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Surrogate Modeling of Single, Multi-Patch and Kirigami Composite Laminates for Design Purposes

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SURROGATE MODELING OF SINGLE, MULTI-PATCH AND KIRIGAMI COMPOSITE LAMINATES FOR DESIGN PURPOSES

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Jebin Biju
May 2021

Accepted by:
Dr. Georges Fadel, Committee Chair
Dr. Suyi Li
Dr. Oliver J. Myers
Thin bistable composite laminates can be used for shape morphing applications by virtue of their material properties and asymmetric ply layup. These laminates are called bistable because they can be snapped into two or more stable shapes. A single bistable patch can result in simple cylindrical shapes and when multiple such patches are assembled into a single multi-patch laminate they result in more complex and multiple stable shapes that can find wide practical use in shape morphing applications. To be able to design such multi-patch laminates we need to have models that can predict the stable shapes of such laminates based on the input of laminate parameters which includes but is not limited to variables like patch shape and size, number of plies, ply thickness, material properties, etc. These models can then be used to establish a design method based on optimization to solve the inverse problem of solving for the laminate parameters given a specific target shape(s).

The curing and snap through of these laminates could be simulated using Finite Element analysis to solve for the stable shapes. But due to the large computational costs associated with simulating multi-patch laminates, using FEA in the optimization model is not preferred and alternate surrogate models need to be considered. Analytical models exist that can approximate the stable shapes of the laminates from the input of material properties and laminate geometry. These models correlate with FEA and experiments to a satisfactory degree and could be used for design purposes. Additionally, machine learning is also considered as an approach to solve the problem since it is data driven and a computationally cheaper tool as compared to FEA. In this research, the aforementioned surrogate models
are explored and their feasibility to design multi-patch laminates are investigated. The most suited model is then used to design a four-patch grid laminate targeting a specific shape(s).

Additionally, this research delves into the novel idea of designing Kirigami composites. Kirigami is an ancient art of paper cutting which is popularly used to make decorative shapes. Using this method, simple cut patterns can be made on a 2D sheet to yield complex 3D shapes. Thus, the disparate concepts of Kirigami and bistability could be integrated to achieve unprecedented shape morphing capabilities. Current work in this area investigates the geometry and simulation of the curing and snapping process of Kirigami composites using FEA and correlates them with experimental results. In this study, the surrogate models discussed earlier are extended to develop an approach to compute the shapes of these laminates in a computationally cheaper method.
DEDICATION

I dedicate this work to my family and friends in India and the United States. Thank you all for believing in me and having my back throughout my graduate student journey.
ACKNOWLEDGMENTS

Firstly, I would like to thank my advisor Dr. Georges Fadel for the continued support and guidance throughout the duration of my research. I also express my immense gratitude to my committee members Dr. Suyi Li and Dr. Oliver Myers for their valuable input to this work. I would also like to acknowledge the feedback and input to my research from Vishrut Deshpande and Akshay Balasubrahmanian as part of the Kirigami Composite group.

Furthermore, I would like to acknowledge the support of the National Science Foundation (NSF grant 1760943) in funding this research. I would also like to thank Esteco for providing the Modefrontier software license to the University, a tool that was extensively used in this work. Lastly, I would like to thank the Mechanical Engineering Department at Clemson for the state-of-the-art facilities that made working on my research stress free.
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CHAPTER ONE

INTRODUCTION

The structures of a mechanical system are typically designed to provide stiffness, strength, fatigue strength, energy absorption and thermal stability. Traditional structural materials such as steel or aluminum have been used to accomplish some of these functions, but not necessarily all of them. Furthermore, the push for energy minimization challenged engineers to seek lighter materials for such functions. Composite materials came to the forefront for their ability to potentially improve many if not all those functions while often being significantly lighter (Gibson, 2010). The advent of composite materials had generated a lot of interest by different industries, especially in the aeronautical and automotive areas, and inspired them to use composite materials on a large scale (Jones, 1999). The Boeing 787 Dreamliner, which is manufactured at the Boeing plant in Charleston, South Carolina, is one such example of the large scale use of carbon fiber composites in the fuselage, wings and tail of the aircraft (Sloan, 2018). Around 50% of the aircraft is made using advanced composites. Use of a composite primary structure has reduced the scheduled and non-routine maintenance of the aircraft due to the reduced risk of corrosion and fatigue of composites (Hale, 2019).

2.1 Composites Basics and Terminologies

Jones (Jones, 1999) states that composites are a combination of two or more materials on a macroscopic level to form a new material which has the best properties of its constituents like improved strength, stiffness, corrosion resistance, weight, etc. They are
different from alloys in that they are not combined on a microscopic level to result in a homogenous material. Composite materials are commonly classified as fibrous, laminated, particulate or a combination of them. In this work, we are dealing with fiber reinforced composite laminates. Fiber reinforced composite laminates are a combination of layers of fibers and a matrix. The fibers form the load carrying component and thus are stiff, while the matrix holds the fiber together and distributes the load among the fibers. A lamina is an arrangement of fibers in a matrix, which is the building block for laminates. Based on the type of laminae or layer, they can be categorized as unidirectional or woven fiber laminae as shown in Figure 1.1. We will focus on unidirectional composite laminates in this research.

---

Figure 1.1: Types of laminae (Jones, 1999)
When a stack of unidirectional laminae is bonded, it results in a composite laminate (Figure 1.2). The individual layers are called plies and based on their stacking, these laminates can be classified as symmetric or asymmetric laminates. Symmetric laminates, as the name suggests, are those that are symmetric about the middle surface/plane of the stack in both geometry and material properties. Asymmetric laminates lack that symmetry about the middle surface/plane. For this study we deal with laminates that have plies of equal thickness and identical material properties, and the parameter that brings about the symmetry or asymmetry is only the fiber orientations of the plies as shown in Figure 1.3.
2.2 Composites Manufacture

In this section, the manufacturing of fiber reinforced composite laminates is discussed in brief. The manufacture of this laminates can be carried out in three broad steps:

i. **Form of constituent materials:** As stated previously, unidirectional fibers are considered for this research. These fibers are available individually or already infused with epoxy which acts as the matrix. The latter are called preimpregnated fibers or prepregs for short.

ii. **Layup:** This process involves the laying of the fiber or prepregs in the desired shape or form. The fibers are usually precut in the required shape, orientation, and size. This is followed by the molding process where the layers are compressed at elevated temperatures.

iii. **Curing:** It is the process of solidification of the matrix material. It involves heating the laminate and slowly cooling it in an autoclave while applying pressure on it. The heat allows the matrix to melt and be uniformly distributed over the laminate.
This results in cross-linking of the matrix material and the temperature can be gradually lowered.

2.3 Bistable Composites

Thin composite laminates with an asymmetric fiber layout when cured flat in a press or autoclave develop a curvature upon cooling back to room temperature. This occurs due to the thermal expansion mismatch between the plies of different fiber orientation, which induces residual thermal stresses in the laminates thus making the laminate warp and develop the aforementioned curvatures which are cylindrical for a single patch laminate (M. Dano & Hyer, 1998; Schlecht & Schulte, 1999). It is also observed that these laminates have two cylindrical configurations post curing which can be changed by applying an external force or excitation at some locations on the patch. This process is called snap through of the laminate. The configurations are referred to as states, and are called pre-snapped or post-snapped states (M. L. Dano & Hyer, 2000). The cylindrical shape of the asymmetric patch can be altered by imposing boundary conditions on the edges of that patch.

2.4 Multistable Composite Patches

In this context, a patch is referred to as a multi-ply laminate which has a single fiber orientation value for an individual ply. Patches may have different shapes, dimensions, number of plies, ply thicknesses, etc., but may have only plies, each with a fixed fiber angle. The implementation potential of single patch bistable composite laminates would be severely constrained since their external shapes in stable equilibria resemble only
cylindrical surfaces. The lack of sophisticated shape changes between stable states prevents developing other promising adaptive functions that exploit this function. Thus, expanding on the bistability idea, and since boundary conditions can alter the shape of a patch, we should be able to assemble multiple asymmetric and/or symmetric laminates to have a multi-patch laminate as shown in Figure 1.4, which could have $2^n$ shapes, where $n$ is the number of bistable patches in the laminate. Thus, making these laminates multi-stable.

![Single Patch and Multi-patch Laminate](image)

**Figure 1.4:** (a) Single patch (b) Multi-patch laminate

These laminates would be much more suitable for practical shape morphing applications since multiple connected patches which are individually bistable would result in a shape significantly more complex than the cylindrical shape. A popular application in the literature for such morphing potential is the use in aerospace structures, where the bistable laminates are used as the trailing edge or an internal structure to allow shape morphing of the wing (Diaconu et al., 2008; Panesar & Weaver, 2012). More about laminates with multiple asymmetric patches connected is discussed in the *Literature Review.*
2.5 Research Objective

In view of the potential to combine patches to create arbitrary shapes in pre-and post-snap stages, the objective of this research is to create a design method for multi patch composite laminates that can match desired shapes. To accomplish this task, the analytical models that predict the stable shapes of single and multi-patch composite laminates are studied. Since the computational burden of computing multi-patch shapes using finite elements grows significantly with the number of patches, surrogate models used to predict the shapes of single and multiple patch laminates are explored. Additionally, the usability of these surrogate models to predict the shape of Kirigami composites is investigated.

2.6 Thesis Outline

After the first chapter which is a broad introduction to bistability and multi-stable laminates, the rest of the thesis is divided into the following chapters. Chapter 2 reviews the present literature related to the mechanics of composite materials, the analytical models that predict the shapes of bistable laminates. Chapter 3 presents multiple surrogate models and discusses their methodology in detail. In Chapter 4, the usability of the models for multi-patch laminates is discussed and the most suited model is selected, and a design method is established. Chapter 5 discusses the setup and results of the design method. Chapter 6 then extends the chosen model to predict the shapes of Kirigami composites. Finally, Chapter 7 concludes the work done and discusses the future work in for this research.
CHAPTER TWO

LITERATURE REVIEW

3.1 **Mechanics of Composite Materials**

Since the purpose is to design specific shapes of multiple composite patches, it is necessary to understand the mechanics that governs them. The mechanics of these materials as explained by Jones (Jones, 1999) are discussed below.

*Micromechanics of Lamina*

Micromechanics is the study of the behavior of composites by examining the interaction between the constituent materials, i.e., the fiber and the matrix (Figure 2.1). It deals with determining the elastic moduli of the composite material as a function of the elastic moduli of its constituent materials. To do so, some assumptions are made which are that (i) *The lamina is linearly elastic, macroscopically homogeneous and orthotropic, and bonds between fibers and matrix are void free*, (ii) *The fibers are homogeneous, linearly elastic, isotropic, regularly spaced, perfectly aligned and bonded*, (iii) *The matrix is homogeneous, linearly elastic, isotropic and void free*.

![Diagram showing constituents of a lamina](Figure 2.1: Constituents of a lamina (Jones, 1999))
Based on the above assumptions the engineering constants, $E_1, E_2, \nu_{12}$ and $G_{12}$ for the unidirectional composite laminate can be calculated as a function of those of the fiber and matrix $E_f, E_m, \nu_f, \nu_m, G_f, G_m$ and the volume fractions $V_f$ and $V_m$, using the rule of mixtures. The subscript $f$ refers to fiber, the subscript $m$ to matrix, the subscripts 1 and 2 are orthogonal material directions.

Where,

\[ E = \text{Modulus of elasticity} \]
\[ G = \text{Modulus of rigidity} \]
\[ \nu = \text{Poisson’s ratio} \]
\[ V_f = \frac{\text{Volume of fibers}}{\text{Total volume of composite material}} \]
\[ V_m = 1 - V_f \]

**Macromechanics of Lamina**

Macromechanics of the lamina is used to study their response to applied stress. This deals with developing the stiffness and compliance matrices using the engineering constants obtained in the previous section. The important relations and terminologies are described below.

The stress-strain relations for anisotropic materials are described using the generalized Hooke’s law in Equation (2.1).
\[ \sigma_i = C_{ij} \varepsilon_j \quad i, j = 1, 2, \ldots, 6 \]

(2.1)

Where \( \sigma_i \) are the stress components as shown on the three-dimensional cube in Figure 2.2, \( C_{ij} \) is the stiffness matrix and \( \varepsilon_j \) are the strain components.

The tensor versus the contracted notations for the three-dimensional stresses and strains are as shown in Table 2.1

Table 2.1: Contracted notations for Stresses and Strains (Jones, 1999)

<table>
<thead>
<tr>
<th>Stresses</th>
<th>Strains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensor</td>
<td>Contracted</td>
</tr>
<tr>
<td>notation</td>
<td>notation</td>
</tr>
<tr>
<td>( \sigma_{11} (\sigma_1) )</td>
<td>( \sigma_1 )</td>
</tr>
<tr>
<td>( \sigma_{22} (\sigma_2) )</td>
<td>( \sigma_2 )</td>
</tr>
<tr>
<td>( \sigma_{33} (\sigma_3) )</td>
<td>( \sigma_3 )</td>
</tr>
<tr>
<td>( \tau_{23} = \sigma_{32} )</td>
<td>( \sigma_4 )</td>
</tr>
<tr>
<td>( \tau_{31} = \sigma_{31} )</td>
<td>( \sigma_5 )</td>
</tr>
<tr>
<td>( \tau_{12} = \sigma_{12} )</td>
<td>( \sigma_6 )</td>
</tr>
</tbody>
</table>
Thus, in Equation (2.1) the stiffness matrix $C_{ij}$ has 36 constants. Since the stiffness matrix is symmetric, i.e. $C_{ij} = C_{ji}$, the number of independent constants reduces to 21 (Jones, 1999). These materials are called anisotropic materials and their stress-strain relation is given by Equation (2.2).

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
$$

(2.2)

If there is one plane of symmetry, say the 1-2 plane or $z = 0$; these materials are called monoclinic and have 13 independent constants. Their stress-strain relation is given by Equation (2.3).

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 & 0 & 0 & C_{16} \\
C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\
0 & 0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & 0 & C_{45} & C_{55} & 0 \\
C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
$$

(2.3)

The relation for orthotropic materials, where there are two orthogonal planes of material property symmetry is given by Equation (2.4). They have 9 independent constants.

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & C_{44} & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
$$

(2.4)
We are interested in the stress-strain relation for unidirectional composite laminae which is transversely isotropic. The relation for transversely isotropic with plane of isotropy as 2-3 is given by Equation (2.5) (Jones, 1999). They have 5 independent constants.

