldorf, et al., 1998; Manjunathaiah, et al., 2000; Guo, et al., 2005; Long, et al., 2005). Part of workpiece material above the stagnant point is pushed upward by the chamfer or rake face to form chip, and the below part is ploughed down to form machined surface. For some large honed, chamfered, or large negative rake angle tools, the stagnant point may be modeled at the surface of the process-induced built-up edge or dead metal zone using the slip line method (Waldorf, et al., 1998; Abebe, et al., 1981; Fang, et al., 2005; Long, et al., 2005) instead of the cutting tool tip. Figure 2.3 shows the representative modeling approach for a honed tool (Waldorf, et al., 1998) and a chamfered tool (Long, et al., 2005). For worn tools with flank wear which are of interest in this study, if the fresh tool is honed, the stagnant point is modeled at the surface of a stable built-up edge as shown in Figure 2.3(c) (Waldorf, 1996); or if the fresh tool is sharp, the stagnant point is typically taken at the top tip as shown in Figure 2.3(d) (Shi, et al., 1991). A non-zero inclination angle of the flank wear land with respect to the cutting direction was assumed in Figure 2.3(d) and this inclination angle was determined as part of the problem solution (Shi, et al., 1991).
Figure 2.3: Slip line models for (a) honed tool (Waldorf, et al., 1998), (b) chamfered tool (Long, et al., 2005), (c) honed tool with flank wear (Waldorf, 1996), and (d) sharp tool with flank wear (Shi, et al., 1991)

Crater wear was mainly observed with high speed steel (HSS), carbide, and CBN tools. Although the effect of crater wear on the chip formation process was recognized almost forty years ago (Armarego, et al., 1969), there is still no documented analytical approach to investigate this effect except some FEM-based attempts (Komvopoulos, et al., 1991; Li, et al., 2002), which are too computationally intensive and not ready for further 3D implementation. There are several slip line models were proposed to capture
the chip formation process using restricted contact tools (Seethaler, et al., 1997; Fang, 2001) and restricted contact grooved tools (Fang, et al., 2003a; Shi, et al., 1991); however, they are not readily applicable to model the chip formation process when both flank and crater wear are present.

Proposed Worn Tool Slip Line Modeling Approach

(1) Basic assumptions

The following assumptions are introduced in order to capture the cutting process:
(a) The cutting process is 2D steady state cutting and workpiece deformation obeys the plane strain condition, (b) The workpiece is perfectly rigid plastic. It means that the yield and shear stresses do not vary with the temperature, strain, and strain rate, (c) The chip thickness stays constant once formed, (d) The crater wear profile \( LNP \) is circular (Huang, et al., 2005a), (e) The crater land here is considered as a sticking zone \( LN \) and a sliding zone \( NP \). The shear stress \( \tau_s = k \) and the normal stress is uniform as \( \sigma_n = \sigma_l \) for the sticking portion \( LN \) (Childs, et al., 1989; Arsecularatne, 1997; Li, 1997), and the normal stress distribution along the sliding portion \( NP \) is power law-based (Molinari, et al., 2002).

(2) Proposed slip line model

Figure 2.4 shows the proposed slip line model and its associated hodograph for machining with a worn tool. Region \( AB_1B_2C_1C_2DOG_2G_1IH \) depicts the primary shear zone which is represented by three field angles \( \theta_1, \theta_2, \) and \( \delta_1 \). Arc \( LNP \) describes the crater land which is assumed circular for simplicity based on the experimental
observations using different tool materials (Armarego, et al., 1969; Huang, et al., 2005a), and chip leaves the tool rake face at point $P$. After passing through the primary shear zone, back-flow chip moves into the crater wear land and continue to flow along the arc $LNP$ with an angular velocity $\omega$ (Dewhurst, 1978; Fang, 2003). Finally, chip may leave the tool crater wear land with a curvature or tangentially depending on the cutting bending moment (Zhang, et al., 1989). Considering the restricting effect of naturally formed crater during the progressive tool wear formation, the chip is assumed tangentially leaving the tool-chip crater interface at point $P$. The effect of flank and crater wear is coupled since the flank wear land contributes to the formation of the pre-flow region $AB_iH$ and the primary shear band $HIG_iG_2OEDC_2C_1B_2B_1$ (Shi, et al., 1991).

The inclination of the flank wear land $OF$ with respect to the cutting direction is not exactly zero (Shi, et al., 1991), and this inclination angle $\xi_F$ is determined as part of the problem solution.

In capturing the chip flow through the primary shear zone, both Merchant’s simplified straight shear plane model (Merchant, 1944) and the following improved slip line model (Kudo, 1965) are not fully supported by experimental observations. So region $AB_iB_2C_1C_2DOG_2G_iIH$ depicting the primary shear zone is proposed to be represented by three field angles $\theta_1$, $\theta_2$, and $\delta_i$. Also the workpiece material is usually observed emerging from the shear zone $B_2C_1C_2DOG_2G_iIH$ with an angular velocity $\omega$ and Dewhurst (Dewhurst, 1978) further elucidated the range of validity for curling chips. Zone $AB_iH$ represents the pre-flow region at the free surface, forming a transition zone.
between the workpiece and the machined chip, especially under the tool wear conditions (Kudo, 1965; Armarego, et al., 1969; Shi, et al., 1991; Fang, 2003b). Then the chip rotates with an angular velocity \( \omega \) after point H. If assuming the chip thickness is constant once the chip is formed, the chip free surface resembles that of the crater wear land. Line \( AH \) and the free surface of the chip are the coincidence of the streamline and principal stress trajectory.

It should be noted that the total tool-chip contact length may be divided into two parts: a short but straight edge \( OL \) near the tool tip and a crater land \( LNP \) based on experimental observations. There is also a back-flow angle (Fang, et al., 2003) at point L since the chip should flow along the worn rake face in quasi-steady cutting process. Chip flow is retarded by this possible edge \( OL \) which is represented by the two field angles \( \beta_1, \beta_2 \). Figure 2.3 includes a region \( OG_2G_1L \) to balance the friction effect along the edge \( OL \) (Fang, 2003; Shi, et al., 1993) with the both lines \( LG_1 \) and \( G_2G_2O \) are concave downwards. \( LG_2 \) is concave downward too to match the distribution of the normal pressure distribution along \( OL \) (Fang, 2001). Line \( G_1G_2O \) is a single slip line, and a triangular region \( DOE \) is introduced to resolve the singularity at point \( O \). Line \( OF \) defines the flank wear land which typically has an inclined angle \( \varepsilon_f \) with the cutting direction (Shi, et al., 1991).
Figure 2.4: Proposed slip line field (top) and its hodograph (bottom)
Figure 2.4(b) shows the corresponding hodograph and it demonstrates that all the velocity boundary conditions can be satisfied: rigid translation across $HIG, G, O$ and $AB, B, C, C, DEF$, zero normal component of velocity along lines $OL$ and $LNP$, and coincidence of the streamline and principal stress trajectory along line $AH$. Figure 2.5 illustrates the stream lines during the chip formation process.

Model Implementation and Calculation Procedure

For computation convenience in model validation, wear land geometric parameters such as $KT$, $KM$, $KB$ and $VB$ are measured from experiments. Further, as
observed in (Chungchoo, et al., 2002), the $PN$ is assumed as $5/12$ of the whole contact length $LNP$, so the angle $\delta$ is $\frac{7}{6} \gamma$.

(1) Geometric relationships of the proposed hodograph

There are seven intermediate slip line variables $r_1$, $r_2$, $r_3$, $\beta_1$, $\beta_2$, $R$, and $\omega$ from the hodograph of Figure 2.4, and the seven variables can be solved based on the geometric relationships discussed in the following.

![Figure 2.6: Triangle of slip-line $OLG_2$](image)

The radius of $LG_1$ is approximated as the radius of $LG_2$, which is denoted as $r_1$, as shown in Figure 2.6. And the radius $r_3$ of $G_1G_2$ is approximated as $r_3 = r_1 \beta_2$ for a small $\beta_2$ (Childs, 1980). Point O is considered as a chip flow separating point. For the polygon $LG_1G_2O$, $\eta = \beta_2$ as the angular extent of $LG_1$, $LG_2$, and $OG_2$ should be the
same (Childs, 1980), \( \theta_2 = \beta_1 + \beta_2 = \epsilon_o - \epsilon_{\text{g}1} \), and \( \gamma + \alpha = \epsilon_{\text{g}1} + \eta = \epsilon_L \). For the area \( G_1G_2O \), \( r_1^2 + r_2^2 = \frac{(OL_0)^2}{2 - 2 \cos \beta_2} \) and \( r_2 / r_1 = \tan(\epsilon_{\text{g}1} / 2) \). Consider the primary shear zone attributed to the flank wear:

\[
\sqrt{2} \epsilon \sin(\epsilon_H - \pi/4 + \delta_1) = \sqrt{2} V \sin(\epsilon_H - \pi/4 + \delta_1) = VB \sin \epsilon_F \tag{2.1}
\]

where, \( \epsilon = VB \sin \epsilon_f \) and \( R - R' = e \omega \) (Shi, et al., 1991). For the area \( WHLG_1, LG_2 \) and \( LG_1 \) approximated as the same radius \( r_1 \) as follows (Childs, 1980):

\[
r_1 = R \tan \left( \frac{\epsilon_{\text{g}1} + \beta_2}{2} \right) \tag{2.2}
\]

For the polygon \( TG_iG_iW \):

\[
(R_L - r_1) \cos(\epsilon_L) + r_1 \cos(\epsilon_{\text{g}1}) + R \sin(\epsilon_{\text{g}1}) = V \tag{2.3}
\]

where \( R_L = 0.5 \left( KT + \frac{(KM - KB)^2}{KT} \right) \). For the undeformed chip thickness \( t_i \):

\[
R(\sin(\epsilon_H) - \sin(\epsilon_{\text{g}1})) - VB \omega \sin \epsilon_f + r_2(\sin \epsilon_o - \sin \epsilon_{\text{g}1}) + r_3(\sin \epsilon_{\text{g}1} - \sin \epsilon_{\text{g}2}) = t_i \omega \tag{2.4}
\]

(1.1) Modeling of the primary shear zone and flank wear land

In Figure 2.4, the slip line field for the primary shear zone and flank wear land is represented by the region \( AB_1B_2C_1C_2DOG_2G_1IH \) as discussed before. Line \( HIG_1G_2O \) is parallel to the line \( B_2C_1C_2D \), and both are represented by the two field angles \( \theta_1 \) and \( \theta_2 \).

Two transient fan-shaped zones \( B_1HB_2 \) and \( DOE \) with the angles of \( \delta_1 \) and \( \delta_2 \), respectively, are also introduced. The lengths of \( OL \) and \( OF \) are determined as \( 2KB - KM \) and \( VB \), respectively.
The maximum normal stress of along the worn flank face is at point $O$ with a value as specified as follows (Shi, et al., 1991):

$$\sigma_o = k \left( (1 + 2 \delta_1 - 2 \theta_1 + 2 \theta_2 + 2 \delta_2 + \sin(2 \zeta_f) ) \right)$$  (2.5)

where $p_H = k (1 + 2 \delta_1)$, $\theta_1 = \epsilon_H - \epsilon_{G_1}$ is the field angle of line $B_2C_1(C_2)$, $\theta_2 = \epsilon_O - \epsilon_{G_1}$ is the field angle of line $C_2D$.

(1.2) Modeling of the secondary shear zone along $OL$

The secondary shear zone along $OL$ (Region $G_iG_2OL$) contains the retarded workpiece flow during chip formation as seen from Figure 2.4. Such a region can be formed near the tip of worn, hone, chamfered, or large negative rake angle tools. The stresses at point $L$ have a normal stress of $p_L$ and a shear stress of $k$ when assuming the deformation is perfectly plastic.

(1.3) Modeling of the secondary shear zone along $LN$

By assuming a constant angular velocity $\omega$ for the formed chip as shown in Figure 2.4 and a constant chip thickness $t_c$, the velocity at the outer radius of curling chip is as follows:

$$R_o \approx R_L$$  (2.6)

It can be seen from Figure 2.4 that if $\beta_1 > \theta_1$, the velocity around point $L$ is slower than the velocity at point $H$. When $\beta_1 = 0$, it means that built-up edge happens along the edge $OL$ and the slip line $G_iL$ meets the edge $OL$ tangentially (Hitomi, 1961). When $\beta_2 = 0$, the velocity at point $L$ reach a minimum value.

The velocity of the inner radius of curling chip is estimated as:
\[
R_i \approx R_H = \sqrt{V^2 + R^2 - 2VR \cos(\varepsilon_H)}
\]  

(2.7)

Equations of \( R_i \) and \( R_o \) are similar to those defined in the previous studies (Hitomi, 1961; Fang, 2003b).

When tool has a chip-breaker or chip-breaker groove, the actual chip radius is different to the naturally curling radius of the chip (Zhang, et al., 1989). When only the naturally curling chip formation process is considered under the perfectly plastic deformation condition, the chip thickness is given as:

\[
t_c = \frac{(R_o - R_i)}{\omega}
\]

(2.8)

when \( R_i = R_o \), \( \omega \) is zero.

The chip velocity at point \( G_i \) is as follows:

\[
R_{G_i} = \sqrt{V^2 + R^2 - 2VR \cos(\varepsilon_{G_i})}
\]

(2.9)

(1.4) Modeling of the secondary shear zone along \( NP \)

The formed chip may leave the tool-chip interface portion \( NP \) either as rotating or straight, which represents the upper and lower bounds of an actual chip leaving mode. The real profile of edge \( NP \) is between a straight one and a circular curve. Considering the naturally formed crater during the progressive tool wear formation, the chip should tangentially leave the tool-chip crater interface at point \( P \). As assumed before, the shear stress at point \( N \) is equal to \( k \), and both the normal and shear stress distributions along \( NP \) follow the power law distribution decreasing from \( N \) to \( P \), where the both stresses
are zero. Friction angle $\xi_{NP}$ is introduced at point $P$ to accommodate the singularity due to the sliding edge $NP$.

(2) Modeling of forces and moments

Figure 2.7: Schematic of force and moment components

Figure 2.7 gives a general illustration of the force and moment components when both flank and crater wear are present. In the following sections, each force and moment component is discussed and modeled in detail.

(2.1) Modeling of forces and moments along $HIG_1G_2O$

From the Figure 2.4 hodograph the relative velocity $V$ of the workpiece with respect to the tool is represented by the vector $TW$. The positions of points $H$ and $B_1$ are determined by the inclination slip line field angle $\xi_H$, the positions of points $G_1$ and $C_1$
are determined by the inclination angle $\varepsilon_{G_1}$, and the cutting tip point $O$ is represented by the inclination angle $\varepsilon_O$, as illustrated in Figure 2.8.

It is generally accepted that the slip line $HIG_1G_2O$ is modeled as a combination of circular and straight lines and the magnitude of velocity discontinuity is assumed unchanged along this slip line $HIG_1G_2O$. The slip line $HIG_1$ can be modeled as circular with a center at point $W$ as seen from the hodograph, and point $G_1$ overlaps point $I$ in the hodograph for the straight slip line $G_1I$. The slip line $B_2C_1C_2D$ can also be modeled in a similar way with a velocity continuity difference $\omega$ between $HIG_1G_2O$ and $B_2C_1C_2D$.

Along any slip line, there are two independent stress vectors: $p$ and $k$. The hydrostatic pressure of any point along the slip line $HIG_1G_2O$ follows the Hencky
equation as \( p(\varepsilon) = p_H - 2k(\varepsilon_H - \varepsilon) \), and the shear stress is equal to \( k \), where \( p(\varepsilon) \) is the mean compressive stress at a point where the incline angle is \( \varepsilon \). By integrating forces and moments along \( HIG_1G_2O \), the effective cutting forces and moment components along \( HIG_1G_2O \) can be computed as proposed by Kudo (Kudo, 1965), Dewhurst (Dewhurst, et al., 1973), and Childs (Childs, 1980). For the \( \beta \) slip line \( HIG_2G_3O \), the cutting force \( F_{sHG_2G_3O} \) and thrust force \( F_{tHG_2G_3O} \) are:

\[
F_{sHG_2G_3O} = F_{sHG_1} + F_{xG_1G_2} + F_{xG_2O} \tag{2.10}
\]
\[
F_{tHG_2G_3O} = F_{tHG_1} + F_{xG_1G_2} + F_{xG_2O} \tag{2.11}
\]

where \( \varepsilon_H, \varepsilon_{G_1} \) and \( \varepsilon_O \) are the inclination angles of points H, \( G_1 \) and O.

