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An Experimental and Computational Study of Windborne Debris in Severe Storms

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An Experimental and Computational Study of Windborne Debris in Severe Storms

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Civil Engineering

by
Arash Karimpour
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Accepted by:
Dr. Nigel B. Kaye, Committee Chair
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Abstract

This research focuses on an experimental and theoretical investigation of windborne debris emanating from loose gravel on built-up roofs. During severe storms, windborne debris can cause considerable physical harm and property damage. One of the major sources of flying debris in large commercial areas is loose gravel on built-up roofs. Such loose gravel can be responsible for significant property damage. Despite the high risk of windborne debris, their flight mechanics are poorly understood. To better understand windborne debris flight, a series of experiments were conducted in the Clemson University Boundary Layer Wind Tunnel. These experiments were designed to quantify the conditions under which gravel became airborne, the rate at which it was removed, and the resulting flight distance of the debris.

In order to conduct experiments in the Boundary Layer Wind Tunnel it is important to understand how to model the atmospheric boundary layer (ABL). A new curve fitting method is presented for calculating the ABL logarithmic velocity profile parameters i.e. shear velocity, surface roughness and zero plane displacement. The new method uses only the time averaged velocity profile and requires no iteration. Comparison with existing methods shows that the new approach has equal or better accuracy than existing curve fitting and geometric approaches with fewer calculation steps.

Debris flight is a highly stochastic process with uncertainty and variability in the debris particle the turbulent wind field. However, current models are almost entirely deterministic. A series of Monte Carlo simulations based on existing debris flight equations were run to quantify the impact of input uncertainty on flight outcome (flight distance and impact kinetic energy). Results indicate that failure to account for parameter variability will result in under predicting
the mean flight distance and kinetic energy, and ignoring outcome variability / uncertainty. A full quantification of the relationship between input variability and outcome variability is presented for roof gravel blow-off.

A series of new experimental methods have been developed to measure the conditions under which blow-off occurs, the rate of gravel removal, and the downwind flight distance for two-dimensional buildings. The critical condition for blow off is parameterized in terms of the particle densimetric Froude number, particle Reynolds number and building geometry. A series of non-dimensional plots of the critical Froude number versus Reynolds number for different parapet heights are presented. The results indicate that the current approach for scaling result from laboratory to full scale is flawed and that full scale experiments are required to fully understand this process.

The rate of removal, or mass flux, varies over time. The removal process exhibits an initial high mass flux regime followed by a period of reduced blow-off rate. Dimensionless plots of both regimes mass flux versus Particle Froude number for different parapet heights are presented. The results show that increasing the parapet height usually decreases the mass loss rate, though this is not the case for very small parapets. Further, the transition time from the initial to secondary blow-off regimes is independent of the building geometry. Finally, the initial mass flux is approximately four times that of the secondary loss rate, and that this ratio is independent of both the building geometry and the Froude number.

Experimental results indicate that the wake behind the building dominates the downwind transport of debris. The flight distance is a function of the building height, particle Froude number (written in terms of a Tachikawa number), and the parapet geometry. A full characterization of the down-wind debris field requires a detailed analysis of the wake behind
the building that is beyond the experimental capability of the current facility. Further, scaling of the results to full scale is again problematic, and therefore full scale testing is recommended.
DEDICATION

To my mother, Khadijeh

my brothers, Farshid and Farid

and in memory of my father, Falamarz
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I am grateful to Dr. Nigel B. Kaye, my major advisor and committee Chair, for his vision, wit, wisdom and patience. I thank the members of my committee, Dr. Nadim M. Aziz, Dr. Abdul A. Khan, and Dr. Wei C. Pang for support and guidance during my study.

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One person uniquely has been supported me throughout my ambition. My mother, Khadijeh who has never stopped her support, has been always encouraging from the very beginning. The journey of my study from my home, Isfahan, to Clemson was a result of the initiative taken by my late father, Falamarz, my mother, Khadijeh, and my brothers Farid and Farshid.
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$A_1$  dimensionless blow off rates in initial stage
$A_2$  dimensionless blow off rates in second stage
$a_1$  blow off rates in initial stage
$a_2$  blow off rates in second stage
$A_f$  total frontal area of the obstacles
$A_p$  particle cross sectional area
$C$  correction factor
$C_D$  drag coefficient
$C_F$  force coefficient
$C_L$  lift coefficient
$C_M$  moment coefficient
$C_p$  pressure coefficient
$CV$  coefficient of variation
$D$  displaced vertically a distance
$d$  zero plane displacement
$d_p$  particle diameter
$\bar{d}_p$  mean particle diameter
$\hat{d}$  zero plane displacement scaled by direct force balance measurements
$d_{90}$  particle size that 90 percent of particles are finer than
\( d_* \) \hspace{1cm} \text{dimensionless particle diameter}

\( F_B \) \hspace{1cm} \text{buoyancy force}

\( F_D \) \hspace{1cm} \text{drag force}

\( F_L \) \hspace{1cm} \text{lift force}

\( F_R \) \hspace{1cm} \text{resistance force}

\( F_W \) \hspace{1cm} \text{weight}

\( Fr_d \) \hspace{1cm} \text{densimetric particle Froude number}

\( g \) \hspace{1cm} \text{gravity acceleration}

\( g' \) \hspace{1cm} \text{reduced gravity}

\( H \) \hspace{1cm} \text{building height (particle released height from rest)}

\( h \) \hspace{1cm} \text{parapet height}

\( h^* \) \hspace{1cm} \text{mean obstacles height}

\( I \) \hspace{1cm} \text{mass moment of inertia}

\( I_x \) \hspace{1cm} \text{horizontal wind turbulent intensity}

\( I_z \) \hspace{1cm} \text{vertical wind turbulent intensity}

\( Je \) \hspace{1cm} \text{Jensen number}

\( K \) \hspace{1cm} \text{Tachikawa parameter}

\( KE \) \hspace{1cm} \text{dimensionless mean impact kinetic energy}

\( ke \) \hspace{1cm} \text{impact kinetic energy}

\( \overline{ke} \) \hspace{1cm} \text{mean impact kinetic energy}
\( k_s \)  equivalent bed particles roughness

\( L \)  building length

\( L_r \)  separation bubble length

\( l_p \)  particle length

\( M \)  dimensionless mass

\( m \)  mass blow off from roof

\( m_p \)  particle mass

\( m_R \)  recovered mass

\( m_{ref} \)  reference mass

\( P(x/H) \)  probability that a particle flies as far as \( x/H \)

\( p \)  pressure

\( Re_d \)  particle Reynolds number

\( Re_* \)  Reynolds number based on particle size and friction velocity

\( S \)  total plan area

\( T \)  dimensionless time

\( T_t \)  dimensionless transition time

\( t \)  time

\( t_{ref} \)  reference time

\( t_t \)  transition time

\( u \)  fluid velocity
\( \bar{u} \) mean wind velocity

\( u_H \) wind velocity at building height level

\( u_p \) horizontal particle velocity

\( u'_x \) instantaneous horizontal wind velocity fluctuation

\( u_* \) shear velocity

\( u_{*c} \) is the critical shear velocity

\( \bar{u}_* \) shear velocity scaled by direct force balance measurements

\( \forall_p \) particle volume

\( v_p \) vertical particle velocity

\( v_{pT} \) particle terminal velocity

\( W \) building width

\( X \) dimensionless mean flight distance

\( x \) horizontal flight distance

\( \bar{x} \) mean horizontal flight distance

\( x_r \) reattachment length

\( z \) height / vertical flight distance

\( z_0 \) surface roughness

\( \bar{z}_0 \) surface roughness scaled by direct force balance measurements

\( \beta \) angle of the relative wind vector to the horizontal

\( \delta \) boundary layer height
\[ \eta \quad \text{dimensionless height} \]
\[ \theta \quad \text{angular rotation} \]
\[ \kappa \quad \text{Von Karman constant} \]
\[ \lambda_f \quad \text{frontal area density} \]
\[ \mu_a \quad \text{air dynamic viscosity} \]
\[ \nu \quad \text{kinematic viscosity of fluid} \]
\[ \nu_w \quad \text{kinematic viscosity of water} \]
\[ \rho_a \quad \text{air density} \]
\[ \rho_p \quad \text{particle density} \]
\[ \rho_w \quad \text{density of water} \]
\[ \sigma_{d_p} \quad \text{standard deviation of particle diameter} \]
\[ \sigma_{u_x} \quad \text{standard deviation of horizontal velocity fluctuations} \]
\[ \sigma_{u_z} \quad \text{standard deviation of vertical velocity fluctuations} \]
\[ \tau \quad \text{surface shear stress} \]
\[ \tau_c \quad \text{critical shear stress} \]
\[ \tau_s \quad \text{critical Shields parameter} \]
\[ \chi \quad \text{dimensionless flight distance} \]
\[ \omega \quad \text{angular velocity} \]
CHAPTER ONE

INTRODUCTION

Loose particles on the top of a roof present a potential hazard during a severe storm such as hurricanes. As the wind velocity increases during the storm, loose particles can become wind borne debris by leaving the roof. These flying particles can hit objects including human beings and cause a serious life and/or property damage. To better understand windborne debris flight and its potential risk, it is important to understand the conditions under which the debris becomes airborne, the rate at which it is removed from a built-up roof, and its flight distance. This research focuses on an experimental, numerical and analytical investigation of windborne debris during severe storms.

1.1 Motivation

Windborne debris can cause considerable physical harm and property damage. One of the major sources of debris in large commercial areas is loose gravel on built-up roofs. Such loose gravel can be responsible for extensive damage to buildings especially ones that are covered with lots of windows or with glass facades, such as many high rise buildings. For example, during Hurricane Katrina 75% of the windows on the north face of the Hyatt hotel in downtown New Orleans were broken. A post storm investigation found the damage to have been caused by pea gravel, most likely from the roof of the adjacent Amoco building (Kareem and Bashor, 2006). Because of this, the total damage to the hotel, including losses due to limited operation, were estimated to be approximately $100M (Bergen, 2005).

Windborne debris can also cause personal injuries, for instance in an Oklahoma tornado “Two people were killed in Oklahoma City — including a young boy hit by debris in his home”
(www.cbsnews.com, May 11 2010), or in a Mississippi tornado, one person described that “You could just feel the glass and debris flying in and cutting you,” (www.cnn.com, April 24 2010). Penetration of debris into the building envelope, can even lead to the complete collapse of the structure. Once the envelope is breached, the wind raises the internal pressure within the building. This pressure increase will lead to an increase in the net uplift on the building’s roof potentially causing the roof to separate from the walls. If the roof is integrated into the structural bracing of the building, roof separation can cause a complete collapse of the building (NIST, 2006).

The risk of wind-borne debris is not restricted to large commercial structures; family dwellings are also at risk. Post-hurricane surveys by Sparks et al. (1994) found that 64% of houses had at least one window broken during Hurricane Andrew. Such storms can cost billions of dollars, for example hurricane Hugo caused $4.1 billion damage in South Carolina (Knowles, 1989).

The American Society of Civil Engineers (ASCE) standard ASCE 7-05 (2005) “Minimum Design Loads for Buildings and Other Structures” requires that buildings located in “windborne debris regions” meet the following glazing requirements:

“6.5.9.3 Wind-Borne Debris. Glazing in buildings located in wind-borne debris regions shall be protected with an impact resistant covering or impact-resistant glazing according to the requirements specified in ASTM E1886 and ASTM E1996 or other approved test methods and performance criteria. The levels of impact resistance shall be a function of Missile Levels and Wind Zones specified in ASTM E1886 and ASTM E1996."

An exception to this debris standard states that “Glazing in Category II, III, or IV buildings located over 60 ft (18.3 m) above the ground and over 30 ft (9.2 m) above aggregate surface
roofs located within 1,500 ft (458 m) of the building shall be permitted to be unprotected.”.

Although this requirement indirectly addresses the issue of roof gravel blow-off, it does not take into account the design of the up-wind gravel roof, the type of gravel used, or the influence of the surrounding terrain on the wind velocity. The commentary published in appendix C-6 of ASCE 7-05 (2005) regarding wind load provisions, recognizes the simplicity of the current requirements and states “The committee recognizes that there are vastly differing opinions, even within the standards committee, regarding the significance of these parameters that are not fully considered in developing standardized debris regions or referenced impact criteria.” The commentary provides no references for further guidance on how to model wind-borne debris.

Despite the high risk of windborne debris, their flight mechanics are poorly understood. In fact, debris flight has been described as “the forgotten land” of wind engineering (Holmes, 2003). There is a little works in the literature regarding debris lift off (Holmes, 2004). While various debris flight models have been proposed, there are only some experimental works on windborne debris flight trajectories, and just a few experiments have been conducted on windborne debris initiation of motion. In the case of experimental test results, concern could be raised due to an inappropriate scaling. Also, the available analytical models for predicting the flight path such as one derived by Baker (2007), are typically only used once the debris is airborne. Further, these models assume that the flight process is entirely deterministic and that the controlling parameters are known and fixed.

To conduct any experimental research on wind engineering phenomena such as windborne debris, it is necessary to reproduce the atmospheric boundary layer (ABL) accurately at laboratory scale. For that, the logarithmic velocity profile which is widely accepted as an
accurate theoretical mean velocity profile for the lower part of the ABL, needs to be modeled. To model the logarithmic velocity profile, it is important to establish the shear velocity, surface roughness and zero plane displacement. These parameters are estimated either based on upstream train geometry or based on iterative curve fitting through experimental data. Although there are several methods in literature to predict these parameters, accurate parameterization of these parameters is problematic.

This dissertation will develop a new curve fitting method for calculating logarithmic velocity profile parameters i.e. shear velocity, surface roughness and zero plane displacement with high accuracy and without any iteration. Also, it will describe the development of a stochastic model for debris flight that seeks to assess the significance of parameter variability on debris flight distance and impact kinetic energy. Also, results will be presented from a series of wind tunnel experiments that give a fuller understanding of the critical condition under which particles will lift off from a roof, the rate that particles leave the roof and subsequent downstream flight pattern. By investigating the critical condition for particles to lift-off from a roof, the rate that particles would leave the roof, and the resulting flight distance, a methodology could be developed for deriving the critical wind condition and critical distance at which people and properties would be endangered.

1.2 Problem Definition

Of all the potential causes of damage due to severe weather, windborne debris is the least well understood. Aside from one set of studies conducted by Kind and Wardlaw (1977) and Kind (1986), the literature has little research on the mechanics of roof gravel blow-off. The previously published studies have significant shortcomings including:
- The critical velocity for initiation of particle removal: Although Kind measured the velocity at which aggregate was removed, his results were based on visual observation and only presented in form of design curves. This research will follow an inherently qualitative and a more accurate test on removal initiation, and will present results in raw and non-dimensional form.

- The volume and rate of gravel removal: While some critical conditions for gravel removal were measured by Kind, there is no quantification of particle removal rate from building top in the literature. These data are important for calculating the downwind risk of debris impact.

- The downwind flight distance: While there are some studies of the downwind debris field, there is no experimental work on the flight distance of roof gravel.

Each of these parameters is critical in quantifying the risk of wind-borne debris from specific built-up roof installations. To better understand the windborne debris flight and associated potential risk, and in order to prevent physical harm and property damage, to improve design standards to resist the impact of windborne debris, and to better understand the potential economic cost of debris impact, it is necessary to answer the following questions:

- How should the atmospheric boundary layer be modeled at laboratory scale?
- How should uncertainty in debris flight, due to flow turbulence and input parameter uncertainty, be modeled?
- What are the conditions under which debris becomes airborne?
- What is the rate at which debris leaves the roof?
• How far does the debris travels downstream once get airborne?

In order to improve our understanding of these issues, the following research has been conducted:

• The development of the accurate and simple method for the characterization of laboratory boundary layers (wind tunnel and water flume) appropriate for urban fluid mechanics applications.

• The development of a stochastic model for wind borne debris flight to address the influence of parameter uncertainty on flight outcomes (distance and impact kinetic energy).

• An experimental quantification of the critical conditions under which loose particles from built-up roofs can become airborne during strong storms.

• An experimental quantification of the rate at which particles leave the roof and their flight distance downwind.

1.3 Thesis outline

This thesis is presented as four self-contained pieces of research. As such each chapter will contain its’ own literature review, research objectives, outcomes and conclusions. The chapters are as follows:

• Chapter one: General introduction on importance of this research along with research outline are presented in chapter 1.
• Chapter Two: It presents a direct method for calculating the friction velocity, the surface roughness and the zero plane displacement for laboratory scale model atmospheric boundary layers. The approach presented is shown to be easier to use and more accurate than existing approaches.

• Chapter Three: The development of a stochastic model for compact debris flight is presented along with the results of 750,000 Monte Carlo simulations of the flight of roof gravel in a turbulent flow. Experimental results are also presented that compare favorably with model results.

• Chapter Four: A new experimental technique is used for finding the critical velocity at which particles start leaving a built up roof. It is demonstrated that the existing published data is based on flawed scaling and the new results are presented using a more appropriate scaling for such flows.

• Chapter Five: Experimental measurement of mass removal rates and resulting downwind debris field are presented along with a discussion of the appropriate scaling of these results to full scale.

• Chapter Six: General conclusions are stated regarding wind borne debris along with suggestions for future research.
CHAPTER TWO

MODELING THE NEUTRALLY STABLE ATMOSPHERIC BOUNDARY LAYER AT LABORATORY SCALE

Abstract

The properties of the atmospheric boundary layer (ABL) influence a range of physical phenomena in urban areas such as wind loading on buildings, wind driven ventilation flows, pollution dispersion, and the lift off and transport of loose debris. In order to accurately model the atmospheric boundary layer (ABL) velocity profile at laboratory scale, it is important to establish the shear velocity $u_*$, surface roughness $z_0$ and zero plane displacement $d$ in either a wind tunnel or water flume. Current techniques for establishing these parameters are based on either an analysis of the boundary layer surface geometry or iterative curve fitting techniques that use mean velocity and/or turbulent kinetic energy profiles, occasionally combined with empirical correlations for $z_0$. A new curve fitting method for calculating these logarithmic velocity profile parameters, is presented. This new method is able to calculate $u_*$, $z_0$ and $d$ directly from wind or water time averaged velocity profile data in just two steps without any iteration with equal or better accuracy than existing techniques. A comparison between the results of the new method with other available methods applied to a range of velocity profile measurements in air and water shows that, despite the new method requiring less data and fewer steps, it calculates $u_*$, $z_0$ and $d$ with high accuracy for both wind tunnel and water flume data.

Keywords: Logarithmic Velocity Profile, Shear Velocity, Surface Roughness, Zero Plane Displacement, Curve Fitting, Boundary Layer, Wind Tunnel, Open Channel Flow, River Engineering, Water Flume.
2.1 Introduction

The properties of the atmospheric boundary layer (ABL) influence a range of physical phenomena in urban areas such as wind loading on buildings (ASCE 7, 2005), wind driven ventilation flows (Syrios and Hunt, 2008), pollution dispersion (Britter et al., 2003), and the lift off and transport of loose debris (Kordi and Kopp, 2009). Therefore, it is necessary to reproduce the ABL accurately for conducting any research on such phenomena at laboratory scale. The logarithmic velocity profile is widely accepted as an accurate theoretical mean velocity profile for the lower part of the ABL (where the urban canopy is) and is given by

\[
\frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z-d}{z_0} \right)
\]

(2-1)

where \( u \) is velocity at height \( z \), \( u_* = \sqrt{\tau/\rho_a} \) is the shear velocity, also known as the skin friction velocity, \( z_0 \) is the surface roughness height, \( d \) is zero plane displacement, \( \tau \) is the surface shear stress, \( \rho_a \) is the air density, and \( \kappa \approx 0.4 \) is the Von Karman constant. Therefore to model the logarithmic velocity profile, it is important to establish the three parameters \( u_* \), \( z_0 \), and \( d \) at laboratory scale. This fact was recognized by Jensen (1958) who showed that any length scales on the surface roughness length (\( z_0 \)) needed to be matched at full and laboratory scale in order to satisfy geometric similarity. While recreating a rough turbulent boundary layer with a logarithmic profile is relatively simple in boundary layer wind tunnels and water flumes, the accurate parameterization of the flows shear velocity, surface roughness height and displacement height is more problematic.

In order to calculate the velocity at any height using (2-1), three independent parameters must be established, i.e. \( u_* \), \( z_0 \) and \( d \). These parameters cannot be calculated by using traditional curve fitting methods as both \( z_0 \) and \( d \) appear in the log term. To illustrate this,
a velocity profile data set was taken from the Clemson University boundary layer wind tunnel and an equation of the form (2-1) was fitted to it. The curve fitting was done by making an initial estimate of each of the three parameters $u_*, z_0$ and $d$, and then sampling different values of these parameters near the initial estimate in order to find the least squares errors as:

$$E = \sum \left( u_i - \frac{u_*}{\kappa} \ln \left( \frac{z_i - d}{z_0} \right) \right)^2$$  \hfill (2-2)

where $u_i$ is the experimentally measured values of velocity at elevation $z_i$. A total of 36 different initial estimates were made leading to fitted values of $u_*$, $z_0$ and $d$. These were divided into three sets of 12 runs. In the first set different estimates of $u_*$ were made, the second set varied the input $d$ while the third set varied $z_0$. The resulting predictions of $u_*$, $z_0$ and $d$ are plotted in figure 2-1. Each sub-plot shows the resulting value of either $u_*$, $z_0$ or $d$ scaled on the largest value found in that set. While this scaling has no physical significance, it clearly illustrates the range of predicted values that the standard least squares curve fitting approach produces. As the initial estimate of $u_*$ increases all three resulting predictions approach a steady value. However, as either $d$ or $z_0$ are varied, no such steady value is predicted. This indicates that there are multiple local minima in equation (2-2) and that, therefore, a more sophisticated approach is required in order to estimate $u_*$, $z_0$ and $d$. 
Figure 2-1: Calculated $u_*$, $z_0$ and $d$ scaled on the maximum predicted value as functions of the initial estimates these parameters. Columns, from left to right, are the predicted values of $u_*$, $d$ and $z_0$ respectively as functions of the input conditions. Rows, from top to bottom, are plots with varying initial estimates of $u_*$, $d$ and $z_0$ respectively.

The logarithmic profile (2-1) is used in both wind and hydraulic engineering, and each of these areas has its own approach for estimating $u_*$, $z_0$ and $d$. However, all of the methods fall into two general categories, geometrical methods and curve fitting methods. Geometrical methods, estimate the surface roughness height and zero plane displacement based on the shape, dimensions and packing density of the surface roughness elements upstream of the point of interest. Curve fitting methods seek to estimate $u_*$, $z_0$, and $d$ by fitting a curve through measured wind velocity profiles or turbulent intensity profiles. In this approach $u_*$, $z_0$ and $d$ are
calculated directly from velocity or turbulent intensity data without considering the upstream topography.

2.1.2 Geometric methods

Herein, geometric approaches to estimating the surface roughness length scale based on the geometry of objects attached to the surface is reviewed. While, in principle the approaches in the literature are the same for both wind engineering and hydraulics, the nature of surface objects is different. For example, typical suburban environments will include relatively regularly spaced houses of relatively regular angular shape. In rivers and streams, the objects are more chaotically spaced and more rounded with a much larger range of scales from large rocks to small pebbles.

2.1.2.1 Wind engineering geometric methods

Lettau (1969) suggested a simple equation for estimating the surface roughness based on the geometry of the upstream obstacles:

\[ z_0 = 0.5h^* (A_f/S) \]  \hspace{1cm} (2-3)

where \( A_f \) is total frontal area of the obstacles, \( S \) is total plan area which \( z_0 \) is estimated over, and \( h^* \) is the mean obstacles height. While the model of Lettau (1969) provides a first order estimate of \( z_0 \) for sparse arrangements of obstacles, the model does not predict the observed reduction in surface roughness for high roughness element packing density caused by the development of a skimming flow over the top of the obstacles. More sophisticated models have been suggested to account for this and other effects. See Bottema (1996), Kondo and Yamazawa (1986), Theurer et al. (1992), Fang and Sill (1992), Macdonald et al. (1998), and
Tieleman (2003). These models typically assume a constant drag coefficient and then model the drag as a function of surface roughness height, plan area, shape, packing density, orientation, and grid layout.

Despite the increasing sophistication of these geometric approaches, the resulting estimates of $z_0$ show little agreement. To illustrate this, the seven models cited above were used to estimate the surface roughness of the blue foam block fetch in the Clemson University boundary layer wind tunnel (figure 2-2). The results are shown in table 2-1. The blocks are 7.6 cm cubes arranged in a staggered grid with a frontal area density of $\lambda_f = 0.124$. The predicted values of $z_0$ have an average of 7.5mm, but range from 2.3mm to 19.5mm. That is, the estimated surface roughness varies by a full order of magnitude. Further, the variation does not decrease by using more recent models. The most recent models of Macdonald et al. (1998) and Tieleman (2003) give values of 5.4mm and 19.5mm respectively, two of the most extreme values.

Figure 2-2: Blue foam blocks fetch in the Clemson University boundary layer wind tunnel
Table 2-1: Surface roughness value for the Clemson University wind tunnel based on geometric models

<table>
<thead>
<tr>
<th>Method</th>
<th>$z_0$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lettau (1969)</td>
<td>4.7</td>
</tr>
<tr>
<td>Kondo and Yamazawa (1986)</td>
<td>2.3</td>
</tr>
<tr>
<td>Fang and Sill (1992)</td>
<td>4.7</td>
</tr>
<tr>
<td>Theurner (1992)</td>
<td>12.05</td>
</tr>
<tr>
<td>Bottema (1996)</td>
<td>4.2</td>
</tr>
<tr>
<td>Macdonald et al. (1998)</td>
<td>5.4</td>
</tr>
<tr>
<td>Tieleman (2003)</td>
<td>19.5</td>
</tr>
</tbody>
</table>

2.1.2.2 Hydraulic engineering geometric methods

Geometric methods in hydraulic engineering are based primarily on empirical correlations between the size and shape distribution of the upstream aggregate (or soil, grass, etc.) and the surface roughness length scale.

