MODEL-BASED SYSTEM FAULT DIAGNOSIS UTILIZING ADAPTIVE THRESHOLD WITH APPLICATION TO AUTOMOTIVE ELECTRICAL SYSTEMS

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MODEL-BASED SYSTEM FAULT DIAGNOSIS UTILIZING ADAPTIVE THRESHOLD WITH APPLICATION TO AUTOMOTIVE ELECTRICAL SYSTEMS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
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Accepted by:
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ABSTRACT

Recent advancement in the field of automotive industry is largely due to the addition of electrical and electronic equipment; some of the safety features in the vehicle and advanced driver assistance systems are examples of such equipment. Furthermore, safe operation of the vehicle is highly dependent on the electrical power generation and storage system (EPGS). Therefore, to ensure optimal operation of this system, a reliable diagnosis of the system is essential. However, as the complexity of the electrical systems has increased, the identification of a malfunction has become an increasingly difficult task to handle.

In the current work, a model-based diagnostic approach for the EPGS system is formulated using the residual generation and adaptive threshold method. The EPGS system comprises an alternator and a battery. Since the focus of the current work is on the vehicle alternator subsystem of the EPGS system, a mathematical model of the alternator subsystem based on the physics of the processes involved is derived. This model is characterized by time-varying nonlinear ordinary differential equations. To simplify the diagnosis scheme development, an equivalent linear time invariant model based on the behavior of the input/output of the alternator is presented. Afterwards, three typical faults for a vehicle alternator, namely belt slipping fault, open diode fault and voltage regulator fault, are modeled and injected into the model separately to observe the effectiveness of the adaptive threshold-based fault diagnosis scheme for fault detection and isolation (FDI). The proposed adaptive threshold scheme for the EPGS system has
proven to be more sensitive and more robust than previously presented diagnostic schemes for the same system as available in the literature.

In addition to the classical adaptive threshold method, a novel general methodology is presented for the derivation of adaptive thresholds in the case of linear time varying-parameter systems, and Gaussian distributed linear parameter systems. The high order of the threshold dynamics in general is the main drawback of this approach. To overcome this problem, order reduction methods can be used. In this thesis, we explore two approximations, namely the steady state threshold and a first order threshold approximation. The study shows that these approximations are effective in detection and isolation of faults, however, a false alarm rate is introduced.

Moreover, the qualitative modeling of the equivalent system via stochastic automaton is also investigated, and a new approach for the evaluation of the transition probabilities based on the Divergence Theorem is proposed.
DEDICATION

This thesis is dedicated to my family: My beloved mother, Maryam Rezapour, my father, Farshid Hashemi, and my brother Ramin Hashemi.

This thesis is also dedicated to my American family: Dr. Douglas Limbaugh Senior, and Janice Limbaugh.
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Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.
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CHAPTER ONE

BACKGROUND AND INTRODUCTION

1.1. MOTIVATION

This thesis is organized as follows: background and motivation for the current work is given in the first chapter. A complete model description along with the mathematical model of the system is given in chapter 2, and the proposed fault diagnosis scheme is presented in chapter 3. Chapter 4 is devoted to signature derivation of the previously introduced fault diagnosis scheme along with two other methods to obtain the adaptive threshold parameters, namely a novel general methodology for the derivation of adaptive thresholds in the case of linear time varying-parameter systems, and Gaussian distributed linear parameter systems. Quantization of the residual via stochastic modeling is the subject of chapter 5. And we present the concluding remarks in the final chapter.

Recent advancements in modern vehicles are highly dependent on the safe operation of power generation and storage system (EPGS) in the vehicle. Addition of many modern electrical systems and electronic devices such as advanced driver assistance systems (lane keep assist, blind spot assist to name a few), require a reliable power generation system in the vehicle. Moreover, some safety applications such as X-by-wire system (A. S. P. Pisu 2006) necessitate having a reliable power generation source in the vehicle. Consequently, to maintain the optimal performance of the vehicle, a robust, effective diagnosis algorithm for the EPGS system is necessary. An effective diagnostic algorithm has many advantages among which are more efficient and faster repair works since mechanics can check the fault and immediately replace the faulty
component; the engine can potentially be serviced due to the condition of the engine and not due to a service schedule, thus saving service costs. Furthermore, the diagnostic algorithm can make the driver aware of faults that can damage the engine, so that the car can be taken to a repair shop in time. This in turn, increases the reliability. The main issue, however, is the increased complexity of the modern electrical systems which in turn makes the task of fault detection and isolation (FDI) extremely difficult.

Technical systems are inherently exposed to faults. In most applications, it is crucial that these faults are detected and isolated at an early stage and accommodated for. Fault detection aims to recognize abnormal behavior of components and processes through information contained in variables based on measured signals. Faults could be defined as unacceptable deviation of at least one characteristic property or feature from the standard conditions. Faults may or may not lead to a failure, i.e. a permanent interruption of a required function.

Fault detection and diagnosis generally includes three functions:

1) Fault detection: indicate the presence of faults and the time of detection.

2) Fault isolation: determine the location of the faults after their detection.

3) Fault identification: determine the size of the faults and their time variant behavior.

These three functions are referred to as FDI in the current work.

A typical automotive EPGS has to supply enough power to drive numerous electrical and electronic components; it also has to charge the battery. The diagnostic problem focuses on the FDI of certain set of alternator faults, namely, open diode
rectifier, belt slipping and voltage regulator faults. In order to formulate an effective fault diagnostic scheme, a good understanding of the system as well as the origin of the faults is necessary. For this reason, the next chapter is devoted to explain and derive governing equations of the vehicle alternator system along with its modeling procedure. Several studies have been conducted on the modeling of the electrical system, with particular attention given to the generator and the battery (H. Bai 2002), (D. J. V. Caliskan 2003), (G. Henneberger 1995), (Z. M. Salameh 1992). However, none of these studies give particular attention to the problem from a diagnostic perspective.

In the current work, we present a model-based diagnostic approach of an alternator subsystem of the EPGS system. The alternator model is based on an accurate model of the AC synchronous generator (Caliskan 2000) derived using the principle of magnetic induction, a model of the diode bridge rectifier (Marques 1998) and a PI (proportional-integral) control describing the voltage regulator. The load also in modeled by a prescribed load current profile obtained from previous experimental data.

1.2. **Analytical Redundancy**

The simplest method to approach FDI, usually used in many industrial applications, is based on physical redundancy which adds to the production costs. Moreover, it makes the system more complicated. In contrast, analytical redundancy uses the sophisticated model-based techniques for model-based FDI. The main idea is to use the redundancy in information obtained from measurement in combination with a process model. The techniques used are various and include, among others, linear or nonlinear
observer, parameter estimation, parity equations, frequency spectral analysis, and other algorithmic approaches.

Model-based diagnosis has the potential to have the following advantages:

- It provides higher diagnosis performance by detecting smaller faults and by shortening the detection time.
- It can provide larger operating range for diagnostic performance.
- It makes the passive analysis possible.
- It makes the isolation of different faults possible.
- It achieves higher diagnosis performance by compensating for disturbances.
- It requires no extra hardware.

There are two primary steps in all FDI algorithms: residual generation and residual evaluation. The purpose of the first step is to generate a signal, or more precisely a residual, which is supposed to be zero for a faultless system and nonzero otherwise. Generally, the residual is obtained by comparing the plant output with the model(s) output. The two main properties that are desired in a fault detection algorithm are robustness and sensitivity. The first one, in this context, means the fault detection system does not produce incorrect diagnoses due to disturbances and modeling error, whereas the second one should be understood as sensitivity to faults. These two properties are often conflicting. The issues of sensitivity and robustness have been addressed in many ways; a common approach is to optimize the residual generator to be sensitive to faults and insensitive to disturbances utilizing various approaches that time domain and frequency domain methods, as well as deterministic and stochastic approaches.
Generally speaking, for a linear system subject to faults, disturbances and parametric uncertainties, a residual can be represented in the frequency domain as

\[ r(s) = H(s) \cdot \left[ G_f(s) \cdot p(s) + G_d(s) \cdot d(s) + \Delta G_u(s) \cdot u(s) \right] \tag{1.1} \]

where \( G_f(s) \), \( G_d(s) \) are the transfer function matrices associated to the fault vector \( p(s) \) and disturbance vector \( d(s) \) respectively, \( \Delta G_u(s) \cdot u(s) \) represents the effect of the modeling errors on the residual, and \( H(s) \) is an arbitrary transfer function matrix. In this context, the residual generator given by (1.1) is said to be robust to the disturbance vector \( d(s) \) if \( H(s) \cdot G_d(s) = 0 \); or is said to be robust to parameter uncertainties if \( H(s) \cdot \Delta G_u(s) = 0 \). However, even when robustness is achieved, a residual is almost always nonzero even if there is no fault due to noise, unknown disturbances, and model uncertainty such as unmodeled dynamics, etc.

The purpose of the second step of the FDI process is therefore to evaluate the residual to draw conclusions as to the presence of a fault. This is done by comparing some function of the residual, the evaluation signal, to a threshold and then to declare the presence of a fault if the former exceeds the latter.

When the decision-making of FDI is made robust against uncertainty, we can speak of passive robustness in FDI. Active robustness, on the other hand, is based on making the residual generator insensitive to disturbances and modeling error. Here, we assume that available methods to achieve active robustness in the residual generation stage have already been employed.
The goal of robust decision-making is to minimize error rates due to the effects that model uncertainty and unknown disturbances unavoidably have on the residuals. This can be achieved in several ways; for instance, through statistical data processing, averaging, or finding and using the most effective threshold via optimization (Z. Qiu 1993) and (X. Ding, P. M. Frank 1991). When a fixed threshold is used, there is an unavoidable trade-off between robustness and sensitivity. In many applications, the cost of false alarms can be higher than that of missed detections, hence leading to the need to set a rather conservative threshold. Moreover, if the effect of noise is to be considered, the threshold should be chosen above noise level which in turn, increases the chance of misdetection.

1.3. **Literature Review**

Selecting the proper threshold is one of the challenges of the fault diagnostic problem since the effectiveness of the residual evaluation is highly dependent on the threshold value. One of the common methods to select the threshold is based on finding an optimized threshold, (X. Ding, P. M. Frank 1991) and (Z. Qiu 1993), in order to have the minimum probability of false alarm and probability of misdetection possible since these two are mutually exclusive. However, there is always a good chance of having false alarm in this case especially during state transition.

Among the various methods used to achieve passive robustness, the one that has received the most attention is based on adaptive thresholds. In the adaptive threshold approach, residual thresholds are varied according to the control activity of the process and observed measurements. The adaptive threshold is generated through a dynamical
system whose trajectories are an upper-bound of the faultless residual dynamics. This idea was initially proposed by Horak in (Horak 1988) and Emami-Naeini et al. (A. Emami-Naeini 1988). Ding and Frank (P. M. Frank 1991) developed this concept in connection with frequency domain approaches. Seliger et al. (R. Seliger 1993), and later Ding et al. (X. Ding, 2010) developed adaptive threshold concept in the discrete time-domain for observer-based fault detection systems. Recently, based on some results from Zhang et al. (Q. Zhang 2004) on fault diagnosis for a special class of nonlinear systems, and by Pisu et al. (A. Scacchioli, 2007) on hierarchical model-based fault detection and isolation, Pisu et al. (P. Pisu, 2006) have extended the results of adaptive threshold to the case of continuous time-domain with parameter uncertainties and observer-based residual generation, which is used extensively in the current work.

In addition to the classical adaptive threshold method, two novel general approaches are presented for the derivation of adaptive thresholds in the case of linear time varying-parameter systems, and Gaussian distributed linear parameter systems which have never been presented before in the literature.

Fault diagnosis of EPGS for the three types of alternator has been investigated in (G. R. A. Scacchioli 2007) and (G. R. A. Scacchioli 2006) in particular. In the first one, hierarchical model-based approach has been used for the purpose of FDI of alternator and battery subsystems of EPGS system. In (G. R. A. Scacchioli 2006), the idea is to use a parity equation approach and compare the behavior of the alternator with the behavior of the equivalent model to produce residuals. These two approaches are only able to detect the high amount of faults. The proposed adaptive threshold-based approach in this work,
however, is able to detect reasonably small amount of the faults as they occur in the system compared to the previously mentioned two approaches; therefore, making this approach the most powerful of the three.
CHAPTER TWO
MODEL DESCRIPTION

2.1. INTRODUCTION

This chapter presents an overview on the EPGS system. Since we are mainly interested in developing a diagnosis scheme for the alternator portion of the EPGS system, a complete mathematical model of the alternator will be derived. The mathematical model for the battery can be found in (W. L. A. Scacchioli 2009).

As we will see later, the mathematical model of the alternator is nonlinear and complex. To obtain a robust fault diagnosis scheme, a simpler linear model of the alternator will be given which will be used in later chapters to design the fault diagnosis scheme.

2.2. AUTOMOTIVE ELECTRIC-POWER GENERATION STORAGE SYSTEM

An automotive electric-power generation storage system (EPGS) as shown Figure 2.1 in comprises two basic subsystems, the alternator and the battery, which together supply power to the vehicles electrical loads. Figure 2.2 shows a simple diagram of an alternator and a battery along with all other electronic and electrical subsystems lumped together into one current sink in an EPGS system. Since we are interested in diagnosing faults only on the EPGS system and not any of it’s the electrical loads, the loads of the system can therefore be represented as a single, time-varying current sink.

The alternator, which is driven by the engine through a belt, provides power to the electrical loads and charges the 12 Volt lead acid batteries. The battery, on the other
hand, provides power when the engine is not running, or when the electrical power demand exceeds the alternator output.

When the engine is running, the alternator AC voltage is rectified through the three phase bridge. The DC output voltage is regulated to be 14.4V. The role of the excitation field is to produce the field current necessary to excite the three-phase synchronous generator.

The alternator model is based on an accurate model of the AC synchronous generator (Caliskan 2000) and (W. L. A. Scacchioli 2009), and model of the diode bridge rectifier (Marques 1998) and PI control to model the voltage regulator. The battery equations are given in (HanSung 2002) are based on an equivalent second order circuit to characterize the electrical behavior and a first order model that describes the thermal behavior.

Figure 2.1. EPGS system, as reported in (Bosch 2004)
2.3. ALTERNATOR

Since the focus of this thesis is on the alternator portion of EPGS system, we will discuss more about the physical principles behind the vehicle alternator. The mathematical model of the battery is explained in detail in (W. L. A. Scacchioli 2009).

An alternator is an electromechanical device that converts mechanical energy to alternating current electrical energy. It comprises a magnet and a loop of wire which rotates in the magnetic field of the magnet; when the magnetic field around a conductor changes, current is induced in the conductor. Usually, a rotating magnet, which is called the rotor, turns within a stationary set of conductors wound in coils on an iron core, the stator. The field cuts across the
conductors, generating an electrical current, as the mechanical input causes the rotor to turn.

Principle of electromagnetic induction states a current flowing through a wire produces a magnetic field around it. When an electric conductor cuts through a magnetic field, a voltage is induced in it. The opposite case (when the field lines of a moving magnetic field cut through a conductor’s path) gives the same results.

The typical alternator for an automotive electrical system comprises the following components:

1) AC synchronous generator
2) Three phase full bridge diode rectifier
3) Voltage regulator
4) Excitation field.

