Clemson University
TigerPrints

All Theses | Theses

8-2013

Newtonian Semiclassical Gravity In Three Ontological Quantum Theories That Solve The Measurement Problem: Formalisms And Empirical Predictions

Maaneli Derakhshani
Clemson University, maanelid@yahoo.com

Follow this and additional works at: https://tigerprints.clemson.edu/all_theses

Part of the Physics Commons

Recommended Citation
Derakhshani, Maaneli, "Newtonian Semiclassical Gravity In Three Ontological Quantum Theories That Solve The Measurement Problem: Formalisms And Empirical Predictions" (2013). *All Theses*. 1764.
https://tigerprints.clemson.edu/all_theses/1764

This Thesis is brought to you for free and open access by the Theses at TigerPrints. It has been accepted for inclusion in All Theses by an authorized administrator of TigerPrints. For more information, please contact kokeefe@clemson.edu.

Newtonian Semiclassical Gravity In Three Ontological Quantum Theories That Solve The Measurement Problem: Formalisms And Empirical Predictions

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Masters of Science
Physics

by
Maaneli Derakhshani
August 2013

Accepted by:
Dr. Dieter Hartmann, Committee Chair
Dr. Catalina Marinescu
Dr. Sumanta Tewari

Abstract

In this thesis, we consider the implications of solving the quantum measurement problem for the Newtonian description of semiclassical gravity. First we review the formalism of the Newtonian description of semiclassical gravity based on standard quantum mechanics - the Schroedinger-Newton theory - and two well-established predictions that come out of it, namely, gravitational 'cat states' and gravitationally-induced wavepacket collapse. Then we review three quantum theories with 'primitive ontologies' that are well-known known to solve the measurement problem - Schroedinger's many worlds theory, the GRW collapse theory with matter density ontology, and Nelson's stochastic mechanics. We extend the formalisms of these three quantum theories to Newtonian models of semiclassical gravity and evaluate their implications for gravitational cat states and gravitational wavepacket collapse. We find that (1) Newtonian semiclassical gravity based on Schroedinger's many worlds theory is mathematically equivalent to the Schroedinger-Newton theory and makes the same predictions; (2) Newtonian semiclassical gravity based on the GRW theory differs from Schroedinger-Newton only in the use of a stochastic collapse law, but this law allows it to suppress gravitational cat states so as not to be in contradiction with experiment, while allowing for gravitational wavepacket collapse to happen as well; (3) Newtonian semiclassical gravity based on Nelson's stochastic mechanics differs significantly from Schroedinger-Newton, and does not predict gravitational cat states nor gravitational wavepacket collapse. Considering that gravitational cat states are experimentally ruled out, but gravitational wavepacket collapse is testable in the near future, this implies that only the latter two are viable theories of Newtonian semiclassical gravity and that they can be experimentally tested against each other in future molecular interferometry experiments that are anticipated to be capable of testing the gravitational wavepacket collapse prediction.

Acknowledgments

I thank Dieter Hartmann, Roderich Tumulka, Travis Norsen, and Ward Struyve for fruitful discussions over the past year about the topics in this thesis. I also thank Eugen Dumitrescu and Shaun Hampton for moral support and for generously serving as my sounding boards over the past year. I would also like to thank Catalina Marinescu for her support and understanding over the past two years. And last but not least, I thank Amanda Crumpton for considerable help with the administrative component of this thesis.

Table of Contents

Chapter 1

Introduction

The problem of formulating a quantum theory of gravity has been around since the early 1930's, and in the seventy years since, a complete and consistent theory of quantum gravity still seems far off [1]. However, there is an intermediate step to quantum gravity that seems to be more tractable - just as there can be semiclassical theories in electrodynamics [2] which approximate the fully quantum effects of QED, so there could be semiclassical theories of gravity that approximate the fully quantum theory of gravity (whatever that theory turns out to be). By semiclassical, we mean a classical gravitational field coupled to quantized matter. The problem of how to consistently couple a classical gravitational field to quantized matter was first addressed by M¿ller and Rosenfeld, who proposed the modified Einstein equation (also called the "M¿ller-Rosenfeld" equation)

$$G_{nm} = \frac{8\pi G}{c^4} < \hat{T}_{nm} >, \tag{1.1}$$

where $< \hat{T}_{nm} > = < \psi|\hat{T}_{nm}|\psi >$. Using standard/textbook quantum mechanics, this turns out to be the only way to incorporate a quantum description of the right hand side of (1) while keeping the left hand side a classical field [**?**]. This theory clearly implies nonlinearities in quantum mechanics since (1) says that the metric couples to the wavefunction, and vice versa. It is notable also that (1) is predicted from the semiclassical approximation to the Wheeler-deWitt equation in canonical quantum gravity [3]. Moreover, the broad research program of "emergent gravity" is based on the idea that the gravitational field is not quantized and that (1) is a fundamental description of the coupling between quantized matter and gravity [3].

The literature contains a number of criticisms of semiclassical gravity [4,6–9], but none seems decisive [3,5,10,11]. For example, one might argue that measurements with nonquantized gravitational waves could violate the uncertainty principle for quantized matter [7]; but there are intrinsic limitations to the measurement of even a classical gravitational field [5, 12]. As another example, it is possible that the necessary measurement would require an apparatus massive enough to collapse into a black hole [11]. Experimentally, neutron interferometry [13] and microscopic deflection experiments [14] show that quantum matter interacts gravitationally as expected, but these results do not require quantization of the gravitational field itself. More direct experimental tests have been proposed using superpositions in Bose-Einstein condensates [15], as well as gravitational radiation from quantum systems [16], but neither is practical yet. On the observational side, the density perturbations in the CMB spectrum predicted by eternal cosmic inflation (which uses semiclassical gravity effects) may also soon be tested with the Planck Satellite's mapping of the CMB power spectrum [**?**].

However, these various proposals depend on the general relativistic effects predicted by (1). Since (1) is very hard to study analytically or numerically, researchers have also looked at its Newtonian limit, the Schroedinger-Newton (SN) theory, and studied that regime in-depth. As it turns out, the SN theory makes straightforward predictions, one of which has already been experimentally tested (the prediction of 'cat states') [14] and another which may be testable in the next generation of molecular interferometry experiments (gravitational wavepacket collapse) [17].

Despite all this work, very little has been done on the implications of the interpretation of quantum mechanics for semiclassical gravity [18,19]. Nevertheless, there are open questions regarding how (1) solutions to the quantum measurement problem and (2) formulating quantum theory in an ontological[1] way might change the very formulation of a semiclassical gravity theory, as well as the empirical predictions of semiclassical gravity. These are questions we will explore here. To do this in the simplest and most straightforward way, we will make use of nonrelativistic, spinless versions of quantum mechanics.

The paper is outlined as follows. In section 2, we review the SN theory and two of its well-known predictions - cat states and gravitational wavepacket collapse. Section 3 reviews the measurement problem in standard quantum mechanics, and how it might be relevant to semiclassi-

[1]'Ontological' is a word commonly used in the quantum foundations literature to refer to a theory which posits elements of physical reality.

cal gravity. Section 4 motivates the usefulness of quantum theories that have "primitive ontologies" (a term to be defined later), in formulating semiclassical theories of gravity. Section 5 reviews three ontological quantum theories that are known to solve the measurement problem - Schroedinger's many-worlds interpretation, the Ghirardi-Rimini-Weber collapse theory with matter-density ontology, and Nelson's stochastic mechanics. In section 6 we formulate Newtonian models of semiclassical gravity based on these three ontological quantum theories and evaluate their implications for the SN predictions of cat states and gravitational wavepacket collapse. Finally, in section 7, we summarize our findings and discuss their implications for the idea of semiclassical gravity more generally.

Chapter 2

Part I

2.1 Schroedinger-Newton Theory

2.1.1 Formalism

To obtain the Schroedinger-Newton description of the semiclassical gravity theory described by (1), we must take its Newtonian limit [CITE]. Making the approximations $g_{nm} = \eta_{nm} + h_{nm}$, $|T^{nm}|/T^{00} = |T^{nm}|/\rho << 1$, and $v << c$, (1) reduces to the semiclassical Newton-Poisson equation

$$\nabla^2 V(x,t) = -4\pi G m |\psi(x,t)|^2, \tag{2.1}$$

with solution $V(x,t) = -G \int \frac{m|\psi(x',t)|^2}{|x-x'|} d^3x'$, and ψ satisfying the nonlinear integro-differential Schroedinger equation,

$$i\hbar \partial_t \psi(x,t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x,t) + mV(x,t)\psi(x,t). \tag{2.2}$$

Although the SN equations are nonlinear, $|\psi|^2$ still satisfies the quantum continuity equation

$$\frac{\partial |\psi|^2}{\partial t} = -\nabla \cdot \left[-\frac{i\hbar}{2m} \left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) \right], \tag{2.3}$$

and $|\psi|^2$ is interpreted as a probability density per standard quantum mechanics.

The N-body generalizations (ignoring the interaction potential term for simplicity) are as follows:

$$\nabla^2 V(x,t) = -4\pi G \int dx'_1 ... dx'_N |\psi(x'_1...x'_N,t)|^2 \sum_{i=1}^{N} m_i \delta^3(x - x'_i), \tag{2.4}$$

and

$$i\hbar\partial_t \psi(x_1...x_N,t) = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 \psi(x_1...x_N,t) + \sum_{i=1}^{N} m_i V(x_i,t)\psi(x_1...x_N,t), \tag{2.5}$$

with solution

$$V(x_i,t) = -G \sum_{j=1}^{N} \int \frac{m_j |\psi(x'_1...x'_N,t)|^2}{|x_i - x'_j|} dx'_1...dx'_N. \tag{2.6}$$

Moreover, $|\psi|^2$ is conserved via

$$\frac{\partial |\psi|^2}{\partial t} = -\sum_{k=1}^{N} \nabla_k \cdot \left[-\frac{i\hbar}{2m_k} \left(\psi^* \nabla_k \psi - \psi \nabla_k \psi^* \right) \right], \tag{2.7}$$

allowing for the standard quantum mechanical probability interpretation in the N-body case.

The coupled equations defined by (2)-(3) or (5)-(6) are known as the Schroedinger-Newton (SN) equations. They describe a physical world in which the wavefunction in configuration space drives the dynamical evolution of a mass-density field (or a set of N mass-density fields in the N-system case) in 3-space, the evolving mass-density field(s) sources a real classical gravitational potential in 3-space, and this gravitational potential couples back to the wavefunction, thereby altering the dynamical evolution of the mass-density field (i.e. the so-called gravitational 'back-reaction').

Let us now consider two well-established predictions of this theory.

2.1.2 Empirical Predictions

2.1.2.1 Cat States

The above formulation of semiclassical gravity has a well-known prediction that makes it an empirically problematic theory - it admits 'cat state' solutions.

