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STRUCTURAL HEALTH MONITORING OF HISTORIC MASONRY MONUMENTS

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STRUCTURAL HEALTH MONITORING OF HISTORIC MASONRY MONUMENTS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Civil Engineering

by
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August 2011

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ABSTRACT

*Structural Health Monitoring* (SHM) is a well-accepted diagnostic technique being used to evaluate modern structures. This method involves monitoring the vibration response of a structure to detect changes in its structural state. The primary intention of this thesis is to address two practical and technical difficulties encountered in deploying SHM on historic masonry monuments: (i) the selection of suitable low dimensional vibration response features that are highly sensitive to the presence and extent of damage, while having low sensitivity to extraneous noise and (ii) the selection of optimal sensor locations for efficient system identification applied to Gothic Cathedrals. Both of the features of this thesis achieve reduction in the size of the raw data to be analyzed leading to reduced computational as well as monetary effort. Compression of the raw vibration response data acquired from the vibration tests on structures is vital from the standpoint of faster real time monitoring of historic structures.

This thesis is composed of two manuscripts. The first manuscript illustrates the concepts of feature assimilation and noise sensitivity on an arch-like structure using both numerical and experimental analysis. The second study is focused on finding optimal sensor locations for vibration testing of Gothic Cathedrals. A modified version of the Effective Independence Method is used for this purpose. This thesis aims to develop a best-practices guide for effective application of SHM for the use of professionals involved in assessing, preserving and maintaining cultural monuments.
DEDICATION

I would like to dedicate this thesis to my parents Arvind Prabhu and Meena Prabhu and my sister Gauri Prabhu.
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I would like to thank my adviser Dr. Sez Atamturktur and my committee members Dr. Ravi, Dr. Nielson and Mr. Bennett for their constant support and encouragement throughout my degree program. I also want to thank my collaborator, Jordan Supler whose meticulous work has contributed to this thesis. I also would like to thank the PTT Grants program of National Center for Preservation Technology and Training (NCPTT) of Department of Interior, for providing the funds that made this work possible.
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CHAPTER ONE
INTRODUCTION

There are a large number of monumental masonry buildings currently in service that have been built using medieval construction techniques. The U.S. National Register of Historic Places (National Park Service 2009) for instance, lists over 2,000 historic buildings in the continental U.S. built in the 'Gothic' style alone. These historic masonry monuments experience the degrading effects of aging and accumulated damage over their lifetime. If the structural integrity of these monuments is not proactively maintained, these monuments may experience sudden loss of integrity without warning. In the past few decades, numerous historic masonry structures have suddenly lost their structural integrity resulting in partial collapse; some of the more notable examples include the Civic Tower of Pavia, Italy (Binda et al. 1992); the bell tower of St. Magdalena in Goch, Germany (Gantert Engineering Studio 1993); Cathedral of Noto, Italy (Binda et al. 1999); the bell tower of the St. Willibrordus Church in Meldert, Belgium (Ignoul and Van Gemert 2006); “Maagdentoren” in Zichem, Belgium (Ignoul and Van Gemert 2007); Church of Kerksken, Belgium (Verstrynge et al. 2011). These incidents have increased the relevance and awareness of safeguarding culturally significant masonry structures through periodic evaluation and assessment. This is currently accomplished using localized techniques (such as endoscopes, thermographs, sonic tomography, ultrasonic, acoustics, radiographs, and the impact-echo method) that are limited in effectiveness when the vicinity of damage is unknown or inaccessible. Also, localized methods are labor intensive and require highly trained personnel. Structural Health Monitoring (SHM) has the potential to be a fully automated, long-term diagnostic approach for
use in the continuous monitoring of structural behaviors (for a review of the method, see Carden and Fanning 2004 and Doebling et al. 1996).

SHM techniques based on global vibration measurements are particularly advantageous when structural problems occur internally, making visual identification of impending loss of integrity difficult. Through early diagnosis, sudden collapse of the structure can be avoided and, thus, the costs of reconstruction can be reduced significantly in addition to the life-safety of the occupants. It is unlikely that one can perform destructive testing on historic monuments owing to its historic and cultural significance.

The basic concept in SHM is that the vibration response of a structure is dependent upon physical properties such as mass, stiffness, and damping. A change in the structural state of a building will change these properties, which in turn alters the vibration response. The underlying assumption is that changes in vibration response, as measured through in situ measurements, can be related to the structural state of a building. Vibration testing is then typically accomplished by exciting a structure using controlled vibrations (e.g. impact hammers, shakers) and capturing the response of the system over time. These time history responses are then transformed into the frequency domain through a fast Fourier transform (FFT). Estimates of the vibration parameters of the structure i.e. the natural frequencies, mode shapes, and damping ratios are obtained by applying various signal processing techniques.

By monitoring frequency response characteristics, SHM has been successively applied to modern structures. However, the application of this new technology to existing historic masonry monuments is still considered to be an unsolved issue (De Stefano and Ceravolo 2007). The greatest obstacle to the successful application of SHM in existing monumental structures is the lack of guidance in making suitable decisions about the following two aspects:
1. Selection of suitable vibration response features (damage indicator) for symptom-based diagnosis of damage, such as peak response, FRF amplitude, natural frequencies, mode shapes, mode shape curvature, etc. (Farrar et al. 2007).

2. Placement of sensors in optimum locations so that vibration response is sensitive to damage.

This thesis, which aims at addressing these two major issues, is compilation of two manuscripts. The first manuscript introduces the concept of feature assimilation with an intention of early detection of damage in the structure and increased confidence in presence of extraneous noise that corrupts the data. It is emphasized that observing multiple low dimensional features together is more advantageous than focusing on a single feature. This concept is illustrated on an arch, a common structural component of historic masonry construction. The second manuscript presents an elaborate methodology applied for finding optimal sensor locations for deploying SHM on Gothic style Cathedrals. Ultimately, this thesis supplies a best practice guide to be used by engineers for testing Gothic style masonry Cathedrals. Although, demonstrated on Gothic style Cathedral, the methodology applied is deemed suitable for any structure.

With proper guidance, the techniques of SHM can be extended beyond modern buildings to existing historic masonry monuments. This thesis intends to make such guidance available to professionals involved in assessing, preserving and maintaining historic structures.

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CHAPTER TWO

FEATURE ASSIMILATION FOR CONDITION BASED MAINTENANCE

ABSTRACT

Structural health monitoring (SHM) technology for the early detection and mitigation of adverse structural effects, such as degradation or damage, is useful in enhancing the proactive maintenance of civil infrastructure. SHM techniques are advantageous since they eliminate the need for both a priori knowledge of the location of damage and access to the damaged portion of the structure. The underlying principle behind SHM involves measuring changes in a system’s vibration response, which ultimately indicates changes in physical properties due to structural damage. A challenge to the successful application of SHM to civil infrastructure is the selection of suitable vibration response features that are highly sensitive to the presence and extent of damage, while having low sensitivity to extraneous noise. This study reveals that both damage and noise sensitivity of vibration response features vary for different states of structural health; therefore, the selection of optimum features is dependent on the damage severity, which is not known a priori. This study illustrates that assimilating multiple low-dimensional features lessens this dependency and improves the sensitivity of the damage indicators for SHM diagnosis.