Alternators are advantageous over direct-current generators in a sense that they do not use a commutator; this makes them lighter, simpler and less costly. Furthermore, stronger construction of automotive alternators allows them to use a smaller pulley to permit them turn faster than the engine, thus improving output when the engine is idling. Alternators utilize rectifiers (Diode Bridge) in order to convert AC current to DC. They also have a three-phase winding to provide direct current with low ripple. Moreover, the pole-pieces of the rotor are shaped (claw-pole) so as to produce a voltage waveform closer to a square wave; this type of wave, when rectified by the diodes, produces even less ripple than the rectification of three-phase sinusoidal voltages.
The most common alternator type used in the automotive industry is the Lundell (claw-pole) alternator as shown in Figure 2.3, where the field north and south poles are all energized by a single winding, with the poles looking rather like fingers of two hands interlocked with each other. Its rotor consists of two circular bodies with claws protruding against the other body of different polarity. Between the two, the excitation wire is wound. The magnetic fields will be somewhat asymmetric, but the field lines will enter the metal rotor perpendicularly through the South pole claws, and exit perpendicularly through the north pole claws.

Modern automotive alternators have a built in voltage regulator that operates by modulating the small field current in order to produce a constant voltage at the stator output. The field current is much smaller than the output current of the alternator. The field current is supplied to the rotor windings by slip rings and brushes. Longer life and greater reliability of the alternators is ensured through the low current and relatively smooth slip rings; in DC generator, however, this is not achieved due to utilization of commutator and higher current being passed through its brushes.

![Figure 2.3. Claw-Pole (Lundell) Alternator.](image-url)
2.3.1. AC SYNCHRONOUS GENERATOR

The alternator considered is the claw pole alternator (Lundell), which is a three phase wound-field auto-excited synchronous machine (Figure 2.4).

The rotor is driven by the engine through the belt. The relation between the mechanical rotational speed and the electrical rotational speed is a function of the number of poles \( p \) as shown in the equation (2.1).

\[
\omega_m = \frac{p}{2} \omega_e
\]  

Figure 2.4. Structure of a Lundell alternator.

Schematics of the rotor and the stator circuits and the electrical behavior of the two circuits is summarized in Figure 4, where \( V_{abc} \) are the terminal voltages and \( V_f \) is the excitation field voltage, \( R_{abc} \) are the resistances of the stator windings and \( R_f \) is the resistance of the field winding.
Figure 2.5. Three-phase AC synchronous alternator circuit.

$E_{abc}$ are the induced electromotive forces (EMF) of individual stator phases and $E_f$ is the induced EMF of the excitation field. We can calculate the EMFs as the flux linkage derivative, as shown in (2.2):

$$E_{abc} = \frac{d\lambda_{abc}}{dt}$$

$$E_f = -\frac{d\lambda_f}{dt}$$

(2.2)

2.3.2. THREE-PHASE DIODE RECTIFIER

The three-phase diode rectifier transforms the alternating current generated by the AC synchronous generator into direct current. The constant-voltage battery load on the alternator makes the analysis of the system different from the classic case of a diode rectifier with a current-source load, and so to develop an analytical model of this system is not satisfactory.
2.3.3. SWITCHING VOLTAGE REGULATOR

The voltage regulator task is to maintain the alternator’s output voltage at a fixed value (reference voltage), independent of engine speed or loads connected to the electrical system. This is accomplished by controlling the voltage applied to the field coil. The mean field voltage is controlled by a pulse-width modulated switching circuit that connects and disconnects the system voltage to the field coil. Figure 2.6 depicts the switching behavior according to (2.3).

\[
\text{if } V_{dc} > V_{ref} \quad \Lambda = 0 \quad \text{else} \quad \Lambda = 1
\]

\[V_f = \Lambda \cdot V_{dc}\]  \hspace{1cm} (2.3)

Figure 2.6. Schematic of voltage regulator

In (2.3), \(\Lambda\) denotes an on-off switch.

2.4. MATHEMATICAL MODEL OF THE ALTERNATOR

The common structure of claw-pole alternator can be modeled as in Figure 2.7.
The three-phase AC synchronous generator is modeled based on coupled-circuit of the electrical dynamics of the stator and rotor as shown in (2.4).

\[
\begin{align*}
E_a &= R_a I_a + V_a \\
E_b &= R_b I_b + V_b \\
E_c &= R_c I_c + V_c \\
E_f &= R_f I_f + V_f
\end{align*}
\] (2.4)

Where \(V_a, V_b \) and \(V_c\) are the applied terminal voltage and \(V_f\) is the excitation field voltage, \(R_a, R_b\) and \(R_c\) are the resistance of the stator winding, \(R_f\) is the resistance of the field winding and \(E_a, E_b, E_c\) are the induced electromotive forces (emf) of the individual phases and \(E_f\) is the induced electromotive force of the excitation field which are given by

\[
\begin{align*}
E_a &= \frac{d\lambda_a}{dt}, E_b = \frac{d\lambda_b}{dt}, E_c = \frac{d\lambda_c}{dt}, E_f = -\frac{d\lambda_f}{dt}
\end{align*}
\] (2.5)
Where $\lambda_a, \lambda_b, \lambda_c$ and $\lambda_f$ are the flux linkages of the individual phases define as

$$\lambda_a = -L_{a}I_a - L_{ab}I_b - L_{ac}I_c + L_{af}(\theta_e)I_f \quad (2.6)$$

$$\lambda_a = -L_{a}I_a - L_{ab}I_b - L_{ac}I_c + L_{af}(\theta_e)I_f \quad (2.7)$$

$$\lambda_c = -L_{ca}I_a - L_{cb}I_b - L_{c}I_c + L_{cf}(\theta_e)I_f \quad (2.8)$$

$$\lambda_f = -L_{fa}(\theta_e)I_a - L_{fb}(\theta_e)I_b - L_{fc}(\theta_e)I_c + L_fI_f \quad (2.9)$$

Where $L_a, L_b, L_c$ are the self-inductance of the stator related to the three phases and $L_{ab}, L_{ac}, L_{bc}$ are the stator-stator mutual inductance. Stator-rotor mutual-inductances, on the other hand, are described by

$$L_{af}(\theta_e) = L_{fa}(\theta_e) = M \cos(\theta_e) \quad (2.10)$$

$$L_{bf}(\theta_e) = L_{fb}(\theta_e) = M \cos(\theta_e + \phi) \quad (2.11)$$

$$L_{cf}(\theta_e) = L_{fc}(\theta_e) = M \cos(\theta_e - \phi) \quad (2.12)$$

$M$ is the peak stator-rotor mutual inductance, $\theta_e$ is the phase angle of the alternator, and $\phi$ is the angle between stator windings. Equation (2.13) Shows the relationship between phase angle and mechanical angular displacement $\theta_m$ (degrees)

$$\theta_e = \frac{p}{2} \theta_m \quad (2.13)$$

$p$ here is the number of poles in the alternator. If a balanced machine is assumed, the balance equation for the three phase currents can be written as

$$I_a + I_b + I_c = 0 \quad (2.14)$$
The three-phase diode bridge rectifier can be modeled by associating for each of the bridge branches a switching function \((g_a, g_b, g_c)\) to represent the conduction state of the diode presented in the branch. If the diode is active, the switching function is equal to 1, and 0 otherwise. To represent the three-phase diode bridge rectifier in mathematical form, the following equations are used.

\[ V_a = f_a V_{dc}, \quad V_b = f_b V_{dc}, \quad V_c = f_c V_{dc} \]  

\[ f_a = \frac{2g_a - g_b - g_c}{3}, \quad f_b = \frac{2g_b - g_a - g_c}{3}, \quad f_c = \frac{2g_c - g_b - g_a}{3} \]  

\[ I_{dc} = g_a I_a + g_b I_b + g_c I_c \]  

The role of voltage regulator is to maintain the alternator output voltage at a predetermined level across the engine’s complete speed range; this also has to be independent of load and engine speed. Set-point of the regulator might vary as a function of the operating conditions. Without loss of generality, consider the following PI controller voltage

\[ V_f = K_p (V_{ref} - V_{dc}) + K_I \int_{t_0}^{t} (V_{ref} - V_{dc}) dt \]  

Where the field voltage \(V_f\) is saturated at \(V_{dc}\), and \(K_p\) and \(K_I\) are gains.

If we use the following matrix-vector notation:


\[
I = \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}, \quad V = \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}, \quad E = \begin{bmatrix}
E_a \\
E_b \\
E_c
\end{bmatrix}, \quad f = \begin{bmatrix}
f_a \\
f_b \\
f_c
\end{bmatrix}, \quad \lambda = \begin{bmatrix}
\lambda_a \\
\lambda_b \\
\lambda_c
\end{bmatrix}
\]

\[
L_f(\theta_e) = \begin{bmatrix}
L_a(\theta_e) \\
L_f(\theta_e) \\
L_f(\theta_e)
\end{bmatrix}, \quad R = \begin{bmatrix}
R_a & 0 & 0 \\
0 & R_b & 0 \\
0 & 0 & R_c
\end{bmatrix}, \quad L = \begin{bmatrix}
L_{ss} & 0 & 0 \\
0 & L_{ss} & 0 \\
0 & 0 & L_{ss}
\end{bmatrix}
\]

(2.19)

Assuming a balanced three-phase circuit with the following equations

\[
R_a = R_b = R_c = R_{ss}
\]

(2.20)

\[
L_a = L_b = L_c = L_s
\]

(2.21)

\[
L_{ab} = L_{ba} = L_{ac} = L_{bc} = L_{cb} = L_{cs} = L_{ss}
\]

(2.22)

A simpler mathematical expression of the alternator and of the excitation field can be obtained as follows

\[
E = RI + V
\]

(2.23)

\[
E = \frac{d\lambda}{dt}
\]

(2.24)

\[
\lambda = -LI + L_f^T(\theta_e)I_f
\]

(2.25)

\[
V = fV_{dc}
\]

(2.26)

And

\[
E_f = R_f I_f + V_f
\]

(2.27)

\[
E_f = -\frac{d\lambda_f}{dt}
\]

(2.28)

\[
\lambda_f = -L_f^T(\theta_e)I + L_f I_f
\]

(2.29)

\[
V_f = \Lambda(V_{dc})
\]

(2.30)

where \( \Lambda \) denotes the PI controller voltage.
The rotor-stator dynamic is given by

$$\dot{\theta} = -L^{-1}(RI + V) + L^{-1}\left(d\frac{\partial}{\partial \theta_e}L_f(\theta_e)\omega I_f + L_f(\theta_e)\dot{\theta}\right)$$  \hspace{1cm} (2.31)

And the dynamic of the field circuit is given by

$$\dot{I}_f = -\frac{R_f}{L_f}I_f - \frac{1}{L_f}V_f + \frac{1}{L_f}(I + L_f^\tau(\theta_e))\dot{\phi}_f$$  \hspace{1cm} (2.32)

2.5. **Equivalent DC Generator Model**

As we have seen in the previous section, the mathematical model of the alternator and rectifier is highly nonlinear and complex. In order to obtain a robust diagnosis algorithm, an equivalent simpler model that still describes the behavior of the original model in terms of input-output relations will be developed. A closer examination of the alternator subsystem shows that the behavior of the system is functionally similar to that of a DC machine; hence, it can be modeled with an equivalent DC generator model (enclosed in the big rectangle) for the alternator and diode bridge rectifier as shown in Figure 2.8.
After a detailed simulation-based analysis using SABER simulator, EPSsim, made available by GM R&D Center, the input/output behavior of the system has shown to follow the behavior of a DC generator whose equations can be used to derive an equivalent model as described equation (2.33) and with equivalent excitation field as in (2.34), accordingly to what presented by (A. Scacchioli, 2011).

\[
\frac{dI_{dc}}{dt} = -\gamma I_{dc} + \gamma \omega_e + \kappa I_f - \lambda V_{dc}
\]  

(2.33)

\[
\frac{dI_f}{dt} = -\alpha I_f + \beta V_f
\]  

(2.34)

where \(I_f\) is the alternator field current, \(V_f\) is the alternator field voltage, \(I_{dc}\) is the rectified output current, \(\omega_e\) is the angular frequency of the alternator, and \(V_{dc}\) is the rectified output voltage. The parameters \(\alpha, \beta, \gamma, \kappa, \text{ and } \lambda\) are function of \(\omega_e\) and they were determined using system identification methods in a way to best fit the input-output relation of the original system.

Equations (2.33) and (2.34) in observable canonical form can be written as:
The parameters identification was conducted at different constant engine speed in the range between 750 and 3000 rpm as shown in Figure 2.9 for the case under study. This speed profile was chosen to test the alternator’s response to acceleration at low, medium and high engine speed. The following plots (Figure 2.10 to Figure 2.17), present
the values of the parameters verses $\omega_e$. Note that $b_{21}$ is zero by construction as obtained in (2.35).

![Engine Speed Profile](image1)

**Figure 2.9** Engine Speed profile used in the parameters identification.

![Parameter Variation](image2)

**Figure 2.10** Parameter variation of $a_{12}$ versus speed profile
Figure 2.11 parameter variation of $a_{22}$ versus speed profile

Figure 2.12 parameter variation of $b_{11}$ versus speed profile
Figure 2.13 parameter variation of $b_{12}$ versus speed profile

Figure 2.14 parameter variation of $b_{13}$ versus speed profile
Figure 2.15 parameter variation of $b_{21}$ versus speed profile

Figure 2.16 parameter variation of $b_{22}$ versus speed profile
Figure 2.17 parameter variation of $b_{23}$ versus speed profile

The next chapter will discuss the proposed fault diagnosis scheme for the purpose of fault detection and isolation of the alternator subsystem.
CHAPTER THREE

FAULT DIAGNOSIS SCHEME

3.1. INTRODUCTION

In this chapter we propose a fault diagnosis scheme to detect and identify three commonly occurring faults in the alternator system (belt slipping, open diode, and voltage regulator fault), but first, we discuss how the faults and modeling errors are modeled.

3.2. FAULTS AND MODELING ERRORS

Faults are the cause of failure and malfunction in a system. We would like to recognize the performance change of the system in the event of fault occurrence in order to be able to find a diagnostic scheme to resolve the fault or to minimize the its effect on system’s performance.

Faults may be represented as additive or multiplicative, i.e. if it can be modeled as an extra input acting on the system, it is additive fault, and if it can be modeled as corresponding to a change in some plant parameters, it is multiplicative. If possible, we try to model faults as additive since it is easier to model.

In the model for the alternator subsystem in EPGS systems, three types of commonly occurring fault will be considered:

1) Belt slip fault: It is an input fault that occurs when the alternator belt does not have the proper tension to keep the alternator pulley rotating synchronously with the engine shaft. Its effect is a decrease in alternator output voltage, which the voltage
regulator compensates by increasing the field voltage. This fault is described as additive fault through adding a percentage of the electrical frequency i.e. \( \omega_{fe} = \omega_e + \Delta \omega_e \) where \( \omega_{fe} \) denotes the faulty electrical frequency.

2) Open diode rectifier fault: This fault consists of a failure of one of the diodes in the three-phase bridge rectifier, causing unbalance in the bridge by loss of one phase. Characteristics of this type of fault are a large ripple in the output voltage and current. This fault is implemented by changing one parameter or breaking one diode.