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & (C_{22} - C_{23})/2 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
\]

(2.5)

If a material has infinite planes of symmetry, then they are called isotropic materials and their stress-strain relations have only two independent constants in their stiffness matrix as shown in Equation (2.6).

\[
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & (C_{11} - C_{12})/2 & 0 & 0 \\
0 & 0 & 0 & 0 & (C_{11} - C_{12})/2 & 0 \\
0 & 0 & 0 & 0 & 0 & (C_{11} - C_{12})/2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
\]

(2.6)

The respective compliance matrix \( S_{ij} \) which is the inverse of the stiffness matrix \( C_{ij} \) for the types of materials described by Equations 2.2-2.6 are given below,
Anisotropic materials,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\
S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\
S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\
S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\] (2.7)

Monoclinic materials,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\
S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\
S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\
0 & 0 & 0 & S_{44} & S_{45} & 0 \\
0 & 0 & 0 & S_{45} & S_{55} & 0 \\
S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\] (2.8)

Orthotropic materials,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\] (2.9)

Transversely isotropic materials,

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 & 0 & 0 & 0 \\
S_{12} & S_{22} & 0 & 0 & 0 & 0 \\
S_{13} & S_{23} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2(S_{22} - S_{23}) & 0 & 0 \\
0 & 0 & 0 & 0 & S_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix}
\] (2.10)
Isotropic materials,

\[
\begin{bmatrix}
\varepsilon_1 & S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\
\varepsilon_2 & S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\
\varepsilon_3 & S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\
\gamma_{23} & 0 & 0 & 0 & 2(S_{11} - S_{12}) & 0 & 0 \\
\gamma_{31} & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12}) & 0 \\
\gamma_{12} & 0 & 0 & 0 & 0 & 0 & 2(S_{11} - S_{12}) \\
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12} \\
\end{bmatrix} = (2.11)
\]

Now, for an orthotropic material the compliance matrix \( S_{ij} \) can be expressed in terms of the engineering constants directly as shown in Equation (2.12)

\[
[S_{ij}] = \begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \\
\end{bmatrix} (2.12)
\]

Where,

\[E_1, E_2, E_3 = \text{Young’s modulus in 1, 2, 3 directions}\]

\[\nu_{13} = \text{Poisson’s ratio}\]

\[G_{23}, G_{31}, G_{12} = \text{Shear modulus in 2-3, 3-1, 1-2 planes}\]
We know from Equation (2.9) for orthotropic materials that $S_{ij} = S_{ji}$. Therefore,

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \quad i, j = 1,2,3 \quad i \neq j$$

(2.13)

![Figure 2.3: Unidirectionally reinforced lamina (Jones, 1999)](image)

For unidirectional laminae in the 1-2 plane (Figure 2.3), we define plane stress by setting,

$$\sigma_3 = 0 \quad \tau_{23} = 0 \quad \tau_{31} = 0$$

The strain-stress relation for orthotropic material in Equation (2.9) reduces to,

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (2.14)$$

From Equation (2.12) and (2.13) the values of compliance matrix elements are,

$$S_{11} = \frac{1}{E_1} \quad S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} \quad S_{22} = \frac{1}{E_2} \quad S_{66} = \frac{1}{G_{12}}$$

Equation (2.14) can be inverted to get the stress-strain relation,
\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

(2.15)

In Equation (2.15), \(Q_{ij}\) is the reduced stiffness matrix for plane stress condition in plane 1-2. \(Q_{ij}\) in terms of engineering constants are given by,

\[
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}
\]

\[
Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}
\]

Thus, the reduced stiffness matrix can be calculated from the engineering constants as illustrated above.

The stress-strain relation presented above has been defined in the principle material directions. Now, they need to be defined in the x y z coordinate system. Figure 2.4 illustrates the rotated principal material axes w.r.t the x-y axes. \(\theta\) is the angle made by 1-axis with the x-axis.

![Figure 2.4: Rotated principal material axes from x-y axes (Jones, 1999)](image-url)
The stresses and strains in the principle material directions can be transformed to the global coordinate system by using the following transformation.

\[
[T] = \begin{bmatrix}
\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\
\sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\
-\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta
\end{bmatrix}
\]  

(2.16)

Thus, the reduced transformed stiffness matrix is given by,

\[
[\tilde{Q}] = [T]^{-1}[Q][T]^{-t}
\]

(2.17)

\([T]^{-t}\) indicates the transpose of the inverse of the transformation matrix.

Thus, the transformed stress-strain relation becomes,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{16} \\
\tilde{Q}_{12} & \tilde{Q}_{22} & \tilde{Q}_{26} \\
\tilde{Q}_{16} & \tilde{Q}_{26} & \tilde{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(2.18)

And for a laminae with fibers oriented at some angle w.r.t. the principle material directions, the stress-strain relation in the global coordinate system x-y-z can be calculated using Equation (2.18).
As described earlier a laminate is multiple laminae stacked and bonded together. The individual laminae may have fibers oriented in different local principle material directions w.r.t. the global laminate axes as shown in Figure 2.5. Macromechanics of the laminate is used to study the response of the laminate to loading and calculate the stiffnesses and strengths of the laminate using the properties of the constituent laminae. This is done by using the classical lamination theory (CLT), also known as classical thin lamination theory or classical laminated plate theory (CLPT). The assumptions made in CLT as per Jones (Jones 1999) are as follows,

i. The laminate consists of perfectly bonded laminae and the bonds are infinitesimally thin and non-shear-deformable. Thus, the displacements are assumed to be continuous across lamina boundaries and there is no slip between laminae.
ii. For a thin laminate, the normal to the middle surface is assumed to remain straight and perpendicular to the middle surface even after the laminate is deformed. The assumption of normal to be straight after deformation means that the shearing strains in planes perpendicular to the middle surface, $\gamma_{xz} = \gamma_{yz} = 0$ where $z$ is the direction normal to the middle surface (Figure 2.6). Additionally, the normal are assumed to have constant length so that the strain perpendicular to middle surface, $\varepsilon_z = 0$

![Diagram of laminate deformation](image)

Figure 2.6: Deformation in x-z plane of laminate (Jones, 1999)

Using the above assumptions, the in-plane displacements $u$, $v$ and the out-of-plane displacement $w$ are derived using the laminate cross-section in x-z plane (Figure 2.6).

The displacement of point C in the x-direction can be given by Equation (2.19) because of the assumption that line ABCD remains straight on deformation.

$$u_c = u_o - z_c \beta$$  \hspace{1cm} (2.19)

From Figure 2.6, we see that the slope $\beta$ can be given by,
\[ \beta = \frac{\partial w_o}{\partial x} \]  \hspace{1cm} (2.20)

Substituting Equation (2.20) in Equation (2.19), displacement \( u \) at any point along the laminate thickness can be calculated by,

\[ u = u_o - z \frac{\partial w_o}{\partial x} \]  \hspace{1cm} (2.21)

Similarly, displacement \( v \) can be given by,

\[ v = v_o - z \frac{\partial w_o}{\partial y} \]  \hspace{1cm} (2.22)

According to the assumptions made, \( \varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0 \). Therefore, the remaining strains can be given by,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{bmatrix}
\]  \hspace{1cm} (2.23)

Using Equation (2.21) and (2.22) to derive and substitute in Equation (2.23)

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial u_o}{\partial x} - z \frac{\partial^2 w_o}{\partial x^2} \\
\frac{\partial v_o}{\partial y} - z \frac{\partial^2 w_o}{\partial y^2} \\
\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} - 2z \frac{\partial^2 w_o}{\partial x \partial y}
\end{bmatrix}
\]  \hspace{1cm} (2.24)
\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_x^o \\
\varepsilon_y^o \\
\gamma_{xy}^o
\end{bmatrix} + \begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
\]

Where,

\(\varepsilon_x, \varepsilon_y, \gamma_{xy}\) = Strains at any point along the thickness

\(\varepsilon_x^o, \varepsilon_y^o, \gamma_{xy}^o\) = Middle-surface strains

\(\kappa_x, \kappa_y, \kappa_{xy}\) = Curvatures of laminate due to bending (x,y) and twisting (xy)

Now, the transformed stress-strain relation in Equation (2.18) for a \(k^{th}\) layer can be expressed in terms of the laminate mid-surface strains and curvatures as,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
_k = \begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix}_k \begin{bmatrix}
\varepsilon_x^o \\
\varepsilon_y^o \\
\gamma_{xy}^o
\end{bmatrix} + \begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
\]

(2.25)

Force and Moment Calculation:

The forces and moments (Figure 2.7, Figure 2.8) for a lamina are obtained by integrating the stresses through the lamina thickness from \(z_k\) to \(z_{k-1}\) (Figure 2.9). For an N-layered laminate the same can be obtained by the summation of the forces and moments for each lamina and is given by Equation (2.26) and (2.27) respectively.
Figure 2.7: Forces in x-y plane of laminate (Jones, 1999)

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k \, dz
\]

(2.26)

Figure 2.8: Moments on laminate (Jones, 1999)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k \, z \, dz
\]

(2.27)
When the laminar stress-strain relation Equation (2.25) is substituted into the force and moment equations, they become

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{x}^o & \varepsilon_{y}^o & \gamma_{xy}^o \\
\gamma_{xy}^o & \gamma_{xy}^o & \gamma_{xy}^o
\end{bmatrix} \begin{bmatrix}
\int_{z_{k-1}}^{z_k} dz \\
\int_{z_{k-1}}^{z_k} z \, dz
\end{bmatrix} + \sum_{k=1}^{N} \begin{bmatrix}
K_x & \kappa_y & \kappa_{xy} \\
\kappa_y & \kappa_y & \kappa_{xy}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{x}^o & \varepsilon_{y}^o & \gamma_{xy}^o \\
\gamma_{xy}^o & \gamma_{xy}^o & \gamma_{xy}^o
\end{bmatrix} \begin{bmatrix}
\frac{t_k}{2} & \frac{t_k}{2} & \frac{t_k}{2} \frac{t_k}{2}
\end{bmatrix} \begin{bmatrix}
\int_{z_{k-1}}^{z_k} dz \\
\int_{z_{k-1}}^{z_k} z \, dz
\end{bmatrix} \quad (2.28)
\]

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{x}^o & \varepsilon_{y}^o & \gamma_{xy}^o \\
\gamma_{xy}^o & \gamma_{xy}^o & \gamma_{xy}^o
\end{bmatrix} \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \frac{z^2}{2} \, dz \\
\int_{z_{k-1}}^{z_k} \frac{z^3}{3} \, dz
\end{bmatrix} + \sum_{k=1}^{N} \begin{bmatrix}
K_x & \kappa_y & \kappa_{xy} \\
\kappa_y & \kappa_y & \kappa_{xy}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{x}^o & \varepsilon_{y}^o & \gamma_{xy}^o \\
\gamma_{xy}^o & \gamma_{xy}^o & \gamma_{xy}^o
\end{bmatrix} \begin{bmatrix}
\frac{t_k}{2} & \frac{t_k}{2} & \frac{t_k}{2} \frac{t_k}{2}
\end{bmatrix} \begin{bmatrix}
\int_{z_{k-1}}^{z_k} \frac{z^2}{2} \, dz \\
\int_{z_{k-1}}^{z_k} \frac{z^3}{3} \, dz
\end{bmatrix} \quad (2.29)
\]

Since \(\varepsilon_{x}^o, \varepsilon_{y}^o, \gamma_{xy}^o, \kappa_x, \kappa_y, \kappa_{xy}\) are middle-surface values and are not dependent on \(z\), they can be removed from the integral and the summation in Equation (2.28) and (2.29) and be written as,
\[ A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k} - z_{k-1}) \]

\[ B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k}^2 - z_{k-1}^2) \]  
(2.30)

\[ D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k}^3 - z_{k-1}^3) \]

In Equation (2.30) \( A_{ij} \) is the extensional stiffness, \( B_{ij} \) is the bending-extension coupling stiffness, and \( D_{ij} \) is the bending stiffness. If \( B_{ij} \) exists, it implies that there exists a coupling between the extension and the bending of the laminate, which means that on application of an extensional force it will extend and bend the laminate at the same time. Thus, for a symmetric laminate \( B_{ij} = 0 \) and the extensional and bending stiffnesses are not coupled. While for an asymmetric laminate, \( B_{ij} \neq 0 \), hence post curing these laminates deform due to coupled extension and bending.

### 3.2 Analytical models

As stated earlier, Hyer (Hyer, 1981) started off with the study of square T300/5208 unsymmetric graphite-epoxy laminates with a \([0_n/90_n]_T\) type of cross-ply layup. He observed that the cylindrical shapes that this family of laminates exhibited did not conform with those by classical laminate theory which predicted a saddle shape at room temperature (Figure 2.10). To explain the cylindrical shapes, Hyer extended on the classical theory by incorporating geometric non linearities by using polynomial approximations for
displacements. He assumed that the out-of-plane deflections develop due to the difference in thermal expansion properties of the individual lamina.

Figure 2.10: Laminate shapes, (a) flat shape pre-curing, and at room temperature (b) unstable saddle shape, (c) pre-snapped shape, (d) post-snapped shape (Hyer, 1981)

Hyer and his colleagues’ work (M. Dano & Hyer, 1998; M. L. Dano & Hyer, 2000; Hyer, 1981) used quadratic polynomials to approximate the midplane strains and the out of plane displacement. Further, the Rayleigh-Ritz technique in conjunction with Classical Laminate Plate theory is deployed to minimize the total strain energy of the laminate and solve for the coefficients of the polynomial, thereby predicting the shape of the laminate post curing. Hyer and Dano’s subsequent work (M. L. Dano & Hyer, 2000) focuses more on the snap through of these laminates along with the forces required to snap. Hyer’s initial model which dealt with cross ply laminates was extended to a more general layup of unsymmetric laminates by Dang and Tang (1986).
We can summarize Hyer’s contributions by listing down the following formulations used in his model. The in-plane strains ($\epsilon_x^0, \epsilon_y^0$) and out-of-plane displacement ($w$) are assumed to be a polynomial function of $x$ and $y$. Since the stable shapes are observed to be cylindrical, the polynomial for $w$ is assumed to be,

$$\epsilon_x^0 = c_1 + c_2 x^2 + c_3 y^2 + c_4 xy$$

$$\epsilon_y^0 = c_5 + c_6 x^2 + c_7 y^2 + c_8 xy$$

$$w = \frac{1}{2} (c_9 x^2 + c_{10} y^2 + c_{11} xy)$$

The non-linear strain by Von-Karman is defined by,

$$\epsilon_x = \epsilon_x^0 + z\kappa_x^0$$

$$\epsilon_y = \epsilon_y^0 + z\kappa_y^0$$

$$\gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy}^0$$

And the midplane strains and curvatures in the above equations are given by,

$$\epsilon_x^0 = \frac{\partial v^o}{\partial x} + \frac{1}{2} \left( \frac{\partial w^o}{\partial x} \right)^2$$

$$\epsilon_y^0 = \frac{\partial v^o}{\partial y} + \frac{1}{2} \left( \frac{\partial w^o}{\partial y} \right)^2$$

$$\gamma_{xy}^0 = \frac{\partial u^o}{\partial y} + \frac{\partial v^o}{\partial x} + \frac{\partial^2 w^o}{\partial x \partial y}$$

Which can be rearranged as,
\[
\frac{\partial u^0}{\partial x} = \varepsilon_x^0 - \frac{1}{2} \left( \frac{\partial w^0}{\partial x} \right)^2 \\
\frac{\partial v^0}{\partial y} = \varepsilon_y^0 - \frac{1}{2} \left( \frac{\partial w^0}{\partial y} \right)^2
\] (2.34)

Thus, by using the approximations for the in-plane strains \( \varepsilon_x^0, \varepsilon_y^0 \) in Equation (2.31) we can integrate Equation (2.34) to obtain the displacements \( u^0(x, y), v^0(x, y) \).

