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Star-Disk Interaction in Herbig Ae/Be Stars

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Star-Disk Interaction in Herbig Ae/Be Stars

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Astrophysics

by
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Abstract

The question of the mechanism of certain types of stars is important. Classical T Tauri (CTTS) stars accrete magnetospherically, and Herbig Ae/Be stars (higher-mass analogs to CTTS) are thought to also accrete magnetospherically, but the source of a kG magnetic field is unknown, since these stars have radiative interiors. For magnetospheric accretion, an equation has been derived (Hartmann, 2001) which relates the truncation radius, stellar radius, stellar mass, mass accretion rate and magnetic field strength.

Currently the magnetic field of Herbig stars is known to be somewhere between 0.1 kG and 10 kG. One goal of this research is to further constrain the magnetic field. In order to do that, I use the magnetospheric accretion equation. For CTTS, all of the variables used in the equation can be measured, so I gather this data from the literature and test the equation and find that it is consistent. Then I apply the equation to Herbig Ae stars and find that the error introduced from using random inclinations is too large to lower the current upper limit of the magnetic field range. If Herbig Ae stars are higher-mass analogs to CTTS, then they should have a similar magnetic field distribution. I compare the calculated Herbig Ae magnetic field distribution to several typical magnetic field distributions using the Kolmogorov-Smirnov test, and find that the data distribution does not match any of the distributions used. This means that Herbig Ae stars do not have well ordered kG fields like CTTS.
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Chapter 1

Introduction

Stars form when a giant molecular cloud collapses. As it collapses, a disk is formed around the star due to conservation of angular momentum. As material from the cloud falls on to the disk and then the star, gravitational potential energy is converted to kinetic energy and ultimately into thermal energy which is radiated away (Figure 1.1. The luminosity of this emission is referred to as the accretion luminosity. Derived in Hartmann (2001), it scales with the mass accretion rate as

\[ L_{\text{acc}} = \frac{GM_\star \dot{M}}{2R_\star}. \]  

(1.1)

Forming stars are subdivided into three classes on the basis of mass. High mass stars \((M_\star \geq 10 \, \text{M}_\odot)\) evolve onto the main sequence and dissipate their disk before they emerge from their parent envelope. Low mass stars \((M_\star \leq 1.5\, \text{M}_\odot)\), referred to as T Tauri stars, evolve on to the main sequence over 10’s of millions of years and are the precursors to most of the stars in our galaxy. As these stars descend onto the main sequence, they are fully convective resulting in typical magnetic field strengths of several kilogauss (Johns-Krull et al., 2004). Herbig Ae/Be stars (HAeBes) are
How are single stars born?

Cloud collapse  Rotating disk

Planet formation  Mature solar system

Scenario largely from indirect tracers.

Fig. by McCaughrean
intermediate mass stars, \((2M\odot \leq M_* \leq 10M\odot)\), higher mass analogs to classical T Tauri stars (CTTS) (see Figure 1.2). They evolve differently from low mass and high mass stars. Once they have finished contracting quasistatically they follow fully radiative tracks, unlike low-mass stars which are convective longer, and high-mass stars which are obscured objects until they reach the main sequence. HAeBe stars have also been shown to have circumstellar disks (e.g. Waters & Waelkens (1998); Beckwith & Sargent (1996); Weinberger et al. (1999); Grady et al. (2005)), and there is emerging evidence of ongoing planet formation in disks around Herbig Be stars (e.g. HD 100546, Grady et al. (2001); Acke & van den Ancker (2006); Quanz et al. (2011); Brittain et al. (2012)) Furthermore, gas giant planets have been imaged directly around Beta Pic (Lagrange et al., 2010) and Fomalhaut (Kalas et al., 2008). So while it is clear that planets form around these stars, there is much about their birthplace that remains unclear. One such unknown is the process by which material in the disks dissipates. Disks around CTTSs have a half-life of about 3 Myr. Most of the material accretes onto the star, some is lost in the form of photoevaporation or from stellar outflows, \((\sim 10\%)\), and some finds its way into planets. It is not clear how disks around HAeBe stars dissipate. The lifetime of the disk of these isolated systems is highly uncertain, because the outflows tend to be weaker (though this could be an effect of observational bias), and very little is known about the accretion of material onto the star. Because they are higher mass analogs to CTTS, it might be expected for them to accrete mass the same way: magnetospherically.

CTTS have been shown to accrete magnetospherically (Bouvier et al. (2007) and references therein). CTTS have a number of observational characteristics that have been successfully accounted for by the magnetospheric accretion paradigm. Firstly these late type stars show a far UV continuum and soft X-ray emission that dissipates with the disk (Valenti et al., 2000). Secondly, the spectral energy density
Figure 1.2: HAeBe stars are higher mass analogs to CTTS (Berdyugina, 2009)

(SED) indicates that the disks have holes of order 0.05 AU (D’Alessio et al., 1997). Thirdly, the Balmer emission lines show absorption components red-shifted several hundred km/s - near the free-fall velocity of these stars (Deleuil et al., 2004). These observational facts have led theorists to appropriate models developed to describe accretion onto neutron stars (e.g. Ghosh et al. (1977)) to describe accretion onto CTTS. If CTTS have strong dipole magnetic fields, then at the point where the ram pressure of the disk equals the pressure from the stellar magnetic field, the disk truncates, and the ionized gas falls to the star via funnel flows along magnetic field lines. This material thermalizes and creates an accretion shock. The accretion shock emits a large fraction in the ultraviolet, and since the typical CTTS has a late spectral type (K), any UV flux must be from the accretion shock. Thus the measurement of the UV flux from these sources is a convenient way to measure the accretion luminosity (Figure 1.3).

