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Dynamics of a Vibration Driven Bristle Bot

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DYNAMICS OF A VIBRATION DRIVEN BRISTLE BOT

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Chandravamsi Gandra
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Accepted by:
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Abstract

Vibration driven robots utilize periodic forced vibration of an internal mass to achieve directed locomotion. Bristle bots are a class of vibration driven robots which are characterized by the presence of bristles or cilia on their surface and contain an internal mass that is driven to oscillate at a high frequency. Besides well known applications in investigating swarming behavior, such robots have potential applications in rescue operations in rubble, inspections of pipes and other inaccessible confined areas and in medical devices where conventional means of locomotion is ineffective. Bristle bot or its commercially available variants such as hexbugs are popular toy robots. Despite the apparent simplicity of these robots, their dynamic behavior is very complex. Vibration robots have attracted surprisingly few analytical models all which can only explain some regimes of locomotion. In this work, a wide range of motion dynamics of a bristlebot is explored using a mathematical model which accounts for slip-stick motion of the bristles with the substrate. Analytical conditions for the system to exhibit a particular type of motion are formulated and the system of equations defining the motion are solved numerically using these conditions. The numerical simulations show transitions in the kinds of locomotion of a bristlebot as a function of the forcing frequency and amplitude. These different kinds of locomotion include stick-slip and pure slip motions along with the important phenomenon of the reversal of the direction of motion of the robot. In certain ranges of frequencies, the robot can lose contact with the ground and ‘jump’. These different regimes of locomotion are a result of the nonlinear vibrations of the robot and the friction between the robot’s bristles and the ground. The results of this work can potentially lead to more versatile vibration robots with predictable and controllable dynamics.

Dedication

I would like to dedicate this thesis to my parents who have always been my source of strength and motivation.

Acknowledgments

First of all, I would like to thank my uncle **Mr. Narsing Rao Panagari** for encouraging me to pursue my dream of higher education. Next, I would like to express my sincere gratitude to my advisor **Dr. Phanindra Tallapragada** for his constant support and guidance to complete my research. I would like to thank my committee members **Dr. Yue Wang** and **Dr. Suyi Li** for their time in reviewing this work. I also would like to thank Clemson University and the Mechanical Engineering department for providing all the required facilities to carry out my research, especially the Palmetto cluster. Lastly, I would like to thank my family and friends for their unwavering support and encouragement. A special thanks to my roommates here at Clemson, who have been like a family away from home.

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Chapter 1

Introduction

Mobile robots employ different locomotion strategies in order to move through its environment. Majority of the robots utilize legged, wheeled or track type locomotion systems and sometimes a combination of these depending on the functionality of the robot. However, these conventional solutions for locomotion are not easily scalable at a micro-level as they involve more number of moving parts. For applications in inspection technology, rescue operations and in medical devices; where the robots have to navigate through narrow passages, tubes, or blood vessels, vibration driven locomotion systems are a potential solution. Such systems are simple to design and employ minimum number of moving parts. They however achieve directed locomotion owing to the periodic excitation of internal masses in the presence of asymmetric interactive forces between the system and the substrate.

1.1 Motion Principle

Vibration driven systems are characterized by the presence of an internal point mass excited by a motor and a power source. The forces produced due to the excitation of the internal point mass are transferred to the main body. As the main body interacts with the surrounding environment, governed by some resistance law, it results in interactive forces between the two. It is the asymmetry in these interactive forces that causes the system to propel in one direction.

1.1.1 Asymmetry in resistive forces

The asymmetric interactive forces can be a result of either anisotropic resistive property of the environment or an asymmetry in physical property of the system. Figure 1.1 provides an example of a two body vibration driven system placed on a surface with anisotropic friction coefficients. Here the internal point mass moves back and forth along a horizontal line within the main body mass and generates periodic forces parallel to its motion. The resistive forces offered by the surface are relatively lower in one direction compared to the other and as a result the system advances in a direction with lower resistive force.

Figure 1.1: An example of two body vibration driven system in the presence of anisotropic dry friction

An example of a vibration driven system using an asymmetry in its physical property to achieve directed locomotion is presented in Figure . Here the asymmetry is provided by the bristles or cilia attached at an angle to the main body mass. Under the influence of an internal point mass vibrating in the vertical direction, the bristles interact with the surface to generate asymmetric normal forces over one cycle. Since the kinetic friction offered by a surface is proportional to the normal reaction ($f = \mu_k N$), they consequently lead to the generation of asymmetric frictional forces over one cycle and hence the system achieves locomotion. The bristles or cilia can also mechanically interlock with the surface to provide anisotropy in friction coefficients. However, the asymmetry in normal reaction caused by the bristle angle alone can be responsible for achieving locomotion.

Figure 1.2: An example of two body vibration driven system with bristles

1.2 Slip-Stick dynamics

Vibration driven robots exhibit Slip-Stick motion behaviour under certain conditions. Over one full cycle motion of the internal vibrating mass, the relative motion between the system and the surrounding exists only some part of the cycle. As an example, for the system presented in Figure 1.1 if the friction coefficients μ_+ and μ_- are such that the inertial forces due to the vibrating mass never exceed the maximum static friction ($\mu_+ N$) offered by the surface only in the positive direction, the system remains at rest throughout the negative directional motion of the internal point mass. However, the system begins to slide backwards when the internal point mass changes its direction of motion and moves forward resulting in a net negative locomotion.

For a system with bristles presented in Figure 1.2 the Slip-Stick motion is a bit more complex as the magnitude of maximum static friction ($\mu_s N$) offered by the surface varies at each instant, proportional to the Normal reaction. The bristles Slip or Stick to the surface depending on the relative values of maximum static friction and the horizontal component of the force applied by the system on the surface. The motion becomes even more involved when the system is excited by a rotating mass with acceleration components in X and Y directions. The aim of this study is to model and explore the Slip-Stick motion dynamics of such a system across a forcing frequency and amplitude spectrum.

1.3 Applications of vibration driven bots

Vibration driven bots find applications where the conventional means of locomotion cannot be employed or is ineffective. They can be used as pipeline inspection robots to detect small cracks and leakages. Robots with long bristles or cilia can easily navigate through curvatures in their path and pipelines of varying diameters. Since these robots can be easily scaled to a micro-level, they have a potential application in medical devices for target delivery of drugs, navigating through narrow tubes or blood vessels. Research on using these robots in search and rescue operations as an active scope camera is also being carried out. Finally, these robots can also be employed in military operations to navigate through harsh environments.

1.4 Thesis outline

In chapter-2, existing models of vibration driven robots and the state of the art research in this field are outlined. In chapter-3, an analytical model to describe the motion dynamics of the bristle bot is presented. Slip and stick type behaviour of the bristles is explained and system of equations defining the slip-stick motion behaviour of the bristle bot are developed. Chapter-4 presents the frequency and amplitude dependent transition in dynamic behaviour of the bristle bot. Periodicity of vibrations of the system are discussed. In chapter-5, a slightly modified bristle bot model which can explain the dynamics of vertical pipe climbing robots with bristles is presented. Chapter-6 summarizes the work done in this research and highlights target areas for future work.

Chapter 2

Literature Review

Vibration driven locomotion is inspired by the motion of some limbless creatures like snakes and worms. The surface of these organisms have some micro/nano scales oriented at some angle causing mechanical anisotropy. This anisotropy leads to the generation of asymmetric contact forces with the substrate when they periodically shift their centre of mass or change their shape [1, 11, 15]. A classic example of systems utilizing this principle are worm like locomotion systems (WLLS) [17, 12]. Such systems have multiple segments, with bristles planted on their surface, connected to each other by linear actuators. The surface offers high resistance to the motion of a body segment in the backward direction due to the orientation of the bristles. Thus a net forward locomotion is achieved when the actuators periodically move the segments relative to each other producing the effect of internal periodic excitation.

Vibration driven locomotion can also be realized using an internal vibrating mass [6]. An internal point mass forced to periodically oscillate at a high frequency generates inertial forces which are transferred to the main body. Interaction of the main body with the surroundings induces reactive forces between them. Therefore, it is apparent that one can control the reactive forces of environment on the system by controlling the motion of internal vibrating mass. For a body moving on a rough horizontal plane excited by periodic oscillations of the internal mass, the normal reaction offered by the surface can be controlled by vertical actuation of the internal mass. It consequently effects the kinetic friction offered by the surface. Whereas, the inertial forces due to horizontal actuation of the internal mass act along the direction of motion of the main body. The direction

of motion and the magnitude of average velocity of such a system can be controlled by controlling the phase shift in oscillations of the internal point masses. Furthermore, the magnitude of average velocity of the system is also dependent on the excitation frequency.

Figure 2.1: A model of a vibration driven system by N.N Bolotnik et al.[6]

Another vibration driven model along the same lines was developed by Zimmerman et al. in [20]. Two identical bodies with an unbalanced rotor mounted on each are connected together by a linear spring to form an oscillatory system characterized by its natural frequency. The steady-state motion of the system was investigated using a mathematical model for the case of low friction. The system is excited at nearly resonant frequencies with force of excitation relatively very small compared to the maximum elastic force developed in the spring. Analogous to a single module system, the average velocity of this system can also be controlled by the phase difference between the two rotors. However, the the direction of motion of this system can be reversed by increasing the excitation frequency from a pre-resonant to post-resonant level. The directional shift phenomenon and the numerical computational results were validated using a physical model.

Figure 2.2: A physical model of a vibration driven system by K. Zimmermann et al. [20]

Figure 2.3: An analytical model of a vibration driven system by K. Zimmermann et al.[20]

As mentioned above, the motion of an internal point mass in a vibration driven system controls the reactive forces offered by the surroundings and thus effects its locomotion. Therefore, the motion parameters of the point mass can be treated as control variables to maximize the average velocity of the system. An optimization problem for the motion of a rigid body, in the presence of dry friction, governed by the periodic motion of an internal point mass moving along a straight line on a horizontal plane was formulated by Chernous'ko in [7]. Two classes of periodic relative motions, two phase and three phase motion, of the internal point mass were considered. In the two-phase motion, the relative velocity is piece-wise constant and the period includes two intervals of constant

velocity. In the three-phase motion, the relative acceleration is piece-wise constant and the period includes three intervals of constant acceleration. The optimal motion of the internal point mass in order for the system to achieve a maximum average velocity was determined in both the cases. The same optimization problem was solved for a wide class of resistive environments governed by non linear resistance laws in [5].