The thermal forces \( N^{th} \) and moments \( M^{th} \) are given by,

\[
[N^{th}] = \Delta T \sum_{k=1}^{N} (Q_{ij})_k \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} (z_k - z_{k-1})
\]

\[
[M^{th}] = \frac{1}{2} \Delta T \sum_{k=1}^{N} (Q_{ij})_k \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} (z_k^2 - z_{k-1}^2)
\] (2.35)

CLT formulation is used in conjunction with Equation (2.31) and (2.35) to obtain the equation for the total strain energy. The total strain energy of the laminate taking into consideration the thermal effects is given by,

\[
\Pi = \int_{-L_y/2}^{L_y/2} \int_{-L_x/2}^{L_x/2} \frac{1}{2} [\varepsilon^0_k] \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} [\varepsilon^0] - [N^{th}] [\varepsilon^0] dxdy
\] (2.36)

The strain energy \( \Pi \) in Equation (2.36) is minimized to obtain the coefficients of the approximations in Equation (2.31). In the above equation, \( L_x, L_y \) are the x and y dimensions of the patch, \( \varepsilon^0, \kappa^0 \) are the mid-plane strains and curvatures and \( A_{ij}, B_{ij}, D_{ij} \) are the
stiffnesses. The formulation will be discussed at length in Chapter 3. This process of minimizing the total strain energy is called the Rayleigh Ritz approach. Thus, the stable shapes of the bistable laminate can be plotted using \( u^o, v^o, w^o \) which are functions of \( x, y \) and the coefficients of the approximations. The strain energy is a function of the composite laminate parameters fiber orientation, no. of plies, ply thickness, patch dimension and the material properties.

Mattioni’s model (Mattioni et al., 2009) goes a step further and predicts the shape of two-patch laminates. The basis for this model is the same as Hyer’s but instead of having free boundary conditions at the edges, boundary conditions to account for the interaction between the two patches are introduced at the common edge. These are geometric boundary conditions which state that the out of plane displacements \( w \) and the in-plane displacements \( u, v \) along with the first derivatives of the out of plane displacements \( \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} \) are equal at the common edge between the two patches. In addition to the boundary conditions, another change from Hyer’s single patch model is that the approximation for the out-of-plane displacement is changed to a higher order polynomial as shown in Equation (2.37), since the assumption of constant curvature may not hold for two patches connected the constant.

\[
w = w_{00} + w_{10}x + w_{01}y + w_{20}x^2 + w_{02}y^2 + w_{11}xy + w_{12}xy^2 + w_{21}x^2y + w_{22}x^2y^2 \tag{2.37}
\]

Some of the shortcomings in this paper not allowing it to be used in a design method are that; one of the patches is symmetric and the other asymmetric and cases where both patches are asymmetric have not been discussed. Another issue with this study was that
since one of the patches was symmetric, the laminate only offered two shapes despite having two patches. Also, according to Arrieta et al., 2014 for any practical application, both ends of the laminate would have to be clamped; this made the laminate monostable, since both of the patches were being clamped as shown in Figure 2.11.

Figure 2.11: Free edge clamping of bistable two-patch laminates (Arrieta et al., 2014)

They furthered the research by presenting a variable stiffness laminate using multiple asymmetric laminates, connected by symmetric ones; which could be embedded in larger structures effectively. Udani & Arrieta, 2019 also studied its use in a variable stiffness airfoil wing.

Cui & Santer, 2015 furthered the work by studying the characteristics of laminates with multiple connected asymmetric patches. They presented a continuous compound laminate with multiple bistable patches connected, which was multistable.

The model by Algmuni et al., 2020 paved the way for use of these models for the design of large compound laminates with multiple bistable patches connected. Algmuni’s
model predicts the shape of a four-patch grid composite laminate. It is an elaborate extension of Mattioni’s and Arrieta’s models whereby the geometric boundary conditions referred to in Mattioni’s model are applied at the common edges and the total strain energy which is the sum of the energies of each patch is minimized subject to those constraints. This model could be used to design four-patch laminates to target specific shapes. Also, this model can be easily extended to more than four patches by considering the boundary conditions at all common edges.

3.3 **Finite Element Method & Experiment**

In their work, Hyer and Dano (M. Dano & Hyer, 1998) presented the model using the Rayleigh Ritz technique and made comparisons with FEA and experimental data for single patch cross-ply laminates. The analytical model captured the shape for the most part, except for deviations at the edges as shown in Figure 2.12.

Schlecht and Schulte (Schlecht & Schulte, 1999) performed an FEA study for single patch \([0_2/90_2]\) laminates comparing them with the results from the analytical model. They observed “edge effects” which make the laminate take up a slight saddle shape at the edges. Betts et al., 2010 also had similar observations in their work. They carried out an experimental study to correlate the results from the analytical model and experiment. Deviation from the analytical results were observed at the corners and edges. Studies were conducted for the multi-patch laminates with similar deviations at the edges (Algmuni et al., 2020; Mattioni et al., 2009).
Figure 2.12: Comparison of analytical, FEA and experimental shapes for (a) $[90_4/0_4]_T$ (b) $[-30_4/30_4]_T$ (c) $[60_4/30_4]_T$ laminates (M. Dano & Hyer, 1998)

From the literature, the consensus is that the overall shape of the laminate correlates relatively well with both FEA and experiments. The only differences are the discrepancies in the z-displacements at the edges. Since we need an approximate representation to be able to design the desired shapes, we can use these models as a tool to design multi-patch laminates; but we need to do so with caution and use the model in conjunction with FEA to converge to the targeted shape.

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CHAPTER THREE

DEVELOPMENT OF SURROGATE MODELS

As stated earlier, the goal of this research project is to establish a design method for multi-patch composite laminates. Since the post-cure shape of the multi-patch laminate is a function of the individual patch geometries and of the orientation of the fibers in each layer, a possible approach to determine a desired shape would be to deploy an ‘All at Once’ optimization workflow with the FEA model as the simulation component and all the input variables like fiber orientation, no. of plies, ply thickness, no. of patches, patch size, etc. set as design variables and converge to the required target shape.

*All at Once Optimization problem*

Design Variables: \([\theta_F, n_{plies}, t_{ply}, n_{patches}, l_{patch}, w_{patch}]\)

Objective function: \(|S_{FE} - S_{Target}|\)

Constraints: *Manufacturing constraints related to D.V. if any*

Where,

\[\theta_F = \text{Fiber orientations of patches}\]

\[n_{plies} = \text{No. of plies}\]

\[t_{ply} = \text{Ply thickness}\]
\[ n_{\text{patches}} = \text{No. of patches} \]

\[ l_{\text{patch}} = \text{Length of patch} \]

\[ w_{\text{patch}} = \text{Width of patch} \]

\[ S_{FE} = \text{FEA predicted shape of laminate} \]

\[ S_{Target} = \text{Target shape of laminate} \]

It is clear from the optimization problem above that this would be a brute force optimization, where all the variables that affect the shape of the laminate are parameterized and run through FEA. Simulating the curing and snapping of a single patch asymmetric laminate on ABAQUS takes roughly five minutes or more. Thus, modeling an \( n \) patch multistable laminate would be computationally more expensive, and using FEA for the optimization of these laminates would not be practical.

![Figure 3.1: Boundary conditions in multi-patch layout](image-url)
Thus, a different strategy needed to be deployed that makes use of the information of the boundary conditions that exist at the common patch edges as illustrated in the Figure 3.1 to resolve the shape of each patch subject to those BC’s and its design variable values. Also, as seen in the literature the room temperature shapes of the single and two patch laminates had visible deviations at the edges of the laminate. To overcome these drawbacks the use of Artificial Neural Networks (ANN’s) was considered to aid in predicting the stable shapes. Use of ANN’s would be a data driven approach that could bypass the underlying physics of these laminates and identify patterns in the data and be able to approximate the shape of the laminate. The data referred to here would be the results of the stable shapes obtained from an FEA package, for a sufficiently large sample for the ANN to train on. Artificial neural networks and their possible application to solve this problem is discussed in the following sections.

4.1 ANN to Predict Two-Patch Laminate Shapes

Artificial neural networks (ANNs) are a machine learning technique which is popularly being used in engineering applications related to composite laminates, like fatigue prediction, wear simulation, manufacturing processes, curing analysis, etc. (Ye et al., 2005). Machine learning methods can solve complex problems without explicitly modeling the physical laws governing the model. It does so by recognizing patterns in the input and output data of a model, and bypasses the expensive physics-based calculations, making the optimization of larger systems efficient(Gu et al., 2018). The ANN architecture takes inspiration from the human nervous system with multiple nerves connected to each other.
The architecture of an ANN as depicted in Figure 3.2 usually has an input and output layer which represent the input and output variables, respectively. The input and output layers are connected via one or more hidden layers. If a neural network has more than one hidden layer, it is called a deep neural network. Every layer is constituted of building blocks called neurons. Every neuron in the hidden and output layers has a weight and an activation function associated with them. The output from each neuron can be given as follows,

$$X_j^{(n+1)} = F \left( \sum_i W^{(n)}_{ji} X_i^{(n)} \right)$$  \hspace{1cm} (3.1)

$F(x) = Activation\ function$

$W_{ji}^{(n)} = Weight\ for\ connection\ between\ output\ of\ unit\ i\ in\ n^{th}\ layer\ to\ unit\ j\ in$
(n + 1)\textsuperscript{th} layer

The input parameters and the initial weights are fed in by the user. At every node in the next layer the inputs are weighted and summed. Then it is passed through the activation function which basically squashes the values and limits the amplitude of the output node. Thus, the output of each neuron is computed using the weighted sum equation described in Equation (3.1).

Deploying an ANN helps in establishing an implicit relation between the input and output parameters without exploring the constitutive relation between the parameters. They do so by learning from examples like a human brain does. ANN’s are trained to identify the functional relationships in the model without prior assumptions. The size of the sample data for training can vary according to the application. The ANN uses the training set to update the weights to get a closer fit with the outputs. By progressively modifying the weights, the algorithm learns and becomes able to then mimic the behavior of an unknown model when inputs are modified (Sapuan & Mujtaba, 2010; Ye et al., 2005).

In the design of multistable composite laminates, we know from the literature review that the final shapes of the laminates which is the output depends on input parameters like fiber orientation, no. of plies, ply thickness, etc. ANN’s could be used in this case to establish relationships between the input and the output parameters of this model. The scope of this section of the research deals with understanding the dependence of fiber orientation on the shape of the laminates in pre-snapped and post-snapped states.
After reviewing the work on single patch laminates from earlier theses of our alumni (Annamalai, 2016; A. G. Lele, 2018) and analytical models by Hyer and Mattioni, the need to predict shapes of dual-patched laminates was identified. Hyer’s and Mattioni’s models predicted the shapes of single and dual-patch laminates, respectively. Thus, the initial approach consisted in implementing a machine learning/neural network algorithm to predict the shapes of these laminates because this approach would not only be useful in predicting the shapes across the whole design space, but also, once trained, would be much faster than FEA.

![Figure 3.3: A 2-ply, dual-patch laminate. The left half (L) has asymmetric fiber layout, and the right half (R) has symmetric fiber layout](image)

In the initial part of the study, a dual-patch two ply laminate as shown in Figure 3.3 was considered; where the right patch has a symmetric layup, and the left one is asymmetric. The fiber orientations for the right patch are, $\theta_3 = \theta_4 = 0^\circ$, and the design variables (input variables) are $\theta_1, \theta_2$.

To predict the shape of the laminate, we chose 16 points on the laminate which the Neural Network would have to predict. The reason behind choosing 16 points was to be able to
approximate the surface by using bicubic interpolation which is an effective method to represent a surface as a polynomial which is given by the following equation.

\[ z = \sum a_{ij} x^i y^j \quad \text{where } i, j = 0,1,2,3 \quad (3.2) \]

The bicubic surface would give a close prediction of the shape, while higher order approximations could result in artificial “oscillations” of the surface.

The number of output variables are 48 (coordinates of the 16 points in space). Thus, the initial plan was to feed in the two fiber orientations corresponding to the asymmetric side to the ANN and obtain 48 outputs that correspond to the location of the 16 points on the post cured surface of the laminate as shown in Figure 3.4.

**Sample data:**

To generate the sample data datasets, different fiber orientation combinations were simulated on ABAQUS. The range for the fiber orientations was \(-90^\circ < \theta < 90^\circ\). And a Uniform Latin hypercube was used to randomize the inputs with a step of 5° for the fiber orientations. The designs were simulated by parametrizing the input variables in the
ABAQUS code simulating the shape of a dual-patch laminate and the size of the sample dataset was 110 designs. The sample data is shown in Table 3.1. In the table, COX_1, COY_1, COZ_1 denote the x, y, z coordinate values of the first out of the 16 fit-points and so on.

Table 3.1: Sample data for two-patch laminate prediction

<table>
<thead>
<tr>
<th>&lt;ID&gt;</th>
<th>Fiber angle (°)</th>
<th>Nodal displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>θ₁, θ₂</td>
<td>COX₁</td>
</tr>
<tr>
<td>0</td>
<td>75.00</td>
<td>-35.00</td>
</tr>
<tr>
<td>1</td>
<td>-5.00</td>
<td>10.00</td>
</tr>
<tr>
<td>2</td>
<td>45.00</td>
<td>60.00</td>
</tr>
<tr>
<td>3</td>
<td>-90.00</td>
<td>-25.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>107</td>
<td>55.00</td>
<td>0.00</td>
</tr>
<tr>
<td>108</td>
<td>50.00</td>
<td>70.00</td>
</tr>
<tr>
<td>109</td>
<td>-70.00</td>
<td>-55.00</td>
</tr>
</tbody>
</table>

To improve the performance of the neural network the sample data was normalized using minmax normalization as shown in Equation (3.3) which transforms the data to a range from 0 to 1, thus transforming the data to the same range of magnitude.

\[
x_{new} = \frac{x_{old} - x_{min}}{x_{max} - x_{min}}
\]

(3.3)

To aid the ANN in improving the performance, the initial location of the 16 points on the flat laminate before curing were fed additionally as the input. Thus, finally having 50 inputs (2 angles and 48 coordinates) and 48 outputs.
**Results**

MATLAB’s neural network module was used to create the ANN’s and the approach to solve the problem was by trial and error. Multiple neural network architectures with different numbers of hidden layers, and number of neurons in each layer were used and the sample data was used to train the network and simulate the results. The metric to compare the results of the multiple trials was the ‘Performance’ value which was chosen as the Mean squared error.

