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THEORETICAL AND COMPUTATIONAL DESIGN ANALYSIS OF A HARMONICA TYPE AEROELASTIC ENERGY HARVESTER

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THEORETICAL AND COMPUTATIONAL DESIGN ANALYSIS OF A HARMONICA TYPE AEROELASTIC ENERGY HARVESTER

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Masters of Science
Mechanical Engineering

by
Songkai Wang
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Accepted by:
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Abstract

A wind energy harvester inspired by music playing harmonicas was proposed for micro-power generation. The energy harvester utilizes flow-excited self-sustained oscillations of a piezoelectric cantilever beam mounted in a wind pipe to generate electric power. The dependence of the energy harvester’s power generation performance on a set of the design parameters such as the chamber volume, aperture width and beam dimensions has been studied previously. However, the performance of the nonlinear multi-physics system with two-way fluid structure coupling also depends on other design parameters such as the geometry of the oscillatory beam, the beam mounting configuration and the geometry of the pipe outlet. A systematic design analysis of the effects of these parameters is necessary for the optimization of the energy harvester.

In this study, theoretical and computational modeling and design analysis are performed to investigate the influence of the design parameters of beam geometry and pipe outlet structure on the performance of the wind energy harvester. It is known that the increase of the beam bending displacement induces a larger strain in the piezoelectric layer and a higher electric energy output of the wind harvester. In addition, the decrease of the threshold wind velocity and pressure will enhance the energy harvester’s adaptability. The beam bending displacement and the threshold wind velocity are thus taken as the performance measures in this work. Theoretical models are developed to take into account different beam geometries. The analysis results show that the beam shape has a significant effect
on the performance. 3-D finite element models are constructed for various designs. The theoretical analysis results are verified by the numerical simulations. In addition, by using the finite element model, the effect of several pipe outlet design parameters are studied. It is shown that the device performance can be improved with an optimal beam mounting position and a smooth pipe outlet wall.
Dedication

To My Parents
Acknowledgments

I would like to thank Dr. Gang Li for all his support and guidance during my stay at Clemson. He taught me many things over the last three years. This thesis would be impossible to complete without his expertise and instruction. I earned rich knowledge and experience from working in Dr. Lis research group. Apart from the thesis, I had an opportunity to develop GUI of a FEA application with Java language.

I would like to thank Dr. Daqaq and Dr. Zhao for accepting to be my masters thesis committee members. I would also like to thank Dr. Daqaq, Dr. Mocko, Dr. Thompson, Dr. Miller and Dr. Coutris for offering challenging course work.

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Chapter 1

Introduction

Wind is the movement of air from an area of high pressure to an area of low pressure. It is a form of solar energy, caused by the uneven heating of the atmosphere by the sun, the irregularities of the earth’s surface, and rotation of the earth. The wind motion energy, or called shortly wind energy, can be used to generate mechanical power or electricity. Wind energy is one of the fastest growing sources of energy in the world today. It is clean, renewable, and economic. The electricity produced from wind power brings no pollution or greenhouse gases. It is inexhaustible and requires nothing but wind that blows across the earth surface. As technology develops, wind power has also become a cost-competitive source of electricity. At the end of 2013, according to global wind energy council, the total wind power produced globally was 318,105 MW, representing a strong annual growth of more than 12.5 percent [1]. In the American wind energy association 2012 annual market report, 13,131 MW amount of new wind capacity installed during 2012, standing for a 28 percentage annual growth rate in the U.S. [2]. The current operating fleet of wind capacity in the US can power the equivalent of more than 15 million average American homes.

In daily life, the most common device that converts wind kinetic energy into electrical power is wind turbine. A wind turbine is a wind-driven turbine for generating elec-
tricity. Large turbine arrays in wind farms are being used across the world to reduce global reliance on fossil fuels. Medium turbines can be utilized for making contributions to a domestic power supply via electrical grids. Small turbines are applied for battery charging to power LED traffic warning signs or other electric applications. Although wind turbines provide an effective way of producing clean power, their disadvantages are non-negligible. For instance, wind turbines are usually noisy. While wind energy is preferred as it does not pollute the environment, the noise produced in the process of generating electricity can cause adverse health effects. This alone is the reason that wind farms are not built near residential areas. Wind turbines also bring negative visual impact to surrounding neighborhoods and potential threat to wildlife. Many people believe wind turbines actually appealing to eyes but some of others disagree. This fact limits the areas where they can be built. In addition, wind turbines are known to kill birds that fly over them. Last but not the least, due to the availability of wind and the unpredictable output amount, large urban areas rarely utilize wind turbines to generate electricity.

The advances in fabrication technology and electronics continue to produce smaller lower-power consumption devices. For instance, many types of lower-power wireless sensors have been developed for remote environmental and health monitoring. For such devices, replacement or recharging of batteries is a very cumbersome process. Scavenging otherwise wasted energy from the ambient environment, such as the wind energy, provides a solution to lower dependence on batteries. Unfortunately, traditional small-scale wind energy harvesters suffer from critical scalability issues because their performance drops significantly with their size [4]. And due to the complex structure and expensive material, the cost could be impractical for small-scale applications.

At small scale, another type of energy harvesters, piezoelectric harvesters, have attracted much interest. Piezoelectric harvesters can convert mechanical strain into electric current or voltage by using piezoelectric materials. The piezoelectric effect is understood
as the linear electromechanical interaction between the mechanical and the electrical state in crystalline materials with no inversion symmetry [3]. Ceramic lead zirconate titanate, commonly known as PZT, is the most widely used piezoelectric material. Key advantages of PZT material include resistant to high temperatures and ability to be manufactured in any shape or size. They are also chemically inert. Most vibration-based piezoelectric harvesters produce power on the order of milli-watt, small for large system applications, but sufficient for micro-scale devices such as remote controls, wireless sensors, wearable electronics, and condition monitoring. If properly designed and installed, devices that utilize piezoelectric harvesters can become self-sustaining and battery-free. As a result, over the past decade, there has been intense interest in developing piezoelectric energy harvesters that use ambient external sources, such as vibrational kinetic energy or fluid energy. However, these vibration-based piezoelectric energy harvesters have a critical shortcoming in their operation concept. These harvesters operate efficiently only within a narrow frequency bandwidth where the excitation frequency is very close to the fundamental frequency of the harvester (Resonance Condition). If the vibration frequency falls out of the range of a particular bandwidth, the energy harvesting process becomes inefficient and low in power density. As such, many viable excitation sources, such as structural and machine vibrations, ocean waves, acoustic excitations, running, walking, among other motions are considered impractical due to their inherent randomness or non-stationarities.

Motivated by the need for portable yet scalable high power density and low maintenance energy harvesters, a concept for a micro-power generator that uses self-excited limit-cycle oscillations of a piezoelectric beam to harness wind energy and maintain low-power consumption devices is proposed [5]. Inspired by music playing harmonica, the harvester shown in figure 1.1 consists of a piezoelectric cantilever unimorph structure embedded within a cavity to mimic the vibration of the reeds in a harmonica when subjected to air blow. The operation principle of the harvester is simple. Wind blows into the chamber
and the air pressure in the chamber increases. The increased air pressure bends the beam and opens an air path between the chamber and the environment. As the air passes through the aperture, the pressure in the chamber decreases. The mechanical restoring force pulls the beam back decreasing the aperture area and the process is repeated. These periodic fluctuations in the pressure cause the beam to undergo self-sustained oscillations. The resulting periodic strain in the piezoelectric layer produces an electric field which can be channeled as a current to an electric device.

The key advantage of this concept for micro-power generation stems from its ability to eliminate the shortcomings of traditional vibration-based energy harvesters and small size wind turbines while, at the same time, combining aerodynamics with vibrations to generate the necessary power. On one hand, this concept is based on transforming vibrations to electricity but does not require an external vibration source eliminating the bandwidth issues associated with resonant vibratory energy harvesters. On the other hand, while this
Figure 1.2: (a) Experimental setup, (b) Output voltage of the energy harvester device depends on the presence of an aerodynamic energy field, it does not suffer from the scalability issues that hinder the efficiency of small size wind turbines. The employed experimental configuration and its output voltage are shown in Figure 1.2.

Although the wind energy harvester has showed its capability of electric power generation and advantage of less structural complexities, the design parameters used in the experiments were chosen arbitrarily and are far from being the optimal parameters necessary to maximize the performance and efficiency. In another study [6], the effects of the chamber volume, aperture width, and beam dimensions are investigated. It is found that the cut-in wind speed can be reduced significantly if the device is designed with an optimal chamber volume, which is shown to be inversely proportional to the square of the first modal frequency of the beam. What’s more, the cut-in wind speed corresponding to the optimal volume is shown to depend on the relative dimensions of the beam. Minimizing the aperture width is also shown to significantly reduce the cut-in speed. However, due to the reduced strain rate in the piezoelectric layer, it is observed that minimizing the wind speed does not always yield an increase in the output power.

To explore the possibility of further improving the performance, in this work, we
aim to investigate a set of additional design parameters of the device. Specifically, we study three sets of the design parameters as described below:

1. Geometry of the beam. The effect of dimensions of the rectangle cantilever beam has been investigated in the previous study. In this work, we develop theoretical and computational models for wind energy harvesters containing cantilever beams with different shapes. For a given wind pressure/velocity, the dynamic response of trapezoid, T-shaped, bowl-shaped, and block-top beams are calculated.