KEYWORDS: Vibration Testing, Experimental Modal Analysis, Damage Detection, System Identification, Feature Extraction, Masonry Arches.
1 INTRODUCTION

Structural Health Monitoring (SHM) based on vibration measurements, has emerged as a global monitoring technique to diagnose damage in a system prior to the structural condition reaching a critical stage. The underlying principle of SHM is straightforward: the onset of damage in a built system modifies the structural properties, such as mass, stiffness and damping, which in turn alters the vibration response of the system. Focusing on this indirect relationship between the structural damage and changes in the measured vibration response, SHM aims to detect the onset of damage as well as determine damage characteristics [1]. In an ideal situation, the changes in the measured vibration response are directly correlated with the desired damage characteristics, such as the severity, type and location of damage [2]. However, attempts to correlate the changes in the measured raw time domain vibration response to damage characteristics are hindered by two factors: (i) the difficulty in monitoring the trends in the oversized measurement data and (ii) the high sensitivity of the time domain measurements to extraneous factors caused by the natural variations in the operational and environmental conditions. Thus, low-dimensional vibration response features are extracted from the raw time domain measurements through data processing and interrogation [3]. While the most common vibration response features include natural frequencies, mode shapes, and basic properties of the time history response, such as peak acceleration, many other forms of vibration response features can be extracted from the raw time domain vibration measurements. Ideally, the extracted vibration response features should be sensitive to damage, but insensitive to extraneous noise factors [4]. For practical applications of SHM on civil structures, there has been much debate over whether vibration response features can be sufficiently sensitive to damage, while remaining insensitive to noise [5-8]. For instance, while some studies [9-12] found vibration response to be
insensitive to damage; other studies [13-17] observed vibration features to be reliable damage indicators.

Figure 1 schematically illustrates the difference between a damage-sensitive and -insensitive vibration response feature. In Figure 1, the unitless damage indicator represents the measured change in a vibration response feature corresponding to the unitless damage index. The damage index, which quantitatively represents the extent of damage, ranges from a value of zero, representing an undamaged structure, to a value that represents the most severe damage condition of interest. The slope of the plot, denoted by $\alpha$, defines the damage sensitivity of a given feature. A steeper slope, $\alpha$, of the plot means a more sensitive feature. Hypothetically, in a worst case scenario, as the damage sensitivity of a feature approaches zero, the feature has insignificant sensitivity; and in a best case scenario, as the damage sensitivity approaches infinity, the feature has significant sensitivity to damage.

Figure 1: Sensitivity of damage indicators to varying levels of damage.
The presence of extraneous noise during vibration testing decreases the ability of vibration features to indicate changes due to damage. Therefore, the damage sensitivity must be considered in light of the noise sensitivity of the feature, which also varies for varying structural states. The schematic illustration in Figure 1 represents the noise sensitivity level gradually increasing for increasing stages of damage. As Figure 1 illustrates, insensitive damage indicators may remain below their noise level even for very high damage indices. On the other hand, sensitive indicators are those that yield values significantly above noise levels at early stages of damage. The schematic illustration of Figure 1 is, of course, an idealized approximation. In practice, both damage and noise sensitivity of vibration response features may vary nonlinearly for different states of structural health; therefore, the selection of optimum features may become dependent upon the damage severity. Moreover, the difficulty in selection of optimum features is compounded with the fact that the feature sensitivity may also change for different damage types. Of course, neither the damage type nor the severity is known \textit{a priori}; therefore, it becomes necessary to use \textit{multiple} vibration response features to account for the non-uniform relationship of a feature’s sensitivity to varying damage type and severity.

The objective of this study is to investigate if damage sensitivity of SHM diagnostics can be improved by assimilating multiple low dimensional vibration response features instead of focusing on a single feature. An experimental study is completed on a scaled semi-circular arch. Section 2 introduces the case study structure and Section 3 overviews the design and execution of the experimental campaign. Section 4 investigates discusses the damage sensitivity of commonly used features. Section 5 overviews the practical application of the assimilation concept and presents the noise sensitivity of selected features. Finally in Section 6, overview of the main findings, discussions on the underlying premises and suggestions for future directions are given.
2  CASE STUDY APPLICATION: ARCH PROTOTYPE

The present study investigates the vibration response of an idealized arch model, a common structural form found in masonry construction, and mimics a typical failure mechanism of masonry arches. If a masonry arch is loaded beyond its capacity, cracks incrementally develop within the arch assembly. A crack propagating through the entire depth of the arch forms a hinge. According to mechanism analysis, the development of four hinges is needed to ensure the failure of an arch with a fixed support [18] (Figure 2), while the location of these four hinges depends upon the loading condition. Herein, a numerical model is used to determine the precise locations of the cracks under a static concentrated load applied at quarter span (Figure 3). Four distinct locations with the highest von Mises stress are approximated as the locations of the four hinges, which agree well with those obtained through a nonlinear FE analysis by Ramos [19] (Figure 3).

![Figure 2: Crack locations in an arch due to concentrated load at the quarter span using the inverted chain analogy, reprinted from Heyman (1997) with permission.](image-url)
For the experimental campaign, a PVC arch with a 31.8 cm radius, 64 cm depth and 2.5 cm thickness is used (Figure 4). The arch is damaged to four levels in succession, with a 2 cm deep crack at each of the four hinge locations as shown in Figure 3a. A total of four damage states are obtained with varying levels of structural damage.

3 EXPERIMENTAL PROCEDURE

The response is measured at 33 equidistant measurement points with three across the width of the arch and 11 on the perimeter. Hammer impact tests are performed exciting two points (points 21 and 23) on the arch, allowing the excitation of both bending and torsional modes (Figure 4). Point 23, located through the centerline of the arch, primarily excites the bending modes, while Point 21, located at the edge, primarily excites the torsional modes. Piezoelectric IEPE accelerometers with a sensitivity of 500 mV/g are used to measure the vibrations, while an impact hammer with a sensitivity of 2.27 mV/N and a maximum force capacity of 2200 N is used to excite the structure. The impact hammer is used with the softest plastic tip available.
Impact hammer tests are performed with a frequency range of 0 to 1.6 kHz. The frequency resolution is set to 1 Hz and the time resolution is set at 244 μsec. The acceleration response is measured for 1 second, within which the response of the arch is attenuated; therefore, no windowing function is applied. To reduce the degrading effects of noise and to increase statistical reliability, a total of five averages are obtained. Anti-aliasing filters are used to prevent higher frequencies from contaminating the measurements.

Using the Rational Fraction Polynomial algorithm, Frequency Response Functions (FRFs) collected for each damage state, are analyzed to identify the natural frequency and mode shapes of the arch. The modes are selected using both the Summation and Multivariate Mode Indicator Functions (MMIF) [20]. These functions make the resonance peaks in the FRFs more evident. The first four modes of the model arch are identified. Next, the tests are repeated in the presence of artificial, random noise to investigate the effect of damage levels on the noise sensitivity of FRFs.
4 EXTRACTING VIBRATION RESPONSE FEATURES

Rarely, multiple vibration response features are objectively and quantitatively compared in their ability to indicate damage [6]. In this study, an extensive list of vibration response features is evaluated, including Frequency Response Assurance Criterion (FRAC) [21], Root-Mean-Square (RMS) Time Domain Response [22], Modal Assurance Criterion (MAC) [23], and Coordinate Modal Assurance Criterion (COMAC) [24]. For brevity only a select few of the most common of these features are reported in detail, while a brief summary of the results for the rest are provided in Section 6.

4.1 NATURAL FREQUENCY CHANGES

Natural frequencies (also known as resonant frequencies) supply convenient, low-dimensional and physically meaningful vibration response features [7, 25-27]. In earlier SHM related studies, higher order natural frequencies are reported to be more sensitive to damage than lower order natural frequencies [28-29]. For instance in a recent study, the first three natural frequencies are observed to be identical for a damaged and undamaged masonry vault of Beverley Minster, a masonry cathedral located in the U.K. [30]. In contrast with earlier studies, for the arch studied herein, the natural frequencies are noted to exhibit sensitivity to the propagation of cracks (Figure 5).

