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Enhancement of Inertial Response of Inverter Based Energy System and Its Application for Dynamic Performance Improvement of a Microgrid

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ENHANCEMENT OF INERTIAL RESPONSE OF INVERTER BASED ENERGY SYSTEM AND ITS APPLICATION FOR DYNAMIC PERFORMANCE IMPROVEMENT OF A MICROGRID

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the Graduate School of
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In Partial Fulfillment
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Puspal Hazra
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Abstract

Application of power electronic converters in energy systems application have been increased due to the present national renewable energy portfolio. Photovoltaic, Battery Energy Storage Systems (BESS), Doubly Fed Induction Generator based Wind System, FACTS devices etc. are some of the applications in energy systems having power electronic converters. Power Electronic converters with its fast electronic switching and standard control configuration have low inertia compared to electro-mechanical device based energy systems.

Motivation of this research work is to develop control system configuration with photovoltaic source as intermittent source and battery storage / ultra capacitor as energy storage unit as auxiliary source. Inverter of energy storage unit is integrated with synchronous generator emulator. This control unit helps in regulating power supplied or absorbed by energy storage unit to improve inertial response of photovoltaic unit. This unit is named as PVSG (Photo-voltaic Synchronous Generator) unit.

In this work, dynamic characteristics of analytic model of PVSG unit is assessed and parameter sensitivity analysis is presented. Based on the results, few recommendations have been made for effective design of PVSG unit.
Acknowledgments

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Chapter 1

Introduction

Inventions and commercialization of energy system technologies have changed our lives in recent history. With more advancement in computer science and communication technologies have given us an effective and unique tools to analyze and improve the decision we make for our day to day activities and make our life smarter, more efficient.

Initial days of energy system innovations were focused mostly on developing efficient energy conversion technologies to generate electricity to help improve quality of life and to explore new opportunities to make significant progress in our society. With successful commercialization [1] of the energy system technologies, more technologies dependent on electricity have been developed and it is considered as a significant and critical infrastructure of many developed and developing economies.

Recent advancement in computer science and communication technologies have enabled us to develop more detailed analytic tools for power system analytics. With improved measurement and sensing technologies and better communication technologies, more
system specific events have been measured and cause of such events have been identified, which was not identified before.

Energy resource technologies for power system have been evolved from fossil fuel based energy sources to renewable energy and its energy conversion system is evolved significantly. Dynamic characteristic of these energy systems have been changed due to innovation of new energy conversion processes and its inclusion to power system. Intermittent renewable energy sources at low inertia power system causes disturbances which may lead to mechanical stresses to electro-mechanical loads and sources due to vibrations, caused by changes in electrical quantities at point of connections of power system.

Dynamic disturbances due to lack of system inertia may also lead to variation of system parameters and inadvertent tripping of protective relays, which may lead to cascaded failure of the system [2].

1.1 Motivation

A review in history of technological evolution of power system have shown that, energy conversion technologies has been developed independently and in many instances, selection of available energy technologies were not economical, as it is today. In many instances, dynamic characteristics are not considered in system resource planning stages. Most cases, controller gains of individual generation unit are tuned based on dynamic characteristics for different generators. In early days, due to high inertia nature of power system, it observed many inter area oscillation events [3] at high voltage network. Power system stabilizer tuning methods to mitigate such oscillations [4] have been proposed by many system operation standardization organizations.
In [5], control function of microgrid is standardized to ensure seamless operation of microgrid in grid connected and islanded mode. These standards helps in improving interoperability of DER units with different energy management systems, independent of different platforms. Primary, secondary and tertiary control functions are standardized based on type of load and time scale of microgrid operations considering device physics and information and communication system (ICT) requirements. These standardization does not consider any system specific nonlinear interaction issues due to virtual inertia.

Main purpose of this research is to formulate an analytic method to estimate nonlinear interaction issues with multiple DERs with virtual inertia and study its impact on sizing of battery energy storage. At present, sizing of BESS are decided by power demand at specific location of installation, based on static system operation criteria. Virtual inertia functions helps in improving grid operations, subject to intermittent grid disturbance events. An analytic method to quantify this would help in improving future power system investments.

1.2 Research Contribution

Quantification of inertial response is the major objective for this study. A linearized model of PVSG scheme has been presented here. This model will help in developing numeric model to assess interactions issues with multiple virtual inertia scheme in a system. Such numeric model for more system study cases need to be developed. The linearized model is also validated with time domain model of the study.
1.3 Thesis Statement

Thesis is organized as follows.

Literature review on virtual inertia modeling for converter based DERs has been presented in chapter 2. A brief review on analytic control scheme design for dynamic performance improvement of power system has also been included in this chapter.

In chapter 3, dynamic model of distributed energy resources (DERs) are introduced. Also transmission and distribution system models are explained, which are included in differential and algebraic equation model of microgrid systems.

Time domain simulation results of microgrid system, with PVSG unit as DER is presented in chapter 4.

In chapter 5, a linearized model of PVSG scheme has been derived and small signal stability of the microgrid system, with and without SG emulator unit has been presented.

Chapter 6 includes concluding remarks and future research directions based on the analysis presented in thesis.
Chapter 2

A review to virtual inertia in power electronic converter based energy systems and its application on different system integration locations

2.1 Literature Review

Inertial response of generators in power systems can be defined by inertia to dynamic phenomena in a generator unit, subject to imbalance in power supply and demand, due to any dynamic event in the system. Synchronous machine or other electro-mechanical devices have high inertia due to large rotating mass. Power electronic converter consists of fast electronic switches. Converter with standard control scheme lacks
inertia, compared to electro-mechanical devices. Dominance of electro-mechanical
generation unit in bulk system provides large inertia to system dynamics. However,
converter based DER units in a smaller system like microgrid cause a weak inertia
system. Synthetic modeling of inertia in inverter control scheme will be reviewed and
a new scheme for inertial response improvement will be proposed in this research.

2.1.1 Inertia of electro-mechanical devices

Dynamic model of electro-mechanical generator based power system components
is presented in [6]. Synchronous generator based DERs consists of speed governor,
turbine, voltage control/exciter and synchronous machine. Synchronous machine can
be modeled as detailed three-damper-winding model, considering linear and non-linear
magnetic circuits [6]. Rotating mass of any such electro-mechanical device provide
inertia to system dynamics. Detail and reduced mathematical model of such dynamics
are explained in [6], [7].

2.1.2 Concept of Virtual Inertia by Four Quadrant Power
Converters

Response time of fast acting inverter can be modulated by effective control algorithm.
Slow dynamics of synchronous machine or other electro-mechanical device can be
mimicked by inverter with virtual modeling equations, integrated in inverter control
module. Energy system required for virtual inertia applications, should have the
capability to work in four quadrant mode. In four quadrant mode, energy system should be designed to operate as receptor and generator [8]. Rectifier mode of operation of converter is required when there is a disturbance in the system due to power imbalance, caused by any event and there are excessive energy generated than power demand. There are two possible solutions for energy absorption, while applying in case of grid inertia enhancements. Excess energy can be stored locally using storage technology and used at a later time when energy demand exceeds energy generated or it can be dissipated locally. In generator mode, converter should work as an inverter.

2.1.3 System Inertia: case study

System inertia of power system always played critical role in power system operations. Many of such oscillations in system are originated by equipment and control system failure and any unplanned operating condition change [9]. Oscillation monitoring of such systems have been improved by PMUs. Sustained oscillation can cause mechanical vibration, cascading failure in power system. Mechanical vibration reduces lifespan of equipment [9].

2.1.4 Selection of storage technologies

Energy storage technologies have been applied to power system for different grid support functions. Storage technologies can be classified based on different performance matrices. Discharge time is one of the important matrix for grid support functions for
inertia applications under power deficient conditions in the grid.

2.1.5 Virtual inertia modeling

Virtual modeling of slower dynamics of electro-mechanical devices to mimic its inertial behavior, which is interfaced to the control algorithm of power electronic converter, has been studied in different literature. Most of these studies focused on mimicking dynamic electro-mechanical behavior of existing electro-mechanical devices, integrated
to power system, and studying interaction of such functions with power electronic converters for different hardware topology. Most of these algorithms able to reduce frequency variations (RoCoF) in the system and effectively improve inertia of microgrid system.

Dynamic model of electro-mechanical generator based power system components is presented in [6]. Synchronous generator based DERs consists of speed governor, turbine, voltage control/exciter and synchronous machine. Synchronous machine can be modeled as detailed three-damper-winding model, considering linear and non-linear magnetic circuits [6]. Rotating mass of any such electro-mechanical device provide inertia to system dynamics. Detail and reduced mathematical model of such dynamics are explained in [6], [7].

Response time of fast acting inverter can be modulated by effective control algorithm. Slow dynamics of synchronous machine or other electro-mechanical device can be mimicked by inverter with virtual modeling equations, integrated in inverter control module. Virtual modeling of synchronous machine in inverter was first reported in [10]. Dynamic properties of virtual inertial response was experimentally analyzed in [11]. A d-q frame based simplified three phase synchronous machine model is considered. A virtual synchronous generator, independent of current and voltage reference tracking, was proposed in [12]. In [12], more detailed modeling of synchronous generator is considered compared to its previous solutions. Synchronverter have the capability to emulate under-excitation and hunting. Inertia, field inductance, friction coefficients etc. can be modeled in synchronverter.

Synchronverter application for power systems have been developed recently considering its performance at different power grid operation scenarios. In [13], phase-lock-loop (PLL) block for grid synchronization has been replaced by a self-synchronization unit.
so that synchronverter can be synchronized, before connected to grid. Synchronverter system based HVDC control has been proposed in [14].

Active power control dynamic response of such system are adjusted by variation in frequency droop coefficients, which is not allowed as per grid code. A damping correction loop is added in [15] to control active power, without any variation in steady state frequency droop co-efficient. Analysis of synchronverter, subject to fault is presented in [16] and a fault ride through mechanism is proposed. Region of convergence of equilibrium of a synchronverter is analyzed in [17] for reliable voltage and frequency control. Virtual field current control for robust response to fault current is proposed in [18]. In [18], global stability of synchronverter was improved by virtual increase in filter inductor. Synchronverter model also been modified for accurate active power tracking.

Virtual Oscillator control of inverter was introduced in [19] and experimentally validated in [20]. Synchronization conditions for such oscillator bases sources in power network had been analyzed in [21] and [22]. A virtual oscillator controlled inverter gives satisfactory dynamic response and stability under system dynamics. Virtual oscillator in inverter is modeled as a dead zone oscillator. Inductance and capacitance of the oscillator are designed based on oscillator frequency (60 Hz). Resistance and other oscillator parameter are designed based on stability criteria ([19] - [22]). A new reactive power control scheme based on Virtual Oscillator Control is proposed in [23]. An inverter control topology of PVSG inverter unit is proposed in [24]. A virtual inertial control loop is integrated in inverter frequency control unit. Also a dual droop control is integrated in battery storage unit of DC microgrid of the system. In this paper, a synchronverter system is applied to a microgrid with gas-turbine based DER. Inertial response of synchronverter system is analyzed and compared with standard
real-reactive power control of inverter. Gas-turbine (GT) governor was modeled as per [25] and physical constraints (acceleration, temperature limits etc.) are considered for governor modeling. EMTP model of GT based DER and Synchronverter are modeled in PSCAD.

Some of the applications of virtual inertia algorithms with DERs will be reviewed in this section. In a recent study in [26], a case study of a microgrid with three inverters and synchronous machine is presented with different virtual inertia of inverters to represent dynamic interactions in a microgrid due to different virtual inertia gains, at islanded and grid connected mode of operations. In this work, active and reactive power control of inverter is considered decoupled due to inductive output impedance of inverter. If, inverter output impedance is not inductive, then it is adjusted [27] to inductive value. Voltage control loop also has desired bandwidth to ensure inverter as ideal controlled voltage source. Under such assumptions, transfer functions have been derived for influence of load power changes on phase angle of voltage at common bus of microgrid, power and frequency of inverter. Bode diagram and step response is used as analytic tool to analyze impact of virtual inertia on frequency and interactions of DERs with different virtual inertia. In this study, for interaction analysis, two inverters have identical inertia constant to emulate high and low inertia microgrid. Inverter 1 in this system is set at two group of inertia values in small and large value region to analyze the interactions. From the interaction study, it is observed that inverter 1 frequency in high frequency region is determined by inertia constant, grid has negligible influence. Inertia constants of other inertia mainly influences inverter 1 response at low frequency region. In the case when inverter 1 inertia is from a small inertia group and inverter 2 and 3, which emulates grid is from a large inertia group, due to load change, sudden change in inverter 1 RoCoF due to load change is decreased by high inertia of other
inverters. In other case study, when inverter 2 and 3 are from small inertia group, the RoCoF of inverter 1 is accelerated. So, this paper clearly presents the inverter inertia interaction issues and how it impacts dynamic response of associated DERs.

A new control strategy by readjusting multiple microgrid (MMG) unit based on for system operations with high renewable penetration is proposed in [28]. A center of gravity based formulation is proposed here to define calculation of set points for frequency deviation reduction. As per the concept of center of gravity (COG) as expressed in this paper, strategic MMG islanding to improve RoCoF and frequency nadir. A fictitious reactance value is adjusted based on a minimization problem, which affects frequency dynamics. This computation strategy helps in determining energy storage dispatch or microgrid islanding, which helps in accommodate high penetration level of DER. This analytic study gives a system level approach to inertia improvement and there is no study on sizing of energy storage for different system operation scenario.

In [29], an analogy has been established between stored energy in mechanical rotor of electromechanical devices and capacitor energy. An equivalent inertia co-efficient for capacitor has been derived here.

2.2 Dynamic Performance Improvement of Power System: A Review

2.2.1 Linear Control Design

The purpose of a control system design is to obtain desired dynamic performance of selected signals/outputs of a physical systems. Any dynamic model of system
component may be inaccurate or change with time. Worst-case system uncertainty need to be considered for control system design. In literature of control system design, robust performance (RS) and robust stability (RS) assess system performance and stability conditions respectively, considering worst-case system uncertainty.

Nonlinear model of power system can be formulated based on math model of device physics of individual system components. A steady state operating point of such dynamic system need to be determined, with respect to which, model is to be linearized. Equations need to be linearized using taylor series expansion and variables need to be replaced by deviation variables [30, Chapter 1].

System transfer function and frequency response gives useful property of system dynamics. Loop-shape of feedback controller can be designed [30, Chapter 2] for disturbance rejection, noise mitigation, system stability and performance improvement. Feedback control design for MIMO system and its trade off are well defined in [30, Chapter 9]. Singular values of matrix transfer functions are shaped for dynamic performance improvement. Linear Quadratic Gaussian (LQG) control provides poor performance under robustness. $\mathcal{H}_\infty$ optimization based control technique is developed to improve controller design performance under robustness.

2.2.1.1 Small signal stability analysis of dynamical systems

Differential and algebraic equations can be linearized using taylor series. Linearized algebraic variables can be represented as linear function of state variable and following
linear equation can be formulated,-

\[ \Delta \dot{x} = A \Delta x + B \Delta u \]  

(2.1)

Where \( \Delta x, \Delta u \) is small perturbation from steady state operating point \( x_0 \) and \( u_0 \).

Significance of eigenvalue of matrix \( A \) on small signal stability are well presented in [7].

2.2.2 Nonlinear Control Design

Nonlinear control system design techniques deals with exact nonlinear dynamical system. Different control design techniques and stability conditions are presented in [31].

Sufficient condition for stability for nonlinear dynamical system is given by Lyapunov stability [31, Chapter 4]. Stability of a steady state operating point of power system can be determined by Lyapunov’s indirect method (Theorem 4.7, [31, Chapter 4]).

Global stability conditions can be assessed by Lyapunov’s direct method (Theorem 4.1, [31, Chapter 4]). Feedback linearization control, sliding mode control and many other nonlinear control schemes are presented in [31]).

2.2.3 Data-driven Control Design

Data-driven control scheme e.g. adaptive control [32] can be applied for power system control scheme design. Adaptive control algorithm tunes controller gains in real time for desired performance of controller gains. In direct adaptive control algorithm,
the error between reference model and actual system model is forced to zero by parameter adjustment algorithm. On-line parameter estimation and computation of controller parameters based on estimated plant model is implemented in indirect adaptive control algorithm [32, Chapter 1]. Parameter adaptation algorithm can be applied for deterministic and stochastic environment. System stability under model uncertainty can be maintained by robust control design [32, Chapter 8].
Chapter 3

Modeling of microgrid system

3.1 Dynamic Modeling of Power System: A Review

Power system dynamics can be mathematically modeled as set of nonlinear differential and algebraic equations (DAEs). In this section, preliminary theory on dynamic modeling of power system will be introduced. Mathematical model of electromagnetic and electromechanical distributed energy sources (DERs) will be presented along with dynamic model and approximate phasor model of distribution system. Preliminary software tools for power system dynamic analysis will also be presented here.
3.1.1 DAE model of power system

Power system dynamics can be modeled as,-

\[
\frac{dx}{dt} = f(x, y, u) \tag{3.1}
\]

\[
g(x, y) = 0 \tag{3.2}
\]

Where \(x\) and \(y\) are state and algebraic variables respectively. \(u\) is input variables. This model is autonomous system, considering equations are not explicit functions of time \(t\). Dynamic state variable \(x\) can be defined based on dynamic model of generators and loads in the system. Equation 3.1 and 3.2 are solved as initial value problem to find solution of this model. The solution, solved using numeric integration techniques, simulates approximate dynamic response of power system. Approximation in the solution consists of approximation in dynamic model of power systems components (generators, loads, transmission/distribution lines etc.) and approximation in numeric integration technique. More details on numeric integration techniques can be explored in ([33], [34]).

3.1.2 Transmission and Distribution System model

In this section transmission and distribution system modeling and analysis techniques are introduced. Fast electromagnetic impact of transmission and distribution systems are neglected for slow dynamic simulations.

1. Line Segment Model
(a) **Exact line segment model**

Figure 3.1 represents exact line segment model between node $m$ and $n$ [35]. Formulation of $Z_{abc}$ and $Y_{abc}$ for some standard overhead line and underground cable configurations are presented in [35].

![Figure 3.1: Exact model of line](image)

(b) **Modified Line Segment Model**

Line segments with small shunt admittance can be approximated by modified line model as shown in figure 3.2.

![Figure 3.2: Modified model of line](image)
(c) **Approximate line segment model**

Line segment data can be available in terms of only positive \((Z_+)\) and zero \((Z_0)\) sequence impedance values. In this model, imbalance effect due to untransposed lines are neglected. In bulk transmission line, system is balanced due to transposed lines. For distribution system in a microgrid scenario, balanced system is considered for controller design in this study.

The sequence impedance matrix of a line can be formulated as:

\[
[Z_{seq}] = \begin{bmatrix}
Z_0 & 0 & 0 \\
0 & Z_+ & 0 \\
0 & 0 & Z_+
\end{bmatrix} \tag{3.3}
\]

\[
[Z_{abc}] = [A_s] \cdot [Z_{seq}] \cdot [A_s]^{-1} \tag{3.4}
\]

\[
[A_s] = \begin{bmatrix}
1 & 1 & 1 \\
1 & a_s^2 & a_s \\
0 & a_s & a_s^2
\end{bmatrix}, a_s = 1\angle120 \tag{3.5}
\]

2. **Nodal Admittance Matrix Calculation**

In this section, nodal admittance matrix calculation methods for distribution/transmission system will be formulated. Nodal admittance matrix is formulated neglecting system network unbalance, since this nodal admittance matrix is used for control scheme design of DERs in microgrid. Actual system configuration (balanced/unbalanced system) should be considered for dynamic simulations. Impact of unbalance at DER terminal (e.g. PCC of Inverter for a battery energy storage) can be reduced using phase balancing and phase prediction algorithm [36] or power quality technology solutions [37] to comply with distribution system standards. Based on positive sequence admittance of line segments, nodal admittance
matrix $Y_N$ can be formed applying KCL. Network matrix can be reduced at DER bused/nodes applying kron reduction. For this study load models are approximated as constant impedance load. More detail dynamic load model can be introduced for more accurate system dynamic study. Network algebraic equations for a balanced network, approximated as phasor equivalent of network positive sequence voltages and currents can be formulated as,-

$$I = Y_{Node}V \quad (3.6)$$

Here, $Y_{Node}$ is a nodal admittance matrix for a $N$ bus system. For $i, j \in N$,

$$Y_{Node}(i, j) = -y(i, j) \quad (3.7)$$

Where $y(i, j)$ is positive sequence admittance of line connected between node $i$ and $j$.

$$y(i, j) = \frac{1}{Z_+(i, j)} \quad (3.8)$$

Diagonal elements of nodal admittance matrix can be formulated as,-

$$Y_{Node}(i, i) = \sum_{j \in N_i} y(i, j) \quad (3.9)$$

Where $N_i$ is set of nodes connected to node $i$ through lines. Three phase nodal
admittance matrix can be formulated for a unbalanced distribution system, based on Kirchoff’s current law, based on similar formulations as in positive sequence nodal admittance matrix. Zero current injection nodes can me removed by reducing positive sequence and three phase nodal admittance matrix as per kron reduction [35].

3.1.3 Dynamic model of DERs

1. Photovoltaic Unit

Differential and algebraic equation model of photovoltaic unit is presented here. Figure 3.3 shows the photovoltaic power delivery system consisting VSC and MPPT scheme. DC/DC converter unit is neglected due to its fast dynamics.

![Photovoltaic Power Delivery System](image)

Figure 3.3: Photovoltaic Power Delivery System

3.1.3.1 Dynamic Equations

- Photovoltaic Array Dynamics:

  Photo-voltaic panel dynamics can be modeled as [38],

\[\text{Photovoltaic} \]

\[\text{Figure 3.3: Photovoltaic Power Delivery System} \]
\[ i_{pv} = n_p I_{ph} - n_p I_{rs} \left[ \exp \left( \frac{q n_s}{k B A} \frac{v_{dc}}{n_s} \right) - 1 \right] \] (3.10)

- **DC Link Capacitor Dynamics:**

Dynamic behaviour of inverter DC link capacitor can be modeled as:

\[ \frac{C}{2} \frac{dv_{dc}^2}{dt} = P_{PV} - P_{AC} \] (3.11)

\[ P_{AC} = v_d i_d + v_q i_q \] (3.12)

where \( P_{PV} = i_{pv} v_{dc} \)

- **DC Voltage Controller Dynamic Model:**

Dynamic model of proportional-integral controller of DC voltage control can be modeled as:

\[ a_{dc} = K_{P_{dc}} \left( V_{dc_{ref}} - v_{dc} \right) \] (3.13)

\[ \frac{ds_{dc}}{dt} = K_{I_{dc}} \left( V_{dc_{ref}} - v_{dc} \right) \] (3.14)

\[ i_{d_{ref}} = a_{dc} + s_{dc} \] (3.15)

\( V_{dc_{ref}} \) is estimated by maximum power point tracking scheme.

- **Decoupled Current Controller:**

Decoupled current control ensure decoupled dynamics in d-q axis current control.
The controller can be modeled as follows,

\[
\frac{di_d}{dt} = -\frac{1}{\tau_i} i_d + \frac{1}{\tau_i} i_{d,ref} \\
\frac{di_q}{dt} = -\frac{1}{\tau_q} i_q + \frac{1}{\tau_q} i_{q,ref}
\]

\[
i_{q,ref} = \frac{Q_{ref}}{v_d}
\]

**Phase Lock Loop:**

Phase lock loop (PLL) in inverter estimates frequency and phase angle. Phase angle estimation is required for \(d-q\) transformation of alternating voltage and current signals. Dynamic model of PLL can be modeled as:

\[
\frac{dX_{pll}}{dt} = K_I v_q \\
a_{pll} = K_p v_q
\]

\[
\omega_{pll} = a_{pll} + X_{pll}
\]

\[
\frac{d\delta_{pll}}{dt} = \omega_{pll}
\]
2. BESS Unit

DAE model of Battery Energy Storage System (BESS) connected to microgrid is presented. Figure 3.5 shows the BESS power delivery system consisting DC-DC bidirectional converter and Voltage source converter. The model presented here is applicable for both charge and discharge mode of BESS due regenerative configuration of sinusoidal pulse width modulation (SPWM) based VSC.
3.1.3.2 Dynamic Equations

- **Battery Storage Dynamics:**

- **DC Link Capacitor Dynamics:**

Dynamic behavior of inverter DC link capacitor can be modeled as:

\[
\frac{C}{2} \frac{d v_{dc}^2}{dt} = P_{BESS} - P_{AC} \tag{3.23}
\]

\[
P_{AC} = v_d i_d + v_q i_q \tag{3.24}
\]

- **Battery Energy Storage Dynamic Model:**

Charging model of BESS can be modeled as [39],

*Lead - Acid:*

Discharge: \( V_{\text{batt}} = E_0 - R \cdot i - K \frac{Q}{Q - it} (it + i^* + 1) + \text{Exp}(t) \)

Charge: \( V_{\text{batt}} = E_0 - R \cdot i - K \frac{Q}{it - 0.1Q} i^* + K \frac{Q}{Q - it} i^* + \text{Exp}(t) \)

Where \( \text{Exp}(t) \) is exponential zone voltage.