<table>
<thead>
<tr>
<th>Table 3.2: ANN specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of NN</td>
</tr>
<tr>
<td>Hidden Layers</td>
</tr>
<tr>
<td>No. of neurons</td>
</tr>
<tr>
<td>Performance function</td>
</tr>
<tr>
<td>Data division</td>
</tr>
</tbody>
</table>

The neural network specifications are shown in Table 3.2 and the architecture is shown in Figure 3.5.

The performance function was selected as the mean squared error. Different combinations of no. of layers and neurons were used, and the least error was obtained in the above stated
configuration. The training set error was successfully minimized, and the best validation performance was obtained with an MSE of $7.35 \times 10^{-6}$ as illustrated in Figure 3.6.

![Best Validation Performance is 7.3522e-06 at epoch 1](image)

Figure 3.6: Mean Squared Error

The simulated outputs of the ANN were then denormalized and plotted on MATLAB. It was observed that the ANN was able to predict the nature of the surfaces quite well, as illustrated in Figure 3.7 where the results for a two-patch laminate with configurations (a) $[80^\circ/-5^\circ]$, $[0^\circ/0^\circ]$ and (b) $[-30^\circ/90^\circ]$, $[0^\circ/0^\circ]$ are plotted. The magnitude of the errors in the outputs ranged from 0-2.5mm. It was also observed that the z-coordinates had higher errors than the x and y coordinates because the maximum displacements are in the z direction.
The ANN has a clear advantage over FEA in terms of computational cost. The simulation for a 2-patch laminate on ABAQUS takes 8-15 minutes to run depending on the processor used. Though the ANN requires a long time to train, a trained network can predict the outputs and approximate the shape instantly.

In this work the ANN is used to predict a single shape and not both the pre and post snapped shapes. This is because predicting both shapes would require a more complicated approach to the neural network setup and training, since the number of outputs would double. Thus, it made sense to verify the working of the ANN to predict the first state before moving to the post snapped state. Future work would include predicting the post

Figure 3.7: FEA (green) and ANN (red) predicted surface and fit-points of post cured laminates
snapped shapes of the laminates and extending this approach to multi patch laminates (more than two).

4.2 ANN to Predict Shapes of Single Patch Laminates

As explained in the start of this section (Figure 3.1) to create a design method for multi-patch laminates a simplified model that can predict the shape of a single patch while specifying the edge conditions was necessary. Hyer’s model (M. L. Dano & Hyer, 2000) is capable of predicting the stable states of a single patch laminate using an analytical approach making use of Classical Laminate Theory. The drawback of this model is its inability to converge to the solution without a correct initial guess. Thus, to explore other ways to approximate the stable shapes, an ANN approach as explained in the previous section was considered. Additionally, geometric modeling techniques like parametric cubic surface approximation were coupled with the ANN approach in a bid to account for the edge conditions.

As illustrated in Figure 3.8, the goal was to create an ANN that would tie the input parameters of fiber orientation and aspect ratio to the bistable shapes of the laminate. It
was created for a two-ply single patch laminate. For simplicity, the fiber orientation of the lower ply was fixed as 0° and the aspect ratio was fixed as one. The output parameters would be the fit-points that would help create the surface patch. Shape prediction of single patch laminates have been done through two approaches:

i. Bicubic Interpolation

In this approach the input parameters would be unchanged, but the output parameters would now be the coefficients of the bicubic equation given by,

\[
z = a_{33}x^3y^3 + a_{32}x^3y^2 + a_{31}x^3y + a_{30}x^3 + a_{23}x^2y^3 + a_{22}x^2y^2 + a_{21}x^2y \\
+ a_{20}x^2 + a_{13}xy^3 + a_{12}xy^2 + a_{11}xy + a_{10}x + a_{03}y^3 + a_{02}y^2 + a_{01}y + a_{00}
\]

(3.4)

Figure 3.9: Bicubic patch

Since the bicubic equation has 16 unknowns, 16 points across the laminate are required to solve for the coefficients. The pre-processing involves extracting those 16 points from the FEA data for a patch (Figure 3.9). This is used to solve for the 16 unknown coefficients in the bicubic equation. The 16 coefficients (output) along with the fiber orientations (input) constitute the sample data for the ANN.
**Results**

The ANN architecture is as shown in Table 3.3. The input variables are the top and bottom ply angles of the single patch laminate, while the output variables are the values of the 16 coefficients from the bicubic interpolation as shown in Equation (3.4). The architecture of the layer recurrent type network is as shown in Figure 3.10. The dataset for training the ANN consists of the fiber orientations with a range of $-90^\circ < \theta < 90^\circ$. And uniform Latin hypercube was used to randomize the inputs with a step of $5^\circ$ for the fiber orientations. A dataset of size 50 is used to train the ANN. As compared to the two-patch case discussed previously, for the shape prediction of single patch laminates a simpler network with fewer hidden layers, number of neurons and a smaller training set is sufficient. This could be attributed to the fact that, unlike the single-patch laminate where all outer edges are free, the ANN must deal with the edge conditions that exist at the common boundaries of a two-patch laminate discussed previously.

<table>
<thead>
<tr>
<th>Table 3.3: ANN architecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: $\theta_1$ &amp; $\theta_2$ (fiber angle of top and bottom ply)</td>
</tr>
<tr>
<td>Output: 16 coefficients</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Figure 3.10: ANN Structure (MATLAB)

The performance graph for the above architecture post training is illustrated in Figure 3.11. As seen from the graph, the MSE for the trained network is 6.0384e-9 which is higher than that for the two-patch case. The better performance can again be attributed to the simplicity of the problem as compared to the two-patch data.

Figure 3.11: ANN Performance

Figure 3.12: [60°/0°] Laminate; Red-Target and Green-Predicted shape

In Figure 3.11 the results of the ANN simulated for [60°/0°] laminate is plotted. The graph depicts the superimposed surfaces of the FEA, or target (Red) and the ANN predicted shape.
(Green). From the results we see that the ANN can predict post cured shapes of single patch, two-ply laminates using the bicubic interpolation approach.

ii. Parametric Cubic Patch Formulation

To explore other ways to approximate the shape of the laminate based on the ANN’s outputs, geometric modeling techniques like parametric cubic surface approximation was considered. Some of the drawbacks of the bicubic interpolation method was that the outer edges of the laminate could not be accurately plotted, since just plotting a bicubic polynomial equation within the x, y bounds would not define the edges of the surface. Secondly as explained earlier, individual patches had to be defined and eventually build the whole laminate. Thus, defining the edges became important. Parametric cubic approximation was a better fit to achieve that target. Additionally, it gave information of the parametric tangent and curvature values which could be leveraged in a meaningful way.

Using geometric modeling techniques, surfaces can be approximated by dividing them into patches, where each patch in the xyz space is mapped to a unit patch in uv space (Figure 3.12). And the coordinates x, y and z are expressed as a parametric cubic polynomial in u and v. The general equation for parametric cubic formulation can be given by Equation (3.5) where Ø represents the three coordinates.
\[
\emptyset(u,v) = [F_1(u) \ F_2(u) \ F_3(u) \ F_4(u)] \begin{bmatrix} \emptyset(0,0) & \emptyset(0,1) & \emptyset_v(0,0) & \emptyset_v(0,1) \\ \emptyset(1,0) & \emptyset(1,1) & \emptyset_v(1,0) & \emptyset_v(1,1) \\ \emptyset_u(0,0) & \emptyset_u(0,1) & \emptyset_{uv}(0,0) & \emptyset_{uv}(0,1) \\ \emptyset_u(1,0) & \emptyset_u(1,1) & \emptyset_{uv}(1,0) & \emptyset_{uv}(1,1) \end{bmatrix} \begin{bmatrix} F_1(v) \\ F_2(v) \\ F_3(v) \\ F_4(v) \end{bmatrix}
\]

\[
\emptyset(u,v) = F_u \ P \ F_v^T \tag{3.5}
\]

In the above equation \(F_u\) and \(F_v\) are the blending functions in \(u\) and \(v\). And in the \(P\) matrix, the terms in the top left represent the four corner points as seen in Figure 3.12. The top right and the bottom left terms represent the \(u\) and \(v\) tangents at the corner points, and the bottom right is the twist vector. Thus, Equation (3.5) can now be written as shown below.

\[
\emptyset = \begin{bmatrix} \text{Corner points} & \text{v tangent vectors} \\ \text{u tangent vectors} & \text{Twist vectors} \end{bmatrix} \{\text{Blending function in v}\} \tag{3.6}
\]

Where the blending functions can be given as the following,

\[
F_u = UN \quad F_v = NV \tag{3.7}
\]

In Equation (3.7) \(U\), \(V\) and \(N\) are given by,

\[
U = [1 \quad u \quad u^2 \quad u^3] \quad V = [1 \quad v \quad v^2 \quad v^3]^T
\]

\[
N = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \tag{3.8}
\]
Thus, as shown in Figure 3.12 a larger surface can be divided into patches and the surface can be approximated by using the parametric cubic formulation stated above. For the single patch we can make use of 9 points across the laminate to create the surface adequately. Figure 3.13 illustrates the single-patch laminate where the surface is divided into four sub-patches to be approximated using the parametric cubic equation. The surface with control points 1-2-5-4 represents Patch-1, 2-3-6-5 for patch-2, 4-5-8-7 for patch-3 and 5-6-9-8 for patch-4. The points would be extracted from the FEA data for that patch. The sample data for the ANN would consist of the fiber orientations as the input and the fit-points as the output.
Results

The ANN architecture is as given in Table 3.4. The ANN for this approach consists of 2 input variables which are the top and bottom ply angles for the single patch laminate, and the output variables are the coordinates of the nine fit points, thus a total of 27 outputs. The architecture of the layer recurrent type network is as shown in Figure 3.15. The dataset for training is like the one previously used where the fiber orientations have a range of $-90^\circ < \theta < 90^\circ$ with a step of $5^\circ$ and uniform Latin hypercube was used to randomize the inputs.

Table 3.4: ANN architecture

<table>
<thead>
<tr>
<th>Input: $\theta_1$ &amp; $\theta_2$ (fiber angles of top and bottom ply)</th>
<th>Network: Layer Recurrent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output: 27 (3 coordinates x 9 points)</td>
<td>No. of hidden layers: 1</td>
</tr>
<tr>
<td></td>
<td>No. of neurons: 20</td>
</tr>
</tbody>
</table>
The performance graph for the above architecture post training is illustrated in Figure 3.16 and the ANN simulated result for \([-30^\circ/0^\circ]\) is plotted in Figure 3.17. As seen from the graph, the MSE for the trained network is \(9.945 \times 10^{-8}\) which is comparable to the performance of the bicubic interpolation case. These results again indicate that the ANN can predict post cured shapes of single patch, two-ply laminates using the parametric cubic approach.
4.3 Analytical Model for Four-Patch Grid Laminate

As discussed earlier, Hyer’s model predicts the shapes of single patch laminates. A recent addition to the literature is the model by Algmuni et. al. (Algmuni et al., 2020) for a four-patch grid laminate. It is an extension of Hyer’s model (M. Dano & Hyer, 1998) but with added continuity constraints at the common edges of the patches, like those by Mattioni (Mattioni et al., 2009) to tie the four patches. The equations to setup the model are listed below (Algmuni et al., 2020).

Algmuni Model Formulation

We know from Hyer’s model discussed in the literature review that the total strain energy for an $n^{th}$ patch can be given by the following,

$$
\Pi_n = \int_{-D_{yn}/2}^{D_{yn}/2} \int_{-D_{xn}/2}^{D_{xn}/2} \frac{1}{2} \epsilon^0 \left[ A_{ij} \right] \left[ B_{ij} \right] \epsilon^0 \left[ K^0 \right]^{-1} \left[ N_{Mth}^{th} \right] \epsilon^0 \left[ K^0 \right] dx dy
$$

(3.9)

Where,

$n$ = Patch number

$\Pi_n$ = Strain energy of patch

$D_{xn}$ = x dimension of patch

$D_{yn}$ = y dimension of patch

$\epsilon^0$ = mid-plane strain
\[ \kappa^0 = \text{Laminate curvature} \]

\[ A_{ij}, B_{ij}, D_{ij} = \text{extensional, bending-extension and bending stiffness respectively} \]

\[ N^{th} = \text{Resultant force matrix} \]

\[ M^{th} = \text{Resultant moment matrix} \]

As discussed in the micromechanics of a laminate in the literature review, \( A_{ij}, B_{ij}, D_{ij} \) can be given by Equation (2.30). All the remaining terms in Equation (3.9) are stated below.

The force and moment matrices \( (N^{th}, M^{th}) \) due to thermal effects can be given by,

\[
[N^{th}] = \Delta T \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (z_k - z_{k-1})
\]

\[
[M^{th}] = \frac{1}{2} \Delta T \sum_{k=1}^{N} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k (z_k^2 - z_{k-1}^2)
\]

(3.10)

Where, \( (\bar{Q}_{ij})_k \) is the reduced transformed stiffness matrix, \( \alpha_x, \alpha_y, \alpha_{xy} \) are the transformed thermal expansion coefficients and \( \Delta T \) is the temperature drop during curing.

