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Valuation of timberland under price uncertainty

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Abstract

In the first essay, a critical examination of three commonly used stochastic price processes is presented. Each process is described and rejected as a possible model of lumber futures prices. A mean reverting generalized autoregressive conditional heteroskedasticity (GARCH) model, developed by Bollerslev (1986), is proposed as a stochastic process for lumber futures prices. The essay provides the steps that should be taken to ensure that a proper price process is used in each application.

In the second essay, a flexible harvesting strategy known as the "reservation price" strategy is presented. When the current price is below the reservation price, the forest owner delays the harvest. An optimal stopping model is used to derive an expression for the optimal sequence of reservation prices under price uncertainty. A solution method using a Monte Carlo backward recursion algorithm is presented. The Monte Carlo simulation procedure may be applied when analytical solutions are difficult or intractable.

In the third essay, a simulation model is used to estimate the per acre value of land devoted to timber production under different harvesting strategies, stumpage price processes, and site qualities. By following the reservation price strategy, forest owners can increase the expected land value and reduce the variability in land values relative to a fixed rotation strategy. For an estimated mean reverting GARCH process, the reservation price strategy increases the value of timberland by 33.0 percent for a site index of 90 and by 22.1 percent for a site index of 60 relative to a fixed rotation strategy.

Dedication

To my parents, Bill and Janie Campbell, for believing that a Ph.D. in Economics was a good idea; my grandmother, Edwina Alexander, for suggesting that I choose a topic related to forestry; Aunt Cheryl and Joe, for not letting me quit; and my friends at Clemson, for making the whole process highly enjoyable.

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1 Introduction

1.1 Types of forest owners

Timberland is defined as "forests capable of producing 20 cubic feet per acre of industrial wood annually and not legally reserved from timber harvest" (Smith et al., 2007). In the United States, ownership of forest land is classified into four groups: national forest, other public, private corporate, and private noncorporate. National forest land is managed by the U.S. Forest Service. In 2007, the Forest Service managed approximately 19 percent of the forest land in the United States (Smith et al., 2007). The "other public" ownership class represents forest land managed by the Bureau of Land Management and state and local governments. These owners managed 11 percent of forest land in 2007 (Smith et al., 2007). Private corporate forest owners include timber investment management organizations (TIMOs) and industrial forest owners. Industrial forest owners both manage forest land and operate wood using plants. Approximately 21 percent of forest land was owned by TIMOs and industrial corporations in 2007 (Smith et al., 2007). Nonindustrial private forest (NIPF) owners are a heterogeneous group and have a diverse set of forest management objectives. Private noncorporate forest owners include individuals and family partnerships. Private noncorporate owners managed 49 percent of the forest land in the United States in 2007 - more than any other ownership class (Smith et al., 2007).

1.2 Objectives of forest owners

Of course, timber is not the only output from forest land. To some extent, each ownership classes uses forest land to produce both timber and non-timber goods. Use of land for timber production often conflicts with other management objectives, such as scenic value. In practice, profit maximization from timber production does not apply to all forest owners.

Clearly, the management behavior of the National Forest Service is not profit maximizing. Forest managers are highly limited by regulation. Two acts have largely defined the management strategy of the Forest Service: the Multiple-Use Sustained Yield Act of 1960 and the National Forest Management Act of 1976 (Gorte, 1998). These acts require that Forest Service balance the production of multiple outputs and consider resource preservation in decision making (Gorte, 1998). Managers must specify an annual maximum allowable harvest according to recreational objectives, forest stewardship objectives, and timber supply objectives. Timber supply from National Forest land is increasingly restricted (Gorte, 2004). Forest Service timber sale levels have declined from a peak of 11.3 million board feet in 1987 to 1.6 million board feet in 2002 and bills have been proposed to eliminate timber harvesting on National Forest land (Gorte, 2004).

Although not as constrained by regulation as the National Forest Service, the majority of private noncorporate forest owners do not claim to be profit oriented. According to the 2006 National Woodland Owner Survey (NWOS), only 13 percent of private noncorporate owners in the U.S. with less than fifty acres of land considered timber production as the most important use of their land (USDA, Forest Service, 2006). On average, owners of small tracts of land placed greater importance on amenity values than revenues from timber production (USDA, Forest Service, 2006). In 2006, the majority of private noncorporate forest owners owned small tracts of land: nearly 90 percent of private noncorporate owners owned fewer than 50 acres of forest land (Butler, 2008). However, private noncorporate owners with large tracts of land were more likely to be profit maximizing. In 2006, fifty-one percent of owners with over 1,000 acres of forest land considered timber production to be the most important use of their land (USDA, Forest Service, 2006).

Even industrial forest owners are not simply focused on maximizing the value of production from their timber stands. Convenience yield is an important objective determining timber harvesting activity among industrial forest owners (Provencher, 1995a). Convenience yield arises from the need to have a constant flow of timber to keep mill equipment running; underutilized workers and equipment are costly. Because of the fixed costs of operating timber mills, industrial owners have incentives to maintain a certain level of production even during periods of low timber prices (Provencher, 1995a).

Timber investment management organizations (TIMOs) are typically expected to maximize profits from timber production (Binkley et al., 1996). Timberland has become a popular asset class among investors seeking a stable investment with high dividend yields (Binkley et al., 1996). With capital from hedge funds, pension funds, and individual investors, organizations such as TIMOs and REITs were formed with the purpose of managing forest land for profit. Through timber harvesting, TIMOs use forest land to generate dividends and capital gains for shareholders. These nonindustrial corporate owners manage more than half of the land formerly owned by industrial corporations: as of 2006, TIMOs managed 22.4 million acres of timberland, worth approximately eighteen billion dollars (USDA, Forest Service, 2011).

Profit maximizing agents are most likely to be interested in the topics discussed in this dissertation. The first essay, an analysis of lumber futures prices, primarily applies to large-scale forest owners that might wish to hedge against timber price risk. The flexible harvesting strategy presented in the second and third essays reflects a profit maximizing approach to forest management. The strategy requires active management of timberland specifically for timber production. As a result, the strategy may not coincide with the objectives of the Forest Service or small-scale, private noncorporate owners. The flexible harvesting strategy primarily applies to large-scale private noncorporate owners and TIMOs. These owners are likely to be profit maximizing and have the freedom to set rotation intervals without being restricted by maximum allowable cuts or convenience yield.

2 Choosing a stochastic price process

A stochastic process is a sequence of random variables that describes the evolution of a system over time (Clapham and Nicholson, 2009). Stochastic processes are often used in dynamic models of uncertainty, including price uncertainty (Dixit and Pindyck, 1994, page 12). The choice of a particular stochastic price process is often critical to the results of the study. However, many studies in forestry economics have examined the effects of fluctuating prices in forestry without devoting attention to accurately modeling the stochastic price process (Brazee and Mendelsohn, 1988; Thomson, 1992; Lu and Gong, 2003; Alvarez and Koskela, 2006, 2007; Manley and Niquidet, 2010). The main objective of this essay is to provide a price process that can be used for simulation of lumber futures prices and to describe the steps that should be taken to ensure that a proper price process is used in each application.

In this essay, three commonly used price processes in forestry economics - independent draws from a normal distribution, geometric Brownian motion, and the Ornstein-Uhlenbeck processes - are described and rejected as a price model for lumber futures price data. Multiple hypotheses are tested regarding features of the price process and distribution of price changes. A generalized autoregressive heteroskedasticity (GARCH) model, developed by Bollerslev (1986), is proposed as a stochastic process for prices. GARCH models imply clustering volatility and heavy tails in the distribution of price changes - features that are consistent with observed lumber futures prices.

2.1 Data

The monthly average price for random length lumber futures from November, 1972 to December, 2011 will be used for analysis (Chicago Mercantile Exchange, 2012). Monthly average prices were calculated from a set of daily opening prices using an algorithm presented in the Appendix, page 72. There are 470 observations in the monthly average price series. These prices represent the front month contract for two inch by four inch lumber, eight to twenty feet long, in dollars per thousand board feet ($/MBF). The front month contract refers to the futures contract with the shortest duration relative to the current date (Chicago Mercantile Exchange, 2012). Real prices were calculated using the consumer price index (CPI) as a measure of inflation (United States Department of Labor, Bureau of Labor Statistics, 2012). The real price at the beginning of month t is

$$P_t^R = P_t \times \left[\frac{\mathrm{CPI}_0}{\mathrm{CPI}_t}\right], \tag{2.1}$$

where CPI_0 is the value of the CPI in January, 1972 and CPI_t is the value of the CPI at the beginning of month t. The time series of monthly average prices and corresponding real prices of lumber futures is presented in Figure 1. Here, real prices are in constant January, 1972 dollars.

2.2 Stochastic price models

An overview of three commonly used stochastic processes is presented below. Each type of price process is characterized by a rational expectations forecast of future prices. Rational expectations imply that forest owners know the properties of the price process and forecasts do not systematically deviate from realized prices (Muth, 1961). The parameters of each model are estimated using lumber futures data and plots of simulated processes are presented. The computer code for the estimation and simulation of each process is presented in the Appendix, pages 72 to 74.

2.2.1 Independent draws from a probability distribution

Properties

Consider a price process consisting of a sequence of prices drawn randomly from a fixed probability density function, $f(P)$. The distribution function may be discrete or continuous. Given $f(P) \sim N\left(\mu, \sigma^2\right)$, an independent normal price process is simulated as

$$P_t = \mu + \sigma\epsilon_t, \tag{2.2}$$

where $\epsilon_t \sim N(0,1)$. The rational expectations forecast of the price for an independent price process is

$$E[P_{t+s}|P_t] \quad = \quad \mu. \tag{2.3}$$

The conditional expectation is the same as the unconditional expectation of all future prices; current and past prices do not affect the forecast of future prices. The variance of the process is σ^2. The independence of prices implies that the correlation between P_t and P_{t+1} (price autocorrelation) is zero.

Estimation

The estimated mean and standard deviation for the series of monthly real prices are $\hat{\mu} = 80.0356$ and $\hat{\sigma} = 28.41$ dollars per thousand board feet. A simulated series of prices and the corresponding percentage price changes, using the mean and standard deviation estimated from the lumber futures data, is presented in Figure 2. The simulation equation is

$$P_t = 80.0356 + 28.41\epsilon_t \tag{2.4}$$

where $P_0 = 80.0356$ and $\epsilon_t \overset{iid.}{\sim} N(0,1)$.

2.2.2 Geometric Brownian motion

Properties

Geometric Brownian motion is defined by the stochastic differential equation

$$dP = \mu P dt + \sigma P dW, \tag{2.5}$$

where μ is a drift parameter, dt is the change in time, σ is a volatility parameter and

$$dW = \epsilon(t) \times \sqrt{dt}, \tag{2.6}$$

is the increment of a Wiener process (Dixit and Pindyck, 1994, page 71). Here, $\epsilon(t)$ is a white noise error process assumed to follow a standard normal distribution. In discrete time, Equation 2.5 can be expressed as

$$P_t = (1+\mu) P_{t-1} + \sigma P_{t-1} \epsilon_t, \tag{2.7}$$

where $dt = 1$ and $\epsilon_t \overset{iid.}{\sim} N(0,1)$ (Dixit and Pindyck, 1994, page 72). The parameter μ represents the average (expected) percentage change in prices over one time period and σ controls the variability of the process.

The conditional forecast of the price is

$$E[P_{t+s}|P_t] = P_t e^{\mu s} \tag{2.8}$$

and the conditional variance of the price is

$$\text{Var}(P_{t+s}|P_t) = P_t^2 e^{2\mu s} \left(e^{\sigma^2 s} - 1\right) \tag{2.9}$$

(Dixit and Pindyck, 1994, page 72). When $\mu = 0$, geometric Brownian motion is a martingale process; the best forecast of all future prices is the current price (Mandelbrot, 1971). Note that the variance of the process increases over time without bound:

$$\lim_{s\to\infty} \text{Var}(P_{t+s}|P_t) = \infty. \tag{2.10}$$

As a result, the expected range of prices grows over time.

Estimation

Let

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2.11}$$

represent the percentage price change from period $t-1$ to period t. Note that Equation 2.7 can be expressed as

$$R_t = \mu + \sigma \epsilon_t. \tag{2.12}$$

Consistent parameter estimates for μ and σ are

$$\begin{aligned} \hat{\mu} &= E\left[R_t - \sigma \epsilon_t\right] & (2.13) \\ &= \bar{R} & (2.14) \end{aligned}$$

and

$$\hat{\sigma} = s_R \tag{2.15}$$

where $\bar{R}$ is the sample mean of R and s_R is the sample standard deviation of R. For the lumber futures prices, the estimated parameters are $\hat{\mu} = 0.0005083$ and $\hat{\sigma} = 0.07458$. A simulated price series using the estimated coefficients is presented in Figure 3. The simulation equation is

$$P_t = (1 + 0.0005083)\, P_{t-1} + 0.07458 P_{t-1} \epsilon_t, \tag{2.16}$$

where $P_0 = 80.0356$ and $\epsilon_t \overset{iid.}{\sim} N(0,1)$. The property of increasing variance over time can lead to unrealistic values in a simulated price series - from zero to thousands of times the original price.

2.2.3 The geometric Ornstein-Uhlenbeck process

Properties

Suppose that lumber futures prices converge to a long run mean price - the marginal cost of lumber. The Ornstein-Uhlenbeck process allows for mean reversion in prices. The standard Ornstein-Uhlenbeck process is

$$dP = \eta(\mu - P)\,dt + \sigma dW, \tag{2.17}$$

where $\eta \geq 0$ measures the speed of mean reversion, μ is the equilibrium price, and dW is the increment of a Wiener process, defined in Equation 2.6 (Dixit and Pindyck, 1994, page 74). Many applications of stochastic prices in forestry economics have used a variation of the Ornstein-Uhlenbeck process that allows the volatility of the price to depend upon the price level (Gjolberg and Guttormsen, 2002; Insley and Rollins, 2005; Insley and Chen, 2012). The process is defined as

$$dP = \eta(\mu - P)\,dt + \sigma P dW. \tag{2.18}$$

The process with volatility term $\sigma P dW$ was suggested by Dixit and Pindyck (1994, page 77) and has become known as the "geometric Ornstein-Uhlenbeck process." An approximate representation of Equation 2.18 in discrete time is

$$P_t = P_{t-1} + \eta(\mu - P_{t-1}) + \sigma P_{t-1}\epsilon_t, \tag{2.19}$$

where $dt = 1$ and $\epsilon_t \overset{iid.}{\sim} N(0,1)$ (Insley and Rollins, 2005). For $\eta = 0$, the process is equivalent to geometric Brownian motion with a drift parameter equal to zero - a random walk.

The conditional forecast is

$$E[P_{t+s}|P_t] = \mu + e^{-\eta s}(P_t - \mu). \tag{2.20}$$

Equation 2.20 implies that if $P_t > \mu$, then $E[P_{t+1}|P_t] < P_t$: if the current price is above μ, the price in the next period is expected to be lower than the current price. For the geometric Ornstein-Uhlenbeck process, the variability of future prices is a function of the mean reversion parameter η as well as the volatility parameter σ. The conditional variance of the price s periods into the future is

$$\operatorname{Var}\left(P_{t+s}|P_t\right) = \frac{\sigma^2 P_t}{2\eta}\left(1 - \exp\left(-2\eta s\right)\right). \tag{2.21}$$

An increase in mean reversion (higher value of η) implies a decrease in variance of future prices:

$$\frac{\partial \operatorname{Var}\left[P_{t+s}|P_t\right]}{\partial \eta} = \frac{-\sigma^2 P_t}{2\eta^2} + \frac{\sigma^2 P_t}{2\eta^2}\exp\left(-2\eta s\right) - \frac{2s\sigma^2 P_t}{2\eta}\exp\left(-2\eta s\right) < 0. \tag{2.22}$$

Therefore, the expected range of prices decreases as the level of mean reversion increases. Lower values for η indicate weaker mean reversion, resulting in a process similar to geometric Brownian motion. The variance of the process increases over time, but reaches a long-run limit. The long-run limiting variance of the process is

$$\lim_{s\to\infty} \operatorname{Var}\left(P_{t+s}|P_t\right) = \frac{\sigma^2 P_t}{2\eta}. \tag{2.23}$$

Estimation

Equation 2.19 can be expressed as

$$R_t = -\eta + \frac{1}{P_{t-1}}\eta\mu + \sigma\epsilon_t, \tag{2.24}$$

where R_t is defined in Equation 2.11. The parameters of the process can be estimated using the regression

$$R_t = \alpha + \beta\frac{1}{P_{t-1}} + e_t, \tag{2.25}$$

where $\alpha = -\eta$, $\beta = \eta\mu$, and $e_t = \sigma\epsilon_t$ (Insley and Rollins, 2005). For the lumber futures data, $\hat{\alpha} = -0.02376$ and $\hat{\beta} = 1.72750$. The estimate σ is the standard deviation of the regression residuals: $\hat{\sigma}_t = \sqrt{\operatorname{Var}\left(\hat{e}_t\right)}$. The estimated parameters of the geometric Ornstein-Uhlenbeck process are $\hat{\eta} = -\hat{\alpha} = 0.02618$, $\hat{\mu} = -\hat{\beta}/\hat{\alpha} = 72.70623$, and $\hat{\sigma} = 0.07419$. A simulated geometric Ornstein-Uhlenbeck process using the estimated coefficients is presented in Figure 4. The simulation equation is

$$P_t = P_{t-1} + 0.02618\,(72.70623 - P_{t-1}) + 0.07419 P_{t-1}\epsilon_t, \tag{2.26}$$

where $P_0 = 72.70623$ and $\epsilon_t \overset{iid.}{\sim} N\,(0,1)$.