Then the magnetic energy density of the stellar field is given by
Figure 1.3: Accretion Luminosity from BP Tau (Bertout et al., 1988).
\[ u_m = \frac{B^2}{8\pi}, \]  
\[ \text{(1.2)} \]

and the kinetic energy density is

\[ u_k = \frac{1}{2}\rho v^2. \]  
\[ \text{(1.3)} \]

At the truncation radius, these two quantities are equal, so, equating 1.2 and 1.3 yields

\[ \frac{B^2}{8\pi} = \frac{1}{2}\rho v^2. \]  
\[ \text{(1.4)} \]

The next assumption is that the material accretes spherically and comes from infinity. This allows the free fall velocity to be used

\[ v = \sqrt{\frac{2GM}{R_t}}. \]  
\[ \text{(1.5)} \]

Another needed quantity is the mass accretion rate. This can be found from the continuity equation.

\[ dM = \rho4\pi R_t^2 v dt \rightarrow \dot{M} = 4\pi R_t^2 \rho v. \]  
\[ \text{(1.6)} \]

Assuming a dipole field gives

\[ B(R_t) = B_*(\frac{R_*}{R_t})^3. \]  
\[ \text{(1.7)} \]

Plugging these into 1.4 yields

\[ \frac{B^2 R_*^6}{8\pi R_t^6} = \frac{\rho G M_*}{R_t}. \]  
\[ \text{(1.8)} \]
But, from 1.5 and 1.6,

$$\rho = \frac{\dot{M}}{4\pi R_t^2} \sqrt{\frac{R_t}{2GM_*}}. \tag{1.9}$$

This gives

$$\frac{B_*^2 R_*^6}{8\pi R_t^6} = \frac{GM_* \dot{M}}{4\pi R_t^2} \sqrt{\frac{R_t}{2GM_*}}, \tag{1.10}$$

which reduces to

$$R_t = 3.7 B^{4/7} \dot{M}^{-2/7} M^{-1/7} R_*^{5/7} \tag{1.11}$$

(Hartmann, 2001), when $R_t$ is measured in $R_*$, $R_*$ is measured in $2R_\bigodot$, $\dot{M}$ is measured in $10^{-7}M_\bigodot$ yr$^{-1}$, and $M$ is measured in $M_\bigodot$. Figure 1.4 illustrates magnetospheric accretion.

But, do HAeBes even accrete? Yes, they do. From infrared excess and scattered light evidence, we expect to see accretion disks around HAeBes. A few people have tried to model HAeBe accretion, starting with Hillenbrand et al. (1992). That model did not work because the mass accretion rate was too high ($\dot{M} \sim 10^{-7} - 10^{-6}M_\bigodot$ yr$^{-1}$), which means the inner disk would be optically thick, and that is not what is observed. Natta et al. (2001) explained the origin of IR excess emission in HAeBe stars as the stars irradiating the disk. This puffs up the inner edge of the disk. The inner disk location and emission from this model are consistent with observations. Muzerolle et al. (2004) successfully modeled the Balmer lines observed toward UX Ori, a HAe star, by assuming that the star accretes magnetospherically. First they looked at the spectra and noticed high-velocity redshifted lines, which are evidence of mass falling onto the star (examples of redshifted lines are shown in Figure.
Figure 1.4: Magnetospheric Accretion Model Illustration from Wood (2004).
Then they matched the observed profiles to magnetospheric accretion models. These model the magnetic field of the star as a dipole magnetic field, with gas from the disk falls on the star along field lines. The inner and outer flow radii are free parameters, but the outer radius must be within the disk co-rotation radius.

The Balmer line profile shape was also matched to the model. It depends on inclination and $\dot{M}$. Large inclination produces asymmetric, broad, low-velocity redshifted absorption lines. When this absorption is seen with a strong blue emission peak, it is evidence for accreting material. The higher $\dot{M}$ is, the broader the emission wings and absorption from continuum opacity. This model constrains the mass accretion between $10^{-7} M_\odot \text{ yr}^{-1}$ and $10^{-8} M_\odot \text{ yr}^{-1}$, $T_* > 8000 \text{ K}$, $i= 75^\circ$, and $V_* \sim 70, \text{ km s}^{-1}$. Assuming a dipole field, the constraints on $\dot{M}$ and inclination ($i$) found from the model are consistent with observations. Another interesting aspect of UX Ori is that it dims by several magnitudes periodically, which changes the line profile shape. This is explained as an extinction event. Grinin & Tambovtseva (1995) modeled it and assumed that the gas in the inner disk had a similar velocity distribution as the magnetospheric accretion model. Muzerolle et al. (2004) added an obscuration in their model, and it correctly models the observations. This also supports magnetospheric
accretion. The authors also give the details for their accretion shock model for early A accreting stars. In this model, material merges with the star through accretion shock. There is excess accretion flux which is explained by the emission from the accretion column. The optically thick hot photosphere is what produces most of the visible to near-UV flux, but unless the energy flux ($F$) of the accretion column emission is high, the heated photosphere emission is not a measure of accretion energy. To measure the accretion energy, the Balmer jump, which depends on $\dot{M}$ and $F$, is used. But the Balmer jump is not an optimal way to measure the mass accretion rate because obscuration events make it difficult to measure accurately.