*Li-Ion:*

Discharge: \( V_{\text{batt}} = E_0 - R \cdot i - K \frac{Q}{Q - it} (it + i^*) + A \cdot \text{Exp}(-B \cdot it) \)

Charge: \( V_{\text{batt}} = E_0 - R \cdot i - K \frac{Q}{it - 0.1Q} i^* + K \frac{Q}{Q - it} i^* + \text{Exp}(t) \)

Charge and discharge behavior of such models and its experimental validation can be found in [39].

- **Inverter control scheme**
Inverter control units such as DC link voltage control, decoupled current control and PLL block are also part of BESS control scheme and its dynamics is modeled as shown in dynamic model of photovoltaic systems.

3. GT based DER

Gas Turbine(GT) based distributed energy resource(DER) consists of GT based governor, exciters and synchronous machine. Generic CIGRE model is used for analysis [25].

3.1.3.3 Dynamic Equations

- CIGRE Generic Gas Turbine Governor Model:

  CIGRE generic gas turbine governor model schematic is presented in figure 3.6. Dynamic model of governor is developed based on CIGRE model consisting temperature control, acceleration control and speed control unit. Acceleration and temperature control units are activated due to fast system operating point change and overheating caused by overload condition. For state space linearized model with respect to steady state operating point, acceleration and temperature control models are neglected from dynamic equations.
Figure 3.6: CIGRE Generic Gas Turbine Governor Model

\[
T_{dg} \frac{dX_1}{dt} = -X_1 - K_{dg} \frac{\omega_s}{2H} \left[ P_{mech} - E' q I_q - \left( X_q - X'_d \right) I_d I_q - D (\omega - \omega_s) \right]
\]

(3.25)

\[
\frac{dX_2}{dt} = K_{ig} (\omega_{ref} - \omega)
\]

(3.26)

\[
\frac{dX_3}{dt} = -X_3 + X_1 + X_2 + K_{pg} (\omega_{ref} - \omega)
\]

(3.27)

\[
T_{td} \frac{dX_4}{dt} = -X_4 + K_t (F_m X_3 - W_{f0})
\]

(3.28)

\[
P_{mech} = X_4
\]

(3.29)

- **Synchronous Machine Model:**

  The one axis (flux decay) machine model is used here for system dynamic analysis.
\[ T_d \frac{dE'_q}{dt} = E_{f_d} - E'_q - \left( X_d - X'_d \right) I_d \quad (3.30) \]

\[ \frac{d\delta}{dt} = \omega - \omega_{ref} \quad (3.31) \]

\[ \frac{2H}{\omega_s} \frac{d\omega}{dt} = P_{mech} - E'_q I_q - \left( X_q - X'_q \right) I_d I_q - D (\omega - \omega_s) \quad (3.32) \]

4. Photo-voltaic Synchronous Generator

Super-capacitor has high energy density [40] compared to battery storage. A hybrid energy storage system (HESS) based on photo-voltaic, battery and ultra-capacitor is proposed here. Control scheme of inverter, connected to ultra-capacitor and battery storage have inertial emulation scheme, which enhances system inertia. Photovoltaic system has standard inverter control scheme.

In the following sections, PVSG system schematic and system integration study will be presented. Inertial response of PVSG unit will be compared for system disturbances. IEEE 13 node distribution system is used for system integration study.
4.1 PVSG System Schematic

Figure 3.7: PVSG Scheme

Figure 3.7 shows the PVSG scheme [41]. Photovoltaic unit control scheme has maximum power point tracking scheme, reactive power and inverter current control scheme integrated in control block. Synchronous generator emulator is integrated to battery storage and ultra-capacitor unit.
Synchronous generator inertia emulator control scheme is presented in figure 3.8. \( P_{set} \) can be set to zero for battery storage / ultra-capacitor participation for enhancement of inertia. \( P_{set} \) can be set to a predefined value based on secondary frequency response.

5. Power System Interface

Power system interface model includes algebraic model of distribution system connecting generators and loads. Dynamic model of the system and control system design only considers positive sequence model of system. Impact of unbalance is neglected. Positive sequence admittance matrix is estimated from system model, as presented in previous chapter.
3.1.3.4 Interface Parameter Calculation

Interface parameters of any DERs and loads are terminal voltage and current variables. Modeling of interface parameters of individual DER dynamic model consists closed form expression of terminal voltage and currents as a function of state and algebraic variable of dynamical system.

• Synchronous Machine:

  Assumptions: $X'_d = X_q$

  Machine terminal voltage phasor: $\overrightarrow{V_{tm}} = V_{tm} \angle \theta_{tm} = (v_{dm} + jv_{qm}) e^{j(\delta_m - \frac{\pi}{2})}$

  Machine internal voltage: $\overrightarrow{V_{int_m}} = jE'_q \cdot e^{j(\delta_m - \frac{\pi}{2})}$

  Current injected by machine phasor: $\overrightarrow{I_{tm}} = I_{tm} \angle \gamma_{tm} = (i_{dm} + ji_{qm}) e^{j(\delta_m - \frac{\pi}{2})}$

  Interface equation:

  $$\overrightarrow{V_{tm}} + (R_S + jX'_d) \overrightarrow{I_{tm}} = jE'_q \cdot e^{j(\delta_m - \frac{\pi}{2})} \tag{3.33}$$

• Photovoltaic Unit:

  Inverter PCC Voltage phasor: $\overrightarrow{V_{PV\text{pcc}}} = V_{PV\text{pcc}} \angle \theta_{PV\text{pcc}}$

  Inverter PCC Current phasor: $\overrightarrow{I_{PV\text{pcc}}} = I_{PV\text{pcc}} \angle \gamma_{PV\text{pcc}}$

  Interface equations:

  $$v_d^{PV} + jv_q^{PV} = \overrightarrow{V_{PV\text{pcc}}} \cdot e^{-j\delta_{PLL}^{PV}} \tag{3.34}$$

  $$i_d^{PV} + ji_q^{PV} = \overrightarrow{I_{PV\text{pcc}}} \cdot e^{-j\delta_{PLL}^{PV}} \tag{3.35}$$

• BESS Unit:

  Inverter PCC Voltage phasor: $\overrightarrow{V_{Batt\text{pcc}}} = V_{Batt\text{pcc}} \angle \theta_{Batt\text{pcc}}$
Inverter PCC Current phasor: \( \overrightarrow{I_{\text{Batt}}^\text{pcc}} = I_{\text{Batt}}^\text{pcc} \angle \gamma_{\text{Batt}}^\text{pcc} \)

Interface equations:

\[
\begin{align*}
v_d^{\text{Batt}} + jv_q^{\text{Batt}} & = \overrightarrow{V_{\text{Batt}}^\text{pcc}} \cdot e^{-j\delta_{PLL}^\text{Batt}} & (3.36) \\
i_d^{\text{Batt}} + ji_q^{\text{Batt}} & = \overrightarrow{I_{\text{Batt}}^\text{pcc}} \cdot e^{-j\delta_{PLL}^\text{Batt}} & (3.37)
\end{align*}
\]

- **Interface:**

Positive sequence kron reduced admittance matrix of the system: \( Y_{\text{red}} \)

Node voltages at DER terminals can be defined by \( V_{\text{node}} \). Terminal voltage for inverter based DERs are defined by voltage at point of common coupling. Terminal voltage of synchronous machine based DER is defined in equation 1.60. Voltage and current at DER terminals are coupled by follow equation,-

\[
I_{\text{node}} = Y_{\text{red}} V_{\text{node}}
\]

(3.38)

This formulation can be extended for three phase network modeling for unbalanced distribution system, by considering reduced three phase admittance matrix and voltage and current for all three phases.
Chapter 4

System integration study with photo-voltaic synchronous generator (PVSG)

4.1 Introduction

Time domain analysis of system with PVSG unit is presented here to study inertia response and non-linear interactions of virtual inertia scheme.

4.2 System Integration Study
4.2.1 2 Node system

2 node system is presented in figure 4.1. The synchronous machine is rated at 52.5 kVA, 460 V L-L RMS, 1800 RPM and PVSG unit is rated at 40 kVA. The photovoltaic unit has same rating as combined PVSG unit, and energy storage is rated at 20 kVA.
In this system, dynamic response of PVSG unit is analyzed with different loads at bus $B_2$ in figure 4.1. At bus B1, the load values are set at 20 kW and time domain responses are analyzed. Exciter dynamics and voltage regulator are neglected in this time domain analysis.

Figure 4.2, 4.3, 4.4, 4.5, 4.6 and 4.7 shows the power response of PVSG units in the 2-bus system under study, subject to power set point change in photovoltaic inverter. For operating point 1, 2 and 3, load at bus B2 has been changed at 20 kW, 30 kW and 40 kW.

Figure 4.2: Time Domain Response: Op. Pt. 1 (load at bus B2 at 20 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): under-frequency
Figure 4.3: Time Domain Response: Op. Pt. 1 (load at bus B2 at 20 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): over-frequency

Figure 4.4: Time Domain Response: Op. Pt. 2 (load at bus B2 at 30 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): under-frequency
Figure 4.5: Time Domain Response: Op. Pt. 2 (load at bus B2 at 30 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): over-frequency

Figure 4.6: Time Domain Response: Op. Pt. 3 (load at bus B2 at 40 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): under-frequency
Figure 4.7: Time Domain Response: Op. Pt. 3 (load at bus B2 at 40 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA): over-frequency

Figure 4.8, 4.9,4.10,4.11,4.12 and 4.13 shows the frequency response of PVSG units in the 2-bus system under study, subject to power set point change in photovoltaic inverter, for operating point 1, 2 and 3.

Figure 4.8: Time Domain Response: Op. Pt. 1 (load at bus B2 at 20 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA)
Figure 4.9: Time Domain Response: Op. Pt. 2 (load at bus B2 at 30 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA)

Figure 4.10: Time Domain Response: Op. Pt. 3 (load at bus B2 at 40 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA)
Figure 4.11: Time Domain Response: Op. Pt. 1 (load at bus B2 at 20 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA)

Figure 4.12: Time Domain Response: Op. Pt. 2 (load at bus B2 at 30 kW, dynamic event: solar inverter power set point change of 0.1 p.u. @ 40 kVA)
4.3 IEEE 13 Node System

Time domain response of IEEE 13 node system is studied here to see the impact of PVSG unit for inertia improvement of the system. The system model is approximated as phasor model and the system admittance matrix is reduced at DER buses for the integration to algebraic equation of system DAE model for dynamic analysis. All loads are approximated as constant impedance load. Exciter dynamics is neglected for time domain analysis of this study case. A proportional-integral control based voltage regulator is considered for the study.
For test case 1, load at bus 13 and 14 for system under study as in figure 4.15 and 4.16, are set at 100 kW, having total system load of 200 kW. Load at bus 13 and 14 is set at 400 kW each, with total system load of 800 kW, for the system under study for case 2.
Figure 4.15: Time domain response: Case 1: under-frequency (load at bus 13 and 14 at 100 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)

Figure 4.16: Time domain response: Case 1: over-frequency (load at bus 13 and 14 at 100 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)
Figure 4.17: Time domain response: Case 2: under-frequency (load at bus 13 and 14 at 400 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)
Figure 4.18: Time domain response: Case 2: over-frequency (load at bus 13 and 14 at 400 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)

Figure 4.19 and 4.20 shows impact of inertial response of PVSG unit to system inertia.

Figure 4.19: Frequency response: Case 2: under-frequency (load at bus 13 and 14 at 400 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)
Figure 4.20: Frequency response: Case 2: over-frequency (load at bus 13 and 14 at 400 kw each, dynamic event: solar inverter power set point change of 0.02 p.u. @ 1 MVA)
Chapter 5

State Space Modeling of microgrid systems

5.1 2 Node System

This system is modeled based on line data obtained from a benchmark low voltage microgrid system as adopted by CIGRE Task Force C6.04.02 [42]. Modeling for generation and load dynamics are adjusted as per the system rating and requirement of case study.
5.2 Synchronous Generator Emulator

Figure 5.1: PVSG Scheme

Figure 5.1 shows schematic of photo-voltaic synchronous generator. Figure 5.2 shows synchronous generator emulator schematic. This system is integrated to standard low voltage distribution system. GT-DER of the microgrid system consists of CIGRE generic speed governor model with speed, acceleration and temperature control unit.
Differential and algebraic equation model of the above systems can be modeled as follows,

\[
\begin{align*}
\frac{d\delta}{dt} &= \omega_p \quad (5.1) \\
\frac{d\delta_p}{dt} &= K_{ip}(P_{set} - P) \quad (5.2) \\
\frac{d\omega_p}{dt} &= K_{i\omega}(P_{set} - P) \quad (5.3) \\
\delta_r &= \delta + \delta_p \quad (5.4) \\
\frac{dV_{rq}}{dt} &= K_{iq}(Q_{set} - Q) \quad (5.5) \\
V_r &= V_{rn} + V_{rq} \quad (5.6) \\
\overrightarrow{V_r} &= V_r \cdot e^{j\delta_r} \quad (5.7) \\
\overrightarrow{I_r} &= I_r \cdot e^{j\delta_i} \quad (5.8) \\
P &= real\{\overrightarrow{V_r} \cdot \overrightarrow{I_r}^*\} \quad (5.9) \\
Q &= imag\{\overrightarrow{V_r} \cdot \overrightarrow{I_r}^*\} \quad (5.10)
\end{align*}
\]
δ_i is an algebraic function of δ_r.

Here \( \delta_i = \delta_r + \theta \)

Difference between \( \delta_r \) and \( \delta_i \) i.e. \( \theta \) can be calculated based on \( P_{set} \) and \( Q_{set} \). \( \theta \) is a constant.

Here \( \delta_i = \delta_r + \theta \)

## 5.3 Photovoltaic Systems

Differential and algebraic equation model of photo-voltaic system can be modeled as-

### 5.3.1 P-Q Controller Dynamics

\[
\frac{dS_p}{dt} = K_{I_p}(P_{ref} - P_{PV}) \tag{5.11}
\]

\[
\frac{dS_q}{dt} = K_{I_q}(Q_{ref} - Q_{PV}) \tag{5.12}
\]

\[i_{dref} = S_p + K_{P_p}(P_{ref} - P_{PV})\]

\[i_{qref} = S_q + K_{P_q}(Q_{ref} - Q_{PV})\]
5.3.2 Decoupled Current Control

\[
\frac{d i_{d_{PV}}}{dt} = -\frac{1}{\tau_i} i_{d_{PV}} + \frac{1}{\tau_i} i_{d_{ref}} \tag{5.13}
\]

\[
\frac{d i_{q_{PV}}}{dt} = -\frac{1}{\tau_i} i_{q_{PV}} + \frac{1}{\tau_i} i_{q_{ref}} \tag{5.14}
\]

\[
i_{q_{ref}} = \frac{Q_{ref}}{v_{d_{PV}}} \tag{5.15}
\]

5.3.3 Phase Lock Loop

\[
\frac{d X_{pll}}{dt} = K_I v_{q_{PV}} \tag{5.16}
\]

\[
a_{pll} = K_p v_{q_{PV}} \tag{5.17}
\]

\[
\omega_{pll} = a_{pll} + X_{pll} \tag{5.18}
\]

\[
\frac{d \delta_{pll}}{dt} = \omega_{pll} \tag{5.19}
\]

5.3.4 Inverter PCC Voltage and Currents

Voltage and current at inverter point of common coupling can be modeled as:

\[
v_{d_{PV}} + j v_{q_{PV}} = \overrightarrow{V_{inv}} e^{-j \delta_{pll}} \tag{5.20}
\]

\[
\overrightarrow{I_{inv}} = (i_{d_{PV}} + j i_{q_{PV}}) e^{j \delta_{pll}} \tag{5.21}
\]

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5.4 GT-DER DAE Model

Synchronous machine of GT-DER unit can be modeled as,-

\[ T_{d0} \frac{dE'}{dt} = E_f - E' - \left( X_d - X_d' \right) I_d \] (5.22)

\[ \frac{d\delta_m}{dt} = \omega - \omega_{ref} \] (5.23)

\[ 2H \frac{d\omega}{\omega_s} = P_{mech} - E' I_q - \left( X_q - X_d' \right) I_d I_q - D (\omega - \omega_s) \] (5.24)

CIGRE gas turbine governor can be modeled as,-

\[ T_{dg} \frac{dX_1}{dt} = -X_1 - K_{dg} \frac{\omega_s}{2H} [P_{mech} - E' I_q - \left( X_q - X_d' \right) I_d I_q - D (\omega - \omega_s)] \] (5.25)

\[ \frac{dX_2}{dt} = K_{ig} \left( \omega_{ref} - \omega \right) \] (5.26)

\[ \frac{dX_3}{dt} = -X_3 + X_1 + X_2 + K_{pg} \left( \omega_{ref} - \omega \right) \] (5.27)

\[ T_{td} \frac{dX_4}{dt} = -X_4 + K_t \left( F_m X_3 - W_f \right) \] (5.28)

\[ P_{mech} = X_4 \] (5.29)

Network interface for synchronous machine can be formulated as,-

Assumptions: \( X_d' = X_q \)

Machine terminal voltage phasor: \( \vec{V_m} = V_m \angle \theta_{lm} = (v_{d_m} + jv_{q_m}) e^{j(\delta_m - \frac{\pi}{2})} \)

Machine internal voltage: \( \vec{V_{int_m}} = jE' q \cdot e^{j(\delta_m - \frac{\pi}{2})} \)
Current injected by machine phasor: $\vec{I}_m = I_m \angle \gamma_m = (i_{d_m} + j i_{q_m}) e^{j(\delta_m - \frac{\pi}{2})}$

Interface equation:

$$\vec{V}_m + (R_S + j X_d') \vec{I}_m = j E'_q \cdot e^{j(\delta_m - \frac{\pi}{2})} \quad (5.30)$$

### 5.5 Network DAE Model

In this test case, two PVSG unit and one GT-DER unit are connected to CIGRE system for dynamic study.

If $\vec{V}_m$ being voltage phasor for GT-DER then,-

$$\begin{bmatrix} V_t \\ I_t \end{bmatrix} = \begin{bmatrix} \vec{V}_m & \vec{V}_{inv1} & \vec{V}_{r1} & \vec{V}_{inv2} & \vec{V}_{r2} \end{bmatrix} \begin{bmatrix} V_t \\ I_t \end{bmatrix}$$

$$\begin{bmatrix} V_t \\ I_t \end{bmatrix} = \begin{bmatrix} \vec{I}_m & \vec{I}_{inv1} & \vec{I}_{r1} & \vec{I}_{inv2} & \vec{I}_{r2} \end{bmatrix} \begin{bmatrix} I_t \end{bmatrix} = [Y_{aug}][V_t] \quad (5.33)$$

Where $Y_{aug}$ is augmented admittance matrix, including interface reactance of SG emulator inverter unit and synchronous machine interface reactance with system nodal admittance matrix.

#### 5.5.1 Inclusion of Inverter and Machine Reactance

$Y_{BUS}$ can be defined as reduced nodal admittance matrix of system. Most loads are modeled as constant impedance load and included in nodal admittance matrix for simplicity.
Machine interface admittance can be modeled as,-
\[ y_m = \frac{1}{R_S + jX'_d} \]

Inverter interface admittance can be modeled as,-
\[ y_{c1} = \frac{1}{g_{c1} + jb_{c1}} \]
\[ y_{c2} = \frac{1}{g_{c2} + jb_{c2}} \]

Reduced system admittance matrix, including loads, modeled as impedance, can be presented as,-
\[
Y_{red_1} = \begin{bmatrix}
g_{11}^0 + j\cdot b_{11}^0 & g_{12}^0 + j\cdot b_{12}^0 \\
g_{21}^0 + j\cdot b_{21}^0 & g_{22}^0 + j\cdot b_{22}^0 \\
\end{bmatrix} \text{ p.u.}
\]

(5.34)

Nodal admittance matrix is reduced to GT-DER and PVSG node. Augmented matrix can be formulated as,-
\[
Y_{aug} = \begin{bmatrix}
g_{11}^0 + j\cdot b_{11}^0 + y_m & g_{12}^0 + j\cdot b_{12}^0 & 0 & -y_m \\
g_{21}^0 + j\cdot b_{21}^0 & g_{22}^0 + j\cdot b_{22}^0 + y_c & -y_c & 0 \\
0 & -y_c & y_c & 0 \\
-y_m & 0 & 0 & y_m \\
\end{bmatrix} \text{ p.u.}
\]

(5.35)
Here node 3 is SG emulator inverter terminal node and node 4 is synchronous machine internal terminal of GT based DER. Rearranging node 1 with node 4, -

\[
Y^R_{aug} = \begin{bmatrix}
    y_m & 0 & 0 & -y_m \\
    0 & g_{22}^0 + j \cdot b_{22}^0 + y_c & -y_c & g_{21}^0 + j \cdot b_{21}^0 \\
    0 & -y_c & y_c & 0 \\
    -y_m & g_{12}^0 + j \cdot b_{12}^0 & 0 & g_{11}^0 + j \cdot b_{11}^0 + y_m
\end{bmatrix}_{p.u.} (5.36)
\]

Applying Kron’s reduction to node 4, -

\[
Y^K_{red_1} = \begin{bmatrix}
    y_m & 0 & 0 \\
    0 & g_{22}^0 + j \cdot b_{22}^0 + y_c & -y_c \\
    0 & -y_c & y_c \\
\end{bmatrix} - \frac{1}{g_{11}^0 + j \cdot b_{11}^0 + y_m} \begin{bmatrix}
    -y_m & g_{12}^0 + j \cdot b_{12}^0 & 0
\end{bmatrix} (5.37)
\]

(5.38)
\[
Y_{\text{red}}^{K} = Y_{\text{red,SMG}}^{K} + Y_{\text{aug}}
\]

Here

\[
Y_{\text{red,SMG}}^{K} = \begin{bmatrix}
\frac{y_{m} - \frac{y_{m}^{2}}{g_{11}^{0} + j \cdot b_{11}^{0} + y_{m}}}{g_{11}^{0} + j \cdot b_{11}^{0} + y_{m}} & \frac{y_{m} \cdot (g_{12}^{0} + j \cdot b_{12}^{0})}{g_{11}^{0} + j \cdot b_{11}^{0} + y_{m}} & 0 \\
\frac{(g_{21}^{0} + j \cdot b_{21}^{0}) \cdot y_{m}}{g_{11}^{0} + j \cdot b_{11}^{0} + y_{m}} & \frac{(g_{22}^{0} + j \cdot b_{22}^{0}) - (g_{21}^{0} + j \cdot b_{21}^{0}) \cdot (g_{12}^{0} + j \cdot b_{12}^{0})}{g_{11}^{0} + j \cdot b_{11}^{0} + y_{m}} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
Y_{\text{aug}} = \begin{bmatrix}
0 & 0 & 0 \\
0 & y_{c} & -y_{c} \\
0 & -y_{c} & y_{c}
\end{bmatrix}
\]

\(Y_{\text{red}}^{K}\) can be reformulated with respect to reduced matrix of standard microgrid case as follows: \(Y_{\text{red}}^{K} = Y_{\text{red,SMG}}^{K} + Y_{\text{aug}}\)

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System DAE model can be formulated as,-

\[
\begin{bmatrix}
\mathbf{i}_m^\rightarrow \\
\mathbf{i}_{inv}^\rightarrow \\
\mathbf{i}_r^\rightarrow 
\end{bmatrix}
= Y_{red}^K \cdot 
\begin{bmatrix}
\mathbf{v}_{int_m}^\rightarrow \\
\mathbf{v}_{inv}^\rightarrow \\
\mathbf{v}_r^\rightarrow 
\end{bmatrix}
\]

(5.41)

5.6 Linearization of system DAE

From equation 40,-

\[
\mathbf{i}_m^\rightarrow = (G_{11} + jB_{11}) \cdot \mathbf{v}_{int_m}^\rightarrow + (G_{12} + jB_{12}) \cdot \mathbf{v}_{inv}^\rightarrow + (G_{13} + jB_{13}) \cdot \mathbf{v}_r^\rightarrow \\
= (g_{11} + jb_{11}) \cdot \mathbf{v}_{int_m}^\rightarrow + (g_{12} + jb_{12}) \cdot \mathbf{v}_{inv}^\rightarrow 
\]

(5.42)

\[
\mathbf{i}_{inv}^\rightarrow = (G_{21} + jB_{21}) \cdot \mathbf{v}_{int_m}^\rightarrow + (G_{22} + jB_{22}) \cdot \mathbf{v}_{inv}^\rightarrow + (G_{23} + jB_{23}) \cdot \mathbf{v}_r^\rightarrow \\
= (g_{21} + jb_{21}) \cdot \mathbf{v}_{int_m}^\rightarrow + (g_{22} + jb_{22}) \cdot \mathbf{v}_{inv}^\rightarrow + y_c \cdot \mathbf{v}_{inv}^\rightarrow - y_c \cdot \mathbf{v}_r^\rightarrow 
\]

