12-2023

Analysis of Length Shortening Effect on Bistability and Curvature in a Rectangular Bistable Composite Laminate with a Rigid Tab on One End Integrated to Its Geometry

Rizwana Akter
rakter@clemson.edu

Follow this and additional works at: https://tigerprints.clemson.edu/all_theses

Recommended Citation
Akter, Rizwana, "Analysis of Length Shortening Effect on Bistability and Curvature in a Rectangular Bistable Composite Laminate with a Rigid Tab on One End Integrated to Its Geometry" (2023). All Theses. 4207.
https://tigerprints.clemson.edu/all_theses/4207

This Thesis is brought to you for free and open access by the Theses at TigerPrints. It has been accepted for inclusion in All Theses by an authorized administrator of TigerPrints. For more information, please contact kokeefe@clemson.edu.
ANALYSIS OF LENGTH SHORTENING EFFECTS ON BISTABILITY AND CURVATURE IN A RECTANGULAR BISTABLE COMPOSITE LAMINATE WITH A RIGID TAB ON ONE END INTEGRATED TO ITS GEOMETRY

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Rizwana Akter
December 2023

Accepted by:
Dr. Oliver J Myers, Committee Chair
Dr. Gang Li
Dr. Zhaoxu Meng
ABSTRACT

Bistable composite laminates gained significant attention due to their distinct feature of maintaining multiple stable equilibrium states. The phenomenon stems from thermal stress during the curing process of systematically laid-up Carbon Fiber Prepreg (CFRP), resulting in snap-through behavior with dual cylindrical shapes characterized by opposite signs and perpendicular axes of curvature. In addition to exploring the fundamental characterization of bistable behavior and equilibrium shapes, investigating the impact of boundary conditions on such laminates poses great potential. In this research, the shape-changing behavior of a cross-ply bistable laminate with varying lengths under the boundary condition of a fixed tab on one edge inherent to its structure has been numerically analyzed. Bistable laminates with such restraint incorporated in their geometry lose the ability to acquire two cylindrical stable shapes, and after a critical length is reached, it also loses bistability. Critical length is defined as the threshold point in two opposite side lengths of a rectangle with the other two sides constant, below which the structure is not bistable anymore. The change in curvature due to the imposed dimensional alterations has been observed. FEA analysis was done on ABAQUSTM Multiphysics software, and results were validated by experimentation on specimens fabricated with carbon-fiber-epoxy unidirectional prepgs with desired geometric structure.
DEDICATION

Abbu, Ammu, Sabbir, Ayaan, Alisha, Onon.
ACKNOWLEDGMENTS

I would like to take this opportunity to extend my gratitude to all the individuals who played a direct or indirect role in supporting me throughout my journey towards this accomplishment. Firstly, I want to express my deepest appreciation to my advisor, Dr. Oliver J Myers, for his continuous guidance and support. Additionally, I am obliged to Dr. Gang Li and Dr. Zhaoxu Meng for their valuable insights and suggestions that improved this thesis. I am very grateful to my fellow colleague and friend, Dr. Shoab Ahmed Chowdhury for consistently providing me with technical advice. I want to thank my colleagues and friends at Clemson University who continuously motivated me and provided invaluable mental support whenever I encountered obstacles. A special note of gratitude goes to Irina Kharitonova for her assistance with every administrative task in the Department of Mechanical Engineering at Clemson University. Finally, my husband, Dr. Sabbir Salek, for his incredible support in organizing this work. This journey towards obtaining my MS degree was challenging, but it would not have been possible without the support of these remarkable individuals.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE PAGE</td>
<td>i</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I.  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction to Composite Materials</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Bistable Materials</td>
<td>16</td>
</tr>
<tr>
<td>1.3 Research Objectives</td>
<td>18</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>19</td>
</tr>
<tr>
<td>2.1 Review of Bistable Composite Laminate Characterizations</td>
<td>19</td>
</tr>
<tr>
<td>2.2 Review of Boundary Conditions of Bistable Composite Laminate</td>
<td>24</td>
</tr>
<tr>
<td>2.3 Research Gaps Addressed in This Study</td>
<td>28</td>
</tr>
<tr>
<td>III. SIMULATIONS AND EXPERIMENTS</td>
<td>29</td>
</tr>
<tr>
<td>3.1 Model Development</td>
<td>29</td>
</tr>
<tr>
<td>3.2 Finite Element Analysis</td>
<td>34</td>
</tr>
<tr>
<td>3.3 Fabrication of the Specimen</td>
<td>36</td>
</tr>
<tr>
<td>3.4 ADMET Universal Test Setup</td>
<td>43</td>
</tr>
<tr>
<td>IV. ANALYSES AND RESULTS</td>
<td>45</td>
</tr>
<tr>
<td>4.1 Finite Element Analysis and Results</td>
<td>45</td>
</tr>
<tr>
<td>4.2 Experimental Results</td>
<td>51</td>
</tr>
</tbody>
</table>
Table of Contents (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>V. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>57</td>
</tr>
<tr>
<td>5.1 Conclusions</td>
<td>57</td>
</tr>
<tr>
<td>5.2 Recommendations</td>
<td>58</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>60</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Comparison in moduli between DA 409U/G35-150 &amp; AS4-8552 [20,36]</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>Elastic and expansion properties of orthotropic laminae</td>
<td>31</td>
</tr>
<tr>
<td>5.1</td>
<td>Snap through load for 300 mm x 230 mm and 300 mm x 240 mm samples</td>
<td>58</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A CFRP is a composite made of a matrix and fibers</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Expanded view of a laminate that is formed by stacking 5 laminae with different fiber orientation</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Uncured unidirectional carbon fiber-epoxy prepreg roll</td>
<td>4</td>
</tr>
<tr>
<td>1.4</td>
<td>A 3D stress element in a general state of stress</td>
<td>8</td>
</tr>
<tr>
<td>1.5</td>
<td>Rotation of the fibers within a lamina</td>
<td>12</td>
</tr>
<tr>
<td>1.6</td>
<td>Angle between principal axis and fiber direction denoted by $\theta$</td>
<td>15</td>
</tr>
<tr>
<td>1.7</td>
<td>Breakdown of the stacking sequence of a [0/45/90/-45/0] laminate</td>
<td>15</td>
</tr>
<tr>
<td>1.8</td>
<td>Ply-orientation breakdown for [90/0], [90/0/45/90/0], and [902/02] respectively</td>
<td>16</td>
</tr>
<tr>
<td>1.9</td>
<td>Laminated shapes: (a) the flat shape, (b) the unstable saddle shape, (c) and (d) two stable cylindrical shapes</td>
<td>17</td>
</tr>
<tr>
<td>2.1</td>
<td>(a) Cylindrical shape and (b) cylindrical shape of opposite sign and perpendicular axis of curvature</td>
<td>20</td>
</tr>
<tr>
<td>2.2</td>
<td>(a) Saddle shape and (b) Cylindrical shape</td>
<td>21</td>
</tr>
<tr>
<td>2.3</td>
<td>Bifurcation diagrams of [02/902] and [04/904] laminates</td>
<td>21</td>
</tr>
<tr>
<td>2.4</td>
<td>Typical boundary conditions</td>
<td>25</td>
</tr>
<tr>
<td>2.5</td>
<td>Experimental setup used in [21] to enforce boundary conditions</td>
<td>26</td>
</tr>
<tr>
<td>2.6</td>
<td>Experimental setup used in [20] to enforce boundary conditions</td>
<td>26</td>
</tr>
<tr>
<td>2.7</td>
<td>Experimental setup used in [19] to enforce boundary conditions</td>
<td>27</td>
</tr>
<tr>
<td>2.8</td>
<td>Experimental setup used in [22] to enforce boundary conditions</td>
<td>27</td>
</tr>
</tbody>
</table>
### List of Figures (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Geometry of the square laminate of [0/90] drawn on ABAQUS™ interface where x is the top ply fiber direction.......................30</td>
</tr>
<tr>
<td>3.2</td>
<td>Set created for left edge which was fixed for the entire simulation representing the restraining tab on one edge.................................33</td>
</tr>
<tr>
<td>3.3</td>
<td>Set created for right edge center node which is also the load application node..........................................................33</td>
</tr>
<tr>
<td>3.4</td>
<td>Boundary conditions for left edge..........................................................35</td>
</tr>
<tr>
<td>3.5</td>
<td>Fixed tab layup [0]...............37</td>
</tr>
<tr>
<td>3.6</td>
<td>Tab laid on bottom lamina before laying top lamina.................................38</td>
</tr>
<tr>
<td>3.7</td>
<td>[0/90] cross-ply sample before curing..................................................38</td>
</tr>
<tr>
<td>3.8</td>
<td>Steps of vacuum bagging process........................................................41</td>
</tr>
<tr>
<td>3.9</td>
<td>Vacuum bag apparatus [24]..........................................................41</td>
</tr>
<tr>
<td>3.10</td>
<td>Vacuum bag setup connected to a vacuum pump through a connector nozzle placed in the oven ........................................42</td>
</tr>
<tr>
<td>3.11</td>
<td>Test fixtures indicated by arrows......................................................44</td>
</tr>
<tr>
<td>3.12</td>
<td>Vacuum bag apparatus [24]..........................................................41</td>
</tr>
<tr>
<td>4.1</td>
<td>Load vs. displacement curve for samples of 300 mm x 300mm and 300 mm x 310 mm......................................................46</td>
</tr>
<tr>
<td>4.2</td>
<td>(a) Post cured shape and (b) post snap shape of 300mm x 300mm..............46</td>
</tr>
<tr>
<td>4.3</td>
<td>Load vs. displacement curve for samples of 300 mm x 226 mm, 300 mm x 230 mm, and 300 mm x 240 mm........................................47</td>
</tr>
<tr>
<td>4.4</td>
<td>(a) Post cured shape and (b) post snap shape of 300 mm x 226 mm...........48</td>
</tr>
<tr>
<td>4.5</td>
<td>Load vs. Displacement curve for samples of 300 mm x 190 mm and 300 mm x 200 mm ......................................................48</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.6</td>
<td>Cured shape of 300 mm x 190 mm</td>
</tr>
<tr>
<td>4.7</td>
<td>Snap through load vs. side length to take [0/90] laminates of various side lengths from first stable shape to second stable shape</td>
</tr>
<tr>
<td>4.8</td>
<td>Bifurcation curve showing the critical length ($L_c$)</td>
</tr>
<tr>
<td>4.9</td>
<td><em>(a)</em> 300 mm x 230 mm laminate of [0/90] shows no snap through going from smaller curvature shape to larger curvature shape <em>(b)</em> 300 mm x 230 mm laminate of [90/0] shows a snap through region going from larger curvature shape to smaller curvature shape but is unstable</td>
</tr>
<tr>
<td>4.10</td>
<td><em>(a)</em> Post cured shape/ first stable shape/smaller curvature stable shape of a 300mm x 230mm specimen of ply orientation [90/0] unloaded <em>(b) snapped but unstable shape after load application</em></td>
</tr>
<tr>
<td>4.11</td>
<td><em>(a)</em> 300 mm x 240 mm laminate of [0/90] shows a snap through phenomena going from larger curvature shape to smaller curvature shape <em>(b)</em> 300 mm x 240 mm laminate of [90/0] snaps through from a stable smaller curvature shape to a second larger curvature shape</td>
</tr>
<tr>
<td>4.12</td>
<td><em>(a)</em> Post cured shape/ first stable shape/larger curvature stable shape of a 300mm x 240mm specimen of ply orientation [0/90] unloaded <em>(b) a snapped but unstable shape after load application</em></td>
</tr>
<tr>
<td>5.1</td>
<td><em>(a)</em> Post cured shape of specimens of 260mm, 270mm and, 280mm, respectively</td>
</tr>
<tr>
<td>5.2</td>
<td><em>(a)</em> Post cured shape of specimens of 290mm, 310mm and, 330mm, respectively</td>
</tr>
</tbody>
</table>
CHAPTER ONE