The magnetospheric accretion model correctly models the line profiles of UX Ori by an optically thin, gaseous inner disk. But this does not prove that magnetospheric accretion occurs. This is not the only evidence - there is even more evidence for magnetospheric accretion: redshifted absorption lines and the Br$\gamma$ - $\dot{M}$ relationship. The redshifted absorption lines seen come from free falling material with a large terminal velocity. Br$\gamma$ lines can form in the funnel flows during the accretion process (Muzerolle et al., 2001). There is a tight connection between Br$\gamma$ luminosity and accretion luminosity for CTTS (Muzerolle et al., 1998), so this connection is expected for HAeBes as well. In fact, Donehew & Brittain (2011) found that this relationship between Br$\gamma$ luminosity and accretion luminosity holds for Herbig Ae stars (HAes) but not for Herbig Be stars (HBes) (Figure 1.6). And since the accretion luminosity is related to the mass accretion rate through equation 1.1, Br$\gamma$ flux and $\dot{M}$ are related for HAes. The reason it does not hold for HBes is uncertain, but may be that there is a strong wind component in Be stars where Br$\gamma$ is also made.

But, without a magnetic field strength of at least 1 kG, stars cannot accrete magnetospherically. This is where the problem lies; the interior of HAeBes is radiative, not convective, and strong magnetic fields are thought to be generated by
Figure 1.6: Logarithm of the accretion luminosity vs. the logarithm of the Br$\gamma$ luminosity (adapted from Donehew & Brittain (2011))
convective interiors (Feigelson & Montmerle, 1999). CTTS can accrete magnetospherically, since they have convective cores which produce strong magnetic fields; however, since HAeBes have radiative cores, they should not have strong magnetic fields. So it seems that HAeBes cannot accrete magnetospherically - they do not have the convective interior necessary to generate the strong magnetic fields. But there is evidence that they do have strong magnetic fields: jets, and X-ray emission. A strong magnetic field may be the launching mechanism for the jets that we see from HAeBes, as it is in active galactic nuclei (Pudritz et al., 2012), and may be in CTTS (Ray et al., 2007). Figure 1.5 shows a jet coming from HD 163296.

X-rays usually come from the chromospheres or coronae of stars. When a star has a chromosphere or corona, it is magnetically active (Skinner & Yamauchi, 1996; Stelzer et al., 2006). This evidence is why magnetospheric accretion is still considered for HAeBes. Whether HAeBes accrete through a boundary layer or through magnetospheric accretion depends on the strength of the magnetic field. To measure the magnetic field for a HAeBe is not as easy as it is for a CTTS. There are two main ways used to measure the magnetic field: polarization and Zeeman broadening. Figure 1.8 shows Zeeman broadening measurements for the CTTS star TW Hya.

These cannot be used for HAeBes for the following reasons: polarization can cancel itself out, so is not a good measure of global magnetic field strength, and HAeBes are rapid rotators, so the rotational broadening masks the Zeeman broadening. Figure 1.9 shows the typical amount of rotational broadening for a HAe. To try to measure the small amount of Zeeman broadening in that line is very difficult, and when the area of the line is conserved (not shown in the figure), it is even harder, if not impossible, to measure the Zeeman broadening. There is no good way to measure the magnetic field strength of HAeBes.

By looking for synchrotron radiation, an upper limit can be put on the mag-
Figure 1.7: Jets from HD 163296 (Wassell et al., 2006)

Figure 1.8: Zeeman Broadening from the accreting CTTS TW Hya. Adapted from Valenti & Johns-Krull (2001).
Figure 1.9: Zeeman Broadening compared to typical Rotational Broadening for HAes. Adapted from Valenti & Johns-Krull (2001).

magnetic field. Skinner & Yamauchi (1996) used this method to find an upper limit for HAeBes at 10 kG, but since the magnetic field of CTTS is around 1-2 kG, and a magnetic field strength on the order of 1 kG is what is needed to accrete magnetospherically, so an upper limit of 10 kG is not extremely helpful.

This is what this project sets out to do: find a way to measure, or at least put further constraints on, the magnetic field strength of HAeBes and see if it is consistent with magnetospheric accretion. One reason that this is an important question is that the evolution of the disk depends on the accretion mechanism, so without understanding the accretion mechanism, the evolution of HAeBes and their disks cannot be correctly modeled. Another reason that this is important is that others have assumed that HAeBes accrete magnetospherically, and if that basic assumption is wrong, then the conclusions drawn from that research may not be valid.
Chapter 2

Analogy to T Tauri Stars

For the reasons listed in the previous chapter, magnetospheric accretion is assumed, and equation 1.11 is applied to HAeBes to narrow the possible range of the magnetic field strength. But before it is applied to HAeBes, the relationship was checked to determine if it was self-consistent for CTTS, since all the variables are known.

2.1 Checking the Relationship using CTTS

The first step was go through the literature to find $B$, $M$, $\dot{M}$, $R$ and $R_t$ for CTTS. A complete set of parameters was found for 13 stars: AA Tau, BP Tau, CY Tau, DE Tau, DF Tau, DH Tau, DK Tau, DN Tau, GG Tau A, GK Tau, IQ Tau, LkCa15, and TW Hya (Table 2.1).

After collecting the data and entering it in Excel, the truncation radius was calculated using equation 1.11. Since the magnetic field is the desired variable, the equation was solved for $B$ (equation 2.1) and then $B_{calc}$ and $B_{meas}$ were compared. Like Bouvier et al. (2007), random inclinations were corrected for by multiplying $B_{calc}$ by

\begin{align*}
B_{calc} \times \cos(\theta)
\end{align*}
Table 2.1: Parameters used to find an order of magnitude estimate of $B$.