(5.43)

\[
\mathbf{i}_r^\rightarrow = (G_{32} + jB_{32}) \cdot \mathbf{v}_{inv}^\rightarrow + (G_{33} + jB_{33}) \cdot \mathbf{v}_r^\rightarrow \\
= -y_c \cdot \mathbf{v}_{inv}^\rightarrow + y_c \cdot \mathbf{v}_r^\rightarrow 
\]

(5.44)

In the above set of equations, variables \( \mathbf{i}_m^\rightarrow, \mathbf{v}_{inv}^\rightarrow \) and \( \mathbf{i}_r^\rightarrow \) depends on algebraic variables. Remaining variables are formulated based on state variables in the DAE model. These algebraic variables need to be formulated in terms of state variables and linearized to form state space model of the system with respect to DAE model, as formulated before.
Phasor quantities in equation 46 can be formulated as,-

\[ \vec{V}_{\text{inv}} = (v_{dPV} + jv_{qPV}) \cdot e^{\delta_{PLL}} \]

\[ \vec{I}_{\text{inv}} = (i_{dPV} + ji_{qPV}) \cdot e^{j\delta_{PLL}} \]

\[ \vec{V}_{\text{int}} = jE'_{q} \cdot e^{j(\delta_{m} - \frac{\pi}{2})} \]

\[ \vec{V}_{r} = V_{r} \cdot e^{j\delta_{r}} \]

As per equation 42

\[ \vec{I}_{\text{inv}} = (g_{21} + jb_{21}) \cdot \vec{V}_{\text{int}} + (g_{22} + jb_{22}) \cdot \vec{V}_{\text{inv}} + y_{c} \cdot \vec{V}_{\text{inv}} - y_{c} \cdot \vec{V}_{r} \]

Simplifying the above equation gives,-

\[ i_{dPV} + ji_{qPV} = (g_{21} + j \cdot b_{21}) \left[ jE'_{q} \cdot e^{j(\delta_{m} - \delta_{PLL} - \frac{\pi}{2})} \right] + (g_{22} + j \cdot b_{22}) \cdot (v_{dPV} + jv_{qPV}) \]

\[ + y_{c} \cdot (v_{dPV} + jv_{qPV}) - y_{c} \cdot V_{r} \cdot e^{j(\delta_{r} - \delta_{PLL})} \]

\[ = (g_{21} + j \cdot b_{21}) \left[ jE'_{q} \cdot \left\{ \cos \left( \delta_{m} - \delta_{PLL} - \frac{\pi}{2} \right) + j \cdot \sin \left( \delta_{m} - \delta_{PLL} - \frac{\pi}{2} \right) \right\} \right] \]

\[ + (g_{22} + j \cdot b_{22}) \cdot (v_{dPV} + jv_{qPV}) + y_{c} \cdot (v_{dPV} + jv_{qPV}) \]

\[ - y_{c} \cdot V_{r} \cdot [\cos (\delta_{r} - \delta_{PLL}) + j \cdot \sin (\delta_{r} - \delta_{PLL})] \]

\[ = (g_{21} + j \cdot b_{21}) \left[ E'_{q} \cdot \left\{ j \cdot \cos \left( \delta_{m} - \delta_{PLL} - \frac{\pi}{2} \right) - \sin \left( \delta_{m} - \delta_{PLL} - \frac{\pi}{2} \right) \right\} \right] \]

\[ + (g_{22} + j \cdot b_{22}) \cdot (v_{dPV} + jv_{qPV}) + y_{c} \cdot (v_{dPV} + jv_{qPV}) \]

\[ - y_{c} \cdot V_{r} \cdot [\cos (\delta_{r} - \delta_{PLL}) + j \cdot \sin (\delta_{r} - \delta_{PLL})] \]

Linearizing the above equation,-

Linearizing equation (27),
\[
\Delta i_{dpv} + j \cdot \Delta i_{qpv} \\
= (g_{21} + j \cdot b_{21}) \left[ E'_{q0} \cdot \left\{ -j \cdot \sin \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) - \cos \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) \right\} \right] (\Delta \delta_m - \Delta \delta_{PLL}) \\
+ (g_{21} + j \cdot b_{21}) \left[ \Delta E'_{q} \cdot \left\{ j \cdot \cos \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) - \sin \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) \right\} \right] \\
+ (g_{22} + j \cdot b_{22} + y_c) (\Delta v_{dpv} + j \Delta v_{qpv}) \\
- y_c \cdot \Delta V_r \cdot [\cos (\delta_{r0} - \delta_{PLL0}) + j \cdot \sin (\delta_{r0} - \delta_{PLL0})] \\
- y_c V_{r0} \{ -\sin (\delta_{r0} - \delta_{PLL0}) + j \cdot \cos (\delta_{r0} - \delta_{PLL0}) \} ((\Delta \delta_r - \Delta \delta_{PLL})) \quad (5.45)
\]

Simplifying the above equation we get,

\[
(\Delta v_{dpv} + j \Delta v_{qpv}) \\
= \frac{1}{(g_{22} + j \cdot b_{22} + y_c)} (\Delta i_{dpv} + j \cdot \Delta i_{qpv}) \\
- \frac{(g_{21} + j \cdot b_{21})}{(g_{22} + j \cdot b_{22}) + y_c} \left[ E'_{q0} \cdot \left\{ -j \cdot \sin \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) - \cos \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) \right\} \right] (\Delta \delta_m - \Delta \delta_{PLL}) \\
- \frac{(g_{21} + j \cdot b_{21})}{(g_{22} + j \cdot b_{22} + y_c)} \left[ \Delta E'_{q} \cdot \left\{ j \cdot \cos \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) - \sin \left( \delta_{m0} - \delta_{PLL0} - \frac{\pi}{2} \right) \right\} \right] \\
- \frac{y_c}{(g_{22} + j \cdot b_{22} + y_c)} \cdot [\cos (\delta_{r0} - \delta_{PLL0}) + j \cdot \sin (\delta_{r0} - \delta_{PLL0})] \cdot \Delta V_r \\
- \frac{y_c}{(g_{22} + j \cdot b_{22} + y_c)} V_{r0} \{ -\sin (\delta_{r0} - \delta_{PLL0}) + j \cdot \cos (\delta_{r0} - \delta_{PLL0}) \} ((\Delta \delta_r - \Delta \delta_{PLL})) \\
\quad (5.46)
\]

Simplifying the above equation we get,
\[ \Delta v_{dPV} = p'_{11} \cdot \Delta i_{dPV} + p'_{12} \cdot \Delta i_{qPV} + p'_{13} \cdot \Delta \delta_m + \left( p'_{14} + p''_{14} \right) \cdot \Delta \delta_{PLL} \]

\[ + p'_{15} \cdot \Delta E'_q + p'_{16} \cdot \Delta V_r - p'_{14} \Delta \delta_r \]  
\[ (5.47) \]

\[ \Delta v_{qPV} = p'_{21} \cdot \Delta i_{dPV} + p'_{22} \cdot \Delta i_{qPV} + p'_{23} \cdot \Delta \delta_m + \left( p'_{24} + p''_{24} \right) \cdot \Delta \delta_{PLL} \]

\[ + p'_{25} \cdot \Delta E'_q + p'_{26} \cdot \Delta V_r - p''_{24} \Delta \delta_r \]  
\[ (5.48) \]

As per equation 41

\[ \vec{I}_m = (g_{11} + j b_{11}) \cdot \vec{V}_{int} + (g_{12} + j b_{12}) \cdot \vec{V}_{inv} \]

\[ (i_{d_m} + j i_{q_m}) = (g_{11} + j b_{11}) \cdot j E'_q + (g_{12} + j b_{12}) \cdot (v_{dPV} + j v_{qPV}) \cdot e^{j(\delta_{PLL} - \delta_m + \frac{\pi}{2})} \]

\[ = (g_{11} + j b_{11}) \cdot j E'_q \]

\[ + (g_{12} + j b_{12}) \cdot (v_{dPV} + j v_{qPV}) \cdot \left\{ \cos \left( \delta_{PLL} - \delta_m + \frac{\pi}{2} \right) + j \cdot \sin \left( \delta_{PLL} - \delta_m + \frac{\pi}{2} \right) \right\} \]

\[ (5.49) \]

Linearizing the above equation we get,
\[
(\Delta i_{d_m} + j\Delta i_{q_m}) = (g_{11} + j\cdot b_{11}) \cdot j\Delta E'_q \\
+ (g_{12} + j\cdot b_{12}) \cdot (\Delta v_{d_{PV}} + j\Delta v_{q_{PV}}) \cdot \left\{ \cos \left( \delta_{PLL_0} - \delta_{m0} + \frac{\pi}{2} \right) + j\cdot \sin \left( \delta_{PLL_0} - \delta_{m0} + \frac{\pi}{2} \right) \right\} \\
+ (g_{12} + j\cdot b_{12}) \cdot (v_{d_0} + jv_{q_0}) \cdot \left\{ -\sin \left( \delta_{PLL_0} - \delta_{m0} + \frac{\pi}{2} \right) + j\cdot \cos \left( \delta_{PLL_0} - \delta_{m0} + \frac{\pi}{2} \right) \right\} \cdot (\Delta \delta_{PLL} - \Delta \delta_m) \quad (5.50)
\]

Equation can be simplified as following analytic form,-

\[
\begin{align*}
\Delta i_{d_m} &= a_{11} \cdot \Delta v_{d_{PV}} + a_{12} \cdot \Delta v_{q_{PV}} + a_{13} \cdot \Delta \delta_m + a_{14} \cdot \Delta \delta_{PLL} + a_{15} \cdot \Delta E'_q \\
\Delta i_{q_m} &= a_{21} \cdot \Delta v_{d_{PV}} + a_{22} \cdot \Delta v_{q_{PV}} + a_{23} \cdot \Delta \delta_m + a_{24} \cdot \Delta \delta_{PLL} + a_{25} \cdot \Delta E'_q
\end{align*}
\quad (5.51, 5.52)
\]
Simplifying the above equation becomes,-

\[
\Delta i_{dm} \\
= \left( a_{11} \cdot p'_{11} + a_{12} \cdot p'_{21} \right) \Delta i_{dPV} + \left( a_{11} \cdot p'_{12} + a_{12} \cdot p'_{22} \right) \Delta i_{qPV} \\
+ \left( a_{13} + a_{11} \cdot p'_{13} + a_{12} \cdot p'_{23} \right) \cdot \Delta \delta_m + \left( a_{14} + a_{11} \cdot \left( p'_{14} + p'_{14} \right) + a_{12} \cdot \left( p'_{24} + p'_{24} \right) \right) \cdot \Delta \delta_{PLL} \\
+ \left( a_{15} + a_{11} \cdot p'_{15} + a_{12} \cdot p'_{25} \right) \cdot \Delta E'_q + \left( a_{11} \cdot p'_{16} + a_{12} \cdot p'_{26} \right) \Delta V_{rq} \\
- \left( a_{11} \cdot p''_{14} + a_{12} \cdot p''_{24} \right) \Delta \delta_r
\]

\[
\Delta i_{qm} \\
= \left( a_{21} \cdot p'_{11} + a_{22} \cdot p'_{21} \right) \Delta i_{dPV} + \left( a_{21} \cdot p'_{12} + a_{22} \cdot p'_{22} \right) \Delta i_{qPV} \\
+ \left( a_{23} + a_{21} \cdot p'_{13} + a_{22} \cdot p'_{23} \right) \cdot \Delta \delta_m + \left( a_{24} + a_{21} \cdot \left( p'_{14} + p'_{14} \right) + a_{22} \cdot \left( p'_{24} + p'_{24} \right) \right) \cdot \Delta \delta_{PLL} \\
+ \left( a_{25} + a_{21} \cdot p'_{15} + a_{22} \cdot p'_{25} \right) \cdot \Delta E'_q + \left( a_{21} \cdot p'_{16} + a_{22} \cdot p'_{26} \right) \Delta V_{rq} \\
- \left( a_{21} \cdot p''_{14} + a_{22} \cdot p''_{24} \right) \Delta \delta_r
\]

(5.53)

Simplifying the above equation we get,-

\[
\Delta i_{dm} = k_{11} \cdot \Delta i_{dPV} + k_{12} \cdot \Delta i_{qPV} + k_{13} \cdot \Delta \delta_m + k_{14} \cdot \Delta \delta_{PLL} + k_{15} \cdot \Delta E'_q + k_{16} \cdot \Delta V_{rq} + k_{17} \cdot \Delta \delta_r
\]

\[
\Delta i_{qm} = k_{21} \cdot \Delta i_{dPV} + k_{22} \cdot \Delta i_{qPV} + k_{23} \cdot \Delta \delta_m + k_{24} \cdot \Delta \delta_{PLL} + k_{25} \cdot \Delta E'_q + k_{16} \cdot \Delta V_{rq} + k_{27} \cdot \Delta \delta_r
\]

As per equation 43

\[
\vec{I}_r \rightarrow = -y_c \vec{V}_{inv} + y_c \vec{V}_r
\]
The above equation can be simplified as,

\[ I_{rd} + jI_{rq} = -y_c \cdot (v_{dpv} + jv_{qpv}) \cdot e^{j(\delta_{PLL} - \delta_i)} + y_c \cdot V_r \cdot e^{j(\delta_r - \delta_i)} \]

\[ = -y_c \cdot (v_{dpv} + jv_{qpv}) \cdot [\cos(\delta_{PLL} - \delta_i) + jsin(\delta_{PLL} - \delta_i)] \]

\[ + y_c \cdot V_r \cdot [\cos(\delta_r - \delta_i) + jsin(\delta_r - \delta_i)] \]

(5.54)

### 5.7 State Space Formation

State variable definitions,

\[ x = [\delta \quad \delta_p \quad \omega_p \quad V_{rq} \quad S_p \quad S_q \quad i_{dpv} \quad i_{qpv} \quad X_{pil} \quad \delta_{pil} \quad X_1 \quad X_3 \quad X_4 \quad E'_q \quad \delta_m \quad \omega] \]

**Derivation of linearized term in state space model**

**Linearization of state space equation**

Linearized model of state space equation can be written as,
**PVSG Unit**

\[
\begin{align*}
\frac{d\Delta \delta}{dt} &= \Delta \omega_p \\
\frac{d\Delta \delta_p}{dt} &= -K_{i_p} \Delta P \\
\frac{d\Delta \omega_p}{dt} &= -K_{i_\omega} \Delta P \\
\Delta \delta_r &= \Delta \delta + \Delta \delta_p \\
\frac{d\Delta V_{rq}}{dt} &= -K_{i_q} \Delta Q \\
\Delta V_r &= \Delta V_{rq}
\end{align*}
\]

**Photovoltaic System DAE**

\[
\begin{align*}
\frac{d\Delta S_p}{dt} &= -K_{I_p} \Delta P_{PV} \\
\frac{d\Delta S_q}{dt} &= -K_{I_q} \Delta Q_{PV} \\
\Delta i_{d_{ref}} &= -K_{P_p} \Delta P_{PV} + \Delta S_p \\
\Delta i_{q_{ref}} &= -K_{P_q} \Delta Q_{PV} + \Delta S_q \\
\frac{d\Delta i_{d_{PV}}}{dt} &= -\frac{1}{\tau_i} \Delta i_{d_{PV}} + \frac{1}{\tau_i} \Delta i_{d_{ref}} \\
\frac{d\Delta i_{q_{PV}}}{dt} &= -\frac{1}{\tau_i} \Delta i_{q_{PV}} + \frac{1}{\tau_i} \Delta i_{q_{ref}} \\
\Delta i_{q_{ref}} &= 0
\end{align*}
\]
\[
\frac{d\Delta X_{\text{pll}}}{dt} = K_I \Delta v_{\text{qPV}} \\
\Delta a_{\text{pll}} = K_p \Delta v_{\text{qPV}} \\
\Delta \omega_{\text{PLL}} = \Delta a_{\text{PLL}} + \Delta X_{\text{pll}} \\
\frac{d\Delta \delta_{\text{PLL}}}{dt} = \Delta \omega_{\text{PLL}}
\]

\[
\Delta P_{\text{PV}} = v_{\text{dPV}0} \Delta i_{\text{dPV}} + v_{\text{qPV}0} \Delta i_{\text{qPV}} + i_{\text{dPV}0} \Delta v_{\text{dPV}} + i_{\text{qPV}0} \Delta v_{\text{qPV}} \\
\Delta Q_{\text{PV}} = v_{\text{dPV}0} \Delta i_{\text{qPV}} - v_{\text{qPV}0} \Delta i_{\text{dPV}} + i_{\text{qPV}0} \Delta v_{\text{dPV}} - i_{\text{dPV}0} \Delta v_{\text{qPV}}
\]

**Gas-Turbine based DER Model**

Linearized model of GT based DER unit can be represented as,-

\[
T_{\text{dg}} \frac{d\Delta X_1}{dt} = -\Delta X_1 - K_{\text{dg}} \frac{\omega_s}{2H} [\Delta X_4 - \Delta E'_q i_{qmA} - \Delta i_{qmA} E'_q0 - (X_q - X'_d) \Delta i_{dM} i_{qmA} - (X_q - X'_d) \Delta i_{qmA} i_{dMA} - D \Delta \omega] \\
\frac{d\Delta X_2}{dt} = -K_{\text{ig}} \Delta \omega \\
\frac{d\Delta X_3}{dt} = -\Delta X_3 + \Delta X_1 + \Delta X_2 - K_{pq} \Delta \omega \\
T_{\text{td}} \frac{d\Delta X_4}{dt} = -\Delta X_4 + K_t F_m \Delta X_3
\]
\[
T_{d0} \frac{d\Delta E'_q}{dt} = -\Delta E'_q - (X_d - X'_d) \Delta i_{d_m}
\]
\[
\frac{d\Delta \delta_m}{dt} = \Delta \omega
\]
\[
2H \frac{d\Delta \omega}{\omega_s} = \Delta X_1 - \Delta E'_q i_{q_m} - \Delta i_{q_m} E'_q
- (X_q - X'_d) \Delta i_{d_m} i_{q_m}\]
\[
= \Delta X_1 - \Delta E'_q i_{q_m} - \Delta i_{q_m} E'_q
- (X_q - X'_d) \Delta i_{d_m} i_{q_m} - D \Delta \omega
\]

**Linearization of real and reactive power**

As per equation 53,

\[
(I_r + jI_q) e^{j\delta} = -y_c (v_{d PV} + jv_{q PV}) e^{j\delta_{PLL}} + y_c V_r e^{j\delta_r}
= -y_c (v_{d PV} + jv_{q PV}) [\cos (\delta_{PLL}) + j\sin (\delta_{PLL})]
+ y_c V_r [\cos (\delta_r) + j\sin (\delta_r)]
\]

Real and reactive power can be formulated as,-

\[\vec{V}_r = V_r e^{j\delta_r}\]
\[\vec{I}_r = I_r e^{j\delta_i}\]
\[y_c = g_c + jb_c\]
\[ P = \text{real}\left\{ \overrightarrow{V_r} \cdot \overrightarrow{I_r}^* \right\} \]
\[ = \text{real}\left\{ -V_r y_c^*(v_{dpv} - jv_{qpv})e^{j(\delta_r - \delta_{PLL})} + y_c^* V_r^2 \right\} \]
\[ = \text{real}\left\{ -V_r (g_c - jb_c) (v_{dpv} - jv_{qpv}) \cdot [\cos (\delta_r - \delta_{PLL}) + j\sin (\delta_r - \delta_{PLL})] \right\} \]
\[ + \text{real}\left\{ (g_c - jb_c) V_r^2 \right\} \]
\[ = \text{real}\left\{ -V_r [(g_c \cdot v_{dpv} - b_c \cdot v_{qpv}) - j(b_c \cdot v_{dpv} + g_c \cdot v_{qpv})] \cdot [\cos (\delta_r - \delta_{PLL}) + j\sin (\delta_r - \delta_{PLL})] \right\} \]
\[ + \text{real}\left\{ (g_c - jb_c) V_r^2 \right\} \]
\[ = -V_r [(g_c \cdot v_{dpv} - b_c \cdot v_{qpv}) \cdot \cos (\delta_r - \delta_{PLL}) + (b_c \cdot v_{dpv} + g_c \cdot v_{qpv}) \cdot \sin (\delta_r - \delta_{PLL})] + g_c V_r^2 \]

\[ \Delta P = -\Delta V_r \left[ (g_c \cdot v_{dpv0} - b_c \cdot v_{qpv0}) \cdot \cos (\delta_{r0} - \delta_{PLL0}) + (b_c \cdot v_{dpv0} + g_c \cdot v_{qpv0}) \cdot \sin (\delta_{r0} - \delta_{PLL0}) \right] \]
\[ - V_{r0} \left[ (g_c \cdot \Delta v_{dpv} - b_c \cdot \Delta v_{qpv}) \cdot \cos (\delta_{ro} - \delta_{PLL0}) + (b_c \cdot \Delta v_{dpv} + g_c \cdot \Delta v_{qpv}) \cdot \sin (\delta_{ro} - \delta_{PLL0}) \right] \]
\[ - V_{r0} \left[ -(g_c \cdot v_{dpv0} - b_c \cdot v_{qpv0}) \cdot \sin (\delta_{ro} - \delta_{PLL0}) + (b_c \cdot v_{dpv0} + g_c \cdot v_{qpv0}) \cdot \cos (\delta_{ro} - \delta_{PLL0}) \right] \]
\[ \cdot (\Delta \delta_r - \Delta \delta_{PLL}) + 2 \cdot g_c V_{r0} \cdot \Delta V_r \] (5.55)
\[ Q = \text{imag}\{\mathbf{V}_r \cdot \mathbf{I}_r^*\} \]
\[ = \text{imag}\{-V_r \cdot y_c^* (v_{dpv} - jv_{qpv}) e^{j(\delta_r - \delta_{PLL})} + y_c^* V_r^2\} \]
\[ = \text{imag}\{-V_r \cdot (g_c - j b_c) (v_{dpv} - jv_{qpv}) \cdot [\cos (\delta_r - \delta_{PLL}) + j \sin (\delta_r - \delta_{PLL})]\} \]
\[ + \text{imag}\{(g_c - j b_c) V_r^2\} \]
\[ = \text{imag}\{-V_r [(g_c \cdot v_{dpv} - b_c \cdot v_{qpv}) - j (b_c \cdot v_{dpv} + g_c \cdot v_{qpv})] \cdot [\cos (\delta_r - \delta_{PLL}) + j \sin (\delta_r - \delta_{PLL})]\} \]
\[ + \text{imag}\{(g_c - j b_c) V_r^2\} \]
\[ = -V_r [(g_c \cdot v_{dpv} - b_c \cdot v_{qpv}) \cdot \sin (\delta_r - \delta_{PLL}) - (b_c \cdot v_{dpv} + g_c \cdot v_{qpv}) \cdot \cos (\delta_r - \delta_{PLL})] - b_c V_r^2 \]

\[ \Delta Q = -\Delta V_r [(g_c \cdot v_{dpv0} - b_c \cdot v_{qpv0}) \cdot \sin (\delta_{r0} - \delta_{PLL0}) - (b_c \cdot v_{dpv0} + g_c \cdot v_{qpv0}) \cdot \cos (\delta_{r0} - \delta_{PLL0})] \]
\[ - V_{r0} [(g_c \cdot \Delta v_{dpv} - b_c \cdot \Delta v_{qpv}) \cdot \sin (\delta_{r0} - \delta_{PLL0}) - (b_c \cdot \Delta v_{dpv} + g_c \cdot \Delta v_{qpv}) \cdot \cos (\delta_{r0} - \delta_{PLL0})] \]
\[ - V_{r0} [(g_c \cdot v_{dpv0} - b_c \cdot v_{qpv0}) \cdot \cos (\delta_{r0} - \delta_{PLL0}) + (b_c \cdot v_{dpv0} + g_c \cdot v_{qpv0}) \cdot \sin (\delta_{r0} - \delta_{PLL0})] \]
\[ \cdot (\Delta \delta_r - \Delta \delta_{PLL}) - 2 \cdot b_c \cdot V_{r0} \cdot \Delta V_r \] 

(5.56)

\[ \Delta P \text{ and } \Delta Q \text{ can be reformulated as,} \]

\[ \Delta P = d_{11} \cdot \Delta V_{rq} + d_{12} \cdot \Delta v_{dpv} + d_{13} \cdot \Delta v_{qpv} + d_{14} \cdot \Delta \delta_{PLL} \]
\[ + d_{15} \cdot \Delta \delta_r \]
\[ \Delta Q = d_{21} \Delta V_{rq} + d_{22} \Delta v_{d\rho V} + d_{23} \Delta v_{q\rho V} + d_{24} \Delta \delta_{PLL} + d_{25} \Delta \delta_r \]

The above equation can be simplified as:

\[ \Delta P = \left( d_{11} + d_{12} \cdot p_{16}' + d_{13} \cdot p_{26}' \right) \cdot \Delta V_{rq} \]
\[ + \left( d_{12} \cdot p_{11}' + d_{13} \cdot p_{21}' \right) \cdot \Delta i_{d\rho V} \]
\[ + \left( d_{12} \cdot p_{12}' + d_{13} \cdot p_{22}' \right) \cdot \Delta i_{q\rho V} \]
\[ + \left( d_{14} + d_{12} \left( p_{14}' + p_{14}'' \right) + d_{13} \left( p_{24}' + p_{24}'' \right) \right) \cdot \Delta \delta_{PLL} \]
\[ + \left( d_{15} - d_{12} \cdot p_{14}'' - d_{13} \cdot p_{24}'' \right) \cdot \Delta \delta_r \]
\[ + \left( d_{12} \cdot p_{15}' + d_{13} \cdot p_{25}' \right) \Delta E_q' \]
\[ + \left( d_{12} \cdot p_{13}' + d_{13} \cdot p_{23}' \right) \Delta \delta_m \]
\[ \Delta Q = \left( d_{21} + d_{22} \cdot p'_{16} + d_{23} \cdot p'_{26} \right) \cdot \Delta V_{rq} \]
\[ \quad + \left( d_{22} \cdot p'_{11} + d_{23} \cdot p'_{21} \right) \cdot \Delta i_{d_{PV}} \]
\[ \quad + \left( d_{22} \cdot p'_{12} + d_{23} \cdot p'_{22} \right) \cdot \Delta i_{q_{PV}} \]
\[ \quad + \left( d_{24} + d_{22} \cdot \left( p'_{14} + p''_{14} \right) + d_{23} \cdot \left( p'_{24} + p''_{24} \right) \right) \cdot \Delta \delta_{PLL} \]
\[ \quad + \left( d_{25} - d_{22} \cdot p''_{14} - d_{23} \cdot p''_{24} \right) \cdot \Delta \delta_r \]
\[ \quad + \left( d_{22} \cdot p'_{15} + d_{23} \cdot p'_{25} \right) \Delta E'_q \]
\[ \quad + \left( d_{22} \cdot p'_{13} + d_{23} \cdot p'_{23} \right) \Delta \delta_m \]

State space model formulation is presented in appendix A.