Approximations for the out-of-plane displacement \( w^0 \) and mid-plane strains \( \varepsilon^0_x, \varepsilon^0_y \)

\[
w^{0(n)} = w_{22}^{(n)} x^2 y^2 + w_{21}^{(n)} x^2 y + w_{12}^{(n)} xy^2 + w_{20}^{(n)} x^2 + w_{02}^{(n)} y^2 \\
+ w_{11}^{(n)} xy + w_{10}^{(n)} x + w_{01}^{(n)} y + w_{00}^{(n)}
\]

\[
\varepsilon_x^{0(n)} = \varepsilon_{x00}^{(n)} + \varepsilon_{x11}^{(n)} xy + \varepsilon_{x20}^{(n)} x^2 + \varepsilon_{x02}^{(n)} y^2
\]

(3.11)
\[ \varepsilon_{y}^{(n)} = \varepsilon_{y00}^{(n)} + \varepsilon_{y11}^{(n)} xy + \varepsilon_{y20}^{(n)} x^2 + \varepsilon_{y02}^{(n)} y^2 \]

The in-plane displacements can now be calculated and are given by,

\[ u_0^{(n)} = \int \left( \varepsilon_x^{(n)} - \frac{1}{2} \left( \frac{\partial w_0^{(n)}}{\partial x} \right)^2 \right) dx + h^{(n)}(y) \]

\[ v_0^{(n)} = \int \left( \varepsilon_y^{(n)} - \frac{1}{2} \left( \frac{\partial w_0^{(n)}}{\partial y} \right)^2 \right) dy + g^{(n)}(x) \]  

\[ (3.12) \]

Where, \( h^{(n)}(y) \) and \( g^{(n)}(x) \) are added because of partial integration to suppress rigid body rotation

\[ h^{(n)}(y) = u_{01}^{(n)} y + u_{03}^{(n)} y^3 \]

\[ g^{(n)}(x) = v_{10}^{(n)} x + v_{30}^{(n)} x^3 \] 

\[ (3.13) \]

The shear strain can now be defined by,

\[ \gamma_{xy}^0 = \frac{\partial u_0^0}{\partial y} + \frac{\partial v_0^0}{\partial x} + \frac{\partial w_0^0}{\partial x} \frac{\partial w_0^0}{\partial y} \]  

\[ (3.14) \]

Now the mid-plane strain matrix can be formed,

\[ \varepsilon^0 = \begin{bmatrix} \varepsilon_x^0 & \varepsilon_y^0 & \gamma_{xy}^0 \end{bmatrix}^T \] 

\[ (3.15) \]

The curvatures in the x, y and twist directions are given by,
\[
\kappa^0_x = -\frac{\partial^2 w^0}{\partial x^2} \quad \kappa^0_y = -\frac{\partial^2 w^0}{\partial y^2} \quad \kappa^0_{xy} = -2 \frac{\partial^2 w^0}{\partial x \partial y}
\]

Thus, the curvature matrix \(\kappa^0\) can now be formed

\[
\kappa^0 = \begin{bmatrix} \kappa^0_x & \kappa^0_y & \kappa^0_{xy} \end{bmatrix}^T
\]

In summary the design variables for this level are the unknown coefficients used for the approximations in Equation (3.11)(3.13). The set of unknown coefficients for a particular patch are denoted by \(c_n\) where \(n\) is the patch number in the laminate.

\[
c_n = [w_{22}, w_{21}, w_{12}, w_{20}, w_{02}, w_{11}, w_{10}, w_{01}, w_{00}, \varepsilon_{x00}, \varepsilon_{x11}, \varepsilon_{x20}, \varepsilon_{x02},
\varepsilon_{y00}, \varepsilon_{y11}, \varepsilon_{y20}, \varepsilon_{y02}, u_{01}, u_{03}, v_{10}, v_{30}]^{(n)}
\]

Thus, for a four-patch laminate the sets of unknown coefficients would be \(c_1, c_2, c_3, c_4\).

Total strain energy for a laminate with \(n\) patches is given by the sum of energies of all patches.

\[
\Pi_{total} = \sum_{n=1}^{P} \Pi_{(n)}
\]

The total strain energy of the laminate is thus a function of the coefficients of each patch. From Eqns(3.9)(3.10)(3.18) we see that the strain energy is a function of the following physical quantities (i) the reduced transformed stiffness matrix and thus a function of the fiber orientations (ii) ply thickness, (iii) patch dimensions, and finally (iv) material properties of the composite laminate. Thus, the strain energy could be minimized to solve
for the coefficients of the approximations \((c_n)\) and thereby predict the shape of the laminate.

Since we are solving for a multi-patch laminate, we need to enforce continuity constraints between the patch edges to ensure that all the patches are connected. From Algmuni et al., 2020 we know that the continuity constraints can be defined as shown in Table 3.5.

![Four-patch grid geometry and coordinate system](image)

**Figure 3.18: Four-patch grid geometry and coordinate system**

Figure 3.16 represents the four-patch grid geometry and the different coordinate systems used. The coordinate system for the integration while calculating the strain energy of an individual patch is local; while that for setting the BC’s and plotting the surface is global (denoted in red). The continuity constraints to be enforced at the common edges of the patches are to equate the displacements \(u, v, w\) and the first differentials of the out-of-plane displacements \(\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\) along the boundaries. They are as follows,
Table 3.5: Continuity constraints at common boundaries for four-patch grid laminate

<table>
<thead>
<tr>
<th>Along y-axis (n= 1, 3)</th>
<th>Along x-axis (n= 1, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^{(n)}(0, y) = u^{(n+1)}(0, y)$</td>
<td>$u^{(n)}(x, 0) = u^{(n+2)}(x, 0)$</td>
</tr>
<tr>
<td>$v^{(n)}(0, y) = v^{(n+1)}(0, y)$</td>
<td>$v^{(n)}(x, 0) = v^{(n+2)}(x, 0)$</td>
</tr>
<tr>
<td>$w^{(n)}(0, y) = w^{(n+1)}(0, y)$</td>
<td>$w^{(n)}(x, 0) = w^{(n+2)}(x, 0)$</td>
</tr>
<tr>
<td>$\frac{\partial w^{(n)}}{\partial x}(0, y) = \frac{\partial w^{(n+1)}}{\partial x}(0, y)$</td>
<td>$\frac{\partial w^{(n)}}{\partial x}(x, 0) = \frac{\partial w^{(n+2)}}{\partial x}(x, 0)$</td>
</tr>
<tr>
<td>$\frac{\partial w^{(n)}}{\partial y}(0, y) = \frac{\partial w^{(n+1)}}{\partial y}(0, y)$</td>
<td>$\frac{\partial w^{(n)}}{\partial y}(x, 0) = \frac{\partial w^{(n+2)}}{\partial y}(x, 0)$</td>
</tr>
</tbody>
</table>

*Results*

As discussed in the sections above, the analytical model for a four-patch grid laminate presented in Algmuni et al., 2020 is considered as a design tool for targeting specific shapes from a shape morphing context. It is thus necessary to gauge the reliability of the model by comparing the analytical shape with FEA. In their paper Algmuni et al compare their analytically obtained results with FEA and experiment. Although the fiber orientations for which the results were validated were restricted to 0° and 90° plies. For the scope of this research, it was necessary to establish the reliability of the model for a wider range of fiber orientations, hence the model was used to solve for a variety of configurations and the results have been compared with FEA. ABAQUS has been used to simulate the curing of these laminates.

The finite element process for simulating the curing and snapping of asymmetric laminates have been described at length in the work by our alumni and fellow researchers.
at Clemson University (Annamalai, 2016; A. G. Lele, 2018; Deshpande et al., 2020). The analysis is done on ABAQUS 6.14, static structural solver. The composite laminate is then modeled with required geometry and fiber layup. The laminate under consideration here is a four-patch two-ply laminate with a total size of 200*200mm and size of each patch being 100*100mm and a ply thickness of 0.15mm. The material properties of the material required for the simulation include the modulus of elasticity, modulus of rigidity, Poisson’s ratio, and the coefficient of thermal expansion. The material properties of AS4 8552 carbon composite prepregs used in this study are listed below,

\[
\begin{align*}
E_1 &= 135e9 \text{ Pa} \\
E_2 &= 9.5e9 \text{ Pa} \\
\nu_{12} &= 0.3 \\
G_{12} &= 5e9 \text{ Pa} \\
\alpha_{11} &= -2e-8K^{-1} \\
\alpha_{22} &= \alpha_{33} = 3e-5K^{-1}
\end{align*}
\]

After defining the laminate geometry, configuration, and material properties the two main steps that follow are: simulating the curing process where the laminate fixed at its initial flat shape and a temperature difference \( \Delta T = -125K \) is applied, which recreates the curing process where the laminate is cooled to room temperature after heating inside a furnace. The second step is the that of snapping, where the laminate can be fixed at the center and a displacement can be applied at the corner point to snap the laminate into its multiple stable states.

To compare the analytical with the FEA shape, 9 fit-points are selected on the analytical result, one at each corner of the four patches, and are compared with the fit-points from the finite element result. The laminates being compared have a total size of
200*200mm and the individual patches are 100*100mm. All the processing has been carried out on an Intel i7-8750H processor with 8GB RAM and 6 cores.

As seen in Figure 3.17, the analytically obtained shape for the [90/0], [0/90], [90/0], [0/90] laminate matches almost exactly with the finite element shape. This is in line with Algmuni’s results. The model is solved for orientations other than 90° and the results are shown in the figures below.

Figure 3.19: [90/0], [0/90], [90/0], [0/90] Patch results, (a) Analytical shape, (b) Comparing analytical with FEA shape
Figure 3.20: [0/0], [0/0], [60/0], [60/0] Patch results, (a) Analytical shape, (b) Comparing analytical with FEA shape
Figure 3.21: [90/0], [30/0], [50/0], [60/0] Patch results, (a) Analytical shape, (b) Comparing analytical with FEA shape

As seen from Figure 3.18 and Figure 3.19 the analytical results deviate from the finite element ones at the corner points. The deviation from the FEA results stand at a maximum of 4% relative to the side of the laminate which is 200mm. These deviations could be attributed to the flawed assumptions made in the Classical Laminate theory which is the foundation for this approach. CLT does not take the interlaminar stresses $\sigma_z$, $\tau_{xz}$, $\tau_{yz}$ into
consideration which are responsible for causing delamination at the edges of the laminate. This causes a contradiction while balancing the stresses at the boundary (Jones, 1999). Additionally, as seen in Equation (3.11), the polynomial approximation for the out of plane displacement is a bicubic equation in x and y and increasing the order of the polynomial does not improve the fit of the surface. Thus, it can be concluded that the reason behind the deviation is the inherent issues in the model which are due to the assumptions made in CLT. Despite these issues we can make use of the analytical model to target specific shapes since it captures the overall nature of the surface relatively well. Also, the analytical model predicts the shapes of the four-patch laminates in under a minute, while FEA usually takes 7-10 minutes to simulate the curing and snap through of the laminate.

In this section, different surrogate models to predict the shape of a single and multi-patch laminate were discussed. The machine learning approach to predict shapes being one that is not explored in the literature. Although there is some merit in using ANN’s to predict the shapes of single patch laminates, extending this approach to a multi-patch problem has proved to be problematic. This is because the boundary conditions between connected patches have not been correlated with the laminate parameters and physical properties responsible for the shape morphing capability of these laminates. While in the analytical model, the boundary conditions which are the connectivity constraints between connected patches (Table 3.5) are correlated with the laminate properties via the minimization of the strain energy equation for the laminate in Equation (3.9). Thus, to establish a design method for multi-patch laminates we decided to move ahead with the analytical model in the following chapter.
CHAPTER FOUR

DESIGN METHOD FOR FOUR-PATCH GRID LAMINATES

5.1 Nested Optimization Setup

As stated earlier, the goal is to optimize to a specific target shape for a surface made of multiple patches. From the discussion above we see that Algmuni’s model can predict the shape of a four-patch grid laminate from the input of the laminate parameters like fiber orientation, patch dimensions, etc. We can thus set up an optimization model to solve the reverse problem which would solve for the laminate properties which would result in a specific target shape. The analytical model described above would be the simulation component for the optimization. The workflow should have a nested optimization setup, where the lower-level or the inner loop would predict the shape of the laminate by minimizing its total strain energy, while the higher-level optimization or the outer loop would minimize the fitness function that describes the difference between the obtained shape from lower-level (analytical shape) and the target shape (user input).

The optimization setup for both the levels is as follows,

**Lower level or Inner loop:** In the case of a four-patch laminate the total strain energy would be as follows,

\[
\Pi_{total} = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4
\]  
(4.1)

\(\Pi_{total}\) is the quantity to be minimized, while \(\Pi_1, \Pi_2, \Pi_3, \Pi_4\) are the strain energies of the individual patches.
The lower-level optimization can thus be setup as follows,

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Objective</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>(<a href="%5Ctext%7Bcoeff%7D">c_1, c_2, c_3, c_4</a>)</td>
<td>Minimize (F_2 = \Pi_{\text{total}})</td>
<td>Continuity constraints</td>
</tr>
</tbody>
</table>

Higher level or Outer loop: As stated above, the lower-level optimization solves for the unknown coefficients \((c_1, c_2, c_3, c_4)\) and hence the shape of the four-patch laminates for a given configuration. The shape of the laminate can now be obtained from the displacement equations in Eqns (3.11) & (3.12). Using the shape obtained from the lower level we minimized a fitness function that compares the obtained shape with the target shape. To simplify the problem, initially we consider a two-ply four-patch laminate where the lower ply is considered to have a fiber orientation of \(0^\circ\). The orientations of the top four-patches \(\theta_1, \theta_2, \theta_3, \theta_4\) are considered as design variables with bounds from \(-90^\circ \leq \theta \leq 90^\circ\). Additionally, the laminate size is kept constant at 200*200 mm; and to parameterize the individual patch sizes, the dimensions of the top right or second patch, \(x'\) & \(y'\) are also set as variables (Figure 4.1). Thus, we can tap into a wider design space with the possibility of having unequal patch sizes.

![Figure 4.1: Design variables for higher level](image-url)
To compare the analytical and the target shapes, fit points are selected across the laminate (Figure 4.2). The fitness function comparing the two shapes involves calculating the deviation in the fit-points of the two surfaces. The objective function could thus be to minimize the sum of squares of the difference between the analytical and the target coordinates as shown in Equation (4.2).

\[
\text{fitness function} = \sum_{i=1}^{9} (x_i - x_i')^2 + \sum_{i=1}^{9} (y_i - y_i')^2 + \sum_{i=1}^{9} (z_i - z_i')^2 \tag{4.2}
\]

Where \(x_i, y_i, z_i\) are the coordinates of the fit-points of the obtained shape, while \(x_i', y_i', z_i'\) are those of the target shape.
To ensure manufacturability of the laminates, manufacturing constraints for minimum
patch size are enforced. As seen in Figure 4.1, $x', y'$ are the variables that control the
individual patch sizes. The following bounds are assigned,

$$50\text{mm} \leq x' \leq 150\text{mm}$$

$$50\text{mm} \leq y' \leq 150\text{mm}$$

Since the size of the four-patch laminate is 200*200mm, these bounds ensure a minimum
patch size of 50*50mm and maximum of 150*150mm.

The higher-level optimization can thus be setup as follows,

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Objective</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1, \theta_2, \theta_3, \theta_4, x', y'$</td>
<td>$F_1 = \min(\text{fitness function})$</td>
<td>Manufacturing constraints</td>
</tr>
</tbody>
</table>

Before discussing the problem setup and workflow for the optimization, lets discuss
snapping of the laminate to its different states using the analytical model. When an
asymmetric laminate has multiple stable shapes, it means that on minimizing the total strain
energy for the laminate, multiple solutions exist that correspond to the respective stable
shapes. Due to the complexity of the problem, which is in part due to the non-linear nature
of the design space and the large number of design variables involved, it is tedious to
converge to multiple solutions in one run. Hence, in the following sections two approaches
are discussed: (i) *Single-state optimization*, which deals with targeting a single state and,
(ii) *Two-state optimization*, which deals with targeting two shapes: pre-snapped and post-
snapped states. In this research, we limit the number of shapes to be targeted to two, in spite the fact that an $n$ patch laminate can have a maximum of $2^n$ stable states. This is because solving for more than two states is extremely tedious since the solution is very sensitive to the initial guess.