Einstein and El Samni (1949) found that for a bed covered with spheres, the velocity profile origin should be displaced vertically a distance $D = 0.2d_p$ below the top of the particles, where $d_p$ is particle diameter. $D$ was reported to be about 0.25 of the roughness height by Jackson (1981) and between 0.15 and 0.35 of roughness height by Bayazit (1983). It is reported by De Bruin and Moore (1985) that $D$ is between $2.8z_0$ to $4.5z_0$.

Nikuradse (1933) found that $k_s = 30z_0$, while $k_s$ is equivalent bed particles roughness. Van Rijn (1982) showed that $k_s$ is between $d_{90}$ to $10d_{90}$, where $d_{90}$ represents the particle size.
that 90 percent of particles are finer than. Bray (1982) presented data that showed $z_0$ is about 0.25$d_{90}$ while Smart (1999) concluded that $z_0$ would be between 0.03$d_{90}$ to 1.25$d_{90}$. Clearly the variability in estimates of $z_0$ based on bed geometry have at least the same level of variability as those presented in the wind engineering literature.

2.1.3 Curve fitting methods

Given the wide range of $z_0$ values predicted by geometric methods, there has been a variety of curve fitting approaches proposed to establish $u_*, z_0$ and $d$. As demonstrated in section 1, standard least squares approaches do not lead to unique results so alternate approaches have been examined both in the wind and hydraulic engineering fields.

2.1.3.1 Wind engineering curve fitting methods

There are many different methods for estimating the surface roughness $z_0$ from measured wind velocity and turbulence intensity profiles. These methods often have an iterative solution procedure and/or require some empirical correlation for turbulence intensity. Several of these approaches were reviewed by Petersen (1997) which is summarized below. Counihan (1975) proposed the following relation for the turbulence intensity at a height of $30m + d$.

$$\left( \frac{\sqrt{u'^2}}{u} \right)_{z=30m+d} = 0.24 + 0.096 \log_{10}(z_0) + 0.016 \left( \log_{10}(z_0) \right)^2$$

(2-4)

The surface roughness can be found using any iterative non-linear equation solver. However, in order to measure the turbulence intensity at the correct height, the origin offset parameter $d$ must be known or estimated.
De Bruin and Moore (1985) used conservation of mass to find $u_*$, $z_0$ and $d$ as:

$$\int_0^{z_f} u(z)dz = \int_{d+z_0}^{z_f} \frac{u^*}{K} \ln \left( \frac{z-d}{z_0} \right) dz$$

(2-5)

where $z_f$ is a point inside the inertial sub layer ($z_f > z_*, z_* = d + 20z_0$). The integration leads to:

$$d = z_f - \frac{z_f - (d+z_0)}{\ln[(z_f-d)/z_0]} - z_m$$

(2-6)

where $z_m$ can be calculated from

$$z_m = \int_0^{z_f} \frac{u(z)}{u(z_f)} dz.$$  

(2-7)

To solve the above equation, the following relationship is assumed

$$z_0 = \lambda(h - d)$$

(2-8)

where $h$ is characteristic obstacle height. Again, $d$ must be known or estimated prior to solution and $z_0$ is estimated using a simple empirical geometric equation. Moore (1974) proposed $\lambda=0.26\pm0.07$, while Thorn (1971) had proposed $\lambda=0.36$. Equation (2-6) can be rewritten as:

$$z_m = z_f - \frac{z_f - (d+\lambda(h-d))}{\ln[(z_f-d)/\lambda(h-d)]} - d$$

(2-9)

By solving equations (2-7), (2-8) and (2-9) iteratively, the unknown parameters $u_*$, $z_0$ and $d$ can be found.

A combination of the EPA on-site Meteorological Program Guidance (1987) and Counihan (1975) leads to another equation for calculating surface roughness based on the turbulence intensity.

$$z_0 = \frac{z}{e^{(u'/u^*)}}$$

(2-10)
The EPA states that the $z$ should be between $20z_0$ and $100z_0$, which again requires that some other approach be used to make a reasonable initial estimate of $z_0$ prior to solution.

Iyengar and Farell (2001) conducted a series of wind tunnel experiments and estimated $u_*$, $z_0$, and $d$. They estimated $u_*$ by measuring the surface shear stress using two independent methods. First they used a direct force balance measurement, and second a Reynolds stress profile measurement. The wind tunnel used had 12m of fetch with 28 mm cubes as roughness elements arranged in staggered format. A floating 0.45 by 0.289 m plate was connected to force sensors to directly measure the shear force and from there, shear velocity. The direct force balance measurement was compared to the shear velocity estimate obtained from Reynolds shear stress profile measurements using an X-wire probe and found to be within 15% of each other.

In their method, once $u_*$ had been established experimentally, they used a curve fitting method to estimate $z_0$ and $d$ by rearranging the logarithmic law as follow:

$$z = z_0e^{\left(\frac{k\mu(z)}{u_*}\right)} + d$$

(2-11)

The value of $u_*$ used in the equation was taken from the direct force balance measurement. Then they plot $z$ versus $exp(ku(z)/u_*)$ and fitted a straight line to the data to estimate $z_0$ and $d$. Iyengar and Farell (2001) also considered two trial and error methods for evaluating $u_*$, $z_0$ and $d$. First, they used the equation developed by Hama (1954) and called it Hama’s method and second, they matched the power law and logarithmic velocity profile to find $u_*$, $z_0$ and $d$ and called it the log-power law method.

Liu et al. (2002) used an empirical expression given by Engineering Science Data Unit (ESDU) to find the wind profile parameters. The expression gives the variation of turbulent intensity up to a height of 100 meters as:
\[
\frac{\sigma_u}{u} = \left( \frac{0.867 + 0.556 \log_{10} z - 0.246 \log_{10} z^2}{\ln(z/z_0)} \right)^B
\]  

(2-12)

where \( B = 1 \) for \( z_0 < 0.02m \), \( B = 0.76z_0^{-0.07} \) for \( 0.02m < z_0 < 1m \) and \( B = 0.76 \) for \( z_0 > 1m \). In their method, first they plotted \( z \) versus \( u \) while using a natural log scale on the \( z \) axis. By using linear extrapolation on the linear part of the wind profile, they calculated the value of \( d + z_0 \) at the point where the extrapolated line meets the vertical axis i.e. \( u = 0 \). Then they converted the wind tunnel data to field data by using a length scale factor of 1:100. Three sets of profiles, \( \sigma_u/u \), \( 1.15\sigma_u/u \) and \( 0.85\sigma_u/u \), were plotted for chosen range of \( z_0 \). By repeating the procedure with different \( z_0 \), the best value of \( z_0 \) could be found when the 90% of measured turbulence intensity profile lay between the \( 1.15\sigma_u/u \) and \( 0.85\sigma_u/u \) profiles. It should be pointed out that the EDSU equation is based on field data in which the surface roughness height was estimated. Therefore, the quality of any estimate of surface roughness, or any other parameter, based on this equation is limited by the quality of the EDSU estimate of the surface roughness from the original study.

The methods DeBruin & Moore (1985), Iyengar & Farell (2001), and Liu et al. (2002) were applied to profile measurements made in the Clemson University boundary layer wind tunnel in order to estimate the surface roughness of the blocks described above, and to compare the results to the geometric method results of table 2-1. The resulting estimates are given in table 2-2. While there is less spread in the estimates compared to the geometric approaches (0.5 - 2mm compared to 2.3 - 19.5mm), the largest estimate is still four times the smallest. Further, all of the estimates based on curve fitting methods are smaller than any of the geometric estimates.
Table 2-2: Surface roughness value for the Clemson University wind tunnel based on curve fitting models

<table>
<thead>
<tr>
<th>Method</th>
<th>$z_0$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>De Bruin and Moore (1985)</td>
<td>0.5</td>
</tr>
<tr>
<td>EPA (1987)</td>
<td>1.1</td>
</tr>
<tr>
<td>Liu et al. (2002)</td>
<td>1.3</td>
</tr>
</tbody>
</table>

2.1.3.2 Hydraulic Engineering Curve Fitting Methods

In hydraulics engineering, generally $z_0$ and $d$ are evaluated based on the particle sizes and distribution, by using an empirical correlation between particle size or particle equivalent roughness with $z_0$ and $d$. Smarts (1999) introduced a trial and error method to fit the logarithmic velocity profile to experimental data. He considered the velocity profile (2-1) without any zero plane displacement $d$ as

$$\frac{u(z)}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right)$$ (2-13)

By fitting (2-13) to experimental data, he calculated $u_*$ and $z_0$. Then by assuming a value for zero plane displacement $d$, he defined a new origin location and from there he recalculated height values $z$ from new origin location. This procedure is be repeated by considering a series of values for $d$, and continues until the best correlation between experimental data and estimated velocity based on the calculated $u_*$ and $z_0$ and assumed $d$ is obtained.

While there are several different curve fitting methods to estimate $u_*$, $z_0$ and $d$, they either require good initial estimates of these parameters or additional experimental data (such
as surface shear stress or turbulence intensity profiles) and are often iterative. Additionally, they also provide a wide range of resulting estimates. The goal of this chapter is to present a simple, non-iterative method for fitting a curve through laboratory velocity profile data that requires only velocity versus height measurements, and does not require initial estimates of $u_*$, $z_0$ and $d$, measurements of turbulence intensity profiles, or surface shear stress.

The remainder of this chapter is organized as follows. In section 2.2 a new curve fitting methodology that is non-iterative and uses only the mean velocity profile data is presented. The predicted values of $u_*$, $z_0$ and $d$ using the new method are compared to the reviewed methods (section 2.3). It is shown that the new approach provides good estimates of $u_*$ and $z_0$ compared to existing approaches as well as reasonable estimates of $d$. Conclusions are drawn in section 2.4.

2.2 New Curve Fitting Methodology

A new method for estimating $u_*$, $z_0$ and $d$ is presented based on fitting a logarithmic velocity profile equation (2-1) through measured velocity data. The procedure involves rearranging the velocity profile equation to estimate the shear velocity $u_*$ and then calculating the surface roughness $z_0$ and zero plane displacement $d$.

Prandtl (1925) developed a velocity defect law for the outer region of turbulent boundary layers

$$\frac{u_{\text{max}} - u}{u_*} = f\left(\frac{z}{\delta}\right) = -\frac{1}{\kappa} \ln \left(\frac{z}{\delta}\right)$$

(2-14)

where $\delta$ is the boundary layer height. Hinze (1975) showed it could be considered for both smooth and rough walls. Clauser (1956) added a correction term equal to 2.5 to the (14), while Coles (1956) proposed the Law of the Wake for considering the deviation of (14) from
measured data. At laboratory scale, the boundary layer height $\delta$ can be taken as the flow depth in a water flume or the height of the maximum velocity in a boundary layer wind tunnel (provided the upstream fetch is substantially rougher than the wind tunnel side walls and ceiling). Equation (2-14) effectively assumes that the zero plane displacement is small compared to the boundary layer height and can therefore be neglected far from the ground.

2.2.1 First step, calculating $u_*$

As a first step in this method, values of $(u_{max} - u)$ versus $(z/\delta)$ for $z/\delta > 0.2$ are plotted and a natural log curve $u_{max} - u = -\frac{u_*}{\kappa} \ln \left( \frac{z}{\delta} \right)$ will be fitted through the data. From this, the shear velocity $u_*$ would be calculated from the coefficient of the logarithmic term $(u_*/\kappa)$. Figure 2-3 shows a schematic plot of $(u_{max} - u)$ versus $(z/\delta)$ and a natural log curve fitted through the experimental data for calculating $u_*$. 

![Figure 2-3: Schematic plot of fitted natural log curve through experimental data for calculating $u_*$](image)
Estimated values of \( u_* \) from this method will be sensitive to the range of \( z/\delta \) used. Since the velocity defect law was originally developed for the outer region of a turbulent boundary layer, considering a value within the internal region will cause an underestimation of \( u_* \). On the other hand, considering data only from outer region will cause an overestimation of \( u_* \) because of the effect of the law of the wake. To find out the best range of \( z/\delta \) for estimating \( u_* \), series of different ranges were investigated using the data from Iyengar and Farell (2001) for which \( u_* \) was measured independently. The extent of \( z/\delta \) used was parameterized by a range:

\[
\Delta = (z/\delta)_{Upper\ Limit} - (z/\delta)_{Lower\ Limit}
\]  \hspace{1cm} (2-15)

and central point

\[
\mu = \frac{(z/\delta)_{Upper\ Limit} + (z/\delta)_{Lower\ Limit}}{2}
\]  \hspace{1cm} (2-16)

A plot of central point (mean) against the ratio of the estimated to measured value of the skin friction velocity \( (U_*, = (Estimated\ Value)/(Measured\ Value)) \) is shown in figure 2-4.

\[ \text{Figure 2-4: Effect of } z/\delta \text{ range on estimated } u_* \text{ value for different } \Delta \]
For $< 0.3$, any range of $\Delta$ leads to under estimation of the $u_*$, while $\mu > 0.3$ cause over estimation of $u_*$. On the other hand, $\Delta > 0.5$ leads to over estimation of the $u_*$, which leaves us with $\Delta \leq 0.4$ for $u_*$ estimation. For $\mu = 0.3$, and $\Delta \leq 0.4$, considering $\Delta = 0.4$ gives the best fit, while it estimates the $u_*$ with high accuracy, it uses wider range of data compared to the $\Delta = 0.2, 0.3$ and it only uses data from outer region, since $\mu = 0.3$ and $\Delta = 0.4$ is equivalent to $0.1 < z/\delta < 0.5$ which it is a match with the defect law region.

2.2.2 Second step, calculating $z_0$ and $d$

After calculating the shear velocity $u_*$, equation (1) is rearranged to read:

$$e^{u_{(\frac{z}{u_*})}} = \frac{1}{z_0} z - \frac{d}{z_0}$$  \hspace{1cm} (2-17)

The values of $e^{u_{(\frac{z}{u_*})}}$ are plotted versus $z$ and straight line $e^{u_{(\frac{z}{u_*})}} = \frac{1}{z_0} z - \frac{d}{z_0}$ is fitted to the data. From this, the surface roughness height $z_0$ would be calculated from line slope ($1/z_0$), and the zero plane displacement $d$ would be calculated from vertical axis intercept ($d/z_0$).

Figure 2-5 shows a schematic plot of $e^{u_{(\frac{z}{u_*})}}$ versus $z$ with a straight line fitted through the experimental data for calculating $z_0$ and $d$.  

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Figure 2-5: Schematic plot of fitted straight line through experimental data for calculating $z_0$ and $d$

2.3 Comparison with previously published methods

In order to evaluate the applicability of the new curve fitting approach, five separate test were conducted. First, the new approach is compared with the De Bruin and Moore (1985) method. Second, the mean velocity profile parameters from the new method were compared with data from Iyengar and Farell (2001) that was calculated using direct force balance measurements. Then the new method is compared to the turbulence intensity method of Liu et al. (2003). Forth, it is compared with the Smart (1999) method proposed for hydraulics application. In this comparison, data from other researchers, data from the Clemson University boundary layer wind tunnel and data from a water flume in the Clemson University fluid mechanics laboratory is used. Finally, the curve fitting method is used to predict values of $z_0$ for the wind tunnel and flume, and are compared to those predicted using geometric methods.
2.3.1 Comparison with De Bruin and Moore method (mass conservation)

To evaluate the new method with other curve fitting method, first it is compared with the De Bruin and Moore (1985) method (equations (2-7), (2-8) and (2-9)). Velocity profiles were fitted to a data set taken from the Clemson University boundary layer wind tunnel using the two methods. Results along with experimental data are plotted in figure 2-6. The new method provides a better representation of the experimental data as measured by the RMS error

$$RMSE = \sqrt{\frac{(u_{Exp, i} - u_{Cat, i})^2}{n}}$$  \hspace{1cm} (2-18)

The RMS error for the new method is 0.18 m/s while it is 0.60 m/s for the De Bruin & Moore (1985) method.

Figure 2-6: Log. velocity profile based on De Bruin and Moore (1985) and new method for the Clemson University wind tunnel data
2.3.2 Comparison with Iyengar and Farell method (direct force measurement)

Iyengar and Farell (2001) calculate the logarithmic velocity profile parameters $u_*$, $z_0$ and $d$ using a direct force balance measurement and compared them with parameters calculated from Hama’s law fit (Iyengar and Farell, 2001), a log-power law fit, and data from Reynolds shear stress profile measurements. Velocity profile parameters calculated based on the new method presented above are compared with the parameters calculated by Iyengar and Farell (2001) for 3 boundary layers, BL1, BL2 and BL3 with free stream velocity equal to 30 m/s. The results are shown in figure 2-7 to 2-9. These figures show the parameters $u_*$, $z_0$, and $d$ scaled on the value gained from the direct force balance measurements of Iyengar and Farell (2001), that is:

$$\tilde{u}_* = \frac{u_*}{u_*(FB)}, \quad \tilde{z}_0 = \frac{z_0}{z_0(FB)} \quad \text{and} \quad \tilde{d} = \frac{d}{d(FB)} \tag{2-19}$$

The subscript (FB) stands for force balance and indicates the value established through direct measurement of the surface shear stress using a force balance.

Figure 2-7 shows that the new approach slightly over-estimates the skin friction velocity (except for BL3) while the other three approaches under-estimate $u_*$. For each of the three boundary layer profiles fitted, the new method’s estimate is closest or second closest to the measured $u_*$ than the other three methods based on the mean percentage difference between the calculated values and the values based on the force balance measurement technique. This trend is repeated in the predictions of $z_0$ shown in figure 2-8 in which the new method provides the best estimate of the surface roughness height for BL1 and BL2 and second best for BL3, though in this case all other curve fitting approaches under-estimate the value calculated based on a measurement of $u_*$. In the estimate of the zero plane displacement $d$, the new method
provides the second best estimate for BL2 and best for BL3 (figure 2-9), while all other curve fitting approaches over-estimate \( \delta \).

Figure 2-7: Dimensionless shear velocity \( (\bar{u}_*) \) from Iyengar and Farell (2001) data and new method data. The dashed line is on the direct force balance measurement and is taken as the reference value.
**Figure 2-8:** Dimensionless surface roughness ($\overline{\varepsilon_0}$) from Iyengar and Farell (2001) data and new method data. Refer to figure 6 for dashed lines definition.

**Figure 2-9:** Dimensionless zero plane displacement ($\overline{d}$) from Iyengar and Farell (2001) data and new method data. Refer to figure 6 for dashed lines definition.
The mean percentage difference between the force balance measurement method, the three comparison methods and the newly proposed method are presented in tables 2-3 to 2-5. Based on the data in these tables it is clear that the method presented in this chapter does the best job of predicting the skin friction velocity and surface roughness height. The mean difference between direct measurement method and the new model predictions for \( u_* \) and \( z_0 \) are approximately an order of magnitude less than the difference between the direct measurement method and the three comparison methods. The difference is less stark in the predictions of the zero plane displacement \( d \) in which the new and comparison methods all exhibited similar discrepancies compared to the direct measurement method.

**Table 2-3:** Difference calculation for shear velocity \( u_* \) (Iyengar and Farell (2001) data and new method data)

<table>
<thead>
<tr>
<th></th>
<th>BL1</th>
<th></th>
<th>BL2</th>
<th></th>
<th>BL3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( u_* )</td>
<td>Difference</td>
<td>( u_* )</td>
<td>Difference</td>
<td>( u_* )</td>
<td>Difference</td>
</tr>
<tr>
<td></td>
<td>(m/s)</td>
<td>(%)</td>
<td>(m/s)</td>
<td>(%)</td>
<td>(m/s)</td>
<td>(%)</td>
</tr>
<tr>
<td>Hama's law fit</td>
<td>1.63</td>
<td>-7.80</td>
<td>1.53</td>
<td>-14.36</td>
<td>1.56</td>
<td>-22.39</td>
</tr>
<tr>
<td>Log-Power law fit</td>
<td>1.54</td>
<td>-12.71</td>
<td>1.65</td>
<td>-8.18</td>
<td>1.87</td>
<td>-6.87</td>
</tr>
<tr>
<td>Reynolds Shear Stress measurement</td>
<td>1.46</td>
<td>-17.29</td>
<td>0.78</td>
<td>-56.09</td>
<td>1.66</td>
<td>-17.16</td>
</tr>
<tr>
<td>New Method</td>
<td>1.80</td>
<td>+2.37</td>
<td>2.03</td>
<td>+12.85</td>
<td>1.79</td>
<td>-11.04</td>
</tr>
</tbody>
</table>
Table 2-4: Difference calculation for surface roughness $z_0$ (Iyengar and Farell (2001) data and new method data)

<table>
<thead>
<tr>
<th></th>
<th>BL1</th>
<th></th>
<th>BL2</th>
<th></th>
<th>BL3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0$</td>
<td>Difference</td>
<td>(%)</td>
<td>$z_0$</td>
<td>Difference</td>
<td>(%)</td>
<td>$z_0$</td>
</tr>
<tr>
<td>Hama's law fit</td>
<td>1.7</td>
<td>-34.62</td>
<td>0.8</td>
<td>-52.94</td>
<td>0.5</td>
<td>-68.75</td>
</tr>
<tr>
<td>Log-Power law fit</td>
<td>1.4</td>
<td>-46.15</td>
<td>1.2</td>
<td>-29.41</td>
<td>1.2</td>
<td>-25.00</td>
</tr>
<tr>
<td>Reynolds Shear Stress measurement</td>
<td>1.08</td>
<td>-58.46</td>
<td>0.8</td>
<td>-52.94</td>
<td>0.62</td>
<td>-61.25</td>
</tr>
<tr>
<td>New Method</td>
<td>2.9</td>
<td>+11.54</td>
<td>2.1</td>
<td>+23.53</td>
<td>0.55</td>
<td>-65.63</td>
</tr>
</tbody>
</table>

Table 2-5: Difference calculation for zero plane displacement $d$ (Iyengar and Farell (2001) data and new method data)

<table>
<thead>
<tr>
<th></th>
<th>BL2</th>
<th></th>
<th>BL3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>Difference</td>
<td>(%)</td>
<td>$d$</td>
<td>Difference</td>
</tr>
<tr>
<td>Hama's law fit</td>
<td>18</td>
<td>+157</td>
<td>18</td>
<td>+200</td>
</tr>
<tr>
<td>Log-Power law fit</td>
<td>10</td>
<td>+43</td>
<td>10</td>
<td>+67</td>
</tr>
<tr>
<td>Reynolds Shear Stress measurement</td>
<td>15</td>
<td>+114</td>
<td>17</td>
<td>+183</td>
</tr>
<tr>
<td>New Method</td>
<td>2.1</td>
<td>-70</td>
<td>2.7</td>
<td>-55</td>
</tr>
</tbody>
</table>
2.3.3 Comparison with Liu et al. method (turbulence intensity)

As described above, Liu et al. (2003) used vertical turbulence intensity profile data and an iterative curve fitting technique to calculate the mean wind profile parameters. The method presented in this paper is compared with the method of Liu et al. (2003). Figure 2-10 shows wind velocity profile data from Liu et al. (2003), the logarithmic wind velocity profile based on the Liu et al. (2003) method and the new method. The Liu et al. (2003) method has an RMS error 0.04 m/s compared to the method presented above which has an RMS error 0.11 m/s. Figure 2-11 shows wind velocity profile data from the Clemson University wind tunnel, with the curves fitted in the same manner as figure 2-10. For this data set, the RMS error for the Liu et al. method is 1.04 m/s compared an RMS error of 1.07 m/s for the new method. In both cases the RMS errors are approximately the same and there is little justification in selecting one technique over the other on the basis of the quality of fit. However, the Liu et al. (2003) method requires turbulence intensity profile information and is iterative, whereas the method presented above requires only mean velocity profile data and does not require iterations. Therefore, a similar quality of fit is achieved with less data and fewer calculation steps.
**Figure 2-10:** Log. velocity profile based on Liu et al. (2003) and new method for Liu et al data

**Figure 2-11:** Log. velocity profile based on Liu et al. (2003) and new method for the Clemson University wind tunnel data
2.3.4 Comparison with Smart method (Water flume measurements)

There are many examples in the literature that water flumes being used to model atmospheric flows in urban areas (see Syrios & Hunt, 2008 and Macdonald et al., 2000). Therefore, the new method is compared to the method of Smart (1999) using data from the water flume in the Clemson University civil engineering department’s fluid mechanics laboratory (figures 2-11 to 2-13). In this comparison, the height of the boundary layer was taken to be the water depth in the flume. The fitted curves along with the original data for three different bed gravel sizes $d_{b4}$ equal to 17.5 mm, 11.35 mm, and 8.25mm are shown in figures 2-12 to 2-14 with a summary table of the RMS error for each profile in table 2-6.

As the results from Smart (1999) was compared with results from new method, it was found that both method’s results are very close together, although the new method has a lower RMS error and provide a better fit through the data for all three cases (table 2-6). This means that, the new method could be used to calculate $u_*$, $z_0$ and $d$ for a logarithmic velocity profile model atmospheric boundary layer in a water flume, without any iteration.