Bristle bots are another class of vibration driven robots. They are primarily used as pipeline inspection robots or active scope cameras [8, 10, 13, 19, 18]. They are characterized by the presence of bristles or cilia on the surface of their main body and are driven by an internal point mass forced to oscillate at a high frequency. The bristles interact with the substrate to provide anisotropy in friction coefficient whereas the vertical actuation of the internal point mass is responsible for generating asymmetric normal reaction. Therefore, the driving force (asymmetric frictional force) can be a result of either frictional anisotropy and/or asymmetry in normal reaction. In recent years, many researchers have developed analytical models and prototype designs of bristle bots for a wide variety of applications.

In [4], the motion of a bristle bot is studied by modelling it as a multi-body system. The system considered has a main body supported by a number of weightless elements which are attached to the main body by means of torsional springs. Two internal masses, one vibrating in the horizontal direction and the other in vertical direction excite the system. Small bending deformations of the support elements are assumed and they are modelled as rigid beams in a first approximation instead of elastic body modelling. Assuming dry Coulomb friction between the contact points of the supports and the surface it is shown that the system exhibits locomotion even in the case of isotropic friction. This is an expected result as in the case of isotropic friction, asymmetry in frictional forces can be realized as a result of asymmetric normal reaction generated by the vertical actuating mass. A physical moving model is developed for locomotion through pipes of diameter 25mm. Experiments were carried out for different inclination angles of the pipe and the average locomotion speed of the prototype for each angle is recorded. Also, the results indicated that the prototype can exhibit locomotion in pipes with an inclination angle of upto 45° . Finally, a mathematical model developed was solved numerically and the simulation results were compared with the prototype results.

Figure 2.4: A prototype of a vibration driven robot by Felix Becker et al. [4]

A prototype of bristle bot for spying and pipeline inspection purposes is presented in [2]. The robot has a main body enclosed in a cylindrical chassis with evenly distributed bristles on it. The chassis is designed to inspect pipes of 25 mm diameter but it can be easily replaced in order to inspect pipes of different diameters. The study also analyzed the motion behaviour of the prototype using an analytical model which is a slight modification of the model presented in [4]. The difference is that the current model has to overcome a constant resistive force F_c (in addition to dry friction) while moving and it is driven only by a single point mass oscillating in the vertical direction.

Figure 2.5: A Bristle bot for spying and inspection purposes by Felix Becker et al. [2]

The system of equations defining the motion are solved numerically to study the influence of selected parameters on the locomotion velocity. In the case of anisotropic friction, the average velocity of the system decreases as the friction coefficient in the forward direction (μ_+) is increased relative to the friction coefficient in the backward direction (μ_-). Moreover, it is also observed that the system has a net forward displacement even when the value of μ_+ is less than the value of μ_- . An analysis on the influence of leg length on the locomotion velocity, in dependence of the torsional stiffness, indicated that for legs shorter than a certain threshold value, lower torsional stiffness results in higher velocities. A completely opposite behaviour is observed for leg lengths longer than the threshold value.

Two models of bristle bots are presented in [3]. The models are classified based on the mechanical properties of the bristle and the direction of excitation. The first model comprises of short stiff bristles and is excited by a mass vibrating in the horizontal direction. The second model is described by long elastic bristles and is excited by a vertical actuating mass. The influence of system parameters on the locomotion velocity is studied for both the models. In Model-I, the locomotion is purely dependent on anisotropic friction coefficients and the excitation frequency. This is due to the fact that the inclination of stiff bristles is not effected by horizontal excitation and hence the normal reaction almost remains constant. The velocity of this system increases with an increase in excitation frequency and a decrease in μ_+ for a fixed value of μ_- . Other parameters defining the system have no effect on the locomotion velocity. Whereas the locomotion velocity of Model-II is highly influenced by the system characteristics. The locomotion velocity is inversely proportional to the bristle inclination angle and directly proportional to its stiffness. The velocity increases with an increase in leg length until a certain value and then decreases. An increase in mass of the robot reduces in locomotion speed for both the models.

A physical model is developed and several experiments were carried out varying the parameters discussed above. Two motors rotating in opposite directions with a phase shift of 180° excite the system. Due to the phase shift, the horizontal component of centrifugal force generated by the motors is balanced out and thus they generate the effect of vertical excitation. In order to avoid tilting, the motors are positioned such that the vertical component of forcing passes through the centre of mass of the system. The experimental results seem to agree qualitatively with the

numerical simulation results.

Figure 2.6: A two model system of Bristle bot [3]

Bristle bots are also used to investigate the collective motion behaviour of organisms [9, 16]. A large number of bristle bots are confined to a small area with soft boundaries and the parameters effecting the transition from random swirling to collective organized motion of the bristle bots are investigated. Within [9] the motion of an individual bristle bot is studied by modelling a group of bristles as an elastic beam subjected to tip loading and horizontal friction force. The Slip-Stick dynamics of the system is explained in detail. The dependence of design parameters on the linear velocity of the bristle bot are also studied using experiments. Results indicated that the velocity increases linearly with an increase in frequency of excitation and the bristle length; and decreases exponentially with an increase in mass and bristle inclination angle. It was also observed that short stiff bristles lead to noisier dynamics associated with rebounds and jumps and the bristle bots tend to move in a circle. Whereas bristle bots with longer bristles are in contact with the ground for a

longer period of time and hence move in a straight line or a weakly curved orbit.

Planar motion of bristle bots has not been studied extensively as most of the applications of bristle bots are in confined spaces. However, some studies also explored the planar locomotion of Bristle bots to some extent. One of the first steerable vibration driven microbot model was developed by Kiyoshi Ioi in [14]. The proposed microbot had many brush fibres planted beneath the main body. Two coreless motors carrying eccentric weights on their rotors were installed on either side of the main body to maintain static equilibrium. The angle made by the output axis of the motor with the horizontal can be tuned and is referred as the 'setting angle' of the motor. A translational moving model, assuming very small deflection of the brush fibres under the influence of centrifugal forces was derived. Under this assumption, the brush bundles were modelled as linear springs. A prototype was built and experiments were carried out to compare the results with numerical simulations. Also, the setting angle of the motor was varied in these experiments to study its effect on the average velocity of the bot. Results indicated that the velocity is maximum when the direction of centrifugal force agrees to the contracting direction of the brush bundle. The robot can be made steerable by maintaining a difference in angular velocity of the two motors. This turning property of the robot on a plane was explained using a yawing model.

Figure 2.7: Mobile microbot by Kiyoshi Ioi [14]

In all the previous study on Bristle bots, although mathematical models describing the motion of Bristle bots were developed, they did not account for the slip-stick motion of the bristle. Depending on the frequency and amplitude of excitation the bristles can either always slip or always stick or exhibit a complex slip-stick dynamics. This has not been studied in detail in any of the existing work on the mechanics of bristle bots. In the present study we aim to address this gap. Analytical conditions for the system to exhibit a particular type of motion are formulated and system of equations defining the Slip-Stick dynamics of a bristle bot are developed. The equations are solved numerically for different forcing frequency and amplitude settings. The direction of motion and the magnitude of average velocity of a bristlebot can be controlled by controlling the frequency and amplitude of the forced oscillations of the internal point masses. In this work we show that our model of the bristlebot shows either 'forward' or 'backward' motion depending on the value of forcing parameters. The feedback control of bristlebot like robots is left for future work.

Chapter 3

Analytical Model

In this chapter, we develop an analytical model to explain the complex motion behaviour of the bristle bot. The rotor is placed exactly at the centre of mass of the bristle bot to prevent tilting. The bristles planted beneath the surface of the main body are assumed to be deflected by the same angle under the influence of centrifugal forces. The eccentric mass rotates in a vertical plane parallel to the direction of motion of the bristle bot. Therefore, there is no forcing component perpendicular to the plane of motion of the eccentric mass and hence the bristle bot moves almost along a straight line without any lateral drift. Under these conditions, a two dimensional planar model is sufficient to study the bi-directional motion of a bristle bot moving along a straight line.

A schematic of the simplified two dimensional model of the bristle bot is presented in Fig. 3.1. It comprises of a main body supported on uniformly distributed bristles and is driven by an internal mass attached to a motor rotating with a frequency ω . The total mass of the main body, bristles, vibration motor sans the eccentric weight and the button cell is m_0 . The mass of the eccentric weight on the motor shaft is m and the center of mass of the bristle bot is denoted by (x, y) . The bristles are of length l and are inclined at an angle θ_0 with respect to the vertical in the undeformed state. The centrifugal forces produced by the rotating mass lead to bending of the bristles. Assuming that the distance between the two ends of a bristle does not change significantly during the tip deflection, the action of the bristles can be thought of as that of torsional springs instead of elastic body modelling. The torsional stiffness of the bristles is denoted by K .

Figure 3.1: Model of a bristlebot

3.1 Equations of Motion

Equations of motion are developed for the two dimensional model presented above. The coordinates describing the configuration of the system are (x, y, θ) . When the bristlebot is in contact with the ground, the holonomic constraint $y = L \cos \theta$, reduces the number of independent generalized coordinates which are then $q = (x, \theta)$. The constraint force associated with this holonomic constraint, the normal reaction force, as well as the friction force can be obtained using the method of Lagrange multipliers.

Position vector of COM of main body (m_o) is given by

$$\mathbf{R}_{\mathbf{m}_o} = x\mathbf{i} + y\mathbf{j}$$

Position vector of COM of rotating mass (m) is given by

$$\mathbf{R}_m = (x + \xi_x)\mathbf{i} + (y + \xi_y)\mathbf{j}$$

Velocity of the main body (m_o) is given by

$$\mathbf{V}_{m_o} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

Velocity of the rotating mass m is given by

$$\mathbf{V}_m = (\dot{x} + \dot{\xi}_x)\mathbf{i} + (\dot{y} + \dot{\xi}_y)\mathbf{j}$$

Therefore, the kinetic energy of the system is

$$T = \frac{1}{2}m_o(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{\xi}_x^2 + \dot{\xi}_y^2 + 2\dot{x}\dot{\xi}_x + 2\dot{y}\dot{\xi}_y) \quad (3.1)$$

where $\xi_x = A \sin \omega t$ and $\xi_y = A \cos \omega t$ are the periodic horizontal and vertical displacements of the rotating mass relative to the center (x, y) of the bristlebot.

