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COST AND EMBODIED ENERGY OPTIMIZATION OF RECTANGULAR REINFORCED CONCRETE BEAMS

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COST AND EMBODIED ENERGY OPTIMIZATION OF RECTANGULAR REINFORCED CONCRETE BEAMS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Civil Engineering

by
Alexander Lawrence Kosloski
December 2013

Accepted by:
Dr. Brandon Ross, Committee Chair
Dr. Leidy Klotz
Dr. Weichiang Pang
ABSTRACT

Concrete has a large environmental footprint so it is desirable to find ways to efficiently design structural members. Engineers can exercise their abilities early on in the design phase of construction projects to reduce the environmental footprint by minimizing the amount of materials required. One way to achieve these results is to optimize the design of structural concrete.

In this study, simple-span rectangular reinforced concrete (RC) beams with a range of different bending moments were analyzed. The primary goal was to combine life cycle analysis (LCA), numerical optimization, and reinforced concrete mechanics to create a framework for designing efficient RC beams. In particular, the method was developed to quantitatively compare the environmental preferability of RC materials with different properties, such as high-strength reinforcement, high-strength concrete, and lightweight (LW) concrete. The method utilizes ratios of unit cost and/or unit embodied energy. This approach makes the method more general, and facilitates application of the method to a wide variety of circumstances. In addition to guiding material selection, the method also provides designers a means for quickly selecting near optimum cross-section properties.

Trade-offs between optimum economic cost and optimum environmental footprint were also evaluated through multiobjective optimization. The results showed that cost-optimized beams have up to 10% more embodied energy than do energy-optimized beams but are up to 5% cheaper than energy-optimized beams.
The products of this thesis will be useful in rationally selecting materials and designing efficient beams in terms of cost and energy.
DEDICATION

I would like to dedicate this thesis to my parents, Larry and Evelyn, and the rest of my family, Claire and Alex. I hold my parents in high esteem, for they have loved and supported me through every single beat. I take pride in this work, knowing that they, too, are proud of my accomplishments. Claire and Alex are my best friends and I am so lucky to have them in my life. None of what I’ve achieved would feel as momentous were it not for these people.
ACKNOWLEDGMENTS

First, I would like to thank my advisor, Dr. Brandon Ross, who is the creative mind behind the foundation of this research. He presented the topic to me and gave me the opportunity to work with him. Throughout the last year, he has given me all the help I have needed and has provided me with a new understanding of civil engineering through a structural and a sustainable point of view. Without his expertise and deep knowledge of reinforced concrete, I could not have completed this research. I would also like to thank Dr. Leidy Klotz and Dr. Weichiang Pang for being a part of my committee. Their suggestions have helped refine my research by offering new perspectives, creating a more well-rounded product.

I would like to thank two fellow graduate students whose assistance should not go unnoticed. Michael Willis, another student of Dr. Ross, helped review my thesis and gave some very good advice. Abby Liu, an expert in MATLAB, answered any questions I had in regard to coding as well as general formatting in MS Word.

Lastly, I want to thank my mother; as an English teacher, she had some helpful tips in terms of grammar and syntax, making my paper flow more easily. Everyone played a key role in shaping my thesis and giving me the resources I needed to enhance the final result.
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CHAPTER ONE
INTRODUCTION AND BACKGROUND

1.1 Introduction

Construction projects require costly investments of time and money. In the last decade, environmental costs of construction have also become a primary concern. One example of an environmental cost is carbon dioxide and other greenhouse gases emitted into the atmosphere as a result of construction activities. Therefore, engineers exercise their abilities early on in the design phase of construction projects to help reduce the cost and carbon footprint by lessening the amount of materials required. One way to achieve these results is to optimize the design of structural members.

Because concrete is the most commonly used construction material on the planet, and with the cement industry responsible for 5% of the world’s carbon dioxide emissions (Worrell, 2001), the current research focuses on the optimization of reinforced concrete (RC) members. By optimizing these RC structures, engineers can scale down the volume of concrete and/or steel used in a structure, consequently lowering the discharge of carbon dioxide emissions, as well as other environmental costs, and the economic costs associated with construction. This study will focus on optimizing RC beams for embodied energy, which is defined as the quantity of energy needed to develop and manufacture a product, as if that energy were manifested within the product itself.

Different techniques have been utilized in this research to conduct optimization on RC members; these methods are discussed in the next chapter. Each method defines a set number of design variables that are modified within the optimization process, such as
the height and width of the member and the area of steel reinforcement placed within the member. These design variables are modified in a fashion that would minimize the objective function; examples of such objective functions are the total cost of the structure and the energy consumed by all the materials and processes necessary to produce the structure. Constraints are also applied to ensure that the optimized structure satisfies code while prevailing as a dependable and durable structural component; these constraints can be the flexural or shear capacity of the member but are not limited to these two principles of structural engineering.

1.2 Research Objectives

While previous research has studied optimization of multiple RC elements, this research is being performed specifically on RC beams, optimizing them for economic and environmental costs. The overall purpose is to combine life cycle analysis (LCA), optimization techniques, and reinforced concrete mechanics to create a quantitative framework for designing sustainable RC structures. Specific goals include the creation of a methodology for rationally comparing the environmental preferability of different RC materials and the creation of a procedure for designing environmentally optimum RC beams.

To accomplish these goals, various parametric studies have been developed to evaluate how high-performance materials can be used in practice by designers to improve the efficiency of RC beams. These parametric studies utilized optimization methods and considered a wide range of design variables, reinforcement configurations, and material properties.
To assist designers in the design of optimized members, the parametric studies have been shaped so that different materials can be selected based on environmental or cost efficiency. After it has been established which material is best suited for their application, the designer can utilize results of supplementary parametric studies to determine the optimal cross-sectional design.

While embodied energy is the only environmental metric considered in this thesis, there are many other metrics that can be applied to the impact assessment of reinforced concrete. Other metrics used in the life-cycle inventory of construction materials include mineral depletion, land use, and human toxicity. Human toxicity is a concern when dealing with chemical additives, such as fly ash.

1.3 Reinforced Concrete Materials

1.3.1 Concrete

Concrete consists of four primary ingredients: cement, coarse aggregate, fine aggregate, and water. Additives are also typically included in the mix to improve properties of the concrete, such as durability, workability, or set-time (Mehta & Monteiro, 1993). A list of definitions has been given for commonly used materials in concrete as well as additives and replacements to enhance its functionality.

Cementitious Materials. Cement is a powdery material that has cementing value when used in concrete either by itself, such as Portland cement, or in combination with products such as fly ash, silica fume, and/or ground granulated blast-furnace slag (ACI Committee 318, 2011).
Aggregate. Aggregate is a granular material, such as sand, gravel, crushed stone, and iron blast-furnace slag, used with a cementing medium to form a hydraulic cement concrete or mortar (ACI Committee 318, 2011). Typical raw materials that are regularly used as aggregate are quartz, basalt, granite, marble, and limestone. The majority of the contents within a concrete mix are aggregate while that aggregate’s physical properties significantly influence the “workability, durability, strength, weight, and shrinkage of the concrete” (The Concrete Countertop Institute, 2006).

Aggregate is broken up into two primary categories: coarse aggregate and fine aggregate. Coarse aggregate is larger than \(\frac{1}{4}\) inch while fine aggregate is smaller than \(\frac{1}{4}\) inch. Fine aggregate is used to fill in gaps between larger particles, thus minimizing the demand for cement paste in the mix; this also mitigates any shrinking within the concrete (TCCI, 2006).

While normal weight (NW) aggregate is used to produce NW concrete for general purposes, lightweight (LW) aggregate is an alternative to reduce the self-weight of the beam. LW aggregate is typically expanded shale, clay, and slate materials, which have been ignited in a rotary kiln to give them a porous consistency. These aggregates are used to produce a LW concrete (National Ready Mixed Concrete Association, 2003), which has a unit weight between 90 pcf and 115 pcf. NW concrete has a greater unit weight, between 135 pcf and 160 pcf (ACI Committee 318, 2011).

Recycled materials have also been given attention to reduce environmental and economic costs. One is recycled glass aggregate (RGA), which is cullet milled from recycled glass products and used as aggregate replacement; an alkali-silica reaction
(ASR) occurs between the cement and glass (when particles are large enough to be considered aggregate) that may cause degradation in the concrete over time. Another recycled material is recycled concrete aggregate (RCA), which is concrete from demolition and renovation projects that is crushed and stripped of all reinforcement so it can be used as aggregate replacement.

Water. Water is added to concrete mixes to activate the cement and to create a mix that is more workable; hydraulic cements require water to harden.

Admixtures. Admixtures are materials other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties (ACI Committee 318, 2011). Seven classic admixtures are listed and defined:

1. A set-retarding admixture impedes the chemical reaction that occurs when the concrete begins to harden, or set. In doing this, more time is given to install concrete, especially in hot climates, which tend to expedite the setting process (Mehta & Monteiro, 1993).

2. Accelerators refer to two properties: high early strength and increased setting time (Mehta & Monteiro, 1993).

3. Air-entrainment improves the durability of the concrete exposed to extreme temperatures; extreme temperatures induce a cycle of freezing and thawing that causes the concrete to expand and contract and eventually form cracks. Air-entrainment also enhances the workability of the concrete (Mehta & Monteiro, 1993).
4. **Water-Reducers** are used to acquire a certain strength while expending less cement by lowering the water-cement ratio required to achieve an acceptable slump (Mehta & Monteiro, 1993).

5. **Shrinkage-reducing** admixtures reduce short- and long-term drying shrinkage that results in the degradation of the concrete due to cracking (Rodriguez).

6. **Superplasticizers** increase the workability of the concrete while yielding a concrete with a high slump; this allows for the placement of concrete in densely reinforced structures or areas where sufficient consolidation is not easily attainable (Rodriguez).

7. **Corrosion-inhibiting** admixtures hinder the effects of corrosion on reinforcing steel present in concrete (Rodriguez).

**Supplementary Cementitious Materials (SCM).** SCM are used to replace the Portland cement to reduce the economic and environmental costs. The four most common are fly ash (FA), ground granulated blast-furnace slag (GGBS), silica fume (SF) and glass powder (GLP). FA is a by-product of the combustion of coal during the generation of electricity. Before GGBS can be used as a cement replacement in concrete, blast-furnace slag, a by-product of the production of iron ore, must be ground into a powder (Ali & Fiaz, 2009). SF is a by-product of silicon metal and ferrosilicon alloy production (Mehta & Monteiro, 1993). GLP is pulverized glass cullet but does not experience the same degradation found in concrete with RGA; this is due to the pulverizing of glass into a fine powder, which mitigates any effects initiated by ASR.
Grades. The grade of concrete refers to the compressive strength ($f'_c$) of the concrete. Different grades of concrete are used in different conditions where higher strength may be required. By including admixtures and adjusting the proportions of water, cement, and aggregate within the concrete mix, increased compressive strength can be achieved. High-strength concrete is described as having a compressive strength greater than 6,000 psi (ACI Committee 363, 1992).

1.3.2 Steel (Reinforcement)

While concrete is strong in compression, it is weak in tension. It is also brittle and therefore liable to break without warning. As a simply supported RC beam deflects, it forms a U-shape with the bottom in tension. Cracks begin to develop at the bottom, so longitudinal reinforcement is placed on the tension side of the beam to carry forces as the concrete cracks, creating a hybrid between concrete and steel that is strong in both compression and tension, as well as ductile. Different coatings are also applied to steel reinforcement to protect it against corrosion. A list of definitions has been given below for common variations in steel reinforcement.

Recycled Materials. Steel reinforcement can either be virgin or recycled. Virgin steel is pure or has never been recycled. However, reinforcement used today is at least 97% recycled material from other steel products (Concrete Joint Sustainability Initiative, 2009); recycled steel reinforcement has a unit embodied-energy coefficient that is typically around 8.9 MJ/kg while a typical unit embodied-energy coefficient of virgin steel reinforcement is 32.0 MJ/kg, more than three times that of recycled material (Table of embodied energy coefficients, 2007).
**Types/Coatings.** Most vendors that sell steel reinforcement offer an assortment with distinct practical characteristics. Black reinforcement is uncoated steel that is cheap but subject to corrosion. Epoxy-coated reinforcement (ECR) bears a membrane that has been applied to defend it against corrosive elements in marine and other harsh environments. Galvanized reinforcement, on the other hand, is coated with several layers of zinc oxide to protect it against corrosion. The zinc forgoes oxidation to spare the reinforcement of any degradation. Stainless steel is a more expensive option but is very resistant against corrosion; this element can either be applied as a coating or adopted as an alternative material for the composition of the reinforcement (Johnson, 2010).

**Grades.** The grade of steel refers to the yield strength ($f_y$) of the steel reinforcement. 60 ksi steel is the most common grade of reinforcement, but other yield strengths (40, 75, 90, 100, and 120 ksi) are also available. High yield strength steel can be used to reduce a structure’s demand for reinforcement and therefore cut back on the congestion in each structural member, as well as the labor required to install the reinforcement.

### 1.4 Optimization Basics

Optimization programs are divided into two functions: the objective function and the constraint function; design variables and other parameters are then defined for each of these actions. The objective function is the value being minimized in the optimization; common examples are cost, embodied energy, and CO$_2$ emissions. Constraints are the maximum and minimum values, calculated based on standard engineering code, that each set of design variables must satisfy in order to be a feasible solution. Design variables are
adjusted to optimize the objective function and must be included in the calculation of that objective function. Other parameters not included in the direct design of the structure are kept constant during each optimization routine, such as beam span and cost and embodied energy of materials.

There is a standard form for writing optimization problems as algorithms. The objective function and constraints are written as two separate functions with every variable previously defined. All constraints must be set to zero; the larger value is subtracted from the smaller value. For example, if the width of the beam must be greater than a minimum value, $b_{\text{min}}$, then the standard form would be $b_{\text{min}} - b = 0$. In discrete optimization problems, each design variable consists of a range of integers that are arranged into a vector. These values are then mapped through the constraint function and the objective function. Other parameters are defined as constants for each optimization.

1.5 Green Concrete Strategies

A number of green strategies have been adopted in the production of reinforced concrete to create more efficient members, reducing the amount of raw materials that the structure demands. One strategy is to utilize recycled materials, which are by-products or waste materials and are used in reinforced concrete to reduce the cost and/or carbon footprint of the structure. Recycled materials help to reduce the carbon footprint otherwise caused by virgin materials, such as Portland cement and steel.

Industrial by-products are commonly used as cement replacements in concrete to enhance its performance as well as reduce the environmental footprint. Fly ash, a by-product of coal during the combustion of electricity, can act as a partial replacement for
Portland cement. Ground granulated blast-furnace slag and glass powder are two more alternatives to Portland cement that possess a much lower unit embodied-energy coefficient than the cement, especially if classified as wastes.

Substituting some of the components within reinforced concrete with high-strength or LW materials may have greater unit costs, but they can also be used to minimize the materials necessary to satisfy the flexural capacity of the member, thus reducing the overall cost. One product that is becoming more popular in the U.S. is MMFX₂ steel reinforcement. These bars are uncoated, but they possess unique chemical and mechanical properties that enhance their performance as reinforcing steel. Although most steel forfeits brittleness in the name of strength, MMFX₂ steel actually manages to be both stronger and tougher. According to MMFX Technologies Corporation (2012), “Structural systems reinforced with MMFX₂ rebar have been shown to achieve design service lives in excess of 75 years.” As high-strength and corrosion-resistant steel, MMFX₂ steel reinforcement meets or surpasses the specifications of ASTM A615 Grades 75 and 80.

In the North American market, #3 through #11, #14, and #18 standard bars are sold. Forty- and sixty-foot bundles are available, as well as custom-mill-cut lengths of up to 72 ft for all sizes and 80 ft for #11, #14, and #18 bars. MMFX vendors provide two grades of steel: Grade 100 and Grade 120. Both of these are certified and are suitable for construction of reinforced concrete.

MMFX₂’s excellent resistance to corrosion has also proven to reduce the cost of repairs. While initial costs are sometimes greater than initial costs of conventional
reinforcement, the long-term costs are greatly reduced. Therefore, high-strength steel can be a cost-effective alternative while also reducing construction time by cutting down on the demand for steel (MMFX Technologies Corporation, 2012).

LW concrete is another product that has been promoted to reduce the environmental costs of RC structures. LW concrete is achieved by using LW aggregate, which has a more porous consistency than does NW aggregate.

