5-2019

Analytical Modeling of Melting of Levitated Ice Particles and Freezing of Water Films on Cold Surfaces

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ANALYTICAL MODELING OF
MELTING OF LEVITATED ICE PARTICLES AND FREEZING
OF WATER FILMS ON COLD SURFACES

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Shivuday Kala
May 2019

Accepted by:
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Dr. Joshua Bostwick
ABSTRACT

Aircraft engine icing has emerged as one of the greatest threats to safe operation of aircrafts today contributing to more than half of all the aircraft icing accidents reported between 1990-2000. Engine icing mostly occurs in the low-pressure compressor section of the engine where ice particles entering the engine undergo partial or complete melting on encountering warm surfaces. These partially melted crystals cool the engine parts until the freezing point of water is reached. Further accumulation of ice inside the engine leads to ice accretion and shedding. This situation may lead to complete power loss in the engine and acts as an operational hazard for the aircraft. This study aims at creating mathematical models for the two fundamental processes that occur during aircraft engine icing namely: melting of ice particles and freezing of water films. To understand the melting behavior of ice particles, a two-layer model was developed and the governing equations for the heat balance at the surface of the particle and the surface of the melting ice core were defined. The physical parameters affecting the melting behavior of the ice particle were identified and their unsteady effects introduced in the model to obtain a final relation between radius reduction ratio and melt time. The analytical model was then validated by comparing the melting rate and melt time data obtained from an experiment conducted by Dr. Jose Palacios and Sihong Yan at Pennsylvania State University for observing melting of isolated levitated ice particles ranging from 300μm to 1200μm in size. From the comparison it was found that the two-phase model developed in this research correctly predicts melting rate
and melt times for ice particles greater than 800μm with an average absolute melt time error of 4%.

The goal of the next part of the research was to obtain a mathematical expression for the latent heat flux at the surface on which freezing of a water film takes place. For this the two-phase Stefan problem approach was applied to a water film undergoing freezing at a constant temperature freezing surface and an insulated water surface. The explicit solution obtained for the Stefan problem was then used to obtain temperature profiles for the varying ice and water layers. The latent heat flux at the freezing surface was derived and validated against a heat flux measurement reading from a similar experiment.
DEDICATION

To my parents, sister and all the teachers who taught me over the years.
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude advisor Dr. Yiqiang Han for his immense support and encouragement throughout the period of my thesis research. His technical inputs and dynamic guidance helped me give my best towards this research and go forward with publishing my research work.

I am thankful to Dr. John R. Saylor for agreeing to chair my thesis committee and Dr. Joshua Bostwick for agreeing to part of my thesis committee. Their valuable feedback throughout the entire duration helped me finetune my research. I would like to thank Dr. Joshua Bostwick for giving me an opportunity to present my research in the weekly Graduate Student Seminar of the Mechanical Engineering department at Clemson university.

I would lastly like to thank Dr. Jose Palacios and Shihong Yan from Department of Aerospace Engineering at Pennsylvania State University for providing experimental data for this research and additional help when required by me.

Lastly, I would like to thank the Clemson University and the Clemson Mechanical Engineering Department for giving me an opportunity to apply myself in the field of engineering.
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NOMENCLATURE

\( \alpha_s \)  
thermal diffusivity of solid

\( \alpha_l \)  
thermal diffusivity of liquid

\( D_{va} \)  
diffusivity of water vapor in air (m\(^2\)/s)

\( f \)  
mean ventilation coefficient

\( k_a \)  
thermal conductivity of air (W/m.K)

\( k_s \)  
thermal conductivity of solid (W/m.K)

\( k_l \)  
thermal conductivity of liquid (W/m.K)

\( k_w \)  
thermal conductivity of water (W/m.K)

\( k_s \)  
thermal conductivity of solid

\( k_l \)  
thermal conductivity of liquid

\( L_e \)  
latent heat of evaporation (J/kg)

\( L_m \)  
latent heat of melting (J/kg)

\( \Phi \)  
relative humidity of environments (\%)

\( \rho_{vrw} \)  
water vapor density at T\(_r\)(r\(_i\)) (kg/m\(^3\))

\( \rho_\infty \)  
water vapor density at T\(_\infty\) (kg/m\(^3\))

\( \rho_{v,sat,\infty} \)  
saturation vapor density at T\(_\infty\) (kg/m\(^3\))

\( r_w \)  
radius of melting ice particle (m)

\( r_i \)  
radius of ice core (m)
<table>
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<tr>
<td>( t )</td>
<td>time (s)</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature variable (K)</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>constant equal to 273.16 K</td>
</tr>
<tr>
<td>( T_{r(r_i)} )</td>
<td>temperature of particle surface as a function of radius (K)</td>
</tr>
<tr>
<td>( T_\infty )</td>
<td>temperature of environment (K)</td>
</tr>
<tr>
<td>( T_s )</td>
<td>temperature of solid</td>
</tr>
<tr>
<td>( T_l )</td>
<td>temperature of liquid</td>
</tr>
<tr>
<td>( y )</td>
<td>fractional radius of ice particle</td>
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Chapter 1:

INTRODUCTION

1.1 Aircraft Engine Icing Overview

Aircraft icing is a major operational hazard facing the aviation industry today. According to the NTSB online database, from 1978 to 2005, 645 in-flight accidents and incidents were reported in the US alone\(^1\). Another 299 in-flight icing incidents were reported by the NASA Aviation Safety Reporting System (ASRS) during the same period\(^1\). Accumulation of ice on aircraft at high altitudes is referred to as ice accretion. Aircraft icing may occur on structural parts such as the airframe, wings, windshield or ice may be ingested by the aircraft engine. The cumulative effect of both types of icing is the reduction in aircraft efficiency by increased weight, reduced lift, decreased thrust, increased drag and power loss and stalling. Other icing effects may include false indication on flight instruments, loss of radio communications and loss of operational control surfaces and landing gears\(^2\). The in-flight ice formation and accretion are highly dependent on weather conditions, which include liquid water content, atmospheric temperature and the droplet size in the cloud\(^3\).

Induction system icing or aircraft engine icing forms a major part of the aircraft icing problem. This type of icing can occur in both piston and jet engines\(^2\). According to the AOPA Air safety Foundation accident database, induction icing contributed to more
than half of the total accidents reported between 1990 and 2000. The ingestion of ice particles in jet engines can cause loss of thrust (rollback), stalling, surge, and engine damage due to ice shedding.

