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Modelling and Characterization of Magnetic Microfibers

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Modelling and Characterization of Magnetic Microfibers

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Electrical Engineering

by
Harshwardhan Dilip Karve
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Accepted by:
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Abstract

Polymer fibers of varying microfluidic properties can be fabricated in a lab setting. Fibers coated with paramagnetic particles act like slender paramagnetic beams. These fiber moves in a magnetic field. Thus polymer fibers coated with paramagnetic particles can be used as actuators in various microfluidic applications, such as DNA separation, droplet manipulation and liquid transport. In order to use the fibers as actuators, it is necessary to model the fiber and develop control strategies.

A static model of the paramagnetic fiber based on energy methods is presented in [1]. The model relies on a demagnetizing factors approximation to determine the magnetic field inside the fiber. The first part of the thesis examines the conditions under which the demagnetizing factors approximation holds.

The model allows for implementation of simple feedforward control strategies to control the position of the fiber. For implementation of better control algorithms, methods to sense the shape and the tip position of the fiber are required. These sensing methods are also presented here.

The model depends on the bending rigidity and magnetic susceptibility of the fiber. Since the fibers can be synthesized in a lab setting, these properties are usually not known. This thesis also presents methods to characterize the bending rigidity of the fiber, based on the sensing methods.
The bending rigidity and the magnetic susceptibility of the fiber, along with the model can be used to implement a basic feedforward control strategy to accurately position the tip of the fiber. This enables the use of the fiber as a microfluidic actuator.
Dedication

This work is dedicated to my parents and my brother and sister-in-law, in appreciation of their support, love and guidance through the years.
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Chapter 1

Introduction

1.1 Motivation

Paramagnetic materials are of particular interest for use in magnetic actuators. Paramagnetic and superparamagnetic materials are magnetized in a magnetic field, but unlike ferromagnetic materials, they cannot be permanently magnetized. This means the magnetic field induced in a paramagnetic body does not depend on the position of the body in the magnetic field at any past instant. Thus the magnetic traction force on the paramagnetic body depends only on the current position of the body in the magnetic field.

Polymer fibers coated with paramagnetic particles act like slender paramagnetic beams. This thesis presents an energy methods based approach to characterizing such paramagnetic fibers and simulating their behavior in a magnetic field. It also presents image processing techniques that are required to use the fiber as an actuator. Fibers of very small lengths and diameters can be fabricated. Fibers of varying cross sections can also be fabricated. These variations allow for fabrication of fibers of varying microfluidic properties. By coating fibers with desirable microfluidic proper-
ties with paramagnetic particles, it is possible to make actuators that are suitable for a variety of microfluidic applications, such as DNA separation, droplet manipulation and liquid transport.

Figure 1.1: A paramagnetic fiber bends in a magnetic field. It’s position can be controlled precisely using an electromagnet. The tip of the fiber can be used as an end effector for microfluidic applications.

Figure 1.1 shows the arrangement of the fiber and the magnet. The fiber is cantilevered at it’s base. The magnetic field is generated using an electromagnet. The magnetic field can be easily varied by varying the coil current. When the electromagnet is active, the fiber bends towards the magnet. Thus the tip of the fiber can be used as an end effector to manipulate objects.

1.2 Literature Review

The literature presents different types of magnetic actuators. Some magnetic actuators rely on electromagnets embedded in the actuator itself [2]. Using electromagnets is prohibitive at small length scales. Also, the electromagnet has to be powered externally. This means there has to be direct contact between the power source and the actuator. Magnetic actuators at small length scales usually take the form of magnetic material deposited on or embedded inside a solid body. The solid body bends or twists in response to a magnetic field. For example, cantilevered beams and torsional beams with Permalloy coatings have been used as micro-actuators [3].
In some designs, the magnetic actuator is suspended in fluid, but otherwise free to move. Superparamagnetic flagella formed by linking paramagnetic colloids using DNA strands have been used to propel red blood cells in microscopic artificial swimmers [4]. The microscopic artificial swimmers were modeled as a series of dipoles attached by springs [5]. Magnetic nanocoils in rotating electromagnetic fields have been used for propulsion of microbots [6]. Anisotropic doublets of paramagnetic particles, linked using DNA were used as microscopic swimmers with two degrees of freedom [7].

In all these works, the magnetic actuator is subjected to a rotating uniform magnetic field. Also, the paramagnetic beads are modeled as dipoles. Flexible Magnetic Filaments made by linking magnetic colloids were used as Micromechanical Sensors for measuring bending rigidity at the micromolecular level [8]. Bending of Flexible Magnetic Rods was analyzed by Cebers [9]. All of the actuators described above are subjected to uniform magnetic fields. The actuators rely on rotation of the magnetic field to generate the torques required. The angle between the actuator and the magnetic field determines the behavior of the actuator. Polymer fibers with embedded paramagnetic particles behave like paramagnetic cantilevered beams. This allows for potentially infinite degrees of freedom. The force on a paramagnetic particle in a magnetic field depends on the gradient of the field [10]. Since nonuniform fields vanish faster than uniform field, a paramagnetic actuator in a nonuniform field experiences higher traction force. Nonuniform magnetic fields are easier to generate. Any commercially available electromagnet generates a nonuniform magnetic field. Also, since the field is nonuniform, actuation occurs even in static magnetic fields. However, the value of the magnetic field cannot be expressed as a function of its position in closed form. This is not a major limitation as the magnetic field can be computed numerically using Finite Element Methods package, such as FEMM [10].
Vibration and buckling of cylinders and plates in uniform magnetic fields was analyzed using force methods in [11]. Buckling of cantilevered ferromagnetic plates in uniform magnetic fields was analyzed using force methods in [12]. The ferromagnetic plates were placed in a uniform magnetic field. The magnetic field inside the plates was computed by applying the boundary conditions on the field at the surface of the plate and solving for the magnetic scalar potential inside and outside the plate. The scalar potential was then used to compute the force on the plate. The same method was applied to study vibration and dynamic stability of ferromagnetic plates in uniform magnetic fields [13, 14]. The chaotic behavior of an externally excited ferromagnetic plate in a non uniform magnetic field was examined in [15]. The plate was assumed to be magnetized only at its tip and the magnetic field was assumed to be locally uniform.

An energy methods approach to model paramagnetic bodies is presented by Cebers [16, 17, 18]. The magnetic fields inside the magnetic bodies were approximated using demagnetizing factors [19, 20, 21]. Demagnetizing factors are derived on the assumption that an infinitely long cylinder can be approximated by an infinitely long ellipsoid. It is also assumed that the magnetic field is uniform, since the field inside an ellipsoid in a uniform magnetic field is easier to compute. This demagnetizing factors approximation needs to be tested for an infinitely long curved magnetic body in a nonuniform field.

A position sensing method is required in order to develop feedback control for the actuator. A camera can sense position of a large number of points. It is possible to construct a continuous and complex shaped curve from an image of the bending beam [22, 23]. The fiber is a continuous, open curve. The strain energy developed in the fiber is a major factor in the behavior of the fiber. The strain energy depends on the first derivative of curvature. Hence it is necessary that the method used to sense
the shape of the fiber returns the correct curvature at all point on the fiber. The fiber has to be fit as an open, continuous curve. The problem of fitting open curves to image data is common in computer vision [24, 25] and in continuum robotics [26]. The solutions used in computer vision do not give the curvature of the curve fit. Continuum robotics usually deals with objects that have constant curvature. If the curve has constant curvature, the strain developed at any point on the curve is constant. The moments that cause the strain cannot be the same at all points along the fiber. Therefore, these approaches cannot be used to sense the shape of the fiber.

Bending rigidity of the fiber is a major factor in the bending behavior of the fiber. Bending rigidity can be determined by studying the deformation of the wire under a known load. Usually, this involves physically measuring the deflection of a beam [27, 28]. Most position sensors can only measure the deflection of the beam at a limited number of points. Also, the bending has to be under controlled conditions that involve a complicated apparatus [27]. The computation of bending rigidity from the deflection is done using small linear deflection beam theory or by solving the elastica equation for the tip displacement, [29]. If the deflection is small compared to the resolution of the sensor used to measure tip position, the tip displacement measurement can be unreliable. Also, these methods assume that the beam is perfectly straight before the load is applied, which may not be the case.