2.2.4 Generalized autoregressive conditional heteroskedasticity models

Properties

Engle (1982) developed a model of asset price changes, known as autoregressive conditional heteroskedasticity (ARCH), to enable fluctuations in price volatility. Unlike geometric Brownian motion and the geometric Ornstein-Uhlenbeck process, ARCH models allow the variance parameter to change over time according to an autoregressive model. Building upon the ARCH model, Bollerslev (1986) developed a generalized ARCH model (GARCH) where the conditional variance of the process is modeled as an autoregressive moving average (ARMA) process. The general form of a GARCH (p, q) model is

$$P_t = E\,[P_t|P_{t-1}] + \sigma_t^2\epsilon_t, \tag{2.27}$$

where $\epsilon_t \overset{iid.}{\sim} N\,(0,1)$ and

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 \tag{2.28}$$

(Bollerslev, 1986, page 309). In Equation 2.28, α_i represent q autoregressive terms and β_i represent p moving average terms. The conditional expectation term, $E\,[P_t|P_{t-1}]$, can represent any discrete stochastic process including geometric Brownian motion and the geometric Ornstein-Uhlenbeck process.

The GARCH process can take many different forms. Here, assume that prices follow a geometric Ornstein-Uhlenbeck process and σ_t follows an ARMA$(1,1)$ process. In discrete time, the mean reverting GARCH process is simulated as

$$P_t \quad = \quad P_{t-1} + \eta\left(\mu - P_{t-1}\right) + P_{t-1}\sigma_t^2\epsilon_t, \tag{2.29}$$

where

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2.30}$$

and $\epsilon_t \overset{iid.}{\sim} N\left(0,1\right)$. The average variance of the process (the unconditional variance) is

$$\mathrm{Var}\left(P_t\right) = \frac{\omega}{1-\alpha-\beta} \tag{2.31}$$

(Bollerslev, 1986). For $\alpha = \beta = 0$, Equation 2.29 is equivalent to a geometric Ornstein-Uhlenbeck process with constant percentage volatility equal to ω.

Estimation

Estimates of the mean and mean reversion parameters remain the same as in the geometric Ornstein-Uhlenbeck process. The log-likelihood function for a GARCH model is given by

$$\ln LF\left(\omega,\alpha,\beta\right) = \frac{1}{2}\sum_{t=1}^{T}\left[-\ln 2\pi - \ln\sigma_t^2 + \frac{R_t^2}{\sigma_t^2}\right], \tag{2.32}$$

where R_t is the percentage price change at time t (Bollerslev, 1986). The GARCH$\left(1,1\right)$ model was chosen to minimize Akaike's Information Criterion (AIC). The estimated parameters for the GARCH model of monthly percentage price changes are $\hat{\omega} = 2.842 \times 10^{-4}$, $\hat{\alpha} = 0.0931879$, and $\hat{\beta} = 0.8580083$ with p values of 0.0732, 4.33×10^{-5}, and 2×10^{-16} respectively. The simulation equation is

$$P_t = P_{t-1} + 0.02618\left(72.70623 - P_{t-1}\right) + P_{t-1}\sigma_t^2\epsilon_t, \tag{2.33}$$

where $P_0 = 72.70623$, $\sigma_0 = 0$,

$$\sigma_t^2 = 2.842 \times 10^{-4} + 0.0931879\epsilon_{t-1}^2 + 0.8580083\sigma_{t-1}^2, \tag{2.34}$$

and $\epsilon_t \overset{iid.}{\sim} N(0,1)$. A plot of a simulated mean reverting GARCH process is presented in Figure 5. When prices are modeled using a mean reverting GARCH process, percentage price changes have regions of high and low volatility and are characterized by heavy tails relative to the normal distribution: see the normal quantile plot in Figure 6.

2.3 Features of observed lumber futures prices

If prices follow an independent normal price process, *prices* must be both independent and normally distributed. If prices follow either geometric Brownian motion or the geometric Ornstein-Uhlenbeck process, *percentage price changes* must be

1. normally distributed,

2. independent,

3. and have constant variance (Lo and Mackinlay, 1999).

Each of these assumptions can be empirically tested. For monthly lumber futures prices, none of these properties are satisfied at any reasonable level of significance.

2.3.1 Prices are not independent or normally distributed

The independent normal price process is rejected as a possible price process on two conditions. First, the correlation coefficient between prices one period apart (the autocorrelation coefficient) is 0.976 - different from zero at a 1×10^{-16} level of significance. Not surprisingly, the price in the current month is highly correlated with the price in the next month. The presence of autocorrelation in prices violates the assumptions of any independent price process. Second, even if prices were not autocorrelated, the distribution of prices is not normal. The Shapiro-Wilk test for normality indicates that prices are not drawn from a normal distribution $\left(p = 7.6 \times 10^{-14}\right)$. Although simple to implement, a price process involving independent draws from a stationary distribution cannot be considered a realistic process for lumber futures prices.

2.3.2 Percentage price changes are not normally distributed

The normal probability (quantile) plot, presented in Figure 8, suggests that the distribution of percentage price changes is characterized by heavy tails and non-normality. Here, the ordered data values are plotted against the theoretical quantiles of the normal distribution. For data drawn from a normal distribution, the points would fall along the line. The points that fall away from the line represent outliers relative to the estimated normal distribution. The Shapiro-Wilk test for normality provides additional evidence for lack of normality for the distribution of monthly percentage price changes. The null hypothesis of normality is rejected: $p = 0.00494$.

Skewness and kurtosis are important properties of the distribution of percentage price changes. For a normal distribution, skewness is zero and kurtosis is three. A skewness of zero implies that the distribution is symmetric. For the set of lumber futures prices, the skewness of percentage price changes is 0.218, indicating positive skewness in percentage price changes within the sample. Examining the lumber futures data, there are 15 months with price increases greater than 20% but only eight months with price drops greater than 20%. However, the asymmetry is not significant; according to the Wilcoxon rank-sum test, we cannot conclude that the distribution of percentage price changes is asymmetric about the mean ($p = 0.887$). The Wilcoxon rank-sum test was used because it does not make assumptions regarding the underlying distribution of price changes (Higgins, 2004).

The sample kurtosis of percentage price changes is 3.836. The presence of excess kurtosis implies that large price changes are not effectively captured by the normal distribution. The largest monthly percentage change of 41.42% is 4.34 standard deviations away from the mean. Using geometric Brownian motion or an O-U process, this price change would be expected to occur in less than one out of one hundred thousand months $\left(p = 7.09 \times 10^{-6}\right)$.

2.3.3 Percentage price changes are not independent or identically distributed

ARCH effects imply clustering volatility (squared percentage price changes); large price changes are more likely to be followed by large price changes (Engle, 1982). As a graphical check, a time series plot of volatility for monthly lumber futures prices prices is presented in Figure 7. Price volatility does not appear to be white noise - there are periods of high and low volatility.

Engle (1982) provides a statistical test for ARCH effects. ARCH effects are present in a time series process if the squared residuals in an estimated AR model are correlated. To perform the test, first estimate a qth order autoregressive (AR (q)) model for the series of prices. Next, estimate the least squares regression model

$$\hat{\epsilon}_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_i \hat{\epsilon}_{t-i}^2, \tag{2.35}$$

where $\hat{\epsilon}_t$ are the residuals from an AR (q) model of lumber futures prices. The hypothesis test for ARCH effects is

$$H_0 \quad : \quad \beta_i = 0 \, \forall i \tag{2.36}$$

$$H_1 \quad : \quad \text{at least one } \beta_i \neq 0. \tag{2.37}$$

Here, an AR (2) model was chosen to minimize AIC. Using the residuals from an estimated AR (2) model, β_1 and β_2 are significantly different from zero: $p = 0.012$ and $p = 1.39 \times 10^{-8}$, respectively. Percentage price changes have ARCH effects because the squared residuals of the process are significantly autocorrelated. The presence of ARCH effects implies that price variability is not independent or constant over time.

2.3.4 Prices are mean reverting

Variance ratios may be used to determine whether or not a process is mean reverting (Lo and MacKinlay, 1988, page 60). The qth variance ratio is defined as

$$\mathrm{VR} = \frac{\hat{\sigma}_b^2}{\hat{\sigma}_a^2}, \tag{2.38}$$

where

$$\hat{\sigma}_a^2 = \frac{1}{n-1}\sum_{i=2}^{n}\left(P_i - P_{i-1} - \hat{\mu}\right)^2, \tag{2.39}$$

$$\hat{\sigma}_b^2 = \frac{1}{nq-q}\sum_{i=3}^{n}\left(P_i - P_{i-q} - 2\hat{\mu}\right)^2, \tag{2.40}$$

and $\hat{\mu}$ is the mean percentage price change. For each value of q, if the variance ratio is equal to 1, the process is a random walk. If the variance ratio is less than 1, the process is mean reverting. A variance ratio of 1 indicates a random walk process and a ratio greater than 1 indicates mean aversion. A plot of variance ratios is presented in Figure 9. Except for the first five lags, the variance ratios for the first 100 lags were all less than 1, indicating mean reversion.

Price forecasts can provide insight into the mean reversion of the price process. Let $\mathcal{F}(P_t)$ be a forecast of the price in period t given the value of all previous prices. The mean squared forecast error is

$$\mathrm{MSE}(\mathcal{F}) = T^{-1}\sum_{t=1}^{T}\left[P_t - \mathcal{F}(P_t)\right]^2. \tag{2.41}$$

An optimal forecast minimizes Equation 2.41. Forecasts for each model can be tested using the series of observed monthly prices. The process that provides the best forecast is the best model of prices. If geometric Brownian motion provides the best fit for the data, the price in the previous period is the most accurate prediction of the current price. If a forecast using a mean reverting model provides a more accurate prediction, there is some degree of mean reversion in the price process.

When the historical mean is used as the expected price in the next period, the forecast error is $5,893.33$. The error when forecasting all future prices as the historical mean is unaffected by the forecast horizon.Using a random walk forecast, the mean squared error forecast is 606.51. The mean squared forecast error associated with the mean reverting forecast is 589.76. The improvement of the mean reverting forecast over the random walk forecast implies that there is likely some degree of mean reversion in prices. The one month ahead mean reverting forecast results in an improvement of only 2.81 percent relative to the martingale forecast. However, when prices are forecast more than one month into the future, the performance of the mean reverting forecast improves relative to the martingale forecast. The two, three, and four month ahead mean reverting forecasts provide improvements of 4.98, 7.28, and 9.04 percent, respectively. Clearly, forecasting all future prices as the historical mean is not the optimal forecast.

Schwartz (1997) and Andersson (2007) present theoretical and empirical evidence for mean reversion in commodity prices. Andersson (2007) examined the mean reversion of over 300 commodity futures prices. According to Andersson (2007), “if we believe in the mechanics of a market economy, prices of standardized goods should in the long-run revert towards the marginal cost of production as a result of competition among the producers.” Likewise, in a competitive timber market, the price of a timber should revert to the marginal cost of production in the long run. If the marginal cost of timber production is constant over time, lumber futures prices may be expected to to revert to a mean price.

There are three compelling reasons to choose a mean reverting process over geometric Brownian motion. First, the variance ratios suggest that the process is mean reverting over time horizons of longer than five months. Second, the mean reverting forecast represents an improvement over the martingale forecast. Third, mean reversion in commodity prices can be justified and explained on theoretical grounds.

2.4 Discussion

Although several studies have examined the common features of a wide range of commodity prices (Mandelbrot, 1963; Andersson, 2007), few studies have specifically analyzed lumber futures prices. Insley and Chen (2012) examined the use of a regime switching model as a process for lumber futures prices. The regime switching model alternated between two geometric Ornstein-Uhlenbeck processes with different degrees of variability. Insley and Chen (2012) find that the regime switching model provides a better fit for lumber futures prices than the geometric Ornstein-Uhlenbeck process. However, each regime model requires percentage price changes to be normally distributed. As demonstrated, processes with normally distributed percentage price changes provide inadequate models of lumber futures prices. Additionally, the forest manager must be able to determine when the regime has switched from a high volatility to a low volatility state.

In this study, several features of lumber futures prices have been identified: price changes are not normally distributed, price variability is not constant, and prices are mean reverting. A mean reverting GARCH process can be used to model each of these features. The use of a ARMA process to characterize the variability of prices relaxes some of the unrealistic assumptions of geometric Brownian motion and the geometric Ornstein-Uhlenbeck process. A simulation-based study involving lumber futures prices would require an accurate characterization of the price process. For example, lumber futures could be incorporated into a flexible harvesting strategy as a method of hedging.

3 Flexible harvesting strategies in forestry

The Faustmann model is commonly used to predict the behavior of profit maximizing agents in forestry and to determine the net present value of land devoted to timber production. In the Faustmann model, forest owners choose the rotation length - the amount of time that trees are allowed to grow in between harvests - to maximize the net present value of land over an infinite number of timber harvests. The rotation length determines the harvested volume of timber and the net present value of land devoted to timber production. Harvesting costs, planting costs, and the timber price are constant and the volume of timber per acre changes according to a specified deterministic growth function. In a deterministic model with fixed parameters, the optimal rotation length is constant and land value is known.

In practice, none of the parameters that determine the optimal rotation age are known with certainty. Fluctuations in prices, costs, interest rates, and timber growth and the potential for catastrophic losses (forest fires, pest infestations, etc.) cause profits from future harvests to be uncertain. When any model parameters are stochastic, the value of land devoted to forestry cannot be expressed with certainty.

In principle, forest managers can time timber harvests in response to changing market conditions. Forest owners could increase profits by harvesting timber when prices are high and delaying the harvest when prices are low. Multiple authors have considered the value and feasibility of flexible harvesting strategies as an alternative to fixing a rotation length at the time of planting. By using a flexible rotation length, forest owners might avoid uneconomical harvests in search of higher prices. Norstrom (1975) and Brazee and Mendelsohn (1988) were among the first authors to model a timber harvesting strategy in which forest owners incorporate price fluctuations into the final harvest decision. Each suggested that forest owners adopt a reservation price harvesting strategy. In each time period, a forest owner sets a reservation price that depends upon both biological and economic factors. The reservation price is defined as the minimum price in the current period that would induce a forest owner to harvest timber. If the current price is below the reservation price, a forest owner will delay the final harvest. If the current price is above the reservation price, a forest owner will harvest timber and replant in the current period.

More recent research has adopted the terminology and models used to value financial options (see Thomson, 1992; Plantinga, 1998; Insley and Rollins, 2005; Manley and Niquidet, 2010). An American style financial option gives the buyer the right to buy or sell an asset on or before a specified future date (Dixit and Pindyck, 1994). Planting timber is the equivalent of purchasing an American put option. By planting trees, the forest owner purchases an option to sell a growing asset over a range of years. The decision to harvest represents the exercise of the option. Option value exists in forestry for three reasons. First, timber harvesting is irreversible. Second, timber harvesting can be delayed; forest owners can choose to harvest timber over a wide range of years. Third the profits from future timber harvests are uncertain at the time of planting. Option value would not exist if one or more of these three conditions were not satisfied (Dixit and Pindyck, 1994, page 3). Long term investments are undervalued if option value is ignored (Laughton and Jacoby, 1993; Dixit and Pindyck, 1994).

Plantinga (1998) described the relationship between option value and the reservation price strategy: option value is equal to the difference between the value of a fixed rotation length policy and the expected value of a flexible harvesting policy. In the forest economics literature, option value has been calculated in several ways including the binomial option pricing model (Thomson, 1992), a discrete time dynamic programming approach (Haight and Holmes, 1991; Provencher, 1995a; Plantinga, 1998), the Black-Scholes option pricing model (Hughes, 2000), and continuous time optimal stopping models (Insley, 2002; Insley and Rollins, 2005; Rocha et al., 2006). In this essay, an discrete time optimal stopping model is used to estimate reservation prices when stumpage prices are stochastic.

3.1 Discrete time optimal stopping model

3.1.1 State and control variables

Timber management can be formulated as a discrete time optimal stopping problem in which there is a choice to stop (harvest) or continue (delay the harvest) in each period. Optimal stopping problems are characterized by state variables and control variables (Bertsekas, 1987). In each period, the forest owner sets the value of the control variable, defined as

$$x_t = \begin{cases} 0 & \text{delay the timber harvest} \\ 1 & \text{harvest timber at the beginning of period } t \end{cases} . \tag{3.1}$$

The use of a binary control variable implies that the final timber harvest is characterized as an all-or-nothing harvest; partial harvests and thinnings are not considered. Here, assume that revenues from thinning cancel out any periodic management costs not included in the model.

Let X be the sequence of harvesting decisions made by the forest owner: $X = \{x_t\}_{t=0}^{\infty}$. Not all sets of harvesting decisions are possible - for example, the forest owner cannot clear cut the same plot two periods in a row. Here, the set of feasible controls, $\mathcal{X}$, is defined by a minimum and maximum harvest age.