Elaborating the example by Ford [?], suppose we have a quantum state $\psi = \frac{1}{\sqrt{2}}[\phi_1 + \phi_2]$, where each state in the superposition corresponds to a macroscopic mass distribution in a distinct location (e.g. a 1000 kg mass occupying a volume located on the left or right side of a room). Inserting ψ into (2) gives

$$\nabla^2 V = -4\pi G \left[\frac{m}{2}|\phi_1|^2 + \frac{m}{2}|\phi_2|^2 \right], \tag{2.8}$$

or the prediction of a semiclassical gravitational field which is an average of the fields due to the two distributions separately (in this case, the gravitational field is the sum effect of two 500 kg masses on opposite sides of the room). However, we would expect that an actual measurement of the gravitational field should correspond to a single 1000 kg mass density source occupying a single location, but in different locations in different measurement trials. Unfortunately, such a measurement outcome is not predicted by anything in the SN equations. Moreover, Page and Geilker's torson balance pendulum experiment[1] has already disconfirmed the gravitational field predicted by (6) [**?**, **?**]. However, we don't really need the Page and Geilker experiment to tell us this; if we trust our perceptual experiences of the physical world, it is obvious that this prediction can't be right since we don't (for example) feel the gravitational pull of a messy smear of suns occupying every possible 3-space volume.

It should be remarked that incorporating the effects of quantum decoherence does not get rid of these cat states (for essentially the same reason that decoherence doesn't solve the measurement problem); all decoherence can do is ensure that $\phi_1(q)\cdot\phi_2(q) \approx 0$ (i.e. ϕ_1 and ϕ_2 have disjoint supports in configuration space) for all $q = (x_1, ..., x_N)$ so that there are no interference terms contributing to the r.h.s. of (6).

Hence, some other modification of the SN equations is needed in order to rid the theory of cat state solutions and make it empirically adequate.

[1]Their experiment tested the gravitational response of a torsion balance to the presence of macroscopic masses. The quantum aspect of the experiment was entirely in the method of choosing the locations of these masses. The choice was determined by a quantum random number generator so that, depending on the value of some quantum variable, the masses would be sent either to the left or the right of the balance. Page and Geilker found that the balance responded only to the presence of a mass and not the expectation value of where the mass would go.

2.1.2.2 Free Particle Wavepacket

It turns out the nonlinearities in (3) also lead to observable consequences. Consider a free wavepacket of mass m with initial Gaussian form

$$\psi(r,0) = \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\alpha r^2/2}, \tag{2.9}$$

with width $\alpha^{-1/2}$. As first shown by Salzman and Carlip [CITE], the time-evolution of ψ will depend on two competing effects, the quantum mechanical spreading of the wave function and its Newtonian 'self-gravitation'. The latter arises from because semiclassical gravity treats a wavefunction as a distributed source. For a very low mass, self-gravitation should be negligible, while for a high enough mass, the wavefunction should undergo 'gravitational wavepacket collapse.'

We can estimate the critical mass at the boundary between wave packet spreading and collapse by first noting that the peak probability density for a free particle occurs at

$$r_p(t) \sim (\alpha)^{-1/2}\left(1 + \frac{\alpha^2\hbar^2}{m^2}t^2\right). \tag{2.10}$$

This peak probability location accelerates outward at a rate $a_{out} = \ddot{r}_p \sim \hbar^2/m^2 r_p^3$, and balances the inward gravitational acceleration $a_{in} \sim Gm/r_p^2$ at $t = 0$ when

$$m \sim \left(\frac{\hbar^2\sqrt{\alpha}}{G}\right)^{1/3}. \tag{2.11}$$

For an initial width of $\alpha = 5 \times 10^{16} meters^{-2}$ (equivalently, 0.5 microns), the mass (12) is on the order of $10^{10} amu$. Salzman and Carlip were the first to numerically test this prediction and found that "collapse" occurred at $10^4 amu$ instead. However, Giulini and Grossardt [CITE] re-did their numerics and found that the mass scale at which collapse occurred was in fact about $10^{10} amu$. More precisely, they found that for the initial width of 0.5 microns, a Gaussian wavepacket will start shrinking, reaching a minimum of 0.4 microns in 30,000 seconds, and dispersing again thereafter [?]. This finding has since been confirmed by Van Meter [CITE].

It should be remarked that this gravitational wavepacket collapse effect observed by Salzman & Carlip and Giulini & Grossardt does not solve the cat states problem - all the wavepacket collapse effect potentially[2] does is ensure that each state in the superposition will localize separate, 500 kg

[2] We say "potentially" because, as we've noted, Giulini and Grossardt observe a rebound effect after the collapsing wavepacket reaches its minimum width, and it's not clear that this rebound effect goes away for larger masses

mass distributions around their respective locations in 3-space.

The special focus on the width of 0.5 microns comes from actual molecular interferometry experiments in which the wave nature of complex molecules (e.g. carbon fluorofullerene $C_{60}F_{48}$) has been demonstrated (see, e.g. [[?, ?, ?]] for an overview). It has also been suggested that the next generation of molecular interferometry experiments with macromolecule clusters [CITE] may be able to reach the mass scale of $10^{10} amu$, thereby allowing for the possibility of an experimental test of gravitationally-induced wavepacket collapse.

2.2 The Measurement Problem

Here we briefly review the measurement problem of quantum mechanics and examine how it's relevant to the empirical predictions of the SN theory considered in 2.2.1 and 2.2.2. We prefer the formulation of Maudlin [CITE], who first notes that the following three claims are mutually inconsistent:

1. The wavefunction of a system is complete, i.e. the wavefunction specifies (directly or indirectly) all of the physical properties of a system.

2. The wavefunction always evolves in accordance with a linear dynamical equation (the Schroedinger equation).

3. Measurements of, e.g. the spin of an electron always (or at least usually) have determinate outcomes [...].

Now, consider a two-valued observable S with eigenvectors ψ_1 and ψ_2, and let Φ_0 denote its wave-function in the "ready-state" and Φ_1 (Φ_2) the state of the apparatus if the measurement yields ψ_1 or ψ_2. Then the time-evolution of the combined system $\hat{U}\left(\psi_i \otimes \Phi_0\right) = \psi_i \otimes \Phi_i$ holds, where $i \epsilon \{1, 2\}$. So for the general superposition state

$$\psi = c_1\psi_1 + c_2\psi_2, \tag{2.12}$$

the action of $\hat{U}$ on it gives

$$\hat{U}\left(\psi \otimes \Phi_0\right) = c_1\psi_1 \otimes \Phi_1 + c_2\psi_2\Phi_2. \tag{2.13}$$

[Grossardt, personal communication].

Whereas this is a superposition of different pointer states, individual measurements always result in *either* Φ_1 *or* Φ_2. Thus, in contrast to our experience, standard quantum mechanics does not explain how the joint object-apparatus system ends up in a definite state[3].

We note that this conclusion is at odds with claims 1 and 2. Thus, assuming claims 1 and 2 contradicts claim 3. Any proposed resolution to this problem has to therefore deny at least one of the three claims. Denial of claim 1 amounts to assuming the necessity of "hidden-variables", i.e. additional physical variables or parameters that would make it possible in principle to predict the result of a single measurement on a single quantum system. Formulations of quantum mechanics that deny claim 1 are, unsurprisingly, called "hidden-variable" theories. Denial of claim 2 amounts to assuming some process during measurement that interrupts the linear time-evolution of quantum systems, and causes the wavefunction to "collapse" into a definite state. Formulations of quantum mechanics that deny claim 2 are thusly called "collapse theories". Finally, denial of claim 3 amounts to assuming a "many-worlds interpretation" of the wavefunction and its unitary time-evolution.

Interestingly, the SN theory denies claim 2 but still suffers from the measurement problem - the predicted cat state solutions clearly contradict claim 3, and the gravitational wavepacket "collapse" effect caused by the nonlinearities of the theory is, as we showed earlier, inadequate to suppress those cat state solutions. So this tells us that not all methods of denying claims 1 and/or 2 lead to a resolution of the measurement problem. Nevertheless, from these observations, it seems reasonable to suggest that the cat states problem of the SN theory may be a *consequence* of the measurement problem. We would therefore like to know if the three main approaches to solving the measurement problem - hidden-variable theories, collapse theories, and many-worlds theories - eliminate the cat state solutions or can reinterpret the cat state solutions in a way that's consistent with claim 3. We would also like to know if the different approaches to solving the measurement problem, when extended to Newtonian descriptions of semiclassical gravity, will make the same prediction for the evolution of a free particle wavepacket as the SN theory.

[3]Although this argument used the simplifying assumptions of ideal measurements and pure states for both object and apparatus, the conclusion remains effectively unchanged in the completely general case of non-ideal measurements, mixed states, interactions with the environment, etc. [CITE].

Chapter 3

Part II

3.1 Quantum Theories With Primitive Ontologies

In choosing theories of quantum mechanics based on the three solutions to the measurement problem, we would like each theory to also have a clear *primitive ontology* (PO). By PO, we simply mean "variables describing the distribution of matter in space-time" [CITE]. Note that a quantum theory in which only the wavefunction is ontological (i.e. an element of physical reality) does not have a PO, because the wavefunction lives in configuration space rather than space-time. In addition to having physical clarity regarding what they are fundamentally *about*, theories with the appropriate PO's allow us to derive the familiar macroscopic image of (fermionic) matter distributions in space-time like tables, chairs, cats, etc. Additionally, quantum theories with PO's (which we will call "quantum POT's") allow for the derivation of precise empirical predictions.

The SN theory, as it turns out, is a quantum POT. For both the single and many particle case, the theory fundamentally describes the dynamical evolution of a mass-density in space-time, $m|\psi(x,t)|^2$, and the classical gravitational potential $V(x,t)$ sourced by this mass-density. Moreover, the empirical predictions of cat states and gravitational wavepacket collapse were derived directly from the dynamics of this PO. In a sense, the assumption of a PO is almost required in formulating a semiclassical theory of gravity; a classical gravitational field is a field in space-time, and by far the most straightforward possibility for a source for such a field is a mass-density that also lives in space-time. As we will see, two of the three ontological quantum theories we will consider make use of the same PO as the SN theory. The one that doesn't leads to different empirical predictions from the former two and the SN theory.

3.2 Three Quantum POTs That Solve the Measurement Problem

Here we review the theoretical structures of three quantum POTs that solve the measurement problem - (1) Schroedinger's many-worlds, (2) the Ghirardi-Rimini-Weber collapse theory with matter-density ontology, and (3) Nelson's stochastic mechanics - and out of which we will construct Newtonian models of semiclassical gravity. Each of these theories represents one of the three main approaches to solving the measurement problem. Additionally, these three theories have unambiguous PO's defined in terms of the mass parameter of quantum systems. Consequently, it is straightforward to show how they can be extended to Newtonian models of semiclassical gravity. For simplicity, we will restrict ourselves to the nonrelativistic versions of these theories without spin.

3.2.1 Schroedinger's Many-Worlds Theory

The many-worlds interpretation of nonrelativistic quantum mechanics has several variants [CITE], but only one so far that employs a primitive ontology. That version, called "Sm" (where S is for the Schroedinger equation and m is for the mass-density function, to be defined later) [CITE], assumes a matter-density field $m(x,t)$ in space-time whose dynamical evolution is tied to the Schroedinger evolution of the wavefunction. In Sm, the wavefunction is taken to be an ontic field, but it is the ontic mass-density field in space-time which composes physical objects and from which we derive the familiar macroscopic image of (fermionic) matter distributions like tables, chairs, cats, etc. (hence why it's the primitive ontology). The ontic wavefunction is always hidden from direct observation, living as it does in configuration space instead of space-time.