The potential energy of the system is

$$V = m_o g y + m g (y + \xi_y) + \frac{1}{2}K(\theta - \theta_o)^2 \quad (3.2)$$

with the Lagrangian being $\mathcal{L} = T - V$. The Euler-Lagrange equations are

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q^j} = \mathbf{Q}_j + \lambda_j \quad (3.3)$$

Where $\mathbf{Q}_j$ are the generalized forces and λ_j are the constraint forces.

Further analysis of the equations of motion require a formulation of the criteria for different types of motion. These types of motion are (a) *stick*, where the bristlebot sways back and forth with the bristle tips not having any displacement, (b) *slip*, where the bristle tips have a velocity relative to

the substrate and (c) *stick-slip*, where the bristle tips are at rest for some time and move for some time. In case (a) a holonomic constraint relates x and θ and the system behaves as a one degree of freedom inverted pendulum with torsional resistance. In case (b) no such holonomic constraint exists, while in case (c) the stick-slip motion leads to a piecewise holonomic system. A fourth case exists where the bristles lose contact with the ground. The system assumes one of the four states depending on the frequency of excitation, design parameters, initial configuration of the system and the surface properties.

Assuming dry friction between the contact points of the bristles and the substrate, an analytic condition is formulated to describe the motion behaviour of the bristles and the system. When the bristles flex periodically under the influence of centrifugal forces produced by the rotating mass, the magnitude and direction of the force exerted by the bristles on the substrate varies with bristle angle (θ), angular velocity ($\dot{\theta}$) and angular acceleration ($\ddot{\theta}$). As a result, the horizontal component of the reactive force (λ) and the normal reaction (N) offered by the substrate varies at each instant. If f_s denotes the maximum static friction offered by the surface, the system exhibits slip, stick or slip-stick dynamics based on the magnitude of the Lagrange multiplier λ relative to f_s . As the maximum allowable static friction ($f_s = \mu_s N$) is proportional to the magnitude of normal reaction, f_s also varies at each instant with $(\theta, \dot{\theta}, \ddot{\theta})$. Hence, all the three forces λ , N and f_s vary continuously throughout one time period of rotation of the rotor. The condition on the Lagrange multiplier λ associated with the friction that leads to the different kinds of motion are described in the following sections.

3.2 Stick Dynamics

As discussed above, the bristlebot just oscillates about the bristle contact point with the surface without any significant locomotion while exhibiting stick type behaviour. The stick type behaviour is observed when the design parameters, frequency of excitation, amplitude of rotation and surface interaction properties are chosen such that

$$\lambda < f_s \quad \forall \quad t \in [nT, (n+1)T] \quad (3.4)$$

where T is the time period of excitation.

Since the bristle tip always sticks to the surface in this phase, the system has only degree of freedom with the holonomic constraint

$$\begin{aligned} x &= P - a + l \sin \theta & y &= l \cos \theta + b \\ \dot{x} &= l \dot{\theta} \cos \theta & \dot{y} &= -l \dot{\theta} \sin \theta \\ \ddot{x} &= l \ddot{\theta} \cos \theta - l \dot{\theta}^2 \sin \theta & \ddot{y} &= -l \ddot{\theta} \sin \theta - l \dot{\theta}^2 \cos \theta \end{aligned}$$

The governing equation of the motion of the system in the stick phase is

$$\ddot{\theta} - \omega_1^2 \sin \theta + \epsilon \delta \omega^2 \sin(\theta - \omega t) + \omega_2^2(\theta - \theta_o) = 0 \quad (3.5)$$

$$\text{Where, } \omega_1^2 = \frac{g}{l}, \quad \omega_2^2 = \frac{K}{(m + m_o)l^2}, \quad \epsilon = \frac{m}{(m + m_o)}, \quad \delta = \frac{A}{l}$$

The normal reaction and friction forces acting on the system are given by

$$N = (m + m_o)(g - l \ddot{\theta} \sin \theta - l \dot{\theta}^2 \cos \theta) + m \ddot{\xi}_y \quad (3.6)$$

$$f = (m + m_o)l(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + m \ddot{\xi}_x \quad (3.7)$$

3.3 Slip Dynamics

In the slip motion the bristle tip slips in both the forward and backward directions throughout one cycle. It slips in the backward direction when the induced forces due to the rotating mass compress the torsional spring; in the forward direction when the compressed spring tries to attain its natural state. However, a net displacement of the centre of mass may be possible in one cycle resulting in net locomotion of the bot. This is due to the fact that the bristle could slip more in one direction as opposed to the other due to asymmetry in kinetic friction ($\mu_k N$) generated as a result of asymmetry in normal reaction provided by the bristles. The system exhibits slip type dynamics when its parameters, frequency of excitation, and surface interaction properties are such that:

$$\lambda > f_s \quad \forall \quad t \in [nT, (n+1)T] \quad (3.8)$$

Assuming the system always remains in contact with the surface the only holonomic constraint is $y = l \cos \theta + b$. The equations defining the motion of system in the slip phase are

$$(m + m_0)(\ddot{x} - \mu l \ddot{\theta} \sin \theta - \mu l \dot{\theta}^2 \cos \theta + \mu g) + m(\ddot{\xi}_x + \mu \ddot{\xi}_y) = 0 \quad (3.9)$$

$$\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta + \epsilon \delta \omega^2 \cos(\omega t) - \omega_1^2 + \frac{\omega_2^2(\theta - \theta_o)}{(\sin \theta + \mu \cos \theta)} = 0 \quad (3.10)$$

The Normal reaction and friction forces acting on the system are given by

$$N = (m + m_0)(g - l \ddot{\theta} \sin \theta - l \dot{\theta}^2 \cos \theta) + m \ddot{\xi}_y \quad (3.11)$$

$$f = -\mu N \text{sign}(\dot{x} - l \dot{\theta} \cos \theta) \quad (3.12)$$

3.4 Stick-Slip Dynamics

The bristles may slip or stick at any instant t depending on the magnitude of λ relative to the allowable maximum static friction f_s . Both these forces are non-linear functions of the state variables and hence vary at each instant. A stick-slip locomotion is observed when the bristles exhibit both stick and slip dynamics over one cycle. A transition in dynamics from stick to slip is observed when the frictional force required to enforce the stick behavior exceeds the maximum allowable value of $\mu_s N$. Once the bristles enter the slip phase dynamics, a return to the stick phase is possible when the bristle tip velocity becomes zero and simultaneously the friction force $\lambda < \mu_s N$. Both these transitions in dynamics need to occur over one time period in order for the system to exhibit a periodic slip-stick behaviour.

The condition for the system to exhibit stick-slip dynamics is

$$\begin{aligned} \text{Transition from stick to slip} \quad \lambda &> f_s & \text{for} \quad t &\in [nT, (n+1)T] \\ \text{Transition from slip to stick} \quad \lambda &< f_s \quad \wedge \quad \dot{x} - l \dot{\theta} \cos \theta &= 0 & \text{for} \quad t &\in [nT, (n+1)T]. \end{aligned} \quad (3.13)$$

Chapter 4

Numerical Simulations

The equations of motion derived in the previous section are numerically solved in Matlab. The Matlab solver uses the slip-stick conditions formulated above to determine the state of the system at each instant and perform piece-wise integration of corresponding equations. A bristle bot model with the following design parameters is chosen for the study. The effect of parameters influencing the periodic force acting on the system: forcing frequency and amplitude of rotation on the motion behaviour of the bristles is explored.

Main body mass (M_o)= $60gm$

Rotating eccentric mass (m)= $4gm$

Torsional Spring constant (K)= $0.3Nm/rad$

Length of the Bristle (L)= $10mm$

Undeformed angle of the spring= $30deg$

4.0.1 Static equilibrium

When the bristle bot is placed on a substrate, the torsional springs are compressed by a certain angle due to the system's own weight. Since the bristles are distributed uniformly, all the bristles are assumed to be compressed by the same angle. The static equilibrium angle of the bristles (θ_i) is evaluated in order to determine the initial condition of the state variable θ .

Figure 4.1: Static equilibrium

Let N' be the normal reaction acting on each bristle tip, K' be the torsional stiffness of an individual bristle and i is the total number of bristles. From Fig. 4.1, the conditions for static equilibrium are obtained as:

$$(m + m_o)g = iN'$$

$$K'(\theta_i - \theta_o) = N'l \sin \theta_i$$

The above two equations can be reduced to obtain a transcendental equation, which can be solved to find the static equilibrium angle θ_i :

$$K(\theta_i - \theta_o) = Nl \sin \theta_i$$

4.1 Frequency and amplitude dependent transitions in dynamics

Frequency and amplitude dependent motion behaviour is investigated for two different models of the bristle bot. In the first model, the system is driven by an eccentric mass attached to a rotor which generates periodic internal forcing in both X and Y directions. The horizontal component of periodic forcing is given by $m\ddot{\xi}_x$ and the vertical component is given by $m\ddot{\xi}_y$. The second model is driven by a vertical actuating mass. As a result, the periodic internal forcing is only along vertical direction i.e., $m\ddot{\xi}_x = 0$.

A bristle bot in its static equilibrium position is excited by a periodic internal forcing due to the vibrating mass. The system can exhibit some or all of the types of motion described in the previous chapter depending on the frequency and amplitude of the vibrations of the eccentric mass. Numerical simulations indicate the existence of up to four threshold frequencies at which a transition in the type of the motion of the bristle bot is observed for both the models. These amplitude dependent threshold frequencies are denoted by η_1 , η_2 , η_3 , and η_4 .