1.3 Organization of the Thesis

The main goal of the thesis is to characterize the properties that govern the behavior of the paramagnetic fibers in a magnetic field and to verify the model presented in [1]. The organization of the thesis is visualized in Figure 1.2. The thesis is divided into four main Chapters, as follows.
Chapter 2 presents the model and the modeling assumptions. The model is presented in Section 2.1 for convenience. In order to get better insight into the behavior of the fiber, a simpler version of the model is studied in Section 2.3. Section 2.2 examines the demagnetizing factors approximation used to compute the field inside the fiber.

Chapter 3 examines the characterization of bending rigidity of the fiber. In order to study the behavior of the fiber in detail, a shape sensing method that can accurately measure the curvature at any point on the fiber is required. A new method to sense the shape of the fiber is proposed in Section 3.1. In Section 3.2, a robust method to compute the bending rigidity of a loaded cantilever beam using shapes sensed from a camera is presented and tested.

The model is validated by comparing the tip deflection predicted by the model with the experimentally observed values of tip deflection in Chapter 4. Section 4.1
describes the algorithm used to sense the position of the tip of the fiber.

Finally, Chapter 5 presents a summary of the results and ideas for further development of the paramagnetic fiber actuators.
Chapter 2

Theoretical Model

An energy methods approach similar to the approach used by Cebers to model magnetic droplets [16, 18] and flexible magnetic filaments [8] is used to model the fiber in [1]. The flexible magnetic filaments are free to move in the magnetic field. The paramagnetic fibers, on the other hand, are cantilevered and placed in nonuniform magnetic field. The model of the fiber, as described by [1] is repeated in Section 2.1 for convenience. The model uses energy methods to predict static equilibria of the fiber in a magnetic field.

The second part of the chapter (Section 2.2) examines the demagnetizing factors approximation used in the model. Demagnetizing factors are derived on the assumption that an infinitely long cylinder can be approximated by an infinitely long ellipsoid. It is also assumed that the magnetic field is uniform, since the field inside an ellipsoid in a uniform magnetic field is easier to compute. This demagnetizing factors approximation needs to be tested for an infinitely long curved magnetic body in a nonuniform field. COMSOL [30] simulations were used to verify the approximation.

The last part of the chapter (Section 2.3 examines a simplified model consisting of a rigid paramagnetic bar with a torsional spring at its base. This model was
simulated in the field of a magnetic dipole using the energy methods approach. This simplified system gives better insight into the energy methods used and the hysteresis behavior of the fiber in a magnetic field.

### 2.1 Fiber Model

The co-ordinate system used here is shown in Figure 3.1. The shape of the beam is represented by a parametrized curve. The parameter \( s \) is defined as \( s \in [0, L] \), where \( s = 0 \) at the base of the beam and \( s = L \) at the tip of the beam. Shape of the beam is represented by the function \( \theta(s) \). \( \theta(s) \) is the angle between the axis of the beam at \( s \) and the \( X \)-axis. \( x \) and \( y \) co-ordinates of the axis of the beam can be obtained from \( \theta(s) \) as

\[
\begin{bmatrix}
x(s) \\
y(s)
\end{bmatrix} = \begin{bmatrix}
x_0 \\
y_0
\end{bmatrix} + \int_0^L \begin{bmatrix}
\cos(\theta(s)) \\
\sin(\theta(s))
\end{bmatrix} ds
\]  

(2.1)

The fiber is modeled as a flexible cylinder. When the fiber is at a static equilibrium, potential energy of the actuator is minimum. The configuration of the
actuator at the static equilibrium in a known field can be determined by solving for the configuration that has the minimum energy. The potential energy has two components, strain energy in the fiber due to the bending and the magnetic energy due to the applied field.

Strain energy at any point in a bending cantilever beam \[31\] is a function of the moment of bending at that point and the bending rigidity of the beam \(EI\). \(E\) is the Young’s modulus of the material of the beam and \(I\) is the second moment of area of the beam. Let \(\theta(s)\) be the shape of the beam with the load after the load was applied and \(\theta_0(s)\) be the shape of the beam before the load was applied. Let \(\alpha\) be the parameters that represent \(\theta(s)\) and let \(\alpha_0\) be the parameters that represent \(\theta_0(s)\). If the fiber is not straight, by computing the strain energy of the difference between the initial shape and the shape after the load is applied, \(\theta - \theta_0\), the strain energy of the initial shape of the fiber can be compensated for. For a beam with low curvature, the bending moment \(M\) is directly proportional to the curvature \[31\]

\[
M = EI \frac{d}{ds} (\theta - \theta_0)
\]  

Total strain energy of the beam is the integral of the strain energy at one point over the length of the beam.

\[
U_s = \frac{1}{2EI} \int_0^L [M(s)]^2 ds = \frac{EI}{2} \int_0^L \left( \frac{d}{ds} (\theta(s) - \theta_0(s)) \right)^2 ds
\]  

The magnetic energy of the fiber can be computed as follows. It is assumed that the material of the fiber is linear and isotropic and the fiber is moving in air.
For a linear, isotropic material, magnetic flux density $B$ is related to the magnetic field intensity $H$ by a scalar parameter $\mu$ i.e. $B = \mu H$, where $\mu$ is called the magnetic permeability. The permeability is a material dependent constant for linearly magnetic materials. The permeability may be rewritten as $\mu = (1 + \chi)\mu_0$ where $\mu_0 = 4\pi \times 10^{-7}H.m^{-1}$ is the permeability of free space. $\chi$ is called the magnetic susceptibility. The magnetic susceptibility controls the magnetization of the material. Total energy stored in a magnetic field is given by

$$T = \frac{1}{2} \int_V H \cdot B dV = \frac{1}{2} \int_V \mu |H|^2 dV,$$

where the volume $V$ is all space and the fields $H$ and $B$ are position dependent, but static. When the fiber is placed in the magnetic field, the permeability of the volume occupied by the fiber $V_f$ changes to $\mu_f$, without changing the permeability anywhere else. The currents that produced $H$ are held constant. The introduction of the magnetic material causes the resulting field $H'$ and flux density $B'$ to differ from the original field $H$ and flux density $B$ both inside and outside of volume $V_f$. Analogous to Equation 2.4, the total energy stored in the new magnetic field is

$$T' = \frac{1}{2} \int_V H' \cdot B' dV,$$

The magnetic energy of the fiber is the change in energy

$$\Delta T = T' - T$$
It is not feasible to calculate the change in energy of the field over all space. Instead, it is more convenient to express the energy change in terms of change in field values over the volume of the fiber. As shown by Stratton [32], the change in total energy when the fiber is placed in the field can be expressed conveniently as:

\[
\Delta T = T' - T = \frac{1}{2} \int_{V_f} (H \cdot B' - H' \cdot B) dV
\]

\[
= \frac{1}{2} \int_{V_f} (\mu_f - \mu) H' \cdot H dV
\]

Where \( V_f \) is the volume of the fiber. Assuming that the radius of the wire is very small, we can assume that the field is constant throughout a given cross section of the fiber. Therefore, we can take the cross sectional area of the fiber \( A \) outside the integral and change the variable of integration to \( s \):

\[
\Delta T = T' - T = \frac{1}{2} A \int_{0}^{L} (H \cdot B' - H' \cdot B) ds
\]

This equation can be written in terms of \( \chi_f \) and \( \mu_0 \) as

\[
\Delta T = \frac{1}{2} A \mu_0 \int_{0}^{L} ((1 + \chi_f) H \cdot H' - H' \cdot H) ds \quad (2.4)
\]

\[
= \frac{1}{2} A \chi_f \mu_0 \int_{0}^{L} (H \cdot H') ds \quad (2.5)
\]
Similar to [16, 17, 18] the value of the field vector $H'$ is approximated using geometry dependent demagnetizing factors [19, 20, 21] as follows.

$$
H'_\parallel = H_\parallel - N_\parallel M_\parallel \\
H'_\perp = H_\perp - N_\perp M_\perp 
$$