The exact mechanism of icing inside aircraft jet engines is not known but there have been several attempts to hypothesize the phenomenon. Glenn Research Center (NASA) attempted to explain how the engine icing phenomenon unfolds inside a jet engine passing through a mixed phase ice cloud based on engine and compression system modelling and simulation\(^5\). According to the study, mixed phase ice clouds are scooped inside the engine ice core. The presence of liquid water in the core due to previously ingested ice particles and supercooled liquid from the atmosphere, slows down the incoming ice particles.

Figure 1-1. Percentage breakup of causes leading to aircraft accidents from 1990-2000 (AOPA Air Safety Foundation Safety Advisor Weather No.1, 2008)
allowing heat transfer to occur between them and the warm engine components. The heat removed from the metal surfaces reduces its temperature until the freezing point of water is reached. Further mixed phase ice impingement causes accumulation of ice inside the engine, leading to ice shedding and power loss. This shows that during engine icing both melting and freezing phenomena takes place in tandem. Jet engine icing mainly affects the low-pressure compressor region of the engine. Figure 1-2 shows the region inside a jet engine which is prone to inflight ice particle accretion.

Figure 1-2 Typical turbofan engine compression system schematic, with potential ice accretion sites noted.
1.2 Mixed Phase Icing Conditions

Icing can be caused by supercooled droplets which appear in the atmosphere at temperatures ranging from -40°C to 0°C. These droplets are considered thermodynamically metastable and can undergo phase change to solid ice by local mechanical disturbances such as collision with an aircraft passing over a cloud of supercooled liquid droplets. Most of the early research related to aircraft icing was focused on supercooled droplets icing conditions. But after analysis of several in flight icing incidents and power loss incidents a renewed focus has occurred on another type of icing condition referred to as mixed phase icing. Mixed phase icing has also been included in air worthiness certification for the very same reason. Mixed phase ice crystals are more adhesive to aircraft engine surfaces than ice crystals. The ice crystals do not merge with each other and thus can be removed easily. Meanwhile partially melted ice crystals coalesce with other ice crystals and firmly attach to surfaces leading to ice accretion which is harder to remove. It is for this reason aircraft engine deicing is more complicated to implement and achieve. One of the earliest attempts to numerically model mixed phase icing was done by Nilameen et al by solving the governing equations for multiphase ice. Villedieu et al and Trontin et al performed a detailed study of the mixed phase icing condition regime which included detailed accounts of models for particle trajectories, heat transfer regimes and phase change, impingement, accretion, and erosion.

The investigation of in flight aircraft icing incidents and accidents are useful in understanding how different atmospheric icing conditions lead to engine icing. These investigations have prompted Federal Aviation Administration (FAA) to publish Appendix
C Part 25 of the Federal Aviation Regulation (FAR) specifying an Icing envelope based on altitude and ambient temperature. Figure 1-3 shows the Icing envelope. It applies to jet aircrafts with a minimum of 10 seats or maximum take-off weight (MTOW) greater than 5,670 kg or prop-airplanes with at least 19 seats or a MTOW greater than 8,618 kg. The icing envelope shows the range of altitude and ambient temperature at which in flight engine icing can occur. It was observed that most of the engine icing incidents occur at altitudes between 12,500 ft to 40,000 ft and at temperatures warmer than the International Standard Atmosphere (ISA).

Figure 1-3 Temperature vs Altitude envelope for jet engine icing and engine icing incidents\textsuperscript{12} (Icing Design Envelopes (14 CFR Parts 25 and 29, Appendix C))

1.3 Ice particle melting
1.3.1 Experimental study of ice particle melting

Studies have been conducted to reproduce mixed-phase icing conditions in wind tunnels to better understand the fundamental physics of engine icing due to partially melted crystals. Griffin et al.\textsuperscript{13} and Oliver et al.\textsuperscript{14} upgraded and validated the NASA Propulsion System Laboratory with engine icing test ability to perform engine inlet ice crystal testing in an altitude environment. A jet engine was placed inside the test rig to observe the behavior when the engine encountered the icing cloud formed by frozen water droplets. The PSL has reproduced and validated several types of abnormal engine events for varying icing conditions. NASA Glenn Research Center and the National Research Council of Canada, NRC, collaboratively conducted ice accretion tests at the NRC’s Research Altitude Test Facility\textsuperscript{15, 16, 17, 18, 19}. In these tests, the air temperature and humidity were observed to change when the icing cloud was activated. These changes indicated heat interaction between the air and the ice particles that eventually became partially melted. In this project, Currie et al.\textsuperscript{15, 16} and Struk et al.\textsuperscript{17, 18, 19} experimentally observed that the accretion rate of ice is positively proportional to the ratio between liquid water content (LWC) and total water content (TWC). The accretion rate reached its maximum value when LWC/TWC was between 10% and 25%. For engine icing, the LWC/TWC ratio represents the melting ratio of ice crystals, or in other words, the partial melting state of the crystals. If the melting ratio can be quantified and controlled in a wind tunnel test, ground test facilities could reproduce the full envelope of engine icing conditions. Understanding the residence time
required to reach these controllable partial melting states of ice crystals is critical for the validation of engine icing models and ultimately, to design engines that prevent such adverse conditions.