**Table 2-6: RMS error comparison between new method and Smart (1999)**

<table>
<thead>
<tr>
<th>$d_{b4}$</th>
<th>RMS Error (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{b4} = 17.5$ mm</td>
<td>0.60</td>
</tr>
<tr>
<td>$d_{b4} = 11.35$ mm</td>
<td>0.65</td>
</tr>
<tr>
<td>$d_{b4} = 8.25$ mm</td>
<td>1.40</td>
</tr>
</tbody>
</table>
Figure 2-12: Log. velocity profile based on Smart (1999) and new method for the Clemson University fluid lab. flume data, bed gravel size $d_{84} = 17.5\text{mm}$

Figure 2-13: Log. velocity profile based on Smart (1999) and new method for the Clemson University fluid lab. flume data, bed gravel size $d_{84} = 11.35\text{mm}$
2.3.5 Comparison of surface roughness estimates

Jensen (1958) established that the surface roughness height $z_0$, is a geometric length that must be matched between full and laboratory scale when modeling the ABL regardless of whether a flume or wind tunnel is being used. This was standardized in terms of a dimensionless parameter called the Jensen number (Cook, 1986) as:

$$Je = \frac{Object \ Height}{z_0}$$

(2-20)

Jensen (1958) showed that $Je$ for a small scale test is required to be equal with $Je$ for full scale model. To apply a Je similarity, the surface roughness length scale should be estimated accurately for wind tunnel or water flume. As discussed above, there is a significant range of surface roughness length scale predictions for both the wind tunnel and water flume data sets. For the wind tunnel there is an order of magnitude range of estimates (table 2-8), while there is
a factor of two difference between the upper and lower estimates for the water flume (table 2-7). For both the wind tunnel and the water flume, the curve fitting methods result in smaller estimates of \( z_0 \) compared to the geometric methods. The variability in estimating \( z_0 \) is to some extent recognized in the ASCE wind code (ASCE-7) by the use of exposure categories to quantify the upstream roughness. The code uses 4 exposure categories with ranges of \( z_0 < 1 cm \) (D), \( 1 cm < z_0 < 15 cm \) (C), \( 15 cm < z_0 < 70 cm \) (B), and \( 70 cm < z_0 \) (A). As the new method results in estimates of \( z_0 \) that are compatible with existing estimation techniques, the method could be used for parameterizing \( z_0 \) for ABL models in either a wind tunnel or water flume.

**Table 2-7:** Surface roughness \( z_0 \) estimation for the Clemson University fluid laboratory flume

<table>
<thead>
<tr>
<th>( d_{94} ) ( (mm) )</th>
<th>( z_0 ) ( (mm) )</th>
<th>Geometric Bray (1982)</th>
<th>Curve fit Smart (1999)</th>
<th>Curve Fit New Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.25</td>
<td>2.0</td>
<td>1.5</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>11.35</td>
<td>2.8</td>
<td>2.3</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>17.5</td>
<td>4.3</td>
<td>2.3</td>
<td>3.3</td>
<td></td>
</tr>
</tbody>
</table>
Table 2-8: Surface roughness $z_0$ estimation for the Clemson University wind tunnel

<table>
<thead>
<tr>
<th>Method</th>
<th>$z_0 \text{ (mm)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometric methods</strong></td>
<td></td>
</tr>
<tr>
<td>Lettau (1969)</td>
<td>4.7</td>
</tr>
<tr>
<td>Kondo and Yamazawa (1986)</td>
<td>2.3</td>
</tr>
<tr>
<td>Fang and Sill (1992)</td>
<td>4.7</td>
</tr>
<tr>
<td>Theurner (1992)</td>
<td>12.05</td>
</tr>
<tr>
<td>Bottema (1996)</td>
<td>4.2</td>
</tr>
<tr>
<td>Macdonald et al. (1998)</td>
<td>5.4</td>
</tr>
<tr>
<td>Tieleman (2003)</td>
<td>19.5</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>7.55</td>
</tr>
<tr>
<td><strong>Curve fitting methods</strong></td>
<td></td>
</tr>
<tr>
<td>De Bruin and Moore (1985)</td>
<td>0.5</td>
</tr>
<tr>
<td>EPA (1987)</td>
<td>1.1</td>
</tr>
<tr>
<td>Liu et al. (2002)</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.0</td>
</tr>
<tr>
<td><strong>New method</strong></td>
<td>2.6</td>
</tr>
</tbody>
</table>

2.4 Conclusion

Estimating the surface roughness height for laboratory scale turbulent boundary layers, whether in wind or water, is essential to accurately modeling urban wind flows and dispersion at small scale. However, establishing this parameter is often difficult and involves estimates either based on fetch geometry that have wide variability (table 2-2) or that require curve fitting
through the measured velocity profile data. Existing curve fitting techniques are either iterative in nature, rely on empirical correlations between turbulence intensity and surface roughness, or both (Liu et al., 2002).

To avoid this complexity, a new curve fitting method for calculating logarithmic velocity profile parameters i.e. shear velocity $u_*$, surface roughness $z_0$ and zero plane displacement $d$, for laboratory work is introduced and compared with previously published estimation methods. The new two-step method is able to calculate $u_*$, $z_0$ and $d$ directly based solely on mean velocity profile data rather than using iterative calculations, turbulence intensity measurements, or direct measurement of surface shear stress.

The new method was tested by comparing its estimation of $u_*$, $z_0$ and $d$ with previously published iterative curve fitting methods. Comparison between the new method and the De Bruin and Moore (1985) method showed that the new method gives a better fit to experimental data. The new method, along with the results from other curve fitting methods, were compared with results from the direct force balance measurement of surface shear stress by Iyengar and Farell (2001). Comparing the results with measured values, the new method gave the best or second best estimation of $u_*$, $z_0$ and $d$ in comparison with previously published estimating techniques. Comparison between the new method and the turbulent intensity method of Liu et al. (2003) shows a slightly improved quality of fit with substantially fewer computations, no iteration, and no need for turbulence intensity profile measurements. Comparison with Smart (1999) (developed for hydraulics applications) showed that the new method is also able to predict the $u_*$, $z_0$ and $d$ for water flow.
A summary of the surface roughness estimates derived from both wind tunnel and water flume profile measurements are compared to existing geometric and curve-fitting methods. The wide range of estimated $z_0$ is not surprising and is recognized in the broad $z_0$ categories used in wind load design codes.

The new method provides good estimates of $u_*$ (table 2-3), $z_0$ (tables 2-4, 2-7 and 2-8) and $d$ (table 2-5). The strength of the new method is its accuracy (very close estimation to measured values), simplicity (two steps and no iteration), the limited data needed to make estimates (mean velocity profile and no turbulence intensity data), and applicability for both wind and water modeling.
CHAPTER THREE

STOCHASTIC MODELING OF COMPACT DEBRIS FLIGHT

Abstract

The stochastic nature of debris flight is investigated through a series of Monte Carlo simulations based on the debris flight equations for compact debris presented by Holmes (2004). For any given debris flight situation, there are a number of uncertainties such as the size of the piece of debris and the time varying turbulent wind flow. Current debris flight models are deterministic and fail to account for such input parameter uncertainty. The simulations presented in this chapter model the flight of a single spherical particle whose diameter is given by a probability distribution function driven by a turbulent wind with velocity fluctuations appropriate to the atmospheric boundary layer. The model predicts the mean and standard deviation of the particle flight distance and impact kinetic energy.

Results show that introducing uncertainty in any of particle diameter, horizontal turbulence intensity and vertical turbulence intensity, leads to larger mean value for flight distance and impact kinetic energy, compared to the condition where there is no variability in input parameters. The mean flight distance and mean impact kinetic energy increase with increasing input parameter variability. Introducing input parameter variability also leads to variability in flight distance and impact kinetic energy. The extent of the outcome variability and its relationship to particle size variability and turbulence intensity are quantified through the simulations.

Detailed analysis of the effect of input variability is not possible as there are no general analytic solutions to the debris flight equations. However, a number of analytical approaches to
understanding and quantifying the stochastic nature of debris flight are presented based on approximate solutions to the flight equations. The analysis presented explains the broad trends observed in the data. The simulation results are compared to a series of wind tunnel experiments in which a spherical particle is released into a turbulent wind field. The simulations accurately predicted both the mean and standard deviation of the measured flight distance.

**Keywords:** Windborne Debris Flight, Monte Carlo Simulation, Stochastic, Uncertainty, Flight Distance, Impact Kinetic Energy

### 3.1 Introduction

The stochastic nature of debris flight is investigated through the series of Monte Carlo simulations using the debris flight equations of Holmes (2004). The investigation focuses on the flight path of roof gravel blown off 1, 2, 5, 10, 20, and 50m high built up roofs. The simulations investigate the variation in flight distance and impact kinetic energy due to variations in the particle diameter and horizontal and vertical turbulent fluctuations about the mean wind velocity.

#### 3.1.1 Risk from Debris flight

Wind borne debris penetrating a building’s envelope can result in significant damage, varying from simple broken windows and resulting rain inundation to total destruction of the building. Such damage occurred at the Hyatt hotel in downtown New Orleans after Katrina. A post-Katrina assessment found that pea gravel, most likely from the roof of the adjacent Amoco building, broke 75% of the windows on the north face of the hotel (Kareem and Bashor, 2006).
Because of this substantial damage, hotel operations were restricted to significantly reduced capacities during subsequent repairs. Total damages were initially estimated at $100M (Bergen, 2005).

Debris penetration of the building envelope can also result in the entire collapse of a structure. Once windows are broken, the wind raises the internal pressure within the building. This internal pressure increase will increase of the net uplift on the building’s roof, potentially leading to roof separation. If the roof is actually integrated into the structural bracing of the building, roof separation can cause a complete collapse of the building. Post-storm forensic investigations (Sparks, 1998) have found a number of such structural failures in buildings with large open internal spaces and un-reinforced walls (e.g. large churches and big box stores). This is shown recently in full scale test in the Insurance Institute for Business & Home Safety (IBHS) wind tunnel in Chester County, South Carolina. Tests showed a building collapse immediately following the front door failing due to high wind pressure.

The risk of wind-borne debris is not restricted to large commercial structures; family dwellings are also at risk. A forensic investigation of 466 houses following Hurricane Andrew (Sparks et al., 1994) found that 64% had at least one broken window whereas only 2% of walls sustained moderate to severe damage.

3.1.2 Existing debris flight models

Existing debris flight models are based on Newtonian mechanics. They consist of equations of motion for a particular particle in which the forces acting on the particle are the gravitational body force $F_W$, drag $F_D$ and lift $F_L$ forces (figure 3-1). Such a model is presented by Tachikawa (1983, 1988) who was the first to derive the non-dimensional equations for the
trajectories of wind born debris. Based on Newton’s second law, Tachikawa presented three equations of motion for horizontal, vertical and rotational motion respectively as:

\[
\frac{d^2\bar{x}}{dt^2} = \frac{d\bar{u}_p}{dt} = K \left[ (1 - \bar{u}_p)^2 + \bar{v}_p^2 \right] \left( C_D \cos \beta - C_L \sin \beta \right) \tag{3-1}
\]

\[
\frac{d^2\bar{z}}{dt^2} = \frac{d\bar{v}_p}{dt} = K \left[ (1 - \bar{u}_p)^2 + \bar{v}_p^2 \right] \left( C_D \sin \beta + C_L \cos \beta \right) - 1 \tag{3-2}
\]

\[
\frac{d^2\bar{\theta}}{dt^2} = K \Delta F_{\text{Fr}}^2 \left[ (1 - \bar{u}_p)^2 + \bar{v}_p^2 \right] C_M \tag{3-3}
\]

where

\[
\bar{x} = \frac{gx}{u^2}, \quad \bar{z} = \frac{gx}{u^2}, \quad \bar{\ell} = \frac{gx}{u}, \quad \bar{u}_p = \frac{u_p}{u}, \quad \bar{v}_p = \frac{v_p}{u}, \quad K = \frac{\rho u^2 A_p}{2 \rho_p \nu_p g}, \quad \Delta = \frac{ml^2}{I_m} \quad \text{and} \quad F_{\text{Fr}} = \frac{u}{\sqrt{g} l_p} \tag{3-4}
\]

The parameters \( m_p, l_p, A_p, \nu_p \) and \( \rho_p \) are particle mass, length, cross sectional area, volume and density respectively. \( g \) is the gravity acceleration, \( I \) is the mass moment of inertia, \( x \) is horizontal and \( z \) is vertical displacement, \( \theta \) is angular rotation, \( u_p \) is horizontal and \( v_p \) is vertical particle velocity, \( u \) is the horizontal wind velocity, \( \rho_a \) is air density. \( C_D, C_L, \) and \( C_M \) are drag, lift, and moment coefficients respectively. \( \beta \) is the angle of the relative wind vector to the horizontal and \( t \) is time. \( K \) is the Tachikawa parameter which is later called Tachikawa number by Holmes et al. (2006a).

**Figure 3-1:** Schematic drag \( F_D \), lift \( F_L \) and weight \( F_W \) forces acting on flying debris
Wills et al. (2002) developed a model to describe the damage of wind borne debris to a building during periods of high wind velocity. For their flight model, they assumed that particle would lift off if the lift force exceeds a fixity force, that is, the force that fixes or keeps the object in place such as weight. By assuming that, drag in horizontal direction and gravity in vertical direction are the only driving force (that is, ignoring vertical drag), they solved (3-5) and (3-6) for the motion of cubic particle.

\[
\begin{align*}
    u_p &= \left( \frac{1}{2} \left( \frac{p_a}{\rho_p} \right) \left( \frac{c_D}{l_p} \right) u^2 t \right) / \left( 1 + \frac{1}{2} \left( \frac{p_a}{\rho_p} \right) \left( \frac{c_D}{l_p} \right) u_p t \right) \\
    t &= \sqrt{\frac{2}{g z}} 
\end{align*}
\]

Comparing their wind tunnel data with their model results, shows up to a 29% error.

Holmes (2004), wrote the equations of motion for the vertical \(z\) and horizontal \(x\) accelerations in terms of the mean horizontal wind speed \(u\) and the horizontal \(u_p\) and vertical \(v_p\) components of the particle velocity. Accounting for both vertical and horizontal drag leads to a set of coupled second order ordinary differential equations

\[
\begin{align*}
    \frac{d^2x}{dt^2} &= \frac{\rho_a c_D A_p}{2 \rho_p v_p} (u - u_p) \sqrt{(u - u_p)^2 + v_p^2} \\
    \frac{d^2z}{dt^2} &= \frac{\rho_a c_D A_p}{2 \rho_p v_p} (-v_p) \sqrt{(u - u_p)^2 + v_p^2 - g}
\end{align*}
\]

By considering longitudinal turbulence intensity equal to 0.2 and vertical turbulence intensity equal to 0.12, Holmes showed that considering the vertical air resistance will cause a slight increase in the flight time, decrease in the vertical acceleration, increase the horizontal velocity and displacement and decrease in vertical velocity when the object impacts the ground.

Holmes et al. (2006b) developed a numerical model for the trajectories of square plates in strong wind, by introducing a normal force coefficient. Comparison of the author’s own data
with their numerical model showed that without considering the Magnus effect, the calculated trajectories do not confirm the experimental results. By considering the Magnus effect the model showed better agreement but still does not match completely with the experimental data.

Lin et al. (2006) carried out experiments to determine the flight characteristics of plate debris. Their analysis of the non-dimensional horizontal trajectories led to the following empirical equations for the horizontal velocity and displacement of plate-like debris:

\[ \bar{u}_p \approx 1 - e^{-\sqrt{1.8K\bar{x}}} \] (3-9)

\[ K\bar{x} \approx 0.456(K\bar{t})^2 - 0.148(K\bar{t})^3 + 0.024(K\bar{t})^4 - 0.0014(K\bar{t})^5 \] (3-10)

Lin et al. (2007) considered the non-dimensional equations proposed by Tachikawa (1983), and simplified them as:

\[ \bar{u}_p = C_D K\bar{t} \; , \; \bar{x} = \frac{1}{2} C_D K\bar{t}^2 \; \text{and} \; \bar{u}_p = \sqrt{2C_D K\bar{x}} \] (3-11)

\[ \bar{v}_p = (C_L K - 1)\bar{t} \; , \; \bar{z} = \frac{1}{2} (C_L K - 1)\bar{t}^2 \; \text{and} \; \bar{v}_p = \sqrt{2(C_L K - 1)\bar{z}} \] (3-12)

Based on their numerical and experimental results, they suggested the equations for particle velocity \( \bar{u}_p \), and particle flight distance \( \bar{x} \) as:

For cubes

\[ \bar{u}_p = 1 - e^{-\sqrt{1.6K\bar{x}}} \; , \; K\bar{x} \approx 0.405(K\bar{t})^2 - 0.036(K\bar{t})^3 - 0.052(K\bar{t})^4 + 0.008(K\bar{t})^5 \] (3-13)

For spheres:

\[ \bar{u}_p = 1 - e^{-\sqrt{1.4K\bar{x}}} \; , \; K\bar{x} \approx 0.248(K\bar{t})^2 + 0.084(K\bar{t})^3 - 0.1(K\bar{t})^4 + 0.006(K\bar{t})^5 \] (3-14)

For rods initially perpendicular to wind

\[ \bar{u}_p = 1 - e^{-\sqrt{1.6K\bar{x}}} \; , \; K\bar{x} \approx 0.4005(K\bar{t})^2 - 0.16(K\bar{t})^3 + 0.036(K\bar{t})^4 - 0.0032(K\bar{t})^5 \] (3-15)
For rods initially parallel to wind:

$$\bar{u}_p = 1 - e^{-\sqrt{1.6K\bar{x}}}, K\bar{x} \approx 0.4005(K\bar{v})^2 - 0.294(K\bar{v})^3 + 0.088(K\bar{v})^4 - 0.0082(K\bar{v})^5$$  \hspace{1cm} (3-16)

Baker (2007) presented a mathematical analysis for the debris flight equations in 2 dimensions using a slightly different non-dimensional scheme. For compact debris, the equations are:

$$\frac{d^2\bar{x}}{dt^2} = \frac{d\bar{u}_p}{dt} = (1 - \bar{u}_p)(1 - \bar{u}_p)^2 + (\bar{v}_p)^2)^{0.5}$$  \hspace{1cm} (3-17)

$$\frac{d^2\bar{y}}{dt^2} = \frac{d\bar{v}_p}{dt} = C_D(-\bar{v}_p)(1 - \bar{u}_p)^2 + (\bar{v}_p)^2)^{0.5} - \Omega$$  \hspace{1cm} (3-18)

and for plate-like object as:

$$\frac{d^2\bar{x}}{dt^2} = \frac{d\bar{u}_p}{dt} = (C_D(1 - \bar{u}_p) - (C_L + C_LA)\bar{v}_p)((1 - \bar{u}_p)^2 + \bar{v}_p^2)^{0.5}$$  \hspace{1cm} (3-19)

$$\frac{d^2\bar{y}}{dt^2} = \frac{d\bar{v}_p}{dt} = (-C_D\bar{v}_p + (C_L + C_LA)(1 - \bar{u}_p))(1 - \bar{u}_p)^2 + \bar{v}_p^2)^{0.5} - \Omega$$  \hspace{1cm} (3-20)

$$\frac{d^2\bar{\theta}}{dt^2} = \frac{d\bar{w}}{dt} = \Delta (C_m + C_{MA})(1 - \bar{u}_p)^2 + \bar{v}_p^2$$  \hspace{1cm} (3-21)

where:

$$\bar{u}_p = \frac{u_p}{u}, \quad \bar{v}_p = \frac{v_p}{u}, \quad \bar{w}_p = \frac{\omega_p}{u}, \quad \bar{\omega}_p = \frac{\omega_p l_p}{u}, \quad \bar{\bar{x}} = \frac{x}{l_p \frac{0.5\rho_A A p l_p}{m_p}}, \quad \bar{\bar{y}} = \frac{y}{l_p \frac{0.5\rho_A A p l_p}{m_p}}, \quad \bar{\bar{\theta}} = \frac{\theta}{0.5\rho_A A p l_p \frac{m_p}{m_p}}, \quad \bar{t} = \frac{tu}{l_p \frac{0.5\rho_A A p l_p}{m_p}}, \quad \Omega = \frac{m_p \theta}{0.5\rho_A A p u^2}$$  \hspace{1cm} (3-22)

\(\theta\) is angular displacement, \(\omega\) is angular velocity and \(C_M\) is pitching moment coefficient.

Baker showed that for compact debris which is allowed to fall for a long enough time, the particle will travel horizontally at the mean wind speed and vertically at its terminal velocity (figure 3-2). This work was extended by Richards et al. (2008) who presented a 3D model with 6 degrees of freedom for plate-like debris.
Debris flight models are becoming increasingly sophisticated at accounting for turbulence in the atmospheric boundary layer (Holmes, 2004), lift forces (Baker, 2007) and three dimensional motions including rotation (Richards, 2008). With the exception of Holmes (2004) who accounts for variation in horizontal and vertical wind velocities due to atmospheric turbulence, the debris flight models are entirely deterministic. These models have constant parameter inputs and solve for the flight of a single compact object. But, debris flight is not a deterministic phenomenon. Debris size of any given particle diameter will follow some probability distribution function. Turbulent fluctuations in the wind velocity will lead to variations in the flight distance and impact kinetic energy. Therefore, compact debris flight is a stochastic process in which there is statistical uncertainty for a range of input parameters that will results in uncertainty in the output parameters. The goal of this chapter is to investigate the stochastic nature of debris flight and the role of input uncertainty in the classical debris flight models through a series of Monte Carlo simulations based on the debris flight equations of Holmes (2004). In particular, the effect of statistical distributions of particle diameter, horizontal
turbulence intensity, and vertical turbulence intensity on the flight distance and impact kinetic energy are investigated.

While all care has been taken to make reasonable assessments of the appropriate input distributions to describe the variability in particle diameter and wind turbulence intensity, a detailed investigation of these distributions is beyond the scope of this study. Further work is needed to accurately parameterize these distributions. Rather, this paper focuses on quantifying the impact of input parameter uncertainty on the mean and variance in the flight path and impact kinetic energy.

The remainder of the chapter is structured as follows. Section 3.2 describes the model developed to investigate the effect of atmospheric turbulence and particle size uncertainty on the flight path. Section 3.3 presents results from numerical solutions of the debris flight equations for different atmospheric turbulence intensities, and appropriate distributions for gravel size. In section 3.4, various analytical approaches for estimating the impact of input parameter uncertainty on flight outcomes is presented. The results of a series of wind tunnel particle flight tests are shown in section 3.5 that demonstrate the validity of the stochastic modeling approach. The significance of the results and a more general discussion of the stochastic nature of debris flight are presented in section 3.6, along with conclusions.

3.2 Model development

This chapter considers the role of uncertainty in input parameters on debris flight. The complexity of turbulent atmospheric boundary layer (ABL) flows causes a significant variability/uncertainty in the value of wind velocity. Moreover, there is uncertainty in the size of
individual pieces of roof gravel whose size and shape are distributed over some range depending on the gravel size used (see figure 3-3).

**Figure 3-3**: Distribution of typical roof gravel particle diameter based on ASTM D 1863-05 (ASTM, 2005)

In order to model this uncertainty, the debris flight equations of Holmes (2004) (equations 3-7 and 3-8) were solved numerically by developing a code with Matlab. A series of Monte Carlo simulations were run to examine the effects of variability in gravel diameter $d_p$, horizontal wind turbulent intensity $I_x$ and vertical wind turbulent intensity $I_z$, on the mean flight distance and mean impact kinetic energy of a gravel particle released from rest at a height of $H$. Simulations were run randomly varying the particle diameter $d_p$, the horizontal wind turbulence intensity $I_x$ and vertical turbulence intensity $I_z$. Simulations were run keeping all three parameters constant, keeping two of the three parameters constant while varying the third,
keeping one constant and varying two parameters, and varying all three parameters (75 simulation series). Each set was run for a height of 50m with data on distance and impact kinetic energy reported at vertical distance of 1, 2, 5, 10, 20, and 50 m below the release height. This is equivalent to having 6 different release heights for each run. Each simulation consisted of 10,000 runs (total of 750,000 runs) with input variables generated using the MATLAB random number generation functions.

To verify that 10,000 runs is large enough to give accurate values of the mean flight distance and mean impact kinetic energy, one simulation was repeated 5 times with 10, 100, 500, 1000, 2000, 5000, and 10000 runs. As it can be seen in figure 3-4, by increasing the number simulations per set, the discrepancy in simulated mean flight distance decreases. When 10,000 simulations were run, the variability in the mean flight distance was less than 1%.

![Figure 3-4: Effect of number of runs on calculated result](image)

**Figure 3-4:** Effect of number of runs on calculated result
Following Holmes (2004), horizontal turbulence intensity is considered as \( I_x = \sigma_{u_x}/\overline{u} \) and vertical turbulence intensity is considered as \( I_z = \sigma_{u_z}/\overline{u} \), where \( \sigma_{u_x} \) and \( \sigma_{u_z} \) are the standard deviation of the horizontal and vertical turbulent fluctuations of wind velocity, respectively (figure 3-5). Wind velocity data sets were generated using the method of Holmes (1978) in which the wind velocities are generated in frequency space with power spectra appropriate for the ABL and phase off-sets generated using a uniform random distribution between 0 and \( 2\pi \). The only difference between the simulations of Holmes and the ones presented here is that the vertical and horizontal turbulent fluctuations were assumed to be uncorrelated. Different values of both horizontal and vertical turbulent intensities were used to establish a relationship between the extent of the fluctuations and the mean and standard deviation of the flight outcomes.

![Figure 3-5: Typical wind velocity time series](image)
The exact distribution function appropriate for gravel diameter $d_p$, is not known. For gravel, the size range is based on the largest and smallest sieves used to categorize the stone. Considering a general distribution function as shown schematically in figure 3-6(a), it can be seen that a small sampled portion will have an approximately linear distribution. However, a uniform distribution also gives a good first approximation, see figure 3-6(b), and requires no data on the statistical properties of the gravel source or manufacturing processes, only the largest and smallest stone size. For the simulations presented in this paper, $d_p$ was taken to be uniformly distributed between the maximum and minimum values for each size range as listed in ASTM D 1863-05 (ASTM, 2005). The mean and standard deviation are therefore given by

$$\overline{d_p} = \frac{d_{p_{\text{max}}} + d_{p_{\text{min}}}}{2}, \quad \sigma_{d_p} = \frac{d_{p_{\text{max}}} - d_{p_{\text{min}}}}{\sqrt{12}}$$

(3-23)

respectively (Harris and Stocker, 1998).