Using point \( G_1 \) as the reference point for the \( \beta \) line \( HIG_1 \), the hydrostatic stress \( p(\varepsilon) = p_H + 2k(\varepsilon - \varepsilon_H) \) is larger than \( p_{G_1} \), and the force components can be represented as follows:

\[
F_{sHG_1} = \int_{\alpha}^{\beta} ((p(\varepsilon) \cdot \cos \varepsilon + k \cdot \sin \varepsilon) R / \omega) \, d\varepsilon
= R \left[ (p_H - 2k \varepsilon_H) (\sin \varepsilon_H - \sin \varepsilon_{G_1}) - k (\cos \varepsilon_H - \cos \varepsilon_{G_1}) + 2k (\varepsilon_H \sin \varepsilon_H - \varepsilon_{G_1} \sin \varepsilon_{G_1}) + 2k (\cos \varepsilon_H - \cos \varepsilon_{G_1}) \right] / \omega \tag{2.12}
\]

\[
F_{tHG_1} = \int_{\alpha}^{\beta} ((p(\varepsilon) \cdot \sin \varepsilon - k \cdot \cos \varepsilon) R / \omega) \, d\varepsilon
= R \left[ -p_H + 2k \varepsilon_H (\cos \varepsilon_H - \cos \varepsilon_{G_1}) - k (\sin \varepsilon_H - \sin \varepsilon_{G_1}) - 2k (\varepsilon_H \cos \varepsilon_H - \varepsilon_{G_1} \cos \varepsilon_{G_1}) + 2k (\sin \varepsilon_H - \sin \varepsilon_{G_1}) \right] / \omega \tag{2.13}
\]

Moment along \( HIG_1 \) is calculated based on point \( G_1 \) using the clockwise direction as positive:

\[
M_{HG_1} = \int_{\alpha}^{\beta} (p(\varepsilon) R / \omega) \sin (\varepsilon - \varepsilon_{G_1}) + k (R / \omega) [1 - \cos (\varepsilon_{G_1} - \varepsilon)] \, d\varepsilon
= R (R / \omega) \left[ (p_H - 2k \varepsilon_H (1 - \cos (\varepsilon_H - \varepsilon_{G_1})) + \varepsilon_H - \varepsilon_{G_1}) - k \sin (\varepsilon_H - \varepsilon_{G_1}) - 2k (\varepsilon_H \cos (\varepsilon_H - \varepsilon_{G_1}) - \varepsilon_{G_1}) + 2k \sin (\varepsilon_H - \varepsilon_{G_1}) \right] \tag{2.14}
\]

Using point \( G_1 \) as the reference point for the \( \beta \) line \( G_1G_2O \), it is found that:
\[
p(\varepsilon) = p_{\alpha} + 2k(\varepsilon - \varepsilon_{\alpha}) = p_H + 2k(\varepsilon - \varepsilon_H) + 2k(\varepsilon - \varepsilon_G) = p_H + 2k(\varepsilon - \varepsilon_H) \tag{2.15}
\]

The force components along \(G_1G_2O\) can be estimated as follows and the moment along \(G_1G_2O\) does not contribute in chip equilibrium modeling.

\[
F_{xG_1G_2O} = F_{xG_1G_2} + F_{xG_2O} = \int_{\alpha_1}^{\alpha_2} \left( \left( p(\varepsilon) \cos \eta + k \sin \eta \right) \sigma_{y} / \omega \right) d\varepsilon + \int_{\eta_1}^{\eta_2} \left( \left( p(\varepsilon) \cos \eta + k \sin \eta \right) \sigma_{\eta} / \omega \right) d\varepsilon \tag{2.16}
\]

\[
F_{xG_1G_2} = \frac{r_1}{\omega} \left[ \left( p_{\alpha} - 2k\varepsilon_{\alpha} \right) \sin \varepsilon_{\alpha} - \sin \varepsilon_{\alpha} - k \left( \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} \right) + 2k \left( \varepsilon_{\alpha} \sin \varepsilon_{\alpha} - \varepsilon_{\alpha} \sin \varepsilon_{\alpha} \right) + 2k \left( \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} \right) \right] \tag{2.17}
\]

\[
F_{xG_2O} = \frac{r_1}{\omega} \left[ \left( p_{\alpha} - 2k\varepsilon_{\alpha} \right) \sin \varepsilon_{\alpha} - \sin \varepsilon_{\alpha} - k \left( \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} \right) + 2k \left( \varepsilon_{\alpha} \sin \varepsilon_{\alpha} - \varepsilon_{\alpha} \sin \varepsilon_{\alpha} \right) + 2k \left( \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} \right) \right] \tag{2.18}
\]

\[
F_{zG_1G_2O} = F_{zG_1G_2} + F_{zG_2O} = \int_{\alpha_1}^{\alpha_2} \left( \left( p(\varepsilon) \sin \eta - k \cos \eta \right) \sigma_{y} / \omega \right) d\varepsilon + \int_{\eta_1}^{\eta_2} \left( \left( p(\varepsilon) \sin \eta - k \cos \eta \right) \sigma_{\eta} / \omega \right) d\varepsilon \tag{2.19}
\]

\[
F_{zG_1G_2} = \frac{r_1}{\omega} \left[ \left( p_{\alpha} - 2k\varepsilon_{\alpha} \right) \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} - k \left( \sin \varepsilon_{\alpha} + \sin \varepsilon_{\alpha} \right) - 2k \left( \varepsilon_{\alpha} \cos \varepsilon_{\alpha} - \varepsilon_{\alpha} \cos \varepsilon_{\alpha} \right) + 2k \left( \sin \varepsilon_{\alpha} - \sin \varepsilon_{\alpha} \right) / \omega \right] \tag{2.20}
\]

\[
F_{zG_2O} = \frac{r_1}{\omega} \left[ \left( p_{\alpha} - 2k\varepsilon_{\alpha} \right) \cos \varepsilon_{\alpha} - \cos \varepsilon_{\alpha} - k \left( \sin \varepsilon_{\alpha} + \sin \varepsilon_{\alpha} \right) + 2k \left( \varepsilon_{\alpha} \cos \varepsilon_{\alpha} - \varepsilon_{\alpha} \cos \varepsilon_{\alpha} \right) + 2k \left( \sin \varepsilon_{\alpha} - \sin \varepsilon_{\alpha} \right) / \omega \right] \tag{2.21}
\]

(2.2) Modeling of forces and moments along the tool-chip interface

The edge OL is generally observed before the worn tool crater for different cutting tool materials under various cutting conditions, and it is assumed as straight with a rake angle \(\alpha\) in this study.

Along \(LN\), the normal stress distribution is taken as \(\sigma_n = \sigma_L\), and the shear stress is constant as \(\tau_s = k\), where \(\sigma_L = p_L = p_{\alpha} - 2k\eta\). Along \(NP\), the normal and shear stress distributions along the chip-tool interface is taken as power law-based (Molinari, et al., 2002) as follows:

\[
\sigma_n = \sigma_L \left( 1 - \frac{\varepsilon}{2\gamma - \delta} \right)^2, \quad \tau_s = k \left( 1 - \frac{\varepsilon}{2\gamma - \delta} \right)^2 \tag{2.22}
\]
where, \( \varepsilon \) is from 0 at point \( N \) to \( 2\gamma - \delta \) at point \( P \). Then cutting force along the tool-chip interface can be predicted as follows:

\[
F_{sc} = F_{sgcl} + F_{slN} + F_{snp} = \int_{0}^{\varepsilon} \left( (p(\varepsilon)\sin\varepsilon + k\cos\varepsilon)\right) / \omega d\varepsilon + \int_{0}^{\varepsilon} \left( (\sigma_n \sin(\varepsilon + \varepsilon) + \tau_s \cos(\varepsilon + \varepsilon))R_L / \omega \right) d\varepsilon
+ \int_{0}^{2\gamma-\delta} \left( (\sigma_n \sin(\varepsilon + \delta + \varepsilon) + \tau_s \cos(\varepsilon + \delta + \varepsilon))R_L / \omega \right) d\varepsilon
\]  

(2.23)

where

\[
F_{sgcl} = R_l \left[ \left( p(\varepsilon) - 2k\varepsilon(\cos\varepsilon - \cos\varepsilon) + k(\sin\varepsilon - \varepsilon) \right) - 2k(\varepsilon \cos\varepsilon - \varepsilon \cos\varepsilon) + 2k(\varepsilon - \sin\varepsilon) \right] / \omega
\]  

(2.24)

\[
F_{slN} = -R_l(\cos(\varepsilon + \delta) - \cos\varepsilon) / \omega + R_l \left( \sin(\varepsilon + \delta) - \sin\varepsilon \right) / \omega
\]  

(2.25)

\[
F_{snp} = \int_{0}^{2\gamma-\delta} \left( (\sigma_n \sin(\varepsilon + \delta + \varepsilon) + \tau_s \cos(\varepsilon + \delta + \varepsilon))R_L / \omega \right) d\varepsilon
\]  

(2.26)

Thrust force at the tool-chip interface is:

\[
F_T = F_{sgcl} + F_{slN} + F_{snp} = \int_{0}^{\varepsilon} \left( (p(\varepsilon)\cos\varepsilon + k\sin\varepsilon) \right) / \omega d\varepsilon + \int_{0}^{\varepsilon} \left( (\sigma_n \cos(\varepsilon + \varepsilon) + \tau_s \sin(\varepsilon + \varepsilon))R_L / \omega \right) d\varepsilon
+ \int_{0}^{2\gamma-\delta} \left( (\sigma_n \cos(\varepsilon + \delta + \varepsilon) + \tau_s \sin(\varepsilon + \delta + \varepsilon))R_L / \omega \right) d\varepsilon
\]  

(2.27)

and each force component can be further expressed as follows:

\[
F_{sgcl} = R_l \left[ \left( p(\varepsilon) - 2k\varepsilon(\sin\varepsilon - \varepsilon) - 2k(\varepsilon \sin\varepsilon - \varepsilon \sin\varepsilon) + 2k(\varepsilon - \sin\varepsilon) \right) \right] / \omega
\]  

(2.28)

\[
F_{slN} = \sigma_n R_L \left( \sin(\varepsilon + \delta) - \sin\varepsilon \right) + kR_L / \omega \left( \cos(\varepsilon + \delta) \right) - \cos\varepsilon) / \omega
\]  

(2.29)

\[
F_{snp} = \int_{0}^{2\gamma-\delta} \left( (\sigma_n \cos(\varepsilon + \delta + \varepsilon) + \tau_s \sin(\varepsilon + \delta + \varepsilon))R_L / \omega \right) d\varepsilon
\]  

(2.30)

Moment applied to chip by the tool-chip interface referring to point \( G_1 \) is:

\[
M_{G_1} = M_{sGcl} + M_{slN} + M_{snp}
\]  

(2.31)

where:
\[ M_{\text{cl}} = \left( \frac{\pi}{2} \right)^2 \int_0^\pi \left( p_L + 2k\varepsilon - \varepsilon_\alpha \right) \sin(\varepsilon - \varepsilon_\alpha) - k(1 - \cos(\varepsilon - \varepsilon_\alpha)) \, d\varepsilon \]

\[ = \left( \frac{\pi}{2} \right)^2 \left( -p_L + 2k\varepsilon_\alpha \cos(\varepsilon_\alpha - \varepsilon_\alpha) - 1 \right) - k\varepsilon_\alpha^2 \cos(\varepsilon_\alpha - \varepsilon_\alpha) - 2k (\varepsilon_\alpha \cos(\varepsilon_\alpha - \varepsilon_\alpha) - \varepsilon_\alpha) + 2k \sin(\varepsilon_\alpha - \varepsilon_\alpha) \right) \]

\[ M_{\text{cl,xy}} = \int_0^\pi \left( \left( R_x / \omega \right) \left( \sigma_x R_{\alpha x} \sin(\beta_1 + \varepsilon) - \tau_x \left( R_x - R_{\alpha x} \cos(\beta_1 + \varepsilon) \right) / \omega \right) \right) \, d\varepsilon \]

\[ = \int_0^\pi \left( \left( R_x / \omega \right) \left( \sigma_x R_{\alpha x} \sin(\beta_1 + \varepsilon) - \tau_x \left( R_x - R_{\alpha x} \cos(\beta_1 + \varepsilon) \right) / \omega \right) \right) \, d\varepsilon \]

\[ \beta_3 \] is the included angle of LTG\(_1\) and equal to

\[ \varepsilon_\alpha + \beta_1 - \frac{1}{2} \sin^{-1} \left( \frac{R \cdot \sin \left( \frac{\pi}{2} - \varepsilon_\alpha \right)}{R_{\alpha x}} \right) \]

(2.3) Modeling of force components along the worn flank face

The normal stress \( \sigma_n \) along the worn flank face is modeled using a power law distribution and it is assumed that the flank wear land has not reached a critical plastic deformation length as follows (Waldorf, 1996; Smithey, et al., 2001):

\[ \sigma_n = \sigma_o \left( 1 - \frac{l}{V_B} \right)^2, \quad 0 \leq l \leq V_B \]

(2.35)

where \( \sigma_o = k \left( 1 + 2\delta_1 - 2\theta_1 + 2\theta_2 + 2\delta_2 + \sin(2\gamma_f) \right) \) and the angle of the fan shaped zone DOE is \( \delta_2 = \frac{\pi}{2} - \varepsilon_\sigma - \varepsilon_\gamma + \gamma_f \) (Shi, et al., 1991).

The shear stress is treated uniform (Shi, et al., 1991; Huang, 2002):

\[ \tau_s = k \cos(2\xi_f) \]

(2.36)

where, \( \cos(2\xi_f) = 0.9 \).

Therefore, the cutting force \( F_{zF} \) and thrust force \( F_{yF} \) can be calculated as follows:
\[ F_{xF} = \int_{0}^{b} \sigma_x \sin \varepsilon_x \, dl + \int_{0}^{b} \tau_y \cos \varepsilon_y \, dl \]  
(2.37)

\[ F_{yF} = \int_{0}^{b} \sigma_x \cos \varepsilon_x \, dl - \int_{0}^{b} \tau_y \sin \varepsilon_y \, dl \]  
(2.38)

The moment along the flank face does not contribute in chip equilibrium modeling.

(2.4) Model implementation

To satisfy the equilibrium of the chip formed, the forces and bending moments from the worn tool tool-chip interface should be equal to those from the primary shear zone. The forces are decomposed into two directions, namely, cutting direction along the X axis: \( F_{xHG_i} \) and \( F_{xLNP} + F_{xG_iL} \) and thrust direction along the Y axis: \( F_{yHG_i} \) and \( F_{yLNP} + F_{yG_iL} \). The bending moments are calculated regarding point \( G_i \), which are \( M_{HG_i} \) and \( M_{cG_i} \) from the primary shear zone and crater face, respectively. The following equations must be satisfied to maintain a steady state cutting status:

\[
\begin{align*}
F_{xHG_i} - (F_{xLNP} + F_{xG_iL}) &= 0 \\
F_{yHG_i} - (F_{yLNP} + F_{yG_iL}) &= 0 \\
M_{HG1} - M_{cG1} &= 0
\end{align*}
\]  
(2.39)

Further, the cutting force \( F_c \) and thrust force \( F_t \) can be calculated based on the solved forces information from Equations (2.12), (2.13), (2.16), (2.19), (2.37), (2.38) and (2.39) as follows:

\[ F_c = F_{xHG_i} + F_{xG_iG_o} + F_{xF} \]  
(2.41)

\[ F_t = F_{yHG_i} + F_{yG_iG_o} + F_{yF} \]  
(2.42)
Implementation of the Analytical Force Model

The proposed slip line field-based worn tool force model can be implemented as shown in Figure 2.9. The necessary model inputs are: cutting conditions, workpiece flow stress, and worn tool geometry (VB, KT, KM, KB), and friction coefficients ($\varsigma_f$ and $\varsigma_{ol}$). The tool wear geometry can be either predicted (Huang, et al., 2004; Huang, et al., 2005a) or measured. The model outputs can be the slip line field variables $\theta_1$, $\theta_2$, $\delta_1$, $\delta_2$, $\beta_1$, $\beta_2$, $R$, $r_1$, $r_2$, $r_3$, and $\omega$ as well as cutting force $F_c$ and thrust forces $F_t$.

Figure 2.9: Computational flow chart of 2D worn tool force model
Modelling Validation with Bayesian Approach Considering Uncertainty

As well recognized, machining process is a process full of uncertainties, and such uncertainties may be due to material property uncertainties in workpiece materials and cutting tools, machine tool spindle rotation variation, cutting configuration variation after each tool replacement/change, and measurement system uncertainty, to name a few. As a result, the force measurement under the same cutting conditions can be different from time to time depending on the degree of machining uncertainties. Some machining research studies have paid attention to the effect of machining uncertainties. For example, in studying the worn tool cutting forces due to the flank wear, the prediction interval has been introduced in modeling cutting forces variation, and such variation can be as large as 300 N over a 500 N median in cutting cast iron (Smithey, et al., 2000).

Conventional validation of analytical force model is to simply compare the model predictions with some experimental measurements and draw conclusions based on the visual comparisons. Due to different machining uncertainties, even three repetitive machining measurements cannot fully represent possible force information due to possible machining uncertainties. To better evaluate a new force model, it is necessary to find an efficient and effective methodology for the model validation purpose. Fortunately, the Bayesian approach-based validation methodology provides a promising alternative to the conventional model validation methodology.
Bayesian Linear Regression and Credible Interval Estimation

The study here aims to verify the modeling accuracy by comparing the modeling results with the Bayesian credible intervals, which are estimated based on a set of measurements using a Bayesian linear regression model with first order terms.

(1) Bayesian linear regression

\[
X = \begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2 \\
\vdots \\
\bar{x}_n \\
\end{bmatrix} = \begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1m} \\
x_{21} & x_{22} & \cdots & x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{nm} \\
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n \\
\end{bmatrix} = \bar{y}
\]

Figure 2.10: Definition of matrix X and vector \( \bar{y} \)

This section discusses how to find a Bayesian linear regression model with first order terms based on force measurements in machining and further estimate the associated Bayesian credible interval for each model-based force prediction. This interval will be used to evaluate the proposed slip-line based worn tool force model.

The first step in this validation study is to define the regression function between the inputs and output(s). Since it is a common practice to model cutting forces as a linear function of cutting specific energy and cross sectional chip load area in machining, a linear model is adopted in this study for simplicity. For future studies, more complex models can be used for this statistical modeling purpose; however, it is not the main purpose of this study.
As shown in Figure 2.10, the input variables \( x_{ij} \) \((i = 1,2,\cdots,n \text{ and } j = 1,2,\cdots,m)\) include the cutting conditions (cutting speed, depth of cut, feed rate), worn tool geometry (flank wear length VB, crater wear depth KT, crater wear length KM), machining time, and workpiece shear flow stress, where \( i \) represents the \( i \)th experimental condition, and \( j \) represents \( j \)th input variable. In this study, there are the total eight input variable, and \( m \) is 8. All \( x_{ij} \) is organized as an \( n \times m \) input matrix \( X \), and each row vector \( \bar{x}_i \) (the \( i \)th experimental condition inputs) stands for \( x_{i1}, x_{i2}, \ldots, \) and \( x_{im} \). For this linear regression, the output variable \( y_i \) \((i = 1,2,\cdots,m)\) can be cutting or thrust forces, which are treated statistically independent, and \( y_i \) stands for the force measurement corresponding to the \( i \)th experimental condition. The vector \( \bar{y}(y_1,y_2,\ldots,y_n) \) denotes the measurement output corresponding to the whole input matrix \( X \). The linear regression model is determined as follows: \( \bar{y} = X\bar{\beta} \) or \( y_i = \bar{\beta}_0 + \bar{\beta}_1 x_{i1} + \bar{\beta}_2 x_{i2} + \cdots + \bar{\beta}_m x_{im} \), which is linear with first order terms, where the regression model coefficients \( \bar{\beta}_0, \bar{\beta}_1, \ldots, \bar{\beta}_m \) are to be estimated based on a conventional least square errors estimation. The higher order \( x_{ij} \) terms have also been selected for this machining force regression modeling study. Since there is negligible improvement observed, the study has used the first order \( x_{ij} \) terms in the following model evaluation discussion.

(2) Bayesian credible interval estimation

Since the machining modeling errors may come from all different sources such as the cutting configuration setup and material property variation during machining, the
normal distribution is assumed for \( p(y | \beta, \sigma^2, X) \) based on the central limit theory. That is, conditional on the parameter \( \beta \) and the variance \( \sigma^2 \), \( p(y | \beta, \sigma^2, X) \) follows a normal distribution with a mean of \( X\beta \) and a variance of \( \sigma^2 I \) as follows:

\[
p(y | \beta, \sigma^2, X) = N(X\beta, \sigma^2 I)
\]

(2.43)

where, \( p(\cdot | \cdot) \) denotes a conditional probability density/distribution with the arguments determined by the context, \( N(\cdot \cdot) \) is the normal distribution, and \( I \) is the identity matrix. For notational convenience, the dependence on \( X \) will be suppressed in the following sections.