4.1.1 Simulation results for Model-I

All the threshold frequencies for each amplitude of rotation are captured on an average velocity plot as a function of forcing frequency Fig.4.2. The plot presented provides a basic understanding of the locomotion direction of a bristle bot operating within a certain frequency range. For very low forcing frequencies $\omega < \eta_1(A)$ the bristle bot has a zero average velocity and hence doesn't exhibit any locomotion. The bristle bot only sways back and forth as an inverted pendulum and remains in the pure stick phase. The vibrations of the eccentric mass do not provide enough forcing

to overcome the friction force. For a forcing frequency $\omega > \eta_1(A)$ the bristle bot begins to move in the negative direction. For small amplitudes, the velocity increases initially with increasing forcing frequency and then begins to decrease eventually becoming zero at the frequency $\eta_2(A)$. The bristle bot remains in the stick phase until the forcing frequency reaches the next critical threshold value denoted by η_3 . However, for higher amplitudes of rotation, this transition to the stick phase does not occur. For certain forcing frequencies in the range $\omega \in (\eta_2, \eta_3)$, the normal reaction becomes negative periodically indicating that the holonomic constraint $y = l \sin \theta$ is being violated. This is shown in fig.4.3 for the case where $A = 5\text{mm}$. As a result, the system begins to exhibit small hopping and chattering vibrations. The current system of equations cannot describe such repeated jump motion behaviour of the bristle bot and hence the average velocity is not evaluated for that frequency range.

Figure 4.2: Average velocity of a bristle bot plotted as a function of forcing frequency for multiple amplitudes of rotation (A). $\eta_1, \eta_2, \eta_3, \eta_4$ are the forcing frequencies at which a transition in motion behaviour of the bristle bot is observed.

For a forcing frequency $\omega > \eta_3$, a reversal in direction of locomotion of the bristle bot is observed. Beyond this frequency, the system begins to move in the forward direction and its locomotion velocity increases with an increase in forcing frequency. With an increase in forcing frequency beyond η_3 the duration of stick phase in a forcing time period continues to decrease and the bristle tips are purely in the slip motion for $\omega > \eta_4$. All the above transitions can be attributed to the difference in type of motion behaviour of the bristles as we vary the forcing frequency across a threshold value.

Figure 4.3: Normal reaction plotted as a function of time for forcing parameters $\omega = 34\pi$ rad/sec and $A = 5\text{mm}$. The plot indicates that the system takes off since the normal reaction reaches zero.

Figure 4.4: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time. The graph indicates that the magnitude of λ is always less than the maximum static friction offered by the surface. As a result the bristle tip doesn't slip in either direction and hence exhibits pure stick behaviour. Forcing parameters are $\omega = 16\pi$ rad/sec and $A = 4$ mm.

Fig. 4.4 illustrates the motion of the bristle tip and the forces acting on it when the system exhibits pure stick motion behaviour i.e., when the frequency of forcing $\omega \in (0, \eta_1)$ or $\omega \in (\eta_2, \eta_3)$ (for lower amplitudes). The bristle tip doesn't slip in either direction since the magnitude of λ is always less than the maximum allowable static friction ($\mu_s N$). As a result, the centre of mass of the main body just oscillates about the bristle contact point with the substrate and has no net displacement over one cycle. The vertical dotted lines correspond to a sudden change in direction of the frictional force. The direction of friction is positive when the bristles are being compressed and negative when the torsional springs uncoil.

Figure 4.5: Displacement vs time in pure stick phase. Forcing parameters are $\omega = 16\pi$ rad/sec and $A = 4\text{mm}$

Figure 4.6: θ vs time in pure stick phase. Forcing parameters are $\omega = 16\pi$ rad/sec and $A = 4\text{mm}$

Figure 4.7: A plot of friction and velocity of the bristle as a function of time when the bristlebot exhibits pure slip behaviour. Forcing parameters are $\omega = 60\pi$ rad/sec and $A = 4$ mm.

Fig. 4.7 illustrates pure slip dynamics of the bristle bot which can be realized for forcing frequency values greater than η_4 . The bristle continuously slips in either direction throughout one cycle. The graph of the friction force shown in fig. 4.7 shows that the effective friction is anisotropic, it is larger in magnitude when the bristles slip in the negative x - (backward) direction and significantly smaller when the bristles in the positive x - (forward) direction. This effective frictional anisotropy produces a net motion in the direction in which the friction is smaller. It is to be emphasized that the friction governed by Coulomb's law is isotropic, but the effective anisotropy is produced by the asymmetry in the normal reaction force induced by the asymmetry of the bristle geometry itself.

Figure 4.8: Displacement vs time when the system exhibits pure slip behaviour . Forcing parameters are $\omega = 60\pi$ rad/sec and $A = 4\text{mm}$

Figure 4.9: θ vs time when the system exhibits pure slip behaviour. Forcing parameters are $\omega = 60\pi$ rad/sec and $A = 4\text{mm}$

Figure 4.10: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time. As we observe $|\lambda| > |\mu_s N|$ for some part of the cycle when the bristles tend to slip in the negative direction. As a result the system acquires a net negative displacement in each cycle. Forcing parameters are $\omega = 24\pi$ rad/sec

The slip-stick locomotion of the bristles observed for a forcing frequency $\omega \in (\eta_1, \eta_2)$ is illustrated in Fig. 4.10. As indicated by the velocity of the bristle tip plot in Fig.4.10, the bristles only slip in the backward direction for a portion of the time they are being compressed in each cycle. They return to the stick phase at point 'B' where the velocity of the bristle tip reaches zero and simultaneously the magnitude of λ becomes less than the maximum allowable static friction. This is the point where the friction reverses its direction i.e., the compressed torsional springs begin to uncoil. Throughout the stick phase the friction is clearly seen to be smaller in magnitude than $\mu_s N$. The static friction exceeds the maximum allowable static friction at point 'C' where the bristles begin to slip again. A result of this kind of slip-stick behaviour would be a net locomotion of the bristle bot in the backward direction.

Figure 4.11: Displacement vs time when the system exhibits slip-stick behaviour with a periodic bristle slip in the negative direction. Forcing parameters are $\omega = 24\pi$ rad/sec and $A = 4$ mm.

Figure 4.12: θ vs time when the system exhibits slip-stick behaviour with a periodic bristle slip in the negative direction. Forcing parameters are $\omega = 24\pi$ rad/sec and $A = 4$ mm.

Figure 4.13: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time. Forcing parameters are $\omega = 40\pi$ rad/sec and $A = 4$ mm

A strikingly different slip-stick motion behaviour is observed when the bristle bot reverses its direction of locomotion at forcing frequency η_3 . For a very small increase in forcing frequency beyond η_3 , the slip-stick behaviour is as observed in fig. 4.13. In this case the bristles slip only in the positive direction for a portion of the time the torsional springs are being uncoiled in each forcing cycle. They return to the stick phase at point 'D' where the slip to stick transition conditions are satisfied. At this point the friction reverses its direction from negative to positive indicating a beginning of the torsional spring compression. The magnitude of friction is found to be smaller than the maximum allowable static friction throughout the spring compression cycle. Therefore, the bristles won't slip in the negative direction. This kind of slip-stick motion behaviour results in a net positive displacement in each cycle and hence the system executes forward locomotion.

Figure 4.14: Displacement vs time when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $\omega = 40\pi$ rad/sec and $A = 4\text{mm}$.

Figure 4.15: θ vs time when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $\omega = 40\pi$ rad/sec and $A = 4\text{mm}$.

Figure 4.16: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time. Forcing parameters are $\omega = 50\pi$ rad/sec and $A = 4$ mm.

More complex stick-slip motion can occur as the forcing frequency, ω , is increased significantly beyond η_3 . Figure 4.16 shows the velocity of bristle tip for one such case. In this scenario stick-slip motion of the bristle occurs such that the slip velocity of the bristle is alternately in the forward and backward directions. The transitions between two such slip motions may be interspersed with the stick phase of the motion. For instance in fig. 4.16 when the negative slip velocity of the bristle tip becomes zero at point *B*, the friction force $\lambda < \mu_s N$ and remains so until point *C* in the figure. At this point bristle tip overcomes the friction and slips forward. The slip velocity decreases and becomes zero at point *D*, however the required friction force λ to transition into a stick phase is more than $\mu_s N$. Therefore at point *C* the slip phase continues, but with the slip velocity reversing. Such sequences of (negative slip velocity-stick-positive slip velocity) and (positive slip velocity-negative slip velocity) alternate periodically. Nevertheless the bristlebot has a net locomotion since the bristle tip slip velocity is larger in the forward direction.

Figure 4.17: A map representing different motion behaviour regions of model-I bristle bot

Figure 4.17 represents the type of motion behaviour exhibited by the bristle bot for different forcing frequency and δ combinations. The threshold frequencies at each amplitude of excitation form the contours for different motion behaviour regions shown on the map. As observed in fig. 4.17, the threshold frequencies at which the system begins to exhibit slip-stick behaviour: η_1 and η_3 decrease with an increase in magnitude of δ . Also, for lower magnitudes of δ , the slip-stick behaviour with a resultant motion in the negative direction occurs for a very small range of forcing frequency and ceases to exist for $\delta < 0.08$. The system exhibits repeated jump motion behaviour for forcing frequency $\omega \in (\eta_2, \eta_3)$ for higher values of δ . The range $\eta_3 - \eta_2$ increases with an increase in magnitude of δ . Moreover, the slip-stick motion with a net forward displacement is not observed for magnitudes of $\delta > 0.55$. The system directly transitions to pure slip state from the repeated jump state.

4.1.2 Simulation results for Model-II

In this section, the frequency and amplitude dependent motion behaviour of a bristle bot model driven by a vertical actuating mass is illustrated. Model-II undergoes similar transitions in dynamic behaviour as in Model-I. However, due to the absence of horizontal component of forcing in Model-II, it traverses with a much lower speed for the same amplitude and frequency combination. Also, model-II bristle bot is found to exhibit pure slip behaviour with a net displacement in the negative direction which is observed in fig 4.24.

Figure 4.18: Average velocity of model-II bristle bot plotted as a function of forcing frequency for multiple amplitudes of rotation (A). $\eta_1, \eta_2, \eta_3, \eta_4$ are the forcing frequencies at which a transition in motion behaviour of the bristle bot is observed.

Figure 4.19: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time for Model-II. The graph indicates that the magnitude of λ is always less than the maximum static friction offered by the surface. As a result the bristle tip doesn't slip in either direction and hence exhibits pure stick behaviour. Forcing parameters are $\omega = 16\pi$ rad/sec and $A = 4$ mm.

Figure 4.20: A plot of friction and velocity of the bristle as a function of time when the bristle bot (Model-II) exhibits pure slip behaviour. Forcing parameters are $\omega = 60\pi$ rad/sec and $A = 4$ mm.