2. Geometry of the air chamber outlet. It was assumed in the previous study that the air chamber cross section is uniform in the longitudinal direction. Therefore, only the chamber volume is taken to be a parameter in the theoretical study. However, the geometry of the air chamber, specially the part close to the outlet, which determines the fluid contraction coefficient of the air flow going through the aperture, can have a significant influence on the performance. This effect of the air chamber geometry is investigated in this work.

3. Mounting position of the beam. The wind energy harvester generates electricity by taking advantage of self-sustained oscillation. This is a fluid-structure interaction behavior which is obviously affected by the structure configuration, the connection type of the beam and the pipe in particular. Thus a discussion of the relative position becomes unavoidable.

We use two approaches to perform the analysis. First, a reduced order theoretical model is utilized and extended to take into account the design parameters described above. By using the reduced order theoretical model, the maximum beam displacement, which is directly related to maximum beam strain, is calculated. With higher beam strain, piezoelectric material produces higher output voltage and more electric power. Under controlled conditions, more electric power output indicates better energy conversion efficiency. In addition, the decrease of the threshold wind velocity and pressure will enhance the energy
harvester’s adaptability. Therefore, the beam bending displacement and the threshold wind velocity are thus taken as the performance measures in this work. The reduced order theoretical model can quickly provide results. It is a helpful tool to provide physical insights of the fundamental mechanism of the fluid-structure interaction in the harvester. However, it also has disadvantages. One limitation is that its results serve merely as rough estimations, since geometric and physical approximations are made in the derivation of the model. The other shortcoming is that only semi-3D plane beam structures can be analyzed in this theoretical model. Analysis of full 3-D beam structures with irregular geometries is beyond its capacity. Second, along with the reduced order theoretical model, the three-dimensional two-way coupled fluid-structure interaction problem is simulated by using the finite element analysis (FEA). The FEA is employed for tow purposes: (1) to validate the reduced order theoretical models and (2) to investigate designs for which the theoretical model is not applicable. FEA is a numerical technique for finding approximate solutions to boundary value problems for differential equations [7]. It uses calculus of variations to minimize an error function and produce a stable solution. In this study, the fluid-structure interaction problem is solved through a two-way coupling simulation. ANSYS Transient Structure and ANSYS Fluent are used to solve the structure and fluid parts, respectively. The coupling algorithm iterates between the structure and fluent solvers until residuals converge within a specified tolerance or the configured iteration number reached. After appropriate convergence study, this approach can produce relatively accurate results for both semi-3-D and fully 3-D devices, with the price of largely increased computational cost.

The rest of the thesis is organized as follows. In Chapter 2, reduced order theoretical models are derived for different beam shapes. The associated assumptions and simplifications are described in the derivation. Chapter 3 presents the finite element modeling of the energy harvesting device. In Chapter 4, the effects of a set of design parameters, such as beam geometric shape, configuration of pipe outlet area, and the mounting position of the
beam, are presented. Conclusions are given in Chapter 5.
Chapter 2

Theoretical Modeling

In this chapter, a reduced order theoretical model of the wind energy harvester with a rectangular cantilever beam is introduced. The model is then extended to take into account different beam geometries. Through the derivation, we describe the assumptions and simplifications that are associated with the reduced order models.

Before presenting the equations, it is necessary to define the pressure situation in the theoretical model. Although the dynamic behavior of the pressure-controlled beam vibration is complex, it is helpful to consider three simplified situations [8]. In these three situations, the motion of the beam can be described by a single displacement parameter $x$ measuring the beam tip opening. We can specify the behavior of the beam by a two-element parameter $(\sigma_1, \sigma_2)$ in which $\sigma_1 = +1$ if a steady positive pressure applied to the upstream or inlet port of the beam tends to increase the beam opening $x$, and $\sigma_1 = -1$ if this pressure tends to close the beam. The second parameter $\sigma_2$ is defined similarly for a pressure applied to the downstream or exit port of the beam [9]. Valves of type $(+1, -1)$ are like outward-swinging doors or doors that are blown open. This is the situation as the wind energy harvester we investigate. As the other situations are not related to the device under consideration, they are briefly explained as follows. Valves of type $(-1, +1)$ can be
pictured as inward-swinging doors or doors that are blown closed. Beams of configuration 
(+1, +1) are like sliding doors or doors that are blown sideways. The remaining beam 
type, described by (−1, −1), does not appear to occur naturally or to have any practical 
utility. The threshold for self-excited oscillation of the three simple beam classes in the 
absence of an attached resonator has been examined theoretically [8]. It is shown that, 
if the acoustic impedance presented to the inlet of the beam is \( Z_1 = R_1 + jX_1 \) and that 
presented to the outlet is \( Z_2 = R_2 + jX_2 \), then a necessary condition for the initiation of 
self oscillation is that \( \sigma_1 X_1 - \sigma_2 X_2 < 0 \). Note that there is another necessary condition 
on blowing pressure that depends in detail upon beam geometry and internal damping. 
For beams of configuration (+1, −1), which are considered in this thesis, the requirement 
\( X_1 + X_2 < 0 \) can be achieved by supplying the beam from a reservoir of volume \( V \) and 
allowing the beam to exhaust to the open air.

2.1 Reduced Order Theoretical Model for Rectangular Cantilever Beams

We consider a structural design of the wind energy harvester as shown in Figure 2.1. The air chamber/pipe has an inlet and an outlet. A cantilever beam is mounted over the 
outlet of the chamber/pipe. The beam is clamped across the bottom end. The beam is taken 
to be a flexible rectangular plate of length \( L \), width \( W \), and thickness \( h \). \( s \) is defined as the 
distance measured from the clamped end of the beam as shown in the figure. Let \( \psi(s)_n \) be 
the mode shapes of the cantilever beam [11],

\[
\psi(s)_n = A_n \left[ \cosh \beta_n s - \cos \beta_n s + \frac{(\cos \beta_n L + \cosh \beta_n L)(\sin \beta_n s - \sinh \beta_n s)}{\sin \beta_n L + \sinh \beta_n L} \right], \quad (2.1)
\]
where $n$ is the mode shape order, $\beta_n$ and $A_n$ are both coefficients. $\beta_n$ can be obtained from $\cosh(\beta_nL)\cos(\beta_nL) + 1 = 0$, and $A_n$ remains arbitrary as the magnitude of beam displacement is unknown for free vibrations. To simplify the problem, only the first mode shape is taken into account. Then $\psi(s)$ is

$$\psi(s) = A \left[ \cosh \beta s - \cos \beta s + \frac{(\cos \beta L + \cosh \beta L)(\sin \beta s - \sinh \beta s)}{\sin \beta L + \sinh \beta L} \right], \quad (2.2)$$

where $\beta = 1.875/L$, and $A$ is 0.5, an appropriate value to normalize $\psi(L) = 1$. From Equation (2.2), it is easy to obtain

$$\int_0^L \psi(s) ds = 0.3915, \quad (2.3)$$

$$\int_0^L \psi(s)^2 ds = 0.25, \quad (2.4)$$
and
\[ \gamma = \frac{\int_0^L \psi(s) ds}{\int_0^L \psi(s)^2 ds} \approx 1.5, \quad (2.5) \]
where \( \gamma \) is a constant to be used later. We can evaluate the horizontal component of the area of the side opening behind the beam. For a planar beam, it is obtained as
\[ a(x) = x_0 + (x - x_0) \int_0^L \psi(s) ds \approx 0.6x_0 + 0.4x, \quad (2.6) \]
where \( x \) is the beam tip opening and \( x_0 \) is the static opening of the beam with no applied pressure. The total air escaping area is then approximately
\[ F(x) = W \left[ x^2 + b^2 \right]^{\frac{1}{2}} + 2L \left[ a(x)^2 + b^2 \right]^{\frac{1}{2}}, \quad (2.7) \]
where \( a(x) \) is the average length of opening along the sides of the beam, and \( b \) is the clearance gap between the beam and the air pipe outlet wall.

As mentioned previously, the beam is assumed to be fed from an air pipe of volume \( V \), which is supplied with a steady volume inflow \( U_0 \) from a high impedance source, and to exhaust to free space so that the downstream impedance is zero. The volume of outflow going over the beam is \( U(t) \). Here \( U(t) \) is modified by a flow contraction coefficient \( C \). In our model, as the outflow goes over a sharp-edged outlet wall, we set \( C = 0.6 \) \([12]\). The average pressure in the air pipe \( p(t) \) must also be supplemented by a small term to represent the inertia of the air in the channel of length \( \delta \) at the tip of the flap \([8,9]\). It is assumed that the average \( \delta \) equals to the thickness of the beam \( h \). By using the Euler-Bernoulli equation, it is easy to show that the equation for the pressure \( p(t) \) can be obtained as \([9]\)
\[ p(t) = \frac{\rho_a U^2}{2C^2 F(x)^2} + \frac{\partial}{\partial t} \left[ \frac{\rho U \delta}{CF(x)} \right], \quad (2.8) \]
where $\rho_a$ is the density of air. Assuming the operation of the device is under standard atmosphere pressure and at environmental temperature which is $25^\circ C$ or $77^\circ F$, $\rho_a$ is $1.1839\, kg/m^3$.