Since there is no prior information on \( \beta \) and \( \sigma^2 \), the noninformative prior distribution is generally taken for the joint distribution of \( \beta \) and \( \sigma^2 \) because it usually gives acceptable results while taking less effort than specifying the prior knowledge in a probabilistic form (Gelman, et al., 2003). For this linear regression-based machining modeling study, this non-informative prior distribution is represented as follows (Gelman, et al., 2003):

\[
p(\beta, \sigma^2) \propto \frac{1}{\sigma^2}
\]

(2.44)

Note that Equation (2.44) is equivalent to \( p(\beta, \log \sigma) \propto 1 \). The model determined by Equations (2.43) and (2.44) is called the noninformative Bayesian linear regression model here.

For the aforementioned noninformative Bayesian linear regression model, the posterior predictive distribution of each cutting force prediction \( \hat{y}_i \), given the
measurements $y$ has a closed form for a new input vector $\tilde{x}_i$ (similar definition as the aforementioned $\bar{x}_i$) as follows (Gelman, et al., 2003):

$$p(\tilde{y}_i | y) = t(n - m, \bar{x}_i \hat{\beta}, (1 + \bar{x}_i V_\beta \tilde{x}_i^T) s^2)$$

(2.45)

where $t(\cdot, \cdot)$ is the student’s t distribution centered at $\bar{x}_i \hat{\beta}$ with the squared scale matrix $(1 + \bar{x}_i V_\beta \tilde{x}_i^T) s^2$ and the degrees of freedom $n - m$, $\hat{\beta} = (X^T X)^{-1} X^T y$, and $V_\beta = (X^T X)^{-1}$.

The scale factor $s^2$ is as follows $s^2 = \frac{1}{n - m} (y - X\hat{\beta})^T (y - X\hat{\beta})$, where $y - X\hat{\beta}$ is the vector of residuals of cutting forces based on the inputs.

Figure 2.11: Bayesian linear regression and confidence interval prediction

For any input vector $\tilde{x}_i$, the predicted force is $\tilde{x}_i \hat{\beta}$, the two-side $100(1 - \alpha)\%$ Bayesian credible interval with the upper Bayesian credible limit (UCL) and lower Bayesian credible limit (LCL) is as follows:

$$[LCL, UCL] = \left[ \tilde{x}_i \hat{\beta} - t_{\frac{\alpha}{2}, n-m} \sqrt{\left(1 + \bar{x}_i V_\beta \tilde{x}_i^T\right) s^2}, \tilde{x}_i \hat{\beta} + t_{\frac{\alpha}{2}, n-m} \sqrt{\left(1 + \bar{x}_i V_\beta \tilde{x}_i^T\right) s^2} \right]$$

(2.46)
which means \[ p\left(\tilde{x}, \tilde{\beta} - t_{\frac{\alpha}{2}, n-m} \sqrt{\frac{1}{n} \sum (\tilde{x}_i - \bar{x})^2} < \tilde{y} < \tilde{x}, \tilde{\beta} + t_{\frac{\alpha}{2}, n-m} \sqrt{\frac{1}{n} \sum (\tilde{x}_i - \bar{x})^2}\right) = 1 - \alpha \] as

\[ p(LCL < \tilde{y} < UCL | y) = \int_{Y_{LCL}}^{Y_{UCL}} p(\tilde{y} | y) d\tilde{y}, \] here, where \( \alpha \) is the significance level, and \( t_{\frac{\alpha}{2}, n-m} \) is the \( \frac{\alpha}{2} \)-upper percentile of t-distribution with the \( n-m \) degrees of freedom. For illustration, Figure 2.11 depicts the whole modeling process.

It should be noted that Equation (2.45) is derived from the noninformative Bayesian linear regression model determined by Equations (2.43) and (2.44). If the distribution of \( y \) given \( \tilde{\beta} \) and \( \sigma^2 \) is not represented by Equation (2.43) or the prior distribution \( p(\tilde{\beta}, \sigma^2) \) is not represented by Equation (2.44), the posterior predictive distribution \( p(\tilde{y} | y) \) should be predicted as \( \int p(\tilde{y} | \tilde{\beta}, \sigma^2) p(\tilde{\beta}, \sigma^2 | y) d\tilde{\beta} d\sigma^2 \), and based on Bayes’ rule, the joint posterior distribution of \( p(\tilde{\beta}, \sigma^2 | y) \) is proportional to the product of \( p(y | \tilde{\beta}, \sigma^2) \) and \( p(\tilde{\beta}, \sigma^2) \).

**Bayesian Approach-based Model Validation**

Generally speaking, the worn tool cutting forces increase as the result of progressive tool wear. However, the cutting forces might increase or decrease under the relative contributions from the tool flank and crater wear. The proposed worn tool analytical force model is dedicated to predict the force variation tendency during the tool wear progression. The experimental measurements from a previous study in machining annealed CK45 (DIN 17200) steel (Mesquita, 1988), which included the force measurements as well as the worn tool geometry measurements, are utilized to verify the
proposed force model in this study. As discussed, the first step is to determine the Bayesian linear regression model and the associated the credible intervals. Then, the analytical force results are compared with the force ranges defined by the regression model and intervals.

The CK45 steel machining measurement results (Mesquita, 1988) are shown in Table 2.1. The cutting tools were the sintered high speed steel inserts with a rake angle $6^\circ$, the workpiece was the annealed CK45 steel with a hardness of 195 HB, and the cutting configuration was assumed as orthogonal for the tool and tool holder used (Mesquita, 1988). All the tests were performed using a cutting speed of 47 m/min, feed rate 0.18 mm/rev and depth of cut 2.5 mm. The time varying flow stress (MPa) of workpiece was fitted by experimental data with a polynomial function as

$$k = a_1 + a_2 t + a_3 t^2 + a_4 t^3,$$

where $t$ is the time in minutes, $a_1 = 595.12$ MPa, $a_2 = -0.39$, $a_3 = -0.01$, and $a_4 = 0$ (Mesquita, 1988).

(1) Determination of Bayesian linear regression model and credible interval

Since the Table 2.1 information was prepared using the same cutting conditions, the input variables for the regression model are simplified as: worn tool geometry VB, KT, KB, machining time, and flow stress, and the contribution of cutting conditions is fully represented by $\vec{\beta}_0$ to be determined. The output variable $y$ is the cutting or thrust force, and the cutting and thrust forces are modeled using two uncoupled regression models individually per the force independent assumption in the previous section. The linear regression model to be determined is

$$y_i = \hat{x}_i \vec{\beta}.$$
<table>
<thead>
<tr>
<th>Case No.</th>
<th>Time (min)</th>
<th>VB (mm)</th>
<th>KT (mm)</th>
<th>KM (mm)</th>
<th>Cutting force Fc (N)</th>
<th>Thrust force Ft (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>REN121</td>
<td>6.1</td>
<td>0.120</td>
<td>0.035</td>
<td>1.04</td>
<td>996</td>
<td>476</td>
</tr>
<tr>
<td></td>
<td>11.9</td>
<td>0.197</td>
<td>0.057</td>
<td>1.10</td>
<td>955</td>
<td>416</td>
</tr>
<tr>
<td></td>
<td>17.6</td>
<td>0.226</td>
<td>0.095</td>
<td>1.22</td>
<td>918</td>
<td>420</td>
</tr>
<tr>
<td></td>
<td>28.0</td>
<td>0.257</td>
<td>0.125</td>
<td>1.29</td>
<td>910</td>
<td>386</td>
</tr>
<tr>
<td></td>
<td>41.4</td>
<td>0.268</td>
<td>0.170</td>
<td>1.35</td>
<td>881</td>
<td>356</td>
</tr>
<tr>
<td></td>
<td>54.0</td>
<td>0.283</td>
<td>0.225</td>
<td>1.38</td>
<td>856</td>
<td>361</td>
</tr>
<tr>
<td></td>
<td>59.8</td>
<td>0.310</td>
<td>0.240</td>
<td>1.38</td>
<td>885</td>
<td>414</td>
</tr>
</tbody>
</table>

Table 2.1: (b) Experimental measurement of REN 131 (Mesquita, 1988)

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Time (min)</th>
<th>VB (mm)</th>
<th>KT (mm)</th>
<th>KM (mm)</th>
<th>Cutting force Fc (N)</th>
<th>Thrust force Ft (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>REN131</td>
<td>6.4</td>
<td>0.14</td>
<td>0.09</td>
<td>1.23</td>
<td>951</td>
<td>441</td>
</tr>
<tr>
<td></td>
<td>13.2</td>
<td>0.244</td>
<td>0.140</td>
<td>1.26</td>
<td>910</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>20.7</td>
<td>0.333</td>
<td>0.200</td>
<td>1.3</td>
<td>910</td>
<td>378</td>
</tr>
<tr>
<td></td>
<td>27.1</td>
<td>0.393</td>
<td>0.220</td>
<td>1.36</td>
<td>997</td>
<td>508</td>
</tr>
<tr>
<td></td>
<td>33.7</td>
<td>0.482</td>
<td>0.258</td>
<td>1.42</td>
<td>1054</td>
<td>550</td>
</tr>
<tr>
<td></td>
<td>46.5</td>
<td>0.714</td>
<td>0.320</td>
<td>1.55</td>
<td>1146</td>
<td>586</td>
</tr>
<tr>
<td></td>
<td>53.0</td>
<td>0.879</td>
<td>0.325</td>
<td>1.64</td>
<td>1219</td>
<td>610</td>
</tr>
</tbody>
</table>
To validate the accuracy of the determined regression model, the regression model is checked using a conventional (n-1, n) cross validation process as shown in Figure 2.12. Starting with zero order polynomial terms, any (n-1) data pairs out of the n available measured data pairs are used to get the posterior distribution of cutting forces, and the unused nth data pair is used to check the model validity. If the predicted nth force data falls within a predetermined significance level-based credible interval, which is determined from the posterior predictive distribution, then this distribution is considered acceptable for this data pair. If there is no satisfied regression model for the polynomial term order used, it means that the higher order terms should be included in the regression model.
Figure 2.12: (n-1, n) cross validation

Through a cross-validation process, it is found that the regression model

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} \]

provides sufficient modeling accuracy, where \( y_i \) is the \( i \)th measurement of the cutting or thrust force, \( x_{i1} \) stands for KT, \( x_{i2} \) stands for KM, \( x_{i3} \) stands for VB, \( x_{i4} \) is the machining time, \( x_{i5} \) is the flow stress value, \( \beta_0, \beta_1, \beta_2, \beta_3, \beta_4 \) and \( \beta_5 \) are the regression coefficients and determined as 1204.0501, 7.0341, -0.9085, -2.6738, -2.9203, and -0.0136 for the cutting force and 895.4043, 4.4205, 6.5089, -4.5104, -3.156, and -0.014 for the thrust force.
Figure 2.13: Force prediction based on the regression model and its credible intervals: (a) cutting force and (b) thrust force under the significance level of 75%.

Figure 2.14: Force prediction based on the regression model and its credible intervals: (a) cutting force and (b) thrust force under the significance level of 90%.

Figures 2.13 and 2.14 show the regression force model results as well as the force credible intervals based on the significance level of 75% and 90%. In addition to the
regression model results, the UCL and LCL predictions are drawn to illustrate the possibility of measurement data ranges considering possible measurement and process uncertainties in machining, which is turning in this study. It can be seen that the 90% significance level interval covers a wider force measurement range.

(2) Validation of proposed analytical model

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Analytical Force Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>LCL</td>
</tr>
<tr>
<td>Time (min)</td>
<td>0</td>
</tr>
<tr>
<td>Forces (N)</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>950</td>
</tr>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>1050</td>
</tr>
<tr>
<td></td>
<td>1100</td>
</tr>
</tbody>
</table>

Figure 2.15: Force model validation under the 75% significance level (REN 121): (left) cutting force and (right) thrust force

The main purpose of this study is to develop a new analytical 2D force model to capture the force variation under the worn tool conditions and subsequently verify the proposed force model against the statistically processed force measurement credible ranges. In using the proposed analytical model for cutting forces predictions, the friction coefficients along the edge OL and flank wear land are set as 0.9 in simulation (Waldorf, 1996), and the rake angle of the edge OL is assumed zero degree here. The length ratio \((PN/LN)\) of the elastic region versus the plastic region along the rake face is set as a
constant \( \frac{5}{7} \) based on a previous study (Chungchoo, et al., 2002). KB/KM is assumed here as changing linearly from 0.70 to 0.55 for simplicity based on a previous measurement in turning harden steels (Poulachon, et al., 2004).

![Graphs showing force model validation](image)

Figure 2.16: Force model validation under the 90% significance level (REN 121): (left) cutting force and (right) thrust force

As seen from Table 2.1, VB increased rapidly to 0.197 mm and slowed down until reaching 0.283 mm during the steady state tool wear period for Case REN 121. After that, VB increased quickly again. Meanwhile, KT increased steadily with the cutting time. The predicted cutting and thrust forces are compared with the measurements as well as the Bayesian model predictions as shown in Figures 2.15 and 2.16, and the predictions agree with the measurements with a satisfactory accuracy. Similar simulation results with the cases REN 131 and REN 981 have been observed as shown in Figure 2.17. To avoid confusion, UCL and LCL are not drawn in this figure.
Figure 2.17: Comparison of analytical force model prediction with experiments: (left) case REN 131, (right) case REN 981

It should be pointed out that the Bayesian prediction mean is closer to the proposed 2D analytical force model prediction than that of the experimental measurement. The observation indicates that it may not be sufficient to just validate the analytical model predictions against a set of experiment measurement since any particular measurement may involve some process and/or material uncertainties and provide some misleading information. With the help of a Bayesian modeling approach, the experimental data can be further presented in terms of the measurement credible limits/ranges to help scientifically validate any proposed analytical model(s). As expected, the 75% significance level provides a more confined force prediction ranges, which any force measurements have a 75% probability falling within these credible ranges (defined by LCL and UCL). For this study, the analytical force model-based predictions fall well within the 75% credible ranges determined by the Bayesian
approach, which means a satisfactory modeling capacity of the proposed analytical model.

Discussion

After the force model is built, it is useful to investigate effects of some worn tool geometric parameters on the cutting force. For the following discussion, if not specified, the cutting conditions are as follows: cutting velocity is 220 m/min, feed rate is 0.18 mm, and depth of cut is 2.5 mm. The worn tool geometry is as follows: VB = 0, KM = 395 µm, KB = 220 µm, rake angle = 5°. Both frictional coefficients of OL and flank wear land are set as 0.9 for simplicity. Although for most cases, KT is set as a variable to appreciate its contribution, the effect of VB is appreciated in this section too. The effectiveness of the proposed worn force model is further compared with that of AdvantEdge®4.5, FEM-based commercial machining simulation software of Third Wave Systems, MN., for a steady state orthogonal cutting process with continuous chip formation. Information about how to use this software is referred to Appendix B.

Effect of Crater Wear and Flank Wear on Forces

Experimental data of the case REN121 are listed in Table 2.1. During the tool wear running in period, VB increased rapidly to 0.197 mm and slowed down until reaching 0.283 mm during the steady state tool wear period. After that, VB increased quickly again. Meanwhile, KT increased steadily with cutting time. The forces generally decrease first due to the dominant effect of crater wear then increase little at the end of
machining due to the increasing dominant effect of flank wear. The forces due to both flank and crater wear are presented in Figures. 2.18(a) and 2.18(b) to appreciate the decoupled tool wear contributions. It is found that the cutting forces decrease almost linearly under the crater wear induced tool sharpening effect as seen from Figure 2.18(a). Also the cutting forces increase as the increasing tendency of the flank wear land as seen from Figure 2.18(b), which increase rapidly before 20 minutes, then smoothly, finally rapidly again. While comparing the force magnitudes due to crater and flank wear, it is found that the forces due to crater wear is almost two times larger than those due to flank wear, and the forces increase due to flank wear is overshadowed by the forces decrease due to crater wear during the most cutting period. As the result, the total cutting forces decrease during the most cutting period while the crater wear induced sharpening effect dominates. Similar simulation results with the case REN221 in (Mesquita, 1988) have been observed.

Figure 2.18: (a) predicted forces due to crater wear (slip line $G_1 LN P$), and (b) predicted forces due to flank wear (slip line $G_1 G_2 OF$) (REN121)
For the case REN321, flank wear increased significantly faster as shown in Table 2.2. The predictions capture the force measurements very well and it is found both cutting forces increase rapidly as seen from Figure 2.19. Although the cutting forces due to crater wear still decrease, the effect of rapidly increasing flank wear dominates the cutting forces tendency as seen from Figures 2.19(b) and 2.19(c). The force increasing contribution due to flank wear overshadows that of crater wear. As the result, the total cutting forces increase while the flank wear induced effect dominates.

Table 2.2: (a) Case REN321 experimental data (Mesquita, 1988)

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>11.4</th>
<th>22.7</th>
<th>33.1</th>
<th>43.4</th>
<th>57.1</th>
<th>70.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>VB (mm)</td>
<td>0.19</td>
<td>0.316</td>
<td>0.538</td>
<td>0.601</td>
<td>0.854</td>
<td>1.11</td>
</tr>
<tr>
<td>KT (mm)</td>
<td>0.085</td>
<td>0.14</td>
<td>0.205</td>
<td>0.235</td>
<td>0.293</td>
<td>0.38</td>
</tr>
<tr>
<td>KM (mm)</td>
<td>1.29</td>
<td>1.29</td>
<td>1.42</td>
<td>1.48</td>
<td>1.61</td>
<td>1.77</td>
</tr>
</tbody>
</table>

(b) The measured shear flow stress constants

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>a3</th>
<th>a4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>599.36 (MPa)</td>
<td>-1.17</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

By combine the study for the cases REN 121, REN 221, and REN 321, it is concluded that the total cutting forces may increase or decrease depending on the relative contribution from both crater and flank wear. The proposed worn force model can reasonably capture the total cutting forces as well as the force contributions from both
crater and tool wear, which helps determine the force variance tendency during the tool wear progression.

Figure 2.19: (a) Cutting (left) and thrust (right) forces comparison, (b) predicted forces due to crater wear (slip line $G_1LNP$), and (c) predicted forces due to flank wear (slip line $G_1G_2OF$) (REN321)
Figure 2.20: Effect of crater wear depth KT (a) cutting forces, (b) slip line field angles $\theta_1$, $\theta_2$, $\delta_1$, $\beta_1$, and $\beta_2$, (c) velocity discontinuities, and (d) chip rotation velocity $\omega$

Effect of Crater Wear Depth KT

The sole effect of crater wear depth (KT) without considering flank wear and assuming constant KM (395 µm) and KB (220 µm) is studied here to appreciate the changes of cutting forces and slip-line intermediate variables. As seen from Figure 2.20(a), all forces contributed by the slip-lines/interface $G_iG_zO$ and $G_iLNP$ decrease as
KT increases while the cutting forces due to $G_1G_2O$ is less affected by the KT progression.