The state variables represent the information available to the forest owner. The forest owner decides to harvest or delay harvest in each period based on the values of the state variables. Here, the state variables are P_t and V_t: the stumpage price and the per acre volume of merchantable timber at the beginning of period t, respectively. The stumpage price, $P = P_H - C_H$, represents the payment that forest owners receive from a timber firm in exchange for the right to harvest standing timber. The stumpage price has two components: the price of timber, P_H, and harvesting costs, C_H. Timber prices vary by wood quality and the forces of supply and demand. Harvesting costs vary by region, tract size, distance from timber mills, and geography of the tract. All else equal, the stumpage price for a plot of land near a mill will be higher than a plot far away from a mill. The stumpage price is represented as a discrete stochastic process:

$$P_{t+1} = E\left[P_{t+1} | P_t\right] + \epsilon_{t+1}, \tag{3.2}$$

where $\epsilon_{t+1} \sim N\left(0, \sigma^2_{t+1}\right)$. Volume is represented as a deterministic function of time:

$$V_{t+1}\left(x_t\right) = \left(g\left(V_t\right) + V_t\right)\left(1 - x_t\right) + x_t \times g\left(V_0\right), \tag{3.3}$$

where $g\left(V_t\right)$ is the change in the per acre volume of timber stock that occurs from the beginning of period t to the beginning of period $t+1$. The volume at the beginning of period $t+1$ is a function of the control variable x_t. The volume changes according to a known function that can be estimated from timber stand data.

3.1.2 Model of wealth maximization

Assume that the forest owner's objective is to maximize the discounted stream of payoffs from an infinite number of harvest rotations and each rotation begins with tree planting on bare ground. The forest owner chooses a sequence of harvesting decisions to maximize wealth from land subject to Equations 3.2 and 3.3. The objective is to define a rule for choosing the values of the control variable that maximize the wealth of the bare land.

To determine the optimal strategy, the infinite sequence of rotations can be broken down into a decision in each quarter. At the beginning of each quarter, forest owners are faced with two choices: harvest timber or delay the harvest to any future period. At the beginning of period t, the value of land with planted timber has two components: the value of the standing timber, $P_t V_t$, and the value of bare land, λ. The discounted expected value of delaying harvest to any future date is defined as

$$\beta E\left[J\left(P_{t+1}, V_{t+1}\right)\right], \tag{3.4}$$

where $\beta = 1/(1 + r/4)$ is a constant, quarterly discount factor. Equation 3.4 can be interpreted as the discounted value of the option to harvest or delay harvest in the next period. The cost of delaying harvest involves the loss of an interest payment that could have been earned on harvest revenue as well as the opportunity cost from delaying all future rotations. The benefits of delaying harvest are timber growth and the possibility of a higher price in the next period.

At the beginning of each period, the forest owner compares the value of the immediate land value with the discounted expected land value from harvesting in any future period and chooses between the maximum of the two values. The maximization problem at time t is

$$J(P_t, V_t) = \max\left[P_t V_t + \lambda, \beta E\left[J(P_{t+1}, V_{t+1}) | P_t\right]\right]. \tag{3.5}$$

The expected value of delay is equal to the expected discounted value function in the next period. The Bellman equation for the optimal stopping model is

$$J(P, V) = \max\left[PV + \lambda, \beta E\left[J\left(P', V'\right) | P\right]\right], \tag{3.6}$$

where P' and V' represent the price and volume at the beginning of the next period. The function $J(P, V)$ is known as the "value function" and can be interpreted as the value of a forest stand with price P and volume V assuming that forest owners follow an optimal policy in all future periods. The Bellman equation allows the value function to be expressed independent of time (Bertsekas, 1987).

3.2 The reservation price strategy

A policy consists of a sequence of single-period decision rules that specify the value of the control variable given the values of the state variables. In the context of forestry, a policy determines choice to harvest or delay harvest in each period. The forest owner's objective is to determine the optimal harvest policy.

The problem is similar to the asset sale model presented by Bertsekas (1987), page 78.

The optimal policy in asset sale models is to set a reservation price at the beginning of each period (Bertsekas, 1987). The reservation price policy implies that the control variable is a function of the current price and current reservation price:

$$x_t\left(P_t, RP_t\right) = \begin{cases} 1 & P_t \geq RP_t \\ 0 & P_t < RP_t \end{cases}. \tag{3.7}$$

When the current stumpage price is below the current reservation price, harvest is delayed. The values of the reservation price and stumpage price determine the optimal decision at each time period. Under the reservation price strategy, the forest owner can update the path of the control variable as new price information arrives.

3.3 Solving for reservation prices using a backward recursion algorithm

3.3.1 Assumptions

Backward recursion is commonly used to solve sequential decision processes and has been widely applied in the forest economics literature (Brazee and Mendelsohn, 1988; Haight and Holmes, 1991; Plantinga, 1998). To calculate the reservation price using backward recursion, the Markov property must hold: the future values of the state variables depend only upon the current values of the state variables, not past values (Dixit and Pindyck, 1994, page 62). Here, the Markov property implies that the expected value of the price in any future period depends only upon the current price and the timber volume in all future periods depends only upon the current timber volume. Additionally, forest owners must know the properties of the price process and the parameters of the price process must not change over time.

To apply the backward recursion algorithm, a boundary date and value must be defined. Let T be the boundary date. The boundary value at the beginning of period T is equal to the value of standing timber plus the value of bare land: $P_T V_T + \lambda$. At the beginning of period T, the forest owner must harvest timber or sell the land with the standing timber - the value is the same either way. Land will be sold at the current stumpage value plus the value of all future rotations. However, if a forest owner reaches the terminal period without harvesting, the decision to harvest or sell timber at the beginning of period T may not be optimal. Therefore, T should be far enough into the future to insure that harvests near T are highly unlikely.

By defining a terminal value, previous value equations can be solved using a backward recursion algorithm. The boundary value implies that $RP_T = 0$. The goal of the backward recursion algorithm is to use the information in the terminal period to solve the sequence of reservation prices $RP_{T-1}, RP_{T-2} \ldots RP_0$.

3.3.2 General solution

Recall the Bellman equation,

$$J\left(P, V\right) = \max\left[PV + \lambda, \beta E\left[J\left(P', V'\right)|P\right]\right]. \tag{3.8}$$

In each period, the forest owner sets a reservation price by comparing the value of an immediate harvest with the discounted expected value from delaying the harvest to any future time period. The reservation price is the value of P that makes the forest owner indifferent between harvesting at the beginning of the current period and delaying the harvest. Equating the two values,

$$PV + \lambda = \beta E\left[J\left(P', V'\right)|P\right]. \tag{3.9}$$

Solving Equation 3.9 for P yields the reservation price at the beginning of the current period. Following the reservation price strategy, a forest owner should harvest at the beginning of the current period if

$$P \geq RP = \frac{\beta E\left[J\left(P', V'\right)|P\right] - \lambda}{V}, \tag{3.10}$$

where the expected value in the next period is expressed as

$$\begin{aligned} E\left[J\left(P',V'\right)|P,V\right] &= \Pr\left(P' \geq RP'\right) \times \left(E\left[P'|P' \geq RP'\right]V' + \lambda\right) \\ &+ \left(\Pr\left(P' < RP'\right) \times \beta E\left[J\left(P'',V''\right)|P'\right]\right). \end{aligned} \tag{3.11}$$

The components of Equation 3.11 can be interpreted as follows.

- $\Pr\left(P' \geq RP'\right)$: the probability that timber is harvested at the beginning of the next period.

- $E\left[P'|P' \geq RP'\right]V' + \lambda$: the expected value of a harvest at the beginning of the next period, given that the price exceeds the reservation price. Because V' is known and λ is assumed to be a constant value, the expectation operator can be distributed. The conditional price expectation is calculated as

  $$E\left[P'|P' \geq RP'\right] = \frac{\int_{RP'}^{\infty} P' f\left(P'|P\right)dP'}{1 - F\left(P'|P\right)}, \tag{3.12}$$

  where $f\left(P'|P\right)$ is the density function of the stumpage price in the next period conditional on the price in the current period. The calculation of this expectation is presented in the Appendix, page 74.

- $\Pr\left(P' < RP'\right)$: the probability that a harvest does not take place at the beginning of the next period.

- $\beta E\left[J\left(P'',V''\right)|P'\right]$: the value of following an optimal strategy in the future if a harvest does not occur at the beginning of the next period. The expected value of the option to delay harvest at the beginning of the next period.

Given that

$$RP' = \frac{\beta E\left[J\left(P'',V''\right)|P'\right] - \lambda}{V'}, \tag{3.13}$$

it follows that

$$E\left[J\left(P'',V''\right)|P'\right]=\frac{RP'\times V'+\lambda}{\beta}. \tag{3.14}$$

Therefore, Equation 3.11 can be expressed as

$$\begin{aligned} E\left[J\left(P',V'\right)|P\right] &= \Pr\left(P'\geq RP'\right)\times\left(E\left[P'|P'\geq RP'\right]V'+\lambda\right) \\ &+ \Pr\left(P'<RP'\right)\left(RP'\times V'+\lambda\right). \end{aligned} \tag{3.15}$$

This step was essential for computational purposes. By substituting a known reservation price, calculated in the previous step, recursive calls to the value function were avoided. The expectation of the value function, $E\left[J\left(P',V'\right)|P\right]$, can be solved without the expectation in the following period, $E\left[J\left(P'',V''\right)|P'\right]$.

3.3.3 Independently and identically distributed normal prices

Assume that the stumpage price in each period is an independent draw from a normal distribution: $P_t\sim N\left(\mu,\sigma^2\right)$. The value function at the beginning of the final period is

$$J\left(P_T,V_T\right) = P_TV_T+\lambda \tag{3.16}$$

and the reservation price is $RP_T=0$. Using these boundary conditions, the sequence of reservation prices can be found by solving a series of recursive equations starting from period $T-1$ and working backwards. The value function at the beginning of period $T-1$ is

$$J\left(P_{T-1},V_{T-1}\right) = \max\left[P_{T-1}V_{T-1}+\lambda,\beta E\left[J\left(P_T,V_T\right)|RP_T\right]\right] \tag{3.17}$$

$$= \max\left[P_{T-1}V_{T-1}+\lambda,\beta\left(\mu\times V_T+\lambda\right)\right]. \tag{3.18}$$

Solving the right hand side of Equation 3.18 for P_{T-1}, the reservation price at the beginning of period $T-1$ is

$$\begin{aligned} RP_{T-1} &= \frac{\beta E\left[J\left(P_T, V_T\right)|RP_T\right] - \lambda}{V_{T-1}} & (3.19) \\ &= \frac{\beta\left(\mu \times V_T + \lambda\right) - \lambda}{V_{T-1}}. & (3.20) \end{aligned}$$

To add one more step, the reservation price at the beginning of the period $T-2$ is

$$RP_{T-2} = \frac{\beta E\left[J\left(P_{T-1}, V_{T-1}\right)|RP_{T-1}\right] - \lambda}{V_{T-2}}, \tag{3.21}$$

where the expected value of delaying harvest to period $T-1$ is

$$\begin{aligned} E\left[J\left(P_{T-1}, V_{T-1}\right)|RP_{T-1}\right] &= \left(1 - \Phi\left(RP_{T-1}\right)\right) \times \left(E\left[P_{T-1}|RP_{T-1}\right] V_{T-1} + \lambda\right) \\ &+ \beta\left(\Phi\left(RP_{T-1}\right) \times E\left[J\left(P_T, V_T\right)\right]\right) & (3.22) \\ &= \left(1 - \Phi\left(RP_{T-1}\right)\right) \times \left(E\left[P_{T-1}|RP_{T-1}\right] V_{T-1} + \lambda\right) \\ &+ \left(\Phi\left(RP_{T-1}\right) \times RP_{T-1}V_{T-1} + \lambda\right). & (3.23) \end{aligned}$$

Here, $\Phi(\cdot)$ represents the normal cumulative distribution function.

A summary of the steps for the backward recursion algorithm is given below.

1. Write out the value function for period $T-1$.
2. Equate the values from harvesting and delaying harvest. Solving for P_{T-1} yields the reservation price at the beginning of period $T-1$.
3. Calculate the expected value function at the beginning of period $T-1$, conditional upon the value of RP_{T-1} calculated in step 2.
4. Write out the value function for period $T-2$.
5. Equating values and solving for P_{T-2} yields RP_{T-2}.

6. Steps (4) and (5) are followed until $t = 0$.

The computer code for the solution procedure is presented in the Appendix, page 75.

3.3.4 Monte Carlo backward recursion algorithm

Although the standard backward recursion algorithm is a well known procedure for solving sequential models, this approach is not always feasible. For example, a GARCH process does not satisfy the Markov assumption because the future distribution of prices depends upon the values of all past prices. The Monte Carlo simulation method provides an approach to estimating reservation prices when one does not know or cannot derive the distribution of expected prices (Ibáñez and Zapatero, 2004). Here, the Monte Carlo simulation method is applied to both the geometric Ornstein-Uhlenbeck process and the mean reverting GARCH process. The backward recursion algorithm remains the same, only the calculation of the expectation of the value function, $E[J(\cdot)]$, changes.

Before beginning the backward recursion algorithm, simulate N price processes for a given set of price parameters. Let P_t^n be the value of the nth simulated price series at the beginning of period t. Given $RP_T = 0$, the first step in the backward recursion algorithm is to calculate RP_{T-1}. Recall that the value function at the beginning of period $T-1$ is

$$J(P_{T-1}, V_{T-1}) = \max\left[P_{T-1}V_{T-1} + \lambda, \beta E\left[J(P_T, V_T) | P_{T-1}, RP_T\right]\right]. \tag{3.24}$$

When $E[J(P_T, V_T)|P_{T-1}, RP_T]$ cannot be derived analytically, it may be estimated through numerical simulation. In particular,

$$E[J(P_T, V_T)|P_{T-1}, RP_T] = \frac{1}{N}\sum_{n=1}^{N}(P_T^n V_T + \lambda) \tag{3.25}$$

represents the expected value of a harvest in the final period. A forest owner is indifferent between harvesting at the beginning of period $T-1$ or at the beginning of T if

$$P_{T-1}V_{T-1}+\lambda=\beta\frac{1}{N}\sum_{n=1}^{N}\left(P_T^nV_T+\lambda\right). \tag{3.26}$$

The reservation price at the beginning of period $T-1$ is

$$RP_{T-1}=\frac{\beta\frac{1}{N}\sum_{n=1}^{N}\left(P_T^nV_T+\lambda\right)-\lambda}{V_{T-1}}. \tag{3.27}$$

This solution to the reservation price does not hold for any value of t other than $T-1$.

Stepping back one period, the value function at the beginning of period $T-2$ is

$$J\left(P_{T-2},V_{T-2}\right)=\max\left[P_{T-2}V_{T-2}+\lambda,\beta E\left[J\left(P_{T-1},V_{T-1}\right)|P_{T-2},RP_{T-1}\right]\right], \tag{3.28}$$

where

$$\begin{aligned} E\left[J\left(P_{T-1},V_{T-1}\right)|P_{T-2},RP_{T-1}\right] &= \frac{1}{N}\sum_{n=1}^{N}I\left(P_{T-1}^n\geq RP_{T-1}\right)\left(P_{T-1}^nV_{T-1}+\lambda\right) \\ &+ \beta\frac{1}{N}\sum_{n=1}^{N}I\left(P_{T-1}^n<RP_{T-1}\right)\left(P_T^nV_T+\lambda\right) \end{aligned} \tag{3.29}$$

represents the expected value from applying the reservation price strategy in period $T-1$ and period T. Here, $I(\cdot)$ is an indicator function, equal to one if the condition in parentheses is true and zero otherwise. At the beginning of period $T-1$, there are only two periods during which the forest owner could harvest: $T-1$ or T. The control variable for the nth simulated series, x^n, is chosen according to the reservation price strategy and determines whether the harvest takes place at the beginning of period$T-1$ or at the beginning of period T. For example, if the observed price at the beginning of period $T-1$ of the nth price series is less than RP_{T-1}, $x_{T-1}^n=0$ and the harvest is delayed until period T. The reservation price at the beginning of period $T-2$ is

$$RP_{T-2}=\frac{\beta E\left[J\left(P_{T-1},V_{T-1}\right)|P_{T-2},RP_{T-1}\right]-\lambda}{V_{T-2}}. \tag{3.30}$$

As t becomes smaller, the calculation of the reservation price becomes increasingly complex because there are more future periods during which the forest owner could harvest. The computer code for the Monte Carlo simulation procedure is presented in the Appendix, page 75.

The Monte Carlo simulation procedure was first applied in the forestry economics literature by Petrášek and Perez-Garcia (2010) to solve a flexible harvesting problem with variability in both stumpage prices and carbon prices. The Monte Carlo method does not require predictions of future prices or price volatility and represents a completely simulated approach to estimating reservation prices. A drawback to the method is the computational intensity required to run the simulations. Given that the number of simulations, N, should be large (at least 5,000), this approach was certainly not feasible for early authors in the reservation price literature.