Formulating this theory more precisely, the matter-density field for a single system is defined as

$$m(x,t) = m|\psi(x,t)|^2, \tag{3.1}$$

with ψ evolving by the usual linear Schroedinger equation of quantum mechanics,

$$i\hbar\partial_t\psi(x,t) = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t) + V(x,t)\psi(x,t). \tag{3.2}$$

In the generalization to an N-body system,

$$m(x,t) = \sum_{i=1}^{N} m_i \int dx_1...dx_N \delta^3(x-x_i)|\psi(x_1,...,x_N,t)|^2, \tag{3.3}$$

and

$$i\hbar\partial_t\psi(x_1,...,x_N,t) = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2\psi(x_1...x_N,t) + V(x_1...x_N,t)\psi(x_1...x_N,t). \tag{3.4}$$

The function (15) is the most natural matter-density field in 3-space that one can define from the $|\psi|^2$ distribution in configuration space. The formula says that, starting from $|\psi|^2$, one integrates out the positions of N$-$1 particles to obtain a density in 3-space. Since the number i of the particle that was not integrated out is arbitrary, it gets averaged over. The weights m_i are just the masses associated with the variables x_i.

This theory is in fact equivalent to Erwin Schroedinger's first quantum theory, which he soon after rejected because he thought it was inconsistent with experiment. After all, the spreading of the continuous mass density arising from the Schroedinger evolution in (16) would appear to contradict the familiar localized detection events for quantum particles, such as in the two-slit experiment. Yet, it appears that Schroedinger's rejection may have been premature - it turns out, as Allori et al. show [CITE], Sm was the first many-worlds theory.

To see why, consider the Schroedinger-cat wavefunction $\psi = \frac{1}{\sqrt{2}}[\phi_{alive} + \phi_{dead}]$. Since ϕ_{alive} and ϕ_{dead} are macroscopic states with disjoint support, we have $m_{alive}(x,t)$ which behaves like the mass density of a live cat, and $m_{dead}(x,t)$ which behaves like the mass density of a dead cat. Note also that, since the linearity of the Schroedinger equation means ϕ_{alive} and ϕ_{dead} evolve independently of each other, the live cat and the dead cat, i.e. $m_{alive}(x,t)$ and $m_{dead}(x,t)$, do not interact with each other. More generally, whenever the configuration space wavefunction consists of disjoint packets $\phi_1, ..., \phi_{\mathcal{L}}$,

$$\psi = \sum_{\ell=1}^{\mathcal{L}} \phi_\ell, \tag{3.5}$$

it follows that

$$m(x) = \sum_{\ell=1}^{\mathcal{L}} m_\ell(x), \tag{3.6}$$

where $m_\ell(x)$ is defined in terms of ϕ_ℓ via (13). Moreover, time-evolution via the Schroedinger equation preserves the disjoint support of the ϕ_ℓ's (up to Poincare recurrence times) so that

$$m(x,t) = \sum_{\ell=1}^{\mathcal{L}} m_\ell(x,t). \tag{3.7}$$

In other words, (21) says that there are $\mathcal{L}$ dynamically evolving mass-density fields superimposed on a single space-time. These mass density fields can be regarded as parallel "worlds" in the sense that each field gives a macroscopic image of the dynamics of a physical object in space-time, corresponding to all the possible states in the Hilbert space of ψ. And, as in the Schroedinger cat example, these parallel worlds don't interact each other due to the linearity of the Schroedinger evolution.

So the "many-worlds" here are the many contributions m_ℓ , and $\mathcal{L}$ is the number of the different worlds. However, we must realize that the concept of a "world" does not enter in the definition of the theory. The theory is merely defined by the postulate that $m(x,t)$ means the density of matter together with the laws (17) and (18) for ψ and m. The concept of a "world" is just a practical matter, useful in comparing the m function provided by the theory to our observations.

We note that, although there is no wavefunction collapse in Sm, this is not in contradiction with experiments. When the cat wavefunction interacts with the wavefunction of a measurement apparatus (i.e. a 'pointer' that points to a live or dead cat), it also interacts with the wavefunction of the experimenter so that, for an ideal measurement, the wavefunction of the experimenter also splits into two macroscopically disjoint copies, one of which entangles with the state of the live cat, the other with the dead cat; So each of the copies of the experimenter will 'see' either a live cat or a dead cat. In this way. the measurement problem also gets solved (or 'dissolved').

Before addressing the question of how Sm ensures that experimenters will see statistics that match those of standard quantum mechanics, there is the question Sm addresses the "incoherence problem" or the problem of how it can make sense to talk of probabilities when all possible outcomes are realized in different worlds. Allori et al. say they prefer the approach of Everett, who denied that the incoherence problem is a genuine problem and appealed to the statistical mechanical notion of *typicality* to interpret probabilities. To motivate typicality for Sm, they begin by making claim(1):

> *The relative frequencies for the results of experiments that a typical observer sees agree, within appropriate limits, with the probabilities specified by the quantum formalism.*

By what a "typical observer sees," e.g. relative frequencies corresponding to some property P, it is meant that P occurs in "most" worlds. When this is true, it is said that the behavior is typical, or that P typically holds, or that P is typical. Allori et al.'s definition of "typical" is that each world m_ℓ is assigned a weight

$$\mu_\ell = \int d^3x m_\ell(x,t) = ||\psi_\ell||^2 \sum_{i=1}^{N} m_i. \tag{3.8}$$

They then make claim(2):

> *A property P holds typically (or, for most worlds) if and only if the sum of the weights μ_k, given by (18), of those worlds for which P holds is very near the sum of the weights of all worlds.*

This is shown with a simple example. Consider an observer performing a large number n of independent Stern–Gerlach experiments for which quantum mechanics predicts "spin up" with probability p and "spin down" with probability $q = 1 - p$. Let this n-part experiment begin at time t_0 and end at time t; now consider just one world at time t_0. Assume that the sequence of outcomes, e.g. ↑↓↓↑ ... ↓↑↑↑ gets recorded macroscopically and thus in $m_\ell(\cdot, t)$. The one world at time t_0 splits into $\mathcal{L} \geq 2^n$ worlds at time t, or

$$\psi(t) = \sum_{\ell=1}^{\mathcal{L}} \phi_\ell(t), \tag{3.9}$$

and

$$m(x,t) = \sum_{\ell=1}^{\mathcal{L}} m_\ell(x,t). \tag{3.10}$$

Now some of the worlds at time t feature a sequence in which the relative frequencies of the outcomes agree, within appropriate limits, with the quantum probabilities p and q. However, this is true only of some worlds, but not all. It is a property P that a world may have or not have.

So we want to ask if P is typical. Let $L(k)$ be the set of those ℓ such that the world m_ℓ features a sequence of k spins up and $n-k$ spins down. All together, these worlds have weight

$$\sum_{\ell \epsilon L(k)} \mu_\ell = \left(\sum_i m_i\right) \sum_{\ell \epsilon L(k)} ||\psi_\ell||^2 = \left(\sum_i m_i\right) \begin{pmatrix} n \\ k \end{pmatrix} p^k q^{n-k}. \tag{3.11}$$

Because n is large, the weight is overwhelmingly concentrated on those worlds for which the relative frequency k/n of "up" is close to p. This follows from the law of large numbers, which ensures that, if we generated a sequence of n independent random outcomes, each "up" with probability p or "down" with probability q, then the relative frequency of "up" will be close to p with probability close to 1. Thus the total weight of the worlds with $k/n \approx p$ is close to the total weight. Thus, this example shows how claim(2) yields claim(1), and how Sm is empirically equivalent to standard quantum mechanics.

Finally, we note that Allori et al. show how Sm has a straightforward formal generalization to relativistic quantum field theory, thereby making it a viable competitor to both standard quantum mechanics and (in principle) standard relativistic quantum field theory.

3.2.2 Ghirardi-Rimini-Weber Collapse Theory with Matter-Density Ontology

The Ghirardi-Rimini-Weber (GRW) collapse theory with matter-density ontology is a dynamical collapse theory which keeps the equations of the original GRW theory [CITE] and adds a primitive ontology that takes the form of a continuous matter-density field $m(x,t)$ in space-time whose dynamical evolution is tied to the GRW evolution of the wavefunction [CITE]. The most common acronym for such a theory is GRWm, where "m" stands for "mass". In GRWm, the wavefunction is taken to be an ontic field, but it is the ontic matter-density field in space-time from which we derive the familiar macroscopic image of (fermionic) matter distributions like tables, chairs, cats, etc. (hence why it's the primitive ontology). The ontic wavefunction is always hidden from direct observation, living as it does in configuration space instead of space-time. Let's formulate this theory more precisely.

For a single system, the GRWm mass-density field is defined as

$$m(x,t) = m|\psi(x,t)|^2, \tag{3.12}$$

with ψ evolving by the usual linear Schroedinger equation of quantum mechanics until it undergoes discrete, instantaneous, intermittent collapses according to the GRW collapse law. The GRW collapse law says that the wavefunction collapse time T occurs randomly with constant rate per system of $N\lambda = \lambda = 10^{-16}s^{-1}$, where the post-collapse wavefunction $\psi_{T+} = lim_{t\searrow T}\psi_t$ is obtained from the pre-collapse wavefunction $\psi_{T-} = lim_{t\nearrow T}\psi_t$ through multiplication by the Gaussian function

$$\psi_{T+}(x) = \frac{1}{C}g(x-X)^{1/2}\psi_{T-}(x), \tag{3.13}$$

where

$$g(x) = \frac{1}{(2\pi\sigma^2)^{3/2}}e^{-\frac{x^2}{2\sigma^2}} \tag{3.14}$$

is the 3-D Gaussian function of width $\sigma = 10^{-7}m$, and

$$C = C(X) = \left(\int d^3x g(x-X)|\psi_{T-}(x)|^2\right)^{1/2} \tag{3.15}$$

is the normalization factor. The collapse center X is chosen randomly with probability density $\rho(x) = C(x)^2$, and the spacetime locations of the collapses are given by the ordered pair (X_k, T_k). In the generalization to an N-body system,

$$m(x,t) = \sum_{i=1}^{N} m_i \int dx'_1...dx'_N \delta^3(x'_i - X)|\psi(x'_1,...,x'_N,t)|^2, \tag{3.16}$$

where the N-body ψ evolves by the N-body linear Schroedinger equation and is subject to the GRW collapse law

$$\psi_{T+}(x_1,...,x_N) = \frac{1}{C}g(x_i - X)^{1/2}\psi_{T-}(x_1,...,x_N), \tag{3.17}$$

where

$$C = C(X) = \left(\int dx_1'...dx_N' g(x_i' - X) |\psi_{T-}(x_1', ..., x_N')|^2 \right)^{1/2}, \tag{3.18}$$

and i is chosen randomly from $1, ..., N$.

The equations of GRWm for a single system say the following - a wavefunction in 3-space, which evolves by the linear Schroedinger equation until it undergoes the random collapse process in (27), drives the dynamical evolution of a mass-density field in 3-space via (26). When the wavefunction collapses, it localizes the mass-density field around a randomly chosen point in 3-space, with width $10^{-7}m$, and the probability of the randomly chosen point is largest where the mod-squared of the uncollapsed wavefunction is largest, as indicated by (29).

For N-systems, the wavefunction lives in configuration space $\mathbb{R}^{3N}$ and evolves by the N-body linear Schroedinger equation until it undergoes the collapse process in (31); this wavefunction drives the dynamical evolution of N mass density fields in 3-space via (30) so that when the wavefunction collapses, it randomly localizes the mass density fields around randomly chosen (non-overlapping) points in 3-space, each of width $10^{-7}m$, and with probability density given by the square of (32).