3) Voltage regulator fault: This fault consists of a reduction in the reference voltage that produces a reduction in the alternator output current. This fault also can be described as additive fault through adding a percentage of voltage regulator to the nominal value of \( V_{ref} \) i.e. \( V_{ref}' = V_{ref} + \Delta V_{ref} \) where \( V_{ref}' \) denotes the faulty voltage regulator.

Figure 3.1 summarizes the faults considered in the system. In the process of developing the fault diagnosis scheme, it is assumed that the faults occur separately. Moreover, to design the observer-based adaptive threshold, the measurable inputs and outputs of the system are defined. The inputs are \( V_{dc}, V_f, \) and \( \omega_e, \) and the output is \( I_{dc}. \)
3.3. PROPOSED FAULT DIAGNOSIS SCHEME

The proposed diagnostic scheme combines observer design and adaptive thresholds in order to detect and isolate the three types of alternator faults (belt slip, open diode, and voltage regulator). Figure 3.2 shows the overall diagnosis scheme for FDI, and Figure 3.3 shows the diagnosis scheme with more details regarding the inputs and outputs.
Figure 3.2. Fault diagnosis scheme

Figure 3.3. Fault diagnosis scheme with input-output relations
The diagnostic scheme is comprised of three stages: a primary residual generation, a secondary residual generation, and a residual evaluation. The primary residual generation is constituted by the two observers generating two residuals $e_1$ and $e_2$. A third residual is generated from $e_1$ by a moving standard deviation algorithm which constitutes the secondary residual generation stage. Finally, the three residuals are compared with thresholds to generate the three signatures $S_1$, $S_2$, and $S_3$ that represent the residual evaluation stage. The signature $S_1$ is obtained by comparing the adaptive threshold with the residual $e_1$ from the first observer. Table 1 summarizes the fault isolation logic for the alternator fault diagnosis scheme. The main assumption in this fault diagnosis scheme is that faults are not occurring concurrently.

<table>
<thead>
<tr>
<th>Fault</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Fault</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Belt Slip</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Open Rectifier Diode Fault</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Voltage Regulator Fault</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Fault Diagnosis Scheme for the Alternator System.

Within Table 1, a “zero” means ‘residual does not cross the threshold’; while a “one” means ‘residual crosses the threshold’.

As it can be seen from Table 1, Adaptive threshold alone can detect all the three faults in the alternator system. However, in order to isolate the faults, two other
signatures are introduced. Details on how to derive these signatures are the subject of the next chapter.
4.1. **INTRODUCTION**

In the previous chapter, a fault diagnosis scheme was defined. In this chapter, the details on how the signatures are derived will be presented. The main focus of this chapter, however, is on classical adaptive threshold method (signature $S_T$) along with a novel general methodology for the derivation of adaptive thresholds in the case of linear time varying-parameter systems, and Gaussian distributed linear parameter systems.

It is noteworthy to state why we preferred to use adaptive threshold in lieu of a fixed threshold. Adaptive threshold changes according to the inputs to the system; thus, it has many advantages over the fixed threshold. In case of the fixed threshold, if the threshold is set too high, sensitivity to fault detection will decrease. In contrast, if the threshold is set too low, false alarm rate will increase. Adaptive threshold, however, does not have these problems as shown in Figure 4.1. Furthermore, the use of adaptive threshold prevents the possibility of having false alarm in transition state of the system. One downside of using adaptive threshold is, in general, its high order. Adaptive threshold was successfully implemented in (P. Pisu et al., 2006) for diagnosis of steer-by-wire system, but the trade-off between complexity and false alarm rate was not investigated.
In Figure 4.1, false alarm means the event that an alarm is generated even though no faults are present. A missed alarm, on the other hand, means the event that an alarm is not generated in spite of that a fault has occurred; this event might also be called missed detection.

This chapter is organized as follow: first, the model based adaptive threshold design will be presented. Two approximations (steady state threshold and adaptive threshold with lesser order) for the first adaptive threshold will be calculated. Subsequently, two more models of adaptive threshold in the case of time-varying parameters and Gaussian distributed parameters will be derived. And finally, details on how $S_2$ and $S_3$ are derived will be shown.

The proposed fault diagnosis scheme shows to be more sensitive to fault detection and to be faster in fault isolation compared to the other methods presented in the literature (G. R. A. Scacchioli 2007), (G. R. A. Scacchioli 2006)). Furthermore, the two novel methodologies, namely the derivation of adaptive thresholds in the case of linear time varying-parameter systems, and Gaussian distributed linear parameter systems, have never been investigated in the literature before.
4.2. MODEL BASED ADAPTIVE THRESHOLD DESIGN

In order to obtain the signature $S_1$, an observer-based adaptive threshold is designed based on the state space representation of the equivalent DC generator equations (2.33) and (2.34) are given here for easy reading.

\[
\frac{dI_{dc}}{dt} = -\gamma I_{dc} + \gamma \omega_c + \kappa I_f - \lambda V_{dc} \tag{4.1}
\]

\[
\frac{dI_f}{dt} = -\alpha I_f + \beta V_f \tag{4.2}
\]

Consider a general state space presentation of a system with $n$ states in observable canonical form:

\[
\frac{dz}{dt} = \begin{pmatrix}
0 & 0 & K & 0 & -a_0 \\
1 & 0 & L & 0 & -a_1 \\
M & M & M & M & M \\
0 & 0 & L & 0 & -a_{n-2} \\
\end{pmatrix}
\begin{pmatrix}
z_0 \\
z_1 \\
z_2 \\
z_3 \\
\end{pmatrix} + 
\begin{pmatrix}
b_{00} & b_{01} & L & b_{0,m-1} \\
b_{10} & b_{11} & L & b_{1,m-1} \\
L & L & L & L \\
\end{pmatrix}
\begin{pmatrix}
u_0 \\
u_1 \\
u_2 \\
u_3 \\
\end{pmatrix} \tag{4.3}
\]

\[
y = \begin{bmatrix}
0 \\
44 \\
2 \\
14 \\
43
\end{bmatrix}z 
\]

Where $z \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the system input, $y \in \mathbb{R}$ is the system output, $A_0 \in \mathbb{R}^{n \times n}$, $B_0 \in \mathbb{R}^{n \times m}$, and $C_0 \in \mathbb{R}^{1 \times n}$ are the system matrices. Assuming parameters uncertainties (4.3) and (4.4) can be written as:

\[
\Delta x = (A_0 + \Delta A_0)z + (B_0 + \Delta B_0)u 
\]

\[
y = C_0 x \tag{4.5}
\]
Notice that $\Delta A_0 \hat{z} = \Delta a y$ with $\Delta a = [\Delta a_0 \quad \Delta a_1 \quad L \quad \Delta a_{n-1}]^T$. A Luenberger observer can be designed for (4.5) as below:

$$\begin{align*}
\dot{\hat{z}} &= A_0 \hat{z} + B_0 u + L(y - \hat{y}) \\
\hat{y} &= C_0 \hat{z}
\end{align*}$$  \hspace{1cm} (4.6)

With $L$ to be defined so that the eigenvalues of $A_u + LC_u$ are all negative and real for stability.

By defining the error by $e = z - \hat{z}$, the error dynamics can be written as:

$$\begin{align*}
\dot{e} &= (A_0 + LC_0) e - \Delta a y + \Delta B_0 u \\
e_1 &= y - \hat{y} = C_0 e
\end{align*}$$  \hspace{1cm} (4.7)

The output residual dynamics can be obtained by integrating the differential equation of the error dynamics given in (4.7):

$$e_1(t) = C_0 e^{(A_0 + LC_0) \tau} e(0) - \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} \Delta a y(\tau) d\tau + \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} \Delta B_0 u(\tau) d\tau$$  \hspace{1cm} (4.8)

Now define vectors $E_i$ to simplify the notation such that:

$$E_i = \left[ \begin{array}{ccc}
1 & 0 & L & 0 \\
0 & 1 & 4 & 0 \\
0 & 1 & 4 & 0 \\
1 & 4 & 0 & 3 \\
\end{array} \right], i = 2, K , n$$  \hspace{1cm} (4.9)

Using (4.9), the parameter uncertainties can be written as:

$$\begin{align*}
\Delta a &= \sum_{i=1}^n E_i \Delta a_{i-1} \\
\Delta B_0 u &= \sum_{i=1}^n \sum_{j=0}^{m-1} E_i \Delta b_{i-1,j} u_j
\end{align*}$$  \hspace{1cm} (4.10) \hspace{1cm} (4.11)

Hence, (4.8) can be rewritten as
\[ e_1(t) = C_0 e^{(A_0 + LC_0)\tau} e(0) + \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i \Delta a_{i-1,1} y(\tau) d\tau + \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} \sum_{j=0}^{m-1} E_i \Delta b_{i-1,j} \mu_j(\tau) d\tau \] (4.12)

The summations can be taken out of the integral since they are independent of the integration variable. Therefore

\[ e_1(t) = C_0 e^{(A_0 + LC_0)\tau} e(0) + \sum_{i=1}^n \Delta a_{i-1,1} \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i y(\tau) d\tau + \sum_{i=1}^n \sum_{j=0}^{m-1} \Delta b_{i-1,j} \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i \mu_j(\tau) d\tau \] (4.13)

Assuming known upper bounds for (4.10) and (4.11), such that\[ |\Delta a_{i-1,j}| \leq \delta_{a_{i-1,j}}, |\Delta b_{i-1,j}| \leq \delta_{b_{i-1,j}}, \quad i = 1, K, n; j = 0, K, m - 1,\] an upper bound of the output residual in the absence of faults from (4.13) can be derived:

\[ |e_1(t)| \leq \varepsilon_0 + \sum_{i=1}^n \delta_{a_{i-1,1}} \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i y(\tau) d\tau + \sum_{i=1}^n \sum_{j=0}^{m-1} \delta_{b_{i-1,j}} \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i \mu_j(\tau) d\tau \]

\[ \sum_{j=0}^{m-1} \sum_{i=1}^n \delta_{b_{i-1,j}} \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i \mu_j(\tau) d\tau := Z_{th}(t) \]

**\[ \varepsilon_0 = C_0 e^{(A_0 + LC_0)\tau} e(0) \]** (4.15)

\( \varepsilon_0 \) is the term pertinent to the transient response of the adaptive threshold dynamics. An equivalent state space representation of the adaptive threshold dynamics can be written as:

\[ \xi_i(t) = (A_0 + LC_0)\xi_i(t) + E_i y(t) \]

\[ \xi_{i,1} = C_0 \xi_i \]

\[ \psi_{ij} = (A_0 + LC_0)\psi_{ij} + E_i \mu_j(t) \] (4.16)

\[ \psi_{ij,1} = C_0 \psi_{ij} \]

\[ Z_{th}(t) = \varepsilon_0 + \sum_{i=1}^n \delta_{a_{i-1,1}} |\xi_{i,1}| + \sum_{j=0}^{m-1} \delta_{b_{i-1,j}} |\psi_{ij,1}| \]
where  $\xi_i \in \mathbb{R}^n, \psi_{ij} \in \mathbb{R}^n, \forall i = 1...n, j = 0...m-1$, and $\xi_i(0) = 0, \psi_{ij}(0) = 0$. A fault is declared if $|e_1(t)| > |Z_{th}(t)|$, which corresponds to signature $S_f=1$. The threshold just derived can be seen as $(m+1)xn$ filters of order $n$. The high order of the threshold dynamics is the main drawback adaptive threshold. The order can be further reduced to $m+1$ filters of order $n$ by transforming the equations from observable form into controllable form, and combining the equations with the same input as shown in (4.17).

\[
\begin{align*}
\dot{\xi}_i(t) &= (A_0 + LC_0)\dot{y} + C_0 \psi_i(t) \\
\psi_{ij}(t) &= (A_0 + LC_0)^T \psi_j + C_0 ^T u_j(t) \\
\psi_{ij,1} &= E_i^T \psi_j, \quad i = 1K n; j = 0K m-1 \\
Z_{th}(t) &= \xi_0 + \sum_{i=1}^{n} \left( \delta_{ij} \xi_{i,1} \right) + \sum_{j=0}^{m-1} \delta_{i-1,j} \psi_{ij,1}
\end{align*}
\]

(4.17)

Where $\gamma \in \mathbb{R}^n, \psi_j \in \mathbb{R}^n$. As an example, we will obtain the adaptive threshold dynamics for the vehicle alternator system presented in (4.1) and (4.2) by following the procedure explained above for the general system of state space equations. These equations can be transformed in observable form as follows.

\[
\begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & -\alpha \gamma \\
1 & 44 \alpha + \frac{4\beta}{2}
\end{bmatrix}
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} +
\begin{bmatrix}
\kappa \beta & \alpha \gamma & -\alpha \lambda \\
0 & 4 & 4 & 4 & 4 & 3
\end{bmatrix}
\begin{bmatrix}
V_f \\
\omega_e \\
V_{dc}
\end{bmatrix}
\]

(4.18)

\[
y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{z}_1 \\
\dot{z}_2 \end{bmatrix}
\]

(4.19)
Where the relationship between the new and old state variables is given by

\[
z_1 = \kappa I_f + \alpha I_{dc}
\]
\[
z_2 = I_{dc}
\]

(4.20)

For the purpose of being consistent with the parameters identification and to be consistent with the notation in the Gaussian distributed parameters section, we use the following notation for (4.18) and (4.19),

\[
\begin{bmatrix}
  \frac{\partial \Phi}{\partial z_1} \\
  \frac{\partial \Phi}{\partial z_2}
\end{bmatrix}
= 
\begin{bmatrix}
  0 & a_{12} \\
  1 & 2
\end{bmatrix}
\begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix}
+ 
\begin{bmatrix}
  b_{11} & b_{12} & b_{13} \\
  b_{21} & b_{22} & b_{23}
\end{bmatrix}
\begin{bmatrix}
  V_f \\
  \omega \\
  V_{dc}
\end{bmatrix}
\]

(4.21)

\[
y = \begin{bmatrix}
  0 \\
  C_0
\end{bmatrix}
\begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix}
\]

(4.22)

Now, assuming parameters uncertainties, we can write

\[
\begin{bmatrix}
  \frac{\partial \Phi}{\partial z_1} \\
  \frac{\partial \Phi}{\partial z_2}
\end{bmatrix}
= (A_0 + \Delta A_0)
\begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix}
+ (B_0 + \Delta B_0)
\begin{bmatrix}
  u_0 \\
  u_1 \\
  u_2
\end{bmatrix}
\]

(4.23)

We can design an observer as

\[
\begin{aligned}
\hat{z}_1 &= A_0 \hat{z}_1 + B_0 u + L(y - \hat{y}) \\
\hat{y} &= C_0 \hat{z}_2
\end{aligned}
\]

(4.24)

The error dynamics are given by

\[
\begin{aligned}
\dot{e}_1 &= (A_0 + LC_0)e_1 - \Delta \alpha y + \Delta B_0 u \\
e_1 &= y - \hat{y} = C_0 e
\end{aligned}
\]

(4.25)