3.4 Advantages of the discrete time model

In forestry, discrete time models are more appropriate than continuous time financial option models for several reasons. First, forest owners do not observe prices on a continuous time scale. Second, there is a limit to the speed of decision making; unlike trading in financial options, a sealed bid auction process cannot be executed instantaneously. NIPF owners often experience a significant lag between the decision to harvest and the end of the bidding process (Haight and Holmes, 1991). Third, the time scale can be adjusted - a daily, monthly, or quarterly model can be derived by changing the parameters of the same model. Fourth, unlike the binomial option pricing model used by Thomson (1992), the backward recursion approach allows a full range of prices and a variety of price processes to be considered. Fifth, discrete time allows a wider range of assumptions to be incorporated in the same model. For example, different price processes, stochastic discount rates, and other variables can be considered in the same model. The binomial and Black-Scholes option pricing models are restrictive in the sense that they both imply that prices are characterized by geometric Brownian motion.

3.5 Conclusion

This study describes a method for applying a flexible harvesting strategy in forestry. The optimal harvesting strategy is characterized by a reservation (threshold) price: if the current stumpage price is below the current reservation price, the forest owner delays the harvest. The sequence of reservation prices can be derived from an optimal stopping model of the timber harvesting decision. The model implicitly assumes risk neutrality - variability in wealth does not enter the calculation of the value equation, $J(P, V)$. Future work should incorporate landowner preferences into the calculation of a reservation price or reservation utility.

4 What is the value of flexibly harvested timberland?

In this essay, two harvesting strategies are compared: a fixed rotation length policy and an flexible harvest strategy known as the "reservation price policy." Both harvesting strategies require forecasts of prices, costs, interest rates, and timber volumes to estimate the value of bare land for timber production. The fixed rotation policy ignores the properties of the stumpage price process and the ability to delay harvest when prices are low. The flexible harvest policy allows for updating - changing decisions based on new information.

Plantinga (1998) defined the increase in net present value from a flexible rotation policy relative to a fixed rotation policy as a "real option value". Because option value can never be negative, the expected profits from a fixed rotation harvesting strategy can never be greater than the expected profits from a flexible strategy. Therefore, the Faustmann model can be used to estimate a lower bound on the expected value of a timber investment. Additionally, the flexible harvest approach shows why forest owners do not abandon timber management when prices are low - they can delay unprofitable harvests in search of higher profits. When current prices are low, a flexible harvesting approach may result in a positive net present value whereas the Faustmann model would suggest that timber management should be abandoned (Thomson, 1992).

The aim of this study is to use a simulation model to estimate the value of applying a flexible harvesting strategy. First, assumptions regarding prices and timber growth are presented. Three types of price processes and two volume functions will be considered. Second, the value of the fixed rotation length strategy is simulated. Third, optimal reservation prices are derived using the model presented in the previous essay. A numeric solution procedure based on Monte Carlo simulation is applied when prices are represented by a geometric Ornstein-Uhlenbeck process or a mean reverting GARCH process. Fourth, the sequence of reservation prices is used in a simulation model to estimate the value of land devoted to timber production. Finally, expected profits from a fixed rotation length policy are compared with the expected profits from a reservation price harvesting strategy. The simulation model indicates that by applying a flexible strategy, a forest owner may be able increase the net present value of land devoted to timber production relative to a fixed rotation strategy.

4.1 Stochastic processes for stumpage prices

The percentage increase in land value varies greatly for each study and each price process. The estimated increase in net present value over the Faustmann model ranges from zero (Clarke and Reed, 1989) to more than six times the Faustmann net present value (Insley and Rollins, 2005). The calculation of option value is heavily dependent upon the assumptions of the model - particularly the specification of the stochastic process for stumpage prices. Manley and Niquidet (2010, page 305) argue that "the sensitivity of results to the underlying price model is one reason why forest valuers (and their clients) have not adopted option valuation techniques."

4.1.1 Independent price process

Early models of stumpage price variability assumed that the stumpage prices were drawn independently from a known price distribution (Norstrom, 1975; Brazee and Mendelsohn, 1988). Norstrom (1975) used a discrete probability distribution in which prices could move to one of five states in each period. Brazee and Mendelsohn (1988) and Brazee and Bulte (2000) are among the studies which assume that prices in each period are independent draws from a normal distribution, $P \overset{iid}{\sim} N\left(\mu, \sigma^2\right)$. Each found that the reservation price strategy greatly increased the value of bare land relative to a fixed rotation strategy. If stumpage prices are independent draws from a normal distribution, the reservation price strategy can increase the net present value of timberland by more than 100 percent relative to the Faustmann model (Brazee and Mendelsohn, 1988). However, the simplistic price models used by Norstrom (1975) and Brazee and Mendelsohn (1988) are no longer considered to be reasonable approximations of the true price process. In particular, the assumption that prices one period apart are uncorrelated is nearly always violated.

4.1.2 Geometric Brownian motion

Clarke and Reed (1989), Thomson (1992), Insley (2002), and Manley and Niquidet (2010) are among the studies that have used geometric Brownian motion as a stochastic process for prices in a flexible harvesting model. The popularity of geometric Brownian motion stems from its analytical tractability and its use in the Black-Scholes option pricing model (Black and Scholes, 1973). Additionally, authors often justify geometric Brownian motion using market efficiency arguments. Clarke and Reed (1989), Washburn and Binkley (1990), and Thomson (1992) argue that the use of geometric Brownian motion is consistent with timber market efficiency. In contrast, mean reversion implies that price changes are forecastable. However, Fama (1970) demonstrates that a price process characterized by geometric Brownian motion is sufficient, but not necessary for market efficiency. Additionally, McGough et al. (2004) show that autocorrelated prices can exist in efficient timber markets given supply and demand shocks. Therefore, market efficiency provides no guidance for choosing a stochastic process for prices.

One drawback to the use of geometric Brownian motion in a simulation study is the property of increasing price variance over time. To keep prices from becoming zero or excessively large, the majority of studies using geometric Brownian motion have applied upper and lower bounds on the prices (Thomson, 1992; Paarsch and Rust, 2004; Manley and Niquidet, 2010). However, bounding the process results in a process that is no longer geometric Brownian motion.

Using single rotation models, Clarke and Reed (1989) and Haight and Holmes (1991) demonstrate that the expected profits from the reservation price strategy and a fixed rotation length policy are nearly equivalent when prices are characterized by geometric Brownian motion. In a multiple rotation model incorporating fixed land management costs, Thomson (1992) demonstrates that the reservation price strategy increases the net present value of land even when prices are characterized by geometric Brownian motion. According to Thomson (1992), the increase in the value of land when prices are characterized by geometric Brownian motion is a result of the ability to delay unprofitable harvests.

4.1.3 Mean reverting processes

Mean reverting prices are attractive from a simulation perspective. Unlike geometric Brownian motion, mean reverting processes do not have the tendency to go to zero or infinity over time. Because the variance reaches a long-run limit and the process reverts to a fixed value, the process stays within what most authors define as a "reasonable" range of prices - no artificial price bounds are required for simulation.

A wide range of studies in forestry have used mean reverting processes to model stumpage prices. The mean reverting models have included a first order autoregressive process (Haight and Holmes, 1991), the Ornstein-Uhlenbeck process (Plantinga, 1998), and the geometric Ornstein-Uhlenbeck process (Gjolberg and Guttormsen, 2002; Insley and Rollins, 2005; Yoshimoto, 2009) to model prices. In the standard Ornstein-Uhlenbeck process, the parameter σ represents the absolute volatility of prices, whereas σ represents percentage volatility in the geometric Ornstein-Uhlenbeck process.

Theoretical arguments have been presented in favor of mean reversion in commodity prices (Schwartz, 1997). Additionally, empirical evidence has been presented to justify mean reversion for stumpage prices. Using the variance ratio test developed by Lo and MacKinlay (1988), Gjolberg and Guttormsen (2002) find evidence of mean reversion in stumpage prices for time intervals greater than one year.

In the forest economics literature, evidence has been presented for both stationarity (Insley and Rollins, 2005) and nonstationarity of prices (Manley and Niquidet, 2010). Second order (covariance) stationary implies that the unconditional mean and variance of the price series do not depend upon t: $E\left[P_t\right] = \mu \, \forall t$ and $\mathrm{Var}\left(P_t\right) = \sigma^2 \, \forall t$ (Hamilton, 1994, page 45). For stationary prices, the expected range and variability of prices are constant. Haight and Holmes (1991) find that monthly prices are stationary and autocorrelated, rejecting the assumptions of both Brazee and Mendelsohn (1988) (independent normal prices) and Clarke and Reed (1989) (geometric Brownian motion). However, although stationarity implies mean reversion, lack of stationarity does not imply that prices are not mean reverting.

Haight and Holmes (1991) were the first to demonstrate that the reservation price strategy increases profits when prices are mean reverting. For different assumptions, the reservation price strategy increased profits by 20 to 30 percent relative to a fixed rotation strategy (Haight and Holmes, 1991). Gjolberg and Guttormsen (2002) demonstrate that the degree of mean reversion impacts the value of land; stronger mean reversion results in lower land values because of the reduction in price variability. In contrast with Gjolberg and Guttormsen (2002), Plantinga (1998) found that an increase in mean reversion increases profits from a flexible harvesting strategy.

4.1.4 Generalized autoregressive conditional heteroskedasticity models

Both geometric Brownian motion and the geometric Ornstein-Uhlenbeck process assume that percentage price changes are normally distributed with constant mean and variance. Mandelbrot (1963), Peters (1994), Cont (2001), and others have pointed out the drawbacks to using a stationary, normal distribution to characterize asset price changes. The use of a GARCH volatility model relaxes this assumption. GARCH models imply clustering volatility; large price changes are likely to be followed by large price changes. Additionally, GARCH models can be used to model heavy tails in the distribution of price changes. Both features are common to a wide range of commodity prices (Mandelbrot, 1963). GARCH models have been widely used in finance, but have not been applied in the forestry option value literature.

GARCH can be applied as a volatility model for a geometric mean-reverting, or geometric Ornstein-Uhlenbeck, process. The use of a mean reverting GARCH process allows the variability of the process to change over time according to an autoregressive moving average process. Saphores et al. (2002) found that ARCH effects were present in four different monthly time series of stumpage prices. Similarly, Insley (2002) and Insley and Rollins (2005) found significant ARCH effects in monthly stumpage prices and suggested the use of a GARCH process in future research. However, the GARCH process has not been implemented in the reservation price literature because of analytical difficulties.

4.2 Assumptions for numerical simulation

4.2.1 Stumpage prices

Three stochastic processes for stumpage prices are considered. Two of the processes have been previously applied in the literature: prices characterized by independent draws from a normal distribution (Brazee and Mendelsohn, 1988) and the geometric Ornstein-Uhlenbeck process (Insley, 2002). The use of a mean reverting GARCH process is new to the reservation price literature.

For pine sawtimber, the mean stumpage price from the first quarter of 1992 to the fourth quarter of 2012 for the upstate region of South Carolina was \$317.88 per thousand board feet and the estimated standard deviation is $\hat{\sigma} = \$74.23$ (Timber Mart South, 2012). Prices were adjusted to constant fourth quarter 2012 prices using the Consumer Price Index (United States Department of Labor, Bureau of Labor Statistics, 2012). The *iid.* normal price process is simulated as

$$P_t = 317.8848 + 74.23383\epsilon_t, \tag{4.1}$$

where $\epsilon_t \overset{iid.}{\sim} N(0,1)$. For the geometric Ornstein-Uhlenbeck process, stumpage prices are expected to revert to a long run equilibrium level of \$306.15 per thousand board feet. Note that the estimated long run equilibrium price is not equal to the sample mean of stumpage prices. The estimated level of mean reversion is $\hat{\eta} = 0.07192$ and the estimated percentage variance is $\hat{\sigma} = 0.131$. The simulation equation for the geometric Ornstein-Uhlenbeck process is

$$P_t = P_{t-1} + 0.07192\,(306.1527 - P_{t-1}) + 0.131P_{t-1}\epsilon_t, \tag{4.2}$$

where $P_0 = 306.1527$ and $\epsilon_t \overset{iid.}{\sim} N(0,1)$. The parameters of the GARCH process are $\hat{\omega} = 0.008007$, $\hat{\alpha} = 0.3508$, and $\hat{\beta} = 3.44 \times 10^{-18}$. The simulation equation for the mean reverting GARCH process is

$$P_t = P_{t-1} + 0.07192\,(306.1527 - P_{t-1}) + P_{t-1}\sigma_t^2\epsilon_t, \tag{4.3}$$

where $P_0 = 306.1527$,

$$\sigma_t^2 = 0.008007 + 0.3508\epsilon_{t-1}^2 + 3.44 \times 10^{-18}\sigma_{t-1}^2, \tag{4.4}$$

and $\epsilon_t \overset{iid.}{\sim} N\,(0,1)$.

4.2.2 Timber volume functions

By assumption, the volume of merchantable sawtimber is a deterministic function of time; volume in every quarter is known with certainty. The yearly volume of loblolly pine sawtimber was modeled using the SiMS plantation simulator (ForesTech International, 2006). Quarterly volumes were estimated from yearly simulated data using linear interpolation. Each site index assumed a planting density of 726 stems per acre and a 90 percent initial survival rate. For a site index of 90 (base age 25), the stand was thinned to 70 square feet of basal area per acre at age 12. For a site index of 60 (base age 25), the stand was thinned to 70 square feet of basal area per acre at age 15. A plot of each timber volume function is presented in Figure 10. The points represent simulated yearly volumes and the lines represent interpolated volumes.

4.2.3 Minimum and maximum harvest ages

Sawtimber is defined as timber that can be processed into lumber (Cunningham et al., 2000). Although a tree can be commercially harvested for pulpwood at any time after it reaches five inches diameter at breast height (DBH), forest owners typically delay the sawtimber harvest until the diameter of trees in a stand are greater than fourteen inches DBH. At this diameter, trees can be harvested as sawtimber, receiving higher stumpage prices than timber harvested for pulpwood (Cunningham et al., 2000). Although merchantable timber products exist at younger stand ages, trees cannot be harvested as sawtimber before a certain age.

In the reservation price model, an exceptionally high price could cause a forest owner to harvest before timber is mature. To produce sawtimber, the tree must be mature enough to produce logs with a minimum length of eight feet (Cunningham et al., 2000). Let t_{min} represent the date when timber reaches sufficient biological maturity to be harvested as sawtimber. Before the beginning of period t_{min}, timber can only be harvested for pulpwood. The value of t_{min} depends upon the species of tree and the productivity of the land. On a high quality site managed for timber production, loblolly pine trees will require a minimum of 25 years to be harvested as quality sawtimber (Cunningham et al., 2000). For a site index of 90, $t_{min} = 25$ years (100 quarters). For a site index of 60, $t_{min} = 30$ years (120 quarters). For $t < t_{min}$, the reservation price is set to infinity - no stumpage price can induce a forest owner to harvest.

Derivation of the reservation price using the backward recursion algorithm requires the specification of a maximum harvest date, t_{max}. Here, the maximum harvest date (the expiration of the option) reflects the date beyond which significant decline in the timber stock begins to occur (Gjolberg and Guttormsen, 2002). Beyond t_{max}, timber may lose value from mortality. Although the volume functions used in this study do not have a region of declining timber stock, an artificial boundary needs to be set. Here, the maximum harvest age is set to 50 years (200 quarters) for a site index of 90 and 65 years (260 quarters) for a site index of 60. Although the choice is somewhat arbitrary, the maximum harvest date can be chosen to be sufficiently far into the future so that the probability of a harvest near t_{max} is nearly zero. Although the reservation price declines sharply to zero near t_{max}, reservation prices over the majority of the rotation interval are not sensitive to the specification of the maximum harvest date. Additionally, the simulation model demonstrates that forest owners will not harvest near t_{max} given an optimal series of reservation prices.

The constraint requiring minimum and maximum harvest ages can be expressed as

$$x_t = \begin{cases} 0 & t < t_{min} \\ 0,1 & t_{min} \leq t < t_{max} \\ 1 & t = t_{max} \end{cases}, \tag{4.5}$$

where $x_t = 0$ represents the decision not harvest at the beginning of period t. The minimum and maximum harvest ages define the set of feasible controls, $\mathcal{X}$.

4.2.4 The value of bare land

The value of bare land, λ, is a random variable that is endogenous to the calculation of the reservation price. Applying the reservation price strategy affects the initial value of land, which in turn, affects the calculation of reservation prices. Land should be valued not at the constant rotation value, but at the expected value using a flexible harvesting strategy.

An iterative procedure can be used to solve for λ. The lower bound on the value of bare land can be estimated using a discrete version of the Faustmann model. The discrete Faustmann model specifies that the net present value of timber land with an infinite number of rotation intervals, each of length t, is

$$\text{NPV}(t) = \frac{\beta^t P_t V_t - C_p}{1 - \beta^t}, \tag{4.6}$$

where C_p represents planting costs (assumed to be constant) (Kennedy, 1986). Equation 4.6 can be maximized by solving the equation for $t = t_{min}, t_{min+1}, \ldots, t_{max}$ and finding the maximum net present value out of all possible choices of t. The solution to Equation 4.6 provides the starting value for λ in the simulation model.