5.7.1 State Space Model Validation with DAE Model

Linearized model, developed based on state space model, is given a step change in input (for this study, exciter voltage of GT-DER unit is set as input variable) and its dynamic response is compared with time domain response of the output variable (system frequency) for the input as applied to transfer function. The validation responses are presented in the following figures.
Figure 5.3: Transfer Function Validation: Operating point 1 (load at bus B2 at 20 kW)

Figure 5.4: Transfer Function Validation: Operating point 2 (load at bus B2 at 30 kW)
Figure 5.5: Transfer Function Validation: Operating point 3 (load at bus B2 at 40 kW)

5.7.2 Measure of small signal stability

In figure 5.6, 5.7 and 5.8, eigenvalues of the 2-node system is presented. Frequency of the oscillatory modes as shown in figure 5.6 are 3.34 Hz and 0.2 Hz. These modes can be identified from the time domain response as shown in figure 5.3. Such modes can be identified for the same test system for other operating points as per the eigenvalues shown in figure 5.7, 5.8 and transfer function validation response in figure 5.4 and 5.5.
Figure 5.6: Eigenvalues for Op. Pt. 1 (load at bus B2 at 20 kW)

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Figure 5.7: Eigenvalues for Op. Pt. 2 (load at bus B2 at 30 kW)

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Figure 5.8: Eigenvalues for Op. Pt. 3 (load at bus B2 at 40 kW)

Eigenvalues of 2-node system without SG emulator are calculated as shown below,-
Figure 5.9: Eigenvalues for Op. Pt. 1 (load at bus B2 at 20 kW)

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Figure 5.10: Eigenvalues for Op. Pt. 2 (load at bus B2 at 30 kW)

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5.8 IEEE 13 Node System: Dynamic model and state space with SG emulator unit

Small signal stability of IEEE 13 node system is studied here to see the impact of PVSG unit for inertia improvement of the system. The system model is approximated as phasor model and the system admittance matrix is reduced at DER buses for the integration to algebraic equation of system DAE model for dynamic analysis. All loads are approximated as constant impedance load.
5.8.1 Validation of state space model

The system is reduced to node 10 and 12 for time domain analysis and state space formulation of PVSG unit, integrated to IEEE 13 node system. Linearized model for IEEE 13 node system are as per the formulation shown in 2-node test system.
Figure 5.13: Transfer Function Validation: Set Point 1 (load at bus 13 and 14 at 100 kw each)

Figure 5.14: Transfer Function Validation: Set Point 2 (load at bus 13 and 14 at 400 kw each)

5.8.2 Measure of small signal stability

Figure 5.15 and 5.16 shows eigenvalues of PVSG system, integrated to IEEE 13 node system. Operating point 1 has modes at 3.32 Hz and 0.19 Hz. Oscillatory modes are observed in figure 5.13 and 5.14 and calculated by eigenvalues of the system. Details on different points are explained in chapter 4.
Figure 5.15: Eigenvalues for Set Point 1 (load at bus 13 and 14 at 100 kw each)

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Figure 5.16: Eigenvalues for Set Point 2 (load at bus 13 and 14 at 400 kw each)

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5.8.3 Root locus and parametric sensitivity of PVSG control gains on system oscillatory modes

Figure 5.17 - 5.23 shows effectiveness of PVSG control scheme on small signal stability of the system. PVSG unit gains $k_{ip}$, $k_{iq}$ and $k_{i\omega}$ (as modeled in equation 5.2, 5.3 and 5.5) have been changes from $k_{ip} = 0$, $k_{iq} = 0$ and $k_{i\omega} = 0$ to $k_{ip} = 1$, $k_{iq} = -0.1$ and $k_{i\omega} = 1$ at an increment of 0.02 p.u. and eigenvalues have been calculated for each gain values as per the state space derived in this chapter. System data for 2-node system is used for this root locus study.

Figure 5.17: Impact of PVSG scheme on system modes
Figure 5.18: Impact of PVSG scheme on system modes

Figure 5.19: Impact of PVSG scheme on system modes
Figure 5.20: Impact of PVSG scheme on system modes

Figure 5.21
Figure 5.22: Impact of PVSG scheme on system modes

Figure 5.23: Impact of PVSG scheme on system modes
Chapter 6

Conclusions and Discussion

In this research work, time domain simulation model and state space analysis of PVSG scheme have been presented. Dynamic response have been analyzed based on the numeric model. PVSG scheme has a significant improvement on system inertia as shown in time domain response (Chapter 4). This scheme also improves small signal stability of the system significantly (as per the root locus, presented in chapter 5). Based on this model, as presented in chapter 4 and 5, nonlinear interaction of multiple PVSG unit can be studied and a quantification method for energy storage sizing can be developed. Integration of such model to a distribution network to observe interactions and its impact on sizing of energy storage based on the study are some of the future research topics, where this study can be extended.
Appendix A  Formulation of state space matrix of microgrid system

A.1 Case 1: 2-bus system with photovoltaic and gas-turbine based DER

\[ \dot{x} = A\cdot \Delta x + B\Delta u \]  

(1)

Where, 

\[ \Delta x = \begin{bmatrix} \Delta v_{dc} & \Delta s_{dc} & \Delta i_{dB} & \Delta i_{qB} & \Delta X_{PLL} & \Delta \delta_{PLL_B} & \Delta X_1 & \Delta X_2 & \Delta X_3 & \Delta X_4 & \Delta E' & \Delta \delta_m & \Delta \omega \end{bmatrix} \]

For this study, there is no variation in inputs \( u \).

\[
A(1,3) = -\frac{1}{CV_{dc0}}(v_{dB0} - i_{dB0}\cdot p_{11} - i_{qB0}\cdot p_{21})
\]

\[
A(1,4) = -\frac{1}{CV_{dc0}}(v_{qB0} - i_{dB0}\cdot p_{12} - i_{qB0}\cdot p_{22})
\]

\[
A(1,6) = -\frac{1}{CV_{dc0}}(i_{dB0}\cdot p_{14} + i_{qB0}\cdot p_{24})
\]

\[
A(1,11) = -\frac{1}{CV_{dc0}}(i_{dB0}\cdot p_{15} + i_{qB0}\cdot p_{25})
\]

\[
A(1,12) = -\frac{1}{CV_{dc0}}(i_{dB0}\cdot p_{13} + i_{qB0}\cdot p_{23})
\]

\[
A(2,1) = K_{i_{dc}}
\]

\[
A(3,1) = \frac{K_{P_{dc}}}{\tau_i}
\]
\[ A(3, 2) = \frac{1}{\tau_i} \]
\[ A(3, 3) = -\frac{1}{\tau_i} \]

\[ A(4, 4) = -\frac{1}{\tau_i} \]
\[ A(5, 3) = K_I \cdot p_{21} \]
\[ A(5, 4) = K_I \cdot p_{22} \]
\[ A(5, 6) = K_I \cdot p_{24} \]
\[ A(5, 11) = K_I \cdot p_{25} \]
\[ A(5, 12) = K_I \cdot p_{23} \]

\[ A(6, 3) = K_p \cdot p_{21} \]
\[ A(6, 4) = K_p \cdot p_{22} \]
\[ A(6, 5) = 1 \]
\[ A(6, 6) = K_p \cdot p_{24} \]
\[ A(6, 11) = K_p \cdot p_{25} \]
\[ A(6, 12) = K_p \cdot p_{23} \]

\[ A(7, 3) = \frac{K_{d_g} \omega_s}{T_{d_g} 2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{11} + i_{d_{m0}} \cdot d_{21} \} + E' q_0 \cdot d_{21} \right] \]
\[ A(7, 4) = \frac{K_{d_g} \omega_s}{T_{d_g} 2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{12} + i_{d_{m0}} \cdot d_{22} \} + E' q_0 \cdot d_{22} \right] \]
\[ A(7, 6) = \frac{K_{d_g} \omega_s}{T_{d_g} 2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{14} + i_{d_{m0}} \cdot d_{24} \} + E' q_0 \cdot d_{24} \right] \]
\[ A(7, 7) = -\frac{1}{T_{d_g}} \]
\[ A(7, 10) = -\frac{K_{d_g} \omega_s}{T_{d_g} 2H} \]
\[ A(7, 11) = \frac{K_{d_g} \omega_s}{T_{d_g} 2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{15} + i_{d_{m0}} \cdot d_{25} \} + E' q_0 \cdot d_{25} + i_{q_{m0}} \right] \]
\[
A(7, 12) = \frac{K_{dq}}{T_{dq}} \frac{\omega_s}{2H} \left[ (X_q - X'_d) \{i_{q_m0} \cdot d_{13} + i_{d_m0} \cdot d_{23}\} + E'_q o \cdot d_{23} \right] \\
A(7, 13) = \frac{K_{dq}}{T_{dq}} \frac{\omega_s}{2H} D \\
A(8, 13) = -K_{ig} \\
A(9, 7) = \frac{1}{T_v} \\
A(9, 8) = \frac{1}{T_v} \\
A(9, 9) = -\frac{1}{T_v} \\
A(9, 13) = -\frac{K_{pg}}{T_v} \\
A(10, 9) = \frac{K_s F_m}{T_{ld}} \\
A(10, 10) = -\frac{1}{T_{ld}} \\
A(11, 3) = -(X_d - X'_d) \cdot d_{11} \cdot \frac{1}{T_{do}} \\
A(11, 4) = -(X_d - X'_d) \cdot d_{12} \cdot \frac{1}{T_{do}} \\
A(11, 6) = -(X_d - X'_d) \cdot d_{14} \cdot \frac{1}{T_{do}} \\
A(11, 10) = [1 - (X_d - X'_d) \cdot d_{15}] \cdot \frac{1}{T_{do}} \\
A(11, 12) = -(X_d - X'_d) \cdot d_{13} \cdot \frac{1}{T_{do}} \\
A(12, 13) = 1 \\
A(13, 3) = -\frac{\omega_s}{2H} \left[ (X_q - X'_d) \{i_{q_m0} \cdot d_{11} + i_{d_m0} \cdot d_{21}\} + E'_q o \cdot d_{21} \right] \\
A(13, 4) = -\frac{\omega_s}{2H} \left[ (X_q - X'_d) \{i_{q_m0} \cdot d_{12} + i_{d_m0} \cdot d_{22}\} + E'_q o \cdot d_{22} \right] \\
A(13, 6) = -\frac{\omega_s}{2H} \left[ (X_q - X'_d) \{i_{q_m0} \cdot d_{14} + i_{d_m0} \cdot d_{24}\} + E'_q o \cdot d_{24} \right] \\
A(13, 10) = \frac{\omega_s}{2H} \\
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\[ A(13, 11) = -\frac{\omega_s}{2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{15} + i_{d_{m0}} \cdot d_{25} \} + E'_{q_0} \cdot d_{25} + i_{q_{m0}} \right] \]
\[ A(13, 12) = -\frac{\omega_s}{2H} \left[ (X_q - X'_d) \{ i_{q_{m0}} \cdot d_{13} + i_{d_{m0}} \cdot d_{23} \} + E'_{q_0} \cdot d_{23} \right] \]
\[ A(13, 13) = -\frac{\omega_s}{2H} D \]

A.2 Case 2: 2-bus system with PVSG and gas-turbine based DER

Definition of state variables,

\[ \Delta x = [\Delta \delta \quad \Delta \delta_p \quad \Delta \omega_p \quad \Delta V_{rq} \quad \Delta S_p \quad \Delta S_q \quad \Delta i_{d_{pv}} \quad \Delta i_{q_{pv}} \quad \Delta X_{pll} \quad \Delta \delta_{pll} \quad \Delta X_1 \quad \Delta X_1 \quad \Delta X_3 \quad \Delta X_4 \quad \Delta E'_q \quad \Delta \delta_m \quad \Delta \omega] \]

\[ \Delta \delta_r = \Delta \delta + \Delta \delta_p \]

Definition of A matrix elements,
\[ A(1, 3) = 1 \]
\[ A(2, 1) = -K_{ip'} \left( d_{15} - d_{12} \cdot p_{14}'' - d_{13} \cdot p_{24}'' \right) \]
\[ A(2, 2) = -K_{ip'} \left( d_{15} - d_{12} \cdot p_{14}'' - d_{13} \cdot p_{24}'' \right) \]
\[ A(2, 4) = -K_{ip'} \left( d_{11} + d_{12} \cdot p_{16}' + d_{13} \cdot p_{26}' \right) \]
\[ A(2, 7) = -K_{ip'} \left( d_{12} \cdot p_{11}' + d_{13} \cdot p_{21}' \right) \]
\[ A(2, 8) = -K_{ip'} \left( d_{12} \cdot p_{12}' + d_{13} \cdot p_{22}' \right) \]
\[ A(2, 10) = -K_{ip'} \left( d_{14} + d_{12} \cdot \left( p_{14}' + p_{14}'' \right) + d_{13} \cdot \left( p_{24}' + p_{24}'' \right) \right) \]
\[ A(2, 15) = -K_{ip'} \left( d_{12} \cdot p_{15}' + d_{13} \cdot p_{25}' \right) \]
\[ A(2, 16) = -K_{ip'} \left( d_{12} \cdot p_{13}' + d_{13} \cdot p_{23}' \right) \]
\[ A(3, 1) = -K_{iw'} \left( d_{15} - d_{12} \cdot p_{14}'' - d_{13} \cdot p_{24}'' \right) \]
\[ A(3, 2) = -K_{iw'} \left( d_{15} - d_{12} \cdot p_{14}'' - d_{13} \cdot p_{24}'' \right) \]
\[ A(3, 4) = -K_{iw'} \left( d_{11} + d_{12} \cdot p_{16}' + d_{13} \cdot p_{26}' \right) \]
\[ A(3, 7) = -K_{iw'} \left( d_{12} \cdot p_{11}' + d_{13} \cdot p_{21}' \right) \]
\[ A(3, 8) = -K_{iw'} \left( d_{12} \cdot p_{12}' + d_{13} \cdot p_{22}' \right) \]
\[ A(3, 10) = -K_{iw'} \left( d_{14} + d_{12} \cdot \left( p_{14}' + p_{14}'' \right) + d_{13} \cdot \left( p_{24}' + p_{24}'' \right) \right) \]
\[ A(3, 15) = -K_{iw'} \left( d_{12} \cdot p_{15}' + d_{13} \cdot p_{25}' \right) \]
\[ A(3, 16) = -K_{iw'} \left( d_{12} \cdot p_{13}' + d_{13} \cdot p_{23}' \right) \]
\begin{align}
A(4, 1) &= -K_{iq}(d_{25} - d_{22}p_{14} \cdot d_{23}p_{24}) \\
A(4, 2) &= -K_{iq}(d_{25} - d_{22}p_{14}'' \cdot d_{23}p_{24}'' ) \\
A(4, 4) &= -K_{iq}(d_{21} + d_{22}p_{16}^{'}, d_{23}p_{26}^{'}) \\
A(4, 7) &= -K_{iq}(d_{22}p_{11}^{'}, d_{23}p_{21}^{'}) \\
A(4, 8) &= -K_{iq}(d_{22}p_{12}^{'}, d_{23}p_{22}^{'}) \\
A(4, 10) &= -K_{iq}(d_{24} + d_{22}(p_{14}^{'}, p_{14}^{'}) + d_{23}(p_{24} + p_{24}')) \\
\end{align}

\begin{align}
A(4, 15) &= -K_{iq}(d_{22}p_{15}^{'}, d_{23}p_{25}^{'}) \\
A(4, 16) &= -K_{iq}(d_{22}p_{13}^{'}, d_{23}p_{23}^{'}) \\
A(5, 1) &= K_{I_p}i_{dpv0}p_{14}^{'}, K_{I_p}i_{qpv0}p_{24}^{'}) \\
A(5, 2) &= K_{I_p}i_{dpv0}p_{14}'' , K_{I_p}i_{qpv0}p_{24}'' \\
A(5, 4) &= -K_{I_p}i_{dpv0}p_{16} - K_{I_p}i_{qpv0}p_{26} \\
A(5, 7) &= -K_{I_p}v_{dpv0} - K_{I_p}i_{dpv0}p_{11}^{'}, K_{I_p}i_{qpv0}p_{21} \\
A(5, 8) &= -K_{I_p}v_{qpv0} - K_{I_p}i_{dpv0}p_{12} - K_{I_p}i_{qpv0}p_{22}^{'}) \\
A(5, 10) &= -K_{I_p}i_{dpv0}(p_{14}^{'}, p_{14}'' ) - K_{I_p}i_{qpv0}(p_{24} + p_{24}'' ) \\
A(5, 15) &= -K_{I_p}i_{dpv0}p_{15} - K_{I_p}i_{qpv0}p_{25}^{'}) \\
A(5, 16) &= -K_{I_p}i_{dpv0}p_{13} - K_{I_p}i_{qpv0}p_{23}^{'}) \\
\end{align}

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\[ A(6, 1) = K_{I_q} \cdot i_{qPV_0} \cdot p''_{14} - K_{I_q} \cdot i_{dPV_0} \cdot p''_{24} \]

\[ A(6, 2) = K_{I_q} \cdot i_{qPV_0} \cdot p''_{14} - K_{I_q} \cdot i_{dPV_0} \cdot p''_{24} \]

\[ A(6, 4) = -K_{I_q} \cdot i_{qPV_0} \cdot p'_{16} + K_{I_q} \cdot i_{dPV_0} \cdot p'_{26} \]

\[ A(6, 7) = K_{I_q} \cdot v_{qPV_0} - K_{I_q} \cdot i_{qPV_0} \cdot p'_{11} + K_{I_q} \cdot i_{dPV_0} \cdot p'_{21} \]

\[ A(6, 8) = -K_{I_q} \cdot v_{dPV_0} - K_{I_q} \cdot i_{qPV_0} \cdot p'_{12} + K_{I_q} \cdot i_{dPV_0} \cdot p'_{22} \]

\[ A(6, 10) = -K_{I_q} \cdot i_{qPV_0} \cdot (p'_{14} + p''_{14}) + K_{I_q} \cdot i_{dPV_0} \cdot (p'_{24} + p''_{24}) \]

\[ A(6, 15) = -K_{I_q} \cdot i_{qPV_0} \cdot p'_{15} + K_{I_q} \cdot i_{dPV_0} \cdot p'_{25} \]

\[ A(6, 16) = -K_{I_q} \cdot i_{qPV_0} \cdot p'_{13} + K_{I_q} \cdot i_{dPV_0} \cdot p'_{23} \]

\[ A(7, 1) = \frac{K_{P_p}}{\tau_i} (i_{dPV_0} \cdot p''_{14} + i_{qPV_0} \cdot p''_{24}) \]

\[ A(7, 2) = \frac{K_{P_p}}{\tau_i} (i_{dPV_0} \cdot p''_{14} + i_{qPV_0} \cdot p''_{24}) \]

\[ A(7, 4) = -\frac{K_{P_p}}{\tau_i} (i_{dPV_0} \cdot p'_{16} + i_{qPV_0} \cdot p'_{26}) \]

\[ A(7, 5) = \frac{1}{\tau_i} \]

\[ A(7, 7) = -\frac{1}{\tau_i} - \frac{K_{P_p}}{\tau_i} (v_{dPV_0} + i_{dPV_0} \cdot p'_{11} + i_{qPV_0} \cdot p'_{21}) \]

\[ A(7, 8) = -\frac{K_{P_p}}{\tau_i} (v_{qPV_0} + i_{dPV_0} \cdot p'_{12} + i_{qPV_0} \cdot p'_{22}) \]
\[ A(7, 10) = -\frac{K_P}{\tau_i} \left( i_{dq_{PV0}} \cdot (p'_{14} + p''_{14}) + i_{qp_{PV0}} \cdot (p'_{24} + p''_{24}) \right) \]

\[ A(7, 15) = -\frac{K_P}{\tau_i} \left( i_{dq_{PV0}} \cdot p'_{15} + i_{qp_{PV0}} \cdot p'_{25} \right) \]

\[ A(7, 16) = -\frac{K_P}{\tau_i} \left( i_{dq_{PV0}} \cdot p'_{13} + i_{qp_{PV0}} \cdot p'_{23} \right) \]

\[ A(8, 1) = K_P \cdot \left( \frac{1}{\tau_i} \right) \left( i_{qp_{PV0}} \cdot p''_{14} - i_{dp_{PV0}} \cdot p''_{24} \right) \]

\[ A(8, 2) = \frac{K_P}{\tau_i} \left( i_{qp_{PV0}} \cdot p''_{14} - i_{dp_{PV0}} \cdot p''_{24} \right) \]

\[ A(8, 4) = \frac{K_P}{\tau_i} \left( i_{qp_{PV0}} \cdot p''_{16} - i_{dp_{PV0}} \cdot p''_{26} \right) \]

\[ A(8, 6) = \frac{1}{\tau_i} \]

\[ A(8, 7) = \frac{K_P}{\tau_i} \left( -v_{qp_{PV0}} + i_{qp_{PV0}} \cdot p'_{11} - i_{dp_{PV0}} \cdot p'_{21} \right) \]

\[ A(8, 8) = \frac{1}{\tau_i} - \frac{K_P}{\tau_i} \left( v_{dp_{PV0}} + i_{qp_{PV0}} \cdot p'_{12} - i_{dp_{PV0}} \cdot p'_{22} \right) \]
\( A(8, 10) \)
\[
= -\frac{K_{P_q}}{\tau_i} \left( i_{q_{PV_0}} \cdot (p^\prime_{14} + p''_{14}) - i_{d_{PV_0}} \cdot (p^\prime_{24} + p''_{24}) \right)
\]
\( A(8, 15) \)
\[
= -\frac{K_{P_q}}{\tau_i} \left( i_{q_{PV_0}} \cdot p^\prime_{15} - i_{d_{PV_0}} \cdot p^\prime_{25} \right)
\]
\( A(8, 16) \)
\[
= -\frac{K_{P_q}}{\tau_i} \left( i_{q_{PV_0}} \cdot p^\prime_{13} - i_{d_{PV_0}} \cdot p^\prime_{23} \right)
\]
\( A(9, 1) = -K_I \cdot p^\prime_{24} \)
\[ A(9, 2) = -K_I \cdot p_{24}^\prime \]
\[ A(9, 4) = K_I \cdot p_{26}^\prime \]
\[ A(9, 7) = K_I \cdot p_{21}^\prime \]
\[ A(9, 8) = K_I \cdot p_{22}^\prime \]
\[ A(9, 10) = K_I \cdot \left( p_{24}^\prime + p_{24}^\ddagger \right) \]
\[ A(9, 15) = K_I \cdot p_{25}^\prime \]
\[ A(9, 16) = K_I \cdot p_{23}^\prime \]
\[ A(10, 1) = -K_P \cdot p_{24}^\ddagger \]
\[ A(10, 2) = -K_P \cdot p_{24}^\ddagger \]
\[ A(10, 4) = K_P \cdot p_{26}^\prime \]
\[ A(10, 7) = K_P \cdot p_{21}^\prime \]
\[ A(10, 8) = K_P \cdot p_{22}^\prime \]
\[ A(10, 9) = 1 \]
\[ A(10, 10) = K_P \cdot \left( p_{24}^\prime + p_{24}^\ddagger \right) \]
\[ A(10, 15) = K_P \cdot p_{25}^\prime \]
\[ A(10, 16) = K_P \cdot p_{23}^\prime \]
\[ A(11, 1) = \frac{K_{dg}}{T_{dg}} \frac{\omega_s}{2H} \left[ \left( X_q - X_d^\prime \right) \{ i_{q_m0} \cdot k_{17} + i_{d_m0} \cdot k_{27} \} + E_{q0}^\prime \cdot k_{27} \right] \]
\[ A(11, 2) = \frac{K_{dg}}{T_{dg}} \frac{\omega_s}{2H} \left[ \left( X_q - X_d^\prime \right) \{ i_{q_m0} \cdot k_{17} + i_{d_m0} \cdot k_{27} \} + E_{q0}^\prime \cdot k_{27} \right] \]
\[ A(11, 4) = \frac{K_{dg}}{T_{dg}} \frac{\omega_s}{2H} \left[ \left( X_q - X_d^\prime \right) \{ i_{q_m0} \cdot k_{16} + i_{d_m0} \cdot k_{26} \} + E_{q0}^\prime \cdot k_{26} \right] \]
\[ A(11, 7) = \frac{K_{dg}}{T_{dg}} \frac{\omega_s}{2H} \left[ \left( X_q - X_d^\prime \right) \{ i_{q_m0} \cdot k_{11} + i_{d_m0} \cdot k_{21} \} + E_{q0}^\prime \cdot k_{21} \right] \]
\[ A(11, 8) = \frac{K_{dg} \omega_s}{T_{dg} 2H} \left( \left( X_q - X'_d \right) \{ i_{q_{mo}} \cdot k_{12} + i_{d_{mo}} \cdot k_{22} \} + E' q_0 \cdot k_{22} \right) \]