For each set of simulations, a set of random input variables were generated using the Matlab random number generator functions. These random numbers were then used as inputs for a 4th order Runge- Kutta code with a fixed time step that solved (3-7) and (3-8) numerically. Once the flight path had been calculated, the horizontal distance and particle kinetic energy were calculated, using cubic spline interpolation, for vertical displacements of 1, 2, 5, 10, 20, and 50 m. For each simulation 10,000 random input sets were generated and solved. The results of these simulations are presented below.
Figure 3-6: (a): Generic distribution function for a particular variable such as gravel size. (b): Sampled variable (for example due to sieving of gravel) showing uniform distribution approximation

3.3 Simulation results

Simulations were first run for the case in which all three input standard deviations, i.e. particle diameter $\sigma_d$, horizontal wind turbulence intensity $I_x$, and vertical wind turbulence intensity $I_z$ were zero. This provided a reference case for comparing the impact of input variability on both the mean and standard deviation of the flight distance and impact kinetic energy. This was followed by a set of Monte Carlo simulations in which simulations were run keeping two of the three parameters constant while varying the third, keeping one constant and varying two parameters, and varying all three parameters (75 simulations). A summary of the input parameters used is given in table 3-1. The different cases are listed in table A in appendix A. Based on 10,000 simulations per case, the mean and standard deviation of the flight distance and impact kinetic energy were calculated and compared to the results of a simulation using the
mean of the input values for that particular case. In all cases, the mean horizontal velocity is assumed to be uniform with height. This is a reasonable assumption for short flight times such as are typical for debris being blown off buildings in which the flight time is significantly less than the averaging time required to establish a smooth mean boundary layer profile. This assumption may be inappropriate for debris flight that has longer flight times such as ember flight from wild fires in which embers can be lofted high into the atmosphere. Ignoring mean velocity variation with height is a conservative assumption in that it leads to a worse case (longer flight distance), see Karimpour and Kaye (2010).

**Table 3-1:** List of input parameters and distributions used for the Monte Carlo simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>$d_p$ Range / $\sigma_{d_p}$ Range</th>
<th>$I_x$</th>
<th>$I_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Diameter $d_p$</td>
<td>7.125mm</td>
<td>4.75-9.5/1.37-3.10 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.625mm</td>
<td>4.75-12.5 / 2.23 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.875mm</td>
<td>4.75-19 / 4.11 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.25mm</td>
<td>9.5-19 / 2.74 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind Velocity $u$</td>
<td>18.87 m/s</td>
<td>0.05, 0.1, 0.03, 0.06, 0.09, 0.15, 0.2, 0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.76 m/s</td>
<td>0.2</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>24.36 m/s</td>
<td>0.2</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.69 m/s</td>
<td>0.2</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Initial Height $H$</td>
<td>1, 2, 5, 10, 20, 50 m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As mentioned in section 3.2, initial uncertainty was introduced into the model by considering a standard deviation for particle diameter $\sigma_{d_p}$, horizontal fluctuation of wind velocity $\sigma_{u_x}$, and vertical fluctuation of wind velocity $\sigma_{u_z}$. The dimensionless standard deviation, or coefficient of variation $CV$, for input parameters is defined as:

$$CV_{d_p} = \frac{\sigma_{d_p}}{\bar{d}_p}, \quad I_x = \frac{\sigma_{u_x}}{\bar{u}}, \quad I_z = \frac{\sigma_{u_z}}{\bar{u}}$$

(3-24)

where an over bar represents the mean value of the parameter.

Output results of the simulations were recorded as mean value of particle flight distance $\bar{x}$, standard deviation of particle mean flight distance $\sigma_{\bar{x}}$, mean value of kinetic energy at the time of impact $\bar{ke}$ and standard deviation of mean kinetic energy at the time of impact $\sigma_{\bar{ke}}$. The coefficient of variation (dimensionless standard deviation) for flight distance and impact kinetic energy are defined as:

$$CV_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\bar{x}}, \quad CV_{\bar{ke}} = \frac{\sigma_{\bar{ke}}}{\bar{ke}}$$

(3-25)

To compare the stochastic condition with deterministic one, flight distance and impact kinetic energy were simulated by using the mean value for particle diameter $\bar{d}_p$ and for velocity $\bar{u}$, without considering any standard deviation for them, i.e. $\sigma_{d_p} = 0$ and $\sigma_{u_x} = \sigma_{u_z} = 0$. The flight distance and impact kinetic energy for this deterministic condition is denoted $x(\bar{d}_p)$ and $ke(\bar{d}_p)$. Then, dimensionless values of mean flight distance and mean impact kinetic energy are defined as:

$$X = \frac{\bar{x}}{x(\bar{d}_p)}, \quad KE = \frac{\bar{ke}}{ke(\bar{d}_p)}$$

(3-26)

These parameters represent the mean outcome (distance or kinetic energy) divided by the outcome resulting from the mean input condition. A value of $X$ or $KE$ other than 1 indicates
that the introduction of variability into the problem, either through particle size uncertainty or atmospheric turbulence, alters the mean outcomes as well as the standard deviation.

3.3.1 Effect of input parameter uncertainty on mean flight distance and mean impact kinetic energy

Effect of uncertainty in input parameters i.e. particle diameter $d_p$, horizontal turbulence intensity $l_x$ and vertical turbulence intensity $l_z$ on the mean flight distance $\bar{x}$ and mean impact kinetic energy $\bar{ke}$ was investigated. All simulations in this section were run for $\bar{d}_p = 7.125\ mm$, $\bar{u} = 18.87\ m/s$ and $H = 50\ m$.

Results are presented in form of dimensionless flight distance $X = \bar{x}/x(\bar{d}_p)$ and dimensionless impact kinetic energy $KE = \bar{ke}/ke(\bar{d}_p)$ versus the dimensionless standard deviation for input parameters as $\sigma_n/\bar{n}$ where $n$ represents $d_p$, $u_x$, or $u_z$ (figures 3-7 and 3-8).

By introducing an uncertainty into the system, the dimensionless flight distance and dimensionless impact kinetic energy increase. For the case of flight distance, results show that the mean flight distance is increasing (up to 13 percent) as $\sigma_{\bar{n}}/\bar{n}$ increases. This implies that ignoring an uncertainty in the input parameters will lead to an under estimation of the flight distance. That is, the mean flight distance is greater than the flight distance of the particle with mean diameter.

A similar trend is observed for the mean impact kinetic energy and it increases up to 64 percent by introducing uncertainty in the input parameters. However, in this case the results show the mean impact kinetic energy is highly sensitive to the uncertainty of particle diameter $\sigma_{\bar{d}_p}/\bar{d}_p$ but is significantly less sensitive to variation of $l_x$ and $l_z$. Again, the mean impact kinetic
energy is greater than the impact kinetic energy of the particle with mean diameter. A detailed discussion of the physical significance of these results and approximate analytical descriptions of the variability are presented in section 3.4.

**Figure 3-7:** Effect of input parameters variation on mean flight distance

**Figure 3-8:** Effect of input parameters variation on mean impact kinetic energy
3.3.2 Effect of input parameter uncertainty on output results distribution

To investigate the effect of uncertainty in input parameters i.e. particle diameter $d_p$, horizontal turbulence intensity $l_x$ and vertical turbulence intensity $l_z$ on the distribution about mean flight distance $\bar{x}$ and mean impact kinetic energy $\bar{ke}$, a series of simulations were run by keeping two of the three parameters constant while varying the third. All simulations in this section were run for $\bar{d}_p = 7.125 \, mm$, $\bar{u} = 18.87 \, m/s$ and $H = 50 \, m$.

The effect of introducing uncertainty in input parameters on flight distance and impact kinetic energy are presented as histograms in figures 3-9 and 3-10. Results are shown first for $\sigma_{d_p} = 1.37 \, mm$, second for $l_x = 0.2$ and third for $l_z = 0.12$ and forth for all three parameters varying at the same time. Introducing an uncertainty in any of $d_p$, $l_x$ and $l_z$ cause an uncertainty in both flight distance and impact kinetic energy. Introducing uncertainty into all three ($d_p$, $l_x$ and $l_z$) at the same time causes a wider range of outcomes compared to varying only one at a time.

From these histograms, the dimensionless standard deviation for flight distance $\sigma_{\bar{x}}/\bar{x}$ and impact kinetic energy $\sigma_{\bar{ke}}/\bar{ke}$ versus the dimensionless standard deviation of the input parameters, $\sigma_{\bar{n}}/\bar{n}$ where $n$ represents $d_p$, $u_x$, or $u_z$ were calculated (figures 3-11 and 3-12). By introducing uncertainty into the system, uncertainty appears in the predicted results. Also, as the value of dimensionless standard deviation of input parameters increases, the uncertainty within the results increases too. Dimensionless standard deviation for flight distance increases up to 0.37 while for the impact kinetic energy it increases up to 1. For the case of flight distance, results show that flight distance is more sensitive to the $\sigma_{d_p}/\bar{d}_p$ for small value of $\sigma_{\bar{n}}/\bar{n}$, but as $\sigma_{\bar{n}}/\bar{n}$ increases, $d_p$, $l_x$ and $l_z$ have a similar impact on $\sigma_{\bar{x}}/\bar{x}$. For the impact kinetic energy,
the results show the same trend. Introducing the uncertainty into the system will result in uncertainty within the predicted results, and the larger $\sigma_n/\bar{n}$ leads to the larger $\sigma_{\text{ke}}/\bar{\text{ke}}$. For the impact kinetic energy, uncertainty in the $\sigma_{d_p}/\bar{d}_p$ has much more effect compared to the $I_x$ and $I_z$.

![Histograms showing flight distance distribution](image)

**Figure 3-9:** Effect of uncertainty in input parameters on flight distance distribution. Uncertainty in $d_p$ (top right), in $I_x$ (top left), in $I_z$ (bottom right), in all $d_p$, $I_x$ and $I_z$ (bottom left)
Figure 3-10: Effect of uncertainty in input parameters on impact kinetic energy distribution.
Uncertainty in $d_p$ (top right), in $I_x$ (top left), in $I_z$ (bottom right), in all $d_p$, $I_x$ and $I_z$ (bottom left)

Figure 3-11: Effect of input parameters variation on mean flight distance variation
3.3.3 Combination of input parameter uncertainty on mean flight distance and mean impact kinetic energy

In sections 3.3.1 and 3.3.2, effect of introducing uncertainty only in one input parameter at a time was investigated. That is, variation in \( d_p \), \( I_x \), and \( I_z \) was studied separately. In this section the combination effect of introducing uncertainty in the input parameters on the mean flight distance and mean impact kinetic energy is studied. In this case, simulations were run by keeping two of the three parameters constant while varying the third, and by varying all three parameters at the same time. The simulations were run for 6 different initial heights \( H \) equal to 1, 2, 5, 10, 20, and 50 m. All simulations in this section were run for \( \bar{d}_p = 7.125 \text{ mm} \) and \( \bar{u} = 18.87 \text{ m/s} \). In these simulation, inputs standard deviation were considered as \( \sigma_{d_p} = 1.37 \text{ mm} \), \( I_x = 0.2 \) and \( I_z = 0.12 \). Results are presented in the form of plots of dimensionless
flight distance $X = \bar{x}/(\bar{d}_p)$ and impact kinetic energy $KE = \bar{ke}/ke(\bar{d}_p)$ versus release height $H$ in figures 3-13 and 3-14 respectively.

Figure 3-13 shows the non-dimensional mean flight distance as a function of height for the cases of single parameter variation and multiple parameter variation. Introducing the uncertainty into all input parameters at the same time leads to a much higher mean flight distance (up to 4 percent for the case with $\sigma_{d_p} = 1.37 \text{ mm}$, $I_x = 0.2$ and $I_z = 0.12$) compared to varying nothing or varying one parameter at a time. Although larger dimensionless flight distances $X$ are predicted by introducing uncertainty into the initial parameters, this effect decreases with increasing release height $H$.

The mean impact kinetic energy was calculated varying $d_p$, $I_x$, and $I_z$ individually and all together (figure 3-14). Results show that introducing uncertainty into $d_p$, $I_x$ leads to higher
impact kinetic energy, while varying $I_z$ has minimal impact on the predicted impact kinetic energy. Varying the $d_p$ has a large effect, varying $I_x$ has small effect and varying $I_z$ has almost no effect on the predicted $KE$. Also, introducing uncertainty into all three input parameters at the same time leads to a much higher $KE$ (up to 13 percent for the case with $\sigma_{d_p} = 1.37 \text{ mm}$, $I_x = 0.2$ and $I_z = 0.12$) compared to varying nothing or varying one parameter at a time. The behavior of $KE$ with respect to the initial release height is different from parameter to parameter. The impact kinetic energy $KE$ increases with increasing height when varying $d_p$. However, $KE$ decreases with increasing $H$ when $I_x$ and $I_z$ are introduced. By introducing uncertainty into all $d_p$, $I_x$, and $I_z$, as the initial $H$ increase, $KE$ also increases. For very large initial $H$, $KE$ is only dependent on the variation in $d_p$ while $I_x$ and $I_z$ have little effect.

**Figure 3-14:** Combination effect of input parameter uncertainty on mean impact kinetic energy
3.4 Analysis of outcome variation

As shown above, variation in particle diameter, horizontal wind velocity, and vertical wind velocity increases the mean flight distance and impact kinetic energy. For both these outcome parameters, both the mean and standard deviation of the outcome is altered by input uncertainty. It is the goal of this section to develop an analysis of the debris flight equations that explains this variation. First, an analysis of the increase in the flight distance and impact kinetic energy is presented. This is followed by analysis of the variation in outcome standard deviation as a function of input standard deviation. Finally, a method for calculating the impact of multiple varying parameters is given.

3.4.1 Impact of input variability on outcome mean

The effect of introducing uncertainty in an input parameter on the mean particle flight distance and mean impact kinetic energy is investigated analytically in this section. First, an analytical estimate of flight distance and impact kinetic energy as a function of the variation in the particle diameter keeping all other parameters constant is presented. For this, it is considered that there is only variability in particle diameter between $d_{p_{min}}$ and $d_{p_{max}}$ with mean value of $\bar{d}_p$ and standard deviation of $\sigma_{d_p}$, and there is no turbulence intensity i.e. $I_x = 0$ and $I_z = 0$. For any given particle diameter $d_p$, it could be written:

$$\Delta d_p = d_p - \bar{d}_p$$  \hspace{1cm} (3-27)

If the flight distance for the mean particle diameter for the deterministic condition i.e. $\sigma_{d_p} = 0$, $I_x = 0$ and $I_z = 0$ is equal to $x(\bar{d}_p)$, then the flight distance for particle with diameter of $(\bar{d}_p + \Delta d_p)$, could be approximated by the first 3 terms of a Taylor series as:
\[ x(\bar{d}_p + \Delta d_p) = x(\bar{d}_p) + x'(\bar{d}_p) \times \Delta d_p + \frac{x''(\bar{d}_p)}{2} \times (\Delta d_p)^2 + O(\Delta d_p)^3 \]  

(3-28)

where

\[ x'(\bar{d}_p) = \frac{d(x(\bar{d}_p))}{d(\bar{d}_p)} \]  

(3-29)

\[ x''(\bar{d}_p) = \frac{d^2(x(\bar{d}_p))}{d(\bar{d}_p)^2} \]  

(3-30)

Varying \( d_p \) randomly \( N \) times between \( d_{p_{\text{min}}} \) and \( d_{p_{\text{max}}} \) will give \( N \) estimates for flight distance. The mean value of these \( N \) estimated flight distances could be calculated as:

\[ \bar{x}(\bar{d}_p + \Delta d_p) = \frac{\sum_N x(\bar{d}_p) + x'(\bar{d}_p) \times (\Delta d_p) + \frac{x''(\bar{d}_p)}{2} \times (\Delta d_p)^2}{/N} \]  

(3-31)

\[ \bar{x}(\bar{d}_p + \Delta d_p) = x(\bar{d}_p) + x'(\bar{d}_p) \times \frac{\sum_N (\Delta d_p)}{N} + \frac{x''(\bar{d}_p)}{2} \times \frac{\sum_N (\Delta d_p)^2}{N} \]  

(3-32)

By dividing the (32) by \( x(\bar{d}_p) \) and substituting \( (\bar{d}_p + \Delta d_p) \) with \( d_p \) (see (3-27)), (3-32) becomes:

\[ \frac{\bar{x}(d_p)}{x(d_p)} = 1 + \frac{x'(\bar{d}_p)}{x(d_p)} \times \frac{\sum_N (\Delta d_p)}{N} + \frac{x''(\bar{d}_p)}{2} \times \frac{\sum_N (\Delta d_p)^2}{N} \]  

(3-33)

The standard deviation of particle diameter is written as:

\[ \sigma_{d_p} = \sqrt{\frac{1}{N-1} \sum_1^N \left( (d_p)_i - (\bar{d}_p) \right)^2} = \sqrt{\frac{1}{N-1} \sum_1^N (\Delta d_p)_i^2} \]  

(3-34)

When \( N \) is going toward infinity, it could be said that for any given \( (d_p)_1 \) there would be a \( (d_p)_2 \) which \( (\Delta d_p)_1 = -(\Delta d_p)_2 \). Therefore,

\[ \lim_{N \to \infty} \frac{\sum_N (\Delta d_p)}{N} = 0 \]  

(3-35)

So, by substituting (3-34) and (3-35) into (3-33), one gets:

\[ \frac{\bar{x}(d_p)}{x(d_p)} = 1 + \frac{x'(\bar{d}_p)}{2x(d_p)} \times \sigma_{d_p}^2 \]  

(3-36)
To estimate the value of \( x''(\bar{d}_p)/x(\bar{d}_p) \) we turn to Baker (2007) who showed that compact debris which is allowed to fall for a long enough time, will travel horizontally at the mean wind velocity and vertically at its terminal velocity (figure 2) which means that:

\[
u_p = u \quad \text{and} \quad v_p = v_{p_T} \tag{3-37}\]

where \( v_{p_T} \) is the particle terminal velocity. As a simple assumption, it is assumed that particle flies with constant velocity, horizontally with wind velocity and vertically with terminal velocity throughout its flight. Then, the horizontal and vertical flight distance could be written as:

\[
\frac{x}{z} = \frac{u_p t}{v_{p_T} t} = \frac{ut}{v_{p_T} t} \tag{3-38}\]

The particle terminal velocity \( v_{p_T} \) is calculated by considering equilibrium between drag force and particle weight, \( F_D = F_W \), in vertical direction as:

\[
\frac{1}{2} C_D \rho_a \left( \frac{\pi d_p^2}{4} \right) v_{p_T}^2 = \rho_p \left( \frac{\pi d_p^2}{6} \right) g \tag{3-39}\]

\[
v_{p_T} = \sqrt{\frac{4 \rho_p g}{3 \rho_a C_D}} d_p = \sqrt{\alpha d_p} \tag{3-40}\]

where

\[
\alpha = \frac{4 \rho_p g}{3 \rho_a C_D} \tag{3-41}\]

Substituting (3-40) into (3-38), will give

\[
x(d_p) = \frac{zu}{\sqrt{\alpha d_p}} \tag{3-42}\]

\[
x'(d_p) = -\frac{zu}{2\sqrt{\alpha d_p}}^{\frac{3}{2}} \tag{3-43}\]

\[
x''(d_p) = \frac{3zu}{4\sqrt{\alpha d_p}}^{\frac{5}{2}} \tag{3-44}\]

So by considering (3-42), (3-43) and (3-44), one gets
\[
\frac{x'(d_p)}{x(d_p)} = -\frac{1}{2} \times \frac{1}{d_p}
\]  
(3-45)

and

\[
\frac{x''(d_p)}{x(d_p)} = \frac{3}{4} \times \frac{1}{d_p^2}
\]  
(3-46)

Substituting (3-46) into (3-36), leads to

\[
\frac{x(d_p)}{\bar{x}(d_p)} = 1 + \frac{3}{8} \times \left(\frac{\sigma_\nu}{d_p}\right)^2
\]  
(3-47)

Equation (3-47) is compared with results from simulation in figure 3-15. As the initial height of the particle release \(H\) increases, the results are getting closer to the (3-47). Equation (3-47) was derived using the assumption that particles fly with a constant horizontal velocity equal to the wind velocity and a constant vertical velocity equal to the terminal velocity, so it is expected that if a particle is released from very large \(H\), the simulation results will get much closer to the value predicted by (3-47). The larger discrepancy at higher values of \(\sigma_\nu/d_p\) is also due to ignoring the higher order terms in the Taylor series (3-28).
Figure 3-15: Effect of particle diameter variation on dimensionless flight distance, line represents (3-36)

The same method could be used to develop an analytical solution for the effect of introducing uncertainty on the impact kinetic energy. By the same argument, (3-36) could be written for the impact kinetic energy as:

\[
\frac{\bar{ke}(d_p)}{ke(d_p)} = 1 + ke''(\bar{d}_p) \times \sigma_{d_p}^2 
\]  

(3-48)

Again, by considering (3-37) and (3-40), particle impact kinetic energy would be:

\[
ke(d_p) = \frac{1}{2} m_p (u_p^2 + v_p^2) = \frac{\rho_p \pi}{12} d_p^3 (u^2 + \alpha d_p) 
\]  

(3-49)

And from there:

\[
ke'(d_p) = \frac{\rho_p \pi}{12} d_p^2 (3u^2 + 4\alpha d_p) 
\]  

(3-50)

\[
ke''(d_p) = \frac{6\rho_p \pi}{12} d_p (u^2 + 2\alpha d_p) 
\]  

(3-51)
So that would be:

\[ \frac{ke'(d_p)}{ke(d_p)} = \frac{1}{d_p} \times \frac{(3u^2+4\alpha d_p)}{(u^2+\alpha d_p)} \]  \hspace{1cm} (3-52)

\[ \frac{ke''(d_p)}{ke(d_p)} = \frac{6}{d_p^2} \times \frac{(u^2+2\alpha d_p)}{(u^2+\alpha d_p)} \]  \hspace{1cm} (3-53)

Making the simplifying assumptions

\[ \frac{(3u^2+4\alpha d_p)}{(u^2+\alpha d_p)} \approx 3 \quad \text{and} \quad \frac{ke'(d_p)}{ke(d_p)} \approx \frac{3}{d_p} \]  \hspace{1cm} (3-54)

\[ \frac{(u^2+2\alpha d_p)}{(u^2+\alpha d_p)} \approx 1 \quad \text{and} \quad \frac{ke''(d_p)}{ke(d_p)} \approx \frac{6}{d_p^2} \]  \hspace{1cm} (3-55)

equation (3-48) can be written as:

\[ \frac{ke(d_p)}{ke(d_p)} \approx 1 + 3 \left( \frac{\sigma_p}{d_p} \right)^2 \]  \hspace{1cm} (3-56)

Equation (3-56) is compared with results from simulation in figure 3-16. The proposed equation (3-56) is in reasonable agreement with the results for high initial releasing height \( H \).

The difference is largely due to the simplifying assumptions in (3-54) and (3-55).
Effect of introducing uncertainty in the horizontal turbulence intensity on the mean particle flight distance and mean impact kinetic energy could be investigated analytically by considering the horizontal wind velocity as:

\[ u = \bar{u} + u'_x \]  

(3-57)

where \( u'_x \) is the instantaneous horizontal wind velocity fluctuation. In the drag equation, the velocity is squared so one might expect the appropriate mean velocity (\( \bar{U} \)) to be the mean of the square of the individual velocities. That is

\[ \bar{U} = \sqrt{\bar{u}^2} = \sqrt{\bar{u}^2 + 2(\bar{u}u') + u'^2} \]  

(3-58)

or

\[ \bar{U} = \bar{u} \sqrt{1 + I_x^2} \]  

(3-59)
That is, the flight distance and impact kinetic energy can be calculated by considering a constant wind velocity equal to \( u = \bar{u}\sqrt{1 + I_x^2} \).

Flight distance and kinetic energy results were calculated using a constant wind speed equal given by (3-59) and plotted against the results of the Monte Carlo simulations presented in figures 7 and 8. These results are shown in figures 3-17 and 3-18. As results show in figures 3-17 and 3-18, a constant velocity equal to \( u = \bar{u}(1 + I_x^2) \) will predict the flight distance and impact kinetic energy of particle almost equal to the mean values that calculated from 10,000 run. It means that the \( u = \bar{u}(1 + I_x^2) \) could be used for a fast and simple prediction of mean flight distance and mean impact kinetic energy due to the presence of horizontal turbulence intensity.