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Star & $B$ (kG)\textsuperscript{a,c,g} & $M$ ($M_\odot$)\textsuperscript{a,b,c,d,h} & $\dot{M}$ ($10^{-7} M_\odot$ yr$^{-1}$)\textsuperscript{i} & $R$ ($R_\odot$)\textsuperscript{c,e,f} & $R_t$ ($R_\star$)\textsuperscript{i} \\
\hline
AA Tau & 2.57 & 0.52 & 0.0331 & 1.8 & 5.14 \\
BP Tau & 2.17 & 1.24 & 0.0288 & 1.9 & 4.87 \\
CY Tau & 1.16 & 0.55 & 0.0759 & 2 & 3.66 \\
DE Tau & 1.35 & 0.23 & 0.257 & 2.7 & 4.2 \\
DF Tau & 2.98 & 0.38 & 0.24 & 3.4 & 4.43 \\
DH Tau & 2 & 0.65 & 0.28 & 1.9 & 7.2 \\
DK Tau & 2.58 & 0.52 & 0.38 & 2.49 & 5.3 \\
DN Tau & 2.14 & 0.47 & 0.0347 & 2.14 & 5.02 \\
GG Tau A & 1.24 & 0.8 & 0.316 & 2.31 & 2.14 \\
GK Tau & 2.28 & 0.75 & 0.0646 & 2.15 & 4.5 \\
IQ Tau & 2 & 0.52 & 0.282 & 2 & 4.41 \\
LkCa15 & 2 & 0.97 & 0.0135 & 2 & 10 \\
TW Hya & 2.61 & 0.7 & 0.02 & 1 & 6.3 \\
\hline
\end{tabular}
\end{table}

On average, the ratio of $B_{\text{calc}}$ to $B_{\text{meas}}$ was 0.96, showing that the equation is fairly self-consistent (Table 2.2). One reason for the discrepancy is that $R_t$ and $\dot{M}$ were not taken simultaneously, and $\dot{M}$ is variable, so this factors in errors.

Finding out that the calculated stellar magnetic field of CTTS is close to the measured stellar magnetic field means that it should be possible to calculate an estimate for the magnetic field of HAeBes by measuring the stellar radius, stellar mass, truncation radius, and mass accretion rate of several stars. Even though the assumptions are basic (dipole field, aligned field, and ideal conductor), this method can still be used, because currently, the magnetic field range of HAeBes is known to be between 0.1 kG and 10 kG. Not only is an estimate of the magnetic field strength

\begin{equation}
B = 1.4(3.7)^{-7/4} R_t^{7/4} \dot{M}^{1/2} M^{1/4} R^{-5/4}
\end{equation}

by 1.4.
of HAeBes wanted, but if HAeBes accrete magnetospherically, like CTTS, then the magnetic field strength distributions should be similar. Identification of kG magnetic fields on HAeBe stars will raise fundamental questions about the origin of these fields from stars that are fully radiative. If we rule out the presence of kG fields, this will raise interesting questions about the origin of the high velocity infall onto these stars. Either way, placing a tighter constraint on the magnetic field strength of these stars will open the door to further investigation of these sources.

### 2.2 Applying the Relationship to Herbig Ae Stars

Since HBe stars do not follow the Br\(\gamma\) - \(\dot{M}\) relation, the method listed in the previous section will only be applied to HAes. To estimate the typical magnetic field strength of HAes, the truncation radius, accretion rate, stellar mass, and stellar radius are measured. To measure the truncation radius, the Half Width at Zero Intensity (HWZI) of CO is measured.
Most of the information known about the star is found from spectroscopy. In most cases there is not enough angular resolution to be able to tell where the disk ends, so in order to tell where the disk is truncated, emission lines are used. The assumption is that CO goes all the way to the disk truncation radius. There is some concern that CO will be dissociated exterior to the truncation radius. However, in the case of the Herbig Be star, HD 250550, the OI, CO, and OH lines, which all have different ionization/dissociation energies are related, and it is found that they imply the same inner radius, which suggests that the truncation is not due to ionization/dissociation (Figure 2.1).
Using that assumption, the HWZI of CO is measured, and since the lines are rotationally broadened, the widest part of the line (HWZI) will come from the gas close to the star, which is rotating the fastest. This is why the HWZI is used. There are several steps to go through in order to convert HWZI to $R_t$. The first step is Kepler’s law:

$$P^2 = \frac{4\pi^2}{GM}a^3.$$  

The first assumption is that the gas is in a circular orbit, so that

$$v = \frac{2\pi a}{P}.$$  

That is substituted into the previous equation to get

$$v^2 = \frac{GM}{a}.$$  

Plugging in $GM_\odot = 887 \text{ km}^2 \text{ s}^{-2} \text{ AU} M_\odot^{-1}$, into the previous equation,

$$v^2 = \frac{887M_*}{r}$$  

when $M_*$ is measured in $M_\odot$, $v$ is measured in km/s, and $r$ is measured in AU. The fastest velocity corresponds to the truncation radius, but this depends on the inclination of the system

$$v_{max} = v_t\sin(i).$$  

The HWZI is how $v_{max}$ is measured. This means

$$R_t = 887M_*\frac{(HWZI)^2}{\sin^2(i)},$$
which makes

\[ B = 125.8 \cdot \sin^{7/2}(i) \cdot \frac{(HWZI)^{7/2}}{\dot{M}^{1/2}M^{5/4}R^{-5/4}}. \]

This is where the dependence of the magnetic field on inclination comes in.

Unfortunately, the inclination of many of the sources is unknown. Equation 2.2 can also be written as

\[ B = B' \sin^{3.5}(i), \]

where all of the measurable variables and constants are combined into the quantity \( B' \).

One way to get around the problem of unknown inclinations is to use the average value of \( \sin^{-3.5}(i) \), but unless an infinite number of stars is measured, this will introduce some error.

To calculate the uncertainty in \( B \), equation 2.2 is rewritten setting \( x = \sin^{7/2}(i) \).