\[ A(11, 10) = \frac{K_{dg} \omega_s}{T_{dg} 2H} \left( \left( X_q - X'_d \right) \{ i_{q_{mo}} \cdot k_{14} + i_{d_{mo}} \cdot k_{24} \} + E' q_0 \cdot k_{24} \right) \]

\[ A(11, 11) = -\frac{1}{T_{dg}} \]

\[ A(11, 14) = -\frac{K_{dg} \omega_s}{T_{dg} 2H} \]

\[ A(11, 15) = \frac{K_{dg} \omega_s}{T_{dg} 2H} \left( \left( X_q - X'_d \right) \{ i_{q_{mo}} \cdot k_{15} + i_{d_{mo}} \cdot k_{25} \} + E' q_0 \cdot k_{25} + i_{q_{mo}} \right) \]

\[ A(11, 16) = \frac{K_{dg} \omega_s}{T_{dg} 2H} \left( \left( X_q - X'_d \right) \{ i_{q_{mo}} \cdot k_{13} + i_{d_{mo}} \cdot k_{23} \} + E' q_0 \cdot k_{23} \right) \]

\[ A(11, 17) = \frac{K_{dg} \omega_s}{T_{dg} 2H} DA(12, 17) = -k_{ig} \]

\[ A(13, 11) = 1 \]

\[ A(13, 12) = 1 \]

\[ A(13, 13) = -1 \]

\[ A(13, 17) = -K_{pg} \]

\[ A(14, 13) = \frac{K_i F_m}{T_{td}} \]

\[ A(14, 14) = -\frac{1}{T_{d0}} \]

\[ A(15, 1) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{17} \]

\[ A(15, 2) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{17} \]

\[ A(15, 4) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{16} \]

\[ A(15, 7) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{11} \]

\[ A(15, 8) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{12} \]

\[ A(15, 10) = -\frac{1}{T_{d0}} \cdot \left( X_d - X'_d \right) k_{14} \]
\[ A(15, 15) = -\frac{1}{T_{d0}} \left( X_d - X'_d \right) k_{15} - \frac{1}{T_{d0}} \]
\[ A(15, 16) = -\frac{1}{T_{d0}} \left( X_d - X'_d \right) k_{13} \]
\[ A(16, 17) = 1 \]

\[ A(17, 1) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{17} + i_{d_m} k_{27} \right) + E'_{q0} k_{27} \right\} \]
\[ A(17, 2) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{17} + i_{d_m} k_{27} \right) + E'_{q0} k_{27} \right\} \]
\[ A(17, 4) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{16} + i_{d_m} k_{26} \right) + E'_{q0} k_{26} \right\} \]
\[ A(17, 7) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{11} + i_{d_m} k_{21} \right) + E'_{q0} k_{21} \right\} \]
\[ A(17, 8) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{12} + i_{d_m} k_{22} \right) + E'_{q0} k_{22} \right\} \]
\[ A(17, 10) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{14} + i_{d_m} k_{24} \right) + E'_{q0} k_{24} \right\} \]
\[ A(17, 14) = \frac{\omega_s}{2H} \]
\[ A(17, 15) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{15} + i_{d_m} k_{25} \right) + E'_{q0} k_{25} + i_{q_m} \right\} \]
\[ A(17, 16) = \frac{\omega_s}{2H} \left\{ \left( X_q - X'_d \right) \left( i_{q_m} k_{13} + i_{d_m} k_{23} \right) + E'_{q0} k_{23} \right\} \]
\[ A(17, 17) = -\frac{\omega_s}{2H} D \]
Appendix B  Formulation of DAE and state space model of microgrid system with 2 PVSG units, 1 GT DER in the system

`simulation_mains_T2.m`

```matlab
clc
clear

global Efd para Pset Y_BUS

% Define initial value
% Initial Condition

y0 = [0.814125834043310, -0.00503454070490807,
```

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1.00000000007634, 6.25637231297459e-09,
0.268081425000747, 0.268081426053610,
0.0271221402488888, 0.120943168246826,
4.8829293917291e-12, 0.120943168246830,
4.88292953793010e-12, 5.69302812186028e-13,
0.0309319229773348, 13.5505718540464,
-4.2951480524036e-34, 6.02506985896540e-33,
1.33028610462059e-27, 5.60257079224068e-33,
2.81862800537932e-27, 5.79881954914202e-11,
0.0309319228872744, 10.7093141753662,
0.0415211998477343, 0.140569575743356,
9.70902938913359e-12, -8.84308567706331e-13,
-4.12192770603923, -2.53125857564843,
1.39356394048230e-40, 1.39356393467925e-40,
1.70128038872725e-41, -2.89758150099862e-38,
2.46975827003752e-38, 0.0415211998369982,
41.7898431182231, 0.100000000001684,
2.49742740569165e-13, 0.120943168246658,
4.85795566511599e-12, 3.13174572953928e-11,
3.18867601075788e-11, 5.59555505373137e-33,
1.33028610462059e-27, 1.05782580139669e-11,
6.85664535053871e-11, 0.100000000000942,
3.94650211137767e-12, 0.140569575743262,
9.31437917799582e-12, 2.10601672600769e-11,
2.01758586923714e-11, 2.78712787516154e-40,
1.70128038872725e-41, 2.86977351440965e-11,
0.0415211998656959, 0.118619985874935,
0.0290408081397433, 0.826834631928914,
3.13174572953928e-11, 0.711391490456769,
2.10601672600769e-11, 0.826834631928914,
1.05782580139746e-10, 0.711391490456768,
2.86977351440965e-11, 3.33321278129974e-33,
2.38318296897958e-27, -2.06131479599541e-38,
1.75696503110245e-38];

load('C:\PUSPAL_HAZRA\DAE_IEEE_13_PVSG\model\IEEEpvsg2_data.mat');

% System Parameter Initialization

Efd = 1.1;

params.Efd = Efd;
params.para = para;
params.Pset = [Pset Pset];
params.Y_BUS = Y_BUS;
% Time Domain Simulation

% Span - 1

% Span - 2

params.Pset = [0.8.*Pset 0.8.*Pset];
l = length(t1);
y0 = y1(l,:);
tspan = [500 1000];
M = [eye(35) zeros(35,34); zeros(34,69)];
options = odeset('Mass',M);
[t2,y2] = ode23t(@(t,y)IEEE_dyn2Model_V2(t,y, params),tspan,y0,options);

% 

% 

t = [t1;t2];
\[ y = [y_1; y_2]; \]

1 = \text{length}(y(:,1));

\[ y_0 = y(l,:); \]

\%

plot(t, y); grid on;

subplot(3,1,1);
plot(t, y(:,3), 'LineWidth', 2);
xlim([496,506]);
grid on;
set(gcf, 'Color', [1,1,1]);
set(gca, 'FontSize', 10);
xlabel('Time in sec');
title('System Frequency');

subplot(3,1,2);
plot(t, y(:,36), t, y(:,46), 'LineWidth', 2);
xlim([496,506]);
legend('PVSG Unit–1', 'PVSG Unit–2');
function F = IEEE_dyn2Model_init(y)
% Define system parameters

global Efd para Pset Y_BUS

% Define system parameters
% Machine dynamic parameters

\[ T_{d0p} = \text{para}(1); \; X_{dp} = \text{para}(2); \; X_{q} = \text{para}(3); \; X_{d} = \text{para}(4); \]

\[ W_{s} = \text{para}(5); \; D = 0.2; \; H = \text{para}(7); \]

% GT governor dynamic parameters

\[ T_{dg} = \text{para}(8); \]

\[ K_{dg} = 5; \]
\[ K_{ig} = 50; \]
\[ K_{pg} = 5; \]

\[ T_{td} = \text{para}(12); \; T_{v} = \text{para}(13); \]

\[ K_{t} = \text{para}(14); \; F_{m} = \text{para}(15); \]

% Photovoltaic dynamic parameters

\[ P_{ref} = P_{set}; \; Q_{ref} = 0.0; \]

\[ T_{i} = \text{para}(16); \]
K_I = 0; K_I_P = 10; K_I_Q = 10;

K_P = 0; K_P_P = 0.1; K_P_Q = 0.1;

% SG emulator dynamic parameters

P_set = 0; Q_set = 0; Vrn = 1;

Kip = 0.5; Kiq = 0; Kiw = 0.5;

% Inverter interface line data

para(28) = 1e-4;

para(27) = 1e-3;

R_c = para(27); Lc = para(28);

Z_base = (480/sqrt(3))/(40e3/(sqrt(3)*480));

W = 2*pi*60;
\[ z_{\cdot c} = \left( R_{\cdot c} + 1j \cdot W_{\cdot c} \cdot L_{\cdot c} \right) / Z_{\cdot \text{base}}; \]

\[ y_{\cdot c} = 1 / z_{\cdot c}; \]

\%

\% Network algebraic equations

\% Partition admittance matrix based on machine and inverter model (not generic form)

\% This decoupling of state and algebraic variable will be in matrix form for larger system

\[ y_{11} = Y_{\text{BUS}(1,1)}; \quad y_{12} = Y_{\text{BUS}(1,2)}; \quad y_{13} = Y_{\text{BUS}(1,3)}; \quad y_{14} = Y_{\text{BUS}(1,4)}; \quad y_{15} = Y_{\text{BUS}(1,5)}; \]

\[ y_{21} = Y_{\text{BUS}(2,1)}; \quad y_{22} = Y_{\text{BUS}(2,2)}; \quad y_{23} = Y_{\text{BUS}(2,3)}; \quad y_{24} = Y_{\text{BUS}(2,4)}; \quad y_{25} = Y_{\text{BUS}(2,5)}; \]

\[ y_{31} = Y_{\text{BUS}(3,1)}; \quad y_{32} = Y_{\text{BUS}(3,2)}; \quad y_{33} = Y_{\text{BUS}(3,3)}; \quad y_{34} = Y_{\text{BUS}(3,4)}; \quad y_{35} = Y_{\text{BUS}(3,5)}; \]
y41 = Y_BUS(4,1); y42 = Y_BUS(4,2); y43 = Y_BUS(4,3); y44 = Y_BUS(4,4); y45 = Y_BUS(4,5);
y51 = Y_BUS(5,1); y52 = Y_BUS(5,2); y53 = Y_BUS(5,3); y54 = Y_BUS(5,4); y55 = Y_BUS(5,5);

% Transformed Admittance Matrix

Y_11 = [y11 y12 y13; y21 y22 y23; y31 y32 y33];

Y_12 = [y14 y15; y24 y25; y34 y35];

Y_21 = [y41 y42 y43; y51 y52 y53];

Y_22 = [y44 y45; y54 y55];

D_1 = Y_12/Y_22;

D_2 = Y_11 - (Y_12/Y_22)*Y_21;

D_3 = inv(Y_22);

D_4 = Y_22\Y_21;
% System dynamic model

% Synchronous machine flux decay model

\[ F(1) = \left( \frac{1}{T_{d0p}} \right) \left( E_{fd} - y(1) - \ldots \right) \]
\[ (X_d - X_{dp}) \cdot \text{real} \left( (D_1(1,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \right) \]
\[ D_1(1,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) + \ldots \]
\[ D_2(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(pi/2)))) + \ldots \]
\[ D_2(1,2) \cdot ((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \right) + \ldots \]
\[ D_2(1,3) \cdot ((V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot (y(2)-(pi/2)))) \right) \right) + \ldots \]
\[ F(2) = y(3) - W_s; \]
\[ F(3) = \left( \frac{W_s}{2 \cdot H} \right) \cdot (y(7) - \ldots) \]

\[ y(1) \cdot \text{imag} \left( (D_1(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \right) \]

\[ D_1(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) + \ldots \]

\[ D_2(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2)))) + \ldots \]

\[ D_2(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) + \ldots \]

\[ D_2(1,3) \cdot ((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) + \ldots \]

\[ (X_q - X_{dp}) \cdot \text{real} \left( (D_1(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \right) \]

\[ D_1(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) + \ldots \]

\[ D_2(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2)))) + \ldots \]

\[ D_2(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) + \ldots \]

\[ D_2(1,3) \cdot ((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) + \ldots \]

\[ \cdot \exp(-1j \cdot (y(2) - (\pi / 2))) \]

\[ \cdot \text{imag} \left( (D_1(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \right) \]

\[ D_1(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) + \ldots \]

\[ D_2(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2)))) + \ldots \]
\[ D_2(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(1,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast (y(2)-(pi/2)))) \ldots \]
\[ - D \ast (y(3) - Ws); \]

\% GTGOV Model

\[ F(4) = (1/T_{dg}) \ast (-y(4) - K_{dg} \ast (Ws/(2 \ast H)) \ast (y(7) - \ldots \]
\[ y(1) \ast \text{imag}((D_1(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_1(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23)))) + \ldots \]
\[ D_2(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(pi/2)))) + \ldots \]
\[ D_2(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(1,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast (y(2)-(pi/2)))) - \ldots \]
\[ (X_q - X_{dp}) \ast \text{real}((D_1(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_1(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23)))) + \ldots \]
\[ D_2(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(pi/2)))) + \ldots \]
\[ D_2(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(1,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \]
\[ \exp(-1j \cdot (y(2) - (\pi/2))) \cdot \text{imag}((D_{1,1} \cdot (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \\
D_{1,2} \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) + \ldots \\
D_{2,1} \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) + \ldots \\
D_{2,2} \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) + \\
\ldots \\
D_{2,3} \cdot ((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \exp(-1j \cdot (y(2) - (\pi/2)))) - \ldots \\
D \cdot (y(3) - Ws)); \]

\[ F(5) = K_{ig} \cdot (Ws - y(3)); \]

\[ F(6) = (1/T_v) \cdot (-y(6) + y(4) + y(5) + K_{pg} \cdot (Ws - y(3))); \]

\[ F(7) = (1/T_{td}) \cdot (-y(7) + K_{t} \cdot (F_{m} \cdot y(6) - 0.25)); \]

% PVSG Unit 1

% PV Model

\[ F(8) = K_{I_P} \cdot (Pset - \ldots \\
\text{real}((D_{3,1} \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \\
\ldots \\
D_{3,2} \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \\]
\[D_4(1,1)\ast((1\ast y(1))\ast\exp(1\ast(y(2) - (\pi/2)))) - \ldots\]

\[D_4(1,2)\ast((Vrn + y(17))\ast\exp(1\ast(y(14) + y(15)))\ast\exp(-1\ast y(13)))\ast y(10) + \ldots\]

\[D_3(1,2)\ast((y(20) + 1\ast y(21))\ast\exp(1\ast y(23))) - \ldots\]

\[F(9) = K_{I\ast Q}\ast(\text{Qref} - \ldots\]

\[\text{real}((D_3(1,1)\ast((y(10) + 1\ast y(11))\ast\exp(1\ast y(13)))) + \ldots\]

\[D_3(1,2)\ast((y(20) + 1\ast y(21))\ast\exp(1\ast y(23))) - \ldots\]

\[D_4(1,1)\ast((1\ast y(1))\ast\exp(1\ast(y(2) - (\pi/2)))) - \ldots\]

\[D_4(1,2)\ast((Vrn + y(17))\ast\exp(1\ast(y(14) + y(15))))\ast\exp(-1\ast y(13)))\ast y(11));\]
$$\begin{align*}
&\text{D}_4(1, 3) * (\text{Vrn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \\
&\quad \exp(-1j \cdot y(13)) \cdot y(11) - \\
&\quad (-\text{imag}((\text{D}_3(1, 1) * ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))))
\quad + \\
&\quad \text{D}_3(1, 2) * ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
&\quad \text{D}_4(1, 1) * ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2)))) - \\
&\quad \text{D}_4(1, 2) * ((\text{Vrn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \\
&\quad \text{D}_4(1, 3) * (\text{Vrn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \\
&\quad \exp(-1j \cdot y(13))) \cdot y(10)) ;
\end{align*}$$

$$\begin{align*}
&\text{F}(10) = -(1/T_i) \cdot (y(10)) - \\
&\quad (\text{K}_P \cdot \text{P}_P \cdot \text{P}_b - \\
&\quad \text{real}((\text{D}_3(1, 1) * ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))))
+ \\
&\quad \text{D}_3(1, 2) * ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
&\quad \text{D}_4(1, 1) * ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2)))) - \\
&\quad \text{D}_4(1, 2) * ((\text{Vrn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \\
&\quad ) - \\
&\quad \text{D}_4(1, 3) * (\text{Vrn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \\
&\quad \cdot \exp(-1j \cdot y(13))) \cdot y(10) + \\
&\quad (-\text{imag}((\text{D}_3(1, 1) * ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))
\quad ) + \ldots
\end{align*}$$
\[D_3(1,2)(y(20) + j \cdot y(21)) \cdot \exp(1j \cdot y(23)) - \ldots\]
\[D_4(1,1)((1j \cdot y(1)) \cdot \exp(1j \cdot ((y(2) - (\pi/2))) - \ldots\]
\[D_4(1,2)((Vrn + y(17)) \cdot \exp(1j \cdot ((y(14) + y(15)))) - \ldots\]
\[D_4(1,3)((Vrn + y(27)) \cdot \exp(1j \cdot ((y(24) + y(25)))) \cdot \exp(-1j \cdot y(13))) \cdot y(11)) + y(8));\]

\[\% \text{ d-axis current reference}\]

\[F(11) = -(1/T_i) \cdot (y(11) - \ldots\]
\[(K_{P,Q} \cdot (Qref - \ldots\]
\[(\text{real} \left((D_3(1,1)((y(10) + j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots\]
\[D_3(1,2)((y(20) + j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots\]
\[D_4(1,1)((1j \cdot y(1)) \cdot \exp(1j \cdot ((y(2) - (\pi/2))) - \ldots\]
\[D_4(1,2)((Vrn + y(17)) \cdot \exp(1j \cdot ((y(14) + y(15)))) - \ldots\]
\[D_4(1,3)((Vrn + y(27)) \cdot \exp(1j \cdot ((y(24) + y(25)))) \cdot \exp(-1j \cdot y(13))) \cdot y(11) - \ldots\]
\[(-\text{imag} \left((D_3(1,1)((y(10) + j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots\]
\[D_3(1,2)((y(20) + j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots\]
\[D_4(1,1)((1j \cdot y(1)) \cdot \exp(1j \cdot ((y(2) - (\pi/2))) - \ldots\]
\[D_4(1,2)((Vrn + y(17)) \cdot \exp(1j \cdot ((y(14) + y(15)))) - \ldots\]
\[
\begin{align*}
\text{D}_4(1,3)*(Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13)) \cdot y(10) + y(9)) \quad \% q \\
-\text{axis current reference} \\
F(12) = K_1 \cdot (-\text{imag}\left((\text{D}_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \right) \quad \% q \\
\text{D}_3(1,2)*((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \\
\text{D}_4(1,1)*((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2)))) - \ldots \\
\text{D}_4(1,2)*((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \\
\text{D}_4(1,3)*(Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13))) \right); \\
\text{F}(13) = (K_P \cdot (-\text{imag}\left((\text{D}_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \right) \quad \% q \\
\text{D}_3(1,2)*((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \\
\text{D}_4(1,1)*((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2)))) - \ldots \\
\text{D}_4(1,2)*((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \\
\text{D}_4(1,3)*(Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13))) + \ldots \\
y(12)); \\
\end{align*}
\]
% SG Emulator Model

\[ F(14) = y(16); \]

\[ F(15) = K_i \cdot (P_{set} - \ldots) \]

\[ = \text{real}(V_r + y(17)) \cdot \left( \left( \text{real}(-\text{real}(D_3((1,1) \cdot (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \right) \right) \]

\[ D_3((1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4((1,1) \cdot (1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) \]

\[ - \ldots \]

\[ D_4((1,2) \cdot ((V_r + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \]

\[ - \ldots \]

\[ D_4((1,3) \cdot (V_r + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \]

\[ \cdot \exp(-1j \cdot y(13))) \ldots \]

\[ + 1j \cdot (-\text{imag}(D_3((1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]

\[ D_3((1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4((1,1) \cdot (1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) \]

\[ - \ldots \]

\[ D_4((1,2) \cdot ((V_r + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \]

\[ - \ldots \]

\[ D_4((1,3) \cdot (V_r + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \]
\[
\begin{align*}
& \cdot \exp(-1j \cdot y(13))) \cdot \exp(1j \cdot y(13))) + \text{real} \\
& \quad \left( (Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \right)) \\
& \quad \cdots
\end{align*}
\]

\[
\begin{align*}
& -1j \cdot \left( \text{imag} \left( \left( \text{real} \left( \left( D_3(1,1) \cdot \left( (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)) \right) \right) \right) \right) \right) + \cdots \\
&  \quad D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
& \quad \cdots
\end{align*}
\]

\[
\begin{align*}
& \quad D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \\
& \quad \cdots
\end{align*}
\]

\[
\begin{align*}
& \quad D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
& \quad \cdot \exp(-1j \cdot y(13))) \cdots
\end{align*}
\]

\[
\begin{align*}
& +1j \cdot \left( \text{imag} \left( \left( D_3(1,1) \cdot \left( (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)) \right) \right) \right) \right) + \cdots \\
&  \quad D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
& \quad \cdots
\end{align*}
\]

\[
\begin{align*}
& \quad D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \\
& \quad \cdots
\end{align*}
\]

\[
\begin{align*}
& \quad D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
& \quad \cdot \exp(-1j \cdot y(13))) \cdots
\end{align*}
\]

\[
\begin{align*}
& + \text{imag} \left( \left( \text{real} \left( \left( D_3(1,1) \cdot \left( (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)) \right) \right) \right) \right) \right) + \text{imag} \\
& \quad \left( (Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \right) \\
& \quad \cdot \text{conj}(y_c) \cdot \exp(1j \cdot (y(14) + y(15))) \right); \quad \%
\end{align*}
\]
delta_p

F(16) = Kiw.(P_set - ...

real((Vrn + y(17)).*(((real(-(real((D_3(1,1)*((y
(10)+1j.*y(11)).*exp(1j.*y(13))) + ...

D_3(1,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ...

D_4(1,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ...

D_4(1,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))

) - ...

D_4(1,3)*(Vrn + y(27)).*exp(1j.*(y(24) + y(25))))

.*exp(-1j.*y(13))) ...

+1j.*(-imag((D_3(1,1)*((y(10)+1j.*y(11)).*exp(1j
.*y(13))) + ...

D_3(1,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ...

D_4(1,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ...

D_4(1,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))

) - ...

D_4(1,3)*(Vrn + y(27)).*exp(1j.*(y(24) + y(25))))

.*exp(-1j.*y(13))))).*exp(1j.*y(13)) + real

(((Vrn + y(17)).*exp(1j.*(y(14) + y(15))))))

...
\(-1j.*((real((D_{3}(1,1)\*((y(10)+1j.*y(11))\*exp(1j.*y(13)))) + \ldots
\end{align*}

\begin{align*}
D_{3}(1,2)\*((y(20)+1j.*y(21))\*exp(1j.*y(23))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,1)\*((1j.*y(1))\*exp(1j.*(y(2)-(\pi/2)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,2)\*((Vrn + y(17))\*exp(1j.*(y(14) + y(15)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,3)\*((Vrn + y(27))\*exp(1j.*(y(24) + y(25))))
\end{align*}

\begin{align*}
+1j.*(-imag(((D_{3}(1,1)\*((y(10)+1j.*y(11))\*exp(1j.*y(13)))) + \ldots
\end{align*}

\begin{align*}
D_{3}(1,2)\*((y(20)+1j.*y(21))\*exp(1j.*y(23))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,1)\*((1j.*y(1))\*exp(1j.*(y(2)-(\pi/2)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,2)\*((Vrn + y(17))\*exp(1j.*(y(14) + y(15)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,3)\*((Vrn + y(27))\*exp(1j.*(y(24) + y(25))))
\end{align*}

\begin{align*}
\exp(-1j.*y(13))) \ldots
\end{align*}

\begin{align*}
+1j.*(-imag(((D_{3}(1,1)\*((y(10)+1j.*y(11))\*exp(1j.*y(13)))) + \ldots
\end{align*}

\begin{align*}
D_{3}(1,2)\*((y(20)+1j.*y(21))\*exp(1j.*y(23))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,1)\*((1j.*y(1))\*exp(1j.*(y(2)-(\pi/2)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,2)\*((Vrn + y(17))\*exp(1j.*(y(14) + y(15)))) - \ldots
\end{align*}

\begin{align*}
D_{4}(1,3)\*((Vrn + y(27))\*exp(1j.*(y(24) + y(25))))
\end{align*}

\begin{align*}
\exp(-1j.*y(13)))\*exp(1j.*y(13))) + imag
\end{align*}

\begin{align*}
(((Vrn + y(17))\*exp(1j.*(y(14) + y(15))))\*conj(y_c)\*exp(1j.*(y(14) + y(15))))�;
\end{align*}

\begin{align*}
delta_{W}
\end{align*}

\begin{align*}
F(17) = K_{iq}\*(Q_{set} + \ldots
\end{align*}
imag((Vrn + y(17)).*((real(-(real((D_3(1,1)*(y(10)+1j*y(11)))*exp(1j*y(13)))+...))
D_3(1,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
D_4(1,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(1,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15))))
*exp(-1j*y(13))) ... +1j*(-imag((D_3(1,1)*((y(10)+1j*y(11)))*exp(1j
*y(13)))+...))
D_3(1,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
D_4(1,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(1,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15))))
) - ... D_4(1,3)*((Vrn + y(27))*exp(1j*(y(24) + y(25))))
*exp(-1j*y(13)))) ... \exp((Vrn + y(17))*exp(1j*(y(14) + y(15))))
... -1j*(imag(-(real((D_3(1,1)*((y(10)+1j*y(11)))*
exp(1j*y(13)))+...))
D_3(1,2)*((y(20)+1j*y(21))*exp(1j*y(23))) -
\[
\begin{align*}
D_4(1, 1) & \left( (1 \cdot y(1)) \cdot \exp(1 \cdot (y(2) - (\pi /2))) \right) - \\
D_4(1, 2) & \left( (Vrn + y(17)) \cdot \exp(1 \cdot (y(14) + y(15))) \right) - \ldots \\
D_4(1, 3) & \left( (Vrn + y(27)) \cdot \exp(1 \cdot (y(24) + y(25))) \right) \cdot \exp(-1 \cdot y(13)) \ldots \\
+1j \cdot (\text{imag}((D_3(1, 1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \\
D_3(1, 2) & \left( (y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23)) \right) - \\
D_4(1, 1) & \left( (1 \cdot y(1)) \cdot \exp(1 \cdot (y(2) - (\pi /2))) \right) - \\
D_4(1, 2) & \left( (Vrn + y(17)) \cdot \exp(1 \cdot (y(14) + y(15))) \right) - \\
D_4(1, 3) & \left( (Vrn + y(27)) \cdot \exp(1 \cdot (y(24) + y(25))) \right) \cdot \exp(-1 \cdot y(13))) \cdot \exp(1j \cdot y(13)) + \text{imag}((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \cdot \text{conj}(y_c) \cdot \exp(1j \cdot (y(14) + y(15))); \quad % \\
V_{rq}\end{align*}
\]
\( F(18) = K_{I\_P} \ast (P_{set} - \ldots \)

\[
(\text{real}(D_{3}(2,1) \ast((y(10)+j\cdot y(11)) \ast \exp(j\cdot y(13))))
+ \ldots
D_{3}(2,2) \ast((y(20)+j\cdot y(21)) \ast \exp(j\cdot y(23))) - \ldots
D_{4}(2,1) \ast((j\cdot y(1)) \ast \exp(j\cdot (y(2)-(\pi/2)))) - \ldots
D_{4}(2,2) \ast((V_{rn} + y(17)) \ast \exp(j\cdot (y(14) + y(15)))) - \ldots
D_{4}(2,3) \ast((V_{rn} + y(27)) \ast \exp(j\cdot (y(24) + y(25)))) \ast \exp(-j\cdot y(23))) \ast y(20) + \ldots
(-\text{imag}(D_{3}(2,1) \ast((y(10)+j\cdot y(11)) \ast \exp(j\cdot y(13))))
+ \ldots
D_{3}(2,2) \ast((y(20)+j\cdot y(21)) \ast \exp(j\cdot y(23))) - \ldots
D_{4}(2,1) \ast((j\cdot y(1)) \ast \exp(j\cdot (y(2)-(\pi/2)))) - \ldots
D_{4}(2,2) \ast((V_{rn} + y(17)) \ast \exp(j\cdot (y(14) + y(15)))) - \ldots
D_{4}(2,3) \ast((V_{rn} + y(27)) \ast \exp(j\cdot (y(24) + y(25)))) \ast \exp(-j\cdot y(23))) \ast y(21)));
\]

\( F(19) = K_{I\_Q} \ast (Q_{ref} - \ldots \)

\[
(\text{real}(D_{3}(2,1) \ast((y(10)+j\cdot y(11)) \ast \exp(j\cdot y(13))))
+ \ldots
D_{3}(2,2) \ast((y(20)+j\cdot y(21)) \ast \exp(j\cdot y(23))) - \ldots
D_{4}(2,1) \ast((j\cdot y(1)) \ast \exp(j\cdot (y(2)-(\pi/2)))) - \ldots
D_{4}(2,2) \ast((V_{rn} + y(17)) \ast \exp(j\cdot (y(14) + y(15)))) - \ldots
\]
\[
D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
\exp(-1j \ast y(23)) \ast y(21) - \ldots
\]

\[
(-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]

\[
D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots
\]

\[
D_4(2,2) \ast ((V_{rn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots
\]

\[
D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
\exp(-1j \ast y(23))) \ast y(20) + \ldots
\]

\[
(-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]

\[
F(20) = -(1/T_i) \ast (y(20) - \ldots
\]

\[
((K_{P_P} \ast (P_{set} - \ldots
\]

\[
(\text{real}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]

\[
D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots
\]

\[
D_4(2,2) \ast ((V_{rn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots
\]

\[
D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
\exp(-1j \ast y(23))) \ast y(20) + \ldots
\]

\[
(-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]

\[
F(20) = -(1/T_i) \ast (y(20) - \ldots
\]

\[
((K_{P_P} \ast (P_{set} - \ldots
\]

\[
(\text{real}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]

\[
D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots
\]

\[
D_4(2,2) \ast ((V_{rn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots
\]

\[
D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
\exp(-1j \ast y(23))) \ast y(20) + \ldots
\]

\[
(-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) \\
+ \ldots
\]

\[
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots
\]
\[ D_4(2,1) \ast (1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(21))) + y(18)); \ % d-axis current reference \]

\[ F(21) = -(1/T_i) \ast (y(21) - \ldots \]
\[ ((K_P \ast Q \ast (Qref - \ldots \]
\[ \text{real} ((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) \]
\[ + \ldots \]
\[ D_3(2,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(21) - \ldots \]
\[ (-\text{imag} ((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) \]
\[ + \ldots \]
\[ D_3(2,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(20))) + y(19)); \ % \]
\[ q-\text{axis current reference} \]

352 \[ F(22) = K_I \cdot (-\text{imag}(D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]

353 \[ D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

354 \[ D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \ldots \]

355 \[ D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \]

356 \[ D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \]

357 \[ \cdot \exp(-1j \cdot y(23))) \];

358 \[ F(23) = (K_P \cdot (-\text{imag}(D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]

359 \[ D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

360 \[ D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \ldots \]

361 \[ D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]

362 \[ D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) + \ldots \]

363 \[ y(22)) ; \% \text{ check the variable} \]

364 \%

365 \% \text{SG Emulator Model}

126
\[ F(24) = y(26); \]
\[ F(25) = Kip \times (P_{set} - \ldots \]
\[ \text{real}((Vrn + y(27)) \times ((\text{real}(-\text{real}((D_3(2,1) \times ((y^{(10)}+1j \times y^{(11)}) \times \exp(1j \times y^{(13)})) + \ldots \]
\[ D_3(2,2) \times ((y^{(20)}+1j \times y^{(21)}) \times \exp(1j \times y^{(23)})) - \ldots \]
\[ D_4(2,1) \times ((1j \times y^{(1)}) \times \exp(1j \times (y^{(2)}-(\pi/2))) - \ldots \]
\[ D_4(2,2) \times ((Vrn + y^{(17)}) \times \exp(1j \times (y^{(14)} + y^{(15)}))) - \ldots \]
\[ D_4(2,3) \times (Vrn + y^{(27)}) \times \exp(1j \times (y^{(24)} + y^{(25)}))) \times \exp(-1j \times y^{(23)}) \ldots \]
\[ +1j \times (-\text{imag}((D_3(2,1) \times ((y^{(10)}+1j \times y^{(11)}) \times \exp(1j \times y^{(13)})) + \ldots \]
\[ D_3(2,2) \times ((y^{(20)}+1j \times y^{(21)}) \times \exp(1j \times y^{(23)})) - \ldots \]
\[ D_4(2,1) \times ((1j \times y^{(1)}) \times \exp(1j \times (y^{(2)}-(\pi/2))) - \ldots \]
\[ D_4(2,2) \times ((Vrn + y^{(17)}) \times \exp(1j \times (y^{(14)} + y^{(15)}))) - \ldots \]
\[ D_4(2,3) \times (Vrn + y^{(27)}) \times \exp(1j \times (y^{(24)} + y^{(25)}))) \times \exp(-1j \times y^{(23)}) \times \exp(1j \times (y^{(24)} + y^{(25)}))) + \text{real}(((Vrn + y^{(27)}) \times \exp(1j \times (y^{(24)} + y^{(25)})))) \ldots \]
\[-1j \cdot (\text{imag}(-(\text{real}(D_3(2,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \\exp(1j \cdot y(13)))) + \ldots \\
D_3(2,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \\
D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (pi/2)))) - \ldots \\
D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
) - \ldots \\
D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \\
\cdot \exp(-1j \cdot y(23)) \ldots \\
+1j \cdot (-(\text{imag}(D_3(2,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \\
\cdot y(13)))) + \ldots \\
D_3(2,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \\
D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (pi/2)))) - \ldots \\
D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
) - \ldots \\
D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \\
\cdot \exp(-1j \cdot y(23))) \cdot \exp(1j \cdot y(23))) + \text{imag}(((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \text{conj}(y_c) \cdot \exp(1j \cdot (y(24) + y(25)))); \%
\delta_p
\]

\[F(26) = \text{Kiw} \cdot (P\_set - \ldots )\]
real((Vrn + y(27)))*((real(-(real((D_3(2,1)*((y(10)+1j*y(11)))*exp(1j*y(13)))+ ...)
D_3(2,2)*((y(20)+1j*y(21)))*exp(1j*y(23))) - ...
D_4(2,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(2,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15))))
) - ...
D_4(2,3)*((Vrn + y(27))*exp(1j*(y(24) + y(25))))
*exp(-1j*y(23))) ... 
+1j*(-imag((D_3(2,1)*((y(10)+1j*y(11)))*exp(1j*y(13)))+ ...)
D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
D_4(2,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(2,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15))))
) - ...
D_4(2,3)*((Vrn + y(27))*exp(1j*(y(24) + y(25))))
*exp(-1j*y(23)))*exp(1j*y(23)))+ real
(((Vrn + y(27))*exp(1j*(y(24) + y(25)))))) ...
-1j*(-imag(-(real((D_3(2,1)*((y(10)+1j*y(11)))*
exp(1j*y(13)))+ ...)
D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
\[ \delta_{W} \]

\[ \text{F(27) = Kiq} \ast (Q_{\text{set}} - \ldots) \]

\[ -\text{imag} ((Vrn + y(27)) \ast (((\text{real}(-\text{real} ((D_{3}(2,1)) * ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \text{conj}(y_{c})) \ast \exp(1j \ast (y(24) + y(25))))) \ast \exp(-1j \ast y(23)))) \ast \exp((Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))))) \ast \exp(-1j \ast y(23))) \ast \exp(1j \ast (y(24) + y(25)))); \]

\[ \text{deltaw} \]
\[
D_4(2,1) \ast (\{1 \ast y(1)\} \ast \exp(1 \ast (y(2)-(\pi/2)))) - \\
D_4(2,2) \ast (\{Vrn + y(17)\} \ast \exp(1 \ast (y(14) + y(15)))) \\
D_4(2,3) \ast (\{Vrn + y(27)\} \ast \exp(1 \ast (y(24) + y(25)))) \\
\ast \exp(-1 \ast y(23))) \\
D_3(2,2) \ast (\{y(20)+1 \ast y(21)\} \ast \exp(1 \ast y(23))) - \\
D_4(2,1) \ast (\{1 \ast y(1)\} \ast \exp(1 \ast (y(2)-(\pi/2)))) - \\
D_4(2,2) \ast (\{Vrn + y(17)\} \ast \exp(1 \ast (y(14) + y(15)))) \\
D_4(2,3) \ast (\{Vrn + y(27)\} \ast \exp(1 \ast (y(24) + y(25)))) \\
\ast \exp(-1 \ast y(23))) \ast \exp(-1 \ast y(23))) \ast \exp(-1 \ast y(23))) + \text{real} \\
((\{Vrn + y(27)\} \ast \exp(1 \ast (y(24) + y(25)))))) \\
\text{...} \\
-1 \ast (\text{imag}(-(\text{real}((D_3(2,1) \ast (\{y(10)+1 \ast y(11)\} \ast \\
\exp(1 \ast y(13)))) + ... \\
D_3(2,2) \ast (\{y(20)+1 \ast y(21)\} \ast \exp(1 \ast y(23))) - \\
D_4(2,1) \ast (\{1 \ast y(1)\} \ast \exp(1 \ast (y(2)-(\pi/2)))) - \\
\text{...} \\
\text{131}
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23)) \]
\[ + 1j \ast (-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots) \]
\[ D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23)) \ast \exp(1j \ast y(23))) \ast \text{imag}(((Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25))))))) \ast \text{conj}(y_c) \ast \exp(1j \ast (y(24) + y(25))))\];

\[ V_{rq} \]

% Algebraic Variables

% PVSG Unit 1
% PV Model

\[ y_{28} = \left( \text{real} \left( (D_3(1,1) \ast (y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)) \right) + \ldots \right. \]

\[ D_3(1,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]

\[ D_4(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi /2)))) - \ldots \]

\[ D_4(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \]

\[ \ast \exp(-1j \ast y(13))) \ast y(10) + \ldots \]

\[ (-\text{imag} \left( (D_3(1,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)) \right) \right) + \ldots \]

\[ D_3(1,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]

\[ D_4(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi /2)))) - \ldots \]

\[ D_4(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \]

\[ \ast \exp(-1j \ast y(13))) \ast y(11); \quad \% \text{edit} \]

\[ y_{29} = \left( \text{real} \left( (D_3(1,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)) \right) + \ldots \right. \]
\[ D_3(1,2) * ((y(20) + 1j * y(21)) * \exp(1j * y(23))) - ... \]
\[ D_4(1,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi / 2)))) - ... \]
\[ D_4(1,2) * ((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) - ... \]
\[ D_4(1,3) * (Vrn + y(27)) * \exp(1j * (y(24) + y(25)))) * \exp(-1j * y(13))) * y(11) - ... \]
\[ (-\text{imag}((D_3(1,1) * ((y(10) + 1j * y(11)) * \exp(1j * y(13)))) + ... \]
\[ D_3(1,2) * ((y(20) + 1j * y(21)) * \exp(1j * y(23))) - ... \]
\[ D_4(1,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi / 2)))) - ... \]
\[ D_4(1,2) * ((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) - ... \]
\[ D_4(1,3) * (Vrn + y(27)) * \exp(1j * (y(24) + y(25)))) * \exp(-1j * y(13))) * y(10); \quad % \text{edit} \]

\[ y_{30} = ((K_P * Pset - ... \]
\[ \text{real}((D_3(1,1) * ((y(10) + 1j * y(11)) * \exp(1j * y(13)))) + ... \]
\[ D_3(1,2) * ((y(20) + 1j * y(21)) * \exp(1j * y(23))) - ... \]
\[ D_4(1,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi / 2)))) - ... \]
\[ D_4(1,2) * ((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) ) - ... \]
\[ D_4(1,3) * (Vrn + y(27)) * \exp(1j * (y(24) + y(25)))) \]
\[ \exp(-1j \cdot y(13)) \cdot y(10) + \ldots \]

\[ (-\text{imag}((D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots \]

\[ D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) - \ldots \]

\[ D_4(1,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \exp(-1j \cdot y(13))) \cdot y(11)) + y(8); \]

\[ y_{31} = ((K_{P,Q} \cdot (Q_{ref} - \ldots \]

\[ (\text{real}((D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots \]

\[ D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) - \ldots \]

\[ D_4(1,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \exp(-1j \cdot y(13))) \cdot y(11) - \ldots \]

\[ (-\text{imag}((D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots \]
\[ D_4(1,2) ((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]
\[ D_4(1,3) (V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13))) \cdot y(10))) + y(9); \]
\[ y_{32} = K_P \cdot (-\text{imag} ((D_3(1,1) ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]
\[ D_3(1,2) (((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]
\[ D_4(1,1) ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2))) - \ldots \]
\[ D_4(1,2) ((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]
\[ D_4(1,3) (V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13)))) ; \]
\[ y_{33} = (K_P \cdot (-\text{imag} ((D_3(1,1) ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]
\[ D_3(1,2) (((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]
\[ D_4(1,1) ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2))) - \ldots \]
\[ D_4(1,2) ((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]
\[ D_4(1,3) (V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(13))) + \ldots \]
\[ y(12); \]
% SG Emulator

\[ y_{34} = (y(14) + y(15)) \]

\[ y_{35} = (V_n + y(17)) \]

\[ y_{36} = \text{real}((V_n + y(17)) \cdot ((\text{real}(\text{real}(D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots
\]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots \]

\[ D_4(1,2) \cdot ((V_n + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]

\[ D_4(1,3) \cdot (V_n + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \exp(-1j \cdot y(13))) \ldots \]

\[ +1j \cdot ( - \text{imag}((D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots \]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots \]

\[ D_4(1,2) \cdot ((V_n + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - \ldots \]
\begin{align*}
D_4(1,3) & \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \\
& \quad \cdot \exp(-1j \cdot y(1)) \cdot \exp(1j \cdot y(13))) \cdot \exp(1j \cdot y(13)) + \text{real} \quad \\
& \quad \cdot \exp((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \\
\ldots \quad \quad \quad \quad \quad \\
& -1j \cdot \text{imag}((\text{real}(D_3(1,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \\
& \quad \cdot \exp(1j \cdot y(13))) + \ldots \\
D_3(1,2) & \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23)))) - \\
\ldots \quad \quad \quad \quad \quad \\
D_4(1,1) & \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \\
\ldots \quad \quad \quad \quad \quad \\
D_4(1,3) & \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \\
& \quad \cdot \exp(-1j \cdot y(13)) \ldots \\
+1j \cdot \text{imag}((D_3(1,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \\
& \quad \cdot y(13))) + \ldots \\
D_3(1,2) & \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23)))) - \\
\ldots \quad \quad \quad \quad \quad \\
D_4(1,1) & \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \\
\ldots \quad \quad \quad \quad \quad \\
D_4(1,2) & \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
& \quad \cdot \exp(1j \cdot y(13))) \ldots \\
\end{align*}
\* \text{conj}(y_c) \* \exp(1j \* (y(14) + y(15)))) \}; \quad \% P

\text{formulation error}

530 \quad y_{37} = \text{imag}((Vrn + y(17)) \* (((\text{real}(-\text{real}(D_3(1,1) * ((y(10) + 1j \* y(11)) \* \exp(1j \* y(13)) \)) + ... \\
\quad \text{D}_3(1,2) * ((y(20) + 1j \* y(21)) \* \exp(1j \* y(23)) \)) - ...
\quad \text{D}_4(1,1) * ((1j \* y(1)) \* \exp(1j \* (y(2) - (\pi/2))) \)) - ...
\quad \text{D}_4(1,2) * ((Vrn + y(17)) \* \exp(1j \* (y(14) + y(15))) \) - ... \\
\quad \text{D}_4(1,3) * (Vrn + y(27)) \* \exp(1j \* (y(24) + y(25))) \) \* \exp(-1j \* y(13))) ... \\
\quad +1j \* (-\text{imag}((D_3(1,1) * ((y(10) + 1j \* y(11)) \* \exp(1j \* y(13)))) + ... \\
\quad \text{D}_3(1,2) * ((y(20) + 1j \* y(21)) \* \exp(1j \* y(23))) \) - ...
\quad \text{D}_4(1,1) * ((1j \* y(1)) \* \exp(1j \* (y(2) - (\pi/2))) \)) - ...
\quad \text{D}_4(1,2) * ((Vrn + y(17)) \* \exp(1j \* (y(14) + y(15))) \) - ... \\
\quad \text{D}_4(1,3) * (Vrn + y(27)) \* \exp(1j \* (y(24) + y(25))) \) \* \exp(-1j \* y(13))) \)) \* \exp(1j \* y(13)) \)) + \text{real}(((Vrn + y(17)) \* \exp(1j \* (y(14) + y(15)))) \))) ...
\[-1j \cdot (\text{imag}(-\text{real}(D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \\
\exp(1j \cdot y(13))) + \ldots \\
D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
\ldots \\
D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \\
\ldots \\
D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \\
\cdot \exp(-1j \cdot y(13))) \ldots \\
+1j \cdot (-\text{imag}(D_3(1,1) \cdot ((y(10) + 1j \cdot y(11)) \cdot \exp(1j \\
\cdot y(13))) + \ldots \\
D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
\ldots \\
D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \\
\ldots \\
D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) \\
\cdot \exp(-1j \cdot y(13))) \ldots \\
D_4(1,3) \cdot ((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \\
\cdot \exp(-1j \cdot y(13))) \ldots \\
) + \text{imag}(D_4(1,3) \cdot ((Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \\
\cdot \exp(-1j \cdot y(13))) \ldots \\
) \cdot \text{conj}(y_c) \cdot \exp(1j \cdot (y(14) + y(15))))); \quad \% Q \]

formulation error

\% PVSG Unit 2
\begin{verbatim}
% PV Model

y_38 = (real(D_3(2,1)*((y(10)+1j*y(11))*exp(1j*y(13)))) +
   ...
   D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
   D_4(2,1)*((1j*y(1)*exp(1j*(y(2)-(pi/2)))) - ...
   D_4(2,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15)))) -
   ...
   D_4(2,3)*((Vrn + y(27))*exp(1j*(y(24) + y(25)))).*
      exp(-1j*y(23)))*y(20) + ...
(=imag((D_3(2,1)*((y(10)+1j*y(11))*exp(1j*y(13))))
   + ...) +
   D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
   D_4(2,1)*((1j*y(1)*exp(1j*(y(2)-(pi/2)))) - ...
   D_4(2,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15)))) -
   ...
   D_4(2,3)*((Vrn + y(27))*exp(1j*(y(24) + y(25)))).*
      exp(-1j*y(23)))).*y(21)); % edit

y_39 = (real((D_3(2,1)*((y(10)+1j*y(11))*exp(1j*y(13)))) +
   ...
   D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
   D_4(2,1)*((1j*y(1)*exp(1j*(y(2)-(pi/2)))) - ...
   D_4(2,2)*((Vrn + y(17))*exp(1j*(y(14) + y(15)))) -
\end{verbatim}
\[ \begin{align*}
D_4(2,3) & \cdot (V_{\text{rn}} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23)) \cdot y(21) - \\
& (-\text{imag}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) \\
& + ... \\
D_3(2,2) & \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - ... \\
D_4(2,1) & \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - ... \\
D_4(2,2) & \cdot ((V_{\text{rn}} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - ... \\
D_4(2,3) & \cdot (V_{\text{rn}} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \\
& \exp(-1j \cdot y(23))) \cdot y(20); \% \text{edit} \\
y_{40} & = ((K_{P \cdot P} \cdot (P_{\text{set}} - ... \\
& \left( \text{real} \left((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) \\
& + ... \\
D_3(2,2) & \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - ... \\
D_4(2,1) & \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - ... \\
D_4(2,2) & \cdot ((V_{\text{rn}} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))) - ... \\
D_4(2,3) & \cdot (V_{\text{rn}} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))) \cdot \\
& \exp(-1j \cdot y(23))) \cdot y(20) + ... \\
& (-\text{imag}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) \\
& + ... \\
D_3(2,2) & \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - ... \\
D_4(2,1) & \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - ...
\end{align*} \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(21))) + y(18)); \]
\[ y_{41} = (K_P \ast (Qref - \ldots \real((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) + \ldots \right.
\[ D_3(2,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(21) - \ldots \]
\[ (-\imag((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) + \ldots \right.
\[ D_3(2,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \ldots \]
\[ D_4(2,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_4(2,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23))) \ast y(20))) + y(19)); \]
\[ y_{42} = K_P \ast (-\imag((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) + \ldots \right. \]
\[ \begin{align*}
D_3(2,2) & \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \\
& \ldots \\
D_4(2,1) & \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi /2)))) - \\
& \ldots \\
D_4(2,2) & \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
& \ldots \\
D_4(2,3) & \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
& \ast \exp(-1j \ast y(23))) \\
& \ldots \\
\end{align*} \]