LW concrete also tends to be more costly (National Ready Mixed Concrete Association, 2003), and while it does not increase the strength of the material, it reduces the self-weight of the beam, producing a smaller moment throughout the specimen. This helps lessen the volume of the concrete or steel required to support a heavier beam. LW concrete may affect lower load demands due to its low unit weight, but that is only one of several advantages it enjoys. Lighter weight means a larger number of RC members per truck, especially when weight limits on roads are a factor. Fewer truck loads ensures lower transportation costs and less CO₂ emissions.

With LW aggregate’s ability to absorb higher volumes of water, NW aggregate can be substituted with LW aggregate when concrete is designed with low water-to-cement ratios. This “internal curing” can suppress the “self-desiccation and early-age cracking” that high-cementitious concretes are susceptible to. LW concrete has also proven to have a greater fire tolerance than its NW concrete equivalent due to “a combination of lower thermal conductivity, lower coefficient of thermal expansion, and the inherent thermal stability developed by aggregates that have been already exposed to temperature greater than 2000 degrees Fahrenheit during pyroprocessing.”
When using air-entrained LW concretes, durability is not compromised. Numerous studies regarding LW concrete’s durability have delivered positive results. However, due to the lower aggregate stiffness that is characteristic of LW concrete, LW concrete generally experiences a slightly larger degree of shrinkage than does NW concrete (Holm & Ries, 2006).
CHAPTER TWO

REVIEW OF OPTIMIZATION STUDIES

2.1 Introduction

Optimization of reinforced concrete (RC) structures was first studied in the 1970s by Rozvany (1970), Leroy (1974), and Chou (1977), among others, and has subsequently been the subject of numerous technical papers and reports. Many of these works are listed in the Extended Bibliography section of this thesis (Appendix C). Papers most relevant to the current study are discussed in this chapter. Papers were ruled as relevant based on common links to the current research, such as multiobjective optimization of economic and environmental costs or parametric studies consisting of the reinforcement ratio, the cost ratio, the compressive strength of the concrete, or the external applied moment. Key points from these works are highlighted, and knowledge gaps are identified. The most relevant papers and their features are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Structure</th>
<th>Method</th>
<th>Objective Functions</th>
<th>Beam Design Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam</td>
<td>Frame</td>
<td>Economic</td>
<td>b</td>
</tr>
<tr>
<td>Samman</td>
<td>1995</td>
<td>X</td>
<td>BB-BC</td>
<td>X</td>
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<td>Alreshaid</td>
<td>2004</td>
<td>X</td>
<td>SA</td>
<td>X</td>
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<tr>
<td>Yeo</td>
<td>2011</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Paya³</td>
<td>2008</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Paya-Zaforteza³</td>
<td>2009</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Camp</td>
<td>2013</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Yeo</td>
<td>2013</td>
<td>X</td>
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</table>

1 Width, b, is held constant for each iteration; vector of four values defined.
2 Reinforcement ratio, ρ, is used in place of the area of steel, A_s.
3 Type of steel and type of concrete are also considered design variables.
C: Continuous
D: Discrete
BB-BC: Big bang-big crunch algorithm
SA: Simulated Annealing algorithm
2.2 Optimization Methods

Different numerical methods are available for locating optimum design solutions. Papers discussed in this chapter used the following methods: the direct search (DS) algorithm, the simulated annealing (SA) algorithm, and the big bang-big crunch (BB-BC) algorithm. Another algorithm commonly used is the genetic algorithm (GA), but there are additional algorithms, both heuristic and non-heuristic, used to perform optimization. Unlike non-heuristic methods, “[heuristic] methods are based on the principles of natural biological evolution” (Alqedra et al., 2011).

The process of optimization can be divided into two categories: direct and gradient-based methods. Direct search methods use an objective function and constraints to search for the solution while “gradient-based methods use the first and/or second-order derivatives of the objective function and/or constraints to guide the search process” (Alqedra et al., 2011). During the pursuit of an optimal solution, the direct search algorithm accesses a group of points around a current point, searching for another that possesses a lower objective function than the previous point. Such a method can only be used for non-differentiable objective functions (Mathworks, 2013).

Heuristic methods have become prominent in the optimization of reinforced concrete. Examples of heuristic methods commonly used are GA, SA, and more recently, BB-BC. The primary issue among these algorithms is the conflict of “accuracy, reliability, and computation time” (Erol & Eksin, 2006).

Using Darwin’s theory of evolution, the GA searches a design space consisting of a population of designs. These designs, or solutions, are generated stochastically. Designs
are then selected at random, with better designs having a higher chance of being chosen, echoing Darwin’s survival of the fittest. In order to converge towards the optimum point, mutations and crossovers occur; mutations are small variations in one or more design variables whereas crossovers occur so that individual solutions are combined to produce new and unique solutions (Alqedra et al., 2011; Kohonen, 1999). The GA has been used to study RC structures by Coello Coello et al. (1997), Camp et al. (2003), Govindaraj and Ramasamy (2005), and Alqedra et al. (2011).

Much like the GA, the SA algorithm approaches the optimum solution by modifying the existing solution using the mutation effect. However, the SA algorithm has only one population, and therefore only one solution; thus, crossover does not occur. These differences do not make one algorithm superior over the other but rather create different tools that work better or worse depending upon the formation of the problem. Each of these algorithms shares the idea that desirable solutions can be located close to formerly established suitable points (Kohonen, 1999). The SA algorithm has been used to study RC structures by Leps and Sejnoha (2003), Martinez et al. (2007), Paya et al. (2008), and Paya-Zaforteza et al. (2009).

A more recent development in the realm of optimization is the BB-BC algorithm. Inspired by the theory regarding the evolution of the universe, the BB-BC algorithm relies on a “population-based search procedure that incorporates random variation and selection,” similar to the GA and SA algorithms. Unlike the GA and SA algorithms, it is comprised of two phases: the Big Bang phase and the Big Crunch phase. During the Big Bang phase, random points are created throughout the design space, mimicking the
dissipation of energy; in the Big Crunch phase, those points are then condensed into a single point expressed as a center of mass, mimicking the attraction initiated by gravity. The center of mass is used to distribute new offspring around the known point; once the points converge again, the center of mass is re-evaluated. This process continues through multiple iterations until the stopping criteria are satisfied (Erol & Eksin, 2006). BB-BC has been used to study RC structures by Camp and Akin (2012), Camp and Huq (2013), and Camp and Assadollahi (2013).

In Erol and Eksin (2006), the BB-BC algorithm was shown to be more reliable and efficient than the GA because the GA does not always converge to the global optimum due to its “selective capacity of the fitness function” and because the GA has a higher computational time. The study showed that the BB-BC method reduced the computational time by a large margin while locating the global minimum “within the maximum number of allowed iterations” (Erol & Eksin, 2006).

2.3 Previous Works

This section summarizes papers on the optimization of beams and frames that relate most closely to the studies in this thesis. Each of these papers considered reinforced concrete structures that have been optimized for minimum economic cost, minimum environmental cost, or both.

2.3.1 RC Beams

Samman and Erbatur (1995), Alreshaid et al. (2004), and Yeo and Gabbai (2011) optimized rectangular RC structures with a given length. While Samman and Erbatur (1995) and Alreshaid et al. (2004) minimized economic cost exclusively, Yeo and Gabbai
(2011) minimized economic and environmental costs simultaneously. Each of these papers is discussed in greater detail in the following subsections.

**Samman and Erbatur (1995)**

Samman and Erbatur (1995) approached the problem of RC optimization using a systematic DS algorithm. The width of the beam was given while the height and main reinforcement were calculated so that the nominal moment capacity constraint was satisfied for each given width; therefore, the height and longitudinal steel reinforcement ratio were calculated with a range of continuous values while the width was defined by a vector of discrete variables. A set of moments ranging from 5 k-in to 500,000 k-in was considered to demonstrate low and high loading conditions.

Rather than using fixed costs, a range of steel-to-concrete cost ratios were computed per unit weight and used to establish trends for varying reinforcement ratios. This is shown in Figure 2.1(b), with the cost-optimum reinforcement ratio decreasing exponentially as the cost of the steel rises relative to the cost of concrete. A typical relationship between cost ratio and reinforcement ratio is given in Figure 2.1(a). Each segment of the curve is given a label (R1, R2, etc.) to signify different optimization regions. R1 is the maximum optimal steel-ratio region, consisting of beams with a minimum width and height; R2 is the cost-sensitive region, with beams having a greater volume of concrete due to an increase of beam height, which is directly linked to the reduction of the steel ratio; R3 is the intermediate constant-ratio region, and only pertains to fixed-fixed and fixed-hinged beams; R4 is the cost-insensitive region, which corresponds to overly designed beams. The effects produced by an array of parameters
were examined for notable trends. Particular parameters displayed more influence on the reinforcement ratio than others: end conditions and material costs had substantial effects, applied loads and yield strength had considerable effects, and concrete strength and beam width had negligible effects.

**Figure 2.1:** (a) Typical curve of cost optimum steel ratios, (b) Simply supported beam, $f'_c = 3$ ksi and $f_y = 50$ ksi (Samman & Erbatur, 1995).

*Alreshaid et al. (2004)*

In Alreshaid et al. (2004), RC beams and columns were optimally designed based on three design variables: the width and height of the member and the reinforcement ratio. For this analysis, the height and width were varied in discrete increments. STAAD II was used to calculate adequate cross-sectional designs, reported as safe sections based on ACI code; a quantity take-off was then assembled based on different optimum designs. By plotting the steel ratio against the cost for different breadths, a range of optimum steel ratios was established for beams: 0.01 to 0.02, with an average of 0.01535. When increasing the cost of steel, these bounds of optimal reinforcement ratios narrowed slightly, with a recommended steel ratio between 0.012 and 0.0198; when increasing the cost of concrete, the bounds narrowed further, with a recommended reinforcement ratio
between 0.0129 and 0.0185. The analytical program evaluated optimal solutions for four bending moments to indicate what effect an increased moment would have; an example of this is demonstrated in Figure 2.2 and Figure 2.3, for 500 kN-m and 700 kN-m, respectively. While the progression of the lines displayed a similar pattern, the cost of the beams grew each time the moment increased, as was expected.

Figure 2.2: Total costs for different breadths for cross sections exposed to 500 kN-m (Alreshaid et al., 2004)
Figure 2.3: Total costs for different breadths for cross sections exposed to 700 kN-m (Alreshaid et al., 2004).

_Yeo and Gabbai (2011)_

A rectangular beam was optimized for cost and embodied energy in Yeo and Gabbai (2011). Analysis considered four design variables: width and height of the beam, total area of the longitudinal reinforcement, and total area of the shear reinforcement. While the width and height were defined by discrete variables, the reinforcements were defined by continuous variables. Minimum values were computed through feasible optimized solutions characterized by constraints based on ACI 318-11. By holding the width of the beam constant and varying the height, the design optimized for embodied energy had a higher reinforcement ratio when compared to the design optimized for cost.

A difference in the physical behavior of cost- and energy-optimized members was also observed. At flexural capacity, the tensile strain of the reinforcement surpassed 0.005 in the cost-optimized design while the tensile strain in the embodied-energy
optimized designs was nearly 0.005; therefore, cost-optimized sections had marginally higher ductility.

The authors concluded that “the optimization of embodied energy can achieve around 10% reduction in embodied energy at an added cost of roughly 5%.” In Figure 2.4, the cost ratio of steel to concrete was plotted to show variation in the percent difference between the cost-optimized solution and the embodied-energy optimized solution for the cost and embodied energy of the beam. For example, “as the relative cost of steel reinforcement increases from R=0.6 to R=1.0, the optimized embodied energy design can achieve a reduction in embodied energy up to approximately 16%. Over the same range, the embodied energy-optimized section also increases the cost by approximately 9%.” When adjusting the cost ratio, R, beyond a factor of 1, the “differences between embodied energy reduction and cost addition reduce.”

![Figure 2.4: Variation in percentage difference in cost and embodied energy with R, b equals 400 mm (Yeo & Gabbai, 2011).](image-url)
2.3.2 RC Frames

The papers reviewed in this section optimized RC frames with reinforced rectangular concrete beams and columns having fixed lengths. Paya et al. (2008), Paya-Zaforteza et al. (2009), and Camp and Huq (2013) optimized frames using discrete member size variables; the area of steel was expressed by a combination of different sizes and numbers of rebar. The research of Yeo and Potra (2013), on the other hand, was governed by the use of continuous variables. Each of these papers is discussed in the following subsections.

Paya et al. (2008)

Various objective functions were optimized simultaneously in pairs for an RC frame in Paya et al. (2008); these included cost, constructability, sustainability, and overall safety. Constructability is the measurement of the number of reinforcement bars; fewer bars imply “fewer execution errors, less complex quality control, and faster construction processes.” Overall safety is a measurement of the cost of safety levels; “an overall safety function of 1 implies strict compliance with the concrete code of practice.”

This structure was analyzed using a matrix method while the design was optimized using a multiobjective SA algorithm. The design variables considered included types of steel and concrete, width of the beams and columns, depth of the beams and columns, top and bottom reinforcement in the beams, shear reinforcement in the beams, and longitudinal and transverse reinforcement in the columns. Two different strengths of steel were used as well as six different strengths of concrete. The multiobjective simulated annealing algorithm described in Suppapitnarm et al. (2000), often referred to
in literature as the SMOSA algorithm, was used to make comparisons with the classical simulated annealing method (C-SA). The best solutions were those that only minimally increased the cost while reducing the environmental impact and number of bars. SMOSA solutions resulted in “increase[d] cost by 5.7% in comparison to the C-SA solution while it reduce[d] not only the number of bars from 118 to 78, but also the environmental cost [by 2.4%].” Another instance saw the increase in cost by 10.7% over the C-SA with a reduction of environmental cost by 16.5%, which highlighted the importance of trade-offs between economic and environmental factors.

Paya-Zaforteza et al. (2009)

In Paya-Zaforteza et al. (2009), an identical set of design variables to Paya et al. (2008) was utilized but with various frame sizes; this time, the carbon dioxide emissions and the cost were optimized simultaneously while applying the SA algorithm.

“Approximate best CO₂ solutions are, at most, 2.77% more expensive than the approximate best cost solutions. Alternatively, the approximate best cost solutions worsen CO₂ emissions by 3.8%.” Figure 2.5 shows the multiobjective optimization of different sized frames, and bears a linear relationship between the two objective functions. Each data point consists of the letters “b” and “f.” The letter “b” represents the number of bays and the letter “f” represents the number of floors.
Figure 2.5: Relation between CO₂ emissions and cost (Paya-Zaforteza et al., 2009).

Camp and Huq (2013)

Camp and Huq (2013) proposed a need for benchmark problems which researchers could use to compare the efficiency and accuracy of different optimization algorithms. As a possible benchmark problem, they considered RC frames with a singly reinforced rectangular beam with three discrete design variables: beam width, beam height, and area of longitudinal reinforcement. Rectangular columns were also used with the same three design variables. The objective functions for this problem were the economic and environmental impacts, but in contrast to those cited by Paya et al. (2008) and Paya-Zaforteza et al. (2009), these impacts were not minimized simultaneously. Using the Big Bang-Big Crunch algorithm, 36 combinations of steel reinforcement for the beams and 54 combinations of steel reinforcement for the columns were examined, as well as six different column topologies for each steel reinforcement configuration. The
first example analyzed by the authors demonstrated the increased accuracy of the BB-BC algorithm over the GA, with the best solution given by the BB-BC being 5.2% lower in economic cost than the best solution produced by the GA. Two subsequent example problems were conducted to evaluate the relationship of cost to CO₂ emissions. In the first example, the results demonstrated that “for a modest 2.2% increase in cost over the low-cost design, a 10.2% reduction in CO₂ emissions is achieved from the low cost design.” A similar result was given in the second example.