This allows the uncertainty to be written as

\[
\Delta B = \sqrt{\left(\frac{\partial B}{\partial x} \Delta x\right)^2 + \left(\frac{\partial B}{\partial \dot{M}} \Delta \dot{M}\right)^2 + \left(\frac{\partial B}{\partial \dot{R}} \Delta \dot{R}\right)^2 + \left(\frac{\partial B}{\partial HWZI} \Delta HWZI\right)^2}.
\]

The next thing to do was find the uncertainties in the parameters. Eisner et al. (2005) listed \( \Delta R = 0.3R \). Donehew & Brittain (2011) measured \( \dot{M} \) in HAeBes, and listed the uncertainties in their measurements. These uncertainties were converted into percent uncertainties and averaged to obtain \( \Delta \dot{M} = 0.4 \dot{M} \). Blondel & Djie (2006) listed different values of \( M \) for several Herbig stars from the literature, and using a similar method as above, \( \Delta M = 0.1M \) was calculated. The uncertainty in HWZI is \( \Delta HWZI = 0.05HWZI \). To find the uncertainty in \( x \), a Monte Carlo simulation was used. First, a flat distribution of \( N \) random inclinations between 7° and 85° was
calculated, and then plugged into the function \( x = \sin^{3.5}(i) \). The lower limit is arrived by an analysis of the HAe star with the narrowest emission lines. The HWZI gives \( v \sin(i) \). It is assumed that the gas is at the surface of the star, and is in a Keplarian orbit. From this assumption, the velocity is calculated, and once this is gotten, the inclination can be calculated. The upper limit comes from HAes having flared disks (figure 2.2). First, hydrostatic equilibrium is assumed, giving

\[
B = 125.8 \frac{\sin^{7/2}(i)}{(HWZI)^{7/2}} \frac{\dot{M}^{1/2} M^{5/4}}{R^{5/4}}.
\]

\[
\frac{dP}{dz} = -\rho g_z
\]

Assuming constant temperature and constant mean molecular mass,

\[
\frac{dP}{dz} = kT \frac{dn}{dz}, \text{ and } \rho = \mu m_H n
\]

. From figure 2.3,

\[
g_z = \frac{G M_* z}{(R^2 + z^2)^{3/2}} = \frac{G M_* z}{R^3}
\]

since \( z \) is small compared to \( R \).

Then, once \( \frac{dP}{dz} \), \( \rho \), and \( g_z \) have been calculated, they are plugged into the equation 2.2, which reduces to

\[
\frac{dn}{n} = -\frac{\mu m_H G M_*}{kT R^3} z dz.
\]

This is integrated to get

\[
n_z = n_o e^{-z^2/H^2},
\]
Figure 2.2: Flared disk (Dullmond et al.)

Figure 2.3: How to get $g_z$
where $H = \sqrt{\frac{2kTR^3}{\mu m_HGM_\star}}$. $H$ is the scale height.

Observations show that $\frac{H}{R} \sim 0.085$. This means that in order to see the star, since the disk is flared, $\tan \theta \leq 0.085$ which gives $\theta \sim 85^\circ$, the upper limit used in the Monte Carlo simulation.

Then the average value of $x$ was calculated. The error introduced was found by taking the standard deviation of $x$. This was calculated for different values of $N$ to see if it depended on the sample size, but for values between 16 and 1,000,000, it was fairly constant: 0.34. The reason that 16 was used as the lower limit is that the data has been collected for 16 stars. The standard deviation was converted into a percent uncertainty $\Delta x = \frac{\text{stddev}(x)}{\text{avg}(x)} = 0.9x$. Once this was obtained, the average of $B$ could be calculated.

Using equation 2.2, $\Delta B = 1.48B$. The next step was calculating the average value of $B$ from the gathered data listed in Table 2.3. $\langle B_{\text{calc}} \rangle = 46.1$ kG and $\Delta B = 68.3$ kG. The upper limit of $B$ for HAes is known to be 10 kG, so the results from this sample do not constrain the magnetic field at all.

The next thing to do was to test the distribution of $B'$ for HAes to see if the underlying $B$ field matched the distribution of magnetic field strengths for CTTS, or
<table>
<thead>
<tr>
<th>Star</th>
<th>$M (M_\odot)^{abcdefg}$</th>
<th>$M (10^{-7}M_\odot$ yr$^{-1})^{bhi}$</th>
<th>$R (R_\odot)^{cdfg}$</th>
<th>HWZI (km s$^{-1})^{e j k l m}$</th>
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<tr>
<td>AB Aur</td>
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<td>0.18</td>
<td>2.7</td>
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</tr>
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</tr>
<tr>
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<td>0.35</td>
<td>2</td>
<td>33.5</td>
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<tr>
<td>HD 135344</td>
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<td>0.05</td>
<td>2</td>
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<tr>
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<td>0.04</td>
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<td>22</td>
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<td>0.51</td>
<td>2.1</td>
<td>33.1</td>
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<tr>
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<td>0.69</td>
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<td>50</td>
</tr>
<tr>
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<td>4.3</td>
<td>1.9</td>
<td>4.96</td>
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</tr>
<tr>
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<td>0.19</td>
<td>2</td>
<td>14.3</td>
</tr>
<tr>
<td>HD 37806</td>
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<td>2.1</td>
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</tr>
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<td>2</td>
<td>25</td>
</tr>
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</tr>
<tr>
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<td>3.2</td>
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</tr>
<tr>
<td>V380 Ori</td>
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<td>25</td>
<td>2.8</td>
<td>25.3</td>
</tr>
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</table>

Table 2.3: Parameters used to try to constrain $B$ for HAes.

<table>
<thead>
<tr>
<th>Star</th>
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</thead>
<tbody>
<tr>
<td>AB Aur</td>
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</tr>
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</tr>
<tr>
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</tr>
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</tr>
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<tr>
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<td>UX Ori</td>
<td>13.8</td>
</tr>
<tr>
<td>V380 Ori</td>
<td>243.1</td>
</tr>
</tbody>
</table>

Table 2.4: Calculated magnetic field strength for HAe sample.

some other common $B$ distribution. A Kolmogorov-Smirnov test (KS test) was used for this purpose.