The measurement problem is solved because the GRW law gives a mathematically well-defined prescription for how and when the Schroedinger evolution is interrupted, and why experiments on single quantum systems always result in the observation of point-like objects[1] . Because of (29) resp. (32) the statistical predictions of GRWm are in agreement with those of standard nonrelativistic quantum mechanics for all current experiments, though slight deviations due to the GRW collapse law are predicted and are in principle experimentally testable [CITE]. Recently, the GRWm theory was extended to the case of relativistic quantum field theory [CITE], thereby making it a viable competitor to both standard quantum mechanics and standard quantum field theory.

3.2.3 Nelson's Stochastic Mechanics

In Nelson's stochastic mechanics [CITE], it is hypothesized that, in the vacuum of free space, a point particle of mass m and position 3-vector $\mathbf{x}(t)$ is constantly undergoing diffusive motion with

[1] Here I am implicitly talking about experiments in which the system Hamiltonian changes non-adiabatically so as to not have to address the special case of weak measurements.

drift, as modeled by the stochastic differential equation of motion,

$$d\mathbf{x}(t) = \mathbf{b}(\mathbf{x}(t), t)dt + d\mathbf{W}(t). \tag{3.19}$$

The vector $\mathbf{b}(\mathbf{x}(t), t)$ is the 'mean forward' drift velocity of the particle, and $\mathbf{W}(t)$ is a Wiener process modeling the particle's interaction with a homogeneous and isotropic noise field[2] which is hypothesized to cause the diffusive motion.

The Wiener increment, $d\mathbf{W}(t)$, is assumed to be Gaussian with zero mean, independent of $d\mathbf{q}(s)$ for $s \leq t$, and with covariance,

$$E_t \left[d\mathbf{W}_i(t) d\mathbf{W}_j(t)\right] = 2\nu\delta_{ij}dt, \tag{3.20}$$

where E_t denotes the conditional expectation at time t. It is then assumed that the magnitude of the diffusion coefficient ν is proportional to the reduced Planck's constant, and inversely proportional to the particle mass m so that

$$\nu = \frac{\hbar}{2m}. \tag{3.21}$$

We emphasize that although equations (33)-(35) are formally the same as those used for the kinematical description of classical Brownian motion in the Einstein-Smoluchowski (E-S) theory, the physical meaning here is different; the E-S theory uses (33)-(35) to model the Brownian motion of macroscopic particles in a classical fluid in the large friction limit [CITE], whereas Nelson uses (33)-(35) to model frictionless stochastic motion for elementary particles hypothesized to interact with a noise field permeating the vacuum of free space[3].

In addition to (33), the particle's trajectory $\mathbf{q}(t)$ also satisfies the time-reversed equation,

$$d\mathbf{x}(t) = \mathbf{b}_*(\mathbf{x}(t), t) + d\mathbf{W}_*(t), \tag{3.22}$$

where $\mathbf{b}_*(\mathbf{x}(t), t) = -\mathbf{b}(\mathbf{x}(-t), -t)$ is the mean backward drift velocity, and $d\mathbf{W}_*(t) = d\mathbf{W}(-t)$ is the time-reversed Wiener differential. The $d\mathbf{W}_*(t)$ has all the properties of $d\mathbf{W}(t)$, except that it

[2]Nelson has suggested that his hypothesized noise field could have an electromagnetic origin [CITE]. However, in his original paper, the noise field is taken as a formal assumption of the theory.

[3]On the other hand, Garbaczewski [CITE] has argued that it is also possible to interpret Nelson's use of (33) as the large-friction limit of a dissipative stochastic particle dynamics in phase-space, provided a suitable form of microscopic energy conservation is incorporated into the formalism.

is independent of $d\mathbf{q}(s)$ for $s \geq t$. With these conditions on $d\mathbf{W}(t)$ and $d\mathbf{W}_*(t)$, (33) and (36) respectively define forward and backward Markov processes on $\mathbb{R}^3$.

Corresponding to (33) and (35) are the forward and backward Fokker-Planck equations,

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} = -\nabla \cdot [\mathbf{b}(\mathbf{x},t)\rho(\mathbf{q},t)] + \frac{\hbar}{2m}\nabla^2 \rho(\mathbf{x},t), \tag{3.23}$$

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} = -\nabla \cdot [\mathbf{b}_*(\mathbf{x},t)\rho(\mathbf{x},t)] - \frac{\hbar}{2m}\nabla^2 \rho(\mathbf{x},t), \tag{3.24}$$

where (37) is the time-reversal of (38), and $\rho(\mathbf{x},t)$ is the probability density of $\mathbf{x}(t)$ satisfying the normalization condition,

$$\int \rho_0(\mathbf{x}) d^3\mathbf{x} = 1. \tag{3.25}$$

The average of (37) and (38) results in the continuity equation

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} = -\nabla \cdot \left[\frac{\nabla S(\mathbf{x},t)}{m}\rho(\mathbf{x},t)\right], \tag{3.26}$$

under the assumption of an irrotational 'current velocity' field given by

$$\mathbf{v}(\mathbf{x}.t) = \frac{\nabla S(\mathbf{x},t)}{m} = \frac{1}{2}\left[\mathbf{b}(\mathbf{x},t) + \mathbf{b}_*(\mathbf{x},t)\right]. \tag{3.27}$$

Here, S is to be physically interpreted as the velocity potential for a statistical ensemble of non-interacting identical particles. It is thereby analogous to the S function used in the Hamilton-Jacobi formulation of Liouville statistical mechanics [CITE]. (In fact, we shall see that the dynamical evolution of Nelson's S ends up being governed by the so-called Quantum Hamilton-Jacobi equation.)

By subtracting (37) from (36), we also obtain the 'osmotic velocity',

$$\mathbf{u}(\mathbf{x},t) = \frac{\hbar}{2m}\frac{\nabla \rho(\mathbf{x},t)}{\rho(\mathbf{x},t)} = \frac{1}{2}\left[\mathbf{b}(\mathbf{x},t) - \mathbf{b}_*(\mathbf{x},t)\right], \tag{3.28}$$

which fixes ρ as the common, 'equilibrium' probability density (in analogy with a thermal equilibrium density) for solutions of (33) and (35), even though it is time-dependent.

In our view, the physical meaning of (42) has been misconstrued by some researchers [SMOLIN, KYPRIANIDIS, BOHM & HILEY, SPEKKENS], so we wish to emphasize that this expression for the osmotic velocity also appears in the E-S theory, as Nelson himself points out, and it does not mean that ρ must be interpreted as the physical cause of the osmotic velocity of Nelson's particle. (Indeed, such an interpretation would be logically and physically inconsistent with the earlier interpretation of ρ as a probability density.) Rather, in analogy with the E-S theory, Nelson postulates that an osmotic potential field, $R(\mathbf{x},t)$, imparts to his particle a velocity, $\nabla R(\mathbf{x}(t),t)/m$, which is then counter-balanced by the osmotic pressure, $(\hbar/2m)\,\nabla\rho(\mathbf{x}(t),t)/\rho(\mathbf{x}(t),t)$, due to the noise field that his particle propagates through. Nelson's osmotic velocity is then the equilibrium velocity acquired by his particle when $\nabla R/m = (\hbar/2m)\,\nabla\rho/\rho$, where ρ depends on R as $\rho = e^{2R/\hbar}$. Hence, the physical cause of $\mathbf{u}$ is R, and (10) is just a mathematically equivalent rewriting of this relation.

So far we have only presented the kinematics of Nelson's particle. To present the dynamics, we must first motivate Nelson's analogues of the Ornstein-Uhlenbeck mean derivatives. The mean forward and backward derivatives of $\mathbf{x}(t)$ are defined as follows:

$$D\mathbf{x}(t) = \lim_{\Delta t\to 0^+} E_t \frac{x(t+\Delta t) - x(t)}{\Delta t}, \tag{3.29}$$

$$D_*\mathbf{x}(t) = \lim_{\Delta t\to 0^+} E_t \frac{x(t) - x(t-\Delta t)}{\Delta t}. \tag{3.30}$$

Because $d\mathbf{W}(t)$ and $d\mathbf{W}_*(t)$ are Gaussian with zero mean, it follows that $D\mathbf{x}(t) = \mathbf{b}(\mathbf{x}(t),t)$ and $D_*\mathbf{x}(t) = \mathbf{b}_*(\mathbf{x}(t),t)$, hence the names 'mean forward' and 'mean backward' velocities. To compute the second mean derivative, $D\mathbf{b}(\mathbf{x}(t),t)$ (or $D_*\mathbf{b}(\mathbf{x}(t),t)$), we must expand $\mathbf{b}$ in a Taylor series up to terms of order two in $d\mathbf{x}(t)$:

$$d\mathbf{b}(\mathbf{x}(t),t) = \frac{\partial \mathbf{b}(\mathbf{x}(t),t)}{\partial t}dt + d\mathbf{x}(t)\cdot\nabla\mathbf{b}(\mathbf{x}(t),t) + \frac{1}{2}\sum_{i,j} dx_i(t)dx_j(t)\frac{\partial^2\mathbf{b}(\mathbf{x}(t),t)}{\partial x_i \partial x_j} + \ldots, \tag{3.31}$$

From (33), we can replace $dx_i(t)$ by $dW_i(t)$ in the last term, and when taking the average in (41), we can replace $d\mathbf{x}(t)\cdot\nabla\mathbf{b}(\mathbf{x}(t),t)$ by $\mathbf{b}(\mathbf{x}(t),t)\cdot\nabla\mathbf{b}(\mathbf{x}(t),t)$ since $d\mathbf{W}(t)$ is independent of $\mathbf{x}(t)$ and has mean 0. Using (34), we then obtain

$$D\mathbf{b}(\mathbf{x}(t),t) = \left[\frac{\partial}{\partial t} + \mathbf{b}(\mathbf{x},t)\cdot\nabla + \frac{\hbar}{2m}\nabla^2\right]\mathbf{b}(\mathbf{x}(t),t), \tag{3.32}$$

and likewise

$$D_*\mathbf{b}_*(\mathbf{x}(t),t) = \left[\frac{\partial}{\partial t} + \mathbf{b}_*(\mathbf{x},t)\cdot\nabla - \frac{\hbar}{2m}\nabla^2\right]\mathbf{b}_*(\mathbf{x}(t),t). \tag{3.33}$$

From these properties, and invoking Newton's 2nd law, we can construct Nelson's time-symmetric mean acceleration equation[4],

$$m\mathbf{a}(\mathbf{x}(t),t) = \frac{m}{2}\left[D_*D + DD_*\right]\mathbf{x}(t) = -\nabla V(\mathbf{x}(t),t). \tag{3.34}$$

By applying the mean derivatives to $\mathbf{x}(t)$, using (41) and (42) to obtain $\mathbf{b} = \mathbf{v} + \mathbf{u}$ and $\mathbf{b}_* = \mathbf{v} - \mathbf{u}$, and removing the dependence of the mean acceleration on the actual particle trajectory $\mathbf{x}(t)$ so that $\mathbf{a}(\mathbf{x}(t),t) \rightarrow \mathbf{a}(\mathbf{x},t)$, (48) yields

$$\begin{aligned} m\mathbf{a}(\mathbf{x},t) &= m\left[\frac{\partial\mathbf{v}(\mathbf{x},t)}{\partial t} + \mathbf{v}(\mathbf{x},t)\cdot\nabla\mathbf{v}(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t)\cdot\nabla\mathbf{u}(\mathbf{x},t) - \frac{\hbar}{2m}\nabla^2\mathbf{u}(\mathbf{x},t)\right] \\ &= \nabla\left[\frac{\partial S(\mathbf{x},t)}{\partial t} + \frac{(\nabla S(\mathbf{x},t))^2}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}\right] = -\nabla V(\mathbf{x},t), \end{aligned} \tag{3.35}$$

where $\mathbf{a}(\mathbf{x},t)$ is the acceleration field over the statistical ensemble of point masses when the particle position $\mathbf{x}(t)$ is not known. Integrating both sides of (49), we then obtain the Quantum Hamilton-Jacobi equation,

$$-\frac{\partial S(\mathbf{x},t)}{\partial t} = \frac{(\nabla S(\mathbf{x},t))^2}{2m} + V(\mathbf{x},t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}, \tag{3.36}$$

which describes the total energy field over the ensemble, and upon evaluation at $\mathbf{x} = \mathbf{x}(t)$, the total energy of the actual point mass along its actual trajectory.