1.3.2 Analytical Modeling of ice particle melting

Besides the experimental study of reproducing the partially melting behavior under laboratory conditions, thermal models\textsuperscript{20,21,22,23,24,26} have been developed to explain the freezing/melting process based on limited considerations related to thermal exchange processes between air and crystals. Wright et al.\textsuperscript{20} used the Dalton’s\textsuperscript{23} energy equation to estimate the energy exchange between frozen droplets and air, so the percentage rate of melting can be calculated from the balance of the energy. Veres et al.\textsuperscript{21} applied this model in the GlennICE engine icing model based on Wright’s research to study the ice accretion in engine with respect to partially melting droplets. Similar models are also investigated from the perspective of atmospheric science. Mason\textsuperscript{24} developed a thermal model to simulate melting process for a single frozen droplet at free fall with constant warming rate. This model provided quantification of ice melted as a decrease in the ice core radius with respect to time. But it was developed from the standpoint of atmospheric sciences and hence its application to aircraft engine icing is not suitable. Figure 1-4 shows a plot comparing the decrease in the radius of the ice core inside melting spheres for different ice particle sizes for fixed warming rate.
Rasmussen et al.\textsuperscript{26} conducted melting test on a single droplet in a wind tunnel at a constant warming rate and used a camera to observe the ice core directly to compare experiments with Mason’s results. Despite the absence of an appropriate technique to visualize the ice, Rasmussen concluded that thermal models tend to underestimate the rate of melting.
Bartkus et al.\textsuperscript{22} further developed a coupled air-particle thermal model to simulate the heat interaction based on NASA and NRC’s tests to simulate how ice particles melted and cooled down the hot airflow in their tests. These thermal models were derived from Dalton’s\textsuperscript{23} thermal analysis on a single particle and reasonably good results were achieved. The code was initially designed for a wind tunnel operation point of view, where the icing cloud typically contains ice crystals with a wide range of diameter. To validate thermal models of single ice particles, it is necessary to study the physics of melting behavior of an isolated and stable ice particle. To that end, wind tunnel tests may not sufficiently validate these thermal models as it is difficult to generate and analyze single ice particles in a wind tunnel. With the exception of Bartkus model, previous attempts to model melting behavior

Figure 1-5 Comparison of change in radius of ice core with time for different conditions\textsuperscript{26}
of ice particles was done in the context of atmospheric sciences. There is need to develop melting models which can replicate the physical conditions found inside an aircraft jet engine.

1.4 Ice accretion measurement

In order to create greater understanding of the engine icing event in an aircraft, it is also important to estimate the ice accumulating inside the engine surfaces. It is currently very difficult to measure the incoming ice in an engine due to the complexity of surfaces and the physical environment. Considerable work has been done to create numerical models for ice accretion over stationary surfaces before from the perspective of meteorological sciences\textsuperscript{27}. Makkonen\textsuperscript{28} provided a numerical model for the amount of ice accreted on transmission wires and its dependence on physical conditions in the environment. More recently Sokolov et al.\textsuperscript{29} performed a detailed analytical study and numerical simulations of ice accretion for determining droplet distribution spectra. For measuring aircraft ice accretion, currently experiments have been performed in the NASA Icing research Tunnel to record the ice accretion geometry on 2-D and 3-D airfoils using 3-D laser scanning tools\textsuperscript{30}. Ice accretion presence and thickness is also measured using microwaves, but these are costly devices and their application in measuring aircraft engine ice accretion is limited as of now. There is a need for simpler measurement techniques which can predict the mass and thickness of ice accreted over surfaces.
1.5 Thesis Objectives

The goal of this thesis is to develop mathematical models for melting and freezing behavior of ice particles and water film respectively in the context of aircraft engine icing, thereby attempting to understand the physics of ice accretion in aircraft jet engines. The thesis research is divided into two main parts: the first part is dedicated to creating an analytical model for melting of an ice particle based on an experiment conducted on melting of levitated ice particles while the second part is dedicated to developing a mathematical expression for estimating latent heat flux at a constant temperature surface for a freezing water film with insulated boundary conditions. The strategy for the research is as follows:

1. Define the governing heat equations and boundary conditions for the physical process at hand (melting and freezing).
2. Identify all the external parameters affecting the physical process and introduce their unsteady effects
3. Obtain an expression for the physical process which takes into account the unsteady effects of all physical parameters involved
4. Validate the analytical model obtained with results from previously conducted experiments for the same physical process
1.6 Thesis Overview

The research work for this thesis has been divided into the following chapters:

Chapter 2: Analytical model for melting of ice particles: In this chapter the experiment conducted by Dr. Jose Palacios and Sihong Yan at Pennsylvania State University for observing and quantifying melting of ice particles is discussed. This is followed by defining a new two-layer analytical model for melting of ice particles which takes into account the unsteady effects of ambient temperature and relative humidity around the ice particle. The melting behavior of an ice particle with time is compared with the experimental results and inferences are derived.

Chapter 3: Analytical model for freezing water film: In this chapter an experiment conducted by Dr. Jose Palacios and Dr. Yiqiang Han for measuring latent heat flux at the surface of a cold plate on which freezing of water takes place is discussed. This is followed by modeling the freezing process for the water film using the Stefan moving boundary problem approach. The temperature profiles for the two phases obtained from the model are used to derive a mathematical expression for the surface heat flux. The temperature profiles of the two phases are modeled for different spatial positions and different freezing times to understand the latent heat flux at the freezing surface. The model is validated by comparing the heat flux readings obtained from the experiment.
Chapter 4: Conclusions: This chapter summarizes the two analytical models developed in the previous chapters, discusses their advantages in application and also their shortcomings. The future work required for each of the models to make them more accurate in predicting melting and freezing behavior is also listed in this chapter.
2.1 Ice Particle Melting Experiment

The experiment described below was performed at Penn State University by Dr. Jose Palacios and Sihong Yan. The data obtained was used to understand the melting behavior of an isolated ice particle and subsequently to validate the analytical model developed in this research. The test rig described in this section has been designed to visualize the melting of a single frozen droplet and to quantify the percentage of partial melting during the process. The melting process under natural convection and unsteady heat diffusion are investigated and discussed. During this experiment, a levitating drop is first frozen using cold air. The cold air is then shut down, and the warm environment of the surrounding room forces the transition of the frozen drop to water. High-speed cameras capture that transition, and a luminescence dye is used to measure the percentage melting state of the drop as during the process. The detailed descriptions for the test setup are shown in the following subsections.

2.1.1 Acoustic Levitating Technique

During the literature survey, it was found that in some referenced research, the ice particle is simply attached to a metal needle, which introduces unwanted heat transfer effects, specifically, related to the heat conduction due to the contact of the needle. In this study, an acoustic levitating technique is introduced to suspend the droplet. The
levitator used is a Tec5 levitator with a working frequency of 58 kHz. It has an ultrasonic transducer on the top and a reflector on the bottom. Between the transducer and the reflector, a standing wave is created, and its nodes support the droplet. Since the density of air is influenced by the environmental temperature, the wavelength of the standing wave is also related to the environmental temperature. When the temperature changes, the droplet can start vibrating between different nodes. The height of the reflector is vertically adjusted to provide steady nodes under different temperatures. In this experiment, the position of the reflector is adjusted for different environmental temperatures. For each test, the reflector is pre-optimized to eliminate the movement of the particle in the vertical direction.