**Yeo and Potra (2013)**

Using the MATLAB optimization solver “fmincon,” which is a heuristic optimization algorithm not capable of handling discrete variables, Yeo and Potra (2013) examined a reinforced concrete single frame for lowest cost and CO₂ emissions, each individually. Beam design variables were the height, total area of the longitudinal reinforcement, and spacing of the shear reinforcement; column design variables were height, total area of the axial reinforcement, and spacing of the shear reinforcement. By optimizing for CO₂ emissions, a reduction of 5 to 15% of emissions was computed when compared to the optimization of cost. In Figure 2.6(a), (b), and (c), \( r_{\text{cost}} \) and \( r_{\text{CO}_2} \) represent the “ratio between the cost of the cost-optimized frame and the cost of the CO₂-optimized frame” and “the ratio between the CO₂ footprint of the cost-optimized frame and the CO₂ footprint of the CO₂-optimized frame,” respectively. \( R_C \) and \( R_{\text{CO}_2} \) represent the cost and CO₂ emission ratios, respectively, of steel to concrete. As the relative cost of steel increases, the relative cost of the cost-optimized frame increases, but the relative CO₂ emissions of the cost-optimized frame remain constant beyond a cost ratio of 0.8, as
demonstrated in Figure 2.6(a). The relative CO$_2$ emissions of the steel have little to no impact on the relative cost and relative CO$_2$ emissions of the cost-optimized frame, shown in Figure 2.6(b). Supplementary axial compressive forces of 3000 kN and 6000 kN were applied to the columns to provide a hypothetical gravity load induced by additional floors. In Figure 2.6(c), the dependence upon concrete compressive strength of $r_{cost}$ and $r_{co2}$ is depicted for 30 MPa and 40 MPa, which causes little change between the two.
Figure 2.6: (a) Dependence upon the cost ratio of the percentage of total cost and CO₂ emissions for the cost-optimized frame and the CO₂-optimized frame, (b) Dependence upon the CO₂ ratio of the percentage of total cost and CO₂ emissions for the cost-optimized frame and the CO₂-optimized frame, (c), Dependence upon concrete compressive strength of the percentage of total cost and CO₂ emissions for the cost-optimized frame and the CO₂-optimized frame (Yeo & Potra, 2013).

2.4 Distinctions of Current Study

While each of these publications has a number of things in common with the research being conducted in this paper, this section discusses the approaches that will be used in the current research that are distinct from previous works.
Although algorithms are very powerful tools capable of shortening the computational time considerably, they cannot be relied upon to obtain the global optimum every time, sometimes returning a local minimum instead. By developing a list of every combination of design variables, all permutations can be checked and the global optimum is thus guaranteed for each optimization routine. This is not a novel concept and requires additional computational time that is not always prudent. However, it does promise complete accuracy not prevalent in other works.

The previous works have given only one optimum point for different cost ratios, compressive strengths, steel ratios, and other parameters, but none have considered trends in the optimal solutions. Taking averages rather than points has the potential to depict overarching trends in near-optimized designs. Knowledge of such trends may be of greater service to practicing engineers during the design phase when it is often impractical to conduct rigorous optimization work to locate optimum points because design parameters are still in flux. By enhancing the capacity of the optimization tool, the complexity of the earlier models can be further developed.

Substituting concrete and steel with higher-strength materials is not the only way to amplify the effects of optimization; LW aggregate, which abates the self-weight of the concrete, can generate a smaller flexural moment. LW concrete, while more expensive, reduces the volume of the beam and the materials required to support a known load or moment. Previous works have not evaluated the benefits of LW concrete in optimal RC design.
Although the embodied-energy factors of materials in reinforced concrete do not fluctuate as significantly as cost factors, more efficient methods of producing concrete and steel are being developed, which in turn reduces the embodied energy required to manufacture RC beams. A unit embodied-energy ratio can be applied to complement the unit cost ratio in the analysis of optimized beams. Using a range of these values in various parametric studies, designers can select which ratio to use based on the current status of steel or concrete.

2.5 Literature Review Summary

Based on the literature review of previous works, a few general conclusions can be made about the reinforced concrete optimization.

According to Samman and Erbatur (1995) and Yeo and Potra (2013), the compressive strength of concrete has little to no effect on the optimization of RC beams. This conclusion is consistent with the well-known observation that compressive strength also has minor effects on flexural capacity of beams relative to other design parameters.

Yeo and Gabbai (2011), Paya et al. (2008), Paya-Zaforteza et al. (2009), Camp and Huq (2013), and Yeo and Potra (2013) all optimized for both economic and environmental factors, demonstrating the trade-offs between each aspect. Yeo and Gabbai (2011), Paya-Zaforteza et al. (2009), Camp and Huq (2013), and Yeo and Potra (2013) conducted environmental optimization to show the effects over cost optimization. Yeo and Gabbai (2011) optimized embodied energy, which resulted in a 10% reduction in embodied energy for a 5% added cost; Paya-Zaforteza et al. (2009) optimized CO₂ emissions for a 2.77% added cost; Camp and Huq (2013) optimized CO₂ emissions,
which resulted in a 10.2% reduction in CO₂ emissions for a 2.2% added cost; Yeo and Potra (2013) optimized CO₂ emissions for a 5-15% reduction in CO₂ emissions when compared to the optimized cost design. Each of these works exhibits similar results: for a small increase in cost (around 2-5%), a larger percentage of energy can be saved (about 10%).

This is a partial list of cost and environmental optimization of reinforced concrete; as a literature review, it only highlights what is most relevant to the research presented in this thesis. For an extended bibliography of additional references pertaining to the optimization of reinforced concrete structures, see page 127; this list can be used as a gateway for more detailed analysis of RC structural optimization.
CHAPTER THREE

OPTIMIZATION OF REINFORCED CONCRETE BEAMS METHODOLOGY

3.1 Overview

This research considered rectangular reinforced concrete beams with simple supports optimized for cost and embodied energy. Feasible beam designs were constrained by the provisions of the ACI 318-11 code (ACI Committee 318, 2011). Section 3.2 establishes the nomenclature used throughout this thesis. Wherever possible, nomenclature was kept consistent with ACI 318. Sections 3.3 through 3.7 of this thesis provide general descriptions of all design variables, parameters, and constraints used with their respective ranges, as well as discussion of any assumptions made regarding the configuration of steel reinforcement within the beam. The two computer programs, Excel (Microsoft, 2011) and MATLAB (Mathworks, 2012), used in this research are also discussed and compared. Section 3.8 provides more detail about the ACI code used and how it was employed within the Excel computer program.

3.2 Nomenclature

Reinforced Concrete Design Definitions per ACI 318-11 Section 2.1

\[ a = \text{depth of equivalent rectangular stress block, in.} \]
\[ A_s = \text{area of nonprestressed longitudinal tension reinforcement, in.}^2 \]
\[ A_v = \text{area of shear reinforcement within a distance } s, \text{ in.}^2 \]
\[ b = \text{width of compression face of member, in.} \]
\[ \beta_1 = \text{factor relating depth of equivalent rectangular compressive stress block to neutral axis depth} \]
\( c = \) distance from extreme compression fiber to neutral axis, in.

\( d = \) distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

\( d_t = \) distance from extreme compression fiber to centroid of extreme layer of longitudinal tension steel, in.

\( \varepsilon_t = \) net tensile strain in extreme layer of longitudinal tension steel at nominal strength

\( f'_c = \) specified compressive strength of concrete, psi

\( f_y = \) specified yield strength of reinforcement, psi

\( M_n = \) nominal flexural strength at section, lb-in.

\( \phi M_n = \) design moment strength at section, lb-in.

\( M_u = \) factored moment at section, lb-in.

\( s = \) center-to-center spacing of shear ties measured along longitudinal axis of member

\( V_c = \) nominal shear strength provided by concrete, lb

\( V_n = \) nominal shear strength, lb

\( \phi V_n = \) design shear strength at section, lb

\( V_u = \) factored shear force at section, lb

\( \lambda = \) modification factor reflecting the reduced mechanical properties of LW concrete

\( \phi = \) strength reduction factor

**Other Reinforced Concrete Design Definitions**

\( c_{s_{\text{min}}} = \) minimum clear spacing between longitudinal reinforcement, in.

\( \gamma_{SC} = \) unit cost of concrete, \$/ft^3

\( \gamma_{SS} = \) unit cost of steel, \$/ft^3
\( \rho \) = reinforcement ratio \( [A_s / bh] \n C = \text{total cost of member, $} \n d_b = \text{diameter of longitudinal reinforcement, in.} \n d_{\text{max, agg}} = \text{maximum aggregate size in the mix, in.} \n d_s = \text{diameter of stirrups, in.} \n \gamma_{EC} = \text{unit embodied energy of concrete, MJ/kg (MJ/lb)} \n \gamma_{ES} = \text{unit embodied energy of steel, MJ/kg (MJ/lb)} \n b_{\text{trib}} = \text{tributary width} \n EE = \text{total embodied energy of member, MJ} \n L = \text{span length of beam, ft} \n LW = \text{lightweight} \n n = \text{number of longitudinal reinforcement} \n ns = \text{number of stirrups} \n NW = \text{normal weight} \n V_{\text{str}} = \text{total volume of stirrups, in.}^3 \n w_D = \text{unfactored dead load, plf} \n w_L = \text{unfactored live load, plf} \n w_S = \text{unfactored self-weight, plf} \n w_u = \text{factored distributed load, plf} \n \gamma_c = \text{unit weight of concrete, pcf} \n \gamma_s = \text{unit weight of steel, pcf}
3.3 Problem Statement

A rectangular reinforced concrete beam with simple supports was optimized to find the minimum cost and embodied energy for a given set of spans and applied loads. Thus, the objective functions minimized in this study were the total cost and the total embodied energy of the reinforced concrete beam, shown in Table 3.1. These functions are dependent upon the total volume of concrete and steel, which were each multiplied by the unit cost or unit embodied energy of each corresponding material. The unit cost of concrete and steel may include any combination of material production, product fabrication, labor, and transportation, as well as formwork for concrete. The unit embodied energy of concrete and steel may include any combination of material extraction, plant processes, and transportation. Some studies include all processes while other studies may omit one or more steps to emphasize the impact of a particular process. For this study, only the operations from material extraction to production were considered.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embodied Energy</td>
<td>$EE = \gamma_{EC}\gamma_c ((A_c - A_s) L - V_{str}) + \gamma_{ES}\gamma_s (A_s L + V_{str})$</td>
</tr>
<tr>
<td>Cost</td>
<td>$C = \gamma_{SC} ((A_c - A_s) L - V_{str}) + \gamma_{SS} (A_s L + V_{str})$</td>
</tr>
</tbody>
</table>

Four design variables were used in the optimization of the beam: the height of the beam, the width of the beam, the number of rebar, and the size of the rebar. A typical cross-section with these features is shown in Figure 3.1. Different combinations of these four discrete variables were checked to identify designs that satisfied constraints for shear...
and flexure as well as others. ACI 318 code requirements were used to establish appropriate constraints. Up to 66,000 combinations of variables were considered in each optimization problem, with each permutation consisting of a unique design. When aggregated, this group of possible design permutations is referred to as the design space of the optimization problem. A feasible set was then formed from all combinations that satisfied the constraints (specified in Table 3.5). From the feasible set, the design with the lowest total cost or embodied energy was chosen as the optimum solution.

![Diagram of a rectangular reinforced concrete beam](image)

**Figure 3.1: Typical cross-section of a rectangular reinforced concrete beam.**

### 3.4 Computer Programs

In this analysis, two separate optimization methods were utilized to obtain the global minimum of either the total embodied energy or total cost of the reinforced concrete beam. The first was a genetic algorithm, using the computer software MATLAB. An objective function was written and constraints were defined; given four
discrete variables, a random number generator produced values for the four variables that were fed through the constraint function and the objective function, converging on a beam design that would minimize the objective function while still existing within the limits set forth by the constraints. The design variables were returned for the optimum design. All of this was accomplished through MATLAB’s built-in GA toolbox.

The second method used was the brute force method, which was executed using the computer program Microsoft Excel. Instead of using a random number generator, all 66,000 combinations of the four discrete variables were individually analyzed. Through an evaluation of every permutation, the global optimum was identified for each optimization routine.

Both methods, MATLAB’s genetic algorithm and Excel’s brute force method, were designed to check the accuracy of one another although each program has its advantages and disadvantages. MATLAB is more flexible when adapting the design being tested, whether it be expanding the design variables or altering the backbone of the design altogether. One example of this would be transforming a rectangular cross-section into a T-beam. Excel is much more rigid, requiring additional time and manipulation to adjust the code for different conditions. However, with genetic algorithms, the global optimum cannot always be assured; by going through every design combination in the brute force method, the global optimum is guaranteed. Also, by using MATLAB, only a single point can be obtained. With Excel, a top percentage of points can be determined and listed in descending order. Analysis of multiple points can be helpful in evaluating trends that are not evident by only considering a single optimum point.
Both programs were used in this study to verify one another, but after a preliminary comparison of the MATLAB and Excel calculations, it was concluded that Excel was an appropriate analysis tool for studying the relatively small design space presented in this research. While using MATLAB’s GA would be a more efficient approach in larger design spaces, it was not necessary for the current study. Therefore, all discussions of optimization methodologies and results in this thesis are based on calculations using Excel.

3.5 Design Variables

Design variables are those values that are altered within the design space when locating the optimum solution. Four design variables were considered in this research program: the height of the beam, the width of the beam, the number of rebar, and the size of the rebar. Each variable was represented by a range of discrete values.

The range of values used for beam height, \( h \), was selected based on the span length, \( L \), which is one of the design parameters discussed in the following section. This range was selected based on the span length to ensure that values were realistic, that is, not too large or too small for the span being considered.

For the beam height, a range from \( L/30 \) to \( L/10 \) was employed. The range included twenty discrete values spaced evenly between the upper and lower limits; these values were then rounded to the nearest half inch, as is typical for the cross-sectional dimensions of RC beams.

The range of values for beam width, \( b \), was a function of the range given for beam height; thus, beam widths were also indirectly related to the span length of the beam,
ensuring that realistic values would be chosen for the span being considered. A range of 0.1\(h\) to \(h\) was employed in increments of 0.1, so that for each value of \(h\) there were ten values of \(b\). The width of the beam was also rounded to the nearest half inch but never breached the boundaries defined by an upper and lower bound.

The third variable was the number of rebar, \(n\), which ranged from two to twelve bars. To produce a realistic representation of a reinforced concrete beam cross-section, up to three layers of rebar were considered. A total of 33 configurations consisting of two to twelve bars were used, shown in Figure 3.2. The final variable was the size of rebar, which was based on standard bar sizes given by ASTM International A615, ranging from #3 to #14 bars, listed in Table 3.2. Only a uniform selection of rebar was accepted for each design. This means there could not be a medley of rebar sizes in a single design. Each of the four variables was represented by discrete values, amounting to a design space of 66,000 separate design combinations. Variables and combinations are summarized in Table 3.3.
<table>
<thead>
<tr>
<th>#</th>
<th>Configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
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<td>6</td>
<td></td>
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<td>7</td>
<td></td>
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<td>8</td>
<td></td>
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<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.2: All configurations of longitudinal reinforcement utilized in the optimization of reinforcement concrete beams, categorized by number of rebar.
The ACI 318-11 code provides minimum span-to-height ratios for which member deflections need not be calculated. For some designers, these ratios become de facto limits on member height. To account for this, some of the optimizations conducted in the test program treated beam height as a fixed value (a parameter) based on the ACI ratios rather than as a variable. For example, instead of appointing \( h \) a range from \( L/30 \) to \( L/10 \), the height of the beam was given one constant value equal to the minimum beam height required by the ACI 318-11 code in Table 9.5(a). For simply supported beams, \( L/16 \) is given, but this only applies to NW concrete and 60 ksi steel reinforcement. For other cases, \( L/16 \) was modified, as discussed in Section 3.8.5 of this chapter. In optimizations where height was treated as a fixed parameter, the design space was condensed from 66,000 possible solutions to 3,300 possible solutions (Table 3.3).

The span-to-height ratios found in Table 9.5(a) of ACI 318-11 are the specified minimum heights unless deflections are calculated. If, however, deflections are calculated, it is permissible to design members that are more slender than the limits in Table 9.5(a). While noting this distinction, member heights based on Table 9.5(a) will be referred to as the “ACI minimum” throughout this thesis.
Table 3.2: ASTM standard chart for reinforcing steel bars considered in the optimization (ASTM International A615, 2012).

