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MODELING AND DESIGN STRATEGY OF ONLINE ADVERTISING ECOSYSTEM

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MODELING AND DESIGN STRATEGY OF ONLINE ADVERTISING ECOSYSTEM

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
Qiuchen Wang
August 2014

Accepted by:
Dr. Yue (Sophie) Wang, Committee Chair
Dr. John Wagner
Dr. Haiying Shen
Abstract

The aim of this thesis is to develop an analytical framework to model a large-scale complex online advertising network. Due to the large increasing annual revenues of global online advertising industry in recent years, agents’ behaviors in online advertising networks have recently received attention as an important area of research. For this reason we decided to purpose a framework, with the aim of identifying and investigating the agents’ (users, advertisers) interaction and influence in the online advertising ecosystem.

This thesis first introduces the background and structure of online advertising ecosystem. In a detailed central section the replicator-mutator (RM) dynamics of users’ and advertisers’ behaviors are proposed. Three adaptions to users’ RM dynamics for testing the users’ behaviors are undertaken. Finally, the framework is applied to a real online advertising ecosystem, within a three-stage testing phase. At the end of each stage the RM dynamics are fine-tuned, and experiment results are charted every stage.

The results of the experiment show that the analytical framework is well suited to revealing the agents’ interaction and their allocating strategies. Specific analyses of two advertisers are highlighted and two examples about targeting users are also explained. We recommend further modification to the framework is to develop a numerical analysis model that integrates advertisers’ critical stage.
Dedication

I would like to dedicate this work to my parents for their constant love and support, and to my friends for their unwavering inspiration and encouragement.
Acknowledgments

First and foremost, I would like to thank my advisor, Dr. Yue (Sophie) Wang, for giving me the opportunities to learn systems controls and perform related experiments. Dr. Wang gave me much help and advices on research. Her knowledge, guidance, and advice helped me in more ways than words can describe. I am greatly indebted to my advisor for all of her support, and hope to keep in touch with her for years to come. I would like to thank my advisory committee members Dr. Helen Shen and Dr. John Wagner for their technical expertise and advice.

I also want to thank my lab mates in the Interdisciplinary Intelligence Research (I^2R) Laboratory, namely, Hamed Saeidi, Behzad Sadrafaridpour, David Adam Spencer. They gave me strong support in conducting experiments and simulations, as well as their generous help. I’d like to thank peers in Pervasive Communications Laboratory of electrical and computer engineering department for helping us with data crawling. Thank you for bringing fun and laugh to our laboratory life.
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Nomenclature

$u$ — Index of User

$x^u(t)$ — States of User

$\rho_{uv}$ — Strength of Links of User

$A^u(t)$ — Preference Matrix of User

$a^u_{ij}$ — Preference Component of User

$f_i(t)$ — Fitness (of agent $i$) of User

$\phi(t)$ — Average Fitness of User

$Q^u(t)$ — Mutation Matrix of User

$q^u_{ij}$ — Mutation Component of User

$\lambda_{uv}$ — Hierarchy Parameter of User

$\alpha^u(t)$ — Total Capacity of User

$W^u(t)$ — Weighted Rewards Matrix of User

$v$ — Index of Advertiser

$y^v(t)$ — States of Advertiser

$\xi_{vl}$ — Strength of Links of Advertiser

$\bar{A}^v(t)$ — Preference Matrix of Advertiser

$\bar{a}^v_{ij}$ — Preference Component of Advertiser

$h^v_i(t)$ — Fitness (of Agent $i$) of Advertiser

$\theta(t)$ — Average Fitness of Advertiser
\( \mathbf{P}^v(t) \) — Mutation Matrix of Advertiser

\( p^v_{ij} \) — Mutation Component of Advertiser

\( \gamma^v_{l} \) — Hierarchy Parameter of Advertiser

\( \beta^v(t) \) — Total Capacity of Advertiser

\( \mathbf{M}^v(t) \) — Weighted Rewards Matrix of Advertiser
Chapter 1

Introduction

1.1 Background of Online Advertisement

Internet has undoubtedly become a major and effective medium for advertising, and is likely to soon replace traditional advertising mediums such as television, newspapers and magazines [10]. In every year since 2005, the annual growth rates of online advertising have exceeded those of other advertising media. Internet advertising has experienced double-digit annual growth in every year except 2009; no other media has experienced this growth in any year (please see Figure 1.1). Revenues in online advertising industry in the U.S. hit landmark numbers at $9.6 billion during the first quarter of 2013, which is equal to a 15.6% increase from the $8.3 billion reported during the first quarter of 2012 [24]. Annual revenues in the global online advertising industry are forecasted to reach an estimated $139.8 billion in 2018 with a Compound Annual Growth Rate (CAGR) of 7.3% during the 2013-2018 period [30]. The inclination of users and advertisers towards the online advertising systems stems from several facts. Firstly, users increasingly prefer to use an online convenient purchase system rather than traveling to the real stores. Moreover, they can access to valuable information such as full details of product as well as previous
users’ feedback via ratings and reviews which is not accessible via traditional shopping methods. On the other hand, advertisers can effectively expand their brand influence and credibility via the Internet’s tremendous popularity. They can also reduce the labor cost through Internet sales. Finally, based on users’ behavior, advertisers are able to target and to track potential users in order to evaluate the success of online advertising [23]. The latter is important since it helps advertisers, and also those who have advertising opportunities to sell, to target the users more effectively and efficiently through online or off-line feedbacks [35]. They are a broad range of fascinating problems in the areas of market place design for the mentioned interactions between the users and advertisers, trends/behavioral targeting and performance optimization. However, only few works have looked into the problem of the systematic modeling and analysis of such a network. Previous research in this area are basically, i) targeting users, ii) bidding auctions. For example, to evaluate tar-
targeting effectiveness of a targeted advertising campaign by using click through rate (CTR) and to run a field experiment in the Yahoo! front page, a difference-in-difference estimator is introduced in [16]. Based on two important characteristics of advertising, targeting and noisy information content, the work in [13] builds a signaling model to distribute advertisements to differentiated users by using targeting as a signal on product attributes. Bidding auctions have also drawn much attention in recent years. In [39], the authors propose a niche-based co-evolutionary simulation approach, aiming at computationally evaluating Sponsored Search Advertising (SSA) auction mechanisms based on advertisers’ equilibrium bidding behaviors. The paper [35] models budget-constrained keywords bidding in sponsored search auctions and designs an algorithm to solve the corresponding bidding optimization problem.

The existing literature provides valuable insights in terms of the targeting and bidding, there are almost no results concerning: the interaction of agents’ as well as important model parameters. Although [29] gives a definition of competitiveness between two items and evaluate it by finding the top-k competitors of a given item. Feedback controllers in [27] learns from its mistakes and takes proper reactive and proactive actions to meet advertisers’ goals. As a result, these models lack predictive power about how structures emerge as the result of the interaction of individual rational agents. As an attempt to address the problem, this thesis proposes a framework for modeling users and advertisers. We derive this framework from replicator-mutator dynamics by adding agents’ interaction with each other in online advertising ecosystem. While many important questions can be addressed using this framework, here we only focus on the short-term interaction and agents’ strategy design, which serve as a first step to more elaborate analysis in the future. We then introduce the online advertising ecosystem where our framework will be applied to. Next part online advertising ecosystem and replicator-mutator dynamics will be introduced. Online advertising ecosystem is the place where our framework will be applied to
by using replicator-mutator dynamics to know the interaction of agents inside.

1.2 Online Advertising Ecosystem

In the following sections, structure of online advertising ecosystem and major agents are highlighted. Typical advertisement types and pricing models used by online advertising ecosystem are also introduced.

1.2.1 Ecosystem Structure

In this section, we introduce the structure of the online advertising ecosystem and its key agents. A definition of the online advertising ecosystem is proposed here to describe a community of all the agents in conjunction with the relative components of their environment (i.e., language and web page platforms). Generally, we can divide the agents into four types: users, advertisers, publishers and ad exchanges. Agents in the ecosystem interact with each other and evolve dynamically. For example, a user types the keywords in the publisher’s search engine and does the comparison between advertisers’ products. The publisher distributes advertisements to specific users based on users’ browsing histories and bidding results of different advertisers. The advertisers bid on locations in the publisher’s website for product placement and brand promotion. Before proceeding to the methodology, it is necessary to clarify some concepts and classify all the agents that comprise the online advertising ecosystem. We present the definition of the general view of the online advertising system here using the ecosystem structure shown in Figure 1.2. The characteristics of the four main agents in online advertising ecosystem are introduced in the following part.

i) **User**: A user is the person who performs searching through search engines, browses
the web and consumes media content. Users may or may not acknowledge, view or engage with advertisement in the process of performing these actions. While accessing the online advertising, users will quickly disregard irrelevant information and focus on searching other more relevant information [33]. Meanwhile, the interactive nature of the Internet allows users to opt out of engaging with advertising altogether. Users are separate centrals to online advertising ecosystem. Users are attracted and enticed by advertisers to click and purchase the products on advertisements. The role that users play in the whole system is unique and interconnected.

ii) **Advertiser:** Through advertisements, advertisers aim to trigger potential users’ demand for a product or service. The advertisements can also be used to improve advertisers’ brand awareness by displaying advertisements. Depending on the budget and investment environment, an advertiser can have its advertisements displayed on as many websites as desired. Advertisers include various industries, e.g., New Balance in sports shoes, iPhone in electronics and Estee Lauder in cosmetics. The most critical
tasks for advertisers in the online advertising ecosystem could be keywords selection and bid optimization.

iii) Publisher: What realizes the interconnection of advertisers and users is the publisher. Publishers reserve and sell impressions (spaces of their webpage) for branding or contextual advertisement in the form of contracts or the real-time bidding. A challenge for publishers is to select the optimal contract or estimate the optimal price for advertisement space. The publisher is responsible for delivering a total number of impressions, i.e., webpages, based on what was agreed in the contract. The ultimate challenge for publishers is to maximize their revenue. The most famous publishers are Google, eBay and Amazon. For example, AdWords, created by Google [6]. The AdWords benefits the advertisers by letting them to pay the price of the next highest bid instead of paying for the bid offered. It also provides the advertisers with “quality score” of different users, so the advertisers can improve their bid outcomes. On the other hand, AdWords benefits the users by providing with advertisements that are relevant to their needs. Thus, it significantly saves the users’ time and eases the process of finding appropriate product or information for them [38].

iv) Ad Exchanges: At the beginning phase of online advertising, advertisers can buy impressions directly from publishers. However, as time passes, the numbers of both advertisers and publishers vastly increase. This poses a need for an entity that can connect and serve them. Ad exchanges are places like the stock exchanges. It balances the demands between advertisers and publishers. Ad exchanges also keep price of advertisement between advertisers and publishers stable. Currently, Bing Ads by Bing, Right Media by Yahoo! and DoubleClick by Google are the three giant groups of Ad exchanges.
As shown in Figure 1.2, users spend time reading advertisements and purchase products from the advertisers. Advertisers estimate budgets to buy advertisement impressions from Ad exchanges and publishers. Publishers sell advertisement impressions to advertisers and Ad exchanges, also serve users. Ad exchanges serve as distributors or matchers for advertisements and impressions.

### 1.2.2 Types of Online Advertisement

After knowing the basic agents, we next introduce different categories of online advertisement. Generally, we can categorize online advertisement into three types as follow:

i) **Sponsored search advertisements:** Sponsored searches, allow advertisers to buy certain keywords that exactly or broadly match queries submitted by users to promote their business (please see Figure 1.3). Sponsored search engines act as publishers by displaying advertisements alongside search results.

![Sponsored search advertisement in response to the keyword search “rental car”](image)

(a) Top advertisement  
(b) Side bar advertisement

Figure 1.3: Sponsored search advertisement in response to the keyword search “rental car”

ii) **Branding advertisements:** In this type of online advertisement, advertisers buy “impressions” from publishers and have their advertisements displayed to all visitors
iii) **Contextual advertisements:** Instead of displaying the same advertisement to every user, this category shows different advertisements based on the geography, device, language and other characteristics of users to maximize the utilization of advertising opportunities. Figure 1.5 shows the universities’ advertisements with geography influence.
1.2.3 Types of Pricing Model

Based on trade-impression relationship between publishers and advertisers, all the advertisement categories listed above contain different pricing models. Three typical models are as follow:

i) **Cost-per-mille (CPM).** Mille, which is Latin for “thousand”, refers to the price based on 1,000 impressions. It is also known as cost-per-impression. Almost all publishers prefer to bill on impressions because it is impression based, rather than performance based. In other words, publishers risk nothing on advertising with impression based CPM system and get paid for every impression; this is also the pricing standard for the largest and most well-known publishers such as Casale Media and Conversant [4]. In terms of overall cost, CPM priced media costs the advertisers the least among all the pricing models.

ii) **Cost-per-click (CPC).** The cost-per-click (CPC) is the amount that advertisers pay each time a user clicks on their advertisements. The CPC for any advertisement is determined by the advertiser. Some advertisers may be willing to pay more per click than others, depending on what they are advertising [3]. The basic formula for CPC is [17]:

\[
\text{Cost-per-click (\$)} = \frac{\text{Advertising cost (\$)}}{\text{Clicked numbers}}
\]

It is difficult for publishers to plan an impression demand for a target that uses multiple publishers concurrently based on CPC pricing model. The most aversive characteristic of this pricing model is that click-through-rate on an advertisement cannot be determined or tested by publishers beforehand. As we know, the CPC price increases when click-through-rate decreases. Two advertisers with the same CPC rate might require vastly different levels of impressions for the publisher to bill in full, and this uncer-
tainty results in a high risk scenario for the publisher. For example, Google’s AdSense is the largest CPC based clearing house used in the industry which attracts thousands of advertisers. For the advertisers, CPC is a very low risk way to buy media because they only have to pay for performance, resulting in some level of confidence in their investment return.

iii) **Cost-per-acquisition (CPA).** Also known as cost per action, CPA is the best deal for all advertisers in terms of risk because they only pay for media when their advertisement results in a sale [2]. Using this pricing model, advertisers can choose the publisher that is most likely to sell their product and allocate a portion of their possible profit to cover the advertisement costs. Similar to CPC pricing, this is usually an awful deal for publishers. However, Affiliate Marketing Programs (AMP) [1] as one of the advertisement network supervisors requires the publishers to follow a CPA policy. This results in attracting publishers that are exclusively devoted to selling their products via websites that are more advertorial than anything else.