The measured changes in the first, third and fourth natural frequencies reach approximately 18% for the most severe damage state. The experiments identify the frequency of the second mode as the least sensitive feature. The first, third and fourth natural frequencies exhibit a monotonic increase for increasing levels of damage; while the second natural frequency exhibits false-negatives since the % change in the frequency is reduced from the second damage state to the third. In such a situation, the monitored structure may appear to be at the same (or better)
structural health level, while in fact the damage is propagating, i.e. the structure exhibits a false negative.

![Figure 5: Percentage change in the first four natural frequencies with progressive damage.](image)

4.2 MODE SHAPE DISTORTIONS

The onset of damage in a structural system tends to distort the mode shapes [31]. For SHM purposes, the mode shape distortions can be exploited in a variety of forms, such as MAC correlation or percentage change, as reviewed by Ewins and Ho [32]. In this study, when determining mode shape distortion, the mode shape vectors are first normalized between 0 and 1, and then the percentage change in mode shape difference is calculated according to Equation (1):

$$\frac{\sum_{i=1}^{N}|x_{di}-x_{ui}|}{\sum x_{ui}} \times 100$$

where:

- $x_{d}$ = the modal displacement of damaged structure
- $x_{u}$ = the modal displacement of undamaged structure
- $N$ = the number of measurement points.
This percentage change is reported for the 33 measurement points of the experimental arch model in Figure 6.

![Graph showing percentage change in the first four mode shape vectors with progressive damage.](image)

**Figure 6**: Percentage change in the first four mode shape vectors with progressive damage.

Compared to the natural frequencies, the sensitivity of the mode shape distortion is observed to be significantly higher. The second, third and fourth mode shapes exhibit an approximately 65-85% distortion for the most severe damage state; this value is only 30% for the first mode shape. The experimental findings presented in Figure 6 reveal a nonlinear and non-monotonic relationship between the mode shape distortions and the extent of damage. The sensitivity of mode shape distortion as a damage-indicating feature is observed to decrease at certain damage levels resulting in false-negatives.

### 4.3 Mode Shape Curvature

Mode shape curvature is a localized vibration feature that is inversely related to the stiffness at the location it is calculated [33]. Since the presence of a crack or separation abruptly decreases the stiffness in the vicinity of damage, the mode shape curvature also abruptly changes near the
location of the damage. The mode shape curvature at node \( i \) is calculated according to Equation (2).

\[
MSC_i = \frac{\Phi_{i+1} - 2\Phi_i - \Phi_{i-1}}{h^2}
\]  

(2)

where:

\( \Phi_i \) = Modal displacement at degree of freedom \( i \)

\( h \) = distance between degree of freedom \( i+1 \) and \( i-1 \)

Figure 7 represents the summation of the mode shape curvature changes for the first four modes summed for all 33 measurement points. The sensitivity of mode shape curvature is noted to be slightly lower than that of the mode shape distortions varying between 55-80% for the most severe damage state. The experimental campaign indicates that the second mode shape curvature is the most sensitive to damage and the first mode shape curvature the least sensitive.

![Figure 7: Percentage change in mode shape curvature with progressive damage.](image)

**4.4 STATISTICAL MOMENTS**

Statistical moments, such as mean, standard deviation, skewness and kurtosis, can be used to effectively compress and characterize raw vibration response measurements, \( x_i \). The first statistical moment is the *mean* of the vibration response measurements given in Equation 3 [34],
which describes the central tendency of the data. The second statistical moment is the *standard deviation*, which measures the dispersion of the data from the mean. The standard deviation of the dataset is given in Equation 4 [34]. The third statistical moment is *skewness*, which measures the asymmetry of the probability density function (PDF). The skewness of a time series is given by Equation 5. A zero skewness value means that the values are evenly distributed on both sides of the mean. The fourth statistical moment is the *kurtosis*, which is a measure of the weight of the tails, i.e. the relative amount of data within the tails of a time series. The kurtosis is calculated according to Equation 6 [34]. A higher kurtosis indicates a distribution, where a majority of the variance is caused by a few severe deviations from the mean rather than more frequent modest deviations.

\[
\mu = \frac{\sum_{t=1}^{N} x_t}{N} = E(x_t) \tag{3}
\]

\[
\sigma_x = \sqrt{\frac{\sum_{t=1}^{N} (x_t - \mu_x)^2}{N}} = \frac{E(x_t - \mu_x)}{\sqrt{N}} \tag{4}
\]

\[
S = \frac{E(x_t - \mu_x)^3}{\sigma_x^3} \tag{5}
\]

\[
k = \frac{E(x_t - \mu_x)^4}{\sigma_x^4} \tag{6}
\]

where:

- \(x_t\) = vibration response data
- \(N\) = the number of data points
- \(E\) = expectation operator, calculates the mean of a random quantity

The first four statistical moments are calculated considering all the FRFs for 33 measurement locations on the test arch. The first two statistical moments are observed to be insensitive to damage and thus left out of further discussion. The third (skewness) and the fourth (kurtosis) statistical moments, plotted in Figure 8 are observed to exhibit high sensitivity to damage. The
percentage changes in these two features are plotted for the experimental data in Figure 9. The skewness feature yields a 90% total change for all sensor locations due to damage for the most severe damage state. The kurtosis feature is more than 3 times as sensitive as the skewness and yields a 300% change for the same damage level.

The damage sensitivity of the skewness and kurtosis are significantly higher compared to the natural frequencies and mode shape derivatives. However, as it will be discussed in Section 4.3, one must bear in mind that skewness and kurtosis are also sensitive to environmental noise.

Figure 8: (Top) Skewness and (Bottom) Kurtosis of FRF measurements from the experimental hammer impact test.
Figure 9: Absolute percentage change in the (Left) skewness and (Right) kurtosis for FRFs from the experimental campaign with the dashed lines signifying the noise sensitivity.

4.5 REGRESSION ANALYSIS

Time domain regression analysis aims to train models to fit auto-correlated time-series data. The coefficients of the fitted model, the residuals between the model and the time domain data, or as in our case, the singular values of the fitted model can be used as features. Perhaps, the most common regression analysis is the Autoregressive (AR) model, which is given for an order \( p \) in Equation 7 [35]. Herein, the Root Mean Square Error measure is implemented to find the optimal AR order. Root Mean Square Error is a measure of the total difference between values estimated by the AR model and actual measured values. A maximum order of 22 and a minimum order of 2 are obtained for the FRFs from the experiments. Therefore, to avoid any loss of information and reduce the residuals, a model order of 22 is used for future analyses.

\[
x_t = \sum_{i=1}^{p} \phi_i x(t - i) + e_t \tag{7}
\]

where:

\( x_t \) = the time or frequency domain response under investigation,

\( e_t \) = the residual term
φᵢ = the AR parameters

Herein, an AR model is trained using the least squares estimation of the undamaged arch. The magnitude of the singular values of these trained models act as features, while the changes in the features are calculated based on Euclidean norm of residuals between the undamaged and damaged singular values. The changes in features are normalized and summed for all 33 measurement points to obtain the damage indicator (Figure 10).

![Figure 10: Singular Value Decomposition (SVD) scores for AR model parameters of the FRFs from the experimental campaign.](image)

5 ASSIMILATING VIBRATION RESPONSE FEATURES

The vibration response features described in the previous section are advantageous, since they are low-dimensional and thus make trends in the vibration response readily observable. Moreover, the mathematical model-fitting during feature extraction acts as a filter and to a certain extent, removes the extraneous effects of noise factors from these low-dimensional features. However, while operating with such low dimensional features, there is the danger of excessively reducing the measurement data, which may result in the loss of important information about the
structural damage. This section illustrates an approach to remedy this problem by assimilating multiple low dimensional vibration features.