Figure 2.21: FEM simulation results of fresh tool when KT =0

The field angles of the primary shear zone ($\theta_1$, $\theta_2$, $\delta_1$, $\beta_1$, and $\beta_2$) are studied and the simulation results are shown in Figure 2.20(b). Except that $\beta_1$ and $\delta_1$ are almost constant, all other angles increase as KT increases. $\theta_1$ increases faster than $\theta_2$, which means that the velocity of point $O$ is closer to that of point $H$. When KT is around zero, $\theta_1$ is approximately zero too, which means the shear zone $HIG_i$ becomes straight. In Figure 2.20(b), as the KT near to zero, the $\theta_1$ is approximately zero, that means the primary shear zone becomes straight. At this time, $\theta_2$ is about 0.4~0.5 radian, which is about 25~30 degree, plus the rake angle 5 degree, so the shear angle of primary shear line is about 30~35 degree. It is almost comparable with the result of FEM simulation of fresh tool using AdvantEdge shown in Figure 2.21. After KT=30 um, $\theta_1$ increasing sharply
will be bigger than $\theta_2$. It means the velocity of O point is faster and faster compared to the velocity of point H. So the edge OL will be eliminated gradually by the more and quicker chip motion along this edge.

There is no pronounced velocity discontinuity changes of $r_1$, $r_2$, and $r_3$, which all increase little bit; however R does decrease significantly from 3.5 to 2.5 m/s when KT increases from 0 to 30 µm as seen from Figure 2.20(c). Considering a combined effect of a decreasing $R$ and increasing $\theta_1$, the primary shear zone turns to be more curled when KT increases. The combined effect of an increasing $\theta_2$ and increasing $r_1$, $r_2$, and $r_3$ also lead to a larger $OG_1G_2L$ zone.

![Figure 2.22: Velocity field in chip with worn tool: KT=20µm](image)

The chip angular velocity $\omega$ increases as KT increases as seen from Figure 2.20(d). This is because the chip need move longer distance when assuming KM is constant, but KT keeps increasing. The FEM result of velocity field as shown in Figure
2.22 supports the chip velocity assumption in the proposed slip line model that there is a uniform angular velocity $\omega$, and all the chip rotate with a same center. Near the free surface, the velocity of chip is slowest; the velocity is fastest near the tool-chip interface when ignoring the effect of the thin layer stick to the interface. This result supports the proposed hodograph of the new slip line model.

Effect of Friction Coefficient along OL

![Graphs showing the effect of friction coefficient on slip line field angles](image)

Figure 2.23: Predicted slip line field angles of two different friction coefficients:

(a) 0.5 and (b) 0.9

All the predictions discussed before are simulated based on a constant friction coefficient (0.9) along the edge OL. By varying the different friction coefficient values along OL, the effect of this friction coefficient on the primary shear zone field angles and cutting forces are studied and shown in Figures. 2.23 and 2.24, and the rake angle is taken
as zero for simplicity. From Figure 2.23, it can be seen that only $\theta_2$, $\beta_1$, and $\beta_2$, which are close to the edge OL, are very sensitive to the friction condition along the edge, while there is no effect on $\theta_1$ and $\delta_1$. It is concluded that the friction condition along the transient edge OL only influences the retarded chip area near the tool tip. The primary shear zone ($\theta_1$ and $\delta_1$) where is not close to the interface is not strongly influenced by friction coefficient.

Figure 2.24: Predicted forces with variant friction coefficients (0.5, 0.7, and 0.9) along OL: (a) cutting force, (b) thrust force.

It can be further seen from Figure 2.24 that different friction coefficients have pronounced effect on cutting force while KT is larger than 10 $\mu$m. This means that with the extremely deep crater wear, the retarded area is so large that the most chip formation area is influence by the retarded area, and the effect from the tool transient edge has changed the chip deformation process along the primary shear zone. Usually the retard
area is similar like BUE or DMZ that is sharper than original tool so the cutting force
decrease with the larger friction coefficient of OL, which is equal to large retarded area.
However, there is no obvious effect of friction coefficients on thrust force, this mainly for
the reason that thrust force is not sensitive to the retard area.

Effect of the Ratio of Crater Sticking and Sliding Regions

Based on a previous study (Chungchoo, et al., 2002), the ratio of crater sticking
and sliding regions \( \frac{LN}{NP} \) is selected as \( \frac{7}{5} \), that is, the ratio of crater sliding and whole
crater regions \( \frac{NP}{LNP} \) is \( \frac{5}{12} \). Figure 2.25 shows the sensitivity of cutting forces on this
ratio. Three ratios are selected as follows: \( \frac{4}{12} \), \( \frac{5}{12} \), and \( \frac{6}{12} \). It is found that this ratio
affects the predicted cutting forces significantly. And a smaller sliding region/portion
leads to higher predicted cutting forces, but lower thrust forces. It can be interpreted from
the stress distribution curve along the tool crater wear interface. Considering the whole
length of tool-chip interface is constant, also the maximum stress at tool tip is constant
and only the ratios changes, when sliding region is smaller, it means the area under stress
distribution curve larger. Therefore the cutting force which is obtained by integration
with the stress distribution is larger. The condition for thrust force is different, because
shear stress along tool crater wear land is opposite to the thrust direction. After
integration, the larger the stick shear stress, the smaller the thrust force is. For an
improved modeling accuracy, the ratio of crater sticking and sliding regions must be
carefully measured or calibrated for different tool and workpiece combinations.
Figure 2.25: Effect of the ratio of crater sliding and the whole crater regions:

(a) cutting force, (b) thrust force

Effect of Flank Wear Land VB

Figure 2.26 shows the effect of flank wear land VB on cutting forces. As expected (Waldorf, 1996; Huang, et al., 2005b), larger VB leads to larger cutting forces. It is also found that VB has a more pronounced effect on thrust force than on cutting force, which can be explained by the possible ploughing effect of the worn flank face because the thrust force comes from the deformation of workpiece, but cutting force mainly from the friction force, scratch effect along the machined surface considering the small angle $\varepsilon_F$ of flank wear land.
Effect of Friction Coefficient along OF

All the predictions discussed before are simulated based on a constant friction coefficient (0.9) along the flank land OF as before (Waldorf, 1996; Huang, et al., 2005b). By varying the friction coefficient values along OF (0.85, 0.90, and 0.95), the effect of this friction coefficient on cutting forces are studied and shown in Figure 2.27. It can be seen that the cutting forces vary little under the range of friction coefficient variation. It is because that forces caused by the flank wear land mainly from the plough effect, so the friction coefficient variance does not influence the cutting force obviously. The thrust force increases little bit first when KT is smaller than 5 µm, which is believed due to computational uncertainty when the tool has a larger VB (50 µm). Fortunately, for the small KT condition, it can be treated as fresh tool with/without flank wear which can be simulated by other models (Waldorf, 1996; Long, et al., 2005). Regarding this small KT cases, one interesting finding is that the prediction using Bayesian approach as in Figure

Figure 2.26: Effect of flank wear land VB: (a) cutting force, (b) thrust force
2.13 or 2.14 also expects a small increase first when KT is smaller. So it is probably due to the experimental measurement uncertainty/error that the measured forces are just unfortunately larger than the real force value for small KT condition during that experiment practice.

![Graphs showing predicted forces under different friction coefficients](image)

**Figure 2.27:** Predicted forces under different friction coefficients (0.85, 0.90, and 0.95) along OF (VB=50 μm): (a) cutting force, (b) thrust force

**Conclusions**

The total cutting forces may increase or decrease depending on the relative contribution from both the crater and flank wear. An analytical 2D force model has been proposed to capture the force variation under the worn tool condition, and it is further validated with the experimental measurement directly as well as the credible ranges determined by the noninformative Bayesian linear regression approach. For this study,
the analytical force model-based predictions fall well within the 75% credible ranges determined by the Bayesian approach, which proves the proposed analytical force model has a satisfactory modeling capacity.

The proposed worn tool force model has proved its promising application in modeling an orthogonal cutting process under the worn tool effect. The proposed worn tool force model can reasonably capture the total cutting forces as well as the force contributions from both the crater and tool wear, which helps to determine the force variance tendency during the tool wear progression.
CHAPTER THREE

MODELING OF THE COMBINED EFFECT OF THE CRATER AND FLANK WEAR ON CUTTING FORCES IN OBLIQUE CUTTING

Introduction

Effective cutting geometry of a tool usually changes after tool wear starts. Quantitative understanding and prediction of cutting forces under worn tool conditions is important to cutting process thermal modeling, tool life estimation, chatter prediction, and tool condition monitoring purposes, as discussed in Chapter 2. Particularly, the effect of the flank and crater wear on cutting forces is often of great interest since the flank and crater wear are the most dominant wear patterns for different tool-workpiece combinations under different cutting conditions (Huang, 2002). However, the effect of tool flank and crater wear on cutting forces in oblique cutting should also be carefully studied in addition to 2D force modeling since most machining operations are done in oblique configuration.

Based on the two well accepted assumptions (Adibi-Sedeh, et al., 2002; Molinari, et al., 2005): (1) each elementary chip segment has the same chip flow direction; and (2) the chip formation process at each infinitesimal chip segment is treated as single straight edge oblique cutting with a nonzero inclination angle, this chapter introduces a new approach to model force information in 3D oblique hard turning using a worn chamfered tool. Firstly, the tool cutting geometry is studied. Secondly, the equilibrium of elementary
chip and force based on the discretized chip elements is analyzed. Then, a 2D worn tool force modeling is briefly introduced in order to model the force information of each discretized element, and the overall modeling procedure is summarized. Finally, the proposed modeling approach is verified with the experimental results, and the paper concludes with the discussion and conclusions.

**Proposed Analytical Model**

Cutting Geometry in 3D Oblique Cutting

(1) Worn tool geometry

For illustration, a typical worn chamfered tool geometry measurement is shown in Figure 3.1. Figure 3.2 represents the geometry when the tool is fresh.

![Figure 3.1: Typical tool wear geometry observation in cutting hardened steel using a CBN tool (cutting time is 66.0 minutes with speed = 1.52 m/s, federate = 0.076 mm/rev, depth of cut = 0.102 mm)](image-url)
Figure 3.2: Tool geometry of the fresh chamfered tool (chamfer angle $\alpha_0$ and chamfer length $l_0$)

The cross sectional area along the chip flow direction of crater wear is typically simplified as circular (Armarego, et al., 1969; Li, et al., 2002; Huang, et al., 2005a) based on experimental observations (Hitomi, 1961; Huang, et al., 2005a). The spacing or width of crater wear along the chip flow direction is commonly assumed equal to the tool-chip contact length. Although the tool-chip contact length theoretically is a function of cutting conditions, cutting time, and tool geometry (Abukhshim, et al., 2004), the observed crater wear width changes negligibly during the tool life span in typical cubic boron nitride (CBN) hard turning (Huang, 2002). To simplify the contact length prediction, the contact length is generally assumed linear to the undeformed chip thickness (Usui, et al., 1978b), and the simplification is adopted in this study as well. The flank wear width (length or wear land), VB, is generally assumed uniform along the cutting edge and it increases with the cutting time (Huang, 2002).

(2) Cutting edge discretization
Under the typical hard turning conditions of interest, material removal generally happens within the chamfered tool nose zone based on the cutting tool manufacturers’ recommendation (Huang, 2002). Cutting is limited with the tool nose area, and the scenarios of cutting with the straight side cutting edges are not of interest in this study. The engaged cutting area is shown as the area ABC in Figure 3.3. The actual cutting geometry associated with the tool surface plane $S_b$ in Figure 3.3 is not exactly the same as $d$, $f$, and $C_y$ as specified in cutting conditions since the oblique mounting angles are not zero (Adibi-Sedeh, et al., 2002). The exact values of $d$, $f$, and $C_y$ in 3D oblique cutting should be modified using a transformation matrix given by Adibi-Sedeh et al. (Adibi-Sedeh, et al., 2002). The $d$, $f$, and $C_y$ values in this chapter are all presented after modification in the following sections.

The round cutting edge along the chamfered zone is further divided into $N$ elements with equal arc length, which is parallel to the given chip flow direction as shown in Figure 3.3. The chip flow direction can be determined iteratively using the energy minimum principle as shown in the following sections, and the flow angle is

$$\frac{\pi}{2} - \eta_c^0. $$

The tool-chip contact length is within the chamfered zone under most gentle hard turning conditions as shown in Figure 3.1, which means $l_c^i < l_0$, where $l_0$ is the length of chamfer edge as shown in Figure 3.2. The tool nose radius $r$ in this paper means $r_0^b$ in Figure 3.2. From Figure 3.3, the angle of the whole engaged cutting edge $AB$ is:
\[ \Phi_c = \frac{\pi}{2} + \sin^{-1}\left( \frac{f}{2r} \right) - \sin^{-1}\left( \frac{r-d}{r} \right) \]  

(3.1)

Figure 3.3: Cutting configuration and its discretization ( \( NP = t_{j-1} \), \( MQ = t_j \), and \( r = r_0 \))

After discretization, each element has an included arc angle \( \Delta \theta = \Phi_0 / N \), here \( N \) is the number of discretized elements. The angle defining the element \( j \), where \( j = 1 \sim N \), can be represented as:

\[ \theta_j = \sin^{-1}\left( \frac{r-d}{r} \right) + j \cdot \Delta \theta \]  

(3.2)
The edge length of the arc MN, or the width of the jth elementary chip MNPQ can be approximated as:

\[
w_j \approx 2r \sin \frac{\Delta \theta}{2} \cdot \sin \left( \pi - \eta_{cs}^0 + C_s - \theta_j + \frac{\Delta \theta}{2} \right)
\]

(3.3)

The length of any element along the chip flow direction should be considered for the zones specified by AC and BC individually. Within the circular edge AC zone, the length can be computed as:

\[
t_j = \left( r \sin \theta_j \sin \left( \frac{\pi}{2} - \eta_{cs}^0 + C_s \right) + \left( f + r \cos \theta_j \right) \cos \left( \frac{\pi}{2} - \eta_{cs}^0 + C_s \right) \right)
- \sqrt{(t_j^*)^2 - \left( f^2 + 2fr \cos \theta_j \right)}
\]

(3.4)

where: \( t_j^* = r \sin \theta_j \sin \left( \frac{\pi}{2} - \eta_{cs}^0 + C_s \right) + \left( f + r \cos \theta_j \right) \cos \left( \frac{\pi}{2} - \eta_{cs}^0 + C_s \right) \).

Within the horizontal edge BC zone, the length can be computed as:

\[
t_j = \left( r \sin \theta_j - \left( r - d \right) \right) / \cos(\eta_{cs}^0 - C_s)
\]

(3.5)

As shown in Figure 3.3, \( t_0 = 0 \) at the point B and \( t_N = 0 \) at the point A. Therefore, the underformed chip thickness of the jth element can be determined as:

\[
t_{ej} \approx 0.5\left( t_{j-1} + t_j \right)
\]

(3.6)

As discussed before, the local tool-chip contact length can be simplified as proportional to the underformed chip thickness (Usui, et al., 1978b). It means:

\[
l_c^j = a \cdot t_{ej}
\]

(3.7)

If KM is measured from the deepest cross section of the crater wear along the chip flow direction, the constant \( a \), which is assumed constant along the whole cutting
edge under the same cutting conditions, can be determined based on the largest \( t_{ij} \). Then each cross sectional area of the whole crater as a circular arc can be further determined. It is also naturally assumed that the chip leaves the tool as an integral part. For given \( I_c^j \), \( KT_j \) corresponding to the jth element is simplified as follows based on the geometric similarity:

\[
KT_j = \frac{KT}{KM_j} = \frac{KT}{KM} I_c^j
\]

(3.8)

(3) Process modeling based on the chip Equilibrium

The process is modeled based on the equilibrium of the chip, and this model procedure can be summarized as follows. Firstly, the geometric relationships about the jth elementary chip are derived. Then, two local coordinate systems for the jth element are defined. Based on the interaction force between the neighboring discretized elements, the force balance about the jth element can be determined in order to predict the cutting forces of the element. Finally, the global force balance is used to solve the cutting forces of the whole chip. An energy minimum principle is utilized to pick the chip flow angle which leads to the minimum cutting force \( P_i \). A more detailed description of the model implementation will be shown in Figure 3.7.

**Geometric Information of an Elementary Chip**

Given the global tool geometric parameters \( i_0, r_h^0, r_a^0, \alpha_0, C_s \), cutting conditions (\( d \) and \( f \)), and an initial value of chip flow angle \( \eta_{0cr} \) (typically as 1°), the
local cutting configuration of the jth element, such as $\alpha_i^j$, $i_j$, $t_{ej}$, $w_j$, and $C^j$, can be determined using Figures 3.3, 3.4, and 3.5. The velocity $V_{c}^{j}$ is assumed the same as $V_{c}$. The local chamfer surface (rake surface if no chamfer zone) corresponding to each element is approximated as a flat plane $S_j$ by ignoring the convex curve due to the chamfer edge with a nose radius for simplicity.

