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NUMERICAL ANALYSIS OF THE EXTRUSION OF FIBER OPTIC AND PHOTONIC CRYSTAL FIBER PREFORMS NEAR THE GLASS TRANSITION TEMPERATURE

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NUMERICAL ANALYSIS OF THE EXTRUSION OF FIBER OPTIC AND PHOTONIC CRYSTAL FIBER PREFORMS NEAR THE GLASS TRANSITION TEMPERATURE

A Dissertation
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by
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Accepted by:
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ABSTRACT

Conventional clad core fiber optic technology has relied on a concentric structure of glass of different refraction indices. These conventional fibers suffer from constraints and limitations related to thermal expansion compatibility between the core and the glass. The new fiber technology broadly characterized as Microstructured Optic Fibers (MOFs) is intended to lift the limitations of conventional clad core fibers and also extend the range of application of fiber optics. Photonic Cristal Fibers (PCFs) are a special family of Microstructured Optic Fibers characterized by the presence of holes in the cross section of the fiber that are organized in a hexagonal pattern. In order to manufacture these fibers, a preform with the same cross section has to be prepared which can later be drawn into fiber. For such complex geometry, glass extrusion at a viscosity higher than that for extrusion of solid glass preforms has proven to give better results.

Despite these merits, the numerical modeling of the extrusion of glass at high viscosity has not received much attention in the literature. Thus, in order to study the extrusion of PCF performs at high viscosity, investigation of the extrusion of a solid glass preform must be considered first.

To establish the valid assumptions to model glass extrusion at high viscosity, a numerical study of solid rod extrusion was performed and validated based on five experimental cases. This
study highlighted the importance of including friction effects to validate both the value of ram force and die swell. Since the Navier law is the most widely used friction law in the extrusion literature and has been identified as a key parameter required in modeling, the ring compression test was adapted to obtain the Navier friction coefficient from standard ring compression test data. In addition to the identification of the essential modeling choices base on experimental data, a sensitivity analysis was performed on extrusion parameters for both viscous and viscoelastic material to establish a general idea of the effect and relationships that governs the extrusion process. The results from this study correlate with several experimental observations.

Based on the same assumptions that were validated for solid rod extrusion, the numerical modeling of extrusion of PCF preforms was performed using two Blockage geometries. The model was validated for both Blockages qualitatively based on a preform showing a significantly deformed cross section. To investigate the creation and the distortion of the holes of PCF preforms, a sensitivity analysis was also performed using both Blockages. To quantify and interpret the distortion data the implementation of several algorithms and mathematical methods were developed. In addition, other tools were developed to provide a way to alter the Blockage geometry in order to improve the geometric quality of the preform. An example of this alteration was carried out.
ACKNOWLEDGEMENTS

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INTRODUCTION

I.1. PREFORM EXTRUSION: SCOPE AND STATE OF THE ART

Though very important in modern technology, the principles of light guidance for fiber optics were established long ago. Initially, fiber optics applications were extremely limited due to the inability to control power attenuation, i.e., the loss of the intensity of the signal as the wave travels through the fiber. However in the past few decades several technological advances have promoted this technology to the leading position it now has in modern technology. Optic fiber research has benefited from several advances in broad areas of engineering. Enhanced control of the material composition has led to a significant reduction of the power attenuation, while the development of new manufacturing techniques has allowed the use of a wider range of materials with a variety of novel fiber architectures. These developments have extended the application of fiber optics beyond light guidance and data transfer to use in commutation, signal amplification, real time sensing, among others.
Modern glass fibers are made using several glass types, such as lead silica [3-5] bismuth [1-2], telluride [6-10] and fluoride [11]. To enhance their optical guidance properties, optic fiber can be drawn with a complex geometry in the cross section [1-4], such as multi-rings, nano fiber and exposed core fiber, which makes manufacturing them more challenging. In general fiber light guidance is achieved by creating a variation of the refractive index between the fiber core and the cladding; core and cladding materials have generally similar chemical composition with differences due to doping elements [12] to create a difference in the refractive index. Despite its apparent simplicity, this technique has several limitations related to material compatibility at the core and cladding interface which limits the range of its exploitation. These fiber are usually referred to as core/clad fibers. Microstructured Optical Fibers (MOF) [1], however, relaxes this constraint since a single material is used for the entire cross section [13]. In addition, the optical properties of these fibers can be modified by simply adjusting the geometrical configuration. Examples of a core/clad fiber and an MOF are presented in Figure I-1. Application of this technology includes:

**Figure I-1. Fiber cross section:**

(a, b) Microstructured optical fibers (MOFs) [1-2], (c) Core/clad fiber [23].
• A broader range of numerical apertures than conventional fibers.

• A large effective areas for high power light transport and generation [14],

• A tight confinement of light for the production of strong nonlinear effects [15].

• A possibility of endless single-mode guidance [16],

• A possibility of control properties such as dispersion and birefringence [17].

While polymer fibers are increasingly present in consumer based applications, glass fibers dominate in the high-end optic fiber applications.

Optic fibers are generally made from separately manufactured glass preforms with a diameter on the order of centimeters, drawn to a long fiber of much smaller cross section on the order of microns. Today, extrusion has become a procedure of choice for manufacturing the glass preforms used for both core/clad and MOFs. The interest of computational modeling of the flow behavior during extrusion has been considered for these and other applications. The focus of this study is extrusion of glass preforms used to create MOFs at relatively high viscosity \( (10^{7.5} - 10^9 \text{ Pa·s}) \). The current study examines several aspects of this process including a study on the correct assumptions for extrusion at high viscosity, as well as the impact of processing and material input parameters on the results. Emphasis on the current study is the prediction of die-swell, and more generally, on the distortion of the entire cross section of a preform with a pattern of holes.

The selection of a computational approach to modeling extrusion of glass at high viscosity involves several considerations, such as the geometry of the die, flow behavior at the die/glass interface, as well as the material behavior of glass. As it will be detailed later in the literature review, the study of extrusion of glass preforms at high viscosity involves a
reexamination of several assumptions that are generally used in comparable processes. For instance, extrusion has been widely explored for different models of linear and nonlinear low viscous polymers [18-22]. The modeled dies are usually composed of only a cylindrical reservoir with a straight channel. While these dies may be suitable for the study of a large range of polymer behaviors, the lack of geometric smoothness makes them less appropriate for highly viscous glass applications.

In the case of glass extrusion, however, the estimation of the die-swell as well as the ram force and speed are usually performed using simplified analytical models [23-24]. However at typical extrusion temperatures the mechanical behavior of glass is viscoelastic [25] and simplified material models sometimes fail to predict accurately the physical process. At the range of viscosity considered, glass has been treated and modeled differently depending on the required precision and goal of the study [26-32]. Hence, viscoelasticity and compressibility have been sometimes but not always taken into account [26-38].

Moreover, concerning the die/glass interface conditions, a no-slip or “sticking” boundary condition is a dominant assumption often made within the context of Newtonian flow [18, 20-22]. This assumption is justified for a low viscosity range of the fluid and a low We number. However, in several cases, the possibility of slipping between the material and the die becomes increasingly relevant and should be considered [39-41]. In some studies a slipping condition has been considered to relieve numerical instability [42]. Factors that can signal the presence of slip include presence of several phases of capillary flow, the spurt phenomenon, shark skin, time dependent ram force for viscous flow and resemblance to plug flow in severe cases. Slip was
fundamentally addressed in 1931 by Mooney [43] who developed mathematical ways to detect and estimate the slip velocity for several configurations.

Contrary to the polymer literature, computational simulations of glass extrusion are limited, although some important studies exist [44, 45]. Generally the glass is used at a high temperature above the glass transition temperature (T_g), where viscous behavior and sticking boundary conditions are justified [46, 44, 47]. At a temperature near T_g, also referred to as the super cooled liquid region, glass has a relatively high viscosity that can be modeled as a linear viscoelastic solid depending on the manufacturing procedure time scale [25, 30], as in lens molding. Extrusion of glass at these temperatures is comparable to the lens molding process. In such conditions frictional sliding between the glass and the mold/die has been shown to occur [33, 48].

In this study, a literature review is carried out to address different aspects related to numerical modeling of glass extrusion, high viscous glass processing as well as glass slip modeling in both the solid and rheology literature. The study focuses first on studying the different approaches of numerical modeling of the process and examines the validity of a few basic assumptions such as no-slip. A sensitivity analysis is then carried out to investigate the influence of the extrusion parameters for both the viscous and viscoelastic cases. In both analyses the entire die shape [49] of the die is modeled. However, in the viscous case two dies are examined to study the dependence of the results on the die geometry. In a second step, the computational extrusion of Photonic crystal fiber (PCF) preform is demonstrated and compared to the experimental data for 2 different Blockage geometries. A sensitivity analysis is then carried out to investigate the influence of the extrusion parameters for both Blockage geometries. A set of
variable and mathematical tools to describe and interpret the shape change of the preform is
performed. Finally, based on these results, two step alterations of the Blockage geometry are
tested to reduce the deformation of the holes.

I.2. Motivation and Objective

Waveguide-loss, which is a power loss experienced by the guided signal at the fiber exit,
is a persistent problem when applying traditional manufacturing methods to the processing of
Chalcogenide photonic-crystal fibers (PCFs). Unlike regular glasses, Chalcogenide glasses are not
based on silica, their composition is a mixture of chalcogen elements (sulphur, selenium, and
tellurium) and other elements such as arsenic, germanium, antimony, or gallium. From a
manufacturing perspective, Chalcogenide glasses are very challenging. Nevertheless, compared
to silica glasses, they offer several interesting optical properties such as:

- A transmission window that extends far into the infrared (IR) spectral region.
- An extremely high nonlinear refractive-index coefficient that can be two or three orders
  of magnitude greater than that of silica.

Making PCFs out of chalcogenide glass could be used to serve a number of highly
nonlinear optical purposes.

To manufacture these PCFs, a preform that will lead to the targeted final cross section is
drawn into a long fiber. The resulting fiber generally preserves the preform cross section
geometric pattern at a smaller scale.
The first silica PCFs were made based on "stack and draw" technique. Though this method was widely used on silica PCF, it is poorly suited to chalcogenide glasses. It generates low quality PCFs marked by a great presence of bubbles, refractive-index variation and significant scattering losses.

![Figure I-2. Schematic of extrusion setup and process [23].](image)

The casting method was adopted as an alternative option to overcome the limitations of the former method. To imprint a hexagonal lattice structure within the preform, a silica mold fitted with silica capillaries is used. A glass rod is then allowed to melt on top of the mold inside a silica ampoule. After the end of the cooling stage, the silica capillaries are removed by applying a hydrofluoric acid solution. While making fiber out of casing has shown some success in producing PCFs, it still suffers from limitations related to the mismatch of the thermal expansion of
chalcogenide and silica, which limits the possibility of a known geometrical outcome. It also involves the use of chemicals that might contaminate the glass at the drawing stage if not carefully removed.

Extrusion is another valuable alternative to casting; it provides more geometric flexibility and better surface quality. Extrusion is carried out at high viscosity. The process consists of forcing a glass billet of a large diameter through a constriction carefully designed to improve the flow conditions. The glass then passes through a die channel and finally the die exit (Figure I-2, Figure I-3.a, b). To imprint a hexagonal lattice or any other type of geometrical pattern, the die is fitted with an extra component named “Blockage” placed at the entrance of the channel (Figure I-3.c). This additional component adds complexity to the flow and splits the channel into two compartments. The leading compartment is a pattern of relatively long feeding holes designed to support a pattern of pins. These pins are arranged to imprint a hexagonal lattice within the preform at the subsequent compartment known as “the welding chamber.” The complexity of the flow leads to a pattern of holes within the preform that is not exactly the negative of the pins. A precise pattern of holes in the preform is required, so an understanding of the flow mechanisms that lead to changes in the cross-section is desired. In other words, given the geometry of the Blockage in the die, it is required to use computational modeling to predict the final geometry of the preform.
Die-swell is one of the most commonly experienced phenomena associated with extrusion. It designates the increase in diameter of the resulting preform at the die exit, which is primarily caused by rearrangement of the velocity field. While an increase of the diameter of the cross section of a solid circular preform is not of major consequence, for the case of a PCF preform, the shape of the holes and their positions are affected severely by this same phenomenon (Figure VII-2.A).

The prediction of the final geometry of the cross section of an extruded fiber optic preform is the main topic of this research. This study starts by covering the numerical modeling of...
extrusion from several aspects. Due to the lack of literature covering extrusion of super cooled
liquids, the basic assumptions typically used to model this process will be examined, followed by
a sensitivity analysis that will cover the influence of the extrusion parameters depending on the
assumptions used. Finally, the extrusion of PCFs preform is studied and treated for possible
optimization aspects.


Two computational finite element software packages have been used in this study:
ABAQUS and POLYFLOW. Both have strengths and weaknesses. For the current extrusion
problem, the strength of ABAQUS is the ability to handle time and temperature dependent
thermo-mechanical material properties. The strength of POLYFLOW is its database of rheological
material and slipping models and its ability to follow the free surface at a low numerical cost.

The range of viscosity of glass involved in the extrusion processes considered in this study
are high enough to allow the glass to be treated within the scope of solid mechanics using a
Lagrangian framework. At the start of the project the focus was on ABAQUS due to its ability to
handle the time dependence of the thermo-mechanical material properties of glass. As shown in
the next section, this strength was used to study the importance of structural relaxation, i.e., time
history dependent thermal expansion, on the shape change of a cross section of a preform with
holes. While ABAQUS offers several frameworks designed to cope with and correct for severe
mesh deformation such as Arbitrary Lagrangian Eulerian framework (ALE) and Coupled Eulerian-
Lagrangian (CEL) to handle the actual extrusion process, these and other approaches were tested
for the current extrusion application with limited success. An Eulerian approach based on
rheology was tested and determined to be more suited to this task, and as such, the software POLYFLOW was selected for the computational modeling.

I.4. References


II. LITERATURE REVIEW

II.1. NUMERICAL MODELING OF GLASS

While computational modeling has been applied to many glass forming processes, glass molding has received the most attention. Applications for glass molding cover a variety of products ranging from TV sets and bottles to lenses and fibers. While the required precession is different, the numerical techniques are quite similar. Indeed numerical modeling depends more on factors like the required precision, the deformation rate, and the target of study (like residual stresses, birefringence, or lens profile deviation).

In his paper “A review of research in numerical simulation for the glass pressing process,” M. Brown [1] went through 72 research papers where he detailed the most important aspects of glass numerical simulation, which are glass flow, heat transfer and residual stress. According to this extensive review, glass numerical modeling acquired the attention of the industry in the mid-eighties with the development of IFORM, which was developed through a partnership among several leaders in the glass industry. This software has evolved to become a glass modeling suite, named ELFEN GD and includes several modules such as IFORM. Brown reported that the important software for glass modeling include DEFORM, IFORM and POLYFLOW, with PLOYFLOW having the widest use within the industry.
In most of the work reviewed in [1], which focuses primarily on high temperature applications, glass behavior was assumed to be incompressible and purely viscous and the inertia forces were negligible compared to the viscous forces. Basically, the glass was modeled as fluid with velocity degrees of freedom. The only solid based approach reported was by Olaf Op den Camp et al. [2]. In several studies, the choice of viscous fluid modeling is driven by the fact that inorganic glass melts, unlike polymer melts which have been studies extensively, have a fairly wide range of strain-rate independent behavior and a steep temperature dependency as illustrated by the Fulcher equation [3],

$$\log_{10} \eta = A + \frac{B}{T - T_0}$$ (I-1)

where $A, B, T_0$ are constants experimentally determined. Moreover, glass processing temperatures are usually chosen to avoid major viscoelastic material behavior. In addition, for most applications the full modeling of viscoelastic affects is typically not justified. Exceptions include precision lens molding where dimensional tolerance is a critical consideration [4] and, according to M. Brown [1], studies of residual stress which require more sophisticated material models. In these studies it can be required to include several degrees of complexity such as viscoelastic material behavior, bulk incompressibility [5] as well as temperature dependent material behavior [6-8] and structural relaxation [9]. The interest in residual stresses comes from their impact on glass quality, the undesirable presence of birefringence for optical applications and the possibility of cracking. The presence of residual stresses is created during the cooling phase, where the glass exhibits complex thermal strain and viscoelastic behavior due to the non-uniform temperature distribution and the slow reactivity of the glass to the cooling high rate.
In recent years precision lens molding has provided the glass molding industry with new manufacturing and computational modeling challenges. One key issue is accounting for the lens profile deviation, which is the difference between the mold shape and the final shape of the lens, given that dimensional tolerance is critical in precision optics. This process is a promise of a fast production tool for aspherical lenses. However due to the required high precision, every aspect of the process, including the material properties of glass, could influence the outcome. An important feature of precision lens molding is that the range of glass viscosity [10] involved in this process \( \left(10^{7.6} - 10^9 \text{Pa.s}\right) \) is relatively higher than the viscosity values typically treated in the glass molding literature. As such, this viscosity range falls in the same interval of the extrusion process present in the current study. Due to this particular case of low temperature/high viscosity, it is possible that the glass viscoelastic behavior can no longer be neglected in numerical simulations. From the lens molding literature, an example of this is given by Jain et al [19] where the glass deviatoric stress is modeled with a single Maxwell element. A cylinder compression test was used to evaluate the friction and the viscosity of the glass [10-11], while light scattering was used to evaluate the Young’s modulus. In a second paper Jain et al [12] use a viscoelastic material and structural relaxation to model the lens molding cooling phase. In another study Jain et al [13] have presented a similar model with a generalized Maxwell model and structural relaxation for stress relaxation during the lens forming process.

Glass viscoelasticity was further considered by Na et al [14] in their study of birefringence in optical glass lenses. The optical glass was modeled using four Maxwell elements; in addition, given that a displacement based FEM model was used, it was possible to explicitly adopt an elastic bulk behavior. Using a similar approach, Sellier et al [15] have coupled an optimizer with a
lens model analysis to target a specific lens shape; the glass was treated as viscoelastic with elastic bulk behavior and glass structural relaxation was implemented as well. An expansion of this modeling was performed by Ananthasayanam et al where a lens shape sensitivity analysis was performed using viscoelastic bulk behavior [4, 16].

Though a majority of the studies in the literature make use of linear viscoelastic behavior, there are still a few lens molding studies performed using a viscous material model [17] and also nonlinear viscosity [18]; the material properties and the friction behavior were both obtained from a cylinder compression test. In many of these studies, the cylinder compression [10, 18, 11-19], ring compression [4] and the light scattering [10] test are used to either evaluate the material properties or extract the contact properties.

Recent progress on fiber optic research has triggered the need to extrude glass preforms at higher viscosity as optic fiber research shifted toward non silica glass types. In the late 90’s, Seddon and Funiss, published two studies [20-21], in which they described an extrusion method coupled with die design optimization to make tube and core/clad preforms for multi and mono-mode fiber. Both studies were conducted with similar equipment at the relatively high viscosity of $10^8$ Pa·s. However, the first study exposed the major reasons to select extrusion at the super liquid zone to make preforms for this type of application and glass. In 2005 a similar study was carried out by E.T.Y. Lee et al [22] using two die assemblies to coextrude glasses with dissimilar thermal properties. Due to optical property requirements, a variety of glass types were treated [20-21,23]. High viscosity extrusion was required due to severe crystallization at high temperatures as well as composition instability. Though the technique of co-extrusion may be adjusted to accommodate the material properties of the different glass type combinations, and
though this technique works on single step core clad extrusion and provides several advantages over the classical methods [20], it suffers from major limitations such as the lack of control of the evolution [23] of the clad core diameter ratio and issues related to material behavior mismatch, such as the coefficient of thermal expansion that could lead to fracture.

Microstructured optical fiber (MOF) added flexibility to fiber design and opened the way for a new field of application for optical fibers. Nevertheless, the introduction of this fiber technology has increased the complexity of extrusion and fiber drawing as well. There have been several studies on manufacturing and performance of these fibers [24,25, 26, 27, 28], but few details of the manufacturing procedure have been presented.

There have been a few experimental and theoretical studies of glass extrusion conducted within the past few decades. Typically these studies made use of silica glass, the focus being the extrusion of glass in a viscous state far from possible viscoelastic behavior (from $10^3$ to $10^7$ Poise). Such a study has been performed in 1971. Based on experimental data, Roeder [29] presented several important relations between the extrusion parameters, many of which were linear such as the case for silica glass. At a large window of pressure silica extrusion behaves linearly for some nickel–based alloy dies. Electro graphite dies were used as well and though empirical relations between the extrusion parameters were established, these relations have no fundamental basis. It was also noted that for these extrusion conditions the die-swell for alkali lime silica was independent of the ram speed. The low range of viscosity was marked by the presence of nonlinear behavior at extreme pressure. While these experiments were conducted at relatively high temperature, it was later shown by [26] that some of these relations still hold for higher viscosity.
Later in 1989, Egel-Hess and Roeder [30] presented another study of die-swell. This time the viscosity treated was considerably higher ($10^{7.05}$ Pa·s), although it still lies outside the super cooled liquid window. The experimental part of the study highlighted the presence of maximum die-swell reached at a critical die length ratio (for cylindrical preform a 14% of the diameter-increase was noted). Below this critical value, lower die-swell were observed. However, for a matter of flow stability short channel lengths are seldom used. According to this study, lower glass-die friction could be achieved by using self-lubricant material and a few other non-self-lubricated materials. This reduces the die-swell and could lead to die-swell smaller than what would occur for the case of extrusion through a knife edge opening. The choice of glass composition was reported to have no influence on the friction. Though the authors reported the presence of “a thin reaction layer” that sticks on the die wall, Coulomb friction was used to interpret several aspects of the process. Authors have also considered a numerical study to assist their experimental data and imperial interpretations.

Another interesting study on extrusion was performed by Y. Yue et al [38]; contrary to the more conventional studies, extrusion was used to estimate viscosity in the rage of $10^5$ to $10^{7.5}$ Pa·s. The extrusion was carried out using a regular (no funnel) die, and it was based on several assumptions like the predominance of the effect of Poiseuille flow on the channel region and on the ram force. In 2012, Ebendorff-Heidepriem et al [26] presented a detailed experimental and analytical study of the basic assumptions used to estimate ram speed, applied ram force and viscosity for regular rod and microstructured preform with a hexagonal pattern of several circular holes. The study included several experimental results and showed several linear correlations of a number of extrusion variables. While these assumptions appear to oversimplify the problem, the
results show reasonable agreement with viscosity estimations in the literature. In this study several assumptions made by Y. Yue et al [31] were applied even though the range of viscosity was considerably higher (from $10^7$ to $10^9$ Pa·s). In this study it was also mentioned that a fluctuation of the flow rate could impact the die-swell [26].

Regardless of the viscosity range and despite the complexity of the geometry of the new fiber [34-36], recent theoretical models for glass extrusion are typically processed using very simple assumptions. For example, the calculation of the extrusion force is performed based on a simplified figure of die using an adaptation of Poisseuille flow [26, 31]. In these studies the mathematical formulas relating the ram force, the ram speed, some aspects of the die shape and the viscosity were established for a number of dies. These relations were verified by experimental data. Details of such a study are presented below.

Assuming that the flow in the channel follows a Poisseuille Law [30], and assuming that the extrusion pressure $P$ is dominated by flow in the channel region, the pressure $P$ is given by:

$$P = \frac{128L_1}{\pi D_1^2} \eta Q = \frac{128L_1}{\pi D_1^2} \eta A_0 v_0 = K_{\text{die}} \eta Q$$

(11-2)

Where $L_1$ is the channel length, $D_1$ is the channel diameter, $\eta$ is the viscosity of the glass, $A_0$ is the billet section area, $v_0$ is ram speed and $Q$ is the flow rate. This relation shows that the steady state value of the extrusion pressure $P$ is proportional to the viscosity and the flow rate, with the proportionality constant given by $K_{\text{die}}$. Given this linear relation ship, $K_{\text{die}}$ could be measured experimentally given that
Based on these theoretical results, a number of these dependencies have been tested against experimental data collected from the extrusion of F2 glass in a stainless steel die and NCS glass in both nickel-base alloy die and a graphite die. The data collected from F2 glass extrusion showed a linear dependency of the pressure on the channel length \( L \) as well as the on \( \frac{1}{D_1^4} \). The estimation of the linear dependency was well predicted by the no-slip formula in both cases; it was noted also that this assumption was limited to \( \frac{1}{D_1} > 0.6 \). Nevertheless, in some cases, equation (II-1) over predicts the linear dependency and had to be adjusted to account for slippage (\( \alpha \): the friction factor) to match the data:

\[
P = \frac{128L_1}{\pi D_1^2(D_1 + 8\alpha) \eta} A_0 v_0; \quad K_{\text{die}} = \frac{128L_1}{\pi D_1^2(D_1 + 8\alpha)}
\]

Unfortunately, preforms like the one presented in the study by Belwalkar et al [27], referred to as “wagon wheel preforms,” rely more on empirical data. These studies provided information about several aspects of and the difficulties related to preform extrusion; they also raise the need for numerical study of the extrusion process in the super cooled liquid domain. Given that several linear relationships have been experimentally verified, the choice for future numerical simulations should include material properties that respect these linear relations between the extrusion parameters.