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Diameter (in.)</th>
<th>Area (in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td>#4</td>
<td>0.500</td>
<td>0.20</td>
</tr>
<tr>
<td>#5</td>
<td>0.625</td>
<td>0.31</td>
</tr>
<tr>
<td>#6</td>
<td>0.750</td>
<td>0.44</td>
</tr>
<tr>
<td>#7</td>
<td>0.875</td>
<td>0.60</td>
</tr>
<tr>
<td>#8</td>
<td>1.000</td>
<td>0.79</td>
</tr>
<tr>
<td>#9</td>
<td>1.128</td>
<td>1.00</td>
</tr>
<tr>
<td>#10</td>
<td>1.270</td>
<td>1.27</td>
</tr>
<tr>
<td>#11</td>
<td>1.410</td>
<td>1.56</td>
</tr>
<tr>
<td>#14</td>
<td>1.693</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Table 3.3: Design variables considered in the optimization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Number of Variables within Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Beam ($h$)</td>
<td>$L/30$ to $L/10$</td>
<td>20 (1)</td>
</tr>
<tr>
<td></td>
<td>-or-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ACI minimum)</td>
<td></td>
</tr>
<tr>
<td>Width of Beam ($b$)</td>
<td>$0.1h$ to $h$</td>
<td>10</td>
</tr>
<tr>
<td>Size of Reinforcement Bars</td>
<td>#3 to #14</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>See Table 3.2</td>
<td></td>
</tr>
<tr>
<td>Number of Reinforcement Bars ($n$)</td>
<td>2 to 12 bars</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>In up to 3 layers</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>66,000 (3,300)</td>
</tr>
</tbody>
</table>

3.6 Parameters

Parameters are values that are held constant in an optimization problem. Parameters in the current program included span length of the beam, applied load, unit cost and unit embodied energy of both concrete and steel, and unit weight of the concrete.
Although parameters are fixed for a given optimization calculation, they can be varied from one individual calculation to another. For example, an optimization program could be made to determine the optimal beam design for a given span length. The span length could then be modified and the optimization program could repeat the calculation for the new parameter. Evaluating changes in an optimum design for a single parameter will emphasize the effects of that parameter on the entire system.

Span length and bending moment were among the parameters adjusted in sequential optimization problems. Thus, trends in optimal designs were evaluated for a range of spans and moments. Applied loads, and consequently applied moments, were linked to the span length using the equations shown in Table 3.4. This approach was used to improve the efficiency of sequential problems, and to ensure that the applied moments were reasonable for each span length.

A range of moments from 50 k-ft to 9050 k-ft with increments of 150 k-ft was used in all studies. The distributed loads were related to the span of the beam, also shown in Table 3.4. As the span of the beam increases, the tributary width broadens at a proportionate rate equal to one-fourth of the span length. So at a span of 20 ft, the tributary width is 5 ft, and at a span of 60 ft, the tributary width is 15 ft.
Table 3.4: Parameters varied for each optimization routine.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Input</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span of Beam (L)</td>
<td>( \left( \frac{4M}{DL + LL} \right)^{1/3} )</td>
<td>ft</td>
</tr>
<tr>
<td>Distributed Live Load (w_L)</td>
<td>( \frac{L}{4} (LL) )</td>
<td>plf</td>
</tr>
<tr>
<td>Distributed Dead Load (w_D)</td>
<td>( \frac{L}{4} (DL) )</td>
<td>plf</td>
</tr>
</tbody>
</table>

The dead load was derived from the assumption that a 4” concrete slab with a unit weight of 145 pcf was resting on the beam.

\[
(4”)(145 \text{pcf}) = 48.33 \text{psf} = 50 \text{psf}
\]

The distributed live load was derived from the assumption that the optimized beam was part of a floor system that occupied an office space. According to ASCE 7-10, the uniform distributed load for office use is 50 psf (ASCE 7, 2010). A factored distributed load was then determined:

\[
w_u = 1.2(50 \text{psf}) + 1.6(50 \text{psf}) = 140 \text{psf}
\]

Other parameters were considered in addition to the span length, which included yield strength of reinforcement, compressive strength and unit weight of concrete, and unit embodied energy and cost for concrete and steel. Separate optimization calculations were conducted for each modification of each parameter. This process allowed evaluation of the influence that each parameter has on the optimum design of the beam.
3.7 Constraints

Constraints are properties of the system that must be satisfied in order for a given solution to be deemed as feasible. For example, one constraint was flexural capacity. Beam designs were only considered feasible if they satisfied the constraints of flexural capacity and other constraints required by ACI 318-11. These other constraints included nominal shear capacity, minimum beam width (to fit reinforcement), minimum and maximum required area of steel, and required stirrup spacing. The maximum amount of reinforcement, and by implication the ductility, was constrained by placing limits on tensile strain in the lowest layer of reinforcement. Based on ACI 318-11, a minimum strain of 0.004 was used for optimization problems that considered grade 60 steel reinforcement. A separate net tensile strain constraint was applied to high-strength steel reinforcement. This approach is consistent with the recommendations made by high-strength reinforcement manufacturers (MMFX Technologies Corporation, 2012). All constraints are summarized in Table 3.5.
Table 3.5: Constraints that must be satisfied in the optimization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constraint Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of Steel</td>
<td>$A_s \geq A_{s,\text{min}}$</td>
</tr>
<tr>
<td>Moment Strength</td>
<td>$M_u \leq \phi M_n$</td>
</tr>
<tr>
<td>Shear Strength</td>
<td>$V_u \leq \phi V_n$</td>
</tr>
<tr>
<td>Width of Beam</td>
<td>$b \geq b_{\text{min}}$</td>
</tr>
<tr>
<td>Net Tensile Strain (40, 60 ksi)</td>
<td>$\varepsilon_t \geq 0.004$</td>
</tr>
<tr>
<td>Net Tensile Strain (75, 90, 100, 120 ksi)</td>
<td>$\varepsilon_t \geq \frac{2f_y}{E_s}$</td>
</tr>
</tbody>
</table>

3.8 Code and Methods

3.8.1 Objective Functions

Two objective functions were used to conduct this research: embodied energy and cost, each calculated based on the volume of steel and concrete materials.

\[
EE = \gamma\gamma_{Ec}(A_c - A_s)L - V_{str} + \gamma\gamma_{Es}(A_sL + V_{str}) \quad \text{Equation 3.1}
\]

\[
C = \gamma\gamma_{Sc}(A_c - A_s)L - V_{str} + \gamma\gamma_{SS}(A_sL + V_{str}) \quad \text{Equation 3.2}
\]

3.8.2 Flexure in Beams (Chapter 10, ACI 318-11)

Design Moment Strength in the Beam

To provide adequate design moment strength in the beam, the following constraint was given:

\[
\phi M_n \geq M_u \quad \text{Equation 3.3}
\]
The nominal moment strength was calculated as

\[ M_n = A_s f_y \left( d - \frac{a}{2} \right) \]  

Equation 3.4

The maximum factored moment in the beam was

\[ M_u = \frac{w u L^2}{8} \]  

Equation 3.5

where the factored distributed load is calculated by

\[ w_u = 1.2(w_D + w_S) + 1.6w_L \]  

Equation 3.6

In order to calculate the design moment strength in the beam, several values must be determined:

\[ a = \frac{A_s f_y}{0.85 f'_c b} \]  

Equation 3.7

\[ \beta_1 = 0.85 \geq 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right) \geq 0.65 ; f'_c \text{ is in units of psi} \]  

Equation 3.8

\[ c = \frac{a}{\beta_1} \]  

Equation 3.9

\[ \epsilon_t = \frac{0.003(d_c - c)}{c} \]  

Equation 3.10

The strength reduction factor is calculated for two different ranges but is not to be taken less than 0.004.

\[ \phi = 0.9 \ [\epsilon_t \geq 0.005] \]  

Equation 3.11

\[ \phi = 0.48 + 83\epsilon_t \ [0.004 \leq \epsilon_t \leq 0.005] \]  

Equation 3.12
3.8.3 Shear in Beams (Chapter 11, ACI 318-11)

To provide shear reinforcement, #3 closed looped stirrups were equipped. Stirrup placement was based on the required and maximum permitted spacing of stirrups for shear in beams, which are detailed in Section 11.4 of ACI 318-11. Conditions for shear design are given in Table 3.6 of this thesis.

Table 3.6: Stirrup design.

<table>
<thead>
<tr>
<th>Stirrup Details</th>
<th>Shear Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3.3</td>
<td>If $V_u \leq \frac{\phi V_c}{2}$</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>If $\frac{\phi V_c}{2} &lt; V_u \leq \phi V_c$</td>
</tr>
<tr>
<td>Figure 3.5</td>
<td>If $V_u &gt; \phi V_c$</td>
</tr>
</tbody>
</table>

Required and maximum-permitted stirrup spacing, $s$

The maximum factored shear force was calculated using the equation

$$V_u = \frac{wL}{2}$$

Equation 3.13

while the nominal shear strength of the concrete was calculated using the equation

$$V_c = 2\lambda bd \sqrt{f'_c}$$

Equation 3.14

Stirrup spacing was then determined by one of three “if” statements:

1. If $V_u \leq \frac{\phi V_c}{2}$, then no stirrups were required and thus $A_v = 0$ and $ns = 0$. The shear resistance factor is given as $\phi_s = 0.75$. The beam without stirrups is shown in Figure 3.3.
2. If $\frac{\phi V}{2} < V_u \leq \phi V_c$, then $A_v = A_b$, which is 0.11 in$^2$ for a #3 bar. The spacing was determined by finding the minimum of four values:

$$s_i = \min \left( \frac{A_v f_y}{50b}, \frac{A_v f_y}{0.75b \sqrt{f'c}}, \frac{d}{2}, 24" \right)$$

Equation 3.15

It is assumed that the stirrups extend into 1/6 of the span of the beam from each support while the cover is accounted for at each end. Therefore, the number of stirrups was calculated with the following equation:

$$ns = 2 \left( \frac{L - \text{cover}}{s_i} + 1 \right)$$

Equation 3.16

Figure 3.4 shows the design of the beam with the spacing for the required number of stirrups.
Figure 3.4: Stirrup spacing for $\frac{\phi V_c}{2} < V_u \leq \phi V_c$

3. If $V_u > \phi V_c$, then additional stirrups were implemented into the beam design.

The spacing of the required stirrups was calculated in the same way as the previous design with $A_v = A_s$, or 0.11 in$^2$.

$$s_1 = \min \left( \frac{A_v f_y}{50b}, \frac{A_f f_y}{0.75b f'_c}, \frac{d}{2}, 24'' \right)$$ \hspace{1cm} \text{Equation 3.17}

Based on the shear in the stirrups, the spacing of the maximum stirrups was calculated in regards to two “if” statements.

If $V_s \leq 4bd f'_c \rightarrow s_2 = \min \left( \frac{A_v f_y d}{V_s}, \frac{d}{2}, 24'' \right)$ \hspace{1cm} \text{Equation 3.18}

If $V_s > 4bd f'_c \rightarrow s_2 = \min \left( \frac{A_v f_y d}{V_s}, \frac{d}{4}, 12'' \right)$ \hspace{1cm} \text{Equation 3.19}
The required reinforcement contribution to shear was estimated with the following equation:

$$V_s = \min \left( \frac{V_u - \phi V_c}{\phi}, 8bd\sqrt{f'_c} \right)$$  \hspace{1cm} \text{Equation 3.20}$$

Figure 3.5 shows the design of the beam with both the spacing for the required number of stirrups and the spacing for the maximum number of stirrups, each expressed with $s_1$ and $s_2$, respectively.

![Figure 3.5: Stirrup spacing for $V_u > \phi V_c$](image)

**Total Volume of Stirrups**

The length of one stirrup was derived from the height and width of the beam, with the cover being subtracted from each dimension. That value was then multiplied by the area of a #3 bar and the total number of stirrups within the beam. This is an approximation based on geometry.

$$V_{str} = (2(b - \text{cover}) + 2(h - 2\text{cover}))A_{\#,ns}$$  \hspace{1cm} \text{Equation 3.21}$$
Single leg stirrups were assumed when calculating the spacing requirements for all stirrups while the total volume of each stirrup was based on double leg stirrups. This had the effect of increasing the volume of shear reinforcement calculated in the optimization calculations. This approach was conservative but had very little impact on results. Shear reinforcement typically contributed around 2-7% of the total embodied energy of the beam (Figure 4.14). Also, the approach was applied consistently to all calculations so as to not impede comparisons.

**Design Shear Strength in the Beam**

To ensure adequate design shear strength in the beam,

\[ \phi V_n \geq V_u \]  \hspace{1cm} \text{Equation 3.22}

The nominal shear strength is given by

\[ V_n = 4\sqrt{f'_c b d} \]  \hspace{1cm} \text{Equation 3.23}

The nominal shear strength was not explicitly calculated by \( V_c + V_S \) as provided by the ACI code. Rather, Equation 3.23 was used as a simplified shear capacity calculation. This simplification ensured that feasible solutions had sufficient shear strength but was effectively of little consequence because shear rarely governed the optimum solutions.

**3.8.4 Other Constraints**

To ensure that the width of the beam was sufficient to accommodate the longitudinal and shear reinforcement, a minimum width was computed for each combination of variables using Equation 3.24.

\[ b_{\text{min}} = 2(\text{cover}) + 2d_s + 2(2d_s) + (n-1)(d_n + cs_{\text{min}}) \]  \hspace{1cm} \text{Equation 3.24}
The minimum clear spacing, $c_{s_{\text{min}}}$, between adjacent longitudinal reinforcement was based on the maximum of three values given by Section 7.6 in ACI 318-11:

$$c_{s_{\text{min}}} = \max \left( d_{b}, 1'' , \frac{4}{3} d_{\text{max,agg}} \right) \quad \text{Equation 3.25}$$

The area of steel was subjected to two constraints: minimum and maximum area of steel. A minimum area of steel is required to avoid brittle failure and the formation of cracks in the concrete as the steel begins to yield; the subsequent equation indicates the minimal area of steel permitted by Section 10.5 of ACI 318-11.

$$A_{s_{\text{min}}} = \max \left( \frac{3 \sqrt{f'_{c}bd}}{f_{y}}, \frac{200bd}{f_{y}} \right) \quad \text{Equation 3.26}$$

The maximum area of steel requirement was constrained by the net tensile strain calculation (Equation 3.27) for yield strengths greater than 60,000 psi, which ensures that the steel will yield in tension before the concrete crushes in compression.

$$\varepsilon_{t} \geq \frac{2f_{y}}{E_{s}} \quad \text{Equation 3.27}$$

Otherwise, the maximum area of steel requirement was constrained by a net tensile strain value of 0.004.

### 3.8.5 Minimum Thickness for Nonprestressed Simply Supported Beams

While ACI 318-11 provides minimum span-to-height ratios for which member deflections need not be calculated, a range of $L/30$ to $L/10$ was examined to study the behavior of beams with a thickness below the ACI limits. The following equations are the ACI minimum thicknesses for avoiding deflection calculations for nonprestressed simply supported beams with various material properties:
1. NW, \( f_y = 60 ksi \)

\[
h_{\min} = \frac{L}{16} \tag{Equation 3.28}
\]

2. LW (\( \gamma_c = 90 – 115\text{pcf} \)), \( f_y = 60 ksi \)

\[
h_{\min} = \frac{L}{16} (1.65 - 0.005 \gamma_c) \geq \frac{L}{16} (1.09) \tag{Equation 3.29}
\]

3. NW, \( f_y \neq 60 ksi \)

\[
h_{\min} = \frac{L}{16} \left(0.4 + \frac{f_y}{100,000}\right); f_y \text{ is in units of psi} \tag{Equation 3.30}
\]

So for higher grades of steel, the ACI minimum beam height is greater than for 60 ksi reinforcement. ACI minimums are listed in Table 3.7.

**Table 3.7: ACI minimum height without deflection calculations for different grades of steel, calculated with Equation 3.30.**

<table>
<thead>
<tr>
<th>Grade of Steel</th>
<th>ACI Minimum Height, ( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>( \frac{L}{20} )</td>
</tr>
<tr>
<td>60</td>
<td>( \frac{L}{16} )</td>
</tr>
<tr>
<td>75</td>
<td>( \frac{L}{13.91} )</td>
</tr>
<tr>
<td>90</td>
<td>( \frac{L}{12.31} )</td>
</tr>
<tr>
<td>100</td>
<td>( \frac{L}{11.43} )</td>
</tr>
<tr>
<td>120</td>
<td>( \frac{L}{10} )</td>
</tr>
</tbody>
</table>
ACI minimums also changed for LW concrete relative to NW concrete. These values are listed in Table 3.8.

Table 3.8: ACI minimum height without deflection calculations for different unit weights of concrete, calculated with Equation 3.29.

<table>
<thead>
<tr>
<th>Unit Weight of Concrete</th>
<th>ACI Minimum Height, $h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>$\frac{L}{13.33}$</td>
</tr>
<tr>
<td>102.5</td>
<td>$\frac{L}{14.07}$</td>
</tr>
<tr>
<td>115</td>
<td>$\frac{L}{14.68}$</td>
</tr>
<tr>
<td>145</td>
<td>$\frac{L}{16}$</td>
</tr>
</tbody>
</table>

3.8.6 Comparisons with ACI 318-11

In addition to the discussion of ACI in previous sections, this section offers clarifications regarding the application of ACI provisions to the research program.