![Figure 3.4: Geometry in the local oblique cutting of the jth element](image)

There is an angle $\alpha_o$ between the two planes $S_o$ and $S_0$. As shown in Figure 3.4, the inclination angle of the jth element $i_j$ can be determined as (Molinar, et al., 2005):

$$i_j = \sin^{-1}\left(\cos\psi_{r}^{j} \sin i_o - \sin\psi_{r}^{j} \sin\alpha_o \cos i_o \right) \quad (3.9)$$

where: $\psi_{o}' = \theta_{j,i} + \frac{\Delta \theta}{2}$ in the plane $S_o$ is the angle rotated from the reference plane $T_o$ to the plane $T_j$ where the jth element edge locates, and $\psi_{r}^{j}$ is the projection of $\psi_{r}^{j}$ in the
plane $S_0$, which is parallel to the cutting direction. There is a geometric relationship between these two rotation angles as (Molinari, et al., 2005):

$$\tan \psi'_j = \frac{\cos \theta \tan \psi'_0}{\cos \alpha_0 - \tan \psi'_0 \sin \alpha_0 \sin \theta} \tag{3.10}$$

In the plane $S_0$, it can be seen that:

$$C'_j = \psi'_0 + C_i \tag{3.11}$$

From Figures 3.4 and 3.5, it can be seen that the geometric relationship in the plane $S_b$:

$$\eta'_j = \eta'^0 - \psi'_j \tag{3.12}$$

The $\eta'_j$ is the angle $\eta'_j$ on the plane $S_j$ projected to the plane $S_b$, therefore:

$$\cos \eta'_j = \frac{\sin (\alpha'_n - \alpha'_0)}{\sin (\alpha'_n - \alpha'_0)} \cos \eta'_j = \frac{\sin (\alpha'_n - \alpha'_0)}{\sin \alpha'_n} \cdot \cos \eta'_j \tag{3.13}$$

In Equation (3.13), the local rake angle of the $j$th element $\alpha'_n$ is defined as follows:

$$\alpha'_n = \alpha'^0 + \alpha'_j \tag{3.14}$$

where: $\sin \alpha'_j = \frac{\sin \theta - \cos \psi'_j \sin i'_j}{\sin \psi'_j \cos i'_j}$. The effective rake angle of the $j$th element $\alpha_{ne}'$, is defined as follows:

$$\sin \alpha_{ne}' = \sin i'_j \sin \eta'_j + \cos i'_j \cos \eta'_j \sin \alpha'_j \tag{3.15}$$
Figure 3.5: Local velocity and force relationships of the jth element in oblique cutting

The direction of the cutting forces $F^j_x$, $F^j_y$, $F^j_z$ of the jth element in Figure 3.5 should be determined first. Figure 3.6 illustrates a chip element j leaving from the curved primary shear zone. This jth element is balanced by the forces from the workpiece-chip interface, the chip-tool interface, and the neighbor elements. This balance can be represented as $\vec{R}_{\text{tool-chip}}^j + \vec{R}_{\text{workpiece-chip}}^j + \vec{R}_j = 0$, and $\vec{R}_{\text{tool-chip}}^j$ can be decomposed into two components: the first one is along the chip flow direction on the chamfer (rake) face, the other is normal to the local tool chamfer (rake) face; $\vec{R}_{\text{workpiece-chip}}^j$ can be decomposed into three components $F^j_x$, $F^j_y$, $F^j_z$. It also indicates that $\vec{R}_{\text{tool-chip}}^j + \vec{R}_j + \vec{F}_x^j + \vec{F}_y^j + \vec{F}_z^j = 0$ or $\vec{R}_{\text{tool-chip}}^j + \vec{R}_j + \vec{F}_xe^j + \vec{F}_ye^j + \vec{F}_ze^j = 0$. As shown in Figure 3.5, $F^j_x$ is normal to the local chamfer (rake) face $S_j$, and $F^j_y$ and $F^j_z$ are in the local face $S_j$. $\vec{R}_j$ is also in the local surface $S_j$, which is exerted on the element j by the neighboring elements j-1 and j+1, where $\vec{R}_j = \vec{R}_{j-1,j} + \vec{R}_{j,j+1}$. For the boundary elements 1 and N, $\vec{R}_{0,1} = 0$ and $\vec{R}_{N,N+1} = 0$. 

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There are two local coordinate systems \(((x^j, y^j, z^j) \text{ and } (x^j_c, y^j_c, z^j_c))\) for the jth element shown in Figure 3.5. There is a transformation matrix to connecting the cutting forces between these two coordinate systems (Shaw, et al., 1952):
\[
\begin{bmatrix}
F^j_x \\
F^j_y \\
F^j_z
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha^j_n \cos i_j & \cos \alpha^j_n \sin i_j & -\sin \alpha^j_n \\
-\sin i_j & \cos i_j & 0 \\
\sin \alpha^j_n \cos i_j & \sin \alpha^j_n \sin i_j & \cos \alpha^j_n
\end{bmatrix}
\begin{bmatrix}
F^j_{xe} \\
F^j_{ye} \\
F^j_{ze}
\end{bmatrix}, \quad (3.16)
\]

Since both \( F^j_{xe} \) and \( F^j_{ze} \) are considered independent of the inclinational angle (Oxley, 1989), \( F^j_{xe} \) and \( F^j_{ze} \) of the jth element can be determined using the orthogonal force model proposed in Chapter 2 based on the local cutting configuration of the jth element.

The \( x_j \) axis is normal to the local chamfer (rake) face, in which the plane \( y_jz_j \) locates, and the \( \tilde{R}^j_{\text{tool-chip}} \) is along the chip flow direction and normal to the local chamfer (rake) face. Then, there is another equilibrium equation about the jth element on the local chamfer (rake) face as:

\[
\tilde{F}_z^j \sin \eta^j_c - \tilde{F}_y^j \cos \eta^j_c = \tilde{R}_j \quad (3.17)
\]

It is also further assumed whatever the length of the crater wear land is, the shear stress distributions along the local chamfer (rake) face are the same for each element. This stress distribution is as follows:

\[
\tau(x) = \begin{cases} 
\frac{k}{k\left(l^j_{\text{cl}} \cdot x\right)/(1-c)\left(l^j_{\text{cl}}\right)} & \text{for } x \leq l^j_{\text{cl}} \\
\frac{1}{k\left(l^j_{\text{cl}} \cdot x\right)/(1-c)\left(l^j_{\text{cl}}\right)} & \text{for } l^j_{\text{cl}} < x \leq l^j_v
\end{cases} \quad (3.18)
\]

Then the tangential force along the chip flow direction on the local chamfer (rake) face \( S_j \) can be represented as

\[
w_j \int_0^{l^j_v} \tau(x) dx = w_j \left( \int_0^{l^j_{\text{cl}}} kdx + \int_0^{(1-c)l^j_{\text{cl}}} k\left(\frac{x}{(1-c)l^j_{\text{cl}}}\right)^m dx \right) \times w_j l^j_v = A_j,
\]

which should be equal to \( \tilde{F}_z^j \cos \eta^j_c + \tilde{F}_y^j \sin \eta^j_c \). Therefore, it can be seen that:
\[
\frac{A_{i+1}}{A_i} = \frac{\tilde{F}_{x}^{i+1} \cos \eta_{x}^{i+1} + \tilde{F}_{y}^{i+1} \sin \eta_{x}^{i+1}}{\tilde{F}_{x}^{i} \cos \eta_{x}^{i} + \tilde{F}_{y}^{i} \sin \eta_{x}^{i}} \tag{3.19}
\]

Equation (3.19) is assumed valid even the tool is worn in this study.

Treating \(F_{xe}^{j}, F_{ye}^{j},\) and \(F_{ze}^{j}\) using the local coordinate transformation matrix Equation (16), \(P_{1}^{j}, P_{2}^{j},\) and \(P_{3}^{j}\) can be determined as follows:

\[
\begin{bmatrix}
P_{1}^{j} \\
P_{2}^{j} \\
P_{3}^{j}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos C_{j}^{i} & \sin C_{j}^{i} \\
0 & \sin C_{j}^{i} & -\cos C_{j}^{i}
\end{bmatrix}
\begin{bmatrix}
F_{xe}^{j} \\
F_{ye}^{j} \\
F_{ze}^{j}
\end{bmatrix}
\tag{3.20}
\]

where: \(\begin{bmatrix} P_{1}^{j} \\
P_{2}^{j} \\
P_{3}^{j}\end{bmatrix}\) is the three dimensional forces of the jth element in the global coordinate system in the cutting, feed, and radial directions, respectively.

**Determination of Chip Flow Direction and Cutting Forces**

The equilibrium of the whole chip can be represented as:

\[
\sum_{j=1}^{N} (\tilde{R}_{tool-chip}^{j} + \tilde{R}_{workpiece-chip}^{j} + \tilde{R}_{f}^{j}) = 0 \tag{3.21}
\]

When viewing the whole chip as an integral part, the effect of the internal inter-element forces leads to an equilibrium state as \(\sum_{j=1}^{N} \tilde{R}_{j} = 0\) as shown in Figure 3.6. The chip boundary conditions are treated as zero, which mean \(\tilde{R}_{b,j} = 0\) and \(\tilde{R}_{N,N+1} = 0\) (Molinari, et al., 2005). Equation (3.21) can be rewritten as:

\[
\sum_{j=1}^{N} \tilde{R}_{tool-chip}^{j} = - \sum_{j=1}^{N} \tilde{R}_{workpiece-chip}^{j} \tag{3.22}
\]
Both sides of Equation (3.22) are the collective resultant force. The three dimensional cutting forces $P_1$, $P_2$, and $P_3$ can be determined from the total resultant force $\sum_{j=1}^{N} \vec{R}_{workpiece-chip}^j$ by projecting the resultant force to the cutting, feed, and radial directions, respectively as:

$$\sum_{j=1}^{N} \vec{R}_{workpiece-chip}^j = \sum_{j=1}^{N} (\vec{P}_1^j) + \sum_{j=1}^{N} (\vec{P}_2^j) + \sum_{j=1}^{N} (\vec{P}_3^j)$$

(3.23)

The global cutting forces can be calculated as follows:

$$P_i = \sum_{j=1}^{N} P_i^j$$

(3.24)

where: $i = 1, 2, \text{ and } 3$.

The minimum energy principle (Molinari, et al., 2005) is adopted to determine the global chip flow direction, which means that the projected result force component in the cutting direction, $P_1$, should be the minimum. In simulation, the value of $\eta_{cr}^0$ is iterated from 1 degree to 90 degrees with an interval of 1 degree. The $\eta_{cr}^0$ corresponding to the minimum cutting force $P_1$ is picked up as the chip flow direction.

Summary of Proposed 3D Force Model

(1) 3D force model flow chart

For illustration, Figure 3.7 shows the modeling procedure of the proposed 3D worn tool force model in 3D oblique cutting.
Tool geometry: nose radius, chamfer angle, rake angle, inclination angle, mount angles; Cutting conditions: depth of cut, feed rate, velocity; Tool wear parameters: KM, KT, VB; The material properties.

Assume the chip flow direction is same for all element, set one initial chip flow angle,

The geometric parameters of the jth element: rake angle $\alpha^j$, inclination angle $i^j$, depth of cut $t_{oj}$ and width $w_j$, rotation angles $C^j_s$

Using the orthogonal force model to calculate the 2D forces $F'_{x_j}$, $F'_{y_j}$ of the jth element

Under the help of coordinate transformation, using the equation: $\vec{F}_z' \sin \eta'_z - \vec{F}_y' \cos \eta'_z = \vec{R}_j$
to represent the third dimensional forces of the jth element

Using the equation: $\sum_{j=1}^{n} \vec{R}_j = 0$
to calculate the three dimensional forces, further to get the three dimensional global forces

End of the iteration of chip flow angle, using the minimum energy principle that $P^1$ should be minimum.

After known the chip flow angle, using the equation: $\vec{F}_z' \sin \eta'_z - \vec{F}_y' \cos \eta'_z = \vec{R}_j = \vec{R}_{j-1,j} + \vec{R}_{j,j+1}$
to get each interaction forces between elements

Using the equation about jth element: $\vec{R}_{tool-chip} = -\left(\vec{R}_{workpiece-chip} + \vec{R}_j\right)$ to get the forces from tool-chip interface

Figure 3.7: Calculation flow chart of the 3D force model
Once the chip is discretized along the cutting edge into \( n \) elements, each element is modeled individually using a 2D worn tool force model in Chapter 2 based on the worn tool geometry shown in Figure 3.8. The work material is assumed perfectly plastic in this study in order to apply the 2D worn tool model. The temperature softening and strain rate hardening effect on material behavior is also ignored in the slip line field model. The model inputs are depth of cut, cutting velocity, feed rate, crater wear geometry, and flank wear length. The model outputs for this study are cutting force and thrust force.

![Figure 3.8: 2D worn tool geometry](image)

(2) Reduced \( N=1 \) case

When \( N \) is taken as 1, the modeling approach is reduced into the equivalent cutting geometry approach for 3D oblique cutting. Forces \( F_{xe}^1 \) and \( F_{ze}^1 \) can be determined using the analytical 2D force model with a rake angle of \( \alpha_{ne}^1 \), a width of cut of \( w_i \), and an undeformed chip thickness of \( t_c^1 \) as

\[
w_i = 2r\sin\frac{\Phi_c}{2} \sin\left(\pi - \eta_r + C_r \sin^{-1}\left(\frac{r - d}{r}\right) - \frac{\Phi_c}{2}\right),
\]

and

\[
\left( f \left( \sqrt{r^2 - 0.25f^2} - r + d \right) + f \left( \frac{r}{2} - \frac{f}{2} \sqrt{r^2 - 0.25f^2} \right) \right) / w_i.
\]
It is known that $\vec{R}_i = 0$ since $R_{0,i} = R_{1,i} = 0$ when $N = 1$. Then, the third force $F_{3e}^i$ can be determined following the Eq. (17) as:

$$\vec{F}_{z}^i \sin \eta_i^i - \vec{F}_{y}^i \cos \eta_i^i = \vec{R}_1 = 0$$

(3.25)

Equation (3.25) is further rewritten as follows under another local coordinate system $(x_e, y_e, z_e)$ using the coordinate transformation matrix Equation (3.16):

$$
\left( \sin \alpha_n^i \cos i_i F_{xe}^i + \sin \alpha_n^i \sin i_i F_{ye}^i + \cos \alpha_n^i F_{ze}^i \right) \cdot \sin \eta_i^i = \\
\left( -\sin i_i F_{xe}^i + \cos i_i F_{ye}^i \right) \cdot \cos \eta_i^i
$$

(3.26)

then, it can be seen that:

$$
F_{3e}^i = \frac{-\sin i_i F_{xe}^i + \tan \eta_i^i \left( -\sin \alpha_n^i \cos i_i F_{xe}^i - \cos \alpha_n^i F_{ze}^i \right)}{\sin \alpha_n^i \sin i_i \tan \eta_i^i - \cos i_i}
$$

(3.27)

This equation is the same to that in (Oxley, 1989; Arsecularatne, et al., 1995), just with the sign of the force direction is opposite.

Further, the global cutting forces $P_1$, $P_2$, and $P_3$ are determined based on:

$$
\begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} =
\begin{bmatrix}
P_1^i \\
P_2^i \\
P_3^i
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos C_s^i & \sin C_s^i \\
0 & \sin C_s^i & -\cos C_s^i
\end{bmatrix}
\begin{bmatrix}
F_{3e}^i \\
F_{ye}^i \\
F_{ze}^i
\end{bmatrix}
$$

(3.28)

**Model Validation and Discussion**

The hard turning experiment was performed to verify the proposed modeling approach. Hardened AISI52100 bearing steel with a shear flow stress of 774.8 MPa and a
Rockwell hardness of 62 was dry machined on a horizontal lathe. The tool insert was the KD050 low CBN content insert with a 0.8 mm nose radius and a -20 degrees and 0.1 mm wide chamfer edge. The cutting configuration was as follows: the side cutting edge angle $c_r$ is 5 degrees, the inclination angle $i_o$ is 5 degrees, the normal rake angle is -5 degrees, and $\alpha_o$ is 5 degrees.

Three typical cutting scenarios were chosen based on the tool manufacturer’s recommendation to validate the proposed model while satisfying that cutting happened within the tool chamfer zone along the tool nose. The three cutting scenarios were: (A) cutting speed $= 1.52$ m/s, feed rate $= 0.076$ mm/rev, and depth of cut $= 0.102$ mm, (B) cutting speed $= 1.52$ m/s, feed rate $= 0.076$ mm/rev, and depth of cut $= 0.152$ mm, (C) cutting speed $= 2.29$ m/s, feed rate $= 0.061$ mm/rev, and depth of cut $= 0.203$ mm. The tool wear geometry was measured using an optical profilometer (Zygo NewView 200), and three dimensional cutting forces ($P_1$, $P_2$, and $P_3$) were measured by a Kistler 9257B dynamometer. For comparison, the prediction considers the conditions of two N values as 1 and 10.

Tables 3.1-3.3 show that the experimental measurements of the tool wear information and the flow angle predictions for the investigated three scenarios. Crater wear geometry (KM and KT) were measured from the deepest cross section of crater wear along the chip flow direction of the maximum contact length. When $N = 1$, the effective $KT_i$ and $KM_i$ are taken as KT/2 and KM/2 based on the linear simplification (Equation (3.7)). The flank wear land friction factor of 0.9 is adopted (Waldorf, 1996). The values of VB were measured along the direction normal to the tool edge, so the flank
wear width used for the predictions are further modified as $VB/\cos \alpha$ by including the influence of the inclination angle $i_0$.

Figures 3.9-3.11 show the predicted and experimental cutting forces for the three scenarios. The solid lines with triangle represent the measured cutting forces. For $N=1$, the cutting forces are marked with the dotted lines. For $N=10$, the forces are marked with the dashed lines. Since the proposed model is for the worn tool cases based on a 2D worn tool force model in Chapter 2, it will have the singularity problem when the tool is fresh ($KT = 0$ and $VB = 0$). As a result, the predictions when the tool is fresh are not presented here. Such force information is not of interest in this study, and it can be predicted using an Oxley approach (Oxley, 1989) or other alternatives (Huang, 2002), if needed.

Table 3.1: Experimental measurement and predicted flow angle of scenario A: $KM = 90\degree$

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>15</th>
<th>35</th>
<th>55</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>KT (µm)</td>
<td>3.5</td>
<td>7.5</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>VB (µm)</td>
<td>50</td>
<td>80</td>
<td>140</td>
<td>180</td>
</tr>
<tr>
<td>$\pi/2 - \eta_0$ (degree)</td>
<td>76</td>
<td>76</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>
Figure 3.9: Predicted and experimental forces of scenario A

Table 3.2: Experimental measurements and predicted flow angle of scenario B: KM = 90 μm

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>KT (μm)</td>
<td>5.3</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>VB (μm)</td>
<td>50</td>
<td>100</td>
<td>140</td>
<td>170</td>
</tr>
<tr>
<td>$\frac{\pi}{2} - \eta^p$ (degree)</td>
<td>73</td>
<td>73</td>
<td>73</td>
<td>73</td>
</tr>
</tbody>
</table>
Figure 3.10: Predicted and experimental forces of scenario B

Table 3.3: Experimental measurements and predicted flow angle of scenario C: KM = 80 µm

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>KT (µm)</td>
<td>5</td>
<td>10.5</td>
<td>11</td>
</tr>
<tr>
<td>VB (µm)</td>
<td>63.4</td>
<td>89.6</td>
<td>109.7</td>
</tr>
<tr>
<td>(\frac{\pi}{2} - \eta_c) (degree)</td>
<td>69</td>
<td>68</td>
<td>68</td>
</tr>
</tbody>
</table>
It can be seen from Tables 3.1-3.3 that the predicted flow angles do not change so much as the tool wears progressively once the cutting conditions are fixed. It means that tool wear has the limited effect on the chip flow angle in 3D oblique cutting under the hard turning conditions. The typical flow angles are around 65-80 degrees for the investigated hard turning conditions. When N =1, a = 2 for all the scenarios based on the measurements. When N = 10, a = 1.3 for scenario A, a = 1.3 for scenario B, and a = 1.15 for scenario C.