The pressure $p(t)$ is related to the change of pipe volume caused by the beam vibration, which leads to

$$\frac{dp}{dt} = \frac{\rho_a c^2}{V} \left( U_0 - U - \dot{V} \right), \quad (2.9)$$

where $\dot{V}$ is the volume changed by the cantilever beam bending and $c$ is the speed of sound in air (346.13 $m/s$ at $25^\circ C$). For the rectangular cantilever beam, $\dot{V}$ can be simplified to equal the derivative of $a(x)$ with respect to time multiplied by the cross-sectional area of the beam. With $da/dt \approx 0.4 \dot{x}$, we have

$$\dot{V} = 0.4WL \dot{x}.$$ \hspace{1cm} (2.10)

Thus Equation (2.9) becomes

$$\frac{dp}{dt} = \frac{\rho_a c^2}{V} \left( U_0 - U - 0.4WL \dot{x} \right), \quad (2.11)$$

Next, let $Q$ be the quality factor of the free oscillation damping of the beam. Then the damping coefficient $R$, determined by the combination of internal losses in the beam material and viscous damping in the surrounding air, can be expressed as $R = \omega_0/2Q$, where $\omega_0$ is the natural frequency of the beam. For rectangular beam, $\omega_0 = 1.875^2 \sqrt{EI/(\rho_b AL^4)}$, where $E$ is the Young’s modulus of beam material, $I$ is the moment of inertia of beam cross-section, and $\rho_b$ is the density of beam material. However, Schlieren images of the oscillating beam show that a vortex develops downstream during the closing part of the beam cycle [9]. To include that in $R$, we first define

$$v = \left( \frac{2p}{\rho_a} \right)^{\frac{1}{2}}.$$ \hspace{1cm} (2.12)
where \( u \) is the air jet velocity in compressible flow determined by Bernoulli’s principle [10]. It is a reasonable assumption that the extra damping is proportional to \( \rho u x_0 dx/dt \) per unit vibrating length of edge of the valve flap [9]. Then, for the rectangular cantilever beam, the new damping coefficient \( k \) can be written as

\[
k = \frac{\omega_0}{2Q} + \eta \frac{x_0 (2p p_a)^{1/2} (W + 0.8L)}{m},
\]

(2.13)

where \( m \) is mass of the beam and \( \eta \) is a numerical coefficient which is assumed to be 1. The other symbols have been defined previously.

To obtain the beam’s equation of motion, which is expressed in terms of its tip opening \( x \), let \( \xi(s, t) \) be the displacement function of the beam, where \( s \) measures the distance from the clamped end. The equation of motion then has the form

\[
m(s) \frac{\partial^2 \xi}{\partial t^2} + R \frac{\partial \xi}{\partial t} + K \frac{\partial^4 \xi}{\partial s^4} = W(s)p,
\]

(2.14)

where \( m(s) \) is the beam mass at \( s \), \( R \) is the damping coefficient, \( K \) is the bending stiffness of the beam, \( W(s) \) is the beam width at \( s \), and \( p \) is the average in-pipe fluid pressure.

Since we are concerned with oscillation near the fundamental cantilever mode of the cantilever beam at frequency \( \omega_0 \), we can write \( \xi(s, t) = [x(t) - x_0] \psi(s) \) [9]. Multiplying both sides of Equation (2.14) by \( \psi(s) \) and integrating over the beam length \( L \) gives

\[
\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + \omega_0^2 (x - x_0) = \frac{\gamma WLp}{m}.
\]

(2.15)

where \( x \) is the beam tip displacement, \( k \) is the damping coefficient defined in Equation (2.13), \( \omega_0 \) is the beam natural frequency, \( x_0 \) is the static opening distance of the beam tip with no applied pressure.

The combination of the average pressure equation shown in Equation (2.8), the re-
duced order continuity equation given in Equation (2.11), and the single degree of freedom (SDOF) equation of beam motion shown in Equation (2.15), gives a complete set of ODEs that can be solved iteratively. In the three coupled equations, the unknowns are the fluid pressure \( p(t) \), transient outflow \( U(t) \) and the tip displacement \( x(t) \) of the beam. Denoting \( x_1 = x, x_2 = \dot{x}, x_3 = U, \) and \( x_4 = p, \) then the set of equations can be rewritten in a state space form containing a set of first order ODEs as

\[
\dot{x}_1 = x_2 
\]

\[
\dot{x}_2 = \frac{1.5WLx_4}{m} - 2kx_2 - \omega_0^2(x_1 - x_0) 
\]

\[
\dot{x}_3 = \frac{x_4CF(x_1)}{\rho_a\delta} + \frac{\partial F(x_1)}{\partial t} \frac{x_3}{F(x_1)} - \frac{x_3^2}{2\delta CF(x_1)} 
\]

\[
\dot{x}_4 = \frac{\rho_a c^2}{V} (U_0 - x_3 - 0.4WLx_2) 
\]

The equations are solved by using the ODE solver ode45 in Matlab. The amplitude and frequency of the beam oscillation are determined by the tip displacement.

### 2.2 Reduced Order Theoretical Model for T-shaped Cantilever Beams

Although the reduced order theoretical model provides helpful insights and reasonable predictions of the fluid-structure interaction behavior in the wind energy harvester, it is only applicable to rectangular beam. One of our research goals is to explore the effect
of the shape of the cantilever beam. To achieve this goal, we extend the theoretical model described in the previous section to include other beam shapes.

The first non-rectangular beam to be studied is a T-shaped beam structure as shown in Figure 2.2. It comes from the observation that the T-shaped beam has a larger beam tip with unchanged bottom width and total length. The T-shaped beam is expected to have a greater applied force but similar bending stiffness. In the following, we construct the reduced order theoretical model for the T-shaped beam. Unless otherwise specified, the symbols used in the T-shaped beam model equations have the same meanings defined in the rectangular beam model.

As shown in Figure 2.2, the T-shaped beam is taken to be a flexible T-shaped plate with bottom width $w_1$, side wing width $w_2$, lower part height $l_1$, side wing height $l_2$, and thickness $h$. The total height of the beam is $L = l_1 + l_2$ and the upper part width is $W = w_1 + 2w_2$. The T-shaped beam is also clamped at the bottom. As the T-shaped structure is a non-uniform beam, its mode shapes are difficult to obtain analytically. However, with
the same cross section of the lower part, $0 < s < l_1$, and $l_2 < l_1$, it is reasonable to assume that the first mode shapes of the rectangular and T-shaped beams are close to each other. To simplify the problem, we assume $\psi(s)$, which is the T-shaped beam first mode shape, is the same as the first mode shape of the rectangular beam. Thus

$$a(x) \approx 0.6x_0 + 0.4x,$$  \hspace{1cm} (2.20)

where $a(x)$ is the horizontal component of the area of the side opening behind the beam. Compared to the rectangular beam structure, there are two more side openings allowing air to escape in the T-shaped beam design. To include the side openings of the bottom edges of the side wings into the model, we define

$$a_{l_1}(x) = [1 - \psi(l_1)]x_0 + \psi(l_1)x.$$  \hspace{1cm} (2.21)

where $a_{l_1}(x)$ is the bottom edge opening of the two side wings and $\psi(l_1)$ is the beam displacement at $l_1$. Therefore, the total air escaping area of T-shaped design is given by

$$F(x) = W \left[ x^2 + b^2 \right]^{\frac{1}{2}} + 2L \left[ a(x)^2 + b^2 \right]^{\frac{1}{2}} + 2w_2 \left[ a_{l_1}(x)^2 + b^2 \right]^{\frac{1}{2}}.$$  \hspace{1cm} (2.22)

The average in-pipe fluid pressure $p(t)$ remains the same form

$$p(t) = \frac{\rho_a U^2}{2C^2F(x)^2} + \frac{\partial}{\partial t} \left[ \frac{\rho U\delta}{CF(x)} \right].$$  \hspace{1cm} (2.23)

Similar to $a(x)$,

$$a'(x) = \left[ 1 - \int_{l_1}^{L} \psi(s)ds \right] x_0 + \int_{l_1}^{L} \psi(s)ds x.$$  \hspace{1cm} (2.24)

where $a'(x)$ is the average opening along the sides of the wings and $\psi(s)$ is the first beam.
mode shape. For the T-shaped cantilever beam, \( \dot{V} \), the pipe volume changing rate caused by the beam bending, has an additional term representing the volume changed by two side wings. \( \dot{V} \) can be presented as

\[
\dot{V} = \left[ 0.4w_1 L + 2w_2 l_2 \int_{l_2}^{L} \psi(s) ds \right] \frac{dx}{dt}.
\]

For the T-shaped design, the reduced order continuity equation has the same form as that in the rectangular beam case:

\[
\frac{dp}{dt} = \frac{\rho_a c^2}{V} \left( U_0 - U - \dot{V} \right),
\]

where \( c \) is the sound speed, \( V \) is the pipe volume, and \( U_0 \) is the constant inflow volume. For damping coefficient \( k \), the additional damping term is also proportional to \( \rho_v x_0 dx/dt \) per unit vibrating length of the valve flap edges. Therefore, for the T-shaped cantilever beam, \( k \) can be written as

\[
k = \frac{\omega_0}{2Q} + \eta \frac{x_0(2p\rho_0)^{1/2}(W + 0.8L + 2w_2\psi(l_1))}{m},
\]

where \( \omega_0 \) is the beam natural frequency, \( Q \) is the quality factor of the beam free oscillation damping, and \( \eta \) is a numerical coefficient assumed to be 1.

The equation of motion of the T-shaped beam has the form

\[
m(s) \frac{\partial^2 \xi}{\partial t^2} + R \frac{\partial \xi}{\partial t} + K \frac{\partial^4 \xi}{\partial d^4} = W(s)p.
\]

Multiplying both sides of the equation by the first mode shape \( \psi(s) \) and integrating over
the beam length $L$, we have

$$\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + \omega_0^2 (x - x_0) = \frac{p(w_1 l_1 \int_0^{l_1} \psi(s)ds + W l_2 \int_{l_1}^L \psi(s)ds)}{h \rho_0 (w_1 l_1 \int_0^{l_1} \psi(s)^2 ds) + W l_2 \int_{l_1}^L \psi(s)^2 ds},$$

(2.29)

where $x$ is the beam tip displacement, $k$ is the damping coefficient defined in Equation (2.27), $\omega_0$ is the T-shaped beam natural frequency obtained from finite element method, $x_0$ is the static opening distance of the beam tip with no applied pressure, and $p$ is the average in-pipe fluid pressure. The integration of $\psi(s)$ employs Equation (2.2).