INTRODUCTION

1.1 Introduction to Composite Materials

A composite is a material structure that consists of at least two macroscopically identifiable materials that work together to achieve a better result [1]. A composite material typically consists of a matrix and a reinforcement, with the reinforcing material embedded within the matrix material. Mixing of two components with distinct physical and/or chemical properties results in a unique combination of properties in the composite material that differs from the individual constituent materials. The goal is to utilize the favorable properties of fibers and matrix to the maximum, while offsetting the unfavorable properties of the individual components as much as possible, achieving a structure that could not have been made with either of the separate components. Composite material can be tailored to improve various properties such as strength, weight, resistance to corrosion, ability to withstand adverse environment etc. according to specific needs.

CFRP Laminates

Composite materials can be classified in several ways based on 1. matrix material such as polymer matrix composites (PMCs), metal matrix composites (MMCs), and ceramic matrix composites (CMCs), 2. reinforcement type – fiber, particle, or structural composites, 3. reinforcement geometry – unidirectional, woven, braided etc., 4. manufacturing process such as hand lay-up, filament winding, pultrusion, or compression
molding etc., 5. application temperature - high-temperature composites or cryogenic composites, 6. Functionality such as such as structural composites, aerospace composites, electronic composites, or biomedical composites, etc. These classification methods help in organizing and understanding the various types of composite materials available, making it easier to select the most suitable material for a specific application or engineering requirement. A Carbon Fiber Reinforced Polymer (CFRP), a composite material made of a thermoset or thermoplastic polymer matrix (typically thermosets such as polyester, vinyl-ester, or epoxy resin), reinforced with unidirectional or woven carbon fibers (In this case, unidirectional long fibers), was used in this research (see Figure 1.1). When multiple laminae are bonded together by stacking multiple layers on top of each other, it’s called a laminate (Figure 1.2).

![Figure 1.1](image)

**FIGURE 1.1** A CFRP is a composite made of a matrix and fibers. [2].
FIGURE 1.2 Expanded view of a laminate that is formed by stacking 5 laminae with different fiber orientation.

Prepreg:

Commercially produced CFRP are typically called prepreg (short for "pre-impregnated"). It consists of reinforcing fibers impregnated with a precise amount of matrix material (usually a thermosetting resin) before the actual manufacturing process (Figure 1.3). The reinforcing fibers can be made of materials such as carbon fibers, glass fibers, aramid fibers, or other high-strength fibers. The pre-impregnation process involves carefully coating the fibers with the matrix material, ensuring uniform distribution and optimal resin content. This is done in a controlled environment to avoid premature curing of the resin. Once the fibers are impregnated, the prepreg is usually rolled or stacked into specific shapes or layups, depending on the intended application.
Prepregs offer several advantages in composite manufacturing. Firstly, they provide consistent and controlled resin content, which results in a more predictable and uniform material. Secondly, the fibers are already coated with the resin, which simplifies the manufacturing process and reduces the chances of resin voids or inconsistencies. Finally, prepregs generally have a longer shelf life compared to dry fibers and resins, as the impregnated resin prevents premature curing.

The aerospace, automotive, sports equipment, and other industries frequently use prepregs for manufacturing high-performance composite structures, as they offer exceptional mechanical properties, lightweight characteristics, and superior structural performance. The prepreg technology is widely employed in the production of components such as aircraft wings, automotive parts, wind turbine blades, and sporting goods like tennis rackets and bicycle frames.
The Hand Layup Method for Creating Composite Part:

The hand lay-up method is a traditional and widely used technique for creating composite parts. It involves manually laying down layers of reinforcement fibers, typically in the form of fabrics or mats, and saturating them with a matrix material, usually a liquid resin. The process is labor-intensive and requires skilled craftsmanship to ensure proper fiber alignment and resin distribution.

Steps involved in the hand layup procedure can be describes as follows:

1. Preparation: The first step is to prepare the mold or tooling on which the composite part will be built. The mold is usually made of a rigid material and is polished and treated with a release agent to prevent the part from sticking to it.

2. Cutting and Layering: Reinforcement materials are cut into the desired shape and size, then carefully laid onto the mold in a specific order to achieve the required strength and properties.

3. Resin Application: Once the layers of reinforcement are in place, liquid resin is applied onto the material. The resin can be manually brushed or rolled onto the fabric layers, ensuring that the fibers are fully impregnated with the resin.

4. Debulking and Consolidation: To remove air bubbles and ensure good adhesion between the layers, a process called debulking or consolidation is performed. This involves using rollers or squeegees to remove excess resin and air pockets from the laminate.