### 1.3 Replicator-Mutator Dynamics

Replicator-Mutator Dynamics belong to evolutionary game theory [36] [20]. Evolutionary game theory has proven itself to be invaluable in helping to explain many complex and challenging aspects of biology. Different from classical game theory, evolutionary game theory focuses more on the dynamics of strategy change as influenced not solely by the quality of the various competing strategies but also by the effect of the frequency with which those various competing strategies are found in the population. The replicator is the central actor in an evolutionary system [19]. The replicator here can be a gene, an organism (in the biology aspect), a belief, a technique, a strategy (in the social network aspect) or some more general form having some means of making approximately accurate copies of
itself. A replicator system is a set of replicators in a particular environmental setting with a structured pattern of interaction among agents. Meanwhile, evolutionary systems may generate novelty if random errors (“mutations” or “perturbations”) occur in the replication process, allowing new replicators to emerge and diffuse into the population if they are relatively well adapted to the replicator system. The mutation parameter $\mu$ reflects a trait of an individual. This trait describes how frequently the individual in question is willing to change its mind. We first introduce the individual choice model of replicator-mutator dynamics based on [28]. First, we assume that there are $n$ types, $T_1, T_2, \cdots, T_n$ and all these types undergo selection. The frequency of each type is expressed as $x_1, x_2, \cdots, x_n$. The reproduction rate of each type $T_i$, is determined by its fitness $f_i$. Fitness is a function of all frequencies which can be denoted by $f_i = F_i(x_1, x_2, \cdots, x_n)$. Let $Q_{ji}$ be the probability that type $T_i$ is produced by type $T_j$. Hence, $Q$ is a row-stochastic matrix given by

$$
\sum_{j=1}^{n} Q_{ij} = 1, \quad 1 < i \leq n \tag{1.1}
$$

If we take a polynomial expansion of fitness in terms of $x_i$ and only keep the linear terms, we obtain

$$
f_j = w_j + \sum_{i=1}^{n} a_{ji}x_i + \cdots \text{higher order terms}, \quad 1 \leq j \leq n. \tag{1.2}
$$

Here $a_{ji}$ are non-negative entries of an $n \times n$ matrix, and $w_j$ is the inhomogeneous part of fitness. The changes in the frequencies of types $T_1, T_2, \cdots, T_n$ in time can be described by the following model:

$$
\dot{x}_i = \sum_{j=1}^{n} f_j x_j Q_{ij} - x_i \phi, \tag{1.3}
$$

where

$$
\phi = \sum_{j=1}^{n} f_j x_j \tag{1.4}
$$
is the average fitness of the population. For the simplicity, the fitness only includes homo-
genous part. Then the general system (1.2) will be considered under the following mild assumptions:

\[ w_j = f_0, \quad 1 \leq i, j \leq n, \]  

(1.5)

and

\[ a_{ij} = a_{ji}, \quad 0 \leq a_{ij} \leq 1. \]  

(1.6)

Then social interaction of replicator-mutator dynamics is introduced based on [22] which transitions from individual choice. The social interaction is modeled as a undirected graph, \( G = (V, E) \), consisting of a set of vertices \( V \), and a set of edges \( E \). An edge \( e_{ij} \in E \) connects vertice \( v_i \in V \) with \( v_j \in V \). Following the above assumptions, Equation (1.3) can be written as

\[ \dot{x}^k = (Q^k)^T F^k x^k - \phi^k x^k \]  

(1.7)

where \( k \) is the number of vertices \( V \). The frequency of vertice \( x^k \) is a vector expressed as \( x^k = [x_1^k, x_2^k, \ldots, x_n^k] \). A row stochastic matrix \( W = [w_{ij}] \) is the weighted interaction matrix given by \( w_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}} \). Let \( \bar{f}_j \) be the social fitness parameter associated with individual \( v_j \). Note that we use a bar to differentiate social parameters from their individual counterparts. The parameter \( \bar{\phi}^k \) is introduced to guarantee that \( e^T x^k \) is unity.

\[ \bar{\phi}^k = \sum_{j=1}^{n} \bar{f}_j w_{kj} \]  

(1.8)

Therefore, \( \bar{\phi}^k \) is the weighted average social fitness of all individuals in the social network with respect to individual \( v^k \). Then the individual replicator-mutator dynamic model with
the effects of social interaction can be expressed as

\[ x^k = (Q^k)^T F^k x^k - \phi^k x^k + \sum_{j=1}^{n} \tilde{f}_{kj} x^j - \tilde{\phi}^k x^k, \quad (1.9) \]

After adding the mutator part, the frequency of replicator-mutator dynamics will not only be affected by each type’s reproduction, but also by perturbations occurring in the replication process. In our work, we adapt the replicator-mutator dynamics to model the online advertising ecosystem where the interactions between users and advertisers follow a similar pattern. New definitions of each parameter will be given.

1.4 Contribution

Currently, most research in the field of online advertising ecosystem focus mainly on advertisement bidding auctions and revenue optimization from the economic and mathematical viewpoints. Most of their works remain mostly ad hoc and provide no insight for advertisers’ smart investment. In other words, although they promise to model the online advertising system, they still lack a systematic analytical framework to model the large-scale complex network. In this thesis, we propose to take advantage of stochastic models and evolutionary models to fulfill this need. Stochastic ordinary differential equations (ODEs) have been wildly used to model financial market. Meanwhile, evolutionary dynamics have been successfully used to model the multi-agent networks. To integrate these two modeling methods in the online advertising ecosystem we utilize the Replicator-mutator (RM) dynamics. RM dynamics are a combination of tools from evolutionary dynamics of populations, complex networks, and control theory, which study the evolution of the behavior of agents in social networks [18] [32]. The RM dynamics capture agents’ behaviors as they are affected by two parts: the willingness of agents and the influence
from other agents. This is the first time the online advertising ecosystem is being defined with the combination of economic and evolutionary interacting dynamics aspects. These dynamic interactions in the social network can be used to model the dynamics of the online advertising system, design predictable performance and advertisement strategy for the online advertising ecosystem which is the main motivation of the proposed work. In this thesis, we first model a large-scale complex online advertising ecosystem using an analytical framework. We then perform the model fitting for the proposed model using crawled data from eBay. System-level performance analyses based on this analytical framework are provided. Profitable strategies are also proposed here corresponding to the analysis.

1.5 Structure of Thesis

The organization of this thesis is as follows: In Chapter 2, we first introduce users’ and advertisers’ RM dynamics. A toy problem including three different users’ dynamics is also highlighted in this part. In Chapter 3, the effects of major parameters in users’ and advertisers’ RM dynamics are simulated in Matlab. We then utilize crawled data of each parameter from online shopping websites and normalize them into RM dynamics. To gain a better idea of the online advertising ecosystem, we conduct a detailed analysis based on the crawled data in Chapter 4. Some online advertisement strategies are also proposed. Discussion and future works are provided in Chapter 5.
Chapter 2

Problem Formulation

In the existing works, replicator-mutator (RM) dynamics are used to model a social choice system where there exists a rich set of evolutionary phases including emergence of a single dominant behavioral trend, a few dominant behaviors, and lack of social norms [31]. As a preliminary study, and for the sake of simplicity, the online advertising ecosystem defined in this chapter is a community of all kinds of agents including users, advertisers, and an integrated publisher.

2.1 Problem Scenario

In our problem formation, we simplify the online advertising ecosystem introduced in the first chapter and illustrate the simplified ecosystem in Figure 2.1. In this ecosystem, we combine the ad exchanges and the publisher together, as one type of agent, which we still call ”publisher” for the sake of simplicity. The role of publisher will not only host the search engine website, but also organize the bidding results of all the advertisers. Some real-life examples of this type of publisher are Double Click of “Google” and Right Media of “Yahoo!”. They collect the information from enormous advertisers such as “Nike”, 

15
“Adidas” and distribute advertisement to some specific users who like sports clothes. In our model, the investment of each advertiser, i.e., how much they spend on the advertisement in the publisher’s webpages, depends on their market forecast and variable revenues. The bidding result of advertisers, i.e., the sequence of ranking places, is believed to have a direct impact on users’ purchasing choices. Each advertiser represents a product. Users initially have their own preferences between different advertisers. Normally the product or advertiser appearing in the top of the search results list can attract a higher level of attraction. Therefore, advertisers would like to compete for the higher ranking place so as to attract more users. In order to understand and analyze the trend of the online advertising ecosystem, we here use RM dynamics to model the interaction between advertisers and users. In this thesis, we take eBay as an example due to its unique mechanism. Publishers such as “eBay” have gained significant popularity recently. Users can find anything they want by entering keywords in the search engine. Automatic product listings are then provided for users after a few seconds. In the following study, we ignore products with “sale” marks and limited time bidding in eBay, focusing on only those items in the normal sale condition. To better understand the scenario, let us consider a simple example. Assume
that the search engine provides users with the results of the keyword “camera”. Consider
n users. A user denoted as \( r_u, u \in \{1, 2, \cdots, n\} \) buying a camera from advertiser \( i \) is set as having an allocation percentage of money \( x_{ui} \). This element can also be viewed as the purchase probability. At the same time, users who have brand loyalty for some products can be modeled with a larger initial condition \( x_{ui}(0) \) in our ecosystem, such as “Nike” for sports clothes, and “Apple” for computers. However, even if a brand has a good reputation, other factors can influence the final decision. For example, users might change their minds for a lower price or higher quality of other similar matched products. Users can also make better informed decisions by reading product reviews. Meanwhile, advertisers bid for different advertisements’ ranking places in the publisher’s website. In the following parts, the original RM dynamics introduced in Section 1.4 are applied and extended to model the evolution of users and advertisers successively.

2.2 Evolutionary Dynamics for Users

We first list the main notations used in a user’s dynamics in Table 2.1. Let the users network form a weighted graph \( G_r = (V_r, E_r) \). A set of vertices \( V_r \) represent users \( r_u, u \in \{1, 2, \cdots, n\} \). An edge \( e_{ij} \in E_r \) connects \( r_i \) with \( r_j, i, j \in \{1, \cdots, n\} \). Similarly, we consider a relational graph \( G^u = (V^u, E^u) \) associating a user \( r_u \) with a set of advertisers. Assume each user is capable of searching for and comparing multiple advertisers simultaneously. Let \( \rho_{uv} \) be strength between user \( r_u \) and advertiser \( s_v \) with \( \rho_{uv} = 1 \) if they have historical transaction records and \( \rho_{uv} = 0 \) otherwise. The set of edges in graph \( G^u \) is given by

\[
E^u = \{(s_i, s_j) : a_{ij}^u(t)\}, \quad i, j \in \{1, 2, \cdots, N\}.
\]
Table 2.1: Notations used in user’s dynamics

<table>
<thead>
<tr>
<th>Notations</th>
<th>User</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers of Users</td>
<td>$n$</td>
</tr>
<tr>
<td>User Index</td>
<td>$r_u$</td>
</tr>
<tr>
<td>States of Percentage Allocation</td>
<td>$\mathbf{x}_u(t)$</td>
</tr>
<tr>
<td>Strength of Links</td>
<td>$\rho_{uv}$</td>
</tr>
<tr>
<td>Preference Matrix</td>
<td>$\mathbf{A}_u(t)$</td>
</tr>
<tr>
<td>Component of $\mathbf{A}_u(t)$</td>
<td>$a_{ij}$</td>
</tr>
<tr>
<td>Diagonal Fitness Matrix</td>
<td>$\mathbf{F}_u(t)$</td>
</tr>
<tr>
<td>Fitness (of Agent $i$)</td>
<td>$f_u(t)$</td>
</tr>
<tr>
<td>Average Fitness</td>
<td>$\phi_u(t)$</td>
</tr>
<tr>
<td>Mutation Matrix</td>
<td>$\mathbf{Q}_u(t)$</td>
</tr>
<tr>
<td>Mutation Component</td>
<td>$q_{ij}$</td>
</tr>
<tr>
<td>Hierarchy Parameter</td>
<td>$\lambda_{uv}$</td>
</tr>
<tr>
<td>Total Capacity</td>
<td>$\alpha_u(t)$</td>
</tr>
<tr>
<td>Preference Potential</td>
<td>$d_u(t)$</td>
</tr>
<tr>
<td>Weighted Preference Matrix</td>
<td>$\mathbf{W}_u(t)$</td>
</tr>
<tr>
<td>Mutation Parameter</td>
<td>$\mu_u$</td>
</tr>
</tbody>
</table>

Define user $r_u$’s *preference matrix* as

$$
\mathbf{A}_u(t) = [a_{ij}^u(t)], \quad i, j \in \{1, 2, \cdots, N\},
$$

where element $a_{ij}^u(t)$ represents the preference of user $r_u$ for advertiser $s_j$ over $s_i$ at time $t$.

Due to the effect of users’ own characteristics, local communications or relationships, users have varying levels of preferences when comparing between advertisers. The element $a_{ij}^u$ can also be viewed as the user’s prioritization of advertiser $s_j$ over $s_i$, and can be evaluated by the publisher that collects users’ click numbers (CPC) and purchase histories (CPA) [21].