$H_\parallel$ and $H'_\parallel$ are the components of field vectors $H$ and $H'$ parallel to the axis of the fiber. Similarly, $H_\perp$ and $H'_\perp$ are the components of $H$ and $H'$ perpendicular to the axis of the fiber. $M_\parallel$ and $M_\perp$ are magnetization of the material in the cross section, and $N_\parallel$ and $N_\perp$ are the demagnetizing factors. For an infinitely long cylinder, the demagnetizing factors are $N_\parallel = 0$ and $N_\perp = \frac{1}{2}$. Methods to compute the demagnetizing factors from geometric ratios are presented in [19, 20, 21]. The demagnetizing factors approximation is tested in Section 2.2. Also, magnetization for a magnetic material is given by $M_\parallel = \chi_f H'_\parallel$ and $M_\perp = \chi_f H'_\perp$, where $\chi_f$ is the magnetic susceptibility of the material of the fiber. Thus,

$$
H'_\parallel = \frac{H_\parallel}{1 + \chi_f N_\parallel} = H_\parallel \\
H'_\perp = \frac{H_\perp}{1 + \chi_f N_\perp} = \frac{H_\perp}{1 + \frac{1}{2} \chi_f} 
$$

To determine the change in energy of field for the fiber moving in air, substitute Equation 2.9 in Equation 2.5,
\[ \Delta T = \frac{1}{2} A \chi f \mu_0 \int_0^L \left[ H_\parallel^2 - \frac{1}{1 + \frac{1}{2} \chi f} H_\perp^2 \right] ds \] \quad (2.10)

\( T \) is the work done in order to restore the source currents to their initial intensity [32]. The energy stored in the fiber is equal in magnitude and negative.

\[ U_M = -\Delta T \] \quad (2.11)

The total potential energy of the fiber is thus,

\[ U = U_S + U_M \]

\[ = \int_0^L F(x, y, \theta, \dot{\theta}, s) ds \] \quad (2.12)

where \( \dot{\theta} = \frac{d\theta}{ds} \) and \( F(x, y, \theta, \dot{\theta}, s) \) is given by

\[ F(x, y, \theta, \dot{\theta}, s) = \frac{1}{2} \left[ EI \dot{\theta}(s)^2 \right] A \chi f \mu_0 \left[ H_\parallel^2 + \frac{1}{1 + \frac{1}{2} \chi f} H_\perp^2 \right] \] \quad (2.13)

Because the field \( H \) is non-uniform, the integrand depends on \( x(s) \) and \( y(s) \) through \( H_\parallel \) and \( H_\perp \). This makes a calculus of variations approach difficult to use. Instead, the Rayleigh-Ritz method is employed to find minimum energy fiber shape. In this method, the fiber shape is approximated as a linear combination of basis functions plus a constant:
\[ \theta(s) = \phi_0 + \sum_i \alpha_i \phi_i(s) \]  

(2.15)

It is assumed that the angle at the base of the fiber is constant, i.e., the fiber is cantilevered. Also, the bending \( \dot{\theta}(s) \) at any point \( s \) along the fiber is the result of the total moment of the force acting on the length \( (s, L] \) of the fiber. Therefore, since the moment of the force at the tip of the fiber at the point \( s = L \) is zero, the bending of the fiber at the tip of the fiber must be zero \( \dot{\theta}(L) = 0 \). The basis functions are chosen so that any linear combination will automatically satisfy these boundary conditions. This means the basis functions must satisfy:

\[ \phi_i(0) = \begin{cases} 
\theta_0 & i = 0 \\
0 & i \neq 0 
\end{cases} \]  

(2.16)

\[ \dot{\phi}_i(L) = 0 \]  

(2.17)

Substituting Equation 2.26 into Equation 2.13, changes the problem to a finite dimensional minimization. The infinite dimensional \( \theta(s) \) is reduced to a finite dimensional approximation over the parameter vector \( \alpha \). Because \( \theta(s) \) is obtained as a finite dimensional approximation, the moment balance at any point along the fiber is only approximately satisfied. The quality of the moment balance can be improved by increasing the number of basis functions used. However, this makes the computation slow and time consuming. Also, the non-uniform field has to be computed numerically. Increasing the number of basis functions makes the solution sensitive to discretization effects from the computation of the field. In practice, three to five basis functions were found to provide sufficient accuracy. The basis functions selected were
as follows

\[ \phi_1(s) = s^2 - 2Ls \quad (2.18) \]
\[ \phi_2(s) = s^3 - 3L^2s \quad (2.19) \]
\[ \vdots \quad (2.20) \]
\[ \phi_n(s) = s^{(n+1)} - (n + 1)L^n s \quad (2.21) \]

For convenience, \( s \) is normalized by \( L \)

\[ \hat{\phi}_1(\hat{s}) = \left( \frac{s}{L} \right)^2 - 2\left( \frac{s}{L} \right) \quad (2.22) \]
\[ \hat{\phi}_2(\hat{s}) = \left( \frac{s}{L} \right)^3 - 3\left( \frac{s}{L} \right) \quad (2.23) \]
\[ \vdots \quad (2.24) \]
\[ \hat{\phi}_n(\hat{s}) = \left( \frac{s}{L} \right)^{(n+1)} - (n + 1)\left( \frac{s}{L} \right) \quad (2.25) \]
\[ \hat{\phi}_n(\hat{s}) = \left( \frac{s}{L} \right)^{(n+1)} - (n + 1)\left( \frac{s}{L} \right) \quad (2.26) \]

Where \( \hat{s} = \frac{s}{L}, \hat{s} \in [0, 1] \). Furthermore, the basis functions were orthonormalized using the Gram-Schmidt process. The orthonormalized basis functions still satisfy the boundary conditions that \( \theta(0) = \theta_0 \) and \( \dot{\theta}(L) = 0 \).

## 2.2 Magnetic Field Approximations

The demagnetizing factors used in the magnetic field approximation are derived for ellipsoids in uniform fields. In particular, an infinitely long cylinder is considered to be an infinitely long ellipsoid. Since the fiber is curved and the field is nonuniform, it is important to determine how the magnetic field approximation will work for these conditions. The field approximation was verified using COMSOL
COMSOL was used to compute the magnetic field without the body present in it. The magnetic field inside the body was computed using demagnetizing factors. The literature suggests different methods to compute the demagnetizing factors from geometric ratios [20, 21]. For cylindrical objects, the geometric ratio is the ratio of the length of the cylinder to the diameter of the cylinder. The fiber is 28 mm long and 54 µm in diameter. Thus, the geometric ratio for the fiber comes out to be $n \approx 520$.  

Figure 2.2: Geometry used in COMSOL simulations
(a) Uniform Field

(b) Nonuniform Field
The various methods used to compute demagnetizing factors give approximately the same values of demagnetizing factors, $N_{∥} \approx 0$ and $N_{⊥} \approx 0.5$. Finally, COMSOL was used to compute the magnetic field inside the body using finite element methods. The magnetic field inside the body as predicted by COMSOL was compared with that predicted by the demagnetizing factors. The procedure was repeated for infinitely long, straight and curved cylinders in uniform and non-uniform magnetic fields and for various values of $\chi_f$.

![Figure 2.3](image.png)

Figure 2.3: Error between magnetic field values computed using the demagnetizing factors and COMSOL for a straight paramagnetic cylinder in a uniform field. There is 2.5% average error for $\chi = 2$. This is possibly because of numerical errors in the simulation.

For reference, the demagnetizing factors approximation was tested for a straight wire in a uniform field, as shown in Figure 2.3. The demagnetizing factors approx-
Figure 2.4: Error between magnetic field values computed using the demagnetizing factors and COMSOL for paramagnetic cylinders of varying curvature. The error in the demagnetizing factors approximation increases with increasing curvature.
Figure 2.5: Error between magnetic field values computed using the demagnetizing factors and COMSOL for varying $\chi_f$. The error in the demagnetizing factors approximation increases with increasing $\chi_f$ for a curved cylinder.
imation predicts the field values well for both the values of chi tested. The error observed in the data for $\chi_f = 2$ is possibly because of numerical errors in the simulation. The error between the magnetic field values computed using the demagnetizing factors and the COMSOL simulation for varying curvature is plotted in Figure 2.4. The error for varying $\chi_f$ is plotted in Figure 2.5. The error increases with increasing curvature. The error is also higher for higher values of $\chi_f$.