Where the matrices \(A_0 + LC_0, \Delta \alpha, \) and \(\Delta B_0\) are
\[
A_0 + LC_0 = \begin{bmatrix}
0 & -\alpha \gamma + L_1 \\
1 & -(\alpha + \gamma) + L_2
\end{bmatrix}
\] (4.26)

\[
\Delta a = \begin{bmatrix}
\Delta a_1 \\
\Delta a_2
\end{bmatrix}
\] (4.27)

\[
\Delta B_0 = \begin{bmatrix}
\Delta b_{00} & \Delta b_{01} & \Delta b_{02} \\
\Delta b_{10} & \Delta b_{11} & \Delta b_{12}
\end{bmatrix}
\] (4.28)

By integrating the differential equation given in (4.25), we obtain the output residual dynamics

\[
e_i(t) = C_0 e^{(A_0 + LC_0) t} e(0) + \int_0^t C_0 e^{(A_0 + LC_0) (t - \tau)} \begin{bmatrix}
-\Delta a_1 \\
-\Delta a_2
\end{bmatrix} y(\tau) d\tau + \int_0^t C_0 e^{(A_0 + LC_0) (t - \tau)} \Delta B_0 u(\tau) d\tau
\] (4.29)

By defining

\[
E_1 = \begin{bmatrix}
1 \\
0
\end{bmatrix}, \quad E_2 = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\] (4.30)

We can write

\[
\Delta a = \begin{bmatrix}
\Delta a_1 \\
\Delta a_2
\end{bmatrix} = \sum_{i=1}^2 E_i \Delta a_i
\] (4.31)

\[
\Delta B_0 u = \sum_{i=1}^2 \sum_{j=0}^2 E_i \Delta b_{i-1,j} u_j
\] (4.32)

Assuming \( |\Delta a_i| \leq \delta_{a_i}, |\Delta b_{i-1,j}| \leq \delta_{b_{i-1,j}}, \quad i = 0,1; \quad j = 0,1,2 , \) then we can calculate a bound of the output error dynamics as

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\[ |e_1(t)| \leq \left[ 0, 1 \right] e^{(A_0 + LC_0)(t-\tau)} e(0) + \left| \int_0^t \left[ 0, 1 \right] e^{(A_0 + LC_0)(t-\tau)} \sum_{i=1}^{2} E_i \Delta a_{ij} y(\tau) d\tau \right| + \left| \int_0^t \left[ 0, 1 \right] e^{(A_0 + LC_0)(t-\tau)} \sum_{i=1}^{2} \sum_{j=0}^{2} E_i \Delta b_{i,j} y_j(\tau) d\tau \right| \leq \varepsilon_0 + \sum_{i=1}^{2} \delta_{\varepsilon_i} \left| \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i y(\tau) d\tau \right| + (4.33) \]

\[ \sum_{j=0}^{2} \sum_{i=1}^{2} \delta_{\varepsilon_i} \left| \int_0^t C_0 e^{(A_0 + LC_0)(t-\tau)} E_i y_j(\tau) d\tau \right| = Z_{th}(t) \]

The last expression defines the adaptive threshold dynamics as an upper bound of the output error in absence of faults.

From (4.33), we can derive a state space representation of the adaptive threshold dynamics

\[ \dot{\xi}_i = (A_0 + LC_0)\xi_i + E_i y(t) \]

\[ \dot{\xi}_{i,2} = C_0 \xi_i \]

\[ \psi_{\varepsilon_i} = (A_0 + LC_0)\psi_{\varepsilon_i} + E_i y_j(t) \]

\[ \psi_{\varepsilon_i,2} = C_0 \psi_i \]

\[ Z_{th}(t) = \varepsilon_0 + \sum_{i=1}^{2} \left( \delta_{\varepsilon_i} \right) \left| \xi_{i,2} \right| + \sum_{j=0}^{2} \delta_{\varepsilon_j} \left| \psi_{\varepsilon_j} \right| \]

For this example, each segment of the threshold dynamics can be written as

\[ \xi_1 = \begin{bmatrix} 0 & -\alpha \gamma + L_1 \\ 1 & -(\alpha + \gamma) + L_2 \end{bmatrix} \xi_1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} y(t) \]

\[ \xi_{1,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_1 \]

\[ \xi_2 = \begin{bmatrix} 0 & -\alpha \gamma + L_1 \\ 1 & -(\alpha + \gamma) + L_2 \end{bmatrix} \xi_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t) \]

\[ \xi_{2,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_2 \]
We have a fault if $|\varepsilon_1(t)| > |Z_{th}(t)|$.

Notice that we obtain a state space representation for the threshold with 16 states.

In order to reduce the order of the threshold dynamics equations, with transform them to the controllable form

\[
\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -\alpha\gamma + L_1 & -(\alpha + \gamma) + L_2 \end{bmatrix} y(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_j(t) \quad j = 0, ..., 2
\]

\[
\psi_{1j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)
\]

\[
\psi_{1j,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{1j}
\]

\[
\psi_{2j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{2j}
\]

\[
Z_{th}(t) = e_0 + \sum_{i=1}^{2} (\delta_{i1} |\xi_{i1}| + \sum_{j=0}^{2} \delta_{i1,j} |\psi_{i,j}|)
\]

\[
\psi_{2j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{2j}
\]

\[
\psi_{1j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)
\]

\[
\psi_{1j,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{1j}
\]

\[
\psi_{2j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{2j}
\]

\[
\psi_{2j,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \psi_{2j}
\]

\[
\eta_{j} = \begin{bmatrix} 0 & 1 \\ -\alpha\gamma + L_1 & -(\alpha + \gamma) + L_2 \end{bmatrix} \eta_j + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_j(t) \quad j = 0, ..., 2
\]

\[
\psi_{1j,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_j
\]

\[
\psi_{2j,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_j
\]

\[
(4.35)
\]

\[
(4.36)
\]

\[
(4.37)
\]
We can combine the equations with the same input to create a reduced order system as follows

\[
\frac{d}{dt} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha \gamma + L_1 & -(\alpha + \gamma) + L_2 \end{bmatrix} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)
\]

\[
\begin{bmatrix} \xi_{2,2} \\ \xi_{\gamma,1,2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix}
\]

\[
\frac{d \eta_{ij}}{dt} = \begin{bmatrix} 0 & 1 \\ -\alpha \gamma + L_1 & -(\alpha + \gamma) + L_2 \end{bmatrix} \eta_{ij} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_j(t) \quad j = 0, ..., 2
\]

\[
\begin{bmatrix} \psi_{2,2} \\ \psi_{\gamma,1,2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \eta_{ij}
\]

\[
Z_{th}(t) = e_0 + \sum_{j=1}^{2} \left( \delta_{a_j} \xi_{2,2} + \sum_{j=0}^{2} \delta_{b_{i,j}} \psi_{\gamma,1,2} \right)
\]

The representation of the dynamic threshold in (4.38), (4.39) and (4.40) consists of an 8th order linear time-invariant system.

The following plots show the simulation results obtained from implementing the above adaptive threshold in the Matlab/Simulink environment utilizing a portion of Federal Urban Driving Schedule (FUDS) as shown in Figure 4.2. The amount of belt slipping fault injected to the system is 40 percent of the nominal value of \( \omega_e \) in Figure 4.5. The voltage regulator fault amount is 15 percent of the regulated voltage in Figure 4.7. As we can see in Figure 4.11, adatpive threshold fails to detect the fault when the belt slipping amount is 15% with respect to the nominal \( \omega_e \). Figure 4.9 shows the voltage regulator fault when the fault amount is 8% with respect to the nominal regulated voltage which still can be detected with this method, but the residual stays shorter amount of time above the threshold with respect to that of Figure 4.7. However, it can not be isolated
since $S_3$ signature which will be defined later in this chapter cannot be triggered with this amount of fault.

Figure 4.2. Portion of Federal Urban Driving Schedule (FUDS).

Figure 4.3. Current Load Profile.
Figure 4.4. Residual and adaptive threshold signals of $S_1$ when no fault is injected.

Figure 4.5. Residual and adaptive threshold signals of $S_1$ when belt slipping fault with magnitude of $40\%$ of the nominal $\omega_e$ is injected at $t = 10s$. 

Figure 4.6. Residual and adaptive threshold signals of $S_1$ when open diode fault is injected at $t = 10$s.

Figure 4.7. Residual and adaptive threshold signals of $S_1$ when voltage regulator fault with magnitude of 15% of the nominal voltage reference is injected at $t = 10$s.
Figure 4.8. Residual and adaptive threshold signals of $S_1$ when belt slipping fault with magnitude of 15% of the nominal $\omega_e$ is injected at $t = 10s$.

Figure 4.9. Residual and adaptive threshold signals of $S_1$ when voltage regulator fault with magnitude of 8% of the nominal voltage reference is injected at $t = 10s$.

Figure 4.4 shows the residual and adaptive threshold in the no fault case. As it can be observed, the threshold changes with changes in the system inputs, and stays on top of the residual at all times even when the current load jumps from 10 $A$ to 45 $A$. The other
figures show the power of adaptive threshold method in detecting the injected faults. With the exception of the belt slipping fault, adaptive threshold is capable of detecting the faults in the system throughout the simulation time when they have been occurred. If we were to use a fixed threshold, faults would only be detected when the jump in the current load had occurred. Using the adaptive threshold, however, enables us to detect the faults as soon as they occur in the system, namely at $t=10$ for this case.

Note the residual shape in the case of the open diode fault; it is different from the residual in other fault cases due to a large ripple in the output voltage and current.

The 8th-order adaptive threshold given in (4.40) is computationally expensive to implement, especially if compared with the original system of 2nd-order. Moreover, to find a more robust diagnosis algorithm, it is desirable to approximate the adaptive threshold with lesser order. Thus, two other alternatives were explored: one by neglecting the dynamics of the adaptive threshold equations, and the other one by keeping the dominant poles of the adaptive threshold dynamics and removing the other poles. Each approximation is derived so that the steady state gain is unchanged. It is important to highlight that the approximations do not guarantee the condition given by (4.33) anymore, therefore introducing a false alarm rate. This problem may be eliminated by increasing the term $\epsilon_0$, but this will increase the fault miss detection rate.

### 4.2.1. Zero Order Threshold Approximation (Steady State)

In the zero order threshold case, the threshold dynamics are neglected. The threshold is directly dependent on the input of the system. Consider one of the filters given (4.3).
\[
\frac{d\gamma}{dt} = \begin{pmatrix}
0 & 1 & K & 0 & 0 \\
0 & 0 & L & 0 & 0 \\
M & M & M & M & M \\
0 & 0 & L & 0 & 1 \\
\end{pmatrix}^T \gamma + \begin{pmatrix} 0 \\
0 \\
M \\
1 \\
\end{pmatrix} y(t)
\]

\[
\begin{bmatrix}
\varepsilon_{1,1} \\
\varepsilon_{1,n}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & L & 0 \\
0 & 1 & L & 0 \\
0 & 0 & L & 0 \\
0 & 4^4 & 2 & 4^3 \\
\end{bmatrix} \gamma
\]

Where \( \alpha_i = -a_i + l_i, i = 0K \, n - 1 \). The equivalent transfer functions of can be obtained as below:

\[
\begin{pmatrix}
T_1(s) & T_2(s) & L & T_n(s)
\end{pmatrix}^T = C(sI - A)^{-1} B
\]

\[
\begin{pmatrix}
T_1(s) & T_2(s) & L & T_n(s)
\end{pmatrix}^T = \begin{bmatrix}
1/p(s) & s/p(s) & L & s^{n-1}/p(s)
\end{bmatrix}^T
\]

where \( p(s) = s^n + \alpha_{n-1}s^{n-1} + L + \alpha_1s + \alpha_0 \) is the characteristic equation polynomial.

From (4.40) the steady state gains are \(-1/\alpha_0\) for \( T_j(s) \) and zero for the remaining of the states. Similar analysis can be done for the other sets of equations in (4.38) and (4.39).

At the end, we can write the zero order thresholds as

\[
Z_{th}^{st}(t) = e_0 + \sum_{i=1}^2 (\delta_{a_i} \frac{y(t)}{\alpha_0} + \sum_{j=0}^{m-1} \delta_{b_{j-1,j}} \frac{u_j(t)}{\alpha_0})
\]

For the second order vehicle alternator system, (4.44) can be written as

\[
Z_{th}^{st}(t) = e_0 + \sum_{i=1}^2 (\delta_{a_i} \frac{y(t)}{-431660} + \sum_{j=0}^{2} \delta_{b_{j-1,j}} \frac{u_j(t)}{-431660})
\]
The following plots shows the $S_I$ signature when a steady state threshold is implemented to detect the faults in the system.

![Residual & Threshold-No Fault](image)

**Figure 4.10.** Residual and adaptive threshold signals of $S_I$ when no fault is injected.

![Residual & Threshold-Belt Slipping Fault](image)

**Figure 4.11.** Residual and adaptive threshold signals of $S_I$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$. 
Figure 4.12. Residual and adaptive threshold signals of $S_1$ when open diode fault is injected at $t = 10s$.

Figure 4.13. Residual and adaptive threshold signals of $S_1$ when voltage regulator fault is injected at $t = 10s$ for fault magnitude of 30% with respect to the nominal voltage reference.

As it is apparent from the figures above, the steady state adaptive threshold has the ability to detect the three types of faults as they occur in the system as time $t=10s$. 
However, the simulation time was significantly shorter than that of the adaptive threshold with dynamics considered. The observations mentioned in the previous section about the figures are still valid for the steady state adaptive threshold approximation.

### 4.2.2. Quantifying Error Between Eighth Order and Zero Order Adaptive Threshold

In the following we are analyzing the error between 8th order adaptive threshold and its zero order approximation given in (4.40) and (4.45). As we have mentioned previously, the zero order approximation does not guarantee anymore zero false alarm rate. The purpose of the following calculation is to determine an upper bound on the probability of false alarm associated with the use of the zero order approximation.

We consider the first set of state space equations for the threshold as in (4.38) and (4.39).