Figure 4.21: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time for Model-II. As we observe $|\lambda| > |\mu_s N|$ for some part of the cycle when the bristles tend to slip in the negative direction. As a result the system acquires a net negative displacement in each cycle. Forcing parameters are $\omega = 24\pi$ rad/sec

Figure 4.22: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time for Model-II. Forcing parameters are $\omega = 40\pi$ rad/sec and $A = 4$ mm

Figure 4.23: The value of λ , maximum static friction ($\mu_s N$) and the velocity of bristle tip plotted as a function of time for Model-II. Forcing parameters are $\omega = 50\pi$ rad/sec and $A = 4$ mm.

Figure 4.24: A map representing different motion behaviour regions of model-II bristle bot

4.2 Periodicity of vibrations

In this section, we analyze the transition in periodicity of vibrations of the bristle bot when the system undergoes a transition in motion behaviour. FFT plots and Poincare maps are generated for a sample point in each motion behaviour region shown on fig 4.17. When the bristle bot is in pure stick phase, the system is found to exhibit quasi-periodic oscillations about the bristle contact point with the surface. As can be observed from the Poincare map (fig. 4.35), the periodic solutions of state variables never seem to converge into a finite number of points indicating a quasi-periodic behaviour. Additionally, FFT plots (4.25, 4.26) suggest that the system oscillates with a wide spectrum of frequencies. The two main frequency components are observed at the forcing frequency value and the natural frequency of the system ($\frac{K}{(m+m_o)l^2}$).

As the forcing frequency is increased to a value close to the natural frequency of the system (20π), large amplitude oscillations occur and as a result the bristles begin to slip. Therefore, the threshold frequency η_1 occurs at a value close to the natural frequency of the system for all values of δ (fig 4.17). At η_1 , the system undergoes a transition from quasi-periodic to periodic state and it no longer oscillates with a natural frequency component. It exhibits multiple period oscillations with harmonics at the multipliers of the forcing frequency (4.27, 4.28, 4.29, 4.30, 4.31, 4.32). Poincare maps also indicate that the periodic solutions of the state variables also converge to a finite number of points indicating periodic behaviour.

At very high forcing frequencies beyond η_4 where the system exhibits pure slip oscillations, the harmonics at the multipliers of the forcing frequency diminish and the system begins to oscillate with a single frequency component. This can be observed from the FFT plots (fig 4.33, 4.34) and Poincare maps (fig 4.49, 4.51, 4.52) presented below. The periodic solutions of the state variables converge to a single point indicating single period oscillations.

Figure 4.25: FFT plot of θ when the system exhibits pure stick behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 10\pi$ rad/ sec

Figure 4.26: FFT plot of $\dot{\theta}$ when the system exhibits pure stick behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 10\pi$ rad/ sec

Figure 4.27: FFT plot of θ when the system exhibits slip-stick behaviour with periodic bristle slip in negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi$ rad/ sec

Figure 4.28: FFT plot of $\dot{\theta}$ when the system exhibits slip-stick behaviour with periodic bristle slip in the negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi$ rad/ sec

Figure 4.29: FFT plot of θ when the system exhibits slip-stick behaviour with periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi$ rad/ sec

Figure 4.30: FFT plot of $\dot{\theta}$ when the system exhibits slip-stick behaviour with periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi$ rad/ sec

Figure 4.31: FFT plot of θ when the system exhibits slip-stick behaviour with periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/ sec

Figure 4.32: FFT plot of $\dot{\theta}$ when the system exhibits slip-stick behaviour with periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/ sec

Figure 4.33: FFT plot of θ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 60\pi$ rad/ sec

Figure 4.34: FFT plot of $\dot{\theta}$ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 60\pi$ rad/ sec

Figure 4.35: Poincaré map of $\dot{\theta}$ vs θ when the system exhibits pure stick behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 16\pi \text{ rad/sec}$

Figure 4.36: Poincaré map of θ , $\dot{\theta}$, and $\dot{x}$ when the system exhibits pure stick behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 16\pi \text{ rad/sec}$

Figure 4.37: Poincare map of $\dot{\theta}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi$ rad/ sec

Figure 4.38: Poincare map of $\dot{x}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi$ rad/ sec

Figure 4.39: Poincaré map of $\dot{x}$ vs $\dot{\theta}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi \text{ rad/sec}$

Figure 4.40: Poincaré map of θ , $\dot{\theta}$, and $\dot{x}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 24\pi \text{ rad/sec}$

Figure 4.41: Poincare map of $\dot{\theta}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi$ rad/ sec

Figure 4.42: Poincare map of $\dot{x}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi$ rad/ sec

Figure 4.43: Poincaré map of $\dot{x}$ vs $\dot{\theta}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi \text{ rad/sec}$

Figure 4.44: Poincaré map of θ , $\dot{\theta}$, and $\dot{x}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in the positive direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 40\pi \text{ rad/sec}$

Figure 4.45: Poincaré map of $\dot{\theta}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/ sec

Figure 4.46: Poincaré map of $\dot{x}$ vs θ when the system exhibits slip-stick behaviour with a periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/ sec

Figure 4.47: Poincaré map of $\dot{x}$ vs $\dot{\theta}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/sec

Figure 4.48: Poincaré map of θ , $\dot{\theta}$, and $\dot{x}$ when the system exhibits slip-stick behaviour with a periodic bristle slip in both positive and negative direction. Forcing parameters are $A = 4\text{mm}$ and $\omega = 50\pi$ rad/sec

Figure 4.49: Poincare map of $\dot{\theta}$ vs θ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 60\pi \text{ rad/sec}$

Figure 4.50: Poincare map of $\dot{x}$ vs θ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 60\pi \text{ rad/sec}$

Figure 4.51: Poincaré map of $\dot{x}$ vs $\dot{\theta}$ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 16\pi \text{ rad/sec}$

Figure 4.52: Poincaré map of θ , $\dot{\theta}$, and $\dot{x}$ when the system exhibits pure slip behaviour. Forcing parameters are $A = 4\text{mm}$ and $\omega = 60\pi \text{ rad/sec}$

Chapter 5

A climber bristle bot

In this chapter, the dynamics of a slightly modified version of the simple bristle bot called "The climber" is investigated. The model considered can move vertically upwards between two supporting surfaces above certain forcing frequency. The analysis of the dynamics of such system can be used to predict and control the motion of vertical pipe climbing bristle bots. The model comprises of a main body with uniformly distributed bristles on either side of it. It is driven by the periodic internal forcing generated by an eccentric mass attached to a rotor rotating with angular frequency ω . The rotor is placed exactly at the centre of mass of the main body to prevent tilting.

A schematic of the simplified two dimensional model of the climber is presented in fig 5.1. The total mass of the main body, bristles, vibration motor sans the eccentric weight is m_0 . The mass of the eccentric weight on the motor shaft is m and the center of mass of the bristle bot is denoted by (x, y) . The model is symmetric along a horizontal axis passing through the centre of mass i.e., the bristle length l , undeformed angle of the bristle θ_o , and the torsional spring stiffness K are equal for the bristles on either side of the main body. The bristles are precompressed to an initial angle θ_i to force contact between the bristle tips and the supporting surfaces. The gravitational force is assumed to act along the negative X-axis.

Figure 5.1: A schematic of a climber bristle bot model

5.1 Equations of Motion

Equations of motion are developed for the two dimensional model presented above. The coordinates describing the configuration of the system are $(x, y, \theta_1, \theta_2)$. The torsional springs are precompressed enough such that the bristles on either side of the body won't lose contact with the supporting surface when the system is subjected to periodic internal forcing. Under such conditions, two holonomic constraints $y = L \cos \theta_1 + b$ and $d - y = L \cos \theta_2 + b$ reduces the number of independent generalized coordinates which are then $q = (x, \theta_1)$. The constraint force associated with the holonomic constraints, the normal reaction N_1 and N_2 , as well as the friction force can be obtained using the method of Lagrange multipliers.

Position vector of COM of main body (m_o) is given by

$$\mathbf{R}_{\mathbf{m}_o} = x\mathbf{i} + y\mathbf{j}$$

Position vector of COM of rotating mass (m) is given by

$$\mathbf{R}_{\mathbf{m}} = (x + \xi_x)\mathbf{i} + (y + \xi_y)\mathbf{j}$$

Velocity of the main body (m_o) is given by

$$\mathbf{V}_{\mathbf{m}_o} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

Velocity of the rotating mass m is given by

$$\mathbf{V}_{\mathbf{m}} = (\dot{x} + \dot{\xi}_x)\mathbf{i} + (\dot{y} + \dot{\xi}_y)\mathbf{j}$$

Therefore, the kinetic energy of the system is

$$T = \frac{1}{2}m_o(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{\xi}_x^2 + \dot{\xi}_y^2 + 2\dot{x}\dot{\xi}_x + 2\dot{y}\dot{\xi}_y) \quad (5.1)$$

where $\xi_x = A \sin \omega t$ and $\xi_y = A \cos \omega t$ are the periodic horizontal and vertical displacements of the rotating mass relative to the center (x, y) of the bristlebot.

The potential energy of the system is

$$V = m_o g y + m g (y + \xi_y) + \frac{1}{2}K(\theta - \theta_o)^2 \quad (5.2)$$

with the Lagrangian being $\mathcal{L} = T - V$. The Euler-Lagrange equations are

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q^j} = \mathbf{Q}_j + \lambda_j \quad (5.3)$$

Where $\mathbf{Q}_j$ are the generalized forces and λ_j are the constraint forces.

Similar to a simple bristle bot model, the climber may exhibit different types of motion depending on the forcing and design parameters. However, the slip-stick locomotion for this model is a bit more complex due to the presence of bristles on either sides of the main body. Also, for the same reason, the system cannot exhibit pure stick motion behavior as the system becomes over-constrained and it is not physically feasible. Therefore, only two different possible types of motion for the bristles exist which are : (a) Stick-slip, where the bristles on one side of the body have a velocity relative to the substrate while the bristles on the other side remain at rest. As the model is symmetrical along X-axis, the bristles on either side of the main body exhibit alternating slip-stick behaviour in one forcing time period. (b) Pure slip, where the bristles on both the sides of the main body have a velocity relative to the substrate throughout one forcing time period. Equations of motion are formulated for both the cases in the following subsections.