Let

$$
\mathbf{x}_u(t) = [x_1^u(t), x_2^u(t), \cdots, x_N^u(t)]^T \in \mathbb{R}^N
$$

be a vector of allocation percentage of money of user $r_u$ to varying advertisers $s_v, v = 1, 2, \cdots, N$ at time $t$. In other words, $x_v^u$ represents the proportion of user $r_u$’s purchase
probability with respect to advertiser \( s_i \). The total money of each individual user equals to unity, i.e.,

\[ \sum_{i=1}^{N} x_i^u(t) = 1. \tag{2.4} \]

The proof is explained in detail in Appendix B. Under this model, each user’s choices of allocation percentage are used by the publisher to prioritize and target individual advertisements. In the matrix form, user \( r_u \)’s individual evolutionary replicator-mutator equation with effects from advertisers can be written as

\[
\dot{x}^u(t) = [Q^u(t)]^T F^u(t) x^u(t) - \phi^u(t) x^u(t) + \sum_{v=1}^{N} \lambda_{uv} \rho_{uv} y^v(t) - \sum_{v=1}^{N} \alpha^v(t) y^v(t). \tag{2.5}
\]

According to this model, user \( r_u \)’s allocation percentage of money vector \( x^u(t) \) updates according to its own evaluation of different advertisers as well as the ranking places of respective advertisers. Next, we explain each term in Equation 2.5 and their effects on the evolution of user’s purchasing probability in sequence. For user \( r_u \), define the fitness by switching to advertiser \( s_i \) from all the other advertisers at time \( t \) as

\[
f_i^u(t) \triangleq \sum_{j=1}^{n} a_{ij}^u(t) x_j^u(t), \quad i, j \in \{1, 2, \ldots, N\}.
\]

In the matrix form, the fitness vector associated with user \( r_u \) for all \( N \) advertisers at time \( t \) is given by

\[
f^u(t) \triangleq A^u(t) x^u(t) \in \mathbb{R}^N. \tag{2.6}
\]

with the \( F^u(t) \) defined as

\[
F^u(t) = \text{diag}[f^u(t)]. \tag{2.7}
\]
Define the average fitness $\phi^u(t)$ of user $r_u$'s allocation to all advertisers at time $t$ as

$$
\phi^u(t) \triangleq f^u(t)^T x^u(t) = \sum_{i=1}^{N} x^u_i(t) f^u_i(t).
$$

(2.8)

Let

$$
Q^u(t) = [q^u_{ij}]
$$

(2.9)

be the mutation matrix associated with user $r_u$, which is a row stochastic matrix satisfying

$$
\sum_{j=1}^{N} q^u_{ij}(t) = 1.
$$

(2.10)

The component $q^u_{ij}(t)$ is the likelihood that user $r_u$ reallocates its money from advertiser $s_i$ to advertiser $s_j$, $j \neq i$, at time $t$. The choices for $Q^u(t)$ are consistent with [37], and related to the preference matrix $A^u(t)$. Below, we briefly show a choice of $Q^u(t)$ based on [37].

Define the weighted preference matrix of $A^u$ as

$$
W^u(t) = [w^u_{ij}(t)],
$$

(2.11)

where $w^u_{ij}(t)$ is given by

$$
w^u_{ij}(t) = \frac{a^u_{ij}(t)}{d^u_i(t)}, \quad d^u_i(t) = \sum_{j=1}^{N} a^u_{ij}(t).
$$

where matrix $D^u(t) = \text{diag}[d^u_1, d^u_2, \cdots, d^u_n]$ satisfies equation $W^u(t) = (D^u(t))^{-1} A^u(t)$. The components of the mutation matrix can be explicitly stated as follows:

$$
q^u_{ij}(t) = \begin{cases} 
\mu^u w^u_{ij}(t) & i \neq j, \\
1 - \mu^u[1 - w^u_{ii}(t)] & i = j.
\end{cases}
$$
where \( \mu_u \geq 0 \) is the *mutation parameter* associated with user \( r_u \)'s individual tendency to change between advertisers. We also consider the effect of local interactions between users in the same neighborhood network. Therefore, let us associate a parameter \( \lambda_{uv} > 0 \) with each user \( r_u \) that describes the effects of unique hierarchy of the individual user. This parameter indicates how effective user \( r_u \) is in propagating his/her allocation percentage of money in different advertisers’ products to other users. The parameter \( \beta^v(t) \) is introduced here to guarantee that \( c^T y^v \) is unity. It is given by

\[
\alpha^v(t) = \frac{\sum_{v=1}^{N} \lambda_{uv} \rho_{uv}}{N} \quad (2.12)
\]

where \( \rho_{uv} \) is the strength link parameter which describes the strength of users’ influence on each other.

### 2.3 Evolutionary Dynamics for Advertisers

Similar as Table 2.1, Table 2.2 lists all the parameters used in advertisers’ RM dynamics. Consider an advertiser network that forms a weighted graph \( \tilde{G}_s = (\tilde{V}_s, \tilde{E}_s) \). A set of vertices \( \tilde{V}_s \) are advertisers \( s_v, v \in \{1, 2, \cdots, N\} \) with edges \( \tilde{e}_{ij} \in \tilde{E}_s \) connecting \( s_i \) with \( s_j \). We also consider a relational graph \( \tilde{G}^v = (\tilde{V}^v, \tilde{E}^v) \) associated with advertiser \( s_v \) with a set of nodes representing ranking places \( \mathcal{R}_i, i \in \{1, 2, \cdots, M\} \) in the publisher’s webpage. Assume that each advertiser is capable of bidding for multiple ranking places simultaneously. Define a *preference matrix* for the advertisers.

\[
\tilde{A}^v(t) = [\tilde{a}_{ij}^v(t)], \quad i, j \in \{1, 2, \cdots, M\}. \quad (2.13)
\]
Table 2.2: Notation used in advertisers’ dynamics

<table>
<thead>
<tr>
<th>Advertiser</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
</tr>
<tr>
<td>Index</td>
</tr>
<tr>
<td>States</td>
</tr>
<tr>
<td>Strength Of Links</td>
</tr>
<tr>
<td>Preference Matrix</td>
</tr>
<tr>
<td>Preference Component</td>
</tr>
<tr>
<td>Diagonal Fitness Matrix</td>
</tr>
<tr>
<td>Fitness (of Agent $i$)</td>
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<td>Mutation Matrix</td>
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<td>Mutation Component</td>
</tr>
<tr>
<td>Hierarchy Parameter</td>
</tr>
<tr>
<td>Total Capacity</td>
</tr>
<tr>
<td>Preference Potential</td>
</tr>
<tr>
<td>Weighted Preference Matrix</td>
</tr>
<tr>
<td>Mutation Parameter</td>
</tr>
</tbody>
</table>

and a set of edges in graph $\bar{G}^v$ given by

$$\bar{E}^v = \{ (R_i, R_j) : \bar{a}_{ij} \}.$$ (2.14)

The matrix element $\bar{a}_{ij}^v$ represents the preference advertiser $s_v$ has for ranking place $R_j$ over $R_i$. It can also be viewed as the prioritization of advertiser $R_j$ over $R_i$ evaluated by the publisher based on advertisers’ bidding price and brand image. Note that the matrix $\bar{A}^v(t)$ can be time-varying. Let

$$y^v(t) = [y_1^v(t), y_2^v(t), \ldots, y_M^v(t)]^T \in \mathbb{R}^M$$ (2.15)

be a vector of allocation percentage of advertiser $s_v$ to ranking places

$$R = \{R_1, R_2, \ldots, R_M\}$$ (2.16)
at time $t$. In the other words, $y^v_i$ represents the percentage of budget that advertiser $s_v$ allocates to ranking place $R_i$. Setting the total budget of advertiser $s_v$ equal to 1, we have $c^Ty^v(t) = 1$, where $c = (1, 1, \ldots, 1)^T \in \mathbb{R}^N$. The proof is explained in detail in Appendix A. For each advertiser $s_v$, define the preference fitness of switching to ranking places $R_i$ at time $t$ as

$$h^v_i(t) \equiv \sum_{j=1}^{M} \bar{a}^v_{ij}(t)y^v_j(t), \ i \in \{1, 2, \ldots, M\}.$$ 

In the matrix form, the fitness vector of advertiser $s_v$ associated with ranking places in $R$ at time $t$ can be given as

$$(h^v(t))^T = \bar{A}^v(t)y^v(t) \in \mathbb{R}^M. \quad (2.17)$$

Let

$$H^v(t) = \text{diag}[h^v(t)]. \quad (2.18)$$

Define

$$\theta^v(t) = h^v(t)^Ty^v(t) = \sum_{i=1}^{M} y^v_i(t)h^v_i(t) \quad (2.19)$$

as the average fitness of advertiser $s_v$ via allocation of all the ranking place $R_i, i \in \{1, 2, \ldots, M\}$.

Let

$$P^v(t) = [p^v_{ij}] \quad (2.20)$$

be the mutation matrix associated with advertiser $s_v$, which is a row stochastic matrix satisfying $\sum_{j=1}^{M} p^v_{ij}(t) = 1$. The component $p^v_{ij}(t)$ is the likelihood that advertiser $s_v$ reallocates its budgets from ranking place $R_j$ to advertiser $R_i$ ($j \neq i$) at time $t$. Similarly, we choose $P^v(t)$ using preference matrix $\bar{A}^v(t)$ [37]. Define the weighted preference matrix of $\bar{G}_s$ as

$$M^v(t) = [m^v_{ij}(t)], m^v_{ij}(t) = \frac{\bar{a}^v_{ij}(t)}{b^v_i(t)}. \quad (2.21)$$
At the same time, define the matrix

\[ \mathbf{B}_v(t) = \text{diag}(b_1^v(t), \ldots, b_N^v(t)) \]  

(2.22)

with element

\[ b_i^v(t) = \sum_{j=1}^{M} \bar{a}_{ij}^v(t). \]  

(2.23)

The mutation components can be explicitly stated as follows:

\[ p_{ij}^v(t) = \begin{cases} 
\bar{\mu}^v M_{ij}^v(t) & i \neq j, \\
1 - \bar{\mu}^v [1 - M_{ii}^v(t)] & i = j. 
\end{cases} \]  

(2.24)

where \( \bar{\mu}^v \geq 0 \) is the mutation parameter associated with advertiser \( s_v \)’s individual tendency to change between ranking places. Now we consider the effect of interaction between advertisers in the same online advertising ecosystem. Associate a parameter \( \gamma_{vl} > 0 \) with each advertiser \( s_v \) to describe the hierarchy and/or capability differences of the advertiser. The parameter \( \beta^v(t) \) is introduced here to guarantee that \( \mathbf{c}^T \mathbf{y}^v \) is unity. It is given by

\[ \beta^v(t) = \sum_{l=1}^{M} \gamma_{vl} \xi_{vl} \]  

(2.25)

where \( \xi_{vl} \) is the strength link parameter which describes the strength of advertisers’ influence on each other. It is given by

\[ \xi_{vl} = \frac{1}{|R_i - R_j|} \quad (i \neq j) \]  

(2.26)

Therefore, \( \beta^v(t) \) is the weighted average social fitness of all advertisers in the social network with respect to advertiser \( s_v \), where the weighting is given by the components of the
weighted interaction matrix $\textbf{M}^v_\text{v}(t)$. The dynamics of advertiser $s_\text{v}$ can be written as

$$
\dot{y}^v(t) = [\textbf{P}^v_\text{v}(t)]^T \textbf{H}^v_\text{v}(t) y^v(t) - \theta^v_\text{v}(t) y^v(t) + \sum_{l=1}^{M} \gamma^v_{vl} \xi^v_{vl} y^v(t) - \beta^v_\text{v}(t) y^v(t). \tag{2.27}
$$

Under this model, we assume all advertisers have the same budget. The advertiser’s choice of allocating how much percentage of its budget to bid on a certain ranking place is essentially based on the requirement of enterprise operation as well as the effects of other advertisers, i.e., the advertising market estimation and the actual competitiveness. In the matrix form, the advertisers’ individual-based evolutionary replicator-mutator equation can be written as equation (2.27). Here we can set the publisher as a leader which has the same dynamics as an advertiser; however, it can largely affect other advertisers’ bidding decisions. According to advertisers’ model, $y^v(t)$ updates according to their individual revenue as well as the effects of other advertisers (including publishers).

### 2.3.1 Ecosystem Performance

Let us consider an online ecosystem of 100 users choosing among 2 advertisers with random initial allocation $\textbf{x}^u(0)$ and preference matrix $\textbf{A}^u_{ij}, u \in \{1,2,\ldots,100\}$ and $i,j \in \{1,2\}$. This implies users have random preference about all advertisers before they choose and make a purchase. We only consider users’ dynamics here and assume that advertiser 1 stays at a better advertising ranking place in the website.
Figure 2.2: Allocation ratio of user $r_u, u = 1, \cdots, 100$, for advertiser $s_1$ and $s_2$ in the online advertising ecosystem. Users prefer advertiser 1 instead of advertiser 2 based on the following setting parameters: random initial condition state $x^1(0) \in [0, 1], \ x^2(0) \in [0, 1]$; preference element $a^{11}_{12} > a^{21}_{12}$ in users’ preference matrix $A^u$ with small different numerical values of mutation parameter.

Figure 2.2 shows the evolution of the state of allocation percentage of each user for advertiser $s_1$ and advertiser $s_2$. Each user in Figure 2.2 has different initial allocation to advertiser 1 and advertiser 2. A dominant allocation is achieved after 1.9 time units due to the choice of relatively small mutation parameters $q_{ij}$. It is obvious from the figure that $x^u_1 > x^u_2, u \in \{1, 2, \cdots, 100\}$ showing most of the users will spend much more time and money on advertiser 1, which is consistent with our assumption that upper or better ranking places will attract more users. The specific Matlab data here also verify $\sum_{i=1}^{N} x^u_i(t) = 1$, which the total money of individual user $r_u$ equal to 1.
2.4 Toy Problem

To give a clearer picture, we start with a toy example to see how the proposed dynamic equation (2.5) and (2.27) model the online advertising ecosystem. In this section, we will present three special cases to see how RM dynamics work between users and advertisers. We assume that all the advertisers have the same budget for bidding ranking places. Meanwhile, all the users have same amount of money for choosing different advertisers’ products. The prices and bidding ranking are based on advertisers’ own business strategies. Real market examples will also be given under each case.