5. Curing: After the lay-up process and debulking, the composite part is left to cure. The curing process involves a chemical reaction that transforms the liquid resin into
a solid state, creating a strong and rigid composite structure. Curing can be done at room temperature or with the application of heat, depending on the type of resin used.

6. Demolding and Trimming: Once the curing is complete, the composite part is carefully removed from the mold, and any excess material is trimmed off to achieve the final desired shape and dimensions.

Curing in a Traditional Electric Oven:

Oven baking is a crucial step in the composite material curing process. During curing, the composite undergoes a chemical reaction that transforms the liquid resin into a solid, creating a strong and durable structure. Oven baking ensures controlled and uniform heating, promoting complete resin polymerization and enhancing the mechanical properties of the composite. The precise temperature and time parameters in the oven are vital to prevent defects like voids and to achieve the desired material performance. Proper curing improves the composite's strength, stiffness, and resistance to environmental factors, making it an essential step to produce high-quality, reliable, and structurally sound composite parts.

Mechanics of Composite Materials:

Mechanical behavior of a composite laminate varies vastly from conventional engineering materials. Most engineering materials are homogenous and isotropic in nature, whereas composite materials are often inhomogeneous and orthotropic/anisotropic. An
inhomogeneous material has nonuniform material properties over its body meaning its property changes depending on the position in the body. An orthotropic body has material properties that are different in three mutually perpendicular directions at a point in the body and, further, has three mutually perpendicular planes of material property symmetry. Thus, the properties depend on orientation at a point in the body [2]. An anisotropic material has properties that are different in all direction at a single point in the body. Due to the complexity in determining material type, mechanics of composite are studied in two sections: macro-mechanics and micro-mechanics. Composite material micromechanics is the study of the mechanical behavior of composite materials at a microscopic scale, focusing on how the individual constituents (e.g., fibers and matrix) interact to determine the overall material properties. It involves analyzing the stress and strain distribution within the composite to understand how its microstructure affects its macroscopic performance.

A lamina is the fundamental building block of a CFRP laminate. To understand mechanical behavior of a fiber reinforced composite laminate of multiple layers, it is crucial to understand the macro-mechanical behavior of a lamina first.

**Macro-mechanics of Lamina:**

To understand how a lamina behaves under applied stress, a 3D stress element under all possible stress is considered, as shown in Figure 1.4. Stress-strain relationship for an anisotropic stress-element can be derived using Hook’s law. An anisotropic material has 36 elastic constants due to its different properties in different direction. The stress-strain relationship matrix can be written as follows,
The above equation can be reduced containing 21 independent elastic constants due to plane symmetry and can be written as follows,

\[
\begin{bmatrix}
\sigma_{11} = \sigma_1 \\
\sigma_{22} = \sigma_2 \\
\sigma_{33} = \sigma_3 \\
\tau_{23} = 2\varepsilon_{23} = \sigma_4 \\
\tau_{13} = 2\varepsilon_{13} = \sigma_5 \\
\tau_{12} = 2\varepsilon_{12} = \sigma_6
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} = \varepsilon_4 \\
\gamma_{13} = \varepsilon_5 \\
\gamma_{12} = \varepsilon_6
\end{bmatrix}
\]

In contracted notation, relationship between stress-strain can be expressed as below,

\[
\sigma_i = C_{ij}\varepsilon_j, \text{ where } i, j = 1, 2, \ldots, 6
\]

If there are three mutually perpendicular planes of symmetry, then the stiffness matrix can be further reduced and only 9 independent elastic constants are needed. A
material with 9 independent elastic constants is referred to as an orthotropic material. The stress strain relation for an orthotropic material can thus be written as,

$$\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 \\
\end{bmatrix}$$

(4)

A unidirectional lamina is a transversely isotropic material, meaning at every point in the body there exists a plane in which mechanical properties are equal in all direction. The stiffness matrix can be then expressed with 5 independent elastic constants. If 2-3 plane is that plane of symmetry, the subscripts 2 and 3 of the coefficients should be interchangeable and stress-strain relationship can be written as,

$$\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C_{22} - C_{23}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 \\
\end{bmatrix}$$

(5)

In practice, these engineering constants are obtained by performing characterization test on materials where displacement is recorded upon application of a known load. So, strain-stress relationship is more practical to use. The constants of this matrix can be obtained by taking inverse of stiffness matrix which is called compliance matrix and can be written as follows,
\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & 2(S_{22} - S_{23}) & 0 & 0 \\
0 & 0 & 0 & 0 & S_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\]

(6)

Writing in terms of engineering constants provides,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{12}}{E_2} & -\frac{v_{13}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{12}}{E_2} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & 0 & 0 & 0 \\
-\frac{v_{13}}{E_3} & -\frac{v_{32}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\]

(7)

Composite laminae are typically loaded under plane stress due to its inability to sustain high stress in direction that are not parallel to its fiber. Thus, for a unidirectional orthotropic thin lamina with no out of plane loads, i.e., \(\sigma_3 = 0, \tau_{31} = \tau_{23} = 0\), the 3D stress-strain relationship can be reduced to 2D case and be written as,

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

(8)

Conversing the matrix equation gets,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

(9)

Or,
\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{12}}{E_2} & 0 \\
-\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

(10)

\[ [Q] \text{ matrix is the 2D-reduced stiffness matrix, where its components are defined as,} \]

\[
Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}}
\]

\[
Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{21}\nu_{12}}
\]

\[
Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}}
\]

\[
Q_{66} = G_{12}
\]

(11)

The relationship between the reduced stiffness matrix and reduced compliance matrix is,

\[ [Q] = [S]^{-1} \]

(12)

When the fibers of a lamina are rotated away from the principal directions (i.e., x-y coordinates), a new coordinate system is established, with the 1-direction aligned with the fibers and the 2-direction perpendicular to them at an angle to the x-axis. The 1-2 coordinate axes represent the local coordinate system, while the x-y coordinate axes represent the global coordinate system, as illustrated in Figure 1.5.
Stress transformation from the global to the local coordinate system can be defined by the following transformation matrix \([T]\),

\[
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2 \cos \theta \sin \theta \\
\sin^2 \theta & \cos^2 \theta & -2 \cos \theta \sin \theta \\
-2 \cos \theta \sin \theta & \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}
\] (13)

where the transformations are defined as,

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = [T] \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\] (14)

Or, conversely,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = [T]^{-1} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\] (15)

To convert strains from the global coordinate system to the local coordinate system, a matrix \([R]\) is defined, to account for the shear strain as \(\gamma_{xy}\) and \(\gamma_{12}\) is used instead of \(\varepsilon_{xy}\) and \(\varepsilon_{12}\). The matrix \([R]\) is as follows,
\[
[R] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\] (16)

Using \([R]\), the transformation from global strain to local strain can be written as,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} = [R][T][R]^{-1}\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\] (17)

and the relationship between global stress and global strain can be written as,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1}\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\] (18)

Then, global 2-D stiffness matrix \([\bar{Q}]\) thus can be defines as,

\[
[\bar{Q}] = [T]^{-1}[Q][R][T][R]^{-1}
\] (19)

and the global stress-strain relation can be written as

\[
[\sigma] = [\bar{Q}][\varepsilon]
\] (20)

**Micromechanics of a Lamina:**

To determine the overall elastic properties of a lamina, we need to combine the elastic properties of its constituent materials: the fiber and matrix. In this process, certain assumptions are made. The matrix is assumed to be linear elastic, homogeneous, and isotropic. Meanwhile, the fibers are assumed to be linear elastic, macroscopically homogeneous, and transversely isotropic. The lamina is believed to have evenly spaced fibers with no voids between the fiber and matrix. Consequently, it is considered linear elastic, macroscopically homogeneous, and transversely isotropic as well as in a plane state.
of stress. Based on these assumptions, we can determine the four elastic constants of the lamina:

1. Modulus of elasticity in direction 1 is given by,

\[ E_1 = E_f V_f + E_m V_m \]  \hspace{1cm} (21)

2. Modulus of elasticity in direction 2 is given by,

\[ E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} \]  \hspace{1cm} (22)

3. Poisson’s ratio is given by,

\[ \nu_{12} = \nu_f V_f + \nu_m V_m \]  \hspace{1cm} (23)

4. Shear Modulus is given by,

\[ G_{12} = \frac{G_f G_m}{G_f V_m + G_m V_f} \]  \hspace{1cm} (24)

where,

\( E_f, E_m \) = Modulus of Elasticity of fiber and matrix, respectively,

\( \nu_f, \nu_m \) = Poisson’s ratio of fiber and matrix, respectively,

\( G_f, G_m \) = Modulus of Rigidity of fiber and matrix, respectively, and

\( V_f, V_m \) = Volume fraction of fiber and matrix, respectively.