Bending of the fiber is proportional to the moment of the magnetic force. The total moment of the magnetic force is highest at the base of the fiber and decreases towards the tip. Hence the curvature of the fiber is higher at it’s base than at the tip. The base of the fiber is usually far from the magnet and experiences smaller fields. The part of the fiber with smaller curvature is affected by the strongest magnetic field. Thus the effect of curvature on the demagnetizing factors approximation can be ignored. Also, the magnetic susceptibility of the fiber is very low. For small magnetic susceptibilities, the error between the finite element methods and the demagnetizing factors approximation is within 0.5%. Thus the demagnetizing factors approximation can be used in the energy computation for the static model.

### 2.3 Rigid Bar Model

A simplified version of the continuum model is introduced here. The fiber is represented by a rigid bar with a torsional spring at it’s base. This is placed in the field of a magnetic dipole as shown in Figure 2.6. The torsional spring is assumed to be linear, with spring constant $k_s$. This means the strain energy can be expressed as:

$$U_s = \frac{1}{2} k_s (\theta^2)$$  \hspace{1cm} (2.27)
The dipole field is nonuniform, but the field at any given point \( x \) can be expressed as a function of its position.

\[
H(x) = \frac{1}{4\pi} \frac{3n(n \cdot m) - m}{|x|^3}
\]  

(2.28)

where \( n \) is the normal vector in the direction of \( x \). The shape of this field is as shown in Figure 2.7. Since the bar is straight, \( \theta \) is constant. \( m \) is the magnetic dipole moment, a constant vector. Therefore, \( H(x) \) at any point on the bar depends only on \( s \) and \( \theta \). Magnetic energy is given by Equation 2.10 and Equation 2.11. Therefore, we can write the energy of the bar as

\[
U = \frac{1}{2} k_s \theta^2 - \frac{1}{2} A \chi_f \mu_0 \int_0^L \left[ H^2 \right]_\parallel - \frac{1}{1 + \frac{1}{2} \chi_f} H^2 \right] ds
\]  

(2.29)

We can find \( \theta \) that minimizes this equation by computing the partial derivative with respect to \( \theta \) and equating it to zero. The energy function and its gradient with theta are presented in Figure 2.8 and Figure 2.9 respectively.

Figure 2.10 and Figure 2.11 clearly indicate the existence of one energy mini-
Figure 2.7: The field of a magnetic dipole. The field of a current loop is similar to the field of a dipole.

Figure 2.8: Energy of the Rigid Bar for varying $\theta$ for $m = 0.01T$. There is one minimum near $\theta = 0$. 
Figure 2.9: Gradient of Energy of the Rigid Bar for varying $\theta$ for $m = 0.01T$. The plot crosses zero at the minimum at $\theta = 0$.

Figure 2.10: Energy of the Rigid Bar for varying $\theta$ for $m = 0.08T$. There are two energy minima, one near $\theta = 0$ and one near the dipole. The minima are separated by a maximum.
Figure 2.11: Gradient of Energy of the Rigid Bar for varying $\theta$ for $m = 0.08T$. There are two energy minima, one near $\theta = 0$ and one near the dipole. The minima are separated by a maximum.

Figure 2.12: Energy of the Rigid Bar for varying $\theta$ for $m = 0.1305T$. There is only one minimum near the dipole.
Figure 2.13: Gradient of Energy of the Rigid Bar for varying $\theta$ for $m = 0.1305 T$. There is only one minimum near the dipole.

Figure 2.14: Gradient of Energy of the Rigid Bar for varying $\theta$ and constant $m_x$. There are two possible solutions for $m \in [0.025, 0.125]$. The minimum the bar travels to depends on the previous equilibrium position.
mum close to the dipole. However, there is one more minimum near $\theta = 0$. Thus in general, there are two energy minima, one close to the dipole and one close to $\theta = 0$. There is an energy maximum between the two minima. When the magnetic dipole field is strong, the minimum near $\theta = 0$ and the maximum disappear, leaving only the minimum near the dipole. Conversely, if the dipole field is weak, the minimum near the dipole vanishes. The bar only travels in the direction of decreasing energy. Thus as the field is increased, initially there is one energy minimum for the bar to travel to. As the dipole strength is increased, this minimum moves closer to the dipole, as seen from Figure 2.8 and Figure 2.9. Beyond a certain dipole moment, there are two minima separated by a maximum. The presence of the maximum means the bar moves to the minimum near $\theta = 0$. When this minimum disappears, the bar travels to the minimum near the dipole. Now if the dipole moment is decreased, the bar attains equilibrium positions near the dipole. This results in a hysteresis behavior. This can be seen from the plot of dipole field vs tip positions of the corresponding minima, Figure 2.14. For $m \in [0, 0.025]$, there is one minimum near $\theta = 0$. For $m > 0.125$ there is one minimum near the dipole. For the remaining values of $m$, there are two minima, one minimum near $\theta = 0$ and one minimum near the dipole. As the dipole moment increases from 0, the bar travels to the nearest minimum. At $m = 0.125$ the minimum near $\theta = 0$ vanishes, along with the energy maximum. Thus the bar travels to the energy minimum near the dipole. Now if the dipole moment is decreased, the bar stays at the minimum near the dipole. Down to a certain dipole moment, the maximum between the two minima prevents the bar from traveling to the minimum near $\theta = 0$. At $m = 0.025$, the minimum near the dipole and the maximum vanish and the bar travels to the minimum near $\theta = 0$. Thus for $m \in [0.025, 0.125]$, there are two possible static equilibria to which the bar can travel. The equilibrium the bar travels to depends on past values of dipole moments. The rigid bar exhibits the
highest displacements in this hysteresis region. The behavior of the rigid bar cannot be captured accurately by a static model alone when it is in the hysteresis region. Thus a dynamic model is required for precise control in the hysteresis region. This also poses an interesting nonlinear control problem.
Chapter 3

Characterization of Bending Rigidity of Fibers

The model depends on the bending rigidity of the fiber. Bending rigidity is the product of the Young’s modulus of the fiber and the second moment of area of the fiber. Thus bending rigidity is a property of the material of the fiber and the diameter of the fiber. Bending rigidity can be determined by studying the deformation of the fiber under a known load. Usually, this involves physically measuring the deflection of a beam [27, 28]. Most position sensors can only measure the deflection of the fiber at a limited number of points. Also, the bending has to be under controlled conditions that involve a complicated apparatus [27]. A camera on the other hand, can sense position of a large number of points. It is possible to construct a continuous and complex shaped curve from an image of the bending fiber [22, 23]. A method to compute the bending rigidity is presented here. This algorithm computes bending rigidity by considering the shape of the entire fiber before and after a known load is applied.

Section 3.1 describes a method to compute the shape of the fiber from an
image. Section 3.2 describes a method to compute the bending rigidity of the fiber from the computed shapes.

### 3.1 Shape Fitting

In order to characterize and model the fiber, we first need to be able to sense the shape of the fiber as angle as a continuous function of length along the axis of the fiber, as shown in Figure 3.1. A camera can sense the position at all the points on the fiber. This section presents a way to represent the shape of the fiber and sense the shape of the fiber from an image of the fiber captured by a camera.

The co-ordinate system used here is shown in Figure 3.1. The shape of the beam is represented by a parametrized curve. The parameter $s$ is defined as $s \in [0, L]$, where $s = 0$ at the base of the beam and $s = L$ at the tip of the beam. The shape of the beam is represented by the function $\theta(s)$. $\theta(s)$ is the angle between the axis of the beam at $s$ and the $X$-axis. $x$ and $y$ co-ordinates of the axis of the beam can be obtained from $\theta(s)$ as
As expressed by Equation 2.26, the shape \( \theta(s) \) is a linear combination of basis functions \( \phi_i(s) \). The basis functions expressed by Equation 2.26 are used here. These basis functions satisfy the boundary conditions presented in the Chapter 2, i.e., \( \theta(0) = \theta_0 \) and \( \theta(L) = 0 \). The shape is thus written as:

\[
\theta(s) = \phi_0 + \sum_i \alpha_i \phi_i(s) \tag{3.2}
\]

The shape of the beam is described in terms of the parameters \( \alpha_i \). The algorithm used to determine these parameters from an image of the beam is presented here.