2.1.2 Droplet Visualization Using a Luminescent Dye

To quantify the percentage melting of a frozen droplet, a luminescent technique is used. The technique follows the concept described by Tanaka et al. and has been described in detail in a previous research paper developed at Penn State. In this experiment, a luminescent dye, Rhodamine B is dissolved in the water (less than 0.1% in volume) to trace the water content in a melting frozen drop. When the Rhodamine B is dissolved in water and exposed to a laser beam, the dye molecules are excited. When water becomes ice, these molecules stop responding to the laser excitation due to the lack of mobility of the molecules. This difference in the strength of luminescent light produced by the ink is used to determine the percentage of melting in a single partially melted drop.
2.1.3 Experiment Setup

As shown in Figure 2-1 a single droplet hovers in the acoustic node created by the ultrasonic levitator. The levitator is placed in a thermally isolated room. The temperature in the room can be controlled between -20°C to 35°C. A high-speed camera with a macro lens is aimed at the droplet. Ice crystals between 300µm and 1800µm were tested, and they are melted at room temperatures of 5°C, 15°C, 25°C and 35°C respectively.

A 10W 532nm Continuous Wave Laser (CWL) is aimed at the droplet at a constant total power of 6 Watts. The laser and the line of sight of the high-speed camera has a small angle in between to avoid the laser from entering the high-speed camera lens directly. An Edmund 592 nm CWL, 43nm bandwidth fluorescence filter was attached to the front of the macro lens to block the reflection and refraction of the laser on the droplet. A tube is placed perpendicular to the camera view, and it is used to cool or heat the droplet, depending on the conditions requested. In other words, the tube channels cold or hot air flow to freeze or melt the droplet. The cooling of the air is accomplished by using a liquid nitrogen heat exchanger with a submerged coil. Compressed air flows in the coil and is cooled by the liquid nitrogen. The same process was performed with hot air when the room temperature was below freezing.
2.1.4 High-Speed Camera Visualization

Due to the absence of an appropriate droplet generator, the diameter of the droplet was not controlled but measured after each experiment. The melting process is recorded by a high-speed camera. Figure 2-1 exhibits a typical process of melting. The luminescent portion of the drop indicates water content.

A high-speed camera (Phantom M310) is used at a frame rate of 200 fps, and with an exposure time of 1 ms. The magnification lens used is a combination of a Navitar 1-60135 6.5X adjustable zoom lens, a Navitar 1-62922 2X adapter tube and a Navitar 1-64299 F-mount adapter. The physical resolution used in this experiment is 5µm/pixel. A laser filter is used to eliminates the influence of the 532-nm laser. The frames had a black background given the selected exposure time. The water content emits orange light into the high-speed
camera under green laser excitation. The color is directly converted into 8-bit grayscale image. By this treatment, the value of grayscale can then be directly related to the strength of the emission from the droplet. In the processed frame, the grayscales indicate water content inside the droplet. As shown in Figure 2-2, a fully frozen droplet is invisible under this configuration. When the particle starts melting, the melting part inside a droplet becomes visible and it is recorded by the high-speed camera.

![Figure 2-2. Sample melting process visualized using luminescence technique and high-speed camera.](image)

The combination of levitation and luminescence can simulate the thermal exchange that occurs on an ice crystal traveling inside an engine. The use of a high-speed camera enables measurement of the percentage of melting of the drop for varying icing/heating conditions and residence times.
2.2 Partial melting quantification

The experimentally measured melting percentage information can be extracted from recorded images. The intensity of excitation of Rhodamine B in the drop is proportional to the number of molecules that are dissolved in the water, therefore the total intensity of light of the particle can be used to quantify the percentage of melting. The summation of grayscale value of a single frame is used to represent the intensity of the emission. Light emission is only possible in those portions of the droplet that have melted water molecules mixed with Rhodamine B, allowing for the quantification of the partial melting state. The active molecule concentration of Rhodamine B in the partially melted drop, \( N_{PM} \), over the total amount of Rhodamine B molecules in the water drop, \( N_{FM} \), is defined as the partial melting state, PM, of the particle, as defined by Equation (1):

\[
PM = \frac{N_a}{N_T} = \frac{I_{PM}}{I_{FM}}
\]  

(1)

The percentage melting of an ice particle obtained from the experimental data is converted to an effective radius reduction ratio or fractional radius (y) of the droplet by equating the percentage melting to volumetric reduction of ice in the particle, as shown in Equation (2). For a fully frozen ice particle y is 1 whereas it goes to 0 when the ice particle is completely melted.

\[
PM = \frac{r_w^3 - r_i^3}{r_w^3} = 1 - y^3
\]  

(2)
2.3 Reference thermal model

The percentage melting of the frozen droplet is measured starting when water was first detected in the image frames due to the luminescence of the Rhodamine B. Figure 2-3 shows the comparison of the luminescent measurement of percentage melting\textsuperscript{35} with Bartkus’s prediction\textsuperscript{22}. The diameter of tested particle is 709 µm and the environmental temperature is 29°C. Alvarez’s research showed that the droplet is fully melted at the peak of percentage melting curve\textsuperscript{36}, where in this case, the peak occurred around 130% melt ratio. The reason for this over-melting phenomenon is due to the unsteady effect during the experiments, such as shape change due to eccentric melting and external acoustic field. In prior research, the droplet was assumed to be spherical through the process, but acoustic forces acting on the drop change its shape upon melting. In addition, nonlinear melting rates can be observed in the melting process. On the other hand, due to the assumption that surface temperature of an ice particle under phase change is 0 °C, the melting rate predicted by Bartkus’s model was a constant, as illustrated by the linear dash line in Figure 2-3. Percentage melting comparison between luminescent measurement and Bartkus’s model. The model proposed in this paper allows the surface temperature to increase when the droplet is melting. Therefore, a better match between the proposed model and experimental measurement is anticipated, due to non-ideal conditions being included in the model, as will be explained in the next subsection.
2.4 Analytical Model

2.4.1 Thermal Physics Model

Currently, the proposed thermal physics model is based on two-layer model assumption as illustrated in Figure 2-4. To compare with experimental measurements, a generalized advection-diffusion equation applied at the water film layer is introduced as the governing equation, as shown in Equation (3). First, verification of the model is conducted using ideal circular shapes, to the include the non-ideal testing conditions, such as shape change during eccentric melting, spinning effect under levitation, internal flow circulation etc.
Figure 2-4. Two-layer model

\[
\frac{\partial T}{\partial t} = -\vec{u}\nabla T + k_w \nabla^2 T + \text{source/sink}
\]  \hspace{1cm} (3)

Where \(T\) the temperature in the water film and \(u\) is the internal circulation in water film.