**Laminate Code:**

Laminate code is an easy shorthand method to efficiently describe the layup sequence and ply orientation of laminae within a laminate. In this code, laminae are listed in sequence starting from the top layer. Each lamina is labeled by its ply orientation and
separated by a slash. The orientation angle is determined by degrees rotated by the fiber from the principal direction shown in Figure 1.6 [2,4,5].

**FIGURE 1.6** Angle between principal axis and fiber direction denoted by $\theta$.

Figure 1.7 shows the breakdown of a laminate stacking sequence. The laminate can be described using the code $[0/45/90/-45/0]$, where $x$ represents principal axis. They are $[90/0], [90/0/45/90/0]T$ and $[90/0/0/90]$. The T-subscript stands for total and is used optionally. In addition an S-subscript can be used, such as $[90/0]_{S}$, where the S stands for symmetric. The code $[90/0]_{S}$ is equivalent to $[90/0/0/90]$ [2].

**FIGURE 1.7** Breakdown of the stacking sequence of a $[0/45/90/-45/0]$ laminate.
A few more examples of laminate code are shown in Figure 1.8. From right, the codes of the laminates are [90/0], [90/0/45/90/0]T and [90_2/0_2] respectively. T-subscript used for the second laminate stands for total and is used optionally.

![Ply-orientation breakdown](image)

**FIGURE 1.8** Ply-orientation breakdown for [90/0], [90/0/45/90/0]T and [90_2/0_2] respectively.

An S-subscript is also used to describe laminate that has a symmetric stacking sequence with respect to its mid-plane. For example, [90/0]s, where the S stands for symmetric and [90/0]s is equivalent to [90/0/0/90].

### 1.2 Bistable Materials

Bistable materials, also known as smart materials, possess the unique ability to maintain two distinct stable shapes and transition between them in response to specific triggers such as temperature changes, mechanical loads, or electric fields (see Figure 1.4). Thin Cross ply laminates, non-symmetric to its mid-plane, exhibits mechanical bistability that enables them to assume large deflections from one equilibrium position to another with only small energy input having usually two cylindrical stable shapes and sometimes a 3rd
saddle shape (see Figure 1.9) [6]. Most of the composites are cured in an autoclave or hot press restrained in a vacuum bag attached to a mold, unable to deform during the curing process. When cooled down to room temperature, they deform to a stable shape; because of anisotropic thermal expansions of the composite plies creating a residual stress field during manufacture. This shape changing behavior can be used as morphing structures.

![Saddle shape diagrams](image)

**FIGURE 1.9** Laminated shapes: (a) the flat shape, (b) the unstable saddle shape, (c) and (d) two stable cylindrical shapes [6].

Prepregs are very suitable for fabricating bistable laminates with comparatively simpler geometry since asymmetry is easy to achieve with already laid out fiber in a single direction in a matrix of thermoset. Unidirectional fiber-epoxy roll can be cut to desired size and laid up in a desired sequence to cure a bistable sample.
1.3 Research Objectives

The objective of this study is to incorporate boundary condition in a 2D cross-ply bistable rectangular laminate and observe its behavioral characteristic in accordance with its geometric change due to the imposed restraint. The goals can be summarized as follows,

1. To study the effect of length reduction on bistable behavior in a rectangular bistable laminate with a rigid tab attached to it inherently,

2. To find a critical length at which the specimen loses bistability,

3. To observe the effect of restrained edge on two cylindrical curvatures of a bistable laminate, and

4. To find load of actuation in relation to various side lengths.
CHAPTER TWO
LITERATURE REVIEW

In this chapter, the author presents a review of the existing studies on bistable composite laminates based on the characterizations and boundary conditions.

2.1 Review of Bistable Composite Laminate Characterizations

Among the first studies to characterize bistable composite laminates is the research done by Michael W. Hyer back in 1981 [7]. Hyer’s experimental investigation revealed that asymmetric composite laminates do not comply with the predictions obtained by the CLT. Hyer showed that the cured shapes of asymmetric laminates can take various curvature shapes because of the residual stresses caused by various thermal expansion coefficients. The post-cured asymmetric laminates in [7] were found to exhibit undesirable shapes because of their inherent extension-bending coupling, which motivated Hyer to explore further on potential usage of these shapes. Hyer found that the principal curvature directions of cylindrical bistable laminates are, indeed, predictable and the CLT is useful in predicting behaviors of thicker asymmetric laminates.

In [7], Hyer reported that a 150 mm × 150 mm (or 5.91 in × 5.91 in) [0°/90°]T asymmetric composite laminate exhibited snap-through behavior with two cylindrical shapes of opposite signs and axes perpendicular to each other at room temperature, as shown in Figure 2.1. However, based on the CLT, all asymmetric composite laminates
should exhibit saddle shapes at room temperature. Hyer’s investigation showed that $[0_2/90_2]_T$ and $[0_4/90_4]_T$ square graphite epoxy laminates exhibit cylindrical shape with a bistable behavior at room temperature instead of a saddle shape [8] (see Figure 2.2). To explain this behavior, Hyer extended the CLT in [8] by developing an analytical solution using the Rayleigh-Ritz energy method, including thermal expansion effects. This solution resulted in bifurcation diagrams for both the laminates with a critical side length at which the laminates exhibit bistable behavior. The bistable diagram presented by Hyer is shown in Figure 2.3. Hyer postulated that the factors due to which the internal stress state of the laminate may change, such as moisture absorption and viscoelastic relaxation, help define the laminates’ behavior near the critical length. Hyer also postulated that the laminates’ bistable configurations are affected by asymmetric curing, cooling, and moisture absorption resulting in a higher snap through force on one direction as compared to the snap back force on the opposite direction.

![Diagram](Image)

**FIGURE 2.1** (a) Cylindrical shape and (b) cylindrical shape of opposite sign and perpendicular axis of curvature [7].
FIGURE 2.2 (a) Saddle shape and (b) Cylindrical shape [8].

FIGURE 2.3 Bifurcation diagrams of $[0_2/90_2]_T$ and $[0_4/90_4]_T$ laminates [8].

Later, Hyer conducted numerical analysis based on the theory he developed in [8] for the in-plane residual strains of thin asymmetric composite laminates that exhibit post-cured cylindrical shapes at room temperature, which he presented in [9]. He investigated
the room temperature shapes of four different asymmetric cross-ply configuration: 
\([0/0/0/90]_T\), \([0/0/90/0]_T\), \([0/90/0/90]_T\), and \([0/0/90/90]_T\) and reported that numerical analysis following his theory could predict the room temperature shapes of the laminates. Hyer also reported in [8] that based on the square laminates’ thickness and side length, the laminates may exhibit different post-cured shapes, such as saddle shape, cylindrical shape, and cylindrical shape with a bistable configuration. Increasing the side length of square laminates was found to result in one of the cylindrical curvatures approaching zero while the other complied with the behavior predicted by the CLT.

Further research was conducted by Akira and Hyer in [10] to determine the relationship between temperature and curvature for asymmetric cross-ply laminates. Typically, composite laminates are cured vacuum-sealed at elevated temperatures to prevent the laminates from deformation during cooling. However, Akira and Hyer allowed this deformation by reheating the cured laminates and measured their deformed curvature for a range of temperatures and reported the relationship they observed between temperature and curvature in [10]. This relationship revealed that the temperature range over which the bistability of an asymmetric laminate can be observed is dependent on the size of the laminate, i.e., a bistable composite can return to exhibiting single stability if the temperature is changed accordingly. In addition, fabrication imperfections, such as thickness variation and volume fraction variation, were found to be affecting the bistability of the laminates. Some of the laminates were also reported to change stability shapes irreversibly under heating.
Following [10], many research endeavors led to a point where the room temperature shapes of asymmetric laminates can be predicted based on ply orientation and thickness [11–13]. Jun and Hong [11] showed that, for bistable composite laminates with medium width-to-thickness ratio, thermally induced in-plane shear strains are not insignificant. In a later study [13], Ren et al. used the Rayleigh-Ritz energy method to show that the post-cured shape of a cross-ply laminate varies with stacking sequence, radius, thickness, and size. The model developed in [13] was able to predict the post-cured shapes based on the abovementioned parameters that aligned with the FEM-based predictions and experimental results. Later, Jun and Hong [12] investigates the characteristics of asymmetric laminates with arbitrary lay-up angles and developed a formulation to predict non-cross-ply laminates’ curvature and principal directions of curvature. Their formulation complied with the CLT for the studied length-to-thickness ratios and was able to predict the curvature and the principal directions of curvature given the parameters, such as length-to-thickness ratios, number of layers, and lay-up angles.