Although the last term on the right hand side of (50) is often called the 'quantum potential', note that it arises from the terms in (49) involving $\mathbf{u}$. So it is actually a kinetic energy term arising from the osmotic velocity component of Nelson's particle. Hence, in equation (50), the quantum potential should be physically understood as a kinetic energy field arising from the osmotic velocity field over the ensemble.

The pair of nonlinear equations coupling the evolution of S and ρ, as given by (40) and

[4] A more general definition of the mean acceleration exists due to Davidson [CITE]. However, Nelson's original definition is sufficient for our purposes.

(50), are generally known as the Hamilton-Jacobi-Madelung (HJM) equations, and can be formally identified with the imaginary and real parts of the Schroedinger equation under polar decomposition [CITE]. However, as Takabayasi [CITE] and Wallstrom [CITE] pointed out, (40) and (50) are not mathematically equivalent to the Schroedinger equation unless one imposes a special condition on ∇S, namely that

$$\oint_L \nabla S \cdot d\mathbf{x} = nh. \tag{3.37}$$

As soon as this condition is imposed[5], the solution space of (40) and (50) is identical to the solution space of the Schroedinger equation, and we can combine (40) and (50) into

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V(\mathbf{x},t)\psi(\mathbf{x},t), \tag{3.38}$$

where $\psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{iS(\mathbf{x},t)/\hbar}$ is single-valued. Here the wavefunction is to be interpreted as an epistemic field, or a field encoding information about the possible position and momenta states that the particle can occupy, because it is defined in terms of ρ and S, both of which are fields defined over a statistical ensemble of point masses.

Applying Nelson's dynamics to systems of N-particles and N-particle potentials results in the N-particle generalizations of the equations of motion. The forward stochastic differential equation of motion becomes

$$d\mathbf{x}_i(t) = \mathbf{b}_i(\mathbf{x}_1(t), ..., \mathbf{x}_N(t), , t)dt + d\mathbf{W}_i(t), \tag{3.39}$$

where $\mathbf{b}_i(\mathbf{x}_1(t), ..., \mathbf{x}_N(t), t) = (1/m_i)\,\nabla_i S(\mathbf{x}_1(t), ..., \mathbf{x}_N(t), t) + (\hbar/2m_i)\,\nabla_i ln\rho(\mathbf{x}_1(t), ..., \mathbf{x}_N(t), t)$, while the HJM equations become

$$\frac{\partial\rho}{\partial t} = -\sum_{i=1}^{N}\nabla_i \cdot \left[\left(\frac{\nabla_i S}{m_i}\right)\rho\right], \tag{3.40}$$

[5] Wallstrom [CITE] has argued that the fact that one must add this condition *ad-hoc* means that Nelson's stochastic mechanics fails to derive quantum mechanics. However, Schmelzer [CITE] has argued that one can in fact motivate (26) from imposing boundary conditions on ρ which are natural to the physical assumptions in Nelson's stochastic mechanics. Derakhshani [CITE] has also proposed a reformulation of Nelson's stochastic mechanics which derives this condition without making logically circular reference to the Schroedinger equation.

and

$$-\partial_t S = \sum_{i=1}^{N} \frac{(\nabla_i S)^2}{2m_i} + V - \sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{\nabla_i^2 \sqrt{\rho}}{\sqrt{\rho}}, \tag{3.41}$$

where $V = V(\mathbf{x}_1, ..., \mathbf{x}_N, t)$. Similarly, the mean acceleration equation becomes

$$\begin{aligned} m\mathbf{a}_i \mid_{\mathbf{x}_i = \mathbf{x}_i(t)} &= m \left[\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla_i \mathbf{v}_i - \mathbf{u}_i \cdot \nabla \mathbf{u}_i - \frac{\hbar}{2m} \nabla^2 \mathbf{u}_i \right] \mid_{\mathbf{x}_i = \mathbf{x}_i(t)} \\ &= \nabla_i \left[\frac{\partial S}{\partial t} + \sum_{i=1}^{N} \frac{(\nabla_i S)^2}{2m} - \sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \frac{\nabla_i^2 \sqrt{\rho}}{\sqrt{\rho}} \right] \mid_{\mathbf{x}_i = \mathbf{x}_i(t)} = -\nabla_i V \mid_{\mathbf{x}_i = \mathbf{x}_i(t)} . \end{aligned} \tag{3.42}$$

Upon imposing the N-particle quantization condition

$$\sum_{i=1}^{N} \oint_L \nabla_i S \cdot d\mathbf{q}_i = nh, \tag{3.43}$$

we can combine (54) and (55) to get the N-particle Schroedinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 \psi + V\psi, \tag{3.44}$$

where $\psi(\mathbf{x}_1, ..., \mathbf{x}_N, t) = \sqrt{\rho(\mathbf{x}_1, ..., \mathbf{x}_N, t)} e^{iS(\mathbf{x}_1, ..., \mathbf{x}_N, t)/\hbar}$ is single-valued.

Nelson's stochastic mechanics has generalizations to nonrelativistic spin-1/2 particles [CITE], relativistic derivations of the Klein-Gordon equation [CITE] and Dirac equation [CITE], and even relativistic scalar field theory [CITE]. It is an example of a hidden-variables theory[6] because the wavefunction is supplemented by additional physical variables, namely, point masses in 3-space. The point masses also constitute the PO of the theory since configurations of point masses compose familiar macroscopic (fermonic) matter distributions like tables, chairs, cats, etc. The measurement problem is solved (or rather 'dissolved') in Nelson's theory because the point masses have definite positions in 3-space at all times, and experiments always result in apparatus pointers pointing to the 3-space locations of these point masses.

Nelson's theory is closely related to the more widely known hidden-variables theory called

[6] Not only is it a hidden-variables theory, but it is a *nonlocal* hidden-variables theory because, for non-factorizable (entangled) probability densities which arise in the multi-particle case, the equations of motion (31) and (34) imply that the trajectory of one point mass instantaneously depends on the trajectory of the other point masses. This means Nelson's stochastic mechanics will violate the Bell inequality in the same way standard quantum mechanics does for EPRB type experiments, as demonstrated by Petroni [CITE].

de Broglie-Bohm theory (dBB) [CITE] in that both theories have point particles as their primitive ontology, and the current velocity in Nelson's theory is mathematically the same as the 'guiding equation' in dBB. And like dBB, the statistical predictions of Nelson's stochastic mechanics (as formulated here) reproduce those of standard nonrelativistic quantum mechanics for all times when the probability density for the particles is equivalent to $|\psi|^2$ - indeed, if $\rho_0 = |\psi|^2$, it is easy to see that this density is locally conserved in time, evolving as it does via the continuity equation (40) or (54). Another feature in common with dBB is that deviations from the statistical predictions of standard quantum mechanics are logically possible when $\rho_0 \neq |\psi|^2$, and it can be shown analytically as well as numerically that dynamical relaxation to the 'equilibrium' density $|\psi|^2$ will occur for a restricted class of well-behaved initial 'nonequilibrium' densities [CITE]. In other words, standard quantum mechanics emerges from the 'equilibrium limit' of Nelson's stochasic mechanics. Although the nonequilibrium case allows for a wealth of 'new physics' to be studied, for this paper, we will restrict our analyses to the equilibrium case for simplicity.

It might be asked why Nelson's stochastic mechanics was chosen for this paper over dBB, given their similarities and that dBB is more widely known and accepted in the physics literature (not to mention mathematically much simpler to formulate). First, while dBB's hidden variables are also point particles, it is not clear that these point particles must be interpreted as actual point masses - in fact, it is not even clear that mass should be considered a property carried by the particles as opposed the wavefunction [CITE]. It turns out one can make plausible arguments for both views (though, in our opinion, the arguments are more persuasive for the view that the wavefunction carries the mass)[7]. By contrast, the very formulation of Nelson's theory makes it unambiguous that the particles must be point masses - the stochastic differential equations (33)-(34) describe a point mass undergoing Brownian motion with drift in 3-space.

[7] In a future paper we intend to discuss these arguments and explore the implications of both views for formulating Newton-dBB theories of semiclassical gravity.

Chapter 4

Part III

Here we will formulate the Newtonian theories of semiclassical gravity based on Sm, GRWm, and Nelson's stochastic mechanics; in addition, we will derive the empirical predictions of these theories for cat states and the free particle wavepacket.

4.1 Newtonian Semiclassical Gravity in Schroedinger's Many-Worlds

4.1.1 Formalism

For a single system, the "Sm-Newton" (SmN) equations are defined as follows. The Sm matter-density field

$$m(x,t) = m|\psi(x,t)|^2, \tag{4.1}$$

acts as a source in the Newton-Poisson equation

$$\nabla^2 V(x,t) = -4\pi G m(x,t), \tag{4.2}$$

where $V(x,t) = -G\int \frac{m(x',t)}{|x-x'|}d^3x'$. This classical gravitational self-potential couples back to the wavefunction via the nonlinear integro-differential Schroedinger equation,

$$i\hbar\partial_t\psi(x,t) = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t) + mV(x,t)\psi(x,t). \tag{4.3}$$

In the N-body generalization (ignoring the interaction potential term for simplicity), the matter-density field is

$$m(x,t) = \sum_{i=1}^{N} m_i \int dx_1...dx_N\delta^3(x-x_i)|\psi(x_1,...,x_N,t)|^2, \tag{4.4}$$

which sources the N-body self-potential via

$$\nabla^2 V(x,t) = -4\pi G\int dx_1'...dx_N'|\psi(x_1'...x_N',t)|^2\sum_{i=1}^{N} m_i\delta^3(x-x_i'), \tag{4.5}$$

where

$$V(x_i,t) = -G\sum_{j=1}^{N}\int \frac{m_j|\psi(x_1'...x_N',t)|^2}{|x_i-x_j'|}dx_1'...dx_N'. \tag{4.6}$$

This N-body potential couples to the N-body ψ via

$$i\hbar\partial_t\psi(x_1...x_N,t) = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2\psi(x_1...x_N,t) + \sum_{i=1}^{N} m_i V(x_i,t)\psi(x_1...x_N,t). \tag{4.7}$$

Note that SmN is mathematically identical to SN theory, but differs in its probability interpretation - SN theory presumes the usual Born-rule probability interpretation of standard quantum mechanics whereas SmN uses typicality to argue that "typical" observers will see experimental statistics that match those of standard quantum mechanics. SmN also allows for an interpretation of the matter-density field associate with each branch of the wavefunction as a "world", but this is merely a practical convenience, with no physical distinction from how the matter-density field behaves in SN theory.