2.4.1 Replicator Dynamics: Users’ Own Belief and Preferences

As explained above in the dynamics section, the belief of users or their own preference is modeled by the first two terms of replicator-mutator dynamics. Each user has its own judgment and preference to different advertisers. In this case, we will prove that the advertisers’ ranking places do not affect users’ choices. Following the above assumption, equation (2.5) reduces to

\[
\dot{x}^u(t) = [Q^u(t)]^T F^u(t)x^u(t) - \phi^u(t)x^u(t), \quad u = 1, \ldots, n
\]

means the evolution of \(x^u(t)\) is solely dependent on its own belief and preference, i.e., the term \(a_{ij}^u\) in the preference matrix \(A^u\). The users here can be represented for a group of people who have the same preference to a certain product or advertiser; they will achieve a consistent resource allocation \(x^u(t)\) for the product. The simulation results are shown in Figure 2.4.
Simulation Results

In this section, we simulate an ecosystem of 3 users and 3 advertisers. We assume that the sequence of ranking places are provided based on Figure 2.3.

![Graph of Advertisers' Rankings](image)

Figure 2.3: Users’ belief: Allocation ratio of advertisers with bidding results from 1st to 3rd ranking place: Advertiser 2, Advertiser 1, Advertiser 3 based only on users’ belief and preference.

Advertiser $s_2$ obtains the 1st ranking place $R_1$, advertiser $s_1$ obtains the 2nd ranking place $R_2$, and advertiser $s_3$ obtains the 3rd ranking place $R_3$. At the same time, we assume that users all have a preference to advertiser $s_1$ instead of $s_2$ and $s_3$. According to users’ dynamics that users’ allocation percentage only affect by users’ own beliefs and preferences, we can find the allocation of users to advertiser 1 is much larger than advertiser 2 and advertiser 3 in Figure 2.4. It is obvious that advertisers’ ranking places have no effects on users’ percentage allocation.
2.4.2 Mutator Dynamics: Advertisers’ Influence

We now study the contribution of interaction element $\rho_{uv}$ to the evolution of $x_u(t)$ under this scenario. For the sake of comparison, each user is assumed to have a same hierarchy parameter $\gamma_{uv}, u = 1, 2, \ldots, n$ to all advertisers $s_v, v = 1, 2, 3$. Therefore, no user in the network will have stronger influence than others. The preference element $a_{ij}^u$ for each user is also set to be the same, hence, each advertiser’s product has the same probability to be chosen by users. Following the assumption that the dynamics only include the advertisers’ effect, Equation (2.5) reduces to

$$\dot{x}_u(t) = \sum_{v=1}^{N} \lambda_{uv} \rho_{uv} y^v(t) - \sum_{v=1}^{N} \alpha_{ij}^u(t) y^v(t).$$

As is shown in Figure 2.6, the evolution of $x_u(t)$ is solely dependent on advertisers’ ranking places. Note that different ranking places of advertisers $s_1, s_2$ and $s_3$ shown in Figure 2.5.
Figure 2.5: Advertisers’ influence: Allocation ratio of advertisers with bidding result from 1st to 3rd ranking place: Advertiser 2, Advertiser 1, Advertiser 3.

Advertiser $s_2$ holds a better place than advertiser $s_1$ which means that advertiser $s_2$ can attract more users than advertiser $s_1$. From Figure 2.5 and Figure 2.6, we can find that the advertisers’ ranking places have a large effect on users’ choices. Users will naturally choose the advertiser who holds the best ranking place.

2.4.3 RM dynamics: Individual Preferences and Social Interactions

Combining users’ beliefs and preferences (first two terms in Equation (2.5)) with interaction with advertisers (last two terms in Equation (2.5)), the full RM dynamics show the trend of the percentage allocation of the users’ budgets. In this case, the sequence of advertisers’ ranking places is similar with previous two cases. In Figure 2.7, advertiser 2 is in the 1st ranking place, then advertiser 1 and advertiser 3. However, the users’ results in Figure 2.8 give us a full idea about RM dynamics. Firstly, we can find that users allo-
Figure 2.6: Advertisers’ influence: Allocation ratio of users who have no preference to any advertisers in the online advertising ecosystem.

Figure 2.7: RM dynamics: Allocation ratio of advertisers to the advertisement places.
cate most of their resources to advertiser 2 \( s_2 \) who holds the 1st ranking \( R_1 \). The results were concordant with our assumption. Secondly, users have various allocations to different advertisers this time instead of having the same allocation consistently due to interactions.
Crawl data, means extracting data from certain webpages. It is not easy to crawl data for online advertising networks. Firstly, the large amount of users in the network cannot simply be divided into several types in regards to geography, language, device and other characteristics of users. The reason why we want to do this, however, is because we want to chase the trend of a group of people to target potential users. What are their responses with regards to different products? Which kind of products will they be particularly interested in? Secondly, websites in the online advertising ecosystem have high security systems that protect all the users’ personal data as well as the websites’ operating information. As we know, a website’s operators protect all their users’ documents and information, including the purchase history and related products or advertisers by certain contracts.

Google Adwords, as a branch of the Google empire business, focuses on advertisement. It shares the users’ information from the Google data base. However, others do not have easy access to this data base. User who has a Google account is more likely to receive more advertisement or other information than others. For instance, after deciding to buy a new tablet online, a simple scene could be drawn as follows. You first open two or three online electronics websites, and type keywords into the search box. Then you consider
waiting for a couple of days for potential price drops after selecting and comparing products with each other. In the following days, advertisements of the tablet you have reviewed will begin to show up in different web pages with the format of small floating windows. You cannot avoid taking a peek at them consciously when browsing the main text. It acts as a reminder that you can easily click the window to go back the product’s page immediately and make a purchase. It not only promotes brand, but also stimulates market consumption. However, the interesting notation here is how the advertisers know what product the users want to buy and distribute the exact advertisement for them. Currently, a large part of the advertising industry is self-regulated and still follows the guidelines set out by the Interactive Advertising Bureau (IAB) [7].

Last but not least, another difficulty of collecting data is the hundreds of thousands of products and categories to select. Products that could be used in our work, however, should at least contain two characteristics. The first characteristic is the appropriate amount of sales. We cannot choose cameras like Canon D60, or Nikon 7000D whose price is relatively high for ordinary people to buy several times within a short time. The sales volume of these products do not have an obvious change in order to give us a price trend of the product and preference of users. Therefore, it is hard to collect data of purchasing behavior for such products. On contrary, we cannot choose products with a huge amount of sales volume, like iPhone cases, due to their indifference in brand influence. It is obvious that users seldom have preference for iPhone case’s brand. Thus, the other characteristic products should have is brand influence. The product’s brand should be well known with a great number of people being familiar with the brand and its products or, even being loyal fans.

After long observation and discussion, the category we decided to focus on is a new technological product — the tablet that has shown to have a suitable price range and sales volume with brands frame. We assume that advertisers are maximizing their profits with
respect to the information available to them. The rest of the sections will first introduce the basic effect of major parameters in RM dynamics with Matlab simulation results. Furthermore, the method of data crawling used is introduced, and detailed data and simulation results are listed.

3.1 Four Main Parameters in RM Dynamics

We introduce four major parameters, namely the hierarchy parameter, the preference matrix, the initial condition, and the mutation matrix in users’ and advertisers’ dynamics. Furthermore, by analyzing and discussing the related market applications, we can aggregate the fundamental problems and capture potential prospects.

3.1.1 Users’ Performance Under Different Task Priorities

In the following sections, we will consider users’ behaviors and effects under the environment of eBay. Each parameter in the users’ dynamics plays a unique role which formulates users’ performance in real online market as eBay. Figure 3.1 gives an example of ratings and comments of an online retailer. In the common sense, users prefer to purchase products which have large amount of positive comments and higher ratings. From Figure 3.1, we can easily find the most helpful favorable review in the certain category. There are some keywords in the product feature that make it easier for users to find the products they want. What’s more, the rating evaluation system is visualized; users can get a brief idea about the product’s rating at a glance. The reviews from previous users give a more detailed story about the product. Some of them would attract more users to make a purchase, others on the contrary.
3.1.1.1 Variation of Hierarchy Parameter

Let us first consider a system with various values of users’ hierarchy parameter $\lambda_{uv}$. The value of hierarchy parameter $\lambda_{uv}$ depends on the users’ influence on others for all the products that online advertisers provide. For example, users usually leave comments for the products after purchasing and using. Others will judge these comments whether to be helpful. The more users accept the comments, the larger value of hierarchy parameter users will have for the products. Users with a higher hierarchy parameter are easier to influence other users. These special users will have the ability of affecting more users, no matter positive or negative comments they leave.
Simulation Results

In this section, we simulate an ecosystem of 4 users choosing between 2 advertisers. The hierarchy parameter of 4 users can be presented as the rating star or comments that a user gives to the advertiser’s product. Higher rating star or positive comment would be more attractive and persuasive; they have more a powerful influence than others, very low rating will have powerful influence as well. We assume that user \( r_1 \) always chooses advertiser \( s_1 \). By only increasing hierarchy parameter of user \( r_1 \), \( \lambda_1 \) from 1 to 30, the percentage allocation of users \( r_2, r_3, \) and \( r_4 \) for advertiser \( s_1 \) will correspondingly increase.
The hierarchy parameter of user $r_2$, $r_3$ and $r_4$ have been chosen to satisfy $\lambda_2 + \lambda_3 + \lambda_4 \ll \lambda_1$. The user with the higher hierarchy can be regarded as the leader in the network with the others as followers. If user $\lambda_1$ tends to allocate more money to advertiser $s_1$, users $r_2$, $r_3$ and $r_4$’s choices follow that of user $r_1$ by increasing $x_{11}^2(t)$, $x_{11}^3(t)$, $x_{11}^4(t)$, which is consistent with our prediction. Note that the propagation of a user’s percentage ratio of money still depends on $\xi_{vl}$, its local connectedness with other users in the ecosystem. Figure 3.2 shows the final allocation of users’ money for advertisers in the online advertising ecosystem corresponding to changes in user $r_1$’s hierarchy parameter.

**Related Market Implication**

We assume that users have the same impression for products with the same initial condition. At the same time, users will not easily change their initial choice. Users with larger hierarchy parameter are identified as public figures which have greater influence. Other users will think about these figures’ choice and tend to make a similar decision. We donate user $r_1$ as a public figure. From Fig 3.2, the larger the gap between $\lambda_1$ and $\lambda_2 + \lambda_3 + \lambda_4$, the other users tend more to change their minds to follow $r_1$ and finally adopt $r_1$’s choice.

### 3.1.1.2 Variation of Preference Matrix

We now consider the effect of the preference matrix $A^u$ in the ecosystem of 4 users choosing between 2 advertisers. With the increasing value of element $a_{ij}$ in the preference matrices of users $r_u, u = 1, \ldots, 4$ to advertiser $s_1$, the percentage allocation of all users to advertiser 1 is expected to increase by the time.

**Simulation Results**

The same preference element in a specific matrix $A^u$ means that the price of these advertisers’ products is same. For example, if product $j$ has a lower price than $i$ or its
shipment fee is free, the preference element \( a_{ij}^u \) is larger in the dynamics. The preference element \( a_{ij}^u \) here can be understood as the users’ preference of one product \( j \) over \( i \). This is intuitive because a user will be happier to purchase the same product at a lower price.

![Graph showing final allocation ratio of each user](image)

Figure 3.3: Final allocation ratio of each user \( r_{iu}, u = 1, \ldots, 4 \) to advertiser \( s_1 \) over advertiser \( s_2 \) in the online advertising ecosystem in response to increasing preference elements \( a_{12}^1, a_{12}^2, a_{12}^3, a_{12}^4 \) from 5 to 40. All users have the same initial condition, hierarchy parameter and mutation parameter.

Figure 3.3 shows the simulation results. It is obvious from the figure that by increasing the preference element of all users to advertiser 1, the percentage of their money allocation will correspondingly increase. In other words, products with a lower price attract more users than the same products with a higher price. Another interesting result we
observe from Figure 3.3 is that by increasing $a_{1j}$, the value of the preference element corresponding to the preference of users to advertiser 1, the slope of the allocation evolution also increase. Users become more likely to choose advertiser 1 to make a purchase.

**Related Market Implication**

Assume that users have the same impression for products with the same initial condition. Meanwhile, they give the same ratings for all products so that choices will not be affected by other users. Users with larger element $a_{ij}$ in the preference matrix will choose the product $j$ with a lower price over $i$. The lower price, as we describe at the beginning of this section may be caused by the shipment fee exclusion, membership discount or coupon. Generally, user would like to purchase the product offered by the advertiser that offers lower prices.

**3.1.1.3 Variation of Initial Conditions**

We now consider the case that 4 users have different initial allocation conditions $x^i(0), i = 1, \ldots, 4$ towards choosing between 2 advertisers. Having the same initial condition $x^i(0)$ implies that users have the same impression or belief to advertiser $s_1$ and advertiser $s_2$ at first. That is to say, these users have the same probability to purchase from these two advertisers before they know the price and review rating history. Some of them could have a better belief on, or even be loyal to a certain brand. All these factors may affect their final decisions.

**Simulation Results**

As shown in Figure 3.4, by increasing the initial allocation ratio of all users to advertiser $s_1$, the allocation ratio to advertiser $s_1$ goes up accordingly. For example, user $r_1$ with initial value $x^1(0) = 0.2$ has the final allocation state $x_1^1 = 0.65$. If, user $r_1$ has initial
condition $x^1(0) = 0.6$, it will reach final allocation state $x^1 = 0.79$. The latter represents a loyal user who owns allegiance to the brand or has purchased from this advertiser before.