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ \frac{a_{12}}{4} + \frac{L_2}{4} & \frac{a_{22}}{4} + \frac{L_3}{4} \end{bmatrix} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{b} \end{bmatrix} y(t) \\
\begin{bmatrix} \xi_{2,2} \\ \xi_{1,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} \gamma_{1,1} \\ \gamma_{1,2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} y(t)
\end{align*}
\]

(4.46)

In steady state, the equations reduce to:

\[
\begin{align*}
\frac{d}{dx} \gamma_{11} &= 0 \Rightarrow \gamma_{12} = 0 \\
\frac{d}{dx} \gamma_{12} &= 0 \Rightarrow (a_{12} + L_1)\gamma_{11} + (a_{22} + L_2)\gamma_{12} + y(t) = 0
\end{align*}
\]

(4.47)

The output \( \xi_{12}(t) \) of the second order system (4.46) is given by
The error between the second order and zero order approximation of the threshold is

$$\left| \xi_{1,2}(t) - \xi_{z2}^{(0)} \right| = \left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} e^{A(t)} \gamma_1(0) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \int_0^t e^{A(t-\tau)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(\tau) d\tau \right| - \frac{y(t)}{a_{12} + L_1}$$

First, we find the matrix exponential of the matrix $A$:

$$A = \begin{bmatrix} 0 & 1 \\ a_{12} + L_1 & a_{22} + L_2 \end{bmatrix} \rightarrow A = \begin{bmatrix} 0 & 1 \\ u & v \end{bmatrix}$$

$$e^{At} = L^{-1} \left\{ sI - A \right\}^{-1} = L^{-1} \left\{ \frac{1}{s(s-v)-u} \begin{bmatrix} s-v \\ u \end{bmatrix} \right\} = L^{-1} \left\{ \begin{bmatrix} \frac{s-v}{s^2-vs-u} \\ \frac{1}{s^2-vs-u} \end{bmatrix} \begin{bmatrix} s-v \\ s \end{bmatrix} \right\} = \begin{bmatrix} \frac{s-v}{s^2-vs-u} \\ \frac{1}{s^2-vs-u} \end{bmatrix}$$

Since the inverse Laplace of this matrix is multiplied by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ only the inverse of element $\frac{1}{s^2-vs-u}$ needs to be calculated for the integral portion of the equation. Moreover, since the roots of the $A$ matrix are real, we impose this condition in the inverse Laplace matrix to obtain the final results as in (4.51).

$$s^2-vs-u = (s+a)(s+b) \Rightarrow s^2-vs-u = s^2+(a+b)s+ab \quad \Rightarrow$$

$$a = -\frac{v+\sqrt{v^2+4u}}{2}; b = -\frac{v-\sqrt{v^2+4u}}{2};$$

Therefore, we have

$$\left| \xi_{1,2}(t) - \xi_{z2}^{(0)} \right| = \left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} e^{A(t)} \gamma_1(0) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \int_0^t e^{A(t-\tau)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(\tau) d\tau \right| - \frac{y(t)}{a_{12} + L_1}$$

(4.52)
Integrating (4.53) by parts leads to

\[
\left| \xi_{1,2} - \xi_{1,2}^{zo} \right| \leq k e^{-\beta t} \left\| y_{1}(0) \right\| + \\
\left| \frac{1}{b-a} \left[ \frac{1}{a} - \frac{1}{b} \right] y(t) - \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) y(0) \right| - \int_{0}^{t} \frac{1}{b-a} \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) y'(\tau) d\tau \left| \frac{y(t)}{u} \right| 
\]

(4.54)

If we define a bound for \( |y'(\tau)| \leq \Delta_y \), we can write the above formula as:

\[
\left| \xi_{1,2} - \xi_{1,2}^{zo} \right| \leq k e^{-\beta t} \left\| y_{1}(0) \right\| + \\
\left| \frac{1}{b-a} \left[ \frac{1}{a} - \frac{1}{b} \right] y(t) - \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) y(0) \right| - \int_{0}^{t} \frac{1}{b-a} \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) \Delta_y d\tau \left| \frac{y(t)}{u} \right| 
\]

(4.55)

In the end, by using \( u = ab \), and by considering \( \| y_{1}(0) \| \leq \varepsilon \), the above formula can be written as:

\[
\left| \xi_{1,2} - \xi_{1,2}^{zo} \right| \leq k e^{-\beta t} \varepsilon + \\
\left| \frac{1}{b-a} \left[ \frac{1}{a} - \frac{1}{b} \right] y(t) - \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) y(0) \right| - \left| \int_{0}^{t} \frac{1}{b-a} \left( \frac{e^{-at}}{a} - \frac{e^{-bt}}{b} \right) \Delta_y d\tau \right| \left| \frac{y(t)}{ab} \right| 
\]

(4.56)

With the assumption of a constant bound for derivative of \( y(t) \), we solve the integral:

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\[ |\xi_{i,j}(t)| - |\xi_{i,j}^{0}| \leq ke^{-\xi} + \]
\[
\frac{1}{b-a} \left[ \left( \frac{1}{a} - \frac{1}{b} \right) y(t) - \left( \frac{e^{-at} - e^{-bt}}{a - b} \right) y(0) \right] - \frac{\Delta_s}{b-a} \left( \frac{e^{-at} - e^{-bt}}{a^2} + \frac{b^2 - a^2}{a^2b^2} \right) \left| y(t) \right| \frac{1}{ab}
\]
\[
\leq ke^{-\xi} + \]
\[
\left[ \frac{y(t)}{ab} - \frac{1}{b-a} \left( \frac{e^{-at} - e^{-bt}}{a - b} \right) y(0) \right] - \frac{\Delta_s}{b-a} \left( \frac{e^{-at} - e^{-bt}}{a^2} + \frac{b^2 - a^2}{a^2b^2} \right) \left| y(t) \right| \frac{1}{ab}
\]
\[
\leq ke^{-\xi} + \]
\[
\frac{1}{b-a} \left( \frac{e^{-at} - e^{-bt}}{a - b} \right) y(0) + \frac{\Delta_s}{b-a} \left( \frac{e^{-at} - e^{-bt}}{a^2} + \frac{b^2 - a^2}{a^2b^2} \right) = J_y
\]

The above equation is just for one piece of the adaptive threshold dynamics. In order to have the equation for the whole threshold, we will consider all the segments with corresponding inputs. For the sake of simplicity and saving space, we define the terms on the right of the (4.58) as J; therefore, by following the same procedure for the whole threshold equation, (4.59) will be obtained:

\[
Z_{th}(t) - Z_{th,0} = \varepsilon_0 + \sum_{i=1}^{2} \left( \delta_{a_i} \xi_{i,2} + \sum_{j=0}^{2} \delta_{b_{i,j}} |\psi_{j,2}| \right) - \left( \varepsilon_0 + \delta_{s} \xi_{1,2}^{0} + \sum_{j=0}^{2} \delta_{b_{j}} |\psi_{1,j,2}^{0}| \right)
\]
\[
\leq \delta_{a_i} \xi_{i,2}^{2} + \sum_{j=0}^{2} \delta_{b_{j}} |\psi_{j,2}| + \left( \delta_{s} J_y + \sum_{j=0}^{2} \delta_{b_{j}} J_{j,2} \right)
\]

Now, we find the terms of \(Z_{th}(t)\) which considered zero in the steady state threshold. Consider the other state of the same segment of the threshold:
\[
\xi_{2,2}(t) = [0 \quad 1] e^{A(t)} \gamma_2(0) + [0 \quad 1] \int_0^t e^{A(t-\tau)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(\tau) d\tau
\]

\[
|\xi_{2,2}(t)| = \left| [0 \quad 1] e^{A(t)} \gamma_1(0) + [0 \quad 1] \int_0^t L^{-1}_{t-\tau} \begin{bmatrix} \frac{s-v}{s^2-vs-u} & \frac{1}{s} \\ \frac{u}{s^2-vs-u} & \frac{s}{s^2-vs-u} \end{bmatrix} y(\tau) d\tau \right| \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(\tau) d\tau
\]

(4.60)

Following the same procedure to calculate the power matrix and reinforcing the condition of (4.51) (real poles), this time the inverse Laplace of the element \( \frac{s}{s^2-vs-u} \) needs to be calculated; hence,

\[
|\xi_{1,2}(t)| = \left| [0 \quad 1] e^{A(t)} \gamma_1(0) \right| + \int_0^t \frac{be^{-b(t-\tau)}-ae^{-a(t-\tau)}}{b-a} y(\tau) d\tau \Rightarrow
\]

\[
|\xi_{1,2}(t)| \leq \left| [0 \quad 1] e^{A(t)} \gamma_2(0) \right| + \int_0^t \frac{be^{-b(t-\tau)}-ae^{-a(t-\tau)}}{b-a} y(\tau) d\tau
\]

(4.61)

Performing the integration by parts and assuming a bound \( \Delta_y \) for the derivative of the input \( y(\tau) \) in this case, we find:

\[
y'(\tau) = \Delta_y ;
\]

\[
|\xi_{2,2}(t)| \leq ke^{-bt} \| \gamma_2(0) \| + \left| \int_0^t \frac{1}{b-a} (e^{-b(t-\tau)}-e^{-a(t-\tau)}) y(\tau) d\tau \right| - \int_0^t \frac{1}{b-a} \left( \frac{1}{b} \frac{e^{-bt} - e^{-at}}{a} \right) y(\tau) d\tau
\]

(4.62)

\[
|\xi_{2,2}(t)| \leq ke^{-bt} \| \gamma_2(0) \| + \frac{e^{-at} - e^{-bt}}{b-a} y(0) - \left[ \frac{\Delta_y}{b-a} \left( \frac{1}{b} - \frac{1}{a} \right) + \frac{e^{-bt}}{b} - \frac{e^{-at}}{a} \right]
\]

\[
|\xi_{2,2}(t)| \leq ke^{-bt} \| \gamma_2(0) \| + \frac{\Delta_y}{ab} + \frac{ae^{-bt} - be^{-at}}{ab(b-a)} \Delta_y
\]

In the same way, we can apply the same procedure to obtain the expression for other inputs. In the next section, with these mathematical manipulations in mind, we calculate the false alarm rate.
4.2.3. False Alarm Rate Calculation

In order to calculate the false alarm rate, the following analysis is carried out. First, the difference between the adaptive thresholds defined in (4.17) and (4.44) is computed and its supremum is denoted as $\Delta$.

$$Z_{th}(t) - Z_{th}^{st}(t) \leq \Delta$$

(4.63)

The probability of false alarm for the adaptive threshold defined by (29) is zero by construction:

$$P(r > Z_{th} \mid \text{no fault}) = 0$$

(4.64)

where $r = |e_i(t)|$. From (4.63), in the worst case scenario, the probability of false alarm in the case of zero order threshold approximation can be defined as:

$$P(r > Z_{th} - \Delta)$$

(4.65)

Defining $Z_{th} - r = \chi$, the probability of false alarm can be written as:

$$P(\chi < \Delta)$$

(4.66)

To find the value of the probability in (4.42) the probability distribution function (pdf) of $\chi$ must be calculated. Since $Z_{th}$ and $r$ are independent random variables, pdf of $\chi$ can be computed by convolution as:

$$f_{\chi}(\chi) = \int_{-\infty}^{\infty} f_{Z_{th}}(x) f_r(x - \chi) dx$$

(4.67)

In order to compute the value of the above integral, pdf of the residual and threshold need to be calculated. In the particular example of alternator & rectifier, the value of the integral in (4.43) is computed numerically. The distribution of the convolution integral is shown in Figure 4.14 The final result obtained from carrying out
the numerical computation is 2.3% which shows that for this particular case the gain associated to the 8th order representation is not worth the additional implementation complexity.

Figure 4.14. Probability density distribution of residual $e_1(t)$.

Figure 4.15 Probability density distribution of adaptive threshold $Z_{th}(t)$
The value of the $\Delta$ was computed 6.22 for the chosen vehicle alternator for the current work.

4.2.4. FIRST ORDER ADAPTIVE THRESHOLD APPROXIMATION

To this end, the state space equations of each filter in the adaptive threshold defined by (4.17) are converted to corresponding transfer function representation as in (4.43). If a first order approximation is seek, the biggest time constant $\tau_1$ is kept while the other smaller time constants are neglected as shown in (4.68).

$$
\begin{bmatrix}
T_1(s) & T_2(s) & L & T_n(s)
\end{bmatrix}^T =
\begin{bmatrix}
\frac{K}{\tau_1 s + 1} & \frac{K s}{\tau_1 s + 1} & L & \frac{K s^{n-1}}{\tau_1 s + 1}
\end{bmatrix}
$$

(4.68)

where $K$ is the gain and $\tau_1$ is the largest time constant. Improper transfer functions in (4.68) are then neglected. Finally, the reduced transfer function is converted back to a state space representation for easy implementation. For the other terms of the adaptive
threshold, similar analysis can be carried out. Therefore, the overall threshold can be written as:

\[
\tau_{\text{r},\text{th}} = \lambda + K y(t)
\]

\[
\hat{\xi}_{i,1} = C_i^\prime \lambda + D_i^\prime y
\]

\[
\tau_{\text{r},j} = \eta_j + Ku_j(t) \quad j = 1K m
\]

\[
\eta_{0,i} = C_i^\prime \eta_j + D_i^\prime u_j \quad i = 1K ,n.
\]

\[
Z_{\text{th}}(t) = \varepsilon_0 + \sum_{i=1}^{2} \left( \delta_{a_i} |\hat{\xi}_{i,1}| + \sum_{j=0}^{m-1} \delta_{b_{i-1,j}} |\eta_{ij,1}| \right)
\]

where \( m \) is the number of inputs to the system, \( \lambda \) and \( \eta \) are the system state. For the system (4.21) and (4.22), this approximation will reduce the order of the adaptive threshold dynamics from 8th-order to 4th-order, which in turn reduces the computational time and complexity of the original adaptive threshold equations; however, this threshold approximation similar to the other approximation introduces a non-zero false alarm rate. The probability of false alarm can be calculated in a similar way as in the previous section. The following plots show the residual and adaptive threshold output for the chosen vehicle alternator system.
Figure 4.17. Residual and adaptive threshold signals of $S_1$ when no fault is injected.

Figure 4.18. Residual and adaptive threshold signals of $S_1$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$. 
Figure 4.19. Residual and adaptive threshold signals of $S_1$ when open diode fault is injected at $t = 10s$.

Figure 4.20. Residual and adaptive threshold signals of $S_1$ when voltage regulator fault is injected at $t = 10s$ for fault magnitude of 30\% with respect to the nominal voltage reference.

The above figures show the effectiveness of the reduced order adaptive threshold (each segment of the adaptive threshold dynamics is first order for the example vehicle alternator chosen) to detect the faults as they occur. The adaptive threshold output is a
little smoother than that of the full adaptive threshold since we are neglecting one of the
poles. However, this does not affect the capability of the adaptive threshold in fault
detection.

4.3. **Adaptive Threshold in Case of Time Varying Parameters**

Consider the system given by (4.3) and (4.4), but suppose now that $\Delta A_0$ and $\Delta B_0$
are bounded functions of time; therefore, error dynamics can be written as:

$$e_1 = y - \frac{\partial e}{\partial e} = C_0 e$$  \hspace{1cm} (4.70)

By integrating the differential equation in (4.70), the error dynamics output is obtained.

$$e_i(t) = C_0 e^{(A_i + LC_i)t} e(0) + \int_0^t C_0 e^{(A_i + LC_i)(t-\tau)} \Delta a(\tau) y(\tau) d\tau + \int_0^t C_0 e^{(A_i + LC_i)(t-\tau)} \Delta b(\tau) u(\tau) d\tau$$  \hspace{1cm} (4.71)

where the parameter uncertainties are defined as in (4.10), and (4.11). Assuming a
positive response in the error dynamics, i.e.

$$w_{E_i}(t) = C_0 e^{(A_i + LC_i)t} E_i \geq 0 \quad i = 1, K, n$$  \hspace{1cm} (4.72)

And assuming known constant upper bounds for the parameter uncertainties:

$$|\Delta a(t)| \leq \delta_{a_i}$$  \hspace{1cm} (4.73)

$$|\Delta b_{i-j}(t)| \leq \delta_{b_{i-j}} \quad i = 0, K, n \quad j = 0, K, m \quad \forall t$$  \hspace{1cm} (4.74)

an upper bound of the residual can be obtained as:

$$|e_i(t)| \leq c_0 + \sum_{i=1}^n \int_0^t C_0 e^{(A_i + LC_i)(t-\tau)} E_i \Delta a_i |y(\tau)| d\tau +$$

$$\sum_{i=1}^n \sum_{j=0}^{m-1} \int_0^t C_0 e^{(A_i + LC_i)(t-\tau)} E_i \Delta b_{i-j} |u_j(\tau)| d\tau \otimes z_{th}(t)$$  \hspace{1cm} (4.75)
where $\varepsilon_0$ is previously defined in (4.15). Equation (4.75) is representative of a system whose state space representation is given by (4.76),

$$
\begin{align*}
\dot{\xi}_i &= (A_0 + LC_0)\xi_i + E_i \left| y(t) \right| & i = 1K n \\
\xi_{i,1} &= C_0 \xi_i \\
\psi_j &= (A_0 + LC_0)\psi_j + E_i \left| u_j(t) \right| & i = 1K n; j = 0K m \\
\psi_{j,1} &= C_0 \psi_j \\
\end{align*}
\tag{4.76}
$$

$z_{\alpha}(t) = \varepsilon_0 + \sum_{i=1}^{n} (\delta_{\xi_i,\xi_{i,1}} + \sum_{j=i}^{m-1} \delta_{\psi_{j,1},\psi_{j,1}}) \psi_{j,1}$

A fault is declared if $|e_i(t)| > |z_{\alpha}(t)|$.