4.1.2 Empirical Predictions

Being mathematically identical to SN theory, SmN also admits cat state solutions and thereby makes the same empirically inadequate predictions from such solutions. So SmN is also inconsistent with the Page and Geilker experiment and with our macroscopic experiences of the physical world. SmN also predicts the gravitational wavepacket collapse effect observed in SN theory, and as in the SN theory, this doesn't help solve the cat states problem - all the effect does is further localize each SmN "world" around its location in 3-space (in addition to the localization effects that will arise from quantum decoherence). Thus, the Sm solution to the measurement problem does not help solve the problems that arise in SN theory.

4.2 Newtonian Semiclassical Gravity in the Ghirardi-Rimini-Weber Collapse Theory with Matter-Density Ontology

4.2.1 Formalism

For a single system, the "GRWm-Newton" (GRWmN) equations are defined as follows. Starting from the GRWm matter-density field

$$m(x,t) = m|\psi(x,t)|^2, \tag{4.8}$$

we can use this as a source in the Newton-Poisson equation

$$\nabla^2 V(x,t) = -4\pi G m(x,t), \tag{4.9}$$

where $V(x,t) = -G\int \frac{m(x',t)}{|x-x'|} d^3x'$. This gravitational self-potential couples back to the wavefunction via (3), but now the wavefunction undergoes discrete and instantaneous intermittent collapses according to the GRW collapse law. That is, the collapse time T occurs randomly with constant rate per system of $N\lambda = \lambda = 10^{-16} s^{-1}$, where the post-collapse wavefunction $\psi_{T+} = lim_{t \searrow T} \psi_t$ is obtained from the pre-collapse wavefunction $\psi_{T-} = lim_{t \nearrow T} \psi_t$ through multiplication by the Gaussian function

$$\psi_{T+}(x) = \frac{1}{C} g(x - X)^{1/2} \psi_{T-}(x), \tag{4.10}$$

where

$$g(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{x^2}{2\sigma^2}} \tag{4.11}$$

is the 3-D Gaussian function of width $\sigma = 10^{-7}m$, and

$$C = C(X) = \left(\int d^3x g(x - X) |\psi_{T-}(x)|^2 \right)^{1/2} \tag{4.12}$$

is the normalization factor. The collapse center X is chosen randomly with probability density $\rho(x) = C(x)^2$, and the spacetime locations of the collapses are given by the ordered pair (X_k, T_k). Between collapses, the wavefunction just evolves by the SN equation (61).

The generalization to an N-body system is as follows. The matter-density field becomes

$$m(x,t) = \sum_{i=1}^{N} m_i \int dx'_1 ... dx'_N \delta^3(x'_i - X) |\psi(x'_1, ..., x'_N, t)|^2, \tag{4.13}$$

which sources the N-body self-potential via

$$\nabla^2 V(x_i, t) = -4\pi G \int dx'_1 ... dx'_N |\psi(x'_1 ... x'_N, t)|^2 \sum_{i=1}^{N} m_i \delta^3(x'_i - X), \tag{4.14}$$

where

$$V(x_i, t) = -G \sum_{j=1}^{N} \int \frac{m_j |\psi(x'_1 ... x'_N, t)|^2}{|x_i - x'_j|} dx'_1 ... dx'_N. \tag{4.15}$$

This N-body potential couples to the N-body ψ via (65), and the N-body ψ is subject to the GRW collapse law

$$\psi_{T+}(x_1, ..., x_N) = \frac{1}{C} g(x_i - X)^{1/2} \psi_{T-}(x_1, ..., x_N), \tag{4.16}$$

where

$$C = C(X) = \left(\int dx'_1 ... dx'_N g(x'_i - X) |\psi_{T-}(x'_1, ..., x'_N)|^2 \right)^{1/2}, \tag{4.17}$$

and i is chosen randomly from $1, ..., N$.

The equations of GRWmN for a single system say the following - a wavefunction in 3-space, which evolves by (61) and undergoes the random collapse process in (68), drives the dynamical evolution of a matter-density field in 3-space via (66). When the wavefunction collapses, it localizes the matter density around a randomly chosen point in 3-space to a width of $10^{-7}m$, and with the probability of the randomly chosen point being largest where the mod-squared of the uncollapsed wavefunction is largest, as indicated by (70). This evolving mass density field also sources a real classical gravitational potential in 3-space via (67), which couples back to the wavefunction via (61), which in turn alters the evolution of the mass density field via (66) again.

For N-body systems, the wavefunction lives in configuration space $\mathbb{R}^{3N}$, evolves by (65), and undergoes the collapse process in (75); this wavefunction drives the dynamical evolution of N mass density fields in 3-space via (71) so that when the wavefunction collapses, it randomly localizes the mass density fields around randomly chosen (non-overlapping) points in 3-space, each of width $10^{-7}m$, and with probability density given by (76). As before, each of these mass density fields acts as a source for a gravitational potential in 3-space that couples back to the N-system wavefunction via (65), which in turn alters the evolution of the mass density fields via (71) again.

4.2.2 Empirical Predictions

Like the SN equations, the Schroedinger equation for the GRWmN wavefunction also admits cat states, but because the GRWmN wavefunction undergoes random collapses according to (68) or (74), which scales with the number of systems, those cat states are not macroscopically observable. (Also, the gravitational field produced by a cat state for a single elementary particle is presumably far too weak to be experimentally measured.) For example, for a massive object composed of Avogadro's number of systems, the collapse rate is $\sim 10^7 \frac{1}{s}$. So the individual mass fields composing the massive object will be localized around definite points in space frequently enough to give the appearance of a macroscopic mass distribution occupying a particular volume of space.

Returning then to the example of a 1000 kg mass in the cat state $\psi = \frac{1}{\sqrt{2}} [\phi_1 + \phi_2]$, it is clear that the number of systems needed in practice to compose such a mass distribution would imply an astronomically faster collapse rate. Moreover, when such collapses take place via (74), formula's (71) and (75) say that the result will be the appearance of a single 1000 kg mass localized on either the left or right side of the room (assuming the collapse center X for each system can take a binary outcome - either the left or right side of the room) with equal frequency. Correspondingly, the gravitational field measured with a classical test particle will look like it is due to only one mass density source at one location. In this way, the gravitational field predicted by GRWmN is consistent with that observed in the Page and Geilker experiment, in contrast to SN theory and SmN.

Since the branches of the GRWm wavefunction evolve by the SN equations in between the GRW collapse events, it is clear that those branches can also undergo the gravitational wavepacket collapse effect observed in numerical simulations of the SN equations for a free Gaussian wavepacket, given the same mass and initial width.

One might ask if GRW collapse might also be observable at the mass scale of $10^{10} amu$, and perhaps happen 'on top of' the gravitational collapse effect for a Gaussian wavepacket. If we make the generous assumption that in GRWmN a mass of $10^{10} amu$ corresponds to 10^{10} systems of 1 amu, this gives an approximate collapse rate of $10^{-6} \frac{1}{s}$, or $10^{6} s$ for each collapse. In other words, to have a chance of observing the GRW collapse effect, we would have to maintain the coherence time of the wavepacket for at least ~ 33 times longer than the timescale for the wavepacket to reach the minimum width through gravitational collapse. It remains to be seen whether technological advancements in molecular interferometry that allow for maintaining coherence times of 30,000 seconds will also allow for maintaining coherence times of $10^{6} s$ or greater. Even so, we note that if gravitational collapse is not observed at the mass scale predicted by the dynamics of the SN equations, this will be sufficient to falsify GRWmN as a semiclassical theory of gravity. And if self-localization is observed, it would be strong evidence for GRWmN or some dynamical collapse variant of GRWmN.

4.3 Newtonian Semiclassical Gravity in Nelson's Stochastic Mechanics

4.3.1 Formalism

In "Newton stochastic mechanics" (NSM) for a single particle, the point mass acts as a source for a classical gravitational potential via the Newton-Poisson equation

$$\nabla^2 V_g(x,t) = -4\pi G m \delta(\mathbf{x} - \mathbf{x}(t))\delta(t - t'), \tag{4.18}$$

where $V_g(\mathbf{x},t) = -G\int\int \frac{m\delta(\mathbf{x}-\mathbf{x}(t))\delta(t-t')}{|x-x(t)|}d^3x(t)dt'$. Here $\mathbf{x}(t)$ can either be the stochastic trajectory

$$\mathbf{x}(t) = \mathbf{x}(0) + \int_0^t \mathbf{b}(\mathbf{x}(t),t)dt + \mathbf{W}(t) - \mathbf{W}(0), \tag{4.19}$$

satisfying the forward stochastic differential equation

$$d\mathbf{x}(t) = \mathbf{b}(\mathbf{x}(t),t)dt + d\mathbf{W}(t), \tag{4.20}$$

or the mean trajectory obtained from solving the mean acceleration equation

$$\mathbf{a}(\mathbf{x}(t),t) = \frac{\partial \mathbf{v}(\mathbf{x}(t),t)}{\partial t} + \mathbf{v}(\mathbf{x}(t),t)\cdot\nabla\mathbf{v}(\mathbf{x}(t),t) = -\frac{\nabla}{m}\left[V_{ext}(\mathbf{x}(t),t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x}(t),t)}}{\sqrt{\rho(\mathbf{x}(t),t)}}\right], \tag{4.21}$$

where $\mathbf{v}(\mathbf{x}(t).t) = \nabla S(\mathbf{x}(t),t)/m$. In the former case, the gravitational potential would be a stochastic field because it depends on a stochastic position variable. In the latter case, it would be a 'mean potential', defined as it would be in terms of the mean trajectory.

If we want to couple the gravitational potential back to the HJM equations (and hence the Schroedinger equation) for the point mass, it would first have to enter somehow into the mean acceleration equation (79). In effect, we would be assuming that as the point mass undergoes its mean acceleration, it feels a mean gravitational 'self-force' from the mean gravitational potential it produces[1]. At first sight, we might think to calculate this self-force by computing the term $-\nabla\left[mV_g(\mathbf{x}(t),t)\right]$. However, this doesn't work because the gravitational field obtained from $-\nabla V_g$ will blow up at $\mathbf{x} = \mathbf{x}(t)$[2]. To properly do this calculation, we must appeal to the linearized approximation of classical general relativity [3][CITE]. (The full derivation of the gravitational self-force expression in the Newtonian limit is carried out in the Appendix.)

[1]Note that $V_g(\mathbf{x},t)$ blows up at $\mathbf{x} = \mathbf{x}(t)$. However, as is standardly done in classical electrodynamics for point charges, we can assume that some renormalization method is possible to remove the infinite gravitational self-energy.

[2]In fact, the same problem comes up in classical electrodynamics for a point charge - although a point charge produces a Coulomb potential from the Poisson equation $\nabla^2 V_C(x,t) = 4\pi kq\delta(\mathbf{x}-\mathbf{x}(t))\delta(t-t')$, one cannot compute the Coulomb self-force from the expression $-\nabla\left[qV_C(\mathbf{x}(t),t)\right]$ because the electric field obtained from $-\nabla V_C$ will also blow up at $\mathbf{x} = \mathbf{x}(t)$. Instead, what is commonly done to compute the electrostatic self-force [CITE] is to start from the Lienard-Wiechart Coulomb potential, make the approximation of a slowly moving point charge, and use an energy conservation argument to deduce the presence of a (time-averaged) electrostatic self-force, i.e. the "radiation reaction force" of a point charge.