This study will assume that the expected value of bare land is equal to a long-run expected value that does not depend upon the current price level. The forecast of a mean reverting random variable more than 100 quarters into the future is approximately the mean of the process. Therefore, when prices are mean reverting, λ does not depend upon the price at the time of planting. For additional details in support of removing the dependence of land value on the current stumpage price level, see Brazee and Mendelsohn (1988), Haight and Holmes (1991), Provencher (1995a), and Plantinga (1998). Although λ is not time-dependent, it depends upon the assumptions of the model.

4.3 Fixed rotation length strategy

The reservation price strategy will be compared relative to the value of a fixed rotation strategy. For a fixed rotation length strategy, the forest owner chooses a rotation length to maximize the expected net present value of land. Given constant prices and planting costs, all rotation lengths will be the same. Harvesting at the Faustmann optimal rotation length is one possible harvesting policy. Let $t^\star$ be the rotation length that maximizes Equation 4.6 given a price equal to the mean of the process. The policy implies that

$$x_t\left(t^\star\right)=\begin{cases}1 & t=t^\star,2t^\star,\ldots\\ 0 & \text{otherwise}\end{cases}. \tag{4.7}$$

This rotation policy is determined according to information available in the initial time period and without regard to the distribution of prices or the stumpage price at each harvest date. Following a constant rotation strategy, the expected value of bare land is

$$E\left[\mathrm{NPV}\left(t^\star\right)\right]=\frac{\beta^{t^\star}E\left[P_{t^\star}\right]V_{t^\star}-C_p}{1-\beta^{t^\star}}. \tag{4.8}$$

The computer code for the discrete Faustmann model is presented in the Appendix, page 75.

4.4 Simulation model of land value and harvest dates

Application of the backward recursion algorithm yields a single estimate of the value of bare land: $E\left[\lambda\right]$. To determine the variability in land values that forest owners can expect from applying the reservation price strategy, a simulation model was developed. This simulation model calculates the profits from applying the reservation price strategy over a period of 1,000 quarters (250 years). For any reasonable discount rate, timber harvest profits beyond 250 years into the future become negligible (less than 0.01%) relative to harvesting profits from earlier rotations. The simulation method is described in the following steps and the computer code is presented in the Appendix, page 78.

1. Simulate a price series for a given set of parameters starting with the same initial conditions: $P_0 = \hat{\mu}$ and $V_0 = 0$. Start with the Faustmann model estimate of the value of bare land, $\hat{\lambda}$.

2. Solve for the set of optimal reservation prices using the backward recursion algorithm.

3. Apply the reservation price strategy to each price series. The optimal harvest date, $t^\star$, is defined as the smallest value of t for which the simulated stumpage price is greater than the reservation price:
$$t^\star = \min\left[t|P_t \geq RP_t\right]. \tag{4.9}$$

4. After timber is harvested, the volume function is reset to zero.

5. Repeat steps three and four until the end of 1,000 quarters. At the end of the simulation, each harvest profit is discounted by the appropriate factor and summed, providing a single net present value. for each simulation. The net present value for each simulation is equivalent to the discounted value of managing timber according to the reservation price strategy for 1,000 quarters.

6. Repeat steps one through five 10,000 times.

7. The average value of bare land is from the 10,000 simulations is used as the new estimate of the expected value of bare land. Repeat steps one through six using the updated value of bare land. After $\hat{\lambda}$ is updated a single time, subsequent changes in bare land are very small - less than 0.1 percent of total land value.

This method generates a different net present value and multiple rotation lengths for each price simulated price process. The expected outcomes are the mean of the net present value and the mean of the simulated rotation lengths for all 10,000 simulations. The standard deviation is used as the measure of variability for both variables. The net present value from the simulation model represents the per acre profit of timberland with specified characteristics, starting with a price equal to the mean of the process. For 10,000 simulations, the method is stable - land values change by less than a tenth of a percent when the simulation algorithm is repeated.

4.5 Simulation results

4.5.1 Expected land value under a fixed rotation length strategy

Fixed rotation length harvest values are calculated using a simulation model presented in the Appendix 80. All simulations used a discount rate of $r = 0.05$ and planting costs of \$250 per acre. If prices are draws from an independent normal distribution, the expected value of the price at any future date is \$317.8848 per thousand board feet. If prices follow a mean reverting process, the expected long-run price is \$306.1527 per thousand board feet. In each case, the optimal constant rotation length is 124 quarters (31 years) for a site index of 90 and 174 quarters (43.5 years) for a site index of 60. Here, the rotation length reflects the amount of time that timber is allowed to grow for the production of sawtimber. The change in the expected price was not large enough to affect the discrete rotation length. Other timber products (pulpwood and chip and saw) would have much shorter rotation lengths.

A summary of the results for the constant rotation strategy is presented in Table 1. Ten thousand simulations were conducted for each set of price process and volume parameters. Here, $E\left[\text{NPV}\right]$ represents the mean net present value of the 10,000 simulations $\left(E\left[\hat{\lambda}\right]\right)$ and $\sqrt{\text{Var}\left(\text{NPV}\right)}$ is the standard deviation of the simulated land values. Under the constant rotation length strategy, an increase in price variability increases the standard deviation of simulated land values but does not change the expected land value.

4.5.2 Applying the reservation price harvesting strategy

Selected results for different price processes and parameters are presented in Tables 2 and 3. Here, $E[t^\star]$ represents the mean harvest date when applying the reservation price strategy and $\sqrt{\mathrm{Var}(t^\star)}$ is the standard deviation of the simulated harvest dates.

Result 4.5.1. *The reservation price strategy increases the value of timberland relative to a fixed rotation strategy.*

Applying the reservation price strategy, a forest owner would typically harvest at prices above the mean of the price process. For all chosen parameters of each price process, the reservation price strategy increased the value of land devoted to timber production. For the estimated parameters of each price process, the net present value of land increased between 22.1 and 70.3 percent, depending upon the price process and site index of the land. When prices are independent draws from a normal distribution, the reservation price strategy increases the value of land by 48.0 percent for a site index of 90 and by 70.3 percent for a site index of 60. When prices follow a geometric Ornstein-Uhlenbeck process with fixed volatility, the reservation price strategy increases the value of land by 28.3 percent for a site index of 90 and by 41.4 percent for a site index of 60. When prices follow a mean reverting GARCH process, the reservation price strategy increases the value of land by 33.0 percent for a site index of 90 and by 22.1 percent for a site index of 60.

Result 4.5.2. *The reservation price strategy decreases the standard deviation of land values relative to land values from a fixed rotation strategy.*

For the GARCH model with estimated parameters and a site index of 90, the standard deviation of land values decreases from 27.5 percent of the expected land value under a fixed rotation strategy to 16.3 percent of the expected land value under the reservation price strategy. For a site index of 60, the standard deviation of land values decreases from 45.1 percent of the expected land value under a fixed rotation strategy to 18.1 percent of the expected land value under the reservation price strategy.

Result 4.5.3. *As stumpage price variability increases, the gains from a flexible harvesting strategy increase.*

For the mean reverting GARCH process and a site index of 90, a 25 percent increase in σ_t led to an increase in simulated land value of 30.2 percent relative to the fixed rotation length strategy. A 25 percent decrease in σ_t led to an increase in simulated land value of only 13.5 percent relative to the estimated process. For a site index of 60, a 25 percent increase in σ_t led to an increase in simulated land value of 44.8 percent relative to the fixed rotation length strategy. A 25 percent decrease in σ_t led to an increase in simulated land value of only 19.8 percent relative to the estimated process.

Result 4.5.4. *The reservation price strategy has a greater impact on land value for lower quality sites.*

The reservation price strategy has a greater positive impact on the value of timberland with a lower site index. For the estimated mean reverting GARCH process, the reservation price strategy increases the value of land by an additional 10.9 percent for a site index of 90 relative to a site index of 60.

4.6 Discussion

4.6.1 Properties of reservation prices

A number of general properties of reservation prices hold regardless of the stochastic process of stumpage prices. First, the optimal reservation prices are not a constant over the rotation. The value of delaying the harvest decreases over time because there are fewer periods during which the option can be exercised. Therefore, $J(P,V) \geq J(P',V')$, which implies that reservation prices must decrease over the rotation interval. Additionally, the decrease in the reservation price over the rotation interval reflects the decrease in timber growth relative to timber volume over time. The reservation price is higher for early stages of each rotation but decreases as timber growth slows. Second, the reservation price sharply declines to zero as t approaches t_{max}. At the maximum harvest age, forest owners will, by assumption, sell timber at the going rate; there is little value to delaying harvest in search of higher prices. However, harvests near t_{max} are extremely rare. In the model with estimated price parameters and 10,000 simulations, no harvests occurred at $t = t_{max}$.

Changes in exogenous variables have a consistent effect upon reservation prices regardless of the stochastic process of stumpage prices and the site index. First, site quality impacts the reservation price. For a high quality site, timber growth imposes a greater penalty for delaying harvest, resulting in lower reservation prices (Figure 12). Second, greater price variability increases reservation prices (Figure 11).

4.6.2 Stumpage price volatility

Greater price volatility increases the value of real options because it allows the holder of the option the ability to sell at a wider range of prices (Laughton and Jacoby, 1993; Dixit and Pindyck, 1994). Likewise, previous studies of real option value in forestry have found that higher levels of stumpage price volatility increase the value of flexible harvesting strategies(Thomson, 1992; Insley, 2002). In this study, an increase in price volatility leads to an increase in the net present value of land, implying that forest owners benefit from greater stumpage price volatility. An increase in price volatility also increases the variability in simulated land values.

4.6.3 Variability in land values

Previous studies of flexible harvesting strategies in forestry have only estimated the expected net present value of land devoted to timber production. The simulation model presented in this study allows for the analysis of the variability of land values. Here, variability refers to the standard deviation of simulated land values.

When forest owners follow a constant rotation policy, all of the variability in profits from timber harvesting comes from price volatility. Here, the volume at the optimal rotation length is assumed to be known. Applying the reservation price strategy introduces harvest volume variability because not all harvests occur at the same rotation length. However, variability in harvest prices is greatly reduced. By choosing not harvesting at low stumpage prices, forest owners harvest over a smaller range of prices. The overall effect is a reduction in the variance of profits. Although a risk neutral forest owner is not concerned with profit variability, risk averse forest owners have an additional incentive to apply the reservation price strategy. A density plot of simulated profits for the fixed rotation length strategy and reservation price strategy is presented in Figure 13. The reservation price strategy increases simulated profits and decreases the spread of simulated profits relative to the fixed rotation length strategy.

4.6.4 Effects of GARCH volatility

The mean reverting GARCH model allows for a more flexible estimation of the volatility parameter in the geometric Ornstein-Uhlenbeck process. The stochastic volatility component for prices impacted the net present value of land relative to the geometric Ornstein-Uhlenbeck process. For a site index of 90, the expected net present value of land was 5.5 percent lower than the value when prices were modeled using a geometric Ornstein-Uhlenbeck process. For a site index of 60, the decrease in expected net present value was 5.4 percent. Additionally, the mean reverting GARCH model impacted the variability of expected profits relative to the geometric Ornstein-Uhlenbeck process. For a site index of 90, the standard deviation of expected profits decreased from \$509.01 under the geometric Ornstein-Uhlenbeck process to \$441.13 when prices follow a mean reverting GARCH process. Likewise, for a site index of 60, the standard deviation of expected profits decreased from \$124.56 under the geometric Ornstein-Uhlenbeck process to \$112.46 when prices follow a mean reverting GARCH process.

4.7 Barriers and limitations of flexible harvesting strategies

4.7.1 Selection and estimation of the price process

Land valuation using the reservation price strategy is sensitive to the price process and the parameter estimates for each process. The use of an unrealistic stochastic process for prices can overvalue land. In this study, modeling prices with an independent normal price process results in a 27.1 percent greater land valuation than when prices are characterized by a mean reverting GARCH process. If the stumpage price in the current month is correlated with the stumpage price in the next month, the assumption that stumpage prices are independent draws from a normal distribution is incorrect. The correlation of stumpage prices one quarter apart, 0.86, suggests that the independent normal price process is not a reasonable process for stumpage prices. If prices followed an independent price process, price autocorrelation would be nearly zero. Additionally, the parameters of the price process could change over time, reflecting changes in the timber market. The reservation price strategy presented in this study relies upon a defined (known) stochastic process for prices. If the mean or volatility of prices changes over time, harvest value calculations could be misleading.

4.7.2 Costs of price monitoring and imperfect price information

Although forest owners can attempt to harvest more timber during periods of high prices and harvest less timber during periods of low prices, a variety of barriers exist that may limit the success of market timing. First, measuring stumpage prices is costly. Forest owners can obtain price information through a consulting forester or through a variety of proprietary timber market information services. Provencher (1995b) argues that a simple rotation length model may be appropriate if price information is costly. Fixed rotation lengths have an advantage of eliminating the costs of price monitoring (Haight and Holmes, 1991). Second, price measurement is imperfect. Regional average prices provided by timber market services mask significant variation in local prices. The stumpage price that a forest owner actually receives can vary depending on location and quality.

4.8 Application of the model

The simulated results apply to a single timber product in the upstate region of South Carolina, with specific site characteristics. For the estimated parameters of the mean reverting GARCH process, the gains from following an optimal flexible management strategy range from 22 to 33 percent relative to the net present value under a fixed rotation strategy. Over all assumptions tested, the range of land values is much larger: from 13 to 70 percent larger than the value of a fixed rotation strategy. This suggests that accurate calibration of the model is critical to the results. For each application, the model should be customized for a specific plot by incorporating different volume functions and a price process estimated from local stumpage price data.

5 Conclusion

The first essay discusses an optimal simulation model for lumber futures prices. The mean reverting GARCH process provides a model that could be used in a study concerned with hedging the value of forest assets. Combined with the model of reservation prices in the second essay, the simulation model presented in the third essay can be applied directly to the valuation of timberland. The simulated results suggest that forest owners can increase timber harvesting profits and decrease the variability in profits by following an optimal strategy. The estimation of land value when stumpage prices are characterized by a mean reverting GARCH is new to the forestry option value literature.

Each essay has broader applicability than the field of forestry economics. The first essay highlights common characteristics of commodity prices: price volatility changes over time and the distribution of percentage price changes has heavy tails relative to a normal distribution. The arguments for the use of a mean reverting GARCH process applies to all fields where a stochastic process must be used to model uncertainty, especially fluctuations in commodity prices. The decision model and simulation procedures in the second and third essays have applications in all areas of natural resource management. Many types of resource extraction problems can be viewed as an choice to extract the resource in the current period for a certain price or delay extraction and face uncertain prices.

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Price process	Site index	$E\,[\mathrm{NPV}]$	$\sqrt{\mathrm{Var}\,(\mathrm{NPV})}$	$t^{\star}$
Mean reverting GARCH	60	465.97	210.25	174
Mean reverting GARCH, $\sigma_t \times 1.25$	60	\$465.94	\$303.57	174
Mean reverting GARCH, $\sigma_t \times 0.75$	60	\$466.01	\$160.28	174
Geometric Ornstein-Uhlenbeck	60	\$463.52	\$248.30	174
iid. Normal	60	\$492.04	\$160.34	174
Mean reverting GARCH	90	\$2219.76	\$609.71	124
Mean reverting GARCH, $\sigma_t \times 1.25$	90	\$2244.41	\$821.96	124
Mean reverting GARCH, $\sigma_t \times 0.75$	90	\$2245.52	\$482.82	124
Geometric Ornstein-Uhlenbeck	90	\$2235.91	\$779.17	124
iid. Normal	90	\$2327.26	\$503.93	124

Table 1: Net present value, fixed rotation lengths

Price process	$E\,[\mathrm{NPV}]$	$\sqrt{\mathrm{Var}\,(\mathrm{NPV})}$	$E\,[t^\star]$	$\sqrt{\mathrm{Var}\,(t^\star)}$
Mean reverting GARCH	\$619.82	\$112.46	171.42	11.80
Mean reverting GARCH, $\sigma_t \times 1.25$	\$674.77	\$138.98	172.06	12.79
Mean reverting GARCH, $\sigma_t \times 0.75$	\$558.45	\$83.68	169.97	10.59
Geometric Ornstein-Uhlenbeck	\$655.30	\$124.56	170.42	12.40
iid. Normal	\$838.14	\$84.37	177.30	7.29

Table 2: Net present value applying the reservation price strategy, site index 60

Price process	$E\left[\text{NPV}\right]$	$\sqrt{\text{Var}\left(\text{NPV}\right)}$	$E\left[t^{\star}\right]$	$\sqrt{\text{Var}\left(t^{\star}\right)}$
Mean reverting GARCH	\$2,709.99	441.13	129.81	9.29
Mean reverting GARCH, $\sigma_t \times 1.25$	\$2,923.02	590.20	131.98	9.92
Mean reverting GARCH, $\sigma_t \times 0.75$	\$2,547.53	338.71	129.11	8.56
Geometric Ornstein-Uhlenbeck	\$2,868.44	509.01	130.18	9.58
iid. Normal	\$3,444.26	\$269.60	129.35	5.78

Table 3: Net present value applying the reservation price strategy, site index 90

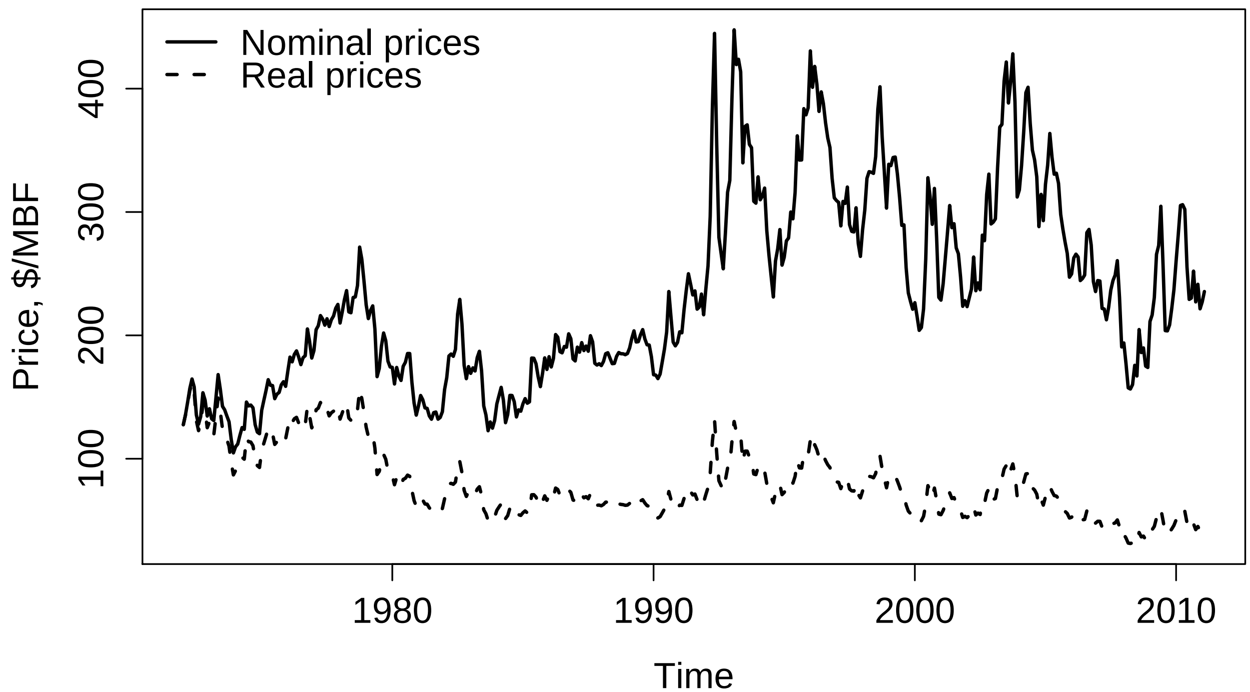

Figure 1: Lumber futures contract prices, monthly averages from November, 1972 to December, 2011

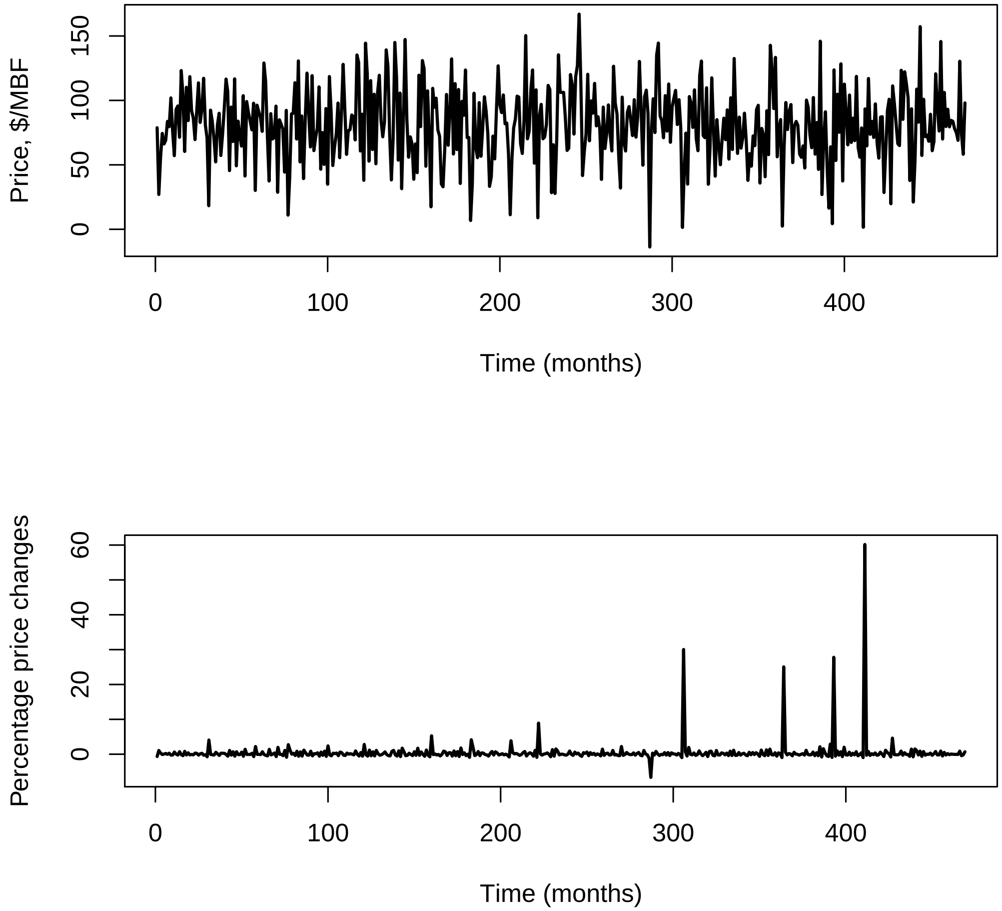

Figure 2: A simulated independent normal price process for lumber futures prices

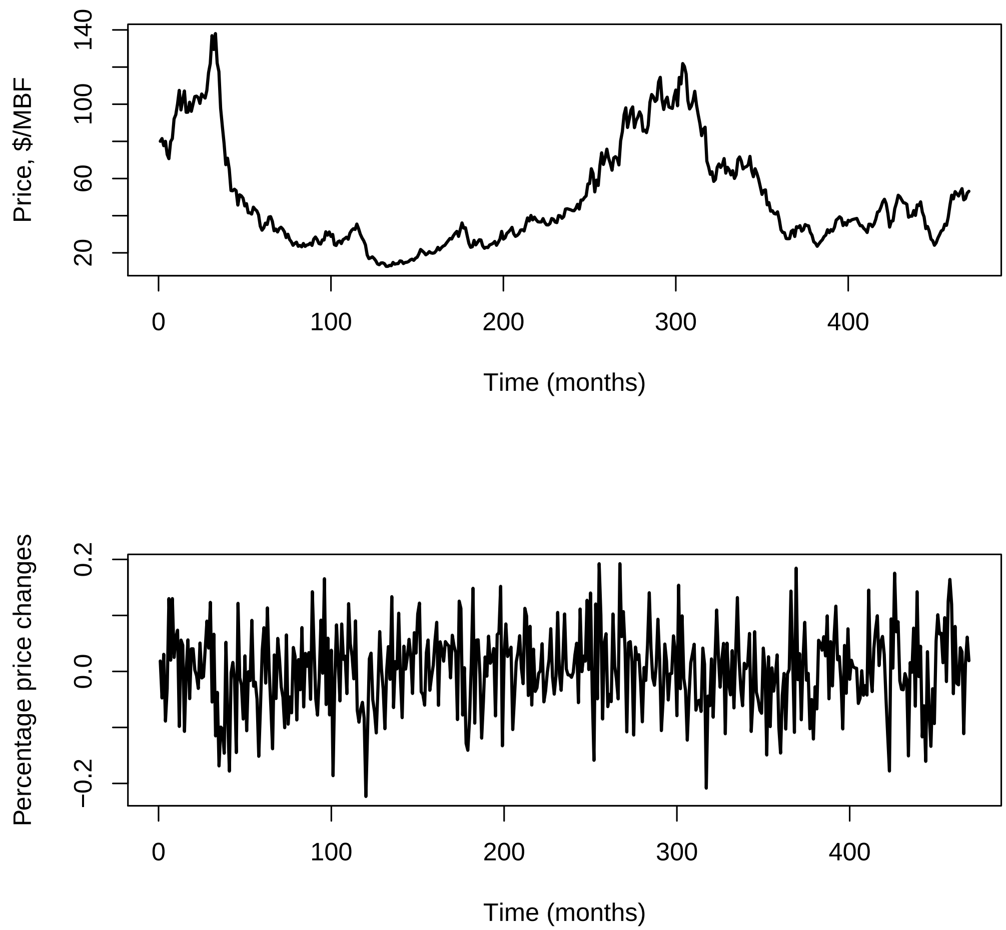

Figure 3: A simulated geometric Brownian motion process for lumber futures prices

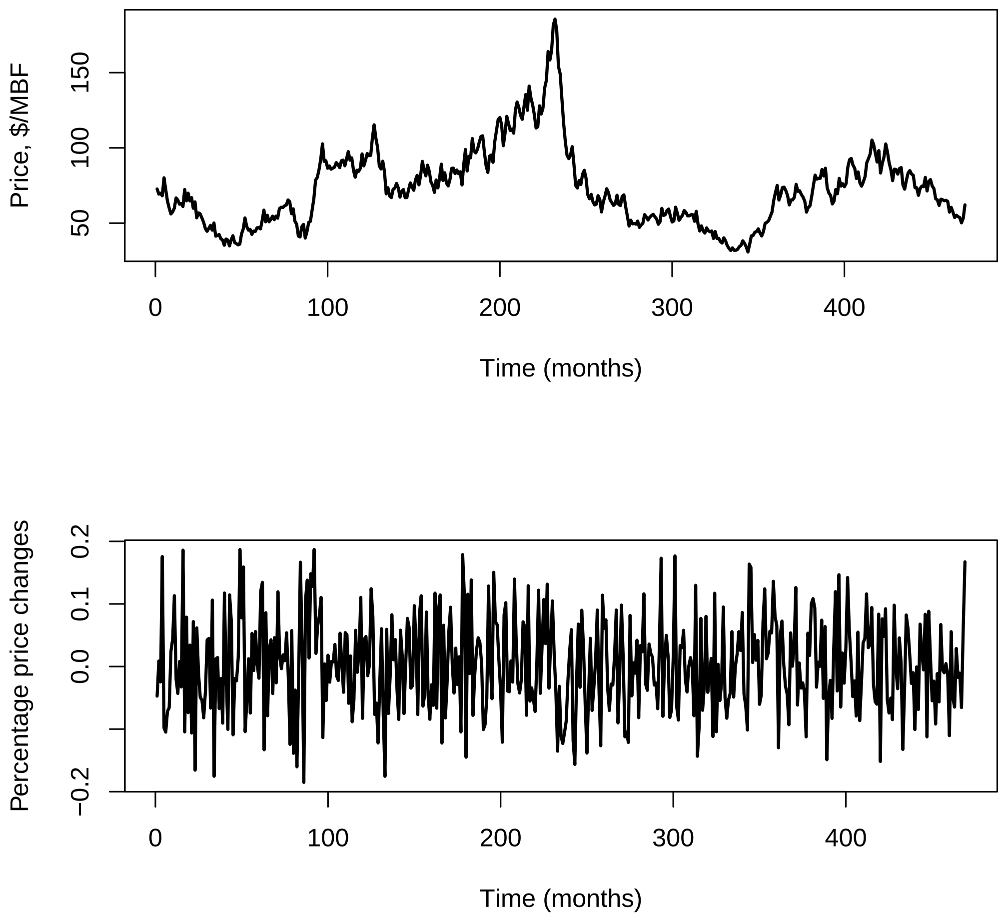

Figure 4: A simulated geometric Ornstein-Uhlenbeck process for lumber futures prices

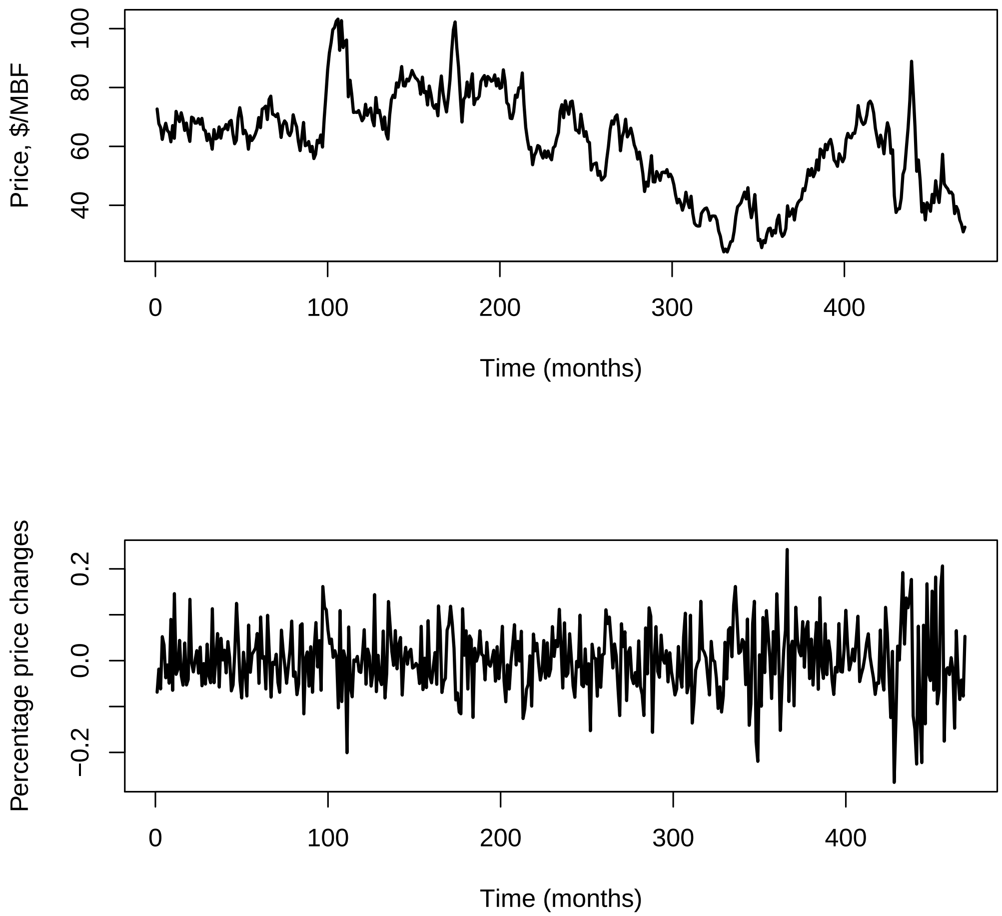

Figure 5: A simulated mean reverting GARCH process for lumber futures prices

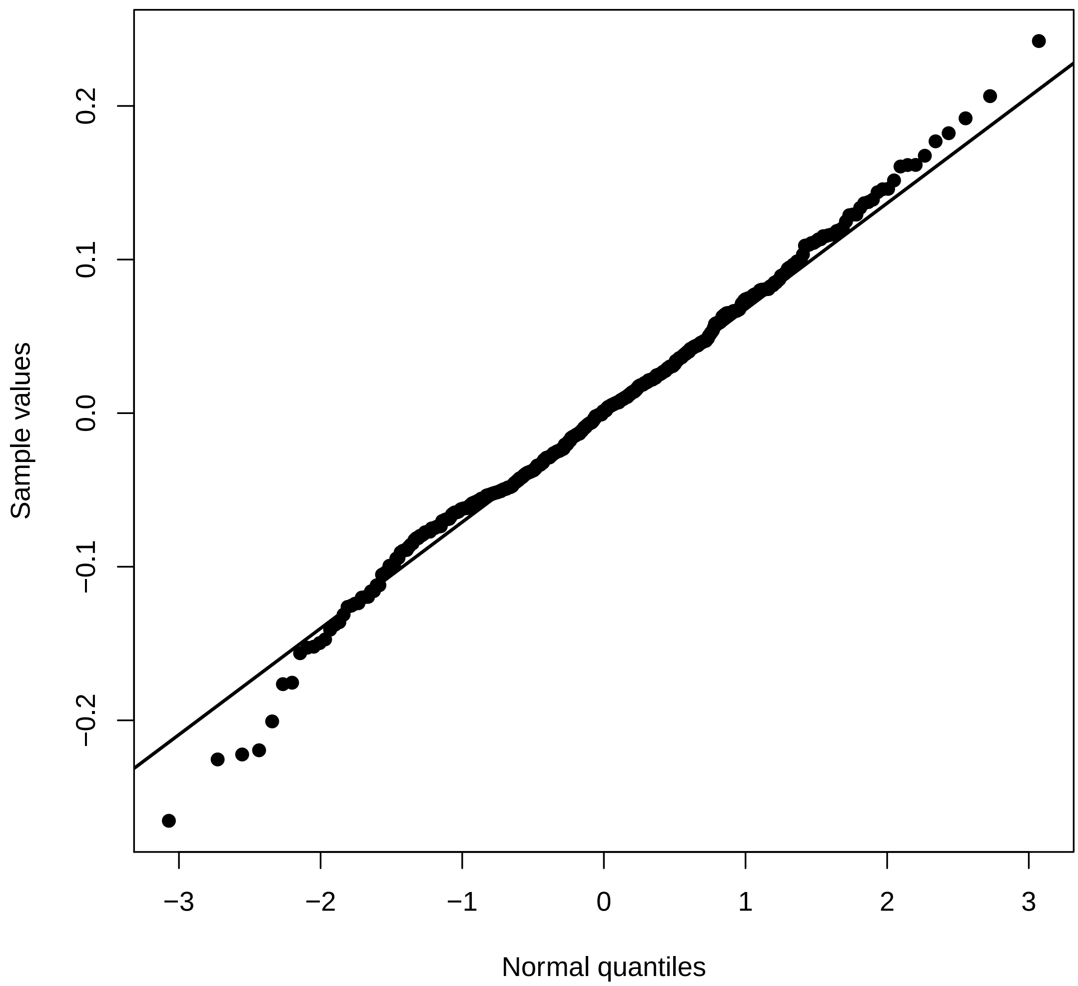

Figure 6: Normal quantile plot of the distribution of percentage price changes for the simulated mean reverting GARCH process