Equations (2.23), (2.26), and (2.29) can be written in a state space form containing a set of first order ODEs. Denoting $x_1 = x$, $x_2 = \dot{x}$, $x_3 = U$, and $x_4 = p$, we obtain

$$\dot{x}_1 = x_2$$

(2.30)

$$\dot{x}_2 = \frac{p(w_1 l_1 \int_0^{l_1} \psi(s)ds + W l_2 \int_{l_1}^L \psi(s)ds)}{h \rho_0 (w_1 l_1 \int_0^{l_1} \psi(s)^2 ds) + W l_2 \int_{l_1}^L \psi(s)^2 ds} - 2kx_2 - \omega_0^2 (x_1 - x_0)$$

(2.31)

$$\dot{x}_3 = \frac{x_4 CF(x_1)}{\rho_0 \delta} + \frac{\partial F(x_1)}{\partial t} F(x_1) - \frac{x_3^2}{2\delta CF(x_1)}$$

(2.32)

$$\dot{x}_4 = \frac{\rho_0 c^2}{V} \left[ U_0 - x_3 - \left( 0.4 w_1 L + 2 w_2 l_2 \frac{\int_{l_1}^L \psi(s)ds}{l_2} \right) x_2 \right]$$

(2.33)

By using the ODE solver ode45 in Matlab, the set of equations are solved for the tip displacement and tip velocity of the T-shaped beam, and the pressure and outflow rate of the fluid. The amplitude and frequency of the beam oscillation are determined by using the beam tip displacement result.
2.3 Reduced Model of Bowl-shaped Cantilever Beam

The second non-rectangular beam design is a bowl-shaped beam structure. It is inspired by the fact that the changing flow pressure causes the vibration of the beam. If the air escaping area changes more rapidly, the force applied on beam will bear more rapid change. Figure 2.3 shows the bowl-shaped beam design.

![Figure 2.3: Schematic diagram of the bowl-shaped beam design](image)

The dimensions of the bowl-shaped beam are shown in Figure 2.3. The total height of the beam is \( L = l_1 + l_2 \). Similar to the T-shaped beam case, we assume that the first mode shape of the bowl-shaped beam, \( \psi(s) \), is the same as that of the rectangular beam with the same width and length. Thus,

\[
a(x) \approx 0.6x_0 + 0.4x.
\]

where \( a(x) \) is the horizontal component of the beam side opening. For the bowl-shaped...
cantilever beam, the total air escaping area, $F(x)$, has two additional terms due to the additional edges of the bowl-shaped beam. We define

$$a'(x) = \left[ 1 - \frac{\int_{l_1}^{L} \psi(s) ds}{L} \right] x_0 + \frac{\int_{l_1}^{L} \psi(s) ds}{L} x, \quad (2.35)$$

and

$$a_{l_1}(x) = [1 - \psi(l_1)] x_0 + \psi(l_1)x, \quad (2.36)$$

where $a'(x)$ is the average horizontal opening size along the vertical inner walls of the bowl shape and $a_{l_1}(x)$ is the horizontal opening size along the bottom edge of the bowl shape. $F(x)$ is then obtained as

$$F(x) = 2w_2 \left[ x^2 + b^2 \right]^\frac{1}{2} + 2L \left[ a(x)^2 + b^2 \right]^\frac{1}{2} + w_1 \left[ a_{l_1}(x)^2 + b^2 \right]^\frac{1}{2} + 2l_2 \left[ a'(x)^2 + b^2 \right]^\frac{1}{2}, \quad (2.37)$$

The equation of the flow pressure $p(t)$ in the air pipe remains the same

$$p(t) = \frac{\rho_a U^2}{2C^2F(x)^2} + \frac{\partial}{\partial t} \left[ \frac{\rho U \delta}{CF(x)} \right]. \quad (2.38)$$

In the bowl-shaped beam design, the pipe volume changing rate caused by beam bending, $\dot{V}$, can be written as the volume changing rate in the rectangular beam model minus the volume changing rate caused by the bowl shape part. Thus

$$\dot{V} = \left[ 0.4WL - w_1l_2 \frac{\int_{l_1}^{L} \psi(s) ds}{L} \right] \frac{dx}{dt}. \quad (2.39)$$

Similar to the rectangular beam model, the reduced order continuity equation is

$$\frac{dp}{dt} = \frac{\rho_a c^2}{V} \left( U_0 - U - \dot{V} \right), \quad (2.40)$$
where $c$ is the sound speed, $V$ is the pipe volume, and $U_0$ is the constant inflow volume. By following the same derivation shown in the previous sections, the damping coefficient $k$, is obtained as

$$k = \frac{\omega_0}{2Q} + \eta \frac{x_0(2\rho\rho_a)^{1/2} \left[ 2w_2 + 0.8L + w_1\psi(l_1) + 2l_2 \frac{\int_{l_2/L}^L \psi(s)ds}{l_2/L} \right]}{m},$$  \hspace{1cm} (2.41)

and the equation of motion describing the beam tip displacement is obtained as

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2(x - x_0) = \frac{p(Wl_1 \int_{l_1}^{l_1} \psi(s)ds + 2w_2l_2 \int_{l_1}^{L} \psi(s)ds)}{h\rho_b(Wl_1 \int_{l_1}^{l_1} \psi(s)^2ds + 2w_2l_2 \int_{l_1}^{L} \psi(s)^2ds)}, \hspace{1cm} (2.42)$$

where $x$ is the beam tip displacement, $k$ is the damping coefficient defined in Equation (2.41), $\omega_0$ is the bowl-shaped beam natural frequency obtained from finite element method, $x_0$ is the static opening distance of the beam tip with no applied pressure, and $p$ is the average in-pipe fluid pressure. The integration of $\psi(s)$ employs Equation (2.2).

The coupled Equations (2.38), (2.40) and (2.42), are rewritten in a state space form as a set of first order ODEs. Denoting $x_1 = x$, $x_2 = \dot{x}$, $x_3 = U$, and $x_4 = p$, we have

$$\dot{x}_1 = x_2$$  \hspace{1cm} (2.43)

$$\dot{x}_2 = \frac{p(Wl_1 \int_{l_1}^{l_1} \psi(s)ds + 2w_2l_2 \int_{l_1}^{L} \psi(s)ds)}{h\rho_b(Wl_1 \int_{l_1}^{l_1} \psi(s)^2ds + 2w_2l_2 \int_{l_1}^{L} \psi(s)^2ds)} - 2kx_2 - \omega_0^2(x_1 - x_0)$$  \hspace{1cm} (2.44)

$$\dot{x}_3 = \frac{x_4CF(x_1)}{\rho a\delta} + \frac{\partial F(x_1)}{\partial t} F(x_1) - \frac{x_3^2}{2\delta CF(x_1)}$$  \hspace{1cm} (2.45)

$$\dot{x}_4 = \rho_c^2 \frac{V}{V} \left[ U_0 - x_3 - \left( 0.4WL - w_1l_2 \frac{\int_{l_2/L}^L \psi(s)ds}{l_2/L} \right) \right]$$  \hspace{1cm} (2.46)
By using the ODE solver ode45 in Matlab, the set of equations are solved for the beam tip displacement and velocity, fluid pressure and outflow rate. The amplitude and frequency of the beam oscillation are determined by using the beam tip displacement result.

2.4 Reduced Order Model of Cantilever Beams with Thick Tip

The third non-rectangular beam design is a beam structure with a thick tip. It is proposed by considering the fact that the drop in air pressure when the beam is opening allows the beam to deflect back. By adding additional thickness to the beam tip, air pressure will drop slower than the rectangular beam, which might lead to a larger tip displacement. Figure 2.4 shows the beam structure and Figure 2.5 is a close-up view of the beam tip. The dimensions of the beam structure are shown in Figure 2.3.