5.2 Single-State Optimization

As discussed, this optimization setup is used to target a single shape. Figure 4.3 illustrates the flowchart of the optimization. As seen in the flowchart, the lower level or inner loop constitutes Algmuni’s analytical model discussed in detail in the literature review. The inner loop solves for the shape of the laminate based on the parameters passed from the higher-level or the outer loop which are the fiber orientations $\theta_1, \theta_2, \theta_3, \theta_4$ and the dimensions $x', y'$ of the laminate. These parameters are taken as the inputs for this level and the total strain energy of the laminate which is the summation of the individual energies of the patches as shown in Equation (4.1) is expressed in terms of the unknown coefficients of the approximations in Equation (3.17). The total strain energy is thus minimized to solve for the unknown coefficients.
The coefficients can then be substituted into the displacement equations (3.11) & (3.12) to obtain the equations of $u$, $v$ and $w$ as $f(x,y)$. The fit-points can now be calculated from these equations and used to calculate the fitness function in Equation (4.2) which is to be minimized in the higher-level or the outer loop.

Figure 4.3: Optimization flowchart for targeting single shape
5.3 Two-State Optimization

In this section the approach to target two shapes: pre-snapped and post-snapped shape, are discussed. Figure 4.4 illustrates the flowchart of the optimization setup to target the two shapes. As discussed on the literature, the stable state of a laminate corresponds to the minimized strain energy for the given configuration. Thus, like the bistable laminate, the four-patch grid laminate will also have multiple minimas. The task here is to solve for the multiple minmas. To solve for the stable shapes, Algmuni’s model has been coded on MATLAB and \textit{fmincon} is used to minimize the strain energy equation. Solving for multiple minima in a single run using \textit{fmincon} has proven unreliable and tedious, because the solver is unable to converge to multiple solutions with a single guess. Thus, we solve for two states by running \textit{fmincon} twice with different guesses. A point to keep in mind is that as discussed earlier an \( n \) patch laminate can have a maximum of \( 2^n \) stable states. But in this research, we limit the number of shapes to be targeted to two, because solving for more than two states is extremely tedious since the solution is very sensitive to the initial guess. This approach is an extension of the previous approach with the addition of the extra block to calculate the shape of the second stable state. As seen in the flowchart, the solution for state-1 is multiplied by -1 and is used as the initial guess for solving for state-2. This is done because the curvatures of the second state are reversed, thus the values of the coefficients can be negated and set as the initial guess for the second optimization.
Thus, restating the optimization problem for the two-state optimization we have,

**Lower level:** The problem formulation largely remains the same. The objective function is the total strain energy of the four-patch grid laminate $\Pi_{\text{total}}$ as shown in Equation (4.1) and the constraints are the continuity constraints as shown in Table 3.5. The difference is that in this case, the two states are solved in sequence as shown in the flowchart. Thus, there are two sets of optimizations in the lower level. The solution of the two optimizations
which are the unknown coefficients are used to plot the two shapes and the two sets of fit points are calculated to pass to the higher-level optimization.

**Higher-level:** The fit points for the two states from the lower level are used to calculate the fitness functions as shown in Equation (4.2) for each stable state. Thus, the multi-objective optimization problem for this level can be written as,

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Objective</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1, \theta_2, \theta_3, \theta_4, x', y'$</td>
<td>$F_1 = \min(fitness\ function\ 1)$</td>
<td>Manufacturing constraints</td>
</tr>
<tr>
<td>$F_2 = \min(fitness\ function\ 2)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, *fitness function* 1 corresponds to state-1 and *fitness function* 2 corresponds to state-2. The manufacturing constraints are the same as that explained earlier.

### 5.4 Kirigami Composites

Kirigami is an ancient paper cutting technique that is popularly used to create decorative shapes. From the work by our alumni (A. Lele et al., 2018; A. Lele, 2018), we are introduced to the idea of integrating the two concepts of bistability of composite laminates and Kirigami to achieve unprecedented shape morphing capabilities that a bistable laminate which results in simple cylindrical shapes cannot achieve. These laminates are referred to as Kirigami composites in their work.

**Kirigami Unit Cell**

In the paper by Lele et al. (A. Lele et al., 2018) a Kirigami composite concept with a simple parallel cut pattern is depicted as shown in Figure 4.5. Lele aimed to study the most elementary part of the structure, which he refers to as Kirigami unit cell. We can see
how it is described in Figure 4.5 (a) and its layup and geometry in (c). In his work, Lele has carried out an in-depth study on the Kirigami unit cell by simulating the curing and snapping process on ABAQUS and validating the FEA model with experimental results. In this section we try to build on his work and recreate his results of the Kirigami unit cell by modifying Algmuni’s model (Algmuni et al., 2020) for a four-patch grid laminate discussed in the previous section.

![Figure 4.5: Multi-stable Kirigami composite concept with parallel cut pattern. (a) Undeformed Kirigami geometry, (b) Deformed Kirigami paper demonstrating snapping between stable states, (c) Configuration and geometry of Kirigami unit cell (A. Lele et al., 2018)](image)

Lele conducted a parametric study with cut sizes of different lengths, but for this work we consider a specific case of Kirigami composite as shown in Figure 4.6. The
laminate consists of two bistable patches with a tab of symmetric layup connecting the two patches. The patch on the left and right have a configuration of \([90°/0°]\) and \([0°/90°]\) respectively, while the center tab has a layup of \([90°/90°]\) and a length of 25.4mm. The FEA simulation of the curing is carried out on ABAQUS and the procedure is as explained in Chapter 3 under the four-patch grid laminate section. The material properties are same as the ones used for the four-patch laminate simulation.

![Figure 4.6: Kirigami unit cell geometry for FEA](image1)

![Figure 4.7: Kirigami unit cell geometry for analytical model](image2)
For the analytical approach, we consider the laminate geometry as shown in Figure 4.7. The total size of the laminate is kept the same as that of the FEA geometry in the previous figure. The tab of symmetric layup is ignored and the Kirigami unit cell is visualized as a four-patch grid laminate with a cut between patches 1 and 2 and the edge that is not connected is highlighted in red in the figure above.

After finalizing the laminate geometry and the location of the cut, the next step is to build the constraints to input to \textit{fmincon}. Recollecting the continuity constraints from Chapter 3, Table 3.5, we know that there will be four sets of constraints, one set for each edge connected. But in this case since we have three connected edges, there will be three sets of continuity constraints which are given below in Table 4.1.

Table 4.1: Continuity constraints for Kirigami unit cell, where superscript denotes patch number

<table>
<thead>
<tr>
<th>Patch 1-3 common edge (Along x-axis)</th>
<th>Patch 2-4 common edge (Along x-axis)</th>
<th>Patch 3-4 common edge (Along y-axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^{(1)}(x, 0) = u^{(3)}(x, 0)$</td>
<td>$u^{(2)}(x, 0) = u^{(4)}(x, 0)$</td>
<td>$u^{(3)}(0, y) = u^{(4)}(0, y)$</td>
</tr>
<tr>
<td>$v^{(1)}(x, 0) = v^{(3)}(x, 0)$</td>
<td>$v^{(2)}(x, 0) = v^{(4)}(x, 0)$</td>
<td>$v^{(3)}(0, y) = v^{(4)}(0, y)$</td>
</tr>
<tr>
<td>$w^{(1)}(x, 0) = w^{(3)}(x, 0)$</td>
<td>$w^{(2)}(x, 0) = w^{(4)}(x, 0)$</td>
<td>$w^{(3)}(0, y) = w^{(4)}(0, y)$</td>
</tr>
<tr>
<td>$\frac{\partial w^{(1)}}{\partial x}(x, 0) = \frac{\partial w^{(3)}}{\partial x}(x, 0)$</td>
<td>$\frac{\partial w^{(2)}}{\partial x}(x, 0) = \frac{\partial w^{(4)}}{\partial x}(x, 0)$</td>
<td>$\frac{\partial w^{(3)}}{\partial x}(0, y) = \frac{\partial w^{(4)}}{\partial x}(0, y)$</td>
</tr>
<tr>
<td>$\frac{\partial w^{(1)}}{\partial y}(x, 0) = \frac{\partial w^{(3)}}{\partial y}(x, 0)$</td>
<td>$\frac{\partial w^{(2)}}{\partial y}(x, 0) = \frac{\partial w^{(4)}}{\partial y}(x, 0)$</td>
<td>$\frac{\partial w^{(3)}}{\partial y}(0, y) = \frac{\partial w^{(4)}}{\partial y}(0, y)$</td>
</tr>
</tbody>
</table>
Recollecting the four-patch formulation discussed earlier, the objective function which is the total strain energy ($\Pi_{total}$) is given by Equation (4.3).

$$ \Pi_{total} = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 $$

(4.3)

In the above equation, $\Pi_n$ which is the strain energy of the patch $n$, is given by Equation (3.9). Now that we have the objective function and the constraints the strain energy minimization problem can be solved to yield the stable shapes of the Kirigami unit cell. The results of this simulation are provided in the next chapter.

*Six-Patch Laminate with Variable Cut Locations*

In the previous section an approach to modify the existing Algmuni’s model used to predict shapes of connected patches to solve for patches with disconnected edges has been detailed. This was done by including the continuity constraints only for those edges that were connected, and the edge where the Kirigami cut was located was left disconnected. The logical next step is to extend the modified model to one where a greater number of patches could be assembled, and cuts could be assigned at multiple locations as shown in the six-patch Kirigami composite concept in Figure 4.8.
The strength of this approach would be that large Kirigami composites could be modeled by correctly specifying the boundary conditions at the common edges independent of the number of patches. In the figure above, a six-patch composite laminate is illustrated with alternate patches of [90°/0°] and [0°/90°] configurations. In this study we focus on patches with only 90° and 0° ply angles; and for the sake of simplicity while establishing the extended Kirigami model, all patches are assigned the same dimensions of 100*100mm. The edges are assigned a number which signifies the possible cut location. Therefore, the seven common edges results in seven possible cut locations. The maximum number of cuts that are practically possible in a six-patch laminate such that no patch is completely disconnected from the laminate are two. Since the cut location is made variable, a binary value is assigned to the variable that defines whether that edge is connected or not. A value of 1 indicates that the edge is connected and a value of 0 indicates that the edge is disconnected. Thus, the model could be solved by turning on or off the constraints at the edges depending on the location of the cut. The flowchart of the process with the major steps is given by the Figure 4.9.
The steps to solve the Kirigami model is largely similar to the ones discussed previously. The first step is to define the material properties and laminate parameters. The second step is to define the cut location(s). The cut locations are inputted through an array of size seven. For example, if the cuts are to be made at locations 1 and 2, the array of the cut locations will be $cut = [0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1]$. Therefore, the top middle patch would not be connected to the patches on either sides. Similarly, other combinations of cuts could be defined to generate different laminate structures. Based on this input of the cut locations, the respective continuity constraints are formulated. This step is followed by the formulation of all the patch equations and the strain energy equation explained in detail in previous sections. Next, the summation of the individual patch energies is minimized subject to the modified constraints, to solve for the coefficients of the polynomial approximations. Finally, the results for the Kirigami composite are plotted.
The Kirigami model could thus be extended for $n$ number of patches with multiple cuts. The objective function would be to minimize the total strain energy of the whole laminate as shown in Equation (4.4) subject to the continuity constraints. And as long as the constraints are setup properly as shown in the previous section, solving for the Kirigami composite would be straightforward.

$$\Pi_{total} = \Pi_1 + \Pi_2 + \Pi_3 + \ldots \ldots + \Pi_n$$

**objective function = minimize** $(\Pi_{total})$

In the next chapter the results and discussions of the design procedure for the single-state, two-state optimizations and the Kirigami composite models are discussed.
CHAPTER FIVE

RESULTS AND DISCUSSION

In this section, the results of the two optimization methods and the different iterations of the problem setup are presented and discussed keeping manufacturability in mind. All computational results presented in this work were done on an Intel i7-8750H processor with 8GB RAM and 6 cores.

6.1 Single-state Optimization

A. Fiber orientation as input variables

This is the first iteration of the design of the four-patch grid laminate. In this setup the fiber orientations of the top ply of the patches $\theta_1, \theta_2, \theta_3, \theta_4$ as shown in Figure 5.1, are set as design variables. The laminate has a common ply of $0^\circ$ at the bottom. The size of the laminate is 200*200mm and that of the individual patches are 100*100mm. The objective functions and constraints are the same as those discussed in the previous section.

For this problem Modefrontier was used to run the optimization. Modefrontier was tied with the MATLAB code that ran the analytical model. The inner loop was minimized
using `fmincon` in MATLAB and the outer loop was minimized by NSGA II algorithm on Modefrontier. The initial population size was set as 30 and the number of generations as 80. The workflow for the optimization is shown below.

![Optimization workflow](image)

Figure 5.2: Optimization workflow

Keeping the manufacturability of the laminate in mind, the input variables were set for different step sizes. This is because plies with very small increments in fiber orientations would be difficult to lay. Hence, the optimization is run for the following cases, where the fiber orientations are (i) continuous, (ii) have a step of 1° and (iii) have a step of 5°. The results for the optimization runs are discussed below.

(i) Continuous
When the fiber orientations are set as continuous, we observe a near perfect match with the target shape as shown in Figure 5.3. The optimized design has the fitness function value of 1.8025mm². The maximum deviation from the target shape in the z direction is observed
to be less than 1mm (Table 5.1). And compared to the 200*200mm size of the laminate, the deviation is deemed acceptable.

Table 5.1: Deviation from target shape

<table>
<thead>
<tr>
<th>Fit-point</th>
<th>Deviation in z-direction (mm)</th>
<th>% Deviation relative to laminate size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8689</td>
<td>0.43%</td>
</tr>
<tr>
<td>2</td>
<td>0.2316</td>
<td>0.12%</td>
</tr>
<tr>
<td>3</td>
<td>0.0565</td>
<td>0.03%</td>
</tr>
<tr>
<td>4</td>
<td>0.0721</td>
<td>0.04%</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.00%</td>
</tr>
<tr>
<td>6</td>
<td>0.0624</td>
<td>0.03%</td>
</tr>
<tr>
<td>7</td>
<td>0.4763</td>
<td>0.24%</td>
</tr>
<tr>
<td>8</td>
<td>0.2316</td>
<td>0.12%</td>
</tr>
<tr>
<td>9</td>
<td>0.4894</td>
<td>0.24%</td>
</tr>
</tbody>
</table>

Since manufacturing laminates of fiber orientations with such small increments is not possible, a step size is added to the fiber orientation and the results are shown below.