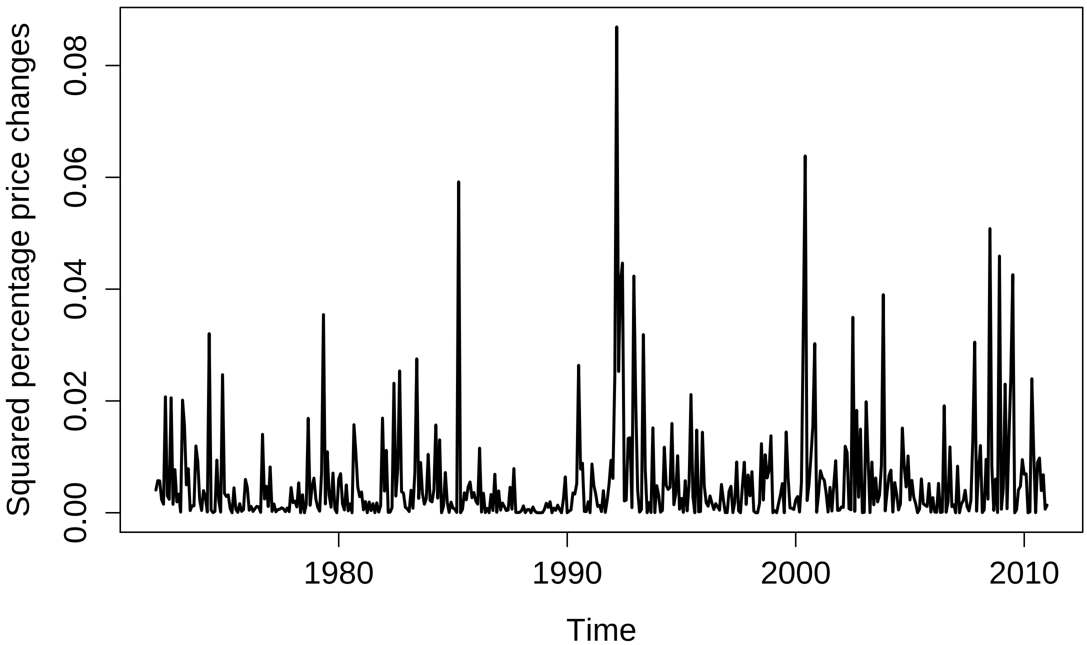

Figure 7: Squared percentage changes in lumber futures prices (monthly volatility)

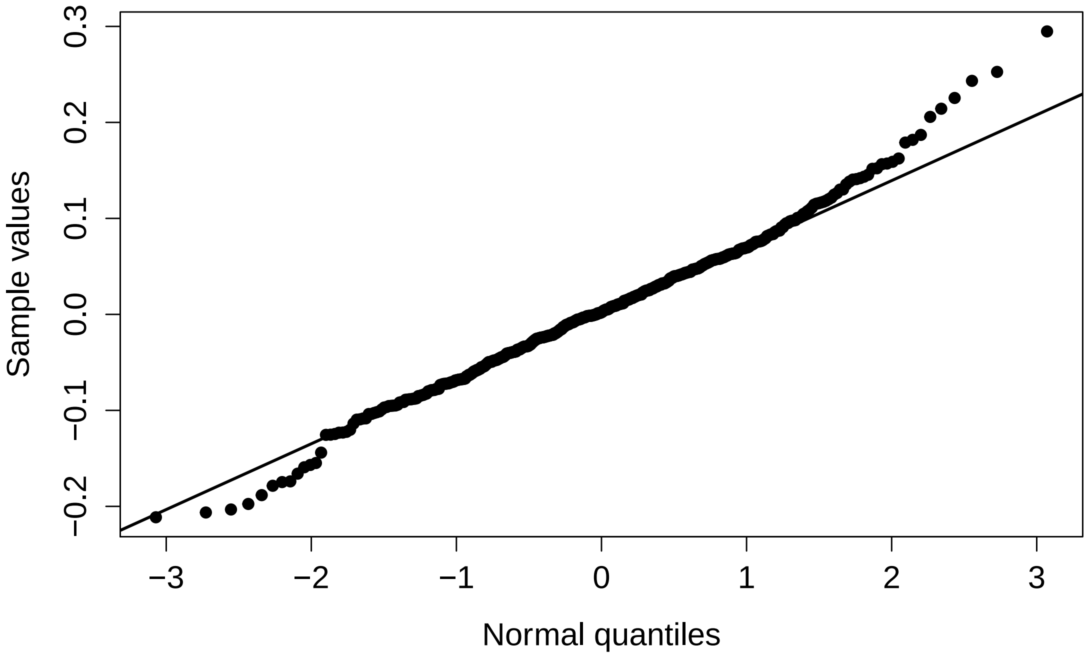

Figure 8: Normal quantile plot for monthly percentage changes in lumber futures prices

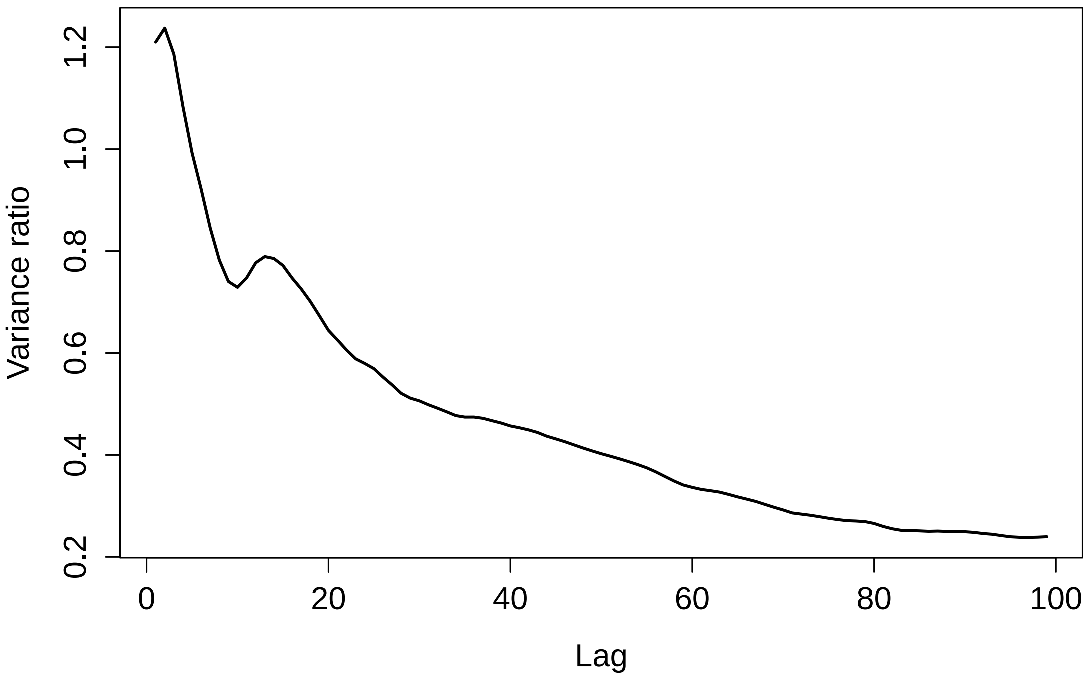

Figure 9: Variance ratios for the first 100 lags, test tor mean reversion of lumber futures

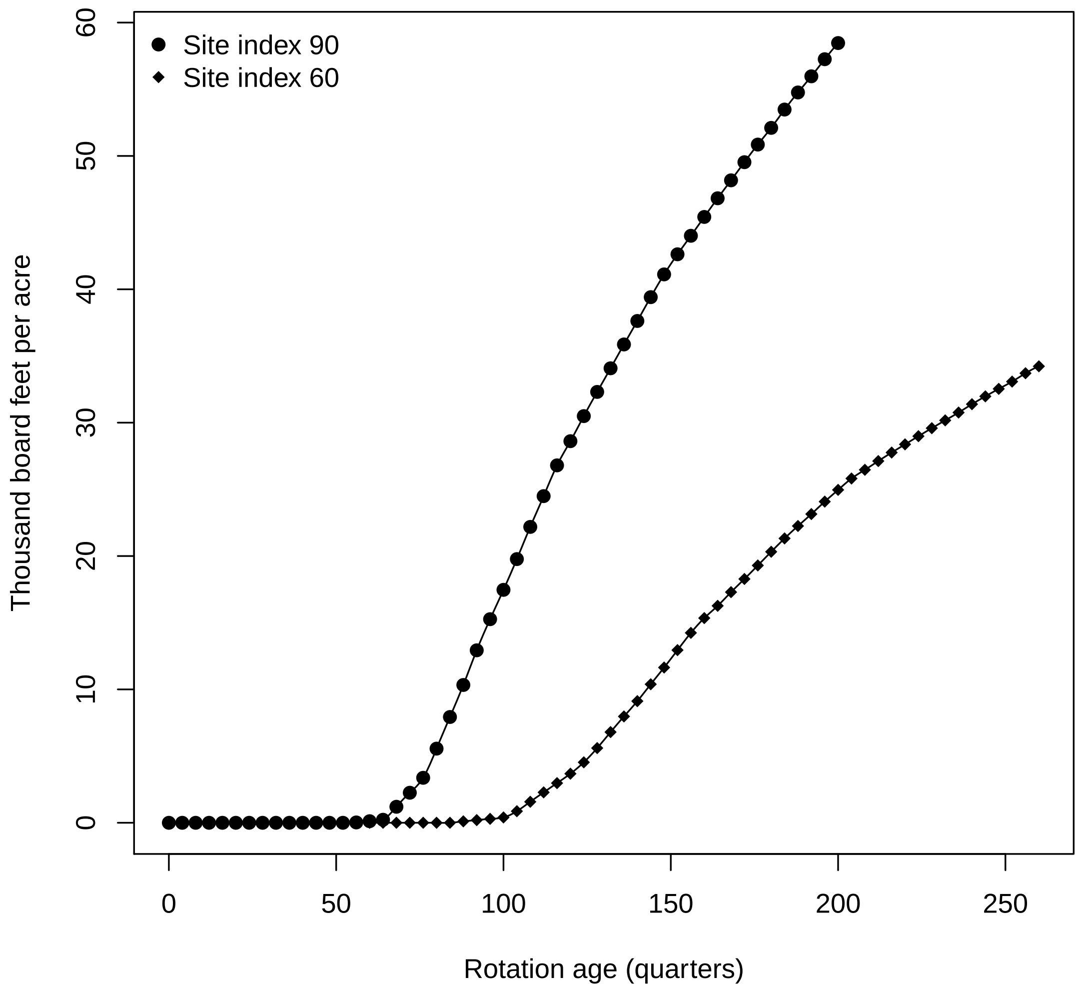

Figure 10: Sawtimber volumes for loblolly pine, upstate region of South Carolina, generated using SiMS plantation simulator (ForesTech International, 2006)

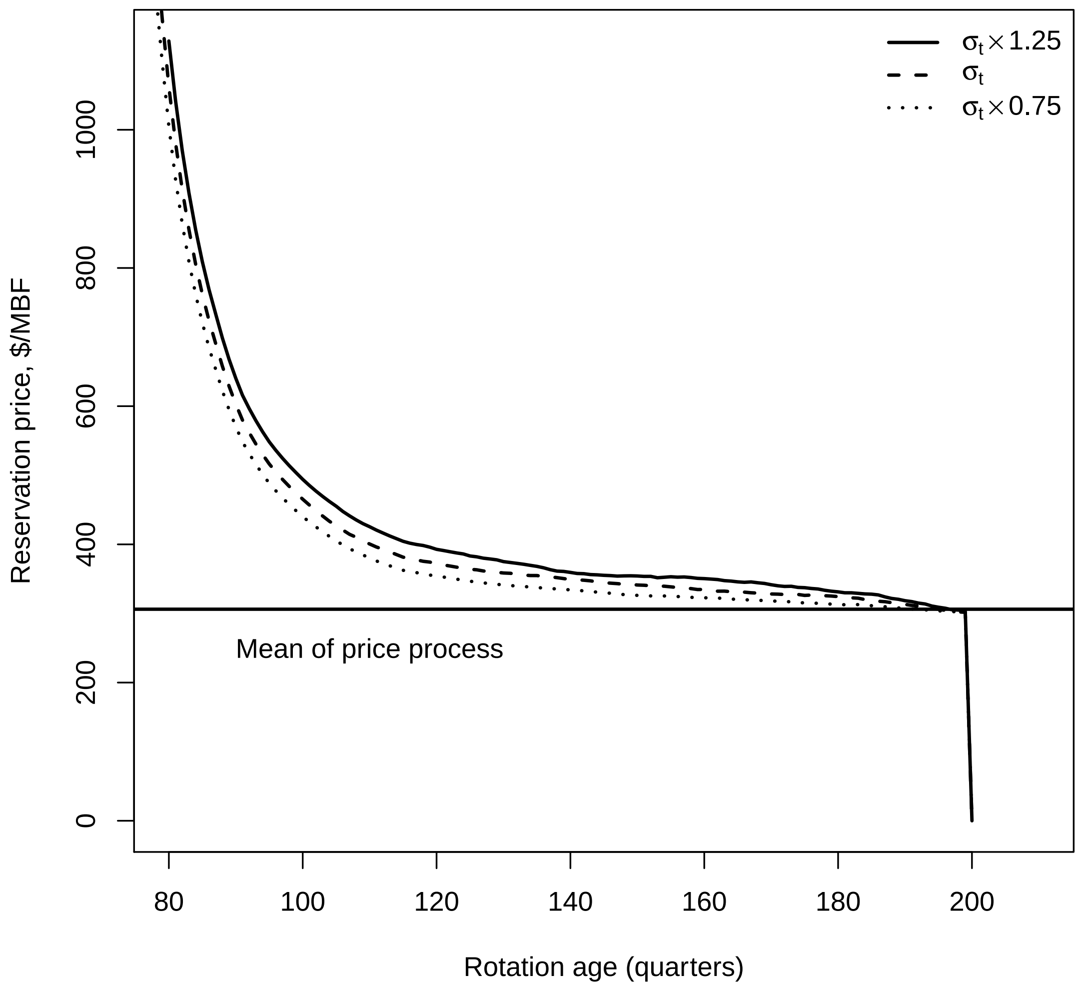

Figure 11: Effect of price volatility on reservation prices, site index 90 and mean reverting GARCH price process

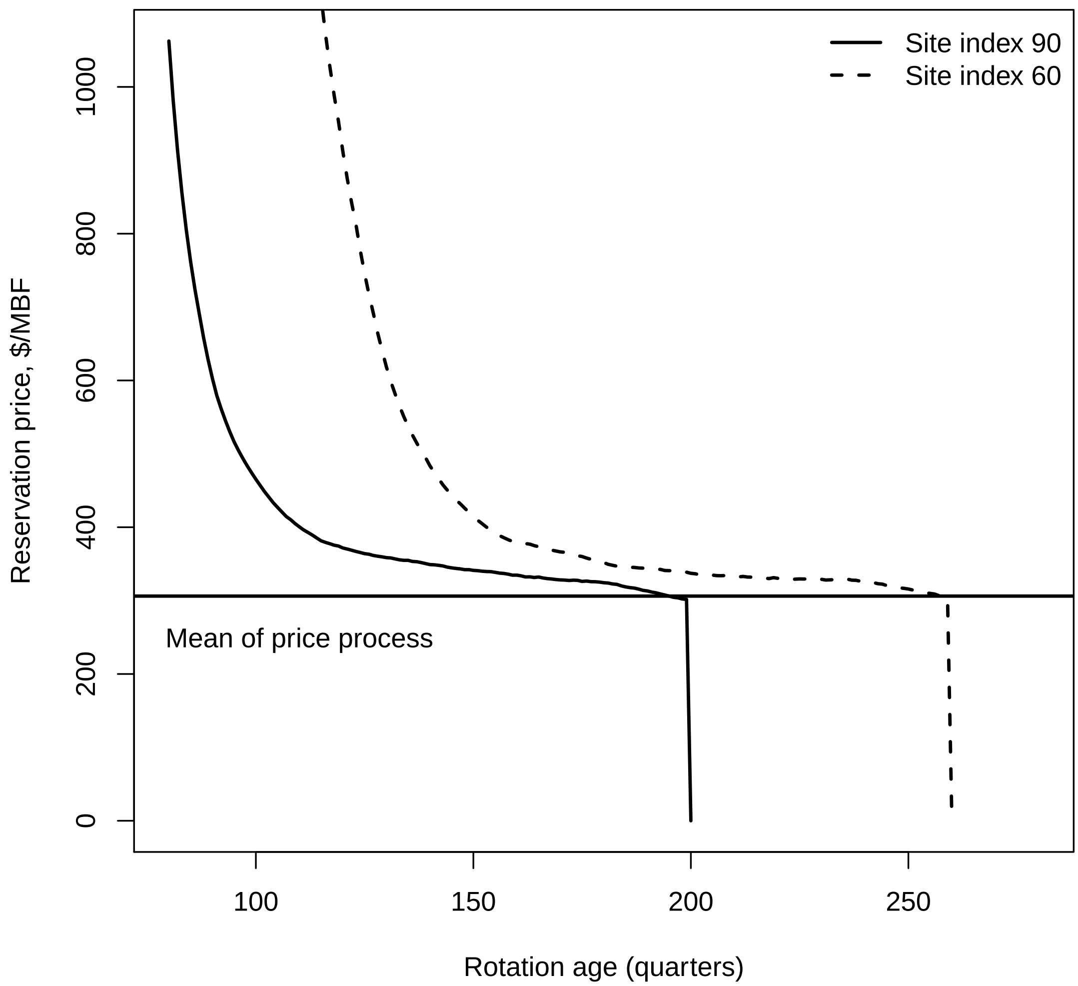

Figure 12: Effect of site quality on reservation prices, site index 90 and mean reverting GARCH price process