A camera captures an image of a scene as intensities as a function of position. Since the camera sensor is actually a grid of discrete light intensity sensors, the image is discretized. Thus data on the image is a set of \([x_i, y_i]\) co-ordinates. When an image of the fiber is captured by a camera, the fiber is sampled as a discrete set of co-ordinates. The angle between any two neighboring points on the image can only have 8 possible values. Also, the position of each point can only be measured accurately up to \( \pm 1 \) pixel. Thus the parametrization information is not captured by the camera, i.e., the value of \( s \) at any point on the image of the fiber cannot be determined. Hence \( \theta(s) \) cannot be measured directly from the image. Also, even though \( \theta(s) \) is linear in parameters \( \alpha_i \), the transformation from \( \theta(s) \) to co-ordinates of the axis of the fiber \([x(s), y(s)]\) is not linear in parameters. Therefore a least squares fit can not be used to find the parameters.
Instead, the parameters are obtained from the image by iteratively comparing points on the image to points on a candidate shape. The image is a map of color values as a function of \([x, y]\) co-ordinates. The darker pixels are on the beam. Therefore, searching the image for dark pixels gives the co-ordinates of points on the beam. Length of the beam in pixels can be obtained by skeletonizing the image and counting the distance in pixels from base to tip. The co-ordinates of the points on the candidate shape can be obtained from the candidate parameters \(\alpha_i\), using the above equations. The co-ordinates of the candidate have to be scaled so that length of the candidate is the same as the length of the beam on the image. An error metric can then be calculated from the two sets of co-ordinates. This error metric is minimized as a function of the parameters to get the shape in the image. The error metric used has to be minimum when the candidate shape is the axis of the fiber on the image. Since the shapes are continuous, a metric on \(C[0,L]\) is the ideal candidate for the error metric. The metric between two shapes \(a, b \in C[0, L]\), written as \(d(a, b)\) is zero if and only if \(a(t) = b(t)\) for all \(t \in [0, L]\). The metric can be computed as

\[
d(a, b) = \int_0^L \|a(x) - b(x)\|^2dx
\]  

(3.3)

If the \(a(s)\) and \(b(s)\) are close and the deflection is small, \(t \approx s\). Thus Equation 3.3 can be rewritten as

\[
d(a, b) = \int_0^L \|a(s) - b(s)\|^2ds
\]  

(3.4)

To find the area, we need to know the parameter \(s\) at all points on the image.
Figure 3.2: The metric $d(a,b)$ is zero if and only if the two curves are equal at all points.

However, since the orientation of the beam in the image is not known, the value of $s$ at any point on the image is not known. Hence it is not possible to compute this area directly.

The error metric used here is the sum of the squared distances between each point on the image and the nearest point on the candidate. The nearest point on the candidate is a good guess for the point that has the same value of $s$ if the shapes are close. Thus this metric is nearly equal to the metric described by Equation 3.4. The error metric can be represented numerically as

$$Error(\alpha) = \sum_{i \in \text{ImagePoints}} \min_{j \in [0,1]} \{(x_i - x_j)^2 + (y_i - y_j)^2\}$$ (3.5)

where $\alpha$ is the vector of parameters of the candidate shape. This error metric approximates the area between the candidate shape and the true shape on the image. If the candidate gets closer to the shape in the image, the error metric decreases. The error metric is minimum when the candidate shape is the same as the axis of the fiber on the image. The convergence of the error metric is not guaranteed for all shapes. However, the error does converge if the initial candidate shape is close to the shape of the fiber on the image. The algorithm fit the shape well for all the images it was
3.2 Bending Rigidity

The total potential energy ($V$) in a cantilever beam under pure bending is the sum of strain energy ($U_s$) in the beam and work done by the deforming force ($U_w$).

$$V = U_s + U_w \quad (3.6)$$

When a deforming force is applied to the beam, the beam comes to rest in the configuration with the minimum potential energy. In this configuration, gradient of the potential energy with the parameters is zero. This means the gradient of strain energy is equal to the gradient of work done.

Strain energy at any point in a bending cantilever beam [31] is a function of the moment of bending at that point and the bending rigidity of the beam $EI$. $E$ is the Young’s modulus of the material of the beam and $I$ is the second moment of area of the beam. Let $\theta(s)$ be the shape of the beam with the load after the load was applied and $\theta_0(s)$ be the shape of the beam before the load was applied. Let $\alpha$ be the parameters that represent $\theta(s)$ and let $\alpha_0$ be the parameters that represent $\theta_0(s)$. If the fiber is not straight, by computing the strain energy of the difference between the initial shape and the shape after the load is applied, $\theta - \theta_0$, the strain energy of the initial shape of the fiber can be compensated for. For a beam with low curvature, the bending moment $M$ is directly proportional to the curvature [31].

\[ M = \frac{d^2 \theta}{ds^2} \]

\[ EI \frac{d^2 \theta}{ds^2} = M \]

\[ \frac{d^2 \theta}{ds^2} = \frac{M}{EI} \]

\[ \frac{d \theta}{ds} = \frac{M}{EI} \cdot s + \text{constant} \]

\[ \theta = \frac{M}{EI} \cdot \frac{s^2}{2} + \text{constant} \]
\[ M = EI \frac{d}{ds}(\theta - \theta_0) \]  

(3.7)

Total strain energy of the beam is the integral of the strain energy at one point over the length of the beam.

\[ U_s = \frac{1}{2} \frac{EI}{L} \int_0^L [M(s)]^2 ds = \frac{EI}{2} \int_0^L \left( \frac{d}{ds}(\theta(s) - \theta_0(s)) \right)^2 ds \]  

(3.8)

The gradient of strain energy is a function of the bending rigidity of the beam and the shape of the beam. The parameter vector \( \alpha \) describing the shape of the beam are determined using the shape sensing algorithm from Section 3.1. On substituting \( \theta(s) \) from (Equation 2.26) and solving, the strain energy term simplifies to a quadratic in \( \alpha \),

\[ U_s = \frac{EI}{2} \alpha \mathbf{S} \alpha^T, \]  

(3.9)

where \( \alpha = \begin{bmatrix} 0 & \alpha_1 & \alpha_2 & \cdots \end{bmatrix} \) and \( \mathbf{S} \) is a matrix with elements

\[ S_{ij} = \int_0^L \left( \frac{d\phi_i}{ds} \right) \left( \frac{d\phi_j}{ds} \right) ds \]  

(3.10)

Where each \( \phi_i(s) \) is a basis function. The gradient of \( U_s \) with respect to the parameter vector \( \alpha \) is
\[ \frac{\partial U_s}{\partial \alpha} = E I S \alpha \] (3.11)

The work done by the deforming force is a function of the vertical displacement and the force applied. The vertical displacement of the tip is the difference between the \( y \) co-ordinate of the tip before the load was applied and after the load was applied.

\[ U_w = mg \int_0^L (\sin \theta(s) - \sin \theta_0(s))ds \] (3.12)

Using Leibniz’s rule and the chain rule of differentiation, the gradient of \( U_w \) with respect to \( \alpha \) is,

\[ \nabla U_w = mg \int_0^L (\cos \theta(s)) \begin{bmatrix} 1 \\ \phi_1(s) \\ \phi_2(s) \\ \vdots \end{bmatrix} ds \] (3.13)

Equating the gradients yields

\[ E I S \alpha = -\nabla U_w \] (3.14)

(3.15)
where $S_i$ is the $i^{th}$ row of $S$ and $\nabla U_{w_i}$ is the partial derivative of $\nabla U_w$ with respect to $\alpha_i$. This is a least squares problem with one parameter. $EI$ is the parameter to be determined. $S\alpha$ is a vector of co-efficients of $EI$. $-\nabla U_w$ is the vector of experimental data. Solving the equation gives estimates for $EI$,

$$EI = \frac{1}{n} \sum_{i=1}^{n} \frac{-\nabla U_{w_i}}{S_i\alpha}$$

(3.17)

$\theta$ and $\alpha$ can be obtained using the shape fitting algorithm described in Section 3.1. $S_i$ can be computed using Equation 3.10. The mass at the tip of the fiber, $m$ is known. Length of the fiber $L$ is also known. $\nabla U_{w_i}$ can be computed using $\theta$, $L$ and $m$ from Equation 3.13. Thus $EI$ is the only unknown in Equation 3.17. The denominator of Equation 3.17 is non-zero as long as the deflection of the fiber is not zero. The number of estimates of bending rigidity obtained from Equation 3.17 is equal to the length of the parameter vector $\alpha$. Thus we can find the bending rigidity of the fiber from this equation by averaging all the estimates.