[3]We could also take the route of assuming a naive gravitational analog of relativistic classical electrodynamics, and then deriving a radiation reaction force expression for a point mass analogous to the radiation reaction force expression for a point charge. However, the linearized approximation of classical general relativity gives an importantly different expression for the radiation reaction force on a point mass, and we would like our treatment to be consistent with general relativity so as to make a future general relativistic extension of NSM possible.

In linearized gravity, a slow moving particle of mass m acted upon by an applied force and radiating gravitational waves will, in a time-averaged sense, experience a radiation reaction force given by

$$\mathbf{F}^{r.r.}(t) = -\frac{2}{5}\frac{Gm}{c^5}\frac{d^5\widetilde{I}_{ij}(t)}{dt^5}\mathbf{x}(t), \tag{4.22}$$

where $\mathbf{x}(t)$ is the trajectory of the particle and $\widetilde{I}^{ij} \equiv I^{ij} - \frac{1}{3}\delta^{ij}I^k_k$ is the quadrupole moment tensor. We can then include this expression in the r.h.s. of the mean acceleration (79):

$$\mathbf{a}(\mathbf{x}(t),t) = \frac{\partial \mathbf{v}(\mathbf{x}(t),t)}{\partial t} + \mathbf{v}(\mathbf{x}(t),t)\cdot\nabla\mathbf{v}(\mathbf{x}(t),t) = -\frac{\mathbf{F}^{r.r.}(\mathbf{x}(t),t)}{m} - \frac{\nabla}{m}\left[V_{ext}(\mathbf{x}(t),t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x}(t),t)}}{\sqrt{\rho(\mathbf{x}(t),t)}}\right]. \tag{4.23}$$

Removing the dependence of the mean acceleration on the actual particle trajectory $\mathbf{x}(t)$ so that $\mathbf{a}(\mathbf{x}(t),t) \to \mathbf{a}(\mathbf{x},t)$, and computing the derivatives on $\mathbf{v}(\mathbf{x},t)$, (81) becomes

$$\mathbf{a}(\mathbf{x},t) = \nabla\left[\frac{\partial S(\mathbf{x},t)}{\partial t} + \frac{(\nabla S(\mathbf{x},t))^2}{2m}\right] = -\frac{\mathbf{F}^{r.r.}(\mathbf{x},t)}{m} - \frac{\nabla}{m}\left[V_{ext}(\mathbf{x},t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}\right]. \tag{4.24}$$

The r.h.s. says that the radiation reaction force is now a force *field* over the statistical ensemble of fictitious point masses. Explicitly,

$$\mathbf{F}^{r.r.}(\mathbf{x},t) = -\frac{2}{5}\frac{Gm}{c^5}\frac{d^5\widetilde{I}_{ij}(\mathbf{x},t)}{dt^5}\mathbf{x}, \tag{4.25}$$

where $\mathbf{x}$ is the location of a particular point in the ensemble and $\widetilde{I}_{ik}(\mathbf{x},t)$ is the quadrupole moment tensor *field* over the ensemble. Integrating both sides of (82), we obtain the Quantum-Hamilton-Jacobi equation

$$-\frac{\partial S(\mathbf{x},t)}{\partial t} = \frac{(\nabla S(\mathbf{x},t))^2}{2m} + \int_0^x \mathbf{F}^{r.r.}(\mathbf{x},t)\cdot d\mathbf{x} + V_{ext}(\mathbf{x},t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}, \tag{4.26}$$

where we notice that the gravitational radiation reaction force field plays the role of a dissipative work *field* term. Alternatively, we can recognize that

$$\int_0^x \mathbf{F}^{r.r.}(\mathbf{x},t)\cdot d\mathbf{x} = \int_0^t \mathbf{F}^{r.r.}(\mathbf{x},t)\cdot \mathbf{v}(\mathbf{x},t)dt = -\frac{1}{5}\frac{G}{c^5}\int_0^t \dddot{\tilde{I}}(\mathbf{x},t)_{ij}\,\dddot{\tilde{I}}(\mathbf{x},t)^{ij}dt = -\int_0^t L_{GW}(\mathbf{x},t)dt, \tag{4.27}$$

and rewrite (84) as

$$-\frac{\partial S(\mathbf{x},t)}{\partial t} = \frac{(\nabla S(\mathbf{x},t))^2}{2m} - \int_0^t L_{GW}(\mathbf{x},t)dt + V_{ext}(\mathbf{x},t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(\mathbf{x},t)}}{\sqrt{\rho(\mathbf{x},t)}}. \tag{4.28}$$

Recalling also that the stochastic trajectory (77) has an associated probability density $\rho(\mathbf{x},t)$ satisfying the continuity equation

$$\frac{\partial\rho(\mathbf{x},t)}{\partial t} = -\nabla\cdot\left[\frac{\nabla S(\mathbf{x},t)}{m}\rho(\mathbf{x},t)\right], \tag{4.29}$$

and imposing the quantization condition

$$\oint_L \nabla S\cdot d\mathbf{x} = nh, \tag{4.30}$$

we can combine (86) and (87) to obtain the Schroedinger equation

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V_{ext}(\mathbf{x},t)\psi(\mathbf{x},t) + -\int_{t_1}^{t_2} L_{GW}(\mathbf{x},t)dt\psi(\mathbf{x},t), \tag{4.31}$$

where $\psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{iS(\mathbf{x},t)/\hbar}$ is single-valued.

For the N-particle case, we would have

$$\mathbf{a}_i(\mathbf{x}_i,t)\mid_{\mathbf{x}_i=\mathbf{x}_i(t)} = \left[\frac{\partial\mathbf{v}_i}{\partial t} + \mathbf{v}_i\cdot\nabla_i\mathbf{v}_i\right]\mid_{\mathbf{x}_i=\mathbf{x}_i(t)} = -\frac{\mathbf{F}_i^{r.r.}}{m_i}\mid_{\mathbf{x}_i=\mathbf{x}_i(t)} -\frac{\nabla_i}{m_i}\left[V_{int} + V_{ext} - \sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\frac{\nabla_i^2\sqrt{\rho}}{\sqrt{\rho}}\right]\mid_{\mathbf{x}_i=\mathbf{x}_i(t)}, \tag{4.32}$$

giving

$$-\frac{\partial S}{\partial t}=\sum_{i=1}^{N}\frac{(\nabla_i S)^2}{2m_i}-\sum_{k=1}^{N}\int_{t_1}^{t_2}L_{GW}^{i}(\mathbf{x}_k,t)dt+V_{int}+V_{ext}-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\frac{\nabla_i^2\sqrt{\rho}}{\sqrt{\rho}}, \tag{4.33}$$

where $\frac{1}{5}\frac{G}{c^5}\dddot{\tilde{I}}(\mathbf{x}_k,t)_{ij}\dddot{\tilde{I}}(\mathbf{x}_k,t)^{ij}=L_{GW}^{k}(\mathbf{x}_k,t)$. Along with

$$\frac{\partial \rho}{\partial t}=-\sum_{i=1}^{N}\nabla_i\cdot\left[\left(\frac{\nabla_i S}{m_i}\right)\rho\right], \tag{4.34}$$

and

$$\sum_{i=1}^{N}\oint_L \nabla_i S\cdot d\mathbf{q}_i=nh, \tag{4.35}$$

we can combine (91) and (92) to get the N-particle Schroedinger equation,

$$i\hbar\frac{\partial\psi}{\partial t}=-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2\psi-\sum_{k=1}^{N}\int_{t_1}^{t_2}L_{GW}^{k}dt\psi+V_{int}\psi+V_{ext}\psi, \tag{4.36}$$

where $\psi(\mathbf{x}_1,...,\mathbf{x}_N,t)=\sqrt{\rho(\mathbf{x}_1,...,\mathbf{x}_N,t)}e^{iS(\mathbf{x}_1,...,\mathbf{x}_N,t)/\hbar}$ is single-valued.

4.3.2 Empirical Predictions

Because NSM has point masses as the PO, it solves the cat state problem rather trivially. This we can see by considering again the general superposition state

$$\psi=c_1\psi_1+c_2\psi_2. \tag{4.37}$$

In terms of this state, we can write the drift for the Nelsonian point mass as

$$\mathbf{b}=\mathbf{v}+\mathbf{u}=\frac{\hbar}{m}\Im(\frac{\nabla\psi}{\psi})+\frac{\hbar}{m}\Re(\frac{\nabla\psi}{\psi}). \tag{4.38}$$

Since $|\psi|^2=|c_1\psi_1|^2+|c_1\psi_2|^2+2|c_1\psi_1||c_1\psi_2|\Re\{cos\left(S_1-S_2\right)/\hbar\}$, this means the dynamics of the particle will depend on both ψ_1 and ψ_2 . Recalling now that in a measurement the action of $\hat{U}$ on

(96) gives

$$\Psi = \hat{U}\left(\psi \otimes \Phi_0\right) = c_1\psi_1 \otimes \Phi_1 + c_2\psi_2 \otimes \Phi_2 = \Psi_1 + \Psi_2, \tag{4.39}$$

which are macrosuperpositions, this implies that the Nelsonian point mass (which contains all the real physical mass in the system) evolving by (78) will depend on either Ψ_1 *or* Ψ_2 via either the drift

$$\mathbf{b}_1 = \mathbf{v}_1 + \mathbf{u}_1 = \frac{\hbar}{m}\Im(\frac{\nabla\Psi_1}{\Psi_1}) + \frac{\hbar}{m}\Re(\frac{\nabla\Psi_1}{\Psi_1}), \tag{4.40}$$

or

$$\mathbf{b}_2 = \mathbf{v}_2 + \mathbf{u}_2 = \frac{\hbar}{m}\Im(\frac{\nabla\Psi_2}{\Psi_2}) + \frac{\hbar}{m}\Re(\frac{\nabla\Psi_2}{\Psi_2}). \tag{4.41}$$

So if, for example, the point mass ends up in the branch Ψ_1, its dynamics will depend on (98) and the component Ψ_2 will evolve by the Schroedinger equation and have virtually no physical influence on the motion of the point mass. Thus, macroscopically, the mass will be either *here* or *there,* thereby being consistent with the Page and Geilker experiment and our perceptual experiences.

For the dynamics of a free Gaussian wavepacket associated with a single particle, we must ask how the radiation reaction work field term $-\int_{t_1}^{t_2} L_{GW}(\mathbf{x},t)dt\psi(\mathbf{x},t)$ changes the dynamics of the wavepacket. The answer is that it makes no change, because this work field term equals zero for this case. The reason is that the quadrupole moment tensor field $\widetilde{I}_{ik}(\mathbf{x},t)$ is equal to zero, since it is defined in terms of the second mass moment (i.e. moment of inertia) I^{ij}, and this is zero for a single point mass moving in a straight-line path. Thus, NSM predicts that a free Gaussian wavepacket for a single point mass should continue to disperse as it does in standard quantum mechanics, and in contrast to the prediction of the SN theory. On the other hand, for a Gaussian wavepacket in the N-particle case, the work field term will be non-zero because then $\widetilde{I}_{ik}(\mathbf{x},t)$ will be non-zero and time-dependent. Physically speaking, there will be some gravitational potential energy between the point masses that will slowly be radiated away. However, because the combination of constants $G/c^5 \sim 10^{-53}s^3/kg * m^2$, the amount of potential energy radiated will be so small that it will make virtually no change to the dynamics of the N-particle wavepacket. So even though there will be some gravitational radiation in the N-particle case, the conclusion remains effectively the same - the wavepacket will continue to disperse as it does in standard quantum mechanics.