Figure 2.4: Schematic diagram of the beam design with a thick tip
Once again, we assume that the first mode shape of the beam with a thick tip, $\psi(s)$, is the same as that of the rectangular beam with the same width and length. For obtaining the air escaping area expression, we define

$$a(x) = 0.6x_0 + 0.4x,$$  \hspace{1cm} (2.47)

$$a'(x) = (1 - \frac{\int^L_0 \psi(s)ds}{l_1/L})x_0 + \frac{\int^L_1 \psi(s)ds}{l_1/L}x,$$  \hspace{1cm} (2.48)

$$a''(x) = (1 - \frac{\int^L_2 \psi(s)ds}{l_2/L})x_0 + \frac{\int^L_{l_1} \psi(s)ds}{l_2/L}x,$$  \hspace{1cm} (2.49)

where $a(x)$ is the horizontal component of the side opening area behind the beam, $a'(x)$ is the averaged horizontal component of the opening along the lower part of the beam, and $a''(x)$ is the averaged horizontal component of the opening along the upper part of the beam. If $x_0$ and $h$ are small enough compared to $L$, when the beam tip passes through
the pipe outlet wall, the beam bending angle can be ignored. Thus, the theoretical model is simplified by assuming the beam tip sides are parallel to the corresponding gaps when the beam passes through the pipe outlet wall. Using \( a(x), a'(x), \) and \( a''(x), \) and assuming small beam bending angle, we obtain the air escaping area

\[
F(x) = \begin{cases} 
W(x^2 + b^2)^{\frac{3}{2}} + 2L [a(x)^2 + b^2]^{\frac{1}{2}} & x \leq 0 \\
Wb + 2l_2b + 2l_1 [a'(x)^2 + b^2]^{\frac{1}{2}} & 0 < x < t \\
W [(x - t)^2 + b^2]^{\frac{3}{2}} + 2l_1 [a'(x)^2 + b^2]^{\frac{1}{2}} + 2l_2 [(a'(x) - t)^2 + b^2]^{\frac{1}{2}} & x \geq t
\end{cases}
\]  

(2.50)

The average fluid pressure equation \( p(t) \), the reduced order continuity equation \( dp/dt \), and the damping coefficient \( k \) have the same forms as those in the rectangular beam model. They are given by

\[
p(t) = -\frac{\rho U^2}{2C^2 F(x)^2} + \frac{\partial}{\partial t} \left[ \frac{\rho U \delta}{CF(x)} \right]. \quad (2.51)
\]

\[
\frac{dp}{dt} = \frac{\rho c^2}{V} \left( U_0 - U - 0.4WL\dot{x} \right), \quad (2.52)
\]

\[
k = \frac{\omega_0}{2Q} + \frac{x_0(2p \rho_a)^{1/2}(W + 0.8L)}{m}, \quad (2.53)
\]

where \( \rho_a \) is the density of air, \( U(t) \) is the outflow volume go over the beam, \( C \) is the flow contraction coefficient, \( F(x) \) is the total air escaping area, \( \delta \) is the inertia of the air at the tip of the flap, \( c \) is the sound speed, \( V \) is the pipe volume, \( U_0 \) is the constant inflow volume, \( \omega_0 \) is the beam natural frequency, \( Q \) is the quality factor of the beam free oscillation damping, and \( \eta \) is a numerical coefficient assumed to be 1.

By following the same derivation shown in the previous sections, the equation of
The motion of the beam is obtained as
\[
\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + \omega_0^2(x - x_0) = \frac{pWL \int_0^L \psi(s) ds}{W \rho_b (h_1 l_1 \int_0^{l_1} \psi(s)^2 ds + h_2 l_2 \int_{l_1}^L \psi(s)^2 ds)}.
\tag{2.54}
\]

where \(x\) is the beam tip displacement, \(k\) is the damping coefficient defined in Equation (2.53), \(\omega_0\) is the block top beam natural frequency obtained from finite element method, \(x_0\) is the static opening distance of the beam tip with no applied pressure, and \(p\) is the average in-pipe fluid pressure. The integration of \(\psi(s)\) employs Equation (2.2).

The coupled Equations (2.51), (2.52) and (2.54), are rewritten in a state space form as a set of first order ODEs. Denoting \(x_1 = x, x_2 = \dot{x}, x_3 = U,\) and \(x_4 = p,\) then we have
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{pWL \int_0^L \psi(s) ds}{W \rho_b (h_1 l_1 \int_0^{l_1} \psi(s)^2 ds + h_2 l_2 \int_{l_1}^L \psi(s)^2 ds)} - 2kx_2 - \omega_0^2(x_1 - x_0) \\
\dot{x}_3 &= \frac{x_4 CF(x_1)}{\rho_o \delta} + \frac{\partial F(x_1)}{\partial t} \frac{x_3}{F(x_1)} - \frac{x_3^2}{2\delta CF(x_1)} \\
\dot{x}_4 &= \frac{\rho_o c^2}{V} (U_0 - x_3 - 0.4WLx_2)
\end{align*}
\tag{2.55-2.58}
\]

Similar to the other models, by using the ODE solver ode45 in Matlab, the set of equations are solved for the beam tip displacement and velocity, fluid pressure and outflow rate. The amplitude and frequency of the beam oscillation are determined by using the beam tip displacement result.
Chapter 3

Finite Element Modeling

3.1 Backgrounds

Other than the reduced order models, full 3-D finite element models are developed for design analysis of the device. The finite element analysis (FEA) is employed for two purposes: (1) to validate the reduced order theoretical models and (2) to investigate designs for which the theoretical model is not applicable.

A coupled-field analysis is a combination of analyses from different engineering disciplines (physics fields) that interact to solve a global engineering problem [13]. Every field may contain several unknown physical quantities that are required to be solved for. When the calculation of one field analysis depends on the results received from other fields, the analyses are coupled. Some analyses are one-way coupling analyses. For instance, in a thermal stress problem, the temperature field transfers thermal strains into the structural field, but the structural strains usually do not affect the temperature field. Thus, there is no need to do iterations between the two field. More complicated problems involve two-way coupling. It means when the coupling is strong in both ways, an iterative procedure is usually required and information needs to be exchanged between the fields until the
solutions of the fields are all converged. Frequently, the coupled responses from the two-way coupling cases are nonlinear. So two-way coupling problems are usually difficult to solve analytically and have to be analyzed utilizing experiments or numerical simulation.

Fluid structure interaction (FSI) is the interaction of some movable or deformable structure with an internal or surrounding fluid flow [14]. It is one of the most frequently encountered multi-physics problems in engineering. Solution strategies for FSI simulations are mainly divided into monolithic and partitioned methods. In the monolithic approach, governing equations of different fields are coupled into a single system of equations to solve the problem simultaneously [15]. The interaction of the fluid and the structure at the mutual interface is treated synchronously. The partitioned method is to solve the problem sequentially with each field solved separately and coupled through a coupling algorithm [16]. The fluid and the structure equations are alternately integrated in time and the interface conditions are enforced asynchronously. The monolithic approach is more suitable for simple problems because it is easy to couple the different governing equations mathematically and then solve the equations set simultaneously. But this approach requires a code developed for each particular combination of the multi-physics problem. The partitioned approach is more convenient for it preserves software modularity. Moreover, the partitioned approach calculates solution of the fluid equations and the structural equations with more efficient methods which are developed specifically. On the other hand, the stable and accurate coupling algorithm is required to be used in partitioned simulations.

Finite element analysis applied in engineering area is a computational tool for performing necessary analysis. It provides detailed information of coupling domains and helpful insights of the relationship between all variables. In our fluid-structure interaction problem, the fluid pressure causes the structure to deform, which in turn causes the fluid solution to change. It requires iterations between the two physics fields for convergence, which implies this is a two-way coupling analysis problem. In addition, due to the differ-
ent descriptions and available solvers for the structural (Lagrangian) and fluid (Eulerian) domains, the partitioned approach is employed for the two-way coupling analysis. In our study, the beam responses are computed by using ANSYS Workbench Transient Structural solver and the fluid equations are solved by using ANSYS Fluent solver. The two solvers are coupled through ANSYS System Coupling.

3.2 Geometrical modeling

In this work, various designs are modeled and simulated via ANSYS Workbench. Figure 3.1 shows a 3-D FEA model of the original wind energy harvester design [5]. A front view of the beam and a side view of the beam (Figure 3.2) are also shown. The fluid domain is modeled as a rectangular parallelepiped representing the air pipe and a large block representing the environment outside of the air pipe outlet. The beam is modeled as a rectangular cantilever beam with a width $W$, length $L$, and thickness $h$. The gap $g$ is the distance between the top and side faces of the beam to the outlet walls of the pipe. The dimension of the gap $g$ is very small compared to the dimensions of the pipe but is comparable to the thickness $h$ of the beam. The dimensions of the structure that are used in the this finite element model are summarized in Table 3.1.

3.3 Material properties

The wind energy harvester design shown in Figure 3.1 uses an aluminum beam. In the finite element model, the pipe is modeled as a rectangular chamber with rigid walls. The environmental temperature is fixed at 25 degree Celsius. The air is modeled as compressible isothermal ideal gas. The material properties of the air and the beam that are used in simulations are listed in Tables 3.2 and 3.3.
Figure 3.1: Isometric view of the whole structure

Figure 3.2: (a) Front view of the beam, (b) side view of the beam.
Table 3.1: Geometric parameters

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of the pipe $W_p$</td>
<td>60 mm</td>
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<tr>
<td>Height of the pipe $H_p$</td>
<td>80 mm</td>
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<tr>
<td>Length of the pipe $L_p$</td>
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</tr>
<tr>
<td>Width of the opening block $W_o$</td>
<td>200 mm</td>
</tr>
<tr>
<td>Height of the opening block $H_o$</td>
<td>200 mm</td>
</tr>
<tr>
<td>Length of the opening block $L_o$</td>
<td>200 mm</td>
</tr>
<tr>
<td>Width of the beam $W_o$</td>
<td>16 mm</td>
</tr>
<tr>
<td>Length of the beam $H_o$</td>
<td>58 mm</td>
</tr>
<tr>
<td>Thickness of the beam $L_o$</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Gap $g$</td>
<td>0.3 mm</td>
</tr>
</tbody>
</table>

Table 3.2: Material properties of the cantilever beam

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho_b$</td>
<td>2700 kg/m$^3$</td>
</tr>
<tr>
<td>Youngs Modulus $E$</td>
<td>69 GPa</td>
</tr>
<tr>
<td>Poissons ratio $\nu$</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 3.3: Material properties of air

<table>
<thead>
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<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Pressure</td>
<td>101325 Pa</td>
</tr>
<tr>
<td>Reference Density</td>
<td>1.184 kg/m$^3$</td>
</tr>
<tr>
<td>Reference Bulk Modulus</td>
<td>101000 Pa</td>
</tr>
<tr>
<td>Density Exponent</td>
<td>1</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>1.831e-05 kg/ms</td>
</tr>
</tbody>
</table>

3.4 Transient Structural Analysis Settings

Due to the simple geometric shape of the beam, the structure meshing is straightforward. Sweep meshing method using all hexahedral elements produces the final structure meshing result. Considering the balance between accuracy and computational cost, the beam is divided into 32 segments along the width direction, 116 segments along the
height direction, and 2 segments along the thickness direction. For boundary conditions, the beam’s bottom surface is fixed, which prevents displacement in all directions. The boundary conditions of other five faces of the beam, which are in contact with the fluid, are set to be fluid-solid interface. The interface is modeled such that the pressure load is received before the iteration of structural calculation and then structure displacement is transferred back to the fluid domain after the iteration. Figure 3.3 shows the beam meshing result and the fixed bottom surface.