The condition (4.72) requires a particular form of impulse response:

$$
W_e(s) = C_0(sI - A_0 - LC_0)^{-1} E_i
\tag{4.77}
$$

A necessary condition is that all poles need to be real and distinct. This condition is difficult to satisfy in general. Instead, the following approach is utilized to ensure (4.77) will be met.

Reconsider the bounds in (4.75)

$$
|e_i(t)| \leq \varepsilon_0 + \sum_{i=1}^{n} W_{e_i}(s) \Delta a_i(t) y(t) + \sum_{i=1}^{n} \sum_{j=0}^{m-1} W_{e_i}(s) \Delta b_{i-1,j}(t) u_j(t)
\tag{4.78}
$$

Assume, with abuse in notation:

$$
\Delta a_i(t) = L_i(s) \Delta \alpha_i(t)
\tag{4.79}
$$

$$
\Delta b_{j,i}(t) = \Pi_i(s) \Delta \beta_{j,i}(t)
\tag{4.80}
$$

where $L_i(s)$ and $\Pi_i(s)$ are stable transfer functions defined in a way to have

$$
L^{-1} \left\{ W_e(s) L_i(s) \right\} = w_L(t) \geq 0 \quad \forall t
\tag{4.81}
$$
\[
L^{-1}\left\{W_{E_i}(s)\Pi_i(s)\right\} = w_{\Pi_i}(t) \geq 0 \quad \forall t \tag{4.82}
\]

\[
W_{E_i}(s)L_i(s) = C_{L_i}(sI - A_{L_i})^{-1}B_{L_i} \tag{4.83}
\]

\[
W_{E_i}(s)\Pi_i(s) = C_{\Pi_i}(sI - A_{\Pi_i})^{-1}B_{\Pi_i} \tag{4.84}
\]

The right hand side of (4.83) and (4.84) define the matrices for the equivalent system.

Assuming known bounds for the new parameter uncertainties

\[
|\Delta \alpha_i| \leq \delta_{\alpha,i}, |\Delta \beta_{j,i}| \leq \delta_{\beta_{j,i}}, \quad i = 0 K n, \quad j = 0 K m \tag{4.85}
\]

The error dynamics can be written as:

\[
|e_i(t)| \leq \varepsilon_0 + \sum_{j=1}^{n} W_{E_i}(s)L_i(s)\Delta \alpha_i(t) y_i(t) + \sum_{j=1}^{n} \sum_{j=0}^{m-1} W_{E_i}(s)\Pi_i(s)\Delta \beta_{i-1,j}(t) u_j(t) \tag{4.86}
\]

Substituting (4.83) and (4.84) into (4.86), and since \( w_{\Pi_i}(t) \) and \( w_{\Pi_i}(t) \) are positive and can be written outside of the absolute value, the following equation for the error dynamics will be obtained:

\[
|e_i(t)| \leq \varepsilon_0 + \sum_{j=1}^{n} \int_{0}^{t} C_{e_i} e^{A_{e_i}(\tau-t)} B_{e_i} |\Delta \alpha_i(\tau)| y_i(\tau) d\tau + \sum_{j=1}^{n} \sum_{j=0}^{m-1} \int_{0}^{t} C_{e_i} e^{A_{e_i}(\tau-t)} B_{e_i,\Pi_i} |\Delta \beta_{i-1,j}(\tau)| u_j(\tau) d\tau \tag{4.87}
\]

Substituting the known upper bounds in (4.87) results in:

\[
|e_i(t)| \leq \varepsilon_0 + \sum_{j=1}^{n} \int_{0}^{t} C_{e_i} e^{A_{e_i}(\tau-t)} B_{e_i} \delta_{\alpha} |y(\tau)| d\tau + \sum_{j=1}^{n} \sum_{j=0}^{m-1} \int_{0}^{t} C_{e_i} e^{A_{e_i}(\tau-t)} B_{e_i,\Pi_i} \delta_{\beta_{j,i}} |u_j(\tau)| d\tau \tag{4.88}
\]

Hence, the equivalent state space representation for the parameter time-varying case is
\[ \mathbf{\dot{\xi}} = A_l \mathbf{\xi}_l + B_l \mathbf{y}(t) \quad i = 1K \ n \]
\[ \mathbf{\xi}_{l,1} = C_l \mathbf{\xi}_l \]
\[ \mathbf{\psi}_y = A_{1i} \mathbf{\psi}_{ij} + B_{1i} \mathbf{u}_j(t) \quad i = 1K \ n \quad j = 0K \ m \]
\[ \mathbf{\psi}_{ij,1} = C_{1i} \mathbf{\psi}_{ij} \]
\[ z_{th}(t) = \mathbf{e}_0 + \sum_{j=0}^{n} (\mathbf{\delta}_{\alpha,ij} \mathbf{\xi}_{ij,1} + \sum_{j=0}^{m-1} (\mathbf{\delta}_{\beta,ij} \mathbf{\psi}_{ij,1})) \]

A fault is declared if \(|e_l(t)| > |z_{th}(t)|\)

Following the procedure explained above, adaptive threshold equations of the alternator system will be obtained as,
\[ \mathbf{\dot{\xi}}_l = -a \mathbf{\xi}_l + |\mathbf{y}(t)| \]
\[ \mathbf{\xi}_{l,1} = \mathbf{\xi}_l \]
\[ \mathbf{\psi}_y = \mathbf{\psi}_{ij} + |\mathbf{u}_j(t)| \quad i = 1K \ 2 \quad j = 0K \ 2 \]
\[ \mathbf{\psi}_{ij,1} = \mathbf{\psi}_{ij} \]
\[ z_{th}(t) = \mathbf{e}_0 + \sum_{j=1}^{2} (\mathbf{\delta}_{\alpha,ij} \mathbf{\xi}_{ij,1} + \sum_{j=0}^{2} (\mathbf{\delta}_{\beta,ij} \mathbf{\psi}_{ij,1})) \]

Where \(a\) is the dominant pole of the matrix \(A\) in the original system and is equal to 2.168913.

The following plots are obtained from implementing the above equations in the Matlab/Simulink environment:
Figure 4.21. Residual and adaptive threshold signals of $S_1$ when no fault is injected.

Figure 4.22. Residual and adaptive threshold signals of $S_1$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 50% with respect to the nominal speed $\omega_e$. 
Based on the above plots, the fault diagnosis scheme table should be modified as...
It can be seen from the figures above this approach only detects the open diode fault when it occurs. For the belt slipping fault, it can only detect the fault between 30s and 40s when the jump in the current load happens. Moreover, the magnitude of fault here is 50%. The simulation has been carried out for 40% of the belt slipping fault. This approach was able to detect the fault, but below the 40%, the approach proved to be unable to detect this type of fault. Even though the voltage regulator is not detected with this signature, we can still detect and isolate this type of fault through the third signature obtained from the second observer which will be given later in this chapter.

4.4. **Adaptive Threshold in the Case of Gaussian Distributed Parameters**

In this case scenario, we assume that the parameters uncertainties defined in (4.10) and (4.11) for the system (4.3) and (4.4) are normally distributed random variables with zero mean and known variance. Define \( p \) (a matrix of parameters) as,

\[
p = [\Delta a_1, \Delta a_2, K, \Delta a_n, \Delta b_{0,0}, \Delta b_{0,1} K, \Delta b_{0,m-1}, \Delta b_{1,0}, K, \Delta b_{1,m-1}, K, \Delta b_{n-1,m-1}] \in N(0, Q) \quad (4.91)
\]

where \( Q \) denotes the covariance matrix defined as

<table>
<thead>
<tr>
<th>Fault</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Fault</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Belt Slip</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Open Rectifier Diode Fault</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Positive Voltage Regulator</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Fault Diagnosis Scheme for the Alternator System in time-varying parameters adaptive threshold.
The solution to equations is obtained as

\[ e_i(t) = C_0 e^{(A_i + LC_i) t} y(0) + \sum_{i=1}^{n} \int_{0}^{t} E_i \Delta \eta_i y(\tau) d\tau + \sum_{i=1}^{m} \sum_{j=0}^{n-1} \int_{0}^{t} E_i \Delta \eta_{i,j} u_j(\tau) d\tau \]  

(4.93)

With \( E_i \) defined as in (4.9)

By switching the summations with the integral, we have

\[ e_i(t) = C_0 e^{(A_i + LC_i) t} y(0) + \sum_{i=1}^{n} C_i \int_{0}^{t} e^{(A_i + LC_i) \tau} E_i \Delta \eta_i y(\tau) d\tau + \sum_{i=1}^{m} \sum_{j=0}^{n-1} C_i \int_{0}^{t} e^{(A_i + LC_i) \tau} E_i \Delta \eta_{i,j} u_j(\tau) d\tau \]  

(4.94)

The expected value of Eq. (4.93) is

\[ E\{ e_i(t) \} = C_0 e^{(A_i + LC_i) t} y(0) = \xi_0 \]  

(4.95)

which can be made vanishing at any desired rated by an appropriate selection of the matrix \( L \).

Define

\[ \xi_{i,n} = \int_{0}^{t} C_0 e^{(A_i + LC_i) \tau} E_i \Delta \eta_i y(\tau) d\tau \]  

(4.96)

\[ \psi_{ij,n} = \int_{0}^{t} C_0 e^{(A_i + LC_i) \tau} E_i \Delta \eta_{i,j} u_j(\tau) d\tau \]  

(4.97)

The variance of Eq. (4.94) can be written as

\[ \text{var}\{ e_i(t) \} = E\left\{ (e_i - E(e_i))^2 \right\} = E\left\{ \left( \sum_{i=1}^{n} \Delta \eta_{i-1,n} \xi_{i,n} + \sum_{i=1}^{m} \sum_{j=0}^{n-1} \Delta \eta_{i,j} \psi_{ij,n} \right)^2 \right\} \]  

(4.98)
If we define \( \Theta^T = \begin{bmatrix} \xi_{1,n} & K \xi_{n,n} & \psi_{10,n} & K \psi_{n,m-1,n} \end{bmatrix} \), according to Rayleigh-Ritz Theorem, an upper bound for (4.98) can be defined using the eigenvalues of the covariance matrix, \( \lambda_{\max} = \max \{ \text{eigenvalue}(Q) \} \).

\[
\text{Var} \{ e_1(t) \} = \left| \Theta^T Q \Theta \right| \leq \lambda_{\max} \left\| \Theta \right\|_2^2
\]  

(4.99)

This upper bound constitutes the adaptive threshold dynamics.

The state space representation of (4.99) can be obtained as

\[
\begin{align*}
\dot{\xi}_i &= (A_0 + LC_0)\xi_i + E_i y(t) \quad i = 1, 2, K, n \\
\dot{\psi}_{ij} &= (A_0 + LC_0)\psi_{ij} + E_i u_j(t) \\
\psi_{ij,n} &= C_0 \psi_{ij}, \quad i = 1, 2, K, n \quad j = 0, 1, 2, K, m - 1
\end{align*}
\]

(4.100)

\[
Z_{th}(t) = \varepsilon_0 + \lambda_{\max} \left( \sum_{i=1}^{n} \xi_{i,n}^2 + \sum_{j=0}^{m-1} \psi_{ij,n}^2 \right)
\]

Where \( \xi_i \in \mathbb{R}^n, \psi_{ij} \in \mathbb{R}^n, \forall i = 1...n, j = 0...m-1, \xi_i(0) = 0, \psi_{ij}(0) = 0 \), and \( \lambda_{\max} \) an upper bound of \( \lambda_{\max} \).

In this case a fault is declared if \( \text{var} \{ e_1(t) \} > Z_{th}(t) \) which corresponds to signature \( S_1=1 \). The threshold just derived can be seen as \((m+1)n\) filters of order \( n \). The high order of the threshold dynamics is the main drawback. The order can be further reduced to \( m+1 \) filters of order \( n \) by transforming the equations from observable form into controllable form, and combining the equations with the same input as shown in (4.101).
\[
\mathbf{x}(t) = (A_0 + LC_0)^T \gamma + C_{0}^T y(t)
\]

\[
\xi_{i,n} = E_i^T \gamma
\]

\[
\psi_{ij,n} = (A_0 + LC_0)^T \psi_j + C_{0}^T u_{j}(t)
\]

\[
\psi_{ij,n} = E_i^T \psi_j \quad i = 1K \; n; \; j = 0K \; m-1
\]

\[
Z_{th}(t) = e_0 + \tau_{\max} \left( \sum_{i=1}^{n} \left( \xi_{i,n} \right)^2 + \sum_{j=1}^{m} \left( \psi_{ij,n} \right)^2 \right)
\]

Where \( \gamma \in \mathbb{R}^n \), \( \psi_j \in \mathbb{R}^m \).

For the second order alternator system, (4.101) can be written as

\[
\mathbf{x}(t) = \begin{bmatrix} -1386613 & -1386603 \\ -636771 & -278372 \end{bmatrix} \gamma + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)
\]

\[
\xi_{i,n} = E_i^T \gamma
\]

\[
\psi_{ij,n} = \begin{bmatrix} -1386613 & -1386603 \\ -636771 & -278372 \end{bmatrix} \psi_j + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{j}(t)
\]

\[
\psi_{ij,n} = E_i^T \psi_j \quad i = 1, 2; \; j = 0, 1, 2
\]

\[
Z_{th}(t) = e_0 + \tau_{\max} \left( \sum_{i=1}^{2} \left( \xi_{i,n} \right)^2 + \sum_{j=0}^{2} \left( \psi_{ij,n} \right)^2 \right)
\]

Note that this method is very useful in calibrating the adaptive threshold parameters since only the variance and the mean of the parameters are needed.

For the vehicle alternator system chosen for this thesis, the following parameters distributions were obtained.
Figure 4.25. Distribution data of parameter $a_{12}$ along with the fitted Gaussian distribution fit.

Figure 4.26. Distribution data of parameter $a_{22}$ along with the fitted Gaussian distribution fit.
Figure 4.27. Distribution data of parameter $b_{11}$ along with the fitted Gaussian distribution fit.