\[ y_{43} = (K_P \ast (-\text{imag}((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))))) + \ldots \\
D_3(2,2) & \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \\
D_4(2,1) & \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi /2)))) - \\
D_4(2,2) & \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \\
D_4(2,3) & \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \\
& \ast \exp(-1j \ast y(23))) \\
& \ldots \\
y(22)) ; \\
\end{align*} \]

\[ \begin{align*}
\% \text{ SG Emulator} \\
y_{44} & = (y(24) + y(25)) ; \\
y_{45} & = (Vrn + y(27)) ; \\
\end{align*} \]
\[ y_{46} = \text{real}((Vrn + y(27)) \cdot (((\text{real}(-(\text{real}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots
\]

\[ D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]

\[ D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \ldots
\]

\[ D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) - \ldots
\]

\[ D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23))) \ldots
\]

\[ +1j \cdot (-\text{imag}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots
\]

\[ D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]

\[ D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) - \ldots
\]

\[ D_4(2,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) - \ldots
\]

\[ D_4(2,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23))) \cdot \exp(1j \cdot (y(24) + y(25))) \ldots
\]

\[ -1j \cdot (\text{imag}(-(\text{real}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots
\]

\[ D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]
\[
\ldots
\]
D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (pi/2)))) - \\
\ldots
D_4(2,2) \ast ((\text{Vrn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
\ldots
D_4(2,3) \ast ((\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \\
\ldots
\ast \exp(-1j \ast y(23))) \\
+1j \ast (-\text{imag}((D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \\
\ldots
D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (pi/2)))) - \\
\ldots
D_4(2,2) \ast ((\text{Vrn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
\ldots
D_4(2,3) \ast ((\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \\
\ldots
\ast \exp(-1j \ast y(23))) \ast \exp(1j \ast y(23))) + \text{imag}(((\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \text{conj}(y_c)) \ast \exp(1j \ast (y(24) + y(25)))); \quad \% \text{P formulation error}
\]
\[
y_47 = -\text{imag}((\text{Vrn} + y(27)) \ast (((\text{real}(-\text{real}((D_3(2,1) \ast ((y(10) \\
+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots
D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \\
\ldots
\]
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D_4(2,1) *((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(2,2) *((Vrn + y(17))*exp(1j*(y(14) + y(15))))
) - ...
D_4(2,3) *((Vrn + y(27))*exp(1j*(y(24) + y(25))))
 *exp(-1j*y(23))) ...
+1j*(-imag((D_3(2,1)*((y(10)+1j*y(11))*exp(1j*y(13))))) + ...
D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
D_4(2,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(2,2) *((Vrn + y(17))*exp(1j*(y(14) + y(15))))
) - ...
D_4(2,3) *((Vrn + y(27))*exp(1j*(y(24) + y(25))))
 *exp(-1j*y(23))) *.exp(1j*y(23))) + real
(((Vrn + y(27))*exp(1j*(y(24) + y(25)))))) ...
-1j*(imag(-real((D_3(2,1)*((y(10)+1j*y(11))*exp(1j*y(13))))) + ... 
D_3(2,2)*((y(20)+1j*y(21))*exp(1j*y(23))) - ...
D_4(2,1)*((1j*y(1))*exp(1j*(y(2)-(pi/2)))) - ...
D_4(2,2) *((Vrn + y(17))*exp(1j*(y(14) + y(15))))
\[ D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25))) \ast \exp(-1j \ast y(23)) \ldots \]

\[ + 1j \ast (-\text{imag}(\ast (D_3(2,1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13))) + \ldots \]

\[ D_3(2,2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]

\[ D_4(2,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi/2)))) - \ldots \]

\[ D_4(2,2) \ast ((V_{rn} + y(17)) \ast \exp(1j \ast (y(14) + y(15))) \ast - \ldots \]

\[ D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25))) \ast \exp(-1j \ast y(23)) \ast \exp(1j \ast y(23)) \ast \text{imag} \ast \text{conj}(y_c) \ast \exp(1j \ast (y(24) + y(25)))) \]

\[ \text{Q formulation error} \]
% State Variables:

\[ I_{\text{inv}_1} = (y(10)+1j*y(11)) \cdot \exp(1j*y(13)); \]

\[ I_{\text{inv}_2} = (y(20)+1j*y(21)) \cdot \exp(1j*y(23)); \]

\[ V_{\text{int}_m} = (1j*y(1)) \cdot \exp(1j*(y(2)-(\pi/2))); \]

\[ V_{r_1} = (Vrn + y(17)) \cdot \exp(1j*(y(14) + y(15))); \]

\[ V_{r_2} = (Vrn + y(27)) \cdot \exp(1j*(y(24) + y(25))); \]

\[ X_1 = [I_{\text{inv}_1}; I_{\text{inv}_2}]; \]

\[ X_2 = [V_{\text{int}_m}; V_{r_1}; V_{r_2}]; \]

\[ A_1 = D_1 \cdot X_1 + D_2 \cdot X_2; \]

\[ A_2 = D_3 \cdot X_1 - D_4 \cdot X_2; \]

% Element by Element approach

\[ y_{48} = \text{real}((D_1(1,1) \cdot ((y(10)+1j*y(11)) \cdot \exp(1j*y(13))) + \ldots)

D_1(1,2) \cdot ((y(20)+1j*y(21)) \cdot \exp(1j*y(23))) + \ldots

D_2(1,1) \cdot ((1j*y(1)) \cdot \exp(1j*(y(2)-(\pi/2)))) + \ldots) \]
\[ D_2(1, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(1, 3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25))) \ast \exp(-1j \ast (y(2) - (\pi / 2))) ; \]

\[ y_{49} = \text{imag}(D_1(1, 1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_1(1, 2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) + \ldots \]
\[ D_2(1, 1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi / 2)))) + \ldots \]
\[ D_2(1, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(1, 3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25))) \ast \exp(-1j \ast (y(2) - (\pi / 2))) ; \]

\[ y_{50} = \text{real}(D_1(2, 1) \ast ((y(10) + 1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_1(2, 2) \ast ((y(20) + 1j \ast y(21)) \ast \exp(1j \ast y(23))) + \ldots \]
\[ D_2(2, 1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2) - (\pi / 2)))) + \ldots \]
\[ D_2(2, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_2(2,3) \times (V_{\text{rn}} + y(27)) \times \exp(1j \times (y(24) + y(25))) \]
\[ \times \exp(-1j \times (y(14) + y(15))) \];

\[ y_{51} = \text{imag}((D_1(2,1) \times ((y(10) + 1j \times y(11)) \times \exp(1j \times y(13))) + \ldots \]
\[ D_1(2,2) \times ((y(20) + 1j \times y(21)) \times \exp(1j \times y(23))) + \ldots \]
\[ D_2(2,1) \times ((1j \times y(1)) \times \exp(1j \times (y(2) - (\pi/2))) + \ldots \]
\[ D_2(2,2) \times ((V_{\text{rn}} + y(17)) \times \exp(1j \times (y(14) + y(15))) \]
\[ \times \exp(-1j \times (y(14) + y(15))) \);\]

\[ y_{52} = \text{real}((D_1(3,1) \times ((y(10) + 1j \times y(11)) \times \exp(1j \times y(13))) + \ldots \]
\[ D_1(3,2) \times ((y(20) + 1j \times y(21)) \times \exp(1j \times y(23))) + \ldots \]
\[ D_2(3,1) \times ((1j \times y(1)) \times \exp(1j \times (y(2) - (\pi/2))) + \ldots \]
\[ D_2(3,2) \times ((V_{\text{rn}} + y(17)) \times \exp(1j \times (y(14) + y(15))) \]
\[ \times \exp(-1j \times ((y(24) + y(25)))) \];
\[ y_{53} = \text{imag} \left( (D_{1}(3,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) \right) + \ldots \]
\[ D_{1}(3,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) + \ldots \]
\[ D_{2}(3,1) \ast (1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2))) + \ldots \]
\[ D_{2}(3,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) + \ldots \]
\[ D_{2}(3,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast ((y(24) + y(25)))) ; \]

\[ y_{54} = \text{real} \left( (D_{3}(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) \right) + \ldots \]
\[ D_{3}(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_{4}(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \ldots \]
\[ D_{4}(1,2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) - \ldots \]
\[ D_{4}(1,3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(13)) ; \]

\[ y_{55} = -\text{imag} \left( (D_{3}(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13))) \right) + \ldots \]
\[ D_{3}(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \]
D_4(1,1)*((1j*y(1)).*exp(1j.*(y(2)-(pi/2)))) -

D_4(1,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))) - ...

D_4(1,3)*(Vrn + y(27)).*exp(1j.*(y(24) + y(25))))
    .*exp(-1j.*y(13)));

y_56 = real((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))) +

D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) -

D_4(2,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) -

D_4(2,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))) - ...

D_4(2,3)*(Vrn + y(27)).*exp(1j.*(y(24) + y(25))))
    .*exp(-1j.*y(23)));

y_57 = -imag((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))) +

D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) -

D_4(2,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) -

...
\[ D_4(2,2) \ast ((V_{rn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
\ast \ldots \]
\[ D_4(2,3) \ast (V_{rn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \ast \exp(-1j \ast y(23)) \];

% Algebraic Variables:

% Int_m = y(48) + 1j \ast y(49)
% I_r_1 = y(50) + 1j \ast y(51)
% I_r_2 = y(52) + 1j \ast y(53)
% V_inv_1 = y(54) + 1j \ast y(55)
% V_inv_2 = y(56) + 1j \ast y(57)
% Check accuracy of equations of this section

\[ y_{58} = \text{real}((\text{Vrn} + y(17)) \cdot (\text{real}(-(\text{real}(D_{3}(1,1) \cdot ((y(10) + 1 \cdot j \cdot y(11)) \cdot \exp(1 \cdot j \cdot y(13)))) + \ldots\]

\[ D_{3}(1,2) \cdot ((y(20) + 1 \cdot j \cdot y(21)) \cdot \exp(1 \cdot j \cdot y(23))) - \ldots\]

\[ D_{4}(1,1) \cdot ((1 \cdot j \cdot y(1)) \cdot \exp(1 \cdot j \cdot (y(2) - (\pi /2)))) - \ldots\]

\[ D_{4}(1,2) \cdot ((\text{Vrn} + y(17)) \cdot \exp(1 \cdot j \cdot (y(14) + y(15)))) \cdot \exp(-1 \cdot j \cdot y(13))) \ldots\]

\[ + 1 \cdot j \cdot (\text{imag}((D_{3}(1,1) \cdot ((y(10) + 1 \cdot j \cdot y(11)) \cdot \exp(1 \cdot j \cdot y(13)))) + \ldots\]

\[ D_{3}(1,2) \cdot ((y(20) + 1 \cdot j \cdot y(21)) \cdot \exp(1 \cdot j \cdot y(23))) - \ldots\]

\[ D_{4}(1,1) \cdot ((1 \cdot j \cdot y(1)) \cdot \exp(1 \cdot j \cdot (y(2) - (\pi /2)))) - \ldots\]

\[ D_{4}(1,2) \cdot ((\text{Vrn} + y(17)) \cdot \exp(1 \cdot j \cdot (y(14) + y(15)))) \]

\[ D_{4}(1,3) \cdot ((\text{Vrn} + y(27)) \cdot \exp(1 \cdot j \cdot (y(24) + y(25)))) \cdot \exp(-1 \cdot j \cdot y(13))) \ldots\]
\begin{align*}
&(((\text{Vrn} + y(17)).\exp(1j.(y(14) + y(15)))))) \\
&\quad \ldots \\
&-1j.(\text{imag}(-((1j.((\text{D}_3(1,1)*((y(10)+1j.*y(11)))*\exp(1j.*y(13))) + \ldots \\
&D_3(1,2)*((y(20)+1j.*y(21)).\exp(1j.*y(23))) - \\
&D_4(1,1)*((1j.*y(1)).\exp(1j.(y(2)-(\pi/2)))) - \\
&D_4(1,2)*((\text{Vrn} + y(17)).\exp(1j.(y(14) + y(15)))) \\
&\quad \ldots \\
&D_4(1,3)*((\text{Vrn} + y(27)).\exp(1j.(y(24) + y(25)))) \\
&\quad .\exp(-1j.*y(13))) \ldots \\
&+1j.(-\text{imag}((-\text{D}_3(1,1)*((y(10)+1j.*y(11)).\exp(1j.*y(13))) + \ldots \\
&D_3(1,2)*((y(20)+1j.*y(21)).\exp(1j.*y(23))) - \\
&D_3(1,1)*((1j.*y(1)).\exp(1j.(y(2)-(\pi/2)))) - \\
&D_4(1,2)*((\text{Vrn} + y(17)).\exp(1j.(y(14) + y(15)))) \\
&\quad \ldots \\
&D_4(1,3)*((\text{Vrn} + y(27)).\exp(1j.(y(24) + y(25)))) \\
&\quad .\exp(-1j.*y(13))) .\exp(1j.*y(13))) + \text{imag} \\
&(((\text{Vrn} + y(17)).\exp(1j.(y(14) + y(15)))))) \\
&\quad .\text{conj}(y_c).\exp(1j.(y(14) + y(15)))) ;
\end{align*}
\[ y_{59} = -\text{imag} ((Vrn + y(17)) \cdot (\text{real} (-\text{real} ((D_3(1,1) \ast (y(10) + 1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots)
\]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots
\]

\[ D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))
\]

\[ \cdot \exp(-1j \cdot y(13))) \ldots
\]

\[ +1j \cdot (-\text{imag} ((D_3(1,1) \ast ((y(10) + 1j \cdot y(11)) \cdot \exp(1j
\]

\[ \cdot y(13)))) + \ldots
\]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]

\[ D_4(1,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi/2)))) - \ldots
\]

\[ D_4(1,2) \cdot ((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))
\]

\[ ) - \ldots
\]

\[ D_4(1,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))
\]

\[ \cdot \exp(-1j \cdot y(13))) \ldots
\]

\[ +1j \cdot (-\text{imag} ((D_3(1,1) \ast ((y(10) + 1j \cdot y(11)) \cdot \exp(1j
\]

\[ \cdot y(13)))) + \ldots
\]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]

\[ \cdot \exp(\exp(1j \cdot y(13))) \cdot \exp((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15)))
\]

\[ ) - \ldots
\]

\[ D_4(1,3) \cdot (Vrn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))
\]

\[ \cdot \exp(-1j \cdot y(13))) \cdot \exp(1j \cdot y(13)) + \text{real}
\]

\[ (((Vrn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))))\ldots
\]

\[ -1j \cdot (\text{imag} (-\text{real} ((D_3(1,1) \ast ((y(10) + 1j \cdot y(11)) \cdot
\]

\[ \exp(1j \cdot y(13)))) + \ldots
\]

\[ D_3(1,2) \cdot ((y(20) + 1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \ldots
\]
\[
\begin{align*}
D_4(1,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi/2)))) & - \\
D_4(1,2) * ((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) & - \\
D_4(1,3) * (Vrn + y(27)) * \exp(1j * (y(24) + y(25)))) & - \\
& * \exp(-1j * y(13))) ... \\
+ 1j * (-\text{imag}((D_3(1,1) * ((y(10)+1j * y(11)) * \exp(1j * y(13))))) + ... \\
D_3(1,2) * ((y(20)+1j * y(21)) * \exp(1j * y(23))) & - \\
D_4(1,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi/2)))) & - \\
D_4(1,2) * ((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) & - \\
D_4(1,3) * (Vrn + y(27)) * \exp(1j * (y(24) + y(25)))) & - \\
& * \exp(-1j * y(13))))) * \exp(1j * y(13)) + \text{imag}(((Vrn + y(17)) * \exp(1j * (y(14) + y(15)))) * \text{conj}(y_c)) * \exp(1j * (y(14) + y(15))) ; \\
y_{60} = \text{real}((Vrn + y(27)) * (((\text{real}(-(\text{real}((D_3(2,1) * ((y(10)+1j * y(11)) * \exp(1j * y(13))))) + ... \\
D_3(2,2) * ((y(20)+1j * y(21)) * \exp(1j * y(23))) & - \\
D_4(2,1) * ((1j * y(1)) * \exp(1j * (y(2) - (\pi/2)))) & -
\end{align*}
\]
\[ D_4(2, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14)+y(15)))) - \ldots \]
\[ D_4(2, 3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24)+y(25)))) \ast \exp(-1j \ast y(23)) \ldots \]
\[ + 1j \ast (-\text{imag}((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_3(2, 2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2, 1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \ldots \]
\[ D_4(2, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14)+y(15)))) - \ldots \]
\[ D_4(2, 3) \ast (Vrn + y(27)) \ast \exp(1j \ast (y(24)+y(25)))) \ast \exp(-1j \ast y(23))) \ast \exp(1j \ast y(23))) + \text{real} \]
\[ (((Vrn + y(27)) \ast \exp(1j \ast (y(24)+y(25))))))) \ldots \]
\[ - 1j \ast (\text{imag}(-(\text{real}((D_3(2,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y(13)))) + \ldots \]
\[ D_3(2, 2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23))) - \ldots \]
\[ D_4(2, 1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \ldots \]
\[ D_4(2, 2) \ast ((Vrn + y(17)) \ast \exp(1j \ast (y(14)+y(15)))) - \ldots \]
\[
D_4(2,3) \cdot (Vn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \\
\cdot \exp(-1j \cdot y(23)) \ldots \\
+1j \cdot (-\text{imag}(D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \\
D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
\ldots \\
D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2)))) - \\
\ldots \\
D_4(2,2) \cdot ((Vn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
) - \ldots \\
D_4(2,3) \cdot (Vn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \\
\cdot \exp(-1j \cdot y(23))) \cdot \exp(1j \cdot y(23))) + \text{imag}((
Vn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25)))))) \cdot \\
\text{conj}(y_c) \cdot \exp(1j \cdot (y(24) + y(25)))
\]

\[
y_{61} = -\text{imag}((Vn + y(27)) \cdot (((\text{real}(-\text{real}(D_3(2,1) \cdot ((y(10) \\
+1j \cdot y(11)) \cdot \exp(1j \cdot y(13))) + \ldots \\
D_3(2,2) \cdot ((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) - \\
\ldots \\
D_4(2,1) \cdot ((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi /2)))) - \\
\ldots \\
D_4(2,2) \cdot ((Vn + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \\
) - \ldots \\
D_4(2,3) \cdot (Vn + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \\
\cdot \exp(-1j \cdot y(23))) \ldots
\]
+1j.*(-imag((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))),) + ...
D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ...
D_4(2,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ...
D_4(2,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))) - ...
D_4(2,3)*((Vrn + y(27)).*exp(1j.*(y(24) + y(25))))
.*exp(-1j.*y(23))).*exp(1j.*y(23)))) + real
(((Vrn + y(27)).*exp(1j.*(y(24) + y(25))))))) ...
-1j.*(imag(-(real((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))),) + ...
D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ...
D_4(2,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ...
D_4(2,2)*((Vrn + y(17)).*exp(1j.*(y(14) + y(15)))) - ...
D_4(2,3)*((Vrn + y(27)).*exp(1j.*(y(24) + y(25))))
.*exp(-1j.*y(23))) ...
+1j.*(-imag((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))),) + ...
D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) -
\[
\begin{align*}
D_4(2,1) &\cdot ((1 \cdot y(1)) \cdot \exp(1 \cdot (y(2) - (\pi/2))) - \ldots \\
D_4(2,2) &\cdot ((Vrn + y(17)) \cdot \exp(1 \cdot (y(14) + y(15))) \\
D_4(2,3) &\cdot (Vrn + y(27)) \cdot \exp(1 \cdot (y(24) + y(25)))) \\
&\cdot \exp(-1 \cdot y(23))) \cdot \exp(1 \cdot y(23))) + \text{imag} \\
&((Vrn + y(27)) \cdot \exp(1 \cdot (y(24) + y(25)))) \\
&\cdot \text{conj}(y_c) \cdot \exp(1 \cdot (y(24) + y(25))) ; \\
\end{align*}
\]

\%

\%

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\[
\begin{align*}
y_{62} &= \text{real}(-\text{real}((D_3(1,1) \cdot ((y(10)+1\cdot y(11)) \cdot \exp(1\cdot y \\
(13))) + \ldots \\
D_3(1,2) &\cdot ((y(20)+1\cdot y(21)) \cdot \exp(1\cdot y(23))) - \ldots \\
D_4(1,1) &\cdot ((1 \cdot y(1)) \cdot \exp(1 \cdot (y(2) - (\pi/2)))) - \ldots
\end{align*}
\]

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\[
D_4(1,2) \ast ((\text{Vrn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
- \ldots \\
D_4(1,3) \ast (\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \\
\ast \exp(-1j \ast y(13))) \ldots \\
+ 1j \ast (-\text{imag}((D_3(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \\
\ast y(13)))) + \ldots \\
D_3(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23)))) - \\
\ldots \\
D_4(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \\
\ldots \\
D_4(1,2) \ast ((\text{Vrn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
) - \ldots \\
D_4(1,3) \ast (\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \\
\ast \exp(-1j \ast y(13))) \ast \exp(1j \ast y(13)));
\]

\[
y_{63} = \text{imag}(-\text{real}((D_3(1,1) \ast ((y(10)+1j \ast y(11)) \ast \exp(1j \ast y \\
(13)))) + \ldots \\
D_3(1,2) \ast ((y(20)+1j \ast y(21)) \ast \exp(1j \ast y(23)))) - \\
\ldots \\
D_4(1,1) \ast ((1j \ast y(1)) \ast \exp(1j \ast (y(2)-(\pi/2)))) - \\
\ldots \\
D_4(1,2) \ast ((\text{Vrn} + y(17)) \ast \exp(1j \ast (y(14) + y(15)))) \\
) - \ldots \\
D_4(1,3) \ast (\text{Vrn} + y(27)) \ast \exp(1j \ast (y(24) + y(25)))) \\
\ast \exp(-1j \ast y(13))) \ldots 
\]
\begin{verbatim}
876  +1j.*(-imag((D_3(1,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))) + ... 
877  D_3(1,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ... 
878  D_4(1,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ... 
879  D_4(1,2)*((Vrn+y(17)).*exp(1j.*(y(14)+y(15)))) 
880  D_4(1,3)*(Vrn+y(27)).*exp(1j.*(y(24)+y(25)))) 
881  *exp(-1j.*y(13))).)*exp(1j.*y(13)));
882  y_64 = real(((Vrn+y(17)).*exp(1j.*(y(14)+y(15)))));
883  y_65 = imag(((Vrn+y(17)).*exp(1j.*(y(14)+y(15)))));
884  % PVSG - 2
885 
886  y_66 = real(-(real((D_3(2,1)*((y(10)+1j.*y(11)).*exp(1j.*y(13)))) + ... 
887  D_3(2,2)*((y(20)+1j.*y(21)).*exp(1j.*y(23))) - ... 
888  D_4(2,1)*((1j.*y(1)).*exp(1j.*(y(2)-(pi/2)))) - ... 
889  D_4(2,2)*((Vrn+y(17)).*exp(1j.*(y(14)+y(15)))) 
890  ) - ... 
\end{verbatim}
\[ D_4(2,3)(V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23)) \ldots \]
\[ + 1j \cdot (-\text{imag}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13)) \ldots \]
\[ D_3(2,2)((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) \ldots \]
\[ D_4(2,1)((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) \ldots \]
\[ D_4(2,2)((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \ldots \]
\[ D_4(2,3)(V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23))) \cdot \exp(1j \cdot y(23))) ; \]
\[ y_{67} = \text{imag}(-(\text{real}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13)))) + \ldots \]
\[ D_3(2,2)((y(20)+1j \cdot y(21)) \cdot \exp(1j \cdot y(23))) \ldots \]
\[ D_4(2,1)((1j \cdot y(1)) \cdot \exp(1j \cdot (y(2)-(\pi/2)))) \ldots \]
\[ D_4(2,2)((V_{rn} + y(17)) \cdot \exp(1j \cdot (y(14) + y(15))) \ldots \]
\[ D_4(2,3)(V_{rn} + y(27)) \cdot \exp(1j \cdot (y(24) + y(25))) \cdot \exp(-1j \cdot y(23))) \ldots \]
\[ + 1j \cdot (-\text{imag}((D_3(2,1) \cdot ((y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13)) \ldots \]
\[ \ldots \]