The good modeling accuracy in terms of feed and cutting forces is observed from Figures 3.9-3.11. The predicted thrust forces have a relatively large deviation from the measurements, and this observation is considered due to the coarse treatment of
interaction forces between the discretized chip elements. The difference between the chamfer geometry and the flat-face geometry of nose radius tools may also contribute to this modeling error since the proposed modeling approach for the chamfered tools is derived from the modeling approach for the flat-faced tools for each discretized element.

Generally, it is considered that a fine discretization (large N) should lead to more accurate modeling results. However, it is not the case as seen from Figures 3.9-3.11. This discrepancy is attributed to that the violation of the plane strain condition for 2D analytical modeling when a fine discretization is performed for a fixed feed rate. If this is the case, there should be an optimal N to be identified for the best modeling accuracy. More studies will be done to resolve this hypothesis.

The force modeling accuracy is expected to be improved by accurate consideration of the flank and crater wear geometry and generalization the interaction forces between the discretized chip elements along the tool nose of the chamfer zone in the future studies.

Conclusions

The chapter presents a modeling approach in 3D oblique cutting using worn chamfered tools with a rounded tool nose under hard turning conditions, which are characterized by small feed rate and small depth of cut using a chamfered nose radius tool. The proposed model is further validated with the experimental hard turning studies. It is found that the chip flow angle does not change noticeably with tool wear and is
around 65-80 degrees for the investigated cutting scenarios. The predicted cutting and feed forces are relatively accurate compared with the predictions of the thrust forces. The force modeling accuracy is expected to be further improved by taking more consideration of the flank and crater wear geometry and generalization the interaction forces between the discretized chip elements along the tool nose of the chamfer zone in the future studies.
CHAPTER FOUR

MODELING OF THE EFFECT OF DEAD METAL ZONE ON CUTTING FORCES IN ORTHOGONAL CUTTING

Introduction

Another tool wear pattern, DMZ, has been reported for cutting with chamfered tools. For better application of chamfered tools in interrupted cutting and advanced materials cutting, there is an increasing need to further the understanding of the chamfered tool cutting mechanism with DMZ effect. The objective of this chapter is to fulfill such a need in modeling the chamfered tool cutting process, since theoretical modeling provides more physical insights on process fundamentals and is much less time consuming than numerical approaches.

This study enhances the previous theoretical efforts (Zhang, et al., 1991; and Ren, et al., 2000) in modeling chamfered tool cutting forces by (1) determining the angle $\gamma$ between the DMZ bottom edge and the cutting direction by minimizing the total energy in cutting instead of assuming a simple relationship between $\gamma$ and the shear angle as in (Zhang, et al., 1991; and Ren, et al., 2000); (2) using the extension of the tool main rake face to define the DMZ cutting edge as observed in (Hirao, et al., 1982; and Jacobson, et al., 1988), which was taken as vertical in (Zhang, et al., 1991; and Ren, et al., 2000); (3) finding the size of the deformation zone under the DMZ bottom edge by using an effective friction angle; and (4) formulating a modeling approach based on the Johnson-
Cook constitutive equation, which is more general than other flow stress models. The following sections discuss the detailed modeling approach and the model validation in cutting P20 mold steel.

**Proposed Analytical Model**

Typical DMZs shown in Figure 4.1 are drawn based on experimental observations in (Hirao, et al., 1982; Jacobson, et al., 1988). The stationary metal under the chamfer edge forms a cutting edge with a stagnation point N at the tip of the DMZ. The cross sectional area of the DMZ is normally taken as triangular in theoretical analysis for simplicity (Zhang, et al., 1991). The workpiece material flows upward above point N, and it flows downward underneath the DMZ if below point N. For typical chamfered tools with a rake angle between 0—±5°, the DMZ cutting edge can be approximated as the extension from the tool rake face based on the observations in (Hirao, et al., 1982; and Jacobson, et al., 1988), and this is taken as an assumption in this study for simplification. This cutting geometry simplification was also adopted in modeling the cutting mechanism of edge honed tools with a stable build-up material adhered to the edge (Waldorf, et al., 1998; and Waldorf, et al., 1999).
Based on the above simplifications, the plastic deformation in cutting is considered from three zones: the primary and the secondary zones due to the tool main rake face and the DMZ cutting edge, and the extrusion zone due to the DMZ boundary below the stagnation point. This proposed cutting geometric model in orthogonal cutting and its hodograph are shown in Figures 4.2 and 4.3 respectively. As pointed in (Ren, et al., 2000), an extrusion analogy for the plastic flow under the DMZ may not be the same as that observed in machining since indentation (Waldorf, et al., 1999) is not considered. However, this approach will give a first approximation to the ploughing process under the DMZ, so the extrusion analogy for the DMZ effect is used in this study as in (Zhang, et al., 1991; and Ren, et al., 2000).

Figure 4.1: (a) Experiment observation of DMZ (Jacobson, et al., 1988), (b) Experiment observation of DMZ (Hirao, et al., 1982).
Plastic Deformation in Primary Shear Zone

The primary shear zone is represented as one plane which is presented by line MN in the Figure 4.2. So the some detailed velocities such as $V_1$ and $V_2$ are ignored (Oxley,
1989; Huang, et al., 2003). After defining one effective underformed material thickness as

$$t_{e1} = t_0 - \lambda_{chamfer} \frac{\sin \gamma \sin(\alpha - \beta)}{\cos(\alpha - \gamma)}$$  \hspace{1cm} (4.1)$$

the basic relationships of the primary shear zone in orthogonal cutting setup can be given as below (Oxley, 1989; Kalpakjian, et al., 2008):

$$t_{chip} = t_{e1} \cos(\phi - \alpha) / \sin \phi$$ \hspace{1cm} (4.2)$$

$$\gamma_{MN} = \frac{\cos \alpha}{2 \sin \phi \cos(\phi - \alpha)} = \sqrt{3} \varepsilon_{MN}$$ \hspace{1cm} (4.3)$$

$$F_{e1} = R \cos(\lambda - \alpha)$$ \hspace{1cm} (4.4)$$

$$F_{e1} = R \sin(\lambda - \alpha)$$ \hspace{1cm} (4.5)$$

$$F_1 = R \sin \lambda$$ \hspace{1cm} (4.6)$$

$$N_1 = R \cos \lambda$$ \hspace{1cm} (4.7)$$

$$R = \frac{k_{MN} t_{e1} w}{\sin \phi \cos \theta}$$ \hspace{1cm} (4.8)$$

$$\lambda = \alpha + \theta - \phi$$ \hspace{1cm} (4.9)$$

$$\tan \theta = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - \frac{\Delta k_{MN}}{\Delta s_2} 2k_{MN} \sin \phi$$ \hspace{1cm} (4.10)$$

where, $\frac{\Delta k_{MN}}{\Delta s_2}$ is the variation of the shear flow stress across the width of the parallel sided shear zone. It can be deduced using partial differential relation. If history effects are neglected, the flow stress varies with strain, strain-rate, and temperature, so

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\[
\frac{\Delta k_{MN}}{\Delta s_2} = \frac{dk_{MN}}{ds_2} = \frac{\partial k_{MN}}{\partial s_2} \frac{\partial s_2}{\partial s_2} + \frac{\partial k_{MN}}{\partial s_2} \frac{\partial s_2}{\partial s_2} + \frac{\partial k_{MN}}{\partial s_2} \frac{\partial T_{MN}}{\partial s_2} \tag{4.12}
\]

Following the assumptions in (Oxley, 1989), the strain-rate term can be neglected as strain-rate can be assumed to pass through a maximum value at shear plane, the temperature term is also neglected because of the difficulties involved in calculating the temperature gradient across shear plane but also because of this should not lead to large errors. Then Equation (4.12) is simplified only left the first term (Huang, 2002), as

\[
\frac{dk_{MN}}{ds_2} = \frac{d\gamma_{MN}}{dt} \frac{d\gamma_{MN}}{ds_2} dt = \frac{d\left(\sigma_{MN}/\sqrt{3}\right)}{d\left(3\frac{\epsilon_{MN}}{\sqrt{3}}\right)} \frac{dt}{ds_2} = \frac{1}{3} \frac{d\left(\sigma_{MN}\right)}{d\left(\epsilon_{MN}\right)} \frac{dt}{ds_2} \tag{4.13}
\]

Using the John-Cook material model \( \sigma = \left(A + B\epsilon^*\right) \left(1 + C\log \frac{\epsilon}{\epsilon_0}\right) \left(1 + D\left(\frac{T_{MN} - T_0}{T_{MN} - T_0}\right)^w\right) \), there is

\[
\frac{d\sigma_{MN}}{d\epsilon} = nB\epsilon^{-1} \left(1 + C\log \frac{\epsilon}{\epsilon_0}\right) \left(1 + D\left(\frac{T_{MN} - T_0}{T_{MN} - T_0}\right)^w\right) \tag{4.14}
\]

And the shear strain rate is approximated (Oxley, 1989), as

\[
\frac{d\gamma_{MN}}{dt} = C_{MN} \frac{V \sin \phi}{t_v} = C_{MN} \frac{V \cos \alpha \sin \phi}{t_v \cos(\phi - \alpha)} \tag{4.15}
\]

The velocity vertical to the primary shear plane is get from slip line field model, as

\[
\frac{dt_{MN}}{ds_2} = \frac{1}{V \sin \phi} \tag{4.16}
\]

Substitute (4.13, 4.14, 4.15, 4.16) into Equation (4.10), get

\[
\tan \theta = 1 + 2 \left(\frac{\pi}{4} - \phi\right) - C_{MN} \frac{\sqrt{3} \cos \alpha B n \epsilon^{-1}}{6 \cos(\phi - \alpha) \sin \phi \left(A + B \epsilon^*\right)} \tag{4.17}
\]
The primary shear zone can be seemed as a straight line MN. Along this straight line, the maximum shear stress happens equal to its shear flow stress $k_{MN}$. And the primary shear zone is usually close to an adiabatic shear band, so the temperature of this primary shear zone $T_{MN}$ can be calculated (Oxley, 1989), as

$$T_{MN} = T_0 + \nu \Delta T_{MN}$$  \hspace{1cm} (4.18)

where, $\Delta T_{MN} = \frac{(1 - \zeta)k_{MN} \cos \alpha}{\rho s \sin \phi \cos (\phi - \alpha)}$, with $\nu$ is often set as 0.7, $\zeta = \begin{cases} 0.5 - 0.35 \log (R_T \tan \phi) \\ 0.5 - 0.15 \log (R_T \tan \phi) \end{cases}$ for $0.04 \leq R_T \tan \phi \leq 100$, $100 \leq R_T \tan \phi$, $0 \leq \zeta \leq 1$, and a non-dimension number $R_T = \rho SV_{\alpha} / K$.

Plastic Deformation in Tool-Chip Interface Zone

It is known that the average normal stress and shear stress on rake face are

$$\sigma_{N1} = N_i / (hw)$$  \hspace{1cm} (4.19)

$$\tau_{int} = F_t / (hw)$$  \hspace{1cm} (4.20)

Assume the primary shear zone meet the tool-chip interface without changing direction, and the primary shear line rotates through the angle $\phi - \alpha$ to meet the interface at right-angles (Oxley, 1989), this is assumed to occur in negligible distance, then

$$\sigma_{N1}' = k_{MN} \left( 1 + \frac{\pi}{2} - 2\alpha - C_{MN} \frac{nB e_{MN}^{n-1} \cos \alpha}{2\sqrt{3}(A + B e_{MN}^{n-1}) \cos (\phi - \alpha) \sin \phi} \right)$$  \hspace{1cm} (4.21)

It is also believed that $\sigma_{N1} = \sigma_{N1}'$ (Oxley, 1989).
Because the contact condition is assumed stick, along the tool-chip interface, there is a relationship: \( \tau_{\text{int}} = k_{\text{chip}} \). The shear flow stress along the tool-chip interface is

\[
k_{\text{chip}} = \frac{1}{\sqrt{3}} \left( A + B \epsilon_{\text{int}}^n \right) \left( 1 + C \log \dot{\epsilon}_{\text{int}} \right) D - E \left( \frac{T_{\text{int}} - T_0}{T_{\text{melt}} - T_0} \right)^{\mu}
\]

(4.22)

where, \( \epsilon_{\text{int}} = \delta t_{\text{chip}} / (\sqrt{3} h) \), \( \dot{\epsilon}_{\text{int}} = V_{\text{chip}} / (\sqrt{3} \delta t_{\text{chip}}) \), \( h \) is tool-chip contact length.

The uniform temperature on the tool-chip interface can be represented as (Oxley, 1989)

\[
T_{\text{int}} = T_0 + \Delta T_{MN} + \psi \cdot \Delta T_{\text{max}}
\]

(4.23)

where, \( \Delta T_{\text{max}} \) is the maximum temperature rise on the tool-chip interface. Assuming the friction energy on this interface are all transformed into heat, get the average temperature rise on the tool-chip interface is (Oxley, 1989)

\[
\Delta T_{\text{chip}} = F_i \sin \phi / \left( \rho S t_{w} \cos (\phi - \alpha) \right)
\]

(4.24)

\( \Delta T_{\text{chip}} \) is not equal to \( \Delta T_{\text{max}} \). And there is no theoretic relation between \( \Delta T_{\text{chip}} \) with \( \Delta T_{\text{max}} \).

So Oxley (Oxley, 1989) introduced \( \delta_i \) by assuming the shear happens in a very thin layer close to the tool face, which is about 5% of total chip thickness, to get an empirical relationship between \( \Delta T_{\text{chip}} \) with \( \Delta T_{\text{max}} \) (Oxley, 1989), as

\[
\log \left( \frac{\Delta T_{\text{max}}}{\Delta T_{\text{chip}}} \right) = 0.06 - 0.195 \delta_i \left( \frac{t_{\text{chip}} R_T}{h} \right)^{0.5} + 0.5 \log \left( \frac{t_{\text{chip}} R_T}{h} \right)
\]

(4.25)

where, \( h \) the tool-chip contact length can be estimated by momentum balance between the primary shear zone and chip-tool interface. Assuming the normal stress distribution along the tool-chip is linear with the maximum value at tool tip (Oxley, 1989), and taking
moments balance of the normal stresses between tool-chip interface and primary shear zone, there is

\[ h = \frac{4t_{c1} \sin \theta}{\cos \lambda \sin \phi} \frac{p_M / 3 + p_N / 6}{p_M + p_N} \]  

(4.26)

where, hydrostatic stress on point M is \( p_M = k_{MN} (1 + 2(\pi / 4 - \phi)) \), and hydrostatic stress on point N is \( p_N = p_M - \frac{\Delta k_{MN}}{\Delta s_2} \frac{t_{c1}}{\sin \phi} \).

Plastic Deformation under the DMZ

Partial workpiece is compressed under the bottom edge ON of DMZ as shown in Figure 4.4. In order to calculate the force along the edge ON, it is necessary to investigate the plastic deformation under the DMZ to get the information about strain, strain-rate, and temperature around the edge. All of these variables are used in the material model, John-Cook model to calculate the shear flow stress.

As explained in Appendix A, if the friction factor on the bottom edge is 1.0, the slip-line field under ON is vanished into one single plane (Hill, 1950) (in the geometry illustration, it seems as one line) along the edge where shear stress is equal to shear flow stress \( k_{work} \). For general case, there is one plastic deformed zone under the edge which can be found in experiments. In order to describe this zone, this chapter proposed to use one effective friction angle \( \eta \), as shown in Figure 4.4.

In the Figure 4.4, \( \eta \) means the equivalent friction angle, which is

\[ \tan \eta = F_2 / N_2 \]  

(4.27)
where, $F_2$ is tangential force on the edge ON, $N_2$ is normal force on ON.

First, assuming the normal stress distribution along ON is uniform, the normal forces can be expresses as,

$$N_2 = \sigma_{N2} \lambda_{\text{chamfer}} \sin(\alpha - \beta)/\cos(\alpha - \gamma)$$  \hspace{5cm} (4.28)

Figure 4.4: Geometry illustration of the zone under the DMZ

Meanwhile, between ON and the primary shear line, the angle is $\pi/2 + \phi - \gamma$. Using the slip line field relationship following the same process in calculating the normal stress on rake face, the normal stress on edge ON is

$$\sigma'_{N2} = k_{MN} \left[ 1 + 2\left(\frac{\pi}{4} - \phi\right) - C_{MN} \frac{\sqrt{3}nB_{e_{MN}} \cos\alpha}{6\cos(\phi - \alpha)\sin(\phi + B_{e_{MN}})} + 2\left(\frac{\pi}{2} + \phi - \gamma\right) \right]$$  \hspace{5cm} (4.29)

These two methods should give the same value normal stress. It means $\sigma'_{N2}$ from slip line model should be equal to the normal stress $\sigma_{N2}$.
It is often assumed that a very thin layer of workpiece exists sticking to the edge ON. Within that layer, the shear stress is equal to shear flow stress of workpiece, so the tangential force $F_2$ acting on ON can be expressed as:

$$F_2 = wk_{work}^2 \csc \alpha (\alpha - \beta) \cos \gamma$$

(4.30)

Therefore, the cutting force coming from the bottom edge of DMZ is

$$F_{c2} = F_2 \cos \gamma + N_2 \sin \gamma$$

(4.31)

The corresponding thrust force is

$$F_{t2} = N_2 \cos \gamma - F_2 \sin \gamma$$

(4.32)

In order to calculate the temperature rise of the material under the DMZ, it is proposed that:

$$\Delta T_{\text{work}} = \xi u_{\text{total}} / (\rho S)$$

(4.33)

where, $\xi$ is usually considered as 0.9 as the ratio of plastic deformation energy transferring to heat. And it is also assumed that the heat is totally absorbed by the workpiece under this DMZ.

Same with the interface between tool and chip, considering the isothermal condition under the DMZ, the highest temperature increasing $\Delta T_{\text{wMax}}$ is calculated by (Zhang, et al., 1991)

$$\log \left( \frac{\Delta T_{\text{wMax}}}{\Delta T_{\text{work}}} \right) = 0.06 - 0.195 \delta \left( \frac{R_{wT}}{\sin \eta} \right)^{0.5} + 0.5 \log \left( \frac{R_{wT}}{\sin \eta} \right)$$

(4.34)

where $R_{wT} = V_{\text{under}} w_{\rho S} / K$, $V_{\text{under}} = V \sin (\gamma + \eta) / \sin \eta$. 