Since the fiber is cylindrical, if the radius of the fiber $r$ is known, the second moment of area $I$ can be computed as,
Thus we can compute the Young's modulus of the fiber $E$ from the bending rigidity $EI$.

The solutions obtained by solving Equation 3.17 were compared with those obtained from linear beam theory. If we assume that the deflection of the fiber is very small, we can reduce Equation 3.15 to a relation between the tip force and the tip deflection [31].

$$y(L) = \frac{FL^3}{3EI}$$ (3.19)

Since the deflection is assumed to be very small, this method cannot be used to measure the bending rigidity if the deflection is large. Linear beam theory also ignores the shape of the fiber. On the other hand, the deflection of the tip is easier to measure and the computation itself is very simple.

3.2.1 Experiment and Results

The algorithm was first tested for accuracy and precision on a material whose exact bending rigidity is known, since the bending rigidity of newly fabricated fibers is unknown. The Young’s modulus of Tungsten and Brass wires is specified to within a tolerance by the manufacturer. Also, the diameter of the wires is accurately known. Thus the Young’s modulus can be calculated from the measured bending rigidity using Equation 3.18. The value of the Young’s modulus measured by the bending
rigidity algorithm can be compared with the empirical value of Young’s modulus of the wires.

Tungsten and Brass wires of different diameters were characterized using the shape fitting algorithm. The bending rigidity of the wires varies with their diameter. The bending of the wire is also a function of its length and the force at the tip. Bending of wires of different lengths under different tip forces was examined.

It was assumed that the wires are firmly cantilevered with no slip at the base. If the wire slips when the load is applied, some of the displacement recorded is not caused by the tip force, introducing an error in the measurements. In order to ensure no slipping, a pin clamp was used to cantilever the Tungsten Wires.

A CMOS USB camera (pixeLINK PL-B742U) with a macro lens (Navitar NMV-M35) was used to capture the images. The base of the wire was just outside the field of view so that only the wire and the load was visible. This made image processing easier. A fiber optic illuminator (Fiber-Lite MI-150) was used to provide a smoothly lit background.

The Matlab Image Acquisition Toolbox was used to acquire the images. The Matlab Image Processing Toolbox was used to threshold the image. The images were stored as vectors, with rows as the vertical axis and columns as the horizontal axis. The image was rotated so that the X-axis of a straight beam would be parallel to the vertical axis. The co-ordinates of each black pixel on the image was added to a vector of \((x, y)\) co-ordinates. These co-ordinates were then translated so that the base of the wire would be at \((0, 0)\).

The candidate shape of the axis is generated using the basis function. The set of \((x, y)\) co-ordinates for the axis are obtained from Equation 3.1. Because the images of the fiber are captured with the origin at the edge of the image, the origin has the y co-ordinate \(y_0 = 0\). The x co-ordinate of the origin can be determined by
Figure 3.3: Shape Fitting Example. A 127 µm Tungsten wire is clamped at the base and loaded using a known weight. Its shape before and after loading is sensed using the algorithm described in Section 3.1.

searching for the intersection of the center line of the wire and the edge of the image. The origin of the candidate is at (0, 0), the same as the base of the set of points on the image. The \((x, y)\) co-ordinates of the candidate are scaled so that the length of the candidate would be equal to the length of the image of the wire in pixels. To avoid errors in the fit due to discretization, the candidate shape of the axis is sampled at ten times the length of the wire in pixels. The error metric is the sum of squared distance between each point in the vector of black pixels and the nearest point on the
The error function and the initial candidate for the parameters were passed to the Matlab’s minimization function `fminsearch`. The generated shape was plotted on the binary image to compare the fit. In all the images, the shape fit by the minimization algorithm was along the axis of the image of the wire. The bending rigidity and Young’s modulus of the wire is then calculated as described in Section 3.2.

![Experimental Results](image)

**Figure 3.4:** Experimental Results. The method outlined in Section 3.2 gives a more precise and accurate estimate of the Young’s modulus in comparison to linear beam theory.

<table>
<thead>
<tr>
<th>Material</th>
<th>Diameter (µm)</th>
<th>E (GPa)</th>
<th>Standard Deviation (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tungsten</td>
<td>178</td>
<td>389.592</td>
<td>5.566</td>
</tr>
<tr>
<td>Tungsten</td>
<td>228.6</td>
<td>403.336</td>
<td>20.347</td>
</tr>
<tr>
<td>Tungsten</td>
<td>254</td>
<td>398.688</td>
<td>18.486</td>
</tr>
<tr>
<td>Brass</td>
<td>508.3</td>
<td>100.362</td>
<td>5.560</td>
</tr>
<tr>
<td>Brass</td>
<td>305</td>
<td>90.897</td>
<td>2.419</td>
</tr>
</tbody>
</table>

**Table 3.1:** Average $E$ and standard deviation
Wires of 178µm, 228.6µm and 254µm diameter were characterized using the algorithm. The standard deviation of the Young’s modulus estimates for the 178µm wire was 5GPa. The standard deviation of the Young’s modulus estimates for 228.6µm and 254µm wire were 20GPa and 18GPa respectively. This is because the bending rigidity of the wire is inversely proportional to the diameter to the forth power as seen from Equation 3.18. This means the larger diameter wire exhibits less deflection than a small diameter wire for the same load. In some cases for the large diameter wires, the images of the loaded and unloaded wire are almost the same. Hence the bending of the large diameter wire is harder to sense. This causes the measurements to be less precise.

Repeatability of the algorithm was tested by taking multiple measurements for the 178µm Tungsten wire and the 305µm Brass wire using a fixed load. The measurements for Tungsten wire averaged 387.14GPa with a standard deviation of 2.0554GPa or 0.53092%. The measurements for Brass wire averaged 87.309GPa with a standard deviation of 1.378GPa or 1.578%.

The experimental data shows that the method outlined here can be used to characterize the bending rigidity of cantilevered beams. The algorithm is accurate to within 5% for small deflections and 2% for large deflections. The algorithm performs better than linear beam theory for large and small deflections. Calculating bending rigidity from linear beam theory requires accurate measurements of the tip position. The calculation from nonlinear beam theory, on the other hand, can be used with a low resolution camera.

The algorithm also compensates for initial curvature of the wires. This is a major advantage as fibers or wires may not always be straight to begin with. The only sensors required for measurement are a camera and calibrated weights. These are commonly available in any laboratory. This makes the characterization of cantilevered
beams very fast and easy.

(a) Straight Wire, measured $E=386\text{GPa}$
(b) Bent wire, measured $E=390.52\text{GPa}$

Figure 3.5: The algorithm estimates the Young’s modulus of straight and bent wires equally well.

From the experiments, it is clear that the algorithm works better for large deflection. The performance for small deflections can be improved by using a camera with higher resolution. If these operating conditions are met, the algorithm presented here can be used to characterize bending rigidity of unknown materials.
Chapter 4

Experimental Results

In order to test the validity of the model, the behavior of a fiber in the field of an electromagnet was simulated using the model. The experimental setup and procedures are described in the second part of the chapter. The tip displacements predicted by the model were compared with tip displacements exhibited by the fiber.

The paramagnetic fiber was cantilevered vertically. An illuminator (Fiber-Lite MI 150) was used to provide a smooth, well lit background for the fiber. A video microscope (PixeLINK PLB-742U) was used to measure the displacement of the tip of the fiber. An electromagnet (Magnetic Sensor Systems E-09-150) was used to generate the magnetic field. The electromagnet coil was excited by current from a linear amplifier (AE Techron 5530). Matlab xPC target with a Quanser Q8 board was used to control the output voltage of the amplifier. This arrangement enables the delivery of sufficiently high power to the magnet coils, while allowing precise control over the coil current. Also, the use of the xPC target system allows automated measurements and implementation of real-time control strategies. The inductance of the coil is of the order of 1mH. The coil resistance is 6.7Ω. Thus the time constant of the coil is a few milliseconds. This means the coil current responds quickly to
changes in the coil voltage. Also, since the coil resistance does not vary significantly over the operating range of temperatures, a linear relationship based on Ohms law holds between coil voltage and coil current.

The arrangement of the magnet and the fiber is as shown in Figure 4.1. The tip positions were measured using the camera. The camera provides measurements accurate up to 1.675µm. The tip position data was gathered for various values of magnetic fields for various positions of the magnet. The process was automated through the use of Simulink models and image processing techniques, which will be described in the next section.
Figure 4.2: Tip Tracking Algorithm. A box filter is used to compute the average intensity at each point on the fiber. The tip of the fiber has the lowest non-zero average intensity of any point on the fiber.

### 4.1 Tip Position Sensing

The images returned by the camera were binarized using background subtraction. Computer Vision literature presents various methods to compute the position of the tip. Since the fiber surface is smooth, the simplest and fastest method that can be used is a linear averaging filter. The tip of the fiber on the binary image has a lower average intensity than other pixels on the fiber as there are least number of “ON” pixels in the neighborhood of the tip of the fiber. The only constraint is that the size of the averaging window has to be larger than the width of the fiber.

The camera resolution used is $6.7\mu m$. Additional error is introduced in the tip of the fiber is flat instead of rounded. This error depends on the width of the fiber. Since the fiber is $50\mu m$ wide, the measurements are accurate to within $50\mu m$. However, since the camera captures a very small portion of the fiber, the orientation of the fiber in the image does not change significantly. Thus the error in the tip measurement is constant. If the first tip position is used as a reference to calculate the displacement of the tip, the error in the measured tip position gets subtracted from the measurement. Hence the measured displacement is accurate to within $6.7\mu m$. 


Since the images of the tip are captured under controlled conditions, as long as the fiber is at the same position, the imaged captured are identical. Given two identical images of the tip, the tip tracking algorithm identifies the same pixel on each image as the tip.

### 4.2 Static Model Simulation

The magnetic field of the electromagnet was simulated using Finite Element Magnetic Methods [10]. The magnetic field is shown in Figure Figure 4.3. FEMM provides 2D magnetic fields. However, fields generated by cylindrical magnets can be generated by axissymmetric simulation providing true 3D results. The magnetic field scales linearly with current. Therefore, if the field of the magnet for a 1A coil current is known, it can be scaled to get the field for any other coil current. The field generated by FEMM was stored as arrays in Matlab. This provides a significant increase in processing speed since calls to FEMM are slow.

The energy of the fiber is given by Equation 2.13 and Equation 2.14. The Matlab Optimization Toolbox was used to find the parameters that give the minimum energy configurations for various values of coil current. The distance between the magnet and the base of the fiber was also varied. The tip displacement of the fiber in a magnetic field was compared with the tip displacement of the shapes predicted by the model. The tip displacements are shown in Figure Figure 4.4.

Since the fiber is hollow, the second moment of area $I$ cannot be measured accurately. Also, the fiber is coated on the inside with paramagnetic particles. Hence the cross sectional area of the fiber $A$ cannot be measured either. Instead, the parameters were fit as the ratios $\frac{0.5x_A\chi_f\mu_0}{EI}$ and $\frac{0.5x_A\chi_f\mu_0}{EI(1+\chi_f)}$. The parameters were fit iteratively by comparing the experimental data with the simulation. Increasing the
Figure 4.3: Electromagnetic Field generated using FEMM

Figure 4.4: Tip positions predicted by the model and experimental observations. The error in the predictions is because the initial shape of the fiber could not be measured accurately. Since the fiber was bent slightly towards the magnet, the experimentally observed tip deflections are higher than those predicted by the model.
parameter values result in increasing tip displacements and vice-versa.

There is some error in the tip displacement as predicted by the model. These errors are because the initial shape of the fiber was not included in the simulation. Since the fiber is initially curved towards the magnet, the strain energy developed in the fiber is smaller for the same curvatures. Hence the fiber has to travel farther to develop the strain energy necessary. This limitation can be removed by including the initial shape of the fiber in the simulation. The strain energy of the fiber can be computed using the initial shape of the fiber by using the difference between initial shape and current shape parameters in the strain energy computation. This way, the strain energy of the fiber when it is in its initial configuration is zero. Thus the simulated fiber behaves like the real fiber.

A hysteresis behavior similar to that observed in the rigid bar in Chapter 2 is predicted by the model for the fiber. However, this hysteresis behavior could not be studied in detail because the magnet becomes hot and there is a possibility of the fiber melting.

The model also predicts hysteresis in the fiber. The range of magnetic fields for which two static equilibria are predicted is very small. This is because the magnetic field of an electromagnet vanishes slowly near the magnet. Therefore, the forces generated at the face of the magnet are not enough to overcome the strain energy needed to keep the fiber attached to the magnet. This means the minimum near the magnet vanishes quickly.

The field farther away from the magnet vanishes quickly. Hence the forces experienced by the fiber at its initial configuration are strong. Thus the first minimum is close to the magnet. The fiber exhibits tip displacements between $0 - 3.5 \text{mm}$ when it is at the first minimum. The simulation and the experiments clearly show the first minima vanishing in each of the configurations used.
The static equilibria can be easily predicted using the model. Also, the forces acting on the fiber in any configuration are a function only of the position of the fiber. Thus the potential energy function used to compute static equilibria can also be used to compute forces on the fiber. Thus the fiber can be used as an actuator using the static equilibria alone. Position control of the fiber is very easy for this configuration, since the static equilibria depend only on the present configuration of the fiber and the change in current.
Chapter 5

Conclusions and Future Work

This goal of this thesis was to present an energy methods based approach to characterizing paramagnetic fibers and simulate their behavior in a magnetic field and to develop image processing techniques that are required to use the fiber as an actuator. These goals were achieved as shown in the preceding Chapters. This Chapter summarizes the results and presents ideas to build on them.

The assumptions under which the demagnetizing factor approximation for an infinitely long cylinder holds were examined in Section 2.2. The demagnetizing factors approximation holds for curved magnetic cylinders with low curvature and small values of $\chi_f$. Since the fiber meets these criteria, the demagnetizing factors approximation predicts the magnetic field inside the fiber accurately.

A simplified version of the static model was used to predict the behavior of a rigid, paramagnetic bar in the field of a magnetic dipole in Section 2.3. The rigid bar energy plots provide a better understanding of the behavior of the fiber in a magnetic field.

A method to sense the shape of a curved fiber was presented in Section 3.1. The shape sensing method and nonlinear beam theory were used to determine the
bending rigidity of Tungsten and Brass wires in Section 3.2. The measurements are accurate to within $2 - 5\%$. These methods can be used for the characterization of bending rigidity of fibers and yarns. A tool to compute the bending rigidity can be developed from the Matlab implementations.

Finally, the static model of the paramagnetic fiber presented by [1] was verified experimentally. The model can predict the stable equilibria of the paramagnetic fiber in a magnetic field. Since the magnetic and strain forces acting on the fiber depend only on the configuration of the fiber, the gradient of the potential energy of the fiber can be used to compute the forces acting on the fiber. Thus the actuation force can be computed from the static model. Also, the fiber can be used as an actuator using only the static equilibria. Moving the fiber to a static equilibrium can be done by simple feed forward control of the coil current.

5.1 Future Work

The work presented in this thesis enables the development of a basic tip position control strategy. This section describes possible extensions and improvements of the work presented in Chapter 2, Chapter 3 and Chapter 4. The possible improvement areas are the modeling and control algorithms, the shape sensing algorithms and better materials for the fiber.

The interactions between the fluid drops and the fiber is complex. It needs to be studied in detail before a control strategy can be developed to use the fiber for fluid transport applications.

The static model is presently limited to 1D motion. It needs to be extended to 2D motion so that the fiber can be used for pick and place operations. This would involve describing the shape as two angles instead of one, i.e. the angle made by the
fiber with the $XZ$ plane and the angle made by the fiber with the $XY$ plane. Also, an arrangement of magnets that will give 2D motion needs to be determined since one magnet can only cause tip deflection in one direction.

The static model of the fiber is enough to develop a basic feedforward control strategy for the fiber. However, the static model only gives static equilibria for less than half of the total range of displacements that can be achieved. There is a large range of tip displacements for which there are no static equilibria. Since larger displacements generate higher strain forces in the fiber, finer control in the large displacement region would be useful in droplet transport applications. In order to develop a better control strategy for the fiber, it is important to develop the dynamics of the fiber. In addition to the position dependent magnetic and strain energy terms, the dynamics would also include the inertia of the fiber. This is a particularly difficult problem as the inertia term depends on the parameter $s$ and time.

In comparison, the inertia of the rigid bar is easier to compute than the inertia of the fiber, since the rigid bar model is a one dimensional system. It behaves in a similar manner to that of the fiber in a magnetic field. The dynamics of the rigid bar would give better insight into the dynamics of the fiber. Also, it is a system for which the development of a nonlinear controller should be relatively easy. Thus a magnetic actuator can be developed using the rigid bar.

On the sensing side, the shape sensing method described in Chapter 3 is slow. In order to use the fiber as an actuator, it is necessary to sense the shape of the fiber in real time. This means using faster shape fitting methods. Alternatively, a better position sensor that has higher resolution can be used to compute angles directly and then fit the shape using least squares approximation.

Higher strain forces can also be generated using stiffer materials like metals. A fiber/wire made from a stiffer material would need stronger magnetic forces to
produce sufficient displacements. Superparamagnetic materials have similar retention characteristics as paramagnetic materials and higher magnetic susceptibilities. This means they produce stronger magnetic forces, but they saturate at very low magnetic fields. To use such materials in the actuator, it is necessary to adapt the magnetic field approximation to include the saturation behavior.

An alternative to using stiffer materials to provide higher tip forces, it is possible to use multiple magnets to generate higher forces. For this reason, it is necessary to analyze the behavior of the fiber in different magnetic fields.

Another possible way of generating higher strain forces is differential physical adsorption as explored by [33, 34]. When one side of a cantilever beam adsorbs some material, the change in surface energy causes the beam to bend. This effect can be used to provide additional strain when the strain force generated in the fiber is insufficient for actuation.
Appendices
Appendix A  Matlab Functions user’s guide

A guideline to use the code used in various experiments this thesis is presented here. This guideline and the code should be sufficient to repeat the experiments. Subsection A.1 outlines the functionality and inputs and output of the shape fitting function described in Section 3.1. Subsection A.2 is a guideline to use the bending rigidity algorithm described in Section 3.2. Subsection A.3 describes the main function used in Section 4.2 to simulate the model.

A.1 Shape Fitting

1. Convert the image to binary so that the fiber/wire appears white (pixels are on) and the background appears black (pixels are off).

2. The binary image should have the base of the fiber at the top edge of the image.

3. The fiber should be positioned vertically.

Function  \([\alpha, \beta, x_{base}, y_{base}, s_{max}] = \]
ShapeFinder(binaryimage, alpha, beta, smaxflag, smax, offset, betaflag, curveflag):

Outputs

- **alpha** - Vector of parameters that describe the shape of the fiber.
- **beta0** - Angle at the base of the fiber.
- **x_{base}** - x co-ordinate of the base of the fiber. Should be 0. If it’s not zero, make sure the binary image follows the guidelines above.
- **y_{base}** - y co-ordinate of the base of the fiber. Should be at the center of the base of the fiber.
• **smax** - Length of the fiber in pixels. Convert actual length (L) of the fiber as
  
  \[
  \text{smax} = \frac{L}{\text{Width of a pixel}}.
  \]

**Inputs**

- **binaryimage** - Input image in binary form.
- **alpha** - Initial guess. Use a vector of zeros for simplicity. The algorithm determines the number of basis functions to use from the length of alpha.
- **beta** - Initial guess for the angle at the origin.
- **smaxflag** - If the length of the fiber in number of pixels is accurately known, set this to 0.
- **smax** - Enter value if smaxflag=0.
- **offset** - If the fiber is very wide, the algorithm guesses a slightly higher length of the fiber. Enter a constant value to subtract from this guess.
- **betaflag** - Set this to 0 if beta is accurately known.

### A.2 Characterization of Young’s Modulus

Use ShapeFinder to get shapes for the loaded and unloaded fiber/wire.

\[
[EI,EI_{\text{lin}}] = \text{GetEI2}(\alpha_0, \alpha, m, L)
\]

**Outputs**

- **EI** - Bending Rigidity.
- **EI_{\text{lin}}** - Bending Rigidity as computed from linear beam theory.

**Inputs**

- **alpha0** - Shape parameters for the unloaded fiber.
• \textbf{alpha} - Shape parameters for the loaded fiber.

• \textbf{m} - Mass loaded on the end of the fiber in kg.

• \textbf{L} - Length of the fiber in meters.

If desired, enter the values of \textit{m} and \textit{L} in CGS units to get bending rigidity in \textit{dynecm}^2. The shape parameters are dimensionless.

\section*{A.3 FiberStaticModel}

This function has nested calls to FEMM and to three other matlab functions. FEMM is used to generate magnetic fields, which are then imported into Matlab before the simulation. The field values are stored in the global variables Fieldx and Fieldy as 2 square matrices, with xrange as the range of the first dimension and yrange as the range of the second dimension. \textit{x0field} and \textit{y0field} are the \textit{x} and \textit{y} co-ordinates of the center of the magnet in the co-ordinate system described in 3.

If a magnetic field is computed using other means and stored in Matlab variables, ensure that the \textit{x} and \textit{y} field variables are in the appropriate format and are labelled Fieldx and Fieldy. Enter all other parameters for the field.

If FEMM is used to generate the field, ensure that xrange and yrange are within the outer boundary of the FEMM model. Change the path variable to the full path and file name of the \textit{.ans} file for the field.

Leave lines 12-15 as they are. A 3d color surface of the imported field is here
generated for convinience.

Leave the optimization settings as they are.

Inputs

- **x0field** - x co-ordinate of the center of the magnet.
- **y0field** - y co-ordinate of the center of the magnet.
- **xrange** - Range of x co-ordinates over which the field is imported.
- **yrange** - Range of y co-ordinates over which the field is imported.
- **path** - Path to the .ans file for the FEMM field.
- **L** - Length of the Fiber.
- **npoints** - The shape generating function uses ode45 to get x and y co-ordinates from $\theta(s)$. This parameter controls the number of discrete points ode45 uses. Reducing this parameter speeds up the minimization a lot, but it also sacrifices accuracy in the estimated shape.
- **I** - The range of coil currents. Set this to 1 if a permanent magnet is used.
- **x0** - x co-ordinate of the center of the magnet. This parameter offsets the field co-ordinates in case different geometries are used.
- **y0** - y co-ordinate of the center of the magnet. This parameter offsets the field co-ordinates in case different geometries are used.
- **alpha0** - Configuration at which the fiber experiences no strain, i.e., configuration at rest.
• **alpha** - Initial seed point. Use alpha0 if a good estimate isn’t available.

• **a** - The Parameter $A\chi_f/EI$.

• **b** - The Parameter $A\chi_f/EI(1 + \chi_f)$.

Nested calls are made to the following functions:

**FiberEnergy** - Compute energy of the fiber.

**Guess2** - Get $x(s)$, $y(s)$ and $\theta(s)$ from the parameter vector $\alpha$.

**IntegralDphiMatrix2** - Compute the strain energy matrix for the given length of the fiber.

These functions must be included in the Matlab path.
Bibliography


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