![Figure 3.4: Final allocation ratio of each user $r_u, u = 1, \cdots, 4$ for advertiser $s_1$ in the online advertising ecosystem with different initial conditions.](image)

**Related Market Implication**

Assume that users can search all products of the same price and same ratings. The user $r_u$ with larger initial condition $x^u(0)$ would more likely to purchase product $i$ by advertiser $s_i$ than other users. Under this situation, there is a larger probability for advertisers to sell products by targeting the specific users instead of common people.
3.1.1.4 Variation of Mutation Matrix

Here we consider the influence of the mutation parameter $\mu^u$ on users’ dynamics. This parameter implies how easily a user tends to change between different advertisers. With the value of mutation parameter lying in certain range, the conclusion — users with larger value of mutation parameter will be more prompt to change their decisions — can be made. That is to say, initial choice will be changed after a shorter comparison.

Simulation Results

In this section, we simulate an ecosystem of 4 users and 2 advertisers. By changing users’ preference matrix with given values of mutation parameter $\mu^u=1$ and $\mu^u=10$, we can gain a brief idea about the effects of the mutation parameter. By comparing Figure 3.5a and 3.5b, the slope, i.e., how rapidly the final effect of the user changes with respect to a change in preference, is greater when $\mu^u=10$. However, when preference element becomes large enough, around $20 \sim 25$, the changes of final state of $r_u, u = 1, \cdots, 4$ are very small. This observation is consistent with the previous assumption in the last section, i.e., a user who has strong willingness to buy a certain product with a low value of the mutation parameter, won’t change his/her mind easily.

Related Market Implication

We assume that if users have the same mutation parameter, they have the same tendency of changing minds between different advertisers. On contrary, the mutative speed of users who have the larger mutation parameter is faster. It is equal to say, users will change minds frequently between products of the similar quality and appearance. Under this condition, they have a lower probability to purchase certain products.
Figure 3.5: Final allocation ratio of each user $r_{u}, u = 1, \cdots, 4$ for advertiser $s_1$ in the online advertising ecosystem with changing users’ preference element of advertiser 1 from 3 to 30.
3.1.2 Advertisers’ Performance Under Different Task Priorities

In the following part, four major parameters of advertisers’ dynamics will be discussed and shown with Matlab simulation results.

3.1.2.1 Variation of Advertisers’ Hierarchy Parameter

In this section, we consider advertisers with different hierarchy parameter $\gamma_{vl}$. Having the same hierarchy parameter amongst the advertisers implies that all advertisers choose the same ranking place to display their advertisements. On the contrary, advertisers with different hierarchy parameters will choose the best matched ranking place. For example, the shoe brand Nike will be more likely to put advertisements either in Amazon or 6PM (6PM is an online sales website focusing on shoes) instead of Target because most of the users prefer buy “Nike” shoes in the specific stores. Advertisements will be distributed to each website with different estimated budgets. Advertisers will allocate resources independently by their own choosing to compete for each ranking place. The total resource percentage of an advertiser to ranking places should be 1.

3.1.2.2 Variation of Advertisers’ Preference Matrix

For advertisers who have the same advertising volume with a lower price or higher quality, they can attract more users and have a bigger preference element for the certain ranking places. Products are provided by different advertisers within various prices. Publisher will provide a better position for an advertiser offering higher price.

Simulation Results

In this section, we consider a case of 2 advertisers bidding between 2 ranking places with a changing value of the preference matrix of advertiser $s_1$ and advertiser $s_2$ to the 2nd
ranking place $R_2$.

![Graph showing final allocation ratio for advertisers $s_1$ and $s_2$ to 2nd ranking place](image)

Figure 3.6: Final allocation ratio for advertiser $s_1$ and $s_2$ to 2nd ranking place in the online advertising ecosystem with different preference matrices. Increasing the preference element of advertiser 1 towards 2nd ranking place over 1st ranking place from 1 to 15, i.e., $a_{12}^1 \in [1, 15]$.

From Figure 3.6, it is obvious that with the increasing value of the preference element to the second ranking place, the allocation of resources of advertiser 1 and advertiser 2 to the second ranking place $R_2$ increases smoothly and eventually overtakes allocation toward $R_1$.

**Related Market Implications**

We assume advertisers with different preference matrices will be treated differently by the publisher. The bigger preference element an advertiser has, the larger probability it will obtain the upper ranking place. In general, we assume that big enterprises which have
a long term relationship with an online retailer will have a bigger preference element.

### 3.1.2.3 Variation of Initial Condition

Each advertiser has its own budget for different ranking places of a publisher’s webpage. Having the same initial condition of advertisers $y^v(0)$ implies that each advertiser plans to distribute the same percentage to bid for those ranking places $R$ initially. However, the real bidding process will keep changing over time due to the effects of the other advertisers’ behavior.

### Simulation Results

In this section, we consider a case of 2 advertisers bidding between 2 ranking places with different initial condition $y^1(0)$ and $y^2(0)$. The final percentage allocations of advertiser $s_1$ and $s_2$ all converge to the same percentage of allocation regardless of the changes in initial condition (as seen in Figures 3.7 and 3.8). Note that state $y^v_1$ is the allocation of advertisers $s_v$ ($v = 1, 2$) to $R_1$ and state $y^v_2$ is the allocation of advertisers $s_v$ ($v = 1, 2$) to $R_2$. In Figure 3.7, the initial condition $y^1(0) = \begin{bmatrix} 0.1 & 0.9 \end{bmatrix}$, $y^2(0) = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}$, separately represent advertiser 1 and 2’s initial percentage allocation to the 1st and 2nd ranking places, respectively. In Figure 3.8, the initial conditions are set as $y^1(0) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$, $y^2(0) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$. In the simulation, we assume that both advertiser 1 and advertiser 2 have preference to the upper advertising place (1st ranking place). However, advertiser 1 has a larger preference element than advertiser 2. Based on Figure 3.8, the final percentage of advertiser 1 to 1st and 2nd ranking place are $y^1_1 = 0.21$ and $y^1_2 = 0.79$. At the same time, $y^2_1 = 0.605$ and $y^2_2 = 0.395$. Because $y^2_1 > y^1_1$, advertiser 2 will successfully obtain the upper location.
Figure 3.7: Allocation ratio for advertiser $s_1$ and $s_2$ in the online advertising ecosystem with initial condition $y^1(0) = [0.1 \ 0.9], y^2(0) = [0.9 \ 0.1]$.

**Related Market Implication**

In the real market, advertisers will collect information about different online retailers and make an initial choice for themselves before the real bidding. However, advertisers’ choices are affected by other reasons such as click numbers and purchase history of users or low prices to ranking places by publishers.
3.1.3 Variation of Mutation Parameter

The mutation parameter $\tilde{\mu}^v$ reflects how easily an advertiser tends to change between different ranking places. Advertisers with larger values of the mutation parameters will tend to change their budget distribution and advertisement places more rapidly.

Simulation Results

In this section, we consider a case of 2 advertisers bidding between 2 ranking places with different mutation parameters $\tilde{\mu}^1$ and $\tilde{\mu}^2$. We assume that both advertisers naturally choose the 1st ranking place. At the same time, we notice that when the mutation value is large enough, the percentage allocation towards the second ranking place increases at a slower rate which verifies the intuition that no advertiser will change its allocation of resources unboundedly. Advertiser $s_1$ has a slightly larger preference element of ranking.
place $R_2$ than advertiser $s_2$. That is to say, advertiser $s_1$ will grow more quickly and have a larger final allocation towards $R_2$ than advertiser $s_2$. From Figure 3.9, when the advertisers’ mutation parameters increase, we observe that the final states of both advertiser $s_1$ and advertiser $s_2$ to 2nd ranking place $R_2$ increases. The simulation result is also consistent with the real market situation.

![Figure 3.9: Final allocation ratio for advertiser $s_1$ and $s_2$ in the online advertising ecosystem with changing values of mutation parameter to second ranking place.](image)

**Related Market Implications**

In the real market, how often an advertiser will change his/her percentage allocation largely depends on the market information collected. Research about the whole market will
help advertisers choose a stable and profitable publisher and a suitable ranking place. In the general case, they should have a low mutation value. In that way, they will not change their minds frequently, which results in wasted resources.

3.2 Data Crawling Method

After knowing how major parameters affect the users’ and advertisers’ RM dynamics, there are three parameters for users and advertisers we need to extract from the real market. They are **hierarchy parameter**, **preference component** and **strength of links** of the user and the advertiser. As we discussed in the previous section, each parameter has different effects on the dynamics. Meanwhile, each parameter represents a unique meaning in the online advertising ecosystem. In this section, data crawled from the real online advertising market are adopted into our RM dynamics to verify our model.

3.2.1 User’s Data

- **Hierarchy Parameter** $\lambda_{uv}$. The hierarchy parameter reflects a user’s own influence. For this parameter, we crawl data that can reflect the relationship between users in eBay. Alongside, the rating star and comments from the users who have bought products, eBay also encourages users to give their opinion of each specific comment (see Figure 3.10). For each comment we can get the number of users who regard the comment helpful and useful. User comments with larger certifying numbers or more recognition will have a bigger hierarchy parameter value due to their larger impact on others.

- **Preference component** $a_{ij}^u$. The preference component reflects users’ attitude towards an advertiser. We collect users’ rating history from the selected advertisers.
Based on the rating system setting in eBay, some users leave comments and rating stars for the advertiser that they just bought products from in order to enhance their degree of integrity. Our calculation of the value of $a_{ij}^u$ (users of a certain advertiser) is mainly depending on these rating stars. User $r_u$ who leaves 4 stars and above will be assumed to have a larger preference value. 2 stars and below are set as negative feedback and correspond to the smaller preference value. 3 stars are treated as neutral feedback. Figure 3.11 is a real user’s profile in eBay. The number of positive, neutral and negative feedback ratings of this user can be found in the bottom left of Figure 3.11.

- **Strength of Links** $\rho_{uv}$. The strength of links represents the relationship between users and advertisers. The value depends on whether a user has previously bought a certain product from an advertiser. We assume that if user $r_u$ has purchased a product from this advertiser $s_v$ before, then in the definition of graph theory for multi-agent social network [15], the user $r_u$ and advertiser $s_v$ connect with each other in the
Figure 3.11: Numbers of feedback rating given by users to the certain advertiser following way:

\[
\rho_{uv} = \begin{cases} 
1 & \text{if user } r_u \text{ purchased product from advertiser } s_v, \\
0 & \text{otherwise.}
\end{cases}
\] (3.1)

### 3.2.2 Advertisers’ Data

- **Hierarchy parameter** $\gamma_{vl}$. Websites like Forbes [5] and Toptenreviews [12] give us the overall scenario about brand ranking in each category. We give a value between zero and one to each advertiser approximately based on those rankings as a representation of their influence. For example, a ranking of the world’s top technology brands reveals that Apple is the most valuable name in its respective category. The growth and profit generated by Apple’s intangible assets, such as its globally recognizable logo, is nearly double than other major technology brands. Figure 3.12 shows an example of the 2014 tablet brand ranking from Toptenview’s website.
Figure 3.12: 2014 Tablet Ranking and Comparison

- **Preference Component** $a_{ij}^p$. This parameter reflects advertisers’ preference to different locations or ranking places. We collect numbers of products sold and their ranking place changes in a period of time to measure this component. The sale volume of this product increases simultaneously when the position or ranking place rises. Otherwise, they both drop. The frequency of place changing of each advertiser corresponds to changes in sales volume. The following figures show the relationship between sales volume and ranking places from the data we have crawled. Figure 3.13 shows the changes of ranking place versus time. Figure 3.14 shows the variation of sales volume against the same time period.
Figure 3.13: Change of ranking places from Feb 28th to March 18th

Figure 3.14: Change of sales volume from Feb 28th to March 18th

Figure 3.15 provides a general comparison of four advertisers, namely Apple, Kindle, Samsung, and Asus. From Figure 3.13, the evolutionary state of four brands or
advertisers can be divided into four stages with each stage described in detail for the four advertisers below.

◊ **Apple.** The Apple tablet has the lowest ranking place during the first stage from Feb 28th to Mar 4th. However, its gap with others is quickly decreasing by the end of this stage. Combined with the Figures 3.13 and 3.14, it is obvious to see that while the ranking is rising, the sale volume is increasing. During the second stage, Apple obtains the first ranking place by changing the strategy used in the first stage. The increase in sales volume is steady and smooth. The battle between all these advertisers reaches the heat of the moment. Although the final ranking place of Apple is the third one, its sales volume is the largest.

◊ **Kindle.** Kindle has the smallest frequency of change and keeps its ranking place in a suitable range during the whole time period. In the beginning, Kindle makes itself in the second ranking place. At the beginning of the second stage, Kindle falls to the 3rd ranking place. The reason could be from two parts, strategy and competition. Kindle does not change its bidding strategy when other competitors start to adjust their strategies for a better sales volume. At the middle of the second stage, after realizing that its place is under the strong threat, Kindle still holds the same strategy and let itself fall down to the last ranking place.

◊ **Samsung.** Samsung holds the highest ranking place during the first stage, and correspondingly gains largest initial sales volume. It then falls to the fourth ranking place during the second stage. From the perspective of Samsung, reasons for these changes might be loss of profit or competition from others. At the end of the second stage, Samsung tries regain back to the top ranking place. By doing so, its sales volume relatively increased.
- **Asus.** The frequency of Asus changes frequently during the time period. Being at the third ranking place, Asus always wants to gain a higher place to increase its sales volume. After a hard struggle with the other advertisers, Asus keeps its third ranking place. Additionally, Figure 3.14 shows the distinct increasing volume of Apple sales, which corresponds to the rising ranking place. There are more interesting results that can be found from Figure 3.15. The plot in the left bottom tells the same story between sales volumes and ranking places of Apple. The sales volumes of other three brands are in the range of 0 to 150.

![Graphs showing sales volume vs ranking place](image)

Figure 3.15: Relationship between sales volume and ranking place changes from Feb 28th to March 18th.