Section 10.6.7 of ACI 318-11 requires that “longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member” when the height of the beam, $h$, is greater than 36 inches. This provision was not considered in the research program because it had minimal impact on the optimization results, which is explained in detail below.
Skin reinforcement need only be distributed along half of the member’s depth.

Spacing is calculated using the following equation:

\[
n = 15 \left( \frac{40,000}{f_s} \right) - 2.5c_c
\]

Equation 3.31

The calculated stress in the reinforcement, \( f_s \), is permitted to be taken as \( 2/3f_y \), or 40,000.

The clear cover is assumed to be 1.5 inches for all optimization problems. With these values defined, the smallest spacing required by ACI 318-11 is 11 inches. Because the largest depth considered in the research program is 87 inches, 8 #3 bars is the maximum number of skin reinforcement bars that is required. In an 87-inch deep member, 8 #3 bars have very little influence on the total cost or total embodied energy of the beam.

The maximum concrete compressive strength considered in the test program was 10,000 psi. Strengths greater than this value trigger additional ACI requirements. For example, Section 11.1.2 states that the values of \( \sqrt{f''c} \) that are used in Chapter 11 “shall not exceed 100 psi” unless minimum shear reinforcement is provided in compliance with provision 11.1.2.1. The additional provisions for higher strengths are mentioned here to assist others in applying the methodology presented in this research.

ACI 318-11 also classifies 2,500 psi as the minimum allowable \( f''c \) for structural concrete, stipulated in provision 1.1.1. Thus, 2,500 psi was the minimum value adopted for concrete in this research.

In provision 10.7.2, deep beams (defined as beams with “clear spans equal to or less than four times the overall member depth, \( h \)”) must meet the requirements set forth
by Section 11.7. By constraining \( h \) to a maximum value of \( L/10 \), the optimized design of the beam could never be categorized as a deep beam, so Section 11.7 did not apply.

Table 3.9 provides a summary of any ACI requirements that were modified in the current research program.

**Table 3.9: Summary of modifications to ACI 318-11**

<table>
<thead>
<tr>
<th>ACI Section</th>
<th>ACI Requirement</th>
<th>Modifications in Research Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.6.7</td>
<td>Skin Reinforcement</td>
<td>Did not consider skin reinforcement</td>
</tr>
<tr>
<td>11.1.2.1</td>
<td>Max ( f'c ) for Shear Unless Minimum Shear Reinforcement Provided</td>
<td>Limited ( f'c ) to 10,000 psi</td>
</tr>
<tr>
<td>11.1.1</td>
<td>Nominal Shear Strength</td>
<td>( V_n = 4\sqrt{f'c} bd )</td>
</tr>
<tr>
<td>Table 9.5(a)</td>
<td>Minimum Thickness without Deflection Calculations</td>
<td>Limited ACI minimum thickness in some parametric studies</td>
</tr>
</tbody>
</table>
CHAPTER FOUR
PARAMETRIC STUDIES WITH EMBODIED ENERGY OPTIMIZED

4.1 Overview

In this chapter, several parametric studies have been conducted to analyze how the flexural moment affects the design of beams optimized for embodied energy. Because there have been fewer studies on the environmental impacts of reinforced concrete design, the primary focus of this thesis centers on the effects of designing for reduced embodied energy. However, cost is also considered in order to present a thorough framework, and to compare results to previous studies for validation; cost-optimized results can be found in Chapter 5.

Parametric studies were conducted to address two questions. First, what are the relationships between different design variables of an RC beam optimized for embodied energy? And second, how can different materials be quantitatively compared to assess environmental preferability?

Explanation of Applied Moment

In all parametric studies, the moment is expressed as $wL^2$. This quantity is proportional to the maximum moment applied to the beam, with the distributed load, $w$, consisting of the unfactored dead and live load. Because this considers the external moment exclusively, the dead load does not include the self-weight of the beam. The self-weight was included in the optimization calculations but not in the value of the distributed load used in the figures. This approach was taken because self-weight is a
function of the design variables, which are not known at the beginning of the design process.

Although all parametric studies in the next section deal with simply supported beams and therefore have a maximum moment equal to $wL^2/8$, results are reported in terms of $wL^2$ to normalize the data for comparison with future studies of beams with alternative boundary conditions. While $wL^2$ is not the flexural moment supported by the beam, it will be referred to as such throughout this thesis. It should be noted that $M_u$ was calculated as $w_uL^2/8$ in all calculations.

**Baseline Assumptions**

A consistent set of baseline values was established for all optimization studies, as listed in Table 4.1. Unless otherwise specified, each study was calculated with these baseline values. Deviations from the baseline values are noted within the figures and/or figure titles. Baseline values consist of the concrete and steel properties and the range of span-to-depth ratios.
Table 4.1: Baseline assumptions and their alternate values for properties of concrete and steel. Baseline assumptions for the height of the beam are also included.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline Values</th>
<th>Alternate Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive Strength of Concrete ($f'_c$)</td>
<td>4350</td>
<td>2500, 6000, 8000, 10000</td>
<td>psi</td>
</tr>
<tr>
<td>Yield Strength of Reinforcement ($f_y$)</td>
<td>60</td>
<td>40, 75, 90, 100, 120</td>
<td>ksi</td>
</tr>
<tr>
<td>Unit Embodied Energy of Concrete ($\gamma_{EC}$)</td>
<td>1.3</td>
<td>0.65 - 3.25</td>
<td>MJ/kg</td>
</tr>
<tr>
<td>Unit Embodied Energy of Steel ($\gamma_{ES}$)</td>
<td>8.9</td>
<td>4.45 - 22.25</td>
<td>MJ/kg</td>
</tr>
<tr>
<td>Unit Cost of Concrete ($C_C$)</td>
<td>245</td>
<td>–</td>
<td>$/\text{ft}^3$</td>
</tr>
<tr>
<td>Unit Cost of Steel ($C_S$)</td>
<td>3.50</td>
<td>–</td>
<td>$/\text{ft}^3$</td>
</tr>
<tr>
<td>Unit Weight of Concrete ($\gamma_C$)</td>
<td>145</td>
<td>90, 102.5, 115</td>
<td>pcf</td>
</tr>
<tr>
<td>Lightweight Modification Factor ($\lambda$)</td>
<td>1</td>
<td>0.75</td>
<td>unitless</td>
</tr>
<tr>
<td>Height of Beam ($h$)</td>
<td>$L/30 - L/10$</td>
<td>ACI minimum</td>
<td>in.</td>
</tr>
</tbody>
</table>

Figure 4.1 is an example of how the baseline values were applied. In this figure, the optimum aspect ratios for six grades of steel were compared for varying flexural moments; these grades are listed in the legend to the right of the figure. While the yield strength of the steel varies from 60 ksi, one of the baseline values, the other parameters remain fixed at the baseline values. The compressive strength of the concrete remains 4,350 psi; the unit embodied energy of concrete and steel remains 1.3 and 8.9 MJ/kg, respectively; the unit weight of concrete remains 145 pcf; and the height of the beam is calculated using the range of $L/30$ to $L/10$. Because this plot was optimized for embodied
energy, the cost of the materials did not contribute to the calculations and therefore does not pertain to this particular study.

Figure 4.1: Example figure describing baseline and alternative values for parameters.

The same approach was applied in all calculations: If one or more of the baseline assumptions is being modified within a figure, all other variables maintain their initial baseline values. The alternate values are listed in Table 4.1 along with the baseline values. Constant values that were assumed throughout all studies are featured in Table 4.2.
Table 4.2: Constant values assumed for all optimization calculations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live Load ((LL))</td>
<td>50</td>
<td>Psf</td>
</tr>
<tr>
<td>Dead Load ((DL))</td>
<td>50</td>
<td>Psf</td>
</tr>
<tr>
<td>Diameter of Stirrup ((d_s))</td>
<td>0.375</td>
<td>in.</td>
</tr>
<tr>
<td>Area of Shear Reinforcement ((A_v))</td>
<td>0.11</td>
<td>in.²</td>
</tr>
<tr>
<td>Cover</td>
<td>1.5</td>
<td>in.</td>
</tr>
<tr>
<td>Diameter of Maximum Aggregate ((d_{max_agg}))</td>
<td>0.8</td>
<td>in.</td>
</tr>
<tr>
<td>Unit Weight of Steel ((\gamma_S))</td>
<td>490</td>
<td>Pcf</td>
</tr>
</tbody>
</table>

**Unit Embodied Energy Values**

A few sources identified the unit embodied energy for corresponding compressive strengths and yield strengths of concrete and steel reinforcement, respectively. The values are given in Table 4.3 with the steel-to-concrete ratios in the rightmost column. For the parametric studies in this section, 1.3 MJ/kg was used as the unit embodied energy for concrete, and 8.9 MJ/kg was used for steel reinforcement. Each of these values was taken from “Table of embodied energy coefficients” (2007), and both correspond with the baseline values used for concrete strength and steel grade. Unit embodied-energy coefficients vary for materials with different properties, but based on limited data, constant values were used throughout this study.
Table 4.3: Unit embodied energy for concrete and steel.\(^1\)

<table>
<thead>
<tr>
<th>Reference</th>
<th>(\gamma_{EC}) (SI)</th>
<th>(\gamma_{ES}) (SI)</th>
<th>(\gamma_{EC}) (US)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alcorn (2003)</td>
<td>1.2 MJ/kg (0.5443 MJ/lb) ((f'_c = 4350 \text{ psi}))</td>
<td>8.6 MJ/kg (3.901 MJ/lb)</td>
<td>7.17</td>
</tr>
<tr>
<td>Table of EE coefficients (2007)</td>
<td>1.3 MJ/kg (0.5897 MJ/lb) ((f'_c = 4350 \text{ psi}))</td>
<td>8.9 MJ/kg (4.037 MJ/lb)</td>
<td>6.85</td>
</tr>
<tr>
<td>Hammond &amp; Jones (2008)</td>
<td>1.39 MJ/kg (0.6305 MJ/lb) (\text{(high-strength)})</td>
<td>8.80 MJ/kg (3.992 MJ/lb)</td>
<td>6.33</td>
</tr>
<tr>
<td>Yeo &amp; Gabbai (2011)</td>
<td>1.37 MJ/kg (0.6210 MJ/lb) ((f'_c = 4930 \text{ psi})) ((f_y = 60,915 \text{ psi}))</td>
<td>8.9 MJ/kg (4.037 MJ/lb) ((f'_c = 4930 \text{ psi})) ((f_y = 60,915 \text{ psi}))</td>
<td>6.50</td>
</tr>
</tbody>
</table>

\(^1\)Units given in SI were converted to U.S. customary

**Methodology for Presenting Parametric Study Results**

Results of parametric studies are presented with four primary types of figures. The first is a contour plot; all data in these plots have been normalized according to the baseline assumptions so that direct comparisons can be made between different materials. In Figure 4.2, for example, 75 ksi steel reinforcement was compared to 60 ksi steel reinforcement (the baseline value) by dividing the unit embodied-energy ratio of 75 ksi steel by the unit embodied-energy ratio of 60 ksi steel. The other three plots are scatter plots which show the optimum cross-section properties as a function of the applied moment. This section discusses how these plots were created and how they can be used to interpret research results. Section 4.3 gives an example demonstrating the application of these types of plots.

The contour plots are useful for comparing the preferability of different types of material. An example of this type of plot is shown in Figure 4.2, which compares the environmental preferability of grade 75 and grade 60 reinforcement. The y-axis of Figure
4.2 is the ratio of unit embodied energy between the materials being compared. For this example, the unit embodied energy of grade 75 reinforcement is divided by the unit embodied energy of 60 ksi steel (the baseline value). Ratios from 0.5 to 2.5 are reflected upon the y-axis of the plot. On the x-axis, the flexural moment supported by the beam is plotted, ranging from 50 to 9050 k-ft. A third value is represented by the different color regions on the plot; this value is the total embodied energy of an optimum beam with 75 ksi steel, divided by the total embodied energy of an optimum beam with 60 ksi steel.

Data in the contour and scatter plots were generated through sequential optimization problems that considered changes in the span-length parameter. Source data from each optimization problem is not a single optimum point but instead is the average of the top ten optimum points. For example, the aspect ratio was calculated for the top ten optimum designs for a specific span length and unit embodied-energy coefficient. These ten aspect ratios were then averaged and plotted as a single point on the scatter plot. The same approach was taken for all studies because the designer may not be interested in an individual point that limits the design of a beam.
Figure 4.2: Example figure comparing the embodied energy ratios for 60 ksi and 75 ksi steel reinforcement.

The different color regions in Figure 4.2 represent a percent increase or decrease in total embodied energy, as defined in the legend. In the red region, the optimum beam with 60 ksi steel has a lower total embodied energy than the optimum beam with 75 ksi steel; the inverse of this is true in the blue region. The approximate border between these two regions is portrayed by a dashed line; when above the dashed line, 60 ksi is preferable to minimize embodied energy, but while below the dashed line, 75 ksi is preferable.

The purpose of this class of plots is to show a large range of unit embodied-energy ratios (Chapter 4) or unit cost ratios (Chapter 5) because the unit embodied-energy and unit cost factors of different types of steel and concrete are uncertain and vary geographically as well as over time. Instead of limiting this data to a specific region or time in history, plots were generated to help designers apply research results to their specific conditions.
The other three plots used to describe the parametric study results are all scatter plots, which describe the aspect \((b/h)\), reinforcement \((A_s/(bh))\), and span-to-depth \((L/h)\) ratios needed to design optimum cross sections. The flexural moment supported by the beam is plotted along the x-axis, and one of the three ratios is plotted along the y-axis. Like the contour plots, the top ten ratio values from each optimization routine were averaged together for each point along the curve. A separate optimization routine was run for each value of moment.

These four types of plots are divided into multiple subcategories. All plots consider either the strength of concrete, the strength of steel, or the unit weight of concrete with either the embodied energy (Chapter 4) or the cost (Chapter 5) optimized. Separate parametric studies were also conducted to evaluate the effects of using the minimum thickness values from ACI table 9.5(a). ACI requires deflection calculations for members not meeting the minimum thickness values. Because the baseline range for thickness \((L/30 \text{ to } L/10)\) included thickness values less than the ACI minimum, the separate studies were conducted by changing the thickness of the beam to a constant value equal to the minimum value prescribed by ACI; this reduced the design space from 66,000 possible solutions to 3,300 possible solutions.

In addition to the studies above, another study was performed to investigate how the optimal values of each ratio (aspect, reinforcement, and span-to-depth) would be affected by modifying unit embodied energy from the baseline value; these studies are presented in both Section 4.3.1.3 (steel reinforcement) and Appendix A (concrete). Modifications included a decrease of the baseline by 50% and an increase by 150%,
allowing evaluations over the same range considered in the contour plots. An example is shown in Figure 4.5, where the baseline unit embodied energy of 8.9 MJ/kg of steel was multiplied by 0.5 and 2.5, producing a unit embodied energy of 4.45 and 22.25 MJ/kg, respectively. If a designer were to know the unit embodied energy of the material being used, he could interpolate between the two plots. While values may not be linearly related, this estimate is a product of two plots that show little variance between each other; therefore, it is reasonable to use linear interpolation since a refined analysis would be unlikely to produce significantly different results.

**Example Application of Parametric Study Results**

To demonstrate the application of each type of plot, a design example is presented. Table 4.4 and Figure 4.3 give the values chosen for this example. In the example, the preferability of 60ksi and 75ksi reinforcement will be evaluated. After the environmentally preferred reinforcement is identified, a cross-section with near-minimum embodied energy will be designed.
Table 4.4: Design example values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_L, w_D$</td>
<td>50 psf</td>
</tr>
<tr>
<td>$L$</td>
<td>50 ft</td>
</tr>
<tr>
<td>$b_{trib}$</td>
<td>12.5 ft</td>
</tr>
<tr>
<td>$f'c$</td>
<td>4,350 psi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>60 ksi, 75 ksi</td>
</tr>
<tr>
<td>$\gamma_{EC}$</td>
<td>1.3 MJ/kg</td>
</tr>
<tr>
<td>$\gamma_{ES}$</td>
<td>10 MJ/kg</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>145 pcf</td>
</tr>
</tbody>
</table>

$w_L = 50 \text{ psf}, w_D = 50 \text{ psf}$

$L = 50 \text{ ft}$

Figure 4.3: Design example of simply supported beam exposed to distributed load.

To interpret the contour plots, the applied moment and unit embodied-energy ratio must be computed. For the example, the moment is calculated as

$$M^* = (w_D + w_L)(b_{trib})L^2 = (100 \text{ psf})(12.5 \text{ ft})(50 \text{ ft}^2)\left(\frac{1k}{1,000 \text{ ft}}\right) = 3,125k \cdot \text{ ft}$$
The unit embodied-energy ratio for the materials being compared is also needed. For the given values of 10 MJ/kg for 75 ksi steel, and 8.9 MJ/kg for 60 ksi steel, the ratio is

$$\frac{\gamma_{E75}}{\gamma_{E60}} = \frac{10 \text{ MJ/kg}}{8.9 \text{ MJ/kg}} = 1.124$$

By presenting the optimization results in this manner, the results can be used to evaluate preferability of materials having a wide range of unit embodied-energy values.

Figure 4.4 is a contour plot that demonstrates the decision-making aspect of this example. This plot was used to establish which type of reinforcement, 75 ksi or 60 ksi, is preferable from an environmental standpoint. The applied moment and unit embodied-energy ratio are represented on the plot by two solid black lines. Because these two lines intersect in the blue region, 75 ksi is the desirable strength of steel reinforcement.

![Contour plot](image)

**Figure 4.4: Design example comparing the embodied-energy ratios for 60 ksi and 75 ksi steel reinforcement.**

Since grade 75 steel was established as the preferable strength of reinforcement, Figure 4.5 through Figure 4.7 were utilized to determine the near-optimum cross-sectional characteristics of reinforced concrete with 75 ksi steel reinforcement. Figure 4.5
depicts two plots: The one on the left is one-half the baseline unit embodied energy, or 4.45 MJ/kg; the one on the right is two-and-a-half times the baseline unit embodied energy, or 22.25 MJ/kg. With a moment of 3125 k-ft, the aspect ratio was determined for each plot (0.243 for Figure 4.5(a) and 0.217 for Figure 4.5(b)). To estimate the optimum aspect ratio proportional to 10 MJ/kg, interpolation was executed utilizing the two values previously identified. This resulted in an aspect ratio of 0.235. It is unknown if the transition occurring between each plot is linear, but because the two values are so close, this interpolated value is a close approximation of the optimum value.

**Figure 4.5:** Design example demonstrating the effects of the unit embodied-energy ratio of steel on the aspect ratio. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

The same approach was used in Figure 4.6 and Figure 4.7. From Figure 4.6, the reinforcement ratio for 10 MJ/kg was 0.0119. From Figure 4.7, the span-to-depth ratio for 10 MJ/kg was 15.34.
Having established all of these ratios, the design variables were calculated as follows:

\[
\frac{L}{h} = 15.34 \rightarrow h = \frac{50 \text{ ft}}{15.34} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) = 39.5 \text{ in}
\]

\[
\frac{b}{h} = 0.235 \rightarrow b = \frac{50 \text{ ft}}{15.34} \left( 0.235 \right) \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) = 9.5 \text{ in}
\]

\[
\rho = \frac{A}{bh} = 0.0119 \rightarrow A_s = \left( \frac{50 \text{ ft}}{15.34} \right)^2 \left( 0.235 \right) \left( 0.0119 \right) \left( \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right) = 4.21 \text{ in}^2 \text{ or } 7 \#7 \text{ bars}
\]

Figure 4.6: Design example demonstrating the effects of the unit embodied-energy ratio of steel on the reinforcement ratio. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

Figure 4.7: Design example demonstrating the effects of the unit embodied-energy ratio of steel on the span-to-depth ratio. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.
The cross-section derived from the design process is shown in Figure 4.8. Because these values were calculated using averages of the top ten optimum points and because some rounding occurred, it is important for the designer to verify that these design variables meet the constraint criteria described in Section 3.8.

![Diagram of cross-section](image)

**Figure 4.8: Cross-section of design example, drawn to scale.**

**Validation**

Where possible, the methodology presented in this study has been compared and validated using results of the previous works discussed in Chapter 2. Optimization of the reinforcement ratio for embodied energy and cost were both considered for validation and are later examined in Section 4.3 and Section 5.2, respectively.

Using 25 MPa (≈ 3,625 psi) concrete and 400 MPa (≈ 58 ksi) steel, Alreshaid et al. (2004) established that the recommended reinforcement ratio for beams is about
0.0154 when optimized for cost, which is close to the 0.0149 ratio obtained in the current study for beams with 4,350 psi concrete and 60 ksi steel optimized for embodied energy. However, when optimized for cost, the recommended steel ratio was only 0.0124. This is in large part due to fluctuating steel-to-concrete unit cost ratios and different problem set-ups. Also, the reinforcement ratio was defined as $A_s/(bh)$ in this study but was defined as $A_s/(bd)$ in Alreshaid et al. (2004). Therefore, the reinforcement ratio results will have slightly smaller values in this study. The recommended ratios are shown in Figure 4.9.

Figure 4.9: Recommended reinforcement ratio comparisons to Alreshaid et al. (2004) for 60 ksi steel reinforcement.

In Section 2.5 of this thesis, the results for the simultaneous optimization of embodied energy and cost are summarized for each study in the literature review. One example is the work of Yeo and Potra (2013), which stated that optimizing for CO$_2$ emissions culminated in a 5-15% reduction in CO$_2$ emissions when compared to the optimized cost design. The results from Yeo and Poltra are similar to the results obtained in the current study as shown in Figure 4.10. This figure presents the ratio of total embodied energy for cost and energy optimum solutions. The percentage difference in energy was between 1% and 11%, which is similar in magnitude to the results from Yeo.
and Potra, as well as other researchers. Further analysis on multiobjective optimization is given in Section 5.2.4.

Figure 4.10: Total embodied energy of the cost-optimized design divided by the total embodied energy of the embodied-energy optimized design with variable span-to-depth ratio

4.2 Effects of Parameters on Problem Set-Up

Section 4.2 presents general information about the optimization results. Information in this section is based on—and provides context for—the parametric studies presented in subsequent sections.

4.2.1 Percentage of Feasible Solutions in each Study

Figure 4.11, Figure 4.12, and Figure 4.13 demonstrate the percentage of feasible solutions within the design space as the span lengths and moments changed. Feasible solution percentages from each parametric study in this research are represented by these six plots: strength of steel, strength of concrete, and unit weight of concrete, each one divided into the two ranges given to the height of the beam throughout this paper. The percentage is based on 66,000 possible solutions for the baseline case of $L/30 < h < L/10$ and 3,300 possible solutions for $h = ACI$ minimum. A higher strength in steel and concrete and a lower unit weight of concrete tend to generate more solutions. As the yield
strength of steel increases, the nominal moment capacity also increases, producing more feasible solutions. Yet as the compressive strength of the concrete increases, the nominal moment capacity increases slightly and the nominal shear capacity increases significantly, producing a slight boost in the percentage of solutions that are feasible; the slight increase is in response to the moment governing over the shear in most circumstances. When the unit weight of concrete is reduced, the factored shear force and the factored moment lessen, resulting in a higher percentage of feasible solutions.

At short spans with less moment, fewer solutions are feasible. This occurred because many of the reinforcement bar sizes and configurations had too much steel to be practical for beams carrying small moments. This condition was manifested in the calculations when the calculated neutral axis depth, \( c \), was greater than the effective flexural depth, \( d_e \). Such conditions are physically meaningless in reinforced concrete mechanics and were just a function of the values used for the variables. At long spans with larger moment, the percentage of feasible solutions began to decline due to the increase in the factored shear force and the factored moment.

![Figure 4.11: Percentage of possible solutions that are feasible for different grades of steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.](image-url)
4.2.2 Contribution of Shear Reinforcement

Several assumptions were made to simplify the calculations necessary for the computation of the shear reinforcement, which have been expressed in detail as part of Section 3.8.3. Although they are all well-informed assumptions based on engineering judgment, it is desirable to know what impact the shear reinforcement has on the total embodied energy and total cost. Figure 4.14(a) and Figure 4.15(a) show that the shear reinforcement contributed to less than 4% of the total embodied energy and less than 7% of the total cost for beams with moments greater than 800 k-ft. In Figure 4.14(b) and Figure 4.15(b), the shear reinforcement contributed to less than 4% of the total embodied energy and less than 7% of the total cost for beams with moments greater than 800 k-ft.
energy and less than 7% of the total cost for beams with moments greater than 500 k-ft. Thus, in most examples, any assumptions made with regard to shear reinforcement have small ramifications on the total output of the problem.

Figure 4.14: Contribution of 60 ksi shear reinforcement to the total embodied energy of the beam. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.15: Contribution of 60 ksi shear reinforcement to the total cost of the beam. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

4.2.3 Average Number of Layers in Optimum Designs

Many variables define the design space in this research. One among these is the number of rebar, arranged in one, two, or three layers. All configurations considered for the optimization problem at hand are shown in Figure 3.2.
Figure 4.16 through Figure 4.18 convey the average number of layers in the top ten embodied energy optimized designs. In most cases, the average number of layers increases as the moment increases. That result suggests that some of the optimum solutions for longer span beams may have benefitted from four or more layers of reinforcement.

Overall, higher grades of steel resulted in optimized designs with fewer layers. This is because it takes fewer high-strength bars (relative to lower-strength bars) to carry tensile forces in the section. Another interpretation could be justified with Equation 3.4, where a greater yield strength could take advantage of the increased effective depth (which is a feature of fewer layers of reinforcement), thereby strengthening the nominal moment capacity.

Compressive strength and unit weight of the concrete have a lesser impact on the number of layers. Stronger concrete does not always minimize the demand for reinforcement; however, this enhanced property does lessen the need for concrete, which oftentimes results in optimum beams with thinner widths. One of the constraints in the optimization routines was minimum beam width, which is governed by the widest layer of reinforcement. When the beam width becomes smaller, more layers are needed to meet the required nominal moment capacity. Therefore, stronger concrete may veer towards an overall trend with additional layers of reinforcement.
Figure 4.16: Average number of layers of reinforcement for different grades of steel reinforcement when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.17: Average number of layers of reinforcement for different grades of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.18: Average number of layers of reinforcement for different unit weights of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
4.3 Parametric Studies

This section is devoted to parametric studies performed for the optimization of embodied energy. For additional parametric studies on the optimization of embodied energy, see Appendix A.

4.3.1 Variation of Steel Yield Strength

4.3.1.1 Selection of Optimum Steel Yield Strength

In this section, contour plots were developed to determine when different grades of reinforcement would be preferable to grade 60 reinforcement when different unit embodied-energy ratios are used. In the left column, results are based on an \( h \) ranging from \( L/30 \) to \( L/10 \); in the right column, \( h \) was set equal to the minimum beam height prescribed by ACI 318-11. Thus, instead of 20 values for \( h \), only one constant value is given for each grade of steel. For clarification, the range of \( h \) is labeled in each figure.

The flexural moment was plotted on the x-axis, with a range of 50 to 9050 k-ft in increments of 150 k-ft. The unit embodied-energy ratio was plotted along the y-axis, with a range of 0.5 to 2.5 to show a wide spectrum of realistic values.

Figure 4.19(a) shows a positive linear relationship between the flexural moment and the unit embodied-energy ratio. As the strength of the steel increases (Figure 4.20(a)), the trend becomes steeper and more erratic, having periods of highs and lows. Both conditions demonstrate that as the moment increases, it is more efficient to use higher-grade steel.

Figure 4.19(b) and Figure 4.20(b) do not share the same trends as their \( L/30 \) to \( L/10 \) counterparts. While an increase in the moment does prove to be a selling point for
75 ksi steel reinforcement (Figure 4.19(b)), 120 ksi steel is not supported by favorable results (Figure 4.20(b)), with 60 ksi steel being the superior choice in most circumstances. (See Appendix A for other standard steel reinforcement grades.)

**Figure 4.19:** Comparison of embodied-energy ratios for 60 ksi and 75 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

**Figure 4.20:** Comparison of embodied-energy ratios for 60 ksi and 120 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Comparing the results in Figure 4.19 and Figure 4.20, the significant effects of problem set-up are evident. Changes that may seem subtle can have great effect on the optimization results. For example, while assigning one value to the height of the beam limits the number of feasible solutions and therefore typically causes the total embodied
energy to increase, the total embodied energy of 60 ksi steel may increase at a higher relative rate; this causes the ACI minimum plots to produce more ideal results, evident in Figure 4.19(b) when greater moments are at play. Generally, though, an $h$ value that is not constrained to strict limits will produce results that are more favorable to higher strengths of steel.

For higher grades of steel, the ACI minimum beam height is greater (Table 3.7), which in turn prompts a greater total embodied energy due to a larger volume of concrete. By showing the two graphs side by side, the effects of using the ACI minimum are made clear.

The plots in the left column show results of a more general optimized reinforced concrete design, but the plots in the right column depict a more practical application by which reinforced concrete beams are often designed. However, because the problem was simplified to one constant value for $h$ in the right column, this does not portray perfect conditions since the minimum beam height will not always control in the optimum design.

By looking at the results in terms of the yield strength of the steel, these plots can be dissected. When $f_y$ increases, the nominal moment capacity also increases. Because moment controls in many instances, this can greatly reduce the amount of materials required to support the beam, especially when larger moments are present.

After establishing the unit embodied energy of the steel and the moment applied to the beam, a designer could select which grade of steel to use. Once the grade of steel is known, the designer could then use the figures in the next section to determine the design
parameters of the beam cross-section based upon the aspect, reinforcement, and span-to-depth ratios.

### 4.3.1.2 Selection of Optimum Cross-Section

Figure 4.21, Figure 4.22, and Figure 4.24 present the parametric study results in terms of three different ratios: aspect ratio, reinforcement ratio, and span-to-depth ratio. Combined, these ratios provide information necessary to design an optimized cross-section. In all cases, the unit embodied-energy coefficient is kept consistent with the baseline value for 60 ksi steel.

Figure 4.21 shows the optimal aspect ratio for minimizing embodied energy. In Figure 4.21(a), as the moment increases, the optimal aspect ratio decreases sharply before leveling out between 0.1 and 0.2. The strength of steel does not have a notable impact on the aspect ratio. Unlike Figure 4.21(a), Figure 4.21(b) demonstrates a strong correlation between the grade of steel and the aspect ratio: as the strength increases, the beam becomes more slender, consistent with Table 3.7. The beam also becomes more slender when the applied moment increases. This observation is consistent for all cases throughout the research.

**Figure 4.21:** Aspect ratio for different grades of steel reinforcement when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Optimal reinforcement ratios for minimizing embodied energy are shown in Figure 4.22. The optimal reinforcement ratio stays relatively constant for a given grade of reinforcement when the moment is greater than 500 k-ft.

An extension of the reinforcement-ratio study was produced to exhibit the recommended reinforcement ratio for each material strength, exemplified in Figure 4.23. Values in the chart were calculated by taking an average of the optimal reinforcement ratios for each type of steel. Values for reinforcement ratios at moments less than 500 k-ft were omitted from the average because of the different behavior observed in Figure 4.22.

The bar chart in Figure 4.23 emphasizes the decrease of the optimal reinforcement ratio as stronger steel is utilized. Equation 3.26 demonstrates that a higher strength of steel lowers the minimum limits needed for the area of longitudinal reinforcement. This concept corresponds with the notion that beams designed with stronger steel require less reinforcement.

In Figure 4.22(b), the differences in the optimum reinforcement ratios between each grade of steel are greater than those in Figure 4.22(a). This result highlights the importance of selecting appropriate constraints and parameters for optimization problems in RC. Because the $h$ calculated for 40 ksi steel is less than the $h$ that governs in Figure 4.22(a), the reinforcement ratio is greater. The opposite of that is true for 60, 75, 90, 100, and 120 ksi steel.
Figure 4.22: Reinforcement ratio for different grades of steel reinforcement when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.23: Recommended reinforcement ratio for different grades of steel reinforcement when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

The span-to-depth ratio was also used to evaluate trends in optimal beam design for minimizing embodied energy. By assigning a constant height established by the ACI minimum, $L/h$ did not change with respect to moment (Figure 4.24(b)). When $h$ was constrained by $L/30$ and $L/10$, the span-to-depth ratio slowly declined as the moment increased (Figure 4.24(a)). This was caused by an increase in the beam height.