5.1.1 Slip-stick

In slip-stick type motion behaviour of the climber, the bristle tips on one of the sides of the main body remain at rest at any instant t in one forcing time period. Therefore, a holonomic constraint relates either x and θ_1 or x and θ_2 and the system behaves as a one degree of freedom system.

For the case where the bristles on the side corresponding to angle θ_1 are in stick phase, the constraint equations are:

$$\begin{aligned} x &= U - ia + l \sin \theta_1 & y &= l \cos \theta_1 + b & y &= d - l \cos \theta_2 - b \\ \dot{x} &= l \dot{\theta}_1 \cos \theta_1 & \dot{y} &= -l \dot{\theta}_1 \sin \theta_1 & \dot{y} &= l \dot{\theta}_2 \sin \theta_2 \\ \ddot{x} &= l \ddot{\theta}_1 \cos \theta_1 - l \dot{\theta}_1^2 \sin \theta_1 & \ddot{y} &= -l \ddot{\theta}_1 \sin \theta_1 - l \dot{\theta}_1^2 \cos \theta_1 & \ddot{y} &= l \ddot{\theta}_2 \sin \theta_2 + l \dot{\theta}_2^2 \cos \theta_2 \end{aligned}$$

The governing equation of motion and the equations for constraint forces can be obtained by reducing

the following equations using the constraints above.

$$K(\theta_1 - \theta_0) + f_1 l \cos \theta_1 - N_1 l \sin \theta_1 = 0 \quad (5.4)$$

$$K(\theta_2 - \theta_0) + N_2 L \sin \theta_2 - \mu N_2 l \cos \theta_2 \cdot \text{sign}(\dot{x} - l\dot{\theta}_2 \cos \theta_2) = 0 \quad (5.5)$$

$$(m + m_0)\ddot{x} + m\ddot{\xi}_x + (m + m_o)g - f_1 + \mu N_2 \cdot \text{sign}(\dot{x} - l\dot{\theta}_2 \cos \theta_2) = 0 \quad (5.6)$$

$$(m + m_0)\ddot{y} + m\ddot{\xi}_y - N_1 - N_2 = 0 \quad (5.7)$$

And for the case where the bristles on the side corresponding to angle θ_2 are in stick phase, the holonomic constraint relating x and θ_2 is:

$$x = V - ia + l \sin \theta_2$$

$$\dot{x} = l\dot{\theta}_2 \cos \theta_2$$

$$\ddot{x} = l\ddot{\theta}_2 \cos \theta_2 - l\dot{\theta}_2^2 \sin \theta_2$$

The governing equation of motion and the equations for constraint forces can be obtained by reducing the following equations using the constraints above.

$$K(\theta_1 - \theta_0) - N_1 l \sin \theta_1 - \mu N_1 l \cos \theta_1 \cdot \text{sign}(\dot{x} - l\dot{\theta}_1 \cos \theta_1) = 0 \quad (5.8)$$

$$K(\theta_2 - \theta_0) + N_2 L \sin \theta_2 + f_2 l \cos \theta_2 = 0 \quad (5.9)$$

$$(m + m_0)\ddot{x} + m\ddot{\xi}_x + (m + m_o)g - f_2 + \mu N_1 \cdot \text{sign}(\dot{x} - l\dot{\theta}_1 \cos \theta_1) = 0 \quad (5.10)$$

$$(m + m_0)\ddot{y} + m\ddot{\xi}_y - N_1 - N_2 = 0 \quad (5.11)$$

5.1.2 Slip

In pure slip type motion behaviour of the climber, the bristle tips on either side of the main body continuously slip throughout one forcing time period. Hence there is no holonomic constraint that relates x and θ_1 or x and θ_2 and the system behaves as a two degree of freedom system. The only holonomic constraints in pure slip phase are the ones which enforce continuous contact between

bristle tips and the surface at all times.

$$y = l \cos \theta_1 + b$$

$$y = d - l \cos \theta_2 - b$$

$$\dot{y} = -l\dot{\theta}_1 \sin \theta_1$$

$$\dot{y} = l\dot{\theta}_2 \sin \theta_2$$

$$\ddot{y} = -l\ddot{\theta}_1 \sin \theta_1 - l\dot{\theta}_1^2 \cos \theta_1$$

$$\ddot{y} = l\ddot{\theta}_2 \sin \theta_2 + l\dot{\theta}_2^2 \cos \theta_2$$

The governing equations of motion in slip phase are given by:

$$(m + m_0)(\ddot{x} + g) + m\ddot{\xi}_x + \mu N_1 \text{sign}(\dot{x} - l\dot{\theta}_1 \cos \theta_1) + \mu N_2 \text{sign}(\dot{x} - l\dot{\theta}_2 \cos \theta_2) = 0 \quad (5.12)$$

$$-(m + m_0)(l\ddot{\theta}_1 \sin \theta_1 + l\dot{\theta}_1^2 \cos \theta_1) + m\ddot{\xi}_y - N_1 - N_2 = 0 \quad (5.13)$$

The Normal reaction N_1 and N_2 are given by:

$$N_1 = \frac{K(\theta_1 - \theta_o)}{l \sin \theta_1 + \mu l \cos \theta_1 \text{sign}(\dot{x} - l\dot{\theta}_1 \cos \theta_1)} \quad (5.14)$$

$$N_2 = -\frac{K(\theta_2 - \theta_o)}{l \sin \theta_2 - \mu l \cos \theta_2 \text{sign}(\dot{x} - l\dot{\theta}_2 \cos \theta_2)} \quad (5.15)$$

The equations of motion developed above are solved numerically in Matlab for a climber model with the following design parameters. Simulation results indicate that the system begins to move vertically upwards against the gravitational force at a very high forcing frequency of $\omega = 3600\pi$ rad/sec. This is illustrated in fig 5.2, 5.3. Also, the average velocity of the system is found to be proportional to the frequency of forcing. Since the torsional springs are precompressed, the bristles on either side of the main body won't lose contact with the supporting surfaces even at such relatively high forcing frequencies. This is illustrated by the normal reaction plots in fig 5.4. The normal reaction N_1 is always positive (pointing upwards) and the normal reaction N_2 is always negative (pointing downwards) indicating that the bristles always remain in contact with the surfaces. Detailed analysis of the slip-stick behaviour of the system and the effect of forcing and design parameters on the motion behaviour of the bristles is left for future work.

Figure 5.2: Displacement of the climber over time indicating that the system moves downwards under the influence of gravitational force for a forcing frequency less than $\omega = 3600\pi$ rad/sec

Figure 5.3: Displacement of the climber over time indicating that the system moves vertically upwards for a forcing frequency at and above $\omega = 3600\pi$ rad/sec.

Figure 5.4: Normal reaction $N1$ and $N2$ over time indicating continuous contact of the bristles with the surface.

Chapter 6

Conclusions and future work

Bristle bots are a class of vibration driven robots which utilize the inertial forces generated by an internal vibrating mass and the asymmetry provided by the bristles to achieve directed locomotion. In this study, the dynamics of a bristle bot is analyzed for two different models. The first model is driven by a rotating internal point mass whereas the second model is excited by a vertical actuating mass. It has been shown that in the presence of dry frictional forces acting between the bristle contact points and the substrate, both the models of bristle bot exhibit different kinds of locomotion such as pure stick, stick-slip and pure slip motions depending on the forcing frequency and amplitude. Also, a possibility of the robot losing contact with the ground for certain frequency and amplitude combinations is identified. Analytical conditions which enforce the type of motion behaviour exhibited by the bristles are formulated. System of equations defining the stick and slip motions are developed and the stick-slip dynamics of the bristle bot is explored.

Upto four threshold amplitude dependent frequencies are identified at which a transition in motion dynamics of the bristle bot occurs. An important transition at which the bristle bot reverses its direction of locomotion is observed. These transitions occur as a result of non-linearity in slip-stick toggle conditions. Furthermore, it has been observed that the bristles begin to slip in the negative direction when the frequency of forcing is close to the natural frequency of the system. Therefore the threshold frequency η_1 at which the bristle bot begins to move backwards is approximately equal to the natural frequency of the system. This is true for both the models of the bristle bot for any ratio of $\delta = \frac{A}{l}$. A map representing different motion behaviour regions of the bristle bot

for different forcing frequency and δ combinations is developed. Finally, the average velocity of the bristle bot is evaluated for different amplitude and frequency combinations.

A slightly modified version of a simple bristle bot which can move vertically upwards between two supporting surfaces is presented in chapter-6. This model can be used to analyze and predict the dynamics of a vertical pipe climbing bristle bot. The system is found to move upwards for a very high forcing frequency.

Future work is focused on exploring the dependency of threshold frequencies on the design parameters of the system. Elastic body modelling of the bristles subjected to tip loading should be considered for prototypes with soft bristles to get more accurate results. Slip-stick motion behaviour of 'the climber' should be explored to accurately predict and control the dynamics of in-pipe inspection bristle bots.

Appendices

Appendix A Matlab code

```
global flag M_o M K L f Omega A Theta_o g

for f=5:1:60

disp(f)

Design Parameters of the system

M_o=0.060; M=0.004; K=0.03; L=0.01; Omega=2*pi*f; A=0.004; Theta_o=0.523599; g=9.8;

Initial conditions

Theta_i=0.6501; Changes with system parameters

Thetadot_i=0; x_i=0; xdot_i=0;

tspan=[0 10];

tstart=tspan(1);

tend=tspan(end);

Define Output arrays

t_out=[]; x_out=[]; xdot_out=[]; theta_out=[]; thetadot_out=[]; thetadd_out=[];

xdd_out=[]; N_out=[]; f_out=[]; f_s_out=[]; Lambda_out=[];

[thetadd_i,xdd_i]=Initialaccn.eval(f);

N_i=(M+M_o)*g+M*(-Omega^2*A)-(M+M_o)*L*thetadd_i*sin(Theta_i);

Lambda_i=(M+M_o)*xdd_i;

f_s_i=0.25*N_out*sign(Lambda_i);

vt=0; Initial value of bristle tip velocity

frn=Lambda_i; Initial value of friction

NR=N_i; Initial value of Normal reaction

options=odeset('Events',@stick_slip,'RelTol',1e-12,'AbsTol',1e-16);

Output vectors to store Slip/Stick displacements per cycle

Dt_stk=[]; Dt_slip=[]; Dx_stk=[]; Dx_slip=[]; Dx_cycle=[];

x_strt=[]; x_end=[];

time=0;
```

```

while (time(end)<tend)

if (abs(Vt)<0.000000001)

if (abs(frn)≤abs(0.25*NR))

disp('Stick Phase')

flag=1;

[time,state_values]=ode45(@BB.Stick_phase,[tstart tend],[Theta_i Thetadot_i x_i xdot_i],options);

theta=state_values(:,1);

thetadot=state_values(:,2);

x=state_values(:,3);

xdot=state_values(:,4);

else

disp('Slip Phase')

flag=-1;

[time,state_values]=ode45(@BB.Slip_phase,[tstart tend],[Theta_i Thetadot_i x_i xdot_i],options);

theta=state_values(:,1);

thetadot=state_values(:,2);

x=state_values(:,3);

xdot=state_values(:,4);

end

else

Slip ODE

disp('Slip Phase')

flag=-1;

[time,state_values]=ode45(@BB.Slip_phase,[tstart tend],[Theta_i Thetadot_i x_i xdot_i],options);

theta=state_values(:,1);

thetadot=state_values(:,2);

x=state_values(:,3);

xdot=state_values(:,4);

end

clear thetadd xdd N friction f_s A1 A2 A3 A4 B1 B2 B3 B4 F1 F2 F3 F4 F1_a F1_b ...

eta_x eta_y eta_xdd eta_ydd

```

Evaluating force vectors in Slip/Stick condition

```

eta_x=A*sin(Omega*time);
eta_y=A*cos(Omega*time);
eta_xdd=-Omega^2*eta_x;
eta_ydd=-Omega^2*eta_y;
Mu=0.2*sign(xdot-L.*thetadot.*cos(theta));
if flag== -1
disp('Slip')
disp(flag)
x_strt=x(1);
dt_slip=time(end)-time(1);
dx_slip=x(end)-x(1);
Dt_slip=[Dt_slip;dt_slip];
Dx_slip=[Dx_slip;dx_slip];

A1=( (M_o+M)*L^2*sin(theta).^2)+(Mu.*(M_o+M)*L^2.*sin(theta).*cos(theta));
B1=0;
A2=-Mu.*(M+M_o)*L.*sin(theta);
B2=M+M_o;
F1_a=(Mu.*(M_o+M)*L^2.*cos(theta).^2+(M_o+M)*L^2.*sin(theta).*cos(theta)).*thetadot.^2;
F1_b=-Mu.*(M_o+M)*g*L.*cos(theta)-Mu.*M*L.*eta_ydd.*cos(theta)-M*L*(eta_ydd).*sin(theta)
-(M_o+M)*g*L*sin(theta)+K*(theta-Theta_o);
F1=-(F1_a+F1_b);
F2=-M*(eta_xdd)-Mu.*M.*(eta_ydd)+Mu.*(M+M_o)*L.*cos(theta).*thetadot.^2-Mu*(M+M_o)*g;

thetadd=(B2.*F1)/(A1.*B2 - A2.*B1) - (B1.*F2)/(A1.*B2 - A2.*B1);
xdd=(A1.*F2)/(A1.*B2 - A2.*B1) - (A2.*F1)/(A1.*B2 - A2.*B1);

```

Friction and Normal reaction

```

N=(-M*L*thetadd.*sin(theta)-M*L*thetadot.^2.*cos(theta)-M_o*L*thetadd.*sin(theta)

```

```

-M_o*L*thetadot.^2.*cos(theta)+M*(eta_ydd)+M*g+M_o*g);

friction=-Mu.*N;

f_s=NaN*time;

Lambda=(M+M_o)*xdd+M*eta_xdd;

else

disp('Stick')

disp(flag)

x_end=x(end);

del_x=x_end-x_strt;

Dx_cycle=[Dx_cycle;del_x];

dt_stk=time(end)-time(1);

dx_stk=x(end)-x(1);

Dt_stk=[Dt_stk;dt_stk];

Dx_stk=[Dx_stk;dx_stk];


A3=(M_o+M)*(L^2*sin(theta).^2);

B3=(M_o+M)*L*cos(theta);

A4=L*cos(theta);

B4=-1;

F3=-(M_o+M)*L^2*sin(theta).*cos(theta).*thetadot.^2+M*L*sin(theta).*(eta_ydd)

+(M+M_o)*g*L*sin(theta)-M*L*(eta_xdd).*cos(theta)-K*(theta-Theta_o);

F4=L*thetadot.^2.*sin(theta);


thetadd=(B4.*F3)./(A3.*B4 - A4.*B3)-(B3.*F4)./(A3.*B4 - A4.*B3);

xdd=(A3.*F4)./(A3.*B4 - A4.*B3)-(A4.*F3)./(A3.*B4 - A4.*B3);


Friction and Normal reaction

N=(-M*L*thetadd.*sin(theta)-M*L*thetadot.^2.*cos(theta)-M_o*L*thetadd.*sin(theta)

-M_o*L*thetadot.^2.*cos(theta)+M*(eta_ydd)+M*g+M_o*g);

Lambda=(M+M_o)*xdd+M*eta_xdd;

friction=Lambda;

```

```
f_s=0.25*N.*sign((M+M_o)*xdd+M*eta_xdd);
end
```

Accumulating vectors

```
t_out=[t_out;time(1:end-1)]; Time output
theta_out=[theta_out;theta(1:end-1)]; Theta output
thetadot_out=[thetadot_out;thetadot(1:end-1)]; Thetadot output
x_out=[x_out;x(1:end-1)]; Displacement output
xdot_out=[xdot_out;xdot(1:end-1)]; Xdot output
N_out=[N_out;N(1:end-1)]; Normal reaction output
f_out=[f_out;friction(1:end-1)];
f_s_out=[f_s_out;f_s(1:end-1)];
Lambda_out=[Lambda_out;Lambda(1:end-1)];
thetadd_out=[thetadd_out;thetadd(1:end-1)];
xdd_out=[xdd_out;xdd(1:end-1)];
```

New initial conditions and tspan for Ode's

```
tstart=time(end);
Theta_i=theta(end);
Thetadot_i=thetadot(end);
x_i=x(end);
xdot_i=xdot(end);
```

Updating vectors for while loop conditions

```
Vt=xdot_out(end)-L*thetadot_out(end).*cos(theta_out(end));
stat_frn=f_s(end);
f_k=-0.2*sign(xdot(end)-L*thetadot(end)*cos(theta(end)))*N_out(end);

[NR,frn]=Lambda_stick_eval(f,time(end),theta(end),thetadot(end)); end

Vtip=xdot_out-L*thetadot_out.*cos(theta_out);
```

```

filename=sprintf('f=0.01f_xi_x=0_A=0.004_M=0.06_m=0.004_K=0.03_mu=0.2_L=0.01_RelTol_e-12.mat',f);
save(filename);
end

```

Events Function

```

function [Value,isterminal,direction]=stick_slip(t,s)

global flag M_o M K L Omega A Theta_o g

isterminal=1;
direction=-1;

Mu=0.2*sign(s(4)-L*s(2)*cos(s(1)));

eta_x=A*sin(Omega*t);
eta_y=A*cos(Omega*t);
eta_xdd=-Omega^2*eta_x;
eta_ydd=-Omega^2*eta_y;

A1=( (M_o+M)*L^2*sin(s(1))^2+(Mu*(M_o+M)*L^2*sin(s(1))*cos(s(1)) );
B1=0;
A2=-Mu*(M+M_o)*L*sin(s(1));
B2=M+M_o;
F1_a=(Mu*(M_o+M)*L^2*cos(s(1))^2+(M_o+M)*L^2*sin(s(1))*cos(s(1)))*s(2)^2;
F1_b=-Mu*(M_o+M)*g*L*cos(s(1))-Mu*M*L*(eta_ydd)*cos(s(1))-M*L*(eta_ydd)*sin(s(1))
-(M_o+M)*g*L*sin(s(1))+K*(s(1)-Theta_o);
F1=-(F1_a+F1_b);
F2=-M*(eta_xdd)-Mu*M*(eta_ydd)+Mu*(M+M_o)*L*cos(s(1))*s(2)^2-Mu*(M+M_o)*g;

EQN1=(B2*F1)/(A1*B2 - A2*B1) - (B1*F2)/(A1*B2 - A2*B1);
EQN2=(A1*F2)/(A1*B2 - A2*B1) - (A2*F1)/(A1*B2 - A2*B1);

A3=(M_o+M)*(L^2*sin(s(1))^2);

```



```

B3=(M_o+M)*L*cos(s(1));
A4=L*cos(s(1));
B4=-1;
F3=-(M_o+M)*L^2*sin(s(1))*cos(s(1))*s(2)^2+M*L*sin(s(1))*(eta_ydd)+(M+M_o)*g*L*sin(s(1))
-M*L*(eta_xdd)*cos(s(1))-K*(s(1)-Theta_o);
F4=L*s(2)^2*sin(s(1));

EQN3=(B4*F3)/(A3*B4 - A4*B3) - (B3*F4)/(A3*B4 - A4*B3);
EQN4=(A3*F4)/(A3*B4 - A4*B3) - (A4*F3)/(A3*B4 - A4*B3);

if flag==1
Value=double(abs(s(4)-L*s(2)*cos(s(1)))<0.000000001 &&
abs((M+M_o)*EQN4+M*eta_xdd)<abs((M+M_o)*(-L*EQN3*sin(s(1))-L*s(2)^2*cos(s(1))
+M*eta_ydd+(M+M_o)*g)*0.25));
if Value==0
velocity_check=abs(s(4)-L*s(2)*cos(s(1)))
Lagrange_multiplier=abs((M+M_o)*EQN4+M*eta_xdd)
Static_friction=abs((M+M_o)*(-L*EQN3*sin(s(1))-L*s(2)^2*cos(s(1)))+M*eta_ydd+(M+M_o)*g)*0.25)
end
else
Value=double((abs(s(4)-L*s(2)*cos(s(1)))>0.000000001) ||
(abs((M+M_o)*EQN2+M*eta_xdd)>abs((-M*L*EQN1*sin(s(1))-M*L*s(2)^2*cos(s(1))
-M_o*L*EQN1*sin(s(1))-M_o*L*s(2)^2*cos(s(1))+M*(eta_ydd)+M*g+M_o*g)*0.2)));
if Value==0
velocity_check=abs(s(4)-L*s(2)*cos(s(1)))
Lagrange_multiplier=abs((M+M_o)*EQN2+M*eta_xdd)
friction_k=abs((-M*L*EQN1*sin(s(1))-M*L*s(2)^2*cos(s(1))-M_o*L*EQN1*sin(s(1))
-M_o*L*s(2)^2*cos(s(1))+M*(eta_ydd)+M*g+M_o*g)*0.2)
end
end
end