Unfortunately, though several advances have been made on glass optical fiber extrusion, few numerical studies have been carried out for glass to further understand the extrusion

\[
K_{\text{die}} = \frac{p}{Q \eta}
\]
procedure. The extrusion literature is mostly dominated by polymer rheology [32, 33]. In fact, in addition to the study by Eagle-Hess et al [30] mentioned earlier, one of the rare numerical glass extrusion studies was conducted in 1996 by Mayer et al [28] to simulate a quasi-static extrusion of a non-circular glass preform. The study reiterated the same assumptions used by Y. Yue et al [31] while the viscosity below viscosity associated with the super cooled liquid domain.

Given higher viscosity of modern glass optic fiber, the issue of slip between the glass and die becomes important. This is considered in the next section.

II.2. SLIPAGE IN EXTRUSION AND GLASS EXPERIMENTAL LITERATURE

II.2.1. SLIP AT THE MOLD/GLASS INTERFACE

Typically in the glass processing literature interface slip is not considered since most glass forming processes are carried out at high temperature where a no-slip assumption is justified or its presence is not needed for the required precision. However, with more computational modeling at higher viscosity, slip should be considered.

The presence of slip during glass extrusion was noted early in the glass extrusion literature. In 1971, in a pure experimental study, Roeder noted that extrusion of Alkali lime silica glass obeys a number of linear empirical equations that link geometrical and input parameters [29]. These relations hold well for metallic dies with a length to diameter ratio large enough to dominate the extremity effects. According to the author, these conditions suggest the presence of a Poiseuille flow like regime. However, the data collected using a graphite die suggest an adjustment of these formulas with a nonlinear exponent. The use of graphite die was followed by
a drop in the extrusion pressure and a reduction of the die-swell which, according to the author, was due to the presence of slip. The presence of slip was later investigated by Egel-Hess and Roeder [30] in 1989. In consideration of their experimental data, the authors noted that friction behavior has a considerable impact on die-swell. It was noted that this behavior depends mostly on the die material composition and not the glass type. The proposed interpretation was based on a stick slip transition of the Coulomb friction law, which also served to explain the variation of the ram force during the extrusion and a few instabilities of the extruded profile (in a more recent study, the interpretation of stick slip behavior was explained using a dynamic friction model [34]). To illustrate these interpretations, the authors conducted a numerical study in which the slip velocity was imposed at the tip of the channel exit. The numerical results were able to illustrate the impact of friction on the die-swell.

An example of glass slip investigation is given by [17], where the Coulomb friction coefficient was determined experimentally based on loss of energy of glass gobs passing though funnel shaped metals. The lens molding literature has explored this subject as well, and though there are a number of studies that assume no-slip at the glass/mold interface, some research papers have explored the possibility of Coulomb friction at molding viscosity such as the work done by Ananthasayanam et al on lens molding [4].

Slip has been also considered in a study by Ebendorff-Heidepriem et al [26], where a simple theoretical interpretation of data was used to interpret experimental data. The authors had to adjust the theoretical model by introducing a slip coefficient to match the experimental data. The adjusted model delivered a wider match to the experimental data set. Unfortunately, the data collected from the NCS glass extrusion using a graphite die does not follow the former
equation. The explanation given by the author for this phenomenon is the dependence of the friction factor $\alpha$ on the geometry.

In an attempt to study slipping velocity, Hee Eon Park et al [35] considers the effect of all die compartments and was able to measure slip velocity using Couette flow and capillarity flow.

### II.2.2. Interface Slip in Extrusion Literature

The rheology literature, and especially the polymer literature, has examined slip phenomena in several experimental and theoretical studies. As it will be detailed later in this section, experimental studies, usually, serve to test empirical and theoretical models as well as reporting experimental observations that might fall outside the scope of current models. Most of the experimental studies are performed on the macroscopic scale. Theoretical models, however, use assumptions based on micro molecular interactions to explain and model experimental observations at the macro scale. Thus, modeling the slip phenomena at the microscopic level is a challenging task. In the eighties in a series of three papers, deGennes [36], [37] and [38] has proposed a theoretical model for slip. And though the model is limited to a specific micro-molecular configuration of the grafted chain at the surface, this study is widely cited in several experimental studies and sometimes it failed to model slip when applied to cases not suited to the original assumptions.

In 2000, in an attempt to propose a general model of polymer slip, Joshi et al [42] have proposed another theoretical model that was tested against a larger set of experimental observations. Theoretical models are based on several micro molecular mechanisms. Several of them have been evoked by Joshi et al [39] in an effort to establish their general micro-molecular
model. According to Joshi et al. [39], long molecular chain materials tend to react differently with a neighboring foreign material dependent on their molecular composition. At the boundary, several different types of molecular interactions have been proposed, the most common being:

- Desorption from the wall and chain disentanglement at the wall [40-42]
- Migration of macromolecules away from the wall under a stress gradient [43-45]

Generally, desorption and disentanglement are most likely to happen and more supported by the literature. According to the two first mechanisms, the flow region could be divided into two separate domains:

- The first domain is composed of a layer close to the wall where chain adsorption and desorption takes place
- The second domain is the main bulk flow domain that is located on top of this layer; chain disentanglement and entanglement take place at the contact region between this domain and the first layer.

Based on micromechanics, models were built to explain numerous complex slip phenomena. However for the sake of simplicity and numerical efficiency, models based on macro observations provide more workable implementation while being compatible with particular cases of the micro models and experimental observations.

As far as extrusion is concerned, the polymer literature has explored the phenomenon thoroughly, mainly because unlike most regular Newtonian fluids, polymer melts slip over solid surfaces. Often when performing experiments to quantify material behavior, unusually large
displacement and/or low force can be explained by interface slip. The early study performed by Mooney [46] constitutes a major reference in this area. In this study the author theoretically established the basic equations and methods to detect and measure slip for several experimental conditions. A review of the current state of the art has been done by Hatzikiriakos [47]. According to Hatzikiriakos one of the most used friction models in the polymer literature is the Navier law [48], which is defined by

\[
\frac{d u}{dy} \bigg|_{y=0} = b \dot{\gamma}_w = \frac{b}{\eta} \sigma_w = k \sigma_w \quad \text{(II-5)}
\]

where \( u_s \) is the slip velocity, \( \dot{\gamma}_w \) is the shear rate at the wall, \( \eta \) is the viscosity of the melt and \( k \) is the friction coefficient. There are different interpretations of the physical meaning of the parameter \( b \), one is given in Figure II-1 where \( b \) is the extra length needed for a Couette flow to reach no-slip; \( b \) and \( k \) could also be interpreted as penalty coefficients. This suggests a viscosity dependence of the friction coefficient \( k \). As will be discussed later this relation has been experimentally verified for several cases. Other studies have suggested a more general relation known as the Navier generalized law where the linear relation between the shear and the slip is maintained on the logarithmic scale [49-51].
The review [47] cites a number of experimental methods for macroscopic detection and measurement of slip. One way to detect slip is the quantification of the dependence of the measured apparent mechanical properties of the material on the geometry of the measurement apparatus. The theoretical basis of these indirect measurement methods has been established in a paper by Mooney [36]. This work has been exploited in several papers to measure and quantify the slip velocity using various experimental setups [35, 49, 52]:

Experimental setups like Couette flow are often used to measure the mechanical properties of fluids. The mathematical foundation governing the experimental setup is involves simple mathematics and can easily be applied to nonlinear viscosity. An adjustment of its mathematical foundation for slip does not increase the complexity of the mathematics but increases the number of required measurements. Assuming that the slip velocity $u_s$ is a function

![Figure II-1. Slip in a Couette flow according to the Navier law.](image-url)
of the wall shear stress and the slip is identical on the upper and lower plate, the real shear rate $\dot{\gamma}$ is linked to the nominal (measured) shear rate $\dot{\gamma}_n$ by [35]:

$$\dot{\gamma} = \dot{\gamma}_n - \frac{2\dot{\gamma}_s}{h}$$  \hspace{1cm} (II-6)

This implicates a linear dependency of the apparent measured viscosity on $1/h$ [35, 46]. Such observations have been reported by Hatzikiriakos and Dealy [53] in their effort to study the slip of a high density polyethylene on a coated and uncoated steel plate in the absence of a pressure gradient. The slip velocity can then be calculated by plotting $\dot{\gamma}_n$ as a function of $1/h$ for constant shear stress in plots that are referred to as the Mooney plot [49, 46]. Using a set of these plots for several shear stresses, the slip velocity can be defined as a function of the shear stress. Hatzikiriakos and Dealy [53] reported that a minimum shear stress of 0.09MPa was required for polymer to slip. Using a similar setup, Park et al [35] were able to establish slip-shear stress curves for a wide range of shear rate. According to the latter study, the use of Couette flow allows slip measurement at shear rates where capillary flow is unstable. However, the test requires special attention, since the use of relatively tall samples could introduce undesirable effects in the final results. For this reason the authors preferred to reduce the number of Couette flow measurements by acquiring the material properties from a different experimental method.

An expansion of Mooney’s work [54] was performed by Yoshimura and Prud’Homme [55] where they have described and validated a similar method to extract the slip velocity and thus correct the viscosity measurements using a coaxial Couette flow. The advantage of their experimental setup is its simplicity and the reduction of boundary effects. The authors were able to reduce the required experimental measurements by imposing constraints on torques and the
inner and outer radii [55]. For the material sample used in their study the slip data reached 1.8cm/s for a maximum shear stress of 250dyne/cm$^2$ and a viscosity as low as 100 poise; the slip velocity also seems to be linearly dependent on the shear stress.

Similar to Couette flow, the parallel disk viscometer can be used to measure slip velocity. Though the flow is more complicated, both experiments use apparent shear rate dependency on the height of the sample. However, relating the slip data at the outer radius to the shear rate is less obvious than the previous cases. This procedure is well detailed in the studies by Yoshimura and Prud’Homme [55] and Mooney [46]. The results obtained [55] were comparable to those obtained earlier using Couette flow. Another study by Henson and Mackay [52] showed that monodisperse polystyrene melts slip at all levels of shear stress for a solid stainless steel metal substrate, which is consistent with the slip theory of deGennes [36], [37] and [38]. The authors used the Navier law to quantify the slip behavior. In a separate study Awati et al. [56] have shown that the slip velocity of PS melts increases linearly with shear stress at very small values of shear stress. The Presence of critical shear for the slip to occur was also debated. Its existence was sometimes reported [53] as well as denied [52]. It was also reported that the pressure has a limited influence on the slip behavior [35].

Though measurement using capillary flow could be adversely affected by several factors and thus requires extreme attention [35], this technique is prevalent in the literature. In an attempt to study slipping velocity, Hee Eon Park et al [35] considers the effect of all die compartments and was able to measure slip velocity using Couette flow and capillarity flow. Indirect measurements of slip involve using a no-slip flow configuration as a reference flow [57]. Other measurement techniques for this flow configuration involve measuring the electrostatic
field generated by slip [58]. Extreme slip cases have been reported where major manifestation of slip could be detected through the change of flow regime toward plug flow and the observation of variations of die-swell (and melt fracture in the unstable zone [54]).

In general experimental study focuses on the effect of polydispersity and molecular weight on the slip behavior, especially at high shear rate. For example, in the case of capillary flow, the slip shear rate curve plotted as a function of wall slip in the logarithmic scale consists of two separate linear portions. Depending on the polydispersity and molecular weight of the polymer, the linear portion of the curve can be either disconnected or connected. A discontinuity usually signals the presence of an unstable region where the slip velocity oscillates between the two linear portions, indicating the presence of two equilibrium states on the curve [47, 35]. Some other studies [49] reported a shear rate-slip curve divided into several regions, most of them are linear on the logarithmic scale.

The previously presented slip measurement methods are based on experimental setups where slip is measured indirectly by extrapolating the variation of a measured entity of a factor related to slip. Thus they are classified as indirect measurement methods. These are relatively simple to implement and do not require advanced equipment. As is discussed below, it is also possible to make direct measurements of slip by monitoring the movement of particles in the contact region.

Direct measurements were also carried using similar experimental setups as for the indirect approaches. These measurements, which are sometimes limited in scale, are more precise. An example of such a study was made by Mhetar VR and Archer in three separate papers
These authors used tracer particles as well as nanosized fluorescent tracer particles to measure the displacement of polymers’ molecular chains at the substrate contact region. Both procedures led to similar conclusions and confirmed the presence of slip at the microscopic level.

To conclude, polymer slip has been widely reported and measured in the literature, and though glass slipping was not as well investigated, a number of papers confirm its presence. The Navier law is referenced in the literature as the slip velocity is always reported as a function of shear stress.
II.3. REFERENCES


COMPUTATIONAL MODELING OF DIE SWELL OF EXTRUDED GLASS PREFORMS AT HIGH VISCOSITY

III.1. INTRODUCTION

Extrusion is a widely spread manufacturing process for making glass and polymer preforms that can be used to make optical fibers. While polymers are the material of choice for many other extrusion applications, for the case of optical fibers the use of specific glasses is often required to achieve targeted optical properties. Microstructured Optical Fibers (MOF), which have complex geometrical patterns of holes in the cross section, are an important example [1-3]. To achieve the desired preform structure and excellent optical quality (in order to achieve ultimately low-loss optical fibers), use of high viscosity combined with low speed is essential. The typical viscosity range is from $10^4 - 10^5$ Pa·s, while the ram speed is reduced to values that are typically on the order of 0.5 mm/min and smaller. Additional contributions that support the need for glass extrusion at high viscosity include Seddon et al.[4], Furniss and Seddon[5] and Lee and Taylor[6].

For the case of MOF the issue of cross-sectional distortion during extrusion is a problem [1] that can potentially be addressed by using computational mechanics. Ideally the extrusion parameters and die geometry can be selected to achieve a targeted precise cross-section that accounts for the distortion. In order to develop such a computational approach it is necessary to
understand the basic modeling choices that must be made. In this regard, extrusion at high viscosity introduces the possibility of two interesting aspects that affect cross sectional distortion: viscoelastic glass behavior and glass/die interface slip. Evidence that slip occurs in this viscosity range was provided for L-BAL35 glass at a viscosity of $10^{8.3}$ Pa·s pressed on a tungsten mold with a diamond-like carbon coating [7]. The ring compression test was used to quantify a very low coefficient of Coulomb friction of 0.05. While interface slip and viscoelastic material behavior are common in studies of Precision Glass Molding [8-10], most glass extrusion studies apply the no-slip boundary condition appropriate at higher temperature [11,12]. Exceptions include the important contribution by Egel-Hess and Roeder [13] who studied die swell both experimentally and computationally at a viscosity of $10^{6.05}$ Pa·s. Die swell is important in extrusion applications as it is global evidence of the deformation mechanisms that cause cross sectional distortion. These authors concluded that interface slip must occur and is a strong function of the die material, while the choice of glass composition in the cases studied had no influence on die swell and therefore of friction. An approximate method based on an inputted wall slipping velocity at the die exit was used to quantify the effect of friction. In another study of die swell for non-circular cross sections [14] both no-slip and full slip were implemented into a computational scheme for a viscosity of $10^{6.86}$ Pa·s.

Ebendorff-Heidepriem and Monro [15] presented a detailed experimental and analytical study of the basic assumptions used to estimate ram speed, applied ram force and viscosity in the range of $10^7$ to $10^9$ Pa·s for extruded solid glass preforms. The study included several experimental results, accounts for interface slip and showed several linear correlations of a number of extrusion variables. While these assumptions appear to oversimplify the problem, the
results show reasonable agreement with viscosity estimations in the literature. In this study several assumptions made by Y. Yue et al. [11] were applied even though the viscosity range was considerably higher.

While there have not been many die swell computational studies for glass that involve viscoelasticity and/or interface slip, such studies are common in the polymer literature. For example, there have been die swell studies that use a no-slip boundary condition with viscoelastic material behavior[16-18] and studies with slip[19,20]. The importance of interface slip in the polymer literature is evident in the review paper by Hatzikiriakos[21], which includes the review of various slip models.

In the current combined computational and experimental study of extrusion of high viscosity glass, the most basic modeling assumptions were used to understand the primary reasons for die swell. This includes material behavior of viscous, upper convective Maxwell model to account for viscoelastic behavior and shear thinning using the Cross law. In addition, the ram speed is considered as a processing parameter, while behavior at the glass/mold interface condition takes into account no-slip and slip making use of the linear form of the Navier law. By comparing results to experimental data within the context of realistic values of the material parameters, conclusions about the validity and importance of these assumptions are made that are specific to glass extrusion at high viscosity.
III.1.1. Governing equations

In the Eulerian framework the governing equations for steady state, axisymmetric, incompressible flow are given by the continuity and conservation of momentum equations as follows:

\[ \nabla \cdot u = 0 \quad (\text{III-1}) \]

\[ \nabla \cdot (\eta \nabla u) - \nabla p + \mathbf{f} = 0 \quad (\text{III-2}) \]

where \( u = u(r, z) \) is the velocity vector in cylindrical coordinates, \( T = \eta \nabla u \) is the deviatoric stress tensor, \( \eta \) is the shear viscosity, \( p \) is the pressure and \( \mathbf{f} \) represents the body force terms. Inertial forces are neglected due to the low rates of deformation. When the viscosity is constant Eqn. (2) simplifies to

\[ \eta \nabla^2 u - \nabla p + \mathbf{f} = 0 \quad (\text{III-3}) \]

The equations for both constant viscosity and shear thinning viscosity are solved using the ANSYS software, POLYFLOW.

III.1.2. Die geometry

The experimental die swell data, which are presented in Table III- 1, where obtained from extrusions of three different glasses under various conditions of viscosity (temperature) and ram speeds. The axisymmetric die used in this process has a geometric composition similar to the one shown in Figure III- 1 [15]. The die shape is composed of three compartments:
Figure III-1. Model of the reference die geometry used in the experiments [15].

- The feeding chamber, which is a reservoir-like compartment where the glass billet is pressed. This part is included in the computational domain with sufficient length for the flow to reorganize before reaching the next chamber. Its length does not have a significant influence on the die swell results.

- The second compartment is referred to as the Interface (or funnel) in Figure III-1, which links the feeding chamber to the channel. In this case the shape of the interface is tangent to the channel and non-perpendicular to the feeding chamber.

- The Die Channel, which is the most basic component in the die, is the region where the flow profile is shaped before the glass reaches the exit.

III.1.3. BOUNDARY CONDITIONS

Boundary conditions for this problem are specified on the four surfaces identified in Figure III-1, which are defined as the Inflow, Outflow, Free surface and Die/Glass interface regions. At the Inflow region the volume flow rate is specified and the radial component of velocity is zero. Both parabolic and uniform velocity profiles were used at the die entrance, leading to the conclusion that the details of the flow profile at the die entrance had negligible effect on the die swell. A parabolic profile was used due to improved convergence. At the
Outflow region, which is taken far enough downstream to achieve a steady state solution, the shear stress is zero and the normal stress is zero in the absence of gravity. On the Free Surface region the glass is free of stress. Finally, at the Die/Glass interface, for the no-slip condition, the velocity is zero, while for the slipping, the normal component of velocity is zero and the tangential component obeys the friction law.

III.1.4. EXPERIMENTAL DIE SWELL DATA

As shown in Table III-1, the die swell and ram force data were obtained for five cases of extrusion performed using the following glasses: F2 (lead-silicate glass from Schott Glass Co.), Bismuth (bismuth borosilicate glass from Asahi Glass Co.) and Tellurite (Na-Zn-tellurite glass made in-house). The die material was stainless steel grade 303 for all the extrusion cases. For each trial a new steel die was used, and the dies were cleaned ultrasonically using detergent to ensure clean, consistent die surface for each trial. The five glass fiber preform cases were extruded at relatively high viscosity ranging from $10^{7.4}$ to $10^{8.8}$ Pa·s, while the ram speed ranged from 0.5 to 0.2 mm/min, which corresponds to a volume flow rate of 6.1 to 2.4 mm³/s. Experimental observations indicate that for log viscosity/(Pa·s) of 7.5 or lower, the preform diameter decreases in the course of extrusion due to the effect of gravity. This preform tapering effect can significantly affect the determination of die swell from the measured preform diameter. For log viscosity/(Pa·s) = 8.5 or greater, no tapering of extruded rods was observed. To avoid the impact of gravity, the experimental die swell values were measured as the preform diameter 10-30 mm from the end of the preform that was extruded first, where the effect of gravity is negligible. The glass extrusion conditions were experimentally determined to be very similar for the three glass types, but the F2 glass was also extruded at a higher ram speed and at lower viscosity.
Experimentally, the extrusion parameters are selected to avoid extrusion failure which include, for example, low preform surface quality (sharkskin surface) or in severe cases a corkscrew preform shape. Details of the experimental procedure are presented in Section 2.1 of the study by Ebendorff-Heidepriem and Monro [15]. The primary cause for error in the measurements was temperature uncertainty of ±5 °C, which had a direct effect on the viscosity. In addition to this potential source of error, conversion from temperature to viscosity was based on curve fits using limited data outside of the range of the extrusion experiments and therefore should be considered as approximate. For example, for F2 glass the manufacturer provides $10^{6.6}$ Pa·s at 594 °C and $10^{12}$ Pa·s at 430 °C. An interesting observation of the data in Table III-1 is that the die swell is not affected by viscosity. Although not presented in Table III-1, this trend of constant die swell was also observed with the Bismuth and Tellurite glasses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Glass</th>
<th>Ram speed (mm/min)</th>
<th>Billet Diameter (mm)</th>
<th>Die exit Diameter (mm)</th>
<th>$\log \frac{\eta}{\text{Pa} \cdot \text{s}}$ (Temp.)</th>
<th>Extruded preform diameter (mm)</th>
<th>Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F2</td>
<td>0.5</td>
<td>30.5</td>
<td>10.2</td>
<td>7.4 (555 °C)</td>
<td>11.0±0.2</td>
<td>6.8</td>
</tr>
<tr>
<td>2</td>
<td>F2</td>
<td>0.2</td>
<td>30.5</td>
<td>10.2</td>
<td>8.1 (529 °C)</td>
<td>11.0±0.2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>F2</td>
<td>0.2</td>
<td>30.5</td>
<td>10.2</td>
<td>8.6 (509 °C)</td>
<td>11.0±0.2</td>
<td>67</td>
</tr>
<tr>
<td>4</td>
<td>Bismuth</td>
<td>0.2</td>
<td>30</td>
<td>10.2</td>
<td>8.8 (460 °C)</td>
<td>10.4±0.2</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>Tellurite</td>
<td>0.2</td>
<td>30</td>
<td>10.2</td>
<td>8.8 (342 °C)</td>
<td>10.5±0.2</td>
<td>25</td>
</tr>
</tbody>
</table>
III.1.5. Mesh presentation and convergence

Mesh convergence studies were performed by running simulations for fourteen mesh configurations for a viscous glass extruded through the die. Though the die had a relatively complex geometry, it was meshed entirely with rectangular axisymmetric elements. Refinement was only necessary near the die exit. The meshes were characterized by two parameters: the minimum mesh size and the refinement coefficient, as provided by the software ANSYS. Combinations of these parameters for each mesh were compared to those of the reference mesh which had the highest density of elements and refinement. The details of the fourteen different meshes used in this mesh convergence study are given in Table III-2. The reference mesh is designated as M5.2 in this table.

The die swell profile was chosen as the primary convergence criterion since it is the most basic measurable outcome of the simulations. In addition, variables related to the velocity and pressure distribution along the centerline were also considered.

Table III-2. Mesh configuration definitions performed on Case 1 in Table III-1 (log viscosity = 7.7 Pa·s, ram speed = 0.5 mm/min).

<table>
<thead>
<tr>
<th>Mesh nomenclature</th>
<th>Refinement coefficient (respective values for meshes on the left)</th>
<th>Minimum mesh size (mm)</th>
<th>Number of Elements</th>
<th>Number of Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1.0, M1.1, M1.2</td>
<td>2,5,8</td>
<td>0.6</td>
<td>1113</td>
<td>1218</td>
</tr>
<tr>
<td>M2.0, M2.1, M2.2</td>
<td>2,5,8</td>
<td>0.4</td>
<td>2431</td>
<td>2587</td>
</tr>
<tr>
<td>M3.0, M3.1, M3.2</td>
<td>2,5,8</td>
<td>0.2</td>
<td>9411</td>
<td>9719</td>
</tr>
<tr>
<td>M4.0, M4.1, M4.2,</td>
<td>2,5,8</td>
<td>0.12</td>
<td>26141</td>
<td>26652</td>
</tr>
<tr>
<td>M5.1, M5.2</td>
<td>5,8</td>
<td>0.09</td>
<td>46121</td>
<td>46800</td>
</tr>
</tbody>
</table>
The extrusion parameters for Case 1 in Table III-1 were used to determine the steady state die swell for each of the fourteen mesh configurations presented in Table III-2. This case has the lowest viscosity and the highest ram-speed. The die swell results are presented in Table III-3 both as absolute values and as percent error when compared to the results from the densest mesh. The data in this table indicate that most of the mesh setups perform similarly, with all but M1.0 agreeing to the first digit.