![Graph](image)

**Figure 3-17:** Comparison of flight distance for \( u = \bar{u}\sqrt{1 + I_x^2} \) with simulation results for randomly generated \( u \). The line has a slope of 1 for comparison.
3.4.2 Impact of input variability on outcome variability

In this section, an analytical estimate of the variation in flight distance and impact kinetic energy as a function of the variation in the particle diameter keeping all other parameters constant is presented. Using Taylor series, the flight distance of particle can be written as:

$$x(\bar{d}_p - \Delta d_p) = x(\bar{d}_p) - x'(\bar{d}_p) \times \Delta d_p + \frac{x''(\bar{d}_p)}{2} \times (\Delta d_p)^2 + O(\Delta d_p)^3$$  \hspace{1cm} (3-60)

or, moving the first term on the right hand side over and dividing by the mean flight distance from multiple realizations one gets

$$\frac{\Delta x(\bar{d}_p)}{\bar{x}(\bar{d}_p)} = -\frac{x'(\bar{d}_p)}{\bar{x}(\bar{d}_p)} \times \Delta d_p + \frac{x''(\bar{d}_p)}{2\bar{x}(\bar{d}_p)} \times (\Delta d_p)^2$$  \hspace{1cm} (3-61)
Substituting (3-47) leads to

\[
\frac{\Delta x(d_p)}{\bar{x}(d_p)} = \left( \frac{1}{1 + \frac{3}{8} \times \frac{(\sigma_p)}{d_p}} \right)^2 \left( -\frac{x'(d_p)}{x(d_p)} \times \Delta d_p + \frac{x''(d_p)}{2x(d_p)} \times (\Delta d_p)^2 \right). \tag{3-62}
\]

Finally, substituting in (3-45) and (3-46), one gets

\[
\frac{\Delta x(d_p)}{\bar{x}(d_p)} = \left( \frac{1}{2} \times \frac{\Delta d_p}{d_p} + \frac{3}{8} \times \left( \frac{\Delta d_p}{d_p} \right)^2 \right) / \left( 1 + \frac{3}{8} \times \left( \frac{\sigma_p}{d_p} \right)^2 \right) \tag{3-63}
\]

The variation in \(d_p\) is taken to be the standard deviation of the uniform distribution \((\sigma_{d_p} = \sqrt{\frac{d_{p,\text{max}} - d_{p,\text{min}}}{12}}, \text{ see Harris and Stocker, 1998})\). Therefore, the variation in flight distance can be found to be:

\[
\frac{\sigma_x}{\bar{x}} \approx \left( \frac{1}{2} \times \frac{\sigma_p}{d_p} + \frac{3}{8} \times \left( \frac{\sigma_p}{d_p} \right)^2 \right) / \left( 1 + \frac{3}{8} \times \left( \frac{\sigma_p}{d_p} \right)^2 \right) \tag{3-64}
\]

Equation (3-64) is compared with the simulation results in figure 3-19. The proposed analytical equation slightly underpredicts the results though exhibits the trend found in the simulations. This is due to the simplified relationship between flight distance and particle diameter, by considering a constant horizontal velocity equal to wind velocity and a constant vertical velocity equal to the terminal velocity. If the particle is released from very high initial height, the simulation results and analytical model would be closer.
The same method could be used to develop an analytical estimate for the effect of particle diameter variation on impact kinetic energy. By the same argument, the impact kinetic energy of the particle is:

$$ke(\bar{d}_p + \Delta d_p) \approx ke(\bar{d}_p) + ke'(\bar{d}_p) \times \Delta d_p + O(\Delta d_p)^2$$  \hspace{1cm} (3-65)

$$\frac{\Delta ke(\bar{d}_p)}{ke(d_p)} \approx \frac{ke'(\bar{d}_p)}{ke(d_p)} \times \Delta d_p$$  \hspace{1cm} (3-66)

Considering (3-56) leads to:

$$\frac{\Delta ke(\bar{d}_p)}{ke(d_p)} \approx \left( \frac{ke'(\bar{d}_p)}{1 + 3\left(\frac{\sigma_p}{\bar{d}_p}\right)^2} \times \Delta d_p \right).$$  \hspace{1cm} (3-67)

Finally, substituting in (3-54), then:

$$\frac{\Delta ke(\bar{d}_p)}{ke(d_p)} \approx \left( 3 \times \frac{\Delta d_p}{\bar{d}_p} \right) / \left( 1 + 3\left(\frac{\sigma_p}{\bar{d}_p}\right)^2 \right)$$  \hspace{1cm} (3-68)
or

\[
\frac{\sigma_{ke}}{ke} \approx \left( 3 \times \frac{\sigma_d}{d_p} \right) / \left( 1 + 3 \left( \frac{\sigma_p}{d_p} \right)^2 \right)
\]

(3-69)

In order to test this model, a series of simulations were run in which the range, and therefore the standard deviation of particle diameters was varied and the standard deviation of the resulting impact kinetic energy calculated. A plot of \(\sigma_{ke}/\bar{ke}\) versus \(\sigma_d/\bar{d}_p\) is shown in figure 3-20 along with a line of the proposed model (3-69) for comparison. Clearly for small variations in \(d_p\), the theory does an excellent job of predicting the variation in impact kinetic energy. However, for larger variations, the theory slightly under predicts the variation. This difference is due to assumptions that \((3u^2 + 4ad_p)/(u^2 + ad_p) \approx 3\) and \((u^2 + 2ad_p)/(u^2 + ad_p) \approx 1\).

Figure 3-20: Effect of particle diameter variation on impact kinetic energy variation, line represents (3-69)
This is an interesting result given that the data in figures 3-19 and 3-20 is derived from simulations of the full equations for an initially stationary particle, whereas the proposed model used a simplified analytic solution that assumed that the particle was travelling at a constant velocity throughout the flight. This suggests that such simple models have value in terms of the appropriate scaling of results and simple uncertainty analysis.

In general, the flight distance and impact kinetic energy will be a function of the wind velocity, diameter, initial height, etc. Therefore, the general variability in the flight distance and impact kinetic energy could be evaluated by considering all these effects at the same time.

3.4.3 Impact of multiple input variations

In order to assess the outcome uncertainty based on the input uncertainty of a particular parameter, for example the particle diameter, one needs to run a simulation examining just that parameter. However, in order to assess the overall outcome variation when two or more of the input parameters have uncertainty, a full simulation with multiple input uncertainties would be needed. If the uncertainty in each input parameter is statistically independent of the others, then the Bienayme formula (Loeve, 1977) can be used to predict the total standard deviation in the outcomes:

$$\sigma_{total}^2 = \sum \sigma_n^2$$

(3-70)

where $\sigma_{total}$ is the standard deviation accounting for the complete stochastic problem and the $\sigma_n$ are the outcome standard deviations due to the individual input parameters. To verify (3-70), the square of standard deviation of calculated mean flight distance when all the input parameters i.e. $d_p$, $l_x$, and $l_z$ are varying at the same time $(\sigma_x^2)_{d_plzlz}$ is plotted versus the sum of the square of the standard deviations of calculated mean flight distance when only...
one of the input parameters is varied, \((\sigma_{d_p}^2) + (\sigma_{I_x}^2) + (\sigma_{I_z}^2)\). See figure 3-21. In a same way, the standard deviation in the impact kinetic energy, \((\sigma_{ke}^2)_{d_p/l, I_z}\) was plotted versus \((\sigma_{ke}^2)_{d_p} + (\sigma_{ke}^2)_{I_x} + (\sigma_{ke}^2)_{I_z}\) in figure 3-22. In both figures 21 and 22, the Bienayme formula can be seen to accurately predict the standard deviation of the flight distance and impact kinetic energy.

Figures 3-21 and 3-22 also indicate that 10,000 simulations per case is adequate, and leads to a good approximation of the outcome standard deviation.

**Figure 3-21:** Variation in outcome estimated using (3-70), line has a slope of 1 for comparison
Though this approach can be used to estimate the total outcome variation given some known individual outcome variations, the individual outcome variations still need to be calculated either analytically or via simulations. Also, the analysis above only leads to predictions of the variance in outcome due to multiple input variances. It does not indicate how the mean outcome will vary due to multiple varying inputs.

3.5 Wind Tunnel Tests

A series of ball-drop experiments were conducted in the Clemson University Boundary Layer Wind Tunnel to assess the validity of the statistical modeling approach presented. A remotely controlled robot hand was used to hold the particle and release it. The flight of the particle was filmed using a high definition video camera. The horizontal flight distance was measured for each test as well as the wind velocity at the release height. In the test, only the

Figure 3-22: Variation in outcome estimated using (3-70), line has a slope of 1 for comparison
horizontal wind velocity was varied and the particle diameter was kept constant. The measured mean and standard deviation of the wind velocity along with the particle diameter from the experiments were used as inputs into a Monte Carlo simulation. A total of 10,000 simulations were run and the resulting distribution of predicted flight distance was compared to the experimentally measured flight distance distribution. The drag coefficient was taken to be $C_D = 0.5$.

In the tests, a ball with a constant diameter of $d_p = 37.8 \text{ mm}$ and a density of $\rho_p = 89.9 \text{ kg/m}^3$ was dropped from a height of 0.8 m. The wind velocity measurements were separated into 1 second bursts, and the distribution of the 1 second average gust was calculated. For the fan speed used in these tests, the mean one second wind velocity was $\bar{u} = 6.54 \text{ m/s}$ and the standard deviation was $\sigma_u = 0.41 \text{ m/s}$. A total of 128 experiments were run with the typical flight time less than one second. The Monte Carlo simulations were run with a fixed particle diameter and constant wind speed during flight. In these simulations the mean wind speed (one second average) was taken to be normally distributed with mean and standard deviation as measured.

Histograms of the experimental and simulated flight distance are shown in figure 3-23. The measured mean flight distance was 0.42 m while the simulated mean was 0.43 m. The measured standard deviation was 0.047 m compared to the simulated standard deviation of 0.049 m. Comparison between experimental and simulation results represents an excellent agreement.
3.6 Discussion and conclusions

Debris flight models are almost exclusively deterministic. These models typically assume known fixed input parameters such as wind velocity and particle size as well as constant coefficients such as the drag coefficient. However, such determinism is very rarely the case and debris flight modeling can be improved by accounting for model input uncertainty. The results presented above indicate that failure to account for uncertainty in the particle size, horizontal turbulence intensity and vertical turbulence intensity will result in an incorrect prediction of the mean flight distance and mean impact kinetic energy, and no information about the spatial distribution of the particle impact location or variation in impact kinetic energy.

The use of Monte Carlo simulations, as described in this chapter, provides a means for quantifying the influence of input uncertainty on the resulting flight characteristics (mean and variance of flight distance and impact kinetic energy). Running a series of simulations varying each input parameter i.e. particle diameter, horizontal turbulence intensity and vertical turbulence intensity separately shows the effect of input parameters variability on the simulation outcome. Introducing uncertainty in any of particle diameter, horizontal turbulence...
intensity and vertical turbulence intensity, leads to larger mean value for flight distance (up to 13 percent) and impact kinetic energy (up to 64 percent), compared to the condition where there is no variability in input parameters. The mean flight distance and mean impact kinetic energy increase with increasing variability in input parameters (figures 7 and 8). Uncertainty in the particle diameter has a significant impact on the results. Introducing horizontal turbulence intensity has considerable effect on results, while vertical turbulence intensity has only a small effect on the results. Also, introducing variability in input parameters leads to variability in flight distance and impact kinetic energy (figures 9-12). By introducing variability in input parameters, dimensionless standard deviation for flight distance increases up to 0.37 while for the impact kinetic energy it increases up to 1. Larger input variability will produce larger output variability.

Varying particle diameter, horizontal turbulence intensity and vertical turbulence intensity at the same time leads to larger flight distance (up to 4 percent for the case with $\sigma_{d_p} = 1.37 \, mm$, $l_x = 0.2$ and $l_z = 0.12$) and impact kinetic energy (up to 13 percent for the case with $\sigma_{d_p} = 1.37 \, mm$, $l_x = 0.2$ and $l_z = 0.12$) compared to condition that varying nothing or varying one parameter at a time (figures 13 and 14). The effect of introducing uncertainty on mean flight distance decreases with increasing initial release height, whereas the impact kinetic energy mean increases with increasing height. For very high release heights, the impact kinetic energy almost only depends on particle diameter variation.

A number of analytical approaches to understanding and quantifying the stochastic nature of debris flight were presented that explain the broad trends observed in the data. However, a full analysis of the impact of parameter uncertainty and variability is better realized through the Monte Carlo simulation approach presented.
The Monte Carlo simulation approach was tested against a series of ball-drop experiments in the Clemson University Boundary Layer Wind Tunnel. The simulations accurately predicted both the mean and standard deviation of the flight distance.

In summary, the results presented above demonstrate the following:

1. By introducing an uncertainty into the system, the mean flight distance and mean impact kinetic energy increase, while ignoring any uncertainty in the input parameters will lead to an under estimation of the flight distance and impact kinetic energy.

2. The mean flight distance is sensitive to uncertainty of $\sigma_{d_p}/\bar{d}_p$, $l_x$ and $l_z$ while the mean impact kinetic energy is more sensitive to the uncertainty of $\sigma_{d_p}/\bar{d}_p$ than to $l_x$ and $l_z$.

3. By introducing uncertainty into the system, uncertainty appears in the predicted results.

4. The $\sigma_{x}/\bar{x}$ is more sensitive to the $\sigma_{d_p}/\bar{d}_p$ than $l_x$ and $l_z$ for small value of $\sigma_{d_p}/\bar{d}_p$, $l_x$ and $l_z$, but they have a similar impact on $\sigma_{x}/\bar{x}$ as the input uncertainty increases.

5. Uncertainty in $\sigma_{d_p}/\bar{d}_p$ has much more effect on $\sigma_{x}/\bar{x}$ compared to the $l_x$ and $l_z$.

6. Introducing the uncertainty into all input parameters at the same time leads to a much higher mean flight distance and mean impact kinetic energy compared to varying nothing or varying one parameter at a time.

7. Analysis revealed that uncertainty in flight distance and impact kinetic energy has the following approximate relationships:

\[ \frac{\bar{x}(d_p)}{\bar{x}(d_p)} \approx 1 + \frac{3}{8} \times \left( \frac{\sigma_p}{\bar{d}_p} \right)^2 \]

\[ \frac{\bar{k}(d_p)}{\bar{k}(d_p)} \approx 1 + 3 \left( \frac{\sigma_p}{\bar{d}_p} \right)^2 \]

\[ \frac{\sigma_{x}}{\bar{x}} \approx \left( \frac{1}{2} \times \frac{\sigma_p}{\bar{d}_p} + \frac{3}{8} \times \left( \frac{\sigma_p}{\bar{d}_p} \right)^2 \right) \sqrt{1 + \frac{3}{8} \times \left( \frac{\sigma_p}{\bar{d}_p} \right)^2} \]
8. The Bienayme formula \( \frac{\sigma_{ke}}{ke} \approx \frac{3 \times \frac{\sigma_{dp}}{d_p}}{1 + 3 \left(\frac{\sigma_{dp}}{d_p}\right)^2} \) can be used to predict the total outcome variations of the flight distance and impact kinetic energy, by using individual outcome variations provided all the inputs are uncorrelated.

9. Comparison between wind tunnel experiments and simulation results showed excellent agreement and confirm the validity of proposed model.

There are many other sources of uncertainty, even in the simple problem of a piece of gravel blowing off a high roof. For example, the launch angle, location on the roof at which flight initiation occurs, and particle shape will all vary about some mean. Further, the wind velocity relative to the particle will change during flight as the particle accelerates resulting in a change in Reynolds number, and therefore drag coefficient. All of these parameters require further investigation. Also, further work is needed to accurately parameterize the appropriate statistical description of each input parameter such as the particle size distribution or launch angle distribution.
CHAPTER FOUR

WIND BORNE DEBRIS FLIGHT INITIATION

Abstract:

A new experimental approach is presented for calculating the critical conditions under which loose debris on a roof becomes airborne. The critical condition for gravel blow-off from the top of a roof depends on the building geometry, particle properties and the wind conditions. A series of two-dimensional wind tunnel tests were run to measure the critical condition for particle removal. The experimental results demonstrate that the critical condition for blow-off, parameterized in terms of a particle densimetric Froude number, is a function of the particle Reynolds number and the building geometry. Results for buildings without a parapet show that the critical particle densimetric Froude number has a power-law relationship with dimensionless particle size as $F_{d}^{2} = 8.06(d_{c})^{-0.44}$, as well as with particle Reynolds number as $F_{d}^{2} = 11.31(Re_{d})^{-0.34}$ for the range of parameters tested. For buildings with a parapet, the densimetric Froude number for critical condition depends on both Reynolds number and parapet height to building height ratio. The experimental results indicate that buildings without a parapet are not always the most prone to blow-off, and that under certain conditions, a small parapet height can increase the risk of gravel removal.

As the critical Froude number is dependent on the Reynolds number, raw data from small scale experiments cannot simply be scaled up by using a Froude number. Further, it is demonstrated that the current approach of using the Shields Diagram (or equivalent data) to scale results from small to large scale are also flawed as the motion initiation mechanism is
different. Therefore, existing design guides should be re-visited and full-scale experiments should be conducted in order to fully analyze the risk of blow-off.

**Key Words:** Wind Borne Debris, Flight Initiation, Critical Condition, Particle Leaving the Roof, Reynolds Number, Particle Densimetric Froude Number

4.1 Introduction:

In order to fully understand the risk associated with wind driven blow off of roof gravel, one needs to understand the conditions under which it becomes airborne, the rate at which it is removed, and the distance it is transported. In this chapter, the conditions under which loose aggregate will become airborne and be removed from a roof are considered. The mechanics of blow-off involve both fluid-structure interaction (the flow over and around the building) and fluid-particle interaction (the scouring of particles from the aggregate bed).

The interaction between loose particles and a fluid flow has been matter of interest for a long time in the areas of water sedimentation and wind erosion. One aspect of particle motion in wind is wind borne debris which occurs during a severe storm. When strong winds occur in storms, they can pick up loose particles and carry them downstream of the particle source. One of the main sources for these loose particles is gravel on top of built-up roofs. Although, research in wind and water sedimentation dates back to the early 20th century, work on wind borne debris did not begin until the 1980s, which Tachikawa (1983, 1988) tried to develop a flight equation for wind borne debris. In this chapter, the fluid mechanics of sediment transport in both wind and water are briefly reviewed, and is related to the problem of roof gravel blow off. This approach takes advantage of the long research history in sediment transport and fluid
mechanics to develop a better wind engineering approach. For this reason, the literature review will go through the both water and wind sedimentation first and then will review wind engineering research on roof gravel blow off.

4.1.1 Sediment Transport

The majority of research that has been done on to the motion of solid particles in fluid flows has been in the area of sediment transport. Research on this area began in the early 20th century. The first method for identifying the threshold of sediment motion initiation and sediment discharge were derived in the 1930s and 1940s by Shields (1936a, b, c) for water and Bagnold (1937, 1941) for wind. Those methods are still being used widely because of their simplicity. In water sedimentation, attention was mostly on sediment discharge estimation while in the field of wind researchers paid attention on soil erosion.

In the field of water sedimentation, Albert Shields (1936a, b, c) identified the critical shear stress for sediment motion by proposing a function between two dimensionless parameters:

\[
\frac{\tau_c}{(\rho_p-\rho_w)g d_p} = f \left( \frac{u_{*} d_p}{v_w} \right)
\]

where

\[
\tau_* = \frac{\tau_c}{(\rho_p-\rho_w)g d_p}
\]

is the so called critical Shields parameter and

\[
Re_* = \frac{u_{*} d_p}{v_w}
\]

Is the Reynolds number based on the particle size and the skin friction velocity given by
\[ u_{sc} = \sqrt{\frac{\tau_c}{\rho_w}} \]  

(4-4)

where \( \tau_c \) is the critical shear stress, \( \rho_p \) is the particle density, \( \rho_w \) is the fluid (water) density, \( g \) is gravity acceleration, \( d_p \) is the particle diameter, \( u_{sc} \) is the critical shear velocity, \( \nu_w \) is the kinematic viscosity of water. This function is plotted in what is widely known as the Shields diagram (Figure 4-1). This plot shows the relationship between the critical shear stress required to initiate motion as a function of the particle Reynolds number. The presented function is implicit as the critical shear velocity is represented in both \( Re_\ast \) and \( \tau_\ast \).

The critical Shields’ parameter \( \tau_\ast \) can be rewritten as:

\[ \frac{\tau_c}{(\rho_p - \rho_w)gd_p} = \frac{\rho_\ast u_{sc}^2}{(\rho_p - \rho_w)gd_p} = \left( \frac{u_{sc}^2}{\rho_p - \rho_w} \right) gd_p = Fr_{\ast d}^2 \]  

(4-5)

That is, the Shields parameter \( \tau_\ast \) which is the square of the densimetric particle Froude number based on the shear velocity. Alternatively, it can be thought of the inverse of the Richardson number widely used in parameterizing the stability of stratified flows (Linden, 1979).

**Figure 4-1:** Shields’ diagram, Source data from Guo (2002)
4.1.2 Wind driven erosion (Aeolian processes)

Bagnold began to work on wind sedimentation at roughly the same time as Shields (Bagnold, 1937, 1941). Bagnold ran experiments on sand movement by wind. The tests were carried out by using sand with a diameter of 0.25 mm in a wind tunnel with 30 cm$^2$ cross section and 10 m length. He derived an equation based on the experimental results for threshold shear velocity as:

$$u_{*c} = A \frac{\rho_p - \rho_a}{\rho_a} g d_p$$

where $\rho_p$ is particle density, $\rho_a$ is the air density and $A$ is an experimental coefficient which is $A = 0.1$ for sand with a uniform grain diameter $d_p > 0.2$ mm. By rewriting the equation as

$$A = \frac{u_{*c}}{\frac{\rho_p - \rho_a}{\rho_a} g d_p} = F r_{*d} = \sqrt{T_s}$$

It can be seen that Bagnold’s equation is equivalent to having a constant Shields parameter (critical Froude number) which is independent of the $Re$ number.

After Bagnold, research has been continued on wind sedimentation critical conditions and Aeolian (wind sedimentation) transport. Zingg (1953) worked on finding the threshold of saltation (motion of a particle through series of short jumps) by relating the threshold of saltation to the grain size. Chepil (1945) plotted the threshold shear velocity versus the maximum equivalent diameter of transported particles which shows that by increasing the grain size, the threshold shear velocity decreases until it gets to a minimum point. Chepil (1959) used a force balance on a grain sitting on the top of two other grains to predict motion initiation. The balance considered the friction forces resulting from the weight a lift force and a drag force. Chepil (1961) found that both lift and drag forces have a role in particle movement. As the
elevation of a particle increases, the lift force begins to decrease until it gets close to zero.

Iversen and White (1982) offered two threshold equations for the initiation of particle movement based on the dimensionless critical shear stress. Anderson and Hallet (1986) developed a simple model for sediment transport by wind. In their model, they considered the acting forces to be gravity, drag force, lift force and Magnus lift forces due to rotation of the particle. Iversen et al. (1987) proposed an empirical relation for the motion threshold based on experimental results. Le Roux (1997) worked on finding a relation between the aerodynamic entrainment threshold and hydrodynamic settling velocity. He pointed out that the threshold relation proposed by Iversen and White (1982) and Iversen et al. (1987) is directly related to \( Re \), which itself is related to \( u_* \). Therefore, Le Roux attempted to present a relationship between settling velocity and the dimensionless critical shear velocity of Shields. Stout (1998) performed a field study on the effect of averaging time on the threshold velocity. Shao and Lu (2000) proposed a threshold based on the shear velocity and Stout (2004) conducted field measurements to define the velocity threshold for aeolian transport.

There are many differences between the study of windborne debris and sediment transport. While much of the physics is the same, the scales are different as well as the parameter ranges that need to be modeled, for example, the dimensionless particle diameter \( d_* \), which is proposed by Gessler (1971) as

\[
d_* = d_p \left( \frac{\rho_p}{\rho} - 1 \right) \frac{g}{\nu^2} \right)^{1/3}
\]

where \( \nu \) is the kinematic viscosity of fluid. To show the scale difference, \( d_* \) is calculated for sand, gravel and a small wooden sphere in water and wind, see table 4-1. The value of \( d_* \) for a piece of roof gravel is 40 times that of a sand particle in a river or stream.
### Table 4-1: $d_*$ for gravel and wooden sphere in water and wind

<table>
<thead>
<tr>
<th>Object</th>
<th>Diameter (mm)</th>
<th>Density $kg/m^3$</th>
<th>$d_*$ in water</th>
<th>$d_*$ in wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>1</td>
<td>2650</td>
<td>25</td>
<td>45</td>
</tr>
<tr>
<td>Gravel</td>
<td>20</td>
<td>2650</td>
<td>505</td>
<td>916</td>
</tr>
<tr>
<td>Wooden Sphere</td>
<td>80</td>
<td>500</td>
<td>-</td>
<td>2101</td>
</tr>
</tbody>
</table>

Sediment transport studies in water are not typically interested in tracking individual particles as it is the effect of many particles that is of concern. In wind engineering, a single particle can break a window. Therefore, the dynamics of a single particle is important. Particle concentrations in terms of particle volume per unit volume of fluid, are also typically much lower in wind driven debris. Further, windborne debris, such as 80mm wooden particle with $d_* = 2101$, are considerably larger than particles typically regarded as sediment. This means that, once airborne, windborne debris is typically not strongly influenced by the Reynolds number.

#### 4.1.3 Wind engineering studies on motion initiation

Loose particles at the top of roofs are one of the main sources of air born missiles. Particles at the top of the roof can experience higher wind velocities compared to particles at ground level. Because of that, these particles are more likely to become wind born missiles. They are also elevated once airborne allowing them to be transported further down wind that loose debris picked up off the ground. Unfortunately, very limited research has been done in
this area. Further, as shown below, the research that has been done is based on a flawed scaling of laboratory results and is, therefore, of questionable value.

Though much research has been done on modeling the trajectories of individual pieces of windborne debris (Tachikawa, 1983, 1988, Wills et al., 2002, Holmes, 2004, 2006, Lin et al., 2006, 2007, Baker, 2007, Richards et al., 2008), existing models typically assume that the particle is already airborne and they have not investigated the critical condition under which particles become airborne. Windborne debris models were discussed in detail in chapter 3.

Kind and Wardlaw (1977) ran a set of experiments to develop design criteria for preventing gravel from blowing off a roof. They ran experiments using a 1/10 length scale and a total of 4 buildings, 2 for low rise building and 2 for high rise building. Experiments were run with the building walls at 45° to the wind direction. Four different roof level critical velocities were defined; $u_{c1}$ - the velocity at which one or two stones move several centimeters, $u_{c2}$ - the velocity at which stones are scouring continuously, $u_{c3}$ - the velocity at which more than 6 stones leave the roof from the upstream edge, and $u_{c4}$ – the velocity at which more than 6 stones leave the roof from the downstream edge. These conditions were estimated by observing the aggregate through a telescope and recording the velocity when various particles were observed to move. There was no direct measurement made of the rate of aggregate removal by the wind.

Kind and Wardlaw used Bagnold’s threshold equation (4-6) and therefore ignored any Reynolds number effect. That is, they assumed that the densimetric Froude number is the dominant dimensionless parameter needed to scale results from laboratory to full scale. This means that the critical Froude number is always constant and independent of the $Re$ number. They assumed that the critical velocity is only proportional of $\sqrt{d_p}$ where $d_p$ is the gravel or
particle diameter. Therefore, to scale up the wind tunnel results to full scale, the velocity results should be multiplied by

\[ \sqrt{\frac{(d_p)_{Full\, Scale}}{(d_p)_{Test}}} \]  

(4-9)

It should be noted that the critical shear velocity, and not the critical velocity, will be proportional to some function of the particle diameter. Therefore, there is an implicit assumption that the wind velocity is directly proportional to the surface shear velocity and that surface shear is the dominant physical process in initiating motion. Kind and Wardlaw also ignored the particle material properties such as particle density though this is somewhat justified given the relatively small range of aggregate densities compared to difference between air and aggregate densities. Results of this research were presented only in the form of series of design curves with no raw or scaled experimental data shown.