5.1 DAMAGE SENSITIVITY OF ASSIMILATED FEATURES

 Earlier in the paper, Figure 1 introduced the concept of determining the damage sensitivity of a vibration feature based on the slope, $\alpha$, between the damage indicator and damage index. Recall that the damage indicator is a unitless entity and thus, can be directly compared against each other. Therefore, once all the damage indicators are normalized and made dimensionless, they can be added together to increase the sensitivity to damage. This approach can be extended to all possible combinations of vibration features, as long as the damage indicators derived from the changes in the vibration features are treated as normalized values.

 Figure 11 illustrates the assimilated damage indicators obtained through the experimental campaigns considering only the frequency, mode shape distortion and mode shape curvature. Here, the damage sensitive indicators of the vibration response features are summed together to obtain a more sensitive damage indicator. As seen, assimilation of various vibration features greatly increases the slope, $\alpha$, of the damage indicator with respect to the damage index. By adding new features through assimilation, this slope, $\alpha$, can be further improved up to an asymptote of infinity. For instance, see the bottom figure in Figure 11, where features obtained from statistical moments and regression analysis are added to those in the top figure in Figure 11 to reach an even more sensitive damage indicator. As seen, the slope, $\alpha$, of the plot increases approximately from seven to 13 by the addition of two new features.
Figure 11: Damage sensitivity comparison of various damage indicators with progressive damage: (Top) experimental campaign with three features, (Bottom) experimental campaign with five features.

Section 4 revealed that the sensitivity of features may vary for different damage levels and exhibit decreasing trends. However, as seen in Figures 11, the assimilation of multiple low-dimensional features yields a monotonic, non-decreasing trend and reduces the risk of false negatives, i.e. interpreting the data as if the structure is maintaining its state, while the damage is in fact propagating.
5.2 NOISE SENSITIVITY

The selection of vibration response features must go beyond damage sensitivity and consider the effects of noise on features. In this study, the vibration response features are identified under controlled excitation forces (i.e., impact excitation); therefore herein noise constitutes the effects of ambient noise in the system on identified features.

This section investigates the degrading effects of random noise by applying an artificial white noise signal with amplitude of 0.25V rms and a frequency range of 0 to 1.6 KHz to the scaled arch structure using electrodynamic shakers. Figure 12 presents the FRFs for the five damage levels obtained with and without artificial noise. As evidenced in Figure 12, the effect of noise on FRFs is not constant for all damage levels. In fact, the noise sensitivity of FRFs increases as the damage level increases. Therefore, as damage progresses, system identification of modal parameters and extraction of other low dimensional features form FRFs become increasingly difficult and the parameters extracted become increasingly uncertain. This implies that in Figure 1 the noise sensitivity of features should in fact be represented as a non-constant variable.
In Figure 9, the dash-lines display the percentage change in the skewness and the kurtosis, respectively, between the FRFs with and without the applied noise for all experimental damage states. The change in noise sensitivity of skewness remains below 30% and exhibits a non-monotonic trend as the damage increases. On the other hand, for increasing levels of damage, the absolute change in kurtosis, i.e. the peakedness of the FRF, remains below 35%. As seen in
Figure 12, the FRFs become noisier as damage a level increase, which makes it more difficult to identify features in the FRFs [36].

In Figure 13, the assimilation of both damage and noise sensitivity plots considering the skewness and kurtosis features is demonstrated. Here, for the most severe damage state, the assimilated damage indicator reaches as high as 400%, while the assimilated noise effects remain as low as 50%, yielding a signal-to-noise ratio of eight to one. However, from Figure 13, it is also evident that the first damage state with a single crack is not diagnosable when skewness and kurtosis features are used. The changes in the selected vibration response features are approximately 6%, which fall below the noise levels of 17%.

![Figure 13: Assimilated damage indicator with the change in skewness and kurtosis and the assimilated noise level for the two features.](image)

6 DISCUSSION

Through the case study structure evaluated herein, it is observed that the noise sensitivity of a structure increases as the damage level increases. Therefore, for varying levels of damage severity, there is a need to determine optimum features, which concurrently exhibit high damage sensitivity and low noise sensitivity, and thus yield high signal-to-noise ratios. A variety of vibration response based features are experimentally evaluated for the semi-circular arch
considering the propagation of cracks due to a hypothetical, gradually increasing concentrated load. The development of four distinct hinges is approximated as cut-outs from the cross-section. Experimentally obtained natural frequencies, mode shape distortions, mode shape curvature, as well as features obtained from statistical moments and regression analysis exhibit monotonic, non-decreasing trends as the damage level increases. For higher order modes, difficulties are observed when calculating the MAC and COMAC features due to the modes swapping order, appearing and disappearing as damage propagates. Owing to the abrupt changes in the sequence of modes, the FRAC are observed to yield significantly high changes. However, it must be emphasized that the noise sensitivity of the FRFs is observed to increases with the increase in damage; therefore, for practical applications a change in FRAC may overestimate the presence of damage. The maximum vibration response of the structure is found to be increasing in a linear relationship to an increase in the damage index, due to the increased flexibility of the structure. However, the RMS vibration response feature is observed to have a highly nonlinear relationship to the severity of damage.

7 CONCLUSION

SHM is a global diagnostic technique for condition-based maintenance of civil infrastructure. In the past, the research community involved in SHM has used a variety of vibration based features. Largely based on the convenience and ease in their identification from measurements, natural frequencies and mode shapes as well as their derivatives have received the most attention in published work. An overview of pertinent literature reveals that optimum damage indicating features vary based on the varied user end requirements of SHM, i.e. the specifics of the structure and damage types to be diagnosed. Therefore, it becomes challenging to select a single, best feature, while maintaining the general applicability of the diagnostic procedure. Our contribution
to the state-of-the-art is the concept of assimilating normalized unitless damage indicators. By first normalizing the damage indicators, one can obtain unitless quantities which cannot only be objectively compared in their damage and noise sensitivity, but also be added together to become better indicators of damage.

The experimental campaign is configured to mimic an impact hammer test, which is a type of test where controlled excitation sources are used. However, the experimental findings of ambient vibration testing can easily be incorporated into the proposed framework. This study presumes the availability of measurements from the undamaged state of the structure of interest and this assumption, in real life applications, may reduce the practicality of the proposed method.

Although the vibration response features are observed to be successful in indicating damage in general, the results presented herein may vary for different structure types and damage scenarios. Therefore, similar studies must be completed for other common forms of masonry structures, such as domes, vaults, buttresses, etc.

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REFERENCES


CHAPTER THREE

VIBRATION TESTING OF GOTHIC CATHEDRALS: OPTIMAL SENSOR LOCATIONS BASED ON MODIFIED EFFECTIVE INFORMATION METHOD

ABSTRACT

This manuscript supplies guidance regarding the optimal regions for placing vibration measurement sensors to properly extract the dynamic characteristics of Gothic style masonry churches based on a study completed on the Cathedral of St. John the Divine. Placing sensors at optimal locations cannot only reduce the cost and time related demands of vibration testing but also lower the amount of measurement data to be post-processed. In this study, first, an accurate finite element model of the Church is built and correlated against in-situ measurement and inspection data. Using this correlated finite element model, optimal sensor locations are determined through a modified version of the Effective Independence Method, in which the goal is to both maximize the relative independence of mode shape vectors of interest and effectively explore the geometry of the structure. A tradeoff between information gain and the visual observability of mode shapes is noted. The relationship between the number of desired modes and the required number of sensors is investigated. The robustness of the method to modeling errors and thus, the validity of the guidelines presented herein are demonstrated. Although focus has been given to large scale testing of Gothic Cathedrals, the methodology and concepts presented herein can be applied to many forms of structural systems.