**Table III-3. Mesh convergence study of the die swell radius.**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>M1.0, M1.1, M1.2</th>
<th>M2.0, M2.1, M2.2</th>
<th>M3.0, M3.1, M3.2</th>
<th>M4.0, M4.1, M4.2</th>
<th>M5.1, M5.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die swell radius (mm)</td>
<td>5.8083</td>
<td>5.7900</td>
<td>5.7707</td>
<td>5.7621</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.7842</td>
<td>5.7733</td>
<td>5.7616</td>
<td>5.7564</td>
<td>5.7527</td>
</tr>
<tr>
<td></td>
<td>5.7751</td>
<td>5.7670</td>
<td>5.7582</td>
<td>5.7542</td>
<td>5.7544</td>
</tr>
<tr>
<td>Percent error of radius – 5.1 mm as compared to results of M5.2.</td>
<td>8.25</td>
<td>5.45</td>
<td>2.50</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.55</td>
<td>2.90</td>
<td>1.11</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>3.17</td>
<td>1.94</td>
<td>0.59</td>
<td>0.02</td>
<td>---</td>
</tr>
</tbody>
</table>

The trend of the results in Table III-3 shows that convergence improves by increasing the refinement coefficient as well as increasing the number of elements, although increasing the refinement coefficient is more effective. Evidence that the results have nearly converged is seen by comparing M3.1 to M4.0, where a small adjustment in the refinement coefficient gives the better result but the deviation is greater than 1%. Another example of the benefits of refinement is shown by comparing the M4.2 and M5.1 results. Based on the die swell results, which are also presented in Figure III-2 for the entire length of the preform, the optimum mesh is M3.2 since it has less than 1% deviation and uses the fewest elements. Before final selection of this mesh the behaviors of the pressure and velocity along the centerline were also considered.
Figure III-2. Die swell profiles (left) and velocity along the centerline (right) from the mesh convergence study.

These results showed that in general the variations of these quantities for the best meshes have good agreement; however, the location of the maximum value of the velocity was the most sensitive to mesh since, also shown in Figure III-2, the maximum occurs near the die exit, which is at a normalized length of "0" in the figure. A detailed investigation of this behavior narrowed the choice to M3.2 over M4.1, although the difference was not significant. To conclude, the mesh, M3.2, which is presented in Figure III-3, gives good agreement with die swell or by the pressure and velocity along the centerline, and thus was selected for all the simulations in this study.
III.2. RESULTS

III.2.1. VISCOUS MATERIAL BEHAVIOR

In this section the viscous material model with a no-slip boundary condition is used to compare the predicted die swell with the experimental data for the four test cases. A summary of the results is presented in Table III-4. Most notable is that the predicted die swell values for this model are independent of the flow rate and viscosity, which is consistent with the results presented by Egel-Hess and Roeder [13]. Furthermore, the numerical simulations overestimated the die swell in all cases, especially for the cases of Bismuth and Tellurite. For the case of F2 glass, though the die swell is again over-predicted, it is better than the other predictions by a factor of two. The differences between the data and the predictions are large enough to discard the possibility of computational error as the explanation.
Table III-4. Die Swell results for viscous material behavior with no-slip boundary condition.

<table>
<thead>
<tr>
<th>Case</th>
<th>Ram speed (mm/min)</th>
<th>Extruded preform diameter (mm)</th>
<th>Data</th>
<th>Numerical (11.516 for $\lambda \rightarrow 0$, all cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\log_\eta$ $\frac{Pa\cdot s}{Pa\cdot s}$</td>
<td>$\lambda = 0.1$</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>7.4</td>
<td>11.0±0.2</td>
<td>11.498</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>8.1</td>
<td>11.0±0.2</td>
<td>11.502</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>8.6</td>
<td>11.0±0.2</td>
<td>11.502</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>8.8</td>
<td>10.4±0.2</td>
<td>11.502</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>8.8</td>
<td>10.5±0.2</td>
<td>11.502</td>
</tr>
</tbody>
</table>

The presented cases cover a considerable range of flow rate and a variety of viscosities; however these variations had no impact on the numerical outcome. It is therefore apparent from the data and these numerical results that an inappropriate material model was used and/or the assumption of a no-slip boundary condition is incorrect.

III.2.2. Time dependent material behavior

The viscous assumption neglects the time dependent response introduced by viscoelastic material behavior and is justified when the extrusion process is extremely slow or the glass is at a temperature above the transition temperature range, where glass behavior transitions from that of an elastic solid to a viscous fluid[22]. Given that the interest in this study is high viscosity extrusion at a temperature within this range, the effect of viscoelastic material behavior should be studied, especially since the viscous assumption failed to describe the outcome of the experimental data. To determine the impact of this model adjustment, the five study cases are considered using the Maxwell upper convected model[23], which is presented below:
\[ T + \lambda \dot{T} = 2\eta D \] 

(III-4)

where \( T \) is the deviatoric stress tensor, \( \lambda \) is the relaxation time, \( D \) is the tensor of the deformation rate, \( \dot{T} \) is the upper-convected time derivative of \( T \) defined as

\[
\dot{T} = \frac{\partial T}{\partial t} + u \cdot \nabla T - \left( (\nabla u)^T \cdot T + T \cdot (\nabla u) \right)
\]

(III-5)

Table III-5 shows die swell results for five relaxation times (\( \lambda = \eta/G \)) for each of the four experimental data sets. Table III-4 provides the \( \lambda = 0 \) limiting case of the viscoelastic solution. It is important to note that another mesh convergence study was conducted for viscoelastic material behavior.

The results in Table III-5 show an initial decrease of the die swell as the relaxation time, \( \lambda \), increases, followed by an increase in the die swell and the computational solution captures the complexity of the response.

Table III-5. Die Swell results for viscoelastic material behavior with no-slip boundary condition.

<table>
<thead>
<tr>
<th>Case</th>
<th>Ram speed (mm/min)</th>
<th>( \log \frac{\eta}{Pa \cdot s} )</th>
<th>Extruded preform diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Data</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>7.4</td>
<td>11.0±0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>8.1</td>
<td>11.0±0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>8.6</td>
<td>11.0±0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>8.8</td>
<td>10.4±0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>8.8</td>
<td>10.5±0.2</td>
</tr>
</tbody>
</table>
In order to understand this, as the relaxation time increases from zero, a recoverable elastic contribution to the stress occurs within the glass. If the relaxation time is high, this stress does not have time to relax until the flow emerges from the die exit which increases the die swell. However, if the relaxation time is low, it appears that there is time for the stress to relax and cause a rearrangement while still in the channel which causes a slight reduction in the die swell. Realistic values of the relaxation time for viscosity in the range of $10^7$ – $10^9$ Pa·s are less than unity since the shear modulus is larger than 1 GPa [24]. Therefore, while the numerical solution shows that die swell can increase for large relaxation times, in the physical range of material behavior there is a small decrease, which is too small to explain the experimental results.

From the results in Table III-5 the time dependence of the material behavior makes the result more sensitive to the process parameters. However, this sensitivity does not produce the correct die swell results observed in the experimental data as the die swell is still over estimated by the model.

III.2.3. Shear thinning

The possibility of using a shear thinning model to target the measured die swell was considered even though the numerical data indicates that the maximum shear rate in the experiments is very low for such behavior. To illustrate this point, an optimization process was used to adjust the material properties for a glass with shear thinning to fit the experimental value of the preform diameter. The Cross law [25,26] was chosen for this task due to its simplicity and limited number of parameters, which is presented below.
\[
\eta = \frac{\eta_0}{1 + (\lambda \dot{\gamma})^m},
\]

(III-6)

where \( \eta_0 \) is the equilibrium viscosity of the glass, \( \dot{\gamma} \) is the rate of shear strain and \( m \) and \( \lambda \) are model parameters. Written in this form the exponent \( m \) can be used as a measure of nonlinearity; the closer this coefficient to 1 the higher the degree of nonlinearity. From a material behavior perspective a high \( m \) means that the material viscosity decreases rapidly for a small increase of the local shear rate. This behavior is bounded in the shear rate domain by the constant \( \lambda \), which defines the value at which the shear rate starts to affect the viscosity. As the \( \lambda \) parameter increases, the viscosity becomes sensitive to shear rate at a lower value of the shear rate. Because of this, for very high values of \( \lambda \) the nominal viscosity \( \eta_0 \) is difficult to measure.

Figure III-4. Die swell profiles (left) and velocity along the center line (right) from the optimization study using the Cross law of shear thinning.
To perform the task of optimization, a number of shear thinning glass property configurations were considered. First a value of $\lambda$ was selected and then an optimizer was coupled with POLYFOW to search for the corresponding value of $m$ required to achieve the experimental value of the die swell. This task was carried out for the Tellurite example, which is Case 5 in Table III-1. The results of this optimization study are presented in Table III-6 and Figure III-4, which includes the extruded preform profiles and the velocity profiles along the centerline for three cases of the Cross law parameters. The results in Figure III-5 show the viscosity distributions in the die channel for the two extreme cases considered in Table III-6 ($m = 0.947, \lambda = 100$) and ($m = 0.632, \lambda = 4000$). The regions are defined as a range of percentage of $\eta_0$. These results indicate a very high degree of shear thinning, especially for the case when $\lambda = 4000$. The results in Figure III-6 summarize the variation in viscosity throughout the die and the channel for all the cases in Table III-6. In these figures the x-axis represents the viscosity normalized by the nominal viscosity $\eta_0$, while the y-axis represents the percentage of the volume of glass that has a viscosity greater than $\eta/\eta_0$. The closer this is to 100%, the less the effect of shear thinning. The figure on the right, which accounts for just the volume of glass in the channel, shows more extreme results since there is no stagnation region.
Figure III-5. Viscosity distributions for two cases (upper: $m = 0.947$, lower: $m = 0.632$) of the Cross law optimization study (see Table 6), which show regions in the die channel where viscosity is within the prescribed range of percentage of $\eta_0$, where $\eta_0$ is $10^{6.8}$ Pa·s. The centerline is on the bottom and the die exit is on the right.

The results presented in Table III-6 indicate a high degree of nonlinearity since the $m$ parameter is greater than 0.6 for a wide range of the $\lambda$ parameter. In order to better appreciate this degree of nonlinearity, a constant “apparent” viscosity can be defined which requires the same ram force as that obtained using the Cross law. Based on the linear relationship between
the ram force and the viscosity established by Ebendorff-Heidepriem and Monro [15], the apparent viscosity can be calculated using the definition of the Relative force[15], which is equal to the ram force calculated for the viscous case normalized by the ram force calculated for the cross law case. Based on this definition, as shown in Table III-6, the apparent viscosity is at most 57% of the nominal viscosity.

Table III-6. Shear thinning optimization study results for Tellurite using a no-slip boundary condition.

<table>
<thead>
<tr>
<th>Cross law Parameters</th>
<th>Extruded preform diameter (mm)</th>
<th>Relative force</th>
<th>ηApparent (Pa·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ηo (Pa·s)</td>
<td>λ</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>6.31E+08</td>
<td>1.00E+02</td>
<td>0.947</td>
<td>10.5</td>
</tr>
<tr>
<td>6.31E+08</td>
<td>2.00E+02</td>
<td>0.823</td>
<td>10.5</td>
</tr>
<tr>
<td>6.31E+08</td>
<td>4.00E+02</td>
<td>0.745</td>
<td>10.5</td>
</tr>
<tr>
<td>6.31E+08</td>
<td>8.00E+02</td>
<td>0.695</td>
<td>10.5</td>
</tr>
<tr>
<td>6.31E+08</td>
<td>2.00E+03</td>
<td>0.653</td>
<td>10.5</td>
</tr>
<tr>
<td>6.31E+08</td>
<td>4.00E+03</td>
<td>0.632</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Glass material behavior can be represented by a constant viscosity for a broad range of shear rate [12]. Furthermore, Scherer[22] reported that inorganic glass starts to deviate from viscous behavior at relatively high stress levels such as 50-100 MPa, which were not reached in these numerical experiments. Therefore, while nonlinear viscosity can explain the die swell results in Table III-1 from a numerical point of view, from a physical point shear thinning is an unlikely explanation.

In a study of friction mechanisms in polymer extrusion, Joshi et al. [27] present a simulation where the influence of slipping of the polymer was considered using a nonlinear viscosity. They showed that interface slip between the polymer and the channel could lead to a
behavior similar to that of nonlinear viscosity. Therefore, while shear thinning can cause a noticeable change in die swell, other factors must be more important. In the next section the no-slip boundary condition, which was enforced throughout this study, is relaxed.

III.3. Interface Slip

Due to the relatively high viscosity of the glass in the extrusion cases of Table III-1, the possibility of slip at the die/glass interface is considered. In this section the linear form of the Navier law of friction [21] is used to account for interface slip, which is defined by

\[
\dot{u}_s = b \frac{du}{dy} \bigg|_{y=0} = b \dot{\gamma}_w = \frac{b}{\eta} \sigma_w = k \sigma_w
\]

(III-7)

where \( u_s \) is the relative slip velocity at the interface, \( b \) is a constant with units of length, \( \dot{\gamma}_w \) is the shear rate in the glass at the interface, \( \eta \) is the viscosity of the glass, and \( k \) is the friction coefficient. Similar to the shear thinning case, an optimizer was used to adjust the friction coefficient to target the measured die swell for the case of viscous material behavior. The results are presented in Table III-7.

Table III-7. Die Swell optimization results for viscous material behavior with interface slip using the Navier law.

<table>
<thead>
<tr>
<th>Case</th>
<th>Ram speed (mm/min)</th>
<th>(\log \frac{\eta}{Pa \cdot s})</th>
<th>Extruded preform diameter (mm)</th>
<th>Numerical diameter from optimizer</th>
<th>Required Navier friction coefficient: Log([k/(\eta m)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>7.4</td>
<td>11.0±0.2</td>
<td>11.007</td>
<td>3.560</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>8.1</td>
<td>11.0±0.2</td>
<td>11.000</td>
<td>3.549</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>8.6</td>
<td>11.0±0.2</td>
<td>11.002</td>
<td>3.551</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>8.8</td>
<td>10.4±0.2</td>
<td>10.399</td>
<td>2.480</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>8.8</td>
<td>10.5±0.2</td>
<td>10.497</td>
<td>2.710</td>
</tr>
</tbody>
</table>

63
As shown in Table III-7 the presence of slip can account for the discrepancy between the data and the results. Furthermore, the values of the log of the normalized coefficient of friction, 
\[ \log \left( \frac{k}{\eta} \times m \right) \], are at a reasonable level and show a nearly constant value for F2 glass. For this viscosity range, additional computations showed that the no friction limit corresponds to about \[ \log \left( \frac{k}{\eta} \times m \right) = -1 \], while the no slip limit occurs at about \( \log[k/(\eta m)] = 8 \). In order to see the effect of including slip and viscoelastic material behavior, the same optimization procedure to determine the friction coefficient to obtain the target value of the die swell was applied for a viscoelastic material. These results are presented in Table III-8 for the same range of relaxation times considered in Table III-5. These results show that the friction coefficient is not sensitive to the relaxation time, indicating that viscoelasticity does not have a significant effect on die swell compared to that of slip.

### Table III-8. Same as Table III-7 for viscoelastic material behavior.

<table>
<thead>
<tr>
<th>Case</th>
<th>Required Navier friction coefficient to achieve the measured preform diameter: ( \log(\frac{k}{\eta} \times m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda \rightarrow 0 )</td>
</tr>
<tr>
<td>4</td>
<td>2.480</td>
</tr>
<tr>
<td>5</td>
<td>2.710</td>
</tr>
</tbody>
</table>
III.4. **Ram Force**

The focus of the current study is the effect of modeling assumptions on the die swell. Since interface slip appears to be the most significant factor in the die swell, it is important to also consider the ram force since for a given viscosity and die geometry, friction plays the primary role in the force required to maintain a given ram speed. In all the previous calculations in Tables III-4 - 8 the ram force is a computational output from the extrusion. These forces in units of kN are summarized in Table 9.
### Table III-9. Plateau values of ram force (kN) for selected trials from Tables III-4 - 8.

<table>
<thead>
<tr>
<th>Case</th>
<th>Force data (kN)</th>
<th>Viscous no-slip (Table III-4)</th>
<th>VE no-slip (Table III-5)</th>
<th>S.T. no-slip (Table III-6)</th>
<th>Viscous slip (Table III-7)</th>
<th>VE slip (Table III-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.8</td>
<td>11.56</td>
<td>(\lambda = 0.1: 11.548) (\lambda = 1: 11.546) (\lambda = 5: 11.61)</td>
<td>-</td>
<td>9.33</td>
<td>(\lambda = 0.1: 9.294) (\lambda = 1: 9.287) (\lambda = 5: 9.095)</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>23.16</td>
<td>(\lambda = 0.1: 23.23) (\lambda = 1: 23.22) (\lambda = 5: 23.24)</td>
<td>-</td>
<td>18.61</td>
<td>(\lambda = 0.1: 18.67) (\lambda = 1: 18.66) (\lambda = 5: 18.69)</td>
</tr>
<tr>
<td>3</td>
<td>67</td>
<td>73.14</td>
<td>(\lambda = 0.1: 73.22) (\lambda = 1: 73.37) (\lambda = 5: 73.35)</td>
<td>-</td>
<td>58.82</td>
<td>(\lambda = 0.1: 58.97) (\lambda = 1: 58.93) (\lambda = 5: 59.05)</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>108.46</td>
<td>(\lambda = 0.1: 108.81) (\lambda = 1: 108.78) (\lambda = 5: 108.87)</td>
<td>-</td>
<td>34.72</td>
<td>(\lambda = 0.1: 35.00) (\lambda = 1: 34.93) (\lambda = 5: 34.82)</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>108.46</td>
<td>(\lambda = 0.1: 108.81) (\lambda = 1: 108.78) (\lambda = 5: 108.87)</td>
<td>(m = 0.947: 62.05) (m = 0.695: 26.18) (m = 0.632: 12.49)</td>
<td>45.43</td>
<td>(\lambda = 0.1: 45.92) (\lambda = 1: 45.87) (\lambda = 5: 45.66)</td>
</tr>
</tbody>
</table>

As a general trend in Table III-9 the computational model over-predicts the ram force.

Similar to the results for the die swell comparisons in Tables III-4 - 6, when no-slip is assumed only shear thinning has the ability to match the data. The shear thinning case of \(m = 0.695\) and \(\lambda = 800\) from Table III-6 matches the die swell and comes close to the ram force as shown in Table III-9. However, as shown in Figure III-5, it is believed that the reduction in viscosity in this shear thinning model is too drastic to be physical. The ram force can also be reduced by allowing for slip, but as the results in Table III-9 show the values of friction used to match the die swell from Tables III-7 and 8 lead to a ram force that is still too high for four of the five study cases.

Concerning Case 3 which is the only case for which the model over-predicts the die swell, the experimental ram force is unusually high compared to Cases 4 and 5, which have higher viscosity.
In order to quantify the discrepancy in force in Table III-9 in terms of the die swell for which the experimental error is known, the measured ram force can be matched by changing the friction coefficient, which leads to a prediction of the die swell. This optimization task for viscous glass behavior was performed and the results are presented in Table III-10.

**Table III-10. Die swell obtained by adjusting friction coefficient to match experimental ram force.**

<table>
<thead>
<tr>
<th>Case</th>
<th>(\log(\eta/(Pa\cdot s)))</th>
<th>Measured preform diameter (mm)</th>
<th>Force data (kN)</th>
<th>Force calculated (kN)</th>
<th>Required (\log((k/\eta) \times m)) to match force</th>
<th>Resulting preform diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.4</td>
<td>11.0±0.2</td>
<td>6.8</td>
<td>6.800</td>
<td>3.050</td>
<td>10.69</td>
</tr>
<tr>
<td>2</td>
<td>8.1</td>
<td>11.0±0.2</td>
<td>14</td>
<td>14.000</td>
<td>3.083</td>
<td>10.71</td>
</tr>
<tr>
<td>3</td>
<td>8.6</td>
<td>11.0±0.2</td>
<td>67</td>
<td>67.000</td>
<td>4.011</td>
<td>11.23</td>
</tr>
<tr>
<td>4</td>
<td>8.8</td>
<td>10.4±0.2</td>
<td>26</td>
<td>26.000</td>
<td>2.236</td>
<td>10.33</td>
</tr>
<tr>
<td>5</td>
<td>8.8</td>
<td>10.5±0.2</td>
<td>25</td>
<td>25.000</td>
<td>2.202</td>
<td>10.32</td>
</tr>
</tbody>
</table>

As seen from the Table III-10 results, by changing the friction coefficient from the values in Table III-7 to those in Table III-10, the die swell is now under-predicted in all cases but Case 3. The Bismuth (Case 4) and Tellurite (Case 5) extruded preform diameters are within experimental measurement error, while the cases of F2 glass (Cases 1, 2 and 3) are not. In terms of percent deviation, the predicted value of Tellurite deviates most from the measured value, which coincides with Tellurite having the lowest normalized friction coefficient. Perhaps the most interesting change from Table III-7 to Table III-10 is that the normalized coefficient of friction is no longer constant for the three F2 glass cases. The constant trend in Table III-7 is believed to be correct, which indicates that matching the die swell, which is insensitive to viscosity, is more reliable for predicting the coefficient of friction than matching the ram force, which is very sensitive to viscosity.
The under-prediction of die swell in Table III-10 can be caused by several reasons. From a computational point of view, two possible explanations for under-predicting the die swell are shear thinning and viscoelastic behavior. Inclusion of either of these behaviors can be used to match both the die swell and the ram force. For example, by adding shear thinning the friction coefficient would have to be higher, which would result in an increase in die swell. From an experimental point of view, the low die swell values in Table III-10 can be explained by viscosity that is lower than what was determined due to a lack of precision in the temperature measurement and/or the conversion from temperature to viscosity. In this case a higher friction coefficient would be needed to match the ram force, which would increase the die swell. As a final study the combinations of friction coefficient and viscosity using viscous glass behavior were determined to match both the die swell and the ram force. These results are presented in Table III-11. The last column of this table provides the difference between the temperature that corresponds to the required viscosity in the optimization study and the measured temperature from Table III-1. Only Case 1, which has the lowest viscosity and highest ram speed, is outside of the ±5 °C range of measurement error.

**Table III-11. Required friction coefficient and viscosity values used in computations to match experimental die swell and ram force.**

<table>
<thead>
<tr>
<th>Case</th>
<th>$\log(\eta/(Pa\cdot s))$</th>
<th>Required friction coefficient: $log((k/\eta) \times m)$</th>
<th>Required viscosity: Log[($\eta/$(Pa·s))]</th>
<th>Corresponding temperature difference (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.4</td>
<td>3.549</td>
<td>7.27</td>
<td>6.2</td>
</tr>
<tr>
<td>2</td>
<td>8.1</td>
<td>3.550</td>
<td>7.98</td>
<td>2.9</td>
</tr>
<tr>
<td>3</td>
<td>8.6</td>
<td>3.551</td>
<td>8.656</td>
<td>-1.4</td>
</tr>
<tr>
<td>4</td>
<td>8.8</td>
<td>2.480</td>
<td>8.674</td>
<td>2.1</td>
</tr>
<tr>
<td>5</td>
<td>8.8</td>
<td>2.710</td>
<td>8.541</td>
<td>2.2</td>
</tr>
</tbody>
</table>
As with the results presented in Table III-7, the results in Table III-11 indicate that the friction coefficient is constant at \( \log((k/\eta) \times m) = 3.55 \) for this die and F2 glass over this temperature range. This result is believed to be a confirmation of the convergence of the computational model and the appropriate choice of the Navier friction model. The interpretation of these viscosity/temperature results is that if the measured temperatures corresponded to the calculated viscosities, adjustment of the friction coefficient to match the die swell would also match the ram force. Furthermore, since the normalized friction coefficient is constant, if only one case had been matched, the other two cases could have been predicted for both die swell and ram force. These results give perhaps the best indication of how the computational and experimental results compare. Furthermore, these results show the importance of a precise determination of the viscosity to rely on computational predictions of extrusion.

III.5. Discussion

Glass forming processes such as Precision Glass Molding and extrusion of microstructured fiber preforms can take place at temperatures below those where a no-slip fluid mechanics boundary condition is justified. Typical viscosity ranges for these processes are \(10^{7.5}\) to \(10^9\) Pa-s, where both interface slip and viscoelastic material behavior complicate processing and modeling. In this study experimental extrusion data in this viscosity range was used to quantify the importance of shear thinning, viscoelastic behavior and slip. Comparisons between model predictions and measurements were presented with reasonable results, understanding there are sources of error from both. From the experimental point of view, the viscosity values reported in Table III-1 are based on limited data and temperature measurements of the outer portion of the die [15] and are believed to be accurate enough to conclude that the current study provides
evidence that slipping occurred in these extrusion cases and is the most significant factor that affects the die swell. From the computational point of view, two important simplifications in the computational model are the assumption of constant viscosity (temperature) throughout the volume of glass and uniform friction throughout the die surface. Each of these assumptions can be the cause for a discrepancy between model predictions and measurements. While the computational results show that slip most likely occurs in these extrusion examples, they do not rule out the presence of mild shear thinning and/or viscoelastic behavior. In order to address further these aspects, independent experiments are required to characterize the material behavior and friction. As discussed by Joshi et al. [28,29] who studied the cylinder compression test, a possible complication in interpreting data is that changes in both friction and viscosity can cancel each other, which requires accurate knowledge of one quantity in order to determine the other with accuracy. However, in the current study a constant normalized friction coefficient was obtained for F2 glass at different temperatures indicating that matching die swell in extrusion might be a very reliable way to estimate the friction coefficient since die swell is insensitive to viscosity. While the combined computational approach of matching both the ram force and the die swell by adjusting the friction coefficient and the viscosity provides a way to determine the viscosity, ideally the viscosity would be determined with high accuracy in an independent test. As a related comment concerning the reliability of the computational approach to predict outcomes of glass forming operations such as extrusion, investigations such as the current study are necessary to identify the key glass and glass/die properties and to quantify the degree of accuracy required in the experimental determination of these properties. Without accurate inputs of the key properties, the computational approach cannot be reliable.
III.6. CONCLUSION

The extrusion of high viscosity glass was studied from a numerical point of view under various assumptions. After demonstrating convergence of the computational solution and providing justifications for the neglect of gravity and the details of the flow profile at the die entrance, a comparison between experimental data and computational results was made using the most basic viscous material model with no-slip boundary conditions. Since the corresponding numerical results were not able to fit the data, the model was adjusted by considering viscoelastic material behavior, shear thinning and slip at the glass/die interface. Based on this study the following conclusions can be made:

- The die swell results are over-predicted using a no-slip boundary condition and a viscoelastic material model for all values of the relaxation time including the viscous limit of zero. For the physical range of relaxation times, $\lambda < 1$, a very small drop in computed die swell is observed. A dramatic increase in die swell occurs as the relaxation time increases, but this is outside of the physical range.