Figure 4.24(b) directly correlates to the ACI minimum for $h$: the value on the $y$-axis is the denominator in Table 3.7. With that in mind, only 40 ksi steel satisfies the ACI minimum at each point along curve while all others breach their minimum, especially at
lower moments. Thus, each grade of steel is affected by the ACI minimum to varying degrees. Note that the height of the beam was rounded up to the nearest half inch, so the lines in Figure 4.24(b) are not perfectly flat and deviate slightly from the ACI minimum.

\[ L < h < L_{30} = \frac{L}{10} \]

Figure 4.24: Span-to-depth ratio for different grades of steel reinforcement when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

4.3.1.3 Effects of Unit Embodied Energy

The following analysis applies to the effects of unit embodied energy of steel reinforcement only. All observations in this section are based upon the results in Figure 4.25 through Figure 4.28.

The previous set of studies expressed the difference between using two distinct designs spaces: one where \( h \) was given a range between \( L/30 \) and \( L/10 \) and one where \( h \) was set equal to the ACI minimum. To highlight the effects of unit embodied energy on optimum design, the minimum and maximum unit embodied-energy factors used to generate the contour plots in Figure 4.19 and Figure 4.20 were implemented. The baseline range for member thickness, \( L/30 < h < L/10 \), was enforced throughout this exercise. Data in the plots in the left column of Figure 4.25 through Figure 4.28 are based on a steel embodied energy of 4.45 MJ/kg, or one-half of the 8.9 MJ/kg used in the previous studies. In the right column, all steel has a unit embodied energy of 22.25
MJ/kg, or two-and-half times the 8.9 MJ/kg formerly used. For clarification, the unit embodied energy is labeled on each graph.

Differences between Figure 4.25(a) and Figure 4.25(b) are subtle. Variation in unit embodied energy of steel did not affect the optimal aspect ratio.

**Figure 4.25:** Aspect ratio for different grades of steel reinforcement when optimized for embodied energy. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

Figure 4.26(a) and Figure 4.26(b) suggest that the unit embodied energy has a greater effect on the reinforcement ratio than it did on the aspect ratio. As the unit embodied energy of steel increases, the optimal reinforcement ratio decreases. If the environmental impacts of steel are more than double the original 8.9 MJ/kg, more concrete and less steel will give way to a greener solution. Figure 4.27(a) and Figure 4.27(b) state the recommended reinforcement ratios for each grade of steel.
Figure 4.26: Reinforcement ratio for different grades of steel reinforcement when optimized for embodied energy. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

Figure 4.27: Recommended reinforcement ratio for different grades of steel reinforcement when optimized for embodied energy. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

Figure 4.28(a) and Figure 4.28(b) present the optimal span-to-depth ratios for low and high unit embodied-energy ratios, respectively. Little distinction can be made between the lower and upper bounds of the unit embodied energy in these figures. At 22.25 MJ/kg, the optimal span-to-depth ratio shifts downward marginally, which upholds the idea that the ratio of steel to concrete decreases when significant environmental repercussions of steel are taken into account. This is most remarkable in grades 40, 60, and 75 when the steel’s strength cannot overcome the environmental burden of a higher unit embodied-energy coefficient.
Figure 4.28: Span-to-depth ratio for different grades of steel reinforcement when optimized for embodied energy. (a) One-half the baseline unit embodied energy of steel, (b) Two-and-a-half times the baseline unit embodied energy of steel.

4.3.2 Variation of Concrete Compressive Strength

4.3.2.1 Selection of Optimum Concrete Compressive Strength

Figure 4.29 and Figure 4.30 are set up in a very similar fashion to the figures in Section 4.3.1.1 of this thesis but are optimized with regard to varying concrete compressive strengths. Again, $h$ has a range from $L/30$ to $L/10$ in the left column while $h$ is set equal to the ACI minimum beam height in the right column. For clarification, the range of $h$ is labeled on each graph.

In Figure 4.29 and Figure 4.30, 6,000 psi and 10,000 psi concrete are compared to the baseline value of 4,350 psi. Like the figures in Section 4.3.1.1, these figures illustrate when it is best to use which strength of material from an environmental perspective. Each plot is divided into two regions, which are separated by a dashed line and labeled appropriately.
The transition in preferability between 6,000 psi and 4,350 psi concrete lies between a unit embodied-energy ratio of 1.00 and 1.25. This means that in many cases 6,000 psi concrete is preferable even if it has greater unit embodied energy. The figures for 10,000 psi concrete feature very similar results to 6,000 psi concrete.

The subtle changes among different concrete strengths can be explained by the equations used to optimize, found in Chapter 3. When the compressive strength increases, the lower bound for the required area of steel shifts upward, which is evident in Equation

**Figure 4.29:** Comparison of embodied energy ratios for 4,350 psi and 6,000 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

**Figure 4.30:** Comparison of embodied-energy ratios for 4,350 psi and 10,000 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
3.26. Higher-strength concrete decreases the depth of the compression block, $a$, given by Equation 3.7. This increases the internal moment arm of the reinforcement and results in a slight increase in moment capacity. Equation 3.4 demonstrates this relationship. A greater compressive strength also directly impacts the nominal shear capacity of the section, given in Equation 3.23. Although shear does not typically control in an optimized section, the slight increase in the nominal moment capacity explains the subtle differences between lines in Figure 4.29 and Figure 4.30.

4.3.2.2 Selection of Optimum Cross-Section

Figure 4.31, Figure 4.32, and Figure 4.34 exhibit results of three individual parametric studies used to design an optimum cross-section. Figure 4.31 shows the optimal aspect ratio for minimizing embodied energy. In Figure 4.31(a), as the moment increases, the aspect ratio decreases sharply before leveling out. The strength of concrete in this figure does not appear to have a significant impact on the aspect ratio; however, higher grades of concrete do spawn designs with greater aspect ratios due to shorter beams. While some of the curves in Figure 4.31(b) are more closely linked, the lower grades of concrete (2,500 psi and 4,350 psi) begin to separate themselves from the others, and are actually products of a reduced aspect ratio. Because the height of the beam is constant and the area of the cross-section increases when lower grades of concrete are utilized, the width of the beam must broaden.
Figure 4.31: Aspect ratio for different grades of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.32 shows the optimal reinforcement ratio for minimizing embodied energy. In this figure, the reinforcement ratio stays relatively constant for a given concrete strength when the moment is greater than 500 k-ft. The bar chart in Figure 4.33 presents the recommended reinforcement ratios for different concrete grades and emphasizes the increase of reinforcement as stronger concrete is used. Recommended values were calculated by averaging optimum results for moments greater than 500 k-ft.

Beams designed with high-strength concrete require a smaller cross-section, hence the uptick in the reinforcement ratio. In Figure 4.32(b), the differences between each grade of steel are minimized. Because the height is constant, concrete strength has less effect on optimum design, and the results are all close to the baseline of 4,350 psi.
Figure 4.32: Reinforcement ratio for different grades of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.33: Recommended reinforcement ratio for different grades of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.34 shows the optimal span-to-depth ratio for minimizing embodied energy. Unlike reinforcement strength, when the strength of concrete increases, the optimum span-to-depth ratio increases, which is due to shorter beams (Figure 4.34(a)).

By assigning the beam with a constant height established by the ACI minimum, $L/h$ did not change with respect to moment (Figure 4.34(b)). When $h$ was constrained by $L/30$ and $L/10$, the span-to-depth ratio slowly declined as the moment increased (Figure 4.34(a)). This is caused by an increase in the height of the beam at larger moments.

Figure 4.34(b) directly correlates to the ACI minimum for $h$: the value on the y-axis is the denominator in Table 3.7. (All beams in this study use grade 60 steel)
With that in mind, only 2,500 psi concrete satisfies the ACI minimum at each point along curve while all others breach their minimum, especially at lower moments. Note that the height of the beam was rounded up to the nearest half inch, so the line in Figure 4.34(b) is not perfectly flat and deviates slightly from the ACI minimum, $L/16$.

**Figure 4.34: Span-to-depth ratio for different grades of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.**

### 4.3.2.3 Effects of Unit Embodied Energy

For results on the effects of unit embodied energy on different grades of concrete, see Figure A.6 through Figure A.9 in Appendix A. While Section 4.3.1.3 analyzes the effects of unit embodied energy on different grades of steel reinforcement, some of the same principles apply. Note that 0.65 MJ/kg and 3.25 MJ/kg are given for the unit embodied energy of concrete and not the 4.45 MJ/kg and 22.25 MJ/kg discussed in Section 4.3.1.3. Each of these values, 0.65 MJ/kg and 3.25 MJ/kg, represents one-half of the baseline unit embodied energy of concrete and two-and-a-half times the baseline unit embodied energy of concrete, respectively.
4.3.3 Variation of Concrete Unit Weight

For all studies labeled with 103 pcf, 102.5 pcf was actually used. This number was rounded to the nearest whole number in all plots to avoid formatting issues.

4.3.3.1 Selection of Optimum Concrete Unit Weight

Figure 4.35 and Figure 4.36 are set up in a very similar fashion to the figures in Section 4.3.1.1, but instead they are optimized with regard to varying concrete unit weights. Again, \( h \) has a range from \( L/30 \) to \( L/10 \) in the left column while \( h \) is set equal to the minimum beam height prescribed by ACI 318-11 in the right column. For clarification, the range of \( h \) is labeled on each graph.

In Figure 4.35 and Figure 4.36, 90 pcf and 115 pcf concrete are compared to the baseline value of 145 pcf. Like the figures in Section 4.3.1.1, these figures illustrate when it is best to use which unit weight of concrete from an environmental perspective. Each plot is divided into two regions, which are separated by a dashed line and labeled appropriately.

In Figure 4.35(a) and Figure 4.36(a), the line indicating the transition between preferability of concrete unit weights is practically horizontal. 90 pcf concrete is recommended for a unit embodied energy within about 1.60 times that of 145 pcf, and 115 pcf concrete is recommended for a unit embodied energy within about 1.25 times that of 145 pcf. Because a lighter beam produces more modest shear forces and moments, lower unit-weight concrete is preferable to NW concrete even if LW has higher unit embodied energy. When the height of the beam, \( h \), is set equal to the ACI minimum in
Figure 4.35(b) and Figure 4.36(b), the results vary from their $L/30$ to $L/10$ counterparts, demonstrating more favorable results for LW concrete when larger moments are applied.

When the ACI minimum is applied, the beam heights are greater for concrete with lower unit weights (Table 3.8).

![Figure 4.35: Comparison of embodied energy ratios for 145 pcf and 90 pcf concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.](image)

Figure 4.36: Comparison of embodied energy ratios for 145 pcf and 115 pcf concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

4.3.3.2 Selection of Optimum Cross-Section

Figure 4.37, Figure 4.38, and Figure 4.40 exhibit three individual parametric studies. Figure 4.37 shows the optimal aspect ratio for minimizing embodied energy. In
Figure 4.37(a), as the moment increases, the aspect ratio decreases sharply before leveling out between 0.1 and 0.2. The unit weight of concrete in this figure does not have any impact on the optimum aspect ratio. When ACI minimum thickness is regarded, the optimum aspect ratio is increased for lighter weight concretes (Figure 4.37(b)).

Figure 4.37: Aspect ratio for different unit weights of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.38 shows the optimal reinforcement ratio for minimizing embodied energy. In this figure, the optimum reinforcement ratio stays relatively constant when the moment is greater than 500 k-ft. The bar chart in Figure 4.39 presents the recommended reinforcement ratios for different unit weights of concrete and emphasizes the decrease of reinforcement as lighter concrete is used. Recommended values were calculated by averaging optimum results for moments greater than 500 k-ft.

Optimized beams with LW concrete don’t require as much steel reinforcement as those with NW concrete, evident in Figure 4.38 and Figure 4.39.
Figure 4.38: Reinforcement ratio for different unit weights of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.39: Recommended reinforcement ratio for different unit weights of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 4.40 shows the optimal span-to-depth ratio for minimizing embodied energy. By assigning the beam with a constant height established by the ACI minimum, $L/h$ did not change with respect to moment (Figure 4.40(b)). When $h$ was constrained by $L/30$ and $L/10$, the optimum span-to-depth ratio slowly declined as the moment increased (Figure 4.40(a)). This is caused by an increase in the height of the beam at larger moments.

Figure 4.40(b) directly correlates to the ACI minimum for $h$: the value on the y-axis is the denominator in Table 3.8. With that in mind, none of the unit weights
considered satisfy the ACI minimum. Note that the height of the beam was rounded up to the nearest half inch, so the lines in Figure 4.40(b) are not perfectly flat and deviate slightly from the ACI minimum.

![Figure 4.40](image)

**Figure 4.40:** Span-to-depth ratio for different unit weights of concrete when optimized for embodied energy. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

### 4.3.3.3 Effects of Unit Embodied Energy

For results on the effects of unit embodied energy on different concrete unit weights, see Figure A.11 through Figure A.14 in Appendix A. While Section 4.3.1.3 analyzes the effects of unit embodied energy on different grades of steel reinforcement, some of the same principles apply. Note that 0.65 MJ/kg and 3.25 MJ/kg are given for the unit embodied energy of concrete and not the 4.45 MJ/kg and 22.25 MJ/kg discussed in Section 4.3.1.3. Each of these values, 0.65 MJ/kg and 3.25 MJ/kg, represent one-half of the baseline unit embodied energy of concrete and two-and-a-half times the baseline unit embodied energy of concrete, respectively.
CHAPTER FIVE

PARAMETRIC STUDIES WITH COST OPTIMIZED

5.1 Overview

In this chapter, several parametric studies have been conducted to analyze how the flexural moment affects the design of beams optimized for cost. Parameters in these studies include yield strength of steel reinforcement, compressive strength and unit weight of concrete, and unit cost. In a few studies, the optimum embodied energy design was computed to determine what effect it would have on the total cost of the section when compared to the total cost of the optimum cost design; the same was done again, but vice versa. When one objective function is optimized for the other, it is commonly referred to as multiobjective optimization. Examples of this are shown in Figure 5.16 through Figure 5.19.

Explanation of Applied Moment

Consistent with Chapter 4 (see Section 4.1), changes in span length and flexural moment were considered in all parametric studies. The applied moment, represented as \( wL^2 \), is plotted on the x-axis for each study in this chapter.

Baseline Assumptions

The baseline assumptions used in Chapter 4 are used again in this chapter. These assumptions are discussed in detail in Section 4.1. Unless labeled otherwise, the figures in this chapter were created using those baseline assumptions.
**Unit Cost Values**

Several sources identified the unit cost for corresponding compressive strengths and yield strengths of concrete and steel reinforcement, respectively. The values are given in Table 5.1 with the steel-to-concrete ratios in the rightmost column. For the parametric studies in this section, $3.50/\text{ft}^3$ was used for concrete, and $245/\text{ft}^3$ was used for steel reinforcement. Each of these values was taken from the 2013 edition of RS Means after interpolating between 4,000 psi and 4,500 psi for the concrete (Waier (Ed.), 2012). Because the cost of concrete includes the cost of delivery, a ten percent reduction was applied to account for this cost. Both of these unit costs correspond with the baseline values. Unit cost coefficients vary for materials with different properties, but based on limited data, constant values were used throughout this study. The values for unit embodied energy listed in Section 4.1 were also used for multiobjective optimization in this chapter.
### Table 5.1: Unit cost for concrete and steel.¹

<table>
<thead>
<tr>
<th>Reference</th>
<th>$γ_{SC}$</th>
<th>$γ_{SS}$</th>
<th>$γ_{SS}$ ¹</th>
<th>$γ_{SC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samman &amp; Erbatur (1995)</td>
<td>-</td>
<td>-</td>
<td>≈ 20</td>
<td></td>
</tr>
<tr>
<td>Rajeev &amp; Krishnamoorthy (1998)</td>
<td>$20.81/\text{ft}^3$ ($f'_c = 3000 \text{ psi}$)</td>
<td>$1578/\text{ft}^3$ ($f_y = 60,000 \text{ psi}$)</td>
<td>75.83</td>
<td></td>
</tr>
<tr>
<td>Sahab et al. ² (2001)</td>
<td>($f'_c = 5075 \text{ psi}$)</td>
<td>($f_y = 66,715 \text{ psi}$)</td>
<td>71.43</td>
<td></td>
</tr>
<tr>
<td>Paya-Zaforteza et al. (2009)</td>
<td>($f'_c = 5075 \text{ psi}$)</td>
<td>($f_y = 58,015 \text{ psi}$)</td>
<td>86.34</td>
<td></td>
</tr>
<tr>
<td>Guerra et al. ² (2011)</td>
<td>($f'_c = 4060 \text{ psi}$)</td>
<td>($f_y = 60,915 \text{ psi}$)</td>
<td>62.79</td>
<td></td>
</tr>
<tr>
<td>Yeo &amp; Gabbai (2011)</td>
<td>$3.68/\text{ft}^3$ ($f'_c = 4930 \text{ psi}$)</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Waier (Ed.) ³ (2012)</td>
<td>$3.78/\text{ft}^3$; $3.89/\text{ft}^3$ ($f'_c = 4000 \text{ psi}$; 4500 psi)</td>
<td>$245/\text{ft}^3$ ($f_y = 60,000 \text{ psi}$)</td>
<td>64.85; 63</td>
<td></td>
</tr>
<tr>
<td>Yeo &amp; Potra (2013)</td>
<td>$3.68/\text{ft}^3$; $3.82/\text{ft}^3$ ($f'_c = 4350 \text{ psi}$; 5800 psi)</td>
<td>$240.04/\text{ft}^3$ ($f_y = 60,915 \text{ psi}$)</td>
<td>65.21; 62.79</td>
<td></td>
</tr>
</tbody>
</table>

¹Units given in SI were converted to U.S. customary
²Includes material and placement costs
³Unit cost of concrete includes delivery

**Methodology for presenting parametric study results**

Results in this chapter are presented using the same general plot types that were used in Chapter 4. Section 4.1 provides information on each plot type and gives an example demonstrating how the plots can be applied in design.
5.2 Parametric Studies

This section contains parametric studies of cost optimization. Multiobjective optimization of cost and embodied energy is also discussed. Results from supplementary parametric studies on cost optimization are presented in Appendix B.