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Then the temperature along the bottom edge of DMZ is known as (Zhang, et al., 1991)

\[ T_{work} = T_0 + \Delta T_{MN} + 0.9 \Delta T_{wMax} \quad (4.35) \]

In order to calculate the temperature rise \( \Delta T_{work} \) under DMZ, it is necessary to find out how to calculate the energy consumed in the zone under DMZ. The method in (Zhang, et al., 1991) which is related to three phenomena: excess, extrusion, friction is applied here to express the total energy consumption \( u_{total} = u_{r1} + u_{r2} + u_{r3} \).

(1) Extrusion deformation energy (unit time and unit volume)

The strain in the extrusion part is calculated as:

\[ \varepsilon_{r1} = \sin \gamma / \sin \eta \quad (4.36) \]

In estimating the strain rate, it is proposed that the workpiece material deforms uniformly during the extrusion deformation (Zhang, et al., 1991), so the strain rate is estimated as

\[ \dot{\varepsilon}_{r1} = \frac{\varepsilon_{r1}}{\Delta \text{time}} \quad (4.37) \]

Where, the time for the change of strain is the time material flows through the edge:

\[ \Delta \text{time} = \frac{\lambda_{chamfer} \sin(\alpha - \beta) \sin \eta}{V \cos(\alpha - \gamma) \sin(\gamma + \eta)} \quad (4.38) \]

Therefore, by considering the constant strain rate and the uniform temperature distribution during deformation, the specific energy for the extrusion deformation is calculated as:
\[ u_{p1} = \int_0^{\varepsilon_p} \sigma d\varepsilon = \left( A\varepsilon_{p1} + B \frac{\dot{\varepsilon}_{p1}^{n+1}}{n+1} \right) \left( 1 + C \log \dot{\varepsilon}_{p1} \right) \left( D - E \left( \frac{T_{\text{work}} - T_0}{T_{\text{mech}} - T_0} \right)^m \right) \]  \hspace{1cm} (4.39)

(2) Excess deformation energy (unit time and unit volume)

Excess deformation in cutting is contributed by the interior shear deformation (Zhang, et al., 1991), and consumes energy in overcoming this interior shear. The strain of excess part is estimated as (Zhang, et al., 1991)

\[ \varepsilon_{p2} = \tan \frac{\gamma}{\sqrt{3}} \]  \hspace{1cm} (4.40)

Similar as in computing the extrusion strain rate, the strain rate of excess is approximated as:

\[ \dot{\varepsilon}_{p2} = \frac{\varepsilon_{p2}}{\Delta \text{time}} \]  \hspace{1cm} (4.41)

And the specific energy of the entrance is calculated as follows:

\[ u_{p2,\text{entrance}} = \frac{1}{t_{\varepsilon_2}} \int_0^{\varepsilon_{p2}} \sigma d\varepsilon \int_0^{t_{\varepsilon_2}} dx = \left( A\varepsilon_{p2} \frac{B \varepsilon_{p2}^{n+1}}{2} + \frac{B \varepsilon_{p2}^{n+1}}{(n+1)(n+2)} \right) \left( 1 + C \log \dot{\varepsilon}_{p2} \right) \left( D - E \left( \frac{T_{\text{work}} - T_0}{T_{\text{mech}} - T_0} \right)^m \right) \]  \hspace{1cm} (4.42)

Further, due to the specific energy of the exit is equal to the specific energy of enterance, the total specific energy of excess is

\[ u_{p2} = 2u_{p2,\text{entrance}} \]  \hspace{1cm} (4.43)

(3) Friction energy (unit time and unit volume)

A thin layer along ON is considered between the DMZ and the workpiece. If the ratio of this layer thickness to the thickness of the whole deformation area is set as \( \delta_z \),
usually it is assumed as 0.10 (Zhang, et al., 1991). And it is also assumed that along edge ON, there is the sticking region and the shear stress is equal to shear flow stress $k_{\text{work}}$.

So, the strain within that thin layer is

$$\varepsilon_{p3} = \delta_2 \sin \eta / \sqrt{3}$$  \hspace{1cm} (4.44)

the strain rate within the friction zone can be represented as

$$\dot{\varepsilon}_{p3} = \varepsilon_{p3} / \text{Delta time}$$  \hspace{1cm} (4.45)

and the shear flow stress is

$$k_{\text{work}} = \frac{1}{\sqrt{3}} \left( A + B \varepsilon_{p3} \right) \left( 1 + C \log(\dot{\varepsilon}_{p3}) \right) \left[ D - E \left( \frac{T_{\text{work}} - T_0}{T_{\text{melt}} - T_0} \right)^m \right]$$  \hspace{1cm} (4.46)

The specific energy of the friction can be expressed as

$$u_{p3} = k_{\text{work}} \sin(\gamma + \eta) / \sin^2 \eta$$  \hspace{1cm} (4.47)

Computation Procedure of the Proposed Model

Before simulation, the program should be given the working conditions and cutting conditions. And $\delta_1$ is 0.05 (Oxley, 1989), and $\delta_2$ equal to 0.1 (Zhang, et al., 1991), and give the angle $\gamma$ one initial value.

Then there are four iterations for $\gamma$, $\eta$, $\phi$, and $C_{MN}$ as shown in Figure 4.5.

The most internal iteration is about shear angle $\phi$ from zero to 45 deg. After get the energy balance on temperature of shear zone and interface of tool-chip, the
relationship on the interface of chip tool $r_{in} = k_{chip}$ is used to find out the value of shear angle $\phi$.

Next, using the force balance between the primary shear zone and interface of tool-chip, the normal stresses on rake face should meet $\sigma_{N1} = \sigma'_{N1}$, which is used to determines the value of $C_{MN}$.

The third iteration is for the friction angle $\eta$. Similar to contact conditions on the rake face, relationship $\sigma_{N2} = \sigma'_{N2}$ exists along the bottom edge of DMZ. Then using the definition $\eta = \tan^{-1}(F_2 / N_2)$ to calculate the friction angle $\eta$.

In the forth iteration, the minimum energy rule is used to judge the value of $\gamma$ within the range of 0 to 45 deg. During the steady state machining, the tool has considered no displacement in the vertical direction, the minimum energy is effectively equal to the minimum cutting force $F_{cTotal} = F_{c1} + F_{c2}$.

When the final value of $\gamma$ is found, the simulation also ends. With all final values of $\gamma$, $\eta$, $\phi$, and $C_{MN}$, the total cutting forces and thrust forces can be calculated based on Equations (4.4, 4.5, 4.30, and 4.31), using $F_{cTotal} = F_{c1} + F_{c2}$, and $F_{nTotal} = F_{n1} + F_{n2}$. 
Given cutting conditions and materials properties. And set $\delta_1 = 0.05$, $\delta_2 = 0.10$

- Iteration $\gamma$
- Iteration $\eta$
- Iteration $C_{MN}$
- Iteration $\phi$
- Calculate the primary shear zone: New $T_{MN} = \text{Old } T_{MN}$?
- Calculate the interface of tool and chip: New $T_{chip} = \text{Old } T_{chip}$?
- Plot $\tau_{\text{int}} \sim k_{\text{chip}}$, to get $\phi$
- Judge $\sigma_{N1}' = \sigma_{N1}$, to get $C_{MN}$
- Calculate the interface of tool and work piece: New $T_{\text{work}} = \text{Old } T_{\text{work}}$?
- From the definition: $\tan^{-1}(F_3/N_2)$, then get New $\eta$
- Plot $F_{\text{total}} \sim \gamma$, to get $\gamma$
- $F_{\text{total}}$ meets min $F_{\text{total}}$
- Plot all results

Figure 4.5: Flowchart of the proposed model
Model Validation and Discussion

Experimental results from (Ren, et al., 2000) are used to verify the proposed force modeling approach. The process conditions for this orthogonal machining were: workpiece material was P20 mold steel (HRc 34) with other physical properties provided in (Ren, et al., 2000). Room temperature $T_0$ is 20°C. Instead of using a power law as the workpiece flow stress model, a Johnson-Cook equation-based model (Shatla, et al., 2001) is used for P20 steel (HRc 30) in this study as follows:

$$
\sigma = \left( A + B \varepsilon^m \right) \left[ 1 + C \log \left( \frac{\dot{e}}{\dot{e}_0} \right) \right] \left[ D + E \left( \frac{T - T_0}{T_{melt} - T_0} \right) \right]^n,
$$

where $A=145$MPa, $B=565.6$MPa, $C=0.03$, $D=1.26$, $E=1.07$, $n=0.154$, $m=1.8$, and $T_{melt}=1480^\circ C$. The flow stress difference between these two P20 mold steels is considered negligible. The other material properties of the P20 mold steel include density $\rho = 7850kg/m^3$, thermal conductivity $K=51.5W/m^\circ C$, and specific heat $S = 470J/Kg^\circ C$.

Effect of Cutting Speed

In order to observe the effect of cutting speed, the conditions are set as undeformed chip thickness is 0.6mm, the cutting width = 2.55mm. And the tool geometry are: rake angle is -5 degree, the chamfer angle is -25 degree, the length of the chamfer edge is 0.1mm. The experimental data with different the cutting speed are at $V = 240m/min$, $600m/min$ and $1000m/min$, respectively.
Figure 4.6: The total cutting force per width

Figure 4.7: (a) Cutting force of the primary shear zone, (b) cutting force under the DMZ

As shown in Figure 4.6, with the speed increasing, there is almost no influence on the cutting force. Instead of traditional model such as (Oxley, 1989) that the cutting force should decrease with speed monotonously which is matched by the force from primary shear zone as in Figure 4.7(a), the total cutting force does not behavior like that. It is
thought for the cutting force contributed by the DMZ increasing with the speed increases as shown in Figure 4.7(b). Opposite to the trends shown by the model developed by Ren et al. (Ren, et al., 2000), the proposed model shows the cutting force slightly decreases with speed increasing.

![Figure 4.8: The total thrust force per width](image)

![Figure 4.9: (a) thrust force of primary shear zone, (b) thrust force under the DMZ](image)
As to the thrust force, it decreases with speed increasing as shown in Figure 4.8. It is because the variance of the thrust force from DMZ is very small. Looking at Figure 4.9, the combined thrust force decreases with speed increases that contributed mainly by the decreasing of thrust force from the primary shear zone.

Figure 4.10: The shear angle regard to the cutting speed

In Figure 4.10, with the cutting speed increases from 300 to 1000 m/min, the predicted shear angle increases a little from 26.6 to 28 degree. It matches the FEA results in (Movahhedy, et al., 2002) very well. Also this simulation result closes experimental results (Ren, et al., 2000) which are about 28 to 32 degree.

Further, as Figure 4.11 shown, the average temperatures on the tool surface have been estimated. With cutting speed increasing, the average temperature also increases.
Compare these two interface, it is easy to find that the temperature on the bottom edge of DMZ is a little higher than that of chip-tool interface.

Figure 4.11: (a) the temperature along the interface of chip and tool, (b) the temperature of the bottom edge of DMZ

Effect of Chamfer Angle

Experiments performed to investigate the effect of chamfer angle following the conditions: undeformed chip thickness $t_0 = 0.1 mm$, width $w = 3.6 mm$, cutting speed $V = 240 m/ min$. There are three different chamfered tools. Their geometries are listed in Table 4.1 as below:

As shown in Figure 4.12, the model prediction of thrust forces matches the experiment measurement better than the cutting force prediction. The thrust force increases quickly with the amplitude of the chamfered angle decreasing from zero to
negative 35 degree. It may be due to the ploughing effect. At the same time, the cutting force increase very slow due to chamfered angle, so rake angle of chamfered edge does not influence cutting force strongly.

Table 4.1: Tool geometry of three chamfered tools

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha$ (deg)</th>
<th>$\beta$ (deg)</th>
<th>$\lambda_{chamfer}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-10</td>
<td>0.0902</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-25</td>
<td>0.0841</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-35</td>
<td>0.0863</td>
</tr>
</tbody>
</table>

Figure 4.12: Effect of chamfer angle on cutting force (left) and thrust force (right)

Effect of feed rate

In order to observe the influence of feed rate on the forces, the feed rate changes from 0.05 to 0.15mm. Other simulation conditions are set as below: cutting speed is 240
m/min, chamfer angle is -35 deg, rake angle is 0 deg, the length of chamfer edge is 0.09mm, cutting width is 3.66mm. The simulation results are shown in Figure 4.13. Similar to what was observed in (Waldorf, et al., 1998), cutting force increases quickly from 100N to 300N as the feed rate increases. But thrust force increases very slowly with only less than 100 N, which can be understood that with the depth of cut increasing, the ploughing effect for large feed rate is not as strong as that for small feed rate.

![Figure 4.13: Effect of feed rate on cutting force (left) and thrust force (right)](image)

**Conclusions**

This chapter presents a new force model considering dead metal zone effect in orthogonal cutting with chamfered tools. A three-zone approach is applied to model the cutting process under the effect of the DMZ. The performance of the new model has been compared with that of Ren et al. (Ren et al., 2000) in orthogonal cutting P20 mold steel, and overall an improved force modeling accuracy is observed. Although the predicted
cutting force is lower than the measured force, the predicted thrust force matches the measurement quite well. After discussion, it is found that cutting force is mainly determined by the primary and secondary deformation zones and thrust force is mainly determined by the deformation zone under the DMZ. Since the proposed approach underestimates the cutting forces, a better understanding of the DMZ and the associated extrusion process needs further developments.

Another finding in the model is when calculating the strain rate, the constant $C_{MN}$ is not simply set by 5.9. Instead, an iteration (from 1 to 10) is used to find the appropriate value. Based on the calculation, the $C_{MN}$ is found depended on the material properties and working condition, instead of what was believed by Oxley (Oxley, 1989) that it should be a constant of 5.9.

Although this new approach has given good results, there are still some limitations for the proposed model, and it needs to be improved in the future. The most important issues to be solved are why the dead metal zone forms, and on what conditions the DMZ exists. There are still no satisfactory answer to these problems.
CHAPTER FIVE

ACHIEVEMENT AND FUTURE WORK

This dissertation is motivated by the need to model the tool flank/crater wear and DMZ effects on metal cutting performance for the purpose of guiding the design of tool/insert geometry, and the optimization of cutting parameters in cutting process which is important for product quality improvement.

Summary

In metal cutting process, it is common to have the tool crater and flank wear and other phenomena such as DMZ near the tool tip. Further, these phenomena influence the cutting performance. So this dissertation is trying to investigate, under these worn tool conditions, how the cutting process performs. As cutting force is the direct factor to measure cutting performance and is useful to model tool wear rate, so force model is of great interest here.

In the first chapter of this dissertation, the fundamentals of metal cutting research are shortly summarized. Firstly, tool wear and its relationship to cutting forces are shortly introduced as the background of this dissertation. Then it turns to the topic of metal plasticity which is the basis of deformation modeling of metal in cutting process. Under plane strain condition, the slip-line field method is introduced because it is capable to model cutting forces. Compared to the FEM method, analytical method has its own
advantages such as easy to implement, ready to extend for the three-dimensional machining modeling study, and cheap in computation resources. Appendix B is the overall summary of slip-line field theory. After the background section, there is a review of the current state of research in the areas of cutting force modeling. The review is firstly covers the two dimensional force modeling including analytical, numerical and mechanistic/empirical models. Secondly, the review introduces the three dimensional force modeling. Most current 3D force models are mechanistic models either obtained from experiments, or extended from 2D force model. Later, the DMZ phenomenon which happens in cutting with chamfered tools is investigated. Finally current force modeling approaches of the DMZ effect in cutting has been reviewed.

Chapter 2 proposes a new analytical force modeling approach for orthogonal cutting under worn tool conditions considering both crater and flank wear effects. The basic assumption of plane strain deformation for a perfect rigid plasticity is first made. Then a new slip line field model is proposed for the 2D cutting condition with both crater and flank wear. The proposed model has been verified based on a published high speed cutting experimental data and it gives satisfactory accuracy. The developed approach is ready to use for force modeling in oblique cutting modeling which is the topic in the next chapter. Considering the uncertainties in the real cutting process, Chapter 2 introduces a model validation method. A non-informative Bayesian linear regression method is implemented in this chapter for cutting force model validation. The simulation result of analytical force model shows very good confidence level.

A further 3D force model is constructed in Chapter 3. In this chapter, the worn
tool geometry is approximated with some assumptions on the profile of wear area. The flank wear land is believed to be a straight edge area with uniform VB, and the crater wear land is treated that for all cross sections, their areas are geometrically similar to each other. So given KT, KM with the width of crater wear, geometry of the whole crater wear land can be approximated. Based on these simplified geometry of worn tool, chip is separated into many segments, and each segment can be equally treated as orthogonal cutting condition, so the 2D force model can be applied to predict the cutting forces for each segment. Bringing the concept of inter-element interaction, further considering the overall force normal to cutting direction should be zero, the 3D force equations are established and solved. The prediction forces under oblique condition match the experimental data very well.

In Chapter 4, cutting under another tool wear pattern, DMZ, has been modeled to predict cutting forces. This chapter begins with the review of DMZ in metal cutting. Based on observation from the experimental flow around the retarded zone, one analytical force model based on slip-line field theory is proposed. Combining thermal effect, strain hardening, and strain-rate effect, equations for each variable have been derived analytically. Then the proposed model is verified by comparing the predicted cutting forces with experimental measurements. Compared to another existed model, the new model shows improved accuracy and gives more reasonable force tendency. So the developed model is suitable for force modeling under DMZ effect.

At last, Chapter 5 summaries the work presented in this dissertation. Three new force models have been proposed and proven successfully for force modeling under worn
tool conditions. The achievements of this study have been highlighted. However, there are still some limitations on current force modeling approaches, recommended future works are presented at the end of this dissertation.

Achievement

In order to model the effect of tool wear such as crater wear, flank wear, and DMZ on cutting performance, the cutting force has been intensively investigated and modeled. The main achievements of this dissertation are listed as below:

1. On 2D force modeling
   - An analytical force model for worn tool is proposed, which is based on slip-line field theory, covering both crater and flank wear. The totally force depends on not only flank wear land, but also the crater wear land. Although the flank wear effect always increases the cutting force, when the effect of crater wear is dominant which causes the cutting force decrease, the totally force may decrease.
   - The effect of DMZ has been investigated. A slip-line field model has been proposed for metal cutting under DMZ conditions. The force prediction is coherent with experimental force data.

2. On force modeling validation using Bayesian linear regression
   - A Bayesian method for model validation has been proposed considering uncertainty in metal cutting process. Model validation has been applied to the
2D force model for metal cutting with both crater and flank wear land on tool face. The new model validation approach provides confidence inference for the prediction result of the 2D force model.