- Shear thinning can explain the measured die swell, but only if local reductions in viscosity are so severe as to be non-physical. Therefore, for reasonable shear thinning behavior using a no-slip boundary condition the die swell is over-predicted.

- Relaxation of the no-slip boundary condition allowed for an easy match of the die swell results using purely viscous material behavior. As such interface slip is a primary mechanism that affects the current die swell results. If either the friction coefficient or the viscoelastic material behavior was known based on independent tests, further refinement in the die swell solution could be obtained.
Die swell was shown to be independent of viscosity for a given glass type which allowed the friction coefficient to be determined by matching the die swell even when the viscosity was not known with significant precision. For F2 glass it was demonstrated that the Navier friction coefficient normalized with respect to viscosity was independent of temperature for the range, $509 \, ^\circ \text{C} \leq T \leq 555 \, ^\circ \text{C}$. Furthermore, both experimental and computational results show that friction is dependent on glass type for the same die.

The computational model using viscous material behavior and interface slip over-predicted the ram force for all but one case, although the results were close enough to be explained by a measurement error in temperature of $\pm 3 \, ^\circ \text{C}$ except for the lowest viscosity case where the error was 6.2 $^\circ \text{C}$.

III.7. REFERENCES


RING COMPRESSION TEST FOR HIGH TEMPERATURE

GLASS USING THE GENERALIZED NAVIER LAW

IV.1. INTRODUCTION

Glass forming processes such as Precision Glass Molding (PGM) and extrusion of Photonic Crystal Fibers (PCF) are often conducted at temperatures that correspond to viscosity above $10^7$ Pa·s [1-4]. At such levels interface slip between the hot glass and mold or die surface is known to occur [5] and the associated level of friction affect, for example, processing time [2], final dimensions of a molded lens [2] and the final diameter of a glass preform through the mechanism of die swell [6]. Computational studies of such processes therefore require a model to account for interface slip with friction. The focus of the current study is to provide an approach to characterize interface slip using the Navier friction model [7,8], which can be an appropriate choice when glass is at the transition between fluid and solid behavior and the contacting counterpart is a highly polished mold or die surface.

The friction behavior of amorphous long molecular chain materials has been considered in several studies [7, 9-11]. These materials, whether considered as viscous or viscoelastic, react with a neighboring foreign material in a way that depends on their molecular composition. In the vicinity of the contact zone, the molecular chains can create a boundary layer with completely different behavior from that of the bulk material. Slipping within this layer can occur due to
several mechanisms [11]. The review paper by Hatzikiriakos [9] summarizes the experimental methods to measure slip, the slip mechanisms involved and the slip models, which unlike the Coulomb friction model, depend on the relative slip velocity as a primary parameter.

The generalized Navier model will be used in the current study. This choice is based on combined computational and experimental work for the prediction of die swell for glass extrusion [12] and other combined studies such as [8], where it was demonstrated that the Navier model was able to account for experimental polymer extrusion data that was affected by interface slip. The generalized form of this model assumes a linear relationship between the logarithm of the shear stress and the logarithm of the relative sliding speed between the contacting pair, given by

$$\tau = k_s \rightarrow \log(\tau) = \log(k) + e\log(\nu_s) \quad (\text{IV-1})$$

where $\tau$ is the interface shear stress and $\nu_s$ is the relative sliding speed. Comparing this model with the complex slip behavior discussed in [9-11], it is clear that this model 1) neglects an initial value of a critical shear stress to allow slip and 2) does not take into account possible complex slip phenomena that have been shown to occur at high levels of relative sliding speed. These simplifications are justified in the current study, since it is dedicated to glass forming processes that favor no interaction between mold and glass surfaces and, more importantly, require very slow processing speeds to avoid phenomena such as spurt or stick-slip behavior [10] that would lead to poor glass surface quality. To summarize, this model, which provides a workable implementation into a large scale computational problem, is based on macro-mechanical observations, while still being compatible with the appropriate features of the micro-mechanical models and experimental observations.
A review of methods to identify slip and, as a consequence, characterize the Navier friction coefficient is presented in [9]. These studies apply to polymers, whose melt temperatures are well below the processing temperatures of different glass types. In addition to the requirement of high temperature testing, the viscosity levels of polymers used in processing are also typically below $10^7$ Pa·s, which also has an effect on the choice of test. Since temperature and viscosity are such important aspects of proper characterization of glass properties [5], experimental approaches that make use of existing and reliable equipment at high viscosity and high temperature are required. Perhaps the best approach is the Ring Compression Test (RCT) that can easily be conducted with a Parallel Plate Viscometer (PPV) or a lens molding machine [5].

The RCT [5, 13-15] is based on a ring-shaped cylindrical glass sample that is deformed in compression between two flat surfaces that are identical to those of the mold or die material of interest. The advantage of the test is that within a certain range of friction behavior a very precise characterization can be made due to the extreme sensitivity of the inner radius of the ring to the level of friction. The important advantage of the RCT for glass is that Ananthasayanam et al. [5] showed that the friction calibration curves (FCCs), which are used to convert experimental data directly to a friction parameter, are independent of material behavior, which makes the FCCs universal. The FCCs properly take into account the complex combination of interface slip and material deformation and therefore, the experimentalist does not need to perform any calculations.

To the knowledge of the authors, the RCT test has only been used with the Coulomb friction model and the friction factor model to account for an upper limit of a shear stress [15]. Motivated by processes such as extrusion of glass preforms and lens molding, in the current
study the Navier friction law will be used to account for the possibility that glass at high
temperature has friction behavior more influenced by relative sliding speed than by pressure,
which has been shown to be true in polymer studies such as [10]. Furthermore, our experience
with the use of the Navier model for prediction of die swell in high viscosity \((10^7\text{-}10^9 \text{ Pa}\cdot\text{s})\)
extrusion [12] is that experimental die swell data over a large range of viscosity is matched using
the same normalized Navier coefficient.

IV.2. Finite Element Model

The study by Ananthasayanam et al. [9] showed that if the ring compression test was
conducted according to a prescribed process of heating, soaking, pressing and cooling, the
friction calibration curves (FCCs) in terms of the Coulomb friction coefficient could be obtained
simply by assigning the viscosity at the pressing temperature in the computational model and
then simulating the entire test at that temperature. This significantly reduces complexity due to:
1) complex time-temperature dependent thermo-mechanical material characterization of the
glass is not required and 2) there is no need to simulate the heating, cooling and unloading of the
ring. Additional requirements for a valid FCC included uniform temperature in the ring at the time
of pressing and measurement of the inner diameter at the centerline. Furthermore, either
constant load or pressing using a constant rate resulted in identical FCCs. Guidelines were
presented for both cases for a glass specimen size with outer diameter (OD) of 19.15 mm, inner
diameter (ID) of 9.59 mm and a height \((H)\) of 6.37 mm, which gives the ratios of \(OD:ID:H = 6:3:2\)
that were recommended by Male and Depierre [1].
Based on the work of Ananthasayanam et al. [9] in the current study it was sufficient to conduct isothermal, rate controlled tests with loading rates of 0.5 mm/min or less using the standard specimen at constant viscosity to produce a valid FCC for the Navier friction model. The constant rates used in the current study range from the realistic level of 0.108 mm/min to an unrealistic level of 43.4 mm/min that is used for the purpose of demonstration in computational experiments. Contrary to Ananthasayanam et al. [9], who used the finite element software, ABAQUS, the numerical simulations of the ring compression test in the current study were performed using the commercial finite element code POLYFLOW. The glass ring was modeled using purely viscous material behavior in an axisymmetric, isothermal analysis. A total of 5349 elements with quadratic velocity and linear pressure representation were used to mesh the ring.

POLYFLOW supports implementation of the Navier linear law for contact problems. The nonlinear form of the law was implemented using the PMAT user subroutine. As required by the large deformation feature of the ring compression test, the model was designed to handle contact of the inner, outer and lateral surfaces of the ring with the upper and lower mold surfaces.

A mesh convergence test was performed using five different meshes with respectively, 44%, 100%, 139%, 179% and 276% elements of the base mesh. The model was run for no-slip boundary conditions and the percentage of deformation of the inner radius was chosen as the validation criterion. The maximum errors compared to the highest mesh density were respectively 1.190%, 0.510%, 0.500% and 0.390%, which shows that convergence of the FCCs is relatively easy to obtain. The base mesh was selected for the simulations since there was very little improvement for the three denser meshes.
IV.3. RESULTS

IV.3.1. PRESSING SPEED DEPENDENCE OF THE GENERALIZED NAVIER LAW

Ananthasayanam et al. [9] showed that the FCCs obtained using the Coulomb friction law were the same for rate and load controlled tests. Since the Navier law is based on relative sliding speed, it is important to study how rate of loading affects the FCCs for a given viscosity. The computational results revealed that the calibration curves for a given nonlinear model ($e \neq 1$) can be very sensitive to pressing speed. The FCC for a specific combination of $k$, $e$ and range of pressing speeds where this sensitivity was very noticeable is presented in Figure IV-1. In this figure the calibration curves are plotted for a viscosity of $10^9$ Pa-s, exponent, $e = 1.2$, Navier friction coefficient, $k = 10^{12}$ Pa/(m/s)$^e$ and the six pressing times of 5 s, 10 s, 20 s, 200 s, 1000 s and 2000 s (to achieve an axial deformation of 3.613 mm, which is about 57% axial deformation). This shows that different FCCs can be achieved using the same generalized Navier friction law. As such, Figure IV-1 shows a case where the FCCs are sensitive to the rate of loading.
Figure IV-1. Dependence of the FCCs on the pressing time (rates range from 0.108 – 43.4 mm/min) for $\eta = 10^5$ Pa·s using a nonlinear form of the Navier law ($e = 1.2$, $k = 10^{12}$ Pa/(m/s)$^e$).

In order to show an example of the importance of the exponent, $e$, in Figure IV-2 the FCCs are presented for $e = 0.5$, 0.75, 1, 1.2 and 1.75 using the same $k$, $\eta$ and loading rates as for Figure IV-1. This result is typical of all nonlinear cases in that for each $e \neq 1$, there is a range of $k$ where the ram speed affects the FCCs. The most important conclusion, which is based on examination of behavior for the full range of viscosity and friction coefficient, is that the FCCs for all $k$ values are independent of loading rate for the linear form of the law.
IV.3.2. Viscosity dependence of the calibration curves

While the FCCs using the linear form of the Navier law are independent of rate, the FCCs depend on the viscosity, which was not the case for the Coulomb law [9]. Therefore it appears that for each viscosity new calibration curves should be generated. Generating FCCs for a specific viscosity is not practical, and compromises one of the most important advantages of the ring
compression test, which is material behavior independence. In order to understand how the calibration curves depend on the value of viscosity, the data for the FCCs must be plotted in a different way. The standard FCC corresponds to plotting the percent decrease in inner radius as a function of axial deformation for a family of friction coefficients for each value of viscosity. In Figure IV-3 the percent decrease in inner radius is instead plotted as a function of the Log of the friction coefficient, \( \log(k \times m/(Pa \cdot s)) \), for a family of viscosity values for each value of axial deformation. While these results show a dependence on viscosity, there appears to be a constant shift from one viscosity curve to another and this shift is constant from one axial deformation level to another.
Figure IV-3. Percent decrease in internal diameter of the ring as a function of $\log(k \times (m/\text{Pa} \cdot \text{s}))$ for $\log(\eta/(\text{Pa} \cdot \text{s})) = 7, 8, 9, 10$ and $11$ for axial deformation of $0.3$ and $0.5$.

This suggests plotting the percent decrease in inner radius instead as a function of $\log((k/\eta) \times m)$ which was done in Figure IV-4 for the two cases of five different viscosity values presented in Figure IV-3. Clearly this simple but important approach, which works for all levels of deformation, eliminates the viscosity dependence of the FCC. The results in Figure IV-4 can be used to characterize the friction coefficient in place of standard FCCs and for this reason the curves for the additional deformation levels of $0.1, 0.2, 0.4$ and $0.45$ are included. These results also clearly show the lower and upper limits of friction behavior that can be detected using the ring compression test, which based on Figure IV-4 are $\log((k/\eta) \times m) = -1$ and $6$, respectively, although
the range where the inner radius is most sensitive to the Navier coefficient ranges from about

$$\log((k/\eta) \times m) = 0.5$$ to 4.

Figure IV-4. The two axial deformation cases from Figure 3 plotted using the normalized form $$\log(k \times m/\eta)$$, which unifies the viscosity curves at each level of deformation. Results for axial deformation levels of 0.1, 0.2, 0.4 and 0.45 are also included.

The results in this study and, in particular, this section are based on the assumptions of constant temperature and pure viscous material behavior. In order for these assumptions to be valid, the process parameters used to conduct the actual test must follow the guidelines presented in [5], which eliminate the possible interference of viscoelastic behavior and nonlinear material behavior, especially at high viscosity. This logic can be related to the Deborah \((De)\)
number, which is the ratio of the characteristic time of the viscoelastic material to the time to conduct the test and to the Weissenberg (Wi) number, which is a product of shear rate and characteristic time. To ensure that the experimental process is simulated correctly by the computational model, both numbers must be kept low so an understanding of the effects of the key parameters in the RCT is required. The characteristic time increases as viscosity increases and the shear rate increases as friction increases, which means that for a given loading rate, the De and Wi numbers increase as viscosity and friction increase. Therefore, for the highest viscosity of interest in a rate controlled test, which is $10^3$ Pa·s, and the highest level of friction, which is no-slip, the objective is to determine a rate at which the FCC are valid using the assumptions of constant temperature, viscous material behavior and no loading or unloading. This was accomplished in [5] to be 0.5 mm/min. At higher viscosity a load controlled test following [5] can be used which will lead to the same FCCs. Because of these results, the FCCs obtained in the current study, including the viscosity shift demonstrated in Figure IV-4, apply to all the glass processing conditions established in [5]. In other words, if viscoelastic effects and thermo-mechanical loading and unloading were included in the current study, the master curves presented in Figure IV-4 would still be valid to within a very small error. Such independence of the results on the complex material behavior of glass in the temperature range of interest is one of the advantages of the ring compression test.

**IV.3.3. FCCs for both the Coulomb and linear Navier models**

FCCs were generated using both the Coulomb friction model (following [9]) as a function of the Coulomb friction coefficient, $\mu$, and the linear Navier model as a function of $\log\left(\frac{k}{\eta}\right) \times m$. Both sets of curves are presented in Figure IV-5, where it is demonstrated that the profiles are
very similar, allowing one RCT to characterize both laws. This similarity is quantified in Table VI-1 for axial deformation levels of 0.3 and 0.5 by converting from the Coulomb coefficient to the normalized Navier coefficient.
Figure IV-5. FCCs using the Navier model in terms of $\log(k \times (m/\eta))$ presented as solid lines. For comparison the FCCs for the Coulomb model in terms of $\mu$ are shown as dashed lines. See also Table IV-1.
Table IV-1. Values of the Navier friction coefficient that coincide with the FCC values for the Coulomb friction coefficient at levels of 0.30 and 0.50 axial deformation (see also Figure IV-5).

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>Decrease of inner D: 0.3</th>
<th>( \log((k/\eta) \times m) ) for 0.3</th>
<th>Decrease of inner D: 0.5</th>
<th>( \log((k/\eta) \times m) ) for 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-19.51</td>
<td>(-\infty)</td>
<td>-41.41</td>
<td>(-\infty)</td>
</tr>
<tr>
<td>0.01</td>
<td>-17.89</td>
<td>0.9477</td>
<td>-36.33</td>
<td>0.9018</td>
</tr>
<tr>
<td>0.02</td>
<td>-16.25</td>
<td>1.2636</td>
<td>-31.26</td>
<td>1.2372</td>
</tr>
<tr>
<td>0.03</td>
<td>-14.59</td>
<td>1.4577</td>
<td>-26.16</td>
<td>1.4483</td>
</tr>
<tr>
<td>0.04</td>
<td>-12.92</td>
<td>1.6107</td>
<td>-21.09</td>
<td>1.6188</td>
</tr>
<tr>
<td>0.06</td>
<td>-9.48</td>
<td>1.8594</td>
<td>-11.70</td>
<td>1.8839</td>
</tr>
<tr>
<td>0.08</td>
<td>-6.025</td>
<td>2.0661</td>
<td>-4.494</td>
<td>2.0648</td>
</tr>
<tr>
<td>0.1</td>
<td>-3.219</td>
<td>2.2211</td>
<td>1.198</td>
<td>2.2023</td>
</tr>
<tr>
<td>0.15</td>
<td>1.569</td>
<td>2.4878</td>
<td>12.07</td>
<td>2.4642</td>
</tr>
<tr>
<td>0.2</td>
<td>5.28</td>
<td>2.7098</td>
<td>20.99</td>
<td>2.6891</td>
</tr>
<tr>
<td>0.3</td>
<td>11.18</td>
<td>3.1583</td>
<td>34.41</td>
<td>3.0703</td>
</tr>
<tr>
<td>0.4</td>
<td>15.24</td>
<td>3.7084</td>
<td>43.17</td>
<td>3.422</td>
</tr>
<tr>
<td>( \infty )</td>
<td>17.47</td>
<td>( \infty )</td>
<td>50.78</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

This similarity of the FCCs was unexpected, and means that in spite of the significant difference in interface shear stresses, the two models produce a nearly equivalent global response in terms of the inner diameter of the ring. Such an equivalent response between the friction parameter pairs in Table VI-1 is also true of the axial force required to maintain a constant rate of axial deformation. The reason for these similarities is tied to the linear nature of the two laws and the near linear relationships among loading rate, interface pressure and relative sliding speed. Essentially as the loading rate is doubled, both interface pressure and sliding speed are approximately doubled, and therefore the levels of shear stress in each model are doubled. Furthermore, computational results show that while the interface shear stress distribution is different for the two models, the resultant radial force that effects sliding, and therefore the behavior of the inner diameter, is approximately the same. The similarity of the FCCs means that the ring compression test results cannot indicate which law is more appropriate (i.e., valid), and
therefore additional experimental or physically based information is required to select the model that best represents the true behavior. For example, basic tests that decouple interface pressure from sliding speed can be conducted to see if either sliding speed or pressure [9,10] have strong affects on interface friction for the given material pair and thermo-mechanical conditions. To summarize, the RCT allows a precise determination of the friction coefficient without the need to know the complex thermo-mechanical material behavior of glass, but cannot identify the correct friction model.

IV.4. DISCUSSION

The RCT has been used for over forty years to characterize the coefficient of Coulomb friction for metals used in forming processes. The complexity of conditions at the interface, which can include slip and/or partial slip as axial deformation progresses, is quantified through computational mechanics by the value of the inner radius as a function of the level of axial deformation, which is easy for the experimentalist to measure. The test is useful in a variety of applications because the inner radius is very sensitive to the level of friction between the ring specimen and the mold surfaces. The test works especially well for glass since it has been shown [5] that the FCCs are insensitive to the complex thermo-mechanical behavior of glass, meaning that one set of master curves can be used for any glass type. This universality of the FCCs for glass is attributed to the extreme sensitivity of the inner radius to friction for this particular application. While other factors affect the inner radius, the effects are of secondary importance and can be neglected when compared to those of friction.
In applying this test to characterize friction between hot glass and a mold or die surface, it is important to select the correct friction model for the application. In solid mechanics the most basic model is Coulomb friction, where the frictional shear stress while slip occurs is proportional to the contact pressure and independent of sliding speed. In fluid mechanics the most basic model at low relative sliding speeds is the linear Navier law used herein [7, 8, 16], where the shear stress is proportional to the relative sliding speed and independent of the pressure. While the two laws have this opposite behavior, for the special case of the RCT the two models produce FCCs that are nearly equivalent for the full range of axial deformation. The interface shear stress along a radial line is different for the two models, but the cumulative effects produce nearly equivalent behavior of the inner radius of the ring. While this has the advantage of allowing one test to provide both characterizations, the choice of the appropriate model depends on the conditions of the actual test, which include, for example, the mold and die surface materials and roughnesses, the loading rate and the temperature/viscosity. As such, representation of the process of interface slip by either the Coulomb or Navier model should be based on further experimental evidence. Typically characterizations of this type will be for an input to a computational study of a particular glass forming process. Experimental data for the specific process should guide the choice of the model. Finally, the glass forming problems of interest, which include PGM and extrusion of PCF, occur at the relatively high viscosities of about $10^7 - 10^9$ Pa·s, which essentially spans the transition between fluid and solid behavior, making choice of the friction model an interesting topic in itself.
IV.5. CONCLUSIONS

The ring compression test can be used to characterize the linear form of the Navier friction law for any glass type using universal FCCs that are loading rate independent. In addition, viscosity independence of the FCCs is achieved by normalizing the Navier coefficient by the viscosity. As such, contrary to the FCCs using the Coulomb friction model, the viscosity must be known to obtain the Navier friction coefficient. The computational results revealed that the FCCs for a given nonlinear model \(e \neq 1\) can be very sensitive to pressing speed, making the nonlinear form very difficult to characterize using the RCT. As an interesting result given the opposite nature of the two laws, the profiles of the FCCs using the linear Navier model are very similar to those of the Coulomb model, indicting there is an approximate one to one correspondence between \(\mu\) and \(k\times(m/\eta)\) for the ring compression test. This behavior is tied to the linear nature of the two laws and the proportional dependence of both interface pressure and relative sliding speed to the rate of loading in the RCT.

IV.6. REFERENCES


SENSITIVITY ANALYSIS AND THE EFFECT OF DIE SHAPE IN EXTRUSION OF VISCOUS GLASS WITH SLIP

V.1. INTRODUCTION

According to the die swell study in Chapter 3, friction is an important factor to consider in extrusion of glass at high viscosity ($10^7 - 10^9$ Pa·s). In addition a viscous material model proved to be very successful, however, it was concluded that factors such as shear thinning and viscoelasticity could possibly play minor roles. In this chapter the viscous analysis from Chapter 3 was extended by performing a sensitivity analysis on the major extrusion parameters. The results were compared to experimental data [1-4] to see if experimentally observed phenomena can be reproduced or if there is a motivation to add complexity to the model. The key features from the experimental literature are: 1) die swell is affected by ram speed [1] and 2) ram force varies linearly with both ram speed and the log of the viscosity[2-4]. As such a goal of this study is to

**Figure V-1. Dies ‘geometries.**
determine the validity and the limits of the basic assumption, which is very common in the literature of low viscosity extrusion, that glass can be modeled as a viscous material even for high viscosity extrusion. To insure that the results are independent of the die geometry, two different dies were considered. These dies are referred to as Die A and Die B and are presented in Figure V-1.

V.2. Analysis parameter choice

This study is based on extrusions performed on F2, HOYA, Bismuth, and Tellurite similar to the study conducted by Ebendorff-Heidepriem et al [4]. The operational viscosity for these glasses ranges from $10^{7.6}$ to $10^{9}$ Pa·s, while the ram speed ranges from 0.5 to 0.05 mm/min, which corresponds to a flow rate of 6.1 to 0.4 mm$^3$/s. The ranges of viscosity and flow rate are large since the combination of flow rate and viscosity must be matched to a specific glass. This is required since a flow rate and viscosity suitable to one glass type can lead to a failure for another glass type. As discussed in Chapter 3, the Navier law is used to model the friction in this study since the shear rate is limited and this law is widely used in the extrusion literature [5]. The viscosity window used in the simulations is stretched outside of the experimental domain to further confirm a number of observations to be discussed in the result section. Following this reasoning, a viscosity range of $10^7$ to $10^{12}$ Pa·s is selected and the full range of friction is used from no friction to no-slip.

V.3. Results: Die-swell profile

The die-swell generated by both dies for different viscosity as a function of friction is presented in Figure V-2. The results in this figure, which summarize results from a large number of simulations,
show several interesting behaviors. First, the results from two very different dies at two different ram speeds are basically the same for each of the three viscosity values considered. This is because the ram speeds are low. As a second observation, the profile of the curves for each viscosity value is the same, and is shifted by the same amount for the same change in viscosity. This is the identical behavior seen in Figure IV-4 from Chapter 4. Clearly the die-swell is affected appreciably by the level of friction, which transitions from a no-friction lower limit to a no-slip upper limit within a relatively narrow band of \( \log(k) \).

![Graph](image)

*Figure V-2. Steady state die-swell as a function of friction parameter for two die shapes and three values of viscosity.*
As a third observation, the die-swell friction profiles show three major phases regardless of the viscosity, the die shape and the ram speed. For conditions of low friction (for example, \( \log(k \times (m/\text{Pa} \cdot \text{s})) < 8 \) for viscosity of \( 10^7 \text{ Pa} \cdot \text{s} \)) the preform radius keeps the nominal value of the die’s exit and does not swell. The die-swell increases smoothly within a range of friction (8 < \( \log(k \times (m/\text{Pa} \cdot \text{s})) < 14 \) for viscosity of \( 10^7 \text{ Pa} \cdot \text{s} \)). Finally, the die-swell reaches a maximum at a given friction level (\( \log(k \times (m/\text{Pa} \cdot \text{s})) \approx 14 \) for viscosity of \( 10^7 \text{ Pa} \cdot \text{s} \)) that corresponds to a no-slip boundary condition. It is clear from this plot that the die-swell is affected by the level of slip, and that the die-swell data in Chapter 3 can be matched using Figure V-2. The die-swell plots show an evolution similar to the ring compression test FCC such as:

- The three friction phases
- The monotonic evolution of the deformation
- The shift of the plots toward higher values of \( \log(k \times (m/\text{Pa} \cdot \text{s})) \) as the viscosity increases

This last feature it practically interesting since like the ring compression test FCC the friction coefficient of the die-swell plots in Figure V-2 can be also normalized the same way to obtain yet one single master plot as in Figure IV-5 from Chapter 4.

V.4. RESULTS: PRESSURE VARIATION PROFILE

The ram force and the pressure are included among the outcomes examined in this study. Plots of normalized ram force as a function of friction for both dies at two rates and three levels of viscosity are presented in Figure V-3. The variation of the maximum normalized pressure value along the centerline measured for both dies is presented in Figure V-4. Both the ram force
and the pressure in these plots are normalized by the viscosity and the ram speed. As seen in the figures, once normalized in this manner, the maximum pressure and the ram force profiles scale to the same order of magnitude and those with the same viscosity are nearly identical and overlap in the figure. This shows a linear equivalence between the plotted outcomes and the normalizing factors. Such linear dependency was experimentally observed [2-4] and in several studies was interpreted using a Poiseuille flow assumption [2, 4], which explains the popularity of this model to interpret extrusion data.

![Diagram of normalized ram force as a function of friction parameter for two die shapes and three values of viscosity.](image)

**Figure V-3. Steady state normalized ram force as a function of friction parameter for two die shapes and three values of viscosity.**

However, unlike die-swell, the maximum pressure responds more to the change of the die shape. Over a range of low friction values, the lowest maximum pressure value is recorded for Die A, whereas for high friction values it is recorded on Die B. Such high friction values, however,
are unlikely since as discussed earlier the maximum experimental preform radius from Chapter 3 measured was 5.5mm (which corresponds to a case of friction far from the no-slip limit and on the order of $\log((k/\eta) \times m) = 3.5$). Similar trends were also observed for the applied ram force.

![Graph](image)

**Figure V-4. Steady state maximum value of the pressure along the centerline as a function of friction parameter for two die shapes and three values of viscosity.**

It appears also that the similarities between the FCCs and the die swell plots apply for the observed variables in this section and a normalization of the friction coefficient by the viscosity makes the plots of each die overlap for both observed variable here.
Figure V-5. Pressure at the centerline for Die A and Die B from the inlet to the exit, viscosity $10^7$ Pa·s, ram speed 0.5 mm/min.
The position of maximum pressure was also monitored in this study. A comparison between the pressure distribution along the centerline for the low friction and no-slip cases in Figure V-5 shows that for Die A the maximum pressure position is stable with respect to the level of friction compared to that of Die B. It could be also observed that Die A has a lower zone of high pressure and a smoother pressure gradient.

V.5. RESULTS: VELOCITY VARIATION PROFILE

![Graph showing velocity variation profile](image)

Figure V-6. Steady state value of the maximum velocity along the centerline as a function of friction parameter for two die shapes and three values of viscosity.

The maximum velocity along the centerline and the velocity along the centerline at the die exit were also considered in this study and the results are presented in Figures V-6 and V-7.
The former helps to illustrate the variation of the velocity profile along the center line (Figure V-6), while the latter measures the correlation between the die-swell and the velocity at the exit (Figure V-7). The velocity values have been normalized by the flow mean value at the inlet. An observation of these profiles reveals that the viscosity and the rate of inflow have an extremely limited impact on the velocity profiles. And though both velocity profiles describe the same trends of the die-swell, the die geometry still has an impact on the results at low values of friction.

![Diagram](image)

**Figure V-7.** Steady state value of the velocity on the centerline at the die exit as a function of friction parameter for two die shapes and three values of viscosity.

Like the FCC a normalization of the friction by the viscosity is possible in this case as well for both dies.
Figure V-8. Velocity profile at the centerline for Die A and Die B, viscosity 10^7 Pa·s, ram speed 0.5 mm/min.
The velocity profile at the centerline presented in Figure V-8 reveals that the effect of the pressure drop at the die exit propagates deep inside the channel. Thus, unlike the results of Poiseuille flow, due to the limited length of the channel a zone of constant velocity profile cannot be observed inside the channel. Even though the flow in both dies experiences similar behavior, the profiles of the variations are quite different. Die A has its maximum velocity farther from the die exit and has a lower velocity gradient throughout the die length.

V.6. Conclusion

Computational studies of glass extrusion at high viscosity has had limited attention in the literature [1-2]. A sensitivity analysis has been carried out using two different die shapes. The influence of ram speed, viscosity and friction coefficient were explored for these two extrusion configurations. The impact of the variation of the extrusion input parameters on the die swell was first studied, and then carried out for other extrusion output parameters. The material behavior was chosen as viscous to test the limits of such a model on the extrusion outcomes by comparing behavior to experimental observations in the literature. The presence of two different dies in the study was used as an additional factor to test if the results are dependent on the die geometry or if the level of the viscosity has an implication on the results. The geometry of the die has been shown to have an impact on the ram force as well as the velocity and pressure at the centerline. However the die swell was not affected by the geometry. The numerical analysis confirmed several experimental observations such as the linear dependency of the ram force on the flow rate and the viscosity [2,4]. However, the die swell was insensitive to the ram speed for both dies, which is a consequence of the viscous flow assumption.
The study of the extrusion parameters revealed several interesting aspects and relations. First of all, the ram force and the maximum axial pressure showed a linear dependence on the viscosity and flow rate. In addition, as expected the choice of die geometry was shown to have a non-negligible effect on the ram force. However, these factors have no noticeable effect on the die swell. The friction coefficient, however, has a significant effect on the die-swell. In fact the friction coefficient appears to be the most influential parameter in affecting distortion of the cross section in extrusion. Finally, a normalization of the friction coefficient by the viscosity reproduces the same behavior observed for FCCs in Figures IV-4 and IV-5.

V.7. REFERENCES


IMPACT OF VISCOELASTICITY ON MODELING GLASS

EXTRUSION

According to the results of Chapters 3 and 5, modeling extrusion of highly viscous materials using the Navier friction law and a viscous glass material model showed excellent success to predict several experimental observations. However, in the temperature range of interest for high viscosity extrusion glass can behave as a viscoelastic material, contrary to higher temperatures where a viscous model is justified. Therefore, it is possible that if the viscoelastic material behavior were known, comparison with experimental results would improve. Therefore in this chapter a more detailed study of the effects of viscoelasticity using the upper-convected Maxwell model will be made. Compared to the results in Chapter 5, now the relaxation time, \( \lambda = \eta G \), which was introduced in Chapter 3 is a new parameter.

VI.1. Results: Die-swell

As a first step, the die swell results from Figure V-2 for the viscous case, which corresponds to \( \lambda \to 0 \), are presented in Figure VI-1 for four non-zero values of the relaxation time. The resulting plots reveal several similarities to the viscous results, which include:

- The presence of three major phases regardless of the viscosity and the ram speed,
- The decoupled effects of the viscosity and of the flow rate on die-swell,
- The increase of viscosity only introduces a shift of the profile and does not change the shape the curves,
- The change of the ram speed introduces the same change to the die-swell profiles regardless of the viscosity,
- The same similarity to the FCC, notably the effect of normalizing the friction by the viscosity, holds for this case as well.

Figure VI-1. Die-swell versus friction.
However, in this viscoelastic case the die-swell shows a dependency on the ram speed for high values of the relaxation time and ram speed. Furthermore, the results in Figure VI-1 show that die swell first decreases and then increases as the relaxation time increases from zero. The effect of viscoelasticity is therefore difficult to predict based on the way the data is presented in Figure VI-1. In order to investigate the rate dependent effect in more detail, a high value of $\lambda = 20$ will be used. First, the die-swell is presented in Figure VI-2, where the tangent of the die-swell profile in the middle of the second phase is drawn. It can easily be seen that a large portion of this curve follows this tangent line which is called the near-linear zone. Therefore, knowing the die swell for two points of this near-linear region could be used to monitor the evolution of a large segment of the die-swell curves.
Figure VI-2. Die-swell versus friction: Choice of inspection friction coefficient.

The die-swell was then plotted as a function of the ram speed in Figure VI-3. Both limiting viscosities of \( \log(k \times (m/Pa \cdot s)) = 7 \) and 9 were presented in this plot and a total of four friction values were used to show the effect of friction. These values were chosen as follows: 1) in the middle of phase two of each profile, 2) at the value where the profile starts to depart from the tangent, 3) an extra friction value was selected between the two previous points and 4) at the no-slip limit.
Figure VI-3. Die-swell versus ram speed. Profiles for set of friction coefficients.

Since the effect of the normalization of the friction coefficient by the viscosity holds in this model, the normalized values of the friction coefficient that were chosen for inspection are the same for all values of the viscosity. An inspection of the results in Figure V-3 reveals that regardless of the viscosity and friction coefficient, all profiles show a small zone of very low dependency of the die-swell on the ram speed. Thus a minimum flow rate is required for the die-swell dependence on viscoelastic properties to be visible even in the high friction regime. The trend of increasing die-swell vis-à-vis the flow rate is confirmed as a general rule. It is interesting to note that the gap between the curves in Figure VI-3 increases as the flow rate increases but a
clear linear relationship to the flow rate could not be asserted. It is also noted that the die-swell in Figure VI-3 expand as the ram speed increases.

![Graph showing die-swell versus relaxation time](image)

**Figure VI-4. Die-swell versus relaxation time \(\lambda [s]\).**

A similar plot of the die-swell as a function of the relaxation time was performed, which is presented in Figure VI-4. Though most trends of the plot were predictable, it was possible to observe the fact that the viscoelastic material component could either increase or decrease the die-swell depending on the choices of other extrusion parameters.
It is important to mention that studies have reported a sensitivity of the die-swell to the ram speed [1]. This observation was only reproducible using an assumption of viscoelastic material behavior.

VI.2. RESULTS: PRESSURE VARIATION PROFILE

The ram force and the maximum pressure along the centerline were plotted against the friction coefficient as previously performed for the die-swell in Figures V-4 and V-3. The recorded results mimicked those previously obtained for the viscous case and are therefore not presented. The results showed an extremely limited dependency on the relaxation time and thus comply with the experimental observations [1-4]. A study of variations of these entities against the ram speed and the relaxation time further confirmed this observation.
Figure VI-5. Normalized maximum axial pressure versus the relaxation time $\lambda$[s]. Profiles for set of friction coefficients.

To illustrate this low dependency on relaxation time, the normalized maximum pressure is plotted against the relaxation time in Figure VI-5 the same way it was presented previously for the die swell in Figure VI-4. It is clear from this Figure VI-5 that the normalized pressure dependence on relaxation time is negligible, and furthermore the influence of the viscosity is very weak. In fact, the maximum normalized pressure behavior here is very similar to the behavior recorded for the viscous case; and when the maximum pressure is normalized by the viscosity and the ram speed and plotted as a function of the log of the friction, all the curves of the same viscosity overlap. Moreover, like the viscous case the normalized ram force in Figure VI-6 follows exactly the same trends of the normalized maximum pressure.
Figure VI-6. Normalized maximum ram force versus the relaxation time λ[s].

In fact, experimental studies suggest a linear dependency between the ram speed and the ram force [2-4]. Theoretically, a viscous approximation of the material mechanical behavior is a good approximation to estimate the ram force. To further illustrate this point a plot of the normalized maximum pressure versus the flow rate is presented in Figure VI-7.
VI.3. RESULTS: VELOCITY VARIATION PROFILE

The velocity profile variations (velocity at the center of the die exit and the maximum velocity at the centerline) as a function of the coefficient of friction share the same common aspect that the die-swell and pressure profiles share: both profiles are composed of three major regimes with a near-linear regime in the middle. In addition, the velocity profiles are sensitive to the relaxation time just as the die-swell is sensitive. To further study the impact of viscoelasticity on these profiles, plots of the variation of the studied velocities as a function of the relaxation time were carried out in Figures VI-8 and VI-9.
As expected, at the lowest ram speed in Figures VI-8 and VI-9 show that both velocity quantities show a negligible impact of the variation of the relaxation time as was the case for the maximum pressure, the ram force and the die swell. For the highest ram speed the trends of both velocity profiles are quite different from that of the die-swell. In this case, regardless of the friction coefficient associated with each curve, all the curves show a presence of a small low variation zone near the smallest value of the relaxation time and comparable monotonic increase; thus, the gap between the curves for both velocities does not display a significant change.
These trends translate to a limited effect of the relaxation time on the shape of the velocity profile for both measured velocity quantities:

- The slope of the near-linear zone at the high friction regime shows a weak increase with relaxation time especially for the maximum axial velocity.
- The limiting velocity at the vicinity of the no-slip region also follows the same behavior especially for values of the relaxation time below 10s.
- These trends, however, are more perceivable on the maximum axial velocity.
Strangely, the trends of the velocity curve versus the relaxation time, especially the axial velocity at the die exit, are not similar to the die-swell trends. It appears that for low relaxation time and high friction (Figure VI-8), the velocity at the die exit was increasing while the die swell was decreasing. Moreover, for high relaxation times the die-swell increases sharply (Figure VI-4) while the velocity at the die exit slowly increases with the relaxation time (Figure VI-8). This suggests that the viscoelastic behavior not only affects the flow pattern but has a significant effect on the stress state at the die exit. A possible explanation for this is that the presence of viscoelastic behavior retards the expansion of the glass at the die exit, however when the relaxation time is high the material stores the stress acquired from the die funnel, which
enhances the amount of die swell. In fact, Figures VI-10 and VI-11 show a near-linear dependency of the velocities on the flow rate while the die-swell profiles show a totally different behavior.

![Graph showing normalized maximum axial velocity versus flow rate.](image)

**Figure VI-11.** Normalized maximum axial velocity versus flow rate.

### VI.4. CONCLUSION

The die used for this study is designed at the University of Adelaide to extrude preforms for use in drawing fiber optic. Using a sensitivity analysis for most of the extrusion parameters, the viscoelastic material model was able to match all the experimental observations previously validated using a viscous glass model. In addition, the use of a viscoelastic material made it possible to observe a dependency of the die-swell on the flow rate for a range of viscoelastic time...
constants which agrees with the experimental observations [1]. This study allowed several correlations and dependencies to be observed between several extrusion parameters..

In this study a single element of the Maxwell upper convected model was used to model the glass material behavior. This choice was motivated by the simplicity of the model, limited amount of shear rate and its compatibility with the glass molding and extrusion literature. Moreover, the choice of including just a single Maxwell element simplified the problem but also isolated the effect of the relaxation time, and that is why a large value of the relaxation time was used in this study. The study highlights the major experimental observation such as:

- The low dependence of the ram force on the relaxation time
- The linear dependence of the ram force on the flow rate
- The sensitivity of the die-swell to the ram-speed

These observations were shown to be valid for a large range of friction coefficients and are likely to be noted on most experimental setups.

The study also showed the impact of viscoelastic behavior, friction and ram speed on several aspects of the extrusion process such as die swell, pressure, ram force and velocity.

VI.5. References


EXTRUSION OF GLASS PCF FIBER PREFORM:

COMPUTATIONAL MODEL AND VALIDATION

VII.1. INTRODUCTION

In Chapters 3, 5 and 6 the extrusion of a solid rod of glass was modeled and the impacts of material and processing parameter inputs were tested in a sensitivity analysis to understand their impacts on the extrusion process. This study was conducted in part to provide the guidance for modeling choices to address PCF preform extrusion. According to the die swell results reported in Chapter 3, modeling slip with friction at the wall of the die is of primary importance and must be accounted for in the simulations. From the point of view of material behavior, while modeling viscoelasticity enhances the precision of the model, purely viscous material behavior was shown to be an accurate first assumption that conserves computational cost. The reduction in computational cost due to neglecting viscoelastic effects is of course offset by the added geometric complexity of PCF extrusion. In addition to adding a series of holes with diameters that are much smaller than the diameter of the channel, an axisymmetric model can no longer be used since the die is fitted with an extra component placed at the entrance of the channel that breaks this symmetry and adds two important compartments to the glass flow (Figure VII-1):

- The Feeding holes which control the flow of glass in the subsequent chamber.
- The welding chamber which consist of an array of pins that are arranged to create a hexagonal pattern of holes in the cross-section.

The resulting pattern of holes that results in the preform is almost the negative of the pins. An understanding of the flow mechanisms that lead to slight changes in the cross section is required. Die swell is the primary mechanism causing this slight distortion.

![Image of welding chamber with feeding holes and blockage]

**Figure VII-1. Schematics of die/ Blockage used to extrude PCF preform.**

**VII.2. Numerical experiment with structural relaxation**

In lens molding structural relaxation (temperature history dependent thermal expansion) was shown to be the dominant mechanism for shape change [1]. In order to understand the role of structural relaxation in extrusion, a numerical experiment was performed in which a stress free preform was cooled from the extrusion temperature to room temperature. The objective was to determine the order of magnitude of the distortion due to thermal effects relative to the experimentally observed cross-sectional distortion, which is presented on the left in Figure VII-2. The computational results to actual scale are presented in the center image of Figure VII-2, while the image on the right exaggerated the displacement field by a factor of 500,000 in order to see a
significant effect. Thus the results show that structural relaxation cannot account for the large shape change observed in such fibers. Therefore it was assumed in this study that extrusion can be conducted at the uniform extrusion temperature without the need for heating and cooling.

Figure VII-2. Numerical Experiment with Structural Relaxation.

VII.3. NUMERICAL MODEL OF PCF PREFORM EXTRUSION

VII.3.1. PRESENTATION OF THE NUMERICAL MODEL: COMPUTATIONAL DOMAIN

To make a PCF preform the feeding holes and the pins on the Blockage are organized in a concentric hexagonal structure. Each group of feeding holes belonging to the same hexagon is called a feeding hole ring. Similarly, each pin group arranged in the same hexagon is simply called a ring. The pins and the feeding holes can be organized in two different ways. Both configurations are presented in Figure VII-3. The basic die/Blockage geometry in these configurations was obtained from [2-3]; Only Blockade B Geometry blueprint has been referenced [2-3]. The rest of the geometry parameters were either deduced based on geometric relations or hypothesized
(see Appendix 1 for more details). In this study these are the only two configurations to be considered.

**Figure VII-3. Extrusion of PCFs: Blockage cross-section.**

PCF preforms made of three rings are the simplest preform structure that captures the major aspects of PCF extrusion since it is the simplest preform structure to have holes outside the planes of symmetry.
Figure VII-4. Determining the minimum length, $L$, of the model relative to the radius, $R$.

By exploiting the symmetry of the structure, a 30° portion of the model is isolated to model the entire extrusion process. However the remaining structure is still fairly complicated due to the presence of either 17 or 13 feeding holes, 6 free surfaces and the intersection of free surfaces with the symmetry planes.
The computational model had to be further simplified to further reduce the numerical complexity. Minimum values of the length, $L$, of the computational domain above the Blockage and the length of the preform were determined to reduce computational cost yet still achieve a sufficient level of convergence. Results from the computational study to determine $L$ are presented in Figure VII-4. The study was performed for a very high value of friction which is the most difficult to converge. The Cases A-E correspond to five choices of $L/R$, where $R$ is the radius of the die exit. Superimposed in each of these images are results at the end of the preform for the current value of $L/R$ and a very long reference length for which convergence was achieved. These results show that A and B are not good enough for such a high coefficient of friction. The rest of the cases give acceptable results with a difference of less than 0.05mm. Knowing that the deformation rises rapidly with friction, the choice of C looks acceptable for this analysis. However at the scale of the holes it is advisable to have better precision. Though cases D and E appear to have enough precision at the local scale, a length around the double of case C was used.

![Mesh refinement locations](image)

Figure VII-5. Modeling domain and mesh refinement.
Figure VII-6. The inner radius of the first ring of feeding holes.

To improve the accuracy of the model a refinement of the mesh is required in several locations. These locations are indicated in Figure VII-5. In addition convergence problems could rise from various meshing and geometric properties. For example, as shown in Figure VII-6, for a given pattern of feeding holes, convergence becomes more difficult as the value of the radius of the pins becomes close to the value of the inner radius of the first ring of feeding holes. This detail is obvious to see if a pin placed at the center of the Blockage. A large pin radius requires the presence of a number of small elements at the boundary of the pins to monitor the boundary layer which makes the convergence of the simulation difficult.

The size of the upper region, which is defined as the portion above the Blockage as shown on the left side of Figure VII-5, is important to include in the model with a sufficient length to study the effect of friction at the die wall and to allow a smooth reorganization of the flow.
VII.3.2. Presentation of the Numerical Model: Constitutive Equations

Due primarily to the high deformation involved in PCF extrusion and the high viscosity of the glass, the Eulerian interpretation is used to simulate the extrusion process. The governing equations for incompressible flow are given by mass and momentum conservation as follows:

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad \text{(VII-1)}
\]

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla T + \mathbf{f} \quad \text{(VII-2)}
\]

where \( \mathbf{u} \) is the velocity, \( \rho \) is the density, \( \mathbf{f} \) is the body force and \( T \) is the stress tensor. When an axisymmetric steady state extrusion of an incompressible Newtonian fluid is considered and when inertia and gravity forces are neglected, the conveting equations are reduced to:

\[
\nabla \cdot \mathbf{u} = 0 \quad \text{(5-3)}
\]

\[
\nabla T - \nabla p = 0 \quad \text{(5-4)}
\]

As shown any regular extrusion problem, the boundary conditions must be defined on four areas: Inflow, Outflow, Free surface and Die glass interface.

\[
\text{Inflow} \quad \begin{cases} 
Q = 2\pi \int_0^R u_r r dr \\
 u_r = 0
\end{cases}
\]

\[
\text{Outflow} \quad \begin{cases} 
T \cdot e_r = 0 \\
T \cdot e_z = 0
\end{cases}
\]

\[
\text{Free surface} \quad \begin{cases} 
T \cdot n = 0
\end{cases}
\]

\[
\text{Axis of symmetry} \ (T \cdot e_z = 0)
\]

132
\[ \text{Inlet: } T_{\text{wall}} = -k \ u_{\text{wall}} \]

Plan of symmetry \( u \cdot e_n = 0 \)

At the inflow a flow rate \( Q \) is applied, the velocity profile is automatically calculated by POLYFLOW based on the friction at the wall.

VII.3.3. Presentation of the numerical model: mesh

The mesh was based entirely on sweeping meshed surfaces from top to bottom for each compartment of the die. Thus the mesh is completely free of tetrahedral elements and dominated by brick elements. Referring to Figure VII-5, due to the complexity of the geometry several levels of refinement are required at regions of high gradient of velocity such as:

- The entrance of the feeding holes,
- The exit of the feeding holes,
- The die exit.

Moreover, referring to the right hand side of Figure VII-5, additional refinements are also required in the following regions where boundary layers develop such as:

- Around the feeding hole boundaries,
- Around the pins and the hole boundaries.

To insure an equilibrated partition of elements all over the model, extra partitions were added along local symmetry planes.
Extra partitions along the local symmetry planes were added to ensure a nearly uniform distribution of elements throughout the cross section (right side of Figure VII-5). The number of elements is high and requires significant computational power to conduct sensitivity analysis.

VII.4. MESH CONVERGENCE STUDY

Unlike the regular glass extrusion, the flow in the PCF extrusion process is divided into several compartments as shown in Figure VII-5. As far as flow is concerned, the flow can be divided into two major steps. The first step is composed by the upper chamber and the feeding holes. The primary focus of this step is to distribute the glass flow among the feeding holes. The subsequent compartments (the welding chamber and the extruded preform) present the zone where flow coming from the feeding holes merges together to form the final preform. Thus, the flow in this compartment depends on convergence of the upstream flow. For that reason, it is advised to monitor the mesh convergence on these parts separately. Thus, the study of the mesh convergence is divided into 2 steps. In the first step only the mesh density at the upper chamber and the feeding holes is studied. To assist the quality of the mesh, the flow rate at the feeding holes is monitored in this step. Based on this study, the subsequent step studies the mesh density of the remaining region to ensure the convergence of the final geometric outcome, i.e., the final shape of the preform. Both Blockage geometries have been studied for mesh convergence; however only the Blockage A results are discussed in the subsequent paragraph.