Kind (1986) extended this work to examine the effect of length scale on the results of the earlier wind tunnel tests. Tests were run for three different buildings at 1/10 length scale, 2.3x2.3x0.46 m as low rise building and 0.92x0.92x2.3 m and 0.92x1.84x2.3 m as the high rise building. Tests were run at a 45° degree wind direction toward the building with 2 different particle sizes (2.3 and 3.8 mm). Kind used 1/60 and 1/120 scale lengths for 5 different particle diameters i.e. 0.2, 0.35, 0.51, 0.63 and 0.72 mm, and found that results for small scale buildings were 30% less than for large scale results. Kind argued that the Reynolds number has an effect on the results for small scale wind tunnel experiments and a Reynolds number correction must be applied when extrapolating the results to full scale. Based on the work of Iversen and White (1982), he suggested a correction factor to scale up the wind tunnel results to full scale. In the proposed method the Froude number is used to scale up the velocity, while the correction
factor is used to eliminate the effect of Reynolds number, however, the author does not apply this correction to their own data. The effect of Reynolds number on this data and the correction factor for Reynolds number effect are discussed in details in section 4.2.

Wills et al. (2002) developed the model to describe the damage of wind borne debris to a building during periods of high wind velocity. For their flight model, they assumed that a particle would lift off if the lift force exceeds the forces that fix the object in place such as weight and friction. Taking gravity as the only fixing force, the critical wind velocity \( u \) for sphere would be

\[
 u_{\text{critical}} = \left( 2 \left( \frac{\rho_p}{\rho_a} \right) \left( \frac{1}{C_F} \right) d_p g \right)^{0.5}
\]

(4-10)

where \( C_F \) is a force coefficient, assumed to be close to unity i.e. \( C_F \sim O(1) \). Again this is equivalent to assuming a constant critical Froude number and ignores any Reynolds number effects.

4.1.4 Building geometry effects on motion initiation

The main parameter that influences the wind flow and pressure distribution on a flat built-up roof is the parapet height. Data on the effect of the parapet height, scaled on the roof height \( h/H \) was collected by Kind (1986) but again, only design guides were reported, not experimental data. The failure mechanism for a flat roof that uses pavers for ballast was investigated by Bienkiewicz and Meroney (1986). Their results show that small parapet heights actually reduce the wind velocity required to remove the pavers. However, as the parapet height increases, they play a protective role. The same trend was reported by Pindado and Meseguer (2003) who examined the effect of parapet height on the pressure distribution on the
roof. The up-lift pressure increases compared to the no parapet case, for a relatively small parapet height and then reduces as the parapet height is increased.

Despite the studies described above, there are still several gaps in our understanding of aggregate removal from flat roofs. These include the role of the Reynolds number on the critical condition under which particles begin to leave the roof, the volume and rate of particle removal, and the downwind debris flight distance. This chapter considers the question of the Reynolds number effect on aggregate removal and presents detailed experimental results for the critical condition for removal. The measurements are made using a newly developed quantitative technique for assessing when aggregate starts to be removed rather than the qualitative approach adopted by Kind and Wardlaw (1977). The remainder of the chapter is arranged as follows. Section 4.2 discusses the Reynolds number effect on aggregate removal, why it is important, why the correction of Kind (1986) is inappropriate for built up roofs, and how experimental results should be presented. Section 4.3 gives a detailed definition of the problem considered in this chapter. Section 4.4 discusses the particle lift off mechanism from roof tops. Description of the experimental setup and technique used in the investigation present is section 4.5. Results of experiments are presented in section 4.6. Discussion is in section 4.7 and conclusions are drawn in section 4.8.

4.2 Reynolds effects on aggregate removal

To make their wind tunnel results applicable for full scale cases, Kind and Wardlaw (1977) used Froude number similarity to scale up the wind tunnel velocity results up to full scale. Other researchers such as Visscher and Kopp (2007) and Kordi et al. (2010) used the same method to scale up their results to full scale. This method will only be correct if the tests are
independent of \( Re \) number. But, wind tunnel data can be sensitive to the \( Re \) number (Richards et al., 2001, Lim et al., 2007), which means that data should be corrected for any \( Re \) number effect. However, it is not clear how such a correction should be done (Lim et al., 2007).

Kind (1986) investigated this issue and suggested a correction factor to scale up a particle scour on a roof based on the work of Iversen and White (1982). In the method, the Froude number is used to scale up the velocity, and then a correction factor is used to account for the effect of the Reynolds number. The method assumes that the ratio of the small scale Froude number to the large scale Froude number is a function of the small scale particle diameter. That is

\[
\frac{Fr_L}{Fr_S} = C \left( \frac{(d_p)_S}{(d_p)_L} \right)
\]

(4-11)

where \( C \) is a correction factor that is a function of the lab scale particle diameter. The subscript \( S \) indicates small scale and the subscript \( L \) indicates large scale. There are a number of underlying assumptions in this approach. First, for the assumption that \( C \) depends only on the small scale particle diameter, it is assumed that the full scale flow is Reynolds number independent. This approach also requires that the small scale and full scale aggregate has the same density.

The full scale critical velocity is calculated by multiplying the lab scale velocity by the square root of the ratio of the diameters at large and small scale and then multiplying by the correction factor \( C \) (see figure 4-2). That is

\[
u_L = u_S C \sqrt{\frac{(d_p)_L}{(d_p)_S}}
\]

(4-12)

where \( u \) is fluid velocity. The correction factor is based on the critical velocity work on wind driven sand motion initiation of Iversen and White (1982). The use of this data implicitly
assumes that the mechanism for motion initiation and removal is surface shear stresses, as is
the case for sand erosion from flat surfaces and sediment motion initiation as studied by
Shields.

\[ \frac{(Fr^2)_L}{(Fr^2)_S} = \frac{(u^2)_L}{g'(d_p)_L} \frac{(u^2)_S}{g'(d_p)_S} = C^2 \]  \hspace{1cm} (4-13)
where \( g' \) is defined as:

\[
g' = \frac{\rho_p - \rho_a}{\rho_a} g
\]  

(4-14)

So for the same fluid and the same particle density,

\[
C = \sqrt{\frac{0.054}{(Fr_e^2)_S}}
\]  

(4-15)

and therefore,

\[
\frac{u_k}{u_S} = C \times \sqrt{\frac{(d_p)_1}{(d_p)_S}}
\]  

(4-16)

which is identical to (4-11). Values of \( C \) based on the Shields’ diagram data, for different particle sizes assuming a particle density of 2650 kg/m\(^3\), as used by Kind, are given in figure 4-3.

Figure 4-3: Correction factor for Reynolds effect based on Shields’ diagram
For comparison purposes, the correction factor of Kind and the correction factor based on the Shields’ diagram are presented together in figure 4-4. Clearly there is very close agreement between the two approaches confirming that the underlying assumption of the Kind correction is that motion initiation is driven by surface shear stresses at the bottom of a fully developed turbulent boundary layer.

![Correction Factor for Reynolds effect based on particle size](image)

**Figure 4-4:** Correction factor for Reynolds effect based on particle size

Although Kind (1986) presented details of this correction factor, the correction was not applied to the data in the paper. The raw and corrected data from Kind (1986) is presented in figure 4-5 and 4-6 respectively. Clearly the correction factor, as applied to this data, does not collapse the data onto a single line. The spread is just as large post correction as pre-correction. Further, the data does not get appreciably closer to the line based on full scale experimental results. Therefore, the correction cannot be used for scaling small scale experimental data to full
scale. This implies that the mechanism for blow off is not surface shear stress. Therefore, other mechanisms must be considered.

Figure 4-5: Scaled up data from wind tunnel (diamonds) and full scale data (line), from Kind (1986)

Figure 4-6: Corrected scaled up data for Reynolds effect from wind tunnel (diamonds) and full scale data (line), from Kind (1986)
In order to find a scheme for accurately scaling up laboratory scale experimental results to full scale the mechanism of aggregate removal must be accurately parameterized. Clearly, based on the results in figures 4-5 and 4-6, aggregate does not begin to move as a result of a fully developed turbulent boundary layer reaching a critical shear stress at the aggregate surface. This is not surprising as the flow on much of the roof will be dominated by the flow separation that occurs at the leading edge of the roof. Motion initiation could therefore be due to either scour from intense vortices due to separation or from pressure variations over the surface of the aggregate bed causing pressure gradients across individual aggregate pieces. These mechanisms are discussed in more detail in section 4.4.

4.3 Problem definition

The critical condition that causes a particle at rest on a roof to begin to move and eventually blow off and leave the roof, is not fully understood. The main goal of this research is to investigate the critical condition under which particles begin to leave the roof. In this research, the critical velocity is considered as the velocity, measured at the top of the parapet, at which particles begin to leave the roof, not the velocity at which particles begin to move within the confines of the parapet. In order to avoid infinite geometric complexity the study is limited to two dimensional buildings of rectangular cross section. The main parameters of the study are shown in figure 4-7 below.
Figure 4-7: Schematic diagram of the model buildings (the building height ($H$), parapet height ($h$) and the side wall width ($W$).

A full quantification of particle blow off from rooftops is beyond theoretical analysis, due to the complex geometry and variability in wind and particle properties. However, some quantification is possible through dimensional analysis. The key parameters influencing particle movement are the wind conditions, the building geometry and the particle properties. Modeling the atmospheric boundary layer requires an understanding of how both the velocity and the turbulence intensity vary with height. This understanding of variance is important since particle lift off is a function of the parapet top wind velocity, the velocity gradient at that height, and the turbulence intensity. Jensen (1958) has shown that the properties of the boundary layer can be largely parameterized by the surface roughness length of the upstream fetch and a reference wind speed at a defined height. The main parameters are, therefore, the ABL properties parameterized in terms of a reference velocity and a surface roughness height, the building geometry (height, width and parapet height), the aggregate properties (size, density), and the air material properties (density and viscosity). This is a total of nine parameters and therefore 6 dimensionless groups can be created. These cover geometric similarity as
\[ \frac{H}{W}, \frac{h}{H}, \text{ and } Je = \frac{H}{s_0} \]  \hspace{1cm} (4-17)

where \( Je \) is the Jensen number. Dynamic similarity is achieved by matching

\[ Re_d = \frac{\rho_a u_H D_p}{\mu_a}, Fr_d = \frac{u_H}{\sqrt{g' d_p}} \text{ and } \frac{\rho_a}{\rho_p} \]  \hspace{1cm} (4-18)

at lab and full scale, where \( u_H \) is wind velocity at building height level and \( \mu_a \) is air dynamic viscosity. The subscript \( a \) indicates air and \( p \) indicates particle.

The density ratio could be written in terms of the reduced gravity \( g' = \frac{\rho_g - \rho_a}{\rho_a} g \). Also, the particle Froude number can be written as a densimetric Froude number.

\[ Fr_d = \frac{u_H}{\sqrt{g' d_p}} = \frac{u_H}{\sqrt{\frac{\rho_g - \rho_a}{\rho_a} g d_p}} \]  \hspace{1cm} (4-19)

In this research, the critical velocity that causes particles to leave a roof is studied based on the particle Reynolds number \( Re_d \), densimetric particle Froude number \( Fr_d \) and parapet height to building height ratio \( h/H \). The Jensen number is not varied in this study as this would both dramatically increase the parameter space considered and also because of the large uncertainty inherent in establishing the surface roughness length (see chapter 2).

Generally, hydraulics engineers prefer to have a critical condition for sediment motion initiation based on the dimensionless particle diameter \( d_\ast \) (Guo, 2002), (4-8), rather than Reynolds number as, in this case, velocity is only presented in one non-dimensional group in the plot. Because of that, the critical condition is also presented in terms of the dimensionless particle diameter \( d_\ast \).
4.4 Particle Lift off Mechanism from Roof Top

The forces acting on a particle on top of a building are drag $F_D$, lift $F_L$, weight $F_W$, buoyancy $F_B$, and resistance $F_R$. Weight $F_W$, buoyancy $F_B$ and drag force $F_D$ could be considered as:

\[ F_W - F_B \sim (\rho_p - \rho_a)gd_p^3 \]  \hspace{1cm} (4-20)

\[ F_D \sim C_D\rho_a u_H^2 d_p^2 \]  \hspace{1cm} (4-21)

where $C_D$ is drag coefficient. Lift force on particle sitting on flat bed is mainly from flow circulation around the particle, which is negligible in creeping flow, and as a first approximation, it is proportional to the drag force in turbulent flow (Julien, 1998). But for the particle sitting on top of the roof, lift force is mainly from pressure difference due to the flow separation and vortex formation on top of the roof, and less from flow circulation around the particle (figure 4-8). Pressure difference is parameterized using a dimensionless pressure coefficient as:

\[ C_p = \frac{\Delta p}{\frac{1}{2} \rho_a u^2} \]  \hspace{1cm} (4-22)

So, the pressure difference $\Delta p$ on top of the building could be written as

\[ \Delta p = \frac{1}{2} C_p \rho_a u_H^2 \]  \hspace{1cm} (4-23)

The lift force $F_L$ applied on the particle would scale as

\[ F_L = \Delta p \times A_p \sim C_p \rho_a u_H^2 d_p^2 \]  \hspace{1cm} (4-24)
4.4.1 Particle Motion Threshold based on Moment Equilibrium:

Forces acting on a single two-dimensional particle sitting on top of two others is shown in figure 4-9. One approach to calculating the particle motion threshold could be reached by considering moment equilibrium about the point of contact between the exposed particle and the upstream particle. The particle will begin to move as the moment of driving force becomes greater than the moment of forces that keep the particle in place. Therefore, at the threshold condition, the driving moment is equal to the resisting moment. By calculating the moment of all forces in figure 4-9 about point \( O \), the driving moment and resisting moment balance would scale like:

\[
\left( F_D \times \frac{d_p}{2} \cos 30 + F_L \times \frac{d_p}{2} \sin 30 \right) \sim \left( F_W - F_B \right) \times \frac{d_p}{2} \sin 30
\]  

(4-25)

Which can be re-written as

\[
\frac{\sqrt{3}}{2} C_D \rho_a u_H^2 d_p^2 + \frac{1}{2} C_p \rho_a u_H^2 d_p^2 \sim \frac{1}{2} \left( \rho_p - \rho_a \right) g d_p^3
\]  

(4-26)

or in terms of the densimetric Froude number

\[
\frac{u_H^2}{((\rho_p-\rho_a)/\rho_a) g d_p} = Fr_H^2 \sim \frac{1}{(\sqrt{3} C_D + C_p)}
\]

(4-27)
Figure 4-9: Force acting on single particle

The pressure coefficient $C_p$ on top of the building is depends on fluid $Re$ number (Richards et al., 2001) and building properties (ASCE 7), also, the drag coefficient $C_d$ is depends on $Re$ number (Cimbala and Cengel, 2008). Therefore (4-27) can be rewritten as:

$$Fr_d^2 = f(Re_d, Building\ geometry)$$  \hspace{1cm} (4-28)

Note that (4-28) assumes that the velocity at the surface of the roof is directly proportional to the velocity at the top of the parapet. This may or may not be the case. However, the ratio of these velocities will be a function of the building geometry and building Reynolds number, leaving (4-28) unchanged.

4.4.2 Particle Motion Threshold based on Vertical Force Equilibrium

Another approach for calculating the particle threshold could be reached by considering vertical force equilibrium due to flow and building geometry induced pressure variations.
Considering the vertical forces acting on the top particle in figure 4-9, vertical force equilibrium could be written as:

\[ F_L \sim (F_W - F_B) \]  

where

\[ C_p \rho_a u_2^2 d_p^2 \sim (\rho_p - \rho_a) g d_p^3 \]  

Which leads to

\[ \frac{u_2^2}{((\rho_p - \rho_a)/\rho_a) gd_p} = Fr^2 \sim \frac{1}{C_p} \]  

Again, \( C_p \) is a function of \( Re \) number (Richards et al., 2001) and building properties (ASCE 7). Therefore, just like the moment equilibrium case, the vertical force equilibrium also confirms that the particle motion threshold, (written in terms of a critical Froude number) is dependent on \( Re \) and the building geometry (4-28). Therefore, for a given building geometry, plots of densimetric Froude number versus Reynolds number should exhibit data collapse. That is, all the data should fall on a single line. However, this does not give significant insight into the actual physical mechanism as both possibilities described above lead to the same scaling.

### 4.4.3 Critical Condition Threshold for aggregate removal

The critical condition mechanism under which a particle will leave the roof could be studied by considering a particle projectile path. Our experimental results indicate that particles on top of a building first move toward the windward wall due to the vortex on top of the building, and they will begin to fly from that windward parapet downstream (figure 4-10).
Figure 4-10: Critical condition for particle to leave the roof

Conditions in 4.4.1 and 4.4.2 parameterize the initiation of motion of a particle on top of the roof, but it cannot guarantee that particles will leave the roof. Particles have to be lifted over the parapet height to leave the roof. By assuming that a particle begins its flight from the windward parapet, it must be at least as high as the parapet height when it reaches the leeward parapet. Therefore, the threshold for a particle to leave the roof refers to the condition that the particle projectile path goes over the leeward parapet. Therefore, the critical condition will be a function of the aspect ratio of the roof cavity $h/W$. Combining this with (4-28) leads to:

$$Fr_d^2 = f \left( Re, \frac{h}{H}, \frac{h}{W} \right)$$

(4-32)

Without loss of generality, this can be re-written in terms of the relative parapet height and the building aspect ratio

$$Fr_d^2 = f \left( Re, \frac{h}{H}, \frac{H}{W} \right)$$

(4-33)
4.5 Experimental Program

4.5.1 Experimental Setup

A series of tests were conducted to measure the critical velocity for different building shapes over a range of $Re_d$ and $Fr_d$. The tests were run using the parameters listed in table 4-2. The main building property investigated was the relative parapet height $h/H$, though some test were run for different values of $H/W$ as well. The parapet height was measured from the top of the particle layer to the top of the parapet (Figure 4-7). Also, a series of test were conducted with $h/H = 0$ by filling the roof with particles up to top of the parapet level. Experiments were conducted in the Clemson University Wind Tunnel, which is 28 m long with a 14.6 m long roughness blocks section and with a 3m wide by 2m high cross section.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_H$</td>
<td>Wind velocity at parapet top</td>
<td>m/s</td>
<td>Measured</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Particle density</td>
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<td></td>
<td>Sand</td>
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<td></td>
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<tr>
<td></td>
<td>Millet</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>Particle mean diameter</td>
<td>mm</td>
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<td>$g$</td>
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</table>
4.5.2 Test Method

A new method was developed for calculating the critical velocity. Unlike the method used by Kind and Wardlaw (1977), this new method does not rely on visual observation, but rather everything is calculated based on measurements of mass loss. For each test, an accurately weighted amount of aggregate was spread uniformly on the roof. Then, the building was exposed to wind for 5 minutes. The average velocity at the top of the parapet height was measured by using a Dwyer pitot tube along with Setra digital monometer and a Measurement Computing data acquisition module, while Matlab was used for velocity data analyzing and calculating during the test. At the end of 5 minutes, the aggregate remaining on the roof was weighted using an Ohaus Adventurer Pro 3102 Precision Balance with precision of 1/100 grams. The removed particles were then replaced, and the test was repeated at a higher velocity. This procedure was repeated four times for each building. The mass loss over each five minute period was then plotted against the wind velocity and a straight line fitted through the data. The critical velocity was calculated to be the point at which the fitted line intersected the velocity axis. That is, the critical velocity was taken to be the velocity at which no mass would be removed. The data analysis procedure is illustrated in figure 4-11. Sample results for four different particle diameters on the same building are presented in figure 4-12.
Figure 4-11: a) Schematic diagram of the critical velocity calculation procedure, b) Experimental data for building with H=W=0.15, h/H=0.002 and d=0.23 mm.
Figure 4-12: Critical velocity results for building with $H = W = 0.15 \text{ m}$ and $h = 0.0025 \text{ m}$ .

Top left: $d_p = 0.11 \text{ mm}$, top right: $d_p = 0.23 \text{ mm}$, bottom left: $d_p = 0.45 \text{ mm}$ and bottom right:

$$d_p = 0.89 \text{ mm}$$

The repeatability of the critical velocity procedure was checked by running a series of five tests with the same parameters. The results of this series are shown in figure 4-13. All of the critical velocities measured were within plus or minus 6% of the mean.
In this section, experimental results are presented. Along with the raw data, results are presented as plots of the square of the densimetric particle Froude number $F_{d}^{2}$ versus particle Reynolds number $Re_{d}$ and dimensionless particle size $d_{*}$ for different relative parapet heights and building aspect ratios. The primary focus of the experiments is to investigate the role of the parapet height on the critical velocity. The first set of data presented is for $h/H = 0$, followed by data for a range of non-zero $h/H$.

4.6.1 Zero Parapet Height Results ($h/H = 0$)

Critical condition results for buildings with $h/H = 0$ (achieved by filling the roof with particles up to the top of the parapet level) are presented in this section. In these tests, the
main goal was eliminating the parapet effect to investigate and focus on the effect of Reynolds number on the critical condition on top of the roof. In addition, four different building aspect ratios were considered, namely \( H/W = 0.5, 0.75, 1, \text{ and } 2 \). As given in table 4-2, sand and millet were used as particles on top of the roof. Raw data for these tests are presented in figures 4-14 to 4-17 for \( H/W \) ratio of 0.5, 0.75, 1, and 2 respectively. As expected, results show that the critical velocity depends on both particle size and density of the particles. To compare the raw data and evaluate the effect of \( H/W \) on critical velocity, results for all \( H/W \) ratios with \( h/H = 0 \) are presented in figure 4-18. As can be seen, the critical velocity is not very sensitive to changes \( H/W \) for the range considered. Experimental error for the raw data was calculated to be 6 percent about the mean value based on the series of test repeated with the same condition (figure 4-13).

![Figure 4-14: Critical velocity versus particle size \( d_p \) for \( h/H = 0 \) and \( H/W = 0.5 \) (Filled marker represents millet)]
Figure 4-15: Critical velocity versus particle size $d_p$ for $h/H = 0$ and $H/W = 0.75$ (Filled marker represents millet)

Figure 4-16: Critical velocity versus particle size $d_p$ for $h/H = 0$ and $H/W = 1$ (Filled marker represents millet)
Figure 4-17: Critical velocity versus particle size $d_p$ for $h/H = 0$ and $H/W = 2$

Figure 4-18: Critical velocity versus particle size $d_p$ for $h/H = 0$ (Filled marker represents millet)

The raw data is presented in dimensionless form in plots of $Fr^2_d$ versus dimensionless particle diameter $d_*$ for $h/H = 0$ (figure 4-19). Hydraulics engineers prefer to have a critical
condition for sediment motion initiation based on the dimensionless particle diameter $d_*$ (Guo, 2002) because then velocity is only presented in one non-dimensional group. The results show that for all $H/W$ considered, $Fr_d^2$ can be represented by a power law function of $d_*$, as represented by a linear relationship on the log-log scale plot. For the range of parameters tested this relationship can be approximated by

$$ Fr_d^2 = 8.06(d_*)^{-0.44} \quad (4-34) $$

Presenting the data in terms of $d_*$, which also accounts for the particle density, should show the millet data being consistent with the sand data. This is clearly seen in figure 4-19 while this is considerably less clear in the raw data plot (figure 4-18). These results indicate the validity of using different density particles in order to increase the parameter range tested. This would not be the case using the data analysis approaches previously published as they failed to account for particle density in their parameterizations.

*Figure 4-19: $Fr_d^2$ versus $d_*$ for $h/H = 0$ and different $H/W$ ratio (Filled marker represents millet)*
Results for the critical condition are presented following the Shields’ diagram approach (Figure 1). In the Shields’ diagram, the square of critical densimetric particle Froude number based on a shear velocity $Fr_d^2$ is plotted versus the particle Reynolds number based on the particle shear velocity $Re_d$. Following the same method, results of this research are presented as $Fr_d^2$ versus $Re_d$ (figure 4-20). However, figure 4-20 is not directly analogous as the velocity in figure 4-20 is the roof top velocity not a surface skin friction velocity.

As results in figure 4-20 show, as $Re_d$ increases, $Fr_d^2$ for critical condition decreases for all $H/W$ ratios considered. By re-plotting the same data in log-log format (figure 4-21), it can be seen that $Fr_d^2$ has a power law relationship with $Re_d$, which means that densimetric particle Froude number has a power-law relationship with particle Reynolds number which can be approximated by

$$Fr_d^2 = 11.31 (Re_d)^{-0.34}$$  \hspace{1cm} (4-35)

**Figure 4-20**: Critical condition for $h/H = 0$ and different $H/W$ ratio (Filled marker represents millet)
In this section the critical condition results for buildings with finite height parapets are presented. All buildings which were considered for these tests have the same height to width ratio $H/W = 1$. To investigate the effect of the parapet on the critical velocity, four different parapet height to building height ratios $h/H = 0.002, 0.02, 0.08$ and $0.15$ were investigated. Again, both sand and millet were used as model gravel on top of the roof.

Raw data for these tests are presented in figures 4-22 to 4-25 for the dimensionless parapet heights ($h/H$) listed above. Again, the results show that the critical velocity depends on both particle size and density of the particles. The results of all four sets of experiments are presented in figure 4-26. This shows that the critical velocity is also sensitive to changes in $h/H$.