Keywords: Experimental Modal Analysis, Vibration Testing, Optimal Sensor Placement, Structural Health Monitoring.
1. INTRODUCTION

Safeguarding heritage structures requires accurate numerical models for structural assessment as well as reliable continuous monitoring systems for damage diagnosis. In both applications, in situ experimental measurements become necessary. In this regard, modal analysis that supplies the natural frequencies and mode shapes of the structure has gained popularity as a monitoring and assessment tool (Atamturktur et al. 2009). Modal analysis can serve two specific purposes (i) experimental evidence to calibrate and validate the numerical models of historic monuments and (ii) non-destructive testing and evaluation method for periodic monitoring of historic monuments.

Implementing modal analysis as experimental evidence for the calibration and validation of numerical models of historic masonry structures falls under the aegis of the concept known as experiment-based validation. The primary concern of experiment-based validation is to ensure that the boundary conditions and material properties of the block and mortar assembly are defined properly. Successful applications of validation include masonry arches (Ramos et al. 2010); domes (Atamturktur and Boothby 2007, Erdogmus 2008, Atamturktur and Sevim 2011); towers (Bayraktar et al. 2009); buildings (Antonacci et al. 2001, De Sortis, Antonacci and Vestroni 2005, Ramos, Laurenco and Costa. 2005); amphitheaters (Zonta 2000); and churches (Turek et al. 2002, Boothby et al. 2006, De Stefano 2007, Erdogmus et al. 2007, Atamturktur and Boothby 2010). Implementation of modal analysis as a non-destructive monitoring tool falls under the aegis of the broader concept of Structural Health Monitoring (SHM) (Farrar and Worden 2007). The basic idea behind vibration based SHM is that a change in a structure’s physical properties, such as stiffness or mass, causes a change in the vibration response of the structure, which can be measured using a variety of vibration measurement techniques. The concept of vibration response
based monitoring of historic structures has gained increased popularity in the past two decades (Armstrong et al. 1995, Ellis 1998, Gentile and Saisi 2007).

For the two applications of modal analysis discussed above, performing full scale testing on a large scale historic masonry monument poses both technical difficulties and practical challenges (Atamturktur et al. 2010 and Atamturktur and Laman 2010). One of the challenges comes from the large size of these monuments. It is of course uneconomical, and more importantly unnecessary, to envelop the entire structure with closely spaced sensors. By finding optimal sensor locations from the perspective of information gain and thus, limiting the necessary number of sensors, it is not only possible to reduce the demands on experimental resources (such as time and budget), but also to reduce the volume of data that must be processed. Moreover, the success of SHM techniques for diagnosing damage has been reported to be dependent upon the proper selection of the sensor locations (Hemez and Farhat 1994); therefore, the optimal sensor placement is crucial to the success of vibration based SHM.

In the earlier published work that involves vibration testing of historic masonry monuments, the sensor locations are selected invariably based upon qualitative engineering judgment and intuition (Atamturktur et al. 2009). The focus of this paper is to supply guidance regarding the optimal sensor locations for Gothic style masonry churches based on quantitative, information gain based criteria. The procedure implemented herein, the modified Effective Independence Method (EIM), selects the optimal sensor locations such that the dynamic characteristics of the structure are properly identified.
This manuscript begins with a detailed explanation of the EIM and the modification introduced to EIM to improve the effective distribution of sensors across the structure. A discussion focused on the details of the case study structure, the Cathedral of Saint John the Divine, follows. The next section provides an overview of the development of the FE model and its correlation with the structure’s dynamic and static response. Then the relationship between the desired mode shapes and the number of necessary sensor locations is demonstrated. Following the discussion on the robustness of the sensor locations, the optimal sensor locations obtained through EIM are provided. Finally, the paper concludes with guidelines for practicing engineers and preservationists on efficiently conducting modal testing experiments on large scale Gothic Cathedrals.

2 MODIFIED EFFECTIVE INDEPENDENCE METHOD

In this section, first an overview of the EIM as originally introduced by Kammer (1991) is given. Next, to ensure that the sensor locations effectively explore the structure, a modification to the EIM by adding a distance based criterion is suggested. It is shown that EIM may exhibit sensitivity to modeling errors. Finally, to remedy this sensitivity and obtain more consistent sensor locations, the error theory proposed by Kammer (1992) is implemented.

2.1 EFFECTIVE INDEPENDENCE METHOD:

The goal of EIM is to retain maximum information about the dynamic behavior of the structural system with a reduced number of sensors through the maximization of the Fisher information matrix (Kammer 1991). It is a relatively simple and rapid method compared to exhaustive processes like those involving neural networks and genetic algorithms; therefore, EIM is ideal for large structures with a high number of possible sensor locations (Kammer 1991).
Successful applications of EIM have been well documented in published literature; see for instance Glassburn (1994), Heo et al. (1997), Meo and Zumpano (2005), and Kammer (1996).

Let us assume a scenario in which the number of candidate sensors is $s$, but due to the resources available for testing it is only possible to use $m \ll s$ sensors. The problem then becomes the optimal placement of $m$ sensors in $s$ possible locations. The EIM iteratively eliminates the sensors that contribute least to the independence of the modal vectors. First, a large enough set of candidate sensors that can clearly identify the desired modes must be selected. EIM initially assumes that the mode shape matrices, obtained from the FE model with the entire set of candidate sensor locations, are linearly independent. Next, EIM chooses $m$ optimum sensors from the $s$ candidate sensors, while maintaining as much linear independence and orthogonality of mode shapes vectors as possible.

Conceptually, the response at any point in an elastic structure can be represented, in the time or frequency domain, as a linear combination of mode shapes (Ewins 2000). The vector of the measured vibration response $y_s$ can be estimated as a combination of $n$ mode shapes and a noise term through the expression:

$$y_s = q\Phi + w$$  \hspace{1cm} (1)

$\Phi$ is the mode shape matrix with $n$ mode shape vectors;
$q$ is the coefficient response vector, a function of time or the natural frequency;
$w$ is the stationary Gaussian white noise with a zero mean value;

To obtain the best estimate of a mode shape, the covariance matrix of the estimate errors must be minimized (see Equation 2). As explained by Udwadia (1994), the covariance matrix of the estimate errors is bound by the Cramer-Rao lower bound.
\[ E[(q - \hat{q})(q - \hat{q})^T] \geq Q^{-1} \]  \hspace{1cm} (2)

Udwadia (1994) explains that for unbiased and efficient estimators, the inequality in Equation 2 becomes an equality. The right hand side of Equation 2 then yields the inverse of the Fisher information matrix, \( Q \). Alternatively, one can derive the Fisher information matrix as given in Equation 3. Accordingly, by maximizing the Fisher information matrix in Equation 3, one can obtain the best estimates of the coefficients of the response vector in Equation 2.

\[
Q = \left[ \left( \frac{\partial y_s}{\partial q} \right)^T \frac{1}{\Psi^2} \left( \frac{\partial y_s}{\partial q} \right) \right] \]  \hspace{1cm} (3)

where,

\( \Psi^2 \) is the Gaussian white noise variance and

\( \hat{q} \) is the efficient unbiased estimator of \( q \)

Next, by substituting Equation 1 into Equation 3, Fisher information matrix is obtained in the following form:

\[
Q = \left[ \Phi^T \frac{1}{\Psi^2} \Phi \right] \]  \hspace{1cm} (4)

Considering the noise as uncorrelated and having identical statistical properties at all locations, the effective independence values (EIV) of the each of the sensors, \( i \), can thus be calculated as:

\[
\text{EIV}_i = \phi_i \cdot Q \cdot \phi_i^T \] \hspace{1cm} i=1,2,...,k  \hspace{1cm} (5)

where \( \phi_i \) is the vector of the target modal co-ordinates of the \( k \)th sensor, and \( k \) is the remaining number of sensors considered in that particular iteration. The EIV of a sensor location lies between 0 and 1, where a value of 0 implies that the target modes are not observable from the sensor location (or that the corresponding row in the mode shape matrix is null). On the other hand, an EIV of 1 implies that the sensor location is vital for maintaining independence of the mode shape matrix and for identifying the target modes. EIM, in an iterative manner, eliminates
the sensor with the minimum EIV from the list of candidate sensors. Both the mode shape and Fisher information matrices are updated after each iteration until the pre-defined number of sensors remains to serve as the optimal set of sensor locations. Because of the iterative nature of the algorithm, the final sensor configuration can be suboptimal; however, Kammer (1991) states that the mode shape estimates from this configuration are close approximations of the actual optimal configuration.