The KS test can be used to determine two different things: if two datasets came from the same parent population, or if one dataset came from a certain specified distribution. It has several benefits: it does not depend on a certain distribution, it can be used for small datasets, and it does not bin data. Certain statistical tests assume that the data follows a certain distribution, so if the distribution is unknown, it is useful to have a test which does not assume an underlying distribution. Most statistical tests can only be used with large datasets, and for extremely small datasets, the KS test is the only alternative (J. V. Wall & C. R. Jenkins, 2003). The chi-square test bins data, but the KS test does not. By not binning data, no information is lost when the test is run. But there are also some limitations: the test is less sensitive at the tails than at the center of the distribution, and if used to test a dataset against a
distribution, the distribution must be continuous and fully specified. In this research, the KS test was used to test a dataset against a specified distribution, and was also used to compare the HAe $B'$ data to the CTTS $B'$ data.

The KS test compares a dataset to a distribution, or two datasets to each other, by contrasting their cumulative distribution functions (CDF). First, the CDF is calculated for the dataset and the distribution, or other dataset, and then the difference between them is found. The maximum difference is called the D statistic. When this is calculated, it is compared to a table of critical D values. If it is above the critical value for the desired significance level, the statistically significant result is that the dataset did not come from the tested distribution (the null hypothesis is rejected). The critical values of the D statistic depend on the size of the datasets and/or distribution, and on the desired significance level.

To run the KS test, the kstwo function in IDL was used. This function has two inputs and two outputs. The dataset and the calculated distribution are put in the function, and the function calculates the D statistic and confidence level. To get the dataset, $B'$ was calculated for 16 stars. Once $B'$ was calculated, the next step was to find the parent distribution that it came from. There are three typical possibilities that were tested: a flat distribution from 1-4 kG (like CTTS), a delta distribution at 2 kG, and a Gaussian distribution peaked around 2 kG ranging from 1-4 kG. These were not the distributions put into the function because even if one of these distributions is the underlying distribution for HAe, the measured values of $B'$ would not match this distribution because random inclinations must be accounted for. The distributions used in the KS test were the listed distributions multiplied by $\sin^{3.5}(i)$ with random inclinations from 7° to 85°. The reasons for that inclination range are listed above. Equation 2.2 is the reason that the listed distributions were convolved with the $\sin^{3.5}(i)$ random inclination distribution. The KS test was run
for each distribution, and the values of D were compared to critical values of D to
determine how well the distributions matched. The code used is given in Appendix
B.

For this research, Table A2.12 from J. V. Wall & C. R. Jenkins (2003) was
used to get the critical D values when comparing HAe $B'$ to the distributions, and
table A2.11 from J. V. Wall & C. R. Jenkins (2003) was used to get the critical D
values used to compare the two datasets. Table 2.3 contains the results of the KS
test. Figure 2.5 is the plot of the CDFs of the distributions compared to the HAe
$B'$ CDF. It is obvious that the data is not from any of these distributions, and that
the distributions are dominated by the inclination term. Figure 2.6 is the plot of the
Figure 2.6: Comparison of the $B'$ datasets from HAes and CTTSs

HAe $B'$ CDF compared to the CTTS $B'$ CDF. These sets of data did not come from the same parent population.
The thought is that HAeBes are higher mass analogs to CTTS. If they are, then they should accrete the same way and have a similar magnetic field distribution. After checking Equation 1.11 to make sure that it was self-consistent for CTTS, we found that it is possible to check the accretion mechanism by assuming that HAes accrete the same way as CTTS (magnetospherically), and then by using equation 1.11 (derived for CTTS) to calculate the magnetic field of HAes. However, the issue is that the magnetic field depends on the inclination, and the uncertainty that random inclinations add to the calculated value of the magnetic field makes it impossible to constrain the magnetic field of HAes using this method. The second research question was if the distribution of $B'$ calculated for HAes was consistent with the distribution of $B'$ calculated for CTTS. The answer to this question is that the data distribution is not consistent with the CTTS distribution, or any other typical magnetic field distribution. Putting these results together, the main conclusion of this project is that HAeBes do not have dominant, well ordered kG fields like CTTS do. They may still accrete magnetospherically, but there may be higher order magnetic fields. Higher order fields fall off faster than dipole fields, so the magnetosphere produced
by these higher order fields is smaller, meaning that equation 1.11 does not apply to HAeBes, even though it applies to their lower mass analogs, CTTS.

This brings up an interesting question: if HAes do not accrete like CTTS do, why the $L_{\text{acc}} - L_{\text{Br}\gamma}$ correlation as found in Donehew & Brittain (2011)? If Br$\gamma$ is formed in the magnetosphere, and the emitting size is smaller for HAes than for CTTS, then the correlation should not be the same for HAes as it is for CTTS. We need to understand the origin of this correlation before we can apply it with confidence. Perhaps Br$\gamma$ forms in the winds, and perhaps the stellar wind depends on the mass accretion rate. Or, perhaps the winds and mass accretion rate depend on something else, like age. One way to test to see if there is a wind - Br$\gamma$ correlation is to use the P Cygni profile to measure the wind strength, and compare it to the Br$\gamma$ flux. If there is a correlation, then Br$\gamma$ may correlate with $\dot{M}$ because it is made in the wind. A second step to do to help discover the origin of the Br$\gamma - \dot{M}$ correlation is to do spectroastrometry of HI lines at 100 microarcsecond scale. Getting X-SHOOTER spectra of multiple accretion diagnostics simultaneously, can also shed light on the correlation. By measuring the UV excess flux of CTTS and the Br$\gamma$ luminosity at the same time, we can test to see how strong the correlation really is. Another thing that can be done to discover the cause of the correlation is the use the CO flux. CO flux depends on $\dot{M}$ (Glassgold et al., 2004). Stevans & Brittain (2010) compared Br$\gamma$ flux to CO 1-0 P(26) or P(30) emission line flux for 25 HAeBes. I continued with that work and brought the sample size up to 36 HAeBes. Once a large enough sample size is obtained, the ratio of Br$\gamma$ flux to CO flux can be plotted against the mass and temperature of the stars in the sample to determine if the ratio depends on either of those. It may be that a higher $\dot{M}$ produces a greater amount of Br$\gamma$ flux, since they correlate, but remember that correlation does not necessarily mean causation. More data must be collected to test this. The origin of the correlation between Br$\gamma$ and
CO luminosity remains a mystery as does the mechanism by which HI is accelerated to several hundred km/s as it falls onto the central star.
Appendices
Appendix A  Error Analysis