Figure 4-21: Critical condition for $h/H = 0$ and different $H/W$ ratio (Filled marker represents millet)

4.6.2 Non-Zero Parapet Height Results ($h/H \neq 0$)
**Figure 4-22:** Critical velocity versus particle size $d_p$ for $h/H = 0.002$ and $H/W = 1$ (Filled marker represents millet)

**Figure 4-23:** Critical velocity versus particle size $d_p$ for $h/H = 0.019$ and $H/W = 1$ (Filled marker represents millet)
Figure 4-24: Critical velocity versus particle size $d_p$ for $h/H = 0.084$ and $H/W = 1$ (Filled marker represents millet)

Figure 4-25: Critical velocity versus particle size $d_p$ for $h/H = 0.15$ and $H/W = 1$ (Filled marker represents millet)
The raw data is presented in dimensionless form in plots of $Fr_d^2$ versus $d_*$ for different $h/H$ ratio (figure 4-27). For $d_* < 20$, parapet height has significant effect on the critical condition, while for larger $d_*$, it seems that critical condition is less sensitive to the parapet height. For $h/H = 0.002$, as $d_*$ increases, the critical $Fr_d^2$ first increases until a peak and then begins to decrease, but for large $h/H$ ratio, critical $Fr_d^2$ always decreases as $d_*$ increases, within the parameter range tested.
The results for the critical condition are presented as plots of $Fr_d^2$ versus $Re_d$ (figure 4-28). For $h/H = 0.002$, as the Reynolds $Re_d$ increases, the critical Froude number first increases until a peak around $Re_d \approx 100$ and then begins to decrease. For larger $h/H$, the $Fr_d^2$ always decreases with increasing $Re_d$ for the parameter range tested. Also, the results show that the critical condition is strongly dependent on $h/H$ for $Re_d < 160$ with the results being less dependent on $h/H$ for $Re_d > 160$. This trend may not continue for $Re_d$ larger than the values tested, and further full scale testing is required.
To compare the critical condition in presence and absence of parapet on top of the building, results for all finite $h/H$ ratios are presented in figure 4-29. The data for $h/H = 0$ is replaced by the fitted power law curve as presented in equation (4-35). Figure 4-29 clearly indicates that, for small Reynolds numbers, the critical Froude number is not lowest for $h/H = 0$. That is, the presence of a parapet will not always act to protect the roof gravel. This result is similar to that found by Meroney & Bienkiewicz (1986), who found that the velocity required to remove paving stones from a roof decreased when a short parapet was added to the building.

**Figure 4-28**: Critical condition for different $h/H$ and $H/W = 1$ (Filled marker represents millet)
Figure 4-29: Critical condition for $H/W = 1$ for both $h/H = 0$ and $h/H \neq 0$ (Filled marker stands for millet)

4.6.3 Parapet height Ratio ($h/H$) Effect

In sections 4.6.1 and 4.6.2, critical $Fr_d$ for buildings with and without parapets were presented for different parapet heights. However, the effect of the parapet height is hard to see in these figures. In this section the data is re-plotted to show the effect of the parapet on the critical condition.

Results are re-plotted showing $Fr_d$ versus $h/H$ for small particles ($d_* = 5.3$ and 10.6 ) in figure 4-30, and large particles ($d_* = 21, 34,$ and 75 ) in figure 4-31. The results for smaller particles (figure 4-30) show that, as the parapet height increases from zero, $Fr_d$ initially decreases indicating that the presence of a relatively short parapet increases the risk of blow-off for small particles. After reaching a minimum, $Fr_d$ then increases with increasing $h/H$. However, the critical $Fr_d$ does not exceed the value for zero parapet height until $h/H \approx 0.1$. For larger
particles (figure 4-31), the results show a different trend. In this case, increasing the $h/H$ ratio, always increases the critical $Fr_d$. Critical $Fr_d$ increases more rapidly for lower $h/H$ ratio, then less rapidly for higher values of $h/H$.

**Figure 4-30**: Critical condition based on $h/H$ ratio for fine particles

**Figure 4-31**: Critical condition based on $h/H$ ratio for coarse particles
4.7 Discussion

This chapter has presented results of an experimental investigation on the effect of Reynolds number and building geometry on aggregate blow-off from built up roofs. By using different size sand particles and millet, \( d_s \), was varied from 5.3 for fine sand particle to 76.6 for millet. However, the largest pea gravel used on full scale buildings has an approximate range of 200 < \( d_s \) < 900. Achieving such high \( d_s \) values are beyond the Clemson wind tunnel’s capability. Further work needs to be done at full scale. However, much can be learned from the results presented.

4.7.1 Zero Parapet Height (\( h/H = 0 \))

As shown in figure 4-18, the critical condition in the absence of a parapet is independent of the building aspect ratio \( H/W \) for the range of parameters tested. It could be argued that for \( h/H = 0 \) the flow is analogous to flow over a flat surface in which the length of the bed does not have any effect on the critical condition of particle motion initiation and initiation is due to boundary layer shear stresses. However, this ignores the flow separation and vortex formation at the leading edge of the roof. Agelinchaab and Tachie (2008) found that the separation bubble length \( L_r \) would be 8.5 times the building height for square objects and 4.1 times of height for rectangular object (figure 4-32). Therefore, for the range of \( H/W \) considered in this research, the whole width of building will be located within the separation bubble. Hence, changes in width while \( W < 4.1H \) will not have any significant influence on particle motion initiation. If \( W > 4.1H \), then the flow will reattach on top of the roof. In that case it is expected that changes in width will influence the critical conditions for blow off.
4.7.2 Non-Zero Parapet Height \((h/H \neq 0)\)

The flow and pressure distribution over a flat built up roof is strongly influenced by the parapet height (Meroney & Bienkiewicz, 1986 and Pindado & Meseguer, 2003). However, as shown in these studies and the results presented above, the parapet does not always reduce the risk of damage to the roof. As seen figure 4-29, the critical condition is highly sensitive to the \(h/H\) ratio for lower \(Re_d\), though it appears to become less sensitive as the \(Re_d\) number increases. As mentioned in section 4.6.2 and 4.6.3, \(h/H = 0\) is not always the worst case (Figure 4-29, 4-30 and 4-31). Buildings with relatively low parapet heights (small \(h/H\)) have lower critical Froude numbers compared to buildings with \(h/H = 0\). This is because the vortex on top of the building which produces a drag and lift forces is stronger for buildings with short parapets than in buildings without a parapet. For higher parapets, the energy required to lift the particles up and over the parapet results in higher wind speeds being needed to cause blow off. The results above are entirely consistent with previous studies that showed that small \(h/H\) lead to higher up-lift forces (Pindado & Meseguer, 2003) and lower wind velocities for paver blow-off (Meroney & Bienkiewicz, 1986) compared to having no parapet.
4.8 Conclusion

The critical condition for roof gravel blow-off from the top of a roof depends on the building geometry, particle characteristics and the wind velocity. To better understand this phenomenon, a series of two-dimensional tests were run using 9 different particle sizes with two different densities. The tests were run for five different dimensionless parapet heights and for three different building aspect ratios.

Results were categorized in 2 group, first the ones which address the critical condition in the absence of a parapet i.e. \( h/H = 0 \), and second the ones which address the critical condition in the presence of a parapet i.e. \( h/H \neq 0 \). A new experimental method was developed to measure the critical condition which eliminates the need for visual judgment in establishing the critical velocity for blow-off. Results were presented both in the form of raw data and dimensionless plots. Raw data was presented for different particle sizes, while dimensionless plots showed the densimetric particle Froude number \( Fr_d^2 \) for critical condition as a function of particle Reynolds number \( Re_d \), dimensionless particle diameter \( d_* \) and also as a function of \( h/H \), \( H/W \) ratio.

Results for buildings with \( h/H = 0 \) show that the critical condition is independent of \( H/W \) for the range of parameters considered. Also, the densimetric particle Froude number has a power-law relationship with dimensionless particle size as \( Fr_d^2 = 8.06(d_*)^{-0.44} \). For all \( H/W \) ratios considered, the \( Fr_d^2 \) for the critical condition decreases as \( Re_d \) increases. Densimetric particle Froude number also shows a power-law relationship with particle Reynolds number as \( Fr_d^2 = 11.31(Re_d)^{-0.34} \).

For buildings with \( h/H \neq 0 \), the critical \( Fr_d^2 \) depends on both \( Re_d \) and \( h/H \) ratio. For \( h/H = 0.002 \), as the Reynolds \( Re_d \) increases, Froude for critical condition \( Fr_d^2 \) first increases
until it reaches a pick, and then begins to decrease. For higher $h/H$ ratio, the $Fr_d^2$ for the critical condition decreases as $Re_d$ increases. The critical condition is more dependent on the $h/H$ ratio for $Re_d < 160$, compared to the $Re_d > 160$ though it is not clear that this trend will continue as the Reynolds increases further beyond the range achievable in the wind tunnel.

The experimental results indicate that $h/H = 0$ is not always the most critical condition. Results for small particles ($d_* = 5.3$ and 10.6) show that, as the parapet height increases from zero, $Fr_d$ initially decreases indicating that the presence of a short parapet increases the risk of blow-off for small particles. After reaching a minimum, $Fr_d$ then increases with increasing $h/H$. However, the critical $Fr_d$ does not exceed the value for zero parapet height until $h/H \approx 0.1$.

For larger particles ($d_* = 5.3$ and 10.6), the results show a different trend. In this case, increasing the $h/H$ ratio, always increases the critical $Fr_d$. Critical $Fr_d$ increases more rapidly for lower $h/H$, then less rapidly for higher values of $h/H$.

As the critical Froude number is dependent on the $Re$ number, the raw data cannot simply be scaled up by using the $Fr$ number similarity without considering the effect of $Re$. Kind (1986) suggested a method for doing such a correction. However, the correction does not work for blow-off from roof tops as the correction is based on data for surface shear stress driven motion initiation which is not the process by which debris is blown off roof tops. Rather, the blow-off is driven by vortices from flow separation and pressure variation over the roof surface. The exact nature of the physical forcing is not revealed by the experiments presented herein as both mechanisms result in the same Froude number Reynolds number scaling for motion initiation.

In summary, the results presented above demonstrate the following:
1. For building with $h/H = 0$, the critical condition is independent of $H/W$ for the tested range.

2. The critical $Fr_d^2$ depends on $Re_d$ and $d_*$ and has a power law relationship with both of them (figures 4-19 and 4-21).

3. For buildings with $h/H \neq 0$, the critical condition $Fr_d^2$ depends on $Re_d$, $d_*$ and $h/H$.

4. A parapet does not always reduce the risk of blow-off (figure 4-30)

5. Existing design guidelines based on small scale wind tunnel tests are flawed as the scaling of the experimental results is incorrect (figure 4-5 and 4-6).

6. Full scale testing is required in order to fully understand the process of roof-gravel blow-off

Finally, this research was conducted for 2 dimensional buildings, without systematically considering the effects of turbulence and atmospheric boundary layer. Future work should be done on:

1. Systematically consider the influence of the turbulent boundary layer properties on the critical condition for blow off

2. Be run at full scale, that is for values of $200 < d_* < 900$

3. Consider three dimensional buildings.
Abstract:

Experimental results are presented for both the rate of removal of particles from a roof top, and the resulting down-wind flight distance. Both sets of results were achieved through the use of newly developed experimental techniques that are accurate and repeatable.

Blow-off rate results indicate that there is an initial period of rapid gravel removal (high mass flux) followed by a longer period in which the mass flux is about a quarter of the original rate. Results are presented for both the initial and secondary mass flux as a function of the particle densimetric Froude number and the building geometry. In general, increasing the wind velocity or decreasing the parapet height increases the mass flux. For a constant Froude number, except for very short parapets, as the parapet height increases, both the initial and second stage dimensionless blow off rates decrease due to the shielding role of parapet. The transition time between the initial rapid and secondary lower mass flux regimes is calculated and shown to be independent of the parapet height and only influenced by the Froude number.

Experiments were run in which the particles blown-off model buildings were captured in a series of bins downwind of the building. From this, the extent of the debris field could be calculated as a function of the wind conditions and building geometry. Experimental results show that much of the mass blow off is deposited within the wake behind building and lands before the wake reattachment point. However, some particles escape the building wake and can fly much further away from the building. The experimental data are compared with Monte Carlo simulations based on the debris flight equation for a sphere by Holmes (2004). Results show that
the predictions of the debris flight model do not match the experimental results. The large discrepancy between the analytical model and experimental results is due to the debris flight model not considering the wake behind the building which dominates the downwind transport of the debris. As with the critical blow-off condition, scaling of results to full scale is non-trivial and scaling considerations are discussed in detail.

Key Words: Wind Borne Debris, Blow-Off Rate, Transition Time, Downwind Fields, Flight Distance, Froude Number

5.1 Introduction

Chapter 4 presented detailed experimental results for the critical condition under which loose gravel will be blown off a built-up roof. Those results focused on the initiation of blow-off. The other two pieces of information required to fully assess the risk of debris flight is the rate at which gravel is removed, and the downwind flight distance of the gravel. This chapter presents experimental results and non-dimensional parameterizations of both the rate of removal and the downwind flight distance.

Tachikawa (1983, 1988) was the first to derive the non-dimensional equations for the trajectories of wind born debris. This work was recognized by the naming of the Takikawa parameter by Holmes et al. (2006a). Wills et al. (2002) developed a model to describe the damage of windborne debris to a building during periods of high wind velocity. Holmes (2004) studied the trajectories of spheres released in strong wind theoretically and numerically assuming no rotation and no lift force. Holmes et al. (2006b) developed a numerical model for the trajectories of square plates in strong winds and investigated the effect of considering the
Magnus force in numerical models. Lin et al. (2006) carried out experiments to determine the flight characteristics of debris by using compact, plate like and rod like debris. They found that particle resultant velocity is the same as particle horizontal velocity in strong winds while the particle resultant velocity is higher than the particle horizontal velocity in mild winds. They said the difference should be because of the lift force. Lin et al. (2007) used the Tachikawa equations and proposed a simple equation for compact debris by ignoring the lift and rotational moment and effect of angle of attack. Baker (2007) presented a mathematical analysis for the debris flight equations and showed that for long enough flight times, the debris horizontal velocity approaches the wind velocity and the vertical velocity approaches the terminal velocity. Richards et al. (2008) used rectangular plates and rods for wind tunnel testing at different angles of attack and tilt angles. Based on the experimental results, the 3D model with 6 degrees of freedom was presented that accounts for lift, drag and moments in all three directions. Kordi and Kopp (2009) developed a model for the flight of windborne plate debris by using a quasi-steady theory. Kordi et al. (2010) conducted research on the effects of wind direction on the flight of roof sheathing panels in extreme wind and found that the flight distance was very sensitive to the initial release angle and was strongly influenced by the flow structure over the building and the building wake.

Although loose particles on top of a building are one of the main sources of wind born debris, its blow off rate and flight distance downstream are not fully understood. The main goal of this research is to investigate first, the rate at which particles leave the roof, and second, the distance that those particles fly downstream of the building. This chapter presents a discussion on the Tachikawa number in section 5.2, while the experimental setup for both studies is described in section 5.3. This is followed by a detailed discussion of the particle blow off
mechanism, the appropriate non-dimensional groups needed to describe it, and the blow off test results in section 5.4. Section 5.5 presents mechanism, scaling and results for downwind debris field along with the comparison of the experimental results with Monte Carlo simulations. Conclusions are drawn in section 5.6.

5.2 Densimetric Froude number versus Tachikawa number

The Tachikawa number was proposed by Holmes et al. (2006a) to name the dimensionless group

\[ K = \frac{\rho_a u^2 A_p}{2m_p g} \]  

(5-1)

where \( \rho_a \) is air density, \( u \) is wind velocity, \( A_p \) is particle cross section area, \( m_p \) is particle mass and \( g \) is gravity acceleration. It was first appeared in the non-dimensional debris flight equations derived by Tachkawa (1983, 1988). The Tachikawa parameter, \( K \) represents the ratio of the aerodynamic drag force to the gravity force, and can be rewritten as follows for sphere, plate and rod shaped particles. For a sphere with diameter \( d \):

\[ K = \frac{\rho_a u^2 A_p}{2m_g} = \frac{\rho_a u^2 A_p}{2(\rho_p - \rho_a)\pi d^2 g} = \frac{3u^2}{4(\rho_p - \rho_a)g d_p} = \frac{3}{4} \frac{u^2}{g d_p} = \frac{3}{4} Fr_d^2 \]  

(5-2)

For Plate with length \( l \), width \( w \), and thickness \( t \)

\[ K = \frac{\rho_a u^2 A_p}{2m_g} = \frac{\rho_a u^2 A_p}{2(\rho_p - \rho_a)lw g} = \frac{u^2}{2(\rho_p - \rho_a)lwt g} = \frac{u^2}{2g t} = \frac{1}{2} Fr_t^2 \]  

(5-3)

For Rod with diameter \( d \), and length \( l \)

\[ K = \frac{\rho_a u^2 A_p}{2m_g} = \frac{\rho_a u^2 (dl)}{2(\rho_p - \rho_a)\pi d^2 g} = \frac{2u^2}{\pi (\rho_p - \rho_a)g d} = \frac{2}{\pi} \frac{u^2}{g d} = \frac{2}{\pi} Fr_d^2 \]  

(5-4)
Essentially, the Tachikawa number can be thought of as a densimetric Froude number based on the smallest particle dimension. Thus, it is similar to the critical Froude number used in the previous chapter to parameterize the conditions for blow-off. In this chapter, we use the Froude number for parameterizing the blow off rate in order to be consistent with the previous chapter’s notation. We also maintain this notation because the blow-off rate is more analogous to sediment bed load modeling than flight modeling. However, the conventional Tachikawa number is used when discussing the resulting flight distance as this involves modeling and tracking individual particles.

5.3 Experimental Program

The main goal of this chapter is to investigate the rate at which particles leave the roof and the distance that those particles fly downwind of the building. A series of tests were conducted to calculate the blow off rate and downwind debris field. The tests address the effect of building shape, wind velocity and particle properties on the blow off rate and flight distance. The experimental setup and test method for conducting the blow off rate and downstream flight distance are described below.

5.3.1 Experimental Setup

The experimental setup used in the blow off rate experiments was identical to that used in the critical velocity tests. The only difference between the tests is the procedure. The primary dimensionless groups are the same as for the critical velocity, namely the particle densimetric Froude number $Fr_d^2$, the particle Reynolds number $Re_d$ and the dimensionless parapet height $h/H$. In addition, there are two new parameters, namely the mass blow off amount $m$, and
downstream distance measured from the building centerline \( x \). A full list of building and particle properties, appropriate dimensionless numbers, and full description of the scaling and non-dimensionalization are given in chapter 4 and section 5.4.

### 5.3.2 Blow off rate test method

A series of tests were conducted to measure the blow off rate of particles on top of the several 2-dimensional buildings. Building dimensions are given in detail in chapter 4. For calculating the particle blow off rate, a measured amount of particles were spread uniformly on the roof. The building was located at the center of wind tunnel, and first, was exposed to wind for 3 minutes. After 3 minutes, the remaining particles were weighted with precision of 1/100 of a gram by using an Ohaus Adventurer Pro 3102 Precision Balance. The removed particles were replaced and spread uniformly, and then the test was repeated 6 more times for 5, 8, 10, 15, 20, and 30 minutes of exposure to the wind. The mass lost during each time period was plotted against time (see figure 5-1). The blow off rate is the slope of the curve marked out by the data points. The average velocity at the top of the parapet height was measured by using a Dwyer pitot tube along with a Setra digital manometer, and a Measurement Computing data acquisition module. Matlab was used for analyzing the velocity data during the test.
5.3.3 Downwind debris field test method

A series of tests were conducted to measure the downwind debris flight distance distribution. To measure the debris flight distance distribution, a series of thin angled slats were placed downwind of the test building on the wind tunnel floor to catch the flying particles that leave the roof. The slats were located downwind of the building at $x = 0.5\text{m}, 1\text{m}, 1.5\text{m}, 2\text{m}, 2.5\text{m}, 3\text{m}, \text{and} 4\text{m}$ from the building center. These 7 slats divided the space located downwind of the building into 7 bins with mean distance of $x = 0.25\text{m}, 0.75\text{m}, 1.25\text{m}, 1.75\text{m}, 2.25\text{m}, 2.75\text{m}, 3.5\text{m}$ from the building centerline (Figure 5-2).

To run the tests, a measured mass of particles was spread uniformly on the roof. The building was located at the center of wind tunnel, and was exposed to wind for 30 minutes. After 30 minutes, the remaining particles were weighted using an Ohaus Adventurer Pro 3102 Precision Balance with precision of 1/100 of grams. The particles that had blown off and landed
in each bin were collected and weighed with same the precision. Results are presented in section 5 as plots of mass that landed in each bin as a percentage of the total mass that was blown off the roof.

![Diagram of grid network for catching particles]

**Figure 5-2: Grid network for catching particles**

### 5.4 Particle blow-off rate

#### 5.4.1 Mechanism and non dimensionalization

The rate at which mass is removed from a built up roof, or mass flux \( \dot{m} \), depends on the wind conditions, the building geometry and the particle properties. Also, as the wind begins to blow, particles will move around the roof to readjust to the new conditions. All these factors make the lift off rate a complicated process. In ideal conditions, it could be expected that as time goes toward infinity, no more particles leave the roof since all the particles already left the roof. But in real condition this never would happen because the parapet will protect some of the particles and does not let them leave the roof. Also, the time taken for this to occur could be quite large. The experimental results presented herein focus on the early stages of blow-off when the mass flux is at its largest.
In the experiments presented below the total mass removed from the roof is measured at different times during the test. That is

\[ m = m(t) = \int_0^t \dot{m} dt \quad (5.5) \]

or conversely, the mass flux is \( \dot{m} = dm/dt \) given by the slope of the curve shown in figure 1.

Tests results also show that the particles leave the roof with high rate at the beginning of the test, while the rate decreases as the time passes. During preliminary analysis of the results, two different methods were used to fit curves through the data and calculate the blow off rate. In the first method, results were represented by two straight lines. The first line was fitted through the early stage data, which shows the blow off rate at the beginning of the tests and the second line was fitted to the rest of data which represented the blow off rate of particles in the later stages of the test. In the second method, a single logarithmic curve was fitted to the data. Figure 5-3 shows the sample data for \( d_p = 0.45 \) mm. Both methods do a good job fitting to the data; however, there are two issues with logarithmic profile. First, the log function cannot pass through the origin (zero mass loss at \( t = 0 \)). Second, the logarithmic line did not fit well in some cases.

![Figure 5-3: Possible fitting curve on blow off rate result, raw data](image)
When the wind first started blowing, there was an adjustment in which the particles were driven upwind toward the windward edge of the building and then lifted up and swept off the building. After some time this adjustment slowed and a consistent, slowly growing scour hole was observed. A series of images of this adjustment are shown in figure 5-4. These observations support the use of the two line mass flux model described above. The first line represents the initial re-arrangement of aggregate on the roof and the second, lower rate, describes the gradual erosion of the remaining particles.

![Image of particles adjustment on roof top](image)

**Figure 5-4:** Particles adjustment on roof top, $h/H = 0.08$, $d_p = 0.45$ mm, wind blows from right to left
For modeling purposes, the blow off process was divided into two stages, an initial high blow of rate stage and second stage with a lower mass flux (figure 5-5). The initial stage refers to the initial time after the wind begins to blow. At this stage particles leave the roof more rapidly and total mass blown off the roof increases rapidly. In the second stage, the blow off process stabilizes and particles leave the roof at a constant rate that is lower compared to the initial stage. The initial mass loss is given by by

\[ m = a_1 t \]  

(5-6)

and the second phase by:

\[ m = a_2 t + b \]  

(5-7)

where \( a_1 \) and \( a_2 \) are the blow off rates for the initial and second stage, and \( m \) is the accumulated mass that left the roof from the beginning of the process up to the time \( t \). The time that (5-6) and (5-7) intersect is taken as the transition time \( t_t \). Blow off rates are calculated based on fitting straight line to the initial and second section of data.

\[ m = \int_0^t \dot{m} \, dt \]

**Figure 5-5:** Schematic blow off rate curve fitting method
To develop a dimensionless group for calculating the blow off rate, a reference mass
and reference time are defined as:

\[ m_{ref} = \rho_p W L d_p \tag{5-8} \]

\[ t_{ref} = \frac{d_p}{u_H} \tag{5-9} \]

The reference mass represents the mass of one layer of particles with thickness of \( d_p \) sitting on top of the roof with area of \( W \times L \). The dimensionless time is the time taken for the wind to pass over a particle of diameter equal to the mean particle diameter on the roof.

Dimensionless mass is therefore given by:

\[ M = \frac{m}{m_{ref}} = \frac{m}{\rho_p W L d_p} \tag{5-10} \]

Dimensionless time is given by:

\[ T = \frac{t}{t_{ref}} = \frac{t}{\frac{d_p}{u_H}} \tag{5-11} \]

Substituting (5-10) and (5-11) into the equations (5-6) and (5-7) for the fitted lines will give

\[ M = \frac{a_1}{\rho_p W L u_H} T \tag{5-12} \]

\[ M = \frac{a_2}{\rho_p W L u_H} T + \frac{b}{\rho_p W L d_p} \tag{5-13} \]

The dimensionless blow off rates is therefore given for the initial stage as:

\[ A_1 = \frac{a_1}{\rho_p W L u_H} \tag{5-14} \]

and for the second stage as:

\[ A_2 = \frac{a_2}{\rho_p W L u_H} \tag{5-15} \]

\[ B = \frac{b}{\rho_p W L d_p} \tag{5-16} \]
The transition time is given by:

\[
T_t = \frac{t_t}{t_{\text{ref}}} = \frac{t_t}{(d_p/u_H)} = \frac{B}{A_1-A_2}
\]  
(5-17)

The dimensionless blow off rates \( A_1 \) and \( A_2 \) will be controlled by the forces that act on the particles on top of the roof and the building geometry. The forces acting on a single particle on top of the building are drag \( F_D \), lift \( F_L \), weight \( F_W \), buoyancy \( F_B \), and resistance \( F_R \). The lift force is mainly from pressure differences due to the vortex circulation on top of the roof, and less from flow circulation around the particle. Therefore, the driving force in vertical direction would be:

\[
F_L \sim C_p \rho_a u_H^2 A_p
\]  
(5-18)

where \( C_p \) is pressure coefficient. The resistance force in the vertical direction would be as the particle weight minus the buoyancy force

\[
F_W - F_B \sim (\rho_p - \rho_a) \gamma_p g.
\]  
(5-19)

The balance between the driving and resisting forces is:

\[
\frac{F_L}{F_W-F_B} \sim \frac{C_p \rho_a u_H^2 A_p}{(\rho_p - \rho_a) \gamma_p g} \sim \frac{1}{2} C_p u_H^2 \frac{\pi d_p^2}{4} \sim C_p \frac{u_H^2}{(\rho_p - \rho_a) g d_p} \sim C_p \frac{u_H^2}{g d_p} \sim C_p F_{R_d}^2
\]  
(5-20)

Based on the results of Chapter 4, the primary building geometric ratio that controls blow-off is the relative parapet height \( h/H \). Therefore, the dimensionless blow-off rates will be given by:

\[
A_1, A_2, B = f \left( \frac{h}{H}, F_{R_d}^2 \right)
\]  
(5-21)

Although the critical condition for particles to leave the roof depends on the \( Re \) number (see Chapter 4), once the particle gets airborne Reynolds number effects are less significant.