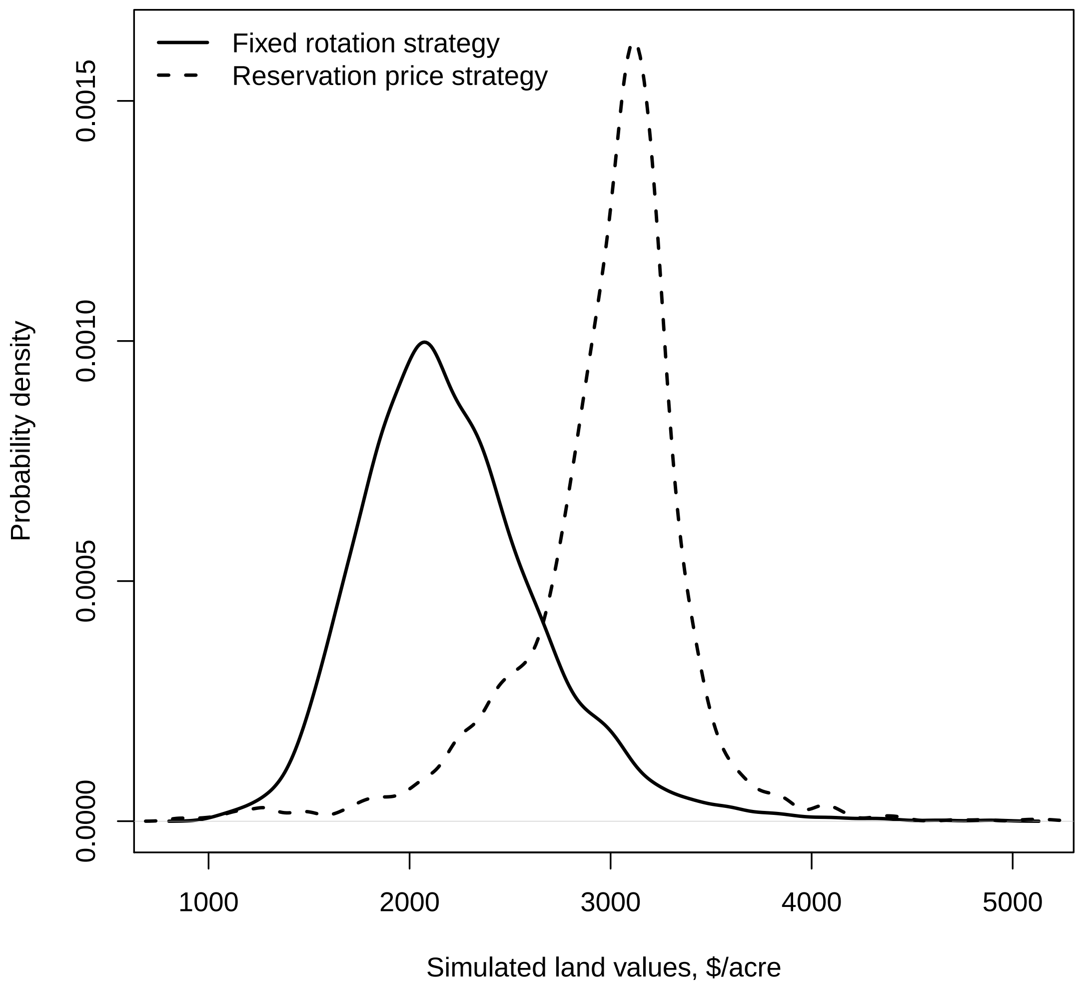

Figure 13: Land values from each harvesting strategy, site index 90 and mean reverting GARCH price process

Appendix

Some of the more difficult programming work required in the dissertation is presented below. I did all of the programming using R. All of the code is completely original. Additional code and a complete set of results are available at http://people.clemson.edu/~campbwa/index.html.

Converting daily prices to monthly average prices

```
mon = month(date)
T = length(price)
#need to know when months change
change_month = rep(0,T) for(t in 2:T){

    if(mon[t] != mon[t-1]){
        change_month[t-1] = 1
    }

}
month_avg = rep(0,T)
total = 0 ; days = 0
for(t in 1:T){

    if(change_month[t] == 0){
        #cumulative sums for each variable
        total = total + price[t]
        days = days + 1
    }
    else{

        #need to include the current month in the calculation
        month_avg[t] = (total + price[t]) / (days + 1)
        #reset the variables     total = 0     days = 0
    }

}
#remove zeros
month_avg = month_avg[month_avg > 0]
```

Estimation and simulation of geometric Brownian motion

```
#function to calculate R_{t}, the sequence of percentage price changes
pct.diff = function(price){
```

```
    PCT = rep(0,length(price))
    d = diff(price)
    for(t in 1:length(price)){
        PCT[t] = d[t] / price[t]
    }
    PCT = PCT[-length(PCT)]
    return(PCT)
}
#percentage price changes
R = pct.diff(price)
#estimation of the drift parameter
mean(R)
#estimate of the variance parameter
sd(R)
#Simulation of the price series
GBM.sim <- function(P_0 = 80, length = 2000, mu = 0.002461,
    sigma = 0.09592){
    P = rep(P_0, length)
    epsilon = rnorm(length) #standard normal
    for(i in 2:length){

        P[i] = P[i-1] * (1 + mu) + sigma * P[i - 1] * epsilon[i]
    }
    return(P)
}
```

Estimation and simulation of the geometric Ornstein-Uhlenbeck process

```
R = pct.diff(price)
Z = 1/price[-470]
#regression equation to find the parametervalues
summary(lm(R ~ Z))
OU.sim = function(length = 2000, mu = 80.0356, eta = 0.03721,
    sigma = 0.09557){
    P_0 = mu #starting price is the mean
    P = rep(P_0,length)
    for(i in 2:length){

        P[i] = P[i-1] + eta * (mu - P[i-1]) + sigma * rnorm(1) * P[i-1]
    }
    return(P)
}
```

Estimation and simulation of a geometric Ornstein-Uhlenbeck process with GARCH price changes

```
require(fGARCH)
garch.model = garch(pct.diff(price))
#this provides the GARCH parameters
summary(garch.model)
GARCH.sim = function(mu = 80.0356, eta = 0.03721, omega = 0.000294,
    alpha = 0.0949, beta = 0.855, length = 1000){
    specs = garchSpec(model = list(omega = omega, alpha = alpha,
        beta = beta))
    sigma = garchSim(spec = specs, n = length)
    P_0 = mu #starting price, known
    P = rep(P_0,length)
    for(i in 2:length){

        P[i] = P[i-1] + eta * (mu - P[i-1]) + sigma[i] * P[i-1]
    }
    return(P)
}
```

Calculation of the expected value of a truncated normal distribution

```
#mu is the mean, stdev is the standard deviation, and
#a is the point of truncation:
#E[x|x>a], the function assumes positive truncation
e.normal = function(x, mean = 427.0379, stdev = 31.28014){
    value = x * ((1 / (stdev * sqrt(2 * pi))) * exp((-0.5)
        * ((x - mean) / stdev)^2))
    return(value)
}
cond.exp.normal = function(mean = 427.0379, stdev = 31.28014, a = 0){

    value = integrate(e.normal, a, Inf, mean = mean, stdev = stdev)$value
        / (1 - pnorm(a, mean = mean, sd = stdev))
    return(value)
}
cond.exp.normal(a = 427.0379)
#the result is 451.9958
```

The discrete Faustmann model

```
#expected value of bare land with given parameter values:
faustmann = function(price = 150, r = 0.05, Cp = 250, vf = hq){

    t = 1:500
    beta = 1 / (1 + r/4)
    t_star = which.max((beta^t * price * vf(t) - Cp)/(1 - beta^t))
    NPV = (beta^t_star * price * vf(t_star) - Cp)/(1 - beta^t_star)
    return(list(Optimal.rotation = t_star, NPV = NPV))

}
```

Backward recursion model for *iid* normal price process

```
mu = 427.0379; stdev = 151.72; r = 0.05
#monthly discount rate
beta = 1 / (1+(r/12))
#set the volume function to either hq or lq
vf = hq
#planting costs
Cp = 250
#initial estimate of the value of bare land
lambda = faustmann(price = mu, Cp = Cp, vf = vf)$NPV
#define the value function, call this function during the RP calculation
J = function(t, RP = 0){

    value = (((1 - pnorm(RP, mean = mu, sd = stdev)) *

        cond.exp.normal(a = RP, mean = mu, stdev = stdev) * vf(t)) +
        (pnorm(RP, mean = mu, sd = stdev) * RP * vf(t))) + lambda
    return(max(value, 0))

}
T = 800
RP = rep(0,T)
t = T - 1
while(t >= 1){

    RP[t] = (beta * J(t + 1, RP[t + 1]) - lambda) / vf(t)
    t = t - 1 #count backwards

}
```

Solving for reservation prices using Monte Carlo simulation, mean reverting GARCH process

```
#Monte Carlo land value function
#the function assumes that harvesting decisions are made quarterly
```

```
#the function has two built-in volume functions: 1 for high quality plots
#and 2 for low quality plots
#prices is a vector of stumpage prices: timber price - harvest costs
land.value = function(initial.time = 1, ending.time = 200,
    volume.function = "hq", r = 0.05, prices, res_price){
    #discount factor
    beta = 1 / (1 + r/4)
    #determine the volume function and minimum and maximum harvest dates
    if(volume.function == "hq"){
        vf = hq
        t_min = 100
        t_max = 200
    } else
    if(volume.function == "lq"){
        vf = lq
        t_min = 30 * 4
        t_max = 65 * 4
    }
    #the sequence of the control variable
    harvest = rep(FALSE, ending.time)
    #vector to store profits
    profit = rep(0, ending.time)
    current.time = initial.time
    while(current.time <= ending.time && harvest[current.time] == FALSE){

        #if timber is below the minimum age, there can be no harvest
        if(current.time < t_min){
            harvest[current.time] = FALSE
            current.time = current.time + 1
        }
        #contract owner may choose to harvest or delay
        else if(current.time >= t_min && current.time < t_max){

            #don't harvest if current price is less than the current RP
            if(prices[current.time] < res_price[current.time]){
              harvest[current.time] = FALSE
              current.time = current.time + 1
            }
            #otherwise, harvest
            else{
              harvest[current.time] = TRUE
              profit[current.time] = prices[current.time] * vf(current.time)
```

```
          * beta^(current.time - initial.time)
        #store the value of the rotation length
        rotation.length = current.time
      }
    }
    #contract owner always harvests when the age of trees is t_max
    else if(current.time == t_max){
      harvest[current.time] = TRUE
      profit[current.time] = prices[current.time] * vf(current.time)
        * beta^(current.time - initial.time)
      #store the value of the rotation length
      rotation.length = current.time
    }
  }
  return(list(Profits = sum(profit), Harvest.times = harvest, profit))
}
nsim = 5000 #number of simulations
N = 200 #number of periods in the model
prices = matrix(nrow = nsim, ncol = N)
for(i in 1:nsim){
  prices[i,] = GARCH.sim(mu = mu, eta = eta, omega = omega,
    alpha = alpha, beta = beta, length = N)
}
beta = 1 / (1 + (r / 4))
lambda = 1610.877 #337.4541 #set to Faustmann value
vf = hq
n = 0 #start at time N
value = rep(0, N)
RP = rep(0,N)
### Start the backward recursion algorithm ###
while((N - n) >= 250){
  v = rep(0,nsim)
  for(i in 1:nsim){
    #the value for the ith simulated price
    v[i] = land.value.MC(initial.time = N - n, ending.time = N,

      volume.function = "hq", prices = prices[i,],
      res_price = RP)$Profits
  }
  #the expected value function is
  value[N-n] = mean(v) + lambda
```

```
        #the reservation price at N-2 is
        RP[N-n-1] = (beta * value[N-n] - lambda) / vf(N-n)
        n = n + 1
    }
```

Simulation model of the reservation price strategy

```
    #Model inputs: price process, discount rate, volume function,
    #and set of reservation prices.
    #Model outputs: profits and rotation lengths from applying
    #the reservation price strategy.
    #duration is the length of the simulation, 1,000 quarters should be enough,
    #anything beyond this will not affect discounted profits
    #the function assumes that harvesting decisions are made quarterly
    land.value = function(duration = 1000, volume.function = "hq", r = 0.05,
        prices, res_price){
        #discount factor
        beta = 1 / (1 + r/4)
        #set NA values of the reservation price to infinity
        for(i in 1:length(res_price)){
            if(is.na(res_price[i])){
                res_price[i] = Inf
            }
        }
        #store values for the volume function in the vector vf
        vf = rep(0, duration)
        if(volume.function == "hq"){
            vf = hq
            t_min = 25 * 4
            t_max = 50 * 4
        } else
        if(volume.function == "lq"){
            vf = lq
            t_min = 30 * 4
            t_max = 65 * 4
        }
        #check to make sure the length of the price vector
        #matches the duration of the contract (in months)
        if(length(prices) != duration){

            print("Length of the price vector needs to match the duration")
            stop()
```

```
}
#the sequence of the control variable
harvest = rep(FALSE, duration)
rotation.time = 0
#starts with planting
total.time = 1
#an object to store all of the rotation lengths
rotation.lengths = 0
#vector to store profits
profit = rep(0, duration)
while(total.time < duration){
    #needs to be incremented here
    rotation.time = rotation.time + 1
    total.time = total.time + 1
    #if timber is below the minimum age, there can be no harvest
    if(rotation.time < t_min){
       harvest[total.time] = FALSE
    }
    #contract owner may choose to harvest or delay
    else if(rotation.time >= t_min && rotation.time < t_max){

       #don't harvest if current price is less than the current RP
       if(prices[total.time] < res_price[rotation.time]){
         harvest[total.time] = FALSE
       }
       #otherwise, harvest
       else{
         harvest[total.time] = TRUE
         profit[total.time] = prices[total.time] * vf(rotation.time) *
           beta^total.time
         #store the value of the rotation length
         rotation.lengths = c(rotation.lengths, rotation.time)
         #reset the time for each rotation
         rotation.time = 0
       }
    }
    #contract owner always harvests when the age of trees is t_max
    else if(rotation.time == t_max){
       harvest[total.time] = TRUE
       profit[total.time] = prices[total.time] * vf(rotation.time) *
         beta^total.time
       #store the value of the rotation length
```

```
            rotation.lengths = c(rotation.lengths, rotation.time)
            #reset the time for each rotation
            rotation.time = 0
        }
    }
    return(list(Profits = sum(profit), Harvest.times = harvest,
        Rotation.lengths = rotation.lengths[-1], profit))
}
```

Simulation model for the valuation of bare land with constant rotation lengths

```
r = 0.05
beta = 1 / (1 + (r/4))
#number of simulations
nsim = 10000
profit = rep(0, nsim)
t_star = faustmann(price = 317.8848, r = 0.05, Cp = 250,
    vf = lq)$Optimal.rotation
#ten rotations
time = seq(from = t_star, to = t_star*10, by = t_star)
#create a matrix of prices
price = matrix(0, nrow = nsim, ncol = t_star * 10)
for(i in 1:nsim){
    price[i,] = GARCH.sim(length = t_star * 10)
}
#low quality site
for(i in 1:nsim){
    for(t in time){

        profit[i] = sum(beta^time * price[i, time] * lq(t_star))
    }
}
lq_profit = profit
#high quality site
t_star = faustmann(price = 317.8848, r = 0.05, Cp = 250,
    vf = hq)$Optimal.rotation
time = seq(from = t_star, to = t_star*10, by = t_star)
for(i in 1:nsim){
    for(t in time){
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        profit[i] = sum(beta^time * price[i, time] * hq(t_star))
    }
}
hq_profit = profit
```