For a function $f(x, y, z, t)$, if we know $\Delta x, \Delta y, \Delta z,$ and $\Delta t$, in order to find $\Delta f$, we use equation 1 (Philip R. Bevington & D. Keith Robinson, 2003).

$$\Delta f = \sqrt{\left( \frac{\partial f}{\partial x} \Delta x \right)^2 + \left( \frac{\partial f}{\partial y} \Delta y \right)^2 + \left( \frac{\partial f}{\partial z} \Delta z \right)^2 + \left( \frac{\partial f}{\partial t} \Delta t \right)^2} \tag{1}$$

To find $\Delta B$, we apply equation 1.

$$\Delta B = \sqrt{\left( \frac{\partial B}{\partial x} \Delta x \right)^2 + \left( \frac{\partial B}{\partial \dot{M}} \Delta \dot{M} \right)^2 + \left( \frac{\partial B}{\partial M} \Delta M \right)^2 + \left( \frac{\partial B}{\partial R} \Delta R \right)^2} \tag{2}$$

The first step was to find the partial derivatives. Since $B = \alpha \left( \frac{x}{(HWZI)^{7/2}} \right) \dot{M}^{1/2} M^{5/4} R^{-5/4}$ ($\alpha$ is the constant),

$$\frac{\partial B}{\partial R} \Delta R = \alpha \left( -\frac{5}{4} \right) x HWZI^{-1/2} M^{1/2} M^{1/4} R^{-9/4} (0.3R) = \Delta R \frac{B}{R}$$

$$\frac{\partial B}{\partial \dot{M}} \Delta \dot{M} = \alpha \left( \frac{1}{2} \right) x HWZI^{-1/2} \dot{M}^{-1/2} M^{1/4} R^{-5/4} (0.4\dot{M}) = \Delta \dot{M} \frac{B}{M}$$

$$\frac{\partial B}{\partial M} \Delta M = \alpha \left( \frac{1}{4} \right) x HWZI^{-1/2} M^{1/2} M^{-3/4} R^{-5/4} (0.1M) = \Delta M \frac{B}{M}$$

$$\frac{\partial B}{\partial HWZI} \Delta HWZI = \alpha \left( \frac{1}{4} \right) x \left( -\frac{1}{2} \right) HWZI^{-3/2} \dot{M}^{1/2} M^{-3/4} R^{-5/4} (0.05M) = \Delta HWZI \frac{B}{HWZI}$$
<table>
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<th>Variable</th>
<th>Uncertainty</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>0.3$R$</td>
<td>(Eisner et al., 2005)</td>
</tr>
<tr>
<td>$M$</td>
<td>0.4$M$</td>
<td>Donehew &amp; Brittain (2011)</td>
</tr>
<tr>
<td>$M$</td>
<td>0.1$M$</td>
<td>Blondel &amp; Djie (2006)</td>
</tr>
<tr>
<td>HWZI</td>
<td>0.05HWZI</td>
<td>This work</td>
</tr>
<tr>
<td>$x$</td>
<td>0.9$x$</td>
<td>This work</td>
</tr>
</tbody>
</table>

Table 1:Uncertainties

\[
\frac{\partial B}{\partial x} \Delta x = \alpha_{HWZI}^{-1/2} \dot{M}^{1/2} M^{1/4} R^{-5/4} \Delta x = \Delta x \frac{B}{x}
\]

Now that we have the partials, we need to find the uncertainties in the variables (Table 1). Notice that all of the partials are in terms of $\Delta$variable $B$/variable. Because of this, the uncertainties were found as percent uncertainties.

Once we got those, we plug them into the partials above to get

\[
\frac{\partial B}{\partial R} = -\frac{5}{4}(0.3)B,
\]
\[
\frac{\partial B}{\partial \dot{M}} = \frac{1}{2}(0.4)B,
\]
\[
\frac{\partial B}{\partial M} = \frac{1}{4}(0.1)B,
\]
\[
\frac{\partial B}{\partial HWZI} = \frac{1}{4}(0.05)B,
\]

and

\[
\frac{\partial B}{\partial x} = (0.9)B.
\]

These were used in equation 2 to calculate the uncertainty in $B$.

\[
\Delta B = \sqrt{(0.9)^2 + (0.05)^2 \left(-\frac{1}{2}\right)^2 + (0.4)^2 \left(\frac{1}{2}\right)^2 + (0.1)^2 \left(\frac{1}{4}\right)^2 + (0.3)^2 \left(-\frac{5}{4}\right)^2} + B = 1.48B.
\]
Appendix B  IDL Code used to run KS Test