2.2 DISTANCE BASED CRITERION:

One major drawback of the EIM, which is especially evident for symmetric structures, is that it can select measurement locations nearly adjacent to each other. This problem occurs when there are several sensors with approximately the same value of effective independence. The closely spaced sensors measure similar (or near-similar) response and supply an incomplete spatial representation of the mode shape of the structure. Therefore, it is of value to maximize the minimum distance between the sensor locations to assure that sensors effectively explore the geometry of the structure. This way, a better visualization of the mode shapes can be obtained, which is particularly relevant when visually correlating analytical and experimental mode shapes. Therefore, a modification is made to the EIM, originally proposed by Kammer (1991), by introducing a distance based criterion (DBC) to the optimal sensor selection as follows:

$$d_{ij} = \sqrt{(x - \hat{x}_j)^2 + (y - \hat{y}_j)^2 + (z - \hat{z}_j)^2} \quad i=1,2,...,m; \quad j=1,2,...,s$$

where, $x$, $y$ and $z$ are the co-ordinates of the $i^{th}$ optimal sensor chosen by the EIM and $\hat{x}_j$, $\hat{y}_j$ and $\hat{z}_j$ are the co-ordinates of the $j^{th}$ sensor locations. If $d_{ij} < \text{DBC}$, the sensor location $j$ is eliminated. This criterion assures that the sensors are not clustered in certain regions.
2.3 ROBUSTNESS TO MODEL ERROR:

The sensor locations chosen by the EIM may be sensitive to the errors in the numerical model predictions. Such errors may be caused by imprecisions in the material properties and boundary conditions. Therefore, it is crucial to correlate the FE model predictions against experimental evidence to ensure the input values of the FE model are realistic. However, even after elaborate and successful test-analysis correlation, a level of uncertainty in model predictions may remain. To remedy the sensitivity of EIM to the uncertainty in model predictions and to obtain more consistent sensor locations, Kammer (1992) suggested the use of error theory, where the error between the mode shapes of the FE model and those obtained by slightly inaccurate model is calculated as:

\[ \delta_s = \Phi_{fs} - \Phi_{rs} \]  

(7)

where, \( \Phi_{fs} \) is the mode shape matrix obtained by the FE model with uncertain input parameters and \( \Phi_{rs} \) is the reference mode shape matrix.

The net information matrix \( I_n \) is then calculated as:

\[ I_n = A_r - D \]  

(8)

where, \( A_r = \Phi_{rs}^T \Phi_{rs} \) and \( D = \delta_s^T \delta_s \). The information matrix, \( A_r \), corresponds to the reference mode shapes and \( D \) is the information matrix of the mode shape errors. If the matrix \( I_n \) is positive definite (Bhatia 2007) for a particular sensor configuration, then \( A_r \) is greater than \( D \), which means that there is more information in the reference modes than in the mode shape errors. The positive definiteness of \( I_n \) is a sufficient condition for the positive definiteness of the reference mode shape information matrix \( A_r \) (Kammer 1992). In the course of the EIM, \( I_n \) is calculated at every iteration and if it is determined to be positive definite, then the sensor eliminated by the EIM at that iteration is not vital to the independence of the reference modes.
3 THE CASE STUDY STRUCTURE

The structure considered for this study is the Cathedral Church of St. John the Divine (SJD) (Figure 1a). Located in the heart of New York City, this Cathedral of Gothic Revival design is currently the fourth largest in the world (Hall 1920). The construction of the Church began in December 1892 and is still under completion. This study focuses on one of the bays along the nave of the Cathedral (Figure 1b). The 37.8m high and 75.6m long nave consists of four bays each 44.5m wide (Wickersham 1998).

![Diagram of the Cathedral Church of St. John the Divine]

Figure 3: (a) The west front of the Cathedral Church of St. John the Divine, New York, (b) sectional elevation drawing of the nave.

The main walls of SJD are made of Maine granite with Mohegan golden granite facing on the outside and Frontenac stone facing on the inside, while the piers and buttresses are in-filled with concrete (Hall 1920). The webbing of the vaults is composed of Guastavino tile (Rossell 1995) and the vault ribs are made of cut schist (Hall 1920).
In this study, a reconnaissance survey of the church building is completed and all possible sensor locations are scouted for (shown later in the highlighted regions of Figure 6a). The walkways between the buttress and the nave, as well as mezzanines inside the nave, allow the placement of sensors at intermediate levels on the piers, walls and buttresses on the inside and outside of the church. The entire nave vault is accessible from the top. The availability of power sources and the length of sensor cables are also considered while selecting the candidate sensor locations.

4 DEVELOPMENT AND VALIDATION OF THE FINITE ELEMENT MODEL

A linear elastic FE model of SJD is built in two systematic phases using the FE software package ANSYS. In the first phase, a substructure consisting of the vaulted section of the naves is built and correlated with experimentally obtained dynamical characteristics of the vaults. In the second phase, the entirety of a single bay is modeled by adding the piers, buttresses and the walls.

4.1 FIRST PHASE: VAULTS OF THE NAVE

Model development: According to available construction drawings, the geometry of the Cathedral is first simplified and idealized preserving the structural properties, such as cross sectional area and the moment of inertia. The vault ribs and the vault webbing are modeled using 20-node SOLID95 brick elements and 8-node SHELL93 shell elements, respectively (ANSYS 2005). Since the stresses in the surcharge are less important, the surcharge material is modeled using lower order, 10-node SOLID92 elements. The entire model of one nave bay consists of 47194 elements.
The initial material property values for the elastic modulus, density and Poisson’s ratio are determined according to historic documentation and engineering judgment (Theodossopoulos 2004, Özen 2006). The vault webbing is built out of Guastavino tile with the thickness varying from 15 cm to 20 cm. The elastic modulus and density of the Guastavino tile is reported by Saliklis, Kurtz and Furnbach (2003). Atamturktur and Sevim (2011) have conducted laboratory experiments on Guastavino tile and mortar specimens and obtained the homogenized material properties for a tile-mortar assembly. The surcharge volume behind the vaults is composed of masonry rubble for which the material properties are determined according to Erdogmus et al. (2007). The initial material properties for the ribs and arches are assigned based upon available documentation on cut schist (URL-1). The element type and material property assignments for different structural components are summarized later in Table 2.

The initial boundary conditions are applied according to a combination of visual observations of support conditions, engineering judgment and the recommendations of Erdogmus (2004). These initial material properties and boundary conditions are then adjusted according to the experimentally measured natural frequencies and mode shapes, respectively. The single bay model is then reflected to obtain a three-vault model as shown in Figure 2.
Test-Analysis Correlation: Since only the vaulted section of SJD is modeled in the first phase, the structural effects of the unmodeled sections of the nave, such as the nave walls, aisles and buttresses are represented as boundary conditions. The initial boundary conditions in the FE model are tuned iteratively by systematically comparing the analytically obtained mode shapes and mode shape sequence with those obtained experimentally (see Table 1). The final boundary conditions consist of the following: (i) displacement and rotational restraint applied in all directions at the base of the vault springing, (ii) longitudinal horizontal restraint to represent the unmodeled adjacent vaults, and (iii) transverse horizontal restrain along the length of the piers where buttresses rest. The initial material properties of the Cathedral are fine-tuned according to the natural frequency agreement. The final values obtained for material properties are given in the first three rows of Table 2 (Boothby, Atamturktur and Hanagan 2006).