VII.4.1. SECTION 1 (UPPER REGION: FLOW CRITERIA)

In this section the upper region shown in Figure VII-5 (upper chamber and the feeding holes) of the model is optimized for mesh convergence. The flow distribution among the feeding
holes is monitored and recorded for each mesh configuration. The aim of this study is to ensure that the flow distribution is mesh independent. The meshes treated in this study are presented in Table VII-1. As shown in the table, it was decided to use meshes of high density since the subsequent mesh study depends on the current study.

Table VII-1. Mesh density configuration (only the upper chamber and feeding holes are counted).

<table>
<thead>
<tr>
<th>Mesh</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements number</td>
<td>37108</td>
<td>75784</td>
<td>129597</td>
</tr>
<tr>
<td>Percentage of Element Increase</td>
<td>100.0%</td>
<td>204.2%</td>
<td>349.2%</td>
</tr>
</tbody>
</table>

For the first mesh convergence criteria, the flow at each feeding hole is calculated for each mesh. Since M3 is by far the densest mesh, it is used as a reference for this study. Thus, the flow rate in each feeding hole of M1 and M2 is compared to that of M3. The maximum and the minimum percentage differences are presented in Table VII-2.

Table VII-2. Difference from M3 in the feeding holes.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate difference: Max value</td>
<td>0.00%</td>
<td>0.00%</td>
<td>/</td>
</tr>
<tr>
<td>Flow rate difference: Min value</td>
<td>0.00%</td>
<td>0.00%</td>
<td>/</td>
</tr>
</tbody>
</table>

The table shows that the flow rate distribution in the feeding holes is completely independent of the mesh configuration event for the lowest mesh density.
The maximum velocity is chosen as a secondary mesh convergence criterion in this section. This criterion is considered as a tool to monitor the flow velocity profile in the feeding holes. In Table VII-3 only the maximum and the minimum percentage differences as well as the mean value of the percentage differences are presented.

Table VII-3. Maximum velocity difference from M3 in the feeding holes

<table>
<thead>
<tr>
<th>Mesh</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max velocity difference: Max value</td>
<td>2.47%</td>
<td>2.44%</td>
<td>/</td>
</tr>
<tr>
<td>Max velocity difference: Min value</td>
<td>0.00%</td>
<td>0.00%</td>
<td>/</td>
</tr>
<tr>
<td>Max velocity difference: Mean value</td>
<td>-0.44%</td>
<td>-0.31%</td>
<td>/</td>
</tr>
</tbody>
</table>

The results in Table VII-3 show that the mean values for M1 and M2 are both under 0.5%, while the maximum values are less than 2.5%. Given the number of elements in the three meshes (Table VII-1), this indicates sufficient convergence and thus M1 was chosen to as a converged mesh for this study.

VII.4.2. SECTION2 (LOWER REGION FORM BENCHMARK)

This section is dedicated to study mesh convergence for the lower region (welding chamber and the preform region) of the model as shown in Figure VII-5. Five meshes of different density are utilized for this task. As shown in Table VII-4, a large number of elements are used even for the lowest mesh configuration.
Table VII-4. Mesh density configuration (only the welding chamber preform are counted).

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes number</td>
<td>88357</td>
<td>114188</td>
<td>155253</td>
<td>210768</td>
<td>234378</td>
</tr>
<tr>
<td>Elements number</td>
<td>83568</td>
<td>107970</td>
<td>147704</td>
<td>201972</td>
<td>224404</td>
</tr>
<tr>
<td>Percentage of Node Increase</td>
<td>100%</td>
<td>129%</td>
<td>176%</td>
<td>239%</td>
<td>265%</td>
</tr>
<tr>
<td>Percentage of Element Increase</td>
<td>100%</td>
<td>129%</td>
<td>177%</td>
<td>242%</td>
<td>269%</td>
</tr>
</tbody>
</table>

The coordinates of the location of the centroid of the holes and the area of the holes in the final preform are selected as convergence criteria. The centroid or gravity center can be calculated either based on the contour or the area of the hole (Further details about the calculation of the coordinates of the centroid is provided in Chapter 8). The Results from both methods are used in this this study.

Table VII-5. Mesh comparison for the lower part of the model.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Difference from M5 (absolute error)</th>
<th>Difference from subsequent refinement (relative error)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M2</td>
</tr>
<tr>
<td>Mean</td>
<td>0.16%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Min</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Max</td>
<td>0.29%</td>
<td>0.14%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.15%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Min</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Max</td>
<td>0.29%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.15%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Min</td>
<td>0.08%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Max</td>
<td>0.29%</td>
<td>0.17%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.11%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Min</td>
<td>0.02%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Max</td>
<td>0.29%</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

Surface of the holes

137
<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>2.72%</th>
<th>1.63%</th>
<th>0.90%</th>
<th>0.32%</th>
<th>1.11%</th>
<th>0.73%</th>
<th>0.59%</th>
<th>0.32%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>2.26%</td>
<td>1.48%</td>
<td>0.83%</td>
<td>0.18%</td>
<td>0.72%</td>
<td>0.66%</td>
<td>0.52%</td>
<td>0.18%</td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>3.55%</td>
<td>2.03%</td>
<td>1.11%</td>
<td>0.49%</td>
<td>1.55%</td>
<td>0.94%</td>
<td>0.71%</td>
<td>0.49%</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table VII-5 two quality measures are used. The first measure compares each mesh to the densest mesh for each of the five holes. Then the average of the maximum and minimum values are collected in the left-hand side of the table for each variable. The second measure is a comparison between each mesh configuration and the subsequent mesh that is one level denser.

Both methods used to calculate the center of gravity give very similar results and for this mesh criterion all the meshes gives an error less than 0.3%. This means that this criterion is satisfied for all the meshes. However, the areas of the holes seem to be more affected by the mesh density. For M1, the minimum and mean absolute errors are above 2% which are acceptable; however the maximum absolute error is high. Moreover the relative error is above the mean and the maximum value is above 1% for a mesh increase of 29%. The absolute errors for the M2 mesh are around 2% and its relative errors are less than 1% for a mesh increase of 36%, which looks a cost effective choice compared to M1 and the other meshes. For these reasons M2 is chosen as the mesh reference for this model.

VII.5. Overview of Experimental and Numerical Results

In this final section of this chapter, the computational results using the converged model will be compared visually to the cross-section of a microstructured optical fiber (MOF) from Ebendorff-Heidepriem and Monro [2-3]. Using both Blockage geometries in Figure VII-3, computational simulations were performed to match the final preform shape. This was
accomplished by adjusting the pin diameter and the friction coefficient. The results shown in Figure VII-7 therefore give only a qualitative validation of the model. Knowing that the simulation has been carried out with two different die and Blockage geometries, the results in Figure VII-7, especially the result from Blockage B, clearly predict the correct trend of the deformation pattern, unlike the results in Figure VII-2 that was generated by the structural relaxation. This validates and confirms that die swell is the primarily cause of the distortion of the holes. These results show that regardless of the geometric details the die swell deformation in a PCF preform seems to follow a general deformation path that can be modified by adjusting the extrusion parameters.
\[
\log\left(\frac{k}{\eta} \times m\right) = 7, \text{ (Closest match)}
\]
Blockage A with adjusted pin diameter

\[
\log\left(\frac{k}{\eta} \times m\right) = 4.63, \text{ (Closest match)}
\]
Blockage B

Figure VII-7. Comparison between the experimental result and the numerical results.
Obviously, there are several other parameters in the Blockage geometry that may be adjusted to have a perfect match. However, these results show that it is hard to obtain a perfect match of the experimental deformation figure without an accurate data about the Blockage geometry. Clearly, for both cases, achieving holes’ deformations close to the experimental data was made at the expense of matching the die swell and the holes’ position. In addition, in the case of Blockage A matching the experimental data required using a high friction coefficient (close to no-slip) which is not in agreement with the experimental data of the extrusion of a solid rod in chapter 3.

To illustrate this point, the size of the welding chamber of Blockage B was adjusted based on the geometry of the Blockage used in the extrusion of the experimental case. Though not all the geometric details were available, a perfect match was obtained by adjusting the coefficient of friction to value of \( \log((k/\eta)\times m) = 4.35 \), which is similar to the friction obtained in the solid rod extrusion. As shown in Figure VII-8 both holes’ shape and position are matched with a high level of accuracy. The exact value of the die diameter was not available, thus the die swell was not matched to the accuracy level of the holes.

A no-slip case was also presented Figure VII-8. In this case the positions of holes remain in good agreement with the experiment; however the deformation of the holes are severe enough to discard the possibility of no-slip boundary condition.
\[ \log \left( \frac{k}{\eta} \times m \right) = 4.35, \text{(Closest match)} \]

Blockage B with actual welding size

\[ \log \left( \frac{k}{\eta} \times m \right) > 8, \text{(Noslip)} \]

Blockage B with actual welding size

*Extraction of complex preforms for microstructured optical fibers. Helke Enderle-Heddegem and Tanya M. Monc.*

Figure VII-8. Comparison between the experimental result and the numerical results.
Next, more detail as to how the computational results in Figure VII-7 were obtained which will help to introduce the sensitivity analysis that follows in Chapter 8. As stated earlier, both pin size and friction had to be adjusted. In Figures VII-9 and VII-10, respectively for Blockages A and B, results for three different friction coefficients are presented for the selected values of pin size. Similar to the results in Chapter 3, this shows the important role of interface slip in all die swell related phenomena such as hole distortion. This also shows that the results in Figure VII-10 for Blockage B a wider range of friction coefficients could be used to approximate the experimental hole deformation pattern in Figure VII-7.
Figure VII-9. Modification of the preform geometry for the validation die geometry as the friction increases (Blockage A).
Concerning the choice of pin size, the hole distortion for Blockage A was very sensitive to the diameter of the pin size, which required a high level of adjustment. The case for Blockage pattern B was much easier to adjust. Thus the results presented in Figure VII-7 and 8 are only used to validate the numerical model. The geometry used in the sensitivity analysis portion of this study has the same pin size as B.
The geometric detail used for the final validation cases are confidential and cannot be used for further investigation.

In summary, comparing the starting geometric configuration and the final configuration shows the importance of a mathematical approach to quantify hole distortion. In the case, a solid rod extrusion the radial velocity profile rearrangement at the die exit causes the flow to swell which is known as the die swell; in the case of the PCF preform the velocity profile is way more complex. As illustrated in Figure VII-11, the hexagonal pattern of a PCF is affected by the complexity of the flow rearrangement at the die exit and this phenomenon causes the holes to drift radially outward and to severely affect the circularity of the holes. To illustrate this point, the velocity distribution at the die exit for both Blockages is presented in Appendix 2, for no slip and no friction vases. These figures show a radial expansion of localized high levels of velocity at the die exit for no slip cases. The radial velocity at the die exit in Appendix 3 also shows a high radial velocity similar to the final shape deformation of holes.

In Appendix 4 and 5, similar plots of the velocity distribution and the radial velocity at the middle of the welding chamber show migrations of the flow to low resistance flow region such as the center of the die which mostly contributes to the distortion of the holes near the perform center.
The results in Figure VII-12 show how dramatic changes in distortion can be due to changing a basic parameter like the friction coefficient. This figure clearly shows a considerable difference between the deformations at high and low friction. In order to quantify the hole distortions seen in Figure VII-11 and Figure VII-12, the most basic quantities to record are:

- the hole centers coordinates,
- the radius function around of each holes,
- the die swell at the symmetry planes,
- the holes areas,
- the preform final cross section area.

Figure VII-11 also provides the hole numbering scheme to be used in the subsequent chapters.
Figure VII-12. Deformed PCF preform for several levels of friction for 2 pin size of Blockage A

In addition the flow rate at the feeding holes could be monitored for any possible correlation with the previous parameters.

VII.6. CONCLUSION

Manufacturing preforms for PCFs has been performed using diverse procedures. Despite the importance of extrusion on making MOFs and PCFs preforms, it has not been thoroughly studied theoretically or numerically due to its complexity. In this chapter the extrusion of PCF preform has been numerically modeled for two different Blockages. And though the symmetry of the model considerably reduces the numerical domain, the computational domain required additional study to further limit the numerical cost of the model. This goal also required the completion of a thorough mesh convergence study based on several convergence criteria.
The model was validated using the deformed shape of an extruded preform [2]. Each of the blockages used in this chapter was adjusted to visually match a validation preform deformed shape. However, Blockage B delivered a better match than Blockage A. The validation of the model required an adjustment of the pin size of the Blockage and the friction coefficient.

A preliminary observation of the results highlighted the importance of the impact of few extrusion parameters, namely the friction and the blockage geometry. Using the numerical model presents an opportunity to explore the effect of the extrusion parameters on the preform final shape. It can also serve to shape a better understanding of the extrusion process to visualize the flow and understand its influence on the final outcome.

VII.7. References


SHAPE DISTORTION IN EXTRUSION OF GLASS PCF

PREFORMS

VIII.1. INTRODUCTION

This chapter is divided into five sections. In the first section various measures that quantify distortion of the holes will be introduced and a pin/hole numbering system will be defined. In the second section results for the distribution of the flow rate for the two Blockage patterns will be presented as a function of the normalized friction coefficient. With this as a background, in the third and fourth sections the results switch to hole distortion in terms of the quantities introduced in Section 1. These two sections differ in terms of the normalization approach being used. In Section 3 both the size and shape of the distortion are studied, while in Section 4 the results focus on shape change which is the most important distortion feature since the preform will be drawn into a fiber. These sections include decisions on the best ways to interpret and normalize the data. In the final section quantification measures will be introduced that are best suited to optimization of the results to search for conditions that lead minimal hole distortion.
VIII.2. QUANTIFICATION MEASURES OF HOLE DISTORTION

Compared to the die swell of a solid cross-section which as an axisymmetric geometry is quantified as a change in diameter, the quantification of cross-sectional deformation of a PCF preform is very complex. Besides a more complex radial dependence, there is dependence in the circumferential direction. The holes in the preform also drift from their original positions, their cross sectional areas change and their shapes become non-circular. It is therefore necessary to select variables to quantify these important changes. Furthermore, these changes are a function of hole position so it is necessary to clearly identify the different feeding holes. After introducing this numbering system, the measures used to quantify hole distortion will be introduced.

VIII.2.1. HOLE NUMBERING SYSTEM

Referring to Figure VII-1 for the Blockage patterns A and B, within a 30 degree sector the models are composed of 17 and 13 feeding holes, respectively, and have five holes. These holes are arranged in a hexagonal pattern to form what is known as the rings of the preform. To simplify the study of the flow distribution, the feeding holes were grouped according to their relative location within these rings. Each Blockage pattern was divided into six sets of feeding holes that are defined below making use of the hole numbering system presented in Figure VII-1:

Blockage A sets of feeding holes:

▲ The feeding holes numbered 1 and 2 located below ring 1
▲ The feeding holes numbered 3, 4 and 5 located below ring 2
▲ The feeding holes numbered 6, 7, 8 and 9 located below ring 3
- The feeding holes numbered 10 and 11 located across ring 3
- The feeding holes numbered 12, 13, 14 and 15 located just above ring 3
- The feeding holes numbered 16 and 17

Blockage B sets of feeding holes:

- The feeding holes numbered 1 and 2 located below ring 1
- The feeding holes numbered 3 and 4 located below ring 2
- The feeding holes numbered 5, 6 and 7 located below ring 3
- The feeding holes numbered 8 and 9 located across ring 3
- The feeding holes numbered 10, and 11 located just above ring 3
- The feeding holes numbered 12 and 13
VIII.2.2. **Die Swell Measures**

For a regular circular cross section, die swell can be calculated based on the change of the diameter of the preform, while for non-circular shapes the area of the cross section is a more useful measure [1]. The second definition is more general and will be used in the current study of PCFs since the final preform cross section is not circular as shown in Figure VIII-2. Measuring the die swell bases on the area alone, however, does not fully describe the die swell and must be supplemented by measures of the preform radius at special locations such as the symmetry planes that are presented in Figure VIII-1. This figure provides three radii that are defined as
follows: \( R_{p0} \) is the die radius, \( R_{p1} \) is the radius at symmetry plane 1 that intersects one hole and \( R_{p2} \) is the radius at symmetry plane 2 that intersects three holes.

\[ \text{Figure VIII-2. Deformed PCF preform: Die swell measures.} \]

\[ \text{VIII.2.3. Hole deformation measures} \]

One of the most important aspects of the extrusion of PCFs is the control of shape change of the holes. Measuring the area of the deformed holes (like the die swell) is necessary but not sufficient. As shown in Figure VIII-3, the holes can exhibit severe form distortions that require the introduction of additional variables. There is no unique way to quantify the shape change of a hole. Observations of the deformed shapes reveal the presence of specific points where the curvature of the deformed shape is maximum or minimum. These points are chosen to measure the shape change for each hole.
Figure VIII-3. Hole deformation measures. The holes on the left and in the middle lie on a plane of symmetry, while the hole on the right does not have a plane of symmetry.

Mathematically, the determination of these special points involves calculating a few derivatives, which is tedious for a discretized contour. However, the presence of a line of symmetry simplifies this task considerably since the first two points, which are identified by the radii, $R_1$ and $R_2$ in Figure VIII-3, are on this line of symmetry. The third and last point is identified by the radius $R_3$, which is defined as the furthest point from the symmetry line. In this study the orthogonal projection of this third point on the symmetry line is defined as the geometric center. By definition the geometric center is the point from where all radii are measured. For the hole geometry in the current study (Figure VII-3), four of the five holes in each 30 degree sector have a symmetry line. A study of the geometrical form of hole 5 revealed that the locations of the points with extreme values of curvature are coincide with the maximum and the minimum radii measured from the centroid.
VIII.2.4. **HOLE DRIFT MEASURES**

Each hole drifts from its original position due to the mechanism of die swell. In order to measure this deformation the center of gravity of each hole must be calculated. The “center of gravity” can be calculated with a surface integral or by considering the hole contour as given by

\[
\bar{X}_{A_i} = \frac{1}{A_i} \int \bar{r} \, dA_i ,
\]

(VIII-1)

\[
\bar{X}_{s_i} = \frac{1}{s_i} \oint \bar{r} \, ds_i .
\]

(VIII-2)

The two approaches gave similar results in the current study. It is important to note that the geometric center and the centroid are not always the same but are usually relatively close to each other.

VIII.2.5. **DISCRETIZATION INACCURACY**

The evaluation of the majority of these variables involves numerical integration and differentiation to obtain maximum values. However the raw data are limited to nodal points, which present the difficulty of locating the position of a maximum or a minimum. Thus the values collected based on the nodal values had to be refined based on a local approximation of the hole shape. An example of the processing of raw data to determine the point identified as \( R_3 \) in Figure VIII-3 is given in Figure VIII-4. The example in Figure VIII-4 demonstrates the calculation of the principal radii for a hole at the symmetry plane. To calculate these radii, the location of the extremum curvature of the hole’s contour has to be determined. This task is carried out based on the discreet data at the nodal points. For each friction level (each line in Figure VIII-4 corresponds to a different level of the friction parameter) the position of the radius is identified at the nodal point that gives the maximum value along with two adjacent nodes. The data is rotated so that
the desired point corresponds to the maximum of the curves in Figure VIII-4. Based on these three points a Lagrange polynomial is used to approximate the local shape of the hole and allows the calculation of the maximum value that corresponds to the point where \( R_3 \) is measured. As shown in Figure VIII-4, if the nodes are used to locate the maximum, then unless a curve fitting procedure is used (to locate the real maximum), the maximum may shift suddenly to an adjacent node at a certain value of friction, which will be visible in extracted data. All these tasks are carried out using script at different levels of the post processing process. Due to the high number of variables and cases studied, an algorithm was developed to:

- Extract raw data
- Treat the data to overcome discretization inaccuracy
- Extract key variables
- Compare and manage several cases
Figure VIII-4. Use of Lagrange polynom curve fit to identify the location of $R_3$ for hole 2 and 3. Each line corresponds to a different value of the friction parameter.


VIII.3. **RESULTS: FLOW IN THE FEEDING HOLES**

In this section the flow rates through the six sets of feeding holes described in Section VIII.2.1 will be presented for the two Blockage patterns. While the focus in this study is hole distortion, it is important to have a basic understanding of how the flow behaves as a function of friction and position. For example, the inflow through a hollow cylinder for the no slip case generates a parabolic velocity profile, whereas for the opposite extreme of no friction a uniform flow is observed. The results in this section are basically the PCF analogy of this simple example.

In order to study the effect of friction on the flow distribution, the total flow rate in each of the feeding holes of Figure VIII-1 is presented in Figure VIII-5 for both Blockages and for low, medium and high values of friction. These flow rates are normalized with respect to the average flow rate in each hole and are presented in terms of their radial position normalized with respect to the outer radius of the Blockage. Note that since the total flow rate is considered for holes that are only partially included in the 30 degree sector, conservation of mass cannot be visualized in these plots. The most notable observation is that friction does not have a strong influence on the flow distribution since the flow rate only increases or decreases by about 5%. For example, for a very high friction value of \( \log(k \times (m/\text{Pa} \cdot s)) = 8 \), there is a higher flow rate near the die wall than for the case of \( \log(k \times (m/\text{Pa} \cdot s)) = 3 \). The flow distribution is clearly dominated by the presence of the Blockage and not by friction.
Figure VIII-5. The flow rate in the feeding holes for both blockages as function of the normalized upper chamber radius for low, medium and high values of friction ($\log((k/\eta) \times m)$=-2, 3 and 8).
In addition to viewing flow through the feeding holes, it is important to see how the flow evolves within the welding chamber. The results in Figure VIII-6 correspond to the flow along the two symmetry planes downstream of Blockage A and halfway down the pin length for the three selected friction values of \( \log(k \times (m/\text{Pa} \cdot \text{s})) = -2\) (no friction), 3 (medium friction) and 8 (no slip). The no slip boundary condition is clearly seen in Figure VIII-6 C, while a slightly non-uniform profile is observed for the no friction case. In addition, for the no friction case the velocity at the surface of the die is significantly different for the two symmetry planes indicating why the die swell is non-uniform in the circumferential direction in Figures VII-7 and 8. Another important observation is the relatively high flow rate in the center of the preform for the high friction case. While Figure VIII-5 indicates the center feeding hole has a flow rate about 6% higher than the average, the results in Figure VIII-6 B indicate a peak of velocity of twice the average velocity, which as will be shown has a significant effect on hole 4. In fact, a comparison between flow in Figure VIII-5 and the velocity in Figure VIII-6 at the center shows that the flow in the feeding chamber increases as the friction increases despite the fact that the flow rate at the feeding hole at the center does not always increase.
Figure VIII-6. The normalized velocity profile at the symmetry planes in the middle of the welding chamber for blockage A.
Another way to study the effect of friction is presented in Figure VIII-7. In this figure the normalized flow rates through each of the 17 feeding holes of Blockage A as a function of the normalized friction coefficient, $k/\eta$, are presented. In similar fashion the case of Blockage B is presented in Figure VIII-8. The results are organized in the 6 sets described in Section VIII.2.1. This classification of the feeding holes is interesting since it shows how similar the flow rate behavior is for the two Blockages. Once again it can be observed that the flow rates are not strong functions of position, but for each hole there is a functional dependence on friction. From Figure VIII-7 and 8 it can be observed that the flow rate at the feeding holes is affected by coefficients of friction up to $\log((k/\eta) \times m) = 5$, including even the smallest values of the friction parameter. However, the friction range where the sensitivity is highest is approximately $1 < \log((k/\eta) \times m) < 5$. The friction interval where the measured output variable is most sensitive to friction will be referred to as the “active friction interval.” As will be later shown, just like in Chapter 4 for die swell, friction plays a key role in the distortion of the holes. Furthermore, the active friction interval for the flow rate at the feeding holes does not exactly match the active friction interval that causes preform deformation, which suggests that the flow rate in the feeding holes is not the primarily mechanism affecting the shape.
Figure VIII-7. Flow rate in the feeding holes as a function of the normalized friction in Blockage A.
Figure VIII-8. Flow rate in the feeding holes as a function of the normalized friction in Blockage B.
VIII.4. Primarily results: first normalization

In this section the impact of the key extrusion parameters on the final shape of the PCF preform is investigated. Material behavior is assumed to be viscous and the same viscosities and flow rates used for extrusion of solid rod in Chapter 3 are applied. The sensitivity analysis conducted in this section is performed the same way as it was performed in Chapters 5 and 6 for the viscous case in solid rod extrusion. To summarize this procedure, for a specific set of flow rate and viscosity, several runs are performed for the full friction range and the results are plotted as a function of the log of the friction coefficient normalized by the viscosity. This study was conducted for both Blockage geometries. In this section all results are normalized with respect to dimensions of fixed value, i.e., quantities that do not change with deformation.