Figure 4.28. Distribution data of parameter $b_{12}$ along with the fitted Gaussian distribution fit.
Figure 4.29. Distribution data of parameter $b_{13}$ along with the fitted Gaussian distribution fit.

Figure 4.30. Distribution data of parameter $b_{22}$ along with the fitted Gaussian distribution fit.
The results of implementation of the above analysis are shown in the plots below; please note due to characteristics of the particular alternator chosen for this simulation, the movement of the threshold is not conspicuous.

Figure 4.31. Distribution data of parameter $b_{23}$ along with the fitted Gaussian distribution fit.

Figure 4.32. Residual and adaptive threshold signals of $S_1$ when no fault is injected.
Figure 4.33. Residual and adaptive threshold signals of $S_1$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$.

Figure 4.34. Residual and adaptive threshold signals of $S_1$ when open diode fault is injected at $t = 10s$. 
The above figures show the result of implementation of adaptive threshold method using Gaussian distributed parameters. This approach is capable in detecting the voltage regulator fault as it occurs whereas the belt slipping fault and open diode fault are detected at time 30s. The threshold seems to move slightly with respect the previously implemented thresholds; the characteristic of the particular vehicle alternator system chosen for this case is the cause of this threshold behavior. All in all, this signature combined with the other signatures defined in the fault diagnosis scheme is capable in detecting and isolating the occurred faults in the vehicle alternator system.

4.5. SECOND AND THIRD SIGNATURES

The second signature $S_2$ is obtained by comparing the standard deviation of the residual $e_1(t)$ with a fixed threshold. For this signature, a one second moving window which contains 100,000 sampling points (this number of sampling points were chosen to
be consistent with the sampling period of the original system) was considered in implementation of the standard deviation (STD) algorithm described by

\[
(\text{STD}_{k+1})^2 = \sum_{i=k+2-N}^{k+1} \frac{(e_{i,k} - \mu_k)^2}{N} = (\text{STD}_k)^2 + \frac{(e_{1,k+1} - \mu_{k+1})^2 - (e_{1,k+1-N} - \mu_{k+1-N})^2}{N} \tag{4.103}
\]

\[
\mu_k = \sum_{i=k+1-N}^{k} \frac{e_{i,k}}{N} = \mu_{k-1} + \frac{e_{1,k-N}}{N} \tag{4.104}
\]

where \( \mu_k \) is the mean value of the residual over the 10s time period, and \( N \) is the window.

\( S_1 \) and \( S_2 \) signatures are capable of detecting all the faults in the alternator but a voltage regulator fault cannot be isolated. The detection and isolation of this last fault is accomplished by means of the signature \( S_3 \). The following analysis demonstrates the method utilized to design a second observer used to isolate the voltage regulator fault.

The alternator voltage regulator is implemented as a PI controller, with saturation on \( V_f \) that cannot be greater than \( V_{dc} \)

\[
V_f = \text{sat} \left( K_p (V_{\text{ref}} - V_{dc}) + \text{sat}(K_i, \int (V_{\text{ref}} - V_{dc})) \right) \tag{4.105}
\]

Where \( K_i \) and \( K_p \) are the integral and proportional controller gains. Saturation is defined as

\[
\text{if } V_{dc} > V_{\text{ref}} \Rightarrow V_f = \Lambda \tag{4.106}
\]

where \( \Lambda \) is a constant number.

By defining \( U = V_{dc} - V_{\text{ref}} \) and the state \( x = K_i \int U(t) \, dt \), Eq. (4.105) away from the saturation of the integral can be represented by

\[
\dot{x} = -K_i (U) \tag{4.107}
\]

81
\[ V_f = \text{sat}(x - K_p(U)) \]

Consider the observer:

\[ \dot{\hat{x}} = L(V_f - \hat{V}_f) - K_x U \]  \hspace{1cm} (4.109)

\[ \hat{V}_f = \dot{x} - K_p U \]  \hspace{1cm} (4.110)

\[ e_2 = V_f - \hat{V}_f \]  \hspace{1cm} (4.111)

By defining the error as \( e = \dot{x} - x \), the error dynamics in absence of faults and away from voltage saturation are

\[ \dot{\epsilon} = L(V_f - \hat{V}_f) = Le_2 = -Le \]  \hspace{1cm} (4.112)

In the presence of a voltage regulator fault, \( \Delta U \) and no saturation conditions, we have

\[ \dot{\epsilon} = Le_2 + K_1 \Delta U = -Le - (LK_p - K_1) \Delta U \]  \hspace{1cm} (4.113)

\[ e_2 = -e - K_p \Delta U \]  \hspace{1cm} (4.114)

which explicitly shows the dependence on the fault. When \( V_f \) saturates, nothing can be said about the presence of a fault.

The following figures show the \( S_2 \) and \( S_3 \) signatures for the implemented vehicle alternator system model. The fixed threshold in both signatures has been chosen in a way to detect and isolate the faults with minimum false alarm rate and misdetection. Additionally, \( S_2 \) signature for the case of Gaussian distributed parameters threshold is also presented since the parameters of the residual are chosen in a way to satisfy the zero mean condition for the Gaussian fit as explained earlier in the adaptive threshold method in the case of Gaussian distributed parameters.
Figure 4.36. $S_2$ when in the no fault case.

Figure 4.37. $S_3$ when in the no fault case.
Figure 4.38. $S_2$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$.

Figure 4.39. $S_3$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$. 
Figure 4.40. $S_2$ when open diode fault is injected at $t = 10s$.

Figure 4.41. $S_3$ when open diode fault is injected at $t = 10s$. 
Figure 4.42. $S_2$ when voltage regulator fault is injected at $t = 10s$ for fault magnitude of 15% with respect to the nominal voltage reference.

Figure 4.43. $S_3$ when voltage regulator fault is injected at $t = 10s$ for fault magnitude of 15% with respect to the nominal voltage reference.
Figure 4.44. $S_2$ when open diode fault is injected at $t = 10s$.

Figure 4.45. $S_2$ when belt slipping fault is injected at $t = 10s$ for fault magnitude of 40% with respect to the nominal speed $\omega_e$. 
Figure 4.46. $S_2$ when open diode fault is injected at $t = 10s$.

Figure 4.47. $S_2$ when voltage regulator fault is injected at $t = 10s$ for fault magnitude of 15% with respect to the nominal voltage reference.

The above figures show the result of implementation of the vehicle alternator system in Matlab/Simulink environment. Figure 4.38 shows that the belt slip fault is
detected at 33s. This in return means the belt slipping fault is isolated at this time according to the defined fault diagnosis scheme. Since open diode fault is only detected and isolated via signature $S_1$, at time $t=33s$, we can distinguish between belt slipping and open diode fault. Signature $S_3$ is effective at time $t=20s$. At this point, we can be certain the voltage regulator fault has been occurred in the system.

In summary, $S_1$ detects the faults when they occur in the system. If voltage regulator fault has been occurred in the system, it can be isolated at time $t=20s$. at time $t=33s$, we can determine whether belt slipping fault or open diode fault has been occurred in the system.

And finally, $S_2$ in the adaptive threshold implementation with Gaussian distributed parameters is able to detect the belt slipping fault at $t=30s$. At this point we can isolate the belt slipping fault from the open diode fault.

4.6. COMPARISON OF THE THREE METHODS OF ADAPTIVE THRESHOLD IMPLEMENTATION

Of the three methods of implementing the adaptive threshold, the first one is the most effective in fault detection. It can detect the fault as it occurs in the alternator system. It can detect the belt slipping fault as low as 30 percent with respect to the nominal value of the belt speed. In contrast, the time-varying parameters adaptive threshold can only detect the belt slipping fault as low as 40 percent with respect to the nominal value of belt speed with considerable delay from the fault injection time. Gaussian distributed parameters method can detect the belt slipping fault with the same amount as in the bounded parameters methods but with a certain time delay. All the
discussed methods are capable of detecting the open diode fault; Gaussian distributed method, however, detects it when the jump in the load current profile occurs.

Since all the methods utilize the same observer to generate the $S_3$ signature for voltage regulator fault, the fault detection capability of this type of fault is the same in all three methods.

To summarize, the first method along with its two approximations proves to be the most effective to detect the occurrence of faults in the system compared with the other methods presented. Table 3 compares the mentioned three methods with the least amount of fault they can detect and isolate. In this table, “YES” means that the method is capable of detecting this amount of fault in the method utilized to obtain the $S_I$ signature; while “NO” means that the method is incapable of detecting fault with the provided amount.

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>Bounded Parameters</th>
<th>Time-varying Parameters</th>
<th>Gaussian-distributed Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belt Slipping</td>
<td>30%(YES)</td>
<td>30%(NO)</td>
<td>30%(YES)</td>
</tr>
<tr>
<td></td>
<td>40%(YES)</td>
<td>40%(YES)</td>
<td>40%(YES)</td>
</tr>
<tr>
<td>Voltage Regulation</td>
<td>11%(YES)</td>
<td>11%(YES)</td>
<td>11%(YES)</td>
</tr>
<tr>
<td>Open Diode</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table 3. Sensitivity of the diagnosis scheme to faults.
4.7. CONCLUSION

In this chapter, the procedure to obtain the fault signatures introduced in chapter 3 was shown. Moreover, the effectiveness of the proposed fault diagnosis scheme for the alternator system was tested through simulation. The proposed diagnostic scheme proved to be effective in detecting belt slipping fault, open diode fault, and voltage regulation fault above certain magnitude as listed in Table 3.
5.1. INTRODUCTION

Jan Lunze in (Lunze 1998) states the modeling problem as, ”For a given dynamical system $S$ and a given set of questions about the behavior $B$ of $S$, find a representation $M$ that helps to answer the given questions. Then, $M$ is called the model of $S$.”

The models formulated to be used to solve a given has to be adapted to the questions to be answered. Hence, there are many different models $M_i$ of a given system $S$. One way to classify the modeling is quantitative modeling and qualitative modeling.

5.2. QUANTITATIVE MODELING

In engineering field, we model systems through differential or difference equations in order to describe the system output $y(k)$ for a given input $u(k)$ and initial state $x_0$. Such a model usually has the form,

$$x(k+1) = f(x(k), u(k)) \quad x(0) = X_0 \quad (5.1)$$

$$y(k) = g(x(k), u(k)) \quad (5.2)$$

where $x \in \mathbb{R}$ is the system state, $u \in \mathbb{R}$ is the input and $y \in \mathbb{R}$ is the output.

The behavior $(B)$ can be described as all pairs $(u(k), y(k))$ that satisfy Eq. (5.1).

The quantitative modeling is widely used in engineering for many reasons among which two are presented here:

1) It makes it possible to predict the future behavior of the system precisely.
2) It can represent many different systems of a given class since they are parameterized (parameter values are changeable).

There is, however, one major problem in quantitative modeling; in order to solve a given problem, parameter values along with the input and quantitative initial state $x_0$ must be known, which is not always possible.

**5.3. Qualitative Modeling**

Despite all the advantages quantitative modeling offers, there are, however, many reasons why it cannot be used as a suitable presentation of a given system.

- If the knowledge on the system is incomplete, quantitative modeling cannot be set up to investigate system behavior.

- If the measurement of the input & initial state is not accurate, this modeling cannot give a precise assessment of the system.

- If we are only interested in qualitative assessment of the system behavior subject to given input or initial state, quantitative modeling in not a suitable way of modeling the system.

- In many practical cases untimed models yield the wanted results.

One of the main advantages of qualitative modeling is the untimed models that are much simpler. In untimed models, the effect of time is neglected and only the order of occurrence of the events (a change in the symbolic values of the input, output or state) is considered.
5.4. *INTRODUCTION TO QUANTIZED SYSTEMS*

Quantized systems are continuous-variable systems; moreover, sensors and actuators signals of this type of systems can only be accessed through quantizers that produce symbolic state sequences, thus giving the system a discrete behavior.

In order to model quantized systems properly, we need to take into account both continuous and discrete phenomena together in hybrid systems. This type of modeling plays a crucial role in diagnosis and fault-tolerant control.

5.4.1. *STRUCTURE OF A QUANTIZED SYSTEM*

Since in this type of system, we are dealing with different ranges of signals, we must find a way to be able to represent signals relations. Quantizer and injector are used to establish these relations.

The quantizer transforms a real-values signal into a sequence of symbols, where the real-values signal or signal vector is denoted by a lower-case letter like $u$ and the corresponding quantized signals by $[u]$. If, in the simplest case, the quantizer decides to which real interval of a given set of intervals the current value $u(t)$ belongs, the value of the quantized signal $[u(t)]$ at the time instant $t$ is the number of the corresponding interval. This interval can be associated with symbolic names like “normal”, “high” or “low”, which give a semantic signal value. As long as the signal does not leave a given interval, the quantized value remains the same. Therefore, a continuous change of $u(t)$ is transformed into a sequence of discrete changes of $[u(t)]$, which indicates the quantizer can act as an interface between real-values and symbolic signals.
The injector, in contrast, carries out the inverse mapping. Its input is a symbolic signal like \([f]\), which is associated with a real-values signal \(f\). The relation between \([f]\) and \(y\) can be either deterministic, where every symbolic value is associated with a unique real value or non-deterministic, where the associated real value is randomly selected from a given set of signal values or may vary within this set as long as the symbolic value does not change. In this manner, injector could be interpreted as the interface from symbolic to real-valued signals.

![Figure 5.1. Structure of a quantized system.](image)

As it can be seen from Figure 5.1, at the center of a quantized system, there is a continuous system with discrete time. The governing equations for this system is given by (5.1) and (5.2). Here, we assume we have a priori knowledge of the \(x(0)\) distribution. For any initial state \(x(0) \in X_0\) and input sequence \(U(0…k_h) = (u(0),u(1),…,u(k))\), equations (5.1) and (5.2) generate a unique state and output.

If the system is linear, (5.1) and (5.2) can be simplified
\[ x(k+1) = Ax(k) + Bu(k), \ x(0) \in X_0 \]  
\[ y(k) = Cx(k) + Du(k) \]  

(5.3)  

(5.4)  

Where \( A, B, C \) and \( D \) are matrices of appropriate dimensions. In this case, a solution can be found for the above equations which are given in (5.5) and (5.6) over the time interval of \([0, k_h]\).  

\[ x(k) = A^k x(0) + \sum_{i=0}^{k-1} A^{k-1-i} Bu(i) \]  

(5.5)  

\[ y(k) = CA^k x(0) + \sum_{i=0}^{k-1} CA^{k-1-i} Bu(i) + Du(k) \]  

(5.6)  

Faults occurring in the system are modeled as additional input sequence to the system.  

\[ E(0..k_h) = (e(0), e(1), ..., e(k_h)) \]  

(5.7)  

We can rewrite equations (5.1) and (5.2) for a faulty system as  

\[ x(k+1) = f(x(k), u(k), e(k)) \ x(0) = X_0 \]  

(5.8)  

\[ y(k) = g(x(k), u(k), e(k)) \]  

(5.9)  

In the modeling problem presented later, we assume the system is faultless. Afterwards, the model will be extended to consider faults in the system.  

