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Effects of Heavy Topping on Vibrational Performance of Cross-Laminated Timber Floor Systems

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EFFECTS OF HEAVY TOPPING ON VIBRATIONAL PERFORMANCE OF CROSS-LAMINATED TIMBER FLOOR SYSTEMS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the
Degree Master of Science
Civil Engineering

by
Benjamin K. Schwendy
August 2020

Accepted by:
Dr. Weichiang Pang, Committee Chair
Dr. Brandon Ross
Dr. Laura Redmond
ABSTRACT

Cross-Laminated Timber (CLT) is gaining momentum as a competitor to steel and concrete in the construction industry. However, with CLT being relatively new to North America, it is being held back from realizing its full potential by a lack of research in various areas, such as vibration serviceability. This has resulted in vague design guidelines, leading to either overly conservative designs, hurting profit margins, or leading to overly lenient designs, resulting in occupancy discomfort. Eliminating these design inefficiencies is paramount to expanding the use of CLT and creating a more sustainable construction industry.

This thesis focuses on the effect of a heavy topping, in this case 2” of concrete over a layer of rigid insulation, on a CLT floor. To this end, modal analysis was performed on two spans of three CLT panels in the Andy Quattlebaum Outdoor Education Center at Clemson University. By performing a series of instrumented heel-drop tests with a roving grid of accelerometers, the natural frequencies, mode shapes, frequency response functions, and damping coefficients were determined. By comparing the results to several different numerical models, the most appropriate model was selected for use in future design. In addition, a walking excitation test was performed to calculate the root mean square acceleration of the floor for comparison to current design standards.

This study found that, with a layer of rigid insulation separating the topping and the panel, the system behaved predictably like a non-composite system. The resultant mode shapes also verified that the boundary conditions behaved very close to “hinged”
and showed that the combination of the surface splines and the continuous topping provide significant transverse continuity in terms of response to vibrations. Lastly, the results of the walking excitation test showed that, with some further study, the current design standards for steel vibration serviceability can be applied to great effect to CLT systems.
DEDICATION

I would like to dedicate this thesis to my grandparents and my parents, without whose hard work and sacrifice I would not be in a position where I could comfortably pursue my M.S.
ACKNOWLEDGMENTS

Firstly, I would like to thank Dr. Weichiang Pang for giving me such a unique opportunity to work with mass timber, advising me, and assisting me in my research. Without his support and guidance, this would not have been possible. I would like to thank my committee members, Dr. Laura Redmond, and Dr. Brandon Ross for their teachings in the classroom, through this process, and in life.

I would like to thank Dr. Patricia Layton for her tireless advocacy of CLT and other mass timber products, spearheading the WU+D group that I so enjoyed being a part of, being instrumental in obtaining the opportunity to perform an in-situ test on a CLT building, and always being willing to help, even though I was not directly her student.

I would like to thank all the wonderful people in Clemson Outdoor Recreation and Education who assisted in making this project happen. Without them, and their willingness to let me drill holes in their ceiling, store things in their closets, meet with them to discuss plans, and just generally be a pain for them, this vital information could not have been gathered.

Lastly, I would like to thank my fellow grad students. I would like to thank Roberts Smith for assisting in the physical testing. Without his help, the deadlines for set up and data acquisition could not have been met. I would like to thank Michael Stoner and Lancelot Reres for always being willing to take time out of their own packed schedules to help me solve problems in mine. And I would like to thank Bibek Bhardwaj, for suffering through many long hours at the lab as we slowly transitioned from helpless undergrads to only mostly helpless graduate students.
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CHAPTER ONE
INTRODUCTION AND OBJECTIVES

Cross Laminated Timber (CLT), made its debut in the early 1990’s in Austria and Germany and has been on the rise ever since. CLT is an engineered mass timber product, in which layers of planks (often 2x6’s or 2x8’s) are pressed together in alternating orientations (usually 90 degrees offset) to create panels that can bear loading out-of-plane and in both in-plane directions. These panels can reach a maximum of 18 feet wide and 96 feet long (APA, 2018), and compete with steel, masonry, and concrete as a major building material for medium and large-scale projects. Being lighter weight and better for the environment than steel and concrete, as well as sharing the advantage of faster erection speed that steel enjoys, CLT has clear potential as a building material of the future. For example, from an environmental perspective, CLT has a net negative carbon balance and saves about 2 metric tons of CO₂ emissions per cubic meter when compared to a building material like concrete (Kuilen et al. 2011). To put this in perspective, the single high-rise (43 stories) building constructed with approximately 80% CLT as discussed in the 2011 Kuilen et al. paper saved the equivalent of 33,000 vehicle emissions for an entire year when compared to more traditional building systems. (Gagnon et al. 2013)

With such benefits, it is easy to see why CLT has become very popular in Europe. However, it is still relatively new to North America, and it is not yet always the most cost-effective option on the market today, which has somewhat hindered its progress (Gagnon et al. 2013). In addition to limited supply chains due to its current relatively
small market share, design inefficiencies due to imprecise design standards cause CLT construction costs to be artificially high. One of, if not the, biggest contributing factors to these problems is a lack of knowledge regarding the abilities of CLT (Schmidt and Griffin, 2013). As industry professionals are not always aware of the benefits, CLT is often not considered in situations for which it may be well suited. (Schmidt and Griffin, 2013). Furthermore, an absence of robust research conducted to precisely characterize the properties and responses of a material requires more conservative designs, and, in turn, drives up cost.

**Main Objective**

The objective of this study was to help eradicate some of the aforementioned inefficiency, particularly as it pertains to the vibrational serviceability of CLT, by adding to the knowledge regarding the characteristics of CLT. Currently, the CLT Handbook recommends ignoring continuous spans, assuming simple spans and calculating for the longest span in the member, and applying a 10% stiffness reduction, while ignoring the increased weight when a heavy topping is added (Hu and Chui, 2013). The intent of this adjustment is likely to be conservative by ignoring the mass, which would otherwise reduce the response of the system, and then reducing the stiffness in order to counteract the artificially inflated frequency that results from using the lower mass. These recommendations are given with the caveat that further research must be done to refine the techniques for both continuous span members and heavy toppings (Hu and Chui, 2013), clearly indicating the need for this study. In order to meet this need, this study
aims to lay the groundwork for a more precise method of calculating the natural
frequency of CLT systems that are more complex than simple-span, bare CLT.

**Sub-Objective 1: Find a Representative Specimen**

In order to accomplish this main objective, several smaller goals were achieved. First, a suitable test subject was required. To this end, the Andy Quattlebaum Outdoor Education and Wellness Center at Clemson University in South Carolina was selected. This two-story structure is constructed of CLT and glulam, with steel columns. In addition, the structure has a 2 inch concrete topping on top of 2 inches of rigid insulation over the CLT, providing a real-world case study of the difference between the actual properties of the CLT with heavy topping, the current CLT Handbook method of approximation, and any alternate approximations designed to account for the addition of topping. The Andy Quattlebaum Center was designed to accommodate classrooms and offices, as well as spaces for a variety of more dynamic assemblies and activities, such as aerobics classes. This gives the building a well-defined governing occupancy in terms of vibration serviceability, as “Offices” is one of the named categories in the ISO standards for vibration and its restrictions take precedent over the other uses (Gu, 2017).

**Sub-Objective 2: Data-Acquisition**

Once a suitable test subject was established, the collection of real-world experimental data was required. The chosen experiment consisted of a series of heel-drop tests performed in the Andy Quattlebaum Center, using a force-plate and a grid of roving accelerometers, and walking excitation tests using just the accelerometers. This setup
allowed for the discernment of the dominant frequency and actual vibrational performance of the panels, which was the main objective of this study. Additionally, the mode shapes, secondary frequencies, damping ratios, and accelerance frequency response functions of the panels were all ascertained from these results.

Sub-Objective 3: Aid Future Research

This supplementary information segues into the next sub-objective, which was to gain information to further future studies. If the knowledge and design procedures for CLT are to be as robust and efficient as those for steel and concrete, many more studies will be required; the more data and information there is available, the better off future studies will be. Without subjective results as to the acceptability of the vibrational performance of this building, not all of the information collected can be used effectively in this study, and the use of some other pieces of data collected is beyond the scope of this study. However, it is still important to collect and document all findings, so that they can be used by others who are trying to understand various aspects of CLT’s response to vibration.

Sub-Objective 4: Summarizing Equation

The next sub-objective was to use the collected information to derive an equation that best accounts for variables that are currently unaccounted for in the CLT Handbook, namely: continuous spans and heavy toppings. This more robust equation will allow designers greater precision in their calculations, and subsequently, more efficiency in their designs. Balancing accuracy and ease of use is vital to ensure that the resulting
method can feasibly be applied in a typical design setting while avoiding the need for special design tools, such as finite element method (FEM) (Hu and Chui, 2013).

**Sub-objective 5: Establish a Procedure**

A single study cannot provide enough confidence to both determine and fully verify a design method; therefore, the final sub-objective was to set a procedure and a precedent for future experiments of this ilk. By documenting the process and data, this study provides a manual of sorts for testing in-situ CLT floor systems to obtain their vibrational characteristics in order to further validate, or further modify, the methods presented in this paper. This documentation will hopefully help future researchers get the most out of experimental opportunities, as well as continue to solidify and refine the design procedures for CLT floor systems to help propel CLT to the forefront of the construction industry.

**Thesis Organization**

In order to accomplish the aforementioned objectives, the following format will be followed. First, to provide the context for this study, a review of existing literature will be presented. Second, to add accountability and credibility to this experiment, the methods and materials used in this study will be explained. Third, the experimental and theoretical results will be presented. Fourth, and finally, conclusions will be drawn from these results, and the impact of these results and conclusions will be discussed.
CHAPTER TWO
LITERATURE REVIEW

Vibration Fundamentals

Simple Systems

A basic understanding of vibration fundamentals is required to assess the expected vibrational performance of a structural material. Due to the complexity of real-world vibration, a number of assumptions and simplifications need to be made in order to efficiently estimate the vibrational performance of the structure. Therefore, it is important to know how various simplifications will affect the results, so one can select the appropriate methods to get an adequately accurate estimate without being too conservative and impacting the cost-effectiveness of design, or being too liberal, which can result in poor performance. The most basic simplification of this problem is the single mass and spring model, represented visually in Figure 1, covered in numerous papers and textbooks including Sundararajan (2009), which gives the exact natural frequency, $\omega$ (in radians/sec) of a defined point mass, $M$, attached to a spring of stiffness, $k$ as:

$$\omega = \sqrt{\frac{k}{M}} \quad (2.1)$$
A point mass, however, is a very poor approximation for a physical system consisting of a beam and associated loads. As such, Sundararajan (2009), along with many others, also presents a formula for natural frequency, $\omega$; used when assuming a uniformly distributed mass along a beam instead of a concentrated mass where:

$$\omega_n^2 = \beta_n^4 \frac{EI}{m/L}$$

(2.2)

where $n$ represents the mode shape, $m$ is the total mass, $L$ is the length of the beam, $E$ is the modulus of elasticity of the beam, and $I$ is the moment of inertia of the beam, while $\beta$ is a tabulated constant dependent on boundary conditions. $\beta$ values can be determined by solving transcendental equations, but that is beyond the scope of this paper. Taking $\beta$ to equal $\pi/l$ (first mode shape of a simply supported beam), this equation can be re-written as:

$$\omega = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{w}}$$

(2.3)

where $w$ represents the mass per unit length.
This reconfiguration, alongside the fact that the natural frequency \( f \) in terms of Hz is equal to \( \omega/2\pi \), presents the same model used by Hu (2007), Chen and Wambsganss (1974), Hu and Gagnon (2012), and the AISC DG11 (2003). It is also notable that this equation is properly dimensioned, yielding a result as a “per second”, or “hertz”, value. Because of this, as long as the units of the input values are consistent, this equation works regardless of the unit system used. A visual representation of this system can be seen in Figure 2.

![Figure 2: Representative Beam with Parameters for Simply Supported Distributed Mass Vibrational Model](image)

The CLT handbook (2013) cites Hu (2007) and Hu and Gagnon (2012) and uses a model that looks extremely similar to the one presented above. A version of this formula with a modified constant is as follows:

\[
f = \frac{2.259}{2L^2} \sqrt{\frac{EI_{app}}{\rho A}}
\]  

(2.4)
where $f$ is the fundamental natural frequency in Hz, $L$ is the length in feet, $EI_{\text{app}}$ is the apparent stiffness for a 1 ft panel in lb-in$^2$, $\rho$ is the specific gravity of the beam, and $A$ is the cross-sectional area in in$^2$. The reason for the modified constant compared to the equation in the CLT Handbook, Second Edition (2013) is shown in Appendix A. As is evident upon closer inspection, the units of this formula come out to be:

$$ft^2 \times \sqrt{lb}$$

meaning that this equation does not hold for different unit systems, as it has “hidden” units in the constant. Furthermore, because it lacks any tabulated values for the constant, it cannot be used for varying conditions or to obtain any natural frequencies beyond the first mode. Therefore, this paper will focus on the variations of the model presented in Equation 2. While this model is far more accurate than the simple mass and spring model, it still relies on several assumptions that simply cannot be attained in the real world. The more notable of these remaining assumptions are the boundary conditions, purely uniform mass, and single span.

Span Conditions

Attempting to come up with a simple equation that accurately models the real boundary conditions would be futile, because there are a theoretically infinite number of possible boundary conditions between “fixed” and “pinned”. However, because fixed and pinned represent the extreme options of boundary conditions, the real result will always be in between those two theoretical values. It is therefore appropriate to use those two conditions as bounds. With a significant number of data points, an empirical weighting system of the two theoretical bounds may be developed, which would allow for a more
accurate calculation of expected vibration behavior. Without such a system yet in place, this paper will evaluate both bounds to determine where within those parameters the real conditions lay. While the pinned end conditions are represented in Figure 2, the fixed condition is displayed in Figure 3.

![Figure 3: Representative Beam with Parameters for Fixed Ends Distributed Mass Vibrational Model](image)

Addressing the single span assumption is a little bit more involved. Again, solving for the natural frequencies of continuous spans can be done to varying degrees of accuracy with various methods, including the finite element method, the Rayleigh-Ritz procedure, or the dynamic three-moment equation (Chen and Wambsgans, 1974). Using any one of these methods is very involved and complex, and is beyond the scope of this paper, but it is still important to select a suitable model, so each method must be investigated. The finite element method can be used with a high degree of accuracy, but requires involved and time-consuming modeling, and as such is not suitable for simple design guides (Oz and Ozkaya, 2005). The Rayleigh Ritz method loses accuracy for
higher mode shapes and is really only reliable for the fundamental natural frequency, making the use of this method rather limited (Stokey, 2001). The three-moment equation, on the other hand, does not have these complications. The three-moment equation is based on the equation of motion and the boundary conditions of the structure, which allows for the inclusion of fixed or pinned ends, so, as discussed previously, the bounds are still valid. This method is centered around the fact that, in an infinite beam supported at regular intervals, the bending moment in one support \( M_j \) is related to the bending moment in the previous support \( M_{j-1} \) by the following equation:

\[
M_j = M_{j-1} e^{i\mu}
\]

where \( \mu \) is the “propagation constant” of the system (Chen and Wambsganss, 1974).

When a vibration is induced in a span of a continuous beam, there are “bands” of frequencies which can propagate from one span to the next without decay (propagation bands), and “bands” of frequencies which decay exponentially at the supports (stop bands). The propagation constant for a given setup is a function of \( \lambda \), for which there are tabulated values, and \( \Gamma \), which is obtained using the equation:

\[
\Gamma = \frac{TL^2}{EI}
\]

where \( T \) is equal to the axial tension in the beam (Chen and Wambsganss, 1974). For the purposes of this paper, axial tension will always be assumed to be equal to zero, yielding a consistent \( \Gamma \) value of 0. Using the concept of wave propagation and the theory of determinants, Chen and Wambsganns (1974) devised a simple graphical method to
determine the first five natural frequencies of a continuous span beam, which will be utilized in this paper. This continuous span case is represented visually in Figure 4.

![Representative Beam with Parameters for Continuous Span Distributed Mass Vibrational Model](image)

Figure 4: Representative Beam with Parameters for Continuous Span Distributed Mass Vibrational Model

**Combined Loads**

Multiple methods are also available for calculating or estimating the natural frequency of a beam with a distributed mass and a concentrated mass. This is significant because, while the self-weight of the beam and the concrete topping are well-modeled with a uniform load, the weight of the tester represents a concentrated load that may not be insignificant. Being able to adjust for the expected effect of the mass of the tester will eliminate, or at least lessen, the impact of this difference on the perceived accuracy of the theoretical estimates of vibrational behaviors as compared to the experimental results. One such method, provided by Stokey (2001), is defined by the following equation:
\[ \omega = \sqrt{\frac{k}{M + m/2}} \quad \text{where} \quad k = \frac{48EI}{L^3} \quad (2.7) \]

where \( M \) is the concentrated mass, and \( m \) is the uniform mass. This method adjusts the formula for a concentrated load in order to account for the additional uniform load. As a result, this method is more accurate when the concentrated load is bigger than the uniform load. Because the weight of the tester in this experiment is significantly smaller than the self-weight of the structure, this model is not ideal. Furthermore, this model only calculates the fundamental natural frequency (Stokey 2001).

In his 2009 paper, Sundararajan used an alternate method, for which he cites the Wachel and Bates 1976 paper titled, "Techniques for controlling piping vibration failures". This method adjusts the natural frequency based on just the uniform mass to account for the concentrated mass, and is defined as follows:

\[ \omega^2 = \frac{\omega_u^2}{1 + cR} \quad \text{where} \quad R = M/m \quad (2.8) \]

with \( \omega_u \) representing the natural frequency of the system ignoring the concentrated mass, and \( c \) being a correction factor. The values for the correction factor for different end conditions and mass locations are listed in Table 1.

Table 1: Wachel and Bates Correction Factor (Taken from Sundararajan, 2009)

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Location of Concentrated Mass</th>
<th>Correction Factor (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both ends simply supported</td>
<td>Mid-span</td>
<td>2.0</td>
</tr>
<tr>
<td>Both ends clamped</td>
<td>Mid-span</td>
<td>2.7</td>
</tr>
<tr>
<td>One end clamped, other end free (cantilever)</td>
<td>Free end</td>
<td>3.9</td>
</tr>
<tr>
<td>One end clamped, other end simply supported</td>
<td>Mid-span</td>
<td>2.3</td>
</tr>
</tbody>
</table>
For the purposes of this paper, this method, while not perfect, has several advantages. As can be seen in Table 1, this combined load method allows for different boundary conditions, permitting the correction to be applied appropriately to the upper and lower bounds previously discussed. Furthermore, because this method relies heavily on the natural frequency of the uniform mass alone, it can be applied to natural frequencies beyond the fundamental frequency, so long as the higher modes can be solved for just the uniform mass, which Chen and Wambsganss (1974) have already shown is possible. This model is not suitable for all situations, as it is an approximation, but it is useful for situations where the concentrated mass is small relative to the uniform mass, with the accuracy increasing as $R$ decreases. This model is adequate when $R$ is less than or equal to 1, so, for this experiment, when $R$ is a couple orders of magnitude smaller than one, it should be plenty accurate. (Sundararajan, 2009) This case is visually represented in Figure 5.
Vibration Control Criteria

Now that the basics of vibration have been explained, the way that it affects how we design buildings must be examined. It is no great revelation that excessive vibration in a building will be uncomfortable for its occupants, but how we determine what is and is not excessive is much more complicated. Luckily, while vibrational responses vary from material to material, people’s responses to those vibrations do not depend on the material—all that matters are the vibrations themselves. Therefore, serviceability criteria that are already in place can be used to determine the maximum allowable vibrations in CLT. This section will look at these criteria, as well as investigate how these criteria may impact CLT design based on what we know about other materials.
**Modified Reiher-Meister Scale**

Vibration serviceability began pushing its way to the forefront of structural design very early in the 20th century, with one of the first major leaps in this field occurring in 1946, when the Reiher-Meister scale was introduced (Lenzen, 1966). This was an important step in the continual improvement of building design, but the scale was still far from perfect. The Reiher-Meister scale assigns frequency-displacement relationships to qualitative categories, such as “annoying”, which, as a highly subjective term, makes it difficult to design for (Kowalska-Koczwara and Stypula, 2016). What is “annoying” for one may not be at all annoying to another. Even more importantly, this scale was designed for steady-state vibrations. If this scale was used in all instances, buildings would be built far too conservatively, as humans are much more tolerant of transient vibrations than they are to steady-state vibrations (Lenzen, 1966). To this end, in 1966, another major development was made with Kenneth Lenzen’s paper *Vibration of Steel Joist and Concrete Slab*, in which he presents a modified version of the Reiher-Meister scale. This scale, which can be seen in Figure 6, alters the parameters to be based on human tolerance of transient vibrations, such as those induced by footsteps, which are more likely to be experienced by a structure. To do this, the peak displacements associated with each curve are increased by a factor of ten (Lenzen, 1966).
Obsoletion of Reiher-Meister Scale

As construction continued to evolve, new practices posed an interesting new problem in terms of vibration serviceability. Namely, as span lengths increased, lighter weight concretes gained popularity, and fewer partitions were needed, the traditional deflection-based methods for limiting vibration issues were failing to satisfy occupants more and more frequently (Allen et al, 1977). The main issue was this: the lowered natural frequency of the members was too close to the forcing frequencies associated with common human activities, particularly those activities often seen in large assembly spaces (Allen et al. 1985). These new long-span concrete systems, as well as long-span steel joist systems with thin concrete toppings, had significant trouble with annoying
vibrations simply from occupants walking through the building (Allen et al, 1977). CLT, therefore, faces an even greater challenge, as bare CLT systems generally have less stiffness than concrete or steel systems (lowering the natural frequency), and have a relatively low damping ratio of 1%, (Hu and Chui, 2013) which, as noted by Allen et al (1977), increases overlap between the vibrations from one footfall and those from subsequent footfalls, further exacerbating the issue, leaving them very susceptible to vibration problems.

**Hu and Chui Criterion**

As a mass timber system, CLT also poses more vibrational problems than light frame timber systems. Light frame systems seldom have a natural frequency below 9Hz, which is the threshold for potential resonance with repeated footfall impacts associated with typical human walking (Hu and Chui, 2001). CLT, however, often has a natural frequency below this threshold (Gu, 2017). Light frame construction also benefits from a higher damping ratio of around 3%, making the vibrations induced by walking somewhat less likely to be problematic (Hu and Chui, 2013). Another concern with CLT is that it is often paired with a heavy topping, a condition which is thus far under-researched, and is the focus of this paper. Heavy toppings have been shown to render some vibration control methods, such as static point load deflection, inadequate for light frame systems, and may well have a similar adverse effect on CLT (Hu and Chui, 2001). Attempts to adequately control timber floor vibrations have included requiring the fundamental natural frequency of the system to be greater than a certain threshold, such as 8Hz in a method proposed by Smith and Chui, but it has been noted that this could severely limit the use of long span,
heavy-topped systems, such as the CLT in question (Hu and Chui, 2001). To remedy this, they propose using a design criterion that includes both natural frequency \( f \) and static deflection \( d \), in the hopes of both allowing long, heavy-topped spans where appropriate and limiting annoying vibrations in a wide variety of floor systems (Hu and Chui, 2001). This criterion is defined by the equation below:

\[
\frac{f}{d^{0.39}} \geq 15.3
\]  

(2.9)

where \( f \) is in Hz, and \( d \) is in mm. An adapted version of this formula was deemed adequate for CLT floor design, in addition to its light frame roots, and can be found in the CLT Handbook (Hu and Chui, 2013) as follows:

\[
\frac{f}{d^{0.7}} \geq 125.1
\]  

(2.10)

where \( f \) remains in Hz, but \( d \) is in inches.

**AISC Design Guide 11**

Another of the more widely used and cited examples of general vibration limits can be found in AISC Design Guide 11, by Murray, Allen, and Ungar. (2003). This Design Guide cites International Standard ISO 2631-2 (1989) recommended acceleration limits, along with modifiers adapting the limits to be used for different occupancies. These limits are functions of the frequency of the vibrations in question, as humans are more perceptive of vibrations in a certain range (about 4 to 9.5 Hz (Gu, 2017)), so weaker vibrations in this range can cause more discomfort than stronger vibrations outside of this range (Murray, Allen, & Ungar, 2003). Figure 7 shows the chart detailing these limits, in terms of peak accelerations. As this study is focused on CLT in a building used for
offices and classroom space, the “Offices, Residences” line is the one of most interest to us. As can be seen, this indicates a maximum allowable peak acceleration of about 0.5% of gravity for about the 4-9 Hz range.

Figure 7: Maximum Allowable Vibrations (Gu, 2017; Murray, Allen, and Ungar, 2003b; ISO 1989)

Clearly, in order to use this figure effectively, a uniform way of determining the peak acceleration is needed. An equation detailing the method for finding this peak is presented by Gu (2017), and can be seen in Equation 1, below:

\[
\frac{P_0 e^{-0.35f_n}}{W\beta} \leq \frac{a_0}{g}
\]

where \(P_0\) represents a constant force, dependent on the type of occupancy (65lbs for Offices and Residences), \(f_n\) is the natural frequency of the system, \(W\) is the unfactored weight of the system (including live loads), \(\beta\) is the effective damping ratio of the system,
and $a_0/g$ is the acceptable peak acceleration. In this way, the left side of this inequality represents the estimated peak acceleration of the system in question and is compared to the limit set by its occupancy as per Figure 7. The limits used in Design Guide 11 (Murray, Allen, & Ungar, 2003) conservatively assume the lowest allowable peak for each respective occupancy type regardless of the resonant frequency of the system in question. This would become extreme in the much higher frequency ranges, but the expected resonant frequency of a non-light frame floor system is expected to be reasonably close to the 4-9.5 Hz range, making this a perfectly valid assumption.

Being that this study focuses primarily on the effects of variables, such as a heavy topping, on the natural frequency, and not directly on the serviceability of the floor system, it is not vital to choose a serviceability criterion, nor do we have the subjective test data to confirm the results of the theoretical check. However, for academic purposes, we will determine the acceptability of the floor in question using both the Hu and Chui (2013) CLT Handbook method, and the Murray, Allen, and Ungar (2003) Design Guide method, using both the theoretical natural frequency and the measured natural frequency. This will allow us to compare how close to reality the theoretical values are when actually applied to a serviceability criterion, as well as see to what extent that comparison is affected by the choice between these two widely accepted models.

**Modal Testing**

There are multiple ways of determining the modal properties (which include the natural frequencies, damping, and mode shapes), of a structure. These methods are collectively known as “modal testing” or “modal analysis” (Barrett, 2006). In order to
select the best method for the particular situation studied in this paper, the various types of modal analysis must be explored and considered. Modal testing can be generally broken down into two main categories: unreferenced modal testing, also known as output-only modal analysis (Batel, 2002), and referenced modal testing (Gu, 2017). Each of these categories has multiple subsections, the most notable of which will be explored in this section.

*Unreferenced Modal Analysis*

Unreferenced modal analysis is modal analysis where the inputs, or applied forces on a structure, are unknown, and only the outputs, or structural responses, are measured (Hermans and Van der Auweraer, 1999). One of the main advantages of this type of analysis is that the testing can be performed without closing the structure, which is why another name for unreferenced modal analysis is “operational modal analysis”. Another advantage is that less equipment is required for unreferenced modal analysis than for referenced modal testing, as only the outputs are measured, as opposed to the outputs and the inputs (Živanović et al. 2005). This type of analysis, however, is not without disadvantages. The major disadvantages of unreferenced modal analysis are that it takes more sophisticated data processing to obtain the relevant information (Batel, 2002); it cannot be used to obtain frequency response functions (FRFs), which correlate the structural response to the excitation forces and will be discussed in more detail later (Barrett, 2006); and it is less accurate than referenced modal analysis (Gu, 2017).
One method of unreferenced modal analysis is the use of a Laser Doppler Vibrometer, or LDV (Stanbridge and Ewins, 1999). An LDV, depicted in Figure 8, uses a focused laser beam to ascertain the velocity of a point by measuring the Doppler shift that occurs when the laser is reflected off the surface. This allows for rapid and precise measurements at the point in question, with extremely little interference with the structure, as only the laser beam actually needs to be in contact with the structure in a lab environment, or, if in a real-world environment, all that is required is something to hold the LDV. This is much less invasive than the installation of accelerometers and an instrumented excitation source. A relatively dense map of data can also be achieved through this method using a Scanning Laser Doppler Vibrometer, or SLDV, which, as the name suggests, implements a laser which scans over the surface collecting data at each point as it goes. However, this level of detail results in high costs both in terms of equipment and the storage and processing of data. Furthermore, the level of detail that the LDV system boasts as its main advantage is not necessary for this study and, due to the fact that its main use is in unreferenced analysis, Frequency Response Functions cannot be generated, which, while not vital to this particular study, may be useful to future research. As such, LDV is not an ideal method for the purposes of this paper. (Stanbridge and Ewins, 1999)
Another option for unreferenced modal analysis is a hand-held spectrum analyzer. This portable device, depicted in Figure 9, allows the researcher to obtain a time history of the acceleration, and the corresponding frequency domain of the signal with limited financial expenditure and almost no interruption to the typical use of the structure. This method requires significantly less data storage and processing than the LDV system, and is cheaper and easier to use, but yields a lower level of detail and less information. While this trade-off makes the hand-held spectrum analyzer more suited to this study, its inability to obtain the information needed to generate frequency response functions make it less useful than a referenced test, which will be shown in the following sections to be both practical and desirable for this study. (Gu, 2017)
Referenced Modal Analysis

Referenced modal analysis involves measuring and recording both the inputs (excitation forces) and the outputs (response of the structure) of a dynamic system (Barrett, 2006). This, as could be expected, requires more equipment, more time, and more invasive procedures than unreferenced modal analysis. Referenced modal analysis most frequently uses accelerometers to measure the dynamic response of the structure, with the differences between methods lying in the method of inducing a measured excitation (Stanbridge and Ewins, 1999).

The fact that referenced modal analysis cannot be carried out while a structure is in use is widely regarded as its main disadvantage (Batel, 2002). The reason for this constraint is that the excitation forces need to be limited, to the extent possible, to only those being measured. This inability to test during normal operation often makes referenced modal analysis unpractical in in-situ systems. Fortunately, the Clemson Outdoor Recreation and Education department was kind enough to work with us to
schedule a time to perform the test when we could ensure no occupants would be using
the building, allowing referenced modal analysis methods to be considered.

**Electrodynamic Shaker**

One of the two main methods of excitation for referenced modal analysis is the
use of an electrodynamic shaker (Gu, 2017), depicted in Figure 10, which is a device
made up of a static core with moveable masses. By raising and lowering the masses at
specific speeds and frequencies, the shaker can induce the desired dynamic excitations.
Small shakers can be used to simulate human footfalls (Barrett, 2006), while much larger
shakers can be used to excite entire structures, simulating more extreme vibrational
events (Gu, 2017). If the shaker does not have a built-in method of measuring and
reporting applied force, the shaker can be mounted on a force plate, which will provide
the necessary data. Within the realm of referenced modal analysis, electrodynamic
shakers offer precision by electronically controlling the applied force, and versatility due
to the scalable size of the device. However, the requirement of a shaker, an expensive
piece of specialty equipment, often renders this method impractical, as it did for this
study. (Barrett, 2006)
Impact Testing

There are several different ways to conduct impact testing, but the basics of the method are as follows: an impact is measured while being used to induce a transient excitation in the specimen, and strategically placed accelerometers are used to measure the response of the specimen. Frequently used methods of applying the impact are striking the specimen with an instrumented hammer (a hammer with a force plate attached to the end in order to measure the impact) or utilizing an instrumented heel drop test, which is depicted in Figure 11. An instrumented heel drop test consists of the researcher standing on a force plate (so as to measure the impact) on the specimen, rising up onto the balls of their feet, and suddenly dropping back down onto their heels. This method can be used to simulate human footfalls, and, as the method selected for this study, will be discussed in more detail in the “Procedures and Equipment” section of this paper. Whichever method of impact is used, impact testing offers the benefits of low
equipment costs as well as the quantity and quality of information that comes with referenced modal testing. The heel-drop test in particular has been found to provide excellent FRF resolution in the range of 2-15Hz. One of the main drawbacks of this method is that it can be difficult or impossible to apply enough of an impact through these means to excite an entire structure. Furthermore, in order to properly control the excitation of the specimen, the structure must be taken out of operation while the testing is underway. For the purposes of this study, only the CLT floor system is of interest, and, as stated previously, temporarily halting the operation of the structure to allow for testing was feasible. As such, impact testing was selected as the optimal method for this study. (Gu, 2017)

Figure 11: Instrumented Heel Drop Test (Barrett, 2006)

**Past Studies**

While there is still much to learn in regard to the vibrational performance of CLT, there have been several studies that paved the way for this experiment. Some of the most notable are summarized, chronologically in this section.
In-situ testing of Timber Floors

The first study to be reviewed is “In situ testing of timber floor vibration properties” by Jarnerö et al. (2010), which looked at the floor system of an 8-story building with CLT floors and Glulam supports and was most interested in how the various stages of construction affected the fundamental frequency and damping ratio of the floor elements. In order to accomplish this, Jarnerö et al. used an electrodynamic shaker and accelerometers to excite and measure the response of a CLT floor panel at various points in time during construction. Additionally, they tested a replica of the panel in a lab environment so they could also measure the response of the panel in isolation to compare to the in-situ performance. They also attempted to use their data to validate a finite element model, but they concluded some of the material properties, most likely stiffness, were not accurately represented in the model, leading to some discrepancies between the real and modeled values.

For the purposes of this study, the most notable conclusion was that the fundamental frequency of the panel increased about 23% from simply supported in a lab setting to in-situ with the floor above it (and no further stories) built. This particular set of conditions is important because the simply supported laboratory condition most nearly mimics the theoretical calculations for natural frequency, while the condition with only the floor above completed most nearly mimics the conditions of the test in this thesis, where the panel in question is on the only elevated story with walls and a roof above it. An interesting finding of their study, which bears less relevance to this thesis due to the building in question only having one elevated story, but is important to note in terms of
CLT design overall, is that the addition of more stories above the story in question does not have a significant effect on the fundamental frequency of the floor. After the addition of walls and a covering (which in the case of Jarnerö et al. were the floor elements of the story above), the fundamental frequency of the floor remains relatively stable. The biggest contributor to the increased fundamental frequency between lab conditions and in-situ was found to be the coupling of the panel to those adjacent. This is not surprising, as it drastically changes the boundary conditions on the sides of the panel and increases its overall stiffness. The second biggest contributor was found to be the addition of the walls. This, again, is unsurprising, as the walls provide a degree of clamping to the ends of the panel, pushing the boundary conditions further from simply supported and closer to fixed. These two factors are both almost ubiquitous in CLT construction, meaning that the observed effects can generally be expected to hold true for most structures, such as the one studied in this thesis. (Jarnerö et al. 2010)

*Controlling CLT Vibration*

The second study to be looked at in this section is “Controlling Cross-Laminated Timber (CLT) Floor Vibrations: Fundamentals and Method” by Lin Hu and Sylvain Gagnon (2012). In their study, Hu and Gagnon focused on the serviceability of a CLT floor in vibration, and tested several methods of determining vibration controlled spans for CLT in order to establish a method which closely mirrored the results of subjective rating of the actual performance of the floor. To accomplish this, they created multiple test floors with several variables, including span length, connections, and boundary conditions, and outfitted them with carpets and furniture in order to mimic a real
environment. This step was important because occupants are not only annoyed by vibrations in a floor but can also be negatively impacted by rattling dishes (represented by placing glassware in a cabinet on the floor being tested) or rippling of water (represented by placing vases of flowers in water on the cabinet). They then had test subjects walk around on the floor, one by one, and then subjectively rate the performance of the floor. These results were used to help set the limit state using theoretical calculations by comparing them to the calculated responses in the following steps.

In addition to the subjective testing, static deflection under a 1kN concentrated load was measured, and a modal analysis was performed. The concentrated load test, while having little direct bearing on the study in this thesis, is an important step, because traditionally, static load deflection is how vibration serviceability was estimated at the time. In order to show that a frequency-based design criterion was more accurate, it needed to be compared to the static load deflection. The modal analysis was used to verify the adequacy of the frequency calculations, and thus propose a design criterion for vibration-controlled spans of CLT with a foundation in the fundamental frequency of the system. This criterion is now used in the CLT Handbook (Second Edition, 2013), and is the current design standard in North America (Hu and Chui, 2013). It is this method which this thesis aims to expand upon by providing a more comprehensive method of estimating the fundamental frequency of an in-situ CLT floor with a heavy topping. (Hu and Gagnon, 2012)
Effect of Concrete on CLT High-Rise Fundamental Frequency

The third study to be looked at in this section is “Ambient Vibration Tests Of A Cross-Laminated Timber Building” by Thomas Reynolds et al. (2015), which looked at a 7-story building constructed using CLT for both the floor system and the wall system. Reynolds et al. were not specifically studying the floor system, but instead were looking at the effect that adding the weight of concrete and other non-structural components had on the fundamental frequency of the entire building when exposed to ambient excitation. With wind forces providing the main source of excitation and shear walls providing the main resistance, the stiffness of the concrete was not nearly as impactful as when the vibrational characteristics of the floor system itself were considered, and was therefore not a focus of their study. In order to see the real effects of the weight of the concrete, as well as observe the effects of real boundary conditions as compared to laboratory settings, Reynolds et al. outfitted the building with accelerometers to observe, among other things, its fundamental frequency when excited by wind forces. They observed this building for two days, with a key difference between the days: on day one, all of the CLT structure was in place, but only Level 1 had a concrete topping, and internal plasterboard was only installed up to Level 3, and on day two, the building was complete with the concrete topping on all floors, plasterboard throughout, and the external cladding installed.

Their study showed that the additional weight of these components reduced the fundamental frequency of the structure by 20%. This is the biggest take-away from their study for the purposes of this thesis, as it illustrates the fundamental problem faced by
CLT; namely, due to its exceptionally light weight compared to its strength, the nonstructural components of a CLT building will make up a much larger percentage of the total weight than for a building made of steel or concrete, meaning that these same components will have a larger effect on the vibrational behavior of the structure. Studies such as this show quite clearly the need for more research on the effects of variants such as a concrete topping on CLT structures, as the unique material properties of CLT lead to unique responses when compared to otherwise traditional aspects of construction. (Reynolds et al. 2015)

*Vibration Properties of Composite CLT-Glulam Beam*

The fourth study of interest is Chapter 7 of Mengzhe Gu’s 2017 dissertation “Strength and Serviceability Performances of Southern Yellow Pine Cross-Laminated Timber (CLT) and CLT-Glulam Composite Beam”. This chapter focuses on the vibrational properties of the beam in question. For this experiment, Gu tested a composite CLT-Glulam beam twice: once with simply supported ends, and once with clamped ends. Gu analyzed the vibrational performance of the beam by conducting modal analysis through an instrumented heel-drop test, using a force plate and a grid of accelerometer locations.

Gu’s test provided the template for the test carried out for this thesis, though the focus of this thesis is not as broad as Gu’s. Gu used the results of this modal analysis to compute fundamental frequencies, frequency response functions, and the damping of the system, and used these results to verify a finite element model, and then assess the
applicability of the AISC Design Guide 11’s design criteria to CLT. The purpose of this thesis is, instead, to improve the accuracy of calculating the fundamental frequency of an in-situ CLT panel so as to better apply the design criteria laid out in the CLT Handbook (Second Edition). Gu’s study concludes that, while Design Guide 11 may be a good place to start, it is not adequate to cover the serviceability design of CLT, and that more work is needed to be able to properly design and account for the unique material properties of CLT, to which end this thesis aims to contribute. (Gu, 2017)
CHAPTER THREE
LONG-TERM VIBRATION MONITORING

Equipment

In order to install a system for long-term monitoring of the vibration of the building, 18 Lord Microstrain G-Link-200 accelerometers were installed at various locations on the underside of the elevated CLT floor, as well as 3 on the underside of the CLT roof. These sensors, depicted in Figure 12, are approximately 1¾” tall and have a diameter of approximately 1¾”. They are powered by ½AA batteries, are housed in a weatherproof plastic shell and can operate in temperatures from -40 to +85°C, making them well suited to both indoor and outdoor use. They have a metal bottom, with a ¼” threaded hole which can be used to mount the sensor to a threaded rod. These sensors are capable of measuring acceleration on three axes.

Figure 12: Microstrain G-Link-200-8G wireless accelerometer

When purchasing ½AA batteries, it is important to note that the battery terminals are not standardized across different manufacturing plants within a company, so it is
important to acquire the right style of ½AA battery. These particular sensors use ½AA batteries from SAFT (Model Number LS 14250), manufactured at their French facility. Batteries with the same model number but manufactured in Singapore are not compatible with these sensors, so extra care must be taken when looking for replacement batteries.

G-Link-200 sensors are completely wireless, have an operational range of ±2g, and can sample at rates in powers of 2 up to 4096Hz and transmit the data back to a gateway (G-Link-200 Data Sheet, 2020). In this system, 3 Lord Microstrain WSDA-200-USB gateways were used. These gateways plug into a standard USB 2.0 port to allow the computer to communicate with the sensors. For this study, the gateways were plugged into laptops which could be left on site to continuously collect data. In order to periodically upload the data to an accessible, cloud-based platform, an AutoHotKey script was written which would, at a set interval, export the data to an Excel file, and save it to a Google Drive folder, which could be accessed by all involved researchers.

Procedure

This section details the process by which the sensors were installed. Due to timeline constraints, the actual long-term monitoring was not feasible for this thesis. However, these sensors are used as an auxiliary measure during the heel drop test, so the procedure for gathering the data is described in the procedure for the heel drop part of the experiment. The same principals apply for long-term testing as for the heel drop testing.

Several factors were considered when deciding where the accelerometers would be situated. The most important factor, which does not need to be considered in lab settings, was the building architect. Due to the fact that semi-permanent fixtures (the
accelerometers) were being attached to visible parts of the CLT, the aesthetic of the structure could have been negatively impacted; therefore, it was vital to work closely with the architecture team to ensure that the accelerometers were placed in inobtrusive locations where they wouldn’t be an eyesore or detract from the overall look of the structure. Within these constraints, the sensors were positioned in close proximity to the center of the panels, where accelerations are expected to be the greatest, while avoiding the centerline itself so as not to be on a modal line. In addition, some accelerometers were placed near panel edges, so that panel to panel interactions and boundary behaviors could be observed. A map of the approximate sensor locations can be seen in Figure 13.

Figure 13: Long-term Accelerometer Layout

Due to the wonderful support of the building managers and aid of the construction team at the Quattlebaum Building, the installation process was relatively straightforward. A scissor lift was used to reach the locations where the sensors were to be installed;
however, a ladder could also be used to reach all locations but those on the underside of the roof. This is important for sensor maintenance, as the batteries can be replaced on the first floor without the help of a scissor lift, making the process far simpler. To install the sensors, a ¼” hole was drilled in the underside of the CLT panel where the sensor was to be placed. Care was taken to ensure the depth did not exceed 2”, so as to not risk drilling all the way through the panel. This installation method may not be appropriate in all situations, as many owners/building managers will not be so amenable to the idea of drilling holes in the ceiling. Once the hole was drilled, a ¼” threaded rod, cut to 2” in length, was screwed into the base plate of the sensor, super glue was spread on the rod, and the rod was inserted into the pre-drilled hole. A rubber mallet was used to gently tap the sensor and rod fully into the pre-drilled hole when it was too tight to press in by hand. When the mallet didn’t provide enough force, the rods could typically be screwed into the panel until the bottom of the sensor was flush against it.

To set up the gateway, it was simply plugged into the computer and SensorConnect, Microstrain’s proprietary software, was used to configure the network. Using this software, each sensor was assigned one of three frequencies (each corresponding to one of the gateways) and set to sample continuously at 128Hz. The laptops were then stored in supply closets within range of their assigned sensors. Despite having the setup in place, due to restrictions associated with the COVID-19 pandemic, the building was closed from when the sensors were made operational until the defense of this thesis.
CHAPTER FOUR  
INSTRUMENTED HEEL-DROP TEST AND WALKING EXCITATION

Equipment

An instrumented heel-drop test requires three categories of equipment: instruments needed to measure the force of the heel-drop itself, instruments needed to measure the dynamic response of the structure, and a system to convey the data from the instruments to the computer. A walking excitation test needs only the components for measuring the response and associated data conveyance, and does not need any additional equipment as compared to the instrumented heel-drop test, making it reasonably simple to perform when a heel-drop is already planned (Brenemen, 2020)

Excitation Measurement

A force plate was selected to measure the force of the heel-drop impact. Force plates are the option typically used in instrumented heel-drop tests, and this method was made even more appealing for this particular study by the fact that a force plate of adequate size was readily available at the lab. This force plate was built by Mengzhe Gu for his 2017 dissertation, and is constructed of three load cells, produced by OMEGA Engineering (Model LC401-1K), with an operational range of ±1000lbf each. The load cells utilize strain gauges with nominal resistance of 350Ω and are wired in a full Wheatstone bridge configuration. The three load cells are arranged in a triangle and sandwiched between two steel plates. The steel plates are 1’ x 1’ and ½” thick and are connected to the load cells with threaded rods. The force plate is depicted in Figure 14.
The force plate was calibrated by incrementally placing steel plates of a known mass on top of it and recording measurements at each increment. A linear equation was fit to these data points and shown to have an $R^2$ value of 1, indicating that the force plate output was scaling linearly, as expected. This calibration showed that, when excited with 2.5V, a change in applied weight of 1lb corresponds to a voltage change of 3.021μV, meaning that a voltage change of 1μV corresponds to a change in force of about 0.331bs. The graph of this calibration can be seen in Figure 15. Once the calibration was complete, the weight of the force plate setup itself was approximated by taking the offset voltage when right-side-up and adding it to the offset voltage when upside-down, then multiplying this voltage by the 0.331 factor. Using this method, the weight of the force plate was estimated to be about 25 pounds, which was about what would be expected based on its size and materials. An approximate value was deemed adequate as this weight is only used in the combined load adjustment and is a relatively small proportion of the point load being considered.
In order to measure the response of the floor system, 12 accelerometers were used. The accelerometers used in this test were PCB Piezotronics “PCB 333B50” accelerometers, depicted in Figure 16. These accelerometers can capture frequencies within a range of 0.5Hz to 3000Hz, which easily encompasses the frequencies of interest (approximately 5Hz to 30Hz) of this study; therefore, these accelerometers were well suited to the task. The operational range of the accelerometers is ±5g, which, again, makes them well suited for this particular study (Gu, 2017). However, due to the very low weight of the accelerometers, they must be fastened down to prevent them from moving around during testing. For this purpose, the manufacturer recommends super glue or beeswax because these adhesives are stiff enough to transfer high frequency vibrations without loss of accuracy (Gu, 2017). However, for the purposes of this test, super glue and beeswax were not appropriate, as super glue increases the time required to move a
sensor and beeswax does not provide adequate adhesion. It was determined that hot glue is sufficiently stiff to transmit the relatively low frequencies of interest without these disadvantages (Gu, 2017). For this particular study, the owners of the building in which the test took place did not want hot glue applied directly to the floor, for fear of staining the finished concrete topping. To circumvent this, a piece of masking tape was adhered to the floor where the sensor was to be attached, and the sensor was glued to the tape. To ensure that this would not impact the accuracy of the study, a comparison of accelerations measured with and without tape was performed using a steel plate. This test showed that the tape did not have a noticeable impact on the performance of the accelerometers (test results are provided in Appendix D).

Figure 16: PCB Piezotronics “PCB 333B50” Accelerometer

An important advantage of the PCB 333B50 accelerometer is that it is an integrated electronics piezoelectric (IEPE) sensor, meaning that it has an integrated amplifier (National Instruments, 2019). This makes it simpler to use due to some compatibility benefits, as will be explained later.
Data Transfer

Several components were required to transfer the data from the instruments to the computer. A National Instruments compact USB chassis (NI cDAQ 9172, depicted in Figure 17) was utilized, with an NI 9237 module in slot one, and NI 9234 modules in slots two through four. The NI cDAQ 9172 has a built-in master time, which allows it to synchronize all of the readings it takes across the various modules. This is extremely helpful in post-processing, as the data from each sensor is already on the same timeline and does not need to be offset for the data points to align. This master time has a tick rate of 13.1072MHz (fm), and the chassis samples data at a rate of fs, where:

\[
fs = \frac{fm}{256 \times n}
\]  

(4.1)

assuming \(n\) is an integer from 1 to 31, meaning that the minimum sampling rate is 1651.6Hz (Gu, 2017). This minimum was used throughout the experiment because the frequencies of interest for this test are significantly below half of even the minimum sampling frequency of the chassis.

Figure 17: NI cDAQ 9172 With No Modules
The 9237 module, depicted in Figure 18, uses an RJ-50 connection and has four analog-input channels. This module was connected to the load cells in the force plate using BNC to RJ-50 cables. The 9234 modules, depicted in Figure 19, are compatible with IEPE sensors (resulting in the compatibility with the PCB 333B50 sensors previously mentioned) and have BNC connections which accept sound and vibration inputs. These modules were connected to the accelerometers using 10-32 to BNC TFE jacketed cables, in order to minimize the external noise gained between the sensors and the cDAQ, so as to make use of the exceptional sensitivity of the sensors themselves. The cDAQ with all modules installed is depicted in Figure 20.
NIMAX v17.5 was used to designate which channels were in use and which inputs each channel should expect, as well as to adjust various settings for each channel on the cDAQ. It is important to note that later versions of NIMAX are not compatible with the model of chassis used in this test. In future tests, the specific model should be checked to ensure that a compatible version of NIMAX is installed. This configuration step is where the type of bridge, signal input range, and nominal resistance of the loadcells is entered, as well as the signal input range and the sensitivity of the accelerometers. These values are listed specifically for each sensor on that particular sensor’s datasheets. Using the generic values given in the product information for each type of sensor (such as (±10%) 1000mV/g for the PCB 333B50s) will impact the accuracy of the readings, as the exact values for each individual sensor vary.
Procedure

This section details the procedure followed for the heel drop test. This section covers only the physical experiment, and not the post-processing of the data. Data processing details are contained in Chapter V of this paper.

Before the actual test could take place, the grid for measurement points was mapped out. For this particular test, the area of interest was three 7’9” wide by 49’6” long panels, laid next to each other along the long sides. Similar to the study presented by Gu (2017), the main mode shapes of interest are in the longitudinal direction because, as can be seen in the calculations in Appendix E, the first mode is the only one of concern in terms of serviceability. This is due to its proximity to the range most disturbing to humans, and, with aspect ratios of about 6, the panels will be significantly stiffer in the transverse direction. For the purpose of capturing the mode shapes in question, accelerometer locations were planned out to be one column every two feet in the longitudinal direction, and each column consisting of twelve accelerometer locations, one 6” from each edge of a panel, and two in the field of each panel, spaced 27” from each other and from the edge accelerometers. Given that twelve accelerometers were used, each column could be measured in one set of heel-drop tests, before moving the accelerometers to the next location. This stipulation made it much easier to keep track of the accelerometer locations and ensure no locations are skipped or tested twice unnecessarily. A schematic of the accelerometer grid can be seen in Figure 21, and the node numbers are displayed in Figure 22, with the filled in node on each indicating the location of the force plate. As these figures illustrate, accelerometer locations were
numbered top to bottom, and left to right. This means that the number of any accelerometer location can be found using the following equation:

\[ \# = Row + 12 \times (Column - 1) \]  \hspace{1cm} (4.2)

assuming that Row and Column are both one-based arrays.

Figure 21: Heel-Drop Accelerometer Layout
With the grid planned out and the equipment calibrated and configured as mentioned in the Equipment section, the physical preparation could begin. At the site, the grid of accelerometer locations was measured out and each location was marked with tape, as depicted in Figure 23. This served the dual purpose of marking the entire grid at once to drastically increase efficiency over measuring each time the accelerometers needed to be moved, and laying down the tape to which the accelerometers would be glued to prevent direct glue-to-floor contact. Once all tape was in place, the force plate was attached, like the accelerometers with tape and glue to prevent any slippage, on the driving point, selected as node 79 (Row 7, Column 7). This location was selected so as to avoid the modal lines for the first several modes of vibration, which occur at the centerlines and third-lines of the panels, because if excitation is applied on a modal line, that mode shape will not be excited (Gu, 2017).
The computer and DAQ chassis were set up off to the side of the panels being tested, so that the weight of the computer operator would not affect the vibrational characteristics of the panels. The three cables from the load cells of the force plate were then connected to the NI 9237 module. With the force plate ready, the twelve accelerometers were glued to the tape marks in Column 1, one at each location, and connected to their respective NI 9234 modules, with Channel 1, Module 1 hooked up to the Row 1 accelerometer, and so on. This configuration was kept throughout the process, so each 9234 Module corresponded to one panel. In addition to the force plate set up, the computer associated with the long-term accelerometers in the location of the heel-drop was brought out. The data from before the heel-drop test was saved, and the network was reset so that the data associated with the heel-drop could be easily isolated. The time stamp on the G-Link-200 readings was noted and compared with the time on a smartphone. With all of the equipment in place, the heel-drop tester stood on the force plate, as depicted in Figure 24.
Once settled on the force plate, the researcher performed five standard heel-drops at 40 second intervals, noting the time on the smartphone just before each drop. This time stamp was used later to match up the data from the G-Link-200s with those from the force plate. This chronological pairing did not need to be exact, as peak matching could be used to align the data sets more precisely. The times of each heel-drop were recorded on whiteboards, depicted in Figure 25. A standard heel-drop test is performed by the person raising onto the balls of their feet so that their heels are about 2.5” off the ground, very similar to how one holds their feet in a typical athletic stance, though considerably closer together, and with the rest of the body upright, as depicted in Figure 26. Then, after pausing for half a second, they relax, allowing themselves to freefall back down onto their heels, striking the floor with their weight as the heels impact. This provides a relatively consistent impact between drops of the same tester, which is typically idealized as 600lb force linearly decreasing to 0lb over a period of 0.05 seconds, resulting in a
67N·s impulse (Gu, 2017). A sample of one set of the heel-drop impulses from this experiment can be seen in Figure 27. This impulse set is from when the accelerometers were on Column 17. Five heel-drops were performed so that an average could be used to improve the quality of data, and outliers from poorly implemented drops could be identified, excluded from the data set, and redone immediately. The 40 second wait between drops is recommended by Gu (2017) to allow the vibration from the last drop to fully dissipate, preventing residual vibrations from affecting the next drop. This separation is important, as the results are presented as the response to transient excitations, and not forced excitations.

Figure 25: Whiteboards Recording Time of Each Heel-Drop
Figure 26: Heels Raised Before Drop

Figure 27: Graph of Heel-Drop Force
While these heel-drops were being performed, the coherence functions comparing the impact and the response were processed immediately. This process is detailed in the “Data Processing” section of this paper, but it is important to mention here, because if a heel drop failed the coherence test, it could be redone immediately. It was important that all of the data be checked for quality before leaving the test site to prevent the need to return and repeat nodes after everything had been packed up, which may not even have been feasible, resulting in holes in the data set. Once the five drops were performed and recorded satisfactorily, the accelerometers were removed from the floor, and any residual glue was peeled off the accelerometers. The accelerometers were then moved to the next row and glued down, and the heel-drop process was repeated.

On column lines 4, 5, 9, and 13, in addition to the heel-drop test, a walking excitation test was performed. For this process, with the accelerometers still in place from the heel-drop, the experimenter used a metronome app on a smartphone to time footfalls such that the dominant frequency of the floor was a harmonic of the footfall frequency. This dominant frequency was obtained by processing the FFTs from the heel drops for each column on-site, the procedure for which can be found in the Data Processing chapter of this thesis, and was found to be about 10.1 Hz for the first span, and about 8.4 Hz for the second span. These frequencies represent the fourth harmonic of 2.525 Hz and 2.1 Hz respectively. This was found by simply dividing the dominant frequency by 4. As such, the metronome was set to 2.525 Hz, or 152 bpm, for all four columns, and then another run was performed with the metronome set to 2.1 Hz, or 126 bpm on Column 13, and footfalls were timed to this metronome as the experimenter walked back and forth near
the line of accelerometers across the panels, depicted in Figure 28, and near the center of the middle panel longitudinal direction, in order to excite the dominant frequency of the panel. A schematic of columns where this was test was carried out can be seen in Figure 29. The accelerometer columns where measurements were taken are highlighted in pink and the walking paths are indicated by blue dotted lines. The accelerometer data from this experiment represents a realistic acceleration that the floor system can be expected to experience from normal walking use and is a direct measure of the floor’s vibrational performance (Brenemen, 2020).

Figure 28: Experimenter Performing Walking Excitation Test
Figure 29: Walking Excitation Schematic
CHAPTER FIVE
DATA PROCESSING

Fortunately, for the purposes of this study, all of the data processing was easily handled in MATLAB. For this study, MATLAB version R2019b with the vibration analysis toolbox was used. All functions referred to in this section are from this version. Using MATLAB built-in functions, all of the processing was done rather conveniently, and without requiring intimate knowledge of the mathematics behind it. That is not to say, of course, that a working understanding of what each function does and what is required to extract the results from the data wasn’t required. This chapter will describe how the data from this study was processed and provide some background as to how these processes extracted the desired results.

Heel-Drop Data Processing

Data gathered during the heel-drop tests consisted of voltages from three channels corresponding to the three load cells on the force plate, and voltages from twelve channels corresponding to the accelerometers. The accelerometer data was converted from Volts to g in the DAQ, using the sensitivity inputs from the accelerometer data sheets used during configuration. The outputs from the force plate were summed and converted to pounds using the equation determined during force plate calibration. Due to how data was acquired, raw data files each contained five heel-drops, which were averaged together to determine the response of one column of accelerometers.
In order to separate and align these heel-drops, the MATLAB function “findpeaks” was utilized. This function locates peaks in data based on parameters such as peak width, peak prominence, and distance from one peak to the next. By setting the parameters to located spikes of at least 100lbs, each heel impact could be easily located and separated, including 2500 data points before the peak to capture the entirety of the heel-drop motion, and 57,499 data points after the peak to fully capture the response. This resulted in conveniently uniform, 60,000-sample windows, separating every individual heel-drop for processing. These same windows were used to assign the appropriate forces to the corresponding responses, which was made simple by the internal clock on the DAQ, meaning that all of the channels were temporally aligned already.

At this point, all the force and acceleration vectors were filtered using “lowpass”, a MATLAB function that filters out frequencies above a user-inputted threshold, in order to reduce the electrical noise. Obviously, not all noise can be eliminated from the signal, as separating the noise from the actual data at the frequencies of interest is a near impossible task, but frequencies outside the area of interest can be eliminated, as it is safe to assume that they are not part of a meaningful response. For this study, the significant response was in the 5-25 Hz range, and largely petered out in the 25-40 Hz range. In order to allow for some cushion, a filter threshold of 60 Hz was selected. Once the data was filtered, each set of five heel-drop force-response pairs was averaged together. The effect of this sort of process is demonstrated on the force readings from Column 17 in Figure 30.
Natural Frequencies

Once all of the data had been averaged into one time history per node, the resonant frequencies, coherence functions, accelerance frequency response functions (FRFs), mode shapes, and damping coefficients were extracted. In order to extract the resonant frequencies, a Fast Fourier Transformation (FFT) was employed in the form of
MATLAB’s built in “fft” function. A Fast Fourier Transformation takes a signal in a discrete time domain and breaks it down into its component frequencies and corresponding powers. This works under the principle that any signal in the time domain can be represented as a summation of an infinite number of sine and cosine waves of different frequency and amplitude. For continuous signals, this can be accomplished using the Fourier Transform, which is written mathematically as:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi ft} \, df$$ \hspace{1cm} (5.1)

To apply this principle to a discretely sampled signal in a bounded time window, it can be represented by a series of sine and cosine waves of varying amplitudes and of frequencies between 0 Hz and $(Fs/2)$ Hz where $Fs$ is the sampling rate. This discrete representation is described by the following summation:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{i2\pi kn/N}$$ \hspace{1cm} (5.2)

known as the Discrete Fourier Transform (DFT) (Cooley, Lewis, & Welch, 1970). Using the DFT, the FFT makes this operation computationally feasible and precise enough for most practical applications. (Barrett, 2006)

Using the “fft” function returns the signal as a series of complex numbers as a function of frequency. The magnitude of these complex numbers represents the relative magnitude of the component of the original signal associated with the corresponding frequency. However, the returned function is a symmetric function ranging from $-Fs/2$ to $+Fs/2$. Conventionally, the results of the FFT are represented only from 0 to $+Fs/2$, and
the magnitudes are doubled. The angle of the complex results represents the phase shift of the represented frequency, but this information is not relevant to this study. From the FFT plot, the peaks can be manually picked by visual inspection, or “findpeaks” can be used to find local maxima, which indicate resonant frequencies of the system. An example of a time history and the associated FFT plot can be seen in Figure 31. Visual peak picking was used to quickly calculate the approximate dominant frequency at each accelerometer column before performing a walking excitation test. This allowed the metronome to be set so that the experimenter walked at a frequency that would excite a resonant response in the system.

![Figure 31: Time History (Top) and FFT (Bottom) of Heel-Drop on Node 79](image)
Coherence Functions

Coherence functions were also calculated on-site to ensure that the data being obtained was adequate. In essence, a coherence function is a measure of how related the output is to the input, or, more practically speaking, how much of the output measurement is actually a result of the measured input force. For this study, magnitude-squared coherence was used to measure the correlation between the input and output. Magnitude-squared coherence is calculated by squaring the absolute value of the cross spectrum of the two signals (calculated as the complex conjugate of the FFT of the input times the FFT of the response), divided by the product of the auto-spectrums (calculated as the complex conjugate of the FFT of the input or the response multiplied by the FFT of the same signal) of the input and the response. This can be represented mathematically as

\[
M.S.\text{ Coherence}(f) = \frac{|FFT_{in}^* \times FFT_{out}|^2}{FFT_{in}^* \times FFT_{in} \times FFT_{out}^* \times FFT_{out}}
\]  

(5.3)

where “*” indicates the complex conjugate of the FFT.

This can be easily implemented manually, but the MATLAB function “mscohere” does the job just as well and does so more cleanly. As can be seen in Figure 32, the magnitude-squared coherence function ranges from 0 to 1, with 0 indicating that there is no correlation at all between the input and response, and 1 indicating that the response is perfectly correlated to the input. The function is in terms of frequency, and therefore gives a measure of how much each response at each frequency is a direct result of the input. For this reason, frequency ranges that resonate with the system, and are therefore more easily transmitted through the structure are expected to display high correlation and
are the subject of interest for studies such as this. Frequencies not expected to be excited by the input are expected to show low correlation. Deviation from unity of the correlation function indicates a response being measured at that frequency which is not originating directly from the measured input. This can be a result of noise from either signal, a non-linear process, or, especially in in-situ experiments, outside forces not being measured as input, such as wind forces, or vibrations from an HVAC unit. (Barrett, 2006)

![Coherence Function Graph](image)

**Figure 32**: Node 91 Coherence Function

**Frequency Response Functions**

With the data deemed to be adequate for the frequency range of interest, the FRFs were calculated, and the mode shapes acquired. The FRFs can be calculated by dividing the cross-spectrum of the two signals by the auto-spectrum of the input (Gu, 2017), as in the following equation:
\[ FRF = \frac{FFT^*_{in} \times FFT_{out}}{FFT^*_{in} \times FFT_{in}} \quad (5.4) \]

Because the data recorded was the acceleration of the floor, the FRF this produces is the accelerance of the floor, or the acceleration of the floor per pound of force applied as a function of frequency. In this study, the FRFs were calculated using a script written by Mengzhe Gu which calculated the FRF and separated the components into convenient variables. This script, along with those written by the author of this thesis for the rest of data processing, can be found in Appendix H. Once all FRFs were calculated, the composite FRF for each span was determined by averaging together all of the FRFs from nodes on that span. It should be noted that the FRFs and coherence can be calculated by the MATLAB function “modalfrf”. These results were calculated individually in this study in order to allow for intermediate checks and develop a deeper understanding of where the results came from.

Mode Shapes

While the accelerance of the floor is of direct interest, it also allows for the determination of mode shapes. As explained by Avitabile, the mode shape can be computed by assigning each node an elevation based on the value of the imaginary part of its FRF at the frequency of interest. This is very easily done by plotting the absolute values of these imaginary components for each node at the frequency determined to be a resonant frequency against the location of each respective node using a “surf” plot.
Damping Coefficients

Damping coefficients were determined from the composite FRF plot using the half power bandwidth method as explained by Mengzhe Gu (2017). The half power bandwidth method, illustrated in Figure 33, is a very simple way to estimate the damping coefficient ($\zeta$) of lightly-damped systems ($\zeta < 5\%$). To implement this method, each peak representing a mode of interest is isolated. The half power band is simply the band of frequencies where acceleration is greater than or equal to 0.707 times the peak acceleration for that mode, or:

$$A_2 = A_1 \sqrt{2}$$

(5.5)

where $A_2$ is the acceleration at the half power, and $A_1$ is the peak acceleration for that mode. The frequencies of $A_2$ and $A_1$ are determined, and plugged into the following equation:

$$\zeta = \frac{f_b - f_a}{2 \times f_r}$$

(5.6)

where $f_b$ is the higher half power frequency, $f_a$ is the lower half power frequency, and $f_r$ is the resonant frequency.
Long-Term Accelerometer Processing

During the heel drop test, data was also gathered on four of the accelerometers in place for long-term monitoring, as mentioned in Chapter IV. While the data gathered from them added little to the current study, the opportunity was used to check the consistency of the readings between the two types of accelerometers. In order to accomplish this, three random heel drops were selected from the long-term accelerometer data, and each channel was processed in similar fashion to the process described above, excepting the mode shape calculations.

Walking Excitation Processing

Processing the data for the walking excitation test was significantly simpler than for that of the heel drop test. This stems mainly from the fact that the walking excitation test is an unreferenced test, and the information of interest is much less detailed. For the
walking excitation test, the only result of interest was the root mean square (RMS) acceleration of the system as a response to walking at a frequency whose harmonic would resonate with the floor. RMS acceleration is determined exactly as the name describes: the acceleration signal is squared, making all values positive, preventing a symmetric signal from canceling out, all of the square values are averaged together, and the square root of this average is taken, returning a measure, in units of acceleration, that accounts for both magnitude and duration of the vibration. MATLAB, again, has a convenient built in function for this, called simply “rms”, which performs the operations described above. The RMS acceleration was calculated in this manner for each accelerometer at each of the columns for which the walking excitation test was performed, and at both frequencies for Column 13. Because the worst case is the subject of interest, the maximum RMS acceleration from all of these data points was considered the RMS acceleration for the system as a whole in serviceability checks.
Coherence Functions

With as many data points as were gathered, and because this experiment was done on an in-situ specimen, introducing unavoidable deviations from unity in coherence, it is not feasible to intelligibly present all of the individual coherence functions. For transparency’s sake, all 204 coherency functions are plotted in Figure 34. However, while each individual coherence was checked by experimenters, Figure 35 depicts the coherence function for the average of all response time histories and the average of all force time histories, which is much easier to digest. While this method may leave out downward spikes at various frequencies in individual coherence functions, it gives a very accurate representation of the coherence of the data as a whole. As can be seen from this graph, the coherence for the frequency range of interest (about 6 Hz to 18 Hz) is very good before starting to tail off as it moves further from the excitable resonant frequencies, as one would expect.
Figure 34: All Coherence Functions Superimposed

Figure 35: Average Magnitude-Squared Coherence
Natural Frequencies and Frequency Response Functions

The natural frequencies of the system can be extracted from multiple different locations. In theory, they should be exactly the same at every node, and are apparent in both the FFT and FRF graphs. While the natural frequencies at each node are very close, experimental data is never exact, so there is some variation. In order to account for this, the natural frequencies of the system are taken from the composite FRF plot, and not from individual node FFTs. Composite FRFs were computed for each span individually, and for the system as a whole. These plots, as well as the FFTs from two representative nodes (Node 18 and Node 78) are shown below.

Figure 36: Node 18 FFT
Figure 37: Node 78 FFT

Figure 38: Composite FRF for Span 1
As is apparent from looking at these plots, the individual node readings are not far from the FRF composite results, adding confidence to using composite FRF as the
representative results for the system. The resultant natural frequencies which were used to determine the rest of the characteristics of the system can be seen in Table 2. In addition to the composite FRFs, all individual FRFs can be seen in 2D in Figure 41 and in 3D in Figure 42.

Table 2: Resonant Frequency of First Six Modes

<table>
<thead>
<tr>
<th></th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Mode 4</th>
<th>Mode 5</th>
<th>Mode 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 18 FFT</td>
<td>8.23</td>
<td>8.65</td>
<td>10.08</td>
<td>12.2</td>
<td>12.78</td>
<td>15.94</td>
</tr>
<tr>
<td>Node 78 FFT</td>
<td>8.32</td>
<td>8.67</td>
<td>10.08</td>
<td>12.14</td>
<td>12.69</td>
<td>15.80</td>
</tr>
<tr>
<td>Span 1 FRF</td>
<td>8.34</td>
<td>8.70</td>
<td>10.10</td>
<td>12.17</td>
<td>12.67</td>
<td>15.86</td>
</tr>
<tr>
<td>Span 2 FRF</td>
<td>8.40</td>
<td>8.78</td>
<td>10.10</td>
<td>12.11</td>
<td>12.67</td>
<td>13.85</td>
</tr>
<tr>
<td>Composite FRF</td>
<td>8.37</td>
<td>8.76</td>
<td>10.10</td>
<td>12.14</td>
<td>12.67</td>
<td>15.83</td>
</tr>
<tr>
<td><strong>Representative</strong></td>
<td><strong>8.37</strong></td>
<td><strong>8.76</strong></td>
<td><strong>10.10</strong></td>
<td><strong>12.14</strong></td>
<td><strong>12.67</strong></td>
<td><strong>15.83</strong></td>
</tr>
</tbody>
</table>

Figure 41: Span 1 FRFs (Top Left); Span 2 FRFs (Top Right); and All FRFs (Bottom)
Mode Shapes

As detailed in Chapter Five of this thesis, the FRFs were used to plot the mode shapes for the first six frequency peaks. Looking at these mode shapes, shown in Figure 44 through Figure 49, several things stand out. For the most part, these mode shapes are reasonably smooth, look much like one would expect, and tell a lot about the behavior of the physical system. However, it is also apparent that Node 79 does not seem to adhere to the same mode shapes as the rest of the nodes. This node represents the driving point, which is typically the most reliable data point in a modal test. It is suspected that the “misbehavior” of the driving point in this setup was due to the complications of attaching the accelerometer.

While at all other nodes the accelerometer could simply be glued to a piece of tape and adhered directly to the floor, at the driving point the accelerometer had to be glued to the inside of the force plate. This meant that between the accelerometer and the floor there was a layer of tape, two layers of glue, and a half inch of steel. Furthermore, the hot glue did not adhere as firmly to the steel as it did to the tape, or as the tape did to
the floor. Also, there is a possibility that the force plate and experimenter represented a semi-independent vibrational system on top of the panels. All of this combined made this data point somewhat unreliable for measuring the response of the floor. Unfortunately, this was not picked up on during testing because the dominant frequencies of the node were still quite similar to the other nodes, and the coherence function, depicted in Figure 43, showed that the data was in fact reliable. It was not until post-processing that it became apparent that while the data may have been a reliable response to the input force, it was not the isolated response of the entire floor system. Fortunately, the general quality of data means that missing this node did not impact the overall results of the study, aside from producing an unfortunate blemish on the mode shape plots.

![Coherence Function](image)

Figure 43: Node 79 Coherence Function

Looking at the mode shape of the first peak, which occurs at 8.37 Hz, it is apparent that this mode of vibration is not a mode dominated by the floor panels, but rather is what would traditionally be expected of a first order bending mode of vibration
of the supporting glulam beams. As is shown in Appendix I, this frequency lies comfortably between the expected natural frequencies of the glulam with hinged and fixed ends. This relatively large amount of fixity is unsurprising due to the very sturdy bolt and steel plate connections of the glulams to the columns.

Vibration associated with the second peak, which occurs at 8.76 Hz, again appears to be largely first order bending of the glulams. The mode shape associated with the third peak, which occurs at 10.1 Hz, is the first mode dominated by the floor panel behavior, and as such is considered the fundamental frequency of the floor panels. This mode also makes it clear that, contrary to expectations, the floor panels are not dominated by the longitudinal modes. This is a result of a combination of the panel splines and the continuous concrete topping providing enough of a lateral connection for the panels to behave, in vibration, as one continuous slab. This mode exhibits first order bending in both the longitudinal and transverse directions. Mode shapes associated with peaks 4 and 5 further emphasize that the splines and topping are enough of a connection for the panels to act as one, and represent the continuous lateral behavior of the floor system. These modes are again very closely spaced at 12.14 Hz and 12.67 Hz, respectively, and are similar-looking second order bending of the floor panels. The mode shape associated with the sixth peak, at 15.83 Hz, is a torsional mode of vibration of the floor system.
Figure 44: Mode Shape 1a (8.37 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
Figure 45: Mode Shape 1b (8.76 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
Figure 46: Mode Shape 2 (10.10 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
Figure 47: Mode Shape 3a (12.14 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
Figure 48: Mode Shape 3b (12.67 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
Figure 49: Mode Shape 4 (15.83 Hz); 3D (Top), X-Z Plane (Middle), and Y-Z Plane (Bottom)
In order to investigate the stability of the modes, and particularly to determine if peaks 1 and 2 and peaks 4 and 5 are indeed distinct modes, the spectrogram of each heel drop at each node was investigated. Some examples of these spectrograms can be seen in Figure 50 through Figure 52. As can be seen in these spectrograms, it is very likely that peaks 1 and 2 represent a single mode. It is the opinion of the author that this “two-peak” effect is the result of the connectivity between the glulams and the CLT panels. When the floor is moving down, gravity is assisting any mechanical fasteners, increasing composite action, but when the floor is moving up, there may be a tendency for the CLT to separate from the glulams slightly, reducing the composite action. This would result in slightly different natural frequencies on the two sides of the same mode and could account for this “two-peak” behavior. Peaks 4 and 5 are harder to discern on these graphs, but the combination of the similarity of their mode shape graphs, and the lack of strong evidence in the spectrograms that they are distinct modes is enough to consider them to be representing the same mode, like peaks 1 and 2. This conclusion means that by investigating the first six peaks, the first four modes of the system have been identified.

Aside from the connectivity strength and lateral transmission of vibration of the panels, perhaps the most important take-away from the mode shapes taken as a group is the boundary conditions. While the boundary conditions are clearly acting much closer to hinged than to fixed, the slight inflection points on the boundaries near X = 0 indicate some of the increased localized stiffness associated with fixed end conditions. There is no evidence of such fixity on the other end of the test area, indicating that that side is behaving effectively fully hinged, as assumed. The big difference between these sides,
physically, is that the side showing some fixity has a partition wall on it, which is providing a clamping effect. However, this assumption of hinged conditions is still considered close enough for the purposes of calculations in this case because the amount of fixity is minimal. This conclusion is supported by the relatively low natural frequencies of the system, which could be expected to be several hertz higher with fixed end conditions, given the other parameters. However, this also provides strong evidence that load bearing walls on the boundaries could provide a reasonable amount of fixity, causing a significant change in the vibrational characteristics of the system.

Figure 50: Node 54 Spectrogram
Figure 51: Node 80 Spectrogram

Figure 52: Node 147 Spectrogram
Damping Coefficients

Damping coefficients for each of the six modes selected from the composite FRF can be seen in Table 3. Wherever possible, these coefficients were calculated from the Span 1 FRF, the Span 2 FRF, and the Composite FRF. The lowest coefficient for each mode is the one that should be used, as it represents the worst case, and is therefore highlighted in yellow for each mode. In some instances, the damping coefficient for a mode could not be properly calculated, as neighboring modes did not allow the FRF to drop below the half power on one or both sides. When this occurred on both sides of the peak, the field was left blank, and no damping coefficient was calculated. When this occurred only on one side, the peak was treated as symmetric using the side that did reach the half power and mirroring it to the side that did not. Values obtained by these means are noted with an asterisk, and should be treated as purely educational, as they related only to artificially isolated modes which, in reality, interact significantly with others.

While Mode 3a has a damping coefficient of only 0.86%, this mode has a relatively low accelerance, as indicated by the $A_1$ column. Mode 3a, along with Mode 3b, can be ignored for calculating the design damping coefficient of the system because of their relatively low accelerance, along with their distance from the range of most human discomfort. With those modes not being considered, the only damping coefficient below 1.5% is Mode 1b, Span 1. While Mode 1b is a high accelerance mode in general, closer inspection shows that it is not a high accelerance mode in Span 1. Furthermore, in Span 2 and in the Composite FRF, it has a damping coefficient of about 2%. Taking all of this into account, a damping coefficient of 1.5% was deemed to be the best design value for
this particular system as, within the confines of modes likely to control, it is a conservative but reasonable value.

Table 3: Damping Coefficient Calculations

<table>
<thead>
<tr>
<th>Mode</th>
<th>FRF</th>
<th>( f_r )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( f_a )</th>
<th>( f_b )</th>
<th>( f_b-f_a )</th>
<th>( \zeta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Span 1</td>
<td>8.37</td>
<td>0.088</td>
<td>0.0622</td>
<td>8.12</td>
<td>8.42</td>
<td>0.3</td>
<td>1.79%</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>8.43</td>
<td>0.1172</td>
<td>0.0829</td>
<td>8.23</td>
<td>8.49</td>
<td>0.26</td>
<td>1.54%</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>8.37</td>
<td>0.099</td>
<td>0.07</td>
<td>8.15</td>
<td>8.48</td>
<td>0.33</td>
<td>1.97%</td>
</tr>
<tr>
<td>1b</td>
<td>Span 1</td>
<td>8.7</td>
<td>0.0629</td>
<td>0.0445</td>
<td>8.57</td>
<td>8.81</td>
<td>0.24</td>
<td>1.38%</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>8.72</td>
<td>0.1079</td>
<td>0.0763</td>
<td>8.66</td>
<td>9.02</td>
<td>0.36</td>
<td>2.06%</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>8.76</td>
<td>0.0815</td>
<td>0.0576</td>
<td>8.61</td>
<td>8.91*</td>
<td>0.30*</td>
<td>1.71%*</td>
</tr>
<tr>
<td>2</td>
<td>Span 1</td>
<td>10.1</td>
<td>0.1882</td>
<td>0.1331</td>
<td>9.87</td>
<td>10.42</td>
<td>0.55</td>
<td>2.72%</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>10.1</td>
<td>0.1046</td>
<td>0.0795</td>
<td>9.56</td>
<td>10.31</td>
<td>0.75</td>
<td>3.71%</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>10.1</td>
<td>0.1497</td>
<td>0.1058</td>
<td>9.75</td>
<td>10.36</td>
<td>0.61</td>
<td>3.02%</td>
</tr>
<tr>
<td>3a</td>
<td>Span 1</td>
<td>12.17</td>
<td>0.0658</td>
<td>0.0465</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>12.14</td>
<td>0.0391</td>
<td>0.0276</td>
<td>11.98</td>
<td>12.19</td>
<td>0.21</td>
<td>0.86%</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>12.14</td>
<td>0.0526</td>
<td>0.0392</td>
<td>12.03*</td>
<td>12.25</td>
<td>0.22*</td>
<td>0.91%</td>
</tr>
<tr>
<td>3b</td>
<td>Span 1</td>
<td>12.67</td>
<td>0.0576</td>
<td>0.0407</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>12.67</td>
<td>0.0588</td>
<td>0.0415</td>
<td>12.48</td>
<td>12.87</td>
<td>0.39</td>
<td>1.54%</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>12.67</td>
<td>0.0605</td>
<td>0.0427</td>
<td>12.4</td>
<td>12.97</td>
<td>0.57</td>
<td>2.25%</td>
</tr>
<tr>
<td>4</td>
<td>Span 1</td>
<td>15.86</td>
<td>0.1058</td>
<td>0.0748</td>
<td>15.46</td>
<td>16.26*</td>
<td>0.60*</td>
<td>1.89%*</td>
</tr>
<tr>
<td></td>
<td>Span 2</td>
<td>15.86</td>
<td>0.0411</td>
<td>0.0291</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Composite</td>
<td>15.83</td>
<td>0.0754</td>
<td>0.05332</td>
<td>15.45</td>
<td>16.21*</td>
<td>0.76</td>
<td>2.40%*</td>
</tr>
</tbody>
</table>

**Walking Excitation**

Results from the walking excitation tests consist only of root mean square accelerations from each trial. These results are summarized in Table 4. These results are qualitatively as would be expected. Column 9 shows the lowest RMS acceleration, because it is over a support. Column 4 shows slightly lower RMS acceleration than Column 5, as Column 5 is midspan and Column 4 is adjacent (one off midspan). Perhaps
the only surprise is that Column 13 responded less to 126 bpm, whose harmonic is 8.4 Hz, than to 152 bpm, whose harmonic is 10.1 Hz, even though the heel-drop tests showed Mode 1 as having a slightly higher accelerance than Mode 2 in Span 2. However, it may be that the dominant mode within a span is dependent on whether the excitation occurs within that span or an adjacent span, meaning that the span in which the excitation occurs is dominated by Mode 2, the fundamental mode of the floor panels, while adjacent spans are dominated by Mode 1, the fundamental mode of the glulams. This would explain the walking excitation results, as the rhythmic walking was performed in the same span as the accelerometers, whereas the heel-drops were always in Span 1.

Table 4: Walking Excitation Resultant RMS Accelerations

<table>
<thead>
<tr>
<th>Column</th>
<th>Frequency</th>
<th>Max RMS Acc</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>152 bpm</td>
<td>0.8</td>
<td>%g</td>
</tr>
<tr>
<td>5</td>
<td>152 bpm</td>
<td>0.93</td>
<td>%g</td>
</tr>
<tr>
<td>9</td>
<td>152 bpm</td>
<td>0.42</td>
<td>%g</td>
</tr>
<tr>
<td>13</td>
<td>152 bpm</td>
<td>0.87</td>
<td>%g</td>
</tr>
<tr>
<td>13</td>
<td>126 bpm</td>
<td>0.75</td>
<td>%g</td>
</tr>
</tbody>
</table>

In Figure 53, the floor’s RMS acceleration was plotted against the frequency on the DG11 chart of serviceability requirements. Two points were plotted, one for 0.93% g at 10.1 Hz, and one for 0.75% g at 8.4 Hz. Both of these points had to be checked, as 8.4 Hz has a lower threshold of unacceptable vibration for each occupancy type. As the graph indicates, this floor response is just above acceptable for offices, but well below the acceptable limit for other occupancy types.
In order to validate the accelerometers in place for long-term use, readings from during the heel-drop tests and walking excitation tests were isolated from the accelerometer approximately in the center of Span 1 in the middle panel being studied.
This comparison presents some difficulties as the long-term accelerometer positions were not precisely chosen or measured, and their sampling rate was 128 Hz, instead of the 1652 Hz used for the other accelerometers and force plate. However, that is not to say that no valid comparisons can be made. 

The two main results of interest from this accelerometer are the fundamental frequency and the RMS acceleration. Even with these measurements, the results are a bit rough (frequency due to the much lower resolution and RMS acceleration due to the sensor not being positioned directly below one of the PCB accelerometers) but are plenty adequate to provide confidence in the readings. As is shown in Figure 54, the FFT graph of this accelerometer (taken from the data corresponding to the Column 8 set of heel-drops) does not have enough clarity to allow for the identification of the later modes, but has clear peaks at 8.44 Hz, 8.62 Hz, and 10.41 Hz. As Table 5 shows, these values are very close to the PCB readings, and certainly adequate for use in monitoring for any shift in natural frequency over an extended period of time.
Figure 54: FFT from Long-term Accelerometer

Table 5: Comparison of Long-term Accelerometer Results to PCB Results

<table>
<thead>
<tr>
<th>Value</th>
<th>Long-Term</th>
<th>PCB</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1a</td>
<td>8.44 Hz</td>
<td>8.37 Hz</td>
<td>0.84%</td>
</tr>
<tr>
<td>Mode 1b</td>
<td>8.62 Hz</td>
<td>8.76 Hz</td>
<td>1.60%</td>
</tr>
<tr>
<td>Mode 2</td>
<td>10.41 Hz</td>
<td>10.10 Hz</td>
<td>3.07%</td>
</tr>
<tr>
<td>RMS Acc</td>
<td>0.83% g</td>
<td>0.90% g</td>
<td>7.78%</td>
</tr>
</tbody>
</table>

Also seen in Table 5 is the RMS acceleration of the long-term sensor. Selecting what PCB accelerometer to compare to the long-term accelerometer is no simple task, and as a result, the value displayed for the PCB accelerometer is the average RMS acceleration of the four PCB accelerometers on the same panel as the long-term one. The difference in RMS acceleration is much larger than the difference in frequency, but this is to be expected, as the acceleration depends on where exactly the accelerometer is
positioned relative to the excitation and the mode shape. The long-term accelerometer was positioned on the opposite side of Column 5 from the majority of the excitation, and thus would experience a lower RMS acceleration.

**Subjective Evaluation**

While the opinions of the experimenters cannot be considered independent, and therefore should be considered with caution, it is the author’s opinion that it is worth noting the experimenters’ feelings about the levels of vibration experienced by the floor system. While performing the walking excitation test, both experimenters were in agreement that the resultant vibrations were noticeable. With the DG11 design criteria in mind for verbiage, and before the results were analyzed so that the opinions would be unbiased, both felt that these vibrations would be annoying in an office setting or residence, but would likely not be an issue for other uses such as shopping malls or footbridges. This lines up well with the measured results, as evidenced above in Figure 53.
CHAPTER SEVEN
THEORETICAL FREQUENCY CALCULATIONS

Selecting a Model

In order to determine the best way to estimate the natural frequency of a CLT floor system with a heavy topping, several variables were investigated, and their impacts determined. As discussed in the literature review earlier in this thesis, these variables are the end conditions of the floor system, the composite action between the CLT and the topping, the continuity of the spans, and load distribution. These variables can be hard to control in a real environment and will never match the idealistic conditions typically used in calculations. Therefore, when determining the governing equation for a real system, it is often desirable to empirically determine the conditions by relating the results of experimentation to theoretically calculated values for the extreme cases of each parameter. Calculations quantifying the impact of each of these parameters can be found in Appendix E. Discussing, in full, all of the calculations in said appendix would be lengthy and cumbersome, and would add little of value to this discussion, so only the set of parameters determined to give the most accurate estimate, and therefore the set recommended for future design, will be discussed at length here.

All of the values presented in this section take into account the combined loading of the uniform mass of the floor system, as well as the point load of the tester’s mass. For this purpose, the floor system mass was taken to be the mass of all 8 panels in the span, due to the mode shapes showing that they behave, effectively, as a single slab. Appendix
E contains the values for the same conditions under only uniform loading as well, but the combined loading is considered to be more insightful for a couple reasons. The main reason in this particular study is because the mass of the tester is, in fact, present, so the combined loading equations more closely match the specific physical system being tested. Another important reason to use the combined loading equation for design purposes is because vibration is a serviceability concern. Human comfort is the reason vibration control on this scale is considered in design, so the vibrational properties of the floor without a human on it is of little consequence. Whenever an occupant is in a position to be bothered by the vibration of the floor system, they themselves will be a part of said system, and accounting for that will lead to the institution of more relevant design criteria.

The fundamental frequencies calculated using various parameters under combined loading can be seen in Table 6. One thing that stands out in this table is that accounting for continuous span behavior makes very little difference when the ends of the beam are hinged. In fact, as Equations (7.1) and (7.2) show, accounting for continuous spans using the graphical method lowers the fundamental frequency by only about 1.8% in both the composite and non-composite cases.
Table 6: Fundamental Frequencies Calculated Using a Variety of Models

* Significant figures expanded for more accurate % change calculation

\[
\frac{9.5 - 9.33}{9.5} = \frac{0.17}{9.5} = 0.018 \text{ or } 1.8\% \\
\frac{26.13 - 25.68}{26.13} = \frac{0.45}{26.13} = 0.017 \text{ or } 1.7\%
\]

This very small difference makes the additional step of graphically determining the frequency factor for a continuous beam, as per Chen and Wambsganss (1974), not worth the time. This statement is supported by Zhang and Kilpatrick’s 2019 article, which shows, analytically, that if two continuous spans are of equal length, the fundamental frequency will be the same as if there was only one span, provided that both ends are pinned. This statement, however, does depend on the fact that the end behavior is much closer to the hinged condition than the fixed condition. As expected, the results of this experiment, which are presented in full in the “Results” section, show that this is the case for the situation studied, but it should be noted that if the ends of the CLT panels are clamped in some manner such that they behave closer to the fixed condition, continuous span behavior may play an important role in the vibrational performance of the system. Fixed end behavior would also make any recommendations made as a result of this study very conservative with regards to frequency calculations, but because truly

<table>
<thead>
<tr>
<th>End Conditions</th>
<th>Bare CLT</th>
<th>CLT Handbook Estimate</th>
<th>Non-composite Estimate</th>
<th>Continuous Non-composite Estimate</th>
<th>Continuous Composite Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinged Ends</td>
<td>13 Hz</td>
<td>12.3 Hz</td>
<td>9.50 Hz*</td>
<td>26.13 Hz*</td>
<td>25.68 Hz*</td>
</tr>
<tr>
<td>Fixed Ends</td>
<td>29.2 Hz</td>
<td>27.8 Hz</td>
<td>21.4 Hz</td>
<td>58.9 Hz</td>
<td>12.2 Hz</td>
</tr>
</tbody>
</table>

Table 6: Fundamental Frequencies Calculated Using a Variety of Models
fixing the ends of the CLT panels is a near impossible task, such a design would need to be treated as a special case.

In the hinged ends condition, however, the very small margin of difference between continuous span and single span assumptions is negligible, because the vibration equations, when applied to a real system, such as a fully constructed and occupied floor, are simply not accurate enough to reliably distinguish between the two. Further adding comfort to the decision to neglect this theoretical small decrease in frequency is the fact that one of the major notable discrepancies between the theoretical calculations and the actual system is that the ends can never be entirely hinged. While the true end conditions are much closer to hinged than to fixed, they do still lie somewhere between the two, and any deviation from truly hinged will tend to increase the fundamental frequency, thus counteracting the decrease potentially caused by the continuous span behavior.

By eliminating the continuous beam models, it can be seen that the experimental fundamental frequency, 10.1 Hz, most closely matches the hinged ends, non-composite model, though the natural frequency is slightly higher than the model anticipates. This small difference can likely be accounted for by some small amount of fixity provided by the partition wall at the boundary conditions, as discussed in Chapter Six. With as small a difference as there is, though, (5.9% difference), a simple mathematical model cannot really be any more reliable, as when dealing with complex real structures, reliability of the calculated vibrational characteristics cannot exceed 90% (Aktan, Lee, Chuntavan, & Aksel, 1994).
Calculation of Fundamental Frequency Using Selected Model

Calculating the non-composite natural frequency of this system is relatively straightforward and relies on values readily available to the design engineer. The equation for the fundamental frequency is as follows:

\[
f = \frac{\lambda}{2\pi L^2} \sqrt{\frac{E I_{\text{app}}}{w + \frac{W_c}{g} \times b \times t}}
\]  

(7.3)

where \(f\) is the natural frequency, \(\lambda\) is a constant (equal to 9.870 for the first natural frequency of a system with hinged ends), \(L\) is the span length, \(w\) is the weight of the CLT panel per unit length, \(W_c\) is the weight of the concrete topping per cubic foot, \(b\) is the width of the topping, \(t\) is the thickness of the topping, resulting in the denominator under the radical taken as a whole being equal to the mass per unit length of the system, and \(E I_{\text{app}}\) is defined as follows:

\[
E I_{\text{app}} = \frac{1}{E I_{\text{eff}}} + \frac{11.52}{G A_{\text{eff}} + L^2}
\]  

(7.4)

where \(E I_{\text{eff}}\) and \(G A_{\text{eff}}\) are obtained directly from the CLT manufacturer. Plugging the values for this experiment into Equation (7.3) yields the following:

\[
f = 9.87 \times \frac{9.6 \times 10^{11}}{2 \times \pi \times (177.6 \text{ in})^2} \sqrt{\frac{13.49 \frac{\text{lb}}{\text{in}} + \left(\frac{110}{\text{ft}^3} \times \frac{386.1}{\text{in}^2}\right) \times 93 \text{ in} \times 2 \text{ in}}{13.49 \frac{\text{lb}}{\text{in}} + \left(\frac{110}{\text{ft}^3} \times \frac{386.1}{\text{in}^2}\right) \times 93 \text{ in} \times 2 \text{ in}}}
\]  

(7.5)
It is important to note that the $EI_{app}$ and total mass terms are frequently done as per unit width of the system. So long as they are consistent (i.e. if one is per unit width, both are per unit width), this cancels out, meaning both methods are acceptable. This simplifies down to:

$$
 f = 4.98 \times 10^{-5} \frac{1}{\text{in}^2} \sqrt{\frac{9.6 \times 10^{11} \frac{\text{lb} \times \text{in}}{s^2} \times \text{in}^2}{25.325 \frac{\text{lb}}{\text{in}}}}
$$

(7.6)

Separating out the constants and units,

$$
 f = 4.98 \times 10^{-5} \sqrt{\frac{9.6 \times 10^{11}}{25.325}} \sqrt{\frac{\text{lb} \times \text{in}^4}{\text{lb} \times \text{in}^4 \times s^2}}
$$

(7.7)

and simplifying,

$$
 f = 9.7 \times \frac{1}{s}
$$

(7.8)

shows that the fundamental frequency of the system accounting only for uniform mass is 9.7 Hz. According to Sundararajan (2009), this can be adjusted to account for the point mass as follows:

$$
 f = \frac{f_u^2}{\sqrt{1 + cR}} \text{ where } R = \frac{M}{m}
$$

(7.9)

where $f_u$ is the fundamental frequency of the system only accounting for the uniform mass, $c$ is a constant defined as 2 for hinged ends, $M$ is the point mass, and $m$ is the total
distributed mass. Because it’s a ratio, these can be entered as lbf or lb. For this system, this becomes:

\[
f = \sqrt{\frac{(9.7 \text{Hz})^2}{1 + 2 \times \frac{185 \text{lbf}}{8 \times 1300 \text{lbf}}}} = 9.5 \text{Hz}
\]  

(7.10)

with the constant “8” being the number of panels whose mass is being considered, and 1300 lbf being the weight of one span of one panel.

**Ramifications**

Overall, this matches expectations very well. DG11 recommends that full composite action can be assumed when using a concrete topping (for steel), so this result may seem a bit odd. The reason for this lack of composite action is apparent when inspecting the cross section sketched in Figure 55. The layer of rigid insulation drastically reduces, if not eliminates, any composite action that would have otherwise been possible. Even though there is plenty of friction between the insulation and the wood and between the insulation and the concrete, the shear strength of the insulation is far too low to effectively transfer the forces required to allow the floor act as a composite section.

Figure 55: CLT Panel Cross-Section
However, even with an accurate estimate of the natural frequency, obtaining a good serviceability estimate from either the CLT Handbook (Second Edition, 2013), or AISC Design Guide 11 (2003) methods proves difficult. Using the CLT Handbook method, which can be seen in Appendix E, results in the conclusion that the floor just misses the threshold for acceptability by about 1%. Based on the subjective analysis and calculations using other methods, this is a bit generous. Furthermore, this method does not account for occupancy type, making it somewhat difficult to use efficiently for different scenarios. For example, this method would deem the floor unacceptable, when other methods and subjective testing deem it acceptable for all but office/residential use.

Conversely, using the parameters of the floor in DG11’s criterion, also shown in Appendix E, requires the mass of the system. This is problematic because RMS acceleration is a much more localized event than the system frequency, which is the same throughout. Therefore, knowing how much of the mass is contributing fully to resisting the local acceleration is difficult. Using the mass of a single panel results in a peak RMS acceleration of 2.8% $g$, which, as demonstrated by Figure 56, puts it well above where the experimental data indicates, and even disqualifies it from acceptability for a shopping mall. Using the mass of all 8 panels, as for the frequency calculation, results in an RMS of 0.35% $g$, which is far below the experimental indication.

However, one parameter which this equation does not take into account is continuous spans. While span continuity may not affect the natural frequency of a system, it will affect the RMS acceleration, because energy is required to excite all, in this case three, spans (Brenemen, 2020). In fact, calculating the RMS acceleration using
the DG11 equation with the mass of 3 spans yields a result of 0.94% g, which is remarkably close to the measured value of 0.93% g. However, as the mode shapes indicate that the acceleration is transmitted to adjacent panels as well as adjacent spans, the system mass is likely to be some combination of the adjacent panels and the adjacent spans. Determining how much each of these parameters contributes could be a great step forward in developing an accurate design equation for CLT but will require much more research to build confidence in any proposed ratio. Another important consideration in determining this mass is the contribution of the glulam beams. If the glulams are being significantly excited by the impulses, then their tributary width is likely to have an appreciable impact on the magnitude of the acceleration.

Figure 56: Acceptability Criteria with DG11 Equation (Murray, Allen, & Ungar, 2003)
CHAPTER EIGHT
CONCLUSIONS AND RECOMMENDATIONS

Summary of Research

As cross-laminated timber becomes a more popular construction material, more detailed knowledge and more efficient design guidelines are vital for its continued success. Due to its lightweight nature, vibration serviceability is of particular concern for ensuring the comfort of occupants. This study focused on improving the methods through which the vibration-controlled spans of heavy-topped CLT are calculated. The results provide five main conclusions pertaining to the future design in this area.

Less of a finding and more of a confirmation, is that this study showed that the current CLT Handbook (Second Edition, 2013) guidelines do not adequately account for the effects of a heavy topping. This is not surprising, as the guidelines themselves say that the current handling of heavy toppings is just an interim measure and more research is needed in order to accurately predict how a heavy-topped CLT system will behave. While this study simply does not supply enough data to fully provide a new method for determining a new design criterion, it provides a strong next step on the path to cost-effective and satisfactory design of mass timber buildings.

Another conclusion which is more reaffirming than it is profound comes from analyzing the mode shapes boundaries. The distinctly hinged behavior of one side, paired with the slight fixity of the side with a partition wall, shows that the assumption of hinged boundary conditions for calculations is good when there are no walls on the boundary, adequate when there is a partition wall, though some adjustment may be considered, but
likely inadequate when dealing with load bearing walls. For the situation being studied, this intermediate conclusion aids in the isolation of the other variables, which in turn lends additional credence to the other, flashier results.

Another insight that the mode shapes provide is in regard to panel to panel connections. The relative smoothness of the mode shapes transversely across the panels shows that, with the continuous concrete topping, the panels are effectively behaving as one continuous member parallel to span for the purposes of vibration serviceability. This cohesiveness may be utilized to increase the dynamic mass responding to excitations, thus reducing the accelerance of the system as a whole, increasing serviceability without affecting the natural frequency. Taking advantage of this behavior could help limit vibrations to acceptable levels at longer spans than would otherwise be feasible, helping to reduce costs and make CLT a more attractive product.

Calculating the expected natural frequency of the system using various parameters showed that the frequency could be accurately predicted using the standard vibration equation found in DG11, the CLT Handbook, and various other publications by assuming there to be no composite action between the panels and the topping. This assumption, though, is strongly tied to how the topping is attached or not attached to the CLT. In this case, there was a layer of rigid insulation between the concrete and the panels, preventing any composite action from occurring. Removing the insulation from the equation and placing the concrete directly on the CLT would likely result in a very high amount of composite action, if not turning the system fully composite for the purposes of vibration serviceability. That said, use of insulation between the CLT panels and heavy topping is
very common, particularly due to CLT’s own poor acoustic insulation abilities. This is, therefore, not considered to be a unique or special case, and is a situation design engineers will encounter frequently, making design provisions for this case a reasonable and useful thing to have.

Finally, from the walking excitation test, there is strong evidence that the vibration serviceability criteria in the AISC Design Guide 11 can be used to good effect for CLT. With the continuous spans and the concrete topping, using these criteria is not quite straightforward, as the total mass being excited is very important, in addition to the weight per unit width, as is required for frequency calculations. While the system in this study displayed an RMS acceleration indicative of a mass triple that of one span of one panel, it is not clear how much of this extra mass was a result of the span continuity, and how much was a result of the transverse connectivity provided by the splines and the topping. Without knowing how to account for these variables individually, it is very difficult to apply these criteria to different cases. Therefore, while this study shows that there is a lot of promise in this method, there is not enough information available yet to put it to use.

**Recommendations**

Based on the results of this study, further knowledge regarding the effect of both continuous spans and the transverse continuity provided by splines and the topping is required. Separating the effects of these two variables would allow for full use of the DG11 design criteria for vibration, which would provide more accurate results which could be used in a larger variety of situations. To further this goal, studies on the specific
effect of each parameter are recommended as the next step in the development of design guidelines for CLT vibration. With enough data, an empirical equation for the effective system mass can be determined, and the DG11 criteria can be implemented, providing a more robust solution to design engineers.

Another potential area for further research is taking into account the supports. Currently, supports are considered to be rigid when performing calculations, but as the mode shapes show, this is not the case. Deflection of the glulams contributed to the overall deflection, reducing serviceability. Treating these supports as springs instead of rigid supports may allow for more comprehensive calculations and more insight into how the structure will respond.

For current design, if using a method that requires the calculation of the fundamental frequency, it is very likely that the presence of layers in between the topping and the CLT surface play a very large role. This study has shown that a fairly accurate estimate can be obtained by treating the system as a non-composite beam, accounting for the added weight of the topping, but assuming the topping does not contribute to stiffness. While that has been shown to be an effective method for the presented case, it likely does not hold true for cases where the topping is directly on top of the CLT. Based on the treatment of concrete toppings on steel decks in DG11, as well as general familiarity with material properties such as surface roughness, it is expected that full composite action could be achieved through friction alone in direct contact cases. This, however, needs further research, but would constitute a significant step forward for the
economic efficiency of CLT, because being able to utilize composite action would significantly improve the vibrational performance of the floor, allowing for longer spans.

Building on this, it is also recommended that if vibrational serviceability is a big concern, this can be mitigated by forcing composite action between the topping and the panel, by adding fixity to the boundary conditions (possibly through the placement of load bearing walls), adding mass, or stiffening the supporting beams. A mechanical connection between the topping and panel would allow the stiffness of the topping to contribute to the overall stiffness of the system, thus raising the natural frequency further away from the range most irritating to humans. Likewise, more fixity of the boundaries adds stiffness to the system, producing similar results. Adding mass (e.g., using normal weight concrete instead of lightweight, or increasing the topping thickness) can reduce the RMS acceleration, increasing serviceability. Stiffening the supporting beams may increase the natural frequency of the system as a whole, and will also reduce the deflection of the system, providing multiple avenues of increasing serviceability. While the benefits of these measures will need further studies to prove and quantify, they are avenues that show significant promise moving forward.
Appendix A

CLT Handbook Frequency Equation Modification

Proof of Difference between CLT Handbook and General Equation

Theorem:

\[
\frac{2.188}{2 \times L^2} \sqrt{\frac{EI}{\rho \times A}} \neq \frac{1}{2} \times \frac{\pi}{L^2} \sqrt{\frac{EI}{m}}
\]

Proof: By contradiction; Assume

\[
f = \frac{2.188}{2 \times L^2} \sqrt{\frac{EI}{\rho \times A}} = \frac{1}{2} \times \frac{\pi}{L^2} \sqrt{\frac{EI}{m}}
\]

A = 1 in²

L = 1 ft

EI = 1 lbf*in²

m = 1 lbm/in

\[
\rho = \frac{m/A}{\gamma_w} = \frac{1 lbm/in^3}{0.036 lbm/in^3} = 27.78
\]

Equation A1: Original Equation

\[
f = \frac{2.188}{2 \times L^2} \sqrt{\frac{EI}{\rho \times A}} = \frac{1}{2} \times \frac{\pi}{L^2} \sqrt{\frac{EI}{m}}
\]

Equation A2: Sub in Values

\[
\frac{2.188}{2 \times ft^2} \sqrt{\frac{lbf \times in^2}{27.78 in^2}} = \frac{1}{2} \times \frac{\pi}{144 in^2} \sqrt{\frac{lbf \times in^2}{lbm/in}}
\]
Equation A3: Cancel 1/2
\[
\frac{2.188}{ft^2} \sqrt{\frac{lb \times in^2}{27.78in^2}} = \frac{\pi}{144 \text{ in}^2} \sqrt{\frac{lb \times in^2}{lbm/in}}
\]

Equation A4: Convert Units
\[
\frac{2.188}{144 \times \text{in}^2} \sqrt{\frac{lb \times in^2}{27.78in^2}} = \frac{\pi}{144 \text{ in}^2} \sqrt{\frac{lb \times in^2}{lb \times \frac{g}{g \times \text{in}}}}
\]

Equation A5: Expand \( g \)
\[
\frac{2.188}{144 \times \text{in}^2} \sqrt{\frac{lb \times in^2}{27.78in^2}} = \frac{\pi}{144 \text{ in}^2} \sqrt{\frac{lb \times in^2}{386 \times \frac{in}{s^2} \times \text{in}}}
\]

Equation A6: Rearrange Fraction
\[
\frac{2.188}{144 \times \text{in}^2} \sqrt{\frac{lb \times in^2}{27.78 \times \text{in}^2}} = \frac{\pi}{144 \text{ in}^2} \sqrt{\frac{386 \times lb \times in^2}{lb \times s^2}}
\]

Equation A7: Simplify Radical
\[
\frac{2.188}{144 \times \text{in}^2} \sqrt{\frac{lb \times in^2}{27.78 \times \text{in}^2}} = \frac{\pi \times \text{in}^2}{144 \text{ in}^2} \sqrt{\frac{386}{s^2}}
\]

Equation A8: Cancel Terms
\[
\frac{2.188}{\text{in}^2} \sqrt{\frac{lb}{27.78}} = \frac{\pi}{1} \sqrt{\frac{386}{s^2}}
\]

Equation A9: Convert Force to Mass
\[
\frac{2.188}{\text{in}^2} \sqrt{\frac{lbm \times 386 \times \text{in}}{27.78 \times s^2}} = \frac{\pi \sqrt{386}}{s}
\]

Equation A10: Separate Radical
\[
\frac{2.188}{\text{in}^2 \times s} \sqrt{\frac{lbm \times \text{in}}{27.78 \times 386}} = \frac{\pi \sqrt{386}}{s}
\]
**Equation A11:** Simplify Radical and Cancel Terms
\[
\frac{2.188}{\text{in}^2} \sqrt{\frac{\text{lbm} \times \text{in}}{27.78}} = \pi
\]

**Equation A12:** Separate Radical
\[
\frac{2.188}{\sqrt{27.78 \times \text{in}^2}} \sqrt{\text{lbm} \times \text{in}} = \pi
\]

**Equation A13:** Reduce
\[
\frac{2.188 \times \sqrt{\text{lbm} \times \text{in}}}{5.27 \times \text{in}^2} = \pi
\]

**Equation A14:** Reduce
\[
0.415 \times \frac{\sqrt{\text{lbm} \times \text{in}}}{\text{in}^2} = \pi
\]

Because
\[
0.415 \times \frac{\sqrt{\text{lbm} \times \text{in}}}{\text{in}^2} \neq \pi
\]

it follows that
\[
\frac{2.188}{2 \times L^2} \sqrt{\frac{E I}{\rho \times A}} \neq \frac{1}{2} \times \frac{\pi}{L^2} \sqrt{\frac{E I}{m}}
\]

**Alternate Derivation of CLT Handbook Constant**

**Equation A15:** Separation of Units
\[
\frac{1}{2} \times \frac{\pi}{L^2} \sqrt{\frac{E I}{m}} = \frac{1}{2} \times \frac{\pi}{144 \text{ in}^2 \times \left(\frac{L}{12}\right)^2} \sqrt{\frac{E I \times \text{lbf} \times \text{in}^2}{\gamma \text{ in}^2 \times A \text{in}^2}}
\]
Equation A16: Expansion of Force
\[
\frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{EI}{\gamma A}} = \frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \text{bm} \times \text{in}^2 \times g}{\gamma \frac{\text{lbm}}{\text{in}^3} \times \frac{\gamma w}{\gamma w} \times \text{Ain}^2}}
\]

Equation A17: Cancellation of Units
\[
\frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \text{bm} \times \text{in}^2 \times 386 \text{in}^2}{\gamma \frac{\text{lbm}}{\text{in}^3} \times \frac{\gamma w}{\gamma w} \times \text{Ain}^2}} = \frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \times 386 \text{in}^2}{\gamma \frac{\text{lbm}}{\text{in}^3} \times \frac{\gamma w}{\gamma w} \times \text{A}}}
\]

Equation A18: Substitution of Unit Weights
\[
\frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \times 386 \text{in}^2}{\gamma \frac{\text{lbm}}{\text{in}^3} \times \frac{\gamma w}{\gamma w} \times \text{A}}} = \frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \times 386 \text{in}^2}{0.036 \times \frac{\text{lbm}}{\text{in}^3} \times \rho \times \text{A}}}
\]

Equation A19: Simplify Units
\[
\frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \times 386 \text{in}^2}{0.036 \times \frac{\gamma w}{\gamma w} \times \rho \times \gamma w \times \text{A}}} = \frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left( \frac{L}{12} \right)^2} \sqrt{\frac{E l \times 386 \text{in}^2}{0.036 \times \rho \times \text{A} \times \text{s}^2}}
\]
\[
\frac{1}{2} \times \frac{\pi}{144 \text{in}^2 \times \left(\frac{L}{12}\right)^2} \sqrt{\frac{E I \times 386 \text{in}^4}{0.036 \times \rho \times A \times s^2}} = \frac{\pi}{144} \sqrt{\frac{386}{0.036}} \times \frac{1}{2 \left(\frac{L}{12}\right)^2} \sqrt{\frac{E I}{\rho \times A}}
\]

\[
\frac{\pi}{144} \sqrt{\frac{386}{0.036}} \times \frac{1}{2 \left(\frac{L}{12}\right)^2} \sqrt{\frac{E I}{\rho \times A}} = 2.259 \times \frac{1}{2 \left(\frac{L}{12}\right)^2} \sqrt{\frac{E I}{\rho \times A}}
\]

\[
\frac{2.259}{2L_f^2} \sqrt{\frac{E I}{\rho A}} \approx \frac{2.188}{2L^2} \sqrt{\frac{E I}{\rho A}}
\]

As can be seen in the comparison, there is only about a 3% difference between these constants, but they are not exactly the same. Most likely, this discrepancy can be attributed to intermediate rounding. For the precision required in design for vibration control, this small difference is insignificant, but for the purposes of this study, the constant is more consistent with other frequency models, such as the model presented in AISC Design Guide 11 (2003), 2.259.

An alternate theory as to where this difference arises is in the rho term. In the CLT Handbook, US Second edition, rho is taken as 1.0625 times the oven dry specific gravity of the timber used in the CLT panel, but in EuroCode 5 (2008), rho is taken as the specific gravity of the panel itself. If the specific gravity of a CLT panel were taken as simply the oven dry specific gravity of the timber used, this 1.0625 term, being under the
radical, would change the result by the same $\sim 3\%$ by which the two equations differ.

While this seems like strong evidence that this may be where the difference arose, a look through three prominent CLT manufacturers’ design guides (values shown in Appendix B) shows that the specific gravity of CLT is typically considered to be higher than that of the oven dry timber. While the percentage difference varies from manufacturer to manufacturer, this strongly indicates that a small increase to the oven dry specific gravity is warranted and should not be cancelled out by an adjustment to the constant. This, again, points to the idea that the discrepancy is caused by intermediate rounding.
Appendix B

CLT Design Manuals
Cross-Laminated Timber Panels (CLT)

3-Ply  4.125” thick  Max Size: 10’ (W) x 41.5’ (L)
5-Ply  6.875” thick  Max Size: 10’ (W) x 41.5’ (L)
7-Ply  9.625” thick  Max Size: 10’ (W) x 41.5’ (L)

* V1, E2 & E2M1 Grades available per APA-PRG320-2018 Certification

* Douglas Fir fiber used on all grades of CLT panels

Panel Specifications —

Manufacturing Certification: APA PRG 320
Product Report PR-L320

Maximum Panel Size: 10’-0” x 42’-0”

Panel Thickness: 4 1/8”, 6 7/8”, 9 5/8”

Moisture Content: 12 % ± 3%

Wood Species: Douglas fir-Larch

Lumber Grades: DFL #2, DFL #3, 2250Fb-2.0E-1600Ft

Density: ± 34 lb/ft3

Glue Specification: MF Resin – Meets LEED Credit EQ 4.4.1263

CLT Grades: V1, E2, E2M1

Thickness Tolerance: ±1/16” or 2% of panel thickness, whichever is greater

Width Tolerance: ±1/8”

Length Tolerance: ±1/4”

Squareness Tolerance: Length of two panel face diagonals shall not differ more than 1/8”

Straightness Tolerance: Deviation of edges from a straight line between adjacent panel corners shall not exceed 1/16”

Flame Spread: 20 – Meets “Class A” requirements

Smoke Development Classification: 50 – Meets “Class A” requirements
## TECHNICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>PRODUCT NAME / BRAND</th>
<th>KLH®</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTHER PRODUCT NAMES</td>
<td>Cross-laminated timber (CLT)</td>
</tr>
<tr>
<td>APPLICATION</td>
<td>Structural elements for walls, floors and roofs</td>
</tr>
<tr>
<td>DURABILITY</td>
<td>Service classes 1 and 2 according to EN 1995-1-1</td>
</tr>
<tr>
<td>WOOD SPECIES</td>
<td>Spruce (pine, fir, stone pine and other wood types on request)</td>
</tr>
<tr>
<td>PANEL BUILD UP</td>
<td>3, 5, 7 or more layers depending on static requirements</td>
</tr>
<tr>
<td>LAMELLÆ</td>
<td>Thickness 20 to 45 mm, technically dried, quality-sorted and finger-jointed</td>
</tr>
<tr>
<td>STRENGTH CLASS</td>
<td>C 24 according to EN 338, maximum 10% C 16 permitted (compare ETA-06/0138)</td>
</tr>
<tr>
<td>ADHESIVE</td>
<td>Formaldehyde-free PUR adhesive, approved for load-bearing and non-load-bearing components indoors and outdoors according to EN 15425</td>
</tr>
<tr>
<td>LAMINATING PRESSURE</td>
<td>At least 0.6 N/mm²</td>
</tr>
<tr>
<td>WOOD MOISTURE</td>
<td>12% (+/- 2%) on delivery</td>
</tr>
<tr>
<td>CONTENT</td>
<td></td>
</tr>
<tr>
<td>MAXIMUM ELEMENT</td>
<td>Length 16.50 m / width 2.95 m / thickness 0.50 m</td>
</tr>
<tr>
<td>DIMENSIONS</td>
<td>2.40 / 2.50 / 2.73 / 2.95 m</td>
</tr>
<tr>
<td>PRODUCED WIDTHS</td>
<td>Non-visual quality (NVQ)</td>
</tr>
<tr>
<td>SPECIAL SURFACES</td>
<td>Special surfaces on request</td>
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<tr>
<td>WEIGHT</td>
<td>5.5 kN/m² according to ÖNORM B 1891-1-1:2011 for structural analysis</td>
</tr>
<tr>
<td></td>
<td>500 kg/m³ for determination of transport weight</td>
</tr>
<tr>
<td>MOISTURE MOVEMENT</td>
<td>In panel plane 0.01% per % change in wood moisture content, perpendicular to panel plane (panel thickness direction)</td>
</tr>
<tr>
<td></td>
<td>0.24% per % change in wood moisture content</td>
</tr>
<tr>
<td>THERMAL CONDUCTIVITY</td>
<td>( \lambda = 0.12 \text{ W/(m*K)} ) according to EN ISO 10456</td>
</tr>
<tr>
<td>HEAT STORAGE</td>
<td>( c_p = 1600 \text{ J/(kg*K)} ) according to EN ISO 10456</td>
</tr>
<tr>
<td>CAPACITY</td>
<td></td>
</tr>
<tr>
<td>VAPOUR RESISTANCE</td>
<td>( \mu = 20 \text{ to 50 according to EN ISO 10456} )</td>
</tr>
<tr>
<td>AIR TIGHTNESS</td>
<td>KLH® solid wood panels can generally be used as airtight layers. Connections to other components, butt joints, penetrations, etc. must be sealed appropriately.</td>
</tr>
<tr>
<td>REACTION TO FIRE</td>
<td>Euro class D-s2, d0</td>
</tr>
<tr>
<td>RESISTANCE TO FIRE</td>
<td>According to ETA - 06/0138</td>
</tr>
</tbody>
</table>
# CrossLam® CLT Panel Characteristics

- **Maximum Panel Size:** 9’10.5” x 40.0’
- **Maximum Thickness:** 12.42”
- **Minimum Thickness:** 3.43”
- **Production Widths:** 7’10.5” & 9’10.5”
- **Moisture Content:** 12% (+/-3%) at time of production
- **Glue Specifications:** Purbond polyurethane adhesive
- **Glue Type:** Weatherproof, formaldehyde free foaming PUR
- **Species:** SPF, Douglas-fir
- **Lumber Grades:** SPF #28, Btr, SPF #3, Dfrr L3, MSR 2100
- **Stress Grades:** V2M1.1, V2.1, E1M4, E1M5
- **Manufacturing Certification:** APA PRG 320 Product Report FR-L314
- **Density:** 30.3 lbs/ft³ (shipping weight at time of manufacturing)
- **Dimensional Stability:** Longitudinal and Transverse 0.01% per % Δ in MC Thickness 0.2% per % Δ in MC
- **Thermal Conductivity:** R value: 1.2 per inch (h·ft²·°F/Btu)
- **CO₂ Sequestration:** 13.7 lbs/ft³

**Dimensional Tolerances:**
- **Thickness:** 1/16” or 2% of CLT thickness, whichever is greater
- **Width:** 1/8” of the CLT width
- **Length:** 1/4” of the CLT Length

**Squareness:** Panel face diagonals shall not differ by more than 0.125”

**Straightness:** Deviation of edges from a straight line between adjacent panel corners shall not exceed 1/16”
Appendix C

Force Plate Calibration Data

Table 7: Force Plate Calibration Data

<table>
<thead>
<tr>
<th>Weight Added (lbs)</th>
<th>Total Superimposed Weight (lbs)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>1.51</td>
</tr>
<tr>
<td>8.82</td>
<td>8.82</td>
<td>2.83</td>
</tr>
<tr>
<td>8.82</td>
<td>17.64</td>
<td>4.16</td>
</tr>
<tr>
<td>8.82</td>
<td>26.46</td>
<td>5.49</td>
</tr>
<tr>
<td>8.80</td>
<td>35.27</td>
<td>6.83</td>
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<tr>
<td>8.82</td>
<td>44.09</td>
<td>8.16</td>
</tr>
<tr>
<td>8.83</td>
<td>52.91</td>
<td>9.49</td>
</tr>
</tbody>
</table>

\[ V = 0.151(lb) + 1.499 \implies \Delta lb = \Delta 0.151V \implies \Delta V = \Delta 6.636lb \]
Appendix D

Verification of Acceptability of Tape Connection with Accelerometers

This section details the experiment carried out to show that the use of a layer of tape between the accelerometer and the test specimen did not have a meaningful effect on the data. To accomplish this goal, a steel plate, whose properties are given in Table 8, was clamped to a desk and two PCB 333B50 accelerometers were affixed to it with varying conditions with hot glue. In order to ensure that any deviation in results seen was indeed from the inclusion of tape, two accelerometers (and associated cables and DAQ channels, which remained consistent throughout) were affixed both with tape and without tape to both sides of the steel plate, as detailed in Table 9. This ensured that any variation due to the accelerometers themselves or due to the placement of the accelerometers could not be attributed to the tape. This set-up, depicted in Figure 57, was not controlled enough to expect the frequency measured to match the theoretical fundamental frequency of the steel plate in use. This was largely due to the loose boundary conditions, particularly the tendency of the desk to rock slightly. However, being that the purpose of this test was to show that the same value was obtained whether or not tape was used, the frequencies themselves do not matter, and only the percent difference between values was of interest.
Table 8: Properties of Steel Plate

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Conditions</td>
<td>Fixed-Free</td>
<td></td>
</tr>
<tr>
<td>Total Length</td>
<td>16.75</td>
<td>in</td>
</tr>
<tr>
<td>Clear Span</td>
<td>12.49</td>
<td>in</td>
</tr>
<tr>
<td>Width</td>
<td>2.99</td>
<td>in</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.1265</td>
<td>in</td>
</tr>
<tr>
<td>Distance from End to Accelerometer</td>
<td>1</td>
<td>in</td>
</tr>
<tr>
<td>Distance from Side to Accelerometer</td>
<td>0.5</td>
<td>in</td>
</tr>
</tbody>
</table>

Table 9: Tape Test Trial Variations

<table>
<thead>
<tr>
<th>Trial</th>
<th>Left Accelerometer</th>
<th>Right Accelerometer</th>
<th>Left Condition</th>
<th>Right Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LW59063</td>
<td>LW59065</td>
<td>bare</td>
<td>tape</td>
</tr>
<tr>
<td>2</td>
<td>LW59063</td>
<td>LW59065</td>
<td>tape</td>
<td>bare</td>
</tr>
<tr>
<td>3</td>
<td>LW59065</td>
<td>LW59063</td>
<td>tape</td>
<td>bare</td>
</tr>
<tr>
<td>4</td>
<td>LW59065</td>
<td>LW59063</td>
<td>bare</td>
<td>tape</td>
</tr>
</tbody>
</table>

Figure 57: Tape Test Set Up: Trial 1

Once everything was in place, the steel plate was manually excited by pulling down the end and suddenly releasing it, allowing it to vibrate freely. The acceleration
data was captured using an NI 9237 module in the NI cDAQ 9172, and Clemson’s proprietary software, Clemson DAQ Scribe. This data was exported to Excel and then processed in MATLAB using a fast Fourier transform to convert the data to a normalized frequency domain. The resulting graph from Trial 1 can be seen in Figure 58. A relatively high amount of noise can be seen; still, five peaks are easily discerned and are indicated by red circles in this figure. Due to their relatively low power and less precise peak, Peaks 3 and 5 were not compared in this study. This was deemed an acceptable exclusion, particularly because the steel plate has a higher fundamental frequency than the CLT floor and only the first two modes are of interest in this study.

![Figure 58: Tape Test Trial 1 Normalized Frequency](image)

The values of peaks 1, 2 and 4 for all four trials, normalized as a percentage of the sampling rate, are tabulated in Table 10, along with the percent differences. Two important conclusions can be drawn from these results. Most importantly, the largest percent difference of any peak in any trial was only 1.37%, the average percent difference
was only 0.29%, and half of the peaks had a percent difference of 0.00%, showing rather plainly that none of the variables in this test had a significant impact on the measurements. Further enforcing the claim that the tape has no significant effect is the second conclusion: of the six times that a difference was observed, three times the accelerometer mounted on tape displayed the higher frequency, and three times the accelerometer mounted directly on the plate displayed the higher frequency, while all six times, accelerometer number LW59063 displayed the higher frequency. This very strongly indicates that even the small discrepancy between results, when present, was a result of differences in the accelerometers themselves, meaning that any impact the addition of tape had on the readings is within the tolerance of the accelerometers. While this shows that it is acceptable to use tape to mount the accelerometers for this experiment, it should be noted that the higher the frequencies being observed are, the greater the impact the bonding method has. Therefore, if much higher frequencies are being studied, the results of this test should not be used without further investigation to justify the use of tape.
Table 10: Tape Test Results

<table>
<thead>
<tr>
<th>Trial</th>
<th>Peak Number</th>
<th>Left Accel. Value</th>
<th>Right Accel. Value</th>
<th>Percent Difference</th>
<th>Conditions of Higher Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.01019</td>
<td>0.01019</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.09450</td>
<td>0.09432</td>
<td>0.19%</td>
<td>Left/Bare/63</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.02624</td>
<td>0.02624</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.01025</td>
<td>0.01025</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.09520</td>
<td>0.09519</td>
<td>0.01%</td>
<td>Left/Tape/63</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.29550</td>
<td>0.29400</td>
<td>0.51%</td>
<td>Left/Tape/63</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.01023</td>
<td>0.01023</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.08353</td>
<td>0.08395</td>
<td>-0.50%</td>
<td>Right/Bare/63</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.29920</td>
<td>0.30190</td>
<td>-0.90%</td>
<td>Right/Bare/63</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.01018</td>
<td>0.01032</td>
<td>-1.37%</td>
<td>Right/Tape/63</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.08705</td>
<td>0.08705</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.30200</td>
<td>0.30200</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Appendix E

Calculation of Fundamental Frequencies
Calculating the Fundamental Natural Frequency by Various Methods

Ben Schwendy

Inputs

All inputs from shop drawings, unless otherwise noted

Table 1. ASD Reference Design Values\(^{(a)}\) for Lumber Laminations Used in IB MAX-CORE CLT (for Use in the U.S.)

<table>
<thead>
<tr>
<th>CLT Layup</th>
<th>F(_1) (psi)</th>
<th>E (10^6 psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V3(^{b})</td>
<td>760</td>
<td>1.4</td>
</tr>
</tbody>
</table>

For SI: 1 psi = 0.006895 MPa

\(^{(a)}\) Tabulated values are allowable design values and not permitted to be increased for the lumber flat use or size factor in accordance with the NDS. The design values shall be used in conjunction with the section properties provided by the CLT manufacturer based on the actual layup used in manufacturing the CLT panel (see Table 2).

Table 2. ASD Reference Design Values\(^{(a)}\) for IB MAX-CORE CLT Listed in Table 1 (for Use in the U.S.)

<table>
<thead>
<tr>
<th>CLT Layup</th>
<th>Layout ID</th>
<th>Thickness, t (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V3(^{b})</td>
<td>3</td>
<td>3/36</td>
</tr>
</tbody>
</table>

For SI: 1 in. = 25.4 mm; 1 ft = 304.8 mm; 1 lb = 4.448 N

\(^{(a)}\) Tabulated values are allowable design values and not permitted to be increased for the lumber flat use or size factor in accordance with the NDS.

\(^{(b)}\) The V3 layup is developed based on ANSI/APA PRG 320, as permitted by the standard.

Tables from IB MAX-CORE CLT Shop Drawings page 3

Table from IB MAX-CORE CLT Shop Drawings page 4

\[ g = 9.81 \frac{m}{s^2} \]

Gravity

\[ L = 16.6 \text{ ft} - 21.5 \text{ in} = 14.8 \text{ ft} \]

Span length

\[ h = 6.875 \text{ in} \]

Panel thickness

\[ b = 7 \text{ ft} + 9 \text{ in} \]

Representative width

\[ A = b \times h \]

Cross-sectional area

\[ EI_{clt} = b \cdot 363000000 \frac{\text{lbf} \cdot \text{in}^2}{\text{ft}} = \left(2.8 \cdot 10^9\right) \frac{\text{lbf} \cdot \text{in}^2}{\text{ft}} \]

Created with Mathcad Express. See www.mathcad.com for more information.
\[ GA_{clt} := b \cdot 980000 \frac{\text{lb}_f}{\text{ft}} = (7.6 \cdot 10^8) \, \text{lb}_f \]

\[ I_{clt} := \frac{b \cdot h^3}{12} \]

Moment of inertia of CLT section

\[ E_{clt} := \frac{EI_{clt}}{I_{clt}} \]

Modulus of elasticity of CLT

\[ w := A \cdot 1.0625 \cdot 0.55 \cdot 62.4 \, \frac{\text{lb}_f}{\text{ft}^3} \cdot \frac{1}{g} = 5 \frac{8^2}{\text{ft}^2} \cdot \text{lb}_f \]

Weight per unit length (Hu and Chui, 2013)

\[ M := 0.185 \, \text{kip} \]

Point mass

\[ m := b \cdot w \cdot g \]

Uniform mass

\[ t := 2 \, \text{in} \]

Topping thickness

\[ f'c := 5760 \]

Concrete strength at 28 days

\[ E_{con} := 57000 \cdot \sqrt{f'c} \, \text{psi} = (4.3 \cdot 10^3) \, \text{ksi} \]

Modulus of elasticity of concrete

\[ W_c := 110 \, \text{pcf} \]

Unit weight of concrete

**EI Calcs**

**Non-Composite EI**

\[ EI_{app} := 1 \frac{1}{1 + \frac{1}{11.52} EI_{clt} \frac{G A_{clt} \cdot (L)^2}{\text{lb}_f \cdot \text{in}^2}} \]

Apparent EI of CLT section

(Hu and Chui, 2013)

**Transform Sections**

\[ n := \frac{E_{con}}{E_{clt}} = 3.9 \]

Transformation ratio

\[ b_{con} := n \cdot b = 30 \, \text{ft} \]

Transformed width of concrete

Created with Mathcad Express. See www.mathcad.com for more information.
\[
y := \frac{A \cdot \frac{h}{2} + b_{\text{con}} \cdot t \cdot \left( h + 2 \, \text{in} + \frac{t}{2} \right)}{A + b_{\text{con}} \cdot t} = 0.6 \, \text{ft}
\]
Height of neutral axis (the concrete is raised on 2" of rigid insulation)

\[
I_{\text{con}} := \frac{b_{\text{con}} \cdot t^3}{12} + \left( b_{\text{con}} \cdot t \cdot \left( h + 2 \, \text{in} + \frac{t}{2} - y \right) \right)^2 = 0.3 \, \text{ft}^4
\]
I of concrete

\[
I_{\text{wood}} := \frac{b \cdot h^3}{12} + A \cdot \left( \frac{h}{2} - y \right)^2 = 0.5 \, \text{ft}^4
\]
I of clt

\[
I_{\text{comp}} := I_{\text{con}} + I_{\text{wood}} = 0.8 \, \text{ft}^4
\]
I composite

\[
EI_{\text{comp}} = E_{\text{clt}} \cdot I_{\text{comp}} = (1.3 \cdot 10^6) \, \text{lbf} \cdot \text{ft}^2
\]
EI composite

---

**Frequency Calculations**

**Naming Convention:**

\[X_{a#bcd}\]

- \(a = c\) -> Combined Loading
  - \(u\) -> Uniform Load Only
- \(\#\) -> Mode shape
- \(b = h\) -> Hinged Ends
  - \(f\) -> Fixed Ends
- \(c = b\) -> Bare CLT
  - \(h\) -> CLT Handbook Approximation
  - \(n\) -> Non-composite (DG11)
  - \(c\) -> Composite (DG11)
- \(d = c\) -> Continuous Span
  - \(s\) -> Single Span

Created with Mathcad Express. See www.mathcad.com for more information.
Bare

Lambda Values for modes 1 and 2 from Chen and Wambsganss (1974)

\[ \lambda_{1h} = 9.870 \quad \lambda_{1f} = 22.374 \]
\[ \lambda_{2h} = 39.479 \quad \lambda_{2f} = 61.674 \]

\[ f_{1hbs} := \frac{\lambda_{1h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E I_{app}}{w}} = 13.3 \text{ Hz} \]
1st Natural Frequency of bare CLT simple span with hinged ends (Chen and Wambsganss, 1974)

\[ f_{2hbs} := \frac{\lambda_{2h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E I_{app}}{w}} = 53 \text{ Hz} \]
2nd Natural Frequency of bare CLT simple span with hinged ends (Chen and Wambsganss, 1974)

\[ f_{1fbs} := \frac{\lambda_{1f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E I_{app}}{w}} = 30 \text{ Hz} \]
1st Natural Frequency of bare CLT simple span with fixed ends (Chen and Wambsganss, 1974)

\[ f_{2fbs} := \frac{\lambda_{2f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E I_{app}}{w}} = 82.8 \text{ Hz} \]
2nd Natural Frequency of bare CLT simple span with fixed ends (Chen and Wambsganss, 1974)

\[ f_{1hbs} := \frac{\lambda_{1h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{0.9 \cdot E I_{app}}{w}} = 12.6 \text{ Hz} \]
1st Natural Frequency using Handbook Approximation with hinged ends (Hu and Chui, 2013)

\[ f_{2hbs} := \frac{\lambda_{2h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{0.9 \cdot E I_{app}}{w}} = 50.3 \text{ Hz} \]
2nd Natural Frequency using Handbook Approximation with hinged ends (Hu and Chui, 2013)

\[ f_{1fbs} := \frac{\lambda_{1f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{0.9 \cdot E I_{app}}{w}} = 28.5 \text{ Hz} \]
1st Natural Frequency using Handbook Approximation with fixed ends (Hu and Chui, 2013)

\[ f_{2fbs} := \frac{\lambda_{2f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{0.9 \cdot E I_{app}}{w}} = 78.5 \text{ Hz} \]
2nd Natural Frequency using Handbook Approximation with fixed ends (Hu and Chui, 2013)
Non-Composite

\[ f_{u1_{hns}} = \frac{\lambda_{1h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{app}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 9.7 \text{ Hz} \]

1st Natural Frequency of non-composite section with hinged ends (Chen and Wambsganss, 1974)

\[ f_{u2_{hns}} = \frac{\lambda_{2h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{app}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 38.7 \text{ Hz} \]

2nd Natural Frequency of non-composite section with hinged ends (Chen and Wambsganss, 1974)

\[ f_{u1_{fns}} = \frac{\lambda_{1f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{app}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 21.9 \text{ Hz} \]

1st Natural Frequency of non-composite section with fixed ends (Chen and Wambsganss, 1974)

\[ f_{u2_{fns}} = \frac{\lambda_{2f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{app}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 60.4 \text{ Hz} \]

2nd Natural Frequency of non-composite section with fixed ends (Chen and Wambsganss, 1974)

Composite

\[ f_{u1_{hcs}} = \frac{\lambda_{1h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{comp}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 26.6 \text{ Hz} \]

1st Natural Frequency of composite section with hinged ends (Murray et al. 1997)

\[ f_{u2_{hcs}} = \frac{\lambda_{2h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{comp}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 106.4 \text{ Hz} \]

2nd Natural Frequency of composite section with hinged ends (Murray et al. 1997)

\[ f_{u1_{fcs}} = \frac{\lambda_{1f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{comp}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 60.3 \text{ Hz} \]

1st Natural Frequency of composite section with fixed ends (Murray et al. 1997)

\[ f_{u2_{fcs}} = \frac{\lambda_{2f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E1_{comp}}{w + \frac{Wc}{g} \cdot b \cdot t}} = 166.3 \text{ Hz} \]

2nd Natural Frequency of composite section with fixed ends (Murray et al. 1997)
Continuous Beam
Frequency Factors, Determined Graphically (Chen and Wambsganss, 1974) as shown in Appendix F

<table>
<thead>
<tr>
<th>Hinged</th>
<th>Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{u1hnc} = 9.7 )</td>
<td>( \lambda_{u1fnc} = 12.8 )</td>
</tr>
<tr>
<td>( \lambda_{u2hnc} = 12.8 )</td>
<td>( \lambda_{u2fnc} = 18.3 )</td>
</tr>
</tbody>
</table>

Frequencies found using factors above (Chen and Wambsganss, 1974)

\[
f_{u1hnc} = \frac{\lambda_{u1hnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{opp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 9.5 \text{ Hz} \\
\text{1st Natural Frequency of noncomposite continuous spans with hinged ends}
\]

\[
f_{u2hnc} = \frac{\lambda_{u2hnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{opp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 12.5 \text{ Hz} \\
\text{2nd Natural Frequency of noncomposite continuous spans with hinged ends}
\]

\[
f_{u1fnc} = \frac{\lambda_{u1fnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{opp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 12.5 \text{ Hz} \\
\text{1st Natural Frequency of noncomposite continuous spans with fixed ends}
\]

\[
f_{u2fnc} = \frac{\lambda_{u2fnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{opp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 17.9 \text{ Hz} \\
\text{2nd Natural Frequency of noncomposite continuous spans with fixed ends}
\]

\[
f_{u1hnc} = \frac{\lambda_{u1hnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{comp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 26.1 \text{ Hz} \\
\text{1st Natural Frequency of composite continuous spans with hinged ends}
\]

\[
f_{u2hnc} = \frac{\lambda_{u2hnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{comp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 34.5 \text{ Hz} \\
\text{2nd Natural Frequency of composite continuous spans with hinged ends}
\]

\[
f_{u1fnc} = \frac{\lambda_{u1fnc}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{EI_{comp}}{w^2 + \frac{W_c}{g} \cdot b \cdot t}} = 34.5 \text{ Hz} \\
\text{1st Natural Frequency of composite continuous spans with fixed ends}
\]
\[ f_{u2fec} := \frac{\lambda_{u2fec}}{2 \cdot \pi \cdot L^2} \cdot \sqrt{\frac{E I_{comp}}{w + \frac{W_c}{g} \cdot b \cdot t}} = 49.3 \text{ Hz} \] 

2nd Natural Frequency of composite continuous spans with fixed ends

**Combined Loads to Check**

(Sundararajan, 2009)

\( c_1 := 2 \quad \text{Correction Factor for simply supported} \)

\( c_2 := 2.7 \quad \text{Correction Factor for fixed ends} \)

\[ R := \frac{M}{8 \cdot m} = 1.8 \times 10^{-2} \]

Convert Hz to radians/s

Adjust for combined load, and convert to Hz

\[ f_{c1hbs} := \sqrt{\frac{f_{u1hbs}^2}{1 + c_1 \cdot R}} = 13 \text{ Hz} \quad \text{Fundamental Frequency of bare CLT with hinged ends under combined loading} \]

\[ f_{c1fbs} := \sqrt{\frac{f_{u1fbs}^2}{1 + c_2 \cdot R}} = 29.3 \text{ Hz} \quad \text{Fundamental Frequency of bare CLT with fixed ends under combined loading} \]

\[ f_{c1hbe} := \sqrt{\frac{f_{u1hbs}^2}{1 + c_1 \cdot R}} = 12.34 \text{ Hz} \quad \text{Fundamental Frequency using Handbook estimate for heavy topping with hinged ends under combined loading} \]

\[ f_{c1fhe} := \sqrt{\frac{f_{u1fbs}^2}{1 + c_2 \cdot R}} = 27.81 \text{ Hz} \quad \text{Fundamental Frequency using Handbook estimate for heavy topping with fixed ends under combined loading} \]

\[ f_{c1hns} := \sqrt{\frac{f_{u1hns}^2}{1 + c_1 \cdot R}} = 9.5 \text{ Hz} \quad \text{Fundamental Frequency of non-composite section with hinged ends under combined loading} \]

\[ f_{c1fns} := \sqrt{\frac{f_{u1fns}^2}{1 + c_2 \cdot R}} = 21.39 \text{ Hz} \quad \text{Fundamental Frequency of non-composite section with fixed ends under combined loading} \]

\[ f_{c1hes} := \sqrt{\frac{f_{u1hes}^2}{1 + c_1 \cdot R}} = 26.13 \text{ Hz} \quad \text{Fundamental Frequency of composite section with hinged ends under combined loading} \]
\( f_{c1\text{fcs}} = \sqrt{\frac{f_{u1\text{fcs}}}{1 + c_2 \cdot R}} = 58.87 \text{ Hz} \) Fundamental Frequency of composite section with fixed ends under combined loading

\( f_{c1\text{hns}} = \sqrt{\frac{f_{u1\text{hnc}}}{1 + c_1 \cdot R}} = 9.33 \text{ Hz} \) Fundamental Frequency of continuous span non-composite section with hinged ends under combined loading

\( f_{c1\text{fnc}} = \sqrt{\frac{f_{u1\text{fnc}}}{1 + c_2 \cdot R}} = 12.24 \text{ Hz} \) Fundamental Frequency of continuous span non-composite section with fixed ends under combined loading

\( f_{c1\text{hnc}} = \sqrt{\frac{f_{u1\text{hnc}}}{1 + c_1 \cdot R}} = 25.68 \text{ Hz} \) Fundamental Frequency of continuous span composite section with hinged ends under combined loading

\( f_{c1\text{fnc}} = \sqrt{\frac{f_{u1\text{fnc}}}{1 + c_2 \cdot R}} = 33.68 \text{ Hz} \) Fundamental Frequency of continuous span composite section with hinged ends under combined loading

**Acceptance Criteria Calculations**

\[
d = \frac{(68.56 \text{ lbf}) \cdot L^3}{48 \cdot 0.9 \cdot \frac{EI_{app}}{b \cdot \left(\frac{1}{ft}\right)}} = (2.8 \cdot 10^{-2}) \text{ in}
d\text{ value calculated per Hu and Chui (2013)}
\]

\[
\left(\frac{f_{c1\text{hns}}}{Hz}\right) = 151.4
\]

\[
151.4 > 125.1 \text{ Hu and Chui Acceptance Criterion (2013)}
\]

\[
\left(\frac{d}{\text{in}}\right)^{0.7} = 123.9
\]

\[
\frac{123.9}{125.1} = 0.99 \text{ Hu and Chui Acceptance Criterion (2013) with real frequency}
\]

\[
\frac{65 \text{ lbf} \cdot e^{-0.35 \cdot 10.1}}{L \cdot (g \cdot w + W_c \cdot b \cdot t) \cdot 0.015} = 2.8 \cdot 10^{-2} \text{ % from DG 11 Acceptance Criterion (Murray et al. 2003) for mass of 1 panel}
\]

\[
\frac{65 \text{ lbf} \cdot e^{-0.35 \cdot 10.1}}{L \cdot 8 \cdot (g \cdot w + W_c \cdot b \cdot t) \cdot 0.015} = 3.5 \cdot 10^{-3} \text{ % from DG 11 Acceptance Criterion (Murray et al. 2003) for mass of 8 panels}
\]

\[
\frac{65 \text{ lbf} \cdot e^{-0.35 \cdot 10.1}}{L \cdot 3 \cdot (g \cdot w + W_c \cdot b \cdot t) \cdot 0.015} = 9.4 \cdot 10^{-3} \text{ % from DG 11 Acceptance Criterion (Murray et al. 2003) for mass of 3 spans}
\]
Appendix F

Graphical Method for Finding Natural Frequencies of Continuous Beam

In this appendix, the use of the graphical method as described by Chen and Wambsganss (1974) is demonstrated to obtain the frequency factor for the first mode assuming both hinged and fixed end conditions of equal span continuous beams. To obtain these factors, the vertical axis is divided into equal parts, one for each span. This results in four horizontal lines, including the top and bottom boundaries of the graph, which are highlighted in red for this example, labeled from bottom to top as 1-4. The projection of the intersection of each horizontal line and the line representing the appropriate $\Gamma$ value (which is a function of the axial tension in the beam, and is, for the purposes of this study, equal to 0) from the first propagation band represents a frequency function. Lines showing these projections are highlighted here in blue. For hinged end conditions the frequency factors are indicated by projections 1-3, while for fixed end conditions the frequency factors are indicated by projections 2-4. As can be seen in the graph on the following page, this method results in frequency factors of 9.7, 12.8, and 18.3 for hinged end conditions, and 12.8, 18.3, and 22.3 for fixed end conditions.
Fig. 3. $p-\lambda$ curves for straight beams; first and second propagation bands
Appendix G

Shop Drawings and Material Specs

Note for 3000 PSI LW 3/8 and 4000 PSI LW 3/8 mixes: Both of these mixes are lightweight mixes. The subcontractor is unsure whether the topping over slab interior mix should be 3000 or 4000. Please which or if we have misunderstood the design intent.

November 1, 2018

P.O. Box 39 Six Mile, SC 29682
PHONE: (864) 868.9882 FAX: (864) 868.9887

Sherman Construction
H & H Concrete
Clemson University Outdoor Fitness and Wellness Center
Clemson, SC

2399 Norris Highway Central, SC 29630
E-mail: dcrosby@metroconinc.net

3000 PSI LW 3/8
Mix Number: 3000 L/W 3/8 Stone

Cement- Type I/II National Cement- Ragland, AL 520 lbs
Fly Ash- Class F Pozzolans Southeastern Fly Ash Cross Station- Cross, SC 120 lbs
Sand- Martin Marietta Materials – Edmund, SC 1385 lbs
*ASTM C-33, ASTM C-40, ASTM C-117, ASTM C-136
3/8 Stalite- Carolina Stalite 850 lbs
Sika 300 GP Water Reducer- Sika Corporation 4 oz per 100# Cement
Water- Six Mile Water District 33 gallons
CONCRETE CYLINDER TEST REPORT

Mix Design No.: 3000LW  
Actual Slump (C143):  
Actual Air Content (C173/C231):  
Air Temperature:  
Unit Weight (C138):  
Admixtures: N/A  
Gallons Water Added On Site: 0

<table>
<thead>
<tr>
<th>Cylinder Sample</th>
<th>Test Date</th>
<th>Age (days)</th>
<th>Load (lbs)</th>
<th>Dia. (in)</th>
<th>Area (sq. in)</th>
<th>Strength (ksi)</th>
<th>Type of Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5/6/15</td>
<td>7</td>
<td>129,780</td>
<td>6.00</td>
<td>28.27</td>
<td>4,590</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>5/27/15</td>
<td>28</td>
<td>162,760</td>
<td>6.00</td>
<td>28.27</td>
<td>5,760</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>5/27/15</td>
<td>28</td>
<td>162,880</td>
<td>6.00</td>
<td>28.27</td>
<td>5,760</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IB MAX-CORE CLT DESIGN VALUES

APA Product Report® PR-L327  
Issued December 3, 2018

Table 1. ASD Reference Design Values(a) for Lumber Laminations Used in IB MAX-CORE CLT (for Use in the U.S.)

<table>
<thead>
<tr>
<th>CLT Layup</th>
<th>F0 (ksi)</th>
<th>E (10^6 psi)</th>
<th>F1 (ksi)</th>
<th>F2 (ksi)</th>
<th>F3 (ksi)</th>
<th>F4 (ksi)</th>
<th>F5 (ksi)</th>
<th>E (10^6 psi)</th>
<th>F6 (ksi)</th>
<th>F7 (ksi)</th>
<th>F8 (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V3</td>
<td>750</td>
<td>1.4</td>
<td>450</td>
<td>1,250</td>
<td>175</td>
<td>55</td>
<td>450</td>
<td>1.3</td>
<td>250</td>
<td>725</td>
<td>175</td>
</tr>
</tbody>
</table>

For SI: 1 ksi = 0.006895 MPa

(a) Tabulated values are allowable design values and not permitted to be increased for the lumber flat use or size factor in accordance with the NDS. The design values shall be used in conjunction with the section properties provided by the CLT manufacturer based on the actual layup used in manufacturing the CLT panel (see Table 2).

Table 2. ASD Reference Design Values(a) for IB MAX-CORE CLT Listed in Table 1 (for Use in the U.S.)

<table>
<thead>
<tr>
<th>CLT Layup</th>
<th>Layup ID(c)</th>
<th>Thickness, t, (in)</th>
<th>Lamination Thickness (in.) in CLT Layup</th>
<th>Major Strength Direction</th>
<th>Minor Strength Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>V3</td>
<td></td>
<td></td>
<td>=</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>4/18 1/38 1/38 1/38</td>
<td>1/38 1/38 1/38 1/38</td>
<td>1,740</td>
<td>95</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>6/78 1/38 1/38 1/38</td>
<td>1/38 1/38 1/38 1/38</td>
<td>4,000</td>
<td>363</td>
</tr>
</tbody>
</table>

For SI: 1 in = 25.4 mm; 1 ft = 0.3048 m; 1 lbf = 4.448N

(a) Tabulated values are allowable design values and not permitted to be increased for the lumber flat use or size factor in accordance with the NDS.

(b) The CLT layup is developed based on ANSI/APA PRG 320, as permitted by the standard.

(c) The layup identification (ID) refers to the number of layers.

(d) The V3 layup is as specified in ANSI/APA PRG 320 using No. 2 Southern pine lumber in the major strength direction and No. 3 Southern pine lumber in the minor strength direction.
<table>
<thead>
<tr>
<th>No. production list</th>
<th>Name</th>
<th>Material</th>
<th>U.V.P.</th>
<th>APPEARANCE</th>
<th>R.T. width</th>
<th>R.T. height</th>
<th>R.T. length</th>
<th>Quantity</th>
<th>Total retail weight [lb]</th>
<th>TRUCK No.</th>
<th>EDGES</th>
<th>Lifting</th>
<th>No. Lifting screes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>10'-4 5/8&quot;</td>
<td>1</td>
<td>2932</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 4</td>
<td>4 (2)</td>
</tr>
<tr>
<td>2</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>49'-0&quot;</td>
<td>1</td>
<td>7554</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 1</td>
<td>8 (4)</td>
</tr>
<tr>
<td>3</td>
<td>TERRACE</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>19'-0&quot;</td>
<td>1</td>
<td>2768</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 5</td>
<td>4 (2)</td>
</tr>
<tr>
<td>4</td>
<td>TERRACE</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>19'-0&quot;</td>
<td>1</td>
<td>2083</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 5</td>
<td>4 (2)</td>
</tr>
<tr>
<td>5</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>49'-0&quot;</td>
<td>1</td>
<td>7642</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 1</td>
<td>8 (4)</td>
</tr>
<tr>
<td>6</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>21'-11 1/8&quot;</td>
<td>1</td>
<td>3302</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 4</td>
<td>4 (2)</td>
</tr>
<tr>
<td>7</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>29'-0&quot;</td>
<td>1</td>
<td>4030</td>
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<td>1/8 EASED</td>
<td>Type 10</td>
<td>8 (4)</td>
</tr>
<tr>
<td>8</td>
<td>ELEVATED FLOOR</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>49'-0&quot;</td>
<td>1</td>
<td>7642</td>
<td>1</td>
<td>1/8 EASED</td>
<td>Type 1</td>
<td>8 (4)</td>
</tr>
<tr>
<td>9</td>
<td>TERRACE</td>
<td>IB MAX-CORE QT</td>
<td>Soh TL</td>
<td>ARCH-1</td>
<td>7'-0&quot;</td>
<td>6 7/8&quot;</td>
<td>19'-0&quot;</td>
<td>1</td>
<td>2933</td>
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</tr>
<tr>
<td>500</td>
<td>Spine</td>
<td>Plywood</td>
<td>CART 1</td>
<td>SHEATHING</td>
<td>5 7/8&quot;</td>
<td>3/4&quot;</td>
<td>25'-10&quot;</td>
<td>1</td>
<td>2723</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ALL STEEL BY OTHERS.
CLT CONNECTORS NOT INCLUDED UNLESS OTHERWISE NOTED.
Appendix H

MATLAB Scripts
%Written by Mengzhe Gu

function [accelerance_magnitude, phase_angle, accelerance_frequency, ... 
  accelerance] = Comp_accelerance(SampleRate, 
  t,signal_nominator,... 
  signal_denominator) 
  T=1/SampleRate; 
  L=length(signal_nominator); 
  
  Y1 = fft(signal_nominator); %acc 
  Y2 = fft(signal_denominator); %force 
  A=Y1./Y2; 
  
  % Calculate phases [-pi pi] 
  phase2=angle(A); 
  phase1=phase2(1:L/2+1); 
  
  % Calculate accelerance 
  P2_acc=abs(Y1/L); 
  P1_acc=P2_acc(1:L/2+1); 
  P1_acc(2:end-1)=2*P1_acc(2:end-1); 
  
  P2_force=abs(Y2/L); 
  P1_force=P2_force(1:L/2+1); 
  P1_force(2:end-1)=2*P1_force(2:end-1); 
  
  % Calculate frequency vector 
  f=SampleRate*(0:(L/2))/L; 
  
  accelerance_frequency=f; 
  accelerance_magnitude=P1_acc./P1_force; 
  phase_angle=phase1; 
  
  % Combine as complex accelerance 
  accelerance=accelerance_magnitude.*exp(phase_angle*1i); 

end

Published with MATLAB® R2019b
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% Script for calculating FFT, FRF, Coherence, and Mode Shapes from
modal
% analysis
% Written by Ben Schwendy
% 6/26/2020 in MATLAB vR2019b

clear
close all
clic

Fs = 1652;       % Sampling frequency
T = 1/Fs;        % Sampling period

Read Data

This section is used to read in and organize the data. Once done, the data can be save as a variable, and
loaded in on future runs to save time

% HeelDrop = struct();
% for i=1:17 % Initiate loop to run for every data set
%    Input = xlsread(['column' num2str(i) '.xlax']); % Read in data
%    Time = Input(:,1); % Create time vector
%    % Separate, combine and calibrate force readings
%    Force = (Input(:,2)+Input(:,4)+Input(:,6))*6.636-9.95;
%    % Locate each individual heel drop
%    [peak,loc] = findpeaks(Force,'MinPeakProminence',100);
%    for j = 1:5 % Initiate loop to isolate each heel drop in the set
%        % trim data to uniform window around heel drop
%        cutoff(1,2*j-1) = (loc(j)-2500);
%        cutoff(1,2*j) = (loc(j)+57499);
%        end
%    end
%    for ii=1:60 % Initiate loop to isolate each accelerometer
%        response to
%        % individual heel drops
% Determine Y location of accelerometer
row = fix((ii-1)/5);
row=row+1;
% Filter and store each individual response in intermediate
struct
  fullHeelDrop((i-1)*60+ii).Acc = lowpass(Input(i,...
    (rem(ii,5)+1)*2-1).cutoff(i,(rem(ii,5)+1)*2),2*row
+6),60,Fs);
  fullHeelDrop((i-1)*60+ii).Force = lowpass(Force(i,...
    (rem(ii,5)+1)*2-1).cutoff(i,(rem(ii,5)+1)*2)),60,Fs);
end
end
% Average response to five heel drops to creat struct with
% accelerometers mean response for processing
for i = 1:204 % Initiate loop for total number of nodes
  HeelDrop(i).Acc = (fullHeelDrop(5*(i-1)+1).Acc
+fullHeelDrop(5*(i-1)+2).Acc+fullHeelDrop(5*(i-1)+3).Acc
+fullHeelDrop(5*(i-1)+4).Acc+fullHeelDrop(5*(i-1)+5).Acc)./5;
  HeelDrop(i).Force = (fullHeelDrop(5*(i-1)+1).Force
+fullHeelDrop(5*(i-1)+2).Force+fullHeelDrop(5*(i-1)+3).Force
+fullHeelDrop(5*(i-1)+4).Force+fullHeelDrop(5*(i-1)+5).Force)./5;
end

Load Data
Speeds up runtime once data has been read once and saved as vars
load('HeelDrop.mat')
load('fullHeelDrop.mat')

Acc fft
Calculate FFTs for determination of natural frequencies
for i=1:length(HeelDrop) % Initiate loop to run for every data set
  temp=(HeelDrop(i).Acc-mean(HeelDrop(i).Acc)); % Signal to
  transform
  L = numel(temp); % Length of signal
  t = (0:L-1)*T; % Time vector
  Y=fft(temp); % Take FFT
  P2 = abs(Y/L); % Magnitude of FFT
  P1 = P2(1:floor(L/2)+1); % Limit to positive frequencies
  P1(2:end-1) = 2*P1(2:end-1); % Double power to account for
  negatives
  f = Fs*(0:(L/2))/L; % Set frequency bins
  HeelDrop(i).f = f; % Store frequency bins
  HeelDrop(i).P1 = P1; % Store formatted magnitude
  HeelDrop(i).ff = Y; % Store full FFT
end

**Coherence Functions**

Calculate and store coherence and frequency bins

```matlab
for i = 1:length(HeelDrop)
    [HeelDrop(i).Coherence,HeelDrop(i).fc] = mscohere(HeelDrop(i).Acc,...
        HeelDrop(i).Force, [], [], [], Fs);
end
```

**FRFs**

Use Mengzhe Gu's function to calculate and store FRF data

```matlab
for i = 1:length(HeelDrop)
    [HeelDrop(i).acceleration_magnitude, HeelDrop(i).phase_angle, ...
        HeelDrop(i).acceleration_frequency, HeelDrop(i).FRF] = ...
    Comp_acceleration(Fs, length(HeelDrop(i).Force), HeelDrop(i).Acc, ...%
        HeelDrop(i).Force);
end
```

**Acquire Peaks**

This section of code can be used to automatically locate peaks for i = 1:length(HeelDrop) [temp,HeelDrop(i).locs] = findpeaks(HeelDrop(i).P.t(57:end), HeelDrop(i).f(57:end),MinPeakDistance', 1, ...
'MinPeakProminence',10*mean(HeelDrop(i).P.t(57:end)), 'NPeaks', 5); end for i = 1:length(HeelDrop)
HeelDrop(i).Peak1 = HeelDrop(i).locs(HeelDrop(i).locs >= 7.5 ... & HeelDrop(i).locs < 9); HeelDrop(i).Peak2 = HeelDrop(i).locs(HeelDrop(i).locs >= 9 ... & HeelDrop(i).locs < 11); HeelDrop(i).Peak3 = HeelDrop(i).locs(HeelDrop(i).locs >= 11 ... & HeelDrop(i).locs < 13); HeelDrop(i).Peak4 = HeelDrop(i).locs(HeelDrop(i).locs >= 14 ... & HeelDrop(i).locs < 16); if isempty(HeelDrop(i).Peak1)
Peak1(i) = 0; else Peak1(i) = HeelDrop(i).Peak1; end if isempty(HeelDrop(i).Peak2)
Peak2(i) = 0; else Peak2(i) = HeelDrop(i).Peak2; end if isempty(HeelDrop(i).Peak3)
Peak3(i) = 0; else Peak3(i) = HeelDrop(i).Peak3; end if isempty(HeelDrop(i).Peak4)
Peak4(i) = 0; else Peak4(i) = HeelDrop(i).Peak4; end

% Peaks =
%    [mean(nonzeros(Peak1));mean(nonzeros(Peak2));mean(nonzeros...
%    (Peak3));mean(nonzeros(Peak4))];

**Select Peaks**

This section of code is used to manually select peaks

```matlab
Peaks = [8.37, 8.756, 10.1, 12.14, 12.67, 15.03]; % Set peaks as natural frequencies
for i = 1:6 % Initiate loop for number of peaks used
    % Find closest frequency bin
    [~, in(i)] = min(abs(HeelDrop(5).acceleration_frequency-Peaks(i)));
end
Peaks = HeelDrop(5).acceleration_frequency(in);
```
Set Geometric Location of Each Node

\[ x = [0:24:384]; \] % X grid lines
\[ y = [0,-27,-54,-81,-120,-147,-174,-186,-213,-240,-267]; \] % Y grid lines

Acquire Modal Positions

```matlab
for i = 1:length(HeelDrop)
    P2 = imag(HeelDrop(i).FRF); % Acquire angle of FRF
    P1 = P2(1:floor(L/2)+1); % Limit to positive frequencies
    P1(end-1) = 2*P1(end-1); % Double power to account for
    negatives
    f = Fs*(0:(L/2))/L; % Set frequency bins
    HeelDrop(i).mode = P1; % Store formatted angle
    HeelDrop(i).fmode = f; % Store frequency bins
    HeelDrop(i).Model1 = HeelDrop(i).mode(in(1)); % Store mode 1
    disp
    HeelDrop(i).Model2 = HeelDrop(i).mode(in(2)); % Store mode 2
    disp
    HeelDrop(i).Model3 = HeelDrop(i).mode(in(3)); % Store mode 3
    disp
    HeelDrop(i).Model4 = HeelDrop(i).mode(in(4)); % Store mode 4
    disp
    HeelDrop(i).Model5 = HeelDrop(i).mode(in(5)); % Store mode 5
    disp
    HeelDrop(i).Model6 = HeelDrop(i).mode(in(6)); % Store mode 6
    disp
end
```

Plot Mode Shape

```matlab
figure; % Open new figure to prevent overwrites

% These lines can be used in conjunction with sections further down to
% plot mode shapes in premade groupings with all three main views in
% tile layout
%tilelayout(3,1)
exttile

% Plot Mode 1
Model1 = surf(x,y,reshape([HeelDrop.Model1],12,17),'FaceAlpha', 0.2,...
    'EdgeColor', 'r'); % Plots surface of Mode 1 after reshaping all
Mode
    % 1 node values as one vector
hold on; surf([-6,390],[6,-87,-180,-273],
    [0,0,0,0,0,0,0,0,0],'FaceAlpha'...
    , 0.1, 'FaceColor', 'k', 'EdgeColor', 'k') % Overlays panel
positions
```
% Can be used to plot undeformed glulams
% plot3([2.5,2.5], [6,-273], [0,0], 'LineWidth', 1, 'Color', 'b')
% plot3([183.5,183.5], [6,-273], [0,0], 'LineWidth', 1, 'Color', 'b')
% plot3([200.5,200.5], [6,-273], [0,0], 'LineWidth', 1, 'Color', 'b')
% plot3([381.5,381.5], [6,-273], [0,0], 'LineWidth', 1, 'Color', 'b')

% Plot deformed glulams by interpolating
plot3([zeros(1,12)+2.5], [y],
[2.5/24 *[HeelDrop(13:24).Model]+21.5/24* ... 
  [HeelDrop(1:12).Model]], 'LineWidth', 1.5, 'Color', 'b')
plot3([zeros(1,12)+183.5], [y],
[15.5/24 *[HeelDrop(97:108).Model]+8.5/24* ... 
  [HeelDrop(85:96).Model]], 'LineWidth', 1.5, 'Color', 'b')
plot3([zeros(1,12)+200.5], [y],
[15.5/24 *[HeelDrop(97:108).Model]+8.5/24* ... 
  [HeelDrop(109:120).Model]], 'LineWidth', 1.5, 'Color', 'b')
plot3([zeros(1,12)+381.5], [y],
[2.5/24 *[HeelDrop(181:192).Model]+21.5/ ... 
24 *[HeelDrop(193:204).Model]], 'LineWidth', 1.5, 'Color', 'b')

% Format plot
xlabel('X Position (in)') % Label X axis
ylabel('Y Position (in)') % Label Y axis
zlabel('Relative Deflection') % Label Z axis
ax = gca; % Open axis properties
ax.FontSize = 12; % Set font size
view(3) % Set view

% These sections can be used in conjunction with lines further up to
% plot mode shapes in premade groupings with all three main views in a
% tile layout
% nexttile
% surf(x,y,reshape([HeelDrop.Model],12,17),'FaceAlpha', 0.2, ...
% 'EdgeColor', 'r');
% hold on; surf([-6,454], [6,-87,-180,-273], [0,0;0,0;0,0;0,0],...
% 'FaceAlpha', 0.1, 'FaceColor', 'k', 'EdgeColor', 'k')
% xlabel('X Position (in)')
% ylabel('Y Position (in)')
% zlabel('Relative Deflection')
% ax = gca;
% ax.FontSize = 10;
% view(90,0)
%
% nexttile
% surf(x,y,reshape([HeelDrop.Model],12,17),'FaceAlpha', 0.2, ...
% 'EdgeColor', 'r');
% hold on; surf([-6,454], [6,-87,-180,-273], [0,0;0,0;0,0;0,0],...
% 'FaceAlpha', 0.1, 'FaceColor', 'k', 'EdgeColor', 'k')
% xlabel('X Position (in)')
% ylabel('Y Position (in)')
% zlabel('Relative Deflection')
% ax = gca;
% ax.FontSize = 10;
% view(0,0)
Plot Remaining Mode Shapes

This section, while arranged slightly differently, is the same as the section above, repeated, to plot the rest of the modes. For comments explaining each line, please refer to previous section.

```matlab
figure; Mode2 = surf(x,y,reshape([HeelDrop.Mode2,12,17]),'FaceAlpha',...
0.2,'FaceColor', 'r');
hold on; surf([-6,350],[6,-87,-180,-273],
[0,0,0,0,0,0,0,0], 'FaceAlpha',... 
0.1,'FaceColor', 'k', 'EdgeColor', 'k')
plot3(izeros(1,12)+2.5, [y],
[HeelDrop(1:12).*Mode2], 'LineWidth',1.5,'Color','b')
plot3([183.5,183.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
plot3(izeros(1,12)+183.5, [y],
[15.5/24*[HeelDrop(97:108).*Mode2]+8.5/24*... 
[HeelDrop(85:96).*Mode2], 'LineWidth',1.5,'Color','b')
plot3([200.5,200.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
plot3(izeros(1,12)+200.5, [y],
[15.5/24*[HeelDrop(97:108).*Mode2]+8.5/24*... 
[HeelDrop(109:120).*Mode2], 'LineWidth',1.5,'Color','b')
plot3([381.5,381.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
plot3(izeros(1,12)+381.5, [y],
[2.5/24*[HeelDrop(181:192).*Mode2]+21.5/... 
24*[HeelDrop(193:204).*Mode2], 'LineWidth',1.5,'Color','b')
xlabel('X Position (in)')
ylabel('Y Position (in)')
zlabel('Relative Deflection')
ax = gca;
ax.FontSize = 12;
view(3);
figure; Mode3 = 
surf(x,y,reshape([HeelDrop.Mode3,12,17]),'FaceAlpha',...
0.2,'EdgeColor', 'r');
hold on; surf([-6,350],[6,-87,-180,-273],
[0,0,0,0,0,0,0,0], 'FaceAlpha',... 
0.1,'FaceColor', 'k', 'EdgeColor', 'k')
plot3(izeros(1,12)+2.5, [y],
[HeelDrop(1:12).*Mode3], 'LineWidth',1.5,'Color','b')
plot3([183.5,183.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
plot3(izeros(1,12)+183.5, [y],
[15.5/24*[HeelDrop(97:108).*Mode3]+8.5/24*... 
[HeelDrop(85:96).*Mode3], 'LineWidth',1.5,'Color','b')
plot3([200.5,200.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
plot3(izeros(1,12)+200.5, [y],
[15.5/24*[HeelDrop(97:108).*Mode3]+8.5/24*... 
[HeelDrop(109:120).*Mode3], 'LineWidth',1.5,'Color','b')
plot3([381.5,381.5], [6,-273], [0,0], 'LineWidth',1,'Color','b')
```
```python
plot3([zeros(1,12)+381.5],[y],
       24*[HeelDrop[193:204].Mode3]],'LineWidth',1.5,'Color','b')
xlabel('X Position (in)')
ylabel('Y Position (in)')
zlabel('Relative Deflection')
ax = gca;
ax.FontSize = 12;
view(3)

figure; Mode4 =
surf(x,y,reshape(HeelDrop.Mode4,12,17), 'FaceAlpha',...
     0.2,'EdgeColor','r');
hold on; surf([-6.390],[6,-87,-180,-273],
      [0,0;0,0;0,0],[y],
      0.1,'FaceColor','k','EdgeColor','k')
plot3([zeros(1,12)+2.5],[y],
       HeelDrop[1:12].Mode4]],'LineWidth',1.5,'Color','b')
plot3([zeros(1,12)+183.5],[y],
       HeelDrop[85:96].Mode4]],'LineWidth',1.5,'Color','b')
plot3([zeros(1,12)+200.5],[y],
       HeelDrop[109:120].Mode4]],'LineWidth',1.5,'Color','b')
plot3([zeros(1,12)+381.5],[y],
       HeelDrop[1:12].Mode5]],'LineWidth',1.5,'Color','b')
plot3([zeros(1,12)+183.5],[y],
       HeelDrop[85:96].Mode5]],'LineWidth',1.5,'Color','b')
```
Composite FRF

This section combines the FRFs of all of the nodes to create composite FRFs. This was done for each span individually, and for the both spans together.

```matlab
for i = 1:108 % Initiate loop for all nodes of one span
    % Create matrix of all FRF data from span 1
    FRF1(:,i) = HeelDrop(i).acceleration_magnitude*386;
    % Create matrix of all FRF data from span 2
    FRF2(:,i) = HeelDrop(i+96).acceleration_magnitude*386;
```
end

for i = 1:204 % Initiate loop for all nodes
    % Create matrix of all FRF data
    FRF(:,i) = HeelDrop{i}.acceleration_magnitude*386;
end

for i = 1:length(FRF1) % Initiate loop for each row of FRF data
    FRF1(i,109) = mean(FRF1(i,:)); % Calculate average FRF for span 1
    FRF2(i,109) = mean(FRF2(i,:)); % Calculate average FRF for span 2
    FRF(i,205) = mean(FRF(i,:)); % Calculate average FRF for all
    end

end

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Determine and print maximum RMS acc experienced from each test ............................. 2

% Script for determining RMS acceleration from set of accelerometer time
% histories
% Written by Ben Schwendy
% 6/26/2020 in MATLAB vR2019b
%
clear
clc

Set sampling rate and time per sample

Fs = 1652; % Sampling rate of data
T = 1/Fs; % Time represented by each sample

Read in Data

This section is used to read in and organize the data. Once done, the data can be save as a variable, and
loaded in on future runs to save time

% WalkingExcite = struct(); % Dimensions WalkingExcite as a structure
% in
% preparation to receive data
%
% for i = 1:5 % Initiates loop to read in data from all five tests
% names = {'4', '5', '9', '13at8Hz', '13at10Hz'};
% Input = xlsread([\'WalkingColumn\' names{i} '.xlsx']);
% Time = Input(:,1); % Creates time vector
%
% for ii=1:12 % Initiates loop to separate each accelerometer
%   FullWalkingExcite((i-1)*12+ii).Acc = lowpass(Input(:,2*ii
% +6)...]
% ,50,Fs); % Stores each accelerometers data in its own
% line of
%   struct
%   WalkingExcite((i-1)*12+ii).Acc = FullWalkingExcite((i-1)*...
% 12+ii).Acc((length(FullWalkingExcite((i-1)*12+ii).Acc)/2-...
% 20000):
% (length(FullWalkingExcite((i-1)*12+ii).Acc)/2+19999));
% % Separates out middle portion of data (time-wise) as
% % representative sample set
% end
% end

Load Data

load('WalkingExcite.mat')

Calculate RMS at each

for i = 1:length(WalkingExcite) % Initiates loop to run for each row
    WalkingExcite(i).rms = rms(WalkingExcite(i).Acc); % Calcs RMS
    RMS{i} = WalkingExcite(i).rms; % Creates separate RMS var from struct
end

Determine and print maximum RMS acc experienced from each test

MRMS = [max(RMS(1:12)), max(RMS(13:24)), max(RMS(25:36)), ..., max(RMS(37:48)), max(RMS(49:60))]
Appendix I

Glulam Frequency Calculations
Calculating the Fundamental Natural Frequency of Glulams

Ben Schwendy

Inputs

Table A1 - Design Values for Structural Glued Laminated Softwood Timber Combinations

(1) Members stressed primarily in bending. (Tabulated design values are for normal load duration and dry service conditions. See Section 3 for a comprehensive description of design value adjustment factors.)

<table>
<thead>
<tr>
<th>Combination</th>
<th>Species</th>
<th>Stress Class</th>
<th>Fy (ksi)</th>
<th>fy (ksi)</th>
<th>Fv,M (ksi)</th>
<th>Fv,Y (ksi)</th>
<th>Fy,V (ksi)</th>
<th>E</th>
<th>G</th>
<th>Specific Gravity for Fascia Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>26F-1.8SF</td>
<td>SPF1</td>
<td>SPF2/3</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
<td>2100</td>
</tr>
</tbody>
</table>

153
\[ E := 1900000 \text{ psi} \]

\[ I := \frac{5 \text{ in} \cdot (27.5 \text{ in})^3}{12} = 0.418 \text{ ft}^4 \]

\[ L := 27.5 \text{ ft} \]

\[ t := 2 \text{ in} \]

\[ W_c := 110 \text{ pcf} \]

\[ \text{trib} := 14.8 \text{ ft} \]

\[ W_{clt} := 19.92 \text{ psf} \]

\[ W_{glu} := 5 \text{ in} \cdot 27.5 \text{ in} \cdot 0.55 \cdot 62.5 \text{ pcf} = 32.823 \text{ plf} \]

\[ w := W_c \cdot \text{trib} + W_{clt} \cdot \text{trib} + W_{glu} = 598.973 \text{ plf} \]

Modulus of elasticity of CLT
Moment of inertia of CLT section
Span length
Topping thickness
Unit weight of concrete
Tributary width
Pounds per square foot of CLT
Pounds per foot of Glulam
Mass per unit length
Frequency Calculations

Lambda Values for modes 1 through 5

hinged ends fixed ends
\( \lambda_{1h} := 9.870 \) \( \lambda_{1f} := 22.374 \)
\( \lambda_{2h} := 39.479 \) \( \lambda_{2f} := 61.674 \)

Natural Frequencies with Hinged Ends

\( f_{1bh} := \frac{\lambda_{1h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E \cdot I}{w}} = 5.148 \text{ Hz} \)
\( f_{2bh} := \frac{\lambda_{2h}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E \cdot I}{w}} = 20.59 \text{ Hz} \)

Natural Frequencies with Fixed Ends

\( f_{1bf} := \frac{\lambda_{1f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E \cdot I}{w}} = 11.669 \text{ Hz} \)
\( f_{2bf} := \frac{\lambda_{2f}}{2 \cdot \pi \cdot L^2} \sqrt{\frac{E \cdot I}{w}} = 32.166 \text{ Hz} \)
References