The fine-tuning of the material properties and boundary conditions are uncoupled, where the boundary conditions are adjusted according to the mode shape agreement and material properties are fine-tuned according to the natural frequency agreement. Atamturktur and Laman (2011) cautions such uncoupling since the relative ratio of material properties may also have an effect on
the mode shape vectors as well as their sequence. In this study, by perturbing the ratio of different material properties, it is verified that the mode shapes are not altered.

Table 1: Comparison of experimental and analytical modal analysis results of the vaults of St John the Divine.

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Experimental</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13.61 Hz</td>
<td>12.31 Hz</td>
</tr>
<tr>
<td></td>
<td>15.84 Hz</td>
<td>15.71 Hz</td>
</tr>
<tr>
<td></td>
<td>16.27 Hz</td>
<td>16.68 Hz</td>
</tr>
<tr>
<td></td>
<td>17.70 Hz</td>
<td>17.13 Hz</td>
</tr>
<tr>
<td></td>
<td>20.24 Hz</td>
<td>20.51 Hz</td>
</tr>
</tbody>
</table>

4.2 SECOND PHASE: ENTIRETY OF THE NAVE

**Model development:** In the second phase, the model of the first phase is extended to represent an entire bay, including full length piers, the buttresses and aisle vaults (Figure 3). The element types as well as the material properties are kept consistent with those used in the first phase. A new material is added to represent the granite walls and piers. For the elastic modulus of granite, a range of 30-55 GPa is suggested by Özen (2006) and 40-100 GPa in Gere and Timoshenko (1997). In this study, 50 GPa for the granite piers and walls is used. The densities used for the piers and walls are based on historic correspondences obtained from the SJD archives (Adamson 1917). When adding the aisle vaults, BEAM188 elements are used for the ribs. The entire model of one nave bay consists of 125,920 elements.
A fully rigid boundary condition, which prevents translation and rotation in all three directions, is applied to the base of each foundation. At the floor level, piers and buttresses are restrained horizontally to mimic the restraining effect of floor slabs. Symmetry boundary conditions are applied to both the east and west face of the bay to simulate the existence of adjacent bays in the nave.

Table 2: Element type and material properties used in the FE model for different parts of the structure.

<table>
<thead>
<tr>
<th>Structural Member</th>
<th>ANSYS Element</th>
<th>Density (kg/m³)</th>
<th>E (GPa)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ribs/Arches</td>
<td>SOLID95</td>
<td>2100</td>
<td>12</td>
<td>0.2</td>
</tr>
<tr>
<td>Vault Webbing</td>
<td>SHELL93</td>
<td>1600</td>
<td>6.5</td>
<td>0.15</td>
</tr>
<tr>
<td>Rubble Surcharge</td>
<td>SOLID92</td>
<td>1000</td>
<td>4.5</td>
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</tr>
<tr>
<td>Piers</td>
<td>SOLID95</td>
<td>2640</td>
<td>50</td>
<td>0.2</td>
</tr>
<tr>
<td>Walls</td>
<td>SOLID92</td>
<td>2700</td>
<td>50</td>
<td>0.2</td>
</tr>
</tbody>
</table>
**Inspection-Analysis Correlation:** During the reconnaissance survey, the structure is closely inspected for cracking and hinging of the masonry structure. Cracks are consistently observed at the rose openings (see Figure 4) and in the walkways between the buttress and nave (see Figure 5). It is important to note that these cracks are symmetric on both sides of the nave in each of the four nave bays. If the FE model is an accurate representation of reality, then the FE model simulations under the self-weight of the structure should yield high stress concentrations at the locations of these existing cracks (Mark and Hutchinson 1986 and Ricart-Nouel 1991). However, one must take care to ensure that the cracks and hinges are indeed caused by the self-weight of the structure, and not by the differential support settlements. Therefore, during the reconnaissance survey, the entirety of the nave is inspected to determine if there are any signs of support settlement. However, no significant geometric distortions are observed in the structure. Therefore, the cracks are concluded to be caused primarily by gravitational forces, such as the self-weight of the structure; and thus the test-analysis correlation of crack locations is deemed appropriate.

![Figure 4: Cracks at the rose opening (highlighted) corresponding to high tensile stress regions in the FE model: (Left) Existing cracks in the structure, (Right) tensile stress concentrations in the FE model.](image)
The FE model is executed to analyze the behavior of SJD under its own self-weight and the regions with high tensile stresses are marked as shown in Figures 4 and 5. A close match is observed between the high tensile stress regions in the FE model and the existing crack locations in the structure. As evidenced by Figure 4 and Figure 5, the cracks near the rose openings and in the outer walkways are well predicted by the FE solution. Therefore, the FE model of the Cathedral is deemed to be an accurate representation of SJD.

5 SENSOR OPTIMIZATION

A sufficient number of candidate sensor locations need to be chosen to ensure that the target modes are well-defined. Accordingly, 205 nodal locations of the FE model are selected on the vault and additional 576 nodal locations are selected on the rest of the structure (Figure 6). The optimal sensor locations for triaxial accelerometers are determined, with the subsequent mode shape matrix formed with 781 candidate sensor locations considering the resultant of the X, Y
and Z modal displacements. If triaxial sensors are unavailable at the time of the test, then uniaxial sensors can be revolved three times at each location to obtain measurements in all three directions. Similarly, to identify modes in only one or two directions, the mode shape matrix can be modified to include only the modal displacements in those respective directions.

Figure 6: (a) The accessible locations on the structure highlighted; (b) the candidate sensor locations shown as dots.

Eleven of the first twenty analytical modes are observed to be local, where the motion is focused on a single structural member (typically the piers). Therefore, nine of the first 20 modes are identified as ‘target’ global modes (Figure 7). These nine global modes form the initial mode shape matrix with the 781 candidate degrees of freedom. To assure equal participation from all nine modes, the mode shapes are normalized between 0 and 1. The degrees of freedom are then iteratively reduced from 781 down to the predefined number of sensor locations (in our case, 80).
by eliminating one location in succession according to their contributions to the independence of the mode shape matrix.

The percentage loss in information as the sensor locations are eliminated can be calculated as the determinant of the Fisher information matrix. As shown in Figure 8, the number of required sensors is increasing with the number of modes to be identified. Also note that the Fisher information matrix determinant is sensitive not only to the number of target modes but also to the number of candidate locations. Therefore, the actual value of % loss of information given in Figure 8 is irrelevant and must be treated as a relative measure.
Figure 8: Fisher information matrix determinant updated with sensors eliminated iteratively.

Figure 9 shows the EIV of the eliminated sensors with respect to the iteratively updated mode shape matrix, i.e. the mode shape matrix with only the remaining candidate locations. To identify nine modes with each sensor having an EIV of more than 0.1, it is possible to eliminate 701 sensors and use only 80 sensors. As indicated in Figure 9, the number of candidate locations and number of target modes dictates the information content of the mode shape matrix.

Figure 9: Effective independence values of the sensors with respect to the iteratively updated mode shape matrix; (inset) zoomed in to 80 optimal sensors.
For this study, a total of 80 optimal sensor locations for triaxial sensors are sought for the entire bay. Figure 10a and 10b show the optimal locations computed by the EIM before and after applying the DBC. Without the minimum distance specified, the EIM picks closely spaced sensors at locations that yield an incomplete visual representation of the mode shapes. However, in Figure 10b, each of the 80 triaxial sensors is placed at least three meters apart in all directions. The sensors are thoroughly distributed on the structure, which is ideal for visualizing the target modes of interest. Note that the sensor locations computed without the DBC contain more information. Therefore, one must accommodate for the tradeoff between mode independence and visual observability, depending upon the application.