### 5.4.2. QUANTIZATION OF THE SIGNAL SPACES FOR THE CONTINUOUS SYSTEMS  

The quantizers introduce partitions of the signal spaces \( u \) and \( y \) into a finite number of disjoint sets \( Q_u(v)(v \in N_u = \{0,1,\ldots,M\}) \) and \( Q_y(w)(w \in N_y = \{0,1,\ldots,N\}) \) where \( Q_u(v) \) and \( Q_y(w) \) denote the sets of input values \( u \) and output values \( y \) with the same quantized values \( v \) and \( w \). The mapping invoked by the quantizer is symbolized by \([.]:\)
\[ [u] = v \iff u \in Q_u(v) \quad (5.10) \]
\[ [y] = w \iff y \in Q_y(w) \quad (5.11) \]

The numbers \( v \) or \( w \) are called the quantized values or qualitative values of the input or output, respectively, and \([u]\) or \([y]\) the qualitative input or qualitative output.

The sets \( Q_u(v)(v \neq 0) \) and \( Q_y(w)(w \neq 0) \) are assumed to be bounded whereas \( Q_u(0) \) and \( Q_y(0) \) are the remaining unbounded subsets of \( m \) and \( n \) respectively.

In order to get a concise model of the quantized system, we introduce a quantization of the state space \( r \) with partitions \( Q_x(z) \) \((z \in R_x = \{0, 1, \ldots, R\})\) where
\[ [x] = z \iff x \in Q_x(z) \]

A qualitative change of state \( x(k) \) from \( z_i \) to \( z_j \) is called event and is denoted by \( e_{ji} \).

### 5.4.3. Behavior of the Quantized Systems

As mentioned previously, the behavior of the quantized system is the set of all input/output (I/O) pairs which are consistent with the system dynamics. Since only qualitative I/O is considered, system behavior is sometimes referred to as qualitative behavior.

As all measurements are qualitative, the initial state of the system is also considered on the qualitative level. Therefore, the following investigation is only concerned with the set of qualitative I/O pairs that the quantized system can generate for a qualitatively given initial state. If the qualitative initial state is known precisely, \( X_0 = Q_x(z(0)) \) i.e. \( z(0) = [x(0)] \).
If $[u(k)] \in N_u = \{0,1,...,M\}$ and $[y(k)] \in N_y = \{0,1,...,N\}$ denote the elements of the I/O sequences, then we will have

$$[U(0...k_h)] \in N_u^{k_h+1} = N_u \times N_u \times ... \times N_u$$

$$[Y(0...k_h)] \in N_y^{k_h+1} = N_y \times N_y \times ... \times N_y$$

Hence,

$$B_{qual}(k_h) \subseteq N_u^{k_h+1} \times N_y^{k_h+1}$$

It is easy to find the number of elements in this product as

$$[(M+1) \times (N+1)]^{k_h+1}$$

$B_{qual}(k_h)$ selects only the elements that are consistent with the system over the given time horizon $k_h$. The main issue here is for a given qualitative initial state and qualitative input sequence, it is impossible to determine the output sequence unambiguously; therefore, the qualitative behavior of the system is non-deterministic.

Now consider the probability

$$\text{Pr}([Y(0...k_h)] | [U(0...k_h)], X_0)$$

Which states how often the I/O pair occurs if the probability of the occurrence of $[U]$ and the probability of the initial state $x(0) \in X_0$ are known. In another words, if $x(0)$ is known, this probability tells us how often $[Y]$ for a given $[U]$ occurs if many experiments are done with the system is brought back to the same initial state $x(0)$. Thus, the system possesses stochastic properties.

A stochastic process is a non-deterministic system for which the state and output sequences are generated with a certain probability.
5.5. MODELING THE QUANTIZED SYSTEM WITH STOCHASTIC AUTOMATON

A quantized system can be described by a stochastic automaton as

$$S = (N_x, N_y, N_w, L, \Pr(z(0)))$$

(5.17)

whose state, input and output sets are identical to the sets of qualitative states, qualitative input values and qualitative output values respectively. The modeling problem here is to find the behavioral relation $L$. The automaton is also called an abstraction of the system with equations (5.1) and (5.2). If we assume an autonomous automaton (not directly dependent on time), $L$ can be described as

$$L(z', w \mid z, v) = \Pr([x(k+1)] = z', [y(k)] = w \mid [x(k)] = z, [u(k)] = v)$$

(5.18)

Note the dynamics of the vehicle alternator system is such that the output is equal to one of the system’s state, i.e. $w = z_2$; therefore, Eq. (5.18) can be simplified to

$$L(z', w \mid z, v) = \Pr([x(k+1)] = z' \mid [u(0)] = v)$$

(5.19)

To find a way to compute the probability mentioned in (5.18), we will be using the Gauss Theorem. Consider the following system of equations in general form given by Eq. (5.20) as

$$f_1(x_1, x_2, K, x_n, u)$$
$$f_2(x_1, x_2, K, x_n, u)$$
$$\ldots$$
$$f_n(x_1, x_2, K, x_n, u)$$

(5.20)

where $x_1 \ldots x_n$ are the states of the system and $u$ in the input vector, $f_1 \ldots f_n$ the appropriate functions and component of the vector field of trajectories in phase portrait defined by

$$\dot{F} = f_1 \hat{h} + f_2 \hat{j}_2 + f_n \hat{i}_n$$

(5.21)
Where $\hat{i}_1, \hat{i}_2, ... \hat{i}_n$ are the coordinates in the phase portrait.

Consider the following figure,

![Figure 5.2 Flow of trajectories in a grid cell](image)

In which $Q_{1x1}$, $Q_{2x1}$, $Q_{1x2}$ and $Q_{2x2}$ are the corners of the grid cell (obtained from design parameters); and $(q_1, q_2)$ and $(q_1, q_2)$ are the coordinates of the points where the trajectories enter and leave the grid cell respectively. The gray area in the figure shows where trajectories are concentrated in the grid cell.

In order to compute the trajectories flow, we will take advantage of the Divergence Theorem in 2D form. The theorem states the sum of all inward flows minus the sum of all outward flows renders the net flow, i.e.

$$\phi_{in} - \phi_{out} = \phi_{net} \quad (5.22)$$

In mathematical form, the Divergence Theorem in 2D can be written as

$$\int \int_A (\nabla \cdot F) dA = \oint_C F \cdot n dr \quad (5.23)$$
Where $\hat{F}$ the vector field, $n$ is the outward pointing unit normal field of the boundary $dA$, $A$ is the area and $c$ is the boundary of the area $A$. In Figure 5.2, $A$ is the area of the rectangle and $c$ is the perimeter of that rectangle.

In order to compute the transition probability of the trajectories, we divide Eq. (5.22) by $\phi_{in}$,

$$1 - \frac{\phi_{out}}{\phi_{in}} = \frac{\phi_{net}}{\phi_{out}}$$  \hspace{1cm} (5.24)

$\frac{\phi_{out}}{\phi_{in}}$ denotes the probability of the trajectories leaving the grid, and $\frac{\phi_{net}}{\phi_{out}}$ denotes the probability of the trajectories staying inside the grid. Note that the sum of these two probabilities equals to one.

The following equations obtained from system equations (5.20) and (5.21) along with Figure 5.2, provide useful information about where the trajectories enter and leave the grid. Moreover, the direction of the trajectories flow can be deduced. For instance, consider the following equations obtained from system equations combined with Figure 5.2.

$$q_1 = f_1(x_1, Q_{2x_2}, u)$$  \hspace{1cm} (5.25)
$$q_2 = f_3(x_1, Q_{2x_2}, u)$$  \hspace{1cm} (5.26)
$$q_1 = f_1(Q_{2x_1}, x_2, u)$$  \hspace{1cm} (5.27)
$$q_2 = f_2(Q_{2x_3}, x_2, u)$$  \hspace{1cm} (5.28)
The right hand side of the Gauss Theorem can be broken into four integrals to find the net flow of trajectories on each side of the rectangle. Moreover, from (5.25) to (5.28), we can deduce the direction of the trajectories flow. For example, \( q_1 \) indicates where the trajectories enter the grid, and the direction of the trajectories flow can be deduced from the sign change of \( q_2 \) by finding its zero. The same analysis can be done for the exit point of the trajectories. Since these equations obtained for an arbitrary trajectories flow and arbitrary grid, they are valid for other trajectories flow and grids as well. Therefore, the transition probability can be computed for each grid.

Furthermore, since both the observation method as well as diagnostic method are based on a consistency check for a given I/O pairs, the model has to represent all I/O pairs that may occur for a quantized system. In other words, the model has to be complete, i.e. a model with the behavior \( M \) which satisfies the relation \( M(k_h) \supseteq B_{\text{qual}}(k_h) \) for all \( k_h \) is complete. A complete model, therefore, include all I/O pairs for a given time horizon \( k_h \) that are consistent with the quantized system. However, there may exist pairs consistent with the model but not with that of quantized system. These pairs are called spurious pairs. Spurious pairs are a common phenomenon in qualitative modeling. The reason for that is qualitative model is less complex than the exact model; therefore, it ignores some information about the properties of the quantized system. Moreover, the qualitative model has the Markov property to provide recursive representation of the behavior \( M \) (shown later in the state observation equations); quantized system, in contrast, does not possess this property.
Completeness is an important property since only complete models are suitable for solving the state observation or the diagnostic problem. In order a model of a quantized system to be complete, the following conditions must be satisfied,

\[ L(z', w \mid z, v) > 0 \iff \Pr([x(1)] = z', [y(0) = w \mid [x(0)] = z, [u(0)] = v) \]  

(5.29)

\[ \Pr(z(0) = z) > 0 \quad \text{for all } z = [x_0] \quad x_0 \in X_0 \]  

(5.30)

To model a faulty quantized system, the description of the stochastic automaton has to be extended so as to refer to the fault as an additional input, i.e.

\[ S = (N_x, N_v, N_w, N_f, L, \Pr(z(0))) \]  

(5.31)

In addition, the new behavioral relation \( L(z', w \mid z, v, f) \) has to satisfy the condition (5.29) for a given fault as in (5.32),

\[ L(z', w \mid z, v, f) = \Pr([x(1)] = z', [y(0) = w \mid [x(0)] = z, [u(0)] = v, [e] = f) \]  

(5.32)

where fault \( f = [e(k)] \) is used here as an additional (unknown) input to the quantized system.

The following theorems from (Mogens Blanke 2006) will assist us in probability computation of a state observation algorithm,

**Theorem 1.** If the stochastic automation \( S \) is a complete model of the quantized system, the qualitative state \([x(k_h)]\) of the quantized system belongs to the set \(Z(k_h \mid k_h)\) defined by \(Z(k_h \mid k_h) := \Pr(z(k_h) \mid W(0\ldots k_h), V(0\ldots k_h))\) where \(\Pr([x(k_h)] \mid k_h)\) is an estimate of the probability with which the quantized system assumes the state \([x(k_h)]\)
Because the stochastic automaton used to model the observation problem is only a complete but not an exact model of the quantized system.

5.5.1. A PRIORI KNOWLEDGE OF THE INITIAL STATE

An a priori probability distribution has to be known to initialize the observation method. As for the stochastic automaton, it is most important to ensure this probability is greater than zero for the true qualitative initial state which is unknown. If nothing is known about the initial state, a good choice of the a priori probability distribution is the uniform distribution over the set \( N_x \) of the qualitative states, i.e.

\[
\Pr([x(0)]) = \frac{1}{N + 1} \text{ for all } [x(0)] \in N_x \text{ where } N + 1 \text{ is the number of qualitative states defined by the state quantizer.}
\]

5.6. STATE OBSERVATION IN RECURSIVE FORM

For the vehicle alternator system, we would like to model the equivalent second order system with a stochastic automaton. Hence, we use the stochastic automaton to represent the state observation of a quantized system. We also assume that the input and output of the system are measurable.

(Mogens Blanke 2006) states a theorem (theorem 8.3) to solve the state observation problem in recursive form as,

*Consider a stochastic automaton with the initial state distribution \( \Pr(z(0)) \). If the I/O pair \((V,W)\) is consistent with the stochastic automaton, the a-posteriori state probability distribution is given by the recursive relations*
\[
\Pr(z(k_h) \mid k_h) = \frac{\sum_{z(k_{h-1})} L(k_h). \Pr(z(k_h) \mid k_h - 1)}{\sum_{z(k_{h-1})} L(k_h). \Pr(z(k_h) \mid k_h - 1)}
\] (5.33)

With

\[
\Pr(z(k_h) \mid k_h - 1) = \frac{\sum_{z(k_{h-2})} L(k_h - 1). \Pr(z(k_h - 1) \mid k_h - 2)}{\sum_{z(k_{h-2})} L(k_h - 1). \Pr(z(k_h - 1) \mid k_h - 2)}
\] (5.34)

\[
\Pr(z(0) \mid -1) := \Pr(z(0))
\] (5.35)

Equation (5.33) gives the probability of state \( z(k_h) \) at time \( k_h \), and equation (5.34) gives the probability of the same state at time \( (k_h - 1) \), and finally, equation (5.35) shows the probability of the initial state. For the quantization approximation we would like to consider, we approximate the probability of the state by

\[
\Pr(z(k_h) \mid k_h - 1) \approx \Pr(z(k_h - 1) \mid k_h - 1)
\] (5.36)

Equation (5.36) neglects the history of the previous trajectories and only considers the current state.

We would expect the direction of the trajectories to be different when certain fault is injected to the system. Based on the direction the trajectories follow, the type of the fault can be deduced.
CHAPTER SIX
CONCLUSION AND FUTURE WORK

In this thesis, we have presented a systematic approach in formulating a fault diagnosis scheme for the EPGS system and in particular the vehicle alternator system. A model-based adaptive threshold method was used as part of the proposed diagnostic scheme to identify and isolate certain types of fault in the vehicle alternator system. The method proved to be very useful to detect the fault as it occurs in the system compared to the fixed threshold. One major drawback of adaptive threshold was its high order which in turn, required more resources to be implemented effectively. Hence, two threshold approximations were considered as a solution to this problem, but these approximations introduced false alarm probability rate.

Two other methods to obtain the adaptive threshold, in the case of systems with linear time varying-parameters, and Gaussian distributed linear parameters were explored, and the results were shown. The simulation result illustrates the power of this approach in FDI. Furthermore, to overcome the high order of the adaptive threshold dynamics, two approximations based upon the steady state and first order threshold dynamics were investigated. One downside of these approximations was the introduction of a false alarm rate. Since the computed probability of false alarm for the alternator system under exam was small, it was a reasonable trade-off to use these approximations instead of the full order adaptive threshold to save computational time.

In the current work, we have shown the implementation of this method for the vehicle alternator system. The outcome of this work expands beyond the application
considered more specifically here. The presented approach can be applied to other
engineering systems in which model-based adaptive threshold can be considered as a way
to formulate a fault diagnosis scheme to identify and isolate the faults of interest possibly
occurring in the system. Furthermore, adaptive threshold method is capable of promptly
detecting a fault in the system; this capability can be extremely helpful in critical
applications where the fast detection of the fault is of utmost importance.

Finally, the qualitative modeling of the alternator system was analyzed, and a new
approach to compute the transition probability was introduced. The implementation of
this approach can be the subject of future work in this area.
REFERENCES


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