The following assumptions are applied to solve the differential equation (Equation (3)) describing the heating of the droplet:

1) the process is assumed to be steady state;
2) the ice core layer is both initially assumed to be spherical and axisymmetric, the water film layer is assumed to be concentric;
3) there is no shedding due to external streaming, no additional heat source or sink in the experimental environment;
4) due to the two-layer assumption, no advection is assumed inside of the water film layer
5) currently, no internal streaming effect, i.e., water circulation, is assumed.

After applying the above assumptions, the governing equation reduces to a Laplace equation in a spherical coordinate system, as shown in Equation (4).

\[
\frac{d^2T}{dr^2} + \frac{2}{r} \frac{dT}{dr} = 0
\] (4)

where freezing temperature is assumed at the interface between ice and water film, whereas at the interface between water film and outer flow, the droplet surface temperature needs further evaluation, as illustrated in Equation (5).

\[
T = T_0 = 273.15K \quad @ \quad r = r_i
\]

\[
T = T_s \quad @ \quad r = r_w
\] (5)

An analytical solution for temperature \(T\) at an arbitrary radial location \(r\) within two layers can be expressed as Equation (6).

\[
T = \frac{(T_0 - T_a) r_w r_i}{r (r_w - r_i)} + \frac{r_i T_0 - r_w T_s}{r_i - r_w}
\] (6)

where the temperature gradient at the interface between water film and ice core is shown in Equation (7).

\[
\left(-\frac{dT}{dr}\right)_{r=r_i} = \frac{(T_0 - T_a) r_w r_i}{r_i^2 (r_w - r_i)}
\] (7)

Correspondingly, the heat transfer driven by this temperature gradient can be expressed as the heat conduction from water film zone to ice core zone. Equation (8) represents the balance between the latent heat of melting and the total heat conduction at the current
interfacial layer, which can be further related to the time rate of change in the ice core radius ($r_i$ at current time $t$):

$$L_m \frac{dm_i}{dt} = L_m \rho_i A \frac{dr_i}{dt} = k_w A \left(\frac{dT}{dr}\right)_{r=r_i}, \quad A = 4\pi r_i^2$$

(8)

To solve for the melting rate ($dm_i/dt$ or $dr_i/dt$), one more equation is required. The outer flow heat transfer must be introduced based on different external streaming conditions. The time rate of change of heat convection and evaporation can be expressed in multiple empirical equations based on different Reynolds numbers, as it was shown in Table 1 of Ref.6. This study proposes to establish a set of equations based on experimental measurement that is directly pertaining to aircraft ice particle melting and freezing physics. With the experimental formulated heat transfer correlations, the governing partial differential equations are solvable. Analytical solutions can be found for the melting rate as a function of key parameters that affect the engine ice accretion, such as:

1) Velocity and Reynolds number

2) Residence time

3) Ambient warming rate

4) Relative Humidity (effect of RH in delay of melting, Critical melting temperature ($T_{melt}$) vs. RH)
2.4.2 Additional Equation to Close the Model

Along with the governing Equation (3), another equation that is used to close the model is the balance of heat transfer by water with heat transfer by forced convection and evaporation:

\[
\frac{4\pi k_w [T_0 - T_r(r_i)] r_w r_i}{(r_w - r_i)} = -4\pi r_w k_d [T_\infty - T_r(r_i)] f - 4\pi a_d L_e D_{va} [\rho_{v\infty} - \rho_{vr_w}] f
\]

where \( L_e \) is the latent heat of evaporation, \( f \) is mean ventilation coefficient, \( D_{va} \) is diffusivity of water vapor in air, \( \rho_{v\infty} \) is the water vapor density far away from the ice particle, \( \rho_{vr_w} \) is the water vapor density at the drop surface, \( k_d \) and \( k_w \) are the thermal conductivities of air and water respectively, \( r_w \) is the radius of the frozen drop and \( r_i \) is the radius of the ice core. The mean ventilation coefficient \( f \) is the ratio of the rate of water mass flux for cases of a moving and a stationary drop. The difference in water vapor density is taken as the following:

\[
\rho_{v\infty} - \rho_{vr_w} = [T_\infty - T_r(r_i)] \left( \frac{\partial \rho}{\partial T} \right)_{sat,i} - (1 - \varphi) \rho_{v,sat,\infty}
\]

where \( \varphi \) is the relative humidity and \( \left( \frac{\partial \rho}{\partial T} \right)_{sat,i} \) is the mean slope of ice saturation vapor density curve over ice from \( T_\infty \) to \( T_0 \).

Separating \( T_r(r_i) \) from Equation (9) and substituting in (8) gives an equation

\[
[Ay + (1 - A)y^2] \frac{dy}{dt} = B \left( T_\infty - \frac{C (1 - \varphi) \rho_{v,sat,\infty}}{A} \right)
\]
where \( y = \frac{r_i}{r_w} \) is the fractional radius of ice particle, 
\[ A = \frac{k_a + L_e D_v a \left( \frac{\partial \rho}{\partial T} \right)_{sat, f}}{k_w} f \]
and 
\[ B = \frac{A k_w}{\rho _1 L_m} \]
are constants depending on temperature of air outside the ice particle.

Integrating Equation (11) gives the relationship between fractional radius \( y \) and time \( t \) as:

\[
B \left( T_\infty - \frac{C (1 - \varphi) \rho_{v, sat, \infty}}{A} \right) t = [(A + 2) - 2(1 - A)y^3 - 3Ay^2] \quad (12)
\]

2.5 Physical parameters captured by experiment

The experiment captures two main physical parameters around the levitated ice particle namely: (a) temperature just outside the particle \( (T_\infty) \) and (b) relative humidity \( (\varphi) \) in the vicinity of the particle. Both these parameters are captured by the thermal model developed in this research as can be seen from Equation (12). The effect of size of the drop was also captured by the model and its effect on melting rate and melting time studied as the temperature of the drop \( (T_r(r_i)) \) was considered as a function of radius of ice core \( (r_i) \) as shown in Equation (9). From the analysis of the experimental data, it was observed that the ambient temperature, relative humidity and subsequently saturation vapor density vary with time throughout the melting process. Therefore, there was need to introduce time variation of these parameters in the thermal model for accurately predicting the melting behavior.
Figure 2-5. Sample time history of ambient temperature and relative humidity variation with time

The thermal model developed captures the transient nature of ambient temperature and relative humidity by inserting the two parameters as a function of time in Equation (11) and integrating with respect to time to obtain an expression analogous to Equation (12). A
typical time history of the two test conditions observed during one of the melting experiments are shown in Figure 2-5. The temperature in the vicinity of the drop varied with time for all test cases. For some cases it was observed that the ambient temperature increased sharply in the initial few seconds from the start of melting and later stabilizing at a constant value. For short melting time cases, the ambient temperature was observed to be relatively constant throughout the melting duration.

![Figure 2-6. Sudden drop in melting rate on leaving supersaturated state](image)

In most cases, the relative humidity at the beginning of melting can be observed to be above 100%, referred to as supersaturation. A sudden drop in melting rate is observed when the relative humidity decreases to 100%, as shown in Figure 2-6 by an ellipse. The ambient temperature and relative humidity as functions of time were determined for each test case to be applied to the thermal model.
The saturation vapor density near the ice particle being a function of ambient temperature needed to be calculated for the changing temperature near the ice particle. The empirical relationship of the saturation vapor density as a function of temperature was determined by curve fitting the saturation vapor density data for water\(^{37}\), shown in Figure 2-7. The mean slope of saturation vapor density \(\left(\frac{\partial \rho_s}{\partial T}\right)_{sat,i}\) is calculated by taking the derivative of the empirical relation.

The ventilation coefficient (f) for the experiment is taken as 1 since the ice particle is levitated and subjected to a steady ambient flow field. The latent heat of melting \((L_m)\), and thermal conductivity of water \((k_w)\) are calculated at 0°C while the thermal conductivity of
air ($k_a$), latent heat of evaporation ($L_e$) and diffusivity of water vapor in air ($D_{va}$) are calculated at the ambient temperature ($T_\infty$).

2.6 Validation of Thermal Model

![Figure 2-8. Comparison between experiment and two-phase analytical model for long melting time case](image)

The fractional radius vs time plot shown in Figure 2-8 shows the comparison of the melting behavior in terms of fractional radius for the experiment with the analytical model proposed in this research. Similar plots were obtained for 14 test cases for ice particles ranging from 300 to 1200 microns and shown in Appendix A. The melting rate for the analytical model is less than that for the test case but becomes comparable after roughly 50% melting has been reached as can be seen from the plot. On obtaining plots for all cases, it was observed that the melting rate for particles smaller than 400 microns in
radius was observed to be much less than that for the experiment. Additionally, the melt time i.e. the time when the ice particle has melted completely was also calculated for all test cases by equating \( y=0 \) in equation (12). The melt time obtained from the analytical model was then compared with the experimental melt time and absolute error calculated. Figure 2-9 shows the absolute error in melt time prediction by the analytical model for all 14 test cases.

Figure 2-9. Comparison between experiment and two-phase analytical model melt time and melt time absolute error
2.7 Results and Discussion

Table 2-1 Comparison of melting times for ice particles

<table>
<thead>
<tr>
<th>Ice Particle Radius</th>
<th>Exp Melt Time</th>
<th>Analytical Melt Time</th>
<th>% Error Analytical Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>538</td>
<td>23.34</td>
<td>22.62</td>
<td>3.2</td>
</tr>
<tr>
<td>556</td>
<td>29.46</td>
<td>29.12</td>
<td>1.2</td>
</tr>
<tr>
<td>546.5</td>
<td>28.06</td>
<td>27.98</td>
<td>0.3</td>
</tr>
<tr>
<td>432</td>
<td>21.51</td>
<td>22.10</td>
<td>2.7</td>
</tr>
<tr>
<td>439</td>
<td>20.12</td>
<td>20.72</td>
<td>2.9</td>
</tr>
<tr>
<td>456.5</td>
<td>15.50</td>
<td>17.98</td>
<td>13.8</td>
</tr>
<tr>
<td>351.5</td>
<td>7.56</td>
<td>8.82</td>
<td>14.3</td>
</tr>
<tr>
<td>317</td>
<td>4.06</td>
<td>5.00</td>
<td>18.8</td>
</tr>
<tr>
<td>354.5</td>
<td>4.56</td>
<td>5.82</td>
<td>21.6</td>
</tr>
<tr>
<td>205</td>
<td>4.00</td>
<td>6.60</td>
<td>39.4</td>
</tr>
<tr>
<td>368</td>
<td>4.28</td>
<td>7.30</td>
<td>41.4</td>
</tr>
<tr>
<td>300.5</td>
<td>3.04</td>
<td>5.20</td>
<td>41.5</td>
</tr>
<tr>
<td>162</td>
<td>1.80</td>
<td>9.18</td>
<td>80.4</td>
</tr>
<tr>
<td>381</td>
<td>4.06</td>
<td>8.08</td>
<td>49.8</td>
</tr>
</tbody>
</table>

It was found that the analytical model predicts the correct rate of melting and the melting time with an average error of 4% for particles with radius greater than 400 microns. For particles less than 400 microns in radius, the rate of melting is correctly predicted by the analytical model, but the melting time is over-predicted with an average error of 38%. This can be attributed to difficulty in capturing the melting behavior of smaller particles due to instability during levitation and frame time resolution in the experimental setup. A summary of the melt time prediction comparisons is shown in Table 2-1. It is clear that the error in predicting melt time using the two-phase model increases drastically as particle size goes below 400 microns.
On comparing the results of the analytical model with melt time obtained from the Bartkus model, it is found that the average error in melt time predicted is 10.6% for particles greater than 400 microns in radius and 40% for particles less than 400 microns in radius which is higher as compared to the two-phase analytical model developed in this research. Both models predict higher melt time for small particle sizes with the two-phase analytical model predicting slightly better than the Bartkus model. The two-phase analytical model is also model consistent in predicting the melt times for the ice particles. A summary of the melt time prediction by Bartkus model and Analytical Model along with error percentages for each is shown in Figure 2-10.
3.1 Reference Experiment

Before attempting to model freezing of water on flat surfaces, it was important to understand the physical process of freezing from the aspect of aircraft engine icing. For this a reference experiment was studied where a similar problem was replicated. The experiment studied was performed at Penn State University by Dr Yiqiang Han and Dr. J Palacios\textsuperscript{38}. The experiment was conducted for obtaining heat flux measurement at the surface of a cold plate on which freezing of a film of water takes place. The experiment demonstrated the use of a heat flux sensor to calculate total mass of water frozen at a surface and is based on measurement of the latent heat flux generated during the phase transition process. The physical process occurring in the experiment was then modeled using the two-phase Stefan problem approach.

The hardware for the experiment to measure heat transfer at a surface included a heat flux sensor, thermocouples, and a thermal-controlled plate. The experimental setup is shown in Figure 3-1. during the experiment, the thermal controlled plate was cooled by Peltier at a constant temperature of -14.5°C. The heat flux sensor and thermocouples are located on the cold plate over which a water drop was applied. An infra-red camera was used to monitor the local surface temperature mapping for calibration data. The
water drop was placed over the heat flux sensor on the cold plate and latent heat flux measurements were taken till the entire drop was frozen.

Figure 3-1. Setup for measuring heat flux at the surface of a freezing water film

3.2 Two phase freezing problem

For modeling the freezing of a film of water on a cold surface, it was needed to take an approach which takes into consideration the moving interface of ice and water layers. The idea was to perform an energy balance on the transient ice and water layers separately in order to determine the unknowns i.e. the temperature field and the location of the interface boundary layer. The problem involved phase change with constant density, latent heat, melt temperature, specific heats, and thermal conductivities. For this reason, two phase Stefan problem was employed.
The following assumptions were made for the model:

i. Heat transfer only due to conduction

ii. Constant latent heat of melting

iii. Fixed freezing temperature $T_f=0^\circ C$

iv. Interface thickness is zero and it is a sharp front that separates the two phases

v. Thermophysical properties i.e. thermal conductivities ($k_s$ and $k_l$) and specific heats ($c_s$ and $c_l$) are different for each phase

vi. Densities of water and ice are considered equal ($\rho_s = \rho_l$)

![Figure 3-2](image)

Figure 3-2. Schematic diagram of freezing water film on a constant temperature cold surface

The water film as shown in Figure 3-2 is initially liquid at an initial temperature ($T_{l0}$) greater than the freezing temperature ($T_f$). It is frozen when it comes in contact with a constant temperature cold surface ($T_p<T_f$). Since a two-layer model is assumed, there are separate differential equations for the ice layer and water layer as shown in equation (13).
At any given moment during the freezing process the thickness of the ice layer is given by $s_i$ and it increases as the freezing proceeds. This thickness is a function of time. The thickness of the water film is given by $s_l$ and has a fixed value. For solving the two-phase problem explicitly, we assume a semi-infinite water film. Therefore, $s_l$ is taken as infinity.

\[
\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial x^2} \quad 0 < x < s_i
\]

\[
\frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial x^2} \quad s_i < x < s_l
\]

At the interface, after the freezing starts the temperature of the ice and the water is same and equal to the freezing temperature $T_f$ as shown in equation (14).

\[
T_s(s_i, t) = T_l(s_i, t) = T_f
\]

The initial condition for both the governing differential equations is given by equation (15).

\[
s_i(0) = 0
\]

\[
T_i(x, 0) = T_{i0} > T_f , x > 0
\]

The boundary conditions for the governing differential equations is given by equation (16).

\[
T_s(0, t) = T_p < T_f , t > 0
\]

\[
T_l(\infty, t) = T_{i0} > T_f , t > 0
\]

To solve equation (13) separately for the ice and water layers, another boundary condition is required. This is obtained by balancing the heat flux by the local temperature gradient of ice and water with the latent heat of freezing ($L$). This heat balance is shown by equation (17).
The two differential equations are solved using a similarity variable and the following solutions are obtained as shown in equation (18)

\[ T_s(x,t) = C_1 + C_2 \cdot \text{erf} \left( \frac{x}{2\sqrt{\alpha_s \cdot t}} \right) , \quad 0 < x < s_l \]

\[ T_l(x,t) = C_3 + C_4 \cdot \text{erfc} \left( \frac{x}{2\sqrt{\alpha_l \cdot t}} \right) , \quad s_l < x < \infty \]

Where \( C_1, C_2, C_3 \) and \( C_4 \) are constants and erfc is the complementary function of the error function. On applying boundary conditions (16), we obtain value for the constants as:

\[ C_1 = T_p \]

\[ C_2 = \frac{T_f - T_p}{\text{erf}(\lambda)} \]

\[ C_3 = T_{l0} \]

\[ C_4 = \frac{T_f - T_{l0}}{\text{erfc} \left( \frac{\sqrt{\alpha_s}}{\sqrt{\alpha_l}} \cdot \lambda \right)} \]

Where \( \lambda \) is a constant independent of time \( t \) and is given as:

\[ \lambda = \frac{s_l(t)}{2\sqrt{\alpha_s \cdot t}} \]
Substituting coefficients from equation (19) into equation (18) we obtain expression for temperature of varying ice and water layers as a function of space and time. This is shown in equations (21) and (22).

\[ T_s(x, t) = T_p + \frac{T_f - T_p}{erf(\lambda)} \cdot erf\left(\frac{x}{2\sqrt{\alpha_s \cdot t}}\right) \]  

(21)

\[ T_l(x, t) = T_{l0} + \frac{T_f - T_{l0}}{erfc\left(\frac{\alpha_s}{\sqrt{\alpha_l}}\lambda\right)} \cdot erfc\left(\frac{x}{2\sqrt{\alpha_l \cdot t}}\right) \]  

(22)

To find the value of constant \( \lambda \), (21) and (22) are substituted in the interface energy balance equation (17) to obtain the following \( \lambda \) equation:

\[ \frac{Stes_s}{e^{\lambda^2} erf(\lambda)} - \frac{Stes_l}{\alpha_{sl} e^{\alpha_{sl} \lambda^2} erf(\alpha_{sl} \lambda)} = L \rho \frac{dx}{dt} \bigg|_{x=s_i} \]

(23)

Where \( \alpha_{sl} = \sqrt{\frac{\alpha_s}{\alpha_l}} \) is the square root of the ratios of thermal diffusivities of the solid and liquid regions and \( Stes_s = \frac{c_s(T_f - T_p)}{L} \) and \( Stes_l = \frac{c_s(T_f - T_{l0})}{L} \) are Stefan numbers for solid and liquid regions respectively.

From the expression of temperature of ice layer, heat flux at the surface can be predicted as:

\[ q_{wall} = k_s \frac{\partial T_s}{\partial x} \bigg|_{x=0} = \frac{k_s (T_f - T_p)}{\sqrt{\pi \alpha_s \cdot erf(\lambda)}} \]  

(24)
3.3 Temperature profiles of two-layer model

From the two-layer freezing model, we obtained two separate temperature profiles for the ice and water layer respectively. To understand the heat flow inside the water film, it is important to know the temperature gradient in the freezing water film as the freezing front moves. The temperature vs time plot at different locations was plotted as shown in Figure 3-3.

![Temperature profile vs time plot for different vertical location above the freezing surface](image)

The temperature recorded below the interface line for a given location shows the temperature profile after freezing to ice occurs at that location. Whereas the temperature recorded above the interface line shows the temperature of liquid water as time progresses. The figure shows that the temperature at a given location parabolically decreases to the
freezing temperature, following which it freezes to the plate temperature parabolically at a steeper rate as compared to cooling in liquid stage.

The temperature gradient in the freezing water film was also plotted at different times during freezing. This is shown in Figure 3-4. It is observed that the temperature gradient remains linear for a given time recording. On the other hand, the temperature gradient gradually decreases with time for the entire spatial domain and for both the solid and liquid layers. There is light decrease in the slope of the temperature in the liquid region as compared to that in the solid region.

Figure 3-4. Temperature gradient at different times
3.4 Moving Freezing Front

![Graph of freezing front (s(t)) with time]

Figure 3-5. Plot of freezing front (s(t)) with time

As the freezing progresses the time dependent interface between the ice and water layers, also called the freezing front, increases parabolically as shown in Figure 3-5. The rate of increase of freezing front increases with Stefan number i.e. with greater temperature difference between the two boundaries. Figure 3-6 shows effect of Stefan number on the freezing front movement rate.
3.5 Heat Flux at Freezing Surface

Once the temperature profiles of the ice layer are obtained, the heat flux at the cold plate (freezing surface) was plotted against heat flux reading from the heat flux sensor as shown in Figure 3-7. A correction factor of 0.1 was applied to the heat flux values obtained from the analytical model. The two heat flux values are negative because heat is being transferred from the ice layer to the cold plate. The analytical model predicts follows the experimental plot closely for the first 30 secs after freezing starts. Following that, the difference in the heat flux experimental and analytical model values increases towards the end of the freezing process as the temperature of the ice adjacent to the freezing surface
equalizes with the temperature of the cold plate. The analytical model fails to predict this behavior.

Figure 3-7. Comparison of heat flux at the surface of cold plate
Chapter 4:

CONCLUSIONS

4.1 Conclusions

This thesis presented two separate analytical models for melting of ice particles and latent heat measurement for freezing water film over a constant temperature surface respectively. The approach for both the models was to first generate governing equation based on heat transfer at extreme and interface surfaces, followed by obtaining explicit solutions for melting behavior in the case of two-layer melting model and temperature profiles in the case of two-phase freezing model and finally validating the model with experimental data from tests that capture the same physical phenomena.

Based on the results of the research, following conclusions can be drawn:

1) A mathematical correlation between partial melting quantified in terms of fractional radius (y) and melt time (t) has been obtained from the two-layer model.

2) Unsteady effects in physical parameters of ambient temperature, relative humidity and saturated vapor density, involved in the melting of ice particles has been introduced. The model also considers non-uniform drop temperature which is a function of radius of melting ice core.

3) The two-layer melting model has been validated by using data from a reference experiment and melting rate and melt time has been compared. The reference
experiment involved 14 test cases of melting behavior of a single levitated ice particle ranging from 300μm to 1200μm.

4) From the validation, it has been determined that the ice particle melting model, developed in this research provides accurate melt times with an average absolute error of 4% for ice particles greater than 800μm. For particles smaller than 800μm, the model under-predicts melting rate and over-predicts melt time with an average error of 38% above actual.

5) From the validation, it was also shown that the ice particle melting model provides accurate melting rate measured in terms of fractional radius for ice particles greater than 800μm.

6) The two-phase Stefan approach for solving temperature profiles shows that rate of temperature fall is higher in the ice layer as compared to that in the water layer. The rate of temperature fall decreases away from the freezing surface.

7) The temperature gradient is marginally higher in the ice layer as compared to the water layer and decreases as freezing progresses.

8) The freezing front or the interface between the ice and water layer moves parabolically as freezing progresses.

9) The freezing front moves at a slower rate as freezing progresses as Stefan number is increased.

10) On validation with experimental, the heat flux at the constant temperature freezing surface obtained from the two-phase freezing model captures the trend of heat flux change but differs in magnitude by an average of 45%.
APPENDIX A

Fractional Radius Vs Time Plots for All Melting Test Cases

Figure A. 1. Fractional Radius vs Time for Case 1
Figure A. 2. Fractional Radius vs Time for Case 2

Figure A. 3. Fractional Radius vs Time for Case 3
Figure A. 4. Fractional Radius vs Time for Case 5

Figure A. 5. Fractional Radius vs Time for Case 6
Figure A. 6. Fractional Radius vs Time for Case 7

Figure A. 7. Fractional Radius vs Time for Case 8
Figure A. 8. Fractional Radius vs Time for Case 10

Figure A. 9. Fractional Radius vs Time for Case 11
Figure A. 10. Fractional Radius vs Time for Case 12

Figure A. 11. Fractional Radius vs Time for Case 13
Figure A. 12. Fractional Radius vs Time for Case 14

Figure A. 13. Fractional Radius vs Time for Case 15
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