(ii) Step = 1°

From Figure 5.4 and Table 5.2, it can be observed that there is a greater deviation from the target shape as compared to the previous case. The fitness function value for this case was 2.4224mm². The maximum z-deviation is around 3mm, which is at a corner point.
Figure 5.4: Optimization results, (a) Target shape [90, 30, 50, 60], (b) Optimized shape [83, -42, 62, 47], (c) Comparing fit-points from surface in (a) and (b)
(iii) Step = 5°

Next, the step is increased to 5° and the results are illustrated in Figure 5.5. The optimized configuration for this case yielded a fitness function value of 20.739mm². This case has the highest deviation from the target shape. From Table 5.3, the maximum z-deviation is comparable to the previous case, yet the fitness function value is greater than the previous case. This is because the deviation in x and y directions have also increased slightly. Thus, adding up to the higher value.
Figure 5.5: Optimization results, (a) Target shape [90, 30, 50, 60], (b) Optimized shape [70, -55, 60, 50], (c) Comparing fit-points from surface in (a) and (b)
Table 5.3: Deviation from target shape for step=5°

<table>
<thead>
<tr>
<th>Fit-point</th>
<th>Deviation in z-direction (mm)</th>
<th>% Deviation relative to laminate size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5618</td>
<td>0.28%</td>
</tr>
<tr>
<td>2</td>
<td>0.6756</td>
<td>0.34%</td>
</tr>
<tr>
<td>3</td>
<td>1.7407</td>
<td>0.87%</td>
</tr>
<tr>
<td>4</td>
<td>0.3429</td>
<td>0.17%</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.00%</td>
</tr>
<tr>
<td>6</td>
<td>0.3170</td>
<td>0.16%</td>
</tr>
<tr>
<td>7</td>
<td>2.4979</td>
<td>1.25%</td>
</tr>
<tr>
<td>8</td>
<td>0.6756</td>
<td>0.34%</td>
</tr>
<tr>
<td>9</td>
<td>1.4340</td>
<td>0.72%</td>
</tr>
</tbody>
</table>

Thus, the following conclusions can be drawn from the results for these cases summarized in Table 5.4. The first observation is that when fiber orientation is set as continuous it results in the best match with the target shape as seen from the fitness function value. And as the step size is increased, the fitness function keeps increasing. But it is also seen that the time duration for the optimization runs increases as the step size decreases. Thus, the step size of 5 is chosen for the future optimization runs, since it gives considerably good results in shorter time. Secondly, the result of the optimization is a non-dominated solution set. This is observed from the solutions of the various cases which are different from the target configurations yet yield a close match with the target shape of the laminate.
Table 5.4: Results for different fiber orientation step sizes

<table>
<thead>
<tr>
<th>Type</th>
<th>Optimized Fiber orientations</th>
<th>Fitness function (mm²)</th>
<th>Average z-deviation (mm)</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Continuous</td>
<td>[79.14, 34.67, 63.86, 62.55]</td>
<td>1.8025</td>
<td>0.2765</td>
<td>24</td>
</tr>
<tr>
<td>2 Step = 1°</td>
<td>[83, -42, 62, 47]</td>
<td>2.4224</td>
<td>0.7946</td>
<td>12</td>
</tr>
<tr>
<td>3 Step = 5°</td>
<td>[70, -55, 60, 50]</td>
<td>20.7390</td>
<td>0.9162</td>
<td>9</td>
</tr>
</tbody>
</table>

B. Fiber orientations and patch dimensions as input variables

As discussed in the last chapter the dimensions of the patches are parameterized by setting the length and width of the top right patch as variables as seen in Figure 5.6. Since the step of 5 for the fiber orientations was sufficiently accurate, in the following iterations the focus is more on exploring other means to improve the match between the analytical and target shape. Some of them are by adding more fit-points to calculate the fitness function, and formulating different fitness functions that improve the fit.

Like the previous setup, the inner loop was minimized using `fmincon` in MATLAB and the outer loop was minimized using the NSGA II algorithm on Modefrontier. The initial
population size was set as 50 and the number of generations as 90. The population and the number of generations were increased due to the increase in number of input variables. The workflow for the optimization is shown above. In the workflow, the fiber orientations are denoted by ‘ang’ and the dimensional parameters are denoted by x, y.

![Optimization workflow](image)

Figure 5.7: Optimization workflow

Various methods to improve the fit between the target and analytical shape are discussed in the following sections.

(i) Using 9 fit points

First, the setup described above is tested for comparing the target and optimized shape using 9 fit points. The results for the same are presented in Figure 5.8.
Figure 5.8: Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
(ii) Using 25 fit points

In this section different methods to improve the fit between the optimized and the target shape are discussed. Firstly, the number of fit points used to calculate the fitness function are increased from 9 to 25. The higher number of fit points would help the optimizer to yield a better fit across the whole laminate. Secondly, multiple fitness functions are deployed in a bid to improve the result. The different fitness functions used with their results are discussed in the following section.

- **Minimizing sum of squares of difference between coordinates**

The fitness function in Equation (4.2) used in the previous optimization runs is restated below.

\[
f_1 = \sum_{i=1}^{9} (x_i - x_i')^2 + \sum_{i=1}^{9} (y_i - y_i')^2 + \sum_{i=1}^{9} (z_i - z_i')^2
\]  

(5.1)

From the results in Figure 5.9, we observe that the solution is different from previous case, but still yields a good fit with the target shape. The minimized fitness function is 28.57mm².
(a) Fiber angles = [90, 30, 50, 60]  
\[ x' = 80\text{mm}, \quad y' = 90\text{mm} \]

(b) Fiber angles = [70, -50, 60, 40]  
\[ x' = 95\text{mm}, \quad y' = 94\text{mm} \]

Figure 5.9: Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
• **Minimizing the maximum distance between target and optimized fit points**

The fitness function comparing the two shapes in this case involves calculating the deviation in the fit points of the two surfaces. The objective function could thus be to minimize the square of the maximum Euler distance between the analytical and the target fit points.

That distance \(d_i\) between each fit point and its respective target is given by,

\[
d_i = \sqrt{(x_i - x_i')^2 + (y_i - y_i')^2 + (z_i - z_i')^2}
\]  

(5.2)

Where \(x_i, y_i, z_i\) are the coordinates of the fit-points of the obtained shape, while \(x_i', y_i', z_i'\) are those of the target shape and \(i\) goes from 1 to 25.

The new fitness function and the objective function could then be given as,

\[
f_2 = \max (d_i)^2
\]

(5.3)

\[
objective = \text{minimize}(f_2)
\]

Using the new objective function would aid the optimizer to focus on minimizing the fit point which deviates the most from the target at every iteration.

From the results in Figure 5.10, we observe that all the centrally located fit points are accurate while those at the edges and the corner points have greater deviation from the target than the previous case.
(a) Fiber angles = [90, 30, 50, 60]
\[x' = 80\text{mm}, y' = 90\text{mm}\]

(b) Fiber angles = [-80 20 50 55]
\[x' = 99\text{mm}, y' = 92\text{mm}\]

Figure 5.10: Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
Minimizing the sum of the maximum distance between target and optimized fit points from each patch

From the results of the previous fitness function, we observed that the deviation between fit points across all laminates was not being improved. Hence, a new fitness function was deployed where the squares of the maximum deviation of the fit points in each patch were added and their mean was minimized. The fitness function is as defined below,

\[ f_3 = \frac{1}{4} \left[ \max (d_{\text{patch}_1})^2 + \max (d_{\text{patch}_2})^2 + \max (d_{\text{patch}_3})^2 + \max (d_{\text{patch}_4})^2 \right] \]

\[ \text{objective} = \text{minimize}(f_3) \] \hspace{1cm} (5.4)

In the above equation, \( d_{\text{patch}_1}, d_{\text{patch}_2}, d_{\text{patch}_3}, d_{\text{patch}_4} \) represent the deviation of fit points in patches 1, 2, 3 and 4, respectively.

The results using this fitness function are illustrated in Figure 5.11 below.
(a) Fiber angles = [90, 30, 50, 60]  
\(x' = 80\text{mm}, y' = 90\text{mm}\)

(b) Fiber angles = [70, -30, 60, 50]  
\(x' = 146\text{mm}, y' = 96\text{mm}\)

Figure 5.11: Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
Thus, different fitness functions were used to solve the higher-level optimization. A summary of the results is shown in Table 5.5. From the results it is observed that fitness function $f_3$ performs the best among the three. This can be seen by comparing the corresponding $f_1$ values in the last column. Thus, the fitness function $f_3$ will be used for all the following analysis.

Table 5.5: Fitness functions summary

<table>
<thead>
<tr>
<th>Fitness function</th>
<th>Optimized Configuration (Target configuration Fiber angles = [90, 30, 50, 60] $x' = 80$mm, $y' = 90$mm)</th>
<th>Objective function value (mm$^2$)</th>
<th>Corresponding $f_i$ value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>Fiber angles = [70, -50, 60, 40] $x' = 95$mm, $y' = 94$mm</td>
<td>28.57</td>
<td>28.57</td>
</tr>
<tr>
<td>$f_2$</td>
<td>Fiber angles = [-80, 20, 50, 55] $x' = 99$mm, $y' = 92$mm</td>
<td>1.98</td>
<td>30.54</td>
</tr>
<tr>
<td>$f_3$</td>
<td>Fiber angles = [70, -30, 60, 50] $x' = 146$mm, $y' = 96$mm</td>
<td>2.26</td>
<td>17.37</td>
</tr>
</tbody>
</table>

C. Alternative Layup

To explore new shapes of the laminate and keeping the manufacturability of the laminate in mind, an alternate layup as shown in Figure 5.12 was considered. In this type of layup two adjacent patches always have the same fiber orientation. In this way every patch would have some form of connection with the other patches. As seen in the figure below, the fiber orientations of the bottom plies are also set as variables which is different from the earlier layup where the bottom ply of the laminate had a $0^\circ$ fiber angle throughout.
and only those of the top patches were set as variables. Incorporating this type of layup is just another way of exploring the design space for the possible shapes that could be obtained.

The design variables for this setup are the two top fiber orientations $\theta_{Top}$, the two bottom ones $\theta_{Bottom}$, the dimensional parameters $x'$ and $y'$, and the layup type which has a value of 1 or 2 for the layup-1 or 2 respectively as shown in the figure below.

From the previous section it is observed that fitness function $f_3$ in Equation (5.4) yielded the most accurate results. Thus, the same objective function is used for this setup.
The workflow for this setup is shown in Figure 5.13. In the figure ‘angT’ and ‘angB’ represents $\theta_{Top}$ and $\theta_{Bottom}$ respectively. The dimensional parameters are denoted by x, y and ‘layup’ represents the variable defining the type of layup. The results for this setup are presented below. The target shape used for the optimization is depicted Figure 5.14 (a).

The minimized objective function value is 4.7245mm$^2$. As seen from the results of the single-state optimization above, the values of the design variables do not match the target values, but the optimized shape matches the target shape as seen in Figure 5.14 (c). This is because the optimization yields a relatively flat solution domain wherein the interplay between the design variables of fiber angles and the dimensions results in solutions that are different from the target values yet have a good match with the target shape.
(a) Fiber angles: $\theta_{Top} = [10, -20]$
$\theta_{Bottom} = [90, 60]$, layup = 1
$x' = 110\text{mm}, y' = 90\text{mm}$

(b) Fiber angles: $\theta_{Top} = [-10, -50]$
$\theta_{Bottom} = [80, 85]$, layup = 1
$x' = 118\text{mm}, y' = 131\text{mm}$

Figure 5.14: Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
6.2 Two-state Optimization

As discussed in the optimization setup in the previous chapter, it is tedious to converge to multiple solutions in one run due to the complexity of the problem. Thus, the *fmincon* solver requires multiple initial guesses to converge to different shapes. While solving the single-state problem it was observed that despite using many random initial guesses, the solver always converged to the same shape. Thus, it required a very nuanced and precise initial guess to obtain a different shape. And in the context of the bi-level optimization where large number of iterations were involved in both the higher and the lower level it became necessary to create a method to the calculation of the random guess. Two such methods are discussed in the following text.

(i) Iterative curvature inversion method

In this method the laminate is snapped by applying a displacement at a corner point and fixing the laminate at the center. These boundary conditions as shown in Figure 5.15 are enforced by including them in the constraints for the strain energy minimization. So, in addition to the continuity constraints between patches, the extra constraints would be that of fixing four points at the center by restricting their out-of-plane displacement and secondly, forcing a displacement value at the corner point. Considering the size of the laminate, 10mm increments are sufficient to achieve the snap-through.
The flowchart in Figure 5.16 illustrates the steps to snap the laminate. Since the focus is on achieving two shapes for the laminate, the displacement can be incrementally increased at the corner point till the curvature of the laminate flips. The solution for that result is used to resolve the problem with the displacement constraint released resulting in the final snapped shape of the laminate. Then the strain energy is minimized to solve for the unknown coefficients.
Figure 5.16: Iterative curvature inversion method

(ii) Using negative of State-1 solution as initial guess for State-2

The main challenge with using the previous approach to solve for the snapped shape is that it is time consuming. Incrementing by some displacement and checking for the curvature flip increases the time duration per iteration in the lower level by almost two to three times. Thus, an alternate approach would be to solve for the first state and multiply its solution ($x_{val}$) by (-1) and use it as the initial guess for solving the second state. This approach is represented in the flowchart in Figure 5.17.
The first approach is a more robust method because by incrementing the displacements, the initial guess can be set as the solution closest to the snapped shape. Thus, reliably converging to the second shape for multiple configurations. Although the first approach is a more robust method to achieve the snap through, the second method is preferred since it successfully solves majority of the configurations and cuts down on the time required to run the inner loop of the bi-level optimization which reduces the total run time.

Results

The fitness function $f_1$ from Equation (5.1) is used to run the two-state optimization and the problem setup and results are discussed in this section. Like the single-state optimization setup, the two-state optimization is also setup on Modefrontier where the inner loop which corresponds to the analytical model is scripted in MATLAB and `fmincon` is used to minimize the strain energy, while the outer loop is minimized using the MOGA.
II algorithm in Modefrontier which is a multi-objective genetic algorithm. The workflow is as shown in Figure 5.18.

![Figure 5.18: Two-state optimization workflow](image)

The population size was kept as 50 and the GA was run for 90 generations. The optimizer took 47.5hrs to complete the job and the optimized results are as shown below.

<table>
<thead>
<tr>
<th>Target configuration</th>
<th>Optimized configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber angles = [90, 30, 50, 60]</td>
<td>Fiber angles = [90, 30, 50, 60]</td>
</tr>
<tr>
<td>x' = 80mm, y' = 90mm</td>
<td>x' = 79mm, y' = 91mm</td>
</tr>
</tbody>
</table>

From the above results, we observe that the optimizer has come closest to the target values in this case.
Figure 5.19: State-1 Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
Figure 5.20: State-2 Optimization results, (a) Target shape, (b) Optimized shape, (c) Comparing fit-points from surface in (a) and (b)
From these results we can observe that the GA is able to reach a solution that is very close to the target values, and the shapes of the two states are illustrated in Figure 5.19 and Figure 5.20.