![Figure 3.3: (a) Beam mesh, (b) fixed bottom.](image)

ANSYS Workbench Transient Structure uses ANSYS mechanical APDL solver, which is a general finite element structural solver capable of solving static, harmonic, transient, linear and nonlinear problems. In this solver, the equilibrium equation on a one element basis is

\[
\left( [K_e] + [K^f_e] \right) \{u\} - \{F_e^{th}\} = [M_e] \{\ddot{u}\} + \{F_e^{pm}\} + \{F_e^{nd}\},
\]

where \([K_e]\) is element stiffness matrix, \([K^f_e]\) is element foundation stiffness matrix, \(u\) is nodal displacement vector, \(\{F_e^{th}\}\) is element thermal load vector, \([M_e]\) is element mass
matrix, \( \{\ddot{u}\} \) is acceleration vector, \( \{F_{pr}^{e}\} \) is element pressure vector, and \( \{F_{nd}^{e}\} \) is nodal forces applied to the element. For the fluid-structural interaction problem of the wind energy harvester, the structural dynamics equation must be considered. The discretized equation is

\[
[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{f\},
\]

(3.2)

where \([M]\) is mass matrix, \(\{\ddot{u}\}\) is acceleration vector, \([C]\) is damping matrix, \(\{\dot{u}\}\) is velocity vector, \([K]\) is stiffness matrix, \(\{u\}\) is displacement vector, and \(\{f\}\) is load vector.

### 3.5 Fluent Settings

The quality of fluid mesh is critical to the fluid domain solution and it is often uneasy to produce a good quality mesh with acceptable element number. In our study, the fluid domain is meshed using tetrahedrons method with patch conforming algorithm. For our fluid structure interaction simulation, the beam tip displacement is so large that remeshing technology has to be utilized. In Fluent, the only supported 3D mesh method for remeshing is tetrahedrons, which leaves us no choice. Patch conforming algorithm is a meshing technique in which all faces and their boundaries (edges and vertices) within a very small tolerance are respected for a given part. Mesh based defeaturing is used to overcome difficulties with small features and dirty geometry. Virtual Topology can lift restrictions on the patches, however the mesher must still respect the boundaries of the Virtual Cells [17]. Note that the gap area is a crucial part in the meshing process. Since the gap is extremely small compared to the other system dimensions, increasing the number of elements inside the gap will significantly increase the total number of elements, and consequently, largely increase the computational cost. However, if very few elements are used in the gap, the accuracy of the solution deteriorates quickly. Based on convergence
study results, the mesh with 3 element layers in gap area is employed, as shown in Figure 3.4. The total number of elements is about 260k and the total number of nodes is about 53k.

Figure 3.4: Fluid domain gap mesh

The inlet boundary condition is constant flow with a given velocity to maintain a steady flow of fluid into the domain, as shown in Figure 3.5. The boundary conditions of the outlet opening block are set as 0 Pa above the atmospheric pressure. To avoid the artificial reflection of the fluid flow from the boundary, the outlet fluid block volume is modeled sufficiently large, as shown in Figure 3.6. For the pipe inner faces, we chose stationary rigid wall as boundary condition, which has non-slip shear, as shown in Figure 3.7. The inflow velocity is relatively small compared to the pipe scale length and calculated corresponding Reynold’s number is less than 100. Therefore, the fluent viscous model type is laminar.

As remeshing the fluid domain is necessary when beam tip displacement becomes large, Fluent dynamic mesh is used. The smoothing method is diffusion with cell-volume
function, and remeshing method is local cell. In fluid domain, the five faces that are in contact with the beam faces, are set to be system coupling dynamic mesh type. The calculated beam deformation is obtained from the structural solver before the fluid analysis iterations.
and the pressure loads are transferred back to the beam after the fluid analysis calculation.

The Fluent dynamic mesh model can be used to model flows where the shape of the domain is changing with time due to motion on the domain boundaries. The dynamic mesh model can be applied to single or multiphase flows. The motion can be a prescribed motion or an unprescribed motion where the subsequent motion is determined based on the solution at the current time. The update of the volume mesh is handled automatically by Fluent at each time step based on the new positions of the boundaries. With respect to dynamic meshes, the integral form of the conservation equation for a general scalar $\phi$, on an arbitrary control volume $V$, whose boundary is moving can be written as

$$
\frac{d}{dt} \int_V \rho \phi dV + \int_{\partial V} \rho \phi (\vec{u} - \vec{u}_g) \cdot d\vec{A} = \int_{\partial V} \Gamma \nabla \phi d\vec{A} + \int_V S_\phi dV,
$$

(3.3)

where $\rho$ is the fluid density, $\partial V$ is the boundary of the control volume $V$, $\vec{u}$ is the flow velocity vector, $\vec{u}_g$ is the mesh velocity of the moving mesh, and $\Gamma$ is the diffusion coefficient.
For diffusion-based smoothing, the mesh motion is governed by the diffusion equation

$$\nabla \cdot (\gamma \nabla \vec{u}) = 0,$$  \hspace{1cm} (3.4)

where $\gamma$ is the diffusion coefficient, and $\vec{u}$ is the mesh displacement velocity.

The boundary conditions for Equation (3.4) can be obtained from the user-prescribed or computed boundary motion. On deforming boundaries, the boundary conditions are such that the mesh motion is tangent to the boundary. Then this equation describes how the prescribed boundary motion diffuses into the interior of the deforming mesh. The diffusion coefficient $\gamma$ can be used to control how the boundary motion affects the interior mesh motion. A constant coefficient means that the boundary motion diffuses uniformly throughout the mesh. With a nonuniform diffusion coefficient, mesh nodes in regions with high diffusivity tend to move together.

The cell-volume formulation allows the diffusion coefficient to be a function of the cell volume, and has the form

$$\gamma = \frac{1}{V^\alpha},$$  \hspace{1cm} (3.5)

where $V$ is the local cell volume, normalized by the average cell volume of all deforming cell zones and $\alpha \geq 0$ is a user input parameter. Equation (3.5) controls how the boundary motion diffuses into the interior of the domain as a function of cell size. Decreasing the diffusivity in larger cells causes those cells to absorb more of mesh motion and therefore better preserves the cell quality of smaller cells [17].

In local cell remeshing method, ANSYS Fluent agglomerates cells based on skewness, size, and height prior to the movement of the boundary. Skewness is one of the primary quality measures for a mesh. A value of 0 indicates an equilateral cell (best) and a value of 1 indicates a completely degenerate cell (worst). The size criteria are specified
with minimum length scale and maximum length scale. Cells with length scales below the
minimum length scale and above the maximum length scale are marked for remeshing. The
value of maximum cell skewness indicates the desired skewness of the mesh.

In our model, pressure-based solver is applied. The pressure-based solver employs
an algorithm which belongs to a general class of methods called the projection method [18].
In the projection method, wherein the constraint of mass conservation (continuity) of the
velocity field is achieved by solving a pressure equation [19]. The pressure equation is de-
derived from the continuity and the momentum equations so that the velocity field, corrected
by the pressure, satisfies the continuity. Since the governing equations are nonlinear and
coupled, the solution process involves iterations wherein all governing equations are solved
repeatedly until a convergence value reached. It converts a general scalar transport equation
to an algebraic equation that can be solved numerically. This technique consists of inte-
grating the transport equation about each control volume, yielding a discrete equation. The
discrete equation expresses the conservation law on a control-volume basis. Discretization
of the governing equations are explained by considering the unsteady conservation equa-
tion for \( \phi \), a scalar quantity transport. The following equation written in integral form for
an arbitrary control volume \( V \) shows the concept:

\[
\int_V \frac{\partial \rho \phi}{\partial t} dV + \oint \rho \phi \vec{v} \cdot d\vec{A} = \oint \Gamma_\phi \nabla \phi \cdot d\vec{A} + \int_V S_\phi dV, \tag{3.6}
\]

where \( \frac{\partial \rho \phi}{\partial t} \) is the conservative form of transient derivative of transported variable \( \phi \), \( \rho \) is
the fluid density, \( \vec{v} \) is the velocity vector, \( \vec{A} \) is the surface area vector, \( \Gamma_\phi \) is the diffusion
coefficient for \( \phi \), \( \nabla \phi \) is the gradient of \( \phi \), \( S_\phi \) is the source of \( \phi \) per unit volume.

The pressure-based solver uses an implicit discretization of the transport equation.
All convective, diffusive, and source terms are evaluated for time step $n + 1$,

$$
\int_V \frac{\partial \rho \phi}{\partial t} dV + \oint \rho^{n+1} \phi^{n+1} \vec{v}^{n+1} \cdot d\vec{A} = \oint \Gamma^{n+1} \nabla \phi^{n+1} \cdot d\vec{A} + \int_V S^{n+1}_\phi dV. \quad (3.7)
$$

### 3.6 System Coupling Settings

The ANSYS Workbench System Coupling component is an all-purpose infrastructure that facilitates comprehensive multidisciplinary simulations between coupling participants [20]. Coupling participants are systems that will provide and consume data in a coupled analysis. System Coupling Service is a runtime component of the System Coupling system. It manages the execution of coupling analyses of any participants. A variety of one-way and two-way data transfers are performed between coupling participants during execution. Thus, the two main roles of the coupling service are: the mapping of data transfers, and coupling management.

Data transfers in System Coupling use one of two data transfer algorithms: profile preserving data transfer and conservative profile preserving data transfer. Profile preserving data transfer algorithm is used to transfer non-conserved quantities like displacements and temperatures. Conservative profile preserving data transfer is used when transferring conserved quantities like mass, momentum, and energy flows. Several mapping algorithms are used when executing data transfers during system couplings. Mapping is performed only at the start of the System Coupling simulation. Because of this, the mesh topology on the data transfer regions cannot change during the simulation. The two mapping algorithms used in System Coupling are bucket surface and general grid interface. For profile preserving data transfer algorithm, the bucket surface mapping algorithm is used to generate mapping weights. In this algorithm, the mesh nodes on the target side of the data transfer interface are mapped onto mesh elements on the source side. Usually, weight-based interpolation
and subsequent under-relaxation are utilized to evaluate the final data applied on the interface target side. For conservative profile preserving data transfer algorithm, the general grid interface mapping algorithm is used to generate mapping weights. Similarly, weight-based interpolation and subsequent under-relaxation are also used to evaluate the final data applied on the target side of the interface.

There are three aspects to coupling management: inter-process communication, convergence management, and process synchronization and analysis evolution. The inter-process communication uses a proprietary, light-weight, TCP/IP based client-server infrastructure and it does not interact with other communication mechanisms. As for convergence management, for both the source and target side of the transfer, the system coupling log file records the root mean square convergence of data transfers. At the end of each coupling iteration, convergence of the current time step is evaluated. If the coupling step has not converged yet, a new coupling iteration will begin. If the coupling step is converged, then a new time step will start if the coupling end time has not been reached. The coupling service and participants perform synchronously through a coupled analysis. The five primary synchronization points used to manage advancement through the coupled analysis are shown in Figure 3.8 [20]. This figure also presents the processing between the five primary synchronization points, shown in dark gray. The coupling step and iteration loop structure are noted int the figure too. Every synchronization points is a gateway beyond which a given process will advance only when all other processes arrive.

Details regarding processing between the solution and check convergence synchronization points are shown in Figure 3.9 [20]. At this analysis stage, the advancement of co-simulation participants is controlled by the coupling service through two secondary synchronization points: data transfer and solve.

After convergence study, in System Coupling, time step size is set to 0.00003 second. Minimum iterations is chosen to be 1 and maximum iterations is 6. The step size
Figure 3.8: Execution sequence diagram
Figure 3.9: Processing details
option specifies the time interval associated with each coupling step. Minimum iterations number is the fewest coupling iterations that have to be executed per coupling time step. And similarly, maximum iterations number is the most coupling iterations that could be executed per coupling time step. Within each iteration, Fluent system coupling face force is firstly transferred from the fluid solver to the structure solver. Then Transient Structure fluid-structure interface incremental displacement is transferred from the structure solver back to the fluid solver.
Chapter 4

Results and Discussions

The fluid-structure interaction analysis of the wind energy harvester is performed by adopting the reduced models developed in Chapter 2 and the FEA procedure described in Chapter 3. Firstly, simulation results of the original rectangular beam model produced by ANSYS with appropriate mesh density and time step size are compared with the theoretical model results to validate the correctness of both approaches. Then, the effects of a set design parameters on the performance of the device is investigated. The key performance measures of the wind energy harvester are the threshold pressure or the threshold velocity that triggers the self-sustained vibration, and the magnitude of the beam tip displacement. The former represents the minimum fluid pressure or the minimum wind speed that starts the operation of the energy harvester. And the latter determines the maximum output voltage. Models with different beam geometric shapes are studied to explore improved designs. The effect of static opening distance of the beam tip is investigated to find the optimal position. Models with different pipe outlet wall thicknesses are also compared for a better design.
4.1 Rectangular Beam Design

The beam tip displacement and the fluid pressure curves from the theoretical model and from the 3-D FEA are compared in this section. The parameters applied in rectangular beam reduced model are set as follows: beam width 16 mm, beam length 58 mm, beam thickness 0.3 mm, gap clearance 0.3 mm, static opening of the beam without pressure load 0 mm, fluid contraction coefficient 0.6, beam Young’s modulus 69 GPa, beam density 2700 kg/m$^3$, quality factor of the beam free oscillation damping 50, air density 1.19 kg/m$^3$, pipe volume $2.4 \times 10^6$ mm$^3$, inflow volume $4.8 \times 10^5$ mm$^3$/s, and sound velocity 340 m/s. The beam tip displacement obtained from the theoretical model is shown in Figure 4.1 and the average fluid pressure is shown in Figure 4.2.

In the 3-D FEA, the air chamber is a 60 mm × 80 mm × 500 mm pipe with constant inflow rate 100 mm$^3$/s. Other parameters used in the 3-D simulation have the same values as those used in the theoretical model. After convergence study, the coarsest acceptable mesh has about 260k elements. For this mesh, the largest appropriate time step...
size is $3 \times 10^{-5}$s. This choice is mainly to reduce the computational cost of the simulation without losing much accuracy of the result. Obtained from the 3-D simulation, The beam displacement result is presented in Figure 4.3 and fluid pressure at an in-pipe point which is 100 mm upstream from the beam is presented in Figure 4.4. Fluid pressure contour and fluid velocity contour near outlet area are showed in Figure 4.5 and Figure 4.6. With the same pipe dimensions and the same inflow rate, the threshold pressure can be compared by the threshold inlet velocity. Employing the 3-D FEA simulation, we get the threshold velocity of this rectangular beam model is 53 mm/s and the vibration frequency is 79 Hz.

It is observed that the curves generated independently by the two approaches are similar. While the theoretical model is reduced order and only provides approximation results, the comparison shows that the theoretical model is a helpful tool that gives reasonably accurate prediction of the dynamic behavior of the system.
4.2 T-shaped Beam Design

In the analysis of the T-shaped beam designs, a narrow T-shaped beam is designed with $w_1 = 16$ mm, $w_2 = 8$ mm, $l_1 = 42$ mm, $l_2 = 16$ mm. For comparison, a wide T-shaped
beam is designed with \( w_1 = 16 \text{ mm}, w_2 = 17 \text{ mm}, l_1 = 42 \text{ mm}, l_2 = 16 \text{ mm}. \) The meaning of the symbols are described in Section 2.2. 2-D views of the two designs are shown in Figure 4.7. All the other parameters have the same value as the corresponding parameters in the
Figure 4.7: (a) 2-D views of a narrow T-shaped beam, (b) 2-D views of a wide T-shaped beam

rectangular beam reduced model. Obtained from the theoretical model, the tip displacement of the narrow T-shaped beam is shown in Figure 4.8 and the corresponding average fluid pressure is shown in Figure 4.9. In comparison, the wide T-shaped tip displacement is shown in Figure 4.10 and the corresponding average fluid pressure is shown in Figure 4.11. The threshold velocity of the narrow T-shaped is 48 mm/s and the wide T-shaped case has a higher threshold velocity of 81 mm/s.

In the FEA analysis, for both T-shaped beam designs, the parameters are the same as those in the theoretical models. Obtained from the 3-D finite element simulation, the narrow T-shaped beam tip displacement is shown in Figure 4.12 and the corresponding fluid pressure at a point in the air pipe which is 50 mm upstream from the beam is shown in Figure 4.13. The wide T-shaped beam tip displacement is shown in Figure 4.14 and the corresponding fluid pressure at the same point in the air pipe is shown in Figure 4.15. The threshold velocity of the narrow T-shaped structure is 57 mm/s, which is higher than the rectangular beam design 53 mm/s. The wide T-shaped case has a higher threshold velocity 73 mm/s. The vibration frequency of the narrow T-shaped is 56 Hz and the vibration
Comparing the results obtained from the theoretical and the finite element models, it is observed that the wide T-shaped beam design has a smaller tip displacement than the narrow T-shaped beam design. The frequency of the wide T-shaped beam is 46 Hz.

Figure 4.8: Narrow T-shaped beam tip displacement from the theoretical model

Figure 4.9: Narrow T-shaped beam design fluid pressure from the theoretical model
the narrow T-shaped beam design. The fluid pressure in the wide T-shaped beam design is slightly lower. In addition, the wide T-shaped beam design has a significantly higher threshold velocity. In the 3-D finite element simulations, burrs on the fluid pressure curves
Figure 4.12: Narrow T-shaped beam tip displacement from 3-D FE simulation

Figure 4.13: Narrow T-shaped beam design fluid pressure from 3-D FE simulation

are caused by remeshing. As the pressure curve shows the fluid pressure at a specific point in the fluid domain, the value of the pressure at the point may change after the data interpolation during the remeshing process. Both models show that the tip displacement
decreases with longer T-shaped beam wings, which is the main factor causing a different performance of the device with the T-shaped beam design. The threshold velocity also increases with longer wings. Both set of results indicate that the T-shaped beam design is
worse than the rectangular beam design.