The KS test is used to compare data to distributions and two sets of data to each other. This code used both applications with respect to HAe magnetic fields. The distributions chosen were convolved with the inclination function to produce the HAe observable magnetic field $B'$. I also compared the HAe measured $B'$ with the CTTS $B'$ (calculated by taking the measured B from Zeeman broadening, and dividing by $\sin^{3.5}(i)$).

\[ B_p = \{2.70, 6.05, 10.6, 10.9, 20.3, 29.3, 37.0, 38.1, 38.7, 40.6, 42.2, 73.0, 173., 456., 477., 715.\} \text{; HAe measured observable magnetic field} \]

\[ CTTS = \{1.92, 2., 3.02, 3.27, 3.69, 4.79, 5.52, 7.37, 8.74, 11.83, 17.95, 121.38, 295.91, 13200.30\} \text{; CTTS B/sin^{3.5}(i)} \]

\[ N = 16. \text{; number of HAe stars} \]
\[ M = 1000. \text{; number of points in the distributions} \]

;defining distribution arrays and inclination array
flat=findgen(M)
gauss=findgen(M)
delta=findgen(M)
inc=findgen(M)
sini=findgen(M)

;a flat random inclination of angles between 7 and 85 degrees
for i=0., M-1 do begin
inc(i) = (randomu(seed, 1., /uniform)*78+7)*pi/180.

sini(i) = sin(inc(i))^(3.5)

endfor

; the distributions convolved with the inclination function
for k=0., M-1 do begin
    flat(k) = (randomu(seed, 1)*3+1)/sini(k); flat function from 1 to 4
    gauss(k) = (randomn(seed, 1,/normal)*.35+2.5)/sini(k); Gaussian from 1-4
    delta(k) = 2./sini(k); delta function at 2
endfor

; the KS tests
kstwo, flat, Bp, Df, pf
kstwo, gauss, Bp, Dg, pg
kstwo, delta, Bp, Dd, pd
kstwo, CTTS, Bp, Dch, pch

; preparing the data/distributions for the CDFs
Bp_sort = Bp(sort(Bp))
Bp_cdf = Bp_sort
FOR i=0, N_ELEMENTS(bp_cdf)-1 DO bp_cdf(i) =
    FLOAT(N_ELEMENTS(bp_sort(0:i)))/FLOAT(N_ELEMENTS(bp_sort))

CTTS_sort = CTTS(sort(CTTS))
CTTS_cdf = CTTS_sort
FOR i=0, N_ELEMENTS(CTTS_cdf)-1 DO CTTS_cdf(i) =
FLOAT(N_ELEMENTS(CTTS_sort(0:i)))/FLOAT(N_ELEMENTS(CTTS_sort))

flat_sort=flat(sort(flat))
flat_cdf=flat_sort
FOR i=0,N_ELEMENTS(flat_cdf)-1 DO flat_cdf(i)=
    FLOAT(N_ELEMENTS(flat_sort(0:i)))/FLOAT(N_ELEMENTS(flat_sort))

gauss_sort=gauss(sort(gauss))
gauss_cdf=gauss_sort
FOR i=0,N_ELEMENTS(gauss_cdf)-1 DO gauss_cdf(i)=
    FLOAT(N_ELEMENTS(gauss_sort(0:i)))/FLOAT(N_ELEMENTS(gauss_sort))

delta_sort=delta(sort(delta))
delta_cdf=delta_sort
FOR i=0,N_ELEMENTS(delta_cdf)-1 DO delta_cdf(i)=
    FLOAT(N_ELEMENTS(delta_sort(0:i)))/FLOAT(N_ELEMENTS(delta_sort))

;setting the colors for the graphs
DEVICE, Decomposed=0
colors = GetColor(/Load, Start=1)

;printing the D values and the critical values
print,'flat from 1-4' & print,Df
print,'gaussian from 1-4 kG' & print,Dg
print,'2 kG' & print,Dd
print,'CTTS' & print,Dch

37
print,'0.100 significance level: ' & print,1.22*sqrt((M+N)/(M*N))
print,'0.050 significance level: ' & print,1.36*sqrt((M+N)/(M*N))
print,'0.010 significance level: ' & print,1.63*sqrt((M+N)/(M*N))
print,'0.005 significance level: ' & print,1.73*sqrt((M+N)/(M*N))
print,'0.001 significance level: ' & print,1.95*sqrt((M+N)/(M*N))

;plotting the CDFs
window,0
plot, Bp_sort, Bp_cdf, title='Comparing HAe Data and Distributions',
   xlog = 1, xrange=[1,10000], charsize=2., linestyle=0, background=1,
   color=24, xtitle='B (kG)', ytitle='Probability' ;HAe data CDF
oplot,flat_sort,flat_cdf,color=31,linestyle=5 ;Flat distribution CDF
oplot,gauss_sort,gauss_cdf,color=68,linestyle=4 ;Gaussian distribution CDF
oplot,delta_sort,delta_cdf,color=88,linestyle=3 ;Delta distribution CDF
legend,[‘Data CDF’,’Flat CDF’,’Gaussian CDF’,’2 kG CDF’],charsize=2.,
   linestyle=[0,5,4,3],Colors=[24,31,68,88],/right,/center,textcolor=24

window,1
plot,Bp_sort,Bp_cdf,title='Comparing HAe CDF and CTTS CDF’,xlog=1,
   xrange=[1,10000],charsize=2., linestyle=0, background=1,color=24,
   xtitle='B (kG)’,ytitle='Probability’ ;HAe data distribution
oplot,CTTS_sort,CTTS_cdf,color=31,linestyle=5 ;CTTS data distribution
legend,[‘HAe CDF’,’CTTS CDF’],charsize=2.,linestyle=[0,5],colors=[24,31],
   /right,/center,textcolor=24

end
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