Chapter 5

Conclusions and Discussion

To recap, we first outlined the theoretical structure of the SN theory and two of its well-known predictions, namely, cat states and gravitational wavepacket collapse. We then noted that the cat states problem seems to be closely related to the measurement problem in quantum mechanics. We then reviewed the measurement problem and the possible approaches to solutions, and asked if quantum theories that solve the measurement problem *and* that have primitive ontologies might solve the cat states problem and/or make any change to the wavepacket collapse prediction. We then reviewed the theoretical structures of three such quantum theories - Sm, GRWm, and Nelson's stochastic mechanics - and constructed Newtonian models of semiclassical gravity from these theories: SmN, GRWmN, and NSM. We then explored the implications of these three models for the cat states problem and gravitational wavepacket collapse. We found that (1) SmN makes no changes to either and is thus empirically inadequate just like the SN theory; (2) GRWmN solves the cat states problem for macroscopic superpositions and retains the gravitational wavepacket collapse effect, but allows for GRW collapse to happen on top of the latter; (3) NSM solves the cat states problem as well, but in a fundamentally different way from GRWmN. Moreover, NSM doesn't predict gravitational wavepacket collapse, in contrast to SN, SmN, and GRWmN.

So we found that the three different approaches to solving the measurement problem in three quantum POTs does lead to different empirical predictions. Our findings indicate that the only two empirically viable theories of Newtonian semiclassical gravity that we considered appear to be GRWmN and NSM, and that these two theories make an (in principle) empirically testable difference regarding gravitational wavepacket collapse. This leads to the exciting possibility that the next generation of molecular interferometry experiments can not only test for possible semiclassical gravitational effects, but in doing so, also perhaps experimentally decide which approach to solving the quantum measurement problem is the correct one, and by extension, tell us which is the correct quantum theory. More precisely, if such interferometry experiments find that there is gravitational wavepacket collapse (and perhaps also GRW or GRW-like collapse) under the predicted conditions, this will rule out NSM and strongly support GRWmN (or dynamical collapse theories like it). On the other hand, if such experiments find no gravitational wavepacket collapse under the predicted conditions, this will rule out GRWmN and all dynamical collapse theories like it, while being consistent with NSM (or hidden-variable theories like it).

The reason for the weaker conclusion in the latter case is because NSM's prediction is a negative one, while for GRWmN the predictions are positive - confirmation of positive predictions is generally regarded as stronger evidence for a theory than confirmation of negative predictions, because the latter seems more likely to be consistent with more than one theory. Indeed, while semiclassical gravity effects are predicted by canonical quantum gravity and emergent gravity - both of which are based on standard quantum mechanics - not all approaches to quantum gravity based on standard quantum mechanics predict semiclassical gravity effects. Most notably, string theory does not [CITE], nor does perturbative quantum gravity [CITE]. Moreover, it is hard to see why a dynamical collapse or hidden-variables version of such approaches might change that. So, finding no gravitational wavepacket collapse (or any other kind of collapse) under the predicted conditions might also indicate that there are just no semiclassical gravitational effects at all. That would be an interesting finding as well because then, it seems to us, this would indirectly rule out the canonical approach to quantum gravity as well as emergent gravity theories (at least insofar as such theories are based on standard quantum mechanics).

It could also be argued that if GRWmN and NSM don't have consistent extensions to general relativistic semiclassical gravity, then these are merely toy models and their nonrelativistic predictions shouldn't be taken seriously. Thus, an interesting research program would be to try to extend GRWmN and NSM to the general relativistic regime, and to see if they might make other experimentally (or observationally) testable differences, such as in the regimes of cosmology and astrophysics.

Appendices

Appendix A Derivation Of Gravitational Radiation Reaction Force In Linearized Gravity

In linearized gravity, the metric tensor $g_{nm}(\mathbf{x},t) = \eta_{nm}(\mathbf{x},t) + h_{nm}(\mathbf{x},t)$, i.e. the flat Minkowski metric plus a small perturbation. Introducing the "trace-reversed" amplitude $\bar{h}_{nm} \equiv h_{nm} - \frac{1}{2}\eta_{nm}h$, where $h = h^{\gamma}_{\gamma}$, the Einstein field equations with source T_{nm} become

$$\Box\bar{h}_{nm} = -\frac{16\pi G}{c^4}T_{nm}, \tag{1}$$

upon imposing the harmonic gauge

$$\frac{\partial\bar{h}^{nm}}{\partial x^m} = 0. \tag{2}$$

Raising the indicies, (100) is a set of ten wave equations for $\bar{h}^{nm}$ with T^{nm} as the source, and the general solution can be written as

$$\bar{h}^{nm}(\mathbf{x},t) = 4G\int d^3\mathbf{x}'\,\frac{[T^{nm}(\mathbf{x}',t')]_{ret}}{|\mathbf{x}-\mathbf{x}'|}, \tag{3}$$

where $[.]_{ret}$ means evaluation at the retarded time $t' = t_{ret} = t - |\mathbf{x}-\mathbf{x}'|$. We can use this solution to calculate the gravitational waves produced at large distances from a source moving with slow velocities. First, we assume that $r \gg R_{source}$ and $\lambda \gg R_{source}$, where $r = |\mathbf{x}-\mathbf{x}'|$, R_{source} is the characteristic size of the source, and $\lambda = 2\pi/\omega$ is the wavelength associated with the angular frequency of variation of the source ω. Then we obtain the asymptotic gravitational wave amplitudes

$$\bar{h}^{nm}(\mathbf{x},t) \underset{r\to\infty}{\to} \frac{4G}{r}\int d^3\mathbf{x}'T^{nm}(\mathbf{x}',t-r). \tag{4}$$

Over a limited angle range about any one direction, the gravitational wave produced is approximately a plane wave at large r. This means that the standard analysis of polarization and energy flux for plane waves can be applied here. This analysis depends only on the spatial components of the metric perturbation, $\bar{h}^{ij}(\mathbf{x},t)$. The corresponding sources of these spatial components are

$$\int d^3\mathbf{x}T^{ij}\left(t-r,\mathbf{x}\right), \tag{5}$$

which can be put into a more useful form by using the flat-space conservation law $\partial_m T^{nm} = 0$. Let us consider one component of this, namely,

$$\frac{\partial T^{tt}}{\partial t} + \frac{\partial T^{kt}}{\partial x^k} = 0. \tag{6}$$

If we differentiate this with respect to time, and use the symmetry $T^{tk} = T^{kt}$ along with the flat-space conservation law again, we obtain the relation

$$\frac{\partial^2 T^{tt}}{\partial t^2} = -\frac{\partial}{\partial t}\left(\frac{\partial T^{tk}}{\partial x^k}\right) = -\frac{\partial}{\partial x^k}\left(\frac{\partial T^{tk}}{\partial t}\right) = \frac{\partial^2 T^{k\ell}}{\partial x^k \partial x^\ell}. \tag{7}$$

Multiplying both sides of this equation by $x^i x^j$ and integrating over space, we find that the integral over the r.h.s. can be carried out by parts; the surface terms vanish because the source is bounded. The result is the identity

$$\int d^3\mathbf{x} T^{ij}(\mathbf{x}, t) = \frac{1}{2}\frac{d^2}{dt^2}\int d^3\mathbf{x} x^i x^j T^{tt}(\mathbf{x}, t). \tag{8}$$

Since these gravitational waves have length wavelengths, this implies the source is moving with low (Newtonian) velocities. Thus, the energy density $T^{tt}(x,t)$ will be dominated by the rest-mass density $T^{00} = \mu(\mathbf{x}, t)$, and the integral in (107) defines the second mass moment (i.e. moment of inertia),

$$I^{ij}(t) \equiv \int d^3\mathbf{x}\mu(\mathbf{x}, t) x^i x^j. \tag{9}$$

So the gravitational wave metric perturbation far from a weak, nonrelativistic source in the long-wavelength approximation becomes

$$\bar{h}^{ij}(\mathbf{x}, t) \underset{r\to\infty}{\rightarrow} \frac{2}{r}\ddot{I}^{ij}(t-r), \tag{10}$$

where the dot means a derivative with respect to time.

We will now use this result to find the quadrupole formula, i.e. the time-averaged radiated power from a harmonically moving mass-density source emitting gravitational waves. Just as the expression for the energy flux for a plane wave is (well-known to be) quadratic in the wave amplitude[1], so we should expect the total radiated power in gravitational radiation (L_{GW}) to be quadratic in I^{ij}

[1]This expression takes the form $f \propto \omega^2 A^2$.

and its time-derivatives. To find the appropriate number of time-derivatives, we can note that the wave amplitude (90) is proportional to $\ddot{I}^{ij}$, and there is an additional factor of ω^2 in the expression for a plane wave, so we should expect an extra time-derivative for each of the two factors of $\ddot{I}^{ij}$. L_{GW} also transforms like a scalar and so must be a quadratic scalar combination of $\ddot{I}^{ij}$. The only two possibilities are $\dddot{I}_{ij}\dddot{I}^{ij}$ and $\left(\dddot{I}^k_k\right)^2$, but the correct one is picked out by the fact that there is no radiation from a spherically symmetric system and, therefore, no energy loss. For a spherically symmetric system x, y, and z are all equivalent, and $\ddot{I}^{ij} \propto \delta^{ij}$. So the quadrupole moment tensor,

$$\widetilde{I}^{ij} \equiv I^{ij} - \frac{1}{3}\delta^{ij} I^k_k, \tag{11}$$

vanishes for spherical symmetry. L_{GW} will therefore be proportional $\dddot{\widetilde{I}}_{ij}\dddot{\widetilde{I}}^{ij}$. The complete quadrupole formula turns out to be

$$< L_{GW} > = < \frac{dE}{dt} >_T = \frac{1}{5}\frac{G}{c^5} < \dddot{\widetilde{I}}_{ij}\dddot{\widetilde{I}}^{ij} >_T, \tag{12}$$

where $< . >$ denotes the time-average over a period of motion. This formula is the gravitational analogue of the formula for the power radiated by an oscillating electric dipole in classical electromagnetism. And, just as in classical electrodynamics, one can associate a radiation reaction force with (111):

$$\frac{1}{5}\frac{G}{c^5} < \dddot{\widetilde{I}}_{ij}\dddot{\widetilde{I}}^{ij} >_T = < \mathbf{F}^{r.r.}(t)\cdot \mathbf{v}(t) >_T . \tag{13}$$

Assuming periodic motion and using integration by parts, one then finds

$$\mathbf{F}^{r.r.}(t) = -\frac{2}{5}\frac{Gm}{c^5}\frac{d^5\widetilde{I}_{ij}(t)}{dt^5}\mathbf{x}(t). \tag{14}$$

This says that the rate at which a particle of mass m with time-varying $\widetilde{I}^{ij}$ loses energy in gravitational waves is the same, in a time-averaged sense, as if it were acted on by the gravitational radiation reaction force (113).