- **Strength of Links $\xi_{vl}$.** The strength of links $\xi_{vl}$ represents the connectedness between different advertisers. The advertisers bid with each other for a suitable loca-
tions or ranking places in the webpage to promote brand or to sell products. Therefore, their locations or ranking places will change with time due to sales volume and other reasons. We therefore assume that the difference between two advertisers’ ranking places is the advertisers’ strength link parameter according to the following equation.

\[ \xi_{vl} = \frac{1}{R_j - R_i} \quad (j \neq i). \]  

From the above equation it can be seen that the larger the difference between two advertisers’ place, the smaller the link parameter they have. Table 3.1 gives us the four advertisers’ ranking places changes from February 28th to March 18th. The difference of these ranking places goes up and down in these days. From Asus’s point of view, we observe some interesting values. The largest difference between Asus and Apple happened on Feb 28th. During the week of March 11th to 16th, there is only one difference in the ranking place between Asus and Apple. This decrease in difference and ranking place results in an increase in the link parameter between Asus and Apple according to Equation (3.2), denting an increase in influence of the two advertisers to each other.

3.3 Simulation Results

Let us assume that the budget allocation runs for one cycle from the 1st stage to the 3rd stage. In this section, we now perform a set of Matlab simulations of an example online advertising ecosystem of 160 users and 4 advertisers using the data crawled in Section 3.2 under RM dynamics based on the definition of all the parameters we mentioned in Section 3.2.2. The evolution of users’ and advertisers’ evolutionary percentage allocation is shown for each stage. Further discussion and analysis are also given.
Table 3.1: Ranking places for each advertiser during time period Feb 28th-March 18th.

<table>
<thead>
<tr>
<th>Date</th>
<th>$R_i$</th>
<th>Asus</th>
<th>Samsung</th>
<th>Apple</th>
<th>Kindle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb 28th</td>
<td>5</td>
<td>1</td>
<td>34</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Mar 1st</td>
<td>10</td>
<td>1</td>
<td>39</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Mar 2nd</td>
<td>9</td>
<td>1</td>
<td>36</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Mar 3rd</td>
<td>5</td>
<td>2</td>
<td>22</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Mar 4th</td>
<td>7</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Mar 5th</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Mar 6th</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Mar 7th</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Mar 8th</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Mar 9th</td>
<td>5</td>
<td>7</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Mar 10th</td>
<td>7</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Mar 11th</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Mar 12th</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Mar 13th</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Mar 14th</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Mar 15th</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Mar 16th</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
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</tr>
<tr>
<td>Mar 17th</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Mar 18th</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

3.3.1 The First Stage

Let us assume the first stage is the initial stage for all the advertisers. The sequence of ranking places is bid by advertisers with limited information about their competitors, i.e., other advertisers, and advertisers obtain their initial ranking places after a bidding auction. Apple has been ranked as the lowest with 34th ranking place with Asus in the 5th place and Kindle in the 3rd place. Samsung obtained the top ranking place at the beginning. We assume that a better position (higher ranking place) corresponds to an increased ability to affect more users, resulting in the potential for a larger sales volume. However, each advertiser needs to determine how to rationally use its budget to maximize profits based on
its own brand and product.

3.3.1.1 Simulation Result

According to the initial order of the ranking places, the initial budget allocation for Asus, Apple, Samsung and Kindle are deliberately chosen as $y^1(0) = [0.1 \ 0.3 \ 0.4 \ 0.2]^T$, $y^2(0) = [0.1 \ 0.3 \ 0.2 \ 0.4]^T$, $y^3(0) = [0.4 \ 0.3 \ 0.2 \ 0.1]^T$, and $y^4(0) = [0.2 \ 0.4 \ 0.3 \ 0.1]^T$ respectively. Every advertiser allocates its money according to its current advertising strategy. That is to say, Asus allocates most of its budget to the third best ranking place, Apple allocates most of its budget to the fourth best ranking place at the outset, Samsung initially choose the first best ranking place, and Kindle allocates more budgets to the second best ranking place. The preference matrix of each advertiser for each ranking place is set as follows:

$$
\bar{A}^1 = \begin{bmatrix}
1.0 & 0.5 & 2.5 & 4.5 \\
4.5 & 1.0 & 4.5 & 4.5 \\
9.0 & 9.0 & 1.0 & 9.0 \\
0.5 & 0.5 & 0.5 & 1.0
\end{bmatrix}, \quad
\bar{A}^2 = \begin{bmatrix}
1.0 & 2.5 & 2.5 & 2.5 \\
0.5 & 1.0 & 2.5 & 4.5 \\
0.5 & 2.5 & 1.0 & 2.5 \\
9.0 & 9.0 & 9.0 & 1.0
\end{bmatrix},
$$

$$
\bar{A}^3 = \begin{bmatrix}
1.0 & 9.0 & 9.0 & 9.0 \\
0.5 & 1.0 & 0.5 & 0.5 \\
2.5 & 0.5 & 1.0 & 4.5 \\
0.5 & 0.5 & 0.0 & 1.0
\end{bmatrix}, \quad
\bar{A}^4 = \begin{bmatrix}
1.0 & 4.5 & 4.5 & 4.5 \\
4.5 & 1.0 & 4.5 & 4.5 \\
4.5 & 4.5 & 1.0 & 4.5 \\
4.5 & 4.5 & 4.5 & 1.0
\end{bmatrix},
$$

Since $\bar{a}_{31}^1 = 9.0$, $\bar{a}_{32}^1 = 9.0$ and $\bar{a}_{34}^1 = 9.0$, this indicates that advertiser Asus’s first bidding target is the third best ranking place. Similarly, we know from $\bar{A}^4$ that Kindle has equal preference for all the ranking places.
Figure 3.16: First Stage: Allocation ratio of advertisers Asus, Apple, Samsung and Kindle to the advertisement places.

The evolution of budget allocation, as determined by our model, is shown in Figure 3.16. It is obvious to see that the sequence of ranking places from the highest to lowest are: Samsung, Kindle, Asus and Apple. Additionally, Figure 3.17 shows the predicted evolution of users’ allocation percentages to 4 advertisers. There is a diverse number of users that have a preference to a certain advertiser. We get this data from real eBay users and normalize it for our dynamics. Samsung, the advertiser who obtains the best ranking place, is chosen by 57 users. Meanwhile, Apple has the lowest ranking place and is chosen by 19 users due to its worst ranking place. At the same time, 38 users have chosen Asus and 46 uses have chosen Kindle. The results are consistent with sale’s percentage in the first stage shown in Figure 3.13.
3.3.2 The Second Stage

The second stage is the modification period for all advertisers to adjust their strategies for a better sales volume. Advertisers such as Apple especially need to reallocate its budget to compete with other advertisers. During this stage, Apple strives to achieve the best ranking place. Asus also has the same target as Apple. At the same time, Samsung, the advertiser that has spent a large amount of budget in the first stage tries to adjust strategy to maximize profits with a minimal amount of investment.

3.3.2.1 Simulation Result

Using the final state allocation of the first stage, the initial budget allocation of the second stage is deliberately chosen as $y^1(0) = [0.12, 0.23, 0.61, 0.04]^T$, $y^2(0) = [0.07, 0.18, 0.16, 0.59]^T$, $y^3(0) = [0.67, 0.06, 0.23, 0.04]^T$, and $y^4(0) = [0.28, 0.3$.
As mentioned above, each of the advertisers has to change their marketing strategy to reach a new target in the next stage. The new preference matrix of advertisers for each ranking place is set as follows:

\[
\bar{A}_1 = \begin{bmatrix} 1.0 & 2.5 & 2.5 & 4.5 \\ 4.5 & 1.0 & 4.5 & 4.5 \\ 9.0 & 9.0 & 1.0 & 9.0 \\ 2.5 & 2.5 & 2.5 & 1.0 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 1.0 & 9.0 & 9.0 & 9.0 \\ 4.5 & 1.0 & 4.5 & 4.5 \\ 0.5 & 2.5 & 1.0 & 2.5 \\ 0.5 & 0.5 & 0.5 & 1.0 \end{bmatrix},
\]

\[
\bar{A}_3 = \begin{bmatrix} 1.0 & 2.5 & 2.5 & 2.5 \\ 0.5 & 9.0 & 9.0 & 9.0 \\ 2.5 & 2.5 & 1.0 & 4.5 \\ 0.5 & 0.5 & 0.5 & 1.0 \end{bmatrix}, \quad \bar{A}_4 = \begin{bmatrix} 1.0 & 2.5 & 2.5 & 2.5 \\ 2.5 & 1.0 & 2.5 & 2.5 \\ 4.5 & 4.5 & 1.0 & 4.5 \\ 9.0 & 9.0 & 9.0 & 1.0 \end{bmatrix},
\]

We also increase advertisers’ mutation parameters from range 0.05–0.01 to range 0.1–0.5, which implies the advertisers are more willing to change ranking places during the modification period. During the second stage, every advertiser struggles to use a suitable strategy to achieve its new target. Figure 3.18 shows the changing ranking places during the second stage, as determined by our model. The new sequence of ranking places from the highest to lowest are: Apple, Asus, Kindle and Samsung. Corresponding to the changing of advertisers’ ranking places, users still prefer the advertisers who take over the best ranking place. Therefore, more and more users start to allocate their money to Apple instead of Samsung. Figure 3.19 shows the resulting evolution of users’ allocation percentage to the 4 advertisers during second stage. The number of users chose Asus, Apple, Samsung and Kindle are separately 46, 98, 11, and 15. There are 27 users prompt to increase their allocation to Apple, while 47 users’ allocation to Samsung fell sharply. Meanwhile, Kindle faces
Figure 3.18: Second Stage: Allocation ratio of advertisers Asus, Apple, Samsung and Kindle to the advertisement places.

Figure 3.19: Second Stage: Allocation ratio of 160 users to the 4 advertisers.
the same problem of decreasing user allocation percentage as Samsung when its ranking place falls behind.

3.3.3 The Third Stage

After a furious competition in the second stage, advertisers not only get a better understanding about their own products in this certain market, but they also have more detailed information about other advertisers. Under this circumstance, they adjust their strategies based on the current budget and sales volume. We assume that all the advertisers hold the same amount of budget. Thus, this is the final stage for all the advertisers to hold the better ranking places. These transfer to be the initial ranking places in next cycle.

3.3.3.1 Simulation Result

Using the final state allocation of each advertiser in the second stage, their initial budget allocation of the third stage is deliberately chosen as $y^1(0) = [0.10 \ 0.69 \ 0.14 \ 0.07]^T$, $y^2(0) = [0.57 \ 0.33 \ 0.06 \ 0.04]^T$ and $x^3(0) = [0.08 \ 0.18 \ 0.20 \ 0.54]^T$, $y^4(0) = [0.08 \ 0.30 \ 0.55 \ 0.07]^T$. Each of the advertisers changes its marketing strategy again to finally reach its target. The new preference matrix of the advertisers for each ranking place is now set as follows:

$$
\bar{A}^1 = \begin{bmatrix}
1.0 & 2.5 & 2.5 & 4.5 \\
4.5 & 1.0 & 4.5 & 4.5 \\
9.0 & 9.0 & 1.0 & 9.0 \\
0.5 & 0.5 & 0.5 & 1.0
\end{bmatrix}, \quad \bar{A}^2 = \begin{bmatrix}
1.0 & 2.5 & 2.5 & 2.5 \\
9.0 & 1.0 & 9.0 & 9.0 \\
4.5 & 4.5 & 1.0 & 4.5 \\
2.5 & 2.5 & 2.5 & 1.0
\end{bmatrix}
$$
\[ \tilde{A}^3 = \begin{bmatrix} 1.0 & 9.0 & 9.0 & 9.0 \\ 2.5 & 2.5 & 2.5 & 9.0 \\ 2.5 & 2.5 & 1.0 & 4.5 \\ 0.5 & 0.5 & 0.5 & 1.0 \end{bmatrix}, \quad \tilde{A}^4 = \begin{bmatrix} 1.0 & 2.5 & 2.5 & 2.5 \\ 2.5 & 1.0 & 2.5 & 2.5 \\ 4.5 & 4.5 & 1.0 & 4.5 \\ 9.0 & 9.0 & 9.0 & 1.0 \end{bmatrix}. \]

Figure 3.20 shows the evolution of advertisers’ final percentage allocation. The final sequence of ranking places from the highest to lowest are: Samsung, Apple, Asus and Kindle. Figure 3.21 is the percentage allocation of users to the 4 advertisers. It is clear from Table 3.1 that ranking places of all the advertisers are close. The results in Figure 3.21 tell the same story, that users’ different allocation between these four advertisers. According to the Matlab simulation results, there are 41 users have chosen Asus, 23 users have chosen Apple, 75 users have chosen Samsung and 21 users have chosen Kindle. The users’ allocation percentage to each advertiser is consisted with the crawled data.

Figure 3.20: Third Stage: Allocation ratio of advertisers Asus, Apple, Samsung and Kindle to the advertisement places.
Figure 3.21: Third Stage: Allocation ratio of 160 users to the 4 advertisers.

Further discussion will be presented in the following chapter. Online advertisement’s strategy design about each advertiser will also be highlighted.
Chapter 4

Design of Advertising Strategy

In the global, digital and rapidly-growing Internet market, great changes have emerged in the aspects of market property, space and time concepts, consumer demands and behavior etc. Marketing represents the relationship between the ranking place (market location) and the advertiser. Knowledge of current and emerging happenings in the ranking place is extremely important in any strategic planning exercise. The role of marketing strategy is the creation of a unique and valuable position, which deals with developing, implementing, and directing companies to achieve designated intentions. This chapter discusses different aspects of marketing strategies based on the crawled data in Chapter 3. Furthermore, this chapter discusses budget management and positioning of local users using two highlighted examples.

Indeed, marketing strategy is the most significant challenge that advertisers of all types and sizes have to face. Within a given environment, marketing strategy deals essentially with the three forces known as the strategic three Cs [26]: the user (the customer), the competition, and the advertiser (the corporation).
The user, competition, and advertiser in Figure 4.1 are considered as being dynamic, living creatures with their own objectives to pursue. If what the user wants do not match the needs of the advertiser, the latter’s long-term viability may be at stake. Good matching of the needs and objectives of users and advertisers is required for a lasting positive relationship. That is to say, it is imperative for the advertiser to develop adaptive online marketing strategies and establish a good relationship with users. The advertiser should exert its strategies from the following aspects in [40]: product strategies, price strategies, distribution strategies and promotion strategies.