Typical Reynolds numbers based on the particle diameter and wind velocity were between 100
to 1000 which is in the constant drag coefficient regime for natural particles such as sand and gravel (Chanson H., 2004).

5.4.2 Blow off rate results

Blow off rate tests were conducted for five different parapet heights \((h/H = 0, 0.002, 0.02, 0.08, 0.15)\) and six sand sizes \(d_p = 0.11, 0.23, 0.45, 0.73, 0.89\) mm and one millet size \(d_p = 2.30\) mm for several wind velocities. Blow off test results are presented as plots of \(A_1, A_2,\) and \(T_t\) versus \(Fr_d^2\), and \(h/H\). Plots of \(A_1, A_2,\) and \(T_t\) versus \(Fr_d^2\) for different \(h/H\) are shown in figures 5-6 to 5-8 respectively. Results show that as the \(Fr_d^2\) increases, the blow off rate increases. Further, the transition time \(T_t\) decreases as \(Fr_d^2\) increases. This is as would be expected. The higher wind velocity leads to higher blow-off rates, and a more rapid adjustment to the roof aggregate and, hence, a shorter transition time.

The results also show that the blow-off rate is also, sensitive to the parapet height \(h/H\). With the exception of very small parapet heights, both \(A_1\) and \(A_2\) decrease with increasing parapet height. This shows the protective role of the parapet in preventing the particles from leaving the roof. This trend does not hold for very small parapet heights for which the blow of rates are marginally higher than for the no parapet case (consistent with the observations of critical Froude number for blow-off to start presented in chapter 4). Results show that \(T_t\) is independent of \(Fr_d^2\) for all \(h/H\) ratios. Also, there appears to be little correlation between the transition time and the parapet height, which would tend to indicate that there is still aggregate movement over the roof surface (the adjustment process) despite the presence of the parapet. Therefore, although the parapet will protect down-wind regions from debris impact, it may be less effective at protecting the roof its-self from damage due to gravel movement. Note also,
that the Millet experiments follow the same trends as the sand experiments despite the significant density and size difference.

**Figure 5-6:** Dimensionless blow off rate $A_1$ for the initial line, filled marker represent millet

**Figure 5-7:** Dimensionless blow off rate $A_2$ for the second line, filled marker represent millet
Figure 5-8: Dimensionless transition time $T_t$ from initial stage to second stage, filled marker represent millet

To compare the dimensionless flow rate in the initial and second stages ($A_1$ with $A_2$), they are plotted against each other for each $h/H$ ratio (figure 5-9). The relationship between $A_1$ and $A_2$ appears independent of $h/H$, therefore, despite $A_1$ and $A_2$ depending on the $h/H$ ratio, $A_2/A_1$ is independent of $h/H$. The relationship between $A_1$ and $A_2$, regardless of $h/H$ ratio is approximately linear and is given by:

$$A_2 \approx 0.21A_1 \quad (5-22)$$
5.5 Downwind debris flight

5.5.1 Mechanisms

During severe storms, particles leave the roof and land downwind of the building. This flight distance is function of building geometry, particle properties, and flow condition. In the majority of research related to debris flight equation modeling, researchers neglect the building’s effect on the flow and resulting debris flight pattern. In those models, it was assumed that each particle is released into the uniform wind velocity, and begins to fly within that condition. However, because of the presence of buildings, the flow will not follow this simple assumption of a uniform velocity profile, and because of that, the flight distance in real world could be significantly different from the prediction of those simple models.

When flow passes over a bluff body such as a building, it will produce a vortex due to the flow separation. Observations indicate that particles mainly get lifted off the roof because of
flow circulation on top of the building. After the particles get airborne, their fate depends on whether they are captured within the wake behind the building, or escape the circulation flow. If the particle flies within the wake, then it would land somewhere before the reattachment point, but if it flies outside the vortex circulation, then it could fly further away from building (Figure 5-10).

Figure 5-10: Flow circulation above a downstream of the building, $x_r$ is reattachment distance

Agelinchaab and Tachie (2008) showed that reattachment length $x_r$ downstream of a square is about 8.5 times of square height ($x_r = 8.5H$). The results presented below show that the majority of particles landed within a short distance downstream of the building, which is inside the reattachment zone. This is consistent with results that other researchers reported such as Visscher and Kopp (2007) and Kordi et al. (2010) for roof sheathing panel. It means that generally, after particles leave the roof, they will get captured by circulation on top of the building and they will fall down somewhere before the reattachment point. However, some of the particles can get outside this re-circulation zone. An exact characterization of the wake behind the buildings used in these experiments is beyond the experimental capability of the wind tunnel. Therefore, the debris field results are presented in terms of downwind flight
distance scaled on the building height and compared to simulations that neglect the building wake. These results form the basis of a detailed discussion of the appropriate scaling of wind tunnel debris flight results to full scale and of the requirements of any future study of gravel blow-off flight distances.

5.5.2 Scaling of results

This section discusses the appropriate scaling approach for relating the flight distance from buildings of different heights. The results presented below scale the flight distance on the building height for different relative parapet heights. However, there is another length scale in the problem, namely the particle diameter. A detailed analysis of the debris flight equations by Baker (2007) showed that, for a large enough initial release height, a particle will travel horizontally at the wind speed and vertically at its terminal velocity. That is, after a particle flies a large enough vertical distance, a steady motion is achieved in which the particle travels in a straight line at a fixed angle to the horizontal. The angle at which the particle falls will be a function of the Tachikawa parameter $K$ discussed above. However, the height at which a particle must be released in order to achieve this steady motion depends on $K$ and the particle diameter. A non-dimensional release height $H$ can be defined as

$$\eta = \frac{\rho_a A_p H}{2 \rho_p V_p}$$  \hspace{1cm} (5-23)

where $A_p$ is the particle cross-sectional area, $V_p$ the particle volume, and $\rho_a$ and $\rho_p$ the air and particle density respectively,
In order to establish the height at which the steady flight angle is established a series of simulations were conducted to establish the ratio of the horizontal flight distance to release height as a function of $K$ and $\eta$. That is,

$$\chi = \frac{x}{H} = \chi(K, \eta)$$

The results of these simulations are presented as a contour plot in figure 5-11. The blue regions represent relatively short flight distances while the red regions represent longer relative flight distances. The longer flight distances occur for higher Tachikawa numbers, that is higher relative wind velocity and for lower relative release heights. Regions where the particle has reached the steady flight (constant angle) are represented by vertical lines in the contour plot. These are only seen for small $K$ or large $\eta$.

Figure 5-11: Contour plot of non-dimensional flight distance ($\chi$) as a function of non-dimensional release height ($\eta$) and Tachikawa number ($K$)
A zoomed-in version of figure 5-11 is given in figure 5-12. The same contour plot is focused on the region covered by the experiments presented in this chapter. The circles represent the \((K, \eta)\) points where experiments were conducted and the squares represent the full scale numerical simulations of Holmes (2004). In this region the lines of constant \(\chi\) are not vertical so that variations in \(\eta\) will result in different relative flight distances \(\chi\). Therefore, although there is continuity in the \(K\) values in the experiments, the use of different materials (sand and millet) and different building heights, means that the experimental flight distance will not all fall on a smooth line. This is discussed in the results section below.

Figure 5-12: Same contour plot as in figure 5-11 zoomed in on the region covered in the wind tunnel experiments. The circles represent the experiments run with millet and sand while the squares represent the full scale simulations of Holmes (2004).
The results presented in figures 5-11 and 5-12 suggest that simple geometric similarity (particle diameter to building height ratio matched) is only appropriate if the aggregate density is also matched, see equation (5-23). Rather, (5-24) represents the appropriate variables that must match to achieve full similarity (Reynolds number aside).

### 5.5.2 Downwind Debris Results

Experiments were conducted to assess the scale and distribution of the downwind debris field that results from the blow-off quantified in the previous section (figure 5-2). The results are presented as histograms of the percentage of the aggregate blown off that was recovered in each bin against the dimensionless flight distance \((x/h)\) (distance scaled on the building height). That is:

\[
\left(\frac{x}{h}\right)_i \propto S \frac{m_{R_i}}{\sum m_{R_i}}
\]

where \(m_{R_i}\) is the mass recovered in bin \(i\). Downwind flight distances tests were done for two different parapet heights \(h/H = 0\) and \(h/H = 0.016\) with one sand size equal to 0.5 mm and one millet size equal to 2.3 mm. Each test was run with three different velocities, which is represented by three different Tachikawa numbers \(K\). Recall that the Tachikawa number is equivalent to the Froude number squared and, therefore, increasing Tachikawa number represents a relative increase in wind velocity. The probability that a particle flies as far as \(x/H\) \((P(x/H)\) versus \(x/H)\) is presented as a function of \(K\) and \(h/H\) ratio for the sand and millet particles in figure 5-13. The last column represent the total mass loss that flew out the back of the wind tunnel. Results also are presented as the cumulative probability that a particle flies up to \(x/H\) (figure 5-14).
Typically, as $x/H$ increases, $P(x/H)$ decreases, though secondary peaks can be seen in some of the plots. The dashed-line on histograms mark the reattachment point as explained in figure 10. Most of the blow off particles land very close to the building before the reattachment point, which indicates that those particles are captured by building wake. For millet, $P(x/H)$ has a large value close to the building compared to the far distance from the building while 30 to 50 percent of blow off millet particles landed right away downstream of building. The particles which escape the building wake flew far away from building. It is why in some cases a considerable amount of particles never landed and flew out the back of the wind tunnel.
Figure 5-13: Probability (Not Cumulative) that particle flies to $x/H$, last column represent the total mass lost out the back of wind tunnel, dashed line represent the reattachment point.

Sand, $h/H = 0, H = 0.15$ m

Millet, $h/H = 0, H = 0.15$ m

Millet, $h/H = 0, H = 0.30$ m

Millet, $h/H = 0.016, H = 0.15$ m

Millet, $h/H = 0.016, H = 0.30$ m
Sand, \( h/H = 0, \ H = 0.15 \ m \)

Millet, \( h/H = 0, \ H = 0.15 \ m \)  
Millet, \( h/H = 0, \ H = 0.30 \ m \)  
Millet, \( h/H = 0.016, \ H = 0.15 \ m \)  
Millet, \( h/H = 0.016, \ H = 0.30 \ m \)

Figure 5-14: Cumulative probability that particle flies up to \( x/H \), dashed-line represent the reattachment point.
The experimental data was compared with the numerical solution of Holmes (2004) debris flight equation for spheres. Monte Carlo simulations similar to those described in chapter 3 with 10000 simulations per run were used to model the experimental results described above. A simulation was run to model millet particles on top of the building with zero parapet height for two cases, \( H = 0.15 \text{ m} \) and \( H = 0.30 \text{ m} \). It was assumed that wind velocity has a uniform profile with horizontal turbulence intensity equal to 0.20 (measured with the constant temperature hot wire anemometer system, TSI IFA-300) at the roof top. Simulations were run once with no initial velocity and once with a randomly generated initial velocity. The initial velocity was taken to be the wind speed with a launch angle randomly generated using a uniform distribution between 0 and \( \pi/2 \). Experimental and numerical results are compared for the probability that particles fly to \( x/H \) in figure 5-15.

![Experimental vs Numerical Results](image)

Millet \((K = 1.3), \ h/H = 0, \ H = 0.15 \text{ m} \)

Millet \((K = 1.6), \ h/H = 0, \ H = 0.30 \text{ m} \)

**Figure 5-15:** Probability (Not Cumulative) that millet particle flies to \( x/H \)

Clearly the Monte-Carlo simulations do not match the experimental results. For the case with no initial velocity, the analytical model predicts that all the particles land very close to the building over a much shorter range of flight distance compared to the experimental data. For
the simulations with an initial velocity and random launch angle, the analytical model over predicts the flight distance compared to the experimental data. The discrepancy between the analytical model and the experimental results is clearly related to the initial flight conditions and is also strongly influenced by the buildings effect on the flow. It might be possible to tune the initial conditions for the simulation to better match the experimental results. However, the experimental facilities needed to validate the selected initial conditions are not available. Further, such a model would still ignore the effect of the building wake that has been shown to strongly influence debris flight.

5.6 Conclusions

Loose particle on top of a roof present a potential hazard during a severe storm such as a hurricane. As the wind speed increases during the storm, loose particles can become wind borne debris by leaving the roof. As these particles fly downstream of building, they can hit objects including the human beings and cause a serious life and/or property damage. To better understand the downwind debris field and potential zones which are endangered, first, particle lift off critical condition which refer to the condition that particles leave the roof was studied in chapter 4. Then, particle blow off rate and particle flight distance is investigated in detail in this chapter.

A model is proposed for describing the mass blow off from the top of a roof. In the model, the whole blow off process was divided into two stages, an initial stage and a second stage. The initial stage refers to the initial time after the wind begins to blow during which particles leave the roof more. The second stage refers to the time period after the initial stage in which the blow off process stabilizes and the rate at which particles leave the roof decreases.
Each of these two stages is represented by straight fitted line as \( m = a_1 \, t \) and \( m = a_2 \, t + b \) for the initial and second stages respectively. \( a_1 \) and \( a_2 \) are the blow off rates for the initial and second stage, and \( m \) is an accumulated mass that has left the roof from the beginning of the process up to the time \( t \). The transition time from the initial to the second stage is denoted by \( t_t \). Based on those, dimensionless blow off rates for initial and second stage \((A_1 \) and \( A_2 \)) and a dimensionless transition time \( T_t \) were developed.

It is discussed that the main driving force in blow off process is the lift due to the pressure difference due to the flow separation and vortex formation on top of the building, while the resistant force would be the buoyant weight of particles. By considering that, it is shown that the dimensionless blow of rates are functions of the parapet height and the Froude number, \( A_1, A_2 = f(h/H, Fr_d^2) \).

Blow off rate tests were conducted for five different parapet heights \( h/H = 0, 0.002, 0.02, 0.08, 0.15 \) with both sand and millet. As the Results show, both dimensionless blow off rate for initial and second stage \((A_1 \) and \( A_2 \)) increase as the \( Fr_d^2 \) increases regardless of \( h/H \) ratio. That is, stronger winds will lead to higher lift and drag forces and higher particle removal rate. Also for constant \( Fr_d^2 \) value, except for very short parapet, as the \( h/H \) ratio is increased (parapet height increased), both \( A_1 \) and \( A_2 \) are decreasing which is because of the obstructing role of parapet height. The short parapet provide less protection compared to the zero parapet height (consistent with observed in chapter 4). For tall parapets, as the parapet height increases, it provides better shelter for particles and prevents them from leaving the roof. Although both \( A_1 \) and \( A_2 \) depend on the \( h/H \) ratio and the Froude number, their ratio \( A_2/A_1 \approx 0.21 \) was found to be a constant.
The transition time $T_e$ is not an independent parameter, and its value is calculated based on the values of $A_1$ and $A_2$ (equation 17). The dimensionless transition time $T'_e$ does not show any significant correlation with $Fr_{d}^2$ and $h/H$.

As the particles leave the roof, it would be a matter of interest to know how far the particles are going to fly and where they are going to land. Experiments were conducted to assess the scale and distribution of the downwind debris field. The results are presented as histograms of the percentage of the aggregate recovered against the dimensionless flight distance ($x/H$). As results show, much of the mass blow off is captured in the wake behind building and lands before the wake reattachment point. Also it shows that, some particles which escape the building wake can fly far away from the building.

The experimental data are compared with Monte Carlo simulations based on the debris flight equation for a sphere (Holmes, 2004) with both zero and non-zero initial particle velocity. Results show that the predictions of the debris flight model are highly sensitive to the initial conditions used in the simulation. The flight distance is also strongly influenced by the building wake. More precise modeling of the downwind flight distance requires detailed experimental measurements of the flight initial conditions and it should consider the building wake. Although the simulations are not at all representative of the measured flight distance, the parameterization based on analysis of the equations of motion (equation (5-25)) provides some insight into the appropriate scaling of results.

The results presented in this chapter demonstrate that:

1. The Tachikawa number is simply the densimetric Froude number based on the smallest particle dimension.
2. The particle blow off rate from roof top can be separated into two stages, initial and second.

3. The dimensionless blow off rate for both the initial and second stage \((A_1 \text{ and } A_2)\) is increasing as the \(Fr_{dA}^2\) increases regardless of \(h/H\) ratio.

4. Except for short parapet, as the \(h/H\) ratio is increased (parapet height increased), the dimensionless blow off rate \((A_1 \text{ and } A_2)\) is decreasing for constant \(Fr_{dA}^2\) value.

5. The ratio of the initial and second dimensionless blow off rates \(r (A_2/A_1)\) is independent of \(h/H\) and \(Fr_{dA}^2\) and is equal to \(A_2/A_1 \approx 0.23\).

6. The dimensionless transition time \(T_t\) from initial stage to second stage shows a decreasing trend as the \(Fr_{dA}^2\) increases, but is independent of \(h/H\) ratio.

7. The downwind particle flight distance is strongly influenced by the wake behind the building and its reattachment point.

8. Debris flight models based on uniform flow do not model the building wake and its effect on flight distance, which leads to a huge error in their flight prediction.

This research was conducted for 2-dimensional building, without considering the effects of turbulence and variations in the atmospheric boundary layer. For future works, the current work should be extended into 3 dimensions. Also, the effect of wind turbulence and the atmospheric boundary layer should be taken into account. Comprehensive full scale tests would be needed for downwind debris field to provide a flight map of different particles under different wind and buildings conditions. Conducting the experiments at full scale would eliminate some of the scaling issues discussed in section 5.5.2. In order to better understand the
wind borne debris blow off mechanism and its downwind flight distance, it is important to investigate and understand the flow structure around the building and its effect on the flight path by using accurate equipment such as PIV or LDV.
6.1 Introduction

Loose particles present a potential hazard during a severe storm such as a hurricane. As the wind velocity increases during the storm, loose particles can become wind borne debris. These flying particles can hit objects including people and cause serious life and/or property damage.

One of the major sources of debris in large commercial areas is loose gravel on built-up roofs. Such loose gravel can be responsible for extensive damage to buildings especially ones that are covered with lots of windows or with glass facades, such as many high rise buildings. Penetration of debris into the building envelope, can even lead to the complete collapse of the structure. The risk of wind-borne debris is not restricted to large commercial structures; family dwellings are also at risk. Windborne debris can also cause personal injuries.

Despite the high risk of windborne debris, their flight mechanics are poorly understood. There is very little in the literature regarding debris lift off (Holmes, 2004). While various debris flight models have been proposed, there are only a few experimental studies on windborne debris flight trajectories, and just a few experiments have been conducted on windborne debris initiation of motion. Also, the available analytical models for predicting the flight path such as that presented by Baker (2007), are typically only used once the debris is airborne. Further, these models assume that the flight process is entirely deterministic and that the controlling parameters are known and fixed.

The research presented in this dissertation focused on an experimental, numerical and analytical investigation of windborne debris during severe storms, such as hurricanes. This
dissertation has developed a new curve fitting method for calculating logarithmic velocity profile parameters i.e. shear velocity, surface roughness and zero plane displacement with high accuracy and without any iteration for laboratory work. Also, it was described the development of a stochastic model for debris flight that sought to address validity of the current simplifying assumptions and assess the significance of parameter variability on debris flight distance and impact kinetic energy. The results of a series of wind tunnel experiments that give a fuller understanding of the critical condition under which particles lift off from a roof, the rate that particles leave the roof and subsequent downstream flight pattern, were also presented.

6.2 Modeling the neutrally stable atmospheric boundary layer at laboratory scale

Estimating the surface roughness height for laboratory scale turbulent boundary layers, whether in wind or water, is essential to accurately modeling urban wind flows and dispersion at small scale. However, establishing this parameter is often difficult and involves estimates either based on fetch geometry that have wide variability or that require curve fitting through the measured velocity profile data. Existing curve fitting techniques are either iterative in nature, rely on empirical correlations between turbulence intensity and surface roughness, or both (Liu et al., 2002).

To avoid this complexity, a new curve fitting method for calculating logarithmic velocity profile parameters i.e. shear velocity $u_*$, surface roughness $z_0$, and zero plane displacement $d$, for laboratory work is introduced and compared with previously published estimation methods. The new two-step method is able to calculate $u_*$, $z_0$ and $d$ directly based solely on mean velocity profile data rather than using iterative calculations, turbulence intensity measurements, or direct measurement of surface shear stress. The first step involves calculating the shear
velocity (skin friction velocity) using a curve fit to mean velocity measurements to the velocity defect equation (2-14). The second step uses this value for $u_*$ to calculate $z_0$ and $d$ using a curve fit of the data to a re-cast log law velocity profile equation (2-15).

Comparison between the new method and previously published iterative curve fitting methods and other experimental measurements showed that the new method provides good estimates of $u_*$, $z_0$ and $d$. The strength of the new method is its accuracy (very close estimation of measured values of $u_*$), simplicity (two steps and no iteration), the limited data needed to make estimates (mean velocity profile and no turbulence intensity data), and applicability for both wind and water modeling.

6.3 Stochastic modeling of compact debris flight

Debris flight models are almost exclusively deterministic. These models typically assume known fixed steady input parameters such as wind velocity and particle size as well as constant coefficients such as the drag coefficient. However, this is very rarely the case and debris flight modeling can be improved by accounting for model input uncertainty. The results from this research indicated that failure to account for uncertainty in the particle size, horizontal turbulence intensity and vertical turbulence intensity will result in incorrect predictions of the mean flight distance and mean impact kinetic energy, and no information about the spatial distribution of the particle impact location or variation in impact kinetic energy.

The use of Monte Carlo simulations provides a means for quantifying the influence of input uncertainty on the resulting flight characteristics. Introducing uncertainty in any of particle diameter, horizontal turbulence intensity and vertical turbulence intensity, leads to larger mean value for flight distance (up to 13 percent) and impact kinetic energy (up to 64 percent),
compared to the condition where there is no variability in input parameters. The mean flight
distance and mean impact kinetic energy increase with increasing variability in input
parameters. Uncertainty in the particle diameter has a significant impact on the results.
Introducing horizontal turbulence intensity has considerable effect on results, while vertical
turbulence intensity has only a small effect on the results. Also, introducing variability in input
parameters leads to variability in flight distance and impact kinetic energy. By introducing
variability in input parameters, dimensionless standard deviation for flight distance increases up
to 0.37 while for the impact kinetic energy it increases up to 1. Larger input variability will
produce larger output variability. Varying particle diameter, horizontal turbulence intensity and
vertical turbulence intensity at the same time leads to larger flight distance (up to 4 percent for
the case with $\sigma_{d_p} = 1.37\ mm$, $l_x = 0.2$ and $l_z = 0.12$) and impact kinetic energy (up to 13
percent for the same case) compared to condition that varying nothing or varying one
parameter at a time.

A number of analytical approaches to understanding and quantifying the stochastic
nature of debris flight were presented that explain the broad trends observed in the data. The
Monte Carlo simulation approach was tested against a series of ball-drop experiments in the
Clemson University boundary layer wind tunnel. The simulations accurately predicted both the
mean and standard deviation of the flight distance for both sets of particles.

6.4 Wind borne debris flight initiation

The critical condition for roof gravel blow-off from top of a roof depends on the building
gometry, particle characteristics and the wind velocity. To better understand this
phenomenon, a series of two-dimensional tests were run. A new experimental method was
developed to measure the critical condition which eliminates the need for visual judgment in establishing the critical velocity for blow-off.

Results for buildings with $h/H = 0$ show that critical condition is independent of $H/W$ ratio for the range of parameters considered. Also, the particle densimetric Froude number has a power-law relationship with dimensionless particle size as $Fr_d^2 = 8.06(d_*)^{-0.44}$. For all $H/W$ ratios considered, the $Fr_d^2$ for the critical condition decreases as $Re_d$ increases. The densimetric particle Froude number also shows a power-law relationship with particle Reynolds number as $Fr_d^2 = 11.31(Re_d)^{-0.34}$.

For buildings with $h/H \neq 0$, the critical $Fr_d^2$ depends on both $Re_d$ and $h/H$ ratio. For $h/H = 0.002$, as the Reynolds $Re_d$ increases the Froude number for critical condition $Fr_d^2$ first increases until it reaches a peak, and then begins to decrease. For higher $h/H$ ratio, the $Fr_d^2$ for the critical condition decreases as $Re_d$ increases.

The experimental results indicate that $h/H = 0$ is not always the most critical condition. Results for small particles ($d_* = 5.3$ and 10.6) show that the presence of a short parapet increases the risk of blow-off for small particles. For larger particles ($d_* = 5.3$ and 10.6), the results show that increasing the $h/H$ ratio always increases the critical $Fr_d$.

As the critical Froude number is dependent on the $Re$ number, the raw data cannot simply be scaled up by using the $Fr$ number similarