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Optimization Techniques for Modern Power Systems Planning, Operation and Control

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OPTIMIZATION TECHNIQUES FOR MODERN POWER SYSTEMS
PLANNING, OPERATION AND CONTROL

A Dissertation
Presented to
the Graduate School of
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In Partial Fulfillment
of the Requirements for the Degree
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Abstract

Recent developments in computing, communication and improvements in optimization techniques have piqued interest in improving the current operational practices and in addressing the challenges of future power grids. This dissertation leverages these new developments for improved quasi-static analysis of power systems for applications in power system planning, operation and control.

The premise of much of the work presented in this dissertation centers around development of better mathematical modeling for optimization problems which are then used to solve current and future challenges of power grid. To this end, the models developed in this research work contributes to the area of renewable integration, demand response, power grid resilience and constrained contiguous and non-contiguous partitioning of power networks.

The emphasis of this dissertation is on finding solutions to system operator level problems in real-time. For instance, multi-period mixed integer linear programming problem for applications in demand response schemes involving more than million variables are solved to optimality in less than 20 seconds of computation time through tighter formulation. A balanced, constrained, contiguous partitioning scheme capable of partitioning 20,000 bus power system in under one minute is developed for use in time sensitive application area such as controlled islanding.
Dedication

To my parents, Balasubramaniam and Vimala Devi, my brother Mahadevan and to my wife Meera for their love and unconditional support.
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I would like to thank my advisor Dr. Elham B. Makram for her guidance and for giving me the opportunity to work with her to make myself better both academically and personally. I would also like to thank all my professors at Clemson University, especially Dr. Ramtin Hadidi for his guidance. Finally, I would like to thank my friends for making the entire experience a memorable one.
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Chapter 1

Introduction

Electric power grids are among the most complex network built by humans. Recent developments in computing, communication and improvements in optimization techniques have piqued interest in improving the current operational practices and in addressing the challenges of future power grids. This dissertation leverages these new developments for improved quasi-static analysis of power systems for applications in power system planning, operation and control.

The goal of this dissertation is to develop, analyze and validate optimization techniques that can be used to address challenges of current and future power grids. The premise of much of the work presented in this dissertation centers around development of better mathematical modeling for optimization problems which are then used to solve current and future challenges of the power grid. Better mathematical models in this context refer to models that,

- Capture properties of the physical system to a higher degree of accuracy.
- Exhibit scalability i.e. solution to large problems should be solvable within the time frame of the intended application.

Special care is taken in this dissertation to ensure that the proposed methods are in
compliance with the current energy policy constraints or require only minimal changes to current operational practices. The dissertation is organized into four parts based on application area.

1. Storage Systems and Power Grid Operation

2. Demand Response

3. Power Grid Resilience

4. Partitioning Power Networks

Problems studied in this dissertation are of the multi-period optimization type. The application area determines the type of optimization problem. For instance, demand response applications require mixed integer programming (MILP) representation, power grid resilience requires non-linear programming model (NLP) while renewable integration requires stochastic optimization model. Different set of challenges posed by each application area along with their solution strategies and contributions are presented below. Each part consists of one or more chapters, wherein work pertaining to topics related to the focus area is presented.

1.1 Storage Systems and Power Grid Operation

Higher penetration levels of renewable energy such as solar and wind farms pose significant challenges to present and future power grids. The inherent variability and uncertainty requires stochastic modeling with the exception of few special cases. One of the preferred ways to address issues with renewable integration is through the use of storage systems such as battery energy storage systems (BESS).

In chapter 2, current key issue of mitigating the so called duck curve phenomenon seen at independent system operators (ISOs) such as California ISO (CAISO) is
addressed. Renewable generation, especially solar generation has an output profile that has decreasing energy production from around 14:00 hours. Incidentally, this is the time of the day, particularly during winter months, when load demand increases sharply. In a power system, the algebraic sum of demand and supply must be zero at all times. Hence, any increase in demand and decrease in renewable generation output must be offset by increased output from controllable generation.

Ramping up energy production adds enormous stress on the system - both in stability context and also in terms of economic operation. Any method proposed to address these issues should fit within the current energy policy framework to be of practical use. A provable optimal mitigation scheme considering uncertainty and energy policy constraints is developed in chapter 2 and its feasibility is demonstrated using actual data from CAISO for the year 2015.

Chapter 3 focuses on sizing of BESS for peak load reduction. The significant improvement over existing literature in this context is the use of non-linear programming formulation and the methodology to model only a part of the system that is of interest. Existing approaches in literature are based on linear programming (LP) or MILP based formulations that neglect the importance of reactive power adequacy. As a result, one cannot preclude the fact that the solution provided by LP or MILP formulation could in fact be infeasible when applied to the real system. The focus of chapter 3 is on sizing an appropriately sized BESS based on system operator defined parameters and use the BESS to mitigate peak loading conditions seen in system operation while constraining the solution to adhere to physical limits of the system.
1.2 Demand Response

In chapter 3, BESS was used to time-shift the load demand curve over a given day thereby reducing the peak load. In chapter 4 a similar end result is achieved by altering the end-user energy consumption pattern. Some of the system operators such as Pennsylvania-Jersey-Maryland (PJM) regional transmission operator (RTO) currently have functional demand response schemes. This chapter takes leverage of the DR programs available with some ISOs/RTOs. The challenge in this case is two folds. 1) Need for detailed mathematical load models that depict the load characteristics of different types of consumer loads. 2) The load model developed in 1) should have a tight linear relaxation in order to have acceptable solution times for large problems.

Different load types such as adjustable loads, constant power loads, dependent loads and electric vehicles are modeled. These load models cover a wide range of end user energy consumption pattern. As an example, consider the operation of a washer and dryer of an end user on a working day. The end user would preferably want the washer and dryer cycle to be completed by the time the end user returns home from work. Hence, the washer and dryer can be scheduled in a way that is beneficial to the utility i.e. at an off-peak time. However, this particular type of load has an added constraint such that operation of dryer cannot commence before the washing cycle is complete. In this case, the dryer is dependent on the washer.

Modeling of dependent loads are especially important with the ensuing boom of internet of things (IOT). A sequence of tasks that are to be completed in a specific order can be automated by the end user with the advent of IOT. Without the ability to model dependent loads, the scheduling of these series of tasks would essentially be a one degree-of-freedom operation as the utility can only control the start time of the
first task. However, with dependent load modeling the utility can now individually schedule each task such that their order of operation is satisfied. For instance, a recursive formulation can now be used such that task 4 is dependent on the completion of task 3 which in turn is dependent on task 2 and so on. This presents the utility with a much higher degree of freedom which in turn results in better load management - the goal of this work.

The emphasis here is also on scale of the problem. In existing literature such as in [1] aggregated load models are used. Using aggregate models allows for faster solution time. However, the trade-off is loss of degree of freedom in the optimization problem i.e. only sub-optimal solution is possible with such representations. Detailed load models with a tight relaxation presented in chapter 4 allows MILP problems with upwards of a million variable to be solved in less than 20 seconds.

### 1.3 Power Grid Resilience

Resiliency represents the ability of power systems to withstand high-impact, low-probability events with least possible interruption of electric supply while enabling quick recovery and restoration to normal operating state [2]. According to Electric Power Research Institute (EPRI), three key aspects of enhancing grid resiliency are prevention, recovery and survivability [3]. While prevention and recovery require infrastructural and operational changes, survivability can be accommodated into existing framework. Hence, the focus of this work is on survivability of microgrids.

In chapter 5, the concept of critical and non-critical loads are introduced and a linear programming formulation to prioritize the load under conditions of uncertainty is formulated. Islanding scenarios occur as a result of high impact event such as cascading failure. In islanded mode of operation, the microgrid remains disconnected.
from the bulk power system. The microgrid is still functional, however, to a lesser extent. Under these conditions the microgrid needs to shed load to maintain the balance between generation and demand.

Given an amount of generation, enough to supply all or part of non-critical load, in addition to supplying all of the critical load at a given dispatch. The decision then is to find whether to serve the non-critical load or to store energy. Storing energy might be beneficial for later dispatches where there is a scarcity of resources to supply the critical load. However, storing energy implies shedding some non-critical load that could otherwise have been served. The decision has to be made in such a way, that the total critical load shed should be minimized over the entire optimization period i.e. the time that microgrid remains in islanded mode and at the same time maximize the amount of non-critical load served.

For completeness, grid connected mode of operation is also formulated in chapter 5 considering the environmental impact of power generation through the use of non-renewable generation sources. Grid connected mode is formulated as a quadratic programming problem whose convexity is theoretically proved to assure global optimality of the solution process.

Chapter 6 is an extension of chapter 5 with one major change. Network constraints are modeled. Hence, reactive power adequacy can also be satisfied using the formulation developed in chapter 6. The developed formulation is adept at capturing both spatial and temporal variations in critical and non-critical load. However, this formulation is non-linear due to the power injection formulation used. In addition, the formulation used to describe complex power injection at each node is non-convex by nature. Instead of non-tight generic relaxations such as semi definite programming (SDP), a local optimizer - IPOPT is used in chapter 6.
1.4 Partitioning Power Networks

Partitioning power networks has many important applications. Controlled islanding of power systems, parallel computation of power system analysis, dynamic equivalence reduction, planning and operations are some of the application areas in power systems that either directly or indirectly use network partitioning. Applications in power systems that require partitioning can broadly be classified into two classes based on graph structure - contiguous and non-contiguous. Contiguous partitioning approaches are used for applications such as controlled islanding, while non-contiguous partitions are used for load balancing for high performance computing based parallel power systems analysis.

Non-contiguous applications, even for dynamic graphs, occur at a time scale of 15 minutes to 1 hour. Contiguous partitioning applications on the other hand occur generally occur at near real-time resolutions. Hence, two different approaches and theories are needed based on the intended application area. Unlike other graphs, graphs that arise in power systems are irregular graphs and their partitioning is constrained due to energy policy decisions, ownership etc. Existing approaches such as multi-level graph partitioning does not provide means to express power system operational constraints.

Chapter 7 deals with non-contiguous constrained partitioning of power networks. The contributions of this chapter are outlined as follows.

- A mathematical way to enforce operational constraints in partitioning scheme.

- Provide a balanced partition. The balance constraint can be explicitly set by the user using a tolerance parameter. A large value of tolerance relaxes the balance constraint while a smaller value provides a tighter bound.
Means to verify optimality of partition.

In chapter 8 fundamental theory pertaining to constrained contiguous partitioning is developed. Existing techniques such as spectral partitioning methods [4] allow constrained partitioning but do not guarantee contiguous partitions. This is particularly significant in the case of applications such as controlled islanding. Following a major disturbance event when synchronization between groups of generators is lost, it is imperative that the group of generators that are in synchronization with each other are separated from group of generators that swing against each other. Failure to do so will result in period of sustained oscillation and eventually lead to a cascading failure.

The purpose of controlled islanding approach is to partition these group of generators in such a way that power system operation can continue with minimal load shed. Under such conditions when a partitioning scheme results in non-contiguous partitions, islands within islands are formed. This leads to increased load shedding and possibly even instability. In chapter 8 a graph theoretic approach is used to develop constraints on fundamental basis cycles of a given power system graph which are then used to partition the system.

It is worth pointing out the focus of this work is on the fundamental partitioning problem itself rather than on the eventual application. As such, the theory developed in this work can be extended to different power system applications. Several such applications are discussed and the formulation for the same is also developed in chapter 8. The applicability of the developed approach for time sensitive applications such as controlled islanding problem is proved experimentally by partitioning several large power networks including a 20,000 node test system in real-time. Real-time in this context refers to a minute interval. The contributions of chapter 8 can be summarized
as follows,

- Unified approach to enforce operational, contiguous and balanced partition constraints.

- Handle very large graphs that are of the size and complexity as seen at the system operator level.

- Suitable for dynamic graph partition in real-time.

- Extensibility of the developed theory for improved performance of existing power system applications.
Part I

Storage Systems and Power Grid Operation
Chapter 2

Mitigating Renewable Intermittency Using BESS Under Conditions of Uncertainty

Nomenclature

Binary Variables

$\beta^i$ Variable to enforce charge/discharge cycle of $i^{th}$ BESS

Continuous Variables

$\Delta P_c$ rate of change of power of controllable generation including reserves

$\Delta P_{pcc}$ rate of change of power at point of common coupling

$E^i_b$ Energy stored in $i^{th}$ BESS

$P^i_{bc}$ Rate of charge of $i^{th}$ BESS

$P^i_{bd}$ Rate of discharge of $i^{th}$ BESS

$P^i_{pcc,s}$ Power at point of common coupling of $i^{th}$ BESS for $s^{th}$ stochastic scenario

$P_c$ Power output of controllable generation including reserves
The purpose of this work is to mitigate the impact of uncertainty and variability of renewable generation on controllable generation and reserves through the use of
battery energy storage systems (BESS). Specifically, the proposed approach addresses the high ramp rate phenomenon seen at independent system operator (ISO) such as California ISO (CAISO). High ramp rates are observed when sharp increase in demand coincides with sharp reduction of non-load following renewable generation. This results in the so called duck curve phenomenon. High ramp rates as much as 10,000 MW in 3 hours has been observed at CAISO in the year 2015 and the number is expected to increase to 13,000 MW by 2020. As an example, duck curve observed at CAISO on January 1, 2015 is shown in fig. 2.1 where steep ramp up of controllable generation (red curve) is observed from 14:00 - 17:00 hrs.

A stochastic multi-period optimization problem is formulated as a mixed integer linear programming for mitigating duck curve phenomenon using actual data from CAISO for the year 2015. Uncertainty is modeled using observed day ahead forecast error distribution of CAISO. Solar and wind farms are modeled individually while considering their geographical distribution. An interesting subproblem of sizing BESS for individual solar and wind farms is also addressed. Furthermore, the proposed approach can be integrated with current operational practices for improved economic and secure operation of grid.

2.1 Introduction

Non-load following generation has a high likelihood of lesser energy production capability during times of peak demand - an often neglected factor while addressing secure and economic operation of power system under high penetration levels of renewable energy. For instance, since California Independent System Operator (CAISO) was formed in 1998, yearly peak demand for every year until the current calendar year of 2015 has occurred between 14:30 to 17:00 hours [5]. Non-load following generation
with significant solar generation tend to ramp down production during this window resulting in high ramp rates of controllable generation.

Under ordinary conditions, high ramp rates require additional flexible reserves which results in increased operating cost. Under extraordinary conditions, high ramp rates could potentially result in system stability issues. Such ordinary and extraordinary scenarios require further investigation to design a mitigation scheme which addresses issues of higher flexible reserves, curtailment, system security and most importantly any suggested mitigation scheme should fit under the current or plausible future energy policy framework.

To this end, it is imperative that the data used be real and/or realistic and that the proposed scheme is either applicable to the current grid code or can be easily accommodated to the existing operational practices. The premise of this work is based on this ideology. Realizing such a scheme presents several challenges. These challenges along with the existing work in literature is presented in literature review.
2.1.1 Literature Review

Reference [6] explores the possibility of reducing the ancillary service cost by limiting the ramp rates of wind power output using storage system. Due to the uncertain nature of the problem a stochastic optimization model is built and is used to find a relationship between financial penalties and ramp rate limit. The paper deals with the ramp rate events from an economic stand point. The problem is solved as a two stage stochastic linear problem with a fixed recourse. A multivariate Gaussian process is used to model wind power forecast errors. The paper considers limiting ramp rate by fixing a single large BESS of size 200 MW and 1200 MWh. Only one type of renewable generation - wind power is considered.

One of the challenges of such an approach is appropriation of investment. For instance, consider CAISO, the two major utilities are Pacific gas and electric (PG&E) and Southern California Edison (SCE). It is difficult to establish a base case calculation to share the investment cost. In addition, there are private entities who own some of the renewable generation. Contrary to [6], a different school of thought is followed in this work, wherein individual entities who own renewable generation will be enforced to adhere to a grid code in order to participate in energy transaction. Although BESS locations are distributed in this approach, global optimal solution can be achieved by a centralized optimization scheme with minimal renewable energy curtailment as will be shown in the following sections.

The authors in [7] propose a scenario switching strategy, where the ramp up/down scenarios are judged based on fuzzy logic controller. The decision process is based on the estimation of three parameters which are functionally represented as fuzzy logic membership functions. The optimization model then makes the control decision based on the scenario. The focus of the work is on wind power ramp rates and does
not consider other forms of renewable generation such as solar. Reference [7] follows a similar approach to [6]. Hence, [7] faces the same challenges as discussed above.

Reference [8] proposes a supervised shut down algorithm to mitigate forced shut-down scenarios of off-shore wind farms. Forced shut down scenarios occur when the wind speed exceeds cut-out speed, forcing the pitch angle to be set to zero degree to avoid damage to wind generator. Although the algorithm proposed in [8] avoids structural damage to wind generators it does so by means of higher wind power curtailment.

Reference [9] studies ramp rate control of a PV plant. A method to smooth PV plant output at PCC is developed. The presented method is promising for individual PV plants, however, as will be shown later such a decentralized scheme where individual plants are controlled without the global knowledge on condition of other non-load following generators will result in extended use of BESS. This results in higher use rate of batteries which have limited number of operational cycles. In addition, the higher use rate also results in additional BESS losses which in turn results in lesser total energy output from PV plants.


All of these approaches presented in [10–20] target control strategy at location of renewable generation. This results in sub-optimal solutions when seen from the
system operator level. For instance, consider a cloud cover event. Such an event is not likely to happen at exactly the same time at all of the solar farms. In fact, the geographical distribution of renewable generators along with the diversity of renewable generation source results in a much smoother total renewable generation curve than any individual PV or wind farm output. This fact can be verified by studying the available renewable generation data from system operators and/or by studying NRELs synthetic wind [21] and solar data set [22]. In addition, as pointed out earlier, smoothing individual renewable generation results in increased use of energy storage. This is in turn results in shorter life span of flexible energy storage systems such as BESS. In addition, increased use of energy storage results in higher energy losses.

Based on the literature review the following observations can be made about existing research. 1) To the best of our knowledge an attempt at mitigating high ramp rates witnessed at ISO/RTOs due to the so called duck curve has not been addressed. 2) Geographical distribution of renewable generation provides a more realistic representation of inter-dispatch variation of renewables - a fact neglected in most studies. 3) An unified approach to sizing BESS at renewable generator farms and utilizing BESS for mitigation of high ramp rates as seen from the system operator perspective has not yet been explored. Based on these observations, contributions of this work is outlined below.

**Contributions**

1. An unified approach to sizing BESS at renewable generator farms to enforce adherence to a predefined grid code.

2. A stochastic optimization scheme which utilizes the available BESS, specifically designed to address high ramp rates seen at ISO/RTOs due to duck curve
phenomenon using actual data from CAISO.

Organization

The rest of the chapter is organized as follows. In Section II, uncertainty and variability modeling used in this work is introduced. Section III covers sizing of battery energy storage systems. The design criteria is to minimize variation between dispatches. Stochastic optimization model is discussed in Section IV. Results and discussion make up Section V while conclusions are drawn in Section VI.

2.2 Uncertainty and Variability Modeling

Renewable and load forecast are the major contributors to uncertainty in power grid operations. Uncertainty of both wind and solar generation forecast is modeled. As discussed previously, to be of practical use the design setup for BESS should be at the wind and solar farms with a centralized coordinated scheduling scheme with system wide information. In addition to being practical, such a setup also provides higher degree of freedom for better scheduling.

System level error distribution for all forecasts seen at the ISO/RTO is used. Reference [23] studied day ahead wind and load forecast error distribution of several ISOs such as New York ISO (NYISO), CAISO and electric reliability council of Texas (ERCOT). The error distributions of CAISO as reported in [23] is used to model wind and load forecast error distribution. In this work, wind forecast error distribution is extended to model solar forecast error distribution as system operator wide solar forecast error distribution was not available in existing studies.
2.2.1 Discretization Interval

A discretization interval of 1 hour often does not represent a realistic representation for problems such as the one in consideration. As a result 5 minute discretization interval in this work. In addition, the intra-area dispatch interval for ISOs such as CAISO is 5 minutes. Thus, $\Delta t = 5$ is also a feasible choice with respect to current energy policy standard of ISOs. Furthermore, 5 minute intervals are also adequate to represent load variation [24].

2.2.2 Data Set

An important observation can be made by studying the renewable generation curve of geographically distributed renewable generation units: variation of renewable generation reduces substantially when units from varyingly different locations are aggregated. This can be attributed to the higher degree of statistical independence between renewable generators that are geographically distributed. Hence, it is important to take this factor into consideration. Furthermore, the generation curve has to time correlated meaning the exact weather pattern observed at different sites should be used in the synthetic generation of renewable generation output.

NRELs wind and solar integration study data set satisfies all of the above mentioned requirements. Specifically, Western wind [21] and solar integration [22] study data sets is used. The obtained data sets include generation information for multiple renewable generation sites for the state of California. Historic generation curve of CAISO was used for accurate distribution of generation among different generation types. The data is obtained from [25].

Although wind generation geographically dispersed in the state of California, major portion of wind generation comes primarily from three regions: Altamont pass,
Tehachapi and San Gorgonio [26]. However, in order to account for current as well as future integration a geographically diverse wind generation is assumed in this work.

### 2.2.3 Modeling Forecast Errors

Each individual renewable generation farm is modeled with error distribution seen at CAISO as given in [23]. A normal distribution with $\mu = -0.004$ and $\sigma = 0.130$ is used to characterize day ahead wind power forecast error. For lack of information on solar generation forecast error data seen at system operator level, the same error distribution as that of wind power forecast is used. Day ahead load forecast error seen at CAISO is characterized with a normal distribution of $\mu = -0.002$ and $\sigma = 0.026$ [23].

In order to represent uncertainty in the optimization problem, one needs to discretize forecast error distribution into finite number of scenarios. Using the above mentioned probability density function, wind and solar forecast error is discretized into six scenarios - low, medium and high for over and under forecast scenarios as shown in Fig. 2.2. For each of the six scenarios, the median value is used as the typical forecast error seen for each of the scenario. The probability of each scenario is defined as the area under the curve defined by the bin representing each scenario.

The bounds defined by $\{-0.15, -0.004\}$, $\{-0.25, -0.15\}$ and $\{-\infty, -0.25\}$ represent the over prediction - low, medium and high scenarios. While $\{-0.004, 0.15\}$, $\{0.15, 0.25\}$ and $\{0.25, \infty\}$ represent under prediction - low, medium and high scenarios. Probability of the above bins for over prediction - low, medium and high scenarios are 0.0292, 0.1012 and 0.3688 respectively. Similarly, probability of under prediction - low, medium and high scenarios are 0.0256, 0.0925 and 0.3824 respectively.
2.2.4 Constrained Linear Least Squares Estimate

Since the focus of this work is on controlling output of individual renewable generation at PCC, it is imperative that realistic data set be used. Since [25] provides only the total renewable generation by generation type, the individual information of each wind/solar farm is lost. Hence, in order to preserve the variability seen at each renewable generator, a representative data set from [21, 22] is used. The data set is then scaled in such a way that the total renewable generation for solar and wind seen at the system operator level as obtained from [25] closely matches with that of the scaled data set of [21, 22] - in a least squares sense.

\[
\begin{bmatrix}
P_i(t) & P_n(t) \\
P_i(t+1) & P_n(t+1) \\
\ldots & \ldots \\
P_i(t_{end}) & P_n(t_{end})
\end{bmatrix}
\begin{bmatrix}
K_1 \\
K_2 \\
\ldots \\
K_n
\end{bmatrix}
= \begin{bmatrix}
P_w(t) \\
P_w(t+1) \\
\ldots \\
P_w(t_{end})
\end{bmatrix}
\]

(2.2.1)
Equation (2.2.1) is the linear least squares representation for scaling output of \( n \) wind farms to represent the wind generation pattern as seen at the ISO. Similar approach is followed for solar generation as well. The values of \( K_1 \) to \( K_n \) are constrained such that the scaling is always positive. This constraint is given in (2.2.2) and is required as the output at PCC cannot be negative. Please note that BESS charging from the grid is not modeled in the studies.

\[
0.5 \leq K_i \leq 2 \quad (2.2.2)
\]

In (2.2.2) the limits are decided based upon the number and size of renewable generators and the amount of installed capacity of renewable generation for a given ISO/RTO.

### 2.3 Optimal Storage Sizing for Renewable Generation

The proposed battery sizing scheme is perfectly general: it holds for PV farm, wind farm and for any non-load following/non-controllable generation. BESS in general can participate in a variety of services such as energy time shift, load following, voltage support, reserve capacity etc [27]. The question of optimal sizing depends on the intended application area and purpose. For instance, storage sizing for energy time-shift is more of a business case analysis than storage sizing to meet a predefined grid code for secure grid operation.

A large BESS would provide additional reliability while incurring a higher capital investment. Hence, an optimally sized BESS that satisfies a prescribed set of operational constraints while incurring lowest possible capital investment is desired. It is this sizing problem that is addressed in this section. This problem needs to be
solved before an attempt is made to utilize the BESS in such a way that the problems associated with the so called Duck curve is mitigated to the best possible extent.

In this work, BESS is designed to regulate ramp up and ramp down events such that the limits for these events are bounded by a grid code. The maximum ramp up and ramp down events are designed to be bounded as given in (2.3.1).

\[ \Delta P^i_{pcc} \leq \Delta P^i_{pcc} \leq \Delta P^i_{pcc} \tag{2.3.1} \]

Where the value of \( \Delta P^i_{pcc} \) and \( \Delta P^i_{pcc} \) are user defined parameters, which in this work is set to 3% of rated capacity of the wind/solar farm. Notice that the value is dependent on discretization interval, which in this work is 5 minutes. The choice of value is chosen such that if the plant were to ramp up at the maximum allowable rate then the plant would take \( \sim \)3 hours to go from zero output to rated capacity.

This design process will enable smoothing of renewable generator output at the point of common coupling such that the variations seen between dispatches are minimized. For instance, a cloud cover event for a PV plant often results in a sudden drop in PV plant output immediately followed by a sudden increase in power output once the cloud cover has passed by. In this case, the designed BESS will limit these sudden variations.

To this end, for each solar and wind farm, the day with highest variability from NRELs synthetic data [21,22] is picked and BESS for individual solar and wind farms is designed such that (2.3.1) is satisfied. Variability in this context is defined as given in (2.3.2).

\[ V_i = \sum_{t \in T} \Delta P^i_{pcc} (t) \forall i \in R \tag{2.3.2} \]
2.3.1 Security Constrained Battery Sizing

Installation of BESS should take into account the secure operation of the grid. For instance, if the maximum power output at PCC is not regulated then the transmission line carrying power from the said renewable source could be overloaded. In addition, justification should be provided for studying the sizing problem in isolation from the rest of the grid.

In this work, the sizing problem is intended for wind and solar farms that are already integrated with grid operation. Under such a scenario the transmission lines and relevant equipment are already properly sized during the planning stage. Hence, secure operation can be justified by limiting the maximum power output at PCC to a value for which the wind/solar farm was designed for. Equation (2.3.3) and (2.3.4) are used to enforce this physical constraint.

\[
P_{\text{pcc}}^i(t) = P_{\text{rg}}^i(t) - P_{\text{bc}}^i(t) + P_{\text{bd}}^i(t) \quad \forall t \in T \tag{2.3.3}
\]

\[
0 \leq P_{\text{pcc}}^i(t) \leq \bar{P}_{\text{pcc}}^i \quad \forall t \in T \tag{2.3.4}
\]

Notice the lower bound on (2.3.4) is set to zero. This implies that the BESS for this design stage cannot draw power from grid in order to charge. It should be pointed out that this is done for isolating the PCC from rest of the grid. Once the BESS is installed, with proper converters installed, BESS could potentially be used to charge from the grid - a likely scenario for energy time-shift applications.

Energy balance equation for the sizing problem is similar to the stochastic optimization problem developed in Section 2.4 with the exception of stochastic scenarios and hence stochastic variables. For clarity and for completeness the energy balance
equation is shown separately for both applications.

\[ P_{bd}^i(t) - \overline{P}_b^i \leq 0 \quad \forall t \in T, \forall i \in R \tag{2.3.5} \]
\[ P_{bc}^i(t) - \overline{P}_b^i \leq 0 \quad \forall t \in T, \forall i \in R \tag{2.3.6} \]
\[ E_b^i(t) - \overline{E}_b^i \leq 0 \quad \forall t \in T, \forall i \in R \tag{2.3.7} \]
\[ E_b^i(t) = E_b^i(t-1) + \eta_{in}^i \cdot P_{bc}^i(t) \cdot \Delta t - \frac{P_{bd}^i(t)}{\eta_{out}^i} \cdot \Delta t \quad \forall t \in T, \forall i \in R \tag{2.3.8} \]

Although (2.3.8) captures the evolution of energy for each BESS, it does not impose the constraint that only charging or discharging can happen in a given dispatch but not both. The algebraic sum of charging and discharging of (2.3.3) is not altered by both charging and discharging at the same instance. However, the amount of energy stored in the battery is altered. This is due to the energy losses that arise as a result of \( \eta_{in} \) and \( \eta_{out} \) being < 1. Charge/discharge constraint is imposed by introducing binary decision variables \( \beta \), a dummy constant \( K \) with a large value, and two constraint equations (2.3.9) and (2.3.10).

\[ \beta^i(t) \cdot K \geq P_{bc}^i(t) \tag{2.3.9} \]
\[ 1 - \beta^i(t) \cdot K \geq P_{bd}^i(t) \tag{2.3.10} \]

The primary objective of this sizing problem is to minimize the maximum output deviation i.e. \( \Delta P_{pcc} \) with minimal sizing of battery with respect to power and energy rating. The maximum permissible deviation value \( \overline{\Delta P}_{pcc} \) is provided by the user, based on operational standards and/or desired threshold. If convergence is not achieved then the value of \( \overline{\Delta P}_{pcc} \) is increased incrementally in an iterative way until
convergence is achieved. The cost function to be minimized is given in (2.3.11). For this work both $C_P$ and $C_E$ are weighted equally.

$$\text{min } C_P^i \cdot \hat{P}_b^i + C_E^i \cdot \hat{E}_b^i$$  \hspace{1cm} (2.3.11)

### 2.4 Stochastic Optimization Model

#### 2.4.1 Static Information Structure

The modeling of stochastic parameters in this work is that of static information structure. The reference to static in this context does not imply lack of dynamics but rather implies that any choice of decision from decision space $U$ at a given time $t$ does not alter the stochastic parameters for future time instances. In other words, stochastic parameters are independent of the decision process. Stochastic parameters in this case are renewable generation and load forecast.

#### 2.4.2 Formulation

Each solar and wind farm are assumed to have a BESS and is sized as discussed in Section 2.3.1. Formulation developed in this section will address the issue of high ramp up and ramp down of controllable generation and reserves while considering stochastic nature of renewable generation and load forecast. It is worth pointing out that the stochastic parameters used in this work are independent of each other. For example, the error distribution of wind forecast does not depend on the error distribution of solar forecast.
2.4.2.1 Energy Balance Equation

All BESS follow energy balance equation as defined in (2.4.1).

\[
E^i_b(t) = E^i_b(t-1) + P^i_{bc}(t) \cdot \eta_{in} \cdot \Delta t - \frac{P^i_{bd}(t)}{\eta_{out}} \cdot \Delta t \quad \forall i \in R, \forall t \in T 
\]

(2.4.1)

\[
\beta^i(t) \cdot P_{\text{max}} \geq P^i_{bc}(t) 
\]

(2.4.2)

\[
(1 - \beta^i(t)) \cdot P_{\text{max}} \geq P^i_{bd}(t) 
\]

(2.4.3)

The rationale for the need for binary variables is the same as in Section 2.3.1. The binary variable \(\beta\) makes the optimization mixed integer linear program (MILP). Limits on decision variables are given in (2.4.4)-(2.4.6).

\[
0 \leq P^i_{bc}(t) \leq \overline{P}^i_b \quad \forall i \in R, \forall t \in T 
\]

(2.4.4)

\[
0 \leq P^i_{bd}(t) \leq \overline{P}^i_b \quad \forall i \in R, \forall t \in T 
\]

(2.4.5)

\[
E^i_b \leq E^i_b(t) \leq \overline{E}^i_b \quad \forall i \in R, \forall t \in T 
\]

(2.4.6)

2.4.2.2 Power at Point of Common Coupling

Power at the point of common coupling is defined as the combined output of BESS and solar/wind farm. It is important to note that the BESS is designed for the entire farm and not for individual units.

\[
P^i_{pc}(t) + P^i_{bc}(t) - P^i_{bd}(t) = \overline{P}^i_{rg}(t) \quad \forall i \in R, s \in S, \forall t \in T 
\]

(2.4.7)

Due to the stochastic nature of (2.4.7) multiple scenarios as defined in the set \(S\) need to be modeled. This scenario set is developed in Section 2.2. Equation (2.4.7)
is applied to all renewable generators, both wind and solar, defined in the set \( R \) and for all dispatches as defined in the set \( T \). This formulation requires only one set of stochastic variables, \( P_{i,s}^{i,s} \).

### 2.4.2.3 Controllable Generation

Controllable generation is defined as those generation units that can be controlled. In this work, controllable generation include conventional generation and reserves.

\[
P_c(t) + \sum_{s \in S} \sum_{i \in R} \rho_s \cdot P_{i,s}^{i,s}(t) = E(P_d(t)) \quad \forall t \in T
\]  

(2.4.8)

### 2.4.2.4 Ramping Constraints

Limits on ramping is time dependent as indicated in (2.4.9). This is due to the fact that available ramping is dependent on committed conventional generation and reserves committed for a given dispatch. Equation (2.4.9) ensures ramping constraint is met for all dispatches.

\[
\Delta P(t) = P_c(t) - P_c(t-1) \leq \Delta P(t) \quad \forall t \in T
\]  

(2.4.9)

### 2.4.2.5 Objective Function

The objective is to minimize the sum of \( P_c \) as given in (2.4.10).

\[
\min \sum_{t \in T} P_c(t)
\]  

(2.4.10)

Equation (2.4.10) while minimizing the amount of controllable generation also maximizes the amount of renewable generation as controllable and renewable generation are complimentary to each other. This fact can be observed through (2.4.8).
2.5 Results and Discussion

2.5.1 Security Constrained Sizing

Figure 2.3 is a succinct representation of the results obtained from BESS sizing. Figure 2.3a and 2.3b shows the power and energy rating of every BESS installation at wind and solar farms in the system. The total power rating of installed BESS is 2766 MW while the total energy rating is 1268 MWh. The total installed capacity of wind and solar generation is assumed to be 9000 MW.

The maximum allowed ramp rate for 5 minute dispatch interval as a percentage of wind/solar farm power rating is shown in fig. 2.3c. The grid code in this case is assumed to limit ramp rate to a maximum rate of zero power to rated capacity in 3 hours. This translates to roughly 3% for 5 minute dispatch interval. Notice however that some of the wind and solar farms do have more than 3% ramp up as limit. This is due to the fact that there exists no sizing of BESS that can limit the ramp rate to 3%.

There are several reasons for such an outcome. 1) While sizing the battery the initial SOC of battery is assumed to be zero. This assumption is justified because the sizing of battery is not known a priori and any assumption on the amount of initial energy stored in BESS easily distorts the solution space. 2) Due to 1) any high variation at the beginning of the optimization horizon cannot be regulated effectively. This results in non feasible solution for 3% ramp limit. However, when the ramp limit is recursively relaxed to higher values an optimal solution is obtained.

2.5.2 Experimental Setup for Duck Curve Mitigation

Ramping rates seen at the ISO/RTO are dependent on the time of the year. Hence, the highest ramp rate day observed in the first and second half of each given month
for the year 2015 are used as test scenarios. This results in 24 test cases as shown in Table 2.1. Highest ramping in this context is defined as the highest hourly ramp up seen in controllable generation. It is worth mentioning that controllable generation in this context also includes imported power.

The daily generation curve for 2015 obtained from [25] is used. Since the generation curves are available only at hourly intervals the data is up sampled to 5 minute intervals. The obtained values from [25] represent the hourly average for energy generation. The inter dispatch power variation is lost due to the lower sampling frequency of the original data. However, this is a practical design choice made as a trade-off for actual data.

2.5.3 Mitigating Duck Curve Phenomenon

The multi-period optimization is discretized at 5 minute intervals over a period of 24 hours totaling 288 dispatches. Six stochastic scenarios are used to model uncertainty. Figure 2.4a shows the semi-monthly maximum five minute ramp rate for every month of the year 2015. High ramp events are observed during the winter months of November through March. As an extreme case, consider the month of November. In the first half of the month, the highest peak event resulted in maximum 5 minute ramp event of 455.3 MW as given in Table 2.1. However, with coordinated control of solar and wind generation using the proposed stochastic optimization model while considering uncertainties resulted in a maximum ramp rate of 187.8 MW. Similarly, for the second half of November, maximum ramp rate of 536.8 MW was observed while the proposed approach resulted in 232.2 MW.

There is a practical trade-off between the level of ramp rate mitigation and loss in energy. BESS used in this work is designed with round trip efficiency of 90%. Same efficiency values are used in other studies such as [6]. Due to the charging and
Figure 2.3: BESS sizing and inter-dispatch variation
Figure 2.4: Ramp rate and controllable generation comparison
| Month | Ramp up (MW) | | | | Energy Loss (MWh) | |
|-------|-------------|---------------|---------------|---------------|------------------|
|       | 5 min max   | 1 hr max      |               |               |                  |
|       | Conventional | Proposed      | Conventional  | Proposed      |                  |
| Jan   | 334.8       | 195           | 3553.5        | 2339.2        | 133              |
|       | 327.7       | 204.3         | 3092.3        | 2451          | 64.3             |
| Feb   | 332.7       | 208.4         | 2812.1        | 2500.3        | 24.6             |
|       | 341         | 200           | 2684.8        | 2399.3        | 65.5             |
| Mar   | 221.6       | 200.6         | 2475.3        | 2406.4        | 5.4              |
|       | 258.8       | 198           | 2777.2        | 2375.8        | 25.4             |
| Apr   | 203.1       | 197.1         | 2257.8        | 2257.8        | 0.2              |
|       | 190.5       | 190.5         | 1994          | 1994          | 0                |
| May   | 152.7       | 152.7         | 1540.1        | 1540.1        | 0                |
|       | 187.9       | 187.9         | 1511.5        | 1511.5        | 0                |
| June  | 193.6       | 193.6         | 1099.8        | 1099.8        | 0                |
|       | 188.6       | 188.6         | 1700.5        | 1700.5        | 0                |
| July  | 305         | 290.7         | 2253.8        | 2253.8        | 0.1              |
|       | 276.7       | 262           | 1910.5        | 1910.5        | 0.1              |
| Aug   | 325.3       | 289.1         | 1928.6        | 1933.6        | 0.6              |
|       | 203.5       | 203.5         | 2320.8        | 2320.8        | 0                |
| Sept  | 223.6       | 223.6         | 2273.6        | 2273.6        | 0                |
|       | 227         | 227           | 2475          | 2475          | 0                |
| Oct   | 206.9       | 206.9         | 2322.2        | 2322.2        | 0                |
|       | 263.5       | 225.5         | 2797.9        | 2706.4        | 6.4              |
| Nov   | 455.3       | 187.8         | 3354.5        | 2254.1        | 160              |
|       | 536.8       | 232.2         | 3822.8        | 2786.5        | 140              |
| Dec   | 447.9       | 238.5         | 3661          | 2861.9        | 106.9            |
|       | 381.4       | 217.5         | 3725.2        | 2609.4        | 138.8            |
discharging losses associated with BESS, the amount of energy lost and subsequent increase in energy requirement increases with level of mitigation. This fact is illustrated in Table 2.1. Winter months November through March exhibit high ramp rates and subsequently higher level of energy losses.

Maximum energy loss of 160 MWh is observed in first half of November. Considering the fact that the total energy requirement on this particular day is \(\sim 547\) GWh, the amount of energy lost as a ratio of total energy requirement of the system over the course of the day is \(2.92 \cdot 10^{-4}\). The economic consequence of producing the extra 160 MWh of energy would result in \(\sim \$4480\) at the average monthly energy cost of \(28\) $/MWh as reported by CAISO for the month of November, 2015 [28]. In comparison, the use of BESS assisted scheduling of renewable generation resulted in reduction of 1100 MW of max hourly peak reduction. This would result in a substantial reduction in the amount of required reserves. A thorough economic analysis of the proposed approach is beyond the scope of this work. The purpose of pointing out these numbers is to illustrate the plausible economic benefits in addition to improved secure
Figure 2.5a helps to validate the claim of reduced usage of BESS. Equation (2.4.10) minimizes energy generated by conventional generation. Minimizing the amount of conventional generation is complimentary to the task of maximizing renewable generation. In other words, the formulation implicitly minimizes renewable energy curtailment. In addition, due to the losses associated with charge/discharge cycle, proposed stochastic optimization scheme uses BESS only in circumstances where it is a necessity. Thereby improving the life cycle of BESS - a high capital investment.

This claim can be verified by analyzing the seemingly uninteresting plot shown in Fig. 2.5b showing 5 minute ramp rates for the test case representing first half of May, 2015. Both the proposed and conventional method result in the exact same 5 minute ramp rates. This is due to the fact that the ramp rates observed for this day is within the permissible limits. As a result energy storage was not used resulting in zero energy loss as can be seen in Table 2.1.

### 2.5.4 Computational Performance

All tests were run on intel-i7 5550u processor with 8 GB RAM using CPLEX version 12.6.0 [29]. Geometric mean time defined as $\sqrt[n]{r_1 \cdot r_2 \cdot \ldots \cdot r_n}$ is used to compute mean computation time. Where $r_1, r_2, ..., r_n$ are the $n$ run times. In this work $n = 24$ corresponding to the 24 cases tested. Geometric mean time of 9.40 seconds was obtained for the tested cases.

### 2.6 Conclusion

A method to mitigate renewable intermittency under conditions of uncertainty is developed. Specifically, the developed method addresses the high ramp events due to the phenomenon known as duck curve. Solution to an interesting subproblem -
sizing of BESS for renewable integration is also addressed. Actual data from CAISO is used to showcase the effectiveness of the proposed method and the results showcase the benefits from an economic as well as security viewpoint. Economic benefits occur at two levels, 1) at system level due to the reduction in reserve requirement 2) at generation company level due to minimal use of BESS, thus increasing the life cycle of BESS while adhering to grid code. As a result of the reduction in reserve requirement higher security margins can be achieved at significantly lower cost.
Chapter 3

Optimal Energy Storage Sizing and Scheduling for Power Systems Operation

Nomenclature

Battery Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Efficiency of Battery</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>Binary decision variable for charging</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>Binary decision variable for discharging</td>
</tr>
<tr>
<td>$P^C_B$</td>
<td>Power consumed by battery while charging</td>
</tr>
<tr>
<td>$P^D_B$</td>
<td>Power output of battery while charging</td>
</tr>
<tr>
<td>$E_B$</td>
<td>Amount of energy stored in battery</td>
</tr>
<tr>
<td>$E_{init}$</td>
<td>Initial energy stored in battery</td>
</tr>
<tr>
<td>$\overline{P_B}$</td>
<td>Upper limit for Power output of battery</td>
</tr>
<tr>
<td>$\overline{E_B}$</td>
<td>Upper limit for Energy storage of battery</td>
</tr>
</tbody>
</table>

Cost Variables
$C_P$  
Cost associated with battery power rating

$C_E$  
Cost associated with battery energy rating

$C_O$  
Cost associated with operating BESS

$C_0, C_1, C_2$  
Coefficients of polynomial cost function

**Generator Variables**

$\Delta P_{\text{gen}}^I$  
Ramp up limit of Generator I

$\Delta P_{\text{gen}}^I$  
Ramp down limit of Generator I

$P_{\text{gen}}^I$  
Upper limit for real power output of Generator I

$P_{\text{gen}}^I$  
Lower limit for real power output of Generator I

$Q_{\text{gen}}^I$  
Upper limit for reactive power output of Generator I

$Q_{\text{gen}}^I$  
Lower limit for reactive power output of Generator I

$P_{\text{gen}}^I$  
Real Power output of Generator I

$Q_{\text{gen}}^I$  
Reactive Power output of Generator I

**Network Variables**

$V_{\text{mag}}^I$  
Voltage magnitude of node I

$\theta_I$  
Voltage angle of node I

$V_{\text{mag}}^I$  
Upper limit for Voltage magnitude of node I

$V_{\text{mag}}^I$  
Lower limit for Voltage magnitude of node I

$P_{IK}$  
Power flow limit of line connecting nodes I and K

$P_{IK}$  
Power flow on line connecting nodes I and K

$P_{\text{pcc}}$  
Power at point of common coupling
\( P_{\text{pcc}} \) Maximum allowed power at point of common coupling

\( \Delta P_{\text{pcc}} \) Change in power at point of common coupling between dispatches

\( \Delta P_{\text{pcc}} \) Maximum allowed change in power at point of common coupling between dispatches

**Generic Variables**

\( \Delta t \) duration between dispatches

\( T_{\text{end}} \) Final dispatch

\( T_{\text{off}} \) Dispatches where generator is offline

A method to optimally size and operate energy storage systems for two different applications, peak load management and intermittency mitigation of renewable energy generation is proposed. Relevant security constraints such as voltage limits, generator real and reactive power limits, ramp up/down rates, generator minimum start up/down times, line flow limits and tie-line flows are formulated in the optimization problem. The developed optimization method provides a means to quantify the sizing requirement of energy storage for different applications and also provides a way to optimally operate energy storage to satisfy relevant security constraints while maximizing profit. The problem of peak load management is cast as non-linear optimization problem while intermittency mitigation of renewable generation is formulated as a mixed integer linear program. The formulations developed are for battery energy storage systems, however, other types of energy storage systems can also be modeled by including relevant physical constraints of energy storage type.
3.1 Introduction

3.1.1 Motivation

It is projected that about 25% of the U.S. coal-fired power plants will be retired by the end of this decade [30]. Several factors such as environmental protection agency’s mercury and air toxins standards (MATS), proposed CO$_2$ regulation standards, low price of natural gas, etc. contribute to this projection. With increased awareness about global warming, market based carbon emission trading mechanisms have been introduced in recent years [31]. European union (EU) has introduced a cap and trade carbon emission trading system since 2004 [32] and the same is soon to be established in Australia [33,34].

With the drive towards clean energy, renewable generation has gained prominence. Renewable generation is intermittent by nature and higher penetration levels of renewable energy results in increased uncertainty and variability. Variability of renewable generation has to be compensated by conventional generation. Conventional generators have physical constraints which translates to a maximum allowable ramp up/down rate. Hence, the likely scenario of large variation with high levels of renewable generation penetration poses a threat to secure operation of the grid. A likely solution is the use of energy storage systems, to smooth the output profile of renewable generators. Battery energy storage system (BESS) in particular is a promising candidate because of its flexibility.

BESS can be used for various applications in power system operation, such as improving control, mitigating volatility and intermittency problems of renewable energy resources, load following, voltage and frequency stability, peak load management (PLM), power quality improvement, and deferment of system upgrades [35]. Any of the above mentioned application raises two key questions. How to appropriately size
a storage system for a given application? and how to optimally operate storage system while enhancing grid security and at the same time maximizing profit? This is an area that requires further research and is the focus of this work. The aim of this work can be summarized as follows.

- To provide a means to quantify the required sizing of BESS for the intended applications of
  - Peak load management (PLM)
  - Intermittency mitigation (IM) of renewable energy resources.
- To provide a means to optimally operate BESS to enhance system security while maximizing profit for the above mentioned applications.

3.1.2 Literature Review

The problem of optimal storage sizing has been explored before in literature. The authors in [35] use mixed integer programming for optimal storage sizing in microgrids. They include reliability modeling by means of loss of load expectation factor (LOLE). They define LOLE as the expected fraction of load unserved in microgrid during the study period. The objective of [35] is to find optimal sizing for one-year while considering the relevant security constraints.

The authors in [36] approach the problem of optimal storage sizing by formulating it in unit commitment problem for both grid-connected and islanded mode of operation. Mixed integer linear programming is then used to solve the optimization problem. Reference [37] uses Tabu search method for optimal storage sizing. In [38], the authors formulated the problem of optimal storage sizing and control as a single problem using fuzzy logic and advanced artificial neural networks (AANN) for a wind
farm data set of 282 days. The fuzzy and the AANN controller finds the optimal storage sizing and optimal power set points based on the entire data set. Genetic algorithm based approach is proposed in [39]. An analytical approach to reliability based storage sizing problem is developed and presented in [40].

Reference [41] proposes a method to mitigate intermittency of wind power generation. The authors use lead-acid battery model from [42] to compute DC-link voltage based on which an appropriately sized BESS is used that could keep the DC-link voltage within specified limits. For maintaining wind farm output at scheduled level, the authors in [43] present a control algorithm for energy-leveling service which tries to maintain the net energy delivered from a wind farm at a scheduled level.

Studies on wide area energy storage have also been explored in literature. For example, [44] studied the balancing of intermittent resources using wide area energy storage, specifically fly wheel energy storage for Bonneville power administration (BPA) and California independent system operator (CAISO) control area. Authors in [45] study battery storage evaluation pertinent to BPA and CAISO practices. In [45], one-direction operation of NaS battery for regulation and real-time dispatch service is explored.

3.1.3 Contributions

The contributions of the chapter is divided into two parts. The main contributions of BESS for peak load management can be summarized as follows,

- Inclusion of time-varying nature of BESS, thus the optimization is not a static optimization or a series of static optimizations without dependency between dispatches but rather an extended-time scale optimization where there is dependency of energy storage value between different dispatches.
• Voltage security constraints are modeled in the optimization problem using the standard power flow equations. The coupling between real and reactive power is maintained i.e. no decoupled power flow type approximations are used. Hence, the optimization formulation is a non-linear programming problem.

• In addition to the optimal sizing formulation, an optimal scheduling formulation that reduces the peak load to a desired value while satisfying relevant operational constraints with minimal operating cost is formulated.

• Provide means to study a part of the system where BESS is located by means of scheduled tie-line flow. For example, a heavy load pocket can be studied in isolation from the rest of the system by incorporating a predefined set of values for import/export over multiple dispatches that span the entire horizon of optimization period.

Contribution one is very similar to unit commitment (UC) problem, while contribution two is an optimal power flow (OPF) problem. OPF or its security constrained version SCOPF consider only a snapshot in time \( t \) while UC problem considers several dispatches spanning an entire day. UC can be represented as a mixed integer linear programming (MILP) formulation, while OPF is formulated as non-linear program (NLP) for its AC version (ACOPF). The proposed formulation can be thought of as a hybrid between UC and ACOPF problem with addition constraints for BESS over an extended time-scale, in this case a 24 hour period spanning 96 dispatches i.e. a dispatch every 15 minutes.

For the case of BESS for IM of renewable generation, the problem is expressed as MILP. The major contributions are,

• A mathematical way to determine optimal BESS sizing for wind/solar farms
while specifying an operational constraint. In this case the operational constraint considered is variation between dispatches.

- In addition to finding the optimal sizing, an optimal scheduling formulation is also provided to accomplish the objective of enforcing variation limit and at the same time maximizing the profit of the wind/solar farm through energy time-shift.

### 3.2 BESS Sizing for Peak Load Management

Since the definition of PLM can vary, the intended use for PLM with respect to this work is first defined. The primary objective is to limit peak load in a given part of the system i.e. in a given subsystem where BESS is installed. Peak load is limited while satisfying relevant operational constraints subject to minimizing the overall operating cost. With the intended application clearly defined, the objective of the mathematical formulation for sizing BESS for PLM can be stated as,

- To find an optimal BESS sizing with respect to power and energy rating, for a given section of the grid under study, while providing means to express operational constraints such as generator scheduling, generator minimum start-up/shut-down time, voltage limits, generator real and reactive power limits, generator ramping limits, line flow limits and import/export flow constraints.

In this work, a modified version of the nine bus test system as given in [46] is used as shown in fig. 3.1. Optimization scheme is discretized with dispatch intervals of 15 minutes for an optimization period of 24 hours. It is assumed that the swing generator (generator one) represents the rest of the system and import/export of power is maintained according to a predefined schedule. Generally, this predefined
schedule comes from UC formulation, adjusted to accommodate forecast errors during real-time operation.

The provision to incorporate tie-line flow at scheduled value is important because this allows the study of a part of the system in isolation from the rest of the system. Battery efficiency, $\eta$, is used to include charging/discharging losses of BESS. A schematic which gives an overview of the optimization process involved in sizing and operating a BESS for PLM is shown in fig. 3.2.

### 3.2.1 Energy Balance Constraint for Battery

Several constraints pertaining to battery needs to be enforced in order to maintain law of conservation of energy. First, the amount of energy stored in BESS is time-varying i.e. the value of storage at current time $t$ depends on previous time $t - 1$. This is given as,

$$E_B(t) = E_B(t - 1) + P^C_B(t) \cdot \eta \cdot \Delta t - \frac{P^D_B(t)}{\eta} \cdot \Delta t$$  (3.2.1)
Figure 3.2: Schematic showing overview of optimization process for sizing and scheduling of BESS for peak load management

Equation (3.2.1) ensures that the amount of energy at the end of $t^{th}$ dispatch period is equal to the algebraic sum of energy stored from $(t - 1)^{th}$ period and the energy dispatched/stored in the current period. Representing efficiency as given in (3.2.1) poses certain problems. In reality, in a given discretized dispatch, BESS can either charge or discharge but not both. There are several ways to impose this constraint including the use of binary decision variables, however, that would make the problem mixed integer NLP (MINLP) which are harder to solve. Hence, equation (3.2.2) which maintains the NLP formulation is used in this work.

$$P_B^D(t) \cdot P_B^C(t) = 0 \ \forall t$$  \hspace{1cm} (3.2.2)

An additional constraint to enforce the complete use of stored energy is accomplished by setting the energy of BESS at the last dispatch to be equal to initial energy
stored in the battery at the beginning of the optimization period.

\[ E_B(t_{end}) = E_{\text{init}} \]  

(3.2.3)

3.2.1.1 Sizing Constraints

The power and energy values at every dispatch period needs to be expressed as a value in relation to maximum power and energy rating of BESS. This is accomplished in the following way,

\[ P_B^D(t) - P_B^C \leq 0 \forall t \]  

(3.2.4)

\[ P_B^C(t) - P_B^D \leq 0 \forall t \]  

(3.2.5)

\[ E_B(t) - E_B \leq 0 \forall t \]  

(3.2.6)

3.2.2 Inequality Constraints

The inequality constraints are used to define range of operation with respect to voltage, power generation, ramp up/down rate and line flow limits. Equation (3.2.12) limits the total output of generators in the area under study to a value less than maximum allowable peak load set by the user defined parameter \( P_B^{\text{max}} \).

\[ V_{\text{mag}}^I \leq V_{\text{mag}}^I \leq \overline{V_{\text{mag}}^I} \]  

(3.2.7)

\[ P_{\text{gen}}^I \leq P_{\text{gen}}^I \leq \overline{P_{\text{gen}}^I} \]  

(3.2.8)

\[ Q_{\text{gen}}^I \leq Q_{\text{gen}}^I \leq \overline{Q_{\text{gen}}^I} \]  

(3.2.9)

\[ \triangle P_{\text{gen}}^I \leq \triangle P_{\text{gen}}^I \leq \overline{\triangle P_{\text{gen}}^I} \]  

(3.2.10)

\[ 0 \leq |P_{IK}| \leq |\overline{P_{IK}}| \]  

(3.2.11)
\[ \sum_{i \in G} P_{gen}^i(t) \leq P_{max}^D \forall t \]  

(3.2.12)

### 3.2.3 Unit Commitment Schedule

The idea of this optimization problem formulation is to size BESS while following current operational procedure and to be able to use the same formulation to optimally schedule BESS for better power systems operation. Hence, there is a necessity to consider relevant operational constraints. One such operational constraint is the start up and shut down constraints of thermal units. In UC problem, provision is made to enforce minimum start up and shut down time of thermal units [47–49]. For example, a particular unit might have a constraint to be online for at least \( n \) number of hours once its committed to power generation. UC is performed in day ahead market and these constraints are enforced in real-time operation.

\[ P_{gen}^I(T_{off}) = 0 \]  

(3.2.13)

Equation (3.2.13) forces the optimization problem to include the UC schedule. It is worth mentioning that UC problem formulation can directly be included in this optimization process, however such an inclusion will be unrealistic because of the operational structure of power systems.