Figure 10: Optimal sensor locations for triaxial sensors on the full bay (a) without DBC, (b) with DBC of 3 meters.
6 DISCUSSIONS

The choice of sensor locations on a functioning monumental structure is governed by many factors (e.g. accessibility, maintaining the aesthetics), in which some of the actual optimal locations are eliminated by default. Therefore, the problem becomes that of selecting the optimal locations out of the feasible candidate locations. In this selection, several important factors must be properly evaluated, such as the threshold value used for the distance based criteria and the robustness of the findings to the uncertainty in model predictions. In the end of this section, inferences regarding the optimal regions for sensor placement in Gothic style structures are made.

6.1 SENSITIVITY TO MINIMUM DISTANCE CRITERIA

While the DBC may eliminate sensors that are mathematically more optimal (Figure 10), the DBC supplies visual observability, which is particularly useful while comparing the analytical and experimental mode shapes. Figure 11 shows the higher rate of information loss as the threshold distance between sensors is increased. Note the exception where DBC=0.5m yields more optimal solutions between 45-80 sensors compared to DBC=0.0m. This can be explained by the inherent sub-optimality due to the iterative nature of EIM.

![Figure 11: Behavior of the Fisher information matrix determinant for different DBC.](image)

Figure 11: Behavior of the Fisher information matrix determinant for different DBC.
6.2 ROBUSTNESS TO MODELING ERROR

The model used in this study is correlated with experimental measurements as well as with on-site inspections. However, slight uncertainties remain in the model predictions. Therefore, it is of value to ensure that such uncertainty does not interfere with the main findings obtained through EIM.

To demonstrate the robustness of the method, assuming a 5% error in the model parameters, six FE models are generated: one reference model with material properties given in Table 2 and the other five, each with a 5% reduction in the elastic modulus of one of the five materials given in Table 2. First, the optimal locations for the six models are plotted without employing a DBC (Figure 12). As seen, the sensor locations are all localized in a few regions on the model, while sensors in some regions are consistently eliminated by the EIM for all five cases. This observation shows that if one considers the regions within the structure and not individual candidate DOFs, the EIM selections are consistent and are robust to errors in model parameters.
The EIM is more sensitive to modeling errors in the presence of a threshold distance between sensors as it is without one. Figure 13 shows the sensor configurations for various DBC from 0m up to 3m. As the distance is increased, the robustness of the EIM diminishes. In an effort to achieve a compromise between the retention of information about the mode shapes, the visual observability of the mode shapes and the robustness of the method to change in input parameters, a DBC of 1.5m is deemed suitable.
To reduce the variability of sensor locations due to changes in material properties, the error theory is applied wherein the sensors that cause the net information matrix to remain positive definite are retained. Figure 14 shows the sensor locations on the six models generated with varying material properties where a DBC of 1.5m is applied to each. There is more than one sensor that is retained by the EIM at every iteration after applying the error theory. Hence, there maybe a few sensors in the final configuration that are spaced less than 1.5m apart.
6.3 OPTIMAL SENSOR LOCATIONS FOR GOTHIC CHURCHES

In Figure 14, the optimal sensor locations are consistently concentrated around (i) ribs on the vaults, (ii) vault webbing around the crown, (iii) top of the nave walls, (iv) buttresses above the vaults and (v) outer edges of the buttresses at level of the first crosswalk.

Figure 14: Optimal sensor locations in the six FE models using a DBC of 1.5m and applying the error theory.

While testing large scale Gothic style churches, placing the sensors at the abovementioned locations are most advantageous from the viewpoint of system identification. The mode shapes derived from these locations ensure maximum independence of the mode shape matrix considering the nine global modes discussed before. Although demonstrated using 80 sensor locations, if the user possesses a fewer number of sensors, it will be most valuable to place them at these general regions. For applications where visualization of the mode shapes is of more
importance, the DBC can be increased by acknowledging the fact that the linear independence of
the mode shapes is sacrificed in the process.

7 CONCLUSIONS

In this paper, the application of an existing analytical procedure, the Effective Independence
Method, is discussed and demonstrated to maximize the information gained during in-situ modal
analysis of a Gothic style masonry Cathedral. Herein, this existing established method is modified
by the addition of a distance based criteria, which assures that the sensors are placed with
sufficient distance from each other to better explore the structure. To reduce the effects of
modeling error on the sensor placement, the error theory is applied that acts as a constraint on the
elimination of critical sensors that are important for the identification of the real modes. Ultimately, through the application of the modified EIM, recommendations are made for selected
regions for sensor placement to expedite the process of and reduce the resources necessary for the
full-scale modal testing of Gothic Cathedrals, such as the Cathedral of St. John the Divine, the
subject of this study.

In this study, a vibration test on the easily accessible vaulted section of SJD is first performed
using previously recommended sensor locations (Atamturktur et al. 2009). A partial FE model is
constructed and calibrated against the experimentally-obtained mode shapes and natural
frequencies. At this phase, the boundary conditions mimicking the structural components
excluded from the model are adjusted and the material properties are fine-tuned. Next, this partial
model is extended to a full bay model by adding the missing components such as the aisles,
buttresses, piers and walls. This full FE model is tested against the visual observations during the
site survey. The high tensile stress regions from a static analysis under self-weight are compared
against visually-observed crack locations in the structure. As a result of the satisfactory agreement, the model is accepted as a useful tool for determining the optimal sensor locations.

The possible candidate sensor locations are determined from a site reconnaissance survey. The mode shape matrix is generated with all of the candidate locations from the FE model and the target modes are chosen. Finally, the candidate locations are reduced to a required number of sensor locations using the EIM modified with the DBC. While the DBC causes a loss in information about the mode shape vectors, it distributes the sensors so that the final optimal set of sensors are not clustered and thus results in better visualization of the mode shapes.

The robustness of the method to uncertainties in FE model parameters is also studied and it is seen that the optimal locations chosen by the EIM localize in specific regions within the geometry even when the material properties are varied. However, the robustness of the method is compromised in the presence of too high a distance criterion, which for SJD is found to be equal or higher than 2m.

Although demonstrated on the Cathedral of St. John the Divine, a historic masonry structure, the concepts presented herein are applicable to many structural systems. The variable introduced by the DBC of the modified EIM, the minimum distance between sensors, requires a measure of engineering judgment that, of course, varies with the objective of the test campaign.
ACKNOWLEDGMENTS

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REFERENCES


CONCLUSIONS

The two manuscripts presented herein to address two of the major problems associated with deploying SHM on historic masonry structures.

The first study deals with the selection and assimilation of optimal response features for damage detection using an FE model of an arch as well as a scaled laboratory model. Features are found to show varying damage sensitivities based on the damage severity as well as location. The noise sensitivity of the structure is also shown to increase with the increase in the level of damage. There is not one particular feature that is robust to the type, location and severity of damage and to the inherent noise. By normalizing the damage indicating features to make them unitless quantities, one can assimilate a number of features to get a better damage indicator that reduces the uncertainties caused due to extraneous noise and the non-linear sensitivities of the features to the damage severity.

In the second study, guidelines for the optimal sensor placement on historic masonry monuments is presented using a modified version of the Effective Independence method. A distance based criteria is enforced that allows the sensors to better explore the structural geometry in order to allow visualization of mode shapes, an important aspect from the testing engineers standpoint. The method uses numerical FE models of the Cathedral of St. John the Divine which is developed in two stages. First, a substructure of the building is modeled and calibrated to the experimental results and then, the entire model of one of the nave bays is created and validated with the on-site inspection of existing cracks. Following the application of a modified Effective independence method, optimal sensor locations are suggested, which are applicable to many Gothic style Cathedral built, which poses typical structural configurations.
Although this thesis is focused on historic structures, professionals involved in testing any structural form can follow the methodologies presented for an efficient deployment of SHM.