Further studies were then conducted to improve modeling of bistable composite laminates. For example, Schlecht et al. [14] used FEA to analyze the snap through forces, stresses, and strains for asymmetric composite laminates. Betts et al. [15] developed a method to map the surface profiles of these laminates and confirmed that the existing modeling strategies were good enough to predict post-cured shapes at room temperature unless there are some fabrication imperfections, which was later attempted by Giddings et al [16]. The authors in [16] included manufacturing imperfections, such as high resin areas and ply-thickness variations, into their FEA analysis to predict the post-cured shapes. Their
analysis was shown to be able to predict the bistable states of imperfect laminates within a 3-7% error compared to an ideal laminate without any imperfections. Tawfik et al. [17] used an FEA model in ABAQUS™ to predict the shapes of asymmetric cross-ply laminates under thermal curing stresses and investigated the instability point of snap-through and the effect of aspect ratio (i.e., length vs. width) on bistability.

Analysis of snap-through behaviors have been studied by many others in the literature. For example, Dano et al. [18] started with an investigation of bistable cross-ply composite laminate using known moments near the edges and later applied the Rayleigh-Ritz method and conducted computational analysis to determine the force required at the moment arms placed on the laminates [19]. The authors in [19] also conducted relevant experiments and showed that predicted results aligned well with the ones observed through experiments. In another study, Wang et al. [20] presented bistability of symmetric laminates through pairs of viscoelastically prestressed polymeric matrix composite (VPPMC) strips combined with non-prestressed strips. Their FEA and experimental results showed similar trends. Then, the authors in [20] used FEA to investigate the effect of the modulus ratio between the non-prestressed fiberglass strips and the VPPMC strips on the snap-through characteristics.

2.2 Review of Boundary Conditions of Bistable Composite Laminate

To bolster bistable composites usage in different functional applications, a good understanding of the effects of different types of boundary conditions on bistable
composite laminates is needed. For bistable composite laminates, the boundary conditions can be thought as the conditions applied to the exterior of the laminates, or they can be considered at the boundary between the symmetric and the asymmetric portion of a laminate.

Various exterior boundary conditions have been researched over the years in the literature. The usual boundary conditions that fit this category can be considered as some variations of the conditions presented in Figure 2.4. As observed from the figure, the four corner points are fixed to keep the laminate fixed in the z-direction while allowing it to move along x- and y-directions. In this configuration, a force applied at the center can snap the laminate through to the other equilibrium position. Tawfik et al. [21] utilized an air table, as shown in Figure 2.5, to recreate these boundary conditions. Wang et al. utilized a three-point bending test setup, as shown in Figure 2.6, to measure the snap-through forces for symmetric bistable composite laminates. Dano and Hyer [19] utilized two protruding supports and fixed the center of a square laminate, as shown in Figure 2.7, to snap the laminate through to the other equilibrium state. Sellitto et al. [22] fixed the four corners of their composite laminates onto a base using rubber-tipped clamps, as shown in Figure 2.8.

![FIGURE 2.4 Typical boundary conditions [21].](image-url)
FIGURE 2.5 Experimental setup used in [21] to enforce boundary conditions.

FIGURE 2.6 Experimental setup used in [20] to enforce boundary conditions.
FIGURE 2.7 Experimental setup used in [19] to enforce boundary conditions.

FIGURE 2.8 Experimental setup used in [22] to enforce boundary conditions.
2.3 Research Gaps Addressed in This Study

To the best of the author’s knowledge, not many studies have been conducted where a simple cross-ply bistable composite laminate has a restraining boundary condition integrated to its geometry during the fabrication process. Due to ease of manufacturing, availability, customizable mechanical properties, and many other advantages over materials of fixed properties, parts made of composite material are getting more and more popular. These parts are often attached to a larger body using various types of fasteners. Addition of fasteners can alter the composite part’s structural properties. Material properties of the fasteners must be accounted for that may end up compromising the advantages of using composite material in the first place. Thus, creating complex parts using composite material that has boundary condition integrated in them and analyzing their behavior is a huge potential research area that needs to be explored. This study takes an attempt to address one of these conditions where a fixed tab was added to a two-ply bistable laminate on its one end before curing. Stress generated during the curing process due to a restraining edge impact its post cure shape and bistable behavior than a laminate with all free ends. Effect of length from its fixed end to free end has been numerically and empirically studied to identify the point where the specimen loses bistability and where the impact of a fixed end becomes negligible is the goal of this research. A curvature study has also been conducted to find out whether it achieves right cylindrical shapes or conforms to classical lamination theory.
CHAPTER THREE
SIMULATIONS AND EXPERIMENTS

This chapter covers the entire simulation process starting from developing finite element model to setting up boundary conditions according to specific needs. The models for the composite laminates were created using the ABAQUS™ analysis software. The simulation process was mainly divided into two steps: curing and snap-through. The curing process aka initial cooling step works as the base step for all simulations. For the second step, i.e., the snap through of the laminate by using actuation, for this case, the actuation being displacement, is performed applying necessary boundary conditions. The following sections discusses the process at large.

3.1 Model Development

The initial idea behind this research was to fabricate a bistable composite laminate with a fixed edge boundary condition integrated to its design and study its behavior. Before running the simulations, the rectangular specimen was fabricated with a thick tab on one end in the lab which will later be discussed in detail in the fabrication section of this chapter. Apart from the fixed end on one edge, the geometry of the specimen is simple. Figure 3.1 illustrates the geometric modeling done in ABAQUS™ sketch interface. In the case of complicated geometry, an individual drawing software can be used to draw the part and be imported to ABAQUS™ for analysis. A 300mm x 300mm square laminate was drawn as the first base model. The width was kept constant at 300 mm. The length of the laminate was then varied between 370 mm to 225 mm.
FIGURE 3.1 Geometry of the square laminate of [0/90] drawn on ABAQUS™ interface where x is the top ply fiber direction.

The ABAQUS™ software requires assigning specific material properties, that includes the modulus of elasticity (E1, E2), modulus of rigidity (G12, G13, G23), Poisson’s ratio (ν12), and coefficient of thermal expansion (α1, α2) for the composite. But prepregs used for fabrication, DA 409U/ G35-150 prepregs, sourced from Adhesive Prepregs for Composite Manufacturers LLC, only provided the tensile modulus and flexural modulus for this material. So, to conduct ABAQUS™ simulations, AS4-8552 [20,36] was identified with properties as close as possible to DA 409U/ G35-150 as the most suitable material due to its similarity in tensile and flexural moduli. The comparison is listed in the Table 3.1. Elastic and expansion properties of the orthotropic lamina used in simulations are listed in the following Table 3.2.
TABLE 3.1 Comparison in moduli between DA 409U/G35-150 & AS4-8552 [20,36]

<table>
<thead>
<tr>
<th>Property/Material</th>
<th>DA 409U/G35-150</th>
<th>AS4-8552</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Modulus</td>
<td>18.8 GPa</td>
<td>19.6 GPa</td>
</tr>
<tr>
<td>Flexural Modulus</td>
<td>17.9 GPa</td>
<td>18.4 GPa</td>
</tr>
</tbody>
</table>

TABLE 3.2 Elastic and expansion properties of orthotropic laminae

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₁</td>
<td>135 GPa</td>
</tr>
<tr>
<td>E₂</td>
<td>9.5 GPa</td>
</tr>
<tr>
<td>ν₁₂</td>
<td>0.3</td>
</tr>
<tr>
<td>G₁₂</td>
<td>5 GPa</td>
</tr>
<tr>
<td>G₁₃</td>
<td>7.17 GPa</td>
</tr>
<tr>
<td>G₂₃</td>
<td>3.97 GPa</td>
</tr>
<tr>
<td>α₁</td>
<td>-2x10⁻⁸/°C</td>
</tr>
<tr>
<td>α₂</td>
<td>-3x10⁻⁵/°C</td>
</tr>
<tr>
<td>Ply Thickness</td>
<td>0.2mm</td>
</tr>
</tbody>
</table>

The layup sequence along with the ply thickness (listed in Table 3.2) also needs to be defined. The model has two layers. Within the Section feature, 0° for top ply, referring
to fiber direction along X-axis, and $90^\circ$ for bottom ply, referring fiber to orientation perpendicular to X-axis, were assigned. This will ensure bistable property in the composite. After defining the layup sequences, the corresponding material properties were assigned to the entire geometry.