```

Slip ODE function

```
function sdot=BB_Slip_phase(t,s)

global M_o M K L Omega A Theta_o g

disp('Slip ODE')

Mu=0.2*sign(s(4)-L*s(2)*cos(s(1)));

eta_x=A*sin(Omega*t);
eta_y=A*cos(Omega*t);
eta_xdd=-Omega^2*eta_x;
eta_ydd=-Omega^2*eta_y;

A1=((M_o+M)*L^2*sin(s(1))^2+(Mu*(M_o+M)*L^2*sin(s(1))*cos(s(1))));
B1=0;
A2=-Mu*(M+M_o)*L*sin(s(1));
B2=M+M_o;
F1_a=(Mu*(M_o+M)*L^2*cos(s(1))^2+(M_o+M)*L^2*sin(s(1))*cos(s(1)))*s(2)^2;
F1_b=-Mu*(M_o+M)*g*L*cos(s(1))-Mu*M*L*eta_ydd*cos(s(1))-M*L*(eta_ydd)*sin(s(1))-
(M_o+M)*g*L*sin(s(1))+K*(s(1)-Theta_o);
F1=-(F1_a+F1_b);
F2=-M*(eta_xdd)-Mu*M*(eta_ydd)+Mu*(M+M_o)*L*cos(s(1))*s(2)^2-Mu*(M+M_o)*g;

EQN1=(B2*F1)/(A1*B2-A2*B1)-(B1*F2)/(A1*B2-A2*B1);
EQN2=(A1*F2)/(A1*B2-A2*B1)-(A2*F1)/(A1*B2-A2*B1);

sdot=[s(2);EQN1;s(4);EQN2];

end
```

Stick ODE function

```
function sdot=BB_Stick_phase(t,s)

global M_o M K L Omega A Theta_o g
```

```

disp('stick Ode')

eta_x=A*sin(Omega*t);
eta_y=A*cos(Omega*t);
eta_xdd=-Omega^2*eta_x;
eta_ydd=-Omega^2*eta_y;

A3=(M_o+M)*(L^2*sin(s(1))^2);
B3=(M_o+M)*L*cos(s(1));
A4=L*cos(s(1));
B4=-1;
F3=-(M_o+M)*L^2*sin(s(1))*cos(s(1))*s(2)^2+M*L*sin(s(1))*(eta_ydd)+(M+M_o)*g*L*sin(s(1))-M*L*(eta_xdd)*cos(s(1))-K*(s(1)-Theta_o);
F4=L*s(2)^2*sin(s(1));

EQN3=(B4*F3)/(A3*B4 - A4*B3) - (B3*F4)/(A3*B4 - A4*B3);
EQN4=(A3*F4)/(A3*B4 - A4*B3) - (A4*F3)/(A3*B4 - A4*B3);
sdd=[s(2);EQN3;s(4);EQN4];
end

```

Function to evaluate the initial acceleration of the system

```

function [thetadd_i,xdd_i] = Initial_accn_eval(f)
global M_o M K L A g Theta_o
Omega=2*pi*f;

```

Initial Conditions

```

Theta_i=0.6501;Thetadot_i=0;x_i=0; xdot_i=0; t=0;

eta_x=A*sin(Omega*t);
eta_y=A*cos(Omega*t);
eta_xdd=-Omega^2*eta_x;

```

```
eta_ydd=-Omega^2*eta_y;
```

Coefficients

```
A3=(M_lo+M)*(L^2*sin(Theta_i)^2);
```

```
B3=(M_lo+M)*L*cos(Theta_i);
```

```
A4=L*cos(Theta_i);
```

```
B4=-1;
```

```
F3=-(M_lo+M)*L^2*sin(Theta_i)*cos(Theta_i)*Thetadot_i^2+M*L*sin(Theta_i)*(eta_ydd)
+(M+M_lo)*g*L*sin(Theta_i)-M*L*(eta_xdd)*cos(Theta_i)-K*(Theta_i-Theta_o);
```

```
F4=L*Thetadot_i^2*sin(Theta_i);
```

```
ICs=inv([A3 B3;A4 B4])*[F3;F4];
```

```
thetadd_i = ICs(1);
```

```
xdd_i = ICs(2);
```

```
end
```

Function to evaluate the constraint forces for Slip-Stick toggle condition.

```
function [NR,frn] = Lambda_stick_eval(f,t,theta,thetadot)
```

```
global M_lo M K L A g Theta_o
```

```
Omega=2*pi*f;
```

```
eta_x=A*sin(Omega*t);
```

```
eta_y=A*cos(Omega*t);
```

```
eta_xdd=-Omega^2*eta_x;
```

```
eta_ydd=-Omega^2*eta_y;
```

```
A3=(M_lo+M)*(L^2*sin(theta)^2);
```

```
B3=(M_lo+M)*L*cos(theta);
```

```
A4=L*cos(theta);
```

```
B4=-1;
```

```
F3=-(M_lo+M)*L^2*sin(theta)*cos(theta)*thetadot^2+M*L*sin(theta)*(eta_ydd)
```

```

+ (M+M_o) * g * L * sin(theta) - M * L * (eta_xdd) * cos(theta) - K * (theta - Theta_o);

F4 = L * thetadot^2 * sin(theta);

qdd = inv([A3 B3; A4 B4]) * [F3; F4];

thetadd = qdd(1);

xdd = qdd(2);

NR = - (M_o + M) * L * thetadd * sin(theta) - (M_o + M) * L * thetadot^2 * cos(theta) + M * eta_ydd + (M_o + M) * g;

frn = (M_o + M) * L * thetadd * cos(theta) - (M_o + M) * L * thetadot^2 * sin(theta) + M * eta_xdd;

end

```

Bibliography

- [1] F. Alexander and N. G. Stanislav. Frictional-anisotropy-based systems in biology: structural diversity and numerical model. *Scientific Reports*, (1240), 2013.
- [2] F. Becker, S. Boerner, T. Kastner, V. Lysenko, I. Zeidis, and K. Zimmermann. Spy bristle bot a vibration-driven robot for the inspection of pipelines. 2014.
- [3] F. Becker, S. Boerner, V. Lysenko, I. Zeidis, and K. Zimmermann. On the mechanics of bristle-bots - modeling, simulation and experiments. In *ISR/Robotik 2014; 41st International Symposium on Robotics*, pages 1–6, 2014.
- [4] F. Becker, V. Lysenko, V. Minchenya, I. Zeidis, and K. Zimmermann. An approach to the dynamics of a vibration-driven robot. In *Romansy 19 – Robot Design, Dynamics and Control*, pages 299–306, Vienna, 2013. Springer Vienna.
- [5] N. N. Bolotnik, F. Chernousko, and T. Figurina. Optimal control of a two-body: Vibration-driven locomotion system in a resistive environment. *IFAC-Papers Online*, 48(25):91–96, 2015.
- [6] N. N. Bolotnik, I. M. Zeidis, K. Zimmermann, and S. F. Yatsun. Dynamics of controlled motion of vibration-driven systems. *Journal of Computer and Systems Sciences International*, 45(5):831–840, Oct 2006.
- [7] F. L. Chernousko. On the motion of a body containing a movable internal mass. *Doklady Physics*, 2005.
- [8] Miroslav Dovica, Michal Gorzs, Jozef Kov, and tefan Ondoko. In-pipe passive smart bristled micromachine. 04 2019.
- [9] L. Gomi, N. Hawley-Weld, and L. Mahadevan. Swarming, swirling and stasis in sequestered bristle-bots. *Proceedings of The Royal Society A Mathematical Physical and Engineering Sciences*, 469, 02 2013.
- [10] K. Hatazaki, M. Konyo, K. Isaki, S. Tadokoro, and F. Takemura. Active scope camera for urban search and rescue. *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2007.
- [11] D. L. Hu, J. Nirody, T. Scott, and M. J. Shelley. The mechanics of slithering locomotion. *Proceedings of the National Academy of Sciences*, (100):10081–10085, 2009.
- [12] P. Hyunjun, K. Byungkyu, and P. Jong-Oh. A crawling based locomotive mechanism using a tiny ultrasonic linear actuator (tula).
- [13] K. Isaki, A. Niitsuma, M. Konyo, F. Takemura, and S. Tadokoro. Development of an active flexible cable by ciliary vibration drive for scope camera. *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2006.

- [14] I. Kiyoshi. A mobile micro-robot using centrifugal forces. In *1999 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*. IEEE, 1999.
- [15] S. J. Newman and B. C. Jayne. Crawling without wiggling: muscular mechanisms and kinematics of rectilinear locomotion in boa constrictors. *Journal of Experimental Biology*, 2018.
- [16] M. Rubenstein, A. Cornejo, and R. Nagpal. Programmable self-assembly in a thousand-robot swarm. *Science*, 345(6198):795–799, 2014.
- [17] M. Schulke, L. Hartmann, and C. Behn. Worm-like locomotion systems: Development of drives and selective anisotropic friction structures. *56th International scientific colloquium*, 2011.
- [18] L. Victor, K. Zimmermann, A. Chigarev, and F. Becker. A mobile vibro-robot for locomotion through pipelines, 2012.
- [19] Z. Wang and H. Gu. A bristle-based pipeline robot for ill-constraint pipes. *IEEE/ASME Transactions on Mechatronics*, 13(3):383–392, June 2008.
- [20] K. Zimmermann, I. Zeidis, N. Bolotnik, and M. Pivovarov. Dynamics of a two-module vibration-driven system moving along a rough horizontal plane. *Multibody System Dynamics*, 2009.