- **Product Strategies.** Product is the good provided by the advertiser to the online market that satisfies a user’s need. Strictly speaking, all products are suitable for sale online. However, different products require different strategies and must take into account a user’s psychology and logistics factors. For some products, such as audio-visual products or furniture, users can experience the function online personally. It is easy for a user to make a final purchase decision. For some other products, such as clothes or electronics, users can only read the describing text and pictures online instead of the feelings. Users may need time to analyze products and may somethings
struggle to make a final decision. To achieve a better sale, these products should exert online promotion, and adopt express delivery service to send merchandise to users. As pointed out in [14], the Internet leads to a faster discovery of customer needs, greater customization of products to the customers’ needs, faster product testing, and shorter product life cycles.

- **Price Strategies.** The Internet produces an increasing standardization of prices across borders, or at least narrowing price differentials as users become more aware of prices in different countries [34]. The convenience of searching online makes it easier for users to swap between advertisers. It is expected that the bargaining skill of users are likely to increase by becoming aware of alternative products elsewhere. In response, advertisers can adopt flexible pricing strategies according to product cost and the supply-demand relationship. Example strategies include free shipping price strategy, auction price strategy, try-using strategy, etc.

- **Personalization Strategies.** Online shopping without geographical constraints provides more options for users. Users can seek satisfactory goods in a global scope in accordance with their individual personality and demands. Dissimilar to decades before only the rich can enjoy the personalized customization, normal users have to choose demands according to their income, status, family grades etc. More and more users start to personalize themselves while making a purchase. They place emphasis on the values of the products and their consumption concept tends to be more rational. Therefore, advertisers should focus marketing ideas on the demands of users and their personalized demands as the starting point to provide more desirable products and services.

- **Promotion Strategies.** Promotion refers to the various ways an advertiser can communicate its products’ merits and persuade targeted users to purchase its products.
Generally, there are two classic ways of online advertising [25]. One is to attract users through advertisements in a website where users can click while browsing; the other way is to send advertisements through emails. At the same time, advertisers’ webpages are the key to promotional strategies. By visiting the website, users can access more detailed information about the products, give feedback, and of course, purchase products. Advertisers should adopt suitable measures to intensify communication with the public and make corresponding adjustments according to the feedback.

### 4.1 Cases Discussion

Two example online advertising strategies will be explained in this section. These two examples are the brands Apple and Samsung, which come from the Chapter 3. We will extend the analysis of their evolution and give specific explanation about their strategies at each step.

#### 4.1.1 Advertiser: Apple

As we know, Apple is a technological company which emphasizes the uniqueness and perfection of their products. This tenet brings Apple increasingly loyal fans from the first Apple products. Overtime these people have associated Apple’s products with personal identification. According to the latest U.S. tablet customer satisfaction survey conducted by J.D. Power, a global marketing information services company [9], Apple has earned a 5-star rating and scored 830 on a 1,000 point scale, edging out Samsung, which scored a second place at 822 [11]. A good reputation is an invisible and effective advertisement, which is not easily forgotten by people. It is the reason why Apple chose the 34th ranking places at the beginning of first stage. Under the assumption of equal budget, Apple first tried to
obtain the maximum economic benefits of its reputation while minimizing expenses.

For this study, we assume that the list of products provided to users are those returned products after typing the keyword “tablet” in Ebay’s search engine. All advertisers know their rank after the first round of bidding, as well as their sales volume per day. Apple falls to 36th place on March 2nd with one product selling per day. We also assume all the strategies used by advertisers here follow the contract between advertisers and the publisher. Apparently, higher ranking place needs be acquired to attract more users to Apple at this time. A large investment is redistributed on March 2nd to gain a higher ranking place. From Figure 3.14, sales volume soars from 693 to 1051 between March 2nd and March 5th. However, other advertisers also readjust their distribution based on their sales volume and other requirements. During the first stage, Apple is the only advertiser whose sales volume is over 100.

When approaching the 2nd stage, Apple has already reached the best ranking place among all advertisers. The sales volume is still increasing rapidly before March 7th. We can see its ranking place changing during this period in Fig 3.13. It has almost the same changing rate as advertiser Asus. Now, it is the best time for Apple to shrink its investment and save budget with the best ranking place holding currently.

We assume that the contracts between advertisers and publisher are time-bound, i.e., the conditions of the contract can be fixed several times within a period of validity. With a limited budget left, the 3rd stage is the last time for all advertisers to increase sales volume and attract more users. From Figure 3.13, Apple depletes its budget to compete with other advertisers for a better ranking place. With the largest sales volume during the first three stages, Apple turns a profit much earlier than expected. It is the reason Apple wants to hold a better ranking place for the next bidding around.

In conclusion, Apple’s strategy changes can be divided into three parts: (1) Conservative market estimation, (2) Promptness of plunging an investment, and (3) Readjustment
for next around.

### 4.1.2 Advertiser: Samsung

It seems redundant for us to point out that Samsung is quickly gaining on Apple in terms of brand affection. The signs that Samsung, which has been investing heavily in brand loyalty is now challenging Apple in this crucial area of marketing, are revealed as analysts look across a wider range of social media and psychometric levels in [8]. A single reason that Samsung is gaining on Apple in brand affection then it lies in Samsung’s substantial investment in social engagement, an area Apple eschews. At the same time, Samsung cares more on sales volume at multiple price points. It is known as the second “Nokia” in the recent five years with blockbuster sales at relatively lower price for its products. It might be one of the reasons why Samsung achieves the best ranking place after the initial bidding. Meanwhile, Samsung holds the largest initial basic sales volume at 957, which might be the result from last bidding cycle.

By taking a look at the first stage in Figure 3.13 and Figure 3.14, we can find that even with the 1st ranking place, the increase of sales volume of Samsung is very slow. This situation could be caused by two reasons: (1) Similar products in the immediate ranking places and (2) a relatively lower rate of the products due to the huge sales. In this case, Samsung needs to shrink its budget for the best ranking place in order to avoid useless investment. Thus, a falling process can be found from March 4th.

The sales volume we crawled for Samsung from March 4th to March 7th increases consistently at 5 per day. However, a slight reduction in sales volume can be observed on March 8th. If Samsung does not change strategy, its ranking place will continue to fall with decreasing sales volume. On March 9th, Samsung changes strategy by placing more money for a better rank to help itself return to its initial ranking place. Sales volume then
dramatically increases from March 9th to March 12th and steadily increases during the third stage with sales volume from 1035 to 1055.

Overall, there are three features of Samsung’s strategies. Firstly, it invests heavily at the beginning for the best ranking place. This strategy focuses more on attracting users rather than reaching huge sales volumes. Secondly, its investment outflows in a planned way, which keeps its ranking place floating in the range of first ten. This strategy help Samsung prepare the final bidding effectively. The last is the readjustment just as Apple.

4.2 Strategy Design

In this section, strategy design for increasing online sales and user data collection are proposed. Firstly, the common online strategies of budget management are applied to two examples. Secondly, another two examples are introduced to look for a better targeting method.

4.2.1 Case Study

From the above two example, we can observe that several interesting points of advertisers’ strategy in the online advertising ecosystem. First, the role of the initial ranking places in the first stage is significant. Each initial ranking place leads to different evolutionary locus. Similarly, each advertiser has its own purpose for different initial ranking place. Furthermore, a rapid increase of ranking place leads to a distinct jump in sales volume, which has a stronger effect than holding the best ranking place the entire time. This result is only hold in our work due to the similar advertisers we chose for simplicity. After knowing the strategies of other competitors for a short while, the important thing for each advertiser turns out to be strategic adjustment at appropriate times. The strategy design for budget management is introduced in the following parts.
Because the product we chose for data crawl is a tablet, its product strategies are implemented easily online. Users can make a quick choice after they browse each product’s function. The price of all the products we have chosen is below 500 dollars; the price of Asus, Samsung, Apple and Kindle are respectively $159.99, $169.99, $439.99, $199.99. For simplicity and veracity of simulation results, we assume that their price does not change during the whole process. However, when we come to the personalization strategies, advertiser Apple creates a new category for their user. On the contrary, advertiser Asus and Samsung lack the marketing ideas to satisfy users’ personalized demands.

We assume that each bidding period is divided into three stages as in Figure 3.13. Stage 4 then becomes Stage 1 in the next bidding period. To achieve a huge sales volume, advertisers need rational budget management. Advertiser such as Apple can first choose a relatively lower ranking place close to 25-35, approximately 15-20 percent of its budget. Then is should try its best to compete for the best ranking place with others after becoming familiar with users. The sales volume is big enough, which should already fit the sales target. In the last round, Apple uses the remaining part of its budget, which is around 20-30 percent, to bid for a better ranking place for the next period. However, advertiser Samsung who uses promotion strategy would like to distribute 35-45 percent of its budget in the first stage. Although it reduces the budget for the second stage, it still struggles for a higher ranking place by using the entire budget during the last stage. Other advertisers without obvious purpose can distribute their budget equally at each stage.

4.2.2 Examples

In this section, two examples of user targeting are presented. IP-based geo-targeting, search targeting, explicit profile data targeting, behavioral targeting and contextual targeting are explored with each example.
4.2.2.1 Online Sales Increase

RadioShack is seeking to increase the online sale of electronics using a promotion on Memorial Day. The promotion is for the prize of an iPhone accessory based on users voting for their favorite products. Apple is sponsoring the promotion alongside regional sponsorships by local electronic stores. Information gathered at contest registration will include e-mail, birthday, gender, and numbers of purchase in physical shop, favorite brand, and address.

- **IP-based Geo-Targeting:** IP targeting will be done to ensure coverage of outlying markets such as suburban and rural areas.

- **Search Targeting:** Search targeting will be used to capture users that may be interested in purchasing iPhone product. Key words such as “iPhone” “Ballistic Cases” and other descriptions will be used. In addition, words related to the contest such as “accessories” “16GB” and “Apple” will be used.

- **Explicit Profile Data Targeting:** RadioShack has an e-mail list of users containing their past purchase and demographic information. This information is deemed admissible because it is supplied by the users themselves. The list has been actively maintained and is considered fresh. Special e-mails will be sent to users who are Apple fans and who have indicated they have purchased a similar product in the last six months. The mailing list will be divided by gender and have different subject lines and content for females and for males.

- **Behavioral Targeting:** RadioShack will use behavioral targeting, which collects information on a user’s web browsing actions, usually through the use of “cookies”, to track all visitors accessing to the power tools section of their Website. Informa-
tion gathered on the user’s footprint will be used to refine the targeting of display advertisements and to better place new advertisements online.

- **Contextual Targeting:** RadioShack will place a series of online display advertisements for the contest on specific electronic web sites such as BestBuy and Newegg (newegg.com). These advertisements will be placed in the areas associated with mobile accessories and other electronics.

4.2.2.2 **Localization and User Profile Data Collection**

RadioShack is looking to continue to evolve a very strong national brand, and to remind users that radioShack is a convenient electronic shop in their neighborhood. They want users to know their products so that they can understand users’ needs and deliver better customer service, in-stock inventory, and a friendly shopping environment.

- **IP-based Geo-Targeting:** IP-based geo-targeting can be used to get blanket coverage of users. For larger direct market access (DMAs), they can improve on reaching more specific users by combining IP-based geo-targeting with other targeting methodologies below.

- **Search Targeting:** RadioShack can tailor advertisement messaging to users depending on their local search criteria. For example, RadioShack can target advertisement creative highlighting all the different electronic brands they carry at the local RadioShack to users searching for open houses in Seneca, SC.

- **Explicit Profile Data** RadioShack can target users on Facebook based on where they’ve indicated they live. Similar to sending marketing by post mail, RadioShack can show users’ advertisements with local messaging based on where a user lives. This data is typically very accurate. Therefore, an effective creative that would res-
onate with the user could be an ad that shows the location of the nearest RadioShack with a friendly face.

− **Behavioral Targeting**: This is slightly less accurate than Explicit Profile data targeting but has a broader reach. RadioShack can create a more generalized message based on things the user loves. For example, RadioShack can have advertisements showing tiger logo products in Auburn, AL, and a similar creative for users in Clemson, SC.

− **Contextual Targeting** RadioShack can tailor messaging based on the local content of the site. For instance, the advertisement of RadioShack near Clemson focuses more on earplugs and iPhone accessory due to the large population of students here.
Chapter 5

Conclusion

Replicator-mutator dynamics are widely used in multi-agents network to mimic the agent’s evolutionary state based on its own decision and environmental effects. In order to obtain the agents’ motion and design suitable strategies for online advertising, we borrow this dynamics into our work. The thesis proposes an analytical framework to model and analyze a large-scale complex online advertising ecosystem. We found a lot of similarities shared by the online advertisement ecosystem and real biological systems. Those similarities not only work on the agents’ characteristics, but also contribute to the entire ecosystem. Each major parameter plays a vital role in our replicator-mutator dynamics. Meanwhile, they have crucial effects on the stability of online advertising ecosystem. Furthermore, we provide system-level performance analysis based on the analytical framework. Strategies are changed with every online advertisement bidding circle. By applying the replicator-mutator dynamics, users’ and advertisers’ influence to each other, even to the whole system, can be analyzed. More specifically, the conclusion and contribution of each chapter are summarized below.

In chapter 2, fundamental scenario of online advertising ecosystem comes up first to illustrate the research background. Both users’ and advertisers’ replicator-mutator dy-
namics are fully introduced. Their evolutionary states depend on each other. Both the experimental observations and simulated results confirmed the variation of four major parameters in replicator-mutator dynamics build unique contributes to the ecosystem separately. These parameters are hierarchy parameter, preference matrix, initial condition and mutation parameter, respectively. Additionally, dynamics of different relationship between users and advertisers are introduced to simulate the online advertising ecosystem. It is shown that only the full version of replicator-mutator dynamics can fit and express our research problem. Experimental results, such as the sale volume and the number of users both increased with the raising ranking places of advertisers, have been demonstrated by our Matlab simulation result.