![Scatter chart for fitness 1 vs fitness 2](image)

**Figure 5.21: Scatter chart for fitness 1 vs fitness 2**

This result is more accurate than the single-state result because the second objective function pushes the GA to search for better designs where the fitness functions for both states are minimized. This can be seen in Figure 5.21 where the optimum design targeting both states converge at (0,0), where an objective function value of 0 indicates perfect match.
between the target and optimum configuration; while the designs close to the y-axis are the optimum designs for state-1. Thus, explaining the presence of multiple minima in the single state optimization.

The results of the two optimization approaches indicate that this approach to designing multi-patch laminates has some merit. As discussed in the four-patch grid formulation section in Chapter 3, slight deviation at the edges between the FEA and analytical shape exists when using the Hyer’s analytical model and these deviations may be attributed to some of the assumptions made in CLT. Despite these issues in the present model, the shape obtained from the analytical model can be deemed acceptable, since the overall shape is captured quite well. Thus, it helps to arrive at a sufficiently small set of approximate laminate configurations which can be narrowed down by using FEA. In the absence of a more accurate analytical model, this approach could be rather effective.

6.3 Kirigami Composites

In the last chapter, the modified model to predict the shapes of Kirigami composites was discussed. In this section, the results of the Kirigami model are presented for both the Kirigami unit cell with a single cut, and a six-patch Kirigami composite concept with up to two cuts.

Kirigami Unit Cell

In Chapter 4 under the Kirigami composite section, the formulation of the Kirigami model has been detailed. Thus, after building the total strain energy and the constraint equations,
the minimization problem was solved on \textit{fmincon} to yield the following results depicted in Figure 5.22.

![Figure 5.22: Results comparing analytical and FEA shape of Kirigami unit cell, where surface represents the analytical shape while the fit-points represent the FEA shape](image)

From the figure above we observe a visible deviation between the analytical and FEA shape at the top edges of patch 1 and 2. At the edges, the maximum deviation is about 10mm. In a bid to understand the reason behind the inaccuracy of the model, we increased the order of the polynomial of the approximation function as shown in Equation (2.37) by adding up
to the $x^3y^3$ term. But this measure did not result in significant improvement in the accuracy of the predicted shape. Thus, as discussed in the previous chapter, the inaccuracy in prediction could again be attributed to the fundamental theory governing the four-patch model (CLT). However, this study does hold some merit because the analytical model was capable in capturing the overall nature of the surface after making modifications in Algmuni’s original four-patch model.

**Six-Patch Laminate with Variable Cut Locations**

In this section, the results of the Kirigami model to predict shapes of a more complex Kirigami composite concept of a six-patch laminate with up to two cuts are presented. As discussed in the last chapter, to define the location of the cuts and formulate the appropriate constraints, a column matrix with size equal to the number of common edges is defined and set up as follows. First, we recollect the implementation of the Kirigami model by defining the patch and cut numbering.

![Figure 5.23: Patch numbering](image)

The patches are numbered as shown in Figure 5.23 and the edges are numbered as shown in Table 5.6. According to this numbering, when defining a Kirigami cut, the array element at index of the edge number is set as 0, while the connected is set to 1.
Using the above methodology, the following combinations of cut locations in a six-patch laminate are implemented. The variable ‘Cut’ is defined to define the Kirigami cuts.

(i) Cut = [0 0 1 1 1 1]

This laminate has a cut at edges 1 and 2, which are the common edges between patch 1-2 and 2-3. The patches are configured with alternate [90°/0°] and [0°/90°] patches as shown in Figure 5.24 (a) and the solution for this case is illustrated in (b). It can be observed from the result that the cuts create a stress relief at the edges of Patch-2 and it takes up a stable state that has the opposite curvature to patches 1 and 3. This is because the fiber angles are reversed in the middle patch as compared to the adjacent patches.
Figure 5.24: Six-patch Kirigami composite with Cut = [0 0 1 1 1 1], (a) Laminate geometry and configuration, (b) Laminate result

(ii) Cut = [1 1 0 0 1 1 1]

Figure 5.25: Six-patch Kirigami composite with Cut = [1 1 0 0 1 1 1], (a) Laminate geometry and configuration, (b) Laminate result
In this laminate the cuts are located at edges 3 and 4, which are the common edges between patch 4-5 and 5-6. From the results illustrated in Figure 5.25, we can see that the curvatures of the individual patches are consistent with the previous result. Patch-5 curls in the upward direction while patches 4 and 6 curl downwards.

(iii) Cut = [0 1 1 0 1 1 1]

![Figure 5.26: Six-patch Kirigami composite with Cut = [0 1 1 0 1 1 1], (a) Laminate geometry and configuration, (b) Laminate result](image)

This is a slightly different case where the cuts are located at edges 1 and 4, which are the common edges between patch 1-2 and 5-6. The solution for this case is illustrated in Figure 5.26.

(iv) Cut = [0 1 1 0 1 1 1]

Now that we know about the results for the different combinations of cut locations, we explore the design space by changing the configurations of the patches. As seen in Figure 5.27 (a), instead of the alternate $[90^\circ/0^\circ]$ and $[0^\circ/90^\circ]$ patches, two adjacent patches on
each row are kept the same. Patches 2 and 3 are assigned $[0°/90°]$ and patches 4 and 5 are assigned $[90°/0°]$ configurations with cuts at edges 1 and 4. The results are shown in Figure 5.27 (b).

![Figure 5.27: Six-patch Kirigami composite with Cut = [0 1 1 0 1 1 1], (a) Laminate geometry and configuration, (b) Analytical shape](image)

(v) Cut = [0 1 1 0 1 1 1]

![Figure 5.28: Six-patch Kirigami composite with Cut = [0 1 1 0 1 1 1], (a) Laminate geometry and configuration, (b) Analytical shape](image)
In this case the cut locations are kept the same, but the configuration of patches 4, 5 and 6 are reversed as seen in Figure 5.28 (a). The results are illustrated in (b).

(vi) \( \text{Cut} = [0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1] \)

<table>
<thead>
<tr>
<th>[0°/90°]</th>
<th>[90°/0°]</th>
<th>[90°/0°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0°/90°]</td>
<td>[0°/90°]</td>
<td>[90°/0°]</td>
</tr>
</tbody>
</table>

Figure 5.29: Six-patch Kirigami composite with \( \text{Cut} = [0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1] \), (a) Laminate geometry and configuration, (b) Analytical shape

Finally, for the last case we have configurations as shown in Figure 5.29 (a) and the results are depicted in (b).

From the results in this section, the implementation potential of Kirigami composites has been demonstrated. We see that by varying the location of the cuts and configurations of the patches, an unprecedented types of shapes can be generated. By modifying Algmuni’s model we now have a model that establishes a relationship between the shape of the laminate and the following design variables: (a) Number of patches, (b) Fiber orientations,
(c) Patch dimensions and (d) Kirigami cut location(s). This new Kirigami model would be a powerful design tool to target complex shapes that are not achievable by laminates that have all connected patches.
CHAPTER SIX

CONCLUSION AND FUTURE WORK

In this chapter the work presented in this thesis is summarized, important conclusions are drawn, and future work is discussed.

7.1 Conclusion

There were three major objectives of this research:

1. Explore surrogate models to predict stable shapes of asymmetric composite laminates
2. Utilize surrogate models to create a design method for multi-patch asymmetric laminates
3. Utilize surrogate models to predict the shape of Kirigami composites

Explore Surrogate Models

First was to explore alternate surrogate models that could predict the shapes of asymmetric composite laminates. Machine learning was used to predict the shapes of these laminates using artificial neural networks. Initially, a two-patch two ply laminate was considered where the right patch was assumed to be symmetric and the top and bottom fiber orientations of the left patch were set as input variables and the bicubic coefficients of the resulting surface were the output variables. A layer recurrent type ANN was used for which the training set comprised of data from FEA where the two-patch laminate was solved and the bicubic coefficients were extracted and fed to the ANN. This approach yielded satisfactory results and low mean square errors for the trained ANN. The maximum
error between the target shape and ANN predicted shapes was around 2.5mm. Secondly, the same machine learning approach was used to predict single-patch two ply laminates. But in this case two methods were used to approximate the surface. The first was the earlier discussed bicubic interpolation where the inputs were the fiber orientations, and the outputs were the bicubic coefficients. The second was by using parametric cubic formulation where the inputs were again the fiber orientations while the outputs were the fit-points used to approximate the parametric cubic surface. These models again yielded good results with very low mean square error values. The key learnings from the machine learning approaches were that artificial neural networks could be trained to capture the inherent physics that governs the shape morphing of asymmetric composite laminates. Despite the inability to directly use these models to design n-patch laminates, this research opens the door to visualizing the problem in a different light and making use of machine learning in this context justifiable. Additionally, existing analytical models especially Algmuni’s model was studied. Algmuni’s model being an extension of Hyer’s initial work on a single bistable patch extended to a four-patch grid laminate. The formulation of this model illustrates the scalability of Hyer’s initial model based on the Classical Lamination Theory to a multi-patch laminate with continuity constraints enforced at the common edges of two patches. Some examples of these laminates are simulated on ABAQUS and are compared with the analytically obtained shape resulting in a satisfactory match between the two shapes. Algmuni’s model successfully captures the shape from FEA with slight deviations at the edges. These deviations can be attributed to the assumptions made in CLT and can
be overlooked since the analytical model captures the overall shape and enables us to better capture the boundary conditions between individual patches and is therefore preferred.

**Utilize Surrogate Models to Create a Design Method**

The second objective was to utilize the surrogate models to create a design method based on optimization that would solve to the composite laminate parameters given a specific target shape(s). Although the machine learning approach yielded more accurate results, it could not be extended to design multi-patch laminates because the boundary conditions at common edges of adjacent patches were still unexplored. Hence, Algmuni’s model was used as the foundation for the optimization model since the multiple patches could be assembled and connected simply by enforcing the boundary conditions at the common edges. Two approaches of design are discussed based on the number of stable shapes that are targeted; first where a single state or the pre-snapped or post-cured shape of the laminate is computed and optimized to match the target shape, and second where two stable states, i.e., both pre- and post-snapped shapes are targeted. It was observed that both approaches yielded accurate results as far as matching the target shape was concerned. But the two-state optimization setup was preferred because it resulted in accurately converging to the same values of the design variables (dimensional and fiber orientations) as the target, while the single-state optimization in spite of yielding a good surface fit could not converge to the same values of the design variables. This was attributed to the presence of multiple minima in the single-objective approach, while in the multi-objective approach the second objective function forced the optimization to the global minima. The formulation of the model, the design method and the results of the optimization have been
discussed in Chapter 4 and 5. Thus, meeting the second objective of this research. Additionally, making use of the analytical model in the optimization setup makes this design method superior due to its computational efficiency when compared to an FEA based approach; because the model predicts the shape of the four-patch laminate about 7 to 10 times faster than FEA, as discussed in the results section of Algmuni’s model.

**Utilize Surrogate Models to Predict the Shape of Kirigami Composites**

Finally, the third objective of this research was to utilize the surrogate models developed by extending them to predict the stable shapes of Kirigami composites. As explained in Chapter 6, a Kirigami unit cell was considered to be solved using the analytical model and was represented by a four-patch laminate with a cut between the top two patches. This was achieved by modifying the continuity constraints in the Algmuni model to account for the cut made. Thus, accounting for the discontinuity between two patches. The strain energy minimization problem is then solved to yield the stable shape of the Kirigami unit cell. From the results of the model, we see that in spite of the visible deviation at the edges of the top two patches, the model captures the overall shape of the laminate and handles the modified constraints to incorporate the cut well. Additionally, the implementation of this Kirigami composite model extended to a multi-patch laminate with multiple cuts is discussed. A six-patch Kirigami composite with up to two Kirigami cuts is considered as a case study to explore the design space of the laminate by predicting their post-cured shapes for different locations of the cuts and different configurations. Thus, with the parameterization of the variables for cut locations and laminate geometry and configuration embedded in the Kirigami composite model, it is ready to be incorporated.
into a design method that solves the inverse problem of defining the said parameters based on the input of target shape. The integration of the Kirigami concept into the design of shape morphing composite laminates is a fairly novel idea with limited literature pertaining to it. Thus, the formulation of an analytical model based on those for fully connected composite patches would be a valuable contribution and a starting point for future research in this field.

7.2 Future Work

The design of multi-patch laminates is an area where there is not a lot of literature to be found and the work done in this thesis would contribute towards taking this research forward. The following would be the avenues for future work to follow.

*Analytical model with better accuracy*

As discussed earlier, the use of analytical models like Algmuni’s that are inspired from Hyer’s model come with their own issues. The issues pertain to the inaccurate shape prediction at the edges of the laminate. Increasing the order of the polynomial approximating the out of plane displacement was considered. But adding higher order terms to the polynomial did not result in considerable improvement in the accuracy. Thus, more work needs to be done in developing a model that provides better accuracy.

*Boundary Conditions at Common Edges*

Out of the surrogate models discussed in this research the machine learning model yielded better results when compared to Hyer’s predicted shapes for the single-patch laminate. But as discussed, since the boundary conditions at the non-free edges were not
explored, this model could not be integrated into the design method. Thus, work needs to be done in this area where the edge effects can be quantified and be made a function of the fiber orientations of the adjacent patches. An alternate approach would be to embed the effects of the boundary conditions at the non-free edges into the sample data itself that the network can try to capture. The biggest challenge in formulating this approach would be to provide the data in a way that the network can capture the information regarding the boundary conditions due to the presence of adjacent patches. In this work, the network has been trained to predict single patch laminates based on the training set consisting of single patch laminates with free boundary conditions at the edges, which is not sufficient information to capture the “boundary effects”. A possible solution would be to create a learning scheme for 9-patch grid laminates where the patch of interest is the one at the center as shown in the figure below. This would provide extra information about the configurations of the adjacent patches which would help in predicting shapes of patches with non-free edges. Additionally, the Kirigami concept could be integrated into the model by adding the variable of cut location at the common edges around the laminate.

Figure 6.1: 9-Patch geometry
Targeting a Greater Number of Stable Shapes

A four-patch laminate may have $2^4$ maximum possible stable states. But as seen in Chapter 5, the maximum number of stables shapes targeted coincidently in this research is two. This is because the two targeted shapes which are the ones with maximum curvature in opposite directions are easier for the optimizer to solve for, while the intermediate states are much more difficult to converge to. Thus, to target a greater number of shapes a more nuanced approach than the one discussed in Figure 4.4 needs to be worked out.

Validate Kirigami Model for Complex Structures

In this research we utilized the methodology discussed in the Kirigami composites section to model more complex structures with multiple cuts like the six-patch Kirigami concept. Placing cuts at one or more locations resulted in multiple permutations of complex structures that could be individually snapped to come up with more shapes. The next step in furthering this research would be the experimental investigation to study the manufacturability and the shape morphing capability of these laminates and to correlate the shapes obtained from experiment and FEA to those obtained through the Kirigami model.
REFERENCES


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