(3) On 3D force modeling

✓ The 3D cutting force model considering both crater and flank wear has been developed and verified with experimental data. The concept of interaction between chip segments works well for the 3D force modeling. Flank wear land is approximated as an area with a uniform length VB along the cutting edge. Crater wear is treated by assuming all cross section along chip flow direction has similar profile, and it can be fully represented by KT, KB, and KM with geometry similarity assumption.

Future Work

After years study in manufacturing processes, focusing on the effect of tool wear, some recommendations are proposed for the future work, which emphasize on modeling and analysis of machining process.

(1) Tool wear geometry and friction

In this dissertation, only approximated smooth profile of crater and flank wear land which is simply defined by KT, KM, KB, and VB, has been considered. But the actual face of wear land is rough due to various mechanisms, such as adhesion, abrasion and diffusion. It is necessary to develop physical models for worn tool, which can
describe the wear initiation, wear evolution before tools fail. And the contact condition of tool and chip is very complicated so that one simplified friction law and stress distribution cannot describe the conditions. Hence more work to investigate the tool wear geometry and friction conditions should be performed in the future, which is critic for force modeling considering to tool wear.

(2) Numerical method for metal cutting simulation

Metal cutting is an extreme plastic deformation process which is too complex to get good analytical solution. By far, only slip-line field theory or some upper/lower bound estimation methods are available for some simplified cases. In the future, numerical methods like finite element method should be a good tool to model the process. Particularly advanced tool wear modeling based on the finite element method is very important, although currently FEM method does not work well for this topic.

(3) Dynamics modeling

Based on the force model in Chapter 2, an analytical lumped parameter dynamic model has been proposed to study the effect of the crater and flank wear on cutting dynamics as a preliminary investigation. The effect on the positive damping coefficient from flank wear length VB improves the stability of metal cutting. The effect from crater wear land is complicated. Each geometric parameter KT, KB, KM has its own effect on the stability. KT and KB may improve the stability, but KM may change cutting process from stable to chatter crossing the stability limit. Future work should combine the 2D force model with process dynamics to discover possible cutting dynamics change due to tool wear.
The research in process dynamics faces big challenges. The traditional lumped parameter equations have been developed by some researchers, but the process dynamics is seldom explored well. Using system response function for the cutting is another traditional method which is based on parameter extraction of the lumped matrix. These transfer functions are approximations with only low modes. So theoretically, it is very difficult to work further on process dynamics using the both traditional approaches.

In the future, metal cutting will be performed in miniature scale, so that a small magnitude oscillation of tool tip during microscale cutting will make the product out of use. So modeling and isolation of cutting vibration will be a huge challenge for cutting process.

(5) Uncertainty in metal cutting

Bayesian approach with calibration and prediction based on reasonable prior distribution will be a powerful tool for future research on uncertainty modeling and model validation. One challenge for the Bayesian approach is how to introduce the multivariate statistics into the metal cutting process, and implement it. Sometimes, it is hard to find a good distribution to fit the uncertainty.

Besides Bayesian approach, stochastic FEM may be another good method to model process uncertainty. This type of finite element theory treats the field variable and/or boundary conditions with distributions which are useful for the metal cutting process modeling considering uncertainties and model validation.

(6) Tool design and process optimization based on tool wear knowledge
Tool wear always exists in metal cutting process which happens around the contact area between tool and workpiece due to high temperature and high stress. The tool geometry has been designed based on some optimization criteria for the best cutting performance under fresh conditions. However, the original tool geometry is changed by the tool crater and flank wear and/or DMZ in cutting. Therefore, in order to maintain good cutting performance, the deeply understanding and modeling for metal cutting considering tool wear in this dissertation are helpful to design optimum tool geometry for specified application. It is expected that the models developed for worn tool conditions can be applied in industry and guide metal cutting in the future.
APPENDICES
Appendix A

Slip Line Field Theory

Metal cutting is believed as plastic deformation process. Currently, there is a vast catalog of solutions to linear elastic boundary and initial value problems. In contrast, there are very few exact solutions to boundary value problems involving plastically deforming solids. Such solutions as exist are usually either for highly simplified geometries (e.g. spherical or axial symmetry) or simplified material models (such as rigid plastic solids).

The largest class of solutions to boundary value problems in plasticity exploits a technique known as *slip line field theory*. The method is not widely used these days because it’s a lot harder to use than an FEM package. But it does provide analytical solutions to a number of very difficult problems (e.g. many metal forming, cutting processes). Many of these solutions would be extremely difficult to obtain numerically, because they involve huge deformations, and contain velocity discontinuities.

Derivation of Slip-Line Field Theory

Assumptions and Governing Equations

Let $\sigma_x, \sigma_y$ and $\sigma_z$ be the normal stress in the $x$, $y$, $z$ directions. Let $\tau_{xy} = -\tau_{yx}$, $\tau_{yz} = -\tau_{zy}$, and $\tau_{zx} = -\tau_{xz}$ be the shear stress, for instance, $\tau_{xy}$ is the shear stress acting in the $x$ direction on planes normal to $y$ axis. Normal stresses are taken
positive when tensile. And shear stresses are taken positive when they exert a clockwise couple on the element on which they act. \( \nu_x, \nu_y \) and \( \nu_z \) are the velocities in the \( x, y, z \) directions.

Slip-line theory makes three restrictive assumptions: (a) Plane strain deformation, A state of plane strain is assumed to exist when the flow is parallel to a given plane. Let this be the \( xy \) plane with no flow in the \( z \) direction. Then \( \nu_z = 0 \) and \( \nu_x, \nu_y \) are independent of \( z \). So \( \dot{\nu}_z = \frac{\partial \nu_z}{\partial z} = 0 \), \( \dot{\nu}_{yz} = \frac{\partial \nu_z}{\partial y} + \frac{\partial \nu_y}{\partial z} = 0 \), and \( \dot{\nu}_{zx} = \frac{\partial \nu_z}{\partial x} + \frac{\partial \nu_x}{\partial z} = 0 \), (b) Quasi-static loading which means the cutting process is in the steady state, (c) The material is idealized as a rigid-perfectly plastic Mises solid.

The workpiece is assumed to be a rigid-plastic material which deforms in according to Levy-Mises relations. It means

\[
\frac{\dot{\epsilon}_x}{\sigma_x - \sigma_m} = \frac{\dot{\epsilon}_y}{\sigma_y - \sigma_m} = \frac{\dot{\epsilon}_z}{\sigma_z - \sigma_m} = \frac{\dot{\epsilon}_{xy}}{\tau_{xy}} = \frac{\dot{\epsilon}_{yz}}{\tau_{yz}} = \frac{\dot{\epsilon}_{zx}}{\tau_{zx}}
\]

Where, \( \sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \) is the mean or hydrostatic stress. Following the plane-strain relationships, \( \tau_{yz} = \tau_{zx} = 0 \), therefore, the \( z \) direction is a principal direction with

\[
\sigma_z = \frac{1}{2}(\sigma_x + \sigma_y) \quad \text{(A.1)}
\]

Following the above equation that both the maximum shear stress and shear strain energy yield criteria can be written as

\[
\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2 = k^2 \quad \text{(A.2)}
\]
Where, $k = Y / \sqrt{3}$ is the shear yield stress (shear flow stress) of the material.

Also, for material is assumed to be an isotropic material which is implicit that the principal directions of stress and strain-rate or the directions of maximum stress and maximum shear strain-rate coincide, so from the Mohr stress and strain-rate circles for plane strain conditions, get

$$\frac{\dot{\varepsilon}_x - \dot{\varepsilon}_y}{\sigma_x - \sigma_y} = \frac{\dot{\varepsilon}_{xy}}{\tau_{xy}}$$

(A.3)

Or, explicitly,

$$\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = \frac{\frac{\partial \nu_x}{\partial x} - \frac{\partial \nu_y}{\partial y}}{\frac{\partial \nu_x}{\partial y} + \frac{\partial \nu_y}{\partial x}}$$

Neglecting body forces, the equilibrium conditions are expressed as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

(A.4)

At last, obviously, for the assumed rigid-plastic material elastic strains are taken as zero and during deformation the volume of an element does not change. So this constant volume condition can be expressed as

$$\dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = 0$$

Further, for plane strain ($\dot{\varepsilon}_z = 0$), it is reduced to

$$\dot{\varepsilon}_x + \dot{\varepsilon}_y = 0$$

(A.5)
Which is equivalent to \( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \)

Solution of Governing Equations by Method of Characteristics

Focusing first on a general solution to the governing equations, it is convenient to start by eliminating some of the stress components using the yield condition. Since the material is at yield, at each point in the solid we could find a basis in which the stress state consists of a shear stress of magnitude \( k \) (the shear yield stress), together with an unknown component of hydrostatic stress \( p \). The stress state is sketched as Figure A.1, below.

Instead of solving for the stress components, recall that the Mohr’s circle of stress that relate stress components \( \sigma_{ij} \) to \( p \), \( \phi \) and \( k \).

From the Mohr’s circle shown in Figure A.2, there are:

\[
\sigma_x = -p - k \sin 2\phi
\]
\[ \sigma_y = -p + k \sin 2\phi \]  
\[ \tau_{xy} = k \cos 2\phi \]  
(A.6)

Where, \( \phi \) is the angular rotation of the \( \alpha \) slinline from the \( x \) axis measured positive as shown in Figure A.3.

![Mohr's circle of stress](image)

Figure A.2: Mohr’s circle of stress

Now, the governing equations including (A.1 to A.5) can be rewritten with \( p \), \( \phi \) and \( k \). Considering the yield criterion (A.2) is satisfied automatically, the other governing equations (A.1, A.3, A.4, A.5) can be written in a uniformed form,

\[ A_y \frac{\partial q_j}{\partial x} + B_y \frac{\partial q_j}{\partial y} = 0 \]  
(A.7)

Where, \( q = \begin{bmatrix} \phi \\ v_x \\ v_y \\ -p \end{bmatrix} \), \( A = \begin{bmatrix} 0 & -2k\cos\phi & -2k\sin\phi & 0 \\ -2k\cos\phi & 0 & 0 & 1 \\ -2k\sin\phi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \), and \( B = \begin{bmatrix} 0 & -2k\sin\phi & 2k\cos\phi & 0 \\ -2k\sin\phi & 0 & 0 & 0 \\ 2k\cos\phi & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \).

This equation (A.7) is a quasi-linear hyperbolic system of PDEs, which may be solved by the method of characteristics.

It can be found that there are two same eigenvalues.
\[ \lambda_{1,2} = \cot \phi \]

with corresponding eigenvectors \( \rho_1 = [1 \ 0 \ 0 \ -2k] \), and \( \rho_2 = [0 \ 1 \ \tan \phi \ 0] \)

Or

\[ \lambda_{1,2} = -\tan \phi \]

with eigenvectors \( \rho_1 = [1 \ 0 \ 0 \ 2k] \), \( \rho_2 = [0 \ 1 \ -\cot \phi \ 0] \)

The solution of equation (A.7) indicates: (a) There are two sets of characteristic lines (one for each eigenvalue); (b) The two sets of characteristics are orthogonal (they therefore define a set of orthogonal curvilinear coordinates in the solid); (c) The characteristic lines are trajectories of maximum shear (to see this, recall the definition of \( \phi \)). For this reason, the characteristics are termed slip lines – the material slips (deforms in shear) along these lines.

**Slip-Lines**

The sliplines consist of two orthogonal families of curves whose directions at every point in the plastic region coincide with the directions of maximum shear stress and maximum shear strain rate. The \( \alpha \) slipline are those on which the shear stress is positive.

Consider a small curvilinear element bounded by two pairs of neighboring sliplines as shown in Figure A.3. The state of stress is represented by the Mohr circle in Figure A.2. The shear stress on the slip lines is \( k \), and the normal stress on the sliplines is the mean compressive (hydrostatic) stress \( p \).
Hencky Equations

There is one common relation along slip-lines: Hencky equations, which express the conditions relating $p$ and $\phi$ along slip lines.

From the Mohr stress circle, the transforming equations from Cartesian coordinate system to the curvilinear slip line system are given by (A.6). Remembering that $\phi = 0$ along the $\alpha$ and $\beta$ slip lines, and substituting equation (A.6) into the equilibrium equations (A.4), and differentiating and collecting terms, gives

\[
\frac{\partial p}{\partial s_1} + 2k \frac{\partial \phi}{\partial s_1} - \frac{\partial k}{\partial s_2} = 0 \text{ along } \alpha \text{ slip line}
\]

\[
\frac{\partial p}{\partial s_2} - 2k \frac{\partial \phi}{\partial s_2} - \frac{\partial k}{\partial s_1} = 0 \text{ along } \beta \text{ slip line}
\]  

(A.8)

Here, $s_1$ and $s_2$ are distances measured along the $\alpha$ and $\beta$ slip lines, respectively.
Further, the material is assumed to be perfectly plastic (non-hardening) with the shear flow stress $k$ constant. So integrating the equation (A.8), it gives the Hencky equations, i.e.

\[ p + 2k\phi = \text{const} \quad \text{along } \alpha \text{ slip line}, \]
\[ p - 2k\phi = \text{const} \quad \text{along } \beta \text{ slip line}. \]  
(A.9)

The Hencky equations are the equilibrium equations acting on slip lines.

**Construction of Slip-Line Field and Hodograph**

**Stress Boundary Conditions**

Two stress conditions which are frequently used in constructing slipline fields are stress free surfaces, and tool surface along which the frictional shear stress opposing motion of the workpiece is known. If the frictional shear stress $\tau$ then it is easily shown that the slip lines must meet the tool-workpiece interface at an angle:

\[ \theta = \frac{1}{2} \cos^{-1}\left(\frac{\tau}{k}\right) \]  
(A.10)

From this it can be seen that if $\tau = 0$, the slip lines meet the interface at an angle of $\frac{\pi}{4}$. If $\tau = k$, the slip lines are normal and tangential to the interface.

**Hodograph**

To check a slipline field for velocity, it is common to construct a velocity diagram, named a hodograph. A hodograph is constructed on the basis that in order to
make the rate of extension along sliplines zero (volume constancy condition), adjacent points on a slipline must have a relative velocity which cuts the line joining them at right angles. Following the volume constancy condition, the normal component of velocity should be continuous across the slipline, shown in Figure A.4 as rule 1.

![Figure A.4: Hodograph construction rule 1](image)

It is general that if small enough steps are considered, then corresponding element of slipline field and hodograph should be orthogonal. Therefore, there are another two construction rules, as shown in Figure A.5 and A.6 as rule 2 and rule 3:

![Figure A.5: Hodograph construction rule 2](image)
Figure A.6: Hodograph construction rule 3
Appendix B

Guide for Using AdvantEdge to Simulate Turning Process

As introduced in Chapter 1, FEA method is a choice for numerical simulate metal cutting processes. AdvantEdge developed by ThirdWaves system is one of the popular FEA software for metal cutting simulation. The theoretical detail of AdvantEdge can be found in (Marusich, et al., 1995). The example shown below instructs through the steps to run a 2D turning process simulation using AdvantEdge. Although the simulation results are not shown here, it is necessary to mention that the predicted cutting forces by AdvantEdge are not very good compared to experimental measurements. The possible reasons are material model is not accurate, the deformation around primary shear zone and material fracture before tool tip is not good modeled, also the contact between tool and workpiece along tool-chip interface and tool flank face-machined workpiece is hard to simulate.

Step 1: Project Setup

1. Open the AdvantEdge by selecting “Start→ Programs→ ThirdWaveSystems AdvantEdge→ AdvantEdge 5.7”
2. Click “Project→ New”
3. Enter “2D Turning” for the project name, also comments for the project
4. Select “Process Type” as “Turning”
5. Select “2D Simulation”
Step 2: Selecting Units:

Using default “SI” for the project options

Step 3: Tool Setup: Create a custom tool or Edit standard tool

For standard tool, there are three tool parameters:

1. Cutting edge radius
2. Rake angle
3. Relief angle

For custom tool, the tool geometry can be drawn or imported by user.

Step 4: Tool Setup: Mechanical Boundary Conditions, Thermal Boundary Conditions

Normally, the right and top most side are fixed in the X and Y direction in “Mechanical boundary conditions”, and constant temperature in “Thermal boundary conditions”.

Step 5: Tool Setup: Material

1. Select “Tool → Material”
2. Select the “Carbide-Grade-K” material and click “OK”

For simple case, there is not coating and wear. Also the cutting is steady state, so do not select “Dynamic Tool”. For more information, there is option to define custom wear model.

Step 6: Workpiece Setup: Create/Edit Workpiece

1. Turning Workpiece: workpiece height, and workpiece length
2. Initial stress: user can prescribe an initial stress in the workpiece.

Step 7: Workpiece Setup: Workpiece Material
To define a standard workpiece material, select “Material” from the “Workpiece” menu. There is a drop down menu of countries and a list of materials.

Step 8: Custom Workpiece:

Go to “Workpiece → Create Custom Workpiece” to bring up the custom workpiece interface

Step 9: Workpiece Setup: Mechanical Boundary Conditions and Thermal Boundary Conditions

1. The workpiece needs to be fixed in both the X and Y direction in some respect,
2. Standard workpiece have the bottom boundary fixed in the X and Y.
3. Avoid prescribing mechanical boundaries where contact will occur.
4. Feed is measured from the top most point on the workpiece
5. Avoid contact with sharp corners.
6. Temperature can be prescribed as well on any length.

Step 10: Process Setup: Process parameters

1. Select “Process → Process Parameters…”
2. Enter the following process parameters: “Feed, Depth of Cut, Length of Cut, Cutting Speed, Initial Temperature”.

Step 11: Friction and Coolant

Each workpiece has its own default friction coefficient; however, user can define the value of 0 to 2.

In this example, it is a dry turning process, so no necessary to define coolant.

Step 12: Simulation Options
Select “Simulation → Simulation Options” will open a window with three tabs: General, Meshing, and Results. In the General tab, choose “Standard mode” for simulation mode, and “Steady state analysis”. Other options in the windows need more understanding of metal cutting. In the Meshing tab, to use all default value if not familiar with FEA knowledge of meshing.

Step 13: Saving and Submitting Projects

1. Select “Project → Save as → Enter ‘TwoD Turning Example’ → Save”
2. Select “Simulation → Submit → Submit Current Job” then click “OK”
3. A job can be stopped/restarted by selecting “Stop” or “Submit Current Job” from the job monitor.

Step 14: Analyzing Results with Tecplot

Tecplot is invoked by choosing the “Results” option in the “Simulation” menu of AdvantEdge. AdvantEdge writes two types of results files during a simulation: a contour file and a force file. The contour file is written as job.twb where job is the name of the project. It contains results data used for creating field plots such as contour, mesh, vector, and animation plots. Force file is written as job_fft.tec, which contains the time history data of the tool cutting and transverse forces. More usage of Tecplot can refer to manual of Tecplot.


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