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D_3(2,2) * ((y(20) + 1j * y(21)) * exp(1j * y(23))) − ...
D_4(2,1) * ((1j * y(1)) * exp(1j * (y(2) − (pi/2)))) − ...
D_4(2,2) * ((Vrn + y(17)) * exp(1j * (y(14) + y(15))))
) − ...
D_4(2,3) * (Vrn + y(27)) * exp(1j * (y(24) + y(25))))
* exp(−1j * y(23))) * exp(1j * y(23)));

y_68 = real(((Vrn + y(27)) * exp(1j * (y(24) + y(25))));

y_69 = imag(((Vrn + y(27)) * exp(1j * (y(24) + y(25))));

% 

end

IEEE_dyn2Model_V2.m
function dy = IEEE.dyn2Model_V2(t,y,params)

% Define system parameters

% vars and params
Efd=params.Efd; Pset=params.Pset; Y_BUS=params.Y_BUS; para=params.para;

% Define system parameters

% Machine dynamic parameters
T_d0_p = para(1); X_dp = para(2); X_q = para(3); X_d = para(4);

Ws = para(5); D = 0.1; H = para(7);

% GT governor dynamic parameters

T_dg = para(8);

K_dg = 5;
K_ig = 50;
K_{pg} = 5;

T_{td} = para(12); T_v = para(13);

K_t = para(14); F_m = para(15);

% Photovoltaic dynamic parameters

Pref = Pset; Qref = 0.0;

T_i = para(16);

K_{I_1} = 0.1; K_{I_2} = 0.1; K_{I_3} = 0.01; K_{I_4} = 0.1;

K_{I_P} = 10; K_{I_Q} = 10;

K_{P_1} = 1; K_{P_2} = 1; K_{P_3} = 0.1; K_{P_4} = 1;

K_{P_P} = 0.1; K_{P_Q} = 0.1;

% SG emulator dynamic parameters

P_set = 0.1; Q_set = 0.0;
Kiq = 0.1;

Kip_1 = 0.1; Kiw_1 = 0.1;

% Inverter interface line data

para(28) = 1e-4;

para(27) = 1e-3;

R_c = para(27); Lc = para(28);

Z_base = (480/sqrt(3))/(1e6/(sqrt(3)*480));

W = 2*pi*60;

z_c = (R_c + 1j*W*Lc)/Z_base;

y_c = 1/z_c;
dy = zeros(69,1);

% System dynamic model

% Synchronous machine flux decay model

dy(1) = (1/T_d0_p) * (Efd - y(1) - (X_d - X_dp) * y(56));
dy(2) = y(3) - Ws;
dy(3) = (Ws/(2*H)) * (y(7) - y(1) * y(57) - (X_q - X_dp) * y(56) * y(57) - D * (y(3) - Ws));

% GTGOV Model

dy(4) = (1/T_dg) * (-y(4) - K_dg * (Ws/(2*H)) * (y(7) - y(1) * y(57) - (X_q - X_dp) * y(56) * y(57) - D * (y(3) - Ws)));
dy(5) = K_{ig} \times (W_s - y(3));

dy(6) = (1/T_{\text{v}}) \times (-y(6) + y(4) + y(5) + K_{pg} \times (W_s - y(3)));

dy(7) = (1/T_{\text{td}}) \times (-y(7) + K_{t} \times (F_{m} \times y(6) - 0.25));

% PVSG Unit 1

% PV Model

dy(8) = K_{I_P} \times (P_{\text{ref}}(1) - y(36));

dy(9) = K_{I_Q} \times (Q_{\text{ref}} - y(37));

dy(10) = -(1/T_{i}) \times (y(10) - y(38)); % d-axis current reference

dy(11) = -(1/T_{i}) \times (y(11) - y(39)); % q-axis current reference

dy(12) = K_{I_1} \times y(59);

dy(13) = y(41);
% SG Emulator Model (rename model variable names)

dy(14) = y(16);

dy(15) = K_i_p_1.*(P_set - y(66));  % delta_p

dy(16) = K_i_w_1.*(P_set - y(66));  % delta_W

dy(17) = K_i_q.*(Q_set - y(67));  % V_rq

dy(18) = -(1/T_i).*((y(18) - y(42)));  % d-axis current reference

dy(19) = -(1/T_i).*((y(19) - y(43)));  % q-axis current reference

dy(20) = K_i_3.*y(63);  % PLL model

dy(21) = y(45);

% PVSG Unit 2

% PV Model
dy(22) = K_I_P.*(Pref(1) - y(46));

dy(23) = K_I_Q.*(Qref - y(47));

dy(24) = -(1/T_i).*(y(24) - y(48)); % d-axis current reference

dy(25) = -(1/T_i).*(y(25) - y(49)); % q-axis current reference

dy(26) = K_I_2.*y(61);

dy(27) = y(51);

% SG Emulator Model (rename model variable names)

dy(28) = y(30);

dy(29) = Kip_1.*(P_set - y(68)); % delta_p

dy(30) = Kiw_1.*(P_set - y(68)); % delta_W

dy(31) = Kiq.*(Q_set - y(69)); % V_rq
\( dy(32) = -(1/T_i) \cdot (y(32) - y(52)); \) \% d-axis current reference

\( dy(33) = -(1/T_i) \cdot (y(33) - y(53)); \) \% q-axis current reference

\( dy(34) = K_{I4} \cdot y(65); \) \% PLL model

\( dy(35) = y(55); \)

% Algebraic Variables

% PVSG Unit 1

% PV Model

\( dy(36) = y(36) - (y(58) \cdot y(10) + y(59) \cdot y(11)); \) \% edit

\( dy(37) = y(37) - (y(58) \cdot y(11) - y(59) \cdot y(10)); \) \% edit

\( dy(38) = y(38) - ((K_P \cdot P \cdot (P_r - y(36))) + y(8)); \)

\( dy(39) = y(39) - ((K_P \cdot Q \cdot (Q_r - y(37))) + y(9)); \)
dy(40) = y(40) - K_P_1.*y(59);

dy(41) = y(41) - (y(40) + y(12));

% SG Emulator

dy(42) = y(42) - (y(15) + y(16));

dy(43) = y(43) - y(17);

dy(44) = y(44) - K_P_3.*y(63);  % P formulation error

dy(45) = y(45) - (y(44) + y(20));  % Q formulation error

% PVSG Unit 2

% PV Model

dy(46) = y(46) - (y(60).*y(24) + y(61).*y(25));  % edit

dy(47) = y(47) - (y(60).*y(25) - y(61).*y(24));  % edit

dy(48) = y(48) - ((K_P_P.*(Pref(2) - y(46))) + y(24));
dy(49) = y(49) - ((K_P_Q.* (Qref - y(47))) + y(25));

dy(50) = y(50) - K_P_2.*y(61);

dy(51) = y(51) - (y(50) + y(26));

% SG Emulator

dy(52) = y(52) - (y(29) + y(30));

dy(53) = y(53) - y(31);

dy(54) = y(54) - K_P_4.*y(65);    % P formulation error

dy(55) = y(55) - (y(54) + y(34));  % Q formulation error

% Network algebraic equations
% Partition admittance matrix based on machine and inverter model (not generic form)

% This decoupling of state and algebraic variable will be in matrix form for larger system

\[
\begin{align*}
  y_{11} &= Y_{BUS}(1,1); & y_{12} &= Y_{BUS}(1,2); & y_{13} &= Y_{BUS}(1,3); & y_{14} &= Y_{BUS}(1,4); & y_{15} &= Y_{BUS}(1,5); \\
  y_{21} &= Y_{BUS}(2,1); & y_{22} &= Y_{BUS}(2,2); & y_{23} &= Y_{BUS}(2,3); & y_{24} &= Y_{BUS}(2,4); & y_{25} &= Y_{BUS}(2,5); \\
  y_{31} &= Y_{BUS}(3,1); & y_{32} &= Y_{BUS}(3,2); & y_{33} &= Y_{BUS}(3,3); & y_{34} &= Y_{BUS}(3,4); & y_{35} &= Y_{BUS}(3,5); \\
  y_{41} &= Y_{BUS}(4,1); & y_{42} &= Y_{BUS}(4,2); & y_{43} &= Y_{BUS}(4,3); & y_{44} &= Y_{BUS}(4,4); & y_{45} &= Y_{BUS}(4,5); \\
  y_{51} &= Y_{BUS}(5,1); & y_{52} &= Y_{BUS}(5,2); & y_{53} &= Y_{BUS}(5,3); & y_{54} &= Y_{BUS}(5,4); & y_{55} &= Y_{BUS}(5,5); \\
\end{align*}
\]

% Transformed Admittance Matrix
$$Y_{11} = y_{11};$$

$$Y_{12} = [y_{12}\ y_{13}\ y_{14}\ y_{15}];$$

$$Y_{21} = [y_{21}; y_{31}; y_{41}; y_{51}];$$

$$Y_{22} = [y_{22} \ y_{23} \ y_{24} \ y_{25}; y_{32} \ y_{33} \ y_{34} \ y_{35}; y_{42} \ y_{43} \ y_{44} \ y_{45}; y_{52} \ y_{53} \ y_{54} \ y_{55}];$$

$$D_1 = Y_{12}/Y_{22};$$

$$D_2 = Y_{11} - (Y_{12}/Y_{22}) \cdot Y_{21};$$

$$D_3 = \text{inv}(Y_{22});$$

$$D_4 = Y_{22} \setminus Y_{21};$$

% % % State Variables :

$$I_{\text{inv}_1} = (y(10)+1j \cdot y(11)) \cdot \exp(1j \cdot y(13));$$
\[ I_{\text{inv}_2} = (y(24) + 1j \cdot y(25)) \cdot \exp(1j \cdot y(27)) \]

\[ V_{\text{int}_m} = (1j \cdot y(1)) \cdot \exp(1j \cdot (y(2) - (\pi / 2))) \]

\[ I_{SI_1} = (y(18) + 1j \cdot y(19)) \cdot \exp(1j \cdot y(21)) \]

\[ I_{SI_2} = (y(32) + 1j \cdot y(33)) \cdot \exp(1j \cdot y(34)) \]

\[ X_1 = [I_{\text{inv}_1}; I_{\text{inv}_2}; I_{SI_1}; I_{SI_2}] \]

\[ X_2 = V_{\text{int}_m} \]

\[ A_1 = D_1 \cdot X_1 + D_2 \cdot X_2 \]

\[ A_2 = Y_{22} \cdot X_1 - D_4 \cdot X_2 \]

% Algebraic Variables:

\[ % I_{\text{int}_m} = y(48) + 1j \cdot y(49) \]

\[ % I_{r_1} = y(50) + 1j \cdot y(51) \]

\[ % I_{r_2} = y(52) + 1j \cdot y(53) \]

\[ % V_{\text{inv}_1} = y(54) + 1j \cdot y(55) \]

\[ % V_{\text{inv}_2} = y(56) + 1j \cdot y(57) \]
\[
\begin{align*}
\text{dy}(56) &= y(56) - \text{real}(A_1(1,1) \cdot \exp(-1j \cdot (y(2) - (\pi /2))) ; \\
\text{dy}(57) &= y(57) - \text{imag}(A_1(1,1) \cdot \exp(-1j \cdot (y(2) - (\pi /2))) ; \\
\text{dy}(58) &= y(58) - \text{real}(A_2(1,1) \cdot \exp(-1j \cdot y(13)) ; \\
\text{dy}(59) &= y(59) - \text{imag}(A_2(1,1) \cdot \exp(-1j \cdot y(13)) ; \\
\text{dy}(60) &= y(60) - \text{real}(A_2(2,1) \cdot \exp(-1j \cdot y(23)) ; \\
\text{dy}(61) &= y(61) - \text{imag}(A_2(2,1) \cdot \exp(-1j \cdot y(23)) ; \\
\text{dy}(62) &= y(62) - \text{real}(A_2(3,1) \cdot \exp(-1j \cdot y(21)) ; \\
\text{dy}(63) &= y(63) - \text{imag}(A_2(3,1) \cdot \exp(-1j \cdot y(21)) ; \\
\text{dy}(64) &= y(64) - \text{real}(A_2(4,1) \cdot \exp(-1j \cdot y(34)) ; \\
\text{dy}(65) &= y(65) - \text{imag}(A_2(4,1) \cdot \exp(-1j \cdot y(34)) ; \\
\end{align*}
\]
% Check accuracy of equations of this section

dy(66) = y(66) - (y(62).*y(18) + y(63).*y(19));

dy(67) = y(67) - (y(62).*y(19) - y(63).*y(18));

dy(68) = y(68) - (y(64).*y(32) + y(65).*y(33));

dy(69) = y(69) - (y(64).*y(33) - y(65).*y(32));

end
Appendix C  Formulation of DAE and state space model of microgrid system with 1 PVSG unit and GT DER in the system

simulation_mains_T1.m

1  clc
2  clear
3
4  
5
6  
7  
8  \% Define initial value vector
9
10  \% Initial Condition
11
12  global Efd para Pset Y_BUS
13
14
15
y0 = [0.944387520576973, 0.197822470838027, 1.00000000248079, 1.02013719104335e-09, 0.232652791899858, -0.0260206260954639, 0.378700094028316, -4.67494378103364e-09, 0.378700094028316, -4.67363548401419e-09, 1.0000004644119, 0.381440805756876, 0.381440328416191, 4.64160646358900e-07, 0.64160646358270e-07, 0.0562448225589245, 0.399999999794439, 0.4578715300302e-12, 0.378700096083925, -4.67508956816729e-09, -1.30451103304310e-08, 1.00000045136625, 0.381440792574498, 1.05624482255891, 1.31366222431897e-08, -5.85769147865778e-06, 0.0645695047541847, -0.0257731462241394, 1.05624478625957, -1.30451103304310e-08, 2.07578486764321e-06, -1.424397506867e-06, 1.31366222431897e-08, -5.85769147865778e-06, -0.980331683463077, -0.393195674779321, 0.980331717481913, 0.393195687474196];

load ('C:\PUSPAL_HAZRA\microgrid_studies\DAE_IEEE_13_PVSG\model\IEEEpvsg_data.mat');
params.Efd = Efd;
params.para = para;
params.Pset = Pset;
params.Y_BUS = Y_BUS;

y_0 = y0(1:17);

options = optimoptions('fsolve', 'TolX', 1e-04, 'TolFun', 1e-04);

F = IEEE_dynModel_init(y_0);

[X_eq, fvec, infor, output, fjac] = fsolve(@IEEE_dynModel_init, y_0, options);

Y_alg = alg_est(X_eq);

Y_init = [X_eq Y_alg];

% Span - 1
tspan = [0 100];

M = [eye(17) zeros(17,22); zeros(22,39)];

options = odeset('Mass',M);

[t1,y1] = ode15s(@(t,y)IEEE_dynModel_V2(t,y,params),tspan,
Y_init,options);

% Span - 2

l = length(t1);

y0 = y1(l,:);

Pset = 0.3;
params.Pset = Pset;

tspan = [100 500];
M = [eye(17) zeros(17,22); zeros(22,39)];

options = odeset ('Mass',M);

[t2, y2] = ode15s (@(t, y)IEEE_dynModel_V2(t, y, params), tspan, y0, options);

% t = [t1; t2];

y = [y1; y2];

l = length(y(:,1));

y0 = y(1,:);
plot(t,y); grid on;

subplot(3,1,1);
plot(t,y(:,18)); grid on;
set(gcf,'Color',[1,1,1]);
set(gca,'FontSize',16);
ylabel('Magnitude');
xlabel('Time in sec');
title('Real power of PV in per unit');

subplot(3,1,2);
plot(t,y(:,34)); grid on;
set(gcf,'Color',[1,1,1]);
set(gca,'FontSize',16);
ylabel('Magnitude');
123  xlabell('Time in sec');
124  title('Real power of SG emulator in per unit');
125
126  subplot(3,1,3);
127  plot(t,y(:,34)+y(:,18));
128  grid on;
129  set(gcf, 'Color', [1,1,1]);
130  set(gca, 'FontSize', 16);
131  ylabel('Magnitude');
132  xlabel('Time in sec');
133  title('Real power of PVSG in per unit');

**IEEE_dynModel.m**

1  function dy = IEEE_dynModel(t,y,params)
2  % Define system parameters
3
4  % vars and params
5  Efd=params.Efd; Pset=params.Pset; Y_BUS=params.Y_BUS;
6  para=params.para;
7
8  % Define system parameters
9
10  % Machine dynamic parameters
11
12  T_d0_p = para(1); X_dp = para(2); X_q = para(3); X_d = para
\( W_s = \text{para}(5); \ D = 5; \ H = \text{para}(7); \)

\% GT governor dynamic parameters

\( T_{\text{dg}} = \text{para}(8); \)

\( K_{\text{dg}} = 20; \)
\( K_{\text{ig}} = 100; \)
\( K_{\text{pg}} = 100; \)

\( T_{\text{td}} = \text{para}(12); \ T_{\text{v}} = \text{para}(13); \)

\( K_{\text{t}} = \text{para}(14); \ F_{\text{m}} = \text{para}(15); \)

\% Photovoltaic dynamic parameters

\( P_{\text{ref}} = P_{\text{set}}; \ Q_{\text{ref}} = 0.0; \)

\( T_{\text{i}} = \text{para}(16); \)

\( K_{\text{I}} = \text{para}(17); \ K_{\text{I}.P} = 1000; \ K_{\text{I}.Q} = 100; \)

\( K_{\text{P}} = \text{para}(18); \ K_{\text{P}.P} = 10; \ K_{\text{P}.Q} = 0.1; \)
% SG emulator dynamic parameters

P_set = 0; Q_set = 0; Vrn = para(23);

Kip = 1; Kiq = -0.1; Kiw = 1;

% Inverter interface line data

para(28) = 1e-4;

Rc = para(27); Lc = para(28);

Z_base = (480/sqrt(3))/(40e3/(sqrt(3)*480));

W = 2*pi*60;

zc = (Rc + 1j*W*Lc)/Z_base;

yc = 1/zc;

%
dy = zeros(39,1);

% System dynamic model

% Synchronous machine flux decay model

dy(1) = (1/T_d0_p)*(Efd - y(1) - (X_d - X_dp)*y(28));

dy(2) = y(3) - Ws;

dy(3) = (Ws/(2*H))*(y(7) - y(1)*y(29) - (X_q - X_dp)*y(28)*y(29) - D*(y(3) - Ws));

% GTGOV Model

dy(4) = (1/T_dg)*(-y(4) - K_dg*(Ws/(2*H))*y(7) - y(1)*y(29) - (X_q - X_dp)*y(28)*y(29) - D*(y(3) - Ws));

dy(5) = K_ig*(Ws - y(3));

dy(6) = (1/T_v)*(-y(6) + y(4) + y(5) + K_pg*(Ws - y(3)));

dy(7) = (1/T_td)*(-y(7) + K_t*(F_m*y(6) - 0.25));
% PV Model (convert it to BESS model)

dy(8) = K_I_P.*(Pref - y(18));

dy(9) = K_I_Q.*(Qref - y(19));

dy(10) = -(1/T_i).*(y(10) - y(20)); % d-axis current reference

dy(11) = -(1/T_i).*(y(11) - y(21)); % q-axis current reference

dy(12) = K_I.*y(31);

dy(13) = y(23)-1;

% SG Emulator Model (rename model variable names)

dy(14) = y(16); % Need to stabilize this (free variables)

dy(15) = Kip.*(P_set - y(26)); % delta_p

dy(16) = Kiw.*(P_set - y(26)); % delta_W
% Algebraic Variables

% PV Model (rename model variable names)

dy(18) = y(18) - (y(30) .* y(10) + y(31) .* y(11));

dy(19) = y(19) - (y(30) .* y(11) - y(31) .* y(10));

dy(20) = y(20) - ((K_P_P .* (Pref - y(18))) + y(8));

dy(21) = y(21) - ((K_P_Q .* (Qref - y(19))) + y(9));

dy(22) = y(22) - K_P .* y(31);

dy(23) = y(23) - (y(22) + y(12));

% SG Emulator

dy(24) = y(24) - (y(14) + y(15));
dy(25) = y(25) - (Vrn + y(17));

dy(26) = y(26) - y(34);  \text{ \% P formulation error}

dy(27) = y(27) - y(35);  \text{ \% Q formulation error}

\text{Network algebraic equations}

\text{Partition admittance matrix based on machine and inverter model (not generic form)}

\text{This decoupling of state and algebraic variable will be in matrix form for larger system}

Y = \text{zeros}(3,3);

y11 = Y\_BUS(1,1);  \text{ y12 = Y\_BUS(1,2);  y13 = Y\_BUS(1,3);}
\[ y_{21} = Y_{BUS(2,1)}; \quad y_{22} = Y_{BUS(2,2)}; \quad y_{23} = Y_{BUS(2,3)}; \]
\[ y_{31} = Y_{BUS(3,1)}; \quad y_{32} = Y_{BUS(3,2)}; \quad y_{33} = Y_{BUS(3,3)}; \]

% Transformed Admittance Matrix (validate)

\[ Y(1,1) = y_{11} - \left( y_{21}/y_{22} \right) y_{12}; \]
\[ Y(1,2) = y_{12}/y_{22}; \]
\[ Y(1,3) = y_{13} - y_{12} \left( y_{23}/y_{22} \right); \]
\[ Y(2,1) = -y_{21}/y_{22}; \]
\[ Y(2,2) = 1/y_{22}; \]
\[ Y(2,3) = -y_{23}/y_{22}; \]
\[ Y(3,1) = y_{31} - \left( y_{21}/y_{22} \right) y_{32}; \]
\[ Y(3,2) = y_{32}/y_{22}; \]
\[ Y(3,3) = y_{33} - y_{32} \times \left( \frac{y_{23}}{y_{22}} \right); \]

% Variable order:
% State: Machine internal voltage, inverter current
% Algebraic Variables: Machine current, inverter voltage
% Note: Issues in DAE model

\[
\begin{align*}
\text{dy}(28) &= y(28) - \text{real} \left( (Y(1,1) \times (1j \times y(1))) \times \exp(1j \times (y(2)-(\pi/2))) + Y(1,2) \times (y(10)+1j \times y(11)) \times \exp(1j \times y(13)) + Y(1,3) \times y(25) \times \exp(1j \times y(24)) \times \exp(-1j \times (y(2)-(\pi/2))) \right); \\
\text{dy}(29) &= y(29) - \text{imag} \left( (Y(1,1) \times (1j \times y(1))) \times \exp(1j \times (y(2)-(\pi/2))) + Y(1,2) \times (y(10)+1j \times y(11)) \times \exp(1j \times y(13)) + Y(1,3) \times y(25) \times \exp(1j \times y(24)) \times \exp(-1j \times (y(2)-(\pi/2))) \right); \\
\text{dy}(30) &= y(30) - \text{real} \left( (Y(2,1) \times (1j \times y(1))) \times \exp(1j \times (y(2)-(\pi/2))) + Y(2,2) \times (y(10)+1j \times y(11)) \times \exp(1j \times y(13)) + Y(2,3) \right);
\end{align*}
\]
dy(31) = y(31) - imag((Y(2,1)*(1j*y(1)).*exp(1j*(y(2)-(pi/2)))) + Y(2,2).(y(10)+1j*y(11)).*exp(1j*y(13)) + Y(2,3).*y(25).*exp(1j*y(24))).*exp(-1j.*(y(13))));

dy(32) = y(32) - real((Y(3,1)*(1j*y(1)).*exp(1j*(y(2)-(pi/2)))) + Y(3,2).(y(10)+1j*y(11)).*exp(1j*y(13)) + Y(3,3).*y(25).*exp(1j*y(24))));

dy(33) = y(33) - imag((Y(3,1)*(1j*y(1)).*exp(1j*(y(2)-(pi/2)))) + Y(3,2).(y(10)+1j*y(11)).*exp(1j*y(13)) + Y(3,3).*y(25).*exp(1j*y(24))));

% create algebraic variable for real and reactive power expression
% (as per the analytic model)

dy(34) = y(34) - real(y(25).*(((y(36)+y(38))-1j.*(y(37)+y(39))).*conj(y_c)).*exp(1j.*y(24)));

dy(35) = y(35) + imag(y(25).*(((y(36)+y(38))-1j.*(y(37)+y(39)))).*exp(1j.*y(24)));
\[(39)) \cdot \text{conj}(y_c) \cdot \exp(1j \cdot y(24));\]

\[
\%	ext{Replicate } I_r \text{ as in the formulation (mismatch in the calculation)}
\]

\[
dy(36) = y(36) - \text{real}(-(y(30)+1j \cdot y(31)) \cdot \exp(1j \cdot y(13)));
\]

\[
dy(37) = y(37) - \text{imag}(-(y(30)+1j \cdot y(31)) \cdot \exp(1j \cdot y(13)));
\]

\[
dy(38) = y(38) - \text{real}((y(25) \cdot \exp(1j \cdot y(24))));
\]

\[
dy(39) = y(39) - \text{imag}((y(25) \cdot \exp(1j \cdot y(24))));
\]

\[
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\]

\[
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\]

\[
\text{end}
\]
Bibliography