Accuracy of the FE simulation results greatly depends on refinement of mesh. So, using the correct mesh size is very important. In this study, a coarse mesh of global element size 10 and 9 and a comparatively finer mesh of global element size 5 were performed on the specimens having largest side length, equal length to width and the smallest length, in this case 150 mm, initially, to compare results. Results for coarse and finer mesh varied only for length way below critical length, and since the laminate acts unstable and the curvature properties are irregular at that stage, performance below critical length were ignored. Ultimately, global size 5 was used to conduct the simulations. This size is also verified by peers of the author’s lab who performed similar experiments [23].

One geometric set at the left edge and a node set at the center of right edge of the model were created in assembly so boundary conditions can be applied in the test process. Figures 3.2 and 3.3 show both the sets on a square laminate of 300 mm x 300 mm that were created.
FIGURE 3.2 Set created for left edge which was fixed for the entire simulation representing the restraining tab on one edge.

FIGURE 3.3 Set created for right edge center node which is also the load application node.
### 3.2 Finite Element Analysis

After meshing is done, the testing part begins. As mentioned before, curing process for every laminate worked as a base model. Defining boundary conditions that resonates with the goal of the research was crucial at this step. Since the structure is supposed to have a fixed part integrated with its geometry, the left side of the laminate were set to be fixed from all movements. The curing process is divided into two steps:

**Initial Condition:** An elevated curing temperature of 140°C is applied to the entire laminate using the "predefined field" tab. The entire geometry is fixed in all degrees of freedom during this step which represents conditions like curing process of the laminate on an aluminum plate under vacuum bagging. Additionally, it suppresses rigid body motions.

**Cooling:** In this step predefined field is modified to bring the temperature down to room temperature i.e., 21°C in a “static, general” step. An important factor to be considered in this step is to activate "NLGEOM" function “ON”. This enables ABAQUS™ to perform nonlinear stress-displacement computation without automatically predicting linear stress-displacement relation in accordance with Classical Laminate Theory that assumes an undesired saddle shape. A small constant damping factor of 2.5e-7 is applied during this step to imitate a pseudo-dynamic behavior. During the cooling step, the laminate was made free to have displacement and rotation in all degree of freedom except for the left edge, the fixed edge. Figure 3.4 represents the boundary condition applied on the fixed edge.
After a post-cure shape has been obtained, snap through and snap back test were carried out on the model. The direction of the displacement applied on the laminate is dependent upon the cured shape. Measures taken to perform the snap through, and snap back test are discussed below.

**Snap-Through Loading (STL):** Boundary condition for cooling was propagated in this step to sustain the post cure shape. Then, a new boundary condition was added to actuate the snap-through process. Instead of applying force, displacement was applied on the right edge center node in the +/- z direction depending upon the first stable shape acquired by the laminate. The magnitudes of displacements applied on various length were determined by trial and error.
**Snap-Through Unloading:** Snap-through unloading is the next step run on the specimen to ensure the laminate sustains the snapped state after the load was removed. Boundary conditions for this step includes propagation of cooling stage, and removal of the applied displacement on the right edge center node. Left edge of the laminate is fixed to move from every direction as done in the previous steps.

**Snap-back Loading:** Once the structure was snapped from its cured shape to its second stable shape, it is necessary to perform a snap back test to bring itself back to its first stable shape. This ensures the laminate’s capability to sustain two different shapes. The snap back process is similar to the snap-through process. The direction of the applied displacement is reversed. All other boundary conditions remain the same.

**Snap-back Unloading:** This is the final step in the simulation. Snap-back load is deactivated, and the laminate is allowed to sustain its first post cured shape upon removal of load. The entire process was repeated for laminates with varying side lengths changing from 340mm to 226mm.

### 3.3 Fabrication of the Specimen

The sample prototypes of fixed tab-laminate with fixed width of 300 mm and varying side lengths were fabricated in lab by typical hand lay-up procedure and were cured in a conventional electric oven using vacuum bagging technique. A roll of unidirectional carbon-epoxy DA 409U/G35-150 prepregs were used to lay-up the laminates. The prepregs are stored in a freezer chamber at a temperature between 0°C to 4°C to prevent immature curing of epoxy. It is easier to cut the prepregs to in a desired size right out of the freezer.
but it is recommended to wait at least half an hour before laying up the laminae so there is condensed water trapped in layers. This section will describe fabrication of a 300mm x 240 mm sample laminate with a fixed tab on end.

**Cutting and layup process:** Unlike in the simulation process, where the fixed tab phenomena could be replicated by applying a boundary condition at all steps, a rigid end was required to incorporate in specimen during the fabrication process. To create this rigid end, 14 thin pieces of prepreg strip of 300mm x 20mm was cut and layered on top of each other which can be expressed by laminate code of [90]_{14}. This is shown in Figure 3.5 below. Figures 3.6 and 3.7 present the laminate layup sequence with the tab and the final sample before curing, respectively.

![Figure 3.5 Fixed tab layup [0]_{14}.](image-url)
To prepare the bistable part, two plies were cut from the roll. The top layer was cut into a rectangle of 300mm x 260 mm, 260mm being the fiber length along principal direction. A second layer was cut to another rectangle of 300mm x 260 mm, where 300mm was fiber length perpendicular to principal direction. An important factor to be noticed here is that, to create a 300mm x 240 mm rectangular sample, 20mm allowance was kept.
accommodating the 20mm tab to ensure a geometry consistent with that of the simulation process. This way, the rigid tab at one end only acts as a restricting factor for one end of the specimen without contributing to its geometry. After the prepregs were cut to the desired shape, hand lay-up procedure was used to stack up the layers.

**Vacuum bagging and curing:** Figures 3.8(a) to 3.8(f) demonstrate the entire layering process for curing by vacuum bagging method. Figure 3.8(a) shows the uncured specimen placed on top off of an aluminum plate with a polyester release sheet in between. Figure 3.8(b) shows the perforated sheet on top of the laminate. Figure 3.8(c) shows another layer of polyester fabric. Figure 3.8(d) shows one layer of breathing fabric which also works as an absorbent layer for overflowing resin matrix. Figure 3.8(e) shows base of the vacuum connector placed before the vacuum seal and a sealant tape is laid around the edges of the aluminum mold. Finally, Figure 3.8(f) shows the vacuum bagging film sealed with silicon tape.

Since the structure involved has a very simple 2D like geometry, no mold was created to cure the sample. A large square aluminum plate was used as the base for holding the pre-cured sample in place for the vacuum bagging process. Figure 3.9 presents a schematic diagram of vacuum bagging layers. First, a release sheet of polyester (10 LYD 3000-D-Econo Ply J Polyester Peel Ply) is laid on the aluminum mold and the composite sample is placed on the polyester sheet. Composite sample can also be placed directly on the aluminum mold by treating it with a release agent, but in this case, a polyester release film was more suitable to use. The specimen is then covered with a perforated plastic sheet (10 LYD 3015-PERF-D- Release Ply- High Temp 450F- Perforated Film). Another layer
of the polyester sheet is placed over the perforated film. An absorbent material 10 LYD 3011-D-Breather Fabric is layered next, and the base of the vacuum connector nozzle is placed on top of this fabric. These layers are then sealed with the vacuum bagging film (10 LYD 3014-D - Stretchlon SL200 Vacuum Bag Film) with the help of a sealant tape and the connector nozzle is connected to its base through this film.
FIGURE 3.8 Steps of vacuum bagging process.

FIGURE 3.9 Vacuum bag apparatus [24].
The aluminum plate mold holding the vacuum bagging setup is then placed into a conventional electric oven (see Figure 3.10). Before turning the heat on, the nozzle is connected to a vacuum pump and perfect seal is ensured. The oven has a thermostat to control internal temperature. Once the aluminum mold setup is set in the oven, it is cured at 140°C for one hour.