### 4.3 Bowl-shaped Beam Design

![Diagram of bowl-shaped beam designs](image)

In the analysis of the bowl-shaped beam designs, a shallow bowl-shaped beam is designed with \( w_1 = 8 \text{ mm}, w_2 = 4 \text{ mm}, l_1 = 50 \text{ mm}, l_2 = 8 \text{ mm} \). In comparison, a deep bowl-shaped beam is defined with \( w_1 = 8 \text{ mm}, w_2 = 4 \text{ mm}, l_1 = 28 \text{ mm}, l_2 = 30 \text{ mm} \). 2-D views of the two designs are shown in Figure 4.16. The meaning of the symbols are described in Section 2.3. The quality factor of the beam free oscillation damping is 7. All the other parameters have the same value as the corresponding parameters in the rectangular beam reduced model. Obtained from the theoretical model, the shallow bowl-shaped tip displacement is shown in Figure 4.17 and the corresponding average fluid pressure is shown in Figure 4.18. In comparison, the deep bowl-shaped tip displacement is shown in Figure 4.19 and the corresponding average fluid pressure is shown in Figure 4.20.

In the 3-D finite element simulation, the deep bowl-shaped design parameters have
Figure 4.17: Shallow bowl-shaped beam tip displacement from the theoretical model

Figure 4.18: Shallow bowl-shaped beam design fluid pressure from the theoretical model

the same values as those in the theoretical model. Obtained from the 3-D FEA simulations, the deep bowl-shaped tip displacement is shown in Figure 4.21 and the corresponding fluid pressure at a point in the air pipe that is 50 mm upstream from the beam is shown in Figure
Figure 4.19: Deep bowl-shaped beam tip displacement from the theoretical model

Figure 4.20: Deep bowl-shaped beam design fluid pressure from the theoretical model

4.22. The vibration frequency of the deep bowl-shaped is 99 Hz.

From both the theoretical model and 3-D simulation results, it clearly shows that the deep bowl-shaped beam design has a smaller maximum tip displacement than the shallow
bow-shaped beam design. In fact, the deep bowl-shaped beam design has a very high threshold velocity. The deep bowl-shaped beam design is not able to start self-sustained vibration at a wind velocity as high as 100 mm/s. For this reason, it is not worth further
investigation. The results show that, with a deeper concavity, the beam tip displacement decreases and the threshold pressure increases. This result indicates that the rectangular beam design is better than the bowl-shaped beam design.

### 4.4 Thick Beam Tip Design

![Figure 4.23: (a) 2-D views of a beam with a thick tip (tip thickness =3mm), (b) 2-D views of a beam with a thick tip (tip thickness= 0.8mm)](image)

In this analysis, the 3 mm thick tip design is defined with \( w = 16 \text{ mm}, \ t_1 = 0.3 \text{ mm}, \ t_2 = 2.7 \text{ mm}, \ l_1 = 38 \text{ mm}, \ l_2 = 20 \text{ mm} \). The 0.8 mm thick tip design is defined with \( w = 16 \text{ mm}, \ t_1 = 0.3 \text{ mm}, \ t_2 = 0.5 \text{ mm}, \ l_1 = 38 \text{ mm}, \ l_2 = 20 \text{ mm} \). 2-D views of the two designs are shown in Figure 4.23. The meaning of the symbols are described in Section 2.4. All the other parameters have the same value as the corresponding parameters in the rectangular beam reduced model. Obtained from the theoretical model, the 3 mm thick tip displacement is shown in Figure 4.24 and the corresponding average fluid pressure is shown in Figure 4.25. In comparison, the 0.8 mm thick tip displacement is shown in Figure 4.26 and the corresponding average fluid pressure is shown in Figure 4.27.
In the 3-D FEA of the designs, the design parameters are the same as those in the theoretical models. Obtained from the 3-D simulation, the 3 mm thick-tip beam tip displacement is shown in Figure 4.28 and the corresponding fluid pressure at an interior
point 50 mm upstream from the beam is shown in Figure 4.29. Similarly, the thin block tip displacement is shown in Figure 4.30 and the corresponding fluid pressure at the point is shown in Figure 4.31. The threshold velocity of the 3 mm thick-tip design is 78 mm/s. The
0.8 mm thick-tip design has a threshold velocity 72 mm/s. The vibration frequency of the 3 mm thick-tip design is 30 Hz and the vibration frequency of the 0.8 mm thick-tip design is 58 Hz.

Figure 4.28: 3mm thick-tip beam tip displacement from 3-D FEA

Figure 4.29: 3mm thick-tip beam design fluid pressure from 3-D FEA
From the results obtained from both the theoretical model and 3-D FEA, it is observed that the 3 mm thick-tip beam gives worse performance than the 0.3 mm thick-tip beam with a smaller tip displacement and a higher threshold pressure. The beam tip dis-
placement decreases with an increase of the tip thickness. This set of results show that the rectangle design is still the best geometric shape for the beam.

4.5 Inverted Trapezoidal Beam Results

In this section, an inverted trapezoidal beam is also modeled and analyzed utilizing the FEA. The lower part of the inverted trapezoidal beam is a 16 mm × 16 mm square and the upper part is an inverted trapezoid with with the dimensions shown in Figure 4.33. The calculated tip displacement of the inverted trapezoidal beam is shown in Figure 4.33. The corresponding fluid pressure at an interior point which is 50 mm upstream from the beam is shown in Figure 4.34. The threshold velocity of the inverted trapezoidal beam design is 46 mm/s.

The results show that the threshold velocity of the inverted trapezoidal structure is lower than the threshold velocity of the rectangular beam structure which is 53 mm/s. However, the maximum tip displacement of the inverted trapezoidal beam design 2.38 mm which is smaller than the rectangular beam tip displacement 2.41 mm.
Figure 4.33: Inverted trapezoidal beam tip displacement obtained from FEA

Figure 4.34: Inverted trapezoidal beam design fluid pressure calculated from FEA

4.6 Effect of Beam Mounted Position

In the theoretical models, $x_0$ is defined as the static opening of the beam without pressure load. It represents the distance between the beam mounted position and the air pipe outlet wall. To analyze its effect on $x$, we continuously change the beam position. When the beam is inside the pipe, $x_0$ has a negative value. When the beam is outside of the pipe, $x_0$ is positive. Keeping all the other parameters unchanged from section 4.1 and using $maxx$ donating the maximum beam tip displacement as a function of $x_0$, the $x_0 - maxx$ curve is obtained by using both the theoretical model and the FEA. The result obtained from the theoretical model is shown in Figure 4.35, and the result produced by the 3-D
FEA is shown in Figure 4.36. Next, using $\text{thresholdv}$ to donate the threshold velocity as a function of $x_0$, the $x_0 - \text{thresholdv}$ curves obtained from the theoretical model and the 3-D FEA are shown in Figure 4.37 and Figure 4.38, respectively.

![Figure 4.35: Beam position effect on tip displacement from the theoretical model](image1)

![Figure 4.36: Beam position effect on tip displacement from the 3-D FEA](image2)
Both the $x_0 - \text{maxx}$ curves generated by the theoretical model and the 3-D simulations show that there exists an optimal mounting position for the beam tip displacement. The $x_0 - \text{thresholdv}$ curves show the optimal position for the threshold velocity. Although
the beam tip displacement and the threshold velocity have different optimal mounting positions for the same beam, they are not far from each other. Depending on the applications and design criteria, the rectangular beam can be mounted at a desired place between the two optimal positions. The obvious advantage of this optimization method is its simplicity.

4.7 Effect of Pipe Outlet Wall Thickness

In the rectangular beam design, the pipe outlet wall is modeled as a rigid plate with the same thickness of the beam. In this section, we study its effect on $x$. The outlet wall is thickened inward gradually while its outer surface is kept level with the outer surface of the beam. The beam is set to have its original thickness 0.3 mm and mounted position $x_0 = 0$ mm. All the other parameters remain the same. Let $\text{wallthickness}$ donate the thickness of the outlet wall. Then the maximum beam tip displacement, $\text{maxx}$, is a function of $\text{wallthickness}$. Utilizing the 3-D FEA, the $\text{wallthickness} – \text{maxx}$ curve is obtained and shown in Figure 4.39. We use $\text{thresholdv}$ to donate the threshold velocity which is a function of $\text{wallthickness}$. Obtained from the 3-D FEA, the $\text{wallthickness} – \text{thresholdv}$ curve is shown in Figure 4.40.

Thicker pipe outlet wall moves the abrupt transition in the cross section area of the air pipe away from its outlet. The $\text{wallthickness} – \text{maxx}$ curve reveals that a smoother outlet cross-sectional area transition can give a larger beam tip displacement. The results also demonstrate that the threshold velocity becomes lower with a thicker wall, but will increase again if the thickness of the wall continue to increase.
Figure 4.39: Outlet wall thickness effect on tip displacement (from 3-D FEA)

Figure 4.40: Outlet wall thickness effect on threshold velocity (from 3-D FEA)
Chapter 5

Conclusion

In this study, theoretical modeling and finite element analysis of fluid-structure interaction in a wind energy harvester are performed. Reduced order theoretical models and 3-D finite element models are developed for the design analysis of the device. Effects of a set of structural design parameters on the self-sustained vibration of the beam are investigated. It is found that the rectangular cantilever beam is the best beam shape among the beam geometries tested in this work. By using the finite element simulations, the effects of various structure parameters predicted by the theoretical models are verified. Important factors that could be incorporated to improve the reduced order models are identified. The results show that there exist optimal mount positions for maximizing the beam tip displacement and minimizing the threshold velocity. In addition, a smoother outlet cross-sectional area transition can give a larger beam tip displacement.
References

10. L.J. Clancy, Aerodynamics, Section 3.11.