In Chapter 3, real data of online retailers are crawled and used to formulate our replicator-mutator dynamics. Major data of each parameter are used to validate our model, which also shed light on the design strategies. The data crawled can be divided into four stages which compose a recycling circle of ecosystem. The simulation we conducted at each stage exactly reflects the advertisers’ evolution in the online advertising ecosystem. Some interesting strategies and targeting design can be obtained by analyzing from our simulation. These strategies enable a fast-processing and precise targeting operation for advertisers and users.

In Chapter 4, the major strategies of each stage are explained, based on a variety of scenarios that advertisers may face in the online advertising ecosystem. Furthermore, two examples are respectively highlighted to give more specific details of budget management and local targeting users.

To conclude, the replicator-mutator dynamics offer a rigorous analytical method to describe the dynamics of complex adaptive systems. A future goal will be designing new self-regulation strategies that eventually minimize the advertisement budget by managing the advertising properties. To fulfill this need, a feedback controller about advertisers’
strategies to certain users would be the future interesting aspect.

On the other hand, users’ behaviors need to be fully studied in regard to the advertisers’ strategies and marketing interactions, as these customized ways of marketing may play an important role in online searching and purchasing operations. Relationship between users and advertisers has been verified in the agents’ behaviors and marketing properties; hence, future work is to develop a numerical analysis model that integrates advertisers’ critical stage with users’ variation feedback and response into a feedback control mechanism. The integration of market economics and optimal control methods may provide a possible novel direction, it can be envisioned that budget maximization may be realized using this combined method.
Appendices
Appendix A The evolution of advertiser $s_v$'s allocated resources in time by equation (2.27) guarantees the assumption that the total resources of user $s_v$ equal to 1. i.e., $c^T y^v(t) = 1$ for all $v = 1, 2, \cdots, N$ given initial condition $c^T y^v(0) = 1$.

**Proof.** Multiply $c^T$ for both sides of Equation (2.27) to get

$$c^T \dot{y}^v(t) = c^T [P^v(t)]^T H^v(t) y^v(t) - \theta^v(t) c^T y^v(t) + \sum_{l=1}^{M} \gamma_{vl} \xi_{vl} c^T y^v(t) - \beta^v(t) c^T y^v(t).$$

As $c^T [P^v(t)]^T = c^T$ and $c^T y^v(0) = 1$, it becomes to

$$c^T \dot{y}^v(t) = h^v(t)^T y^v(t) - \theta^v(t) + \sum_{l=1}^{M} \gamma_{vl} \xi_{vl} c^T y^v(t) - \beta^v(t) c^T y^v(t).$$

By the definition of average fitness $\theta^v(t) = h^v(t)^T y^v(t) = \sum_{i=1}^{M} y^v_i(t) h^v_i(t)$ in Equation (2.19) and total capacity $\beta^v(t) = \sum_{l=1}^{M} \gamma_{vl} \xi_{vl}$ in Equation (2.25), we can get

$$c^T \dot{y}^v(t) = 0 + \sum_{l=1}^{M} \gamma_{vl} \xi_{vl} - \beta^v(t) = 0.$$
Appendix B  The evolution of user $r_u$’s allocated resources in time by Equation (2.5) guarantees the assumption that the total resources of user $r_u$ equal to 1, i.e., $c^T x^u(t) = 1$ for all $u = 1, 2, \ldots, n$ given initial condition $c^T x^u(0) = 1$.

Proof. Multiply $c^T$ for both sides of Equation (2.5) to get

$$c^T \dot{x}^u(t) = c^T [Q^u(t)]^T F^u(t) x^u(t) - \dot{\phi}^u(t) c^T x^u(t)$$

$$+ \sum_{v=1}^{N} \lambda_{uv} \rho_{uv} c^T y^v(t) - \sum_{v=1}^{N} \alpha^v(t) c^T y^v(t).$$

As $c^T [Q^u(t)]^T = c^T$ and $c^T x^u(0) = 1$, it becomes to

$$c^T \dot{x}^u(t) = F^u(t) x^u(t) - \dot{\phi}^u(t)$$

$$+ \sum_{v=1}^{N} \lambda_{uv} \rho_{uv} c^T y^v(t) - \sum_{v=1}^{N} \alpha^v(t) c^T y^v(t).$$

By substituting Equation (2.8) and Equation (2.12) into above equation,

$$\dot{\phi}^u(t) \triangleq f^u(t) x^u(t),$$

$$\alpha^v(t) = \frac{\sum_{v=1}^{N} \lambda_{uv} \rho_{uv}}{N},$$

we can get

$$c^T \dot{x}^u(t) = 0$$
Appendix C The general Matlab codes of users’ and advertisers’ RM dynamics.

Main code:

clear all;
close all;
clc;
tic
global n N M Ap1 Ap2 Q_ap Fs As_a
StarNetPaper1

%%% generate the initial condition for users
rand(’state’, 10);
x0p=0.5*rand(N,n);
x0p=0.5*ones(N,n);
% x0p(1,1)=0.4;x0p(1,2)=0.6;x0p(1,3)=0.3;
% x0p(2,1)=0.2;x0p(2,2)=0.5;x0p(2,3)=0.1;
% x0p(3,1)=0.2;x0p(3,2)=0.3;x0p(3,3)=0.6;
x0=x0p./repmat(sum(x0p,1),N,1); % normalization:
%
% x1p=rand(M,N); % initial values - nenormalized
rand(’state’, 10);
% x1p(1,1)=0.6;x1p(1,2)=0.4;%x0p(1,3)=0.3;
% x1p(2,1)=0.4;x1p(2,2)=0.6;%x0p(2,3)=0.1;
% x1p=0.5*ones(M,N);
% just for debug, to generate the same results
x1=xlp./repmat(sum(xlp,1),M,1);

for k=1:n
    if k==1; Ak=Ap1; % Preference parameter of each user
    else   Ak=Ap2;
    end

    Dk=sum(Ak,2); % sum by rows
    L=eye(N)-Ak.*repmat(1./Dk, 1,N);
    % compress form of above e eqs
    Qk=eye(N)-mu(k)*L;
    Q_a(1+(k-1)*N:N+(k-1)*N,:)=Qk;
    % matrix Q
    Q_ap(:, 1+(k-1)*N:N+(k-1)*N)=Qk;
    % transpose needed in odefun, = Q’
end

for g=1:N
    if g==1; Ag=Ap3;
    else Ag=Ap4;
    end

    Dg=sum(Ag,2); % sum by rows
    R=eye(M)-Ag.*repmat(1./Dg, 1,M);
    % compress form of above e eqs
    Qg=eye(M)-mu1(g)*R;

end
Q_a1(1+(g-1)*M:M+(g-1)*M,:)=Qg;
Q_ap1(:, 1+(g-1)*M:M+(g-1)*M)=Qg;
% transpose needed in odefun;
end

y0=reshape(x0, n*N,1);
y1=reshape(x1, N*M,1);

a=(1e-5)*ones(1,n*N+N*M);
options=odeset('RelTol',1e-4,'AbsTol',a);
[T,Y]= ode45(@La_ODE,t,[y0;y1],options); %options;

Y(end,1:n*N)
Y(end,n*N+1:n*N+N*M)

figure(1)
subplot(1,2,1)
plot(t, Y(:,3:2:n*N))
title('Users to Advertiser 1')
xlabel('time-units'), ylabel('Allocation Ratio')
legend('x^u_{1}(t), u=1,2...100')
ylim([0 1.0]);
grid

subplot(1,2,2)
plot(T, Y(:,4:2:n*N))
title('Users to Advertiser 2')
xlabel('time-units'), ylabel('Allocation Ratio')
legend('x^u_{2}(t), u=1,2...100')
ylim([0 1.0]);
grid

**ODE Function:**

function dy1=La_ODE(t,y)

global N M Q_ap1 Fu Au_a Ap3 Ap4
global n Ap1 Ap2 Q_ap Fs As_a

dy1=zeros(n*N+N*M,1);

%%%%%%%%%%%%%%%% dy(1)= User’s dynamics %%%%%%%%%%%%%%%
for k=1:n
    if k==1; Ak=Ap1;
    else Ak=Ap2; % Preference parameter
    end
    xk=y(1+(k-1)*N:N+(k-1)*N);
    fk=Ak*xk; % fitness parameter
    phiK=sum(fk.*xk); % average fitness
    F_a(1+(k-1)*N:N+(k-1)*N,1)=fk;
    Phi_a(1+(k-1)*N:N+(k-1)*N,1)=phiK;
end
%% User's social interaction
% As and Fs defined in main code
% As_a=ones(n,n)-eye(n,n);
% interaction matrix - defined in main code
Ws=As_a./repmat(sum(As_a,2),1,n);
% weighted matrix

Fs=[1:1:n]’; %defined in main code
FiS=(sum(repmat(Fs’,n,1).*Ws,2));
% interaction fitness

%% obtain the first term in the ODE
F_ay=F_a.*y(1:n*N);
f=reshape(F_ay,N,n);

for i=1:N
    F(:,i:N:n*N+i-1)=f;
    FiS4(i:N:n*N+i-1,1)=FiS;
end

%% term 11
term11=(sum(Q_ap.*F,1))’; % Q’*F*y - Islam
% end term 11

%% term 13
FiS_W=repmat(Fs’,n,1).*Ws;
Ad1 = y(n*N+1);
Ad2 = y(n*N+N+1);
z0 = Ad1*ones(n,1);
z1 = Ad2*ones(n,1);
term3(1:1:n,1) = FiS_W*z0;
term4(1:1:n,1) = FiS_W*z1;
term13 = [term3; term4];

% end term 13

%% obtain the 4th term
% FiS4 - in the above loop
term14 = FiS4.*y(1:n*N);
% end term 4

dy1(1:n*N) = term11 - Phi_a.*y(1:n*N) + term13 - term14;

%%%%%%%%%%%%%%%% dy(2) = Advertiser’s dynamics %%%%%%%%%%%%%%%%%%
for g = 1:N
    if g == 1 Ag = Ap3;
    else Ag = Ap4;
    end

    xg = y(n*N+1+(g-1)*M:n*N+M+(g-1)*M);
    fg = Ag*xg;
    phiG = sum(fg.*xg);
F_a1(1+(g-1)*M:M+(g-1)*M,1)=fg;
Phi_a1(1+(g-1)*M:M+(g-1)*M,1)=phiG;

end

%% advertiser’s social interaction
% As and Fu defined in main code
% Au_a=ones(n,n)-eye(n,n);
% interaction matrix - defined in main code
Wu=Au_a./repmat(sum(Au_a,2),1,N);

Fu=[1:1:n]’; defined in main code
FiU =(sum(repmat(Fu’,N,1).*Wu,2));

%% obtain the first term in the ODE
F_az=F_a1.*y(n*N+1:n*N+N*M);
h=reshape(F_az,M,N);

for i=1:M
H(:,i:M:N*M+i-1)=h;
FiU4(i:M:N*M+i-1,1)=FiU;
%FiD4(i:M:N*M+i-1,1)=FiD;
end

term21=(sum(Q_ap1.*H,1))’;
% Q’*F*z - Islam
% end term1

%% obtain the 24th term
% FiU4 - in the above loop
term24= FiU4.*y(n*N+1:n*N+N*M);
% end term 24

%% term 23
FiU_W=repmat(Fu',N,1).*Wu;

for i=1:M

term23(i:M:N*M+i-1,1)=FiU_W*y(n*N+i:M:N*M+i-1+n*N);

%FiU_W2(:,i:N:n*N+i-1)=FiU_W;
end
% end term 23

dy1(n*N+1:n*N+N*M) = term21-Phi_a1.*y(n*N+1:n*N+N*M)
+term23-term24;
% dy=reshape(dy1,42,1)
end

Setting for other parameters:

% General parameters
n=100; N=2; M=2;
t=[0:0.01:4];
global Fu

%% Users choose different advertisers;
Ap1=rand(N,N);
% user 1 allocate preference would be 3>2>1;
for i=1:N
    Ap1(i,i)=1;
end
Ap1(1,2)=3;
Ap1(2,1)=10;
rand('state', 100);
%
Ap2=rand(N,N);
% user 2 allocate preference would be 3>1>2;
for i=1:N
    Ap1(i,i)=1;
end
Ap2(1,2)=10; %Ap2(1,3)=0.5;
Ap2(2,1)=5; %Ap2(2,3)=0.5;
rand('state', 100);
%Preference would be 3>1>2;

%% Advertisers choose different ranking places;
Ap3=rand(N,N);
% advertisers 1 allocate preference would be 2>1>3;
for i=1:N
    Ap1(i,i)=1;
end

Ap3(1,2)=0.5; % Ap4(1,3)=2.5;
Ap3(2,1)=9.5; % Ap4(2,3)=9;
% Ap4(3,1)=0.5; Ap4(3,2)=0.5;
rand('state', 100);
%
Ap4=rand(N,N);
% advertisers 2 allocate preference would be 1>3>2;
for i=1:N
    Ap1(i,i)=1;
end

Ap4(1,2)=9.5; % Ap5(1,3)=4.5;
Ap4(2,1)=0.5; % Ap5(2,3)=0.5;
% Ap5(3,1)=0.5; Ap5(3,2)=2.5;
rand('state', 100);

%% Mutation parameter
mu=0.1*ones(1,n);
mul=0.1*ones(1,N);

%% Social Interaction
Fs=1+0.1*randn(n,1);
Fu=0.5*ones(N,1);
As_a=rand(n,n);
As_a(1,1)=10;

Au_a=zeros(N,N);
Au_a(1,1:N)=0.6;
Au_a(1:N,1)=1;
Au_a(1,1)=0;

alpha=0.2;
Bibliography