![Vacuum bag setup connected to a vacuum pump through a connector nozzle placed in the oven.](image)

**FIGURE 3.10** Vacuum bag setup connected to a vacuum pump through a connector nozzle placed in the oven.

After an hour the oven and vacuum pump are turned off and the laminate is allowed to cool down to room temperature. The heating followed by cooling allows asymmetric cross-ply composite laminate to induce thermal stress in two-direction generating bistability in the structure. Once the setup reaches room temperature, the laminate is fully cured, and the vacuum packet is taken out of the oven. Once the samples are retrieved the vacuum packet materials are disposed. After the curing process, the laminate already takes
up its one stable state. Samples that achieve bistability are snapped to its second stable state by application of external force.

Applying pressure using the vacuum bagging process has multiple objectives. Initially, it eliminates trapped air among the laminate's layers. Secondly, it consolidates the layers of fibers, preventing any alteration in fiber orientation. Thirdly, it decreases humidity. Lastly, it fine-tunes the fiber-to-resin ratio by expelling excess resin through compression.

3.4 ADMET Universal Test Setup

The ADMET universal test setup was used to conduct snap through load experiment on several laminates to compare simulation data with fabrication data. This section will discuss the experimental setup used to perform the load test. To attach a sample to the test base, a quarter inch hole was drilled through the midpoint of the rigid tab of the specimen. Since the fixed tab made with 14 plys is very strong, the integrity of the structure was assumed to be unaltered. The sample was then attached to the test base with cylindrical dowel pins and blind screw sets. For load application, another hole was drilled on the 1 cm right from the right edge center point of the sample. A nylon thread with 16lbs load capacity was attached with nut and bolt. The other end of the nylon thread was attached to a load cell, which is used as a part of the ADMET Universal testing machine setup. Before running the test, the tension of the thread was adjusted to not have any preload. The load cell was calibrated to generate displacement in mm in the upward vertical direction (Y axis on the machine setup) as a form of load. Various displacements depending on the side length and ply orientation of the samples tested were applied at a rate of 0.6 mm/sec. For
example, for [0/90] 300 mm x 230 mm specimen, a larger displacement of 250 mm was applied, for a sample of same dimension but opposite orientation, i.e. [90/0], 100 mm was sufficient to get results. Figure 3.11 shows the test setup for a single sample.

**FIGURE 3.11** Test fixtures indicated by arrows.
CHAPTER FOUR
ANALYSES AND RESULTS

This chapter will discuss the results and observations obtained from FEA and experiments.

4.1 Finite Element Analysis and Results

The objective of this research was to study the shape change behavior of bistable composite laminates of a specified geometry under a fixed boundary condition. The variable in geometry was the side length of a rectangular laminate. The boundary condition was applied on one of the two opposite sides of a rectangular laminate of fixed width, in this case 300 mm and the variable were the two adjacent sides. Side length for the rectangular laminate were gradually decreased from 300mm until it reached a point below which it cannot sustain its second stable structure. Starting from 300 mm side length, length was gradually decreased by 10 mm in decrement. The detailed procedure for FEA simulation has been discussed in previous chapter. This chapter will focus on the results obtained from the analysis. Rectangular laminate with a fixed boundary condition and a fixed width of 300 mm, sustains bistability at side length of 226 mm. The load vs. displacement curves in Figure 4.1 demonstrate the snap through behavior of composite laminate under a rigid boundary condition. Side lengths were gradually decreased by 10 mm to go from a 300 mm side length. For larger side length the specimen snaps from post cure shape to the second bistable state by a very small amount of load. The laminate reaction to rigid boundary condition and force application in the form of displacement remain similar at 300
mm side length and above. Figure 4.1 shows the snap through by application of a small amount of load in samples 300 mm x 300 mm and 300 mm x 310 mm. Figure 4.2 shows the post cured and post snap shape of a 300mm x 300mm laminate imported from ABAQUS™ viewport.

**FIGURE 4.1** Load vs. displacement curve for samples of 300 mm x 300mm and 300 mm x 310 mm.

**FIGURE 4.2** (a) Post cured shape and (b) post snap shape of 300mm x 300mm
Samples of 300 mm x 290 mm, 300 mm x 280 mm show similar trend in load vs. displacement curve. The data was collected from ABAQUS™ for the node set at right edge corner where displacement was applied as coercive form of load. Due to the post cured shape, the out of plane displacement of the node fluctuates between negative and positive values. To plot load data, absolute value of the reaction force was taken since the negative signs only indicates the direction of load being applied.

Figure 4.3 shows the snap through responses for 300mm x 226mm, 300 mm x 230mm, 300 mm x 240 mm samples. Sample of a side length of 226 mm experienced the snap through process obtaining two different shapes. Samples of side lengths up to 280 mm have similar behavior but only three examples are shown here. To reach to the post snapped state, these laminates go through other small load-drop phenomena so bistability is achieved.

![Figure 4.3 Load vs. displacement curve for samples of 300 mm x 226 mm, 300 mm x 230 mm, and 300 mm x 240 mm.](image-url)

**FIGURE 4.3** Load vs. displacement curve for samples of 300 mm x 226 mm, 300 mm x 230 mm, and 300 mm x 240 mm.
Figure 4.4 shows the post cured and post snap shape of a 300 mm x 226 mm laminate imported from ABAQUS™ viewport.

![Figure 4.4](image)

**FIGURE 4.4** (a) Post cured shape and (b) post snap shape of 300 mm x 226 mm.

Samples below the side length of 226 mm shown no snap through region. It acquires only one cylindrical shape of larger curvature. Curves in Figure 4.5 show exponential rise in load in the attempt to snap the specimen without any snap occurring.

![Figure 4.5](image)

**FIGURE 4.5** Load vs. Displacement curve for samples of 300 mm x 190 mm and 300 mm x 200 mm.
Figure 4.6 shows the post cured shape of a 300 mm x 190 mm laminate imported from ABAQUS™ viewport.

The load required to deflect smaller samples are higher and follows a rising trend until a length is reached where a sudden drop in snap-through load is observed. Curve plotted for snap-through load vs. side length presents such a trend. The drop in the load can be explained due to a transition point in post-cured shape of [0/90] laminates. Snap-through load for any laminate is considered to be the load required for a laminate to go from its first stable state to second stable state. It is found as a byproduct of our study that the post cure shape also varies with side length depending on laminate orientation. As this study’s main goal was to find a relationship between bistability and side length in rectangular laminates, the post-cured shape of laminates with opposite orientations, i.e., [0/90] and [90/0] is left to investigate for future research. In Figure 4.7, Snap-through load vs Side Length is presented where it is seen that between 280mm-290mm side length, there lies a point where the post cure stable shape takes a different curvature. Load was applied at the right edge center of each specimen to snap it to its another stable shape. Load requires to snap a
laminate for specimen with side lengths 280mm and below with a larger curvature, which is also fully cylindrical, is much higher than the load required to snap specimen of side lengths 290mm and above.

**FIGURE 4.7** Snap through load vs. side length to take [0/90] laminates of various side lengths from first stable shape to second stable shape.

As the laminate side length increases the curvature plot bifurcates into two curves. At 226mm the sample takes two stable shapes shown in Figure 4.8. Laminate with a rigid boundary and two stable shapes has only one cylindrical shape, the other curvature does not take cylindrical shape rather takes on an irregular saddle like shape. Depending on the side length and ply orientation, the other stable shape varies. Coordinates of curvature profile were acquired from ABAQUS™ and a polynomial curve fitting was done to get both curvatures. This study did not dive deep into correlation between curvature and the
change of side length with a restricted boundary condition since a lot of other factors are to be considered.