VIII.4.1. Die swell

In this chapter die swell is defined as the ratio of the outer radius of the preform and the radius of the die opening. For a PCF preform the outer radius is a function of the circumferential coordinate since the preform does not preserve its initial circular cross section. As such the following three measures of the die swell (from Figure VIII-2) will be used: the die swell measured on both symmetry planes and the average die swell. The expressions of the three measures of die swell are given by
\[ \bar{D}_{p,k} = \frac{D_{p,k}}{D_{p,0}} \]  

(VIII-3)

where \( k = 1, 2 \), \( A_v \) and \( D_{p,Av} = \sqrt{\frac{A_v}{4\pi}} \). As it is shown in Equation VIII-3, the average diameter is calculated based on the preform area.

Just like the die swell behavior for a regular preform presented in Figure V-2, the die swell behavior for a PCF preform in Figure VIII-9 can be divided into three phases based on the level of friction. However, unlike the case of a solid preform, the die swell value is low but not constant in the first phase. In addition there is no clear active friction interval where the profile is close to a linear relation as it was observed for the solid preform in Figure V-2. It is important to note that the geometry of the Blockage impacts the die swell in various ways. For the case presented in Figure VIII-9 all the measures of the die swell started from a value greater than 1. A change in the geometry of the model can make the die swell dip below 1 for range of friction values. In these presented cases Figure VIII-9 shows that as friction increases the difference between the die swell diameters also increases. Thus, the preform becomes less circular for higher friction; clearly this behavior is a complex function of the feeding hole and pin locations, sizes and numbers. The maximum and the minimum die swell are located at the symmetry planes and do not exchange their positions since the curve presenting these quantities never cross. The highly skewed die swell profile for high friction, where the average diameter value is significantly different from the average of the maximum and the minimum diameters, indicates a more non-uniform increase in the preform radius.
Figure VIII-9. The diameters of the preform normalized by $D_{ro}$ (the Diameter of the die).
VIII.4.2. Areas

The area of the holes is one of the most important outcomes of the extrusion as it has a significant impact on the use and operation of the fiber. As it is shown in equation VIII-4, the areas of the feeding holes are normalized by the initial pin size. This measure is comparable to the average die swell, except that the area is proportional to the square of the average diameter.

\[ \bar{A}_f = \frac{A_i}{A_0} \quad \text{(VIII-4)} \]

The data for the areas of the five holes are presented in Figure VIII-10, which has trends that are similar to those of the die swell of the entire cross section in Figure VIII-9, such as each case has:

- the same active range of friction,
- a constant value at high friction,
- a slight variation for low friction, a non-zero change in the no friction limit.

However, contrary to die swell, a maximum is clearly detectable in a few of the cases in Figure VIII-10. Figure VIII-10 also shows that the difference between the holes’ area grows as the friction increases. However, this difference is not proportional to the distance of the hole from the preform center. The Blockage geometry affects the area of the holes for most levels of friction, especially for the lower levels.
Figure VIII-10. The Areas of the holes normalized by $A_0$ (the pin’s area).
VIII.4.3. Radii

A PCF perform undergoes several types of deformation at the die exit. The holes of a PCF perform become distorted and drift from their initial positions. The deformation of the holes shapes could be summarized using a limited set of variables. As previously stated each deformed hole has 4 major radii (Figure VIII-3), which can be reduced if symmetry is present. Four of the five holes in both Blockage patterns are located on a symmetry plane. The normalized measures of all the radii are given by Equation VIII-5. The radii for each hole are normalized by the initial pin radius, a standard normalization process. In addition, the hole’s average radius is included in the plot which is calculated based on the hole’s area.

\[
\bar{R}_n = \frac{R_n}{R_0}
\]

\[(VIII-5)\]

where \(n = 1, 2, 3, 4, \) and \(R_{LAV} = \frac{A_1}{\sqrt{n}}\)

The radii provide a general idea about the shape of the holes and could also serve to qualitatively evaluate their circularities (how close the shape of a hole is to a circular form). In order to simplify and visualize the effect of variation of the radii, the evolution of the shape of the holes is at specific friction values is presented at Figure VIII-11 and 13.

The presence of symmetry (Figure VIII-1 and 3) simplifies the interpretation of the data. In fact, if a hole lies along a symmetry plane, the difference between its values of \(R_1\) and \(R_2\) (Figure VIII-3) indicates the radial skewness direction (i.e., the direction of the principal radius closest to a point where the radius is equal to the average radius), whereas the difference between \(R_3\) (Figure VIII-3) and the other radii reveals how relatively stretched the hole is in the circumferential direction. It is essential to add the average radius to the plot set as it represents a
baseline value to quantify the level of deformation at the principal radii. In fact, if for example $R_3$ is greater than the average radius, the corresponding hole is stretched along the circumferential direction.
Figure VIII-11. Deformation of the holes in blockage A at several values of $\log \left( \frac{k}{\eta} \times m \right)$. 
Figure VIII-12. The holes’ radii normalized by $R_0$ (the Pin radius) for Blockage A.
Despite the difference in the final shapes of the holes, the results in Figure VIII-12 and 14 show several common deformation patterns. Like the die swell, the friction domain can be divided into the same 3 phases.

Within the low friction domain, the value of friction has a limited effect on the radii. In this domain all holes experience a limited expansion. Based on Figures VIII-12 and 14, the variation of hole radii in this friction domain shows that only hole 2 in Blockage A is highly distorted compared to rest of the holes. This appears to be a local effect since both holes that belong to the same symmetry plane as well as its neighboring holes are not affected. In this friction domain, Blockage B preserves better the circular shape and the size of the holes, whereas Blockage A experiences limited distortions of its holes except for the second hole.

Behavior in the active friction interval is characterized by a significant change in the shape of the holes. In this range some of the radii experience a local maximum while others cross each other. In addition, hole 5 experiences a significant rotation as seen in Figures VIII-12 and 14 on the bottom right. Moreover, since the circularity of the holes is deeply affected by the difference between the average hole radius and the other principal radii, it is probable that at around a friction value $\log((k/\eta) \times m) = 3$ that most of the holes are simultaneously close to a circular shape. Only hole 4 does not follow this observation and experiences a drastic shape change within this friction region. Perhaps the high flow rate at the feeding holes close to the center causes this behavior (see Figure VIII-7 and 8).

At least one of the radii of each hole experiences a maximum, which suggests the presence of at least two competing mechanisms. Two important mechanisms that affect the
shape and size of the holes are the die swell, which tends to expand the preform cross section and the effect of the velocity gradient at the tip of the pins which generates another type of die swell that tends to shrink the holes. Understanding the combined effects of these (and other) flow mechanisms is not simple due to the highly non-homogeneous nature of the flow, which as shown by Figure VIII-6, is more complex for high friction. In general, the die swell mechanism is believed to play an important role in the shape change of the holes but its effect is felt more on the holes close to the boundary (holes 2 and 5). The die swell affect that shrinks the holes applies to all the holes. It is understood that there are other mechanisms, based on the complex interaction of the feeding holes and the pins that lead to hole distortion.
Figure VIII-13. Deformation of the holes in blockage B at several values of $log \left( \frac{k}{\eta} \times m \right)$. 
Figure VIII-14. The holes' radii normalized by $R_0$ (The Pin radius) for Blockage B.
Figures VIII-12 and 14 show that, unlike the die swell, the maximum and the minimum radii do not always occur at a specific location. In fact, radii curves cross each other in several occasions for several holes. The behavior of $R_3$ in hole 2 is particular interesting. In fact, though the $R_3$ curve rises above the average radius in most of the holes, it is only in the case of hole 2 that $R_3$ crosses the average radius thus creating an interesting shape change of this hole. In fact as was shown in Figures VIII-11 and 13, hole 2 becomes stretched in the radial direction, while it was stretched in the circumferential direction at low friction. Finally at high values of friction the no slip condition is approached and while the holes experience their maximum deformation there is not much variation.

The change of the shape of the holes is generally caused by a combined effect of the die swell and the gradient of the velocity at the pin outflow, which follows a behavior similar to that of the die swell. The distinction between the effects of these phenomena is not obvious, however since the die swell generally evolves downstream at a scale relative to the die diameter, it is possible to approximately isolate the effect of the velocity gradient at the pins by studying the flow from the tip of the pins downstream to the point where the gradient of the velocity around the surface of the holes is negligible. At this location most of the effect of the velocity gradient at the pins is rearranged, while the velocity gradient responsible for the die swell has to travel a
longer distance to reach a steady state. An example of this evolution is shown in figure VIII-15

**Figure VIII-15. Evolution of velocity distribution after the die exit.**

**VIII.4.4. Holes’ centers**

A study of the evolution of the positions of the holes can be performed in various ways. The centroid can be used to monitor the drift from the original position. For either choice, the position of the hole can be traced relative to the preform center or relative to the pin center. An absolute position can be normalized by the original hole center coordinates. However, if the pin center is considered as the origin, then the pin’s radius is used for normalization. Both measures are given by Equation VIII-6 and 7, respectively.
\[
\bar{p}_i = \frac{p_i}{p_{i,0}} \quad \text{(VIII-6)}
\]

\[
\bar{q}_i = \frac{q_i}{q_{0}} \quad \text{(VIII-7)}
\]

where \(\epsilon_i = \|\bar{x}_i\|\) and \(\bar{x}_i = \bar{X}_i - \bar{X}_{i,0}\)

The first choice is presented in Figure VIII-16. The positions of the centers of the holes are monitored with respect to the preform center and normalized by their original positions. Most of the holes are located on the symmetry plane and thus have to drift along it. The only hole located outside the symmetry plane has its center bounded with one degree of its original radial plan. For this reason only the normalized radial position is presented. When plotted in the same figure the data give a general description of the location of the holes within the entire cross-section.

Figure VIII-16 shows the same three friction phases demonstrated in the previous figures of this section. Interestingly, low friction does not guarantee a zero deviation; whereas high friction causes the holes along the symmetry planes to distort more the closer they are to the center.

Hole 5, which is not located along a symmetry plane, has more degrees of freedom to locate an equilibrium position. In fact, as the friction level increases, this hole drifts in the circumferential direction by 1 degree (toward symmetry plane 2 for both Blockages).
Figure VIII-16. The normalized drift of the holes at the global (preform) scale.
Based on these observations it is clear that the distance of the hole from the center of
the preform is not a determining factor its drift. Neither case of Blockage geometry shows a
significant difference of results for this particular measure.

Figure VIII-17 shows the holes drift at a local scale by measuring hole drift relative to the
center of the pin. Generally the radial position (the distance between the pin center and the hole
centroid) is sufficient to describe the data. The data in Figure VIII-17 shows that the hole’s
displacement evolution at the local scale is different from the previous presentation in Figure VIII-
16. At low friction the displacement is relatively high for a few cases with a very wide spread for
Blockage A, whereas for Blockage B, the displacement is low with less spread. At high friction, for
Blockage A, the difference between the holes decreases whilst the value of the drift increases.
The case of Blockage B is different since the difference between the holes increases. However,
there is no relevant correlation between the position of the holes and their drift. Generally, holes
2 and 4 are extremely affected by the friction change which may compromise the geometric
integrity of the structure. For Blockage A, the fact that at high friction the drifts come closer to
each other is not helpful to preserve the integrity of the structure since the desired position of
the holes must satisfy several geometric constraints of distances and parallelism that cannot be
satisfied simply by homogeneous drift. The presentation of this data confirms some previous
observations. For example, it is clear that the holes along the symmetry plane tend to be more
affected by friction. The particular case of the symmetry plane that contains more than one hole
is interesting since holes 2 and 4 tend to be affected more by friction than hole 3. It is clear that if
the integrity of the structure is the prime goal, numerous adjustments are required to the
Blockage for either case, especially at high friction, and that the initial design must be improved.
However since the effect of die swell cannot be easily suppressed it is suggested to look at these data using a different normalization that is based on the proportions in the deformed state to assist better the quality of the final geometry (which is treated in the next section).
Figure VIII-17. The normalized drift of the holes at the local (hole) scale.
VIII.5. RESULTS AS PROPORTIONS: SECOND NORMALIZATION

In the previous section the results were presented as a ratio with respect to the characteristic dimensions of the initial geometry of the die and Blockage. This approach is important from a rheological point of view and helps to provide a picture of the impact of the flow on the final shape and size. However, given that the preform will be drawn into a fiber of much smaller radial dimension, it is important to quantify the cross sectional shape relative to the deformed shape since shape is more important than size. In this section the results of the previous section are essentially repeated using such normalization.

VIII.5.1. DIE SWELL

The die swell in this section is normalized by the average preform diameter in Figure VIII-13 instead of the initial diameter at the die exit as was presented in Figure VIII-7. The normalized die swell is given by

$$
\bar{D}_{p,k} = \frac{D_{p,k}}{D_{p,avg}} \quad \text{(VIII-8)}
$$

This presentation shows more clearly how the preform deviates from its original circular shape without being influenced by the original die swell. Figure VIII-18 shows clearly that even in the low friction region the preform is not perfectly circular. As the friction increases (in the second friction phase) the difference increases, although it is still skewed to the same direction since $|D_{p,1} - 1| < |D_{p,2} - 1|$. In comparing Figures VIII-9 and 18, of the current normalization simplifies the comparison of how the two Blockage patterns affect the skewness. In fact, it is clear now that at high friction neither Blockage has a significant advantage over the other.
Figure VIII-18. The Diameter normalized by $D_{PAV}$. 
VIII.5.2. AREAS

Based on the current normalization approach, the area of the holes should be normalized by the preform area. This ratio is further divided by the ratio of the initial values of pin size to die area so that a value of unity corresponds to no change in shape. The expression of the normalized area is given by Equation VIII-9. Equation VIII-9 also shows that the normalized area can be regarded as the ratio of the expansion coefficient of the area of the hole and the expansion coefficient of the area of the preform. The normalized area using this approach is presented in Figure VIII-19.

\[
\frac{A_i}{A_p} = \frac{\left(\frac{A_i}{A_{p,0}}\right)}{\left(\frac{A_0}{A_{p,0}}\right)} = \frac{\left(\frac{A_i}{A_0}\right)}{\left(\frac{A_{p,0}}{A_{p,0}}\right)}
\]  \hspace{1cm} (VIII-9)

\[
\frac{A_i}{A_p} : \text{The deformed preform-hole area ratio for hole } i
\]

\[
\frac{A_0}{A_{p,0}} : \text{The initial preform-hole area ratio}
\]

\[
\frac{A_i}{A_0} : \text{The expansion coefficient of the area of the hole } i
\]

\[
\frac{A_p}{A_{p,0}} : \text{The expansion coefficient of the area of the preform}
\]

Unlike the previous area presentation in Figure VIII-10, no sharp maximum is observed since the die swell compensates for the variation that leads to this maximum. As such, the comparison between Figures VIII-10 and 19 shows the advantage of the current normalization approach. The results in Figure VIII-19 also show that the areas of the holes are relatively close to
the targeted proportion of 1.0 at a low friction rate. As the friction increases the die swell expansion effect is no longer able to prevent the hole area from shrinking. Thus, with a combination of large die swell and smaller holes, the plotted area ratio drops drastically. Here again Blockage B performs better at low friction. At high friction there is no significant difference.
Figure VIII-19. The expansion of the areas of the holes normalized by the expansion of the preform area.
VIII.5.3. Radii

Following the same logic as the preceding sub-section, the principle radii and the average radius for each hole would be logically normalized by a coefficient proportional to the average radius including the die swell. However, the resulting information would be similar to that of the hole area plots in Figure VIII-19 and would not bring additional information about the final shape of the holes. For these reasons the average radius of each hole is used to normalize its radii (see Equation VIII-10). This approach will isolate the shape change from the radius variation due to the area shrinkage.

\[
\overline{R}_n = \frac{R_n}{R_{Ap}} \quad \text{(VIII-10)}
\]

The symmetry of the evolution of the radii of hole 5 presented in Figures VIII-20 and 21 clearly shows the benefits of the current normalization approach. For hole 5, these figures suggest a shape that is very close to elliptical since \( R_1 \approx R_3 \) and \( R_2 \approx R_4 \), which confirms that the geometrical center is indeed the gravity center. The hole becomes nearly circular for values of the normalized friction coefficient below about \( \log \left( \frac{k}{\eta} \times m \right) = 3 \).

Based on the data representation in Figures VIII-20 and 21, it can be seen that the hole 4 shape change is one of the most affected by the increase of friction: the difference between the minimum and the maximum radii increase drastically with friction. This hole’s value of \( R_3 \) increases significantly which signals that this hole is extremely stretched in the circumferential direction. Holes 1 and 3 also show a similar evolution pattern to that of hole 4, but with different proportions.
Figure VIII-20. The principal radii normalized by $R_\omega$, for Blockage A.
Figure VIII-21. The principal radii normalized by $R_A$ for Blockage B.
Hole 2 is the closest hole to the preform boundary. Unlike the other holes along the symmetry plane, as friction increases this hole gradually changes from stretching more in the circumferential direction to stretching more in the radial direction. As this hole goes through this transition a circular shaped hole is observed at a very specific friction value. This hole is the only symmetric hole where the radii curves for \( R_1 \) and \( R_2 \) intersect. Thus this hole's shape is not always stretched toward the preform center like all the other symmetric holes.

The friction dependence of the radii still has the behavior that has three distinct ranges of friction after applying this new normalization, but the results no longer have maxima within the middle range. This indicates that these maximum values were related to the change of area in the holes and has nothing to do with the shape of the holes. Thus, the current normalization shows that an increase of friction causes the shape of each hole to evolve in a smooth and monotonous trend.

It was previously stated that Blockage B has a higher level of hole distortion at high friction. The current normalization shows this more clearly. As such, the current normalization can be considered as a better tool to evaluate the hole shape, leading to the need to consider changes such as lowering the friction of altering the geometry of the Blockage. Though there are a few differences between the hole shape evolution of both Blockages, the major trends are essentially the same.

VIII.5.4. Holes' centers

As far as the geometric integrity is concerned, it is simpler to bring the deformed section of the preform and the cross section of the die to the same scale. This can be achieved by
measuring the coordinates of the centroid of the holes and the centers of the pins as a fraction of the average preform radius and the die radius, respectively. Thus the coordinates of the holes and the pins are normalized by the preform and the die, respectively. Then the drift at the global scale is calculated as given by Equation VIII-12. As indicted in Equation VIII-12, this is equivalent to scaling the center of the pins to the preform size using the expansion coefficient of the preform diameter \( \frac{D_{p,Av}}{D_{p,0}} \) before calculating the drift. This eliminates the use of a comparison between the centers’ positions in order to monitor the geometric integrity of the preform and not the size.

\[
\begin{align*}
\bar{P}_i &= \frac{P_i}{R_{p,Av}} = \frac{P_i}{\left( \frac{D_{p,Av}}{D_{p,0}} P_{i,0} \right)} = \frac{P_i}{\bar{P}_{i,0}} \\
\left( \frac{D_{p,Av}}{D_{p,0}} \right) : \text{The Expansion coefficient of the preform diameter} \\
\bar{P}_{i,0} : \text{The distance between the center of the scaled position of pin } i \text{ and the preform center}
\end{align*}
\]

Compared to the previous normalization for the drift of the center of each hole given in Figure VIII-16, the results in Figure VIII-22 show a very different evolution pattern. This is noticeable especially for hole 5 whose evolution looks much simpler compared to the result of Figure VIII-16. The drift measure in Figure VIII-22 for hole 4 is now 15%, which is closest to the center while the other holes stay below 5%. This behavior can be traced to the results in Figure VIII-6 C, which shows an intensification of flow at the center of the welding chamber. This high drift makes hole 4 a priority to handle at high friction. Finally, this presentation of the data shows
that the preform has a relatively good geometric integrity for a wide range of friction except for one single hole.

The results in Figure VIII-22 can be presented at the local scale (i.e., the scale of the holes) using a similar approach. This is presented in Figure VIII-23. To perform such a task, the initial positions of the holes have to be scaled to the final size of the preform. As it is shown in Equation VIII-12, this is accomplished by multiplying the original position of the holes by the expansion of the preform diameter \( \left( \frac{D_{p,Av}}{D_{p,0}} \right) \). Then the distance between the centers of the holes and the scaled pin centers is calculated and normalized by the original radius of the pin. This procedure is given by

\[
\vec{q}_i = \frac{\vec{q}_i}{R_0} \quad \text{(VIII-12)}
\]

where \( \vec{q}_i = \| \vec{x}_i \| \)

and \( \vec{x}_i = \vec{X}_i - \vec{X}_{i,0} = \vec{X}_i - \left( \frac{D_{p,Av}}{D_{p,0}} \right) \vec{X}_{i,0} \)

\( \vec{q}_i \): The distance between the centroid of the hole \( i \) and the center pin \( i \) after accounting for the die swell

\( \vec{x}_i \): The displacement of the centroid of hole \( i \) after accounting for the die swell

\( \vec{X}_{i,0} \): The Cartesian coordinates of the center of pin \( i \) compensated for the die swell

\( \left( \frac{D_{p,Av}}{D_{p,0}} \right) \): The Expansion coefficient of the preform diameter
The results in Figure VIII-23 show the drift from a different perspective. In general, both figures show that the geometric integrity is difficult to maintain in the extruded preform. It also shows that the closest hole to the center (hole 4) does not follow the trends of the other holes which might be essential to resolve before a full optimization task is carried out. Interestingly for Blockage B the drift of a few holes reaches a minimum within a narrow range of friction, but is also relatively low for all holes at low levels of friction. For the case of Blockage A, however, the amount of drift varies from hole to hole at both high and low friction.
Figure VIII-22. The normalized compensated drift of the holes at the global (preform) scale.
Figure VIII-23. The normalized compensated drift of the holes at the local (holes) scale.
VIII.6. **Additional Quality Measures**

VIII.6.1. **Holes Circularity**

Previously, plots of the normalized radii were used to study the shapes of the holes. Though the previous approaches gave a detailed overview of the holes shape, they did not give a straightforward way to compare and evaluate the shape of the holes from a quantitative point of view. To address this issue, in this section mathematical and statistical tools are used to evaluate the quality of the shapes of the holes.

VIII.6.1.1. **Variance of the radius and standard deviation**

One of the simplest ways to quantify the quality of the shape of the holes is to calculate the variance (or standard deviation) of the radius. A perfect circular hole would have a variance of zero. The value of the radius for any given hole is a continuous function in the circumferential direction. It is possible to use a discrete distribution of the radius to calculate the variance with a high number of data points. To accomplish this task the radius value at each node is fitted with a spline approximation function, and using this function it is possible to generate a large sample size. After gradually increasing the sample size, the variance value is no longer a function of the sample size. This value is then plotted as a function of the level of friction.

The computed variance and standard deviation plots in Figures VIII-24 and 25 present a practical way to compare the shape of the hole. The plots reproduce the radii results of Figures VIII-20 and 21 in a much simpler way. However, it is no longer possible to know the nature of the deformation based on these figures alone.
Figure VIII-24. The radius variance and standard deviation of the holes radius for Blockage A.
These measures present an efficient way to monitor the circularity, which is better than comparing the hole radii. The presence of a minimum corresponds to a friction value where the corresponding hole shape is the closest to a circle. The results in Figure VIII-24 and 25 reveal the presence of a few minima. However, not all the curves experience a minimum (only hole 1 and 2 for Blockage A and hole 1 for Blockage B). Unfortunately, these minima are located at different friction levels and a friction value cannot be easily targeted to improve the overall preform quality.

Figures VIII-24 and 25 also reveal that only hole 2 experiences a relatively high distortion at low friction, especially for Blockage A. It is also noted that distortions of the holes are quite different between Blockages A and B. In fact, clearly hole 4 is the most distorted hole at high friction for Blockage A. However for Blockage B both holes 2 and 4 have a comparably high distortion at high friction. For Blockage A a possible explanation of these variations can be found in the velocity profile at the symmetry planes in Figure VIII-6. The difference between the local velocity maxima at the vicinity of the hole seems to generally correlate with its distortion. For example, hole 4 is the closest to the preform center. Based on Figure VII-6, the influence of the flow rate can be easily detected. In fact, Figure VIII-6 shows a steady increase of the velocity at the center of the preform whose effect correlates with the distortion of hole 1.
Figure VIII-25. The radius Variance and standard deviation of the holes radius for Blockage B.
At the other extreme, hole 2 is the furthest hole from the center of the preform. At this location the shape of the hole is severely affected by the die swell. The hole shape is stretched in the circumferential direction at low friction. As the friction increases the effect of the die swell takes effect and stretches this hole along the radial direction leading to a minimum in the variance curve. Finally, hole 3 exhibits the least variance due to its presence in a highly symmetric position which can be seen in Figures VIII-6 A, B and C.

The variance of the radius offers an acceptable tool to monitor the trend of the effect of friction and allows a simple classification of the quality of the holes. However the numerical value alone does not offer a straightforward measure of the quality of the hole shape, which requires setting subjective level of quality to judge whether or not improvement is necessary.

**VIII.6.1.2. Surface Criterion**

A second way to evaluate the shape of the holes is to calculate the area of either the “disjoint surface,” or the “joint surface,” which are explained in Figure VIII-26. According to Figure VIII-26, to calculate these quantities, which by definition are equal, a circle with a radius equivalent to the average radius of the studied hole is placed at the gravity center of the hole. Based on this circle the surface criterion is calculated according to the schematics in Figure VIII-26. This operation is carried out in polar coordinates using gauss quadrature. The values of the radial function are calculated using a spline function. The final result for the joint surface or the disjoint surface is normalized by the area of the deformed hole. The normalized values for both areas are called Joint Area Percentage (JAP) and Disjoint Area Percentage (DAP), respectively.
Figure VIII-26. Schematic of the surface criteria.

