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Multiscale Regional Liquefaction Hazard Assessment and Mapping

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Multiscale regional liquefaction hazard assessment and mapping

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the Graduate School of
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In Partial Fulfillment
of the Requirements for the Degree
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by
Chaofeng Wang
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Abstract

Soil liquefaction is a major cause of damage during earthquakes that could trigger many kinds of ground failures such as ground settlement, lateral spreading, land slides, etc. These ground failures could cause damage to infrastructures such as buildings, bridges, and lifelines resulting in significant economic losses. Therefore it is of significant importance to assess liquefaction hazard.

The triggering and consequencing ground failure of liquefaction have been well investigated in the past decades. Nowadays, the dominant approach that correlates the observed field behavior with various in-situ “index” tests is able to achieve considerably precise assessments for free field conditions at site-specific scale. Regional scale assessments of liquefaction hazard, however, are still underdeveloped. Issues such as cross-geologic units correlations are still not systematically investigated in regional liquefaction assessment. Therefore, the main objective of this dissertation is to develop a solution framework for reliable regional assessment of earthquake-induced liquefaction hazard. Another objective is to validate this framework by applying it to several earthquake-prone regions so that liquefaction hazard maps of these regions could be added to the literature and guide designers, engineers and researchers. Moreover, the dominant method of estimating liquefaction damages via empirical correlations are not capable for complex site conditions. Therefore another objective of this dissertation is to study alternative approaches for general estimation of liquefaction damages.

To achieve these objectives, a multiscale modeling framework for better estimate of regional liquefaction hazard with material randomness and heterogeneity is developed. One advantage the developed methodology is the extension of conventional random field models to account for soil spatial variability at multiple scales and resolutions. The method allows selectively and adaptively generating random fields at smaller scales around critical areas or around areas where soil properties are known to a great detail from lab or field tests. The process is defined such that spatial correlation is consistent across length scales. Illustrative examples (Marina District in San Francisco, Alameda County in California, and Christchurch in
New Zealand) are presented. Liquefaction hazard is evaluated at multi-scale. Compared with single scale analyses, multi-scale random fields provide more detailed information and higher-resolution soil properties around critical areas. This framework provides a new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment mapping. Furthermore, finite element method is identified as a prominent alternative to traditional approach for liquefaction estimation via empirical correlations. A dynamic FEM model is built upon which an effective stress analysis is performed to estimate liquefaction-induced soil deformation at site-specific scale. It is shown the developed finite element model as a numerical tool can be used in predicting cyclic liquefaction in soils.

This research is expected to shed light on the complete understanding of soil liquefaction during earthquakes in hoping of saving economic losses in the future.
Dedication

To my beloved mother and Yidan.
Acknowledgments

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Chapter 1

Introduction

1.1 Problem statement

In fluid-saturated soil, a sudden change in stress (usually caused by rapid loading such as earthquake) could result in “liquefaction”, a phenomenon in which the pore water pressure increases and the effective stress reduces to zero so that the soil loses its stiffness and strength and behaves like a liquid. Liquefaction could trigger various ground failures, including flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils, and is recognized as a major cause of damage during earthquakes causing significant economic losses in many of the earthquakes.

Modern liquefaction engineering initiated in the wake of the two devastating earthquakes of 1964; the 1964 Niigata (Japan) and 1964 Great Alaska Earthquakes. Since then, liquefaction research has been a continuous topic. Over the four decades that have followed, significant progress has occurred.

In the early stage, researchers focus on the assessment of the likelihood of initiation of liquefaction in clean, sandy soils. With the earthquake database continues growing, increasingly aware of the additional potential problems associated with both silty and gravelly soils, and the important additional issues of post-liquefaction strength and stress-deformation behavior also began to attract increased attention. Nowadays, liquefaction research is a semi-mature field of practice in its own right.

Quantitative assessment of the “triggering”/initiation of liquefaction and the consequencing deformations of soils are two topics with great importance. The dominant approach in common engineering practice is to use correlations of observed field behavior with various in-situ “index” tests including the standard penetration test (SPT), the cone penetration test (CPT), shear wave velocity ($V_s$), etc. These in-situ test
methods have now reached a level of sufficient maturity.

The state-of-the-art developments in liquefaction still predict it at macroscopic scale. To enable a more complete understanding of this devastating instability, it is imperative that the micro-mechanical origins of liquefaction should be understood. Numerical simulation tools such as finite element method (FEM) and discrete element method (DEM) have been used to solve geotechnical problems for decades. Efforts have been made to use FEM and DEM to model liquefaction in soils (Seed et al., 1975; Zienkiewicz et al., 1999; Elgamal et al., 2002; Popescu, 2002; Lu et al., 2011).

Liquefaction is a complex phenomenon—a multiscale, multiphysics problem, originating at the particle-pore scale level but rapidly propagating to the particle cluster and specimen scales. None of the numerical methods such as FEM and DEM for liquefaction analysis could address the cross-scale issue independently. The multiscale issue in the numerical simulation of to granular media has attract attentions in the very recent years. However, few attention has been paid to the multiscale simulation of the liquefaction phenomenon.

Though liquefaction hazards could be assessed using empirical methods (i.e., correlations of observed field behavior with various in-situ “index” tests) or numerical methods (e.g., FEM, DEM, etc) at the site-specific scale, the regional scale assessments have attracted increasing attentions. Most available liquefaction hazard maps are qualitative in nature, describing the liquefaction hazard as “high”, “medium”, or “low” based on the geology of an area of interest. For example, Youd and Perkins (1987) proposed a generic method to develop such maps by classifying the liquefaction susceptibility of numerous geologic units according to their depositional type and age. An probabilistic version of this traditional method for regional assessment of liquefaction potentials is to assign a constant probability of liquefaction (i.e., the ratio of number of liquefied site to number of all sites) to a geologic unit Youd and Perkins (1987) Rix and Romero-Hudock (2006) Holzer et al. (2006a) Heidari and Andrus (2010b). These traditional methods, which do not take into account the spatial dependencies of soil properties, have the potential to be strengthened by taking into consideration of spatial structures of soil properties. Tools developed in random field and geostatistics have received considerable attention in recent years and have been applied to assess liquefaction hazard over large regions Sonmez (2003) Liu and Chen (2006) Vivek and Raychowdhury (2014) van Ballegooy et al. (2015) Chen et al. (2015).

1.2 Related work

Now days, using empirical correlation charts to assess the probability of “triggering” of liquefaction is a mature method. The measured SPT N-values and the cyclic stress ratio CSR are usually required to be
fully adjusted and normalized before using these charts to find “triggering” probability. Three best developed categories, i.e., SPT-, CPT- and $V_s$-based empirical charts methods are surveyed here.

The oldest, and still the most widely used of the four methods mentioned above, is the SPT. The use of the SPT as a tool for evaluation of liquefaction potential first began to evolve in the wake of a pair of devastating earthquakes that occurred in 1964. Numerous additional researchers have made subsequent progress, and these types of SPT-based methods continue to evolve today. CPT-based correlations now represent nearly co-equal status with regard to accuracy and reliability relative to SPT-based correlations, even better than SPT-based correlations. CPT-based correlations have evolved continuously as a number of research teams are working on development of improved CPT-based “triggering” correlations. Because of its attractive form and simplicity, the CPT-based correlation of Robertson and Wride (1998) (shown in Figure 2.6) is increasingly used for liquefaction studies. Shear wave velocity $V_s$-based correlations have been developed for several decades. It is a very attractive method because $V_s$ can be measured with non-intrusive methods. The most popular and best $V_s$-based correlation available is that of Andrus and Stokoe II (2000). This method is based on overburden stress-corrected $V_{s1}$ vs. magnitude-correlated equivalent uniform CSR and is well documented in the NCEER workshop summary papers (Youd et al. (2001)).

Regional assessment of liquefaction hazard has been a continuous topic in earthquake engineering (Holzer et al., 2006a; Lenz and Baise, 2007; Hayati and Andrus, 2008; Heidari and Andrus, 2010a; Juang et al., 2017). To assess the consequences of liquefaction over extended areas or to map the estimated liquefaction settlements to a region, it is necessary to take into account the spatial dependence of soil properties and/or the predicted settlements. Tools developed in geostatistics and random field theory have received considerable attention in recent years (Deutsch and Journel, 1992; Goovaerts, 1997; Fenton and Vanmarcke, 1998; Moysey et al., 2003; Baise et al., 2006; Lenz and Baise, 2007; Thompson et al., 2007; Liu et al., 2016) and have been applied to assess liquefaction hazard over large regions. For instance, Liu and Chen (2006) used CPT measurements to estimate the spatial structure of soil deposits. Then, random field models were coupled with Monte Carlo simulations to assess liquefaction potential in the Yuanlin area of Taiwan. Vivek and Raychowdhury (2014) explicitly considered the spatial variations of soil indices from CPT soundings when evaluating liquefaction potential. It is found that the probability of liquefaction could be significantly underestimated if the spatial dependence of soil indices is not considered. Chen et al. (2015) developed a CPT-based approach to map liquefaction potential across scales, where the soil spatial variability expressed in term of an averaged index, i.e., the Liquefaction Potential Index (LPI) developed by Iwasaki et al. (1978, 1982), is explicitly considered through internally-consistent probabilistic models developed at
multiple scales. LPI has also been adopted and modified to assess liquefaction potentials at individual CPT soundings or over extended region (Sonmez, 2003; Sonmez and Gokceoglu, 2005; Holzer et al., 2006a; Baise et al., 2006; Lenz and Baise, 2007; Thompson et al., 2007; Juang et al., 2008b). In another recent work by van Ballegoooy et al. (2015), four liquefaction vulnerability parameters including the LPI were used to map liquefaction hazard in the Christchurch area using extensive CPT database. In contrast to the extensive efforts incorporating geostatistics tools into liquefaction potential evaluation, relatively fewer studies have addressed the liquefaction-induced settlement over extended areas. In (Hinckley, 2010; Bartlett et al., 2007), classical methods of (Tokimatsu and Seed, 1987; Ishihara and Yoshimine, 1992; Youd et al., 2002; Yoshimine et al., 2006) are employed to calculate liquefaction-induced lateral spread and ground settlement, which are plotted within their respective surficial geologic units. Then, each geologic units are assigned an estimate of ground deformation based on statistical analysis.

1.3 Research approach

In this dissertation, a novel framework for regional liquefaction assessment is proposed which incorporates the multiscale random models with finite element analysis. The spatial variation of soil parameters is modeled with the random field concept. The effect of the spatial variation of soil parameters on regional liquefaction vulnerability could be estimated with finite element method method as well as a force method-based empirical liquefaction model. Furthermore, the developed framework is applied to estimation of liquefaction across several liquefaction-prone regions. In the following chapters, the methodology of this proposed framework is formulated in detail, and the effectiveness of this method is demonstrated with illustrative examples.

A brief outline of this framework is illustrated in Figure 1.1, in which two approaches for assessing regional liquefaction hazard are shown:

1. Evaluate liquefaction potential or liquefaction-induced damages at sampling sites (CPT, SPT, etc) using empirical liquefaction models or numerical approaches (finite element). Then evaluate liquefaction hazard at non-sampled sites using random field models and Monte Carlo simulation.

2. Based on site specific sampling data, predict soil properties at non-sampled sites using random field-based spatial analyses method. With predicted soil properties, liquefaction hazard could be evaluated using empirical liquefaction models or numerical approaches (finite element).

Random field model in this framework
Figure 1.1: Research approach

Figure 1.2: Illustration of the multi-scale nature of soil liquefaction phenomenon and its analyses, modified from Baker et al. (2011)
In mechanics, the erratic patterns of local stresses in a body (such as soil) are represented by locally averaged stresses for the purpose of overall force-deformation analysis. With that saying, details of the ‘lower-scale’ structure of the material affect behavior on the ‘higher-scale’ through their effect on local averages. Based on this principle, continuum mechanics becomes meaningful at the macroscale even for much complex materials such as granular soil. This process applies not only to the micro-macro scale but also to macro-higher scale analysis. For example, geologic units (usually with dimensions of kilometers) are identified and averaged by their age and depositional environment and then characterized in terms of their susceptibility. Within each geologic unit, the properties of soil and its vulnerability to liquefaction vary randomly in space following a specific correlation structure. Moreover, the evaluation of a random variable (such as the liquefaction potential index) is usually a non-linear process, which can not be estimated by the average value since the average value make sense only when the process is linear.

As illustrated in Figure 1.2, liquefaction is a multiscale multiphysics phenomenon and the analyses of it demand tools that can overcome gaps between scales. Accurate regional scale liquefaction analyses are challenging, not only because of the complexity associated with the modeling of triggering or the consequences of liquefaction but also due to the difficulties associated the estimation of heterogeneity and spatial variations of soil properties across different scales. Random fields shed light upon the solving of this problem (Baker et al., 2011). The random field theory is a set of statistical methods of great use in studying natural processes such as spatio-temporal analysis, pattern recognition and other applications. Practical problems such as regional liquefaction involving complex random phenomena can be formulated in terms of either extreme values of a random field or excursions by thresholds dividing scales (Vanmarcke, 2010). The detailed definition, history and mathematical formulation are presented in Chapter 3.

In this dissertation, based on the pioneering works of Chen et al. (2011) and Baker et al. (2011), the random field method is extended and improved to account for multiscale correlations in one simulation. The developed framework is then used for regional liquefaction assessment. A unique feature of the proposed methodology is that it allows selectively and adaptively generating random fields at smaller scales around critical areas or around areas where soil properties are known to a great detail from lab or field tests (such as CPT boreholes). This work provides a new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment with a high computation efficiency.
1.4 Contributions

Soil liquefaction, as a major cause of damage during earthquakes, could trigger ground failures including flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils, resulting in significant economic losses. Hence it is of great importance to assess the triggering and consequencing ground deformations caused by liquefaction, not only at specific sites but also across extended regions. Nowadays, multiscale investigation into liquefaction phenomenon is still in its premature stage: all of them are confined to the limit of specimen-scale tests with idealized boundary conditions; the so-called “multiscale” is confined to particle-specimen cross scale problem; spatial randomness and heterogeneity of material are not accounted, etc. This research is dedicated to extend the current methods to higher scale (i.e., site-specific scale and regional scale) to ultimately predict liquefaction hazards in large regions and is expected to shed light on a more complete understanding of soil liquefaction during earthquakes in hoping of saving economic losses in the future.

In this dissertation, a multiscale modeling framework of regional liquefaction with material randomness and heterogeneity is developed to better understand regional liquefaction phenomenon. A unique feature of the developed methodology is the extension of conventional random field models to account for soil spatial variability at multiple scales and resolutions. The method allows selectively and adaptively generating random fields at smaller scales around critical areas or around areas where soil properties are known to a great detail from lab or field tests. The process is defined such that spatial correlation is consistent across length scales. Illustrative examples (Marina District in San Francisco, Alameda County in California, and Christchurch in New Zealand) are presented. Liquefaction hazard is evaluated at multi-scale. Compared with single scale analyses, multi-scale random fields provide more detailed information and higher-resolution soil properties around critical areas. This work provides a new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment mapping.

A dynamic FEM model is built upon which a effective stress analysis is performed to estimate liquefaction-induced soil deformation at site-specific scale. It is shown the developed finite element model as a numerical tool can be used in predicting cyclic liquefaction in soils. A future work succeeding this dissertation is to integrate the developed FEM model with random field models for prediction of regional liquefaction hazard.
Chapter 2

State of the Art in Liquefaction Assessment and Mapping

2.1 Introduction

Liquefaction is a phenomenon in which the pore water pressure increases and the effective stress reduces to zero so that the soil loses its stiffness and strength and behaves like a liquid in response to a sudden change in stress such as earthquake or other rapid loading. Soil is an assemblage of soil particles which stay in contact with each other. In the nature state, the contact forces produced by the weight of the overlying particles holds individual soil particle in its place and provide strength. When applied rapid loads, the loosely-packed individual soil particles tries to move into a denser configuration. However, there is not enough time for the pore-water of the soil to be squeezed out in case of earthquake. Instead, the water is trapped and prevents the soil particles from moving closer together. Thus, there is an increase in water pressure which reduces the contact forces between the individual soil particles causing softening and weakening of soil deposit. In extreme conditions, the soil particles may lose contact with each other due to the increased pore-water pressure. In such cases, the soil will have very little strength, and will behave more like a liquid than a solid - hence, the name “liquefaction”. This process is illustrated in Figure 2.1.

Although static loads could also lead to soil liquefaction, this research focuses on cyclically-induced liquefaction during earthquake and the closely-related phenomenon of strain-softening or sensitivity. Soil liquefaction is a major cause of damage during earthquakes. Generally, liquefaction-induced ground failures include flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils. These ground
failures have caused significant economic losses in many of the earthquakes (Figure 2.2). Examples include the 1964 earthquake in Niigata, Japan (M 7.5), where landslides and related soil-liquefaction phenomena caused an estimated $US 800 million in damage (Lee et al. (1978)), the 1964 Alaskan earthquake (M 9.2), in which landslides caused an estimated $US 279 million in damage (Keefer (2002)), and the 1970 Peru earthquake (M 7.9) in which landslides buried thousands of residences and other buildings and destroyed substantial parts of the regional transportation and utility systems (Plafker et al. (1971)). In many cases, these landslides are the results of soil liquefaction in saturated sands, gravels, or silts.

“Modern” engineering treatment of liquefaction related issues evolved initially in the wake of the two devastating earthquakes of 1964; the 1964 Niigata (Japan) and 1964 Great Alaska Earthquakes. Seismically-induced soil liquefaction produced spectacular and devastating effects in both of these events, thrusting the issue forcefully to the attention of engineers and researchers. The past decades have seen emergence of the research of soil liquefaction, which can be divided into four aspects (Figure 2.3):

1. Assessment of “liquefaction potential”/“triggering”. This is necessary for most engineering projects relating to liquefaction-induced hazards.

2. Assessment of post-liquefaction strength and the consequencing overall stability of a site/structure.
Figure 2.2: Examples of earthquake-induced soil liquefaction
2.2 Assessment of triggering

Quantitative assessment of the likelihood of “triggering” or initiation of liquefaction is the necessary first step for most projects involving potential seismically-induced liquefaction. There are two general types of approaches available for this: (1) use of laboratory testing of “undisturbed” samples, and (2) use of empirical relationships based on correlation of observed field behavior with various in-situ “index” tests.

The use of laboratory testing (e.g., Nagase and Ishihara (1988) Ishihara and Yoshimine (1992) Tsukamoto et al. (2004)) is complicated by difficulties associated with sample disturbance during both sampling and reconsolidation. It is also difficult and expensive to perform high-quality cyclic simple shear testing, and cyclic triaxial testing poorly represents the loading conditions of principal interest for most seismic problems. Both sets of problems can be ameliorated, to some extent, by use of appropriate “frozen” sampling techniques, and subsequent testing in a high quality cyclic simple shear or torsional shear apparatus. The difficulty and cost of these delicate techniques, however, places their use beyond the budget and scope of most engineering studies. In addition, frozen sampling can be infeasible in soils with significant fines content, as the low permeability of these can lead to ice expansion completely disturbing the soils rather than preventing disturbance.

Accordingly, the use of in-situ “index” testing is the dominant approach in common engineering practice. Four in-situ test methods have now reached a level of sufficient maturity as to represent viable tools for this purpose (Youd et al. (2001)), and these are (1) the Standard Penetration Test (SPT), (2) the cone
penetration test (CPT), (3) measurement of in-situ shear wave velocity ($V_s$), and (4) the Becker penetration test (BPT).

Besides empirical equations, artificial neural network (ANN) is a powerful tool to find the correlations between in-site testing data and liquefaction. Pioneering work can be found in Juang et al. (2000a), Juang et al. (2000b), Juang et al. (2001) and others.

**SPT-Based Triggering Assessment**

The oldest, and still the most widely used of the four methods mentioned above, is the SPT. The use of the SPT as a tool for evaluation of liquefaction potential first began to evolve in the wake of a pair of devastating earthquakes that occurred in 1964: the 1964 Great Alaska Earthquake ($M_w = 8+$) and the 1964 Niigata Earthquake ($M_w \approx 7.5$), both of which produced significant liquefaction-related damage (e.g.: Kishida (1966) Koizumi (1966) Seed and Idriss (1971)). Numerous additional researchers have made subsequent progress, and these types of SPT-based methods continue to evolve today.

As discussed by the NCEER Working Group (Youd and Idriss (1997) Youd et al. (2001)), one of the most widely accepted and widely used SPT-based correlations is the “deterministic” relationship proposed in Seed et al. (1984, 1985). Figure 2.4 shows this relationship, with minor modification at low CSR (as recommended by the NCEER Working Group Youd and Idriss (1997)). This familiar relationship is based on comparison between SPT N-values, corrected for both effective overburden stress and energy, equipment and procedural factors affecting SPT testing (to $N_{1,60}$-values) vs. intensity of cyclic loading, expressed as magnitude-weighted equivalent uniform cyclic stress ratio ($CSR_{eq}$). The relationship between corrected $N_{1,60}$-values and the intensity of cyclic loading required to trigger liquefaction is also a function of fines content in this relationship, as shown in Figure 2.4.

This correlation (Seed et al. (1984)), however, has no formal probabilistic basis, and so provides no insight regarding either uncertainty or probability of liquefaction. Efforts at development of similar, but formally probabilistically-based, correlations have been published by a number of researchers, such as Liao et al. (1988), Liao and Lum (1998), Youd and Noble (1997), Toprak et al. (1999) and Moss et al. (2006). During the past decades, these correlations have evolved in these aspects (1) accumulation of a significantly expanded database of field performance case histories, (2) use of improved knowledge and understanding of factors affecting interpretation of SPT data, (3) incorporation of improved understanding of factors affecting
Figure 2.4: Correlation between equivalent uniform Cyclic Stress Ratio and SPT $N_{1,60}$-value for events of magnitude $M_w = 7.5$ for varying fines contents, with adjustments at low Cyclic Stress Ratio as Recommended by NCEER working group (Modified from Seed et al. (1984))
site-specific ground motions (including directivity effects, site-specific response, etc.), (4) use of improved methods for assessment of in-situ cyclic shear stress ratio (CSR), (5) screening of field data case histories on a quality/uncertainty basis, and (6) use of higher-order probabilistic tools (Bayesian Updating).

Figure 2.5: Probabilistic correlations for evaluation of liquefaction potential for $M_w = 7.5$, $\sigma'_v = 0.65$atm and fines content $\leq 5\%$, modified from Cetin et al. (2004)

Now days, using correlation charts such as the one shown in Figure 2.5 to assess the probability of “triggering” of liquefaction is a mature method. The measured SPT N-values and the cyclic stress ratio CSR are usually required to be fully adjusted and normalized before using these charts to find “triggering” probability. For example, to use the chart in Figure 2.5, it is required in Cetin et al. (2004) that the measured SPT N-values need to be corrected to $N_{1,60}$-values, then be further corrected for fines content to $N_{1,60,cs}$-values and CSR should be converted to in-situ equivalent uniform cyclic stress ratio ($CSR_{eq}$), then be adjusted by the magnitude-correlated Duration Weighting Factor resulting in $CSR_{eq,M=7.5}$, then be further adjusted for

The CPT offers advantages with regard to cost and efficiency (as no borehole is required). A second advantage is consistency, as variability between equipment and operators is small (in contrast to SPT). The most important advantage of CPT, however, is continuity of data over depth. SPT can only be performed in 18-inch increments, and it is necessary to advance and clean out the borehole between tests. Accordingly, SPT can only be performed at vertical spacings of about 30 inches (75 cm) or more. As a result, SPT can completely miss thin (but potentially important) liquefiable strata between test depths. Similarly, with a 12-inch test height and allowance for effects of softer overlying and underlying strata, SPT can fail to suitably characterize strata less than about 3 to 4 feet in thickness. CPT, in contrast, is fully continuous and so “misses” nothing.

Because of its attractive form and simplicity, as well as its endorsement by the NCEER Working Group, the CPT-based correlation of Robertson and Wride (1998) (shown in Figure 2.6) is increasingly used for liquefaction studies. This correlation is well described in the NCEER summary papers (Youd et al. (2001)).

Empirical equations to describe the chart in Figures 2.6 are developed using regression analysis and have been updated with the increase of database. Combining regression analysis with neural networks, Juang et al. (2003) developed an empirical equation for CRR based on the cone penetration test (2.7).
Figure 2.6: CPT-based liquefaction triggering correlation for “Clean” Sands Proposed by Robertson and Wride (1998)
relatively large database consisting of CPT measurements and field liquefaction performance observations of historical earthquakes is analyzed. This database is first used to train an artificial neural network for predicting the occurrence and nonoccurrence of liquefaction based on soil and seismic load parameters. The successfully trained and tested neural network is then used to generate a set of artificial data points that collectively define the liquefaction boundary surface, the limit state function. An empirical equation is further obtained by regression analysis to approximate the unknown limit state function. The empirical equation developed represents a deterministic method for assessing liquefaction resistance using the CPT. Based on this deterministic method, probabilistic analyses of the cases in the database are conducted using the Bayesian mapping function approach. The results of the probabilistic analyses, expressed as a mapping function, provide a simple means for probability-based evaluation of the liquefaction potential. The developed simplified method compares favorably to a widely used existing method.

One of the major challenges associated with the use of CPT data for liquefaction studies is the normalization of measured sleeve and tip resistance for the influence of effective overburden stress. A recent research Moss et al. (2006) readdressed this issue by applying cavity expansion methods to a prior empirically based normalization technique. The results can be expressed as a mean and variance of the seismic demand needed to trigger liquefaction conditional on the penetration resistance, or, alternatively, the mean and variance of the penetration resistance needed for liquefaction conditional on the seismic demand.

**$V_s$-Based Triggering Assessment**

Liquefaction triggering correlations based on measurements of in situ shear wave velocity ($V_s$-based correlations) have been developed for several decades. It is a very attractive method because $V_s$ can be measured with non-intrusive methods (e.g. Spectral Analysis of Surface Waves (SASW)). Another advantage of $V_s$-based method is that $V_s$ can be measured in coarse soils (gravelly soils and coarser) in which SPT and CPT can be obstructed by interference with coarse soils particles. VS-based correlations can provide both a potentially rapid screening method, and a method for assessment of coarse, gravelly soils which cannot be reliably penetrated or reliably characterized with small diameter penetrometers (SPT and CPT). The most popular and best $V_s$-based correlation available is that of Andrus and Stokoe II (2000), the core of which is plotted in Figure 2.8. This method is based on overburden stress-corrected $V_s$ vs. magnitude-correlated equivalent uniform CSR and is well documented in the NCEER workshop summary papers (Youd et al. (2001)).

$V_s$-based correlation is very well built. $V_s$-based field case history database, however, is considerably
Figure 2.7: CPT-based probability and deterministic curves, Juang 2003 Juang et al. (2003) compared with Robertson and Wride (1998)
smaller than that available for SPT and CPT correlation development. In realization of this fact, efforts are being made to improve the resolution and reliability of $V_s$-based correlations by increasing the database around the world.

![Figure 2.8: $V_s$-based liquefaction triggering correlation (Andrus and Stokoe II (2000))](image)

2.3 Post-liquefaction deformations

Assessment and mitigation of liquefaction hazard require a reliable tool for evaluating not only the probability of liquefaction “triggering”, but more importantly, the associated effects within liquefied regions. Generally, liquefaction-induced ground failures include flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils.
2.3.1 Ground settlement

Ground surface settlements due to soil liquefaction have been one of the major causes for infrastructure damages during an earthquake. Among various approaches, use of empirical relationships based on the correlation of observed soil behavior with various in-situ index tests or laboratory tests remains the dominant approach for assessing liquefaction-induced settlement, e.g., (Lee and Albaisa, 1974; Tokimatsu and Seed, 1984, 1987; Nagase and Ishihara, 1988; Ishihara and Yoshimine, 1992; Pradel, 1998; Shamoto et al., 1998; Zhang et al., 2002; Wu and Seed, 2004; Tsukamoto et al., 2004; Lee, 2007; Cetin et al., 2009a,b; Lu et al., 2009; Ueng et al., 2010; Tsukamoto and Ishihara, 2010; Juang et al., 2013; Valverde-Palacios, I and Vidal, F and Valverde-Espinosa, I and Martín-Morales, M, 2014). In particular, using cone penetration test (CPT) data, Zhang et al. (2002) proposed a method for liquefaction-induced settlement coupling the volumetric strain chart by Ishihara and Yoshimine (1992) and the classical CPT-based liquefaction model by Robertson and Wride (1998). Building on the work of Zhang et al. (2002), Juang et al. (2013) proposed a probabilistic approach to estimate probability of exceeding a settlement threshold. Using laboratory data and empirical correlations, probabilistic models were developed in Cetin et al. (2009a,b) for cyclic volumetric and shear strains, which were correlated to typical field index test results, such as the corrected standard penetration test blow counts or the cone penetration test tip resistance, to predict field settlements.

Building on the classical CPT-based liquefaction model, the liquefaction-induced settlement is approximated as a summation of the product of the volumetric strain in each soil layer that is susceptible to liquefaction and the corresponding layer thickness. For a site with level ground and far from any free water surface, such approximation is reasonable since the volumetric strain is approximately equal to the vertical strain. When sloping or near free-face ground is involved in the analysis, the deviatoric- or shearing-induced deformation should be considered when estimating ground settlement. For instance, Wu and Seed (2004) recommended to increase the ground settlement estimation by an amount equal to 10 to 20% of the observed or estimated lateral ground displacement. If the estimated lateral ground displacement is smaller than 0.3 m, the influence of the deviatoric deformation is insignificant. In this study, we assume the level ground condition for our evaluation, where the predicted liquefaction-induced settlement \( S \) is estimated as (Juang et al., 2013)

\[
S = \sum_{i=1}^{n} \varepsilon_{vi} \Delta z_i \text{IND}_i \tag{2.3.1}
\]

where \( \varepsilon_{vi} \) is the volumetric strain of the \( i \)th layer; \( \Delta z_i \) is the thickness of the \( i \)th layer; \( \text{IND}_i \) is an indicator of liquefaction occurrence in the \( i \)th layer; and \( n \) is the total number of layers. In (Zhang et al., 2002), \( \text{IND}_i \)
is taken as 1 for all soil layers.

For sandy-like soils, the volumetric strain $\varepsilon_{vi}$ of the $i$th layer is a function of the factor of safety of the $i$th layer, denoted as $FS_i$, and the clean-sand equivalence of the corrected cone tip resistance, denoted as $q_i$. Following the previous work of (Zhang et al., 2002; Juang et al., 2013) that coupled the CPT-based method (Robertson and Wride, 1998) with the volumetric strain relationship (Ishihara and Yoshimine, 1992), the volumetric strain can be approximated with the following equation

$$
\varepsilon_v(\%) = \begin{cases} 
0 & \text{if } FS \geq 2 \\
\min \left\{ \frac{a_0 + a_1 \ln(q)}{1/(2-FS) - [a_2 + a_3 \ln(q)]}, b_0 + b_1 \ln(q) + b_2 \ln(q)^2 \right\} & \text{if } 2 - \frac{1}{a_2 + a_3 \ln(q)} < FS < 2 \\
b_0 + b_1 \ln(q) + b_2 \ln(q)^2 & \text{if } FS \leq 2 - \frac{1}{a_2 + a_3 \ln(q)}
\end{cases}
$$

(2x3.2)

where $q$ is in kg/cm$^2$ ($\approx$ 100kPa); the constant fitting parameters are

for $q \leq 80$

$$a_0 = 0.1649, \ a_1 = -0.006047, \ a_2 = 1.3009, \ a_3 = -0.1022$$

$$b_0 = 28.45, \ b_1 = -9.3372, \ b_2 = 0.7975$$

for $q > 80$

$$a_0 = 0.3773, \ a_1 = -0.0337, \ a_2 = 1.5672, \ a_3 = -0.1833$$

$$b_0 = 28.45, \ b_1 = -9.3372, \ b_2 = 0.7975$$

Figure 2.9 plots the relation given in Eq. (2x3.2) along with the design chart by Ishihara and Yoshimine (1992), where $q$ is the clean-sand equivalence of the corrected cone tip resistance.

It is noted that, in a deterministic analysis, the indicator function $IND_i$ in Eq. (2x3.1) is equal to 0 if the $i$th layer does not liquefy and equal to 1 if the $i$th layer liquefies. In the case of a probabilistic analysis, $IND_i$ may be treated as a random variable with its expected value $E[IND_i]$ and variance $V[IND_i]$ related to the probability of liquefaction $P_{L_i}$ of the $i$th layer (Juang et al., 2013). Accounting for model uncertainties, the FS and $q$ in Eq. (2x3.2) are now both nominal values calculated from the measurement, not the actual value. As shown by Juang et al. (2013), the nominal values can be estimated from the probability of liquefaction as

$$E[IND_i] = P_{L_i}$$

(2x3.3)
Figure 2.9: Chart for estimating the post-liquefaction volumetric strain of clean sand. Adopted from (Juang et al., 2013; Zhang et al., 2002) with the original source data from (Ishihara and Yoshimine, 1992). Units for the clean-sand equivalence of the corrected cone tip resistance, $q$, is in kg/cm$^2$ ($\approx 100$ kPa).

$$V_{\text{IND}_i} = P_{L_i}(1 - P_{L_i})$$  \hspace{1cm} (8.3.4)

where $P_{L_i}$ may be determined using a $P_L$–FS mapping function (Ku et al., 2012)

$$P_L = 1 - \Phi[0.102 + \ln(\text{FS})]/0.276 \approx 1/[1 + (\text{FS}/0.9)^6]$$  \hspace{1cm} (8.3.5)

The liquefaction probability $P_{L_i}$ of the $i$th layer may be determined using a $P_L$–FS mapping function previously proposed by Ku et al. (2012) as defined in Eq. (8.3.5). Then, the probabilistic version of the predicted liquefaction settlement $S$ at individual CPT sounding is characterized by its mean value $\mu_S$ and variance $\sigma_S^2$ as (Juang et al., 2013)

$$\mu_S = \sum_{i=1}^{n} \varepsilon_{vi} \Delta z_i P_{L_i}$$  \hspace{1cm} (8.3.6)

$$\sigma_S^2 = \sum_{i=1}^{n} \varepsilon_{vi}^2 \Delta z_i^2 P_{L_i}(1 - P_{L_i})$$  \hspace{1cm} (8.3.7)

A further development of the above probabilistic settlement model allows the incorporation and quantification of model error. As suggested by Juang et al. (2013), a multiplicative model bias factor $M$ is...
introduced and the corrected settlement prediction, denoted as \( S_a \), for a future case can be expressed as

\[
S_a = MS = M \sum_{i=1}^{n} \varepsilon_{\text{vi}} \Delta z_i \text{IND}_i
\]  

(\ref{eq:3.8})

Assuming that \( M \) is independent of \( S \), the mean and variance of the corrected settlement is then given as (Juang et al., 2013)

\[
\mu_a = \mu_M \mu_S
\]

(\ref{eq:3.9})

\[
\sigma_a^2 = \mu_M^2 \sigma_S^2 + \sigma_M^2 \mu_S^2 + \sigma_M^2 \sigma_S^2
\]

(\ref{eq:3.10})

where \( \mu_M \) and \( \sigma_M \) are the mean and standard deviation of \( M \); \( \mu_S \) and \( \sigma_S \) are the mean and standard deviation of the predicted settlement \( S \) as defined in Eq. (\ref{eq:3.6}) and Eq. (\ref{eq:3.7}), respectively. The mean and standard deviation of the model bias factor \( M \) may be derived empirically from a database of liquefaction case histories.

### 2.3.2 Lateral ground displacements

Besides ground settlements, liquefaction-induced ground failures include flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils, among which lateral spreads are the pervasive types of liquefaction-induced ground failures for gentle slopes or for nearly level (or gently inclined) ground with a free face (e.g., river banks, road cuts).

Efforts have been made to estimate liquefaction-induced lateral ground displacements: numerical models, laboratory tests, and field-test-based methods. Challenges associated with sampling loose sandy soils limit the applications of numerical and laboratory testing approaches in routine practice. Field-test-based methods are likely best suited to provide simple direct methods to estimate liquefaction-induced ground deformations for low-to medium-risk projects and to provide preliminary estimates for high-risk projects.

Methods using standard penetration test (SPT) data are available for estimating lateral displacements in a liquefaction-induced lateral spread (Rauch, Alan F and Martin III, James R (2000); Bardet et al. (2002); Youd et al. (2002)). These methods are empirical and do not incorporate the extensive knowledge gained from laboratory studies of liquefaction. Further, even though the cone penetration test (CPT) has
greater repeatability and reliability, and provides a continuous profile compared with other field tests, CPT-based method to estimate liquefaction-induced lateral displacements is currently limited.

Some laboratory tests such as shake table tests (Sasaki et al. (1991)) and centrifuge model tests (Abdoun (1997)) have been made to investigate the mechanisms of liquefaction-induced ground lateral spreads. These tests generally support the hypothesis that lateral spreads result from distributed residual shear strains throughout the liquefied layers. The residual shear strains in liquefied layers are primarily a function of maximum cyclic shear strains $\gamma_{max}$ and biased in situ static shear stresses. Biased in situ static shear stresses are mainly controlled by ground topographic parameters at the site (e.g., ground slope, free face height, and the distance to a free face). Currently methods for lateral displacement are mainly to correlate maximum cyclic shear strains $\gamma_{max}$ with factor of safety (FS) and relative density $D_r$. A basic correlation chart is shown in Figure 2.10.

![Figure 2.10: Relationship between maximum cyclic shear strain $\gamma_{max}$ and factor of safety (FS) for different relative densities $D_r$ for clean sands (built on data from Ishihara and Yoshimine (1992) and Seed (1979))](image)

To describe the lateral displacement severity, an index named lateral displacement index (LDI) similar to LPI could be defined as the integration of the calculated maximum cyclic shear strains $\gamma_{max}$ over

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the depth:

\[ LDI = \int_{0}^{Z_{\text{max}}} \gamma_{\text{max}} \, dz \]  \hspace{1cm} (2.3.11)

where \( Z_{\text{max}} \) is the maximum depth below all the potential liquefiable layers with a calculated FS < 2.0

### 2.4 Regional assessment of liquefaction hazards

Regional assessment of liquefaction hazard has been a continuous topic in earthquake engineering (pioneering work could be found in Holzer et al. (2006a); Lenz and Baise (2007); Hayati and Andrus (2008); Heidari and Andrus (2010a); Juang et al. (2017) and more other papers). Methods of regional assessment have predominantly relied on criteria that relate surficial geology to liquefaction susceptibility (Youd and Perkins, 1987). A geologic unit is identified by its age and depositional environment and then characterized in terms of its susceptibility. This approach leads to the identification of liquefaction over large regions. The resulting maps show geologic units that likely contain liquefiable sediments but do not identify the location or extent of liquefiable sediments within the geologic unit. Therefore, within a susceptible unit, maybe only a very small area will actually liquefy given an earthquake (Baise et al., 2006). To assess the triggering or the consequences of liquefaction over extended areas or to map the estimated triggering or the consequences to a region, it is necessary to take into account the spatial dependence of soil properties and/or the predicted settlements.

Tools developed in geostatistics and random field have received considerable attention in recent years and have been applied to assess liquefaction hazard over large regions. For instance, Liu and Chen (2006) used CPT measurements to estimate the spatial structure of soil deposits. Then, random field models were coupled with Monte Carlo simulations to assess liquefaction potential in the Yuanlin area of Taiwan. Vivek and Raychowdhury (2014) explicitly considered the spatial variations of soil indices from CPT soundings when evaluating liquefaction potential. It is found that the probability of liquefaction could be significantly underestimated if the spatial dependence of soil indices is not considered. Chen et al. (2015) developed a CPT-based approach to map liquefaction potential across scales, where the soil spatial variability expressed in term of an averaged index, i.e., the Liquefaction Potential Index (LPI) developed by Iwasaki et al. (1978, 1982), is explicitly considered through internally-consistent probabilistic models developed at multiple scales. LPI has also been adopted and modified by other researchers to assess liquefaction potentials at individual CPT soundings or over extended region (Sonmez, 2003; Sonmez and Gokceoglu, 2005; Holzer et al., 2006a;
Baise et al., 2006; Lenz and Baise, 2007; Thompson et al., 2007; Juang et al., 2008b). In another recent work by van Ballegoooy et al. (2015), four liquefaction vulnerability parameters including the LPI were used to map liquefaction hazard in the Christchurch area using extensive CPT database. In contrast to the extensive efforts incorporating geostatistics tools into liquefaction potential evaluation, relatively fewer studies have addressed the liquefaction-induced settlement over extended areas. In (Hinckley, 2010; Bartlett et al., 2007), classical methods of (Tokimatsu and Seed, 1987; Ishihara and Yoshimine, 1992; Yould et al., 2002; Yoshimine et al., 2006) are employed to calculate liquefaction-induced lateral spread and ground settlement, which are plotted within their respective surficial geologic units. Then, each geologic units are assigned an estimate of ground deformation based on statistical analysis.

2.5 Summary

The mechanism of soil liquefaction phenomenon is explained in this chapter. Soil liquefaction during earthquakes is caused by the sudden increase of pore water pressure. It is a multiscale, multiphysics phenomenon.

Quantitative assessment of the likelihood of “triggering” or initiation of liquefaction is the necessary first step for most projects involving potential seismically-induced liquefaction. The use of in-situ “index” testing is the dominant approach in common engineering practice. Four in-situ test methods have now reached a level of sufficient maturity as to represent viable tools for this purpose (Youd et al. (2001)), and these are (1) the Standard Penetration Test (SPT), (2) the cone penetration test (CPT), (3) measurement of in-situ shear wave velocity ($V_s$), and (4) the Becker penetration test (BPT).

Assessment and mitigation of liquefaction hazard require a reliable tool for evaluating not only the probability of liquefaction “triggering”, but more importantly, the associated effects within liquefied region. Generally, liquefaction-induced ground failures include flow slides, lateral spreads, ground settlements, ground oscillation, and sand boils. Among various approaches, use of empirical relationships based on the correlation of observed soil behavior with various in-situ index tests or laboratory tests remains the dominant approach for assessing liquefaction-induced ground deformations.

Although the liquefaction-induced failures are able to estimate at individual locations using empirical correlations with acceptable accuracy, numerical analyses at site-specific scale and regional scale are still premature. Especially, to assess the triggering or the consequences of liquefaction over extended areas or to map the estimated triggering or the consequences to a region, it is necessary to take into account the spatial
dependence of soil properties.
Chapter 3

Multiscale Random Field Modeling Framework

3.1 Introduction

As summarized in the previous chapter, regional scale liquefaction analyses are challenging, not only because of the complexity associated with the modeling of triggering or the consequences of liquefaction but also due to the difficulties associated the estimation of spatial variations of soil properties. In this chapter, several approaches for spatial prediction are reviewed. A multiscale random field model is developed and integrated into a novel framework for probabilistic assessment of regional liquefaction.

3.2 Spatial prediction: random fields and beyond

3.2.1 Random fields

The definition

A random field is also called a random process or stochastic process. In a laboratory where an investigator can do numerous experiments, observing the outcome of all the experiment in the laboratory is equivalent to observing the realization of the random field. The term ‘field’ indicates that the parameter space is multi-dimensional. Most of the literature deals with random process that depend on a single dimension,
usually time.

The history

Many biological, physical, social and economic systems have attributes which, viewed on an appropriate scale, exhibit complex patterns of variation in space and time. Random field theory seeks to model complex patterns of variation and interdependence in these cases. The methodology of random field is applicable to phenomena occurring on very different scales (temporal or spatial). The time scale may be the interval between molecular collisions, as in the study of Brownian motion, or may be measured in geological units, for example, to describe the variation of surficial soil types.

Much of the early literature on random fields deals with variation in function of a single parameter, usually time. The simplest time series model is the purely random process that assumes that successive observations are statistically independent. When observations are made continuously, the purely random process becomes an ‘ideal white noise’, and its integral is referred to as a Wiener process, after Norbert Wiener (Wiener, 1930, 1949) who fully developed its mathematical foundation. The close relation between white noise and the Poisson process is highlighted by Stephen Rice’s model (Rice, 1944) for ‘shot noise’ in communication systems.

Poisson models also apply in multi-dimensional situations. This gives rise to a large family of stochastic models (Parzen, 1962). Brownian movement refers to the chaotic movement of microscopic particles suspended in a fluid. In the classical analysis of Brownian movement by Einstein (1915), a particle experiences a succession of random impulses caused by collisions with other particles. This excitation is modeled as a purely random process which renders particle velocity Markovian. Einstein (1915) found that the probability density function of particle displacement obeys a simple diffusion equation and that after a sufficiently long period, the probability distribution of the displacement along any given direction becomes Gaussian with mean zero and a variance that only depends on diffusion parameter and length of period.

Ornstein (1930) extended Einstein’s analysis by modeling the transient part of the solution. When external forces are added to the equation of Brownian particle motion, the diffusion equation becomes the Fokker-Planck equation governing the probabilities of transition from state to state of a Markov process. The Markovian assumption is readily understood when a random process depends on time.

In physics, disorder is often seen as defective order (Ziman, 1979). The classical approach to second-order analysis of stationary random fields is based on the work of Wiener (1930, 1949) and Aleksandr and
Khinchin (1949) who established the equivalence of, and relationship between, the autocorrelation function and the spectral density function. Among the most important features of a random field are the patterns of correlation and persistence reflected in the second-order statistics, and much of the literature on applied stochastic process offers one candidate or other as s model for the correlation structure.

Early and influential applications of second-order stochastic theory to multi-dimensional problems occurred in the field of fluid mechanics, where theoretical and experimental work by Taylor (1935) and Theodore von Karman provided information about the correlation structure of the random field of velocities in turbulent fluid flow.

An important practical dimension was added to the theory of random functions by the classical work on optimum linear estimation by Wiener (1949) and others. In its simplest form the problem is to make a ‘best’ prediction of the true value of a random field at one point by linearly combing a set of values observed at other points. The criteria for optimality are that the prediction must be unbiased and the variance of the prediction error minimized. In response to practical needs to estimate spatial patterns occurring in geology and mining, Krige (1951) and others developed a class of linear estimation techniques that have the same mathematical foundation with those developed by Wiener (1949). These estimation techniques have been particularly well developed in geostatistics under the name of kriging and will be detailed in Section 3.2.2.

The brief review above is aimed at emphasizing fundamental contributions and methodological breakthroughs. In the following part of this section, the details of the mathematical formulation of a random field is presented.

**The mathematical formulation**

The random field theory is a body of stochastic methods that serve multiple purposes (Vanmarcke, 2010):

1. in stochastic description they provide a format for efficient characterization of distributed disordered systems;

2. in system analysis they provide the basis for predicting response of distributed disordered systems;

3. in decision situation involving distributed disordered systems they permit assessment of the impact of alternative strategies;

4. etc.
For this dissertation, the interest in random field theory is that it could be used to analyze naturally varying properties in geotechnical engineering in both temporal and spatial domain, such as soil permeability over the scale of meters, concrete strength of the scale of centimeters, or earthquake ground motions varying from point to point in both time and space (Vanmarcke et al., 1993). The stochastic estimation of these properties in geotechnical engineering is the classical optimum linear estimation problem (Wiener, 1949) in random field theory. The rest of this subsection will describe mathematical foundation of optimum linear estimation that are rooted in random field.

In probability theory and statistics, the multivariate Gaussian distribution, is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions. The definition could be that a random vector is said to be m-variate normally distributed if every linear combination of its m components has a univariate normal distribution. The multivariate normal distribution is often used to describe any set of random variables (RVs) each of which clusters around a mean value. For instance, Z is a m dimension vector of RVs \([Z_1, Z_2, ..., Z_i, ..., Z_m]^T\) and is subjected to a multivariate Gaussian distribution, which can be noted as:

\[
Z \sim N(\mu, \Sigma) \tag{3.2.1}
\]

in which \(\mu\) is a m-by-1 vector (each element is the estimate of RV \(Z_i\)); \(\Sigma\) is a m-by-m covariance matrix (\(\Sigma_{i,j} = \text{COV}(Z_i, Z_j)\) where \(\text{COV}\) is the covariance operator). If m-dimensional \(Z\) is partitioned into two parts as \(Z_n\) with size q-by-1 and \(Z_p\) with size (m-q)-by-1, Eq. (3.2.1) becomes

\[
\begin{bmatrix}
Z_n \\
Z_p
\end{bmatrix} \sim N
\begin{bmatrix}
\mu_n \\
\mu_p
\end{bmatrix},
\begin{bmatrix}
\Sigma_{nn} & \Sigma_{np} \\
\Sigma_{pn} & \Sigma_{pp}
\end{bmatrix} \tag{3.2.2}
\]

in which \(\mu_n\) is a q-by-1 vector of expected values of RV \(Z_n\); \(\mu_p\) is a (m-q)-by-1 vector of expected values of RV \(Z_p\); \(\Sigma_{nn}\) is a q-by-q matrix of covariances partitioned from \(\Sigma\); \(\Sigma_{np}\) is a q-by-(m-q) matrix of covariances partitioned from \(\Sigma\); \(\Sigma_{pn} = \Sigma_{np}^T\); \(\Sigma_{pp}\) is a (m-q)-by-(m-q) matrix of covariances partitioned from \(\Sigma\). The conditional distribution of RV \(Z_n\) given \(Z_p\) is a multivariate normal Deutsch (1965)

\[
Z_n|Z_p \sim N(\mu_{Z_n|Z_p}, \Sigma_{Z_n|Z_p}) \tag{3.2.3}
\]

in which

\[
\mu_{Z_n|Z_p} = \mu_n + \Sigma_{np} \Sigma_{pp}^{-1} (Z_p - \mu_p) \tag{3.2.4}
\]
If a random field (RF) $Z$ is second order stationary, (1) the expected value $E\{Z(u)\}$ ($u$ is a location) exists and is constant; and (2) the two-point covariance $COV(h)$ exists and depends only on the separation vector $h$. Now we could define the expected value of the second order stationary RF $Z(u)$ is $E\{Z(u)\} = \mu$ and variance $Var\{Z(u)\}$.

If the measured values at locations $p_{p \times 1}$ are known as $Z_p$, the value at an unmeasured location $Z_n$ could be drawn from the conditional distribution in Gaussian random field:

\[
Z_n \sim N(\mu + \Sigma_{np}\Sigma_{pp}^{-1}(Z_p - \mu), Var\{Z(u)\} - \Sigma_{np}\Sigma_{pp}^{-1}\Sigma_{pn})
\]  

(3.2.6)

Random fields are of great use in studying natural processes by the Monte Carlo method and have been used in signal/image processing and machine learning extensively. In this dissertation, random fields are used to estimate regional liquefaction risk, which will be presented in the following chapters.

### 3.2.2 Geostatistics

Geostatistics is a science for data analysis, spatial prediction, assessment of uncertainty and stochastic imaging (Goovaerts, 1997). Gaussian processes have a long history in the statistics community. They have been particularly well developed in geostatistics under the name of kriging, which is a generic name used by geostatisticians for a family of generalized least-squares regression algorithms in recognition of the pioneering work of a mining engineer Danie Krige Krige (1951). All kriging estimators are but variants of the basic linear regression estimator (Goovaerts, 1997). For instance, a simple kriging estimator can be defined as

\[
Z^*(u_\alpha) = \phi_0 + \phi^T Z(u)
\]  

(3.2.7)

in which $\phi_0$ and $\phi$ are combining coefficients; $Z(u)$ is the measured expected values at location $u$. If $Z(u)$ is second order stationary (i.e., invariant and known expected value exists, noted as $\mu$), the observed values at location $u$ could be expressed as

\[
Z(u) = \mu + \epsilon(u)
\]  

(3.2.8)
where $\epsilon(u)$ is the error vector with zero mean; $\mu$ is the invariant and known mean value of this random field.

Hence the unobserved value at location $u_\alpha$ is

$$Z(u_\alpha) = \mu + \epsilon(u_\alpha) \tag{3.9}$$

Simple kriging estimator should be unbiased:

$$E[Z^*(u_\alpha) - Z(u_\alpha)] = E[\phi_0 + \phi^T Z(u) - \mu - \epsilon(u_\alpha)] = \phi_0 + \phi^T u - \mu = 0 \tag{3.10}$$

This yields

$$\phi_0 = \mu - \phi^T u \tag{3.11}$$

Plug Eq. (3.11) into Eq. (3.7) so that the estimator becomes

$$Z^*(u_\alpha) = \mu + \phi^T (Z(u) - \mu) = \mu + \phi^T \epsilon(u) \tag{3.12}$$

Therefore the variance of the predicted values is

$$\text{Var}\{Z^*(u_\alpha) - Z(u_\alpha)\} = E[(\mu + \phi^T \epsilon(u) - \mu - \epsilon(u_\alpha))^2$$

$$= E[\phi^T \epsilon(u) - \epsilon(u_\alpha)]^2$$

$$= \text{Var}\{\phi^T \epsilon(u)\} + \text{Var}\{\epsilon(u_\alpha)\} - 2\text{COV}\{\phi^T \epsilon(u), \epsilon(u_\alpha)\}$$

$$= \phi^T \Sigma_{u,u} \phi + \text{Var}\{Z(u)\} - 2\phi^T \Sigma_{u,\alpha} \tag{3.13}$$

To optimize $\text{Var}\{Z^*(u_\alpha) - Z(u_\alpha)\}$, let its partial derivatives with respect to the vector of coefficients $\phi$ equate to zero

$$\frac{\partial}{\partial \phi} \left( \phi^T \Sigma_{u,u} \phi + \text{Var}\{Z(u)\} - 2\phi^T \Sigma_{u,\alpha} \right) = 2\Sigma_{u,u} \phi - 2\Sigma_{u,\alpha} = 0 \tag{3.14}$$

Hence

$$\phi = \Sigma_{u,u}^{-1} \Sigma_{u,\alpha} \tag{3.15}$$

Plug 3.15 into 3.12 to obtained the simple kriging predictor

$$Z^*(u_\alpha) = \mu + \Sigma_{u,\alpha}^{-1} T \Sigma_{u,u}^{-1} [Z(u) - \mu] \tag{3.16}$$
and the simple kriging variance is obtained as

\[
\text{Var}\{Z^*(u_\alpha) - Z(u_\alpha)\} = \phi^T \Sigma_{u,u} \phi + \text{Var}\{Z(u)\} - 2\phi^T \Sigma_{u,\alpha}
\]

\[
= \phi^T \Sigma_{u,\alpha} + \text{Var}\{Z(u)\} - 2\phi^T \Sigma_{u,\alpha}
\]

\[
= \text{Var}\{Z(u)\} - \Sigma^T_{u,\alpha} \Sigma_{u,u}^{-1} \Sigma_{u,\alpha}
\]

3.2.3 Artificial neural networks

Artificial neural networks (ANNs) are a form of artificial intelligence which attempt to mimic the behavior of the human brain and nervous system. Many researchers have described the structure and operation of ANNs (e.g. Hecht-Nielsen, R. (1990); Zurada (1992); Fausett (1994)). A typical structure of ANNs consists of a number of nodes (processing elements), that are usually arranged in layers: an input layer, an output layer and one or more hidden layers (Figure 3.1).

The input from each node in the previous layer \((x_i)\) is multiplied by an adjustable connection weight \((w_{ji})\).

At each node, the weighted input signals are summed and a threshold value \((\theta_j)\) is added. This combined input \((I_j)\) is then passed through a non-linear transfer function \((f(.))\) to produce the output of the PE \((y_j)\).

The output of one PE provides the input to the nodes in the next layer. This process is summarized in Eq. (3.2.18) and Eq. (3.2.19) and illustrated in Figure 3.1.

\[
I_j = \sum w_{ji} x_i + \theta_j
\]

\[
y_j = f(I_j)
\]

Figure 3.1: Artificial neural network structure
The ANN modelling philosophy is similar to a number of conventional statistical models in the sense that both are attempting to capture the relationship between a historical set of model inputs and corresponding outputs. ANNs learn from data examples presented to them and use these data to adjust their weights in an attempt to capture the relationship between the model input variables and the corresponding outputs. Consequently, ANNs do not need any prior knowledge about the nature of the relationship between the input/output variables, which is one of the benefits that ANNs have compared with most empirical and statistical methods.

ANNs have been applied in a great deal of research for a variety of purposes including medicine and biology (Malngren et al. (2012); Jayalakshmi and Santhakumaran (2011)); pattern recognition and image analysis (Bishop (1995); Yang et al. (2000); Amini (2008)); geotechnical engineering (Shahin et al. (2008)); decision making and control (Johnson and Rogers (1998); Lou and Brunn (1998); Yanar and Akyüre (2007); Zemouri et al. (2010)); and stock market predictions (Dechpichai (2010)) despite the disadvantages such as the black box nature and the empirical nature of model development (Tu (1996)).

ANNs are also widely used in spatial analysis and predictions of geotechnical and other engineering problems. Sitharam et al. (2008) used ANN to evaluate the spatial variability of rock depth in an extended region. Prasomphan and Mase (2013) develop a scheme to generate prediction map for geostatistical data using ANN. When integrated with GIS geographic information system (GIS), ANN is a very powerful tool to make spatial analysis. For example, Lee et al. (2003) integrated with ANN to predict the regional landslide susceptibility. More works relating to ANN-GIS can be found in Gangopadhyay et al. (1999) Yanar and Akyüre (2007) Van Leeuwen et al. (2008), etc.

Spatial predictions with geostatistic tools (such as kriging methods) usually need a prescribed spatial correlation structure, which should be inferred from measured data. This is impossible when the size of the database is small. One advantage of ANNs is that such a prescribed correlation structure is not needed. Geostatistic tools, however, are still a good and widely used method for they are able to yield relatively precise and spatially smooth predictions. Efforts to combine ANNs with traditional geostatistic tools have been made and can be found in literature Rizzo and Dougherty (1994) Demyanov et al. (1998) Liu et al. (2009).
3.3 Measures of spatial variability

Assume the measure at any location \( u \) is a second order stationary Gaussian RF \( Z(u) \) with expected value \( E\{Z(u)\} = \mu \) and variance \( Var\{Z(u)\} = \sigma = 1 \). If the measured values at locations \( p_{p \times 1} \) are known as \( Z_p \), the value at an unmeasured location \( Z_n \) could be drawn from the conditional distribution in Gaussian random field:

\[
Z_n \sim N(\mu + \Sigma_{np} \Sigma_{pp}^{-1} (Z_p - \mu), 1 - \Sigma_{np} \Sigma_{pp}^{-1} \Sigma_{pn})
\]  

(3.3.1)

each element in \( \Sigma \) is the covariance of two points, say, \( u_\alpha \) and \( u_\beta \)

\[
\text{COV}(u_\alpha, u_\beta) = E\{Z(u_\alpha) \cdot Z(u_\beta)\} - E\{Z(u_\alpha)\} \cdot E\{Z(u_\beta)\}
\]  

(3.3.2)

The correlogram can be written as

\[
\rho(u_\alpha, u_\beta) = \frac{\text{COV}(u_\alpha, u_\beta)}{\sqrt{\text{COV}(u_\alpha, u_\alpha) \cdot \text{COV}(u_\beta, u_\beta)}}
\]  

(3.3.3)

Unlike covariance \( \text{COV} \) and correlogram \( \rho \), which are measures of similarity, semivariogram \( \gamma \) is a variable
that measures dissimilarity between data separated by a distance

\[\gamma = \frac{1}{2} \text{Var}\{Z(u_\alpha) - Z(u_\beta)\}\]

(3.3.4)

In a second order stationary, the two-point covariance, correlogram and semivariogram depend only on the separation vector \(h\) hence can be expressed as

\[\text{COV}(h) = E\{Z(u) \cdot Z(u + h)\} - E\{Z(u)\} \cdot E\{u + h\}\]

(3.3.5)

\[\rho(h) = \frac{\text{COV}(h)}{\text{COV}(0)}\]

(3.3.6)

\[\gamma = \frac{1}{2} \text{Var}\{Z(u) - Z(u + h)\}\]

(3.3.7)

And they are related by

\[\gamma(h) = \text{COV}(0) - \text{COV}(h)\]

(3.3.8)

\[\rho(h) = 1 - \frac{\gamma(h)}{\text{COV}(0)}\]

(3.3.9)

Semivariogram is usually expressed as a linear combination of basic models (Figure 3.3):

- **Nugget effect model**

\[\gamma(h) = \begin{cases} 
0 & \text{if } h = 2 \\
1 & \text{otherwise}
\end{cases}\]

(3.3.10)

- **Spherical model with range a**

\[\gamma(h) = \begin{cases} 
1.5 \frac{h}{a} - 0.5\left(\frac{h}{a}\right)^3 & \text{if } h \leq 2 \\
1 & \text{otherwise}
\end{cases}\]

(3.3.11)

- **Exponential model with range a**

\[\gamma(h) = 1 - \exp\left(-\frac{3h}{a}\right)\]

(3.3.12)
• Gaussian model with range $a$

$$\gamma(h) = 1 - \exp\left(-\frac{3h^2}{a^2}\right)$$

(3.3.13)

Figure 3.3: Basic semivariogram models

The relation between covariance model and semivariogram is illustrated in Figure 3.4. $S$ is the variance of the random field; $S_o$ is the nugget effect of the semivariogram model; $S_o + S$ is the sill of the semivariogram model.

3.4 Multiscale spatial dependence

When characterizing fields of soil properties while accounting for uncertainty, consideration of spatial dependence is of great importance, and much effort has been devoted to this problem. A common approach is to use field measured data to fit a best curve of the semivariogram (Baise et al. (2006); DeGroot and Baecher (1993); Goovaerts (1997)). Most experience with random field models for soil properties is limited to a single spatial scale. Multiscale models for some geotechnical properties have been proposed based on theoretical arguments (Taylor and Burrough (1986); Fenton (1999a)). In the following of this section, a multiscale spatial dependence scheme is developed for the spatial similarity measure correlogram $\rho$. 

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Figure 3.4: The relation between covariance model and semivariogram

Figure 3.5: Graphic representation of material properties at two scales. The superscripts ‘c’ and ‘f’ refer to ‘coarse’ and ‘fine’ scales, respectively. The subscripts refer to the element number. Roman letters I, II...: are used for coarse scale element and Arabic numbers 1, 2, 3... are used for fine scale element.
In the subsequent model development, it is assumed that variables of interest follow the standard Gaussian distribution. For other cases, a normal score transformation can be used to map variables to the desired distribution (Goovaerts, 1997). The derivation of spatial correlation across scales is based on the notion that material properties at the coarser scales are the arithmetically averaged values of the properties over corresponding areas at the finer scales (Chen et al., 2012)

\[
Z_c^i = \frac{1}{N} \sum_{i=1, i \in I}^N Z_f^i
\]

where the superscripts ‘c’ and ‘f’ refer to the coarse and fine scales, respectively; \( N \) is the count of fine scale elements within a coarse scale region (refer to Figure 3.5). Defining the variable of interest at the fine scale, the mean of the variable at coarse scale, denoted as \( \mu_{zc} \), can be derived by taking the expectation of Eq. (79.1) as

\[
\mu_{zc} = E[Z_c^i] = \frac{1}{N} \sum_{i=1}^N \mu_{zf}^i = 0
\]

where \( \mu_{zf}^i \) is the mean of fine scale elements within a coarse element \( I \), which equals to zero for variables following the standard Gaussian distribution. Accordingly, if the variance of fine scale value is unity, the coarse scale variance, denoted as \( \sigma_{zc}^2 \), can be computed as

\[
\sigma_{zc}^2 = E[(Z_c^i)^2] - 0 = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \rho_{zf, zf}^i \sigma_{zf}^i \sigma_{zf}^j
\]

where \( \rho_{zf, zf}^i \) is the correlation between two fine scale element \( Z_f^i \) and \( Z_f^j \) with a standard deviation of \( \sigma_{zf}^i \) and \( \sigma_{zf}^j \), respectively.

The covariance between two elements \( Z_i \) and \( Z_j \) at any scale within the random field is defined as

\[
\text{COV}[Z_i, Z_j] = \rho_{zi, zj} \sigma_{zi} \sigma_{zj}
\]

where \( \rho_{zi, zj} \) is the correlation between two elements \( Z_i \) and \( Z_j \) within the random field at any scale with a standard deviation of \( \sigma_{zi} \) and \( \sigma_{zj} \), respectively.

The correlations between all considered scales can be calculated by rearranging the definition of covariance such that

\[
\rho_{zi, zj} = \frac{\text{COV}[Z_i, Z_j]}{\sigma_z \sigma_{zj}}
\]
By making appropriate substitutions at each scale using (7-2) and (3-5), the correlation between elements at different scales can be obtained as

$$\rho_{x_I^I, x_{II}^{II}} = \frac{\sum_{i=1}^{N} \sum_{k=1}^{N} \rho_{x_i^I, x_k^{II}}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{x_i^I, x_j^I}} \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{x_i^I, x_j^{II}}}}$$  (3-6)

$$\rho_{x_i^I, x_j^I} = \frac{\sum_{i=1}^{N} \rho_{x_i^I, x_i^I}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{x_i^I, x_j^I}}}$$  (3-7)

where $\rho_{x_I^I, x_{II}^{II}}$ is the correlation between two coarse-scale elements $I$ and $II$; $\rho_{x_i^I, x_j^I}$ is the correlation between a fine-scale element and a coarse-scale element $I$; $\rho_{x_i^I, x_k^{II}}$ is the correlation between a fine-scale element $i$ and a fine-scale element $k$, which belong to two different coarse scale elements $I$ and $II$, respectively. An example of the correlations between scales calculated using the above equations is shown in Figure 3.6. Given the multiscale spatial dependence specified by the semivariogram and an inferred probability distribution of a parameter value at a single location, a sequential Gaussian simulation process (Goovaerts, 1997) is implemented in Matlab to generate random field realizations of variables of interest. For further details of the algorithm, the reader is referred to (Baker and Faber, 2008; Chen et al., 2012, 2015).

Figure 3.6: Spatial correlations across different scales
3.5 Numerical implementation of multiscale random field framework

Details of the numerical implementation and some common measures to improve the computational efficiency of random field generation are discussed in this section.

The flowchart of the multiscale random field implementation is shown in Figure 3.7. The program requires as inputs: the random field parameters (e.g., the probability model, the semivariogram model), the geometry of the region and an initial grid, and any available field data for the conditional simulation of unknown data. Defining an initial grid allows convenient refinement of an initial coarse random field into higher resolution fine scale fields following the procedure described in Section 3.4. The random variable is simulated at the centroid of each cell and its value is assigned to the corresponding cell. The end product or output of the program is a multiscale random field realization for quantities of interest within the region of interest.

The conditional sequential simulation process adopted in this work preserves the known field data at their locations and allows all subsequent generated data points to be conditioned upon such known information and any previously generated points. One drawback with this process, however, is that it can be quite computationally demanding. Several common measures are implemented in this work to improve the computational efficiency of the simulation.

First, the fine scale random fields are only selectively generated at locations that are deemed necessary for higher resolution information. Examples of such locations include areas around CPT soundings or near important buildings. The sequential simulation process described above has the benefit of selective and adaptive refinement. If deemed necessary at any point in simulation process, any coarse scale element at any location can be refined into its fine-scale components without consideration of the refinement sequence.

Second, the size of the matrix containing previously simulated data $Z_p$ in Eqs. (3.2.2) will be limited by taking advantage of the screening effect. The screening refers to the phenomenon of dramatically reducing the Kriging weight assigned to a datum that is screened by another nearby data near the location being simulated Leuangthong et al. (2011). After the nearest data, other more distant data will receive little weight even if they are correlated with the location under consideration.

Figure 3.8(a) plots the weights assigned to the neighbours of the data point $Z_n$ to be simulated with and without limiting the size of the matrix $Z_p$ containing previously simulated points. “Screened to 30” means the nearest 30 data points are included in the covariance matrix calculation. In this work, including
Input: random field parameters, geometry, field data

Begin

Map known data points (if any) into Gaussian distribution, and initialize $Z_p$ containing mapped values

Initialize the array of cells to be simulated

Any unsimulated cell in the array?

Yes

Calculate covariance matrices $\Sigma_{np}$, $\Sigma_{pn}$, and $\Sigma_{pp}$

Generate value for current cell, and put generated value to $Z_p$

Remove generated cell from the array

Mark coarse-scale cells to be refined and note the count of them as $M$

$i = 1$

Yes

$i < M$?

$i = i + 1$

No

$i = 1$

$i < N$?

$j = j + 1$

No

Yes

Map simulated values to target distribution

End

Output: multiscale random field with target distribution

Figure 3.7: Flowchart of the multiscale random field model implementation.
the nearest 30 data points is sufficient since more distanced points have been assigned negligible weights. This is consistent with previous experience and recommendation in Leuangthong et al. (2011). Also, it is important to note that limiting the size of the matrix $Z_n$ does not adversely affect the spatial correlation, as shown in Figure 3.8(b). The slight difference between the specified and empirical semivariogram is partially due to the relatively small field size.

![Kriging weights and Semivariogram](image)

Figure 3.8: Screening effect on the data point $Z_n$ to be simulated: (a) Kriging weights assigned to neighbours of $Z_n$ with and without limiting the size of the matrix containing previously simulated points; (b) specified and empirical semivariogram calculated from one realization of random field. The distance is normalized by the correlation length parameter ‘$a$’.

As a third method to further improve computational efficiency, it is noted that the covariance matrices only depend on the relative locations between pairs of data points in the random field. Therefore, if the locations of coarse and fine scale data points are known and fixed between different simulations, those covariance matrices need to be computed only once and can be reused by all subsequent simulations. This will significantly reduce computation time when the random field simulation is used within a Monte Carlo framework, where thousands of simulations are typically performed.

Figure 3.9 shows the comparison of computation time in log scale before and after measures are taken to improve computational efficiency. The computation is performed on one computing node of Clemson University’s Palmetto Cluster, with 1 CPU (Intel(R) Xeon(R) L5420 @ 2.50GHz) and 24 Gb RAM. Orders of magnitude improvement in computational efficiency are achieved. Further improvement can be realized by taking advantage of recent parallel computing capabilities added to Matlab, which will be left for future studies.
Figure 3.9: Computational efficiency comparison. Case 1: all coarse elements are included for refinement, without any optimization; case 2: selective refinement, without any optimization; case 3: selective refinement, with screening to the nearest 30 elements; case 4: selective refinement, pre-calculated covariance matrices and with screening.

3.6 Summary

In this chapter, several approaches for spatial prediction are reviewed. A multiscale random field model is developed and integrated into a novel framework for probabilistic assessment of regional liquefaction. In the following chapters, the applicability and assessment of the developed framework will be demonstrated through the probabilistic and spatial assessment of liquefaction potential in several liquefaction-prone regions, where the liquefaction potential and liquefaction-induced ground failures such as ground settlement are evaluated.
Chapter 4

Assessment of Liquefaction Triggering Using Multiscale Random Field Models

4.1 Introduction

Empirical relations provide liquefaction susceptibility and potential of soil deposits at individual locations. To assess the consequences of liquefaction, it is necessary to understand the potential spatial extent of liquefaction, which requires the spatial dependence of soil properties to be taken into account during the analysis Baker and Faber (2008). Tools developed in geostatistics have received considerable attentions and been applied in modeling heterogeneous soil properties in recent years (e.g., Fenton (1999a); Vivek and Raychowdhury (2014); Popescu et al. (2005); Elkateb et al. (2003); Pokhrel et al. (2013); Uzielli et al. (2005); Ku et al. (2012)). Utilizing CPT-based empirical correlation, Liu and Chen (2006) used random field models and Monte Carlo simulations to assess liquefaction potential in Yuanlin Taiwan, where the spatial structures of soils are estimated from available CPT measurements. Vivek and Raychowdhury (2014) also considered spatial variation of soil indices related to CPT soundings and pointed out that the probability of liquefaction could be significantly underestimated as much as 34% if the spatial dependence of soil indices is not taken

\footnote{A similar form of this section has been published at the time of writing: Chen, Q., Wang, C. and Hsein Juang, C. (2015). CPT-based evaluation of liquefaction potential accounting for soil spatial variability at multiple scales. Journal of Geotechnical and Geoenvironmental Engineering, 142(2), 04015077.}
into account. Baker and Faber (2008) proposed a method to quantify the potential extent of liquefaction by accounting for spatial dependence of soil properties and future earthquake shaking. All the previous methods focus on establishing random fields to represent soil properties and ground motion parameters first, and then employing empirical relations to assess liquefaction potential. An alternative approach (Lenz and Baise (2007); Baise et al. (2006); Thompson et al. (2007)) used an averaged soil property such as the Liquefaction Potential Index (LPI) to assess liquefaction potential at individual location (where field data is available), and then utilize geostatistics tools and random field models to generate LPI at unknown locations across a region.

Due to the nature of soil spatial variability, assessment of liquefaction potential naturally involves multiple scales, ranging from site-specific all the way to regional scales. Incorporating small-scale (local) soil information into large-scale evaluation of liquefaction remains largely unsolved challenge. There have been recent efforts on incorporating soil spatial variability across different length scales for geotechnical applications (e.g., Chen et al. (2011); Baker et al. (2011); Thompson et al. (2007)), but few has addressed the linkage of small-scale soil properties on large-scale liquefaction evaluation.

In this work, the multi-scale random field models is exploited to explicitly account for spatial variability of soil properties across different length scales. Soil properties at large scales are defined by averaging values at the corresponding small-scale locations, which provide a way to derive means, variances and spatial correlations of large scales that are consistent with small scales. Following the approach for evaluating liquefaction potential (Lenz and Baise (2007)), an averaged index such as the LPI at individual locations is generated first and the LPI at other locations are simulated through random field models using Monte Carlo simulations. Utilizing multiscale random field model, the proposed work is able to refine and obtain high-resolution random fields adaptively in areas where necessary, such as underneath critical structures, or where detailed small-scale data is available. This refinement is not arbitrary but is consistent with the coarse scale random fields already generated. The details of the proposed method will be presented in the following sections.

### 4.2 Methodology

Liquefaction risk over extended area is assessed combining empirical models from geotechnical earthquake engineering (e.g., CPT-based liquefaction models) with tools from multiscale random field models. The methodology developed in this work is graphically presented in Figure 4.1 and consists of a number of key steps.
Figure 4.1: Key components in the proposed framework utilizing multi-scale random field models to account for soil spatial variability in liquefaction evaluation across a region. Solid red circles in the regional map correspond to location of boreholes.
Step 1. Identify the region of interest and collect and characterize available field test data relevant for liquefaction assessment (e.g., CPT soundings, water table, soil unit weights). The locations where field data is available are marked as solid red circles in Figure 1. Given field data, two options may be taken to characterize the known information: (1) use empirical models to evaluate liquefaction potential at individual location where field data is available, and then treat those liquefaction potentials as known data points for further characterization of random field models across the region; and (2) characterize field data directly (e.g., tip resistance and side friction from CPT soundings) and then use those data for further characterization of random field models. In either option, the probabilistic distribution and spatial correlation of quantity of interest will need to be characterized. In this work, the first option is adopted, with particular emphasis on spatial dependence across different scales. Step 2. Multi-scale random field models and Monte Carlo simulations are used to realize liquefaction potentials at unknown locations that depend on knowledge at locations with known data and that are consistent with the random-field characterizations of all input values. Step 3. Evaluation of liquefaction potential given results of Monte Carlo simulations is then performed. Any quantity of interest can be evaluated by the probabilistic assessment such as the cumulative frequency of different liquefaction severity across the region, and/or the probability of liquefaction underneath critical buildings. This will be detailed through an example application of the proposed methodology.

4.3 CPT-based liquefaction potential assessment

The CPT-based liquefaction model proposed by Robertson and Wride (1998) and subsequently updated by Robertson (2009) is adopted in this work to evaluate the liquefaction resistance of sandy soils. Main gradients of this classical procedure are summarized herein. The cyclic resistance ratio (CRR) is estimated as

\[
\text{CRR} = \begin{cases} 
0.8333[(q_{c1N})_{cs}/1000] + 0.05 & \text{if } (q_{c1N})_{cs} < 50 \\
93[(q_{c1N})_{cs}/1000]^3 + 0.08 & \text{if } 50 \leq (q_{c1N})_{cs} < 160 
\end{cases} 
\quad (4\text{P}3.1)
\]

The equivalent clean sand normalized penetration resistance, \((q_{c1N})_{cs}\) is given as

\[
(q_{c1N})_{cs} = K_c(q_{c1N}) 
\quad (4\text{P}3.2)
\]
where the conversion factor $K_c$ is calculated from the soil behavior type index $I_c$ as

$$K_c = \begin{cases} 
1 & \text{for } I_c \leq 1.64 \\
-0.403I_c^4 + 5.581I_c^3 - 21.63I_c^2 + 33.75I_c - 17.88 & \text{for } I_c > 1.64 
\end{cases} \tag{4P-3.3}$$

and $q_{c1N}$ is the normalized cone penetration resistance

$$q_{c1N} = \left( \frac{q_c - \sigma_{vo}}{P_{at}} \right) \left( \frac{P_{at}}{\sigma_{vo}} \right)^n \tag{4P-3.4}$$

where $P_{at} = 1$ atm of pressure (100 kPa); $P_{at}/\sigma'_{vo}$ should not exceed a value of 1.7 (Youd et al., 2001); $q_c$ is the measured cone penetration resistance; $n$ is the stress exponent (Robertson, 2009)

$$n = 0.381(I_c) + 0.05 \left( \frac{\sigma'_{vo}}{P_{at}} \right) - 0.15 \text{ where } n \leq 1 \tag{4P-3.5}$$

The soil behavior type index $I_c$ is defined by Robertson and Wride (1998) as

$$I_c = \sqrt{(3.47 - \log Q)^2 + (1.22 + \log F)^2} \tag{4P-3.6}$$

where $Q$ and $F$ are the normalized tip resistance and friction ratio, respectively.

$$Q = \left( \frac{q_c - \sigma_{vo}}{\sigma_{vo}} \right) \tag{4P-3.7}$$

$$F = \left( \frac{f_s}{q_c - \sigma_{vo}} \right) \times 100\% \tag{4P-3.8}$$

For the cyclic stress ratio (CSR), the following adjusted form is used

$$CSR = 0.65 \left( \frac{a_{\text{max}}}{g} \right) \left( \frac{\sigma_{vo}}{\sigma_{vo}} \right) (r_d) (\frac{1}{\text{MSF}}) \left( \frac{1}{K_\sigma} \right) \tag{4P-3.9}$$

where $a_{\text{max}}$ is the peak horizontal acceleration at the ground surface generated by a given earthquake; $g$ is the gravitational acceleration; $\sigma_{vo}$ and $\sigma'_{vo}$ are the total and effective vertical overburden stresses, respectively; and $r_d$ is the depth-dependent shear stress reduction coefficient; MSF is the magnitude scaling factor; $K_\sigma$ is the overburden correction factor for the cyclic stress ratio ($K_\sigma = 1$ for $\sigma'_{vo} < 1$ atm ($1$ atm = $100$ kPa)).

The stress reduction factor $r_d$ is estimated based on the recommendation by Youd et al. (2001), which takes
the following specific form

\[ r_d = \frac{1.0 - 0.4113z^{0.5} + 0.04052z + 0.001753z^{1.5}}{1.0 - 0.4177z^{0.5} + 0.05729z - 0.006205z^{1.5} + 0.001210z^{2}} \]  

(4.3.10)

The magnitude scaling factor MSF also follows the recommendation in Youd et al. (2001) as

\[ MSF = 10^{2.24} \frac{M_w}{M_w^{0.56}} \]  

(4.3.11)

where \( M_w \) is the moment magnitude of the earthquake.

Once CSR and CRR are obtained, the factor of safety against liquefaction can be derived

\[ FS = \frac{CRR}{CSR} \]  

(4.3.12)

### 4.4 Liquefaction potential index

In this work, the potential of liquefaction damage will be quantified by an averaged index property, i.e., the liquefaction potential index (LPI), which was originally proposed by Iwasaki et al. (1978, 1982) and has been subsequently used and calibrated by many investigators, e.g., Sonmez (2003); Toprak and Holzer (2003); Sonmez and Gokceoglu (2005); Holzer et al. (2006a,b); Lenz and Baise (2007); Juang et al. (2008b); van Ballegooy et al. (2015); Chen et al. (2015). As pointed out by Juang et al. (2008b) and more recently by Maurer et al. (2015); van Ballegooy et al. (2015), cautions should be taken when interpreting LPI results based on Iwasaki’s criteria Iwasaki et al. (1978, 1982), which is based on liquefaction evaluation procedures commonly used in Japan in 1978. Nevertheless, in the current framework, the LPI is used as an averaged index to quantify potential liquefaction damage, the implications of using different liquefaction criteria and calibration of LPI models are discussed in more details in Juang et al. (2008b), van Ballegooy et al. (2015).

Following the definition given in Iwasaki et al. (1978, 1982), LPI is usually evaluated for the top 20 m of soil profile as

\[ \text{LPI} = \int_0^{20} F_L w(z)dz \]  

(4.4.1)
where \( z \) denotes the depth in meters and \( w(z) = 10 - 0.5z \); \( F_L \) is defined as Sonmez (2003)

\[
F_L = \begin{cases} 
0 & \text{FS} \geq 1.2 \\
1 - \text{FS} & \text{FS} \leq 0.95 \\
2 \times 10^6 e^{-18.427\text{FS}} & 0.95 < \text{FS} < 1.2 
\end{cases}
\] (4.4.2)

where FS is the factor of safety defined in (7.4.1) for CPT-based liquefaction evaluation. Herein, we adopt a discrete form of the integral to calculate LPI along the depth of a given soil profile Luna and Frost (1998); Lenz and Baise (2007)

\[
\text{LPI} = \sum_{i=1}^{N} w_i F_{Li} H_i
\] (4.4.3)

where \( H_i \) is the thickness of the discrete layer and is determined by the CPT sampling frequency (\( H_i = 0.1 \) m for this study); \( N \) is the number of soil layers. LPI can be used to classify the severity of liquefaction according to categories proposed by Sonmez (2003) as shown in Table 6.1.

<table>
<thead>
<tr>
<th>LPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPI = 0</td>
</tr>
<tr>
<td>0 &lt; LPI ( \leq 2 )</td>
</tr>
<tr>
<td>2 &lt; LPI ( \leq 5 )</td>
</tr>
<tr>
<td>5 &lt; LPI ( \leq 15 )</td>
</tr>
<tr>
<td>LPI &gt; 15</td>
</tr>
</tbody>
</table>

### 4.5 Simulation process and implementation

The methodology in the present work defines the model parameters at the finest scale of consideration. Given the specified correlation model, the simulation starts at the coarse scale and then refines into smaller scales around areas deemed necessary. The area of refinement can be locations close to field data points, or under critical buildings where detailed information is needed. The sequential simulation process described in the previous section has the benefit of adaptive refinement and any element can be refined to its fine scale components without consideration of sequence. The parameters for random field models are always defined at the finest scale and any number of averaging scales can be defined relative to the fine scale. One of the challenges in implementing the sequential simulation process is to minimize the computational expense. Even for two scales, as the matrix \( Z_p \) representing previously generated data points increases, the
computational demand for evaluating next data point grows rapidly, due to the need to invert ever-larger matrices. In this study, the size of the matrix is limited through defining a minimum correlation for cutoff. This effectively limits the maximum distance for which to consider the previously generated realizations. An alternative approach would be limiting the total number of previously simulated realizations to consider correlation with. This alternative approach has been adopted and validated in (Chen et al. (2011)). In either case, the threshold of the minimum correlation or limiting number of elements to consider can be varied to optimize the trade off between gained efficiency and lost accuracy.

Once soil properties or indexes of interest (e.g., LPI) are realized through multi-scale random field models described above, any quantities of interest within the region or in local area can be defined and probabilistic evaluation of liquefaction potential will be calculated through Monte Carlo simulations. The application of this proposed framework will be illustrated through an example in the following sections.

4.6 Example application - Marina District, San Francisco

In this section, the proposed framework is illustrated by applying to an example region - the Marina District in San Francisco, which provides a comprehensive case history of seismic effects at a specific site. It should be noted that, even for extensively analyzed Marina District, the available measured data is still very limited. Therefore, several important model parameters have to be assumed or inferred from limited field data. This example application should be interpreted as a demonstration of the methodology, rather than a validation.

Analysis region and collection of data

Following the 1989 Loma Prieta earthquake, the USGS conducted extensive post-earthquake investigation at Marina District (Bennett (1990); Bonilla (1992)). It is found that most of the liquefaction in the Marina District occurred within the area underlain by hydraulic fill. Only a few of the tests in natural sand deposit indicated that liquefaction may have occurred within the natural deposits. Water table in the district is found to vary between 2.3 m and 5.5 m during the earthquake, with 2.4 m water table at most monitored locations (Bonilla (1992); Zhang et al. (2002)). The moist unit weight of soil above water table is and the saturated unit weight of soil is . A total of eight CPT soundings are collected and used in this section, the locations of which are shown in Figure 4.2. Six of these soundings were conducted by Bennett (1990). All CPT data used in this work are publically available from USGS Earthquake Hazards Program database (http://earthquake.usgs.gov/research cpt/).

Ground motion intensity is characterized by peak ground acceleration ($a_{max}$) and the earthquake's...
Figure 4.2: Marina District map showing locations of eight CPT soundings (adapted from USGS Earthquake Hazards Program).

moment magnitude ($M_w$). The values corresponding to different probabilities of exceedance at eight CPT sounding locations have been reported in the USGS Seismic Hazard Map 2008 (http://earthquake.usgs.gov/hazards/apps/map).

Data in Table 2 shows very small variations of peak ground accelerations among the CPT sounding locations within the Marina district. As noted by Baker and Faber (2008), ground motion intensities can be assumed to be perfectly dependent over relative small scales. For large-scale (over kilometers), spatial variations could be incorporated (Wang and Takada (2005)). In this work, we will not consider the spatial variations of ground motion intensity based on the above data. The proposed framework can be extended to incorporate such variation and will be considered in future studies.

**Liquefaction potential index at individual sounding location**

Following procedure described in the previous section on CPT-based liquefaction potential evaluation, the LPI at each of these eight CPT soundings was first computed corresponding to a given ground motion intensity. Figure 4.3 shows the example plots illustrating the major steps in calculating liquefaction potential at one CPT sounding location (SFO-022) following the procedure in Robertson and Wride (1998) and Robertson (2009). In Figure 4.3, the peak ground acceleration is taken as $a_{max} = 0.3g$ and the moment magnitude of the earthquake is $M_w = 7.0$. The moist unit weight of soil above water table is $\gamma_m = 15.0kN/m^3$ and the saturated unit weight of soil is $\gamma_m = 19.4kN/m^3$. The water table is taken at 2.4m below ground surface.

The gap in the factor of safety (FS) plot is due to the fact that soils in those layers are determined
Figure 4.3: Major steps in calculating liquefaction potential at one CPT sounding location to be non-liquefiable soil based on soil type index (Ic) criteria (Robertson and Wride (1998); Youd et al. (2001)). Therefore, the CRR and FS are not computed for those soil layers. Given the profile of factor of safety along depth, the averaged soil property, i.e., liquefaction potential index (LPI) is then calculated by simply integrating FS along depth using Eq. (4-4.3). The calculated LPIs for eight CPT soundings for a given earthquake intensity ($a_{max} = 0.3g, M_w = 7.0$) is summarized in Table 4.2.

Table 4.2: LPI and classifications for eight CPT sounding locations ($a_{max} = 0.3g, M_w = 7.0$)

<table>
<thead>
<tr>
<th>LPI</th>
<th>LPI</th>
<th>Liquefaction severity class</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFO-017</td>
<td>14.1</td>
<td>High</td>
</tr>
<tr>
<td>SFO-018</td>
<td>3.3</td>
<td>Moderate</td>
</tr>
<tr>
<td>SFO-019</td>
<td>6.2</td>
<td>High</td>
</tr>
<tr>
<td>SFO-020</td>
<td>14.6</td>
<td>High</td>
</tr>
<tr>
<td>SFO-21</td>
<td>8.4</td>
<td>High</td>
</tr>
<tr>
<td>SFO-22</td>
<td>24.8</td>
<td>Very high</td>
</tr>
<tr>
<td>SFO-23</td>
<td>14.4</td>
<td>High</td>
</tr>
<tr>
<td>SFO-24</td>
<td>20.5</td>
<td>Very high</td>
</tr>
</tbody>
</table>

Multiscale random field realizations

Calculations shown in Table 4.2 represent liquefaction susceptibility of soil at individual CPT sounding locations corresponding to a given ground motion intensity. Liquefaction susceptibility at other locations will be realized through random field models described in the methodology section. As previously mentioned, in this work, the lognormal distribution is adopted to describe the probability distribution of LPIs, which has been commonly used to describe the non-negative soil parameters such as shear strength parameters (Chen et al. (2011); Popescu et al. (2005)), CPT tip resistance and side friction (Liu and Chen (2006) and ;
Fenton (1999a)). For each set of known LPIs corresponding to a ground shaking level, model parameters of lognormal distribution are estimated using Maximum Likelihood method. The spatial correlation is discussed in the previous section, where the normalized separation distance $h$ is taken such that the correlation distance corresponds to one-quarter of the total domain of measurement (Liu and Chen (2006)).

Conditional sequential simulation process described in the methodology section is then used to generate LPI values at unknown locations, which depend on the LPIs at eight CPT locations as well as any previously generated data points. A unique and novel aspect of the random field models is the ability to selectively and consistently access and generate small-scale detailed data points in critical areas and/or in areas where soil information is known to greater details (e.g., CPT sounding locations).

Monte Carlo simulations of both single and multi-scale random fields are performed. Figure 4.4 shows sample realizations of random fields of LPIs with different levels of resolution. In this work, multi-scale random fields have been generated around eight CPT locations, as highlighted in the figures. Comparing single scale random fields with multi-scale random fields, it can be seen that on average, the multi-scale random fields show same levels of LPIs as corresponding single scale fields. However, because of the small-scale random field introduced, there are much higher resolutions in selected critical areas. This is a crucially important point about the proposed methodology, since it essentially provides a consistent way of linking small-scale information with large-scale region liquefaction assessment. It allows one to analyze in great details liquefaction susceptibility at selected locations, while consistently maintaining information at much larger scale. This will be illustrated by analyzing liquefaction risk underneath one critical structure showing in the next section.

**Liquefaction assessment**

Each Monte Carlo simulation shown in the previous section represents one realization of soil properties (e.g., LPIs) consistent with random field characterizations of the input parameters, which account for the spatial variability of soil properties across scales as well as known field data at selected locations. When a large number of simulations are performed, those illustrations can be summarized to assist liquefaction assessment. Any function of interest can be defined and then evaluated numerically. As an example, we will first evaluate cumulative frequency distribution of LPI value following the methodology proposed in (Holzer et al. (2006a)). Figure 4.5 compares the cumulative frequency of LPI using both single- and multi-scale random fields for different peak ground acceleration inputs. The earthquake magnitude $M_w$ is kept constant at 7.0. The coarse-to-fine ratio in multi-scale random fields is 36. For each $a_{max}$, a total of 1000 Monte
Figure 4.4: Random field realizations of LPIs in Marina District ($a_{max} = 0.3g, M_w = 7.0$) with different levels of resolution. From top to bottom: one large-scale element is refined into 4, 16 and 36 small-scale elements respectively. The left column is single scale random field and the right column is the corresponding multi-scale random field. The solid red circles are locations of known CPT data points.
Carlo simulations are performed. The error bar (± one standard deviation) is also included in the cumulative frequency plots.

As shown in Figure 4.5, the trends in predicted cumulative frequency are similar and the disparity is consistent among single- and multi-scale results: single-scale random field simulations consistently predict higher cumulative frequency for smaller LPI and lower cumulative frequency for larger LPI. Based on the cumulative frequency distribution, Holzer et al. (2006a) proposed that for a given geologic unit, the percent area predicted to have liquefaction during a given earthquake-shaking scenario could be estimated from the cumulative frequency distributions at LPI ≥ 5, which can be further categorized to indicate minor liquefaction (LPI ≥ 5) and major liquefaction (LPI ≥ 15) (Lenz and Baise (2007)). Table 4 summarizes cumulative frequencies corresponding to minor (LPI ≥ 5) and major liquefaction (LPI ≥ 15) in Marina district for both single-scale and multi-scale random field realizations for five different ground shaking intensities characterized by peak ground acceleration.

While both single- and multi-scale random field simulations predict consistent probability of the percent area over the region likely to have liquefaction during a given earthquake-shaking scenario, multi-scale random field realizations will provide more details on liquefaction susceptibility locally, i.e., at small scales. As shown in sample realizations of LPI in Figure 4.4, around CPT soundings, small-scale random fields provide much higher resolution of LPI. Such information could be particular helpful for assessing liquefaction in critical area or underneath critical structure.

To illustrate this last point, we will zoom in around a particular location within Marina district and utilize multi-scale random field to assist local liquefaction assessment. In the following, liquefaction underneath one critical structure, the Materna medical building, will be assessed and the probability of its severity using criteria summarized in Table 6.1 will be calculated. The location and the zoom-in view of the Materna medical building along with one realization of LPI random field are shown in Figure 4.6. The probability that a certain level of liquefaction severities (based on criteria shown in Table 1) will be exceeded under a specific ground shaking intensity underneath this building will be evaluated numerically from Monte Carlo simulations using multi-scale random field models. The results will be expressed in the form of fragility curves, which have been used extensively in earthquake engineering to describe the seismic vulnerability of structures as a function of earthquake intensity (Popescu et al. (2005); Shinozuka et al. (2000)).

For each Monte Carlo simulation, the maximum LPI value underneath the Materna medical building
Figure 4.5: Cumulative frequency distributions for both single- and multi-scale random field realizations of LPI corresponding to different peak ground accelerations. Error bars (± one standard deviation) are shown along with the cumulative frequency distribution. The coarse-to-fine ratio in multi-scale random fields is 36.
will be used to categorize liquefaction severity, which is considered to be worst-case scenario and more conservative. Then, the value of the fragility curve corresponding to a certain ground shaking level \((a_{\text{max}})\) can be estimated as follows for a particular liquefaction severity level \((S)\)

\[
P_S(a_{\text{max}}) = \frac{n_S(a_{\text{max}})}{N}
\]

where \(n_S(a_{\text{max}})\) is the number of samples that has a maximum LPI underneath Materna medical building exceeding liquefaction severity level \(S\) for a given ground shaking level \((a_{\text{max}})\); and \(N\) is the total number of samples.

Figure 4.7 shows the calculated fragility curves for the Materna medical building corresponding to different liquefaction severity levels. The peak ground acceleration \(a_{\text{max}}\) is increased from 0.10g to 0.75g with 0.05g intervals.

For a given earthquake shaking intensity, Figure 4.7 indicates the probability of the soil underneath the Materna medical building exceeding certain level of liquefaction severity. Such small-scale high resolution LPI underneath a particular critical structure could not be accurately calculated using the single-scale random field due to the fact that each coarser element in the single-scale random field is much larger than individual structure (e.g., the Materna medical building) in order to realistically cover a large region. More importantly, it should be emphasized that the small-scale higher resolution realization of soil properties in the local area is simulated consistently with large-scale random field realization of soil properties over extended area, and such small random fields could be simulated adaptively when needed during the analysis process. This presented framework could open the door to effectively link small-scale site-specific information to large-scale regional evaluation of liquefaction susceptibility.
In this chapter, a novel method is presented for assessing liquefaction potential for a particular region through integration of empirical models from geotechnical earthquake engineering (e.g., CPT-based liquefaction criteria) with tools from random field models. A unique feature of the proposed methodology is the extension of conventional random field models to account for soil spatial variability at multiple scales and resolutions. The method allows selectively and adaptively generating random fields at smaller scales around critical areas or around areas where soil properties are known to a great detail from lab or field tests. The process is defined such that spatial correlation is consistent across length scales. An illustrative example of Marina District in San Francisco is shown. The liquefaction potential is evaluated using both single- and multi-scale random field models. While both single- and multi-scale random field models predict consistent cumulative frequency of percent area to have liquefaction over the entire region, multi-scale random fields provide more detailed information and higher-resolution soil properties around critical areas. This is illustrated through calculating liquefaction fragility curves for soils underneath Materna medical building. This work provides a new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment mapping.
Chapter 5

Assessment of Liquefaction-induced Deformations Using Multiscale Random Field Models

5.0.1 Introduction

Assessment and mitigation of regional liquefaction hazard require a reliable tool for evaluating not only the probability of liquefaction occurrence, but more importantly, the associated effects within a region. Ground surface settlements due to soil liquefaction have been one of the major causes for infrastructure damages during an earthquake. Evaluation of liquefaction-induced settlement at individual locations, where field test such as the cone penetration test (CPT) is performed, can be achieved through classical empirical liquefaction model. To map liquefaction settlement over extended area, however, the spatial dependence of soil properties across the region must be taken into account. This section presents a framework integrating the classical liquefaction model (e.g., CPT-based liquefaction settlement evaluation) and tools from random field theory for the probabilistic and spatial assessment of liquefaction-induced settlements across an earthquake-prone region, from regional and geologic unit scale all the way to local site-specific scale.

Previous work

Evaluation and prediction of liquefaction-induced settlement have been the subject of intensive research for

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1A similar form of this section has been published at the time of writing: Chen, Q., Wang, C. and Juang, C. H. (2016). Probabilistic and spatial assessment of liquefaction-induced settlements through multiscale random field models. Engineering Geology, 211, 135-149.
the past several decades. Among various approaches, use of empirical relationships based on the correlation of observed soil behavior with various in-situ index tests or laboratory tests remains the dominant approach for assessing liquefaction-induced settlement, e.g., (Lee and Albaia, 1974; Tokimatsu and Seed, 1984, 1987; Nagase and Ishihara, 1988; Ishihara and Yoshimine, 1992; Pradel, 1998; Shamoto et al., 1998; Zhang et al., 2002; Wu and Seed, 2004; Tsukamoto et al., 2004; Lee, 2007; Cetin et al., 2009a,b; Lu et al., 2009; Ueng et al., 2010; Tsukamoto and Ishihara, 2010; Juang et al., 2013; Valverde-Palacios, I and Vidal, F and Valverde-Espinosa, I and Martín-Morales, M, 2014). In particular, using cone penetration test (CPT) data, Zhang et al. (2002) proposed a method for liquefaction-induced settlement coupling the volumetric strain chart by Ishihara and Yoshimine (1992) and the classical CPT-based liquefaction model by Robertson and Wride (1998). Building on the work of Zhang et al. (2002), Juang et al. (2013) proposed a probabilistic approach to estimate probability of exceeding a settlement threshold. Using laboratory data and empirical correlations, probabilistic models were developed in Cetin et al. (2009a,b) for cyclic volumetric and shear strains, which were correlated to typical field index test results, such as the corrected standard penetration test blow counts or the cone penetration test tip resistance, to predict field settlements.

All aforementioned work estimate the liquefaction-induced settlement, or the probability of exceeding a particular settlement threshold, at individual locations. To assess the consequences of liquefaction over extended areas or to map the estimated liquefaction settlements to a region, it is necessary to take into account the spatial dependence of soil properties and/or the predicted settlements. Tools developed in geostatistics have received considerable attention in recent years and have been applied to assess liquefaction hazard over large regions. For instance, Liu and Chen (2006) used CPT measurements to estimate the spatial structure of soil deposits. Then, random field models were coupled with Monte Carlo simulations to assess liquefaction potential in the Yuanlin area of Taiwan. Vivek and Raychowdhury (2014) explicitly considered the spatial variations of soil indices from CPT soundings when evaluating liquefaction potential. It is found that the probability of liquefaction could be significantly underestimated if the spatial dependence of soil indices is not considered. Chen et al. (2015) developed a CPT-based approach to map liquefaction potential across scales, where the soil spatial variability expressed in term of an averaged index, i.e., the Liquefaction Potential Index (LPI) developed by Iwasaki et al. (1978, 1982), is explicitly considered through internally-consistent probabilistic models developed at multiple scales. LPI has also been adopted and modified to assess liquefaction potentials at individual CPT soundings or over extended region (Sonmez, 2003; Sonmez and Gokceoglu, 2005; Holzer et al., 2006a; Baise et al., 2006; Lenz and Baise, 2007; Thompson et al., 2007; Juang et al., 2008b). In another recent work by van Ballegooy et al. (2015), four liquefaction vulnerability
parameters including the LPI were used to map liquefaction hazard in the Christchurch area using extensive CPT database. In contrast to the extensive efforts incorporating geostatistics tools into liquefaction potential evaluation, relatively fewer studies have addressed the liquefaction-induced settlement over extended areas. In (Hinckley, 2010; Bartlett et al., 2007), classical methods of (Tokimatsu and Seed, 1987; Ishihara and Yoshimine, 1992; Youd et al., 2002; Yoshimine et al., 2006) are employed to calculate liquefaction-induced lateral spread and ground settlement, which are plotted within their respective surficial geologic units. Then, each geologic units are assigned an estimate of ground deformation based on statistical analysis.

In this work, the CPT-based liquefaction model is coupled with novel random field tools for the probabilistic and spatial assessment of the liquefaction-induced settlements over a region and across different scales.

5.1 CPT-based liquefaction-induced settlement assessment

The CPT-based liquefaction model proposed by Robertson and Wride (1998) and subsequently updated by Robertson (2009) is adopted in this work to evaluate the liquefaction resistance of sandy soils. The detailed procedure can be found in Section 4.3. Building on the classical CPT-based liquefaction model, the liquefaction-induced settlement is approximated as a summation of the product of the volumetric strain in each soil layer that is susceptible to liquefaction and the corresponding layer thickness. For a site with level ground and far from any free water surface, such approximation is reasonable since the volumetric strain is approximately equal to the vertical strain. When sloping or near free-face ground is involved in the analysis, the deviatoric- or shearing-induced deformation should be considered when estimating ground settlement. For instance, Wu and Seed (2004) recommended to increase the ground settlement estimation by an amount equal to 10 ∼ 20% of the observed or estimated lateral ground displacement. If the estimated lateral ground displacement is smaller than 0.3 m, the influence of the deviatoric deformation is insignificant. In this study, the level ground condition is assumed, where the predicted liquefaction-induced settlement $S$ is estimated as (Juang et al., 2013)

$$ S = \sum_{i=1}^{n} \varepsilon_{vi} \Delta z_i \text{IND}_i $$

where $\varepsilon_{vi}$ is the volumetric strain of the $i$th layer; $\Delta z_i$ is the thickness of the $i$th layer; IND$_i$ is an indicator of liquefaction occurrence in the $i$th layer; and $n$ is the total number of layers. In (Zhang et al., 2002), IND is taken as 1 for all soil layers. The detailed method to obtained the volumetric strain $\varepsilon_{vi}$ can be found in Section 2.3.1.
5.2 Characterization of spatial dependence through random field models

The procedure described in Sections 4.3 and 2.3.1 estimates the settlement, either as the nominal value, as in Eq. (2.3.8), or as an random variable with mean and variance given in Eq. (2.3.9) and Eq. (2.3.10). Such estimation is performed at individual CPT sounding locations. To assess the spatial extend of liquefaction damage across a region where the settlement estimations at each location vary, spatial dependence must be taken into account. In this section, multiscale random field models (Baker et al., 2011; Chen et al., 2012, 2015) will be used to characterize spatial dependence and to simulate settlement exceedance at unsampled locations.

Semivariogram for characterization of spatial dependence

In this study, spatial dependence is described using a form of covariance known as the semivariogram, $\gamma(h)$, which is equal to half the variance of two random variables separated by a vector distance $h$

\[
\gamma(h) = \frac{1}{2} \text{Var}(Z(u) - Z(u + h))
\]  

where $Z(u)$ is the variable under consideration as a function of spatial location $u$; $Z(u + h)$ is the lagged version of the variable. Sometimes, $Z(u)$ is referred as the “tail” variable; $Z(u + h)$ is referred as the “head” variable in the geostatistics literature. Under the condition of second-order stationarity (spatially constant mean and variance), the semivariogram is related to other commonly used measures to quantify spatial correlation, i.e., the covariance $\text{COV}(h)$ and the correlation $\rho(h)$, as

\[
\gamma(h) = \text{COV}(0) - \text{COV}(h)
\]  

\[
\gamma(h) = \text{COV}(0) [1 - \rho(h)]
\]  

where $\text{COV}(0)$ is the covariance at $h = 0$ and equals to the variance $\sigma^2$. The semivariogram $\gamma(h)$ is typically preferred by geostatistics community because it only requires the increment $Z(u) - Z(u + h)$ to be second-order stationary, i.e., the intrinsic hypothesis, which is a weaker requirement than the second-order
stationarity of the variable itself.

It is possible to define a vector $\mathbf{h}$ to account for both separation distance and orientation. The most common approach to modeling the geometric anisotropy is to define a scalar distance measure as (Isaaks and Srivastava, 1989)

$$h = \sqrt{\left(\frac{h_x}{a_x}\right)^2 + \left(\frac{h_y}{a_y}\right)^2 + \left(\frac{h_z}{a_z}\right)^2}$$

where $h_x$, $h_y$ and $h_z$ are the scalar component of the vector $\mathbf{h}$ along the field’s principal axes; scalar quantities $a_x$, $a_y$ and $a_z$ are ranges that specify how quickly spatial dependence decreases along those axes.

Characterization and inference of spatial dependence remains a challenging and to some extent controversial task. Specific forms of spatial structure adopted in this study will be discussed in more details in the example application section. The definition of semivariogram presented above is written in the form of original values of the variables of interest. The commonly adopted Gaussian simulation technique requires a prior Gaussian transformation of the data. To this end, the inference of variogram model will be performed on the transformed data, which are obtained by the normal score mapping technique (Goovaerts, 1997). Chen et al. (2012) showed that such transformation does not adversely affect the prescribed spatial structure. Nevertheless, the validity of the correlation should be verified to make sure the desired spatial dependence relationship is upheld after the transformation.

**Spatial correlation across scales**

The previously described spatial correlation has been extended to account for multiscale nature of soil variability (Chen et al., 2012). This extension allows a higher resolution random field to be adaptively generated around areas of high interest, such as adjacent to critical infrastructures, or around areas of abundant field data. The key of this multiscale extension is to consistently represent fine and coarse scale random fields while maintaining appropriate spatial correlation structures across scales.

In this work, two scales of interest are considered and all the subsequent development applies to variables following the standard Gaussian distribution, i.e., variables after normal score transformation. The derivation of spatial correlation across scales is based on the notion that material properties at the coarser scales are the arithmetically averaged values of the properties over corresponding areas at the finer scales

$$Z_{T}^{c} = \frac{1}{N} \sum_{i=1}^{N} Z_{i(T)}^{f}$$

where superscripts ‘c’ and ‘f’ correspond to coarse and fine scales, respectively; $N =$ number of fine scale
points within a coarse scale area (element) $I$. Defining the variable of interest at the fine scale and using such a relation, the expression for the variances and spatial correlations of coarse scale variables of interest can be explicitly derived.

The mean of a coarse scale element $Z_c^I$ can be derived by taking the expectation of Eq. (5p.2.5) as

$$
\mu_{Z_c} = E[Z_c] = \frac{1}{N} \sum_{i=1}^{N} \mu_{z_f}^{I(i)} = 0
$$  \hfill (5p.2.6)

where $\mu_{z_f}^{I(i)}$ is the mean at the fine scale, which equals to zero for variables following the standard Gaussian distribution. Accordingly, if the variance of the fine scale variable is unity, the coarse scale variance can be computed as

$$
\sigma^2_{Z_c} = E[(Z_c^I)^2] - 0 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z_f}^{I(i), z_f} \sigma_{z_f}^{I(i)} \sigma_{z_f}^{I(j)}
$$  \hfill (5p.2.7)

The covariance between any two elements $Z_i$ and $Z_j$ within the random field is defined as

$$
\text{COV}[Z_i, Z_j] = \rho_{z_f, z_f} \sigma_{z_f} \sigma_{z_f}
$$  \hfill (5p.2.8)

The correlations between all considered scales can be calculated by rearranging the definition of covariance such that

$$
\rho_{z_i, z_j} = \frac{\text{COV}[Z_i, Z_j]}{\sigma_{z_i} \sigma_{z_j}}
$$  \hfill (5p.2.9)

where $Z_i$ and $Z_j$ are two elements within the random field at any scale with variance $\sigma^2_{z_i}$ and $\sigma^2_{z_j}$. By making appropriate substitutions at each scale using Eq. (5p.2.8) and Eq. (7p.8.4), the correlation between elements at different scales can be obtained as

$$
\rho_{z_f, z_f} = \frac{\sum_{i=1}^{N} \sum_{k=1}^{N} \rho_{z_f}^{I(i), z_f} \sigma_{z_f}^{I(i)}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z_f}^{I(i), z_f} \sigma_{z_f}^{I(i)}} \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z_f}^{I(j), z_f} \sigma_{z_f}^{I(j)}}}
$$  \hfill (5p.2.10)

$$
\rho_{z_f, z_f} = \frac{\sum_{i=1}^{N} \rho_{z_f}^{I(i), z_f}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z_f}^{I(i), z_f} \sigma_{z_f}^{I(j)}}}
$$  \hfill (5p.2.11)

where the Roman numerals $I, II...$ are used for coarse scale element number; $\rho_{z_f, z_f} = \rho_{z_f, z_f} = \rho_{z_f, z_f}$, where $i$ and $k$ belong to two
different coarse scale elements $I$ and $II$, respectively. Given correlation between elements at different scales, the corresponding semivariogram can be easily obtained through relation Eq. (5.2.3) with $\text{COV}(h) = 1$ for a standard Gaussian field.

Given the multiscale spatial dependence specified by the semivariogram and an inferred or assumed probability distribution of a parameter value at a single location, a sequential Gaussian simulation process (Goovaerts, 1997) is implemented in Matlab to generate random field realizations of variables of interest.

5.3 Case study: Alameda County of California

Analysis region, engineering geology and field data

In this section, the proposed framework is applied for the probabilistic and spatial assessment of liquefaction-induced settlements across a particular earthquake-prone region, the Alameda County of California. A comprehensive digital database of the engineering geology in Alameda County has been compiled by Helley and Graymer (1997) and briefly summarized by Holzer et al. (2006a). As shown in Figure 7.1, the area contains five major surficial geologic units: artificial fill, younger San Francisco bay mud, Holocene alluvial fan deposits, Merritt sand and Pleistocene alluvial fan deposits. The surficial geology divides the studied area into three broad northwest-southeast-trending regions. Bedrock is exposed at the surface of the northeast land. The central area, immediately to the southwest of the bedrock, consists of Holocene and Pleistocene alluvial fan deposits. The area next to the central area - southwest of the original natural shoreline - is primarily underlain by artificial fill that rests on younger San Francisco Bay mud.

CPT penetration indicates the artificial fill thickness ranges from about 11 m to zero. The average thickness is about 3 m. In the central area, the Holocene alluvial fan deposits, which overlap the deposits of Pleistocene age, generally consist of fine grained deposits. This layer was active until modern urban development covered the land surface and channelized the modern streams (Sowers and Richard, 2010). The thickness of the fan deposits ranges from about 14.3 m to zero. The average thickness is about 4.4 m. Beneath the Holocene alluvial fan deposits are the older Pleistocene alluvial fan deposits that were last active during the previous interglacial period (Trask and Rolston, 1951). The wind-blown Merritt sand deposits, resting on the Pleistocene alluvial fan deposits, were chiefly deposited near the end of the Pleistocene epoch when sea level was lower than current. Ground water is found to be 3 m or less in much of the studied area.

The Alameda County is a seismically active region at the boundary of the Pacific Plate and the North American Plate. The most important seismic source is the Hayward Fault system (Holzer et al.,
Figure 5.1: Site map of the study area (Alameda, Berkeley, Emeryville, Oakland, and Piedmont, California) and locations of CPT soundings (modified from (Holzer et al., 2006a))
2006a), as shown in Figure 7.1. In addition, the San Andreas Fault, which stretches roughly 1,300 km through California, lies to the west of this region. Herein, the seismic source for the liquefaction analysis will be chosen from the rupture of the Hayward Fault system. The report by the Working Group on California Earthquake Probabilities (WGCEP) (WGCEP, 2003) predicted the probability in a 30 year period (2002-2031) of one or more earthquakes with magnitude $M_w \geq 6.7$ and $M_w \geq 7.0$ for the Hayward-Rodgers Creek Fault system to be 0.27 and 0.11, respectively. It was also predicted that a rupture of the south segment of the Hayward Fault would produce a magnitude $M_w = 6.6$ earthquake and a rupture of the north segment and the Rodgers Creek would produce a magnitude $M_w = 7.1$ earthquake. The last damaging earthquake on the Hayward Fault was an estimated $M_w = 6.8$ earthquake in 1868, when the southern segment ruptured (WGCEP, 2003). In the following analysis, two earthquake events corresponding to $M_w = 6.6$ and $M_w = 7.1$ will be considered. Previous analysis by Holzer et al. (2006a) mapped liquefaction potential index for this region given the above two earthquake events and will be used to compare results from this study.

The CPT data used in the case study are taken from the U.S. Geological Survey (USGS) Earthquake Hazard Program CPT database (USGS, 2015). A total of 210 CPT sounding data are compiled. The water table information is directly obtained from the CPT sounding record wherever it is available (181 out of the 210 CPT soundings compiled have water table measurement). For CPT soundings without such information, the water table is interpolated. For unit weights of soil, a moist unit weight $\gamma_m = 15.0 \text{ kN/m}^3$ and a saturated unit weight $\gamma_{sat} = 19.4 \text{ kN/m}^3$ are assumed for soils above and below water table, respectively. Locations of CPT soundings, the surficial geologic units and the outline of the studied region are shown in Figure 7.1.

**Deterministic evaluation of liquefaction-induced settlement at CPT soundings**

Liquefaction-induced settlement is calculated at each CPT sounding using methodology presented in Section 2.3.1 Eq. (5.1.1) with $M_w = 6.6$ and $M_w = 7.1$ earthquakes on the nearby Hayward Fault. The peak horizontal ground acceleration $a_{\text{max}}$ are taken as constant of 0.4 g and 0.5 g for $M_w = 6.6$ and $M_w = 7.1$, respectively. The assumption of a constant $a_{\text{max}}$ is justified on the basis that the outcrop area of each surficial geologic unit is generally parallel and is close to the Hayward Fault (Holzer et al., 2006a). Alternatively, at each individual location with known longitude and latitude, the joint distribution of $a_{\text{max}}$ and $M_w$ can be derived using the readily accessible National Seismic Hazard Maps (USGS, 2014) and can be used in conjunction with any liquefaction model. A simplified process incorporating joint distribution of $a_{\text{max}}$ and $M_w$ has been developed in (Juang et al., 2008a) and will be considered in a future study.

Histograms of predicted settlements at CPT soundings for the above two earthquake events are
shown in Figure 5.2.

Figure 5.2: Histogram of liquefaction-induced settlements at 210 CPT soundings in the Alameda County.

Cumulative frequency functions (CDF) of predicted settlements at CPT soundings for the above two earthquake events are shown in Figure 5.3. The CDF is used to transform the settlements into a Gaussian distribution, upon which the random field simulations run. The transformation is performed as $X' = \Phi^{-1}(F(X))$, where $X$ is the settlement, $F(X)$ is the corresponding CDF, $X'$ is the mapped settlement in Gaussian distribution and $\Phi$ is the corresponding CDF in Gaussian distribution.

Figure 5.3: CDF of liquefaction-induced settlements at 210 CPT soundings in the Alameda County.

The liquefaction-induced settlements could be correlated to extend of observed damage and one such correlation was proposed by Ishihara and Yoshimine (1992), shown in the following table.
Table 5.1: Damage extent and approximate settlement (Ishihara and Yoshimine, 1992)

<table>
<thead>
<tr>
<th>Extent of damage</th>
<th>Settlement (cm)</th>
<th>Phenomena on the ground surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light to no damage</td>
<td>0-10</td>
<td>Minor cracks</td>
</tr>
<tr>
<td>Medium damage</td>
<td>10-30</td>
<td>Small cracks, oozing of sand</td>
</tr>
<tr>
<td>Extensive damage</td>
<td>30-70</td>
<td>Large cracks, spouting of sands</td>
</tr>
<tr>
<td></td>
<td></td>
<td>large offsets, lateral movement</td>
</tr>
</tbody>
</table>

To estimate the hazard of liquefaction-induced settlement posed by each geologic unit, settlement values at CPT soundings were grouped by surficial geologic unit. Cumulative frequency distributions of settlements are plotted in Figure 5.4(a) for \( M_w = 6.6 \) earthquake and in Figure 5.5(a) for \( M_w = 7.1 \) earthquake. In a previous analysis by Holzer et al. (2006a), liquefaction potential index (LPI) were grouped by each geologic unit and used as basis to assess liquefaction hazards posed by each geologic unit. Cumulative frequency distributions of LPIs from (Holzer et al., 2006a) are shown in Figure 5.4(b) for \( M_w = 6.6 \) earthquake and Figure 5.5(b) for \( M_w = 7.1 \) earthquake for comparison with the current study.

Figure 5.4: Cumulative frequency distributions of liquefaction hazards for an \( M_w = 6.6 \) earthquake in the Alameda County grouped by surficial geologic units. Number of CPT soundings in each geologic unit is shown in parentheses.

Information presented in Figures 5.4 and 5.5 can be used as an initial quantitative evaluation of liquefaction-induced settlements and associated extent of damage (ref. Table 5.1) in each geologic unit. The percentage of soundings underlain by a geologic unit falls within certain settlement range may be used as an indication of the approximate percentage of the surface area exhibiting the corresponding damage level. The more CPT soundings included, the better such approximation becomes.
Following such an interpretation, Figure 5.4(a) predicts that, for an $M_w = 6.6$ earthquake event caused by the rupture of south segment of the Hayward Fault, 42% of the areas underlain by the artificial fill will exhibit medium to extensive damage, which is 21% less than an $M_w = 7.1$ earthquake (Figure 5.4(a)). Approximately 13% of the areas underlain by the Holocene alluvial fan will exhibit medium to extensive damage.

For an $M_w = 7.1$ earthquake event, Figure 5.5(a) indicates that approximately 53% and 15% of the areas underlain by the artificial fill and the Holocene alluvial fan will exhibit medium to extensive damage. Medium to extensive damage are not anticipated for the areas underlain by most of the Pleistocene alluvial fan deposits since only one of the ten CPT soundings in this geologic unit is predicted to have medium damage. This site (numbered as OAK024 in the USGS database (USGS, 2015) and located at $(37.792^\circ, -122.252^\circ)$) is underlain by the Bull Lake till, which is a softer subunit of Pleistocene fan deposits. Most of the Merritt sand are not anticipated to experience extensive damage either.

Similarly, based on the calibration of LPI with surface manifestations of liquefaction, it is postulated in (Toprak and Holzer, 2003; Holzer et al., 2006a) that surface manifestations of liquefaction in general occur when $LPI > 5$. Figure 5.4(b) indicates that, for an $M_w = 6.6$ earthquake event, only the artificial fill will exhibit surface manifestations of liquefaction and 56% of the area underlain by the fill will be affected ($LPI > 5$), which is 23% less than an $M_w = 7.1$ earthquake event, shown in Figure 5.5(b). This is consistent with results obtained using liquefaction-induced settlement (21% difference in predicted damage between the
$M_w = 6.6$ and $M_w = 7.1$ events). For an $M_w = 7.1$ earthquake event, Figure 5.5(b) indicates that 73% and 3%, respectively, of the areas underlain by the artificial fill and Holocene alluvial fan will exhibit surface manifestations of liquefaction.

It is worth pointing out that, though higher LPI values generally correspond to larger predicted settlements, there is no simple linear relation or one-to-one transformation between those two indices. A surface manifestation of liquefaction (LPI > 5) does not necessarily correspond to medium to extensive damage (settlement > 10 cm). The resemblance of the trends of the cumulative frequency distribution shown in Figures 5.4(a) and 5.5(a) for settlements to that in Figures 5.4(b) and 5.5(b) for LPIs, however, are expected considering the positive correlation between LPI and settlement.

A closer look into Figures 5.4 and 5.5 reveals significant discrepancy between settlement prediction and LPI prediction for the subunit of Merritt sand deposits, location of which is shown in Figure 7.1. This subunit is identified as the area with higher liquefaction potential, approximately 38% of this subunit is predicted to exhibit liquefaction effects (LPI > 5) in (Holzer et al., 2006a). However, two of the five CPT soundings within this subarea do not encounter the water table until the end of the penetration. In this study, those two CPT soundings will predict no liquefaction, corresponding to zero LPI or settlement values. Hence, in Figures 5.4(a) and 5.5(a), the cumulative frequency curves for the Merritt sand subunit do not start from 100% and the percentage of this subunit being subjected to liquefaction damage is significantly less than that predicted in (Holzer et al., 2006a).

Spatial analysis and mapping of liquefaction-induced settlements

Previous analysis focuses on the liquefaction-induced settlements at individual CPT soundings. The probabilistic and cumulative frequency plots are based on information at those isolated locations. To estimate the extend of the liquefaction-induced settlements over a large region, the spatial correlation needs to be considered. In this section, the spatial dependence will be characterized through the semivariogram model, described in details in Section 6.4.1. Multiscale random field models will then be used to generate realizations of settlements throughout the region.

The spatial structure of the liquefaction-induced settlements will be obtained from semivariogram inference. Given settlement predictions at 210 CPT soundings and their spatial coordinates, the sample
The semivariogram \( \hat{\gamma}(h) \) is computed as (Goovaerts, 1997)

\[
\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} [z(u_\alpha) - z(u_\alpha + h)]^2
\]

where \( N(h) \) is the number of pairs of data \( z \) located a vector \( h \) apart (i.e., a lag bin \( h \)). In the actual computation, a small tolerance (e.g., \( 10 \sim 20\% \) of the distance \( h \)) is usually added to lag bins to accommodate unevenly spaced sample points. Also, it is often convenient to use a scalar distance measure \( h \), as defined in Eq. (6.4.2), for the calculation of semivariogram.

Figure 5.6 shows the sample semivariogram based on settlements at 210 CPT soundings for both \( M_w = 6.6 \) and \( M_w = 7.1 \) earthquake events. Given the sample semivariogram, a weighted least square method by Cressie (1985) is implemented to fit an analytical semivariogram model, shown as solid line in the plot.

As shown in Figure 5.6, the empirical semivariograms for \( M_w = 6.6 \) and \( M_w = 7.1 \) earthquake events are almost identical, indicating a negligible influence of earthquake intensity on the spatial structure of predicted settlements. In this study, a common semivariogram model is fitted using an exponential model of the form

\[
\gamma(h) = 1 - \exp \left( -\frac{3h}{a} \right)
\]

where the practical range \( a = 2,400 \) m. This fitted semivariogram will be used in the random field realization of settlements across the region of interest.
In addition to the semivariogram, the random field model requires the probabilistic distribution of the variable of interest. Herein, a piece-wise linear probability density function is fitted to the histogram of predicted settlements in Figure 5.2. A normal score mapping and a sequential Gaussian simulation process are then used to generate random field realizations of variables of interest. Such simulation process has been successfully applied in previous applications and it has been shown that the spatial structure is preserved after normal score mapping and during the simulation process (Baker and Faber, 2008; Baker et al., 2011; Chen et al., 2012, 2015).

Figures 5.7 and 5.8 plot the typical single and multiscale random field realizations of liquefaction-induced settlements for earthquake events \( M_w = 6.6 \) and \( M_w = 7.1 \), respectively. In the multiscale realizations, higher resolutions are introduced in the artificial fill geologic unit, where higher liquefaction hazard is expected. It should be noted that, on average, the higher resolution region in a multiscale field resembles the trend seen in the single scale counterpart but with much more detailed information. Such higher resolution information is important when performing local site-specific analysis, as will be shown later.

![Typical random field realizations of liquefaction-induced settlements](image)

**Figure 5.7:** Typical random field realizations of liquefaction-induced settlements in Alameda County for \( M_w = 6.6 \) earthquake event on the Hayward Fault. Unit of the settlement is in cm. Superimposed grey lines correspond to boundaries of geologic units.

The corresponding histograms of simulated liquefaction-induced settlements are plotted in Figure 5.9. The histograms have preserved the distribution of settlements at 210 CPT locations as previously shown in
Figure 5.8: Typical random field realizations of liquefaction-induced settlements in Alameda County for $M_w = 7.1$ earthquake event on the Hayward Fault. Unit of the settlement is in cm. Superimposed grey lines correspond to boundaries of geologic units.

Figure 5.2. Moreover, the spatial structure is also upheld during the simulation, as demonstrated in Figure 5.10, where the exhaustive empirical semivariogram based on simulated liquefaction-induced settlements are compared well with the specified exponential semivariogram model in Eq. (5.3.2).

Figure 5.9: Typical histograms of simulated liquefaction-induced settlements across the Alameda County.

The random field model can be coupled with Monte Carlo simulations to evaluate various quantities.
of interest and associated uncertainties. As an example, the cumulative frequency distribution of the predicted settlements are evaluated along with uncertainties in the prediction. Figure 5.11 shows the cumulative frequency of the predicted settlements based on a total of 1,000 Monte Carlo simulations. The error bar (± one standard deviation) is also included in the cumulative frequency plots. It can be seen that, for the given earthquake events, less than 30% of the Alameda County area is predicted to have a settlement greater than 10 cm. The percentage of area predicted to have more than 30 cm settlement is very close to zero. A comparison between the single and multiscale results in Figure 5.11 reveals a similar trend but a consistent higher prediction by multiscale. For example, for the $M_w = 7.1$ earthquake, multiscale result indicates that about 27% of the area has a settlement exceeding 10 cm, while the single scale predicts the percentage to be 18%. Since the single (coarse) scale is defined as the average of the corresponding fine scale elements as in Eq. (5.2), the findings in Figure 5.11 indicate that such averaging process might yield unconservative estimation of liquefaction hazard.

To further demonstrate the capability of random field models and explore their potential applications combined with Monte Carlo simulations, in the following, the author re-analyze the cumulative frequency distributions of liquefaction-induced settlements grouped by different geologic units, as previously presented in Section 5.3. Instead of using just settlements at CPT soundings, herein, the cumulative frequency distributions are evaluated using settlements throughout the Alameda County based on results of 1,000 Monte Carlo simulations. The results are summarized in Figure 5.12 for both $M_w = 6.6$ and $M_w = 7.1$ earthquake events.
For $M_w = 6.6$ earthquake, Figure 5.12(a) indicates that 48.2% of the areas underlain by artificial fill and 7.4% of the areas underlain by Holocene alluvial fan will exhibit medium to extensive damage (settlement greater than 10 cm). The combined region underlain by Merritt sand Pleistocene fan and the northeast portion of the Holocene fan is predicted to have 10% of its area exhibiting medium damage. Only 1% of the subarea underlain by Merritt sand has a medium damage. These predictions are consistent with those made in Section 5.3.

For $M_w = 7.1$ earthquake, Figure 5.12(b) indicates that 57.7% and 13.2%, respectively, of the areas underlain by artificial fill and Holocene alluvial fan will exhibit medium to extensive damage. These predictions are very close to the results in Section 5.3, which is 53% and 15%. The combined region underlain by Merritt sand, Pleistocene fan (also including the northeast portion of the Holocene fan) is predicted to have less than 2.4% of its area exhibiting medium damage, which is negligible and again, consistent with predictions made in Section 5.3.

A visualization of the predicted liquefaction-induced settlement mapping averaged from 1,000 Monte Carlo simulations are shown in Figure 5.13(a) along with the LPI mapping obtained from (Holzer et al., 2006a) shown in Figure 5.13(b) for $M_w = 7.1$ earthquake event. As shown in Figure 5.13(a), higher settlements (greater than 10 cm) occurs mainly in the southwest region of the original natural shoreline, which is primarily underlain by the artificial fill that rests on younger San Francisco Bay mud. Similar predictions are made by Holzer et al. (2006a) in Figure 5.13(b), where there is 73% possibility for this region to exhibit liquefaction. Figure 5.13(a) shows that, the region close to the northeast of the shoreline, which is underlain by Holocene
fan deposits, is predicted to experience minor settlements (most of the values do not exceed 10 cm). To the northeast of the Holocene fan, the combined region including a portion of Alameda island, is not anticipated to experience high settlements. This region is mainly underlain by Merritt sand and Pleistocene fan deposits.

Figure 5.12: Cumulative frequency distributions of predicted liquefaction-induced settlements across the Alameda County grouped by surficial geologic units for $M_w = 6.6$ and $M_w = 7.1$ earthquakes on the Hayward Fault.

Figure 5.13: Liquefaction hazard mapping in the Alameda county site for earthquake event $M_w = 7.1$ on the Hayward Fault.
While the results show that the proposed framework predicts consistent liquefaction hazards for the entire region and for individual surficial geologic units, the multiscale random field models provide much more detailed information and are able account for spatial variability of the settlement within each geologic units. This enables the assessment of local site-specific liquefaction hazards. To demonstrate this point, two local sites shown in Figure 5.14 are picked from the Alameda County. The first site is located at the Alameda island, consisting of the Ruby Bridges Elementary School, College of Alameda and a crowded residential area. This site will be referred to as the “Ruby Bridges School” site in the following analysis. The second local site, which includes schools and care centers such as the St Mary’s Center Preschool, Hoover Elementary School, Herbert Hoover Junior High School and Love Always Child Care Center, will be referred to as the “Hoover School” site in the following analysis. The assessment are based on the same 1,000 Monte Carlo simulations performed and used in previous cumulative frequency plots.

Figure 5.14: Two subregions selected within the Alameda County for local site-specific liquefaction hazard analysis
Figure 5.15 shows the percentage of area predicted to exceed certain settlement values. It can be seen that 58% of the Ruby Bridges School site will suffer medium to extensive damage (settlement > 10 cm) for the $M_w = 6.6$ earthquake event, while the percentage is only 3% for the Hoover School site. For an $M_w = 7.1$ earthquake event, 75% of the Ruby Bridges School site and 9% of the Hoover School site will suffer medium to extensive damage.

![Figure 5.15: Fraction of area exceeding a particular settlement for the Ruby Bridges School and Hoover School sites.](image)

Figure 5.16 plots the percentage of area that will suffer medium to extensive damage and the corresponding probabilities for both $M_w = 6.6$ and $M_w = 7.1$ earthquake events for two sites. Again, the Ruby Bridges School site is expected to suffer more liquefaction-induced damage (quantified by the predicted settlements) than the Hoover School site. Such detailed information demonstrates the potential of the multiscale random field models for local site-specific liquefaction hazard analysis.

## 5.4 Summary

In this chapter, a framework is developed that integrates the classical CPT-based liquefaction model with multiscale random field models and Monte Carlo simulations for the probabilistic and spatial assessment of liquefaction-induced settlements over a region and across scales. One critical advantage of the developed framework is its ability to consistently refine and provide settlement estimations across different scales, from regional and surficial geologic unit scale all the way to local site-specific scale. The developed framework is applied to the liquefaction hazard analysis of the Alameda County site of California and is demonstrated to
be a valuable tool for multiscale regional liquefaction hazard analysis. In summary, it is found that

1. Quantitatively consistent liquefaction hazards over the entire studied region and within each surficial geologic units are obtained when verified against existing analysis and knowledge of the studied region.

2. Spatial variability of soil properties within each geologic units at a much smaller and much detailed scale are captured, which provides a way to systematically refine and perform local site-specific liquefaction analysis while preserving the liquefaction hazard prediction at the regional scale.

3. The spatial structure of the predicted settlements inferred from available field data are shown to be relatively insensitive to the earthquake shaking intensity and such inferred spatial structure is preserved during the random field modeling process.

4. In the Alameda County site, the artificial fill is the surficial geologic unit most susceptible to liquefaction hazard (48.2% and 57.7% of the area will exhibit medium to extensive damage for the $M_w = 6.6$ and $M_w = 7.1$ earthquake scenarios considered) followed by the Holocene alluvial fan deposits (the corresponding percentages are 7.4% and 13.2%).

5. Local site-specific analysis shows that the Ruby Bridges School site is expected to suffer more liquefaction-induced damage (quantified by the predicted settlements) than the Hoover School site.

Future work will fully incorporate uncertainties in the liquefaction settlement models, the input parameters, and the earthquake shaking intensities, and will explore the effect of these uncertainties in the
regional liquefaction hazard analysis.
Chapter 6

The Spatial Variability of CPT-based Geotechnical Parameters for Regional Liquefaction Evaluation

6.1 Introduction

Earthquake-induced liquefaction of soils often causes significant damage to infrastructure such as buildings, bridges, and lifelines Kramer (1996). Evaluating the likelihood of liquefaction and the associated geohazards involve analysis of ground shaking hazard and liquefaction susceptibility of the soil deposit on Earthquake Engineering and on Earthquake Engineering Research (1985). In practice, the use of the empirical correlations of the observed field behavior with various in-situ index tests, such as the Cone Penetration Test (CPT), the Standard Penetration Test (SPT) and the shear wave velocity test, remains the dominating approach for assessing liquefaction potential; see Youd et al. (2001) for a summary and recommendation of various in-situ test-based liquefaction models. Building on the empirical models, the direct output of a liquefaction evaluation procedure is typically expressed in terms of factor of safety against liquefaction triggering in a soil stratum at depth. The damage potential of liquefaction can then be linked to the factor of safety through a nonlinear depth integration such as the liquefaction potential index (LPI)

\footnote{A similar form of this section has been published at the time of writing: Wang, C., Chen, Q., Shen, M. and Juang, C. H. (2017). On the spatial variability of CPT-based geotechnical parameters for regional liquefaction evaluation. Soil Dynamics and Earthquake Engineering, 95, 153-166.}
proposed in Iwasaki et al. (1978).

The empirical models evaluate the liquefaction potential at individual locations where field tests are performed. Estimation of liquefaction potential at locations away from the measurement site requires some degrees of spatial continuity of soil properties. In other words, soil properties, as indicated by the representative indices like the tip resistance $q_c$ and the side friction $f_s$ from CPT measurements, are spatially correlated DeGroot and Baecher (1993). Tools in geostatistics Goovaerts (1997); Vanmarcke (2010) have been used to model such spatial variation of soil properties as a random field. Interpolation and stochastic simulation techniques are then used to estimate the spatial distribution of properties at a site. In the context of liquefaction analysis, examples of work along this line include Popescu et al. (1997); Fenton and Vanmarcke (1998); Elkateb et al. (2003); Popescu et al. (2005); Liu and Chen (2006); Lenz and Baise (2007); Baker and Faber (2008); Pokhrel et al. (2013); Vivek and Raychowdhury (2014); Chen et al. (2015).

To account for the spatial variability of geotechnical properties in the liquefaction evaluation procedure, two approaches will be developed and assessed in this work. The first approach, termed the local soil property approach, treats the local field data (e.g., the CPT tip resistance and side friction) as spatially correlated soil properties across the region. Random fields of field data are realized in a layer-by-layer sequence, i.e., only the horizontal correlation is explicitly modeled. Examples of existing efforts along this line include Fenton (1999b); Uzielli et al. (2005); Liu and Chen (2006); Baker and Faber (2008); Vivek and Raychowdhury (2014). On a smaller scale, e.g., a specimen in the lab or for a relatively small area around a field bore-hole location, full three-dimensional random field models have also been proposed to characterize spatially correlated soil properties and applied to liquefaction evaluation Fenton and Vanmarcke (1998); Elkateb et al. (2003); Popescu et al. (2005). However, generating full three-dimensional random field models on a regional scale not only poses increased computational challenge but may not guarantee a more accurate result in liquefaction evaluation since now both vertical and horizontal correlations need to be simultaneously accounted for using available field data. The scales of variations in the vertical and horizontal directions can be different in order of magnitude over a region (e.g., centimeter in the vertical direction vs. meter or kilometer in the horizontal direction). For these reasons, a more sophisticated three-dimensional random field model may not yield better estimation therefore will not be considered here.

An alternative approach, termed the averaged index approach, computes an averaged soil property or index (e.g., the liquefaction potential index or liquefaction probability) at individual locations where field data are available. Then, interpolations or geostatistical tools and random field models are utilized to generate the averaged index properties at unmeasured or unsampled locations across the region. This approach is
much more commonly used in liquefaction mapping over extended area, e.g., Holzer et al. (2006a,b); Baise et al. (2006); Lenz and Baise (2007); Baise et al. (2008); Chen et al. (2015); van Ballegooy et al. (2015), since it only requires characterization and/or random field realizations of an averaged quantity of interest, i.e., the liquefaction potential index, in the horizontal direction. This makes parameter identification more straightforward and is much more computationally efficient when evaluating liquefaction susceptibility over large areas. However, by calculating an averaged index, details of soil properties are no long available for the region and the effect of this averaging process on the estimated liquefaction potential is unknown.

While both approaches have been applied to evaluate liquefaction potential across a region, the implications of these two approaches on the liquefaction risk have yet to be addressed. Moreover, much existing experience with random field models for soil properties is limited to a single spatial scale. When evaluating liquefaction potential over an extended region, spatial variability of geotechnical parameters often manifests at different scales, i.e., boring data (local scale) vs. surficial geological features (regional scale). Field data are oftentimes sparse and the uncertainties away from local boring data can be large. A multiscale consideration is deemed necessary. In this work, a multiscale random field model Baker et al. (2011); Chen et al. (2012, 2015) will be integrated with empirical liquefaction models for the evaluation of liquefaction potential over extended areas.

We will assess how the local soil property and the averaged index property approaches affect the evaluation of liquefaction potential in spatially variable soils. The proposed framework is among the first efforts to integrate multiscale local soil property and averaged index random fields with liquefaction potential evaluation. The applicability and assessment of these random fields will be demonstrated through the probabilistic and spatial assessment of liquefaction potential in a liquefaction-prone region.

6.2 General framework

In this work, the CPT-based empirical liquefaction model is integrated with geostatistical tools to account for the spatial variability of geotechnical properties for regional liquefaction evaluation. The flow of the general framework to account for spatial variability is shown in Figure 6.1. As shown in Figure 6.1, within a liquefaction-prone region, CPT measurements (e.g., the tip resistance and the side friction) and other geotechnical data of interest (e.g., water table, soil unit weight, etc.) are first collected and their geostatistical properties are inferred and characterized (e.g., probabilistic distribution, spatial structure). At a CPT sounding, the empirical liquefaction model described in Section 7.4 will be used to evaluate the damage potential of liquefaction, quantified here by the liquefaction potential index (LPI). The two
Figure 6.1: General framework of the CPT-based liquefaction potential evaluation over extended area: the local soil property approach vs. the averaged index approach.
approaches described in Section 7.1, i.e., the averaged index approach and the local soil property approach, are developed to incorporate soil variability into the evaluation of liquefaction over an extended area. Details of multiscale random field model development and implementation will be discussed in Section 6.4. Finally, Monte Carlo simulations will be used to generate realizations of the random fields and results will be used for the probabilistic and spatial assessment of various quantities of interest for liquefaction evaluation over the region.

6.3 CPT-based liquefaction evaluation

In this work, we adopt the classical procedure proposed by Robertson and Wride (1998) and subsequently updated by Robertson (2009) and Ku et al. (2011) to evaluate the liquefaction resistance of sandy soils based on CPT data. Herein, the liquefaction potential of a soil layer is evaluated using two variables - the cyclic stress ratio (CSR) and the cyclic resistance ratio (CRR). Details of CSR and CRR calculation are summarized here.

For CSR, the following adjusted form is recommended in Youd et al. (2001)

\[
CSR = 0.65 \left( \frac{a_{\text{max}}}{g} \right) \left( \frac{\sigma_{vo}}{\sigma'_{vo}} \right) (r_d) \left( \frac{1}{\text{MSF}} \right) \left( \frac{1}{K_{\sigma}} \right)
\]  

(6.3.1)

where \(a_{\text{max}}\) is the peak horizontal acceleration at the ground surface generated by a given earthquake; \(g\) is the gravitational acceleration; \(\sigma_{vo}\) and \(\sigma'_{vo}\) are the total and effective vertical overburden stresses, respectively; and \(r_d\) is the depth-dependent shear stress reduction coefficient; MSF is the magnitude scaling factor; \(K_{\sigma}\) is the overburden correction factor for the cyclic stress ratio (\(K_{\sigma} = 1\) for \(\sigma'_{vo} < 1\) atm (1 atm = 100 kPa)). CSR defined above is also often denoted as CSR_{7.5} for earthquakes with magnitudes of approximately 7.5.

The stress reduction factor \(r_d\) and the magnitude scaling factor MSF are both estimated based on the recommendations in Youd et al. (2001) as

\[
r_d = \frac{1.000 - 0.4113z^{0.5} + 0.04052z + 0.001753z^{1.5}}{1.000 - 0.4177z^{0.5} + 0.05729z - 0.006205z + 0.001210z^2}
\]  

(6.3.2)

\[
\text{MSF} = \frac{10^{2.24}}{M_w^{0.56}}
\]  

(6.3.3)

where \(z\) is the depth; \(M_w\) is the moment magnitude of the earthquake.

The cyclic resistance ratio, CRR, is estimated from CPT data following the procedure in Robertson
and Wride (1998); Robertson (2009); Ku et al. (2011) as

\[
\text{CRR} = \begin{cases} 
0.8333[(q_{c1N})_{cs}/1000] + 0.05 & \text{if } (q_{c1N})_{cs} < 50 \\
93[(q_{c1N})_{cs}/1000]^3 + 0.08 & \text{if } 50 \leq (q_{c1N})_{cs} < 160 
\end{cases} 
\tag{6\cdot3.4}
\]

The equivalent clean sand normalized penetration resistance, \((q_{c1N})_{cs}\) is given as

\[
(q_{c1N})_{cs} = K_c(q_{c1N}) \tag{6\cdot3.5}
\]

where the conversion factor \(K_c\) is calculated from the soil behavior type index \(I_c\) as

\[
K_c = \begin{cases} 
1 & \text{for } I_c \leq 1.64 \\
-0.403I_c^4 + 5.581I_c^3 - 21.63I_c^2 + 33.75I_c - 17.88 & \text{for } I_c > 1.64 
\end{cases} \tag{6\cdot3.6}
\]

and \(q_{c1N}\) is the normalized cone penetration resistance

\[
q_{c1N} = \left(\frac{q_c - \sigma_{vo}}{P_{at}}\right) \left(\frac{P_{at}}{\sigma_{vo}}\right)^n \tag{6\cdot3.7}
\]

where \(P_{at} = 1\) atm of pressure (100 kPa); \(q_c\) is the measured cone penetration resistance; \(\sigma_{vo}\) and \(\sigma'_{vo}\) are the total and effective vertical overburden stresses, respectively. The stress exponent \(n\) is estimated as Robertson (2009)

\[
n = 0.381(I_c) + 0.05 \left(\frac{\sigma'_{vo}}{P_{at}}\right) - 0.15 \quad \text{where } n \leq 1 \tag{6\cdot3.8}
\]

The soil behavior type index \(I_c\) is defined by Robertson and Wride (1998) as

\[
I_c = \sqrt{(3.47 - \log Q)^2 + (1.22 + \log F)^2} \tag{6\cdot3.9}
\]

where \(Q\) and \(F\) are the normalized tip resistance and friction ratio, respectively.

\[
Q = \left(\frac{q_c - \sigma_{vo}}{\sigma_{vo}}\right) \tag{6\cdot3.10}
\]

\[
F = \left(\frac{f_s}{q_c - \sigma_{vo}}\right) \times 100 \tag{6\cdot3.11}
\]
Once CSR and CRR are obtained, the factor of safety against liquefaction triggering at a particular depth $z$ can be calculated

$$FS = \frac{CRR}{CSR}$$  \hspace{1cm} (6.3.12)

which is used to calculate the liquefaction potential index detailed in the following section.

In this work, the potential of liquefaction damage will be quantified by an averaged index property, i.e., the liquefaction potential index (LPI), which was originally proposed by Iwasaki et al. (1978, 1982) and has been subsequently used and calibrated by many investigators, e.g., Sonmez (2003); Toprak and Holzer (2003); Sonmez and Gokceoglu (2005); Holzer et al. (2006a,b); Lenz and Baise (2007); Juang et al. (2008b); van Ballegooy et al. (2015); Chen et al. (2015). As pointed out by Juang et al. (2008b) and more recently by Maurer et al. (2015); van Ballegooy et al. (2015), cautions should be taken when interpreting LPI results based on Iwasaki’s criteria Iwasaki et al. (1978, 1982), which is based on liquefaction evaluation procedures commonly used in Japan in 1978. Nevertheless, in the current framework, the LPI is used as an averaged index to quantify potential liquefaction damage, the implications of using different liquefaction criteria and calibration of LPI models are discussed in more details in Juang et al. (2008b); Maurer et al. (2015); van Ballegooy et al. (2015). The detailed steps to calculate LPI are listed here.

### 6.3.1 Liquefaction potential index

Following the definition given in Iwasaki et al. (1978, 1982), LPI is usually evaluated for the top 20 m of soil profile as

$$LPI = \int_{0}^{20} F_L w(z) dz$$  \hspace{1cm} (6.3.13)

where $z$ denotes the depth in meters and $w(z) = 10 - 0.5z$; $F_L$ is defined as Sonmez (2003)

$$F_L = \begin{cases} 
0 & \text{FS} \geq 1.2 \\
1 - \text{FS} & \text{FS} \leq 0.95 \\
2 \times 10^6 e^{-18.427FS} & 0.95 < \text{FS} < 1.2 
\end{cases}$$  \hspace{1cm} (6.3.14)

where FS is the factor of safety defined in (6.3.12) for CPT-based liquefaction evaluation. Herein, we adopt a discrete form of the integral to calculate LPI along the depth of a given soil profile Luna and Frost (1998); Lenz and Baise (2007)

$$LPI = \sum_{i=1}^{N} w_i F_{Li} H_i$$  \hspace{1cm} (6.3.15)

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where $H_i$ is the thickness of the discrete layer and is determined by the CPT sampling frequency ($H_i = 0.1$ m for this study); $N$ is the number of soil layers.

LPI can be used to classify the severity of liquefaction according to categories proposed by Sonmez (2003) as shown in Table 6.1.

<table>
<thead>
<tr>
<th>LPI</th>
<th>Liquefaction severity class</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPI = 0</td>
<td>I: Non-liquefiable</td>
</tr>
<tr>
<td>0 &lt; LPI ≤ 2</td>
<td>II: Low</td>
</tr>
<tr>
<td>2 &lt; LPI ≤ 5</td>
<td>III: Moderate</td>
</tr>
<tr>
<td>5 &lt; LPI ≤ 15</td>
<td>IV: High</td>
</tr>
<tr>
<td>LPI &gt; 15</td>
<td>V: Very high</td>
</tr>
</tbody>
</table>

Table 6.1: Liquefaction potential index classification

6.4 Multiscale random field characterization

The CPT-based empirical liquefaction model described in Section 7.4 evaluates the liquefaction potential at individual locations at the small scale, e.g., the borehole scale. To estimate the extent of liquefaction risk over the entire region of interest, multiscale random field models are introduced and implemented in this section.

6.4.1 Spatial correlation

In this study, the spatial correlation of geotechnical parameters is described using the semivariogram, $\gamma(h)$, which is equal to half the variance of the difference of two random variables separated by a vector distance $h$

$$\gamma(h) = \frac{1}{2} \text{Var} [Z(u) - Z(u + h)]$$

(6.4.1)

where $Z(u)$ is a Gaussian random variable at location $u$. The vector $h$ accounts for both separation distance and orientation and therefore can be used to simulate anisotropic random fields. Here, we define a simplified scalar measure $h$

$$h = \sqrt{\left(\frac{h_x}{a_x}\right)^2 + \left(\frac{h_y}{a_y}\right)^2 + \left(\frac{h_z}{a_z}\right)^2}$$

(6.4.2)

where $h_x$, $h_y$ and $h_z$ are the scalar component of the vector $h$ along the field’s principal axes; scalar quantities $a_x$, $a_y$ and $a_z$ specify how quickly spatial dependence decreases along those axes.

In previous studies of liquefaction evaluation, e.g., Liu and Chen (2006); Lenz and Baise (2007); Vivek
and Raychowdhury (2014), the correlation $\rho(h)$ is used to describe the spatial dependence of two parameters separated by $h$ and can be related to the semivariogram as

$$\rho(h) = 1 - \gamma(h)$$  \hspace{1cm} (6.4.3)

In practice, various analytical semivariogram models and their linear combinations are typically fitted to empirical semivariogram that is calculated from field data. Examples of commonly used analytical semivariogram models include the nugget effect model, the linear model, the spherical model, the exponential model and the Gaussian model Goovaerts (1997). Specific model form can be inferred from available field data or assumed based on expert knowledge of properties of interest. As an example, an exponential model is adopted in this work

$$\gamma(h) = 1 - \exp \left( -\frac{h}{a} \right)$$  \hspace{1cm} (6.4.4)

where $a$ is the range parameter and $3a$ is the practical range, i.e., the distance at which the exponential semivariogram levels off Goovaerts (1997).

Usually there is a nugget effect in the empirical semivariogram model due to measurement errors or sparse data. For example, an exponential model combined with a nugget effect is expressed as

$$\gamma(h) = \omega \left[ 1 - \exp \left( -\frac{h}{a} \right) \right] + \tau$$  \hspace{1cm} (6.4.5)

where $\tau$ is the nugget, i.e., the variance at zero distance; $\omega + \tau$ is the sill, i.e., the constant semivariance beyond the range $3a$.

It should be noted that the spatial dependence introduced above is for variables having Gaussian distributions. For non-Gaussian distributions, inference of spatial structure is recommended to be performed on transformed data using a normal score mapping Goovaerts (1997). Such normal score mapping is a common practice in many other applications and has previously been shown to preserve the prescribed spatial structure for lognormally distributed variables Baker et al. (2011); Chen et al. (2012).

6.4.2 Sequential simulation process

Given a specified probability density function and spatial correlation model, a sequential simulation process Goovaerts (1997); Deutsch and Journel (1992) is adopted in this work to generate realizations of the...
random field across the region. Each value is simulated individually conditional upon known information as well as any previously simulated data points.

Denote $Z_p$ as a vector of all known and previously simulated points in the random field and \( Z_n \) as the next point to be simulated, the sequential simulation process can be illustrated by

$$\begin{bmatrix} Z_n \\ Z_p \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ \Sigma_{np} \end{bmatrix}, \begin{bmatrix} \sigma_n^2 & \Sigma_{np} \\ \Sigma_{pn} & \Sigma_{pp} \end{bmatrix} \right)$$

(6.4.6)

where $\sim N(\mu, \Sigma)$ denotes the vector of random variables following a joint normal distribution with mean vector $\mu$ and covariance matrix $\Sigma$; $\sigma_n^2$ is the prior variance of the next simulated point; $\Sigma_{np}$, $\Sigma_{pn}$ and $\Sigma_{pp}$ are the covariance matrices, where the subscripts ‘n’ and ‘p’ represent ‘next’ (as in next point to be simulated) and ‘previous’ (as in all previously simulated points), respectively. The covariance matrices are obtained by

$$\text{COV}[Z_i, Z_j] = \rho_{ij} \cdot \sigma_i \cdot \sigma_j$$

(6.4.7)

where $\rho_{ij}$ is the correlation between random variables $Z_i$ and $Z_j$ with standard deviations of $\sigma_i$ and $\sigma_j$, respectively.

Using the above model for joint distribution, the distribution of $Z_n$ conditional upon all previously simulated data is given by a univariate normal distribution with updated mean and variance

$$(Z_n \mid Z_p = z) \sim N \left( \Sigma_{np} \cdot \Sigma_{pp}^{-1} \cdot z, \sigma_n^2 - \Sigma_{np} \cdot \Sigma_{pp}^{-1} \cdot \Sigma_{pn} \right)$$

(6.4.8)

It is noted that $\Sigma_{np}, \Sigma_{pp}^{-1}$ are essentially the weights assigned in the simple Kriging process Goovaerts (1997). For the realization, one value of $Z_n$ is drawn at random from the posterior univariate normal distribution.

Once simulated, $Z_n$ becomes a data point in the vector $Z_p$ to be conditioned upon by all subsequent data locations. This process is repeated by following a random path to each unknown location until all the values in the field have been simulated. This sequential simulation process is beneficial for the proposed work because: (1) it preserves known information (e.g., field data) precisely at their locations in the simulated random field; and (2) it allows one to first simulate the field at only the coarse scale, then add simulation points at the fine scale probabilistically consistent with the previous coarse-scale realizations, a process we will detail in Sections 6.4.3 and 6.4.4.
6.4.3 Multiscale spatial correlation

The multiscale spatial correlation is based on the notion that material properties at the coarser scales are the arithmetically averaged values of the properties over corresponding areas at the finer scales. This relation allows for the explicit derivation of variances and spatial correlation of quantities of interest between different scales and is visually represented in Figure 6.2.

Figure 6.2: Graphic representation of material properties at two scales. The superscripts ‘c’ and ‘f’ refer to ‘coarse’ and ‘fine’ scales, respectively. The subscripts refer to the element number. Roman letters I, II... are used for coarse scale element and Arabic numbers 1, 2, 3... are used for fine scale element.

For two scales of interest, we denote the coarser scale as scale ‘c’ and the finer scale as scale ‘f’. The cross-scale spatial correlations can then be calculated as Chen et al. (2015)

\[
\rho_{z^c_i, z^c_{II}} = \frac{\sum_{i=1}^{N} \sum_{l=1}^{N} \rho_{z^f_{(I)i}, z^f_{(II)i}}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z^f_{(I)i}, z^f_{(I)j}}} \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z^f_{(II)i}, z^f_{(II)j}}}}
\]

(6.4.9)

\[
\rho_{z^f_i, z^c_{II}} = \frac{\sum_{i=1}^{N} \rho_{z^f_{(I)i}, z^c_{II}}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{z^f_{(I)i}, z^f_{(I)j}}}}
\]

(6.4.10)

where \(\rho_{z^c_i, z^c_{II}}\) = correlation between two coarse-scale elements I and II; \(\rho_{z^f_i, z^c_{II}}\) = correlation between a fine-scale element and a coarse-scale element I.

The geotechnical properties will be simulated using random field models at the coarse scale (e.g., the regional scale), and then adaptively refined into smaller scales (e.g., the borehole or structure scales),
conditional upon the coarse-scale random field simulations. It is worth noting that the cross-scale correlations \((\gamma_{8.5})\) and \((\gamma_{8.6})\) are applicable to general non-uniform arbitrary shaped grids and actual field CPT-based geotechnical measurements will be incorporated into the sequential simulation process.

6.4.4 Numerical implementation

The previously described CPT-based empirical liquefaction model in Section 7.4 as well as the random field models in the current section are implemented in a computer code written in Matlab. Details of the numerical implementation and some common measures to improve the computational efficiency of random field generation are discussed in Section 3.5 of Chapter 3.

6.5 Numerical examples

In this section, the developed multiscale random field model is applied to evaluate liquefaction potential in the city of Christchurch, New Zealand. The purpose of the numerical examples is twofold: (1) to demonstrate the applicability of multiscale random field models in liquefaction evaluation over extended areas; (2) to assess the averaged index approach and the local soil property approach in accounting for the spatial variability of CPT-based geotechnical parameters and their implications on the liquefaction evaluation. We assume the stationary of the random field model and focus mainly on the spatial variability of tip resistance, side friction and the liquefaction potential index. Spatial variability of other geotechnical parameters, e.g., water table and unit weight of soil, are not included in the current study.

6.5.1 Analysis region and field data

Christchurch, New Zealand, is a city founded on the boundary of the alluvial Springston formation and the marine Christchurch formation (Brown and Weeber, 1992). During the period between September 2010 and December 2011, the city of Christchurch was strongly shaken by a sequence of four strong earthquake events known as the Canterbury earthquakes. For the following liquefaction analysis and liquefaction potential mapping, we pick one of the great events, the February 22, 2011 earthquake event. The moment magnitude of the earthquake \(M_w = 6.2\). Typical peak ground surface acceleration \(a_{\text{max}}\) in the the area of study ranging from 0.34 g to 0.50 g (Bradley and Hughes, 2012). A median value of \(a_{\text{max}} = 0.42\) g is used in the following liquefaction potential calculations. A total of 155 CPT profiles with measured water table are collected from the New Zealand Geotechnical Database (NZGD). An averaged moist unit weight
of $\gamma_m = 18.5 \text{ kN/m}^3$ is used for soils above water table and an saturated unit weight of $\gamma_{\text{sat}} = 19.5 \text{ kN/m}^3$ is used for soils below water table following information presented in Khoshnevisan et al. (2015).

Figure 6.3: Map of the area of study in Cristchurch, New Zealand. Red dots represent locations of the 155 CPT soundings.

### 6.5.2 Random field realizations - the averaged index approach

The first set of random field realizations is generated following the the averaged index approach. Herein, the liquefaction potential index (LPI) will be calculated at every CPT sounding location using the CPT-based empirical liquefaction model described in Section 7.4.

Figure 6.4 shows the histogram and semivariogram of the calculated LPIs at 155 CPT sounding locations. As can be seen from Figure 6.4(a), most of the LPI at the 155 CPT soundings are greater than 5, indicating that majority of those CPT sounding sites have a high liquefaction severity class (refer to Table 6.1). The spatial structure of the LPIs is fitted using the exponential model defined in Eq.(6-4.5) with the following fitted parameters: $a = 498.70\text{m}$, $\omega = 0.74$, $\tau = 0.22$. A weighted least square method by Cressie (1985) is used to fit the exponential model parameters.

In the subsequent random field realizations, the LPI will be treated as the random variable of interest and its values throughout the Christchurch site will be generated using the sequential simulation process described in Section 6.4.2. The fitted exponential semivariogram model as shown in Figure 6.4(b) will be used to describe the spatial correlation. Given the site specific model and model parameters, random field
Figure 6.4: Histogram and semivariogram of calculated LPIs at 155 CPT sounding locations.

simulations are performed. Typical realizations of LPIs across the Christchurch site are shown in Figure 6.5 for both single and multiscale random fields. The calculated LPI values at CPT soundings are preserved at those locations. It is shown, as expected, that the multiscale random field captures more local fluctuations of LPIs than the single scale in areas where higher levels of resolutions are taken into account. Such a fine scale resolution is important especially for site-specific liquefaction susceptibility and hazard evaluation as will be illustrated later.

Figure 6.5: Typical random field realizations of LPIs following the averaged index approach.

The histogram and empirical semivariogram corresponding to the LPI random field realizations in Figure 6.5(b) are shown in Figure 6.6. The histogram has a lognormal distribution shape. The empirical semivariogram is calculated from the generated LPI random field, which preserves the specified exponential
spatial structure.

Figure 6.6: Typical histogram and semivariogram of LPIs following the averaged index approach.

6.5.3 Random field realizations - the local soil property approach

In the second set of random field realizations, CPT-based geotechnical parameters, i.e., the CPT tip resistance $q_c$ and the side friction $f_s$, are treated as spatially correlated random variables. Their values will be realized using the sequential simulation process described in Section 6.4.2. Figure 6.7 shows the empirical semivariograms of $q_c$ and $f_s$. The spatial structures of those CPT parameters are fitted using the exponential model defined in Eq. (6.4.5) with the following model parameters: for the tip resistance $q_c$, $a = 339.6 m$, $\omega = 0.60$, $\tau = 0.44$; for the side friction $f_s$, $a = 316.23 m$, $\omega = 0.47$, $\tau = 0.55$. The probability distribution are inferred directly from the CPT data.
Figure 6.7: Semivariograms of the tip resistance $q_c$ and the side friction $f_s$ calculated from 155 CPT soundings.

Random field simulations are performed for each soil layer with a thickness of 0.2 m. Figure 6.8 shows typical multiscale random field realizations of $q_c$ and $f_s$ for one soil layer at 10 m below the ground surface. Typical multiscale realizations of $q_c$ and $f_s$ values across the Christchurch site for all soil layers within the first 20 m below the ground surface are shown in Figure 6.9.

Figure 6.8: Typical multiscale random field realizations of $q_c$ and $f_s$ in one soil layer (at the depth of 10 m).
With the calculated $q_c$ and $f_s$ values at each location within the Christchurch site along with assumed constant unit weights, the LPIs can be calculated following the method stated in Section 7.4. One set of LPI field across the area of study is shown in Figure 6.10. It can be seen that the multiscale random field captures the similar spatial distribution of LPIs across the region to that of the single scale random field. Moreover, there are higher resolutions of LPIs in selected area of interest. Such higher level resolution is important for local liquefaction analysis.

The histogram and semivariogram of the multiscale random field realization in Figure 6.10(b) are shown in Figure 6.11(a) and 6.11(b), respectively. It should be noted that, in the local soil property approach, the simulated LPIs are not expected to preserve the specified spatial correlation structure of $q_c$ and $f_s$ due to the nonlinear transformation in the empirical liquefaction model described in Section 7.4. In the averaged index approach, the simulated LPIs are expected to preserve the spatial structure inferred from calculated LPI values at 155 CPT sounding locations. This is verified by Figure 6.11(b), which shows the empirical semivariogram calculated from simulated LPIs across the site and the specified exponential model.
6.5.4 Probabilistic and spatial assessment of liquefaction potentials

In the previous two sections, we have demonstrated typical random field realizations of LPIs by the averaged index approach and the local soil property approach. In this section, we will perform Monte Carlo simulations and assess the implications of two approaches on the probabilistic and spatial characteristics of liquefaction potentials in the Christchurch site. In addition, utilizing multiscale random field realizations, liquefaction evaluation will be performed for selected local sites to demonstrate the advantage and applicability of proposed methodology.

A total of 1000 Monte Carlo simulations are performed to generate realizations of LPIs across the

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Figure 6.10: Typical random field realizations of LPIs following the local soil property approach.

Figure 6.11: Typical histogram and semivariogram of LPIs following the local soil property approach
Christchurch site. Figure 6.12(a) shows the histogram of the averaged LPIs from 1000 Monte Carlo simulations and Figure 6.13 shows the corresponding maps of the averaged LPIs. To attenuate the effect of ergodic fluctuations, empirical semivariograms are calculated 1000 times, averaged, and plotted in Figure 6.12(b). The solid curve is the specified exponential model with parameters fitted using LPIs at CPT soundings. The red dots and blue squares are the empirical semivariograms calculated using Eq. (6.4.1) with LPI values generated by the averaged index approach and the local soil property approach, respectively. The error bars (± one standard deviation) show the variations of 1000 Monte Carlo simulations. Fluctuating semivariogram is resulted from weak and unsteady correlation of properties between separated places. This is verified in Figure 6.12(b), i.e., great fluctuations (in both the average and standard deviation) of the empirical semivariograms are observed at distance greater than 1000 m. The local soil property approach results in slightly greater fluctuation of semivariogram possibly due to the nonlinear transformation from $q_c$ and $f_s$ to LPI.

Figure 6.12: Histograms and semivariograms of the averaged LPIs from 1000 Monte Carlo simulations. Error bars in semivariogram plot indicate ± one standard deviation.

The generated LPI maps by two different approaches are compared with the map of observed liquefaction phenomenons after the February 22, 2011 earthquake event (Beavan et al., 2012), as shown in Figure 6.13. Comparing the maps of predicted LPIs against the observations, it can be seen that both the averaged index approach and the local soil property approach are able to capture the varying severity levels of liquefaction at most locations across the area of study. For instance, the high LPI value area corresponds well with areas with observed moderate to several liquefaction. The results by the local soil property ap-
proach shows more small scale fluctuations or “noises”. This is due to the high nugget effects in the specified
semivariograms of $q_c$ and $f_s$.

Discrepancies between simulated maps of liquefaction potential and observations are also identified. For example, in the dot-circled areas as shown in Figure 6.13(c) and Figure 6.13(d), the predicted LPI is less than 15 which is not the severest level of liquefaction. In the observation map as shown in Figure 6.13(e), however, moderate to severe lateral spreading are observed in these regions and ejected material are also often observed, which usually indicates highest level of liquefaction. Lateral spreading is a unique form of liquefaction manifestation associated with large lateral permanent ground displacements. The severity of lateral spreading could not be classified by LPI. There are separate criteria for assessing its severity. Therefore, while site profiles with thin liquefiable layers may have low LPI values, these sites may be susceptible to lateral spreading if located near rivers or on sloping ground. Actually, take the Christchurch city for an example, there are about 25% of sites with lateral spreading have LPI values less than 4.5 (Maurer et al., 2014). This means that while LPI is a useful tool for large-scale hazard assessments, it does not eliminate the need for site-specific analyses or consideration of the influence of local conditions on the manifestation of liquefaction (Maurer et al., 2014).
Observations: i. No observed ground cracking or ejected liquefied material; ii. Minor ground cracking but no observed ejected liquefied material; iii. No lateral spreading but minor to moderate quantities of ejected material; iv. No lateral spreading but large quantities of ejected material; v. Moderate to major lateral spreading; ejected material often observed; vi. Severe lateral spreading; ejected material often observed. (Beavan et al., 2012)

Figure 6.13: Mapping of LPI by two approaches compared with the observed liquefaction phenomena.
Results from Monte Carlo simulations can also be used for the probabilistic assessment of liquefaction potentials. Various quantities of interest can be defined. As an example, we evaluate the cumulative frequency distributions of LPIs following the methodology proposed in Holzer et al. (2006a).

Figure 6.14 plots the cumulative frequencies of LPIs in the area of study obtained with the averaged LPI map from 1000 Monte Carlo simulations. The error bars (± one standard deviation) are also included in the cumulative frequency plots. It can be seen that the cumulative frequency curve by the averaged index approach is very close to that of the known LPI obtained from 155 CPT soundings, while a sharp change in the slope of the cumulative frequency curve for the local soil property approach is observed, which can be explained by its histogram in Figure 6.12(a). Most of the LPIs simulated by this approach fall between 10 and 20, resulting in a sharp change in cumulative frequency curve in this range. For the averaged index approach, the percentages of different LPI values are more “spread-out”, which results in smoother cumulative frequency curves.

Comparisons between Figure 6.14(a) and Figure 6.14(b) indicate that multiscale random fields yield consistent results as the single scale counterparts for cumulative frequencies of LPIs. This is to be expected given the notion that properties of a coarse element are the averaged values of the properties over the corresponding areas at the fine scale.

It should be noted that all previous analysis and comparisons are based on relatively sufficient field data to infer random field model parameters. It is expected that the amount of the field will impact the results of the random field-based liquefaction mapping. While this subject itself deserves a future study, for
this Christchurch site, we investigated the effect to data availability on the distribution of predicted LPIs in the area of study. The number of available CPT soundings are varied from 9 to 243. It is found that when the number of CPT soundings is small, the local soil property approach yields higher mean LPI values. As the number of CPT soundings increase, the distributions obtained from the averaged index approach and the local soil approach converge. Figure 6.15 plots the histogram of LPIs obtained with 9 CPT and 69 CPT soundings.

![Histograms of predicted LPIs in the Christchurch site when different number of CPT soundings (N) is used to infer model parameters.](image)

Figure 6.15: Histograms of predicted LPIs in the Christchurch site when different number of CPT soundings (N) is used to infer model parameters.

### 6.5.5 Fine-scale site-specific liquefaction assessment

The final analysis involves the fine-scale site-specific liquefaction assessment. Compared to single scale random field, multiscale random field provides higher resolution at local or site-specific scale, e.g., near a critical infrastructure or a particular building of interest. To illustrate this point, we evaluate the liquefaction potential at two local sites in Christchurch. The site locations are shown in Figure 6.16. Site A is a small region consists of two schools, i.e., the Chisnallwood Intermediate School and the Avondale Primary School. Site B is the location of the Aranui High School. The inset of random fields in Figure 6.16 shows that the multiscale random field could provide much more detailed information than the single scale counterpart, which makes it possible to perform site-specific liquefaction evaluation while consistently maintaining predictions of liquefaction for the entire region over much larger scales.
Based on the cumulative frequency distribution, Holzer et al. (2006a) proposed that for a given geologic unit, the percent area predicted to have liquefaction during a given earthquake-shaking scenario could be estimated from the cumulative frequency distributions at $LPI \geq 5$. This criteria can be further categorized to indicate percentage of area to have minor liquefaction ($5 \leq LPI < 15$) and major liquefaction ($LPI \geq 15$) as shown in Table 6.1. Herein, the percentage of areas in the selected sites with $LPI \geq 15$ under a given earthquake-shaking scenario ($M_w = 6.2, a_{\text{max}} = 0.42$ g) is calculated and plotted against the corresponding cumulative frequency in Figure 6.17.

![Figure 6.16: Locations of two selected sites for fine-scale liquefaction assessment.](image)

**Figure 6.16:** Locations of two selected sites for fine-scale liquefaction assessment.

![Fraction of liquefied area](image)

**Figure 6.17:** Cumulative frequency versus fraction of liquefied area with $LPI \geq 15$ for two local sites A & B selected in the area of study.

Figure 6.17 shows cumulative frequencies of LPIs with respect to the fraction of liquefied area.
The cumulative frequency curve for site B is consistently ‘higher’ than site A for both the averaged index approach and the local soil property approach, indicating that site B is more likely to liquefy than site A in a given earthquake. Comparisons between two approaches show that the cumulative frequency for the average index approach is more “spread-out” than the local soil property approach. This is consistent with the predictions within the whole site of Christchurch, as shown in Figure 6.14.

The above analysis on the percentage of the liquefiable area underneath a particular site gives an indication of the implied reliability of the site with respect to liquefaction resistance during a given earthquake. These site-specific curves can only be obtained if a higher resolution multiscale random field is generated. Moreover, multiscale random fields is able to obtain high-resolution information at local site while consistently maintaining low-resolution information at much larger scales, leading to a computationally efficient process.

Finally, it should be noted that the methodologies discussed in this study is sufficiently general to be applied to other earthquake-prone areas and to evaluate quantities of interest such as liquefaction-induced settlement and lateral spreads. This will be explored in future studies.

6.6 Summary

In this chapter, a classical CPT-based empirical liquefaction model and multiscale random field models are integrated for the assessment of regional liquefaction susceptibility. The study focuses on the spatial variability of CPT-based geotechnical parameters. Two approaches, termed the averaged index approach and the local soil property approach, are developed and analyzed to account for spatial variability of geotechnical parameters. Their implications on liquefaction susceptibility evaluation are discussed through one case study at the Christchurch site in New Zealand. In summary, it is found that

1. Both the averaged index and the local soil property approaches are able to capture the spatial variations of liquefaction potential at different scales over the area of study and predict qualitatively consistent results compared to previously reported liquefaction manifestations.

2. The two approaches yield similar and comparable LPI values for the area of study when sufficient CPT data is available. However, when the CPT data is sparse, the averaged index approach, in general, predicts lower LPI values when compared with the local soil property approach.

3. The averaged index approach is recommended for regional liquefaction evaluation given that it yields consistent results with field observations and is much more computationally efficient.
4. The local soil property approach, though providing more details on soil properties, requires much more field data to characterize random field models, which could introduce additional uncertainties into the solution.

5. Both approaches have been cast within a multiscale random field model, which allows efficient and effective assessment of site-specific liquefaction susceptibility.
Chapter 7

Improve The Accuracy of Random Field Modeling With Hybrid Geotechnical and Geologic Data

7.1 Introduction

Infrastructure such as bridges, buildings and underground utility lines are often threatened by soil liquefaction during earthquakes. Assessment of liquefaction hazard involves ground shaking hazard analysis and estimation of liquefaction resistance of soil deposits, where various in-situ tests such as the cone penetration test (CPT), the standard penetration test (SPT) and the shear wave velocity test are commonly utilized to estimate liquefaction resistance or to classify levels of liquefaction severity at individual locations, e.g., Seed and Idriss (1971); Robertson and Wride (1998); Andrus and Stokoe II (2000); Youd et al. (2001); Cetin et al. (2004); Moss et al. (2006); Idriss and Boulanger (2008); Shen et al. (2016).

When mapping liquefaction hazard over an extended region, classical approaches often rely on surficial geologic data and knowledge of past liquefactions (Youd and Perkins, 1987; Knudsen et al., 2000; Witter et al., 2006). The geologic data is sometimes supplemented with hydrological data and geotechnical data (Holzer et al., 2006a; Hayati and Andrus, 2008; Heidari and Andrus, 2010a) or geomorphological information (Papathanassiou et al., 2017). In these approaches, liquefaction hazard level is assumed to be constant within

\[ \text{C. Wang, and Q. Chen, A hybrid geotechnical and geologic data-based framework for multiscale regional liquefaction hazard mapping, Géotechnique, accepted, 2017.} \]
each surficial geologic unit. Such assumption ignores the inherent spatial variability of soil properties and therefore limits the accuracy and the applicability of the generated liquefaction hazard maps. To improve accuracy and to take advantage of the ever-expanding high quality geotechnical database, geostatistical methods have received increased attention in recent years. Geostatistical methods allow the incorporation of geotechnical data and allow the explicit consideration of the soil property spatial variability. For instance, Lenz and Baise (2007) analyzed the spatial structure of liquefaction potential index (LPI) calculated from both CPT and SPT data and mapped the calculated LPI values over the Alameda County site of California using a kriging method. Chung and Rogers (2010) treated the ground water table as a spatially varying property of interest. They used a kriging method to evaluate the regional variation of depth-to-groundwater, which is then converted to a map of LPI over the St. Louis Area. The work of Vivek and Raychowdhury (2014); Liu and Chen (2006); Juang et al. (2017) all explicitly considered the spatial variations of soil indices from CPT soundings when evaluating liquefaction potential. It is found that the probability of liquefaction could be significantly underestimated if the spatial dependence of soil indices is not considered. Liu et al. (2016) also treated soil indices from CPT soundings as spatially correlated random variables when mapping regional liquefaction-induced lateral spread. In an recent effort by Chen and co-workers (Chen et al., 2015, 2016; Liu et al., 2017), a multiscale random field model is developed to consistently account for soil spatial variability across multiple length scales and is applied to mapping soil properties and liquefaction potentials across a region.

The aforementioned geostatistical or random field methods consider the spatial variability of the mapped properties (e.g., soil property indices, LPIs) derived from geotechnical data. However, the spatial structure of the mapped properties could vary within and across different geologic units. The geologic data, to some extend, provides information on large scale (regional scale) material heterogeneity. Such information is not yet accounted for in current geostatistical or random field method-based regional liquefaction hazard mapping studies. In this chapter, a novel hybrid geotechnical and geologic data-based framework will be developed to address this challenge. The framework consistently accounts for spatial variability of soil properties across scales while simultaneously preserves constraints imposed by geologic boundaries. Both geotechnical and geologic data are integrated into a multiscale random field model through a conditional sequential simulation technique. The framework will be applied to an earthquake prone region to demonstrate its capabilities and advantages.
7.2 Data

The project area is a region prone to earthquakes - the Alameda County site of California shown in Figure 7.1. The availability of engineering data and the extensive past liquefaction studies in this region make it an ideal site to test and validate the proposed methodology. Information on engineering geology in the Alameda County site has been compiled by Helley and Graymer (1997) and briefly summarized in Holzer et al. (2006a). This site is divided into three broad northwest-southeast trending regions, parallel to the Hayward Fault - the most important seismic source in this area. Bedrock is exposed at the surface of the northeast land. The central area, immediately southwest of the bedrock, consists of the Holocene and Pleistocene alluvial fan deposits. The area next to the central area - southwest of the original natural shoreline - is primarily underlain by the Artificial fill that rests on younger San Francisco Bay mud.

A total of 210 CPT sounding profiles are collected from the U.S. Geological Survey online CPT database (USGS, 2015). Water table information is directly obtained from each CPT sounding record. For CPT soundings without water table information, simple interpolation is used. For the unit weight of soil, constant values of $\gamma_m = 15.0 \text{ kN/m}^3$ for moist soil above the water table and $\gamma_{sat} = 19.4 \text{ kN/m}^3$ for saturated soil below the water table are assumed. However, varying unit weights along soil layers with good accuracy are better than approximate constants. A hypothetical magnitude $M_w = 7.1$ earthquake is assumed with a constant peak horizontal ground acceleration, $a_{\text{max}} = 0.5 \text{ g}$. This combination of $a_{\text{max}}$ and $M_w$ has been assumed in a previous liquefaction mapping study (Holzer et al., 2006a) for a hypothetical earthquake event on the nearby Hayward Fault. The assumption of a constant $a_{\text{max}}$ was justified on the basis that the outcrop area of the surficial geologic unit generally parallels and is close to the Hayward Fault (Holzer et al., 2006a). However, it would be more appropriate to use varying $a_{\text{max}}$ form U.S. Geological Survey database or other published reports. The analysis region, its surficial geology and locations of CPT soundings are shown in Figure 7.1.

7.3 Overview of the methodology

A schematic illustration of the proposed framework is shown in Figure 7.2. In this work, liquefaction hazard is quantified and mapped in terms of an averaged index called the liquefaction potential index (LPI), initially proposed by Iwasaki et al. (1978, 1982) and utilized in many liquefaction studies, e.g., Lee et al. (2004); Juang et al. (2008b); Papathanassiou (2008); Maurer et al. (2014, 2015). The proposed methodology, however, is general and can be applied to map other measures of liquefaction hazard such as liquefaction-
induced ground settlement and lateral spread.

As shown in Figure 7.2, the liquefaction hazard map is generated taking into account two types of LPI data. The first type, termed the primary data, is evaluated using a geotechnical-based LPI model, e.g., the Robertson and Wride (1998) CPT-based liquefaction model. As shown in Figure 7.2(a), CPT data are collected within the study area upon which the primary LPI are calculated. The second type, termed the secondary data, is obtained based on secondary information such as a surficial geologic map (Figure 7.1). As shown in Figure 7.2(b), the distribution of primary LPIs within each geologic unit are characterized. Then, as shown in Figure 7.2(c), the secondary LPI data within each geologic unit based are generated based on the characterized parameters of distributions of primary LPIs. The role of the secondary data is to constrain and improve the primary data-based LPI map such that the final map conforms to the large-scale geologic boundaries in the analysis region.

Based on the primary CPT-based LPI data and the secondary geology-based LPI data, multiscale random field models are developed to generate realizations of LPIs across the region of interest, as shown in Figure 7.2(d). Coupled with Monte Carlo simulations, uncertainties associated with the generated liquefaction hazard maps can also be obtained. Various quantities of interest related to liquefaction hazard can be obtained and results will be presented and discussed in the result section.

7.4 CPT-based primary LPI data

In this work, the classical CPT-based liquefaction model by Robertson and Wride (1998) is adopted to calculate the liquefaction potential of a soil layer, where two variables, i.e., the cyclic stress ratio (CSR)
Figure 7.2: Regional mapping of LPI with geologic constraints: (a) collect CPT data and calculate primary LPI data at CPT soundings; (b) characterize the distribution of LPIs within each geologic unit; (c) generate secondary LPI data within each geologic unit based on the characterized parameters of distribution; (d) random field realization of LPI over the whole region conditioned upon both primary and secondary data.
and the cyclic resistance ratio (CRR), are evaluated. Details of this classical liquefaction model have been summarized in a previous study (Wang et al., 2017) and are not repeated here. Once CSR and CRR are obtained, the factor of safety (FS) against liquefaction at a particular depth $z$ can be calculated as

$$FS = \frac{CRR}{CSR} \quad (7.4.1)$$

To quantify liquefaction hazard at a particular location, the factor of safety is integrated over the top 20 m of soil to obtain an averaged index termed the liquefaction potential index (LPI) (Iwasaki et al., 1978, 1982)

$$LPI = \int_{0}^{20} F_L(w(z))dz \quad (7.4.2)$$

where $z$ denotes the depth in meters and $w(z) = 10 - 0.5z; F_L$ is defined as Sonmez (2003)

$$F_L = \begin{cases} 
0 & \text{FS} \geq 1.2 \\
1 - \text{FS} & \text{FS} \leq 0.95 \\
2 \times 10^6 e^{-18.427\text{FS}} & 0.95 < \text{FS} < 1.2
\end{cases} \quad (7.4.3)$$

where FS is the factor of safety defined in (7.4.1). The LPI will be used as the primary variable to be mapped over the project region through multiscale random field models.

For the hypothetical earthquake event with $M_w = 7.1$ and $a_{max} = 0.5$ g, the primary LPI values are calculated at all 210 CPT soundings. The histogram of the calculated 210 LPI values is plotted in Figure 7.3(a). To assess the spatial correlations of the primary LPI data, the empirical semivariogram $\hat{\gamma}(h)$ is calculated as (Goovaerts, 1997)

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{n=1}^{N(h)} [z(u_n) - z(u_n + h)]^2 \quad (7.4.4)$$

where $N(h)$ is the number of pairs of data $z$ located a vector $h$ apart (i.e., a lag bin $h$). In the actual computation, a small tolerance (e.g., 10 $\sim$ 20% of the distance $h$) is usually added to lag bins to accommodate unevenly spaced sample points. Also, it is often convenient to use a scalar distance measure $h$ for the calculation of semivariogram (Wang et al., 2017). Figure 7.3(b) shows the calculated empirical semivariogram based on LPIs at 210 CPT soundings. Given the sample semivariogram, a weighted least square method by Cressie (1985) is implemented to fit an analytical semivariogram model, shown as solid line in the plot.
Figure 7.3: Primary LPI data at 210 CPT soundings in the Alameda County. The fitted model semivariogram is an exponential model $\gamma = 1 - e^{(-h/470)}$ with $h$ being the scalar distance.

7.5 Geology-based secondary LPI data

Knowledge of surficial geology is an important piece of information for regional liquefaction mapping as it typically provides broader area coverage and information on large-scale material heterogeneity. Potentially, any surficial geology-dependent liquefaction information (e.g., previous liquefaction observations, regional geology-based liquefaction hazard map) could be used to derive secondary LPI data that essentially enforce geologic constraint to the generated liquefaction hazard map. In this section, a simplified procedure is presented to utilize the calculated primary LPI values and the surficial geologic map to derive secondary LPI data. The method consists of the following steps:

1. Identify the boundaries of each geologic unit within the study region.

2. Group the primary LPI values by geologic units and characterize their statistical distributions.

3. Generate random variable realizations of secondary LPI values within each geologic unit according to the characterized or assumed statistical distributions. Once generated, the secondary data will be kept constant for the following random field realizations.

4. Assign the generated secondary LPI values to a predefined grid, which will be integrated into the conditional sequential simulation algorithm as secondary data.

For the project area, the surficial geology map of the Alameda County has been shown in Figure 7.1, from which the boundaries of the geologic units are identified. As summarized above, the next step is to group primary LPI values by geologic units and characterize their statistical distribution. Figure 7.4 shows the...
histograms of primary LPIs at 210 CPT soundings grouped by the four main geologic units in the study region, i.e., the Artificial fill, the Pleistocene fan, the Holocene fan and the Merritt sand. It can be seen from Figure 7.4 that the distributions of LPIs within the Artificial fill, the Holocene fan and the Merritt sand geologic units can be fitted with a lognormal distribution, while the primary data are too sparse to conclude a basic distribution for the Pleistocene fan unit. In the following, a lognormal distribution will be assumed for generating secondary LPI data.

In addition to histograms, box plots and cumulative frequency curves are also used to understand the primary LPI values grouped by geologic unit and are shown in Figure 7.5. It can be seen from the box plots that the median LPI values for each geologic unit are ordered from high to low in this manner: Artificial fill, Holocene fan, Merritt sand, Pleistocene fan. The cumulative frequency plots (Figure 7.5(b)) show that for a given LPI value, the Artificial fill unit has the highest cumulative frequency values while the Merritt sand unit has the lowest frequency values. The curve for the Pleistocene fan unit shows some abrupt
changes due to very limited amount of data points. Previously, such cumulative frequency curves have been used to assign a constant probability of liquefaction value to the corresponding geologic unit Holzer et al. (2006a). The cumulative frequency curves shown in Figure 7.5(b) is made in the same style as that of Holzer et al. (2006a). They agree with each other: both indicate the Artificial fill unit has the highest cumulative frequency values compared with the other units. But there are some differences between this dissertation and Holzer et al. (2006a): Firstly, there are 210 CPT data used here while Holzer et al. (2006a) had 202 data analyzed; Secondly, Holzer et al. (2006a) had a subunit of Merritt sand analyzed separately; Thirdly, when calculating the LPI values, this dissertation takes into account a threshold value of factor of safety equal to 1.2 (adopted from Sonmez (2003) as shown in Eq. (7.43)) instead of 1 for Holzer et al. (2006a). It should be noted that different models for LPI calculation could lead to different results. This dissertation focuses on the development of a regional assessment tool which can easily integrate different liquefaction models.

Basic statistical parameters of the primary LPIs for each geologic unit are summarized in Table 7.1. Consistent with the above analysis, Artificial fill has the highest mean value of LPI, followed by the Holocene fan. The parameters for Pleistocene fan and Merritt sand are very close to each other.

<table>
<thead>
<tr>
<th>Geologic unit</th>
<th>mean, $\mu$</th>
<th>standard deviation, $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artificial fill</td>
<td>14.6</td>
<td>9.5</td>
</tr>
<tr>
<td>Pleistocene fan</td>
<td>9.5</td>
<td>8.4</td>
</tr>
<tr>
<td>Holocene fan</td>
<td>10.9</td>
<td>8.9</td>
</tr>
<tr>
<td>Merritt sand</td>
<td>8.4</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Figure 7.5: Box plots and cumulative frequency of primary LPI data for each geologic unit. The number in the bracket is the number of CPT soundings in that geologic unit.
Once the distribution type and statistical parameters are determined, secondary LPI values for each geologic unit are obtained from random field realizations and are assigned to a predefined grid. The results are shown in Figure 7.6 for all geologic units in the study region. Such secondary LPI values will be incorporated into the multiscale random field models described in the next section. It is worth noting that the above analysis groups geotechnical data based on surficial geologic units. Geologic units, however, are three-dimensional features. If the subsurface information is available, it would be more appropriate to group geotechnical data according to three-dimensional features.
7.6 Multiscale random field models

7.7 Conditional sequential simulation algorithm

A conditional sequential simulation algorithm is implemented in this work to generate random field realizations of LPIs. This algorithm integrates and preserves multiple sources of known data (e.g., primary and secondary LPI data). In this algorithm, the realization of a random variable \( Z_n \) is represented by a joint distribution as follows

\[
\begin{pmatrix}
Z_n \\
Z_p \\
Z_s
\end{pmatrix} \sim N
\begin{pmatrix}
\begin{pmatrix}
\mu_n \\
\mu_p \\
\mu_s
\end{pmatrix},
\begin{pmatrix}
\sigma_n^2 & \Sigma_{np} & \Sigma_{ns} \\
\Sigma_{pn} & \Sigma_{pp} & \Sigma_{ps} \\
\Sigma_{sn} & \Sigma_{sp} & \Sigma_{ss}
\end{pmatrix}
\end{pmatrix}
\]  \hspace{1cm} (7.7.1)

where \( \sim N(\mu, \Sigma) \) denotes the vector of random variables following a joint normal distribution with the mean vector \( \mu \) and the covariance matrix \( \Sigma \); \( Z_n \) is the random variable to be generated with the expected value \( \mu_n \); \( Z_p \) is the vector of previously generated or known primary random variables with the vector of expected values \( \mu_p \); \( Z_s \) is a vector of secondary random variables with the vector of expected values \( \mu_s \); \( \sigma_n \) is the standard deviation of \( Z_n \); \( \Sigma \) is the covariance matrix with subscripts “n”, “p” and “s” denoting “next”, “previous primary” and “secondary”, respectively. The individual terms in the covariance matrix is defined as

\[
\text{COV}[Z_i, Z_j] = \rho_{Z_i, Z_j} \sigma_{Z_i} \sigma_{Z_j}
\]  \hspace{1cm} (7.7.2)

where \( \rho_{Z_i, Z_j} \) is the correlation between two elements \( Z_i \) and \( Z_j \) within the random field at any scale with a standard deviation of \( \sigma_{Z_i} \) and \( \sigma_{Z_j} \), respectively.

Given the joint distribution (7.7.1), the distribution of the random variable \( Z_n \), conditional upon all previously simulated and known primary and secondary data, is given by a univariate normal distribution with the updated mean and variance as

\[
(Z_n | (Z_p, Z_s)) \sim N(\tilde{\mu}_n, \tilde{\sigma}_n)
\]  \hspace{1cm} (7.7.3)

with

\[
\tilde{\mu}_n = [\Sigma_{np} \Sigma_{ns}]^{-1} [\Sigma_{pp} \Sigma_{ps} \Sigma_{sp} \Sigma_{ss}]^{-1} [Z_p] [Z_s]
\]  \hspace{1cm} (7.7.4)
\[ \sigma_n^2 = \sigma_n^2 - \left[ \sum_{np} \sum_{ns} \right] \left[ \begin{array}{cc} \Sigma_{pp} & \Sigma_{ps} \\ \Sigma_{sp} & \Sigma_{ss} \end{array} \right]^{-1} \left[ \begin{array}{c} \Sigma_{pn} \\ \Sigma_{sn} \end{array} \right] \]  

(7.5)

where symbols in (7.4) and (7.5) have all been defined after (7.1). The value of a random variable \( Z_n \) at an unsampled location is drawn from the above joint distribution. Once generated, \( Z_n \) becomes a data point in the vector \( Z_p \) to be conditioned upon by all subsequent simulations. This process is repeated by following a random path to each unknown location until all the values in the field have been simulated, i.e., a map of the primary variable for the region of interest is generated.

### 7.8 Characterization of spatial correlations across scales

The derivation of spatial correlation across scales is based on the notion that material properties at the coarser scales are the arithmetically averaged values of the properties over corresponding areas at finer scales (Baker et al., 2011; Chen et al., 2012)

\[ Z_c^I = \frac{1}{N} \sum_{i=1, i \in I} Z_f^i \]  

(8.1)

where the superscripts “c” and “f” refer to the coarse and fine scales, respectively; \( N \) is the count of fine scale elements within a coarse scale region. Defining the variable of interest at the fine scale, the mean of the variable at the coarse scale, denoted as \( \mu_{zc} \), can be derived by taking the expectation of (8.1) as

\[ \mu_{zc} = E[Z_c^I] = \frac{1}{N} \sum_{i=1}^{N} \mu_{zf_i} = 0 \]  

(8.2)

where \( \mu_{zf_i} \) is the mean of the fine scale elements within a coarse element \( I \), which equals to zero for variables following the standard Gaussian distribution. Accordingly, if the variance of the fine scale value is unity, the coarse scale variance, denoted as \( \sigma_{zc}^2 \), can be computed as

\[ \sigma_{zc}^2 = E[(Z_c^I)^2] - 0 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{zf_i, zf_j} \sigma_{zf_i} \sigma_{zf_j} \]  

(8.3)

where \( \rho_{zf_i, zf_j} \) is the correlation between two fine scale elements \( Z_f^i \) and \( Z_f^j \) with standard deviations of \( \sigma_{zf_i} \) and \( \sigma_{zf_j} \), respectively.

The correlations between all considered scales can be calculated by rearranging the definition of the
covariance \((\tau^{7.2})\) such that
\[
\rho_{Z_i, Z_j} = \frac{\text{COV}[Z_i, Z_j]}{\sigma_Z \sigma_Z} \quad (\tau^{8.4})
\]

By making appropriate substitutions at each scale using \((\tau^{7.2})\) and \((\tau^{8.4})\), the correlation between elements at different scales can be obtained as (Chen et al., 2015)

\[
\rho_{Z_I, Z_{II}} = \frac{\sum_{i=1}^{N} \sum_{k=1}^{N} \rho_{Z_{I(i)}, Z_{II(j)}}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{Z_{I(i)}, Z_{I(i)}} \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{Z_{II(j)}, Z_{II(j)}}}}} \quad (\tau^{8.5})
\]

\[
\rho_{Z_{I(i)}, Z_{II(j)}} = \frac{\sum_{i=1}^{N} \rho_{Z_{I(i)}, Z_{I(i)}}}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{Z_{I(i)}, Z_{II(j)}}}} \quad (\tau^{8.6})
\]

where \(\rho_{Z_I, Z_{II}}\) is the correlation between two coarse scale elements \(I\) and \(II\); \(\rho_{Z_{I(i)}, Z_{II(j)}}\) is the correlation between a fine scale element and a coarse scale element \(I\); \(\rho_{Z_{I(i)}, Z_{I(i)}}\) is the correlation between a fine scale element \(i\) and a fine scale element \(k\), which belong to two different coarse scale elements \(I\) and \(II\), respectively. Given the multiscale spatial dependence specified by \((\tau^{8.5})\) and \((\tau^{8.6})\) and an inferred or assumed probability distribution of the random variable, the conditional sequential simulation algorithm is employed to generate random field realizations of variables of interest. In the geostatistics community, the semivariogram \(\gamma(h)\) measuring the dissimilarity of variate separated by a vector distance \(h\) is typically used and can be easily converted to the correlation function \(\rho(h)\) (Remy et al., 2009).

### 7.9 Covariances across data types

To perform the conditional sequential simulation in \((\tau^{7.3})\), three covariances must be determined: one for the primary variable, one for the secondary variable, and a cross-covariance describing the relationship between these variables. With relatively sufficient amount of primary LPI data calculated from CPT soundings, the covariance of the primary variable can be easily obtained from the inferred spatial correlation between primary data. However, direct calculation of the secondary and cross-covariances can be challenging. In this work, one simplified approach is adopted based on the Markov-Bayes hypothesis described by Goovaerts (1997) to derive the secondary and cross-covariances by calibrating them to the primary covariance.
as (Goovaerts, 1997; Moysey et al., 2003)

\[
\text{COV}_s(h) = \begin{cases} 
|B| \cdot \text{COV}_p(h) & \text{for } h = 0 \\
B^2 \cdot \text{COV}_p(h) & \text{for } h > 0
\end{cases}
\]  

(7.9.1)

\[
\text{COV}_{ps}(h) = B \cdot \text{COV}_p(h)
\]  

(7.9.2)

where \(B\) is the Markov-Bayes coefficient; \(\text{COV}_p\) is the covariance for the primary variable, \(\text{COV}_s\) is the covariance for the secondary variable and \(\text{COV}_{ps}\) is the covariance between the primary and the secondary variables; \(h\) is the distance vector separating two random variables. The Markov-Bayes coefficient \(B\) generally varies between 0 and 1 when primary and secondary variables are positively correlated. Its value affects the relative importance of primary data and secondary data and this effect will be illustrated in the result section.

The coefficient \(B\) can be chosen based on a calibration procedure recommended by Deutsch and Journel (1998) such that the value \(B\) is determined as the difference between two conditional expectations as follows

\[
B = E_1 - E_0
\]  

(7.9.3)

where the two conditional expectations are defined as

\[
E_1 = E(\text{Prob}\{Z_s \leq z\}|Z_p \leq z)
\]  

(7.9.4)

\[
E_0 = E(\text{Prob}\{Z_s \leq z\}|Z_p > z)
\]  

(7.9.5)

where \(E\) is the expectation operator; \(E\) is the expectation; \(Z_s\) is the secondary variable (e.g., the geologic data-based LPI value) and \(\text{Prob}\{Z_s \leq z\}\) is the probability of \(Z_s\) less or equal than a threshold value \(z\) (e.g., a given LPI threshold value); \(Z_p\) is the primary variable (e.g., the geotechnical data-based LPI value). The conditional expectation \(E_1\) will be close to 1 if the primary and secondary data support each other, i.e., the two data predict similar liquefaction hazard levels. The conditional expectation \(E_0\) will be close to 1 if the primary and secondary data contradicts each other, i.e., the two data predictions of liquefaction hazard contradict each other.
7.10  Results and discussions

7.10.1  Multiscale random field realizations of LPI

Given both primary and secondary LPI data obtained from CPT and geologic information, random field realizations of LPIs across the Alameda County site are performed. The region is firstly discretized by a relatively coarse grid and an example realization of LPIs on the coarse grid is shown in Figure 7.7(a). Upon the coarse scale realization, the grid can be adaptively refined in subregions of higher interest while maintaining consistent spatial correlations across scales. As an example, a subregion marked by the red box in Figure 7.7 is selected for refinement. The corresponding multiscale realization is shown in Figure 7.7(b). This subregion is identified as an area of high liquefaction potential by Kayen et al. (1998) and is evidenced by the concentration of higher LPI values. This refinement process can be continued down to multiple finer scales. Since the refinement is only performed where necessary, it is computationally efficient to generate multiscale random field realizations of LPIs for very large region. The benefit and application of multiscale LPI realizations will be illustrated in the following section.

Figure 7.7: Example realizations of LPIs across the Alameda County site for a hypothetical earthquake event \( (M_w = 7.1, a_{\text{max}} = 0.5 \text{ g}) \) and the Markov-Bayes coefficient \( B=0 \): (a) Single scale; (b) Multiscale.

7.10.2  Monte Carlo analysis

For regional liquefaction hazard mapping, the multiscale random field models are coupled with Monte Carlo simulations to obtain expected liquefaction hazard across the region and to perform probabilistic analysis on quantities of interest. To investigate the influence of the Markov-Bayes coefficient \( B \) introduced in (7.9.1), six \( B \) values are used, i.e., 0, 0.1, 0.4, 0.5, 0.73, and 0.9. For each \( B \) value, 1,000 Monte Carlo
simulations are performed. A hypothetical earthquake event ($M_w = 7.1$, $a_{max} = 0.5 \text{ g}$) is assumed for all simulations.

Figure 7.8 shows maps of expected LPI values for all six cases of Markov-Bayes coefficient $B$. Each of the six maps is obtained by averaging results from 1,000 Monte Carlo simulations. As shown in (7.9.1), the Markov-Bayes coefficient $B$ is essentially a “scaling” factor between the primary covariance and the secondary covariance matrices. The larger the coefficient $B$, the stronger influence the secondary data has on the generated LPI maps. In this work, the secondary LPI data come from geologic information. Therefore, as the value of $B$ increases, the geologic boundaries become more distinguishable in the resulting LPI maps as evidenced in Figure 7.8(b)-Figure 7.8(f). When no secondary data is incorporated, i.e., $B = 0$, no geologic constraint is applied to the LPI map. Such case is shown in Figure 7.8(a) and the resulting LPI map might be inaccurate. For instance, lacking geologic constraint, the bedrock unit (refer to Figure 7.1 for the geologic unit map of the study region) is predicted to have LPI values around 10, which is usually classified as high liquefaction severity class (Sonmez, 2003). This contradicts the common knowledge that the bedrock is not prone to liquefaction. By applying secondary LPI data, this type of incorrect classification could be mitigated.

The case of $B = 0.73$ is obtained following the calibration procedure described in (7.9.3) based on a LPI threshold of 5, which is the value separating the moderate and the high liquefaction severity class (Sonmez, 2003). It should be noted that the selection of LPI threshold affects the calibrated $B$ value. For the Alameda County site, the threshold value of 5 is an appropriate choice as it appropriately separates liquefaction-prone and non-liquefied units, as evidenced by Figure 7.5. As shown in Figure 7.8(e), the LPI map with the calibrated $B = 0.73$ manifests reasonable spatial variations of LPI that comply with the known data while preserving boundaries of different geologic units. It should be noted that the calibrated $B$ value can be used as a starting point for further adjustments. For instance, if there is a relatively high confidence in the quality of secondary data or if the secondary data is known to have greater impact on liquefaction, the coefficient $B$ could be increased. If there is very little knowledge or low confidence about the secondary data, a lower value of $B$ could be adopted.

Results obtained and shown in Figure 7.8 can be utilized to quantify liquefaction hazard and to calculate various quantities of interest. As an example, following the procedure proposed by Holzer et al. (2006a), cumulative frequency (CF) distributions of LPIs are calculated to assess the liquefaction potentials of each geologic unit. The CF distributions obtained with different $B$ values are plotted in Figure 7.9. For a given geologic unit, Holzer et al. (2006a) related the percent area predicted to undergo liquefaction during a
Figure 7.8: Maps of expected LPI values for all six cases of Markov-Bayes coefficient $B$. $B^* = 0$ is the case without secondary LPI data. $B^* = 0.73$ is the case where the coefficient $B$ is calibrated. Each map is obtained by averaging results from 1,000 Monte Carlo simulations.
given earthquake shaking scenario to the value of CF corresponding to $LPI = 5$. Figure 7.8 shows that the Artificial fill geologic unit has the highest CF values for a given LPI and is therefore most likely to liquefy. The Merritt sand unit has the lowest CF values for a given LPI and is predicted to be the most resistance to liquefaction. The impact of coefficient $B$ on the CF distributions is relatively small.

![Cumulative frequency distributions of liquefaction potential index](image)

**Figure 7.9:** Cumulative frequency distributions of liquefaction potential index.

### 7.10.3 Comparison with prior knowledge

As a modest validation of the framework, maps of the expected liquefaction potential hazards are compared with Holzer et al. (2006a)’s liquefaction hazard map. As shown in Figure 7.10, most of the high liquefaction potential areas (along the coastline and in the Artificial fill unit) are also predicted to have high LPI values by the proposed framework. As previously mentioned, the LPI map with the calibrated Markov-Bayes coefficient $B = 0.73$ predicts the bedrock unit as non-liquefiable ($LPI \approx 0$), which is consistent with Holzer et al. (2006a)’s liquefaction hazard map and the fact that no liquefaction phenomenon was observed during the 1989 Loma Prieta earthquake (Kayen et al., 1998).
7.10.4 Small scale analysis

To gain more insights into the improved predictions when the calibrated coefficient $B = 0.73$ is used, two small areas, denoted as “Area A1” and “Area A2” in Figure 7.10, are investigated in more details. Area A1 is underlain by the Merritt Sand geologic unit (refer to Figure 7.1), where low liquefaction hazard is expected in Holzer et al. (2006a)’s liquefaction hazard map and confirmed by results of the statistical analysis (summarized in Table 7.1 and Figure 7.5). When no secondary data is used as shown in Figure 7.10(b), relatively high LPI values ($\approx 10$) are predicted in Area A1, which are classified as high liquefaction severity by Sonmez (2003) and contradict the knowledge of liquefaction hazard in this area. In contrast, the LPI map with $B = 0.73$ (Figure 7.10(c)) correctly predicts low liquefaction potential in Area A1.

The second small area, denoted as “Area A2”, is expected to suffer high liquefaction hazard in Holzer et al. (2006a)’s liquefaction hazard map and confirmed by the statistical analysis. The LPI map without secondary data (Figure 7.10(b)) incorrectly predicts low liquefaction potential values while the LPI map with calibrated coefficient $B = 0.73$ (Figure 7.10(c)) again correctly predicts the liquefaction potential values, which shows the improved accuracy, especially at small scale, when geologic data is appropriately accounted for in the liquefaction mapping.

A more quantitative comparison is presented in Figure 7.11 in terms of cumulative frequency distributions of LPI. As shown in Figure 7.11(a), the LPI map generated without secondary data predicts that Area A1 and A2 are expected to have similar liquefaction potential hazards, which clearly contradicts prior
knowledge (Holzer et al., 2006a). In contrast, Figure 7.11(b) shows that, when the secondary geologic-based LPI data is appropriately accounted for, the predicted liquefaction hazard in the two areas are consistent with prior knowledge (Holzer et al., 2006a).

![Cumulative frequency distributions of LPIs within Area A1 and A2 for cases without \(B = 0\) and with \(B = 0.73\) secondary data](image)

Figure 7.11: Cumulative frequency distributions of LPIs within Area A1 and A2 for cases without \(B = 0\) and with \(B = 0.73\) secondary data

To further illustrate the advantage of multiscale random field models, the LPI maps are refined to such a high resolution that it is possible to investigate liquefaction hazard at particular local sites. For instance, as shown in Figure 7.10(a), two local sites denoted as P1 \((37°46′8.0″ N 122°15′12.8″ W)\) and P2 \((37°47′50.2″ N 122°17′24.9″ W)\) are selected where high resolution random field is available. Predicted distributions of LPIs at these two sites can be obtained using previously performed Monte Carlo simulations and the distributions are shown in Figure 7.12. As has been previously analyzed, the results with geologic constraint \((B = 0.73)\) are more accurate. Such small scale site specific analysis can be useful information to assist engineering design at these locations.

### 7.11 Summary

In this chapter, a hybrid geotechnical and geologic data-based framework is developed for the multiscale random field-based regional liquefaction hazard mapping. The geotechnical data such as the CPT data are used to calculate primary LPI values, which show clear spatial correlation as evidenced by the calculated empirical semivariogram. A simplified method is proposed to derive secondary LPI values from geologic information which essentially enforces geologic constraints to the generated liquefaction hazard maps. A Markov-Bayes coefficient is introduced to derive covariances related to secondary data. Both primary and
secondary LPI data are then integrated into multiscale random field models through a conditional sequential simulation technique. The proposed framework is applied to map liquefaction hazard at the Alameda County site in California. It is found that the influence of geologic constraints on the generated liquefaction hazard maps is significant. Without geologic constraints and with only geotechnical data, the generated liquefaction hazard map incorrectly predicts high liquefaction hazard in the bedrock geologic unit. As the weight of the geologic data increases, the geologic boundaries become more distinguishable in the generated map. With an appropriately calibrated Markov-Bayes coefficient ($B = 0.73$ for the current study), the accuracy of the liquefaction map is improved when validated against prior knowledge. Moreover, the advantage of the multiscale liquefaction hazard map is demonstrated through a small site specific liquefaction hazard analysis.
Chapter 8

Mechanics-based Numerical Modeling of Soil Liquefaction

8.1 Introduction

8.1.1 Motivations

Liquefaction is a major reason for damages of structures supported by saturated soils during earthquakes. Empirical correlation-based methods have been the dominant approach for liquefaction assessment. For site-specific analysis of liquefaction, mechanics-based method is an alternative to empirical correlations. One of the advantages of mechanics-based method is that it applies to complex site conditions where the liquefaction damage could not be evaluated by empirical correlations. Another advantage is that mechanics-based method is a link between structures and soil damages. The complex analysis of soil-foundation-structure interaction during liquefaction could only be solved by mechanics-based numerical methods. In the last three decades, a body numerical tools have been developed to assess soil liquefaction. However, it still is a challenging task to predict liquefaction by numerical analysis.

8.1.2 Backgrounds

Early research into liquefaction paid more attention to the influencing factors and the likelihood of liquefaction occurring than to deformation prediction of post-liquefaction soils. The destruction induced by liquefaction deformation has highlighted the problem of post-liquefaction soil behavior and has recently
attracted the attention of scholars.

The simplest way of modelling liquefaction which is still used in practice is done by means of a total stress analysis. In this type of analysis a dynamic analysis based on total stresses is performed to identify the areas with high potential of liquefaction. By reducing the strength of material to its residual strength and preforming a second total stress analysis, deformation of soil structure will be obtained. In this type of calculation, generation and dissipation of excess pore water pressures are totally ignored in the numerical analysis. The resulted deformation from this type of analysis is always questionable due to oversimplification of the problem. The second type of analysis, is based on an effective stress analysis in which liquefaction is occurred as a result of excess pore water pressure generation. In an effective stress analysis, all input parameters are effective parameters and the total stress is summation of effective stress and excess pore water pressure. The generation of excess pore water pressure is done under undrained conditions by considering volumetric strain and the bulk modulus of water in pores.

Although, the effective stress analysis is a natural way of modelling liquefaction, but it needs more information about the mechanical behavior of soil and the numerical methods used for the analysis. Due to the complex behavior of soil, a proper constitutive model is needed which should be capable of accumulating volumetric strain and consequently capable of modelling liquefaction. The parameter selection and the use of numerical tools play a significant role in this type of analysis.

The main objective of this section is to show how finite element method as a numerical tool can be used in predicting cyclic liquefaction in soils and how finite element method can be integrated with random field models in predicting regional liquefaction hazard.

8.2 Numerical procedures

This section focuses on simulating the response of saturated soils to seismic excitations in finite element models. Saturated soils consist of the soil skeleton phase and the water phase, as seen in the idealization in Figure 8.1. The deformation of the soil skeleton depends on the pore fluid pressure since these control inter-granular effective stresses. Since the dimensions of interest are large enough compared to the size of the grains and the pores, standard continuum analyses of the soil skeleton-water mixture are used.
Figure 8.1: Soil phases
8.2.1 Analyses of Coupled Deformation and Flow

This section describes the equilibrium and mass conservation coupled equations to be solved for a saturated soil continuum following the formulation of Biot (1956). The formulation assumes small strains in the soil skeleton, while flow is controlled by Darcy’s law: Momentum Balance in the soil-fluid mixture:

\[ \text{div}(\sigma) - \rho \ddot{d} - \rho_f [\dot{w} + w \nabla^T w] + \rho b = 0 \]  \hspace{1cm} (8.2.1)

Moment balance in the fluid:

\[-\nabla u - R - \rho_f \ddot{d} - \frac{\rho_f [\dot{w} + w \nabla^T w]}{n} + \rho_f b = 0 \]  \hspace{1cm} (8.2.2)

Mass balance:

\[ \nabla^T w + \alpha \cdot tr(\epsilon) + \frac{\dot{u}}{Q} + n \cdot \frac{\dot{\rho}_f}{\rho_f} = 0 \]  \hspace{1cm} (8.2.3)

Where \( \nabla \) is the gradient operator, \( \sigma \) is the stress matrix, \( u \) is the pore fluid pressure, \( \rho \) is the total density of the fluid, \( d \) is the vector of displacements for the solid matrix, \( w \) is the vector of the average (Darcy) velocity of the percolating water (relative to the soil skeleton), \( b \) is the body forces vector, \( R \) is the viscous drag forces vector, \( \rho_f \) is the fluid density, \( n \) is the porosity, and \( \epsilon \) is the small strain matrix.

\( Q \) characterizes the compressibility of the fluid and solid particle mixture:

\[ \frac{1}{Q} = \frac{n}{K_f} + \frac{a - n}{K_s} \]  \hspace{1cm} (8.2.4)

where \( K_f \) is the bulk stiffness of the fluid, and \( K_s \) is the soil skeleton bulk modulus, while \( \alpha \) defines the effective stresses (\( \alpha \approx 1 \) in soils):

\[ \sigma' = \sigma + \alpha I u \]  \hspace{1cm} (8.2.5)

In large finite element models, it is convenient to reduce the number of variables. Simplifications are possible, by assuming that the flow of water inside the soil skeleton is relatively slow. We can thus neglect the relative acceleration of water to the soil skeleton but still keep the fluid pressure terms. Under this
assumption, we get the following set of equations, which is referred to as the u-p approximation:

\[ \text{div}(\sigma) - \rho \ddot{d} + \rho \dot{b} = 0 \]  
\hspace{1cm} (8.2.6)

\[ \nabla^T k (-\nabla u - \rho_f \ddot{d} + \rho_f \dot{b}) + tr(\dot{\epsilon}) + \frac{\dot{u}}{Q} = 0 \]  
\hspace{1cm} (8.2.7)

This u-p approximation can be used in consolidation problems, and in coupled dynamic pore pressure displacement analyses with very small errors (Zienkiewicz et al., 1999), and is used throughout this dissertation.

### 8.2.2 Numerical Simulation of Governing Equation

In order to solve the aforementioned system of PDE’s, we use OpenSees, "an object-oriented software framework for simulation applications in earthquake engineering using finite element methods" (Mazzoni et al., 2005). OpenSees is a software framework that extends the tcl/tk interpreter with commands dealing with finite element simulations. Tcl/tk is a scripting language, created in the spring of 1988, commonly used for scripted applications, GUIs, and testing. OpenSees is primarily developed in C++, though it uses many preexisting FORTRAN numerical recipes.

The use of OpenSees is dictated by the capability to simulate the u-p approximation of the mass and momentum conservation equations and diffusion in plane strain. Moreover, its modularity makes it convenient for developing new features, even combining the framework with other components. It is an open source code and hence, developments in this thesis can be accessible to the broader scientific research community. This also means that the code can be easily ported to different computational environments without the cost of expensive licenses for multi-threaded computations. Open source code assumes that research groups produce incremental improvements and refinements by the community, and eventually better simulation tools.

Zhang et al. (2004) have used OpenSees to create a two dimensional model of the soil structure interaction of a bridge and its foundations, using a multi-yield pressure dependent material (Yang, 2003) under fully undrained conditions, using transmitting boundaries and equivalent nodal forces to apply the seismic excitation. Vytiniotis (2009) and Siyahi & Arslan (2007) have performed numerical analyses with OpenSees of a centrifuge model test and an earth dam respectively using the u-p set of equations and the multi-yield models of Yang (2003).
Advantages of OpenSees:

1. Modern developed code
2. Parallel, runs on Windows, Mac, Linux.
3. Open source, large community of users and developers
4. Linear, non-linear materials
5. Plenty of elements, 1-Dimensional 2-Dimensional 3-Dimensional ability

Disadvantages of OpenSees:

1. No preprocessing and postprocessing modules
2. No GUI

To facilitate following analyses, a Python tool is developed for pre- and post-processing the model with OpenSees and connecting with ParaView for visualization.

```python
from FEMtools import FEMtools

# Example: select nodes at the base (z = 0) of a FEM model,
# and apply OpenSees command 'fix' to all selected nodes

# select all nodes and elements
FEM_tool.selectAll()

# select nodes by location, getNbyLoc for Cartesian, getNbyLocCyl for Cylindrical
FEM_tool.getNbyLocCyl(x = (r1,r2), y=(angle1,angle2), z = (0,0), tol=8.001) # x is radius, y is angle

# fix node at z=0, stored in baseBCfile.tcl
FEM_tool.setBCfix('baseBCfile',x=1,y=1,z=1,u=0)

# plot selected nodes
selectedNodes = FEM_tool.selectedNodes
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.scatter(selectedNodes[:,1], selectedNodes[:,2], selectedNodes[:,3])
plt.show()
```

Figure 8.2: Pre- and post-processing in Python

### 8.3 Constitutive modeling

In the past two decades, there have been several constitutive models developed to describe the mechanical behavior of soil under cyclic loading. The most widely studied are: UBCSAND, PDMY02, DAFALIAS-MANZARI and PM4SAND.
UBCSAND Puebla et al. (1997) is based on plasticity theory and the characteristic sand behavior observed in laboratory tests under monotonic and cyclic loading conditions. This model is an attempt of using a simple but powerful plasticity framework to analyze and predict the onset of dynamic induced liquefaction. A 3-D extension of the UBCSAND model is developed later. The 3D formulation, known as UBC3D-PLM, was initially developed and implemented as a user defined soil model in PLAXIS.

In the PDMY02 model Elgamal et al. (2002), the yield criteria are defined by open conical-shaped yield surfaces that have a common apex at the origin of the principal stress space. This model was developed to capture the cyclic response of pressure-dependent soils such as dilatancy and non-flow liquefaction. The model follows a non-associated flow rule to simulate volumetric dilation and contraction under shear. No plastic change of volume is predicted by this model under a constant stress ratio. Further, the model requires parameter calibration separately for different soil relative densities. Using the model requires defining a set of parameters that characterize the soil’s elastic and plastic behavior, the evolution of excess pore pressures with time, and the coupling between shear and volumetric strains.

DAFALIAS-MANZARI model Dafalias and Manzari (2004) is a simple stress-ratio controlled, critical state compatible, sand plasticity model. The model builds upon previous work of Dafalias and Manzari with three novel aspects introduced. The first is a fabric-dilatancy related quantity, scalar valued in the triaxial and tensor valued in generalized stress space, which is instrumental in modeling macroscopically the effect of fabric changes during the dilatant phase of deformation on the subsequent contractant response upon load increment reversals, and the ensuing realistic simulation of the sand behavior under undrained cyclic loading. The second aspect is the dependence of the plastic strain rate direction on a modified Lode angle in the multiaxial generalization, a feature necessary to produce realistic stress-strain simulations in nontriaxial conditions. The third aspect is a very systematic connection between the simple triaxial and the general multiaxial formulation, in order to use correctly the model parameters of the former in the implementation of the latter.

The PM4SAND model follows the basic framework of the stress-ratio controlled, critical state compatible, bounding surface plasticity model for sand presented by Dafalias and Manzari (2004). Modifications to the model were developed and implemented by Boulanger (2010), Boulanger and Ziotopoulou (2012) and Boulanger and Ziotopoulou (2015) to improve its ability to approximate the stress-strain responses important to geotechnical earthquake engineering applications; in essence, the model was calibrated at the equation level to provide for better approximation of the trends observed across a set of experimentally- and case history-based design correlations. These constitutive modifications included: revising the fabric for-
mation/destruction to depend on plastic shear rather than plastic volumetric strains; adding fabric history and cumulative fabric formation terms; modifying the plastic modulus relationship and making it dependent on fabric; modifying the dilatancy relationships to include dependence on fabric and fabric history, and to provide more distinct control of volumetric contraction versus expansion behavior; providing a constraint on the dilatancy during volumetric expansion so that it is consistent with Bolton (1986)’s dilatancy relationship; modifying the elastic modulus relationship to include dependence on stress ratio and fabric history; modifying the logic for tracking previous initial back-stress ratios (i.e., loading history effect); recasting the critical state framework to be in terms of a relative state parameter index; simplifying the formulation by restraining it to plane strain without Lode angle dependency for the bounding and dilatancy surfaces; incorporating a methodology for improved modeling of post-liquefaction reconsolidation strains; and providing default values for all but three primary input parameters. The model is shown to provide reasonable approximations of desired behaviors and to be relatively easy to calibrate.

The finite element method (FEM) based on solid mechanics can accurately simulate the initial small deformation of liquefaction, whereas for the flow stage of liquefaction deformation, the smoothed particle hydrodynamics (SPH) method in the framework of fluid dynamics would be more suitable. This section presents recent advancements in liquefaction deformation analysis using the FEM.

FEM is a simple and useful tool for solving problems of continuous media. It has been applied widely in geotechnical engineering analysis, such as in the fields of deformation prediction, strength analysis, and stability estimation. Huang et al. (2008a, 2009, 2012) have carried out many finite element simulations for small deformation of embankments on liquefiable soils. Because the liquefied soil is a saturated fluid—solid coupling medium, Biot’s dynamic coupled theory for saturated porous media simulation and a cyclic elastoplastic constitutive model were used. The liquefaction resistance and deformation characteristics of embankments under earthquakes were studied, and FEM was validated as a useful tool for seismic assessment and design of embankments. The dynamic interaction of pile-soil-structure in a liquefiable site was also studied by FEM (Huang et al. 2005). A beam-column element, which takes into account the effect of volume and axial force, was selected to represent the dynamic behavior of piles. The vulnerable parts and the deformation laws of the whole system were calculated, providing a theoretical basis for the seismic design of piles in liquefiable sites. The authors also proved the effectiveness of liquefaction mitigation measures that use the FEM method. A 2D-FEM program was used to verify the effect of reinforcement measures, revealing that the FEM method can provide useful results for seismic design of embankments (Huang et al. 2006). An effective stress based finite element-finite differences (FE-FD) method was used in the study of Huang et al.
The pore water pressure was calculated by the FD, while the displacement was obtained by the FE method. The simulation results illustrated that cement grouting cannot prevent soils from liquefying but it can substantially reduce the deformation induced by liquefaction.

8.4 Material inhomogeneity and three-dimensional random field models

It has been demonstrated that minute perturbations in the properties of the soil could trigger different failure pattern (Borja and Andrade (2006) Andrade and Borja (2006) Andrade et al. (2008)). For example, a heterogeneous sample sheared at a given mean density could be simulated to localize into a shear band, even if the imposed heterogeneity is very slight, when the equivalent homogeneous sample with the same density would not manifest any form of instability Borja (2006). Andrade et al. (2008) assessed the influence of heterogeneities in the porosity field on the stability of sand samples. They used Monte Carlo method and geostatistic tool to generate random and spatially variating porosity and then conducted numerical triaxial tests on these random material. Results from parametric studies indicate that the axial strength of a specimen is affected by both the degree and orientation of anisotropy in heterogeneous porosity values with anisotropy orientation having a dominant effect. It may be argued that this could also be true with liquefaction instability simulations since soil liquefaction is such a complex phenomenon-a multiscale, multiphysics problem, originating at the pore scale level but rapidly propagating to the particle cluster and specimen scales.

Deterministic analyses for liquefaction-induced deformations assume uniform soil with the same properties as the average properties of the variable soil. Natural spatial variability of soil properties, however, affects the soil behaviour and can modify the failure mechanism of soil systems. For example, for phenomena involving the presence of a failure surface (such as encountered in landslides or in bearing capacity failures) the actual failure surface can deviate from its theoretical position to pass selectively through weaker soil zones, and thus the average mobilised strength is reduced when compared with that of a corresponding uniform soil (e.g., Duncan (2000)). For the case of seismically induced soil liquefaction, it was shown (e.g., Popescu et al. (1997); Konrad and Dubeau (2003)) that a larger amount of excess pore water pressure (EPWP) is generated in a heterogeneous soil than in the corresponding uniform soil having geomechanical properties equal to the average properties of the variable soil.
Two examples of experimental evidence on the effects of soil heterogeneity on liquefaction resistance are mentioned here. Budiman et al. (1995) performed a series of undrained cyclic triaxial tests on sand specimens containing up to 25% gravel inclusions. They found that the liquefaction resistance of sand with inclusions was lower than that of uniform sand. Moreover, the reduction in liquefaction resistance was more pronounced for samples with a higher content of gravel. Konrad and Dubeau (2003) conducted undrained cyclic triaxial tests on fine Ottawa sand and on much finer silica silt. The study consisted of three series of tests. The first two series were conducted to characterize the cyclic strength of the sand and of the silt respectively; the third series was performed on layered samples, consisting of one horizontal silt layer sandwiched between two sand layers at various elevations in the sample (in the centre, close to the base platen, and close to the top platen). The study results revealed that layering induced a much lower cyclic resistance than that developed in either of the materials in uniform samples. Those authors believed that differential pore pressures generated in the layered soils induced water migration from the sand to the silt layer and caused a strength reduction of the layered samples.

Pioneering work in numerical analysis of the effects of soil heterogeneity on liquefaction resistance was presented by Ohtomo and Shinozuka (1990). Later on, Popescu (1995) and Popescu et al. (1997) Popescu et al. (1998) conducted a systematic study of those effects using Monte Carlo simulations, concluding that both the pattern and the amount of dynamically induced EPWP build-up are strongly affected by the spatial variability of soil properties. It was shown that, for the same average values of soil parameters, more EPWP build-up was predicted in stochastic analyses, accounting for soil heterogeneity, than in so-called ‘deterministic analyses’, assuming uniform soil properties. Characteristic percentiles of soil strength were proposed for use in deterministic analyses, resulting in a response similar to that predicted by more expensive stochastic analyses. Effects of the degree of soil variability and of the seismic loading rate were also investigated. Owing to restrictions imposed at the time by computational resources, only a small number of samples (between 10 and 25) were used in the Monte Carlo simulations, and the study was limited to one single earthquake intensity. Koutsourelakis et al. (2002) analyzed the effects of spatial variability on soil liquefaction for a wide range of earthquake intensities, again using Monte Carlo simulations, and presented fragility curves for simple structures on liquefiable soil.

In realization of the significant influence of material randomness and inhomogeneity on the soil behavior and performance, and failure in soils are usually multiscale problems, Baker et al. (2011) and Chen
et al. (2012) developed a probabilistic framework across multiple scales to efficiently model and simulate multiscale fields of spatially varying material properties and to consistently compute the behavior of the material in a multi-scale model.

The studies mentioned previously were based on two-dimensional analyses of soil liquefaction (in a vertical plane) assuming plane strain behavior. Therefore the correlation distance of soil variability in a direction normal to the plane of analysis was implicitly taken as infinite (i.e. no variability in the third direction). An attempt to address the real 3D aspect of the problem was presented by Fenton and Vanmarcke (1998) with an analysis of liquefaction potential at the Wildlife Site, Imperial Valley, California. The soil properties were modelled as a 3D random field; however, the liquefaction analysis was carried out in 1D vertical columns, without any coupling in the horizontal plane. Consequently, the study could only address the pointwise initiation of liquefaction, and did not include the effects of pore water pressure redistribution before and after initial liquefaction. Through this 3D approach, the authors provided interesting observations on the clustering of liquefied zones in a horizontal plane. Another stochastic analysis of the same site was presented by Elkateb et al. (2003). The soil properties and the probabilistic characteristics of their variability were estimated based on a limited number of cone penetration test (CPT) results. A 3D stochastic analysis was performed, but the liquefaction potential was assessed separately at each point in space using an empirical method (e.g. Seed and Idriss (1971); Robertson and Wride (1998)). Though this approach could not capture any interaction between zones of soil with different liquefaction strengths and different amounts of pore water pressure build-up, the study provided some interesting conclusions on the effects of loose soil zones on the probability of liquefaction failure.

Liquefaction is a multiscale problem in nature because of inhomogeneities existing at different length-scales in geomaterials. Few of the investigations mentioned previously has addresses the multiscale issue of 3D modeling of liquefaction in inhomogeneous geomaterials. The author of this dissertation developed a 3D multiscale random field for spatial modeling of regional soil properties (Figure 8.3), which can be used with the previously developed finite element framework for regional liquefaction assessment.

### 8.5 Free field analysis of liquefiable soils

This section discusses a three-dimensional analysis of liquefaction in a soil profile. The considered site consists of layered cohesionless soil underlain by bedrock. The surface of the profile is assumed even
Figure 8.3: 3D random field realizations of soil property with no slope. The model consists of a single column of 3D solid-fluid fully coupled brick elements supported vertically at the base. Periodic boundary conditions are applied in both horizontal directions. ALysmer and Kuhlemeyer (1969) dashpot is utilized to account for the finite rigidity of an underlying elastic medium, and the loading is applied in a manner consistent with that proposed by Joyner and Chen (1975).

8.5.1 Model Description

The liquefaction analysis discussed in this section is for a soil profile consisting of a 10 m thick layer of loose sand (Dr = 40%) above a 20 m thick layer of more dense sand (Dr = 75%). The entire soil profile is underlain by an elastic half-space which represents the finite rigidity of an underlying bedrock layer. The groundwater table is located at a depth of 2 m. Saturated unit weights are used for the soil below water table and effective stress analysis is considered through the use of eight-node brick elements which are able to simulate fluid-solid coupling.

Mesh Geometry

As shown in Figure 8.4, a single column of 8-node brick elements is created. The vertical direction for this column is oriented with the global y-direction, and the elements are sized equally in the global x- and z-directions. The shaking is applied in the x-direction.

Boundary Conditions

The column of elements is supported vertically at the base. Periodic boundary conditions are applied in the global x- and z-directions using the equalDOF command. Dashpots are applied at the base
Figure 8.4: Finite element mesh of 3D liquefaction analysis of a soil column
of the column in the global x- and z-directions to simulate the underlying bedrock layer after Lysmer and Kuhlemeyer (1969). The viscous uniaxial material is used with zeroLength elements to define the dashpots. Following the method of Joyner and Chen (1975), the dashpot coefficients are defined as the product of the mass density and shear wave velocity of the underlying bedrock. Above the groundwater table, the pore pressure degrees-of-freedom are fixed to allow drainage. The pore pressure degrees-of-freedom for all nodes below the groundwater table are left free to indicate saturated undrained conditions.

**Material and Element Definitions**

The soil constitutive behavior is modeled using the PDMY02 nDMaterial object. The default material parameters are based upon the recommended table of parameters available in Mazzoni et al. (2005) for the appropriate relative densities. It should be noted that the mass density input values for the material objects should be total mass densities, i.e. above the groundwater table, the mass density should reflect dry or moist conditions, and below the groundwater table, the mass density should be the saturated value. Included with the material definitions for each soil layer are additional parameters which are used by the elements. These include the fluid bulk modulus, the voids ratio, the body forces, and the x-, y-, and z-direction permeabilities. For effective stress analysis here, the body forces are the components of gravity. Parameters for different soil layers are list in the Table 8.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Layer 1 (2 m)</th>
<th>Layer 2 (8 m)</th>
<th>Layer 3 (20 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($ton/m^3$)</td>
<td>1.8</td>
<td>2.24</td>
<td>2.45</td>
</tr>
<tr>
<td>Reference low-strain shear modulus (kPa)</td>
<td>9e4</td>
<td>9e4</td>
<td>13e4</td>
</tr>
<tr>
<td>Reference bulk modulus (kPa)</td>
<td>22e4</td>
<td>22e4</td>
<td>26e4</td>
</tr>
<tr>
<td>Friction angle at peak shear strength, in degrees</td>
<td>32</td>
<td>32</td>
<td>39</td>
</tr>
<tr>
<td>Octahedral shear strain</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Reference mean effective confining pressure (kPa)</td>
<td>101</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>Pressure dependent coefficient</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Phase transformation angle, in degrees</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>Rate of shear-induced volume decrease</td>
<td>0.067</td>
<td>0.067</td>
<td>0.01</td>
</tr>
<tr>
<td>Constant reflecting $K\sigma$ effect</td>
<td>0.23</td>
<td>0.23</td>
<td>0</td>
</tr>
<tr>
<td>Rate of shear-induced volume increase</td>
<td>0.06</td>
<td>0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>Dilat3, another constant reflecting $K\sigma$ effect.</td>
<td>0.27</td>
<td>0.27</td>
<td>0</td>
</tr>
<tr>
<td>Initial void ratio</td>
<td>0.8</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Permeability coefficient (m/s)</td>
<td>1.0e-2</td>
<td>1.0e-5</td>
<td>1.0e-2</td>
</tr>
</tbody>
</table>

The SSPbrickUP element is used in this example. This element uses a u-p formulation to consider the coupling between the solid and fluid phases of the soil mixture. The SSPbrickUP element uses stabilized single-point integration for the solid phase, and includes direction stabilization of the fluid phase in the incompressible-impermeable limit via a stabilization parameter. This element provides a computationally-
efficient alternative to a higher-order 3D u-p element such as the 20-8-BrickUP Element.

**Loading and Analysis**

The analysis is split into two phases, (1) an initial gravitational phase and (2) a dynamic excitation phase. Separate recorders are generated for each phase. The gravitational analysis phase serves the purpose of developing the initial state of stress for the soil column, including the static shear stress representing the infinite slope. In the second phase of the analysis, an earthquake ground motion is applied to the base of the soil column in a manner consistent with that proposed by Joyner and Chen (1975). The motion is applied in the horizontal x direction. During the gravitational analysis, the permeability in each direction is set at 1.0 m/s to facilitate development of hydrostatic conditions. After completion of this analysis phase, the permeabilities are updated to their input values using the setParameter command.

![Figure 8.5: Load](image)

Figure 8.5: Load
8.5.2 Results

Several sets of results are presents to demonstrate the types of results which can be obtained from this type of analysis. Firstly the deformation of the soil column could be obtained. As shown in Figure 8.6, the deformed soil column is plotted with contours of pore pressure at several different time.

![Figure 8.6: Deformation of soil column, colors represent pore water pressure](image)

To see the detailed information of the displacements at the top of the soil column and pore water pressure changing with time, several more figures are plotted here. Figure 8.7(a) shows the settlement at the top of the soil column (ground surface) changing with time. Figure 8.7(b) shows the lateral spreading at the top of the soil column in the direction of shaking changing with time. Figure 8.7(c) shows the pore pressure in the middle of the liquefiable soil changing with time.

The soil column is initially stable, then loses stability and begins to move forward and back in the shaking direction (Figure 8.7(b)). As shown in Figure 8.8, this loss of stability is due to liquefaction in the
saturated loose sand layer. If the analysis is continued past the end of the applied ground motion, the pore pressures will eventually dissipate and the column will become stable as the liquefied soil regains strength.

Figure 8.8 shows the excess pore water pressure ratio along the soil depth. When the ratio reaches 1, liquefaction will occur. It is shown that the liquefaction occurs in the soil layers around 5 to 10 meters under the ground surface, which is the weak soil Layer 2 shown in Figure 8.4. As shown in Figure 8.5.2(a) and Figure 8.9(b), the deformations around 5 to 10 meters are also very large, which proves that these layers liquefied during the shaking load.

8.6 Summary

In total stress analysis of liquefaction, generation and dissipation of excess pore water pressures are totally ignored in the numerical analysis. Therefore the resulted deformation from this type of analysis is always questionable due to oversimplification of the problem. In effective stress analysis-based analysis, liquefaction is occurred as a result of excess pore water pressure generation. The generation of excess pore water pressure is done under undrained conditions by considering volumetric strain and the bulk modulus of water in pores.

As shown in this chapter, effective stress analysis using finite element method as a numerical tool can be used in predicting cyclic liquefaction in soils. Although, the effective stress analysis is a natural way of modelling liquefaction, but it needs more information about the mechanical behavior of soil and the numerical methods used for the analysis. Due to the complex behavior of soil, a proper constitutive model is needed which should be capable of accumulating volumetric strain and consequently capable of modelling liquefaction. The parameter selection and the use of numerical tools play a significant role in this type of analysis.

Calibration of the constitutive model with in situ testing data (CPT, SPT, $V_{s30}$) is a promising path leading to mechanics-based regional liquefaction assessment. Once the correlation is built, evaluation of liquefaction-induced damage using FEM at local sites could be made. This could be integrated with random field models to predict regional liquefaction hazard. For example, Monte Carlo method and random fields could be combined to generate random and spatially varying soil properties and then conducted finite element analysis at site-specific scale with these random material. The consideration of material randomness and inhomogeneity is important since strength of a soil is affected by both the degree and orientation of anisotropy. This could also be true with liquefaction instability simulations since soil liquefaction is such a complex phenomenon—a multiscale, multiphysics problem, originating at the pore scale level but rapidly
Figure 8.7: Results of finite element analysis

(a) Vertical settlement at the top surface of the soil column

(b) Lateral spread at the top surface of the soil column

(c) Pore water pressure at the middle of the soil column
Figure 8.8: Excess pore pressure ratio along the depth

Figure 8.9: Liquefaction-induced deformations of soil
Pioneering works in numerical analysis of the effects of soil heterogeneity on liquefaction resistance have been presented decades ago. Owing to restrictions imposed at the time by computational resources, only a small number of samples were used in the Monte Carlo simulations, and the studies were usually limited to one single earthquake intensity. Recent years, with improved computation abilities, researchers started to analyze the effects of spatial variability on soil liquefaction for a wide range of earthquake intensities. For example, one of the ongoing work of the author of this dissertation is trying to analyze soil liquefaction accounting for full earthquake possibilities. A future work succeeding this dissertation is to integrate the developed finite element model with multiscale random field framework for regional assessment of liquefaction hazard.
Chapter 9

Conclusions

9.1 Conclusions

Liquefaction is a major reason for damages of structures supported by saturated soils during earthquakes. In this dissertation, the state-of-the-art of liquefaction engineering is reviewed. It is found that the triggering and consequencing ground failure of liquefaction have been well investigated in the past decades as macroscopic phenomenons and the dominant approach is the macroscopic correlations of the observed field behavior with various in-situ “index” tests. Despite the advances in the site-specific scale assessment, it is unveiled that issues still exist when it comes to large scale assessments, for example, cross-geologic units correlations are still not systematically investigated in regional liquefaction assessment.

Problems involving mechanics-based numerical modeling of liquefiable soil are reviewed. It is found that modelings of failures of liquefied soil are still in the premature stage and issues still exist such as idealized specimen-scale tests with simple boundary conditions, non-addressed spatial randomness and heterogeneity of material, etc. Although idealized tests have been made, liquefaction phenomenon is seldom investigated systematically.

A multiscale modeling framework of regional liquefaction with material randomness and heterogeneity is developed to better understand the liquefaction phenomenon. A unique feature of the developed methodology is the extension of conventional random field models to account for soil spatial variability at multiple scales and resolutions. The method allows selectively and adaptively generating random fields at smaller scales around critical areas or around areas where soil properties are known to a great detail from lab or field tests. The process is defined such that spatial correlation is consistent across length scales. Illus-
trative examples (Marina District in San Francisco, Alameda County in California, and Christchurch in New Zealand) are presented. Liquefaction hazard is evaluated at multi-scale. Compared with single scale analyses, multi-scale random fields provide more detailed information and higher-resolution soil properties around critical areas. This work provides a new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment mapping.

In Chapter 4, the developed random field framework is presented for assessing liquefaction potential for a particular region through integration of empirical models from geotechnical earthquake engineering (e.g., CPT-based liquefaction criteria). An illustrative example of Marina District in San Francisco is shown. The liquefaction potential is evaluated using both single- and multi-scale random field models. While both single- and multi-scale random field models predict consistent cumulative frequency of percent area to have liquefaction over the entire region, multi-scale random fields provide more detailed information and higher-resolution soil properties around critical areas. This is illustrated through calculating liquefaction fragility curves for soils underneath Materna medical building. This work provides demonstrated the new way to consistently incorporating small-scale local liquefaction analysis into large-scale liquefaction assessment mapping.

In Chapter 5, the framework integrated with the classical CPT-based liquefaction model and Monte Carlo simulations is applied to the probabilistic and spatial assessment of liquefaction-induced settlements over a region and across scales. An example is shown by applying this framework to the Alameda County site of California. It is found that: Quantitatively consistent liquefaction hazards over the entire studied region and within each surficial geologic units are obtained when verified against existing analysis and knowledge of the studied region; Spatial variability of soil properties within each geologic units at a much smaller and much detailed scale are captured, which provides a way to systematically refine and perform local site-specific liquefaction analysis while preserving the liquefaction hazard prediction at the regional scale; The spatial structure of the predicted settlements inferred from available field data are shown to be relatively insensitive to the earthquake shaking intensity and such inferred spatial structure is preserved during the random field modeling process; In the Alameda County site, the artificial fill is the surficial geologic unit most susceptible to liquefaction hazard (48.2% and 57.7% of the area will exhibit medium to extensive damage for the $M_w = 6.6$ and $M_w = 7.1$ earthquake scenarios considered) followed by the Holocene alluvial fan deposits (the corresponding percentages are 7.4% and 13.2%); Local site-specific analysis shows that the Ruby Bridges School site is expected to suffer more liquefaction-induced damage (quantified by the predicted settlements) than the Hoover School site.

In Chapter 6, the framework is used for a study which focuses on the spatial variability of CPT-based
geotechnical parameters. Two approaches, termed the averaged index approach and the local soil property approach, are developed and analyzed to account for spatial variability of geotechnical parameters. Their implications on liquefaction susceptibility evaluation are discussed through one case study at the Christchurch site in New Zealand. It is found that: Both the averaged index and the local soil property approaches are able to capture the spatial variations of liquefaction potential at different scales over the area of study and predict qualitatively consistent results compared to previously reported liquefaction manifestations; The two approaches yield similar and comparable LPI values for the area of study when sufficient CPT data is available. However, when the CPT data is sparse, the averaged index approach, in general, predicts lower LPI values when compared with the local soil property approach; The averaged index approach is recommended for regional liquefaction evaluation given that it yields consistent results with field observations and is much more computationally efficient; The local soil property approach, though providing more details on soil properties, requires much more field data to characterize random field models, which could introduce additional uncertainties into the solution; Both approaches have been cast within a multiscale random field model, which allows efficient and effective assessment of site-specific liquefaction susceptibility.

In Chapter 7, a hybrid geotechnical and geologic data-based framework is developed for the multiscale random field-based regional liquefaction hazard mapping. The geotechnical data such as the CPT data are used to calculate primary LPI values, which show clear spatial correlation as evidenced by the calculated empirical semivariogram. A simplified method is proposed to derive secondary LPI values from geologic information which essentially enforces geologic constraints to the generated liquefaction hazard maps. A Markov-Bayes coefficient is introduced to derive covariances related to secondary data. Both primary and secondary LPI data are then integrated into multiscale random field models through a conditional sequential simulation technique. The proposed framework is applied to map liquefaction hazard at the Alameda County site in California. It is found that the influence of geologic constraints on the generated liquefaction hazard maps is significant. Without geologic constraints and with only geotechnical data, the generated liquefaction hazard map incorrectly predicts high liquefaction hazard in the bedrock geologic unit. As the weight of the geologic data increases, the geologic boundaries become more distinguishable in the generated map. With an appropriately calibrated Markov-Bayes coefficient, the accuracy of the liquefaction map is improved when validated against prior knowledge. Moreover, the advantage of the multiscale liquefaction hazard map is demonstrated through a small site specific liquefaction hazard analysis.

In Chapter 8, a dynamic FEM model is built upon which an effective stress analysis is performed to estimate liquefaction-induced soil deformation at site-specific scale. It is shown that effective stress analysis
using finite element method as a numerical tool can be used in predicting cyclic liquefaction in soils. Empirical correlation-based methods have been the dominant approach for liquefaction assessment. For site-specific analysis of liquefaction, mechanics-based method is an alternative to empirical correlations. One of the advantages of mechanics-based method is that it applies to complex site conditions where the liquefaction damage could not be evaluated by empirical correlations. Another advantage is that mechanics-based method is a link between structures and soil damages. The complex analysis of soil-foundation-structure interaction during liquefaction could only be solved by mechanics-based numerical methods.

9.2 Future work

In the examples used for demonstration of the developed framework, liquefaction damages are evaluated through their empirical correlations with insitu testing data such as CPT, SPT and \( V_{30} \). Empirical correlations, however, are only capable for simple site conditions such as free field ground settlement and lateral spreading. FEM is a prominent tool for estimation of liquefaction damages under complex site conditions. In this dissertation, a dynamic FEM model is built upon which an effective stress analysis is performed to estimate liquefaction-induced soil deformation at site-specific scale. It is shown the developed finite element model as a numerical tool can be used in predicting cyclic liquefaction in soils. Although, the effective stress analysis is a natural way of modelling liquefaction, but it needs more information about the mechanical behavior of soil and the numerical methods used for the analysis. Due to the complex behavior of soil, a proper constitutive model is needed which should be capable of accumulating volumetric strain and consequently capable of modelling liquefaction. The parameter selection and the use of numerical tools play a significant role in this type of analysis.

Calibration of the constitutive model with insitu testing data (CPT, SPT, \( V_{30} \)) is a promising path leading to mechanics-based regional liquefaction assessment. Once the correlation is built, evaluation of liquefaction-induced damage using FEM at local sites could be made. This could be integrated with random field models to predict regional liquefaction hazard. For example, Monte Carlo method and random fields could be combined to generate random and spatially varying soil properties and then conducted finite element analysis at site-specific scale with these random material. The consideration of material randomness and inhomogeneity is important since strength of a soil is affected by both the degree and orientation of anisotropy. This could also be true with liquefaction instability simulations since soil liquefaction is such a complex phenomenon—a multiscale, multiphysics problem, originating at the pore scale level but rapidly propagating to the particle cluster and specimen scales.
Pioneering works in numerical analysis of the effects of soil heterogeneity on liquefaction resistance have been presented decades ago. Owing to restrictions imposed at the time by computational resources, only a small number of samples were used in the Monte Carlo simulations, and the studies were usually limited to one single earthquake intensity. Recent years, with improved computation abilities, researchers started to analyze the effects of spatial variability on soil liquefaction for a wide range of earthquake intensities. For example, one of the ongoing work of the author of this dissertation is trying to analyze soil liquefaction accounting for full earthquake possibilities. A future work succeeding this dissertation is to integrate the developed finite element model with multiscale random field framework with the consideration of uncertainties in the soil properties and the earthquake shaking intensities for regional assessment of liquefaction hazard.
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