The primary purpose of the cost optimization studies conducted in this research program is to make comparisons to the embodied energy optimization studies. Differences between the two are dependent upon the steel-to-concrete ratio, which is 70 (Table 5.1) for the unit cost and 23.13 (Table 4.3 after multiplying each unit embodied-energy coefficient by the respective material’s unit weight) for the unit embodied energy. This distinction is marked by the objective functions in Equation 3.1 and Equation 3.2. With a steel-to-concrete ratio for unit cost more than three times that of the steel-to-concrete ratio for unit embodied energy, the parametric studies that populate this chapter have different results than those presented in Chapter 4. Therefore, the discussion in this chapter will concentrate on differences between cost and energy optimization results.

5.2.1 Variation of Steel Yield Strength

5.2.1.1 Selection of Optimum Steel Yield Strength

Making direct comparisons between Figure 5.1 and Figure 4.20, it’s easy to see that when optimizing for cost, 120 ksi is more advantageous than when optimizing for embodied energy. As stated in the previous section, the steel-to-concrete ratio for cost is more than three times larger than it is for embodied energy; therefore, it is desirable to reduce the amount of steel and thus reduce the overall cost. One of the leverage points of using high-strength steel is that it reduces the demand for steel. With that knowledge, 120
ksi steel is of greater benefit in a cost-optimized design than in an embodied-energy optimized design.

Figure 5.1: Comparison of cost ratios for 60 ksi and 120 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

5.2.1.2 Selection of Optimum Cross-Section

Figure 5.2, Figure 5.3, and Figure 5.5 can be compared to Figure 4.21, Figure 4.22, and Figure 4.24 to demonstrate this concept: in cost-optimized beams, it is better to have less steel reinforcement and more concrete (in the form of a greater beam height) when juxtaposed with energy-optimized beams. These relative differences between cost- and energy-optimized beams are more pronounced for lower grades of steel.

Comparing Figure 5.2(a) and Figure 4.21(a), it can be observed that the optimum aspect ratio is lower for cost-optimized beams than for energy-optimized beams.

However, the aspect ratio is practically identical for each of these when $h$ is assigned the ACI minimum since the height is restricted to one value for each yield strength (Figure 5.2(b) and Figure 4.21(b)).
Figure 5.2: Aspect ratio for different grades of steel reinforcement when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Similarly, Figure 5.3 exhibits a drop in the optimum reinforcement ratio relative to Figure 4.22. This occurs because steel is more “expensive” from a cost perspective than from an energy perspective, and thus cost-optimized sections have less steel reinforcement and increased concrete. Smaller moments appear to amplify this effect because less steel is required to support the smaller moments.

Figure 5.3: Reinforcement ratio for different grades of steel reinforcement when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure 5.4: Recommended reinforcement ratio for different grades of steel reinforcement when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Because concrete has a lower impact from a cost perspective than from an embodied-energy perspective, the optimum span-to-depth ratio for cost is lower relative to that of embodied energy. This concept is expressed by Figure 5.5(a) and Figure 4.24(a). Figure 5.5(b) and Figure 4.24(b), on the other hand, are not apt for comparison since the height, $h$, is a function of the length and would therefore be identical for each.

Figure 5.5: Span-to-depth ratio for different grades of steel reinforcement when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

5.2.2 Variation of Concrete Compressive Strength

5.2.2.1 Selection of Optimum Concrete Compressive Strength

Figure 5.6 and Figure 4.30 compare the preferability of different concrete compressive strengths for cost and energy, respectively. There are only subtle differences
between these two figures, suggesting that preferable compressive strength does not vary significantly between cost- and energy-optimized beams. However, stronger concrete can increase the nominal moment capacity of the section, which may produce a design with marginally less steel. This can be observed in Figure 5.6(a) where the dashed line shifts slightly downward relative to Figure 4.30(a). Though it may be counterintuitive at first glance, the steel-to-concrete ratio is actually decreasing as one travels up the y-axis, opposite of that in Figure 5.1. This is because the unit cost of the concrete is being multiplied by a factor ranging from 0.5 to 2.5, so at 2.5, the steel-to-concrete ratio is less than at 0.5. Therefore, high-strength concrete produces designs with less reinforcement.

Figure 5.6: Comparison of cost ratios for 4,350 psi and 10,000 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
5.2.2.2 Selection of Optimum Cross-Section

Figure 5.7, Figure 5.8, and Figure 5.10 show the cost-optimum aspect, reinforcement, and span-to-depth ratios, respectively, for varying strengths of concrete. When compared to the energy-optimized results (Section 4.3.2.2), the cost-optimum ratios have more concrete and less steel. As discussed in Section 5.2.1.2, this occurs because steel is more “expensive” than concrete from a cost perspective than from an energy perspective. However, the relative differences between cost- and energy-optimized beams are more pronounced for higher-strength concrete.

Figure 5.7: Aspect ratio for different grades of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.8: Reinforcement ratio for different grades of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure 5.9: Recommended reinforcement ratio for different grades of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.10: Span-to-depth ratio for different grades of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

5.2.3 Variation of Concrete Unit Weight

5.2.3.1 Selection of Optimum Concrete Unit Weight

Cost and energy comparisons of optimum concrete unit weight can be made using Figure 5.11 and Figure 4.35, respectively. Although the trends are similar in both figures, lightweight concrete is slightly more beneficial from a cost perspective than for energy. This is because LW concrete reduces the load demand, and consequently the required reinforcement, relative to NW concrete. Reduced steel quantity has a greater effect on cost than on embodied energy. Just like Figure 5.6, as one moves upward along the y-axis, the steel-to-concrete ratio is decreasing in Figure 5.11. So because a lighter beam
lessens the factored shear and moment in that member and consequently lowers the demand for steel, the dashed line in Figure 5.11 shifts downward relative to Figure 4.35.

![Figure 5.11: Comparison of cost ratios for 145 pcf and 90 pcf concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.](image)

5.2.3.2 Selection of Optimum Cross-Section

Figure 5.12, Figure 5.13, and Figure 5.15 show the cost-optimum aspect, reinforcement, and span-to-depth ratios, respectively, for varying concrete unit weight. When compared to the studies of embodied energy, each of these figures hinges on the same basic principles discussed in Section 5.2.1.2. However, the various unit weights have little to no bearing on the relative differences between cost- and energy-optimized beams.
Figure 5.12: Aspect ratio for different unit weights of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.13: Reinforcement ratio for different unit weights of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.14: Recommended reinforcement ratio for different unit weights of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure 5.15: Span-to-depth ratio for different unit weights of concrete when optimized for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

5.2.4 Cost and Embodied Energy Multiobjective Optimization

Figure 5.16 and Figure 5.18 show that the relationship between optimized cost and optimized embodied energy are strongly correlated. Results in these figures are consistent with the findings from Paya-Zaforteza et al. (2009) as presented in Figure 2.5. These researchers found a strong correlation between optimized CO\textsubscript{2} emissions and optimized cost. Figure 5.16 and Figure 5.18 also show that the total embodied energy and total cost have a positive linear correlation with the flexural moment. It is apparent that limiting the height of the beam to the ACI minimum lends itself to an enlarged total embodied energy and total cost.

While the differences between the embodied-energy optimized design and the cost-optimized design are not pronounced in Figure 5.16 and Figure 5.18, Figure 5.17 and Figure 5.19 give a clear indication of the influence of simultaneous optimization. Values presented in Figure 5.17 are the average total embodied energy from the top ten cost-optimum designs divided by the average total embodied energy from the top ten embodied-energy-optimum designs. Thus, the percentage of additional energy used could be determined for cost-optimized design when compared to energy-optimized designs.
Between five and ten percent more energy was used for cost-optimized short-span beams, but that difference in energy use approaches zero as the moment grows in magnitude. This phenomenon can be explained by Figure 5.16(a), where the two lines run parallel to each other; the difference between the two does not change a great deal, but larger numbers are being divided and therefore smaller percentages are occurring. When the ACI minimum is applied, the two lines in Figure 5.16(b) are virtually equal to each other, hence the horizontal line running throughout Figure 5.17(b).

Figure 5.18 and Figure 5.19 feature the cost when optimized for total embodied energy and total cost. The figures demonstrate very similar findings to Figure 5.16 and Figure 5.17, albeit with a smaller percent difference between the two.

Results from the multiobjective optimization show that cost-optimized beams have up to 10% more embodied energy than do energy-optimized beams but are up to 5% cheaper than energy-optimized beams (Figure 5.17(a) and Figure 5.19(a)).
Figure 5.16: Total embodied energy of the beam when optimized for embodied energy and for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.17: Total embodied energy of the cost-optimized design divided by the total embodied energy of the embodied energy-optimized design. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure 5.18: Total cost of the beam when optimized for embodied energy and for cost. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure 5.19: Total cost of the embodied energy-optimized design divided by the total cost of the cost-optimized design. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
6.1 Summary and Conclusions

A methodology for optimizing RC beams for minimized cost and/or embodied energy was presented, which utilized concepts of LCA, optimization, and reinforced-concrete analysis. The methodology can be applied to determine environmental or economic preferability of different materials used in RC beams. Once materials are selected, the methodology can also be used to design RC sections for minimized economic cost, environmental cost, or both. The methodology was created by considering a wide range of unit cost or unit embodied-energy coefficients to the optimization process.

After developing parametric studies for aspect, reinforcement, and span-to-depth ratios, several overarching trends were observed. The foremost observation was that the optimal results were highly sensitive to the problem constraints. For example, by constraining the span-to-depth ratio to one constant value prescribed by ACI 318-11 for omitting deflection calculations, significant changes in material preferability and optimum cross-section were observed when compared to an unconstrained span-to-depth ratio.

In studies evaluating the reinforcement ratio, it was observed that the optimal ratio did not vary significantly with the flexural moment. Given this fairly constant trend, it was reasonable to average the values together for a given reinforcement yield strength and provide that as the recommended reinforcement ratio. Au contraire, the optimal
aspect and span-to-depth ratios were not constant for changes in moment. The aspect ratio had a negative exponential relationship with the flexural moment, dipping down sharply before leveling off at larger moments. The span-to-depth ratio decreased at a fairly constant rate as the flexural moment increased. These trends were observed generally even as material property parameters were changed.

Three different material properties were considered in the optimization program: reinforcement yield strength, concrete compressive strength, and concrete unit weight. As the strength of the steel reinforcement increased, the optimum designs had less steel. Similarly, as the weight of the concrete decreased, the optimum designs had less steel. The strength of the concrete had a much less significant role on the influence of the optimal design.

The ratio of steel-to-concrete cost is greater than the ratio of steel-to-concrete embodied energy, which means that steel is more “expensive” from a cost perspective than from an energy perspective. This difference results in cost-optimum designs having less steel and more concrete; knowing this and the aforementioned effects of different materials, high-strength steel and LW concrete have a greater impact on the minimization of cost than they do on the minimization of energy. On the other hand, high-strength concrete makes little difference between the optimization of cost and energy.

Multiobjective optimization results of cost and energy were consistent with previous studies that also showed strong correlation between cost and energy optimum designs. Total cost and total embodied energy are also proportional to the flexural moment. Multiobjective optimization suggests that cost-optimized beams have up to 10%
more embodied energy than do energy-optimized beams but are up to 5% cheaper than energy-optimized beams.

6.2 Recommendations for Future Work

Throughout this study, only simply supported rectangular RC beams were considered. However, the types of problems that could be explored are practically limitless. Other applications to be examined include different boundary conditions, different types of structural systems (as opposed to single components), different environmental metrics, and prestressed concrete.

While simply supported beams are common in design analysis, other types of boundary conditions include fixed-fixed, cantilever, and continuous beams. Changing the support conditions produces different shear and moments; therefore, the design criteria for meeting flexural and shear constraints would change in response to these modified values.

There is value in analyzing only one structural component as was done in this study; designers can break down the design of optimal members into an isolated element, which is not so easily accomplished in more complex structural systems. However, structural systems provide a more realistic representation of how individual members are behaving in a larger scheme, thus supplying the engineer with more applicable design criteria. Thus, a natural extension of this research would be optimization of RC systems.

Embodied energy and carbon dioxide emissions are just two metrics utilized to examine the environmental impacts of reinforced concrete. Both of these metrics are associated with the impacts of climate change and are closely linked to one another.
Other metrics used in the life cycle inventory of construction materials include mineral depletion, land use, and human toxicity. These metrics should also be considered in future analyses.

Prestressed concrete has many beneficial applications and can be used to generate more efficient members. While prestressed concrete design is more involved, it may help advance the capacity of structural concrete optimization. Because concrete is used in many bridges, studies pertaining to prestressed concrete should also include durability and degradation as design criteria.

Integrated design is a collaborative effort that incorporates the multiple parties responsible for design, construction, and maintenance of a building. These groups include mechanical, electrical, and structural engineers, as well as architects and contractors. The integrated design process helps to promote holistic design, which could enhance the optimization of structural concrete systems from a practical standpoint.

Finally, it is suggested that future research in this area should include evaluation of additional concrete and reinforcement materials. Low-carbon cement is a technology that has promise for reducing the environmental footprint of the RC structures. The methodology developed in this thesis would be useful in evaluating the benefits and optimum conditions for using low-carbon cements.
APPENDICES
Appendix A

Supplementary Parametric Studies with Embodied Energy Optimized

Figure A.1: Comparison of embodied energy ratios for 60 ksi and 40 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure A.2: Comparison of embodied energy ratios for 60 ksi and 90 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure A.3: Comparison of embodied energy ratios for 60 ksi and 100 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure A.4: Comparison of embodied energy ratios for 4,350 psi and 2,500 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure A.5: Comparison of embodied energy ratios for 4,350 psi and 8,000 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure A.6: Aspect ratio for different grades of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.

Figure A.7: Reinforcement ratio for different grades of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.

Figure A.8: Recommended reinforcement ratio for different grades of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.
Figure A.9: Span-to-depth ratio for different grades of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.

Figure A.10: Comparison of embodied energy ratios for 145 pcf and 103 pcf concrete. (a) Variable-span-to-depth ratio, (b) ACI minimum depth.

Figure A.11: Aspect ratio for different unit weights of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.
Figure A.12: Reinforcement ratio for different unit weights of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.

Figure A.13: Recommended reinforcement ratio for different unit weights of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.

Figure A.14: Span-to-depth ratio for different unit weights of concrete when optimized for embodied energy. (a) One-half the baseline unit embodied energy of concrete, (b) Two-and-a-half times the baseline unit embodied energy of concrete.
Appendix B

Supplementary Parametric Studies with Cost Optimized

Figure B.1: Comparison of cost ratios for 60 ksi and 40 ksi steel reinforcement. (a) Variable span-to-depth ratio, (b) ACI minimum depth.

Figure B.2: Comparison of cost ratios for 4,350 psi and 2,500 psi concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Figure B.3: Comparison of cost ratios for 145 pcf and 115 pcf concrete. (a) Variable span-to-depth ratio, (b) ACI minimum depth.
Appendix C

Extended Bibliography

Beams


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Frames


**Retaining Structures**


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