Unlike the variance and the standard deviation of the radius, this criterion is bounded between 0% and 100% which not only allows a comparison between the holes but also gives a tangible value of the quality of each hole.
Figure VIII-27. Joint Area Percentage (JAP) and Disjoint Area Percentage (DAP) for Blockage A.
Figure VIII-28. Joint Area Percentage (JAP) and Disjoint Area Percentage (DAP), for Blockage B.
Both the joint and disjoint areas percentages are presented in Figures VIII-27 and 28 and are complimentary measures. They both reproduce the same trends as the variance of the radius. However, it is now possible to confirm that holes 2 and 4 are extremely distorted at high friction and that hole 2 is highly distorted for almost all values of friction for Blockage A. It is also easier to see that the distortion for Blockage B at high friction is more severe than that for Blockage A.

VIII.6.1.3. Correlation criterion

The correlation criterion is based on the following simple fact: If a circle is located at center of the frame than for each point of the circle \( x^2 + y^2 = R^2 \). This means that \( x^2 \) and \( -y^2 \) should be linearly correlated with a positive slope. Thus, it possible to evaluate the circularity of the holes based on the correlation between \( x^2 \) and \( -y^2 \). This will be referred to as the correlation of the square of the coordinates (CSC). Using the same procedure used to calculated the variance of the radius, the CSC for each hole is calculated and plotted in Figure VIII-29.
Figure VIII-29. correlation of the square of the coordinates (CSC) criterion for Blockage A.

Since the correlation is bounded between 0 and 1, this method has both advantages of the surface criterion. In addition, the present criterion is more sensitive to large deviation from a circular shape. Thus it drastically drops when the hole has extremely long or short radii. Again this new measure confirms the general observation about holes 2 and 4, though the amount of distortion given by this measure seems extremely high for these holes. Based on data in Figure VIII_29, hole 4 cannot be classified as circular at high friction for its CSC is far from 1 which again suggests that it should be a priority for any eventual Blockage optimization task. The CSC curve for hole 2 also is low for a wide range of friction and requires attention if any eventual Blockage optimization task is carried out.

Unlike the previous measures the CSC suggests that the rest of the holes are fairly circular with a high CSC value above 0.985. Moreover this new measure conserves a few of the
maximum values observed for the previous criteria, however the holes circularity does not exactly give the same classification obtained using the other measures.

The CSC shows high sensitivity compared to the previous measures. As a measuring tool, this variable can be exploited in multiple ways. Statistical bi-variable measures like the influence, the Leverage or the distance can be used to locate interesting point on the hole shape such as the location of the principle radii.

VIII.6.2. Holes position dispersion (distances and angles)

The last quality measures presented in this chapter are related to the positioning of the center of the holes. Referring to Figure VIII-30, the idea consists of locating the holes centers based on a few independent distances. These distances do not need to be normalized since they are supposed to be equal if no deformation occurs. The variance of these distances is plotted against the friction in Figure VIII-31 A. The same idea is applied using angles in Figure VIII-31 B.
These measures are typically used to simply check the influence of any eventual Blockage optimization on the positioning of the holes. A typical measure for these variables is presented in Figure VIII-31 for Blockage A.
Figure VIII-31. The holes position dispersion measures for Blockage A.
VIII.7. CONCLUSION

In this chapter, a sensitivity analysis was conducted on the process of extrusion of PCFs preforms. The study was conducted using two different Blockage geometries and involved several parameters, such as the flow rate and the viscosity. The study showed that only the variation of the Navier friction coefficient normalized by the viscosity has a significant effect on the final shape of the preform. The shape change of the preform is very complex; therefore, a set of techniques, algorithms and variables had to be defined to develop an efficient way to describe and monitor the important aspects of the shape of the final preform using a minimum set of variables.

The interpretation of the results also required a normalization of these variables. In this chapter two normalization methods were proposed, each interpreting the data in a different way. The first normalization was established by comparing the deformed shape to the initial condition (die/Blockage). This approach combines shape changes and size changes, which can be a disadvantage. For this reason a second normalization was proposed to account for this weakness which was achieved by introducing an expansion confident to the normalization. The evolution of a few variables showed a few similarities to the die swell of a solid rod (such as the presence of three friction zone) and a large amount of complexity that made it difficult to interpret. A study of the flow rate was helpful to explain a few aspects of the shape change of the holes and the preform; it also revealed the presence of several mechanisms that control the flow distribution.

The sensitivity analysis in this study was also used to introduce quality variables to quantify the circularity of the holes using a single variable for each hole. These quality variables
are introduced to simplify the optimization of the Blockage geometry in order to reduce the deformation of the holes. Using these quality variables, a study of the alteration of the Blockage shape is performed in the next chapter.
VIII.8. **NOMENCLATURE**

\( R_{p,k} \) : The radius of the preform at the symmetry plane \( k \) (including the average)

\( R_{p,0} \) : The radius of the die

\( R_{p,Av} \) : The average radius of the preform

\( D_{p,k} \) : The diameters of the preform at the symmetry plane \( k \) (including the average)

\( D_{p,0} \) : The diameter of the die

\( D_{p,Av} \) : The average diameters of the preform

\( A_p \) : The area of the preform

\( A_{p,0} \) : The area of the die

\( R_n \) : The radius number \( n \) of a particular the hole (including the average)

\( R_0 \) : The radius of the pin

\( R_{i,Av} \) : The average radius of the hole \( i \)

\( A_i \) : The area of the hole \( i \)

\( A_0 \) : The area of the pin
\( \tilde{X}_i \) : The Cartesian coordinates of the centroid of hole \( i \)

\( \tilde{X}_{i,0} \) : The Cartesian coordinates of the center of pin \( i \)

\( \tilde{x}_i \) : The displacement of the centroid of hole \( i \)

\( P_i \) : The distance between the centroid of the hole \( i \) and the preform center

\( P_{i,0} \) : The distance between the center of the pin \( i \) and the preform center

\( q_i \) : The distance between the centroid of the hole \( i \) and the center of pin \( i \)

\( \bar{A} \) : The normalized value of the variable \( A \)

\( \bar{\tilde{A}} \) : The value of the variable \( A \) after accounting for the die swell

DAP : Disjoint Area Percentage

JAP : Joint Area Percentage

CSC : Correlation of the Square of the Coordinates

VIII.9. REFERENCES

EXTRUSION OF GLASS PCF FIBER PREFORM:

SIMPLE ADJUSTMENTS OF THE FEEDING HOLE

GEOMETRY

IX.1. INTRODUCTION

The goal of this section is to study and propose a few alterations of the Blockage geometry in order to improve the final outcome of the extrusion. Such alterations fall into two categories: 1) those that make use of circular feeding holes and/or pins, which are easily accomplished in practice, and 2) those that disregard the practical manufacture of the Blockage and modify the circular shapes, especially of the pins. In this chapter the former approach will be used.

In the previous section, the quality of the shape of a hole was discussed based on several criteria. It was concluded that the shape of hole 4 becomes highly distorted for high values of friction and needs to be addressed. Moreover, for this case there is a friction range where different measures of the circularity experience a minimum for all the holes except hole 4. Thus it is interesting to consider simple ways to improve the quality of this hole within this friction range, before considering more complex alterations to the Blockage pattern. To overcome these distortions the basic approach used by the experimentalist is to alter the size of the feeding holes
at the center and/or the border of the Blockage. Based on the study conducted in Chapter 8, the reason hole 4 is so distorted is that too much glass flows through the center. Therefore, it is expected that by reducing hole size in the center hole for should better maintain its circular shape and position.

Since, the priority is first to correct the shape of hole 4, the effect of altering the first ring of the feeding holes is studied in the next paragraph. Based on this study, few other feeding holes will be adjusted to correct the distortion at hole 2 in the subsequent section. Finally, using the same alteration logic, a full optimization task is carried out on Blockage B targeting all the feeding holes to drive the final shape of the perform to the targeted geometry.

**IX.2. Effect of minimizing the first ring of the feeding holes**

In this section a sweep of the entire friction range is performed for several Blockage alterations. These alterations are performed on the radii of the feeding holes 1 and 2 as shown in Figure XI-1. The radii were modified to a set of percentage values of the original hole radius, namely 96%, 92%, 88%, 84% and 82%. This alteration method is common in the extrusion literature [1-2]. The resulting data is then processed and compared in terms of quality variables introduced in the previous chapter such as the CSC, the variance of the radius, the percentage of disjoint surfaces. Only the CSC and the disjoint area percentage are discussed in this section.
The disjoint area percentage (DAP) is plotted for each hole in several figures that are presented in Figure XI-2. Each figure consists of a combination of curves of the DAP for the full range of alteration of the targeted feeding holes. When no alteration is conducted on the feeding holes, only the second hole DAP shows a clear minimum within the active friction zone. As the radii of the feeding holes become smaller, a minimum starts to appear in the active friction zone in all DAP plots. These minima occur at about the same friction interval. The locations of the minima drift differently when the size of the feeding holes becomes smaller. The presence of these minima is particularly useful if the glass/die combination for the extrusion corresponds to this friction interval. For each case, each minimum divides the friction domain into two distinct intervals.
Figure IX-2. Effect of the Alteration of the feeding holes at the center on the DAP.

In fact, the Figure XI-2 shows that in most cases, if the friction coefficient is higher than the value corresponding to the minimum, a reduction of the feeding holes radius steadily improves the shape quality of most holes. More importantly, if the level of friction is about
log((k/η) x m) = 3, the hole quality is excellent, which is a realistic value based on the die swell study in Chapter 3 (see Table 11). An opposite behavior is seen if the friction coefficient is lower. This allows the following two important conclusions to be made: 1) the proposed alteration is only useful for relatively high values of friction and 2) though the proposed alteration steadily improves most of the hole shapes, it comes at the cost of distorting others (hole 2 in this case) so it is important to keep a balance and set a quality threshold for each hole. In this case such a balance is particularly important to consider when the radius of the feeding holes is less than 84%.

To illustrate this phenomenon, in Figure XI-3 the shapes of holes 2 and 4 are plotted for two critical values of alteration alongside the initial deformation for log((k/η) x m) = 4 and 8. The results of Figure XI-3 show that both alteration levels introduce a significant improvement of the shape of hole 2. Figure XI-3 also show a the negative effect of the alteration on hole 4. However, based on this figure alone, it is difficult to visualize the effect of the transition from an alteration of 84% to an alteration of 82%. To better visualize this effect, the DAP and the CSC are plotted as a function of friction in Figure XI-4 for both 84% and 82% alteration. Figure XI-4 shows that below 84% alteration the hole 4 DAP decreases at the expense of hole 2 whose DAP goes above that of hole 4. Since these two holes are the major target of this preliminary alteration task, it is obvious to state that this issue has to be addressed before further alteration of the targeted feeding holes.
Figure IX-3. Effect of minimizing the first ring of the feeding holes on the quality criteria for critical level of alterations.
Figure IX-4. Effect of minimizing the first ring of the feeding holes on the quality criteria for alteration of 84% and 82%.
Finally, it is obvious to state that the holes closer to the center of the preform are most affected by decreasing the size of the holes close to the center. This is important as it indicates the behavior is not highly nonlinear.
Figure IX-5. Effect of the Alteration of the feeding holes at the center on the CSC.

Monitoring the CSC as presented in Figure XI-5 forces the study to focus on holes 2 and 4, which are the most distorted, as this criterion shows excellent results for all the remaining holes.
Like the behavior of the DAP quantity, the selected alteration does not affect all the holes in the same way. Most of the observations recorded for the DAP can be observed for the CSC.

Finally, as is shown in Figures IX-6 and 7 this reduction in the size of the feeding holes improves the geometric integrity of the holes centers for all values of friction.

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**Figure IX-6. Effect of minimizing the first ring of the feeding holes on the PCF preform dispersions for 84% (left) and 82% (right) alteration levels.**
Figure IX-7. Effect of minimizing the first ring of the feeding holes by 82% on the holes dispersions for angle (top) and distance (bottom) dispersion.
IX.3. Altering feeding holes at the periphery of the die to adjust the shape of hole 2

Though the previous alteration of the Blockage geometry was targeted to improve the hole closest to the center (hole 4), the reduction of the feeding holes to an optimal size at the center of the die improved the overall quality of the preform. Obviously, a limit of improvement is reached when the targeted radii are set to above 84% of their original values. The target of this section is to explore the possibility of altering other feeding holes to compensate for the degradation of the quality of hole 2, which was affected by a severe reduction of the feeding holes at the center. Following the same logic, the feeding hole rings close to the problematic holes are altered in order to improve the shape of hole 2. As shown in Figure XI-8 in this study the previous alteration was kept at a value of 82% while the feeding holes located above hole 2 were modified. This includes the two largest rings of feeding holes. A friction of $\log \left( \frac{k}{m} \times \frac{\mu}{\eta} \right) = 4$ is chosen for this study while the radii of the targeted feeding holes are incrementally reduced and their effect is monitored for the quality of the preform holes. This operation is meant to reset the equilibrium between the flow rate at the center and the periphery of the preform as well as affecting the holes that are closest to these feeding holes.
Except for the CSC which will be discussed later, all the other outputs show similar results. First, according to the DAP plot in Figure IX-9, the current alteration seems to affect only the holes at the periphery of the preform and the other holes exhibit a very limited effect. Most of the criteria show that at a radii of 96% the quality of hole 2 improves significantly and becomes better than that of hole 4. Below a 96% reduction in hole size this improvement continues, but at the expense of the quality of hole 5. However, even at 84% the quality of hole 5 is still better than that of hole 4. Thus if an overall quality is targeted, 96% is a good choice and could be a good start of a second iteration of alteration.
Figure IX-9. Effect of the second alteration on the DAP and the CSC.

According to Figure IX-9, if the CSC is selected as the evaluation criterion, then 84% seems as a reasonable value of feeding hole size. That being said, it is clear that the final result depends
on the criteria used for the evaluation. A next step could consist of a new iteration or full scale optimization. Similar to Figure XI-3, Figure XI-10 shows the overall effect of several key alteration levels. This helps to illustrate the effect of the second alteration and may help decide on the choice of the quality criterion for this step.

The major target of this study is to take advantage of finite elements to substitute for laborious and expensive experimental study.

Figure IX-10. Effect of minimizing the last ring of the feeding holes on the quality criteria for several alterations.
IX.4. **Adjustment of all the feeding holes for Blockage B to minimize the hole deformation**

The alteration of the feeding hole of Blockage A reveal that the impact of this operation is generally localized and affect mostly the neighboring holes, such observation could be made by comparing the change of DAP of the hole near and far away from the altered feeding hole in Figure XI-2 and 9. Figure XI-2 also shows that even if the alteration process is nonlinear, effect of the alteration on each hole is generally unchanged for an interval of modification.

![Diagram showing target and actual geometries](image)

**Figure IX-11. Hole distortion and drift for the unmodified feeding holes. 22.6% Maximum DAP.**
For this task a friction coefficient of $\log((k/\eta) \times m) = 4$ is selected, far away from the friction region where alteration of the feeding holes in the center results a minimum value of DAP. In this case, both holes 2 and 4 have similar level of DAP. Moreover, Figure IX-11 shows that the initial geometry is highly distorted and fails on providing both good DAP and good geometric integrity.

![Diagram of target and actual geometry]

Figure IX-12. Hole distortion and drift for the modified feeding holes. 7% Maximum DAP.
In this section, a manual optimization is performed over all feeding holes. This process is generally based on the general observations made earlier for the case of Blockage A. The prime target of optimization is to minimize the maximum DAP. The minimization of the average DAP is considered only when a small reduction of the maximum DAP significantly reduce the average DAP. The process targeted all of the feeding holes of the Blockage; however, the alteration of few holes has proven to reduce the quality of the final geometry. In a number of cases, modifications could result in an improvement of few holes at the expense of few others.

Table IX-1. Effect of the optimization on the DAP values.

<table>
<thead>
<tr>
<th></th>
<th>Hole 1</th>
<th>Hole 2</th>
<th>Hole 3</th>
<th>Hole 4</th>
<th>Hole 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original geometry</td>
<td>17.09%</td>
<td>22.31%</td>
<td>6.20%</td>
<td>22.58%</td>
<td>10.80%</td>
</tr>
<tr>
<td>Final Modified geometry</td>
<td>6.0401%</td>
<td>5.1368%</td>
<td>6.8451%</td>
<td>6.3171%</td>
<td>6.9419%</td>
</tr>
</tbody>
</table>

The end result of this process is shown in Figure IX-12. The figure shape shows a significant improvement of the circularity and also an improvement of the geometric integrity. In fact, not only the resulting holes shapes are very close to a circle but also the hole are located right at their targeted positions. As shown in Table XI-1, using this process, the maximum DAP dropped from 22.6% to below 7%. The impact of this modification on the flow rate is presented in Appendix 6-9. Appendix 6 shows a significant change in the flow profile at the die exit: the effect of the friction still can be observed around the pins and the die boundary; however, high flow regions, like the one located the center and near the boundary of the die of the original case, are no longer present. As shown in appendix 7, this translates in a reduction of the radial flow at the die exit especially at the center of the preform.
Table IX-2. Geometric configuration of the optimized bolckage: feeding holes radii.

<table>
<thead>
<tr>
<th>FH 1</th>
<th>FH 2</th>
<th>FH 3</th>
<th>FH 4</th>
<th>FH 5</th>
<th>FH 6</th>
<th>FH 7</th>
<th>FH 8</th>
<th>FH 9</th>
<th>FH 10</th>
<th>FH 11</th>
<th>FH 12</th>
<th>FH 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>70%</td>
<td>75%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>75%</td>
<td>75%</td>
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</tr>
</tbody>
</table>

Table IX-2 shows the geometric configuration of the optimized Blockage. It is important to note that the level of alteration required to improve the preform shape surpasses the amount of the maximum recorded improvement of the DAP. This again highlights a degree of nonlinearity in the process. Inspecting the flow profile at the middle of the welding chamber in Appendix 8 also reveals a tremendous change in the flow behavior, the radial flow in Appendix 9 shows that the flow reacts against the effect of the alteration. In fact, the radial flow increases significantly compared to the initial case and it is redirecting the flow to the regions where the axial flow has been reduced by the modified Blockage.

**IX.5. CONCLUSION**

The results from the previous chapter revealed the presence of a severe deformation of hole 4 at the vicinity of the center of the preform in a wide range of friction. An inspection of the flow rate in the feeding holes and in the Blockage showed that this phenomenon can be caused by a localized high flow rate at the center of the welding chamber. Based on these observations, a first alteration of the Blockage shape was conducted on the feeding holes at the center of Blockage A.

Using the shape quality criteria introduced in the previous chapter, the impact of several alteration levels of the feeding holes was monitored for the entire friction domain. Inspecting the quality of the holes showed that this alteration effect is dependent on the selected friction coefficient. An overall positive effect is observed at high values of friction. In this friction range, a
positive effect on majority of the holes was observed, especially on hole 4. Consecutive reduction of the size of the feeding holes generally improved the quality of majority of the holes. However, a limit of alteration is reached when the operation resulted on a degradation of hole 2 that surpasses the improvement in the quality of hole 4.

For this reason, a second alteration of the feeding holes at the periphery of the Blockage is introduced to overcome this limit. This operation was performed in a separate alteration step for a friction value of \( \log\left(\frac{k}{n} \times m\right) = 4 \). The study showed that this additional alteration reduces the effect of the first alteration on hole 2. However, this alteration has a negatively impact on the other holes and has to be used with caution.

This study revealed the general trends of the effect of modification of few feeding holes radii on final shape of the preform. This information has been exploited to conduct a manual optimization over all the feeding hole of the Blockage. As a result of optimization of the diameter of feeding hole, the final shape of the preform has been successfully matched close to the target geometry. In addition to reducing the DAP, the preform geometric integrity has been significantly improved. A study of the flow at the welding chamber helped explain few aspects of the nonlinearity of process.

IX.6. References

CONCLUSIONS AND FUTURE WORK

X.1. CONCLUSIONS

Though glass extrusion is an important cornerstone in the fabrication of fiber optics, the numerical modeling of the extrusion of glass at high viscosity has not received a lot of attention in the literature. Thus, in order to study the extrusion of PCF preforms, a simpler problem must be validated. First, the establishment of valid assumptions to model glass extrusion at high viscosity was performed based on five experimental cases. This study highlighted the importance of including friction effects to validate both the value of ram force and die swell. Based on this observation, the ring compression test was adapted to extract the Navier friction coefficient, which is the mostly used friction law in the extrusion literature. The extrusion results provided the motivation for a thorough study of the effect of the extrusion parameters on the extrusion process. This was accomplished using a sensitivity analysis on a range of input variables. Both viscous and viscoelastic material models were included in this study. The results from this study correlate with several experimental observations.

Second, the numerical modeling of extrusion of PCF preforms was introduced using the same assumptions that were validated for solid rod extrusion. Two Blockage geometries were used and qualitatively validated. A sensitivity analysis was also performed on these Blockages to investigate the creation of holes and how the holes can be distorted. This suggested the
implementation of several algorithms and mathematical methods to quantify and interpret the distortion data and to provide a way to alter the Blockage geometry in order to improve the geometric quality of the preform. An example of this alteration was carried out.

This study presented an opportunity to explore different aspects of the extrusion of preforms of high viscous glass. The topics covered in this study allowed to:

- Build and validate a numerical model of extrusion of a solid preform at high viscosity using a set of 5 cases of experimental data
- Lay down assumptions guidelines for preform extrusion
- Highlight the importance of slip using the Navier law
- Explore the effects of friction and material properties such as viscoelasticity and shear thinning viscosity.
- Simulate, study and qualitatively validate a real case of PCF Preform extrusion using 2 different type of Blockages
- Very close match was obtained with the experimental results was obtained by adjusting the welding chamber size to the experimental case
- Explore different scaling procedures to build an understanding of the impact of extrusion parameters on the final shape of the preform using 2 different types of Blockages
- Lay down optimization guidelines to improve the final shape of PCF preform using quality variables
• Explore the possibility and the effect of the alteration of Blockage on the final shape of the PCF preform

X.2. Future Work

Ideally future work would be motivated by additional experimental data for PCFs for different geometry. Furthermore, the friction coefficient should be determined in independent testing to see if the model predictions are correct. While model predictions of hole shape were validated in a qualitative sense in Chapter 7, ideally the friction coefficient would be an input rather than a “fitting” parameter. In the event that such higher level validations were not successful, the following behaviors/effects could be added to the model:

• Gravity
• Non-uniform temperature
• Viscoelastic material behavior
• Surface tension
• Friction model and friction differences on different surfaces
• Blockage and/or pin deformation
• Transient phase of the flow

Without the benefit of additional experimental data the primary future work is with optimization of the model to achieve a targeted hole pattern. Ideally this work should make use of the constraint that only circular feeding holes and pins be used to make the die easier to manufacture. Optimization can include changes in size and/or location of the holes and pins. As such the following options for future work can be listed:

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• Additional sensitivity analysis to develop a better understanding of the effect of the size of the feeding holes on hole shape

• Visual adjustment of feeding hole size based on flow profile near the ends of the pins.

• Implementation of an optimizer
APPENDICES
XI.1. APPENDIX 1: PCF MODELS GEOMETRIC PARAMETERS

Characteristic dimensions for the hexagonal pattern geometry.

Vertical dimensions of the computational model.
XI.2. **APPENDIX 2: FLOW PROFILES FOR THE TWO BLOCKAGES AT THE DIE EXIT**

Blockage A. No slip

Blockage B. No slip
XI.3. **APPENDIX 3: RADIAL FLOW FOR THE TWO BLOCKAGES AT THE DIE EXIT**

![Velocity Diagram](image1)

**Blockage A. No slip.**

![Velocity Diagram](image2)

**Blockage B. No slip.**
XI.4. **APPENDIX 4: FLOW PROFILES FOR THE TWO BLOCKAGES AT THE MIDDLE OF THE WELDING CHAMBER**

**Blockage A. No slip.**

**Blockage B. No slip.**
Blockage A. No friction.

Blockage B. No friction.
XI.5. **APPENDIX 5: RADIAL FLOW FOR THE TWO BLOCKAGES AT THE MIDDLE OF THE WELDING CHAMBER**

**Blockage B. Original geometry.**

**Blockage B. Optimized geometry.**
Blockage B. Original geometry.

Blockage B. Optimized geometry.
APPENDIX 6: FLOW PROFILES FOR THE ORIGINAL AND OPTIMIZED CASE OF

BLOCKAGE B AT THE DIE EXIT

Blockage B. Original geometry.

Blockage B. Optimized geometry.
APPENDIX 7 RADIAL FLOW FOR THE ORIGINAL AND OPTIMIZED CASE AT THE DIE

EXIT

Blockage B. Original geometry.

Blockage B. Optimized geometry.
XI.8. **APPENDIX 8: FLOW PROFILES FOR THE ORIGINAL AND OPTIMIZED CASE AT THE MIDDLE OF THE WELDING CHAMBER**

Blockage B. Original geometry.

Blockage B. Optimized geometry.
Appendix 9: Radial flow for the original and optimized case at the middle of the welding chamber

Blockage B. Original geometry.

Blockage B. Optimized geometry.