### 3.2.4 Cost Function

The objective of the optimization is to solve for cost associated with sizing the battery with respect to power and energy while satisfying operational constraints. The cost function for BESS sizing problem is linear and is expressed as given in (3.2.14).

\[ \min \ C_P \cdot \overline{P_B} + C_E \cdot \overline{E_B} \]  

(3.2.14)
Table 3.1: Generator Limits

<table>
<thead>
<tr>
<th>Gen. No</th>
<th>$\Delta P^{I}_{\text{gen}}$ (PU)</th>
<th>$P^{I}<em>{\text{gen}}, Q^{I}</em>{\text{gen}}$ (PU)</th>
<th>$Q^{I}_{\text{gen}}$ (PU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.25</td>
<td>3, 0.1</td>
<td>3, -3</td>
</tr>
<tr>
<td>3</td>
<td>.25</td>
<td>2.7, 0.1</td>
<td>3, -3</td>
</tr>
</tbody>
</table>

The developed NLP problem is solved using IPOPT, an open source primal-dual interior point solver for large scale non-linear problems [50] using MA57 linear solver from HSL library [51].

3.3 Peak Load Management using BESS

Optimally sizing a battery for a particular application alone does not serve much purpose. It is important to provide means to utilize BESS, whatever be the size, and preferably in a way to comply with operational practices. Also, it is worth mentioning that any planning operation inherently has a certain amount of uncertainty. Hence, the optimally sized BESS unit should be able to operate in an optimal way even with varying degrees of changes that was not expected during the planning process. An example would be an unexpected drastic change to the load curve over time.

In this section, a similar problem formulation to the one developed in section 3.2 is presented. The scheduling problem is designed to operate BESS in an optimal way to serve the purpose of PLM even with unexpected changes to variables. The criteria for BESS scheduling is to achieve desired peak load reduction at minimal operating cost subject to satisfying operational constraints. The cost function can be expressed as,

$$
\min \sum_{i=1}^{nGen} (C_2 \cdot P^{I^2}_{\text{gen}}(t) + C_1 \cdot P^{I}_{\text{gen}}(t)) + C_0) \cdot \Delta t \forall t
$$

(3.3.1)

By switching BESS as a variable generation/load, the optimization reduces peak load demand to a user specified level at the lowest possible cost. Compared to the
sizing problem, the cost function is changed to (3.3.1). In addition, $E_B$ and $P_B$ are no longer variables but are fixed constants from BESS sizing optimization. All other constraints remain the same as in section 3.2.

3.4 BESS for Renewable Generation

Renewable generation inherently has uncertainty and variability. While uncertainty can be quantified using probability distributions [52] and appropriate measures can be taken to address it, variability remains an issue. Variations in power output of renewable generation has to be offset with conventional generation. Convention generation has physical constraints on ramp up/down rates. Hence, even moderate amounts of renewable power variation from one dispatch to another poses considerable operational challenges when penetration levels are high.

By installing a BESS and by coupling the output of renewable generation and BESS in a suitable way, the problem of variation can be minimized. The problem of BESS for renewable generation is two folds, one is to appropriately size BESS for renewable generation and the other is to use the optimally sized BESS to enforce relevant operational constraints while maximizing the profits. These two applications are treated separately in the following subsections. Although the sizing and scheduling applications are different, the formulation is similar. A schematic explaining the optimization process is given in fig. 3.3.

3.4.1 Optimal Storage Sizing for Renewable Generation

For this application, the renewable generation source and BESS can be considered independently from the rest of the system. It is assumed that during planning studies appropriately sized transmission lines and related equipment were designed to
accommodate a given size of renewable source. Hence, by limiting the power at point of common coupling (PCC) as given in (3.4.2) the renewable source along with the BESS can be considered independently from the rest of the system. Only the power delivered at the point of common coupling is of interest. The output at the PCC and its limit is given in (3.4.1) and (3.4.2) respectively.

\[
P_{pcc}(t) = P_{rg}(t) - P_{C}^{B}(t) + P_{D}^{B}(t) \tag{3.4.1}
\]

\[
P_{pcc}(t) \leq P_{pcc} \forall t \tag{3.4.2}
\]

\(P_{rg}\) is renewable generation output and is set to maximum power point tracking (MPPT) value of the solar farm for each time instance \(t\). Energy and power rating constraints and energy conservation equation are expressed as,
\[ P_B^D(t) - P_B^B \leq 0 \ \forall t \] (3.4.3)
\[ P_B^C(t) - P_B^B \leq 0 \ \forall t \] (3.4.4)
\[ E_B(t) - E_B^B \leq 0 \ \forall t \] (3.4.5)
\[ E_B(t) = E_B(t-1) + \eta \cdot P_B^C(t) \cdot \Delta t - \frac{P_B^D(t)}{\eta} \cdot \Delta t \ \forall t \] (3.4.6)

As discussed previously, decision variables needs to be introduced to enforce either charging or discharging at a given time and not both. This is accomplished by introducing binary decision variables \( \beta_C \) and \( \beta_D \), a dummy constant \( K \) with a large value, and three constraint equations.

\[ P_B^C(t) \leq \beta_C(t) \cdot K \ \forall t \] (3.4.7)
\[ P_B^D(t) \leq \beta_D(t) \cdot K \ \forall t \] (3.4.8)
\[ \beta_C(t) + \beta_D(t) = 1 \ \forall t \] (3.4.9)

Equations (3.4.7) through (3.4.9) enforces the constraint that both charging and discharging cannot happen at the same dispatch. An additional constraint to utilize maximum energy production from renewable energy generation can be enforced by setting the energy of BESS at the last dispatch to be equal to initial energy stored in the battery at the beginning of the optimization period.

\[ E_B(t_{end}) = E_{init} \] (3.4.10)

The primary objective is to minimize the maximum output deviation i.e. \( \Delta P_{pcc} \) while doing so with minimal sizing of battery with respect to power and energy rating.
\[-\Delta P_{pcc} \leq \Delta P_{pcc}(t) \leq \Delta P_{pcc} \forall t \]  

(3.4.11)

The maximum permissible deviation value \(\Delta P_{pcc}\) is provided by the user, based on operational standards and/or desired threshold. If convergence is not achieved then the value of \(\Delta P_{pcc}\) is increased incrementally in an iterative way until convergence is achieved. The cost function to be minimized is given in (3.4.12).

\[
\min C_P \cdot \overline{P_B} + C_E \cdot \overline{E_B} \tag{3.4.12}
\]

3.4.2 Optimal Scheduling of BESS for Renewable Generation

With minimal modification of formulation developed in section 3.4.1 it is possible to cast the optimization problem as a generation profile shaping problem i.e. time-shifting energy production to maximize profit while ensuring all the constraints are met. The problem has the same constraints as developed in section 3.4.1, however, \(\overline{P_B}\) and \(\overline{E_B}\) are no longer variables but constants. The objective function is set to,

\[
\max C^T \cdot P_{pcc} \cdot \Delta t \tag{3.4.13}
\]

Where \(C^T\) is the transpose of vector of size \(T\) by 1 containing the selling price for each dispatch period and \(P_{pcc}\) is a vector of same size containing the values of power that is injected to the grid at point of common coupling. The formulation falls under the category of MILP type as before. Equation (3.4.13) ensures that total energy produced throughout the day is dispatched in such a way that the profit is maximized while enforcing the change in power output level to be lesser than or equal to a given \(\Delta P_{pcc}\) for a given battery sizing. If convergence is not achieved then the value of \(\Delta P_{pcc}\) is solved iteratively by increasing the value incrementally until convergence is
3.5 Results and Discussion

3.5.1 Peak Load Management

3.5.1.1 Sizing

The results of BESS sizing application is given in fig. 3.4 (a) and (b). In the 9-bus test system, the swing bus is considered to represent the rest of the system and the imported power is kept at pre-defined scheduled values and the rest of the dispatchable generators are scheduled along with BESS to accomplish a specific task of reducing peak load demand while minimizing the required power and energy rating of BESS.

In order to appropriately size BESS, a heavily loaded day is chosen and the optimization as explained in section 3.2 is solved. A modified load demand curves from [53] is used. In fig. 3.4 (a) the maximum allowable peak load is set to 2.85 PU.
Figure 3.5: Line flow limit and generator power output
Table 3.1 lists the generator limits used. It is assumed that both the generators two and three are available at all times. However, provision is given in the formulation to enforce on/off schedule from UC. A battery round trip efficiency of 64% i.e. 80% for both charging and discharging is used.

A total of 96 dispatches i.e. a dispatch every 15 minutes for an optimization period of 24 hours is used. BESS acts as a load during off-peak period and as a generator during peak period. Fig. 3.4 (b) shows BESS charge and discharge cycle.

3.5.1.2 Scheduling

The scheduling problem formulation is extremely flexible, for example, the cost of operating BESS can be easily incorporated by adding (3.5.1) into the objective function defined in (3.3.1).

$$\min \ C_O \cdot P^C_B(t) + C_O \cdot P^D_B(t) \ \forall t \quad (3.5.1)$$

Line Flow Violation

Figures 3.5 (a)-(d) represent results for scheduling operation. In particular fig. 3.5 (a)-(b) present an interesting case. Generation curve of generators two and three is quite different than the expected profile such as in fig. 3.5 (d). The location of BESS is at bus 6 as shown in fig 3.1. Line flow between buses 6 and 7 show that line flow limit was reached but not exceeded. This is a limiting factor, as more power cannot be pushed through the lines without violating the limits as shown in fig. 3.5 (a).

During the optimization problem setup, it was assumed that the base case line flow results were at 50% of system operation limit (SOL). Hence, the flow limit was set at twice the base case line flow value. However, in the base case there was no BESS and hence the power flow between buses 6 and 7 was approximately 0.24 PU. The flow limit on the particular line i.e. line connecting bus 6 and 7 was later assumed
to be at 40% of SOL during base case and the flow limit was appropriately raised as shown in fig. 3.5 (c). With the flow limit raised, generation curve as shown in fig. 3.5 (d) that results in overall lower cost of operation is obtained.

Depending on the system that is under study, any of the security/operational constraints could be potentially violated. In this case, a simplistic model, without security/operational constraints would have resulted in over-loaded lines during operation. Also, the overload happens at peak or near peak loaded condition - the worst possible time for a line trip. The necessity of a detailed model and its potential benefit is realized in scenarios such as these where potential violations are averted. This allows for continued and secure operation of the system.

### 3.5.2 BESS for Renewable Generation

#### 3.5.2.1 Sizing

The primary objective of BESS sizing for renewable generation is to mitigate output variability. Curtailing renewable energy production is an option to reduce variability, however, this results in either over curtailment and variability reduction only in one direction. For example, an increase in output power with respect to the previous dispatch can be limited, however it is not possible to compensate for a sudden drop in output. Depending on the day ahead forecast, output of renewable generation can be curtailed to mitigate variability to an extent, however this results in higher levels of curtailment and variability can also be reduced only to an extent, with a direct correlation between amount of power curtailed and reduction in variability.

The objective of BESS for renewable generation is to enable operation of renewable generation at maximum production capability. Figures 3.6 (a)-(c) are results for IM of renewable generation sizing application. Figure 3.6 (a) shows the results of generation
Figure 3.6: Effect of BESS on power at PCC
Figure 3.7: Regulation for different sizes of BESS
curve with and without BESS. The impact of BESS on generation curve is hardly tangible while looking at fig. 3.6 (a). However, fig. 3.6 (b) clearly shows the impact of BESS on IM.

The formulation developed in section 3.4.1 is primarily intended to reduce variability between successive dispatches. In the original system i.e. the case without BESS, the generation curve shows a variation of up to 0.44 PU, however with BESS, the variation is limited to 0.05 PU. This is a significant improvement and has an important operational benefit. For example, the reduction in variation helps in scheduling conventional generation in a more cost-effective way. The need for operational reserves will also be reduced and in general the system security will not be compromised even with high levels of renewable penetration.

Figure 3.6 (c) shows the charge/discharge cycle of BESS. The maximum power rating required is 0.3022 PU. While the energy rating is 0.1284 PU. Solar generation profile data for the optimization is obtained from [54]. The data set is for 365 days. Data is normalized based on the highest power output over the period of a year. The highest variable day, defined as maximum absolute cumulative deviation between dispatches is used in sizing application. In general, the results imply that if the solar farm was the size of 1 MW then a BESS with power rating of about 302 kW and energy rating of about 128.4 kWh is sufficient to mitigate variability to a level of 0.05 MW.

3.5.2.2 Scheduling

Any planning operation involves uncertainty. The solar power generation profile can change over time or the utility that owns the solar plant might choose to size BESS differently. In general, optimality as used in the context of this work for sizing applications is optimal only with respect to the data that was used in the optimization
process. However, irrespective of the uncertainties and whatever be the sizing of BESS, means to optimally operate BESS within the operational constraints while maximizing profit is provided. Figures 3.7 (a)-(c) illustrate this.

Figure 3.7 (a) shows how BESS in this case is used for energy time-shifting. In the optimization process, the pricing curve for period between 2 pm to 8 pm was assumed to be 2.5 times more profitable. In order to make the results more tangible, a low energy production day is used. Several size of BESS is compared - the same size as obtained from sizing problem and two scaled version of it, scaled twice and four times. Different sizing schemes result in different amount of energy time-shift and hence different amount of profits. However, as can be seen from fig. 3.7 (b), the amount of variation between dispatches were still maintained at 0.05 PU. If a BESS with smaller sizing is used and resulted in non-convergence, then the constraint on $\Delta P_{pcc}$ would be relaxed to a higher value and the optimization would be run again until convergence is achieved.

3.5.3 Computational Requirements

The sizing operation for both PLM and IM is a offline process, however, scheduling operation can be performed either on a daily basis or as a rolling window i.e. at every dispatch for the next $n$ dispatches. For PLM only a subsystem that is of interest is modeled in detail while the rest of the system is represented as injection sources. The lines connecting boundary buses are treated as tie-lines with predefined import/export schedule from UC. This allows PLM to be solved in real-time i.e. time between two dispatches/discretization interval (5 minute intervals). The results presented in table 3.2 gives the minimum, maximum and mean time for solver to obtain optimal solution for a data set consisting of 10 optimization runs on an Intel i7 4510U processor with 8 GB of RAM.
Table 3.2: Computational Performance

<table>
<thead>
<tr>
<th>Optimization</th>
<th>Sizing</th>
<th>Scheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min, max, mean (s)</td>
<td>min, max, mean (s)</td>
</tr>
<tr>
<td>IM</td>
<td>0.21, 0.27, 0.22</td>
<td>0.031, 0.054, 0.034</td>
</tr>
<tr>
<td>PLM</td>
<td>32.04, 43.09, 35.75</td>
<td>27.66, 33.42, 30.15</td>
</tr>
</tbody>
</table>

3.6 Conclusion

Sizing and scheduling of BESS for peak load management and intermittency mitigation of renewable generation are formulated as an optimization problem. In the case of peak load management, the study is concerned with a section of the grid. A non-linear optimization problem is formulated which takes into account real and reactive power requirements of the system, generator power limits, line flow limits, voltage limits, generator schedules from unit commitment and also a predefined import/export schedule from the rest of the grid. Realistic demand curves with high loading is used to find optimal sizing of BESS for peak load management. The optimally sized BESS is then used to reduce peak load of the section under study while adhering to relevant security constraints at the lowest possible operating cost.

Intermittency mitigation of renewable generation is modeled as a mixed integer linear programming problem. The optimal size, that reduces the maximum variation between successive dispatches of a solar farm to a pre-defined operator specified level or the lowest feasible value is obtained. In order to maximize the profit and at the same time mitigate intermittency, a scheduling problem is formulated. Depending on time of the day price, for a given size of BESS and predefined tolerance for change between successive dispatches, the optimization routine time-shifts the generation curve to maximize profit. Optimality of solution was verified through solver status. Global optimum were obtained for sizing and scheduling problems for both applications. The developed formulation is flexible and can be used to include new constraints and cost
functions, for example, different types of energy storage can easily be incorporated by including constraints that impose physical restrictions of different energy storage types.
Part II

Demand Response
Chapter 4

A Tight MILP Formulation for Utility Scale Optimal Demand Side Response

Nomenclature

Binary Variables

$\alpha_i^d$ Variable to enforce dependent load constraint

$\beta_i^p$ Variable to enforce constant power requirement on constant power loads

Continuous Variables

$E_{i,a}$, $P_{i,a}$ Energy and power consumed by $i^{th}$ adjustable load

$E_{i,d}$ Energy consumed by $i^{th}$ dependent load

$E_{i,p}$ Energy consumed by $i^{th}$ constant power load

$E_{i,ev}$ Energy stored in $i^{th}$ EV

$P_{i,ev}^C$, $P_{i,ev}^D$ Rate of charge and discharge of $i^{th}$ EV

$\overline{P}$ Peak load

Parameters
A detailed mathematical model for optimal utilization of demand side response is formulated. A tight formulation for constrained controllable load (dependent load) is modeled, whose operation is dependent on certain other loads completing their schedule prior to the constrained controllable load’s start time. In addition, several other types of load models, including electric vehicles (EV), adjustable and constant power loads.
The loads are modeled. The ensuing problem is that of mixed integer linear programming (MILP) type. The developed MILP formulation is tight and hence optimal solution to utility scale demand response (DR) scheme is obtained in real-time (duration between dispatches). In particular, the work is aimed at reliability based DR schemes used by independent system operators (ISOs) and regional transmission organizations (RTOs). Applicability to utility scale DR schemes is shown by formulation and solution of large problems with more than one million variables - solved in real-time.

4.1 Introduction

With the ever increasing need for energy, power systems are being operated close to their operating limits. Hence, it has become imperative to optimize management of both supply and demand side. Demand response (DR) programs were introduced in 1990s as pilot programs to ascertain feasibility of demand side management. Over the years, DR programs have been largely successful. One study conducted by electric power research institute (EPRI) suggests that DR schemes has the potential to reduce current US peak demand by 45,000 MW [55].

DR programs can be broadly categorized into three classes - reliability based, economic based and ancillary service based. Multiple programs exist withing these classes. For instance New York independent system operator (NYISO) has five programs while Pennsylvania - New Jersey - Maryland (PJM) regional transmission organization (RTO) has seven DR programs under these three classes. It is beyond the scope of this work to discuss about DR based on energy policy, however, it is important to note that any proposed DR scheme must fit into the existing energy policy framework to be of practical use. Interested readers are referred to [55] for detailed explanation of co-evolution of DR and energy policy over the years.
The purpose of this work is to develop better mathematical models for different types of load devices that can then be solved to optimality in real-time - even when the number of variables is in millions. Here, real-time refers to duration between successive dispatches which are in general, five or fifteen minutes. The developed model is aimed for use in reliability based DR programs with an emphasis on peak load reduction. To better understand the contributions of this chapter, a detailed literature review of existing work is presented, followed by contributions of this work and how this work adds to the body of existing literature.

4.1.1 Literature Review

Reference [1] developed a binary particle swarm based algorithm that is capable of providing near optimal solution within a manageable time frame for interruptible loads. Only one type of load is modeled, where the load has minimum on time and maximum off time. In addition, the size of the problem tested was relatively small with only 19 loads for a period of 16 hours. Reference [56] also developed a similar approach with fuzzy dynamic programming. In both approaches the aim is to minimize operating cost.

Reference [57] developed a hierarchical demand response scheme for peak minimization. The developed problem is solved as a linear programming problem. Only one type of load modeling - continuous loads are used. The authors in [58] developed a real-time demand response framework based on Stackelberg game approach. Both continuous and discrete loads are modeled. However, certain load types such as dependent loads are not modeled.

In [59], the authors present a detailed MILP based formulation for a home energy management system with various load types. Emphasis is on reducing home operational cost. Similarly, [60] presents a smart household operation scheme considering
electric vehicles and energy storage. Although, both [59] and [60] are aimed at home energy management they provide interesting insights at different modeling options.

Reference [61] proposes a game theoretic approach to demand side management. However, different load characteristics are not represented in the model. Similar approaches based on game theory is proposed in [62, 63], however, they do not have comprehensive model to include different load characteristics of residential and industrial loads.

With the continued interest in internet of things (IOT), an increased number of smaller loads with different constraints will be available to demand response schemes. They are especially attractive because of their connectivity. Hence, it is imperative that optimization models for these loads are considered in addition to representing constraints of existing loads in an optimization framework. The aim of this work is to add to the excellent body of literature that already exists with improved load modeling for optimization schemes that better represents the loads of present as well as future smart grid. It is also important that any developed method be adept at handling utility scale problems.

4.1.2 Contributions

• Solution to a particular class of DR scheme - reliability based DR programs with detailed load modeling including development of dependent load model.

• The emphasis is on finding solution to utility scale problems to be solved to optimality in real-time. Hence, a tight MILP formulation is developed and its feasibility is experimentally verified by solving problems involving more than million variables to optimality in real-time in order to optimize peak load reduction.
4.2 Load Classification

Some interruptible loads can be scheduled on or off in discrete intervals, while other types of loads can be operated with a varying power demand such that total energy consumed over a given start and end time is satisfied. In addition, with the boom of internet of things (IOT) and increased connectivity of home appliances it is important to have a wide range of modeling capability for devices that could potentially participate in DR schemes in the future - such as dependent loads. A smart home could have a series of tasks that need to be performed in a given order where the start time of a particular load is dependent on some other load completing its operation. The dependent loads in this smart home have an additional constraint of completing the series of tasks within a given end time. Load models in this work are developed based on these varying constraints and are classified as follows.

4.2.1 Adjustable Loads

Adjustable loads (ALs) are loads whose power demand can be varied between a minimum and maximum limit during its period of operation. Period of operation of adjustable loads is pre-defined with a given start and end time. Within its operational period, the adjustable loads consume a pre-defined amount of energy. Hence, adjustable loads give the operator the flexibility to schedule them in such a way that maximizes benefit for the operator and at the same time satisfies the energy requirement of adjustable loads.

4.2.2 Constant Power Loads

In this work, constant power loads (CPLs) are defined as a special set of adjustable loads with an additional constraint of having a constant power requirement during
dispatches when they are operated. Like adjustable loads, constant power loads have a pre-defined start and end time in addition to a pre-defined energy consumption over its period of operation.

4.2.3 Dependent Loads

In addition to loads in a future smart home, certain type of currently existing loads are inherently dependent on other loads. For instance, consider a programmable dryer. The dryer cannot commence its operation before the washer has completed its washing cycle. Hence, the dryer is dependent on the washer. Such loads are given an end time i.e. the time before which the operation needs to be completed. However, the start time is not specified as it is dependent on some other load completing its cycle. Dependent loads (DLs) can be adjustable or constant power loads with an added constraint of dependence on other load. In this work, the formulations developed are for dependent loads that belong to constant power loads category. It is worth mentioning that the developed formulation can easily be extended for other load types as well.

4.2.4 Electric Vehicles

Electric vehicles are modeled with both vehicle to grid (V2G) and grid to vehicle (G2V) capabilities. The number of available EVs and their initial state of charge (SOC) is modeled in a probabilistic manner. Converter and battery efficiency is modeled as a single unified efficiency value, however, a different efficiency value for charge, $\eta_{in}$ and discharge, $\eta_{out}$ is used. It is important to note that the model developed for EV operation can be modeled exclusive with continuous variable. This drastically reduces the solution time compared to a similar formulation involving binary variables.

Using the above four categories of loads, a wide variety of residential and indus-
trial loads can be modeled and included in DR scheme for any of the three classes: reliability based, economic based and ancillary service based DR programs.

4.3 Problem Formulation

In this section, mathematical model that enforces constraints specific to each load type discussed in section 4.2 is developed. In addition to the constraints, the cost function that enforces the primary objective of the work i.e. reduction in peak load demand is also developed. The ensuing problem formulation is of mixed integer linear programming (MILP) type. The developed formulation is tight i.e. the distance between relaxed and integer feasible solution is small. Tightness of problem formulation defines the search space that the solver needs to explore [48]. Hence, a tighter formulation will result in faster solution times. The developed problem is solved using CPLEX.

4.3.1 Adjustable loads

Adjustable loads consume a specified amount of energy during a given period within a specified power consumption range. A given $i^{th}$ adjustable load for a given operational period $T_i^a$ is modeled using (4.3.1)-(4.3.4). Equation (4.3.1) is the energy consumption equation. Bounds on adjustable power and energy is given by (4.3.2) and (4.3.3). Energy requirement is enforced by setting the bound at end of operation period as given in (4.3.4).

\[
E_i^a(t) = E_i^a(t-1) + P_i^a(t) \cdot \Delta t \quad \forall \, t \in T_i^a
\]  
\[
0 \leq P_i^a(T_i^a) \leq \overline{P_i^a}
\]
\[ 0 \leq E_{i}^{a}(T_{i}^{a}) \leq E_{i,req}^{a} \quad (4.3.3) \]

\[ E_{i}^{a}(t_{i}^{\text{end},a}) = E_{i,req}^{a} \quad (4.3.4) \]

### 4.3.2 Constant Power Loads

Constant power loads in the context of this work are loads that consume power at a pre-defined rate and are interruptible with the constraint of consuming a pre-defined level of energy over its operational period defined by \( T_{i}^{p} \). Where,

\[ t_{i}^{\text{start}}, t_{i}^{\text{start}+1}, ..., t_{i}^{\text{end}} \in T_{i}^{p} \quad (4.3.5) \]

The constant power consumption and energy balance constraint is enforced as in (4.3.6). Bounds on energy are given in (4.3.7). Energy requirement is enforced through (4.3.8).

\[ E_{i}^{p}(t) = E_{i}^{p}(t - 1) + \beta_{i}^{p}(t) \cdot P_{i}^{p} \cdot \Delta t \quad \forall t \in T_{i}^{p} \quad (4.3.6) \]

\[ 0 \leq E_{i}^{p}(T_{i}^{p}) \leq E_{i,req}^{p} \quad (4.3.7) \]

\[ E_{i}^{p}(t_{i}^{\text{end},p}) = E_{i,req}^{p} \quad (4.3.8) \]

### 4.3.3 Dependent Loads

Dependent loads in this work are modeled as constant power loads with an addition constraint of waiting for another constant power load to complete its cycle before commencing its operation. Since the start time is not known a priori, start time for all dependent loads are set to first dispatch. However, dependent loads will only begin their operation after the corresponding constant power load they are dependent upon
has completed its cycle.

\[
(\alpha_i^d(t) \cdot E_{j,req}^p) + (E_j^p(t) - E_{j,req}^p) \geq 0 \quad (4.3.9)
\]

\[
E_i^d(t) = E_i^d(t - 1) + (1 - \alpha_i^d(t)) \cdot P_i^d \cdot \Delta t \quad (4.3.10)
\]

\[
0 \leq E_i^d(T_i^{end,d}) \leq E_i^{d,req} \quad (4.3.11)
\]

\[
E_i^d(t_i^{end,d}) = E_i^{d,req} \quad (4.3.12)
\]

A given \(i^{th}\) dependent load is dependent upon a \(j^{th}\) constant power load. For all periods before the constant power load has completed its cycle, \(\alpha_i^d(t) = 1\) in order to satisfy the constraint (4.3.9). For all periods after the constant power load has completed its cycle \(\alpha_i^d(t)\) can take the value of 0 or 1. For all periods when \(\alpha_i^d(t) = 0\), \(E_i^d\) starts to increase as in (4.3.10). However, \(E_i^d\) has to satisfy (4.3.11) and (4.3.12). Hence, \(\alpha_i^d(t) = 0\) only for the periods the dependent load is active, only after the dependency has been satisfied and only long enough to satisfy the energy requirement. In addition, the entire process will be completed on or before the pre-defined end time.

### 4.3.4 Electric Vehicles

EVs are allowed to be operated in both V2G and G2V mode. The energy balance constraint is given in (4.3.13) subject to the limits in (4.3.14)-(4.3.16).

\[
E_{i,\text{ev}}(t) = E_{i,\text{ev}}(t - 1) + P_i^{\text{C,\text{ev}}}(t) \cdot \eta_{\text{in}} \cdot \Delta t - \frac{P_i^{\text{D,\text{ev}}}(t)}{\eta_{\text{out}}} \cdot \Delta t \quad (4.3.13)
\]

\[
0 \leq P_i^{\text{C,\text{ev}}}(t) \leq P_{i,\text{ev}} \quad (4.3.14)
\]

\[
0 \leq P_i^{\text{D,\text{ev}}}(t) \leq P_{i,\text{ev}} \quad (4.3.15)
\]
Equation (4.3.13) has two continuous variables $P_{i,ev}^C$ and $P_{i,ev}^D$, both of which can take a non zero value in the same dispatch. In reality, in a discretized interval, a given EV can either charge or discharge but not both. This constraint can be enforced using a binary decision variable. However, the use of binary variables would increase computational effort. The need for binary variables is avoided in this work because this constraint is implicitly expressed in the objective function in (4.3.17) and (4.3.18). Scheduling both $P_{i,ev}^C$ or $P_{i,ev}^D$ in a given dispatch will result in lesser energy being utilized due to efficiency losses which would implicitly increase the objective function value.

4.3.5 Objective Function

The objective of this work is to minimize the peak load demand over the course of a given day. To this end, variable $\overline{P}$, which represents peak load, is defined as in (4.3.17). $\overline{P}$ is the maximum value of sum of base load demand and different type of DR loads. The objective is to minimize $\overline{P}$ as in (4.3.18).

\[
\overline{P} \geq P_{BD}(t) + \sum_{i \in A} P_a^i(t) + \sum_{l \in P} \beta_l^p(t) \cdot P_l^p + \sum_{k \in D} (1 - \alpha_d^i(t)) \cdot P_l^d \\
+ \sum_{\forall t \in T} (P_{i,ev}^C(t) - P_{i,ev}^D(t)) \forall t \in T \tag{4.3.17}
\]

\[
\min \overline{P} \tag{4.3.18}
\]

It is important to note that the only contributor to uncertainty in this optimization
model is load forecast. Since the day ahead load forecast errors are relatively small, a
single shot solution to the optimization problem is computed in this work. However,
because of the fast solution time - in the order of tens of seconds even for problems
with more than million variables, a moving window type optimization is possible. In
such a scheme, the forecast will be updated at constant intervals and loads that have
already started their operation will be optimized accordingly and a new schedule will
be computed for loads that have not yet started their operation. Such a scheme can
be realized by extending the model developed in this work.

4.4 Experimental Setup

A set of six test cases were designed to test the proposed method. Each test case has
adjustable loads, constant power loads, dependent loads and EVs (operated both in
V2G and G2V mode) in varying numbers as shown in table 4.1. Base load curve is
obtained from [53] and rescaled. Base load curve is defined as the load consumption
of all loads that do not participate in DR program. DR loads are additional loads
that needs to be scheduled. Hence, total load seen by the utility is the sum of base
load and DR loads. The task is to schedule DR loads in such a way that the peak
load requirement as seen from the utility side is minimized even with additional
energy requirement of DR loads. All test cases were run with a dispatch interval of
15 minutes, over a period of 24 hours representing a total of 96 dispatches - solved
together.

Cases 1 through 4 represent large test cases while cases 5 and 6 represent very
large cases with case 6 requiring more than million variables to represent the problem.
In all six test cases the number of dependent loads were fixed at 5% of the number of
constant power loads. This is because the dependent load type is a rarity. Computa-
tional times are computed using geometric mean, \( \text{geometric mean} = \sqrt[n]{t_1 \cdot t_2 \cdot ... \cdot t_n} \).
Where \( n \) is the number of trials. In this work, for all cases \( n = 10 \) i.e. ten trials were run each time with the same problem but with different random number seed for CPLEX solver - a standard way to compute mean solution times.

### 4.5 Results and Discussion

#### 4.5.1 Large Test Cases

Cases 1 through 4 represent large test cases with 113,092, 228,766, 340,316 and 452,296 variables respectively. In all the four large test cases, solution was obtained in less than 10 seconds as shown in table 4.1. Additional energy requirement because of loads that participate in DR is 11.61 MWh for case 1 and 41.17 MWh for case 4. However, interestingly, the peak load value of case 4 is 246.28 MW compared to 248.30 MW for case 1. Intuitively, one would expect the peak load of case 1 to be lesser than that of case 4 because of the additional 29.56 MWh energy requirement due to DR loads. However, operating the EVs in an optimal manner by switching between V2G and G2V mode allows to bring down the overall peak load while accommodating the addition DR loads.

Fig. 4.1 illustrates the discussed behavior for test case 3. Fig. 4.1a Shows the load curve before and after the addition of DR. An additional 33.71 MWh of energy is required by loads participating in DR scheme. The proposed DR scheme uses the EVs to bring peak load to 246.80 MW compared to 250 MW peak load of base load case by operating EVs in V2G and G2V mode appropriately as shown in fig. 4.1b and by optimally scheduling the addition loads whose operational period is shown in fig. 4.1c.
Figure 4.1: Peak load reduction - comparison results
Table 4.1: Summary of Results

<table>
<thead>
<tr>
<th>Case</th>
<th>No of EVs</th>
<th>No of ALs</th>
<th>No of CPLs</th>
<th>No of Variables</th>
<th>Geometric mean Time (s)</th>
<th>Solver Status</th>
<th>Additional $E_{req}$ (MWh)</th>
<th>Base Peak Load (MW)</th>
<th>DR Peak Load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>13</td>
<td>113,092</td>
<td>0.84</td>
<td>Optimal</td>
<td>11.61</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>25</td>
<td>228,766</td>
<td>1.95</td>
<td>Optimal</td>
<td>21.92</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>750</td>
<td>750</td>
<td>750</td>
<td>38</td>
<td>340,316</td>
<td>3.68</td>
<td>Optimal</td>
<td>33.71</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>50</td>
<td>452,296</td>
<td>5.93</td>
<td>Optimal</td>
<td>41.17</td>
<td>250</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>100</td>
<td>913,224</td>
<td>13.33</td>
<td>Optimal</td>
<td>84.55</td>
<td>400</td>
</tr>
<tr>
<td>6</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>125</td>
<td>1,135,928</td>
<td>19.14</td>
<td>Optimal</td>
<td>107.02</td>
<td>400</td>
</tr>
</tbody>
</table>

4.5.2 Very Large Test Cases

Cases 5 and 6 with 913,224 and 1,135,928 variables represent the very large test cases. In case 6, a total of 7625 DR loads belonging to the four load categories described in section 4.2 is modeled. Each load has its own set of constraints and its own start and end times. These very large cases are assumed to happen in a larger system and hence the base load is scaled accordingly to reflect a large system. The base peak load is adjusted to 400 MW. For both cases 5 and 6, DR scheme reduces peak load as given in table 4.1, while supporting the additional energy requirement of DR loads. Optimal results are obtained with a Geometric mean time of 13.33 and 19.14 seconds for cases 5 and 6.

4.5.3 Computational Performance

In general, MILP problems are $np$-hard. Hence, the worst case solution time could be non-polynomial time. However, in most cases, empirical results are much faster than theoretical worst bound. Empirical results are dependent on problem type, i.e. number of different types of loads, duration of operation of loads, etc.. Hence, the test cases were designed in such a way to have a wide range for all the variables. In addition, all problems were run multiple times with different random number seed to remove solver performance bias based on random number seed. All computation
times are obtained using a *intel i7 – 4550u* processor using 8 GB of *RAM*. As with any MILP problem, it is possible that there exists a problem which represents the theoretical worst case bound. In such a situation, the solution times will drastically differ from other problems. However, in practice, occurrence of such problems are sporadic at best.

### 4.6 Conclusion

A DR scheme with tight MILP formulation for electric vehicles, adjustable, constant power and dependent loads is developed. A wide variety of residential and commercial loads with varying load characteristics can be included in DR scheme using the developed models. In addition, the load models formulated in this work can be combined in different ways to model loads with varying constraints. The developed DR scheme falls under a particular class of DR program - reliability based DR, with an emphasis on peak load management. An utility scale problem with more than 1,000,000 variables is developed and solved to optimality in real-time. The proposed method reduces the peak load of the system while catering to the additional energy requirements of loads that participate in DR scheme.
Part III

Power Grid Resilience
Chapter 5

Optimal Operation of Microgrids Under Conditions of Uncertainty

This chapter proposes a method to optimally operate microgrids under conditions of uncertainty introduced by renewable energy sources, under both grid connected and islanded mode. Uncertainty is quantified with probability distribution and confidence levels are used to establish likelihood of forecast error. The optimization problem is formulated as a quadratic programming problem and the optimality of the solution is shown mathematically by proving the convexity of the problem. The optimization is carried out with the combined objective of minimizing total operating cost and carbon emission. The proposed optimization method is then tested against a priority controller for an extended time-horizon of 24 hours. Furthermore, under islanded mode of operation, for extended time-horizon, a key decision making task of whether to provide energy to non-critical loads or to store excess energy is addressed.
5.1 Introduction

With increased awareness about global warming, the trend in power system operation is to reduce the green house emissions. Market based carbon emission trading mechanisms have been introduced in recent years [31]. European union (EU) has introduced a cap and trade carbon emission trading system since 2004 [32] and the same is soon to be established in Australia [33,34]. The concept of microgrids gives the flexibility to reduce the carbon emissions further by utilizing locally available distributed energy resources (DERs). However, the DERs which are of renewable type introduce uncertainty and variability because of the intermittent nature of renewable resource.

Power scheduling of distributed generation while addressing variation of DERs has been studied in [64] using a modified version of direct search method introduced in [65]. Smart management of energy storage systems, economic load dispatch and operation optimization of DERs are studied in [66] using genetic algorithms. In [67], authors study all possible scenarios off-line and store a look-up table to be used in real-time operation. Others have approached the problem by means of artificial neural networks [68], ant colony optimization [69] and other heuristic techniques.

In [70], the authors suggest that in literature, uncertainty in load and generation profile is handled by means of approaches such as model predictive control. At the system operator level, uncertainty and variability is addressed by means of reserves. In the case of microgrids, in islanded mode of operation, reserves are limited to storage systems such as battery energy storage systems (BESS). It is to be noted that BESS contributes to carbon emission and is quantified by means of cradle to gate battery production emission (CGE) [71].

This goal of this work is to investigate the possibility of addressing optimal op-
eration of microgrids, under conditions of uncertainty and variability by means of quadratic programming (QP) and linear programming (LP). Optimal operation in this case is to minimize fuel cost and emission. Uncertainty and variability is addressed by means of probability distribution and confidence levels, and short dispatch period, respectively. The optimization is carried out over extended-time horizon i.e. numerous dispatches are considered. 5 minute dispatch over a time frame of 24 hours is used.

Theoretically, the optimization is proved to be of convex minimization type, ensuring global optimum. Experimentally, the claim is verified by comparing against a priority based controller, in addition to CPLEX [29] solver’s global optimum status flag. The optimization includes polynomial (quadratic) cost function for fuel and emission of non-renewable generators, linear operation and emission cost (CGE) for BESS, physical ramp constraints for generators, and forecast error profiles for load, solar and wind generation to introduce uncertainty and variability.

The rest of the chapter is divided into four additional sections. In section II, probability distributions for load, wind and solar profiles are built based on seasonal data. The concept of complementary cumulative distribution function (CCDF) with respect to forecast error is introduced. From CCDF, confidence levels can be established and hence uncertainty can be quantified. In Section III, problem formulation is developed. In Section IV, results are presented and discussed. Conclusions are drawn in Section V.

5.2 Quantifying Uncertainty

Optimizing a problem where the variables have uncertainty requires quantifying uncertainty by means of probability distribution. The probability density function is
used to define the range of values a random number can take and the likelihood that a sample falls under a particular interval [72]. In [23], after studying the forecast error distributions of various independent system operators (ISOs), the authors suggest the use of hyperbolic distributions to better fit the error distributions for wind and load forecast error.

In this work, the error distribution for day ahead wind power forecast is obtained from electric reliability council of Texas (ERCOT) data as reported in [23]. The authors in [23] suggest the use of hyperbolic distribution with $\mu = -0.012$ and $\sigma = 0.119$. For solar power, the same error distribution used to characterize day ahead wind forecast error is used. Day ahead load forecast error distribution of New York ISO (NYISO) with $\mu = -0.024$ and $\sigma = 0.036$ from [23] is used to characterize load forecast error. Cumulative distribution function (CDF) and CCDF are defined as,

$$CDF(x) = P(X \leq x)$$

$$CCDF(x) = 1 - CDF(x)$$

CCDF allows defining confidence levels and hence to quantify uncertainty. For example, from Fig. (5.1), 95% confidence level corresponds to $-0.21$ $PU$ deviation for wind power forecast error i.e. on an average, 95% of the time forecast error will be $\leq -0.21$ $PU$. Typical daily wind and solar power generation from Western Wind and Solar Integration Study (WWSIS) [54] is used as daily generation profile. Different confidence levels for daily wind generation profile is shown in Fig. (5.2). Daily load curve is obtained from [53].
Figure 5.1: CCDF of wind power forecast error.

Figure 5.2: Wind power forecast error confidence level
5.3 Optimization

Cost functions associated with various generation types are developed in this section. This includes the usage cost of batteries. Then a quadratic programming problem for grid connected mode and linear programming problem for islanded mode is developed by formulating constraints to impose physical constraints such as ramp rates for generators. Global optimality is then theoretically proved by showing convexity of the problem.

5.4 Test System and Assumptions

The test system has programmable and non-programmable generators. Programmable generators are of conventional generation type i.e. non-renewables. The renewable generators include, wind, solar and BESS. The microgrid system has a peak load of
4000 kW, of which, 70% of the loads are assumed to be critical and the remaining
30% are assumed to be non-critical loads. There are three conventional generators, a
wind turbine (1200 kW), a solar farm (800 kW) and a battery energy storage system
($P_b = 500$ kW and $E_b = 1500$ kWh). Only real power balance is used as constraint,
neglecting losses. This can be justified by the fact that losses in microgrid form a
very small percentage of total power. For example, in [73], total loss was < .25%.

5.5 Cost Functions

5.5.1 Conventional Generator

The conventional way of representing generator cost curve is through either a piece-
wise linear approximation or a polynomial model. The polynomial cost function given
in IEEE 118 bus test system data in [74] is used as shown in Table 5.1. $P_{c,i}$, is the
amount of power generated by the $i^{th}$ non-renewable generator.

$$F_{c,i}(P_{c,i}(t)) = a_{c,i} \cdot P_{c,i}^2(t) \cdot \Delta t + b_{c,i} \cdot P_{c,i}(t) \cdot \Delta t + c_{c,i} \cdot \Delta t \forall i, t \quad (5.5.1)$$

The cost function for emission is adopted from [75]. The authors in [75], have
assumed the cost function of emission to be a quadratic cost function as given in
(5.5.2). They introduce a coefficient, $\alpha$, which makes emission proportional to fuel
cost, and $\beta$, which is the unit emission cost ($/\text{ton}$). Hence, emission cost is given as,

$$E_{c,i}(P_{c,i}(t)) = \beta_i \cdot \alpha_i \cdot \Delta t \cdot (a_{c,i} \cdot P_{c,i}^2(t) + b_{c,i} \cdot P_{c,i}(t) + c_{c,i}) \quad (5.5.2)$$
5.5.2 Battery Energy Storage System

Battery energy storage systems are the reserves for a microgrid. BESS has an associated life-cycle. There are two cost functions for BESS. One is the per charge/discharge cycle usage cost, the second is the CGE which quantifies the amount of $CO_2$ emission in kilogram per kilogram weight of battery. An average value of 3.2 for lead acid batteries from [71] is used. Assuming the battery has life-time charge/discharge cycle of $nCycles$ and a total cost of BESS as $C_b$, cost per cycle for a given percentage use of BESS is assigned as,

$$F_{b,i}(P_{b,i}(t)) = \frac{C_{b,i}}{nCycles_i} \cdot \frac{P_{b,i}(t) \cdot \Delta t}{E_{max,b,i}} \forall i,t \quad (5.5.3)$$

Where $E_{max,b,i}$ is the max capacity of $i^{th}$ BESS and $\frac{P_{b,i}(t) \cdot \Delta t}{E_{max,b,i}}$ is the percentage energy usage during a dispatch. The carbon emission cost can be described as,

$$E_{batt,i}(P_{b,i}(t)) = \beta_{b,i} \cdot \frac{3.2}{nCycles_i} \cdot nKg_i \cdot \frac{P_{b,i}(t) \cdot \Delta t}{E_{max,b,i}} \forall i,t \quad (5.5.4)$$

Where, $\beta_{b,i}$ is the unit emission cost ($$/ton) and $nKg_i$ is the total weight of $i^{th}$ BESS. It is important to note, that quantifying the cost associated with BESS with respect to emission and cost is a complicated subject because of the multitude of variables such as battery technology, standards etc. In this work, care has been taken to provide a general formulation to represent the operation and emission cost of batteries.

5.6 Objective Function and Constraints

The objective is to minimize the combined cost of generation and emission as given in (5.6.1).
\[ \min \sum_{i=1}^{n} F_{c,i}(P_{c,i}(t)) + E_{c,i}(P_{c,i}(t)) + \sum_{i=1}^{m} F_{b,i}(P_{b,i}(t)) + E_{\text{batt},i}(P_{b,i}(t)) \forall i, t \quad (5.6.1) \]

The generic form of the cost function given in (5.6.1) can be written as,

\[ \min \left( \frac{1}{2} \cdot (x^T \cdot C \cdot x) + q^T \cdot x \right) \quad (5.6.2) \]

Equation (5.6.3) represent generation power limits, while (5.6.4) represent ramp rate of generators. \( \rho_{\text{max}} \) and \( \rho_{\text{min}} \) are the maximum ramp up and down rate respectively. Equations (5.6.5)-(5.6.6) are storage capacity power and energy limits. Power balance equation differs with respect to mode of operation i.e. grid connected or islanded mode and hence will be developed later.

\[ P_{\text{min},i} \leq P_i(t) \leq P_{\text{max},i} \forall i, t \quad (5.6.3) \]

\[ \rho_{\text{min},i} \leq \Delta P_i(t) \leq \rho_{\text{max},i} \forall i, t \quad (5.6.4) \]

\[ -P_{\text{max},b,i} \leq P_{b,i}(t) \leq P_{\text{max},b,i} \forall i, t \quad (5.6.5) \]

\[ 0 \leq E_{b,i}(t) \leq E_{\text{max},b,i} \forall i, t \quad (5.6.6) \]

\[ E_{b,i}(t) = E_{b,i}(t-1) - P_{b,i}(t) \cdot \Delta t \forall i, t \quad (5.6.7) \]

\[ E_{b,i}(t_{\text{end}}) = E_{b,i}(t_0) \forall i \quad (5.6.8) \]

Equation (5.6.7) is the energy equation of battery. Equation (5.6.8) forces the battery storage value for the last time instance, \( t_{\text{end}} \), to be the initial value i.e. the value at the start of optimization period. This means, that all the renewable energy produced during the course of optimization horizon will be used by the end of it.
5.7 Global Optimality

A convex minimization problem or conversely a concave maximization problem has the property that any local optimum must also be the global optimum. Convexity provides a means to ascertain whether the solution obtained is optimal or not. The objective function for fuel and emission cost of programmable generator and BESS is of the form,

\[ F(P_c, P_b) = (1 + \alpha \cdot \beta) \cdot (a \cdot P_c^2 \cdot \Delta t + b \cdot P_c \cdot \Delta t + c \cdot \Delta t) + (k_1 \cdot P_b + k_2 \cdot P_b) \]  \hspace{1cm} (5.7.1)

Where \( P_c \) and \( P_b \) are amount of power from programmable generator and BESS respectively. \( \alpha \) and \( \beta \) are defined in Section 5.5. \( k_1 \) and \( k_2 \) are usage and emission cost coefficients of BESS. The Hessian matrix of (5.7.1) is given in (5.7.2).

\[
H = \begin{bmatrix}
(1 + \alpha \cdot \beta) \cdot (2 \cdot a \cdot \Delta t) & 0 \\
0 & 0 \\
\end{bmatrix} \hspace{1cm} (5.7.2)
\]

\[
x = \begin{bmatrix}
P_c \\
P_b \\
\end{bmatrix} \hspace{1cm} (5.7.3)
\]

\[
x^T \cdot H \cdot x = (1 + \alpha \cdot \beta) \cdot (2 \cdot a \cdot \Delta t) \cdot P_c^2 \hspace{1cm} (5.7.4)
\]

Equation (5.7.4) will be always positive semi-definite, a necessary condition for convexity, for non-negative \( \alpha, \beta, a \) and \( \Delta t \). In reality, this is always the case. For a generic case with multiple generators, BESS and dispatches, the Hessian matrix will be a 2 by 2 block matrix. In addition, objective function for islanded mode and all the constraint equations in both grid connected and islanded mode are linear and are known convex functions. Hence, the problem type formulated in this work is convex.
5.8 Results and Discussion

Two different modes of operation are discussed in this section. For grid connected mode, optimality is checked by means of CPLEX solver status. For islanded mode, the objective is to provide power to all critical loads at all times and shed minimal amount of non-critical loads. In order to compare the results from QP solver, a priority/rule-based/greedy algorithm is built and the results are compared. The conceptual framework along with experimental results tested under various scenarios discussed in this section is aimed at establishing the applicability of the proposed QP solver for optimal operation of microgrid.

5.9 Grid Connected Mode

In the grid connected mode, a typical generation mix from California ISO (CAISO) is used to assess the percentage of power imported from the grid that comes from non-renewable sources. Thus, a quadratic cost function is scaled with a coefficient, $\alpha_{grid}$, to compute emission cost. The optimization problem is to find a dispatch that minimizes the combined objective of minimizing fuel cost and emission while satisfying (5.9.1). Here, $P_{b,i}$ and $P_{r,i}$ represent the battery and renewable generation respectively. $P_{b,i}$ is positive while battery is in discharge mode and negative while battery is in charging mode. While $D_{c,i}$ and $D_{nc,i}$ represent critical and non-critical load demand respectively.

\[
\sum_{i=1}^{n} P_{c,i}(t) + \sum_{i=1}^{m} P_{b,i}(t) + \sum_{i=1}^{o} P_{r,i}(t) = \sum_{i=1}^{p} D_{c,i}(t) + \sum_{i=1}^{q} D_{nc,i}(t) \forall i, t \quad (5.9.1)
\]

The optimization problem in grid connected mode is to find the optimum mix
of import from the grid and set points for the programmable generation available within the microgrid, given a load, wind power and solar power profile with a given confidence level. The theory to prove optimality of the solution is described in Section 5.7. In addition, the CPLEX solver on termination of optimization provides flag to ascertain the status of the solver. In all tested cases, global optimum was obtained.

5.10 Islanded Mode

Optimization in islanded mode of operation is different from grid connected mode and the differences are illustrated in Fig. (5.3). If the programmable generation was sufficient to supply the entire load, then the optimization is essentially similar to grid connected mode optimization. To illustrate the capability of the proposed method, it is assumed that the total programmable generation is less than the total critical load.

This leads to an interesting decision making problem. Given an amount of generation, enough to supply all or part of non-critical load, in addition to supplying all of the critical load at a given dispatch. The decision then is to find whether to serve the non-critical load or to store the energy. Storing the energy might be beneficial for later dispatches where there is a scarcity of resources to supply the critical load. However, storing energy implies shedding some non-critical load that could otherwise have been served. The decision has to be made in such a way, that the total critical load shed should be minimized over the entire optimization period i.e. the time the microgrid remains in islanded mode.

The optimization results provided are for a time period of 24 hours, with a dispatch every 5 minutes totaling 288 dispatch scenarios. To facilitate this decision making process, the constraints are altered slightly, while still maintaining the convexity of
the problem.

\[
\sum_{i=1}^{n} P_{c,i}(t) + \sum_{i=1}^{m} P_{b,i}(t) + \sum_{i=1}^{o} P_{r,i}(t) \geq \sum_{i=1}^{p} D_{c,i}(t) \forall i, t \tag{5.10.1}
\]

\[
\sum_{i=1}^{n} P_{c,i}(t) + \sum_{i=1}^{m} P_{b,i}(t) + \sum_{i=1}^{o} P_{r,i}(t) \leq \sum_{i=1}^{p} D_{c,i}(t) + \sum_{i=1}^{q} D_{nc,i}(t) \forall i, t \tag{5.10.2}
\]

The equality constraints pertaining to power balance equation is rewritten into inequality constraints. Equations (5.10.1) and (5.10.2), makes sure that at least all the critical loads are served at all times and at most both the critical loads and non-critical loads are served at all times. The objective function is changed to,

\[
\max \sum_{i=1}^{n} P_{c,i}(t) + \sum_{i=1}^{m} P_{b,i}(t) + \sum_{i=1}^{o} P_{r,i}(t) - \sum_{i=1}^{p} D_{c,i}(t) \tag{5.10.3}
\]

The objective function tries to maximize the amount of power supplied to non-critical load by means of excess energy production while satisfying (5.10.1) and (5.10.2). In order to prove optimality, in an experimental way, a priority-based/rule-based/greedy algorithm is developed to be compared against.

The priority based controller has a simple sequence of decision making, based on rules, looking only at the current dispatch time \(t\) and hence can also be classified as a greedy algorithm. The rules, in the order of priority are as follows,

- Supply critical load
- If excess energy is available after supplying critical load, then supply non-critical load
- If excess energy is available after supplying all loads, store energy in batteries
### Table 5.2: Comparison of QP with Priority Controller

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode</th>
<th>Load Shed (kWh)</th>
<th>Priority Controller Load Shed (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Islanded</td>
<td>0</td>
<td>4.07</td>
</tr>
<tr>
<td>2</td>
<td>Islanded</td>
<td>0</td>
<td>13.93</td>
</tr>
<tr>
<td>3</td>
<td>Islanded</td>
<td>3.88</td>
<td>19.10</td>
</tr>
<tr>
<td>4</td>
<td>Islanded</td>
<td>15</td>
<td>25.80</td>
</tr>
</tbody>
</table>

#### 5.10.1 Comparison of Proposed Method with Priority Controller

Amount of critical load shed in kWh is used as a metric for comparison. All the numerical results presented here are meaningful only within the framework of problem. For example, different load profile, wind and solar generation profile and different confidence levels will result in varying outcomes. However, all of these outcomes can be classified into three different groups. The priority controller can perform as well as the proposed method i.e. QP, or can perform worse, or can perform better.

The last group i.e. when QP is outperformed by priority controller will be addressed in Section 5.10.2. This happens because of the hard constraint imposed by (5.10.1), which results in no feasible solutions under certain circumstances. This can be overcome by relaxing the constraint in (5.10.1). In Section 5.10.2, it is shown that QP still obtains global optimum by means of constraint relaxation. The results are presented in Table. 5.2.

#### 5.10.2 No Feasible Solution

As discussed in Section 5.10.1, the hard constraint in (5.10.1) can result in no feasible solution region for the QP solver. It simply means that no feasible solution exists for the problem with given constraints. Hence, (5.10.1) needs to be relaxed with a relaxation parameter $\epsilon$. The value of $\epsilon$, directly affects the quality of the solution. Hence, when no feasible solution is found, which can be identified by means of CPLEX
solver status, the optimization procedure is solved iteratively as,

$$\sum_{i=1}^{n} P_{c,i}(t) + \sum_{i=1}^{m} P_{b,i}(t) + \sum_{i=1}^{q} P_{r,i}(t) \geq \sum_{i=1}^{p} D_{c,i}(t) - (iter \cdot \epsilon) \forall i, t$$  \hspace{1cm} (5.10.4)

Iterative solution is feasible because the entire optimization process for a 24 hour period with dispatch every 5 minutes only takes about 0.5 seconds of computational time. Hence, the iterative solution can be obtained in a few seconds. In (5.10.4), the value of $\epsilon$, is the amount of critical load in kW that will be shed. Hence, the resolution of $\epsilon$ depends on the physical constraint of size of blocks of critical load that could be shed. It is also possible to rank critical loads, based on their relative importance and shed load based on the rank table. However, the rank based load shedding scheme is not performed in this work. Hence, by employing the approach presented in (5.10.4), based on arbitrary resolution of $\epsilon$ as used in this work or based on rank based load shedding scheme as suggested, the QP solver is guaranteed to find the global optimum. This fact is illustrated in Table. 5.2. In cases 1 and 2, QP outperformed priority controller. In cases 3 and 4, iterative approach as explained in this section is used and it can be seen that QP again outperformed priority controller. Fig. (5.4) illustrates the comparison result for case 4.

5.11 Conclusion

Optimal operation of a microgrid with conditions of uncertainty introduced by renewable generation and to a smaller extent by load forecast errors are optimized over extended time horizon using QP. Uncertainties are quantified using CCDF. Typical load and renewable energy profiles obtained from literature are then adjusted based on required confidence level and used in the optimization process. The optimization
Figure 5.4: QP and Priority controller load shed comparison

problem is formulated with a quadratic cost function for grid connected mode and linear cost function for islanded mode. Different set of constraints are formulated for grid connected and islanded mode of operation. The problem is then proved to be of convex minimization type, which ensures global optimality of solution.

In grid connected mode, QP is set to minimize total cost of operation and emission. In islanded mode, QP is set to make decisions by which all or most critical loads are served throughout the entire time horizon for which the microgrid is in islanded mode. In the QP optimization, means to quantify fuel and emission cost of conventional generation, cost of operation of battery storage and emission cost associated with CGE of batteries are provided. The solutions obtained by means of QP are then compared against a priority controller. QP is shown experimentally to provide the global optimal solution, in addition to the theoretical optimality proof.
Chapter 6

Energy Management System for Enhanced Resiliency of Microgrids During Islanded Operation

Nomenclature

Adjsutable Load Variables

$E_{i,a}$ Energy consumed of adjustable load at $i^{th}$ node

$P_{i,a}$ Power consumption of adjustable load at $i^{th}$ node

Battery Variables

$E_B$ Energy stored in BESS

$P_C^B, P_D^B$ Rate of charge and discharge of BESS

Generator Variables

$P_{i,g}, Q_{i,g}$ $i^{th}$ generator real and reactive power

$P_{i,var}$ Variable generation/load at $i^{th}$ node
\( \Delta P_{i,g} \)  
- ramp rate of \( i^{th} \) generator

**Network Variables**

\( \theta_i \)  
- Voltage angle of \( i^{th} \) node

\( V_i \)  
- Voltage magnitude of \( i^{th} \) node

\( P_{i,inj}, Q_{i,inj} \)  
- Real and reactive Power injection at \( i^{th} \) node

\( P_{i,c}, Q_{i,c} \)  
- Critical real power supplied at \( i^{th} \) node

\( P_{i,nc}, Q_{i,nc} \)  
- Non-critical real and reactive power supplied at \( i^{th} \) node

\( P_{ik} \)  
- Real power flow between nodes i and k

**Parameters**

\( \eta \)  
- Battery efficiency

\( \overline{\theta_i}, \underline{\theta_i} \)  
- Upper and lower limit for Voltage angle of \( i^{th} \) node

\( \overline{V_i}, \underline{V_i} \)  
- Upper and lower limit for Voltage magnitude of \( i^{th} \) node

\( \overline{P_{ik}} \)  
- Real Power flow limit between nodes i and k

\( \overline{P_{i,g}}, \overline{Q_{i,g}} \)  
- Upper and lower limit for \( i^{th} \) generator real power

\( \overline{Q_{i,g}}, \overline{Q_{i,g}} \)  
- Upper and lower limit for \( i^{th} \) generator reactive power

\( P_{i,d}, Q_{i,d} \)  
- Real and reactive power demand at \( i^{th} \) node

\( \Delta P_{i,g}, \Delta P_{i,g} \)  
- Maximum ramp up and minimum ramp down rate of \( i^{th} \) generator

\( \overline{P_{i,a}}, \overline{P_{i,a}} \)  
- Maximum and minimum power consumption of adjustable load at \( i^{th} \) node

\( E_{i,a} \)  
- Energy demand of adjustable load at \( i^{th} \) node

\( a, b \)  
- Linear regression parameters
End time of adjustable load at \( i^{th} \) node

Sets

\( A \) Set of all adjustable loads

\( B \) Set of all buses

\( G \) Set of all generator buses

\( T_{i,a} \) Set of dispatches where adjustable load at \( i^{th} \) node is operational

\( T \) Set of dispatches where microgrid operates in islanded mode

This chapter proposes a method to enhance resiliency of microgrids through survivability. Survivability in this context is to minimize load shed for the duration the microgrid is in islanded mode following a disturbance event. During islanded operation, microgrid loads are prioritized as critical and non-critical loads. The key decision is to ascertain whether to provide energy to non-critical loads after supplying the critical loads or to store excess energy for future dispatches. This task is formulated as a non-linear programming problem. The objective is to minimize the amount of critical load shed while maximizing the amount of non-critical load served for a projected restoration time while adhering to relevant operational and physical constraints. For this extended time-scale problem, uncertainty of renewable generation and load forecast is quantified with probability distribution and confidence levels are used to establish likelihood of forecast error. Distributed generation such as solar and wind farm along with battery energy storage system are modeled. Demand response is implemented through adjustable loads and a fleet of plug in hybrid electric vehicles that can be operated in both grid to vehicle and vehicle to grid mode. Test cases
are studied on a modified CIGRE microgrid benchmark test system and results are compared with a temporal decomposition scheme based energy management system.

6.1 Introduction

Resiliency represents the ability of power systems to withstand high-impact, low-probability events with least possible interruption of electric supply while enabling quick recovery and restoration to normal operating state [2]. According to electric power research institute (EPRI), three key aspects of enhancing grid resiliency are prevention, recovery and survivability [3]. While prevention and recovery require infrastructural and operational changes, survivability can be accommodated into existing framework. Hence, the focus of this work is on survivability of microgrids under islanded mode of operation following a high-impact disturbance event. Survivability in this context is to minimize the amount of load shed following a disturbance.

Microgrids have been an area of active research in power systems community [76–78]. Microgrids are well suited to the concept of survivability of power grid by having the means to operate in islanded mode during a large disturbance. However, depending on the restoration time and the amount of distributed generation available within the microgrid, serving all of the load might be infeasible. In addition, not all loads are equally important and can be prioritized as critical and non-critical loads.

This work is aimed at development of mathematical framework for resiliency based microgrid energy management system (EMS), specifically designed for islanded mode of operation. Relevant operational constraints such as generator operational limits, voltage constraints, reactive power requirement are imposed in the problem formulation. Storage units are modeled with a variable efficiency curve that depends on state of charge (SOC) of battery. The concept of demand response is modeled through
adjustable loads and PHEVs.

The ensuing problem is modeled as a non-linear programming (NLP) problem. The developed resiliency based EMS schedules dispatches in such a way that total amount of critical load unserved during the time the microgrid is in islanded mode of operation is minimal and at the same time tries to maximize the amount of non-critical load served.

6.1.1 Literature Review

A mathematical framework for microgrid resilient operation is developed in [2]. Both grid connected and islanded mode of operation are considered. The aim is to improve the resiliency of the system by lowering the possibility of load shedding. Normal operation problem is formulated as a mixed integer linear programming (MILP) problem while resilient operation is modeled as a linear programming problem. Load modeling used in [2] includes adjustable loads with a pre-defined start and end time. Uncertainty in load and renewable sources is considered. The paper also takes into account the forecast errors of renewable sources by using a robust optimization strategy.

Reference [79] discusses the impact of demand side bidding on microgrid operation taking into account variation in market prices, renewable energy generation and load. The paper presents two scenarios. Scenario one pertains to normal economic operation of microgrid. Second scenario is based on applying an adequacy constraint when a specific section of the microgrid has to be operated in islanded mode in the event of an upstream fault. Load shedding of both critical and non-critical loads are controlled using demand side bidding. Network constraints are not considered in this work.

The authors in [80] discuss reliability and vulnerability of microgrid operation in addition to economic considerations. A vulnerability index in terms of loss of load has been formulated to show the impact of potential outages on a system. The aim
is to study the impact of undesired outages on microgrid and vulnerability index is used as a measure to determine resiliency of the system. Reference [81] studies the impact of extreme weather on power grid resilience.

Coordination of energy storage systems to maintain frequency and voltage of microgrid during islanded mode of operation is studied in [82]. The authors demonstrate how microgrid can be resilient in the moments subsequent to islanding by maintaining its frequency and voltage within limits using storage devices. The focus of the work is limited to restoring stable operation in the moments immediately following an islanding event and not for the entire duration the microgrid remains in islanded mode.

Reference [83] proposes a method to operate microgrids with minimum cost. Additional constraints on power exchange with the grid, generator operating limits and load curtailment of critical and non-critical loads are also considered in this work. However, network and reactive power constraints are not considered.

A multi-agent based self-healing of microgrids is proposed in [84]. A priority hierarchical controller based on a set of predefined rules is used. This work relies on agent cooperation through negotiation and the resulting self-ordering scheme enables transition from emergency to stable operating state. Phase-angle droop control scheme for microgrid is designed in [85] in order to mitigate system vulnerability to storms and hurricanes.

As noted by [2], although the use of microgrid for resiliency is well known, the mathematical modeling of microgrids based on resiliency considerations is limited. The mathematical model developed in this work adds to the existing body of work on resiliency of power systems by including network constraints in the problem formulation while using non-linear, non-convex power injection formulation. Thus, the coupling between real and reactive power is maintained. This enables modeling volt-
age and reactive power constraints of the system. In addition, the problem is solved as a single large NLP optimization problem spanning multiple dispatches instead of a decomposed problem such as first solving a unit commitment (UC) problem and then solving optimal power flow (OPF) at each dispatch.

The problem studied in this paper is fundamentally different from UC and OPF decomposition. It is assumed that the total generation capacity of the microgrid is not adequate at all times to cater to microgrid loads. If not for this assumption, the problem is not much different from economic operation problem of bulk power system. With the given assumption, the need to find schedule for the available generators is not-required as it is expected that all the generators will be running at all times during autonomous mode of operation in order to minimize load shed.

The interesting aspect of the problem is to prioritize between critical and non-critical loads while adhering to relevant operational constraints for a projected restoration time spanning multiple dispatches. An overview of the proposed approach is given in Fig.6.1. Salient contributions of this work are summarized below.

**Contributions**

- Voltage constraints, generator operating limits, line limits and efficiency model for battery energy storage systems (BESS) and plug-in hybrid electric vehicle (PHEV) are included.

- Power injection is modeled in the optimization problem using the standard power flow equations. The coupling between real and reactive power is maintained i.e. no decoupled power flow type approximations are used. Hence, accurate representation of reactive power demand of the system is made possible.
Demand response is modeled through a fleet of PHEVs and adjustable loads.

Uncertainty of renewable generation and load is quantified using probability distribution and confidence levels are used to include uncertainty modeling in the optimization process.

The rest of the chapter is organized in the following manner. Test system, characteristics and modeling aspects such as demand response are covered in section II. Quantifying uncertainty is discussed in section III. Mathematical formulation is developed in section IV. Discussion on exactness of solution, possible relaxation methods and solution methodologies are covered in section V. Experimental setup, results and discussion make up section VI. Conclusions are drawn in section VII.
6.2 Test System and Modeling

A modified version of CIGRE microgrid benchmark test system from [86] is used. The test system has 13 buses with a solar and a wind farm. Both solar and wind farms are equipped with BESS. A fleet of PHEVs are assumed to be available through various times of the day at different nodes with varying probabilities. One line diagram of the test system is shown in Fig. 6.2.

6.2.1 Load Modeling

Test system originally consists of residential, commercial and industrial loads with varying proportions at each load bus. For the purpose of this work, loads are categorized as critical and non-critical loads. A percentage of non-critical loads at each load bus is assumed to be adjustable loads. Adjustable loads have a start and an end time of operation. They are available only within the designated time frame and can be operated as controllable loads within a defined minimum and maximum power range. Adjustable loads have the constraint of a given energy consumption requirement over the course of their active period.
Table 6.1: Distributed Generation Parameters

<table>
<thead>
<tr>
<th>DER Type</th>
<th>Bus No</th>
<th>MVA rating</th>
<th>BESS $P_{\text{rated}}$ (MW)</th>
<th>BESS $E_{\text{rated}}$ (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Wind</td>
<td>5</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 6.2: PHEV Rating

<table>
<thead>
<tr>
<th>Number of PHEVs</th>
<th>$P_{\text{rated}}$ (kW)</th>
<th>$E_{\text{rated}}$ (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>80</td>
<td>24</td>
</tr>
</tbody>
</table>

6.2.2 Distributed Generation

Solar and wind farm are modeled along with BESS. Attributes of distributed energy resources (DERs), including BESS rating and location are given in Table 6.1. Generation curves for solar and wind generation are obtained from [54] and adjusted based on required confidence level. Details on confidence level are discussed in section 6.3. Both solar and wind farm are assumed to provide only real power support.

6.2.3 Demand Response

One of the key recommendations from [3] is the use of PHEV in vehicle to grid (V2G) mode to supply power to the microgrid during islanded operation using demand response. Demand response implementation in this work has two facets - adjustable demand and generation. PHEVs in V2G mode are used as adjustable generation, while PHEVs in grid to vehicle (G2V) mode and adjustable loads act as adjustable demand in the optimization process. PHEV power and energy rating given in Table 6.2 are based on Nissan LEAF’s specifications.
6.3 Quantifying Uncertainty

Problems that involve forecast inherently has uncertainty. Rather than building a forecast model, focus has been given to quantifying uncertainty. In order to have realistic data, NRELs data for different independent system operators (ISOs) wind and load forecast error data are used and scaled accordingly to the given problem. The forecast error distribution for solar, wind and load demand used in this work is similar to the error distribution seen at independent system operator level. This is achieved by reconstructing the error distributions from [23]. In order to quantify uncertainty, a statistical approach described in [52] is used in this work.

The error distribution for day ahead wind power forecast used in this work is obtained from electric reliability council of Texas (ERCOT) data as reported in [23]. The authors in [23] suggest the use of hyperbolic distribution with $\mu = -0.012$ and $\sigma = 0.119$. For solar power, the same error distribution used to characterize day ahead wind forecast error is used. Day ahead load forecast error distribution of New York ISO (NYISO) with $\mu = -0.024$ and $\sigma = 0.036$ from [23] is used to characterize load forecast error.

Cumulative distribution function (CDF) and complementary cumulative distribution function (CCDF) are defined as,

$$CDF(x) = P(X \leq x) \quad (6.3.1)$$

$$CCDF(x) = 1 - CDF(x) \quad (6.3.2)$$

CCDF allows defining confidence levels and hence to quantify uncertainty. For instance, from Fig. 6.3a, 95% confidence level corresponds to $-0.21 PU$ deviation for wind power forecast error i.e. on an average, 95% of the time forecast error will
be between $-0.21$ and $0.45$ $PU$, where $0.45$ is the largest positive forecast error due to under forecasting. Hence, curtailing the forecasted generation based on a given confidence level allows utilizing renewable sources as dispatchable generators with a predefined fixed set of dispatch values whose error over time is bounded by the confidence level. Uncertainty is modeled in the optimization process by using this approach. Different confidence levels for a given wind generation profile is shown in Fig. 6.3b. Load curve is obtained from [53].

6.4 Problem Formulation

The optimization problem as such involves finding dispatch at five minute intervals for the time the microgrid is expected to be in islanded mode of operation. A dispatch interval of five minutes is chosen as it is suitable for capturing typical load fluctuations [24]. The optimization approach can be classified as a quasi-steady state analysis over multiple dispatches. The emphasis is on finding a set of complex node voltages that satisfies the complex power injection for each node of the system while satisfying the operational constraints such as voltage limits, generator limits etc. The remainder of
this section deals with the development of mathematical formulation that models the
system while enforcing these operational constraints. In this work, emphasis is given
to problem formulation rather than modeling different types of DERs. For instance,
DER such as residential fuel cells can be modeled using the same set of equations
developed in this section with parameters pertaining to residential fuel cells.

6.4.1 Adjustable Load Constraint

Adjustable loads consume a specified amount of energy during a given period within
a specified power consumption range. Each node is assumed to have adjustable loads.
Adjustable load at \( i^{th} \) node for a given operational period \( T_{i,a} \) is modeled using (6.4.1)-(6.4.3). Equation (6.4.1) is the energy consumption equation. Bounds on adjustable
power and energy is given by (6.4.2) and (6.4.3).

\[
E_{i,a}(t) = E_{i,a}(t - 1) + P_{i,a}(t) \cdot \Delta t \quad \forall t \in T_{i,a} \tag{6.4.1}
\]

\[
P_{i,a} \leq P_{i,a}(T_{i,a}) \leq \overline{P}_{i,a} \tag{6.4.2}
\]

\[
0 \leq E_{i,a}(T_{i,a}) \leq E_{i,a}^d \tag{6.4.3}
\]

6.4.2 Power Balance Equation

Solution to power flow problem results in a set of complex voltages that satisfies the
complex power injection at every node. Power balance equation at a given \( i^{th} \) node
is written in polar form as in (6.4.4) - (6.4.7).

\[
P_{i,inj} - P_{i,d} = 0 \tag{6.4.4}
\]
\[ Q_{i,\text{inj}} - Q_{i,d} = 0 \] (6.4.5)

Where, \( P_{i,\text{inj}} \) and \( Q_{i,\text{inj}} \) is given as,

\[
P_{i,\text{inj}} = \sum_{k \in B} V_i \cdot V_k \cdot (G_{ik} \cdot \cos\theta_{ik} + B_{ik} \cdot \sin\theta_{ik})
\] (6.4.6)

\[
Q_{i,\text{inj}} = \sum_{k \in B} V_i \cdot V_k \cdot (G_{ik} \cdot \sin\theta_{ik} - B_{ik} \cdot \cos\theta_{ik})
\] (6.4.7)

Where \( \theta_{ik} \) is the voltage angle difference between nodes \( i \) and \( k \). \( G_{ik} \) and \( B_{ik} \) are conductance and susceptance between nodes \( i \) and \( k \) respectively. In (6.4.4) - (6.4.7), loads are not categorized. In a microgrid, during islanded mode of operation, higher priority needs to be given to certain loads such as hospitals that can be categorized as critical loads. Hence, the load distribution within a microgrid is categorized as critical and non-critical loads in this work. Power balance equation can then be rewritten as in (6.4.8) and (6.4.9).

\[
P_{i,\text{inj}}(t) - P_{i,nc}(t) - P_{i,c}(t) - P_{i,\text{var}}(t) - P_{i,a}(t) = 0 \quad \forall \ t \in T
\] (6.4.8)

\[
Q_{i,\text{inj}}(t) - Q_{i,nc}(t) - Q_{i,c}(t) = 0 \quad \forall \ t \in T
\] (6.4.9)

In equation (6.4.8) and (6.4.9) \( P_{i,c}, Q_{i,c}, P_{i,nc} \) and \( Q_{i,nc} \) are modeled as variables with an upper bound of required critical and non-critical, real and reactive power demand at \( i^{th} \) node. \( P_{i,\text{var}} \) in (6.4.8) is the variable generation/load at a given \( i^{th} \) bus. Variable generation include distributed generation sources such as wind and solar farm which provide real power support. PHEVs are modeled as variable generation/load depending on its mode of operation.
6.4.3 Network and Generator Constraints

The constraints in (6.4.10)-(6.4.15) are used to define range of operation with respect to voltage, power generation, ramp up/down rate and line flow limits.

\[
\begin{align*}
V_i &\leq V_i(t) \leq \overline{V}_i \quad \forall t \in T \quad (6.4.10) \\
\underline{\theta}_i &\leq \theta_i(t) \leq \overline{\theta}_i \quad \forall t \in T \quad (6.4.11) \\
P_{i,g} &\leq P_{i,g}(t) \leq \overline{P}_{i,g} \quad \forall t \in T \quad (6.4.12) \\
Q_{i,g} &\leq Q_{i,g}(t) \leq \overline{Q}_{i,g} \quad \forall t \in T \quad (6.4.13) \\
\Delta P_{i,g} &\leq \Delta P_{i,g}(t) \leq \overline{\Delta P}_{i,g} \quad \forall t \in T \quad (6.4.14) \\
-\overline{P}_{ik} &\leq P_{ik}(t) \leq \overline{P}_{ik} \quad \forall t \in T \quad (6.4.15)
\end{align*}
\]

6.4.4 Energy Balance Constraint for Battery

It should be pointed out that solar, wind and PHEV batteries are modeled in the same way with different parameters \(a, b, E_{rated}\) and \(P_{rated}\). Where, \(E_{rated}\) and \(P_{rated}\) are energy and power rating respectively. Hence, only the generic equations are developed in this subsection. Several constraints pertaining to battery needs to be enforced in order to maintain law of conservation of energy. First, the amount of energy stored in BESS is time-varying i.e. the value of storage at current time \(t\) depends on previous time \(t - 1\). This is given as,

\[
E_B(t) = E_B(t - 1) + P^C_B(t) \cdot \eta(t) \cdot \Delta t - \frac{P^D_B(t)}{\eta(t)} \cdot \Delta t \quad \forall t \in T \quad (6.4.16)
\]
\[
\eta(t) = a \cdot \frac{E_B(t - 1)}{E_{rated}} + b \quad \forall t \in T \quad (6.4.17)
\]
\[
0 \leq P^C_B(t) \leq P_{rated} \quad \forall t \in T \quad (6.4.18)
\]
\begin{equation}
0 \leq P_{DB}^B(t) \leq P_{rated} \quad \forall t \in T 
\end{equation}

(6.4.19)

Where \( \triangle t \) and \( \eta \) are dispatch interval and efficiency respectively. Equation (6.4.16) ensures that the amount of energy at the end of \( t^{th} \) dispatch period is equal to the algebraic sum of energy available from \((t-1)^{th}\) period and the energy dispatched/stored in the current period. Representing efficiency as given in (6.4.16) poses certain problems. In a given discretized dispatch, BESS can either charge or discharge but not both. There are several ways to impose this constraint including the use of binary decision variables, however, that would make the problem mixed integer non-linear programming (MINLP) problem - which are much harder to solve. Hence, equation (6.4.20) is used in this work to maintain NLP formulation.

\begin{equation}
P_{DB}^B(t) \cdot P_{BC}^C(t) = 0 \quad \forall t \in T 
\end{equation}

(6.4.20)

6.4.4.1 Efficiency Curve

A simplified efficiency curve for solar, wind BESS and PHEV is used. Efficiency is approximated as a function of SOC.

\[ \eta = f(SOC) \]  

(6.4.21)

\( f(SOC) \) is approximated by linear regression of the form,

\[ y = ax + b \]  

(6.4.22)

The parameters \( a \) and \( b \) can be computed based on experimental data. However, for the lack of available experimental data, parameters \( a \) and \( b \) are chosen based on
engineering judgment. A value of 0.85 for $b$ and 0.1 for $a$ is used. Hence, BESS efficiency will vary from 85% to 95% linearly.

6.4.5 Objective Function

The objective is to maximize the amount of critical, non-critical and adjustable loads served.

$$
\max \sum_{t \in T} \sum_{i \in B} K_c \cdot P_{i,c}(t) \cdot \Delta t + K_{nc} \cdot P_{i,nc}(t) \cdot \Delta t + \sum_{i \in A} E_{i,a}(t_{i,a}^{end}) - E_{i,a}^d \quad (6.4.23)
$$

Equation (6.4.23) will maximize the amount of critical, non-critical and adjustable loads served for the period the microgrid remains in islanded mode. $\sum_{i \in A} E_{i,a}(t_{i,a}^{end}) - E_{i,a}^d$, is the sum of energy mismatch of all adjustable loads after the adjustable loads specified window of operation. Constants $K_c$ and $K_{nc}$ are chosen such that,

$$
K_c \gg K_{nc} \quad (6.4.24)
$$

Equation (6.4.24) ensures that critical loads are shed only when all the non-critical load in the system has been shed.

6.4.6 Temporal Decomposition based EMS

In order to quantitatively compare results, original problem is temporally decomposed i.e. each dispatch is optimized sequentially. Solution to the decomposed problem is obtained via solving the original problem sequentially, one dispatch at a time. The problem formulation has the same constraints as resiliency based EMS formulation. The objective function is modified from (6.4.23) to include an additional term to maximize the amount of generation at a given dispatch. This additional term in
(6.4.25) is relevant only when there is excess energy available in a given dispatch. In such a scenario, the excess energy will be stored subject to storage constraints. It is worth pointing out that temporal decomposition scheme is solved one dispatch at a time and (6.4.25) is the sum of cost of the individual dispatches for the duration the microgrid is in islanded mode of operation.

\[
\max \sum_{t \in T} \sum_{i \in B} K_c \cdot P_{i,c}(t) \cdot \Delta t + K_{nc} \cdot P_{i,nc}(t) \cdot \Delta t + \sum_{i \in A} E_{i,a}(t_{i,a}^{end}) - E_{i,a}^{d} + \sum_{t \in T} \sum_{i \in G} P_{i,g}(t) \cdot \Delta t
\]  

Time dependence of solar and wind BESS as well as PHEV are maintained by setting the initial conditions of every dispatch based on previous dispatch values. Adjustable loads in the case of temporal decomposition problem is assumed to divide its total energy requirement equally over its period of operation. This is a simplifying assumption without which it is not possible to accommodate adjustable load models in temporal decomposition scheme.

### 6.5 Exactness of Solution

Global optimality of solution to an optimization problem is dependent on convexity of problem formulation. If all the constraints and the objective function is convex then global optimality can be ensured as intersection of convex sets is also a convex set. Non-convexity of the developed problem formulation comes from power injection formulation and line flow constraints i.e. ACOPF based formulation. Literature on OPF can be roughly classified into three categories [87].

1. Finding local optimal solutions

2. Convex relaxation for global optimal solution
3. A hybrid of the first two approaches that attempts at finding global optimal solution.

Solution schemes such as MATPOWER [74] belong to category 1 while semi definite programming (SDP) and second order conic programming (SOCP) based relaxation for special cases belong to category 2. Reference [88,89] are notable works in category 2 while [90] belongs to category 3.

Emphasis of this work is on mathematical formulation rather than solution methodology. The developed NLP problem is solved using IPOPT [91], whose solution is globally optimal if the problem formulation is convex. IPOPT belongs to category 1 type solution methodology. Category 1 type solution methodology is used, as both category 2 as well as category 3 have shortcomings. For instance, SDP and its weakly exact case SOCP provide exact solutions only under special circumstances. Reference [88] shows that the SDP relaxation is tight for a resistive network with no reactive loads where demand over-satisfaction is allowed, as long as the dual variables are positive. However, authors in [92] give a simple counterexample with nonzero optimality gap. Hence, category 2 is not chosen as exactness of solution cannot be proved for a generic case [87].

Category 3 is not chosen because of practical inefficiencies associated with such approaches [87]. When category 2 and/or 3 is mature then the formulations developed can either be relaxed or can be efficiently solved to global optimality using category 3 approach.
6.6 Results and Discussion

6.6.1 Experimental Setup

Restoration period varies based on magnitude of disturbance event. Hence, studies are conducted for three different restoration periods - 3, 6 and 8 hours. In addition, demand varies based on time of the day. Hence, the disturbance and the resulting microgrid autonomous mode of operation is assumed to happen at various times of the day. For instance, the six hour restoration period is assumed to happen during 00:00 - 06:00, 06:00 - 12:00, 12:00 - 18:00, 18:00 - 24:00 hours - covering the entire 24 hour span of a day.

During any given restoration window, the diesel generator is operated at its peak capacity of 4 MW as demand exceeds generation. A fleet of 25 PHEVs are modeled, however, their availability, location and amount of initial stored energy vary depending on time of the day. Load demand through the day varies from a minimum value of 6.9 MW to 8.65 MW. Initial conditions of BESS is assumed to vary.

6.6.2 Outage Period

6.6.2.1 Three Hours

Eight three hour restoration periods were tested as given in Table 6.3. Resiliency based EMS shed less critical load in each of the eight cases when compared to temporal decomposition scheme based EMS. During restoration windows 00:00 - 03:00 and 03:00 - 06:00 no critical load was shed by resiliency based EMS while a total of 0.187 MWh was shed by temporal decomposition based EMS. Over the eight cases, resiliency based EMS shed 5.770 MWh of critical load to that of 7.956 MWh by temporal decomposition scheme. However, resiliency based EMS shed less critical load at the expense of shedding higher amount of non-critical load - 71.315 MWh.
compared to 69.124 $MWh$ non-critical load shed by temporal decomposition scheme.

6.6.2.2 Six Hours

Comparison between resiliency based EMS and temporal decomposition based EMS is presented in Fig. 6.4 (a)-(c) for six hour restoration window from 06:00 - 12:00 hours. Three quantities - amount of critical and non-critical load dropped over time and energy profile of storage units are shown. Critical and non-critical load drop is presented as a cumulative sum over time. Hence, any point on the curve in Fig. 6.4 (a) and (b) represents the amount of critical and non-critical energy requirement not met from the start of restoration period till the chosen point. Total stored energy in wind and solar BESS, and PHEVs are given in Fig. 6.4 (c).

The temporal decomposition based EMS is functionally the same as that of resiliency based EMS barring one fundamental difference - the temporally decomposed scheme solves each dispatch sequentially. This results in less flexibility. In addition, adjustable loads are handled in a non-optimal way because each dispatch is solved sequentially.

Different test cases have different ratio of energy production capacity to energy demand. In the 15 test cases given in Table 6.3, this ratio varies from 55 to 65 percent. The amount of critical load is approximately 60 percent of total load. In all cases there are a set of dispatches where energy production capacity is lesser than the total critical energy demand for that period.

In addition, DER outputs are modeled to include unforeseen variations, such as temporary cloud cover resulting in lowered energy output of a solar plant and unforeseen sudden drop in wind velocity. Each dispatch is modeled with a probability of $\frac{1}{12}$ to have lower energy production than the confidence level adjusted renewable forecast. Hence, on an average one five minute dispatch in every hour will have
Figure 6.4: 6 hour window comparison results
Figure 6.5: 8 hour window comparison results
Table 6.3: Performance comparison between resiliency based EMS and temporal decomposition based EMS

<table>
<thead>
<tr>
<th>Time of Day</th>
<th>Restoration Time (hr)</th>
<th>Total Energy Demand (MWh)</th>
<th>Resiliency based EMS</th>
<th>Temporal decomposition based EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Critical Load Shed</td>
<td></td>
<td>Total Critical Load Shed (MWh)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(MWh)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Non-Critical Load</td>
<td>Solver Time (s)</td>
<td>Total Load Shed (MWh)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shed (MWh)</td>
<td>Solver Status</td>
<td></td>
</tr>
<tr>
<td>00 : 00 − 03 : 00</td>
<td>3</td>
<td>21.555</td>
<td>0</td>
<td>7.724</td>
</tr>
<tr>
<td>03 : 00 − 06 : 00</td>
<td>3</td>
<td>21.314</td>
<td>0</td>
<td>7.424</td>
</tr>
<tr>
<td>06 : 00 − 09 : 00</td>
<td>3</td>
<td>22.636</td>
<td>0.170</td>
<td>8.500</td>
</tr>
<tr>
<td>09 : 00 − 12 : 00</td>
<td>3</td>
<td>24.696</td>
<td>0.762</td>
<td>9.153</td>
</tr>
<tr>
<td>12 : 00 − 15 : 00</td>
<td>3</td>
<td>25.694</td>
<td>0.927</td>
<td>9.234</td>
</tr>
<tr>
<td>15 : 00 − 18 : 00</td>
<td>3</td>
<td>26.144</td>
<td>1.563</td>
<td>9.918</td>
</tr>
<tr>
<td>18 : 00 − 21 : 00</td>
<td>3</td>
<td>25.699</td>
<td>1.579</td>
<td>9.953</td>
</tr>
<tr>
<td>21 : 00 − 24 : 00</td>
<td>3</td>
<td>24.428</td>
<td>0.767</td>
<td>9.406</td>
</tr>
<tr>
<td>00 : 00 − 06 : 00</td>
<td>6</td>
<td>42.509</td>
<td>0</td>
<td>14.898</td>
</tr>
<tr>
<td>06 : 00 − 12 : 00</td>
<td>6</td>
<td>46.972</td>
<td>1.410</td>
<td>17.293</td>
</tr>
<tr>
<td>12 : 00 − 18 : 00</td>
<td>6</td>
<td>51.479</td>
<td>3.189</td>
<td>18.792</td>
</tr>
<tr>
<td>18 : 00 − 24 : 00</td>
<td>6</td>
<td>49.737</td>
<td>3.350</td>
<td>19.000</td>
</tr>
<tr>
<td>00 : 00 − 08 : 00</td>
<td>8</td>
<td>57.112</td>
<td>0</td>
<td>20.844</td>
</tr>
<tr>
<td>08 : 00 − 16 : 00</td>
<td>8</td>
<td>66.283</td>
<td>3.141</td>
<td>23.949</td>
</tr>
<tr>
<td>16 : 00 − 24 : 00</td>
<td>8</td>
<td>66.943</td>
<td>5.188</td>
<td>25.456</td>
</tr>
</tbody>
</table>
unforeseen variation in renewable energy output.

Resiliency based EMS sheds less critical load than temporal decomposition scheme based EMS as shown in Fig. 6.4 (a) at the expense of shedding more non-critical load as shown in Fig. 6.4 (b) and by dispatching the stored energy in a better way as in Fig. 6.4 (c). In temporal decomposition scheme, all the stored energy is dispatched within the first hour in order to shed lesser non-critical load. This is due to the lack of look ahead period. A similar trend is seen in all the six hour restoration window periods. Numerical comparisons are given in Table 6.3.

6.6.2.3 Eight Hours

Three eight hour dispatches which span the entire duration of a day is tested. Results for the period from 00:00 - 08:00 hours is show in Fig. 6.5 (a)-(c). In this particular case, probability of unforeseen renewable generation variation is increased to $\frac{1}{6}$ i.e. on an average, output of renewable generation has unforeseen variation in two dispatches during every hour.

The increased variation results in discrete changes in Fig. 6.5 (a) and (c). This case is designed in such a way so that the given microgrid emulates the case where a large amount of generation comes from renewable sources - which inherently includes unforeseen variation.

Given the problem set up, the resiliency based EMS finds the optimal dispatch while satisfying the complex power injection requirements at all nodes across all dispatches, schedules the adjustable load in such a way that they can be served without resulting in critical load drop, incorporates variation in availability, location, SOC and efficiency curve of PHEVs and uses BESS of solar and wind farm optimally to minimize critical load shed and to maximize non-critical load served.

This results in no critical load being shed by resiliency based EMS for the eight
hour restoration window from 00:00 - 08:00 hours. During the same period, temporal decomposition scheme sheds 0.714 MWh of critical load as shown in Fig. 6.5 (a). Comparison results for all eight hour cases are given in Table 6.3. In all the 15 test cases, optimal solution was achieved by resiliency based EMS and it outperformed temporal decomposition scheme. Average computation time of resiliency based EMS for the largest case - eight hour case is 118.28 seconds, which is less than half of the five minute dispatch period.

It is worth noting that it is rather straight forward to design an EMS that only prioritizes in shedding least amount of critical load. This can be achieved by shedding all non-critical loads and storing the energy at all dispatches until the storage devices are full. However, this would result in unnecessary shedding of non-critical load. Hence, the need for resiliency based formulation.

6.7 Conclusion

A mathematical framework for enhancing resiliency of microgrids through survivability is developed. Standard power flow equations are used to preserve coupling between real and reactive power. Demand response is modeled through PHEV V2G mode and adjustable loads. Relevant operational constraints are enforced. The ensuing non-linear, non-convex problem formulation is solved using a local optimizer - IPOPT.

The extended time-scale problem is solved as a single large optimization problem. For this extended time-scale problem, uncertainty of renewable generation and load forecast are quantified using probability distribution and included in the optimization problem. Different scenarios for different restoration times at different time of day are tested. Results are compared against a temporal decomposition scheme based
EMS. Theoretical formulation backed by numerical results suggest applicability of developed mathematical framework for increasing resiliency of microgrids where the objective is minimizing amount of critical load shed while maximizing amount of non-critical load served.
Part IV

Partitioning Power Networks
Chapter 7

Balanced, Non-Contiguous Partitioning of Power Systems Considering Operational Constraints

Nomenclature

Binary Variables

$\alpha^k_i$ decision variable of $i^{th}$ edge that indicates whether both the adjacent vertices belong to $k^{th}$ cluster

$\beta^k_i$ decision variable to indicate if the given $i^{th}$ node/vertex belongs to the $k^{th}$ cluster

$\sigma^k_i$ decision variable of $i^{th}$ edge that indicates if only one of the adjacent vertices belong to $k^{th}$ cluster i.e. an edge cut

Parameters

$k$ Number of partitions

$\gamma$ Ideal partition size

$f$ Edge weight
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i$</td>
<td>Weight of $i^{th}$ node</td>
</tr>
</tbody>
</table>

Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Set of all nodes/buses/vertices in a graph</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of all edges in a graph</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of all cycles in a graph</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of all partitions in a graph</td>
</tr>
</tbody>
</table>

This chapter presents an integer programming based partitioning of dynamic graphs that arise in power systems. The proposed approach allows for explicitly expressing power systems operational constraints in the partitioning algorithm. Balanced, non-contiguous graphs appear in several power systems applications such as network partitioning for high performance computing based parallel transient simulators and in wide area control. Quality of partition obtained using the developed algorithm compares favorably with a well known multi-level graph partitioning approach - METIS. Several test systems, ranging from a 9 bus test system to the 2383 bus western polish test system is used to demonstrate the applicability of proposed approach for power system partitioning problems. In addition, applicability of the proposed approach for a non-stationary system such as power networks is demonstrated by partitioning in real-time. Real-time in this context is defined as the interval between two dispatches as the edge weights of the dynamic graph is expected to change at every dispatch.
7.1 Introduction

Partitioning power networks has many important applications. Controlled islanding of power systems, parallel computation of power system analysis, dynamic equivalence reduction, planning and operations are some of the application areas in power systems that either directly or indirectly use network partitioning. Applications in power systems that require partitioning can broadly be classified into two classes based on graph structure - dense and sparse graphs. Applications such as coherency based clustering often requires a complete graph i.e. every node is connected to every other node. Hence, coherency based network partitioning such as [93,94] can be classified as dense graphs. Clustering methods such as k-means, k-medoid and their extensions are well suited for clustering such dense graphs [93,94].

Sparse graphs occur in controlled islanding applications such as in [95,96]. In such applications, once a coherent set of generators are obtained, the task then is to form boundaries i.e. ascertain cut-sets that would result in minimal load shed across all islands. Typically, graph theoretic approaches such as in [4,96,97] are used. A inherent requirement for these applications is that the formed subgraphs should be contiguous i.e. every node in a given island should be connected to every other node either directly or indirectly.

Within the sparse graph class applications in power system, there are certain applications that do not require contiguous partitions. One such application is the parallel transient simulation of power systems, and most parallel computation of power system analysis in general. A parallel transient simulator such as in [98–100] requires that a given system is split into sub-systems, whose differential algebraic equations are solved parallel in time in a multi-processor environment. One of the bottle neck in such applications is the communication overhead. Consider the following power
injection formula,

\[ P_i = \sum V_i \cdot V_k \cdot (G_{ik} \cdot \cos \theta_{ik} + B_{ik} \cdot \sin \theta_{ik}) \quad \forall k \in V \quad (7.1.1) \]

Computing equation (7.1.1) requires voltage magnitude and angle of all nodes that are connected to the \(i^{th}\) node. It is desirable that all this information is available in the same processor. However, edge-cuts are inevitable as the system is split into several partitions and the information relating to these partitions is in different processors. The focus is to minimize communication overhead by minimizing the amount of inter-processor communication, which in turn is achieved through partitioning schemes whose objective is to minimize edge-cuts. Quality of partition directly affects the performance gain of parallel simulators. Several applications of such simulators are time sensitive and hence improved performance is desired.

The focus of this work is to develop partitioning methods for power system applications whose graph is sparse and the resulting partitions non-contiguous and balanced. Applications such as partitioning networks for parallel transient simulators [98–100], signal selection for decentralized wide area control [101] are some applications that fall under this category.

An often overlooked aspect of partitioning schemes in power systems is the fact that partitioning from an engineering standpoint is often less restrictive than from operational standpoint. For instance, consider the operational hierarchy of an interconnected power system. Each system operator controls a geographical area that is segregated based on ownership. Under these conditions, there are restrictions on partition based on energy policy and also based on ownership rights. As an example, consider a inter-tie where the two nodes at either end of an inter-tie form boundary buses and are in different control areas. For a given partitioning method, one cannot
preclude the possibility of these two boundary buses being partitioned together - an undesirable partition from an operational stand point.

Multi-level graph partitioning techniques provide high quality balanced partitions. However, power system operational constraints cannot be enforced. Please note that multi-level multi-constrained graph partitioning approach such as in [102] implemented in the METIS package [103] is fundamentally different. This is because such approaches allow to partition a graph based on a weighing vector for each node rather than a scalar weight. For instance, a partition for multi-physics simulation might require balance of communication volume and memory requirement. Hence, both of these requirements are expressed as weights in a vector form. The partition is balanced with respect to these two weights.

In the case of power systems, the operational constraints are different as they involve the elements of the partition itself, such as the scenario discussed in the preceding paragraphs. It is worth mentioning that constrained spectral methods provide means to enforce must-link and cannot-link constraints, which can be used to enforce power system operational constraints. Such a scheme is implemented in [4]. However, constrained spectral methods do not provide a balanced partition with respect to cardinality i.e. the number of elements in each partition is not approximately equal. A method which gives balanced partition while enforcing a wide range of operational constraints is desired. Hence, the necessity for this work.

7.1.1 Literature Review

The authors in [104] presented a Tabu search based network partitioning approach. The focus of the paper is to partition the system for parallel computational analysis of power systems. Specifically, the focus of the paper is on parallel load flow computation. The authors minimize cut-sets in order to minimize computation time across
multiple processors. A similar approach is presented in [105] using genetic algorithms. The network partitioning methods presented in [104] and [105] cannot be extended for partitioning problems such as wide area decentralized control that may require enforcing specific operational constraints. Such a restriction, if formulated, can only be achieved by means of a penalty factor in these approaches. Such penalty factors, even for simple constraints arising from energy policy considerations will result in a multi-objective problem - an undesirable formulation that is avoided in the proposed approach by explicitly encoding power systems operational restraints as a set of constraints in the optimization problem.

Reference [106] also proposed genetic algorithm based partitioning of power systems. A sparse graph partitioning scheme using spectral properties of graphs is proposed in [97]. In both papers, partitioning is considered only from an engineering view point. Hence, specific operational constraints are not considered.

Although many network partitioning applications occur in power system research, the focus has mainly been on adapting existing partitioning/clustering techniques for power systems applications. Methods such as k-means, k-medoids clustering and their extensions [93,94,107], spectral clustering [4,97,108] and multi-level graph partitioning schemes [98] are some of the popular partitioning methods used for power system applications.

The focus of this work is to explore the possibility of obtaining high quality partitions for sparsely connected graphs that arise in power systems while considering balance of partition and enforcing operational constraints, where the resulting partitions need not be contiguous. Such graphs appear in parallel computation of power system analysis such as in [98–100,104,109] and in wide area control such as the work in [101].
Contributions

- A mathematical way to enforce operational constraints in partitioning scheme.
- Provide a balanced partition. The balance constraint can be explicitly set by the user using a tolerance parameter. A large value of tolerance relaxes the balance constraint while a smaller value provides a tighter bound.
- Means to verify optimality of partition.

The rest of this chapter is organized as follows. In section II, an brief introduction to graph theory is provided. Problem formulation is formally developed in Section III. The problem of enforcing operational constraints is reduced to fundamental building blocks and their related formulations are developed in section IV. Section V gives a brief introduction to comparison of quality of partition with METIS. Results are presented and discussed in Section VI and conclusions are drawn in Section VII.

7.2 Graph Theory - A Primer

This work is specifically intended for graphs arising out of a given power system’s physical connectivity. Graphs such as susceptance matrix, power flow, nodal connectivity etc. are ideally suited. Such power system graphs are sparse. Hence, the type of graphs used in this work are undirected weighted graphs that are sparse. Equation (7.2.1) indicates the fact that in a power system the ratio of number of transmission lines to buses is above 1 but much less than 2. This fact can be verified through Table 7.1 and 7.2.

\[ 1 \leq \frac{|E|}{|V|} \ll 2 \quad (7.2.1) \]

Where \(|V|\) and \(|E|\) are the cardinality of vertices and edges respectively. When a power system graph that is based on physical connectivity is used, vertices and edges
represent buses and transmission lines in power systems. Each connection i.e. each edge carries a weight of connection - power flow, admittance etc. to name a few.

In a given graph, the algebraic sum of number of vertices, edges, cycles and partitions are related by the equation (7.2.2).

\[ |V| - |E| + |C| = |K| \] (7.2.2)

For a connected graph, number of partitions is equal to one. Hence, (7.2.2) can be reduced to (7.2.3).

\[ |V| - |E| + |C| = 1 \] (7.2.3)

In equation (7.2.3), \(|C|\) is called the cyclomatic number i.e. the number of cycles that needs to be removed to make the graph cycle free i.e. a tree. It is worth pointing out that the complexity of the problem is directly proportional to the number of cycles.

### 7.3 Problem Formulation

Consider a transmission line \(m\), connecting buses \(i\) and \(j\). For a given \(k^{th}\) cluster, we want to enforce the decision variable \(\alpha_{m}^{k}\) to be 1 if and only if both \(\beta_{i}^{k}\) and \(\beta_{j}^{k}\) belong to \(k^{th}\) cluster. That is, both \(\beta_{i}^{k}\) and \(\beta_{j}^{k}\) will take a value of 1. Now consider the scenario where there is an edge-cut i.e. one of the buses belongs to the \(k^{th}\) cluster while the other does not. In such a case, the decision variable \(\sigma_{m}^{k} = 1\). A third possibility is that both \(\beta_{i}^{k}\) and \(\beta_{j}^{k}\) do not belong to \(k^{th}\) cluster and hence \(\alpha_{m}^{k}\) and \(\sigma_{m}^{k}\) should both be zero. This constraint is expressed as in (7.3.1).
\[ \beta_i^k + \beta_j^k - 2 \cdot \alpha_m^k - \sigma_m^k = 0 \forall m \in E, \forall k \in K \quad (7.3.1) \]

When \( \beta_i^k \) and \( \beta_j^k \) take a value of 1, \( \alpha_m^k = 1 \) and \( \sigma_m^k = 0 \) to satisfy (7.3.1). When sum of \( \beta_i^k \) and \( \beta_j^k \) is 1, then \( \sigma_m^k = 1 \) and \( \alpha_m^k = 0 \). While sum of \( \beta_i^k \) and \( \beta_j^k \) is 0, both \( \alpha_m^k \) and \( \sigma_m^k \) will be zero.

It is important to note that for a given \( k \)-way partition, the number of variables required to represent all buses in the system is given by \( |V| \cdot |K| \). A similar logic applies to edge variables \( \alpha \) and \( \sigma \), which require \( |E| \cdot |K| \). For a partition with two clusters, (7.3.1) is expressed as in (7.3.2) and (7.3.3).

\[ \beta_1^i + \beta_2^i - 2 \cdot \alpha_m^1 - \sigma_m^1 = 0 \forall m \in E \quad (7.3.2) \]

\[ \beta_3^i + \beta_4^i - 2 \cdot \alpha_m^2 - \sigma_m^2 = 0 \forall m \in E \quad (7.3.3) \]

Since, there is one binary variable representing a given node in each cluster. Care must be taken that a given node is not placed in multiple clusters. This constraint is enforced as given in (7.3.4).

\[ \sum_{k \in K} \beta_i^k = 1 \forall i \in V \quad (7.3.4) \]

For example, (7.3.4) is expressed as in (7.3.5) for three way partition of a given \( i^{th} \) bus.

\[ \beta_1^i + \beta_2^i + \beta_3^i = 1 \quad (7.3.5) \]

**Equality of Partition**

For network partitioning problems arising in parallel computation of power system analysis such as [98–100], it is desirable that the partitions are balanced. In this
work, a tolerance level is defined for each partition size and cardinality of partition is enforced as in (7.3.6).

\[ \gamma - \epsilon \leq \sum_{i \in V} w_i \cdot \beta_k^i \leq \gamma + \epsilon \; \forall k \in K \] (7.3.6)

Where, \( w_i \) is the weight associated with the vertex and \( \gamma \) is defined as the ideal partition size, given by (7.3.7). A value of \( w_i = 1 \; \forall i \in V \) is used in this work i.e. cardinality is enforced rather than weighted cardinality.

\[ \gamma = \frac{|V|}{|K|} \] (7.3.7)

**Objective**

The objective is to minimize the total weighted edge cut. Where the vector \( f \) is the weight associated with each edge i.e. some parameter relating to transmission line such as average power flow in a transmission line, admittance of the transmission line etc. \( T \) is the transpose operator.

\[ \min f^T \cdot \sigma \] (7.3.8)

**7.4 Enforcing Operational Constraints**

Restrictions that arise in power systems due to operational policies are enforced as constraints in the partitioning problem. Since the possible number of restrictions are many, a reductionist approach is used to reduce the restrictions to fundamental constraint sets in terms of decision variables. These fundamental constraint sets can be combined in different ways to enforce most, if not all operational constraints. The rationale behind this claim is due to the fact that the individual elements of a power
system are used as decision variables in the optimization process. Hence, it is possible to explicitly write any arbitrary constraint as a function of these decision variables.

Operational constraints can be categorized as node and edge based. Node based constraints correspond to restraints that involve buses, while edge based constraints involve transmission lines. We only develop node based constraints since all edge based constraints can also be modeled using node based constraints. For completeness, we show examples of edge based constraints modeled as node based constraints in section 7.6.1.

### 7.4.1 Forced Grouping

Consider a set of buses $S_1$ that should be grouped together in a given $k^{th}$ cluster, where the choice of $k^{th}$ cluster is arbitrary. Equation (7.4.1) is used to forcibly group all buses in the set $S_1$ in $k^{th}$ cluster.

$$\sum \beta_{S_1}^k = |S_1| \quad (7.4.1)$$

Where $|S_1|$ is the cardinality of set $S_1$ i.e. the number of nodes in $S_1$.

### 7.4.2 Mutual Exclusivity

Consider a set of buses $S_1$ that should be grouped together in an arbitrary $k^{th}$ cluster with an additional constraint that the $k^{th}$ cluster should have a null set for a group of nodes in $S_2$. This type of conditions often appear in power systems when a set of nodes belonging to a control area should be partitioned together. However, no set of nodes from another balancing area can appear in the same partition.

$$\sum \beta_{S_1}^k + \sum \beta_{S_2}^k = |S_1| \quad (7.4.2)$$
Equation (7.4.1) along with (7.4.2) enforces mutual exclusivity. Equation (7.4.1) imposes that all buses in set $S_1$ be placed together. While, (7.4.2) enforces that only nodes from $S_1$ is present in $k^{th}$ partition.

7.5 Comparison with METIS

Reference [110] is one of the widely used [111,112] multi-level graph partitioning methods. The algorithm developed in [110] runs two to seven times faster than other well known multi-level graph partitioning algorithms such as [113] and has consistently better partition in terms of cut size [110]. Specific to power systems research, METIS based network partitioning is used in [98,99] for parallel transient simulator. Furthermore, for any comparison to be meaningful, a standard implementation of the algorithm should be available. In the case of METIS, the authors of the algorithm have developed the required partitioning library [103]. Hence, quantitative comparisons can be made. Due to these reasons, we chose METIS as the standard to which the proposed method is compared.

7.5.1 METIS Options

All partitions were computed using $k$-way partitioning strategy using $gpmetis$ function. Flags were set for partitions to be balanced and non-contiguous. METIS provides two options, either to minimize weighted edge-cuts or edge-cut volume. We chose the former, as the later is a special case of weighted edge-cut where all edge-cuts are equally weighted. Furthermore, we wish to demonstrate the performance of the proposed partitioning scheme for a generic weighted edge-cut minimization problem and hence the choice of weighted edge-cut scheme. All other options were set at default values.
7.6 Results and Discussion

Since operational constraints cannot be enforced in METIS, a two step validation process is used. In the first part of validation, operational constraints are enforced on a non-trivial yet tractable 39 bus test system. All the operational constraints developed in section 7.4 are enforced to demonstrate the ability of the proposed method to take in arbitrary set of constraints.

In the second part, quantitative comparisons are drawn against METIS. This is to validate the claim that the proposed approach provides high quality partition comparable to that of METIS in a time limited environment. We emphasize on time limit because without the constraint on time the proposed method will find the global optimal solution. The question on quality of partition do not arise in those cases as global optimality can be achieved. Static graphs such as the topological connectivity of power system are not time limited while dynamic graphs such as the ones that use power flow values require time limit.

7.6.1 Enforcing Operational Constraints

Operational constraints developed in section 7.4 is enforced on 39 bus test system - a non-trivial yet tractable system. Optimal 3 way partition of the 39 bus test system is shown in Fig. 7.1a. Notice that the clusters have 13, 14 and 12 nodes respectively. This is because the ideal partition size $\gamma = 13$ and we used $\epsilon = 0.01 \cdot \gamma$ relative to the ideal partition size. $\epsilon = 1$ when rounded to the next biggest integer value. Hence, the partitions can have between 12 and 14 nodes. In the given partition, nodes 13, 14 belong to cluster 1, while node 15 belongs to cluster 2. We wish to enforce forced grouping scheme on this set. That is, we wish to group set $S_1 = \{13, 14, 15\}$ together. Recalling the constraint developed in (7.4.1), forced grouping for $S_1$ can be written
Figure 7.1: Forced grouping of 39 bus test system

\[
\sum_{i \in \{13, 14, 15\}} \beta_i^1 = 3 \tag{7.6.1}
\]

Since the nodes are to be placed in cluster 1, we assign \( k = 1 \) in (7.6.1). Enforcing (7.6.1) for \( k = 1 \) results in the partition shown in Fig. 7.1b. One can easily verify that the forced grouping for the set \( S_1 \) is achieved as all nodes in set \( S_1 \) are placed together.

In addition, consider the set \( S_2 = \{9\} \) that must remain mutually exclusive with the set \( S_1 \). This is enforced by the additional constraint given in (7.6.2).

\[
\sum_{i \in \{13, 14, 15\}} \beta_i^1 + \sum_{i \in \{9\}} \beta_i^1 = 3 \tag{7.6.2}
\]

Enforcing equations (7.6.1) and (7.6.2) results in desired mutual exclusivity as shown in Fig. 7.2a.

There are cases, when a tie-line connecting two areas should not be grouped
Figure 7.2: Mutual exclusivity based partition

together. This is the case of an edge constraint. However, as discussed earlier, this constraint can be expressed using mutual exclusivity constraint. In the base partition case shown in Fig. 7.1a, assume buses 4 and 5 belong to two operational areas. However, they are partitioned together - an undesirable partition. Hence, we wish to enforce this operational constraint in the partitioning problem. Equation (7.6.3) is used for this purpose.

\[ \sum_{i \in \{4,5\}} \beta_i^1 = 1 \]  

Equation (7.6.3) prohibits both buses 4 and 5 being placed together in cluster 1. The resulting partition is shown in Fig. 7.2b.

Additionally, one may also wish to enforce a constraint that a particular transmission line should not be a part of a cut-set i.e. a given transmission line cannot be a tie-line. In the base case shown in Fig. 7.1a, buses 3 and 4 form a tie-line. It is assumed that this is undesirable. Hence, the forced grouping constraint as given in
\[(7.6.4)\] is used to enforce nodes 3 and 4 to belong to an arbitrary partition, which in this case is chosen as cluster 3. The resulting partition is shown in Fig. 7.2b.

\[\sum_{i \in \{3,4\}} \beta_i^3 = 2 \quad (7.6.4)\]

By using the constraints developed in section 7.4 it is possible to enforce most, if not all power system operational constraints. A general method of enforcing power system operation constraints on the partitioning problem is demonstrated through these examples.

### 7.6.2 Comparison with METIS

#### 7.6.2.1 Experimental Setup

Comparison with METIS is categorized into three classes - small, medium and large, based on size and complexity of the graph used. Properties of graphs are given in Table 7.1 and 7.2. For small systems such as case9, case14, case39 only 2 and 3 way partitioning is used. For all other systems four partitions, namely, 2, 3, 4 and 5 are used. Equality of partition was set to be balanced i.e. each partition will have ideal number of clusters as given in (7.3.7). A small tolerance value of \( \epsilon = 0.01 \cdot \gamma \) i.e. 1% tolerance relative to ideal partition size \( \gamma \) is used for both the proposed method as well as METIS. Average real power flowing through transmission lines from base case power flow are used as edge weights. Test case data and power flow are obtained from MATPOWER [74]. Average real power flow is defined as,

\[P_{i,j} = \frac{|P_{i,j}| + |P_{j,i}|}{2} \quad (7.6.5)\]

The objective of the partition is to separate the system into smaller sub-graphs,
which may or may not be contiguous. In addition, all the partitions should be balanced within the set tolerance level. For all reported performance comparisons, the amount of cumulative weighted edge-cut as defined in (7.3.8) is used to compute the objective of METIS partition as in (7.6.6).

\[ C_m = f^T \cdot \sigma_m \quad (7.6.6) \]

\[ C_p = \frac{f^T \cdot \sigma_p}{C_m} \quad (7.6.7) \]

Equation (7.6.6) and (7.6.7) represent the cost/total cumulative edge-cut from METIS and proposed approach respectively. Total edge cut of proposed approach is expressed in terms of total edge-cut of METIS. Recall that the objective is to minimize the total edge-cut in the system. Hence, \( C_p < 1 \) is indicative of the fact that proposed approach performed better than METIS. For instance, \( C_p = 0.5 \) indicates that the proposed approach has exactly 50% of weighted edge-cut when compared to METIS. Similarly, \( C_p > 1 \) is indicative of the fact that the proposed method has higher total edge-cut when compared to METIS for the \( k \)-way partitioning of a given test system.

All results are computed using geometric mean value as given in (7.6.8).

\[ \text{geometric mean} = \sqrt[n]{r_1 \cdot r_2 \cdot \ldots \cdot r_n} \quad (7.6.8) \]

Where \( r_1, r_2 \) etc. are the values obtained in run 1, run 2, etc. Both edge-cut and solver time are obtained using geometric mean. A value of \( n = 10 \) is used i.e. all reported numerical values are computed over 10 runs. It is important to note that proposed approach is completely deterministic. However, since the CPLEX solver [29] is time limited in the experiments, the solver might not reach optimality in the given time. Hence, the solution obtained is dependent on the random number seed used.
Table 7.1: Small System Graph Properties

<table>
<thead>
<tr>
<th>Casename</th>
<th>No. of Buses</th>
<th>No. of branches</th>
<th>No. of Basis Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>case9</td>
<td>9</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>case14</td>
<td>14</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>case39</td>
<td>39</td>
<td>46</td>
<td>8</td>
</tr>
<tr>
<td>case57</td>
<td>57</td>
<td>78</td>
<td>22</td>
</tr>
<tr>
<td>case118</td>
<td>118</td>
<td>179</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 7.2: Medium and Large Graph Properties

<table>
<thead>
<tr>
<th>Casename</th>
<th>No. of Buses</th>
<th>No. of branches</th>
<th>No. of Basis Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>case300</td>
<td>300</td>
<td>409</td>
<td>110</td>
</tr>
<tr>
<td>case2383wp</td>
<td>2383</td>
<td>2886</td>
<td>504</td>
</tr>
</tbody>
</table>

by CPLEX for branch and bound algorithm. Essentially, this random number seed, among other settings, dictates the order in which the branches are explored by CPLEX solver.

METIS uses algorithms that are random and hence requires multiple runs. The choice of $n = 10$ is used as the performance across the 10 runs for different test systems did not vary appreciably.

7.6.2.2 Small Test System

For small system, 9, 14, 39, 57 and 118 bus test systems are used. Properties of small system graphs are given in Table 7.1. All test runs were limited to a maximum of 15 minutes. Numerical comparison results for the tests are presented in Table 7.3. Please note that $k$ in Table 7.3 represents number of partitions. For all test cases and for all partition sizes the proposed approach obtained solution that is at least as good as METIS. Out of the 14 partitions for which comparisons are drawn, 13 partitions were better than the one obtained by METIS. The case when the proposed approach and METIS had same performance, the proposed approach obtained a solution that was
proven optimal by CPLEX solver. Hence, for this case METIS partitioned the system in an optimal way. For the partition cases presented in Table 7.3, global optimality was achieved for 12 of the 14 cases. Graphical comparisons for 57 and 118 bus test cases are given in Fig. 7.3a and Fig. 7.3b respectively. Computational performance of the proposed approach for small test system cases are listed in Table 7.4. METIS obtained solution for all cases in less than one second.

7.6.2.3 Medium and Large System

The 300 node test system - case300 and the 2383 node test system - case2383wp is used as medium and large test systems respectively. For the larger system i.e. the case2383wp test system, optimization run time was increased from 15 minutes to 60 minutes. The choice of 15 and 60 minutes stems from power systems operation interval. For example, the inter-area dispatch interval at California Independent System Operator (CAISO) is 15 minutes. The schedule for unit commitment at ISO/RTO level is 60 minutes. The implicit assumption in the experimental setup is that the graph is dynamic, meaning, the values of the edge weight can change
Table 7.3: Performance Comparison for Small Test Systems

<table>
<thead>
<tr>
<th>Casename</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k=2</td>
</tr>
<tr>
<td>case9</td>
<td>0.23</td>
</tr>
<tr>
<td>case14</td>
<td>0.53</td>
</tr>
<tr>
<td>case39</td>
<td>0.10</td>
</tr>
<tr>
<td>case57</td>
<td>0.51</td>
</tr>
<tr>
<td>case118</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 7.4: Computation Time for Small Test Systems

<table>
<thead>
<tr>
<th>Casename</th>
<th>Proposed approach solution time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k=2</td>
</tr>
<tr>
<td>case9</td>
<td>0.02</td>
</tr>
<tr>
<td>case14</td>
<td>0.04</td>
</tr>
<tr>
<td>case39</td>
<td>0.04</td>
</tr>
<tr>
<td>case57</td>
<td>0.29</td>
</tr>
<tr>
<td>case118</td>
<td>1.41</td>
</tr>
</tbody>
</table>

over time during each dispatch. This is the case for power flow values. Hence, the constraint on the amount of run time available for the optimization to find either global optimal solution or find the best feasible solution subject to run time constraint. For a static graph, the constraint on run time can be neglected. However, we wish to show the applicability of the developed method for a non-stationary system such as power system. Hence, all partition runs are time limited in this work.

Significant improvements for both medium and large test cases can be noticed in Table 7.5. Graphical results of comparison is shown in Fig. 7.4a and Fig. 7.4b. Case300 with 4 partitions resulted in best comparative performance with a total edge-cut of 26% when compared to METIS, while maintaining partition balance among the resulting subgraphs. Similarly, for the case2383wp test system, two way partition resulted in the best comparative performance with a total cumulative edge-cut of 32% when compared to METIS. All presented results are geometric mean of ten test runs.
Run time for the proposed method is given in Table 7.6. Computational performance of METIS is significantly superior than the proposed method. METIS partitioned all test cases in less than one second of run time. Compared to the proposed approach for large systems, METIS is better by order of magnitude 3 in terms of computational efficiency. However, when compared to minimizing weighted edge-cut, the proposed method outperformed METIS for all test cases ranging from a small 9 bus test case to that of a large 2383 bus case with the additional advantage of being able to enforce operational constraints as shown in section 7.6.1.

Multi-level graph partitioning techniques work by aggregating nodes with similar properties into a single node - a process called coarsening. The actual partition itself happens on the coarsened graph. Once partitioned, the coarsened graph is then uncoarsened to generate the original graph. At each uncoarsening phase, further refinement is applied. Multi-level graph partitioning techniques often deal with graphs whose size is substantially larger than the graphs that appear in power system. For instance, some of the test cases in [110] have more than million edges. A power system graph that stems from physical connectivity of the system will not range in the order of millions but rather in thousands and tens of thousands.

The overall idea of this work is to design a partitioning scheme for partitioning static and dynamic graphs that occur in power systems with the added ability to enforce operational constraints and arbitrary balance constraints on partitions. The experimental verification shown in this section helps to validate these properties of the proposed method.
Figure 7.4: Performance comparison for medium and large test systems. (a) and (b) shows comparison for 2, 3, 4 and 5 partitions of case300 and case2383wp respectively.

Table 7.5: Performance Comparison for Medium and Large Test Systems

<table>
<thead>
<tr>
<th>Casename</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k=2</td>
</tr>
<tr>
<td>case300</td>
<td>0.35</td>
</tr>
<tr>
<td>case2383wp</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 7.6: Computation Time for Medium and Large Test Systems

<table>
<thead>
<tr>
<th>Casename</th>
<th>Proposed approach solution time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k=2</td>
</tr>
<tr>
<td>case300</td>
<td>476.48</td>
</tr>
<tr>
<td>case2383wp</td>
<td>3600</td>
</tr>
</tbody>
</table>

7.6.3 Computational Performance

All the presented results were obtained on Intel Xeon E5-2665 processor with 32 GB memory. Please note that a similar performance can be obtained on a smaller memory machine such as 8 or 16 GB by utilizing the memory optimization techniques in CPLEX. Writing the node log to hard disk instead of memory is one such option.
The results in Fig. 7.4b show that for a large test system with time limit, the solver performance is dependent on the size of the problem. This is especially true for 5 way partition of 2383 bus test system. In such cases where improved performance is desired for dynamic graphs, one can apply coarsening schemes used in multi-level graph partition. This reduces the solution space and high quality partitions can be attained in much faster time. It is also worth noting that even when applying coarsening schemes, the proposed approach can enforce operational constraints by simply not merging nodes that are to remain mutually exclusive. All other constraints can be applied in the same manner but on a smaller equivalent graph.

7.7 Conclusion

An optimization based graph partitioning method for sparse, balanced, non-contiguous graph partitioning applications that occur in power system is developed. Energy policy and ownership based operational constraints can be elegantly encoded in the proposed optimization scheme. This is particularly important as partitions that are of good quality from an engineering perspective might not be a feasible partition from operational view point. Proposed approach compares favorably with state of the art graph partitioning method - METIS, for several test systems ranging from a small 9 node test system to that of a large 2383 node test system. Furthermore, mathematical formulation for a set of fundamental constraints with which most, if not all, operational constraints encountered in power systems can be expressed is developed and validated experimentally. Since power system is a non-stationary system, the graphs encountered are not static but are rather dynamic. Hence, applicability of proposed approach for real-time application is demonstrated by time limiting the graph partition approach based on current power system operational practices.
Chapter 8

Balanced, Contiguous, Multilevel
Partitioning of Large Power Networks
Considering Operational Constraints

Nomenclature

**Binary Variables**

$\alpha_i^k$ \hspace{1cm} decision variable of $i^{th}$ edge that indicates whether both the adjacent vertices belong to $k^{th}$ cluster

$\beta_i^k$ \hspace{1cm} decision variable to indicate if the given $i^{th}$ node/vertex belongs to the $k^{th}$ cluster

$\sigma_i^k$ \hspace{1cm} decision variable of $i^{th}$ edge that indicates if only one of the adjacent vertices belong to $k^{th}$ cluster

$\phi_i$ \hspace{1cm} decision variable of $i^{th}$ cycle that indicates if the cycle is broken

**Sets**

$V$ \hspace{1cm} Set of all vertices/nodes/buses in a graph
Partitioning of power networks play an important role in planning and operations of power grid. Unlike other networks, power networks have operational constraints due to energy policy, ownership and regulatory restrictions. Furthermore, applications such as controlled islanding require a connected/contiguous partition. Hence, theory pertaining to contiguous partitioning problem is developed. The developed theory has direct application to several areas of power systems research. Relation to several such applications is formally derived and also explained. The developed theory is then utilized to build an optimization based multilevel network partitioning scheme that is capable of partitioning large power networks with arbitrary partition balance constraints in real-time. Quality of partition obtained through the proposed method compares favorably with a well known graph partitioning approach - METIS, for several test systems, including a large 2383 bus western polish test system and a very large 20000 bus synthetic test system.

8.1 Introduction

Many practical problems can be represented as a graph, where nodes represent some abstract information and edges represent the connection between these abstract information. Partitioning a graph representing complex data has multiple applications. For instance, solution of a large problem - partition techniques are used in such cases to divide the original problem into smaller portions that could then be solved si-
multaneously across several computing nodes. The web graph is one such example. Scales of such problem, in some cases, is in the order of billion vertices and trillion edges [114].

Graph partition is also used for computational load balancing. Load balancing is used in scientific applications such as parallel multi-physics and multi-phase simulations. Specific to power systems, load balancing schemes are used in parallel computation of power system analysis. Power system parallel transient simulators such as [98–100] is one such application area.

In power systems literature, three broad categories of partitioning methods have been used - multilevel graph partitioning algorithms, spectral partitioning algorithms and clustering techniques. Before the advantages and disadvantages of each method is discussed, we first explain the specific attributes of partitioning methods that are desirable for partitioning power networks.

1) Power systems partition requires several operational constraints due to ownership, regulatory constraints etc. that are to be satisfied by the partitioning method.
2) Certain applications such as controlled islanding [96] require connected/contiguous sub-graphs. By contiguous sub-graph we mean that there should not be an island within an island. Hence, a partition method that guarantees contiguous partition is desirable. 3) Applications such as [98,99] require balanced partition i.e. number of buses in each partition should be approximately equal. This is a desired attribute especially while partitioning the system for planning and analysis purposes.

Now that the desired attributes have been introduced, we now list the advantages and shortcoming of existing methods with respect to these attributes. Multilevel graph partitioning algorithms such as in [110] provide means to enforce contiguous sub-graph constraint and balanced partition. However, operational constraints cannot be enforced.
Spectral partitioning methods in general do not allow to enforce operational constraints or provide contiguous partitions. However, constrained spectral clustering methods allows enforcing operational constraints but contiguous partitioning is not guaranteed. An extension of spectral method reported in [97] allow for contiguous partition but operational constraints cannot be enforced. To the best of our knowledge, a spectral method which handles constraints, contiguous and balanced partitions has not been reported in literature.

Clustering technique in general do not use edge information because of the way the algorithm is designed. Hence, they do not guarantee contiguous partition. In fact, spectral methods which use clustering methods to partition the selected k-eigenvectors also have this drawback.

Hence, the focus of this work is to present a unified extensible approach where operational constraints, contiguous and balanced partitioning constraints can be incorporated into the partitioning scheme. This work is specifically intended for graphs arising out of a given power system’s physical connectivity. Graphs such as susceptance matrix, power flow, nodal connectivity etc. are ideally suited. In addition, the developed partitioning scheme is capable of handling large systems in real-time. A fact we demonstrate by partitioning a 2383 bus western polish system and a synthetic 20,000 bus test system in real-time. In this work we define real-time for these applications as one minute interval.

Literature Review

Based on average connectivity of nodes, a given power system graph can be either dense or sparse. Dense graphs typically arise in coherency based dynamic reduction such as [93,94]. For these problems, k-means, k-medoids and their extensions are well suited and are widely used. We focus our attention exclusively to sparse graphs.
Reference [4] implemented constrained spectral partitioning approach to the problem of controlled islanding. In this approach, a pairwise must-link and cannot-link constraints can be imposed on the partitioning problem by projecting the problem into a subspace where all the must-link constraints are combined to form a single node. Then, a generalized eigenvalue problem is solved and the eigenvectors of interest are chosen. k-medoids clustering is then used to obtain the partitions. k-means, k-medoids and their extensions do not consider the edge-link and hence the resulting partitions are not guaranteed to be contiguous.

In fact, the authors in [97] who also use spectral partitioning acknowledge this fact and propose means to overcome this short-coming. Reference [97] presents a method which addresses the connectivity issue. However, whether the method will work for any generic power system graph is unclear. In addition, constraints are not implementable. A hybrid of the two methods presented in [4] and [97] would be an area worth exploring.

Reference [115] approach the problem of power system partition for use in controlled islanding applications as a two-step process. The authors use an ordered binary decision diagram (OBDD) to obtain a partition that minimizes the power imbalance in each cluster. In the second phase, power flow analysis is used to exclude solutions which result in line flow violations. Complexity of the solution process increases exponentially with system size in such approaches. Hence, applicability for large graphs is questionable. In addition, operational constraints cannot be enforced and contiguous partitions cannot be guaranteed.

Reference [106] uses evolutionary algorithm to solve a multi-attribute partitioning problem of power systems. As in [97], reference [106] also demonstrate the applicability of their methods for large systems through the use a large 2383 bus western polish system. The approach in [106] takes considerable amount of time for partition.
Hence, it is better suited for static graphs. In addition, operational constraints can only be enforced through a penalty function and such a penalty function can easily distort the solution space.

Other strategies for partitioning power systems include heuristics such as particle swarm optimization and their extensions such as in [116]. Tabu search is used in [104] for power system network partitioning. While genetic algorithm and harmony search are used in [105] and [109] respectively. Operational constraints cannot be expressed in any of these methods. Considering the points discussed, the contribution of this work can be summarized as follows,

**Contributions**

- Unified approach to enforce operational, contiguous and balanced partition constraints.
- Handle very large graphs that are of the size and complexity as seen at the system operator level.
- Suitable for dynamic graph partition in real-time.
- Extensibility of the developed theory for improved performance of existing power system applications.

The remainder of the chapter is organized as follows. Problem formulation including the development of set of basis constraints for enforcing operational restraints is developed in Section II and III. Extensibility of the proposed approach for various power system research areas is given in Section IV. Solution methodology is discussed in Section V. In Section VI, experimental setup including test systems and validation
process are discussed. Results and discussion comprise Section VII and conclusions are drawn in Section VIII.

8.2 Problem Formulation

This section is divided into two parts: general partitioning problem and contiguous partitioning problem. The formulation developed in general partition problem is that of a $k$-way partitioning scheme. The problem of $k$-way partition of a graph $G = (V, E)$ is defined as partitioning $V$ into $k$ subsets such that $V_i \cap V_j = \emptyset$ for $i \neq j$ and $\bigcup_i V_i = V$ [110]. In the second part i.e. contiguous partition problem, theory and constraints that enforce contiguous partitions are developed.

8.2.1 General Partitioning Problem

Consider a transmission line $m$, connecting buses $i$ and $j$. For a given $k^{th}$ cluster, we want to enforce the decision variable $\alpha^k_m$ to be 1 if and only if both $\beta_i^k$ and $\beta_j^k$ belong to $k^{th}$ cluster. When there is an edge-cut i.e. one of the buses belong to the $k^{th}$ cluster while the other does not, then the decision variable $\sigma^k_m = 1$. A third possibility is that both $\beta_i^k$ and $\beta_j^k$ do not belong to $k^{th}$ cluster and hence $\alpha^k_m$ and $\sigma^k_m$ should both be zero. This constraint is expressed as in (8.2.1).

$$\beta_i^k + \beta_j^k - 2 \cdot \alpha^k_m - \sigma^k_m = 0 \quad (8.2.1)$$

When $\beta_i^k$ and $\beta_j^k$ take a value of 1, $\alpha^k_m = 1$ to satisfy (8.2.1). When sum of $\beta_i^k$ and $\beta_j^k$ is 1, then $\sigma^k_m = 1$. When sum of $\beta_i^k$ and $\beta_j^k$ is 0 both $\alpha^k_m$ and $\sigma^k_m$ will be 0. Since there is one binary variable representing a given node in each cluster, care must be taken that a given node is not placed in multiple clusters. This constraint is enforced
as given in (8.2.2).

\[ \sum_{k \in K} \beta_i^k = 1 \quad \forall i \in V \]  \hspace{1cm} (8.2.2)

**Cardinality of Partition**

At the system operator level, network is divided into zones for planning and operation purposes. In such cases it is desirable that the partitioned zones are balanced with respect to number of buses in each partition. Network partitioning problems arising in parallel transient simulators such as in [98–100,117] also require balanced partitions. In this work, a tolerance level \( \epsilon \) is defined for each partition size and cardinality of partition is enforced as in (8.2.3). Where, \( \gamma = \frac{|V|}{|K|} \) is defined as the ideal partition size.

\[ \gamma - \epsilon \leq \sum_{i \in V} \beta_i^k \leq \gamma + \epsilon \quad \forall k \in K \]  \hspace{1cm} (8.2.3)

**Objective**

The objective is to minimize the total weighted edge cut.

\[ \min \sum_{k \in K} f^T \cdot \sigma_i^k \forall i \in E \]  \hspace{1cm} (8.2.4)

**8.2.2 Contiguous Partitioning Problem**

For any given graph, the algebraic sum of number of vertices, edges, cycles and partitions are related as in (8.2.5).

\[ |V| - |E| + |C| = |K| \]  \hspace{1cm} (8.2.5)
In order to obtain \( k \) disjoint connected sub-graphs, (8.2.5) should be satisfied. This is a necessary and sufficient condition.

**Theorem 1.** For all chords defined as \( G \setminus T \) of a given graph \( G \) and its corresponding spanning tree \( T \) there exists a strictly fundamental cycle basis.

A chord is defined as edges of \( G \setminus T \) i.e. chords are those edges that are present in \( G \) but not in \( T \). A strictly fundamental cycle basis is one in which there is at least one unique edge in each cycle i.e. the basis cycles are linearly independent. When a chord is added to the spanning tree \( T \), there always exists a path between the adjacent vertices of the chord because every vertex in the tree is connected and every vertex contained in \( G \) is also contained in \( T \). Hence, it is possible to find a strictly fundamental cycle basis by finding a path in \( T \) between the adjacent vertices of a chord. The cycle can then be closed by adding the chord to this path.

**Theorem 2.** A strictly fundamental cycle basis of \( G \) formed using Theorem 1 can be broken if and only if all possible paths between the adjacent vertices of the corresponding chord is broken or if the chord itself forms a cut-set.

Consider a graph realization from \( T \) to \( G \), formed by adding one chord at a time. Recall the relation between number of vertices, edges, basis cycles and number of connected components given in (8.2.5). The spanning tree \( T \) by definition is connected. Hence, adding a new edge i.e. a chord will not change the connectivity and the number of vertices is also the same. As a result, an increase in \( |E| \) should also result in equal increase in \( |C| \) such that \( |V| - |E| + |C| = 1 \). Where the number of connected components is 1. Similarly, removal of any chord from the graph \( G \) will result in decrease of \( |C| \) by 1. In addition, if all paths between the adjacent vertices of a chord is broken by removing edges from \( G \) then no cycle can be formed.
8.2.2.1 Enforcing Contiguous Constraint

Contiguous constraint is enforced by enforcing the \( i^{th} \) chord to form a cut-set if any of the edges that form the \( i^{th} \) basis cycle has an edge-cut. This restrictive constraint directly follows Theorem 2 and enables us to keep track of number of remaining basis cycles in the graph during partition, using which (8.2.5) can be satisfied. Consider a set of vertices \( V_c^i \) that form a given \( i^{th} \) cycle. Similarly consider \( E_c^i \) that contains all edges of the \( i^{th} \) cycle except the \( i^{th} \) chord \( \tau_i \).

\[
2 \cdot \phi_i \leq \sum_{k \in K} \sigma^k_{E_c} \leq 2 \cdot |E_c^i| \cdot \phi_i \quad (8.2.6)
\]

\[
2 \cdot \phi_i \leq \sum_{k \in K} \sigma^k_{\tau_i} \quad (8.2.7)
\]

\[
\sum_{i \in C} (1 - \phi_i) - \sum_{k \in K} \sum_{m \in E} \alpha^k_{m} = -|V| + |K| \quad (8.2.8)
\]

Equation (8.2.6) enforces \( \phi_i \) to take a value of 1 when any of the edges of \( E_c^i \) has an edge-cut. \( \phi_i \) is scaled by 2 as edge-cut variable \( \sigma \) takes a value of 1 for both partitions \( k_1 \) and \( k_2 \) to which the adjacent vertices belong. Equation (8.2.7) forces an edge-cut of \( i^{th} \) chord \( \tau_i \) when \( \phi_i = 1 \). Once again the multiplier 2 is used for the same reason as above. Equation (8.2.8) is a modified version of (8.2.5) where \(|E|\) and \(|C|\) are expressed in terms of \( \alpha \) and \( \phi \) respectively. \(|V|\) and \(|K|\) are constants.

8.3 Enforcing Operational Constraints

In this section, a set of basis constraints to enforce operational restraints is developed. The developed constraints can be combined in various ways to enforce a set of desired operational constraints.
8.3.1 Forced Grouping

Consider a set of buses $S_1$ that should be grouped together in a given $k^{th}$ cluster, where the choice of $k^{th}$ cluster is arbitrary. Equation (8.3.1) is used to forcibly group all buses in the set $S_1$ in $k^{th}$ cluster. Forced grouping situation often arise in systems operation due to ownership rights. Forced grouping constraint can be used in such situations to maintain the exclusivity of a set of critical nodes to a particular partition. Forced grouping is implemented as given in (8.3.1).

$$\sum_{i \in S_1} \beta_i^k = |S_1| \quad (8.3.1)$$

Where $|S_1|$ is the cardinality of set $S_1$ i.e. the number of nodes in $S_1$. Equation (8.3.1) will be satisfied for a chosen $k^{th}$ cluster only when all the buses in $S_1$ is placed in the given $k^{th}$ partition. The constraint that a transmission line should not be cut in the partitioning phase can also be enforced using forced grouping. This can be enforced by simply including the adjacent vertices of the key transmission line in $S_1$.

8.3.2 Mutual Exclusivity

Consider a set of buses $S_1$ that should be grouped together in an arbitrary $k^{th}$ cluster with an additional constraint that the $k^{th}$ cluster should have a null set for a group of nodes in $S_2$. This type of conditions often appear in power systems when a critical set of nodes belonging to a balancing authority should be partitioned together, however, no set of critical nodes from another balancing authority can appear in the same partition.

$$\sum_{i \in S_1} \beta_i^k + \sum_{j \in S_2} \beta_j^k = |S_1| \quad (8.3.2)$$

Equation (8.3.1) along with (8.3.2) enforces mutual exclusivity. Equation (8.3.1)
imposes that all buses in set $S_1$ be placed together. While (8.3.2) enforces that only nodes from $S_1$ is present in $k^{th}$ partition. At the most basic level when $|S_1| = 1$ and $|S_2| = 1$, the mutual exclusivity constraint can be used to enforce edge-cut between the element in $S_1$ and $S_2$. Hence, a particular tie-line that needs to be left as a tie-line during the $k$-way partition can be enforced using this constraint.

8.4 Extensibility

Extensibility of the proposed approach to various power system applications will be shown in this section. Specifically, the problem of controlled islanding, minimal loss configuration of distribution networks and resilient reconfiguration of distribution systems will be discussed. Through these examples we wish to showcase the extensibility and flexibility of the proposed approach either directly as a partitioning scheme or indirectly where an existing application could benefit using the theory developed in this work. It is particularly important to note that radial constraint is required for existing works such as [118, 119]. Both [118, 119] and other such applications are set in MILP based formulations. The difficulty that existed previously is due to enforcing radial constraint within this framework for generic systems. Through section 8.4.2 and 8.4.3 we will show how these works can benefit from the theory developed in this work.

8.4.1 Controlled Islanding

Controlled islanding applications are a two stage process. In the first stage, coherent generator groups are identified. In the second stage a set of load buses that are to be grouped with individual coherent generator groups are identified. The general idea that is followed in the second phase is that minimizing weighted edge-cut will result
in minimizing total load shed. Reference [4,96] follow this approach. Reference [97] defines a term expansion ratio and finds the partition based on that. Please note that [97] is a generic partition method. Given a set of coherent generators, we will develop an exact formulation based on equation (8.3.3) for solving the problem of controlled islanding.

Consider a set of \( k \) coherent generator groups. First, we enforce the forced grouping constraint developed in section 8.3.1 to enforce that each coherent generator group is placed together. Equation (8.4.1) forces each coherent group to be placed together in the same cluster i.e. each \( k^{th} \) cluster will contain all the generators in a given \( k^{th} \) coherent group but none from other coherent groups. The cardinality constraint developed in (8.2.3) is modified to (8.4.2).

\[
\sum_{i \in PV^k} \beta_i^k = |PV^k| \tag{8.4.1}
\]

\[
\sum_{j \in PQ} P_j \cdot \beta_j^k \leq \sum_{i \in PV^k} P_i \cdot \beta_i^k \forall k \in K \setminus K^* \tag{8.4.2}
\]

Where \( PQ \) is the set of load buses and \( PV^k \) is the set of generator buses in \( k^{th} \) coherent cluster. Notice \( k \in K \setminus K^* \) i.e. (8.4.2) is enforced on all partition except for slack partition \( K^* \). The objective function will be modified as,

\[
\min \sum_{j \in PQ} P_j \cdot \beta_j^{K^*} \tag{8.4.3}
\]

In addition, the equations developed in section 8.2 are used to enforce contiguous partition. Notice how (8.4.2) and (8.4.3) solves the problem of assigning discrete load blocks to individual clusters. The proposed algorithm will try to solve the problem of assigning loads such that \( \sum_{j \in PQ} P_j \cdot \beta_j^k \approx \sum_{i \in PV^k} P_i \cdot \beta_i^k \forall k \in K \setminus K^* \) so that
(8.4.3) is minimized. For excess load conditions, the excess load that is placed in the slack cluster can then be shed. This is an exact representation of controlled islanding problem. However, all the load shed will happen in the slack cluster. If this is undesirable and rather a participation factor based load shed among clusters is desired then the following modification can be done.

\[
\sum_{j \in PQ} P_j \cdot \beta_j^k \leq \sum_{i \in PV^k} P_i \cdot \beta_i^k - \rho^k \cdot P_{mis} \forall k \in K\setminus K^* \tag{8.4.4}
\]

Where \( \rho^k \) is the participation factor for load shed and is a number between 0 and 1 such that \( \sum_{k \in K} \rho^k = 1 \). \( P_{mis} \) is the mismatch between load and total generation i.e. excess load.

### 8.4.2 Minimal Loss Configuration

The author in [118] proposes a MILP based optimization framework for minimizing losses in a radial distribution system. Reference [118] describes equation (37) in [118] as cycle opening constraint. For this purpose, the author in [118] finds a subset of all possible cycles and enforces the cycle opening constraint on these cycles. The author remarks that this is done to improve solution convergence speed. The number of possible combinations of cycles is exponentially large than the number of basis cycles. Hence, instead of finding a subset of cycles and then enforcing a constraint for each one of those cycles, one can instead use a single constraint that enforces all chords should have an edge-cut \( \sum_{i \in C} \alpha_{ri} = 0 \).

Please note that this also ensures radial constraint and can replace equations (35) and (36) in [118] subject to corresponding changes to other formulations. This approach reduces the solution space. As a result, performance gain in terms of faster solution time can be expected.
8.4.3 Resiliency

In reference [119] the authors remark that the formulation developed are for radial topology and for other topologies the developed formulation needs modification and is left for future work. We remark that by employing the formulation developed in section 8.2.2 the work in [119] could potentially be extended for any power network as long as modifications for linearizing real and reactive power equations for generic mesh networks are addressed.

8.5 Solution Methodology

Multilevel graph partitioning approach used in this work can be described as a reductionist approach where a given graph $G$ is reduced to a much smaller graph $G^*$. The reduced graph $G^*$ usually has less than hundred vertices. A chosen partitioning algorithm is applied on this reduced graph, which is then projected back to the original graph $G$. The entire process can be described in three stages as given in Fig. 8.1.

The original graph containing 8 nodes and 8 edges is coarsened to a graph with 4 nodes and 4 edges in the first coarsening phase by combining nodes $\{1,2\}, \{3,4\}, \{5,6\}$.

![Figure 8.1: Solution methodology of multilevel graph partitioning algorithm.](image)
and \{7,8\} to form super nodes. The super nodes are again aggregated to form a 2
node, 1 branch graph such that \{1,2,3,4\} and \{5,6,7,8\} form the two super nodes.
Partitioning and uncoarsening phase starts after user defined \(n\) coarsening stages. In
the first partitioning phase, nodes \{1,2,3,4\} and \{5,6,7,8\} are partitioned into two
clusters. The graph is then projected back to the next uncoarsened level. As there
are more degrees of freedom at this level, the partitioning algorithm looks for better
partition - a refinement process. The combined uncoarsening and refinement process
is repeated until the original graph is projected back.

8.5.1 Coarsening

A given graph \(G\) is successively coarsened to \(G^*_{i+1}\) such that \(G^*_{i+1} < G^*_i\). That is, the
graph of each successive coarsening step is smaller than the previous stage. Various
algorithms exist for coarsening a given graph. We use heavy edge matching (HEM)
algorithm in this work. Interested readers are referred to [110] for a broader list of
available algorithms and their descriptions.

8.5.1.1 Heavy Edge Matching

Matching of a given graph \(G\) is defined as the set of edges that have no common
vertices. Each coarser level, \(G^*_{i+1}\) of a graph \(G\) is constructed by collapsing the nodes
of \(G^*_i\) that are a part of matching edges into a single super node [110]. HEM is a
matching scheme where edges that form the matching set are selected in such a way
that the total sum of edge weights contained in the matching set is maximized. An
efficient randomized algorithm as in [110] is used.
8.5.2 Partition

It is in the partitioning phase that the graph is partitioned into \( k \) disjoint sets. However, instead of solving the global problem once, the reductionist approach of multilevel algorithms breaks down the problem to that of several local solutions, one for each coarsening phase.

8.5.3 Uncoarsening

In the uncoarsening phase, the partitioned graph from \( G^*_i = (V^*_i, E^*_i) \) is projected back to \( G^*_{i+1} = (V^*_{i+1}, E^*_{i+1}) \). It is important to note that \( G^*_i = (V^*_i, E^*_i) \) has more degrees of freedom than \( G^*_{i+1} = (V^*_{i+1}, E^*_{i+1}) \). Hence, a refinement algorithm is used to refine the solution from previous coarser level graph. This procedure is repeated all the way up until the origin graph \( G = (V, E) \) is obtained.

8.5.4 Role of Optimization

Efficient partitioning algorithms such as recursive bisection and \( k \)-way partitioning exist in literature. However, it is not possible to encode operational constraints in these algorithms. At this stage, it is important to differentiate between multi-constrained multilevel graph partitioning algorithms such as [102] implemented in [110] and the proposed approach. The constraints that occur in power systems are fundamentally different and cannot be expressed in multi-constrained multilevel graph partitioning algorithms. Interested readers are referred to [102] for more details.

We use binary integer linear program (BILP) for partitioning the graph. Expressing the partitioning problem as an optimization problem provides two advantages,

- A concise mathematical way to represent the problem as well as the operational constraints.
\begin{itemize}
  \item An efficient way to find solutions and verify optimality of local solutions.
\end{itemize}

In order to better understand the process, a brief description of branch and bound (BB) algorithm that is used to find solution of (BILP) is first discussed.

\subsection*{8.5.4.1 Branch and bound algorithm}

Branch and bound algorithm can be described as a general purpose method to evaluate all possible combinations of a combinatorial optimization problem and find an optimal solution. All possible solutions of a problem is represented as a tree. BB evaluates each branch with respect to the constraints and objective. If an infeasible solution is found or if the obtained feasible solution in a particular branch is sub-optimal with respect to the previously known best solution, then that particular branch is pruned i.e. the branch is no longer evaluated.

\subsection*{8.5.4.2 Use of BB in Multilevel Partition}

Every successive stage of uncoarsening phase results in a graph with more degree of freedom. At each uncoarsening phase, we use the solution from the previous stage as a starting point for BB used in CPLEX [29]. BB uses this information to prune branches in a very efficient way. This refinement process is continued till the original graph $G = (V, E)$ is obtained or until the prescribed time limit is reached.

\subsection*{8.5.5 Practical Considerations}

Recall that a spanning tree $T$ of a given graph $G$ is not unique. As a result, when the previous coarser level result is projected to the next uncoarsening phase, the constraints relating to the decision variable $\phi$ might not be satisfied. If this is the case, then no information from previous stage is usable by BB algorithm. To avoid
such situations we propose the following way to recompute a feasible initial condition based on results from previous coarser level.

- Each disjoint set has no edge-cut. Hence, we find a spanning tree for these disjoint sets and construct a strictly fundamental cycle basis. Since there are no edge-cuts, \( \phi = 0 \) for all cycles in these disjoint sets.

- From the edges that forms the cut-set, a set of \(|K| - 1\) edges that connect the different clusters are chosen to connect the graph such that there are no disjoint sets.

- The remaining edges of the cut-set are the chords and we find the associated strictly fundamental basis cycles.

8.6 Experimental Setup

8.6.1 Test System

8.6.1.1 Small, medium and large System

IEEE 39 bus test system is used as a representative case of a small power system. While IEEE 118 and 300 bus test system are used to represent medium sized power system. The 2383 bus, western polish system is used as large power system. All test systems and their power flow results, except the 20000 node synthetic test system are obtained from MATPOWER [74].

8.6.1.2 Very Large System

A synthetic test system with 20,000 nodes and 36,501 edges was created using Havel-Hakimi algorithm [120]. The generated graph is similar to case2383wp with respect to degree distribution. Havel-Hakimi algorithm is a graph realization algorithm that
generates a graph with a given degree distribution, if such a graph is feasible. NetworkX [121] is used to generate the graph. Edge weights are assigned using uniform random distribution. Graph properties of all the test systems used in this work is given in Table 8.1.

### 8.6.2 Validation

We use a two part validation process. In the first part, we take a non-trivial yet tractable 39 bus test system to illustrate how operational constraints developed in section 8.3 is enforced. However, for completeness, we also enforced operational constraints on all test systems and validated the results - the graphical representation of which is not included due to space restriction. In the second part, numerical comparisons are drawn between the proposed method and METIS.

The reasoning behind this two part validation is two folds. 1) When operational constraints are not enforced, the proposed method and METIS both minimize cumulative edge-cut to obtain partition. Hence, they are very similar in that aspect. In addition, a stable and mature software package for METIS is available. Hence, meaningful quantitative analysis can be performed. 2) Methods that can handle operational constraints (constrained spectral method) cannot handle balanced partition and does not guarantee contiguous partition. As a result, meaningful quantitative comparison is not feasible.

<table>
<thead>
<tr>
<th>Casename</th>
<th>No. of Buses</th>
<th>No. of Branches</th>
<th>No. of Basis Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>case39</td>
<td>39</td>
<td>46</td>
<td>8</td>
</tr>
<tr>
<td>case118</td>
<td>118</td>
<td>179</td>
<td>62</td>
</tr>
<tr>
<td>case300</td>
<td>300</td>
<td>409</td>
<td>110</td>
</tr>
<tr>
<td>case2383wp</td>
<td>2383</td>
<td>2886</td>
<td>504</td>
</tr>
<tr>
<td>case20000</td>
<td>20000</td>
<td>36501</td>
<td>16502</td>
</tr>
</tbody>
</table>
8.7 Results and Discussion

8.7.1 Enforcing Operational Constraints

8.7.1.1 Preliminary

When HEM sets are formed during the coarsening phase, one simple rule is followed to allow for transformation of constraints of the original graph $G = (V, E)$ to coarser graph $G_i^* = (V_i, E_i)$. Any set $S_1$ and $S_2$ that should remain in different clusters are never paired to form a super node. This is because it is impossible to find a feasible solution for a given coarse graph $G_i^* = (V_i, E_i)$, when two nodes that are not supposed to be together are fused into a super node. This rule allows all constraints developed in section 8.3 to be implementable at all coarse graph levels. The constraints are merely transformed to the coarse graph $G_i^* = (V_i, E_i)$.

8.7.1.2 Implementation

In order to illustrate the process of enforcing operational constraints for a given graph, a tractable, yet non-trivial 39 bus test system is used. Both operational constraints developed in section 8.3 are enforced on 39 bus test system. Three way optimal partition of the 39 bus test system is shown in Fig. 8.2a.

In the base partition, nodes 13, 14 belong to cluster 1, while node 15 belongs to cluster 2. We wish to enforce forced grouping scheme on this set. That is, we wish to group set $S_1 = \{13, 14, 15\}$ together. Recalling the constraint developed in (8.3.1), forced grouping for $S_1$ can be written as $\sum_{i \in \{13, 14, 15\}} \beta_i^1 = 3$.

Since the nodes are to be placed in cluster 1, we assign $k = 1$. In addition, consider the set $S_2 = \{9\}$ that must remain mutually exclusive with the set $S_1$. This is enforced by the additional constraint $\sum_{i \in \{13, 14, 15\}} \beta_i^1 + \sum_{i \in \{9\}} \beta_i^1 = 3$. Enforcing these constraints results in desired partition as shown in Fig. 8.2b.
Comparison with METIS

Comparison with METIS is categorized into four classes - small, medium, large and very large, based on size of the graph used. Four partition sizes, namely, 2, 3, 4 and 5 are used. Average value of real power flowing through transmission lines from base case power flow is used as edge weights i.e. \( w_{i,j} = \frac{|P_{i,j}| + |P_{j,i}|}{2} \). Same metric is also used by [4,97]. Test case data and power flow is obtained from MATPOWER [74]. METIS allows only integer edge weights. Hence, the per unit (PU) values of real power flow is changed to actual units (MW) to allow for better representation. The amount of cumulative weighted edge-cut as defined in (8.2.4) is used to compute the objective of METIS partition in (8.7.1).

\[
C_m = \sum_{k \in K} f^T \cdot \sigma_m^k \; \forall \; m \in E
\]

Figure 8.2: Constrained partition

(a) Optimal three way partition of 39 bus system.

(b) Forced grouping
\[ C_p = \sum_{k \in K} \frac{f^T \cdot \sigma^k_p}{C_m} \forall p \in E \]  

Equation (8.7.1) and (8.7.2) represent the cost/total cumulative edge-cut from METIS and proposed approach respectively. Total edge cut of proposed approach is expressed in terms of total edge-cut of METIS. Recall that the objective is to minimize the total edge-cut. Hence, \( C_p < 1 \) is indicative of the fact that proposed approach performed better than METIS. For instance, \( C_p = 0.5 \) indicates that the proposed approach has exactly 50% of weighted edge-cut when compared to METIS. Similarly, \( C_p > 1 \) is indicative of the fact that the proposed method has higher total edge-cut when compared to METIS for the \( k \)-way partitioning of a given test system. The parameter \( \epsilon \) for all cases were set at \( 0.1 \cdot |V| \). For example, the 39 bus test system would have a maximum unbalance of 4 with respect to cardinality of partition.

Geometric mean value, geometricmean = \( \sqrt[n]{r_1 \cdot r_2 \cdot \ldots \cdot r_n} \) is used to compute all results. Where \( r_1, r_2 \) etc. are the values obtained in run 1, run 2, etc. A value of \( n = 10 \) is used i.e. all reported numerical values are computed over 10 runs. All partitions for METIS were computed using \( k \)-way partitioning strategy using \textit{gpmetis} program. We set the flag -contig to enforce contiguous partitions and we enforce a maximum imbalance constraint of \( 0.1 \cdot |V| \). All other options were set at default values.

### 8.7.2.1 Small and medium System

IEEE 39 bus test system is used as small test systems, while IEEE 118 and IEEE 300 bus test system is used as medium sized test system. Numerical results for all cases is shown in Table 8.2. Both METIS and the proposed system obtained same result for 3 way partition of 39 bus test system. The proposed method obtained better partition for all other cases of 39, 118 and 300 bus test systems.
Table 8.2: Partition Comparison

<table>
<thead>
<tr>
<th>Casename</th>
<th>Performance of Proposed Method ($C_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nParts=2</td>
</tr>
<tr>
<td>case39</td>
<td>0.71</td>
</tr>
<tr>
<td>case118</td>
<td>0.87</td>
</tr>
<tr>
<td>case300</td>
<td>0.54</td>
</tr>
<tr>
<td>case2383wp</td>
<td>0.91</td>
</tr>
<tr>
<td>case20000</td>
<td>0.36</td>
</tr>
</tbody>
</table>

8.7.2.2 Large System

The 2383 bus western polish test system is used as large test system. Graphical comparison of the results presented in Table 8.2 for 2383 bus test system is shown in Fig. 8.3a. Both METIS and the proposed method result in similar partition quality for two, three, four and five way partition. Lowest $C_p$ value of 0.77 is obtained for five way partitioning. This represents 23% fewer weighted edge-cuts when compared to METIS.

8.7.2.3 Very Large System

A synthetic 20000 nodes test system is created with a similar degree distribution as that of 2383 bus western polish test system. Both METIS and the proposed method are used to partition this very large synthetic test system into 2, 3, 4 and 5 way partitions. The numerical results are presented in Table 8.2, while a graphical representation is shown in Fig. 8.3b. The proposed method performed better than METIS for two, three and five way partition. While a similar performance was obtained for four way partition. Two way partition resulted in the lowest $C_p$ value of 0.36 while four way partition resulted in the highest $C_p$ value of 0.86.
Figure 8.3: Performance comparison for large and very large test systems. (a) and (b) shows comparison for 2,3,4 and 5 partitions of case2383wp and case20000 respectively.

8.7.2.4 Computational Performance

All partition runs using the proposed method were time-limited to a maximum of 1 minute. Partition at the coarsest level with the proposed method took less than 10 seconds even for the very large 20,000 bus test system. However, successive refinement at each uncoarsening phase takes longer time as the number of variables are larger at each successive stage. We note that for the exception of 39 and 118 bus test systems the proposed approach did not reach the final uncoarsening level within the allotted 1 minute time. Under these conditions, the best available solution is projected back to the original graph and the result is validated. Hence, for static problems which are not time limited, the proposed method is likely to perform better when compared to time-limited dynamic graph partition. All reported results were obtained on Intel i7 4550u processor with 8GB RAM. In comparison, METIS took less than one second of computation time for all partitions.
8.8 Conclusion

A multilevel partition technique for partitioning power networks while considering arbitrary operational constraints, contiguity of partition and arbitrary balance of partition is presented. Quantitative comparisons are made with a state of the art multilevel graph partitioning technique - METIS over a wide range of test systems from 39 buses to 20000 buses. The experimental validation illustrates that high quality graph partitions that compare favorably to that of METIS is produced by the proposed method with the advantage of being able to encode power systems operational constraints. A set of basis constraint equations are developed which can be combined in different ways to create complex operational restraints in the partitioning problem. Furthermore, the feasibility of the partitioning scheme for real-time applications is demonstrated for dynamic graphs that occur in power networks by time-limiting the partitioning process to a maximum of one minute.
Chapter 9

Conclusion and Contributions

9.1 Motivation

Recent developments in computing, communication and optimization techniques have piqued interest in improving the current operational practices and in addressing the challenges of future. This dissertation takes leverage of these new developments for improved quasi-static analysis of power systems for applications in the area of power system planning, operation and control.

Specifically the focus is on development of better mathematical models for optimization applications involving storage systems, demand response, resiliency and partitioning. Better mathematical models in this context refer to models that,

- Capture properties of the physical system to a higher degree of accuracy.

- Exhibit scalability i.e. solution to large problems should be solvable within the time frame of the intended application.

For example, consider the area of demand response where the task is to schedule demand responsive loads for improved operational efficiency. There are multiple ways of scheduling the demand responsive loads. Hence, the goal is to find the optimal way
of scheduling by means of an optimization model in order to maximize operational benefits. The results of the optimization process depend on the level on load modeling and the whether loads are represented individually or as an aggregate load.

Aggregate loads reduce the computational complexity of the problem. Hence, the problem can be solved faster. However, the quality of the solution will be inferior than that of a model with individual loads due to reduced degree of freedom. A better model would enable solving problems with individual load representation within required time frame. This is accomplished by having a tighter formulation for the mixed integer programming problem that is used to model demand response optimization model.

On the other hand, there are certain problems that require different solution strategy altogether. For instance, partitioning problems for controlled islanding applications involve graphs with tens of thousands of vertices and edges that need to be partitioned in real-time. For such applications, solution strategy such as multi-level partition is required in order to obtain solutions in real-time.

Hence, the premise of this work is development of better mathematical models and/or faster solution strategies for certain optimization problems that arise in the field of power system planning, operation and control.

9.2 General Conclusion

Role of optimization techniques in power systems planning, operation and control (in a quasi-static sense) has been explored in this dissertation. Different power system application areas such as renewable integration, demand response, resilience and partitioning have been explored. Although the application areas are widely different, the focal point of this research is on better management of available resources both at
supply and at demand side. To this end,

- Better planning schemes such as optimal sizing of battery energy storage systems.
- Address high ramp rate issues due to high penetration of non-controllable generation such as solar and wind generation.
- Efficient power grid operation through programs such as demand response.
- Minimizing customer impact during disturbance events through resilient operation.
- Improved control of bulk power system through better partitioning approaches which seamlessly lead into application areas such as controlled islanding were formulated.

Furthermore, the feasibility of the developed models to address utility scale problems were demonstrated. In particular, very large mixed integer programs with more than million variables were solved to optimality in less than 20 seconds. Large power grid with 20,000 nodes was partitioned subject to operational constraints in under one minute of computation time. The quality of partitions compare favorably against state of the art multi level graph partitioning schemes. Furthermore, the theory developed as part of contiguous partitioning problem has far reaching application areas such as minimal loss configuration and resilience. Particularly, theory pertaining to contiguous partition will lead to faster solution time for the minimal loss configuration problem as the feasible solution space is drastically reduced on applying fundamental basis cycle constraint.
9.3 Contributions

9.3.1 Storage Systems and Power Grid Operation

- Mitigating renewable intermittency
  - An unified approach to sizing BESS at renewable generator farms to enforce adherence to a predefined grid code.
  - A stochastic optimization scheme which utilizes the available BESS, specifically designed to address high ramp rates seen at ISO/RTOs due to duck curve phenomenon using actual data from CAISO.

- Optimal Energy Storage Sizing and Scheduling for Power Systems Operation
  - Inclusion of time-varying nature of BESS, thus the optimization is not a static optimization or a series of static optimizations without dependency between dispatches but rather an extended-time scale optimization where there is dependency of energy storage value between different dispatches.
  - Voltage security constraints are modeled in the optimization problem using the standard power flow equations. The coupling between real and reactive power is maintained i.e. no decoupled power flow type approximations are used. Hence, the optimization formulation is a non-linear programming problem.
  - In addition to the optimal sizing formulation, an optimal scheduling formulation that reduces the peak load to a desired value while satisfying relevant operational constraints with minimal operating cost is formulated.
  - Provide means to study a part of the system where BESS is located by means of scheduled tie-line flow. For example, a heavy load pocket can
be studied in isolation from the rest of the system by incorporating a predefined set of values for import/export over multiple dispatches that span the entire horizon of optimization period.

### 9.3.2 Demand Response

- Solution to a particular class of DR scheme - reliability based DR programs with detailed load modeling including development of dependent load model.

- The emphasis is on finding solution to utility scale problems to be solved to optimality in real-time. Hence, a tight MILP formulation is developed and its feasibility is experimentally verified by solving problems involving more than million variables to optimality in real-time in order to optimize peak load reduction.

### 9.3.3 Power Grid Resilience

- Optimal Operation of Microgrids Under Conditions of Uncertainty
  
  - Method to optimally operate microgrids under conditions of uncertainty introduced by renewable energy sources, under both grid connected and islanded mode.

  - For grid connected mode, optimality of the solution is shown mathematically by proving the convexity of the problem formulation.

  - Under islanded mode of operation, for extended time-horizon, a key decision making task of whether to provide energy to non-critical loads or to store excess energy is addressed.
- Energy Management System for Enhanced Resiliency of Microgrids During Islanded Operation

  - Voltage constraints, generator operating limits, line limits and efficiency model for battery energy storage systems (BESS) and plug-in hybrid electric vehicle (PHEV) are included.

  - Power injection is modeled in the optimization problem using the standard power flow equations. The coupling between real and reactive power is maintained i.e. no decoupled power flow type approximations are used. Hence, accurate representation of reactive power demand of the system is made possible.

  - Demand response is modeled through a fleet of PHEVs and adjustable loads.

  - Uncertainty of renewable generation and load is quantified using probability distribution and confidence levels are used to include uncertainty modeling in the optimization process.

9.3.4 Partitioning Power Networks

- Non-Contiguous Partition

  - A mathematical way to enforce operational constraints in partitioning scheme.

  - Provide a balanced partition. The balance constraint can be explicitly set by the user using a tolerance parameter. A large value of tolerance relaxes the balance constraint while a smaller value provides a tighter bound.

  - Means to verify optimality of partition.
• Contiguous Partition

  – Unified approach to enforce operational, contiguous and balanced partition constraints.

  – Handle very large graphs that are of the size and complexity as seen at the system operator level.

  – Suitable for dynamic graph partition in real-time.

  – Extensibility of the developed theory for improved performance of existing power system applications.
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