![Bifurcation curve showing the critical length ($L_c$).](image)

**FIGURE 4.8** Bifurcation curve showing the critical length ($L_c$).

### 4.2 Experimental Results

Snap through responses of a few laminates were recorded to compare with simulation data. Samples of 300 mm and variable side lengths of 300 mm, 290 mm, 270 mm, 260 mm, 240 mm, 230 mm, 210 mm, 190 mm, 170 mm, and 150 mm were fabricated. But to perform load test, specimens of few dimensions were selected for experiment based on simulation findings. From FEA, critical length for a sample of fixed width of 300 mm was found to be somewhere between 230 mm to 240 mm. It was also observed that significantly larger force is required to snap a laminate from the larger curvature state to smaller curvature state. Whether a specimen will take the first stable state
of larger curvature or smaller curvature post curing, depends on the ply orientation. A [90/0] orientation gives exact opposite curvature after being cured to room temperature. Specimens of 300 mm x 190 mm, 300 mm x 230 mm, 300 mm x 240 mm, 300 mm x 260 mm and 300 mm x 300 mm of laminate orientations [0/90] and [90/0] were tested. Samples of 300 mm x 230 mm and 300 mm x 240 mm give results comparable to simulation results. Since the critical length falls within this region, side by side comparison of two laminates of both sets of ply orientation will be discussed here. Figure 4.9(a) shows loading response for [0/90] laminate of 230 mm side length. Throughout the loading step, no significant drop of load is observed, and the magnitude of load keeps rising.

However, in Figure 4.9(b), for a [90/0] sample of same side length, a snap through region was observed, but when unloaded, the sample went back to its first shape suggesting the sample was not bistable.

![Graphs showing loading response for laminates of different orientations.](image)

**FIGURE 4.9** (a) 300 mm x 230 mm laminate of [0/90] shows no snap through going from smaller curvature shape to larger curvature shape (b) 300 mm x 230 mm
laminate of [90/0] shows a snap through region going from larger curvature shape to smaller curvature shape but is unstable.

Experimental setup and snapped positions of 300 mm x 230 mm laminate [90/0] are shown below in Figures 4.10(a) and 4.10(b).

![Figure 4.10](image_url)

FIGURE 4.10 (a) Post cured shape/first stable shape/smaller curvature stable shape of a 300mm x 230mm specimen of ply orientation [90/0] unloaded (b) snapped but unstable shape after load application.

In Figure 4.11(a) and Figure 4.11(b), a snap through region is observed. In the case of [0/90] laminate, a small, localized snap-through region is observed where the specimen tries to go from larger curvature to smaller curvature. The rise of load beyond that point bends the specimen without conforming to the second stable shape. The specimen went back to its original shape upon load removal, suggesting errors and limitation of hand-
laying a specimen. But [90/0] orientation of same side length snaps and stays in the second shape upon load removal. This phenomenon happens when the specimen tries to go from smaller curvature post cured shape to larger curvature snapped through state. The experimental results suggest a strong relation between ply orientation and post cured shape is there to achieve stability with a fixed tab at one end, but this study did not go further into that.

![Graph](image)

**FIGURE 4.11** (a) 300 mm x 240 mm laminate of [0/90] shows a snap through phenomena going from larger curvature shape to smaller curvature shape (b) 300 mm x 240 mm laminate of [90/0] snaps through from a stable smaller curvature shape to a second larger curvature shape.

Experimental setup and snapped positions of 300mm x 240mm [0/90] orientation is shown below in Figure 4.12(a) and 4.12(b).
FIGURE 4.12 (a) Post cured shape/first stable shape/larger curvature stable shape of a 300mm x 240mm specimen of ply orientation [0/90] unloaded. (b) a snapped but unstable shape after load application.

300 mm x 230 mm [0/90] and 300 mm x 240 mm [0/90] both takes larger curvature shapes as their first stable shape post curing process. so, to snap them a large displacement was applied. From Figure 4.9(a), it is observed that there is no load drop zone in 300 mm x 230 mm specimen curve, suggesting no snap occurred on the specimen. From Figure 4.11(a), specimen of 300 mm x 240 mm shows a small load drop region that indicates snap occurred.

From [90/0] specimen, 300mm x 230mm and 300mm x 240mm both acquired smaller curvature shape as their first stable shape. The load vs. displacement curves for both specimens show a load drop. When unloaded, 300mm x 230mm specimen snaps right back to its first and only stable shape. 300mm x 240mm laminate behavior is interesting. It shows bistable characteristic for a very brief period after curing. But by the time it is
ready for load test, being in the open subjected to absorb moisture from environment affects its bistability [25].
5.1 Conclusions

The goal of the study was to address shape-change behavior of a two-ply rectangular bistable composite laminate when a boundary condition is inherently attached to it and mark a point where bistability is compromised. The steps include in achieving that goal were to create a geometric model with necessary boundary conditions and decide on sample dimensions to conduct FEA simulation on. Then forwarding with fabricating the specimen using available material and replicate simulation scenario in lab. In practice, first a decision was made to fabricate a specimen with industry provided Carbon-Fiber-Epoxy prepreg roll with specific dimensions. That initiated the idea of choosing a sample width of 300mm and go down on other two side lengths to find the impact of decreasing length on bistability. Initially a [0/90] cross-ply square laminate of 300mm x 300mm with 14 layers of thin strips laid on one side [0]14 was cured in a vacuum bagging process to understand physical behavior of the specimen. Based on that, FEA model for side lengths varying from 150mm up to 350mm were created and analyzed. The model properties assigned remained the same for every model except for one change in geometry i.e., the side length of the rectangular laminate. A standard explicit analysis was done in ABAQUS™. All specimen was subjected to a restrained boundary condition on its left edge during the entire process of simulation depicting rigid tab behavior. Steps included curing, snap-through loading, and snap-back loading. Curing process worked as the base
model for every specimen. The snap-through process required a lot of trial and error to make a snap-through happen. Major amount of work went into this process trying to determine the load, or in this case displacement needed to apply to induce required force in the right direction. Something to be noted here that the direction of applied displacement changed between +ve and -ve z-axis depending on the post cured shape of the specimen.

From the analysis, a critical side length of 226mm was found where the laminate stays stable after a snap through and sustains the state upon removal of the load. These results were presented in the result and analysis chapter focusing on the snap-through step of the simulation.

Based on the simulation data, load-test were performed with ADMET Universal testing machine with a tensile test setup. In real, laminate of 240mm side length showed bistability. Table 5.1 compares snap-through load for experimental data vs. simulation data for samples of 300mm x 230mm and 300mm x 240mm with a [0/90] orientation.

**TABLE 5.1** Snap through load for 300 mm x 230 mm and 300 mm x 240 mm samples

<table>
<thead>
<tr>
<th>Snap Through Load (N)</th>
<th>300 mm x 230 mm</th>
<th>300 mm x 240 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA data</td>
<td>2.73N</td>
<td>2.87N</td>
</tr>
<tr>
<td>Experiment data</td>
<td>No Snap Through</td>
<td>0.1933N</td>
</tr>
</tbody>
</table>

The discrepancies in load may be due to error in experimental setup or limitation of expertise in fabricating samples. This is why, side by side comparison of FEA data and experiment data were not elaborated in this study and left for future work. Despite these
limitations, this research makes some important contribution in developing and analyzing a composite structure with integrated boundary restrain a good idea of dimensional requirement was generated for a bistable composite laminate to sustain its structural characteristics.

5.2 Recommendations

Assumptions and compromises made to present the results of this study has potentials for improvement.

First, the properties for DA 409U/ G35-150 used in this research was not available and AS4-8552 was used to run the simulations. Mechanical properties could be obtained for DA 409U/ G35-150 prepregs by conducting ensile (tension), flexural, impact, shear, and compression tests and documented for future refences. This would provide comparable results between FEA and experiments.

Second, FEA results were presented for laminates with [0/90] ply orientation. From analysis done with [90/0] laminates by this author but not presented in this work due to lack of comparable data, it can be said there is a strong correlation between a structures ability to sustain two different shapes with restrictive boundary especially near critical length and the lamina layup sequence during curing process. Further study can be conducted by comparing samples of both orientations.

Post cured shapes of laminates were observed to vary based on laminate orientation and side length. The relation between side length, ply-orientation, and bistability can be a
very potential study derived from this work. Figure 5.1 and Figure 5.2 show different post cure shapes of laminates varying in side lengths and of [0/90] orientation.

**FIGURE 5.1** Post cured shape of specimens of 260mm, 270mm and, 280mm, respectively.

**FIGURE 5.2** Post cure shape of specimens of 290mm, 310mm and, 330mm respectively.

Third, non-dimensional study can be done taking ply orientation in factor to determine critical length for any given dimension of a rectangular composite laminate with such boundary conditions. This author aims to continue further research towards this goal. This will facilitate future researchers for taking decisions and advancing to develop more complex structure of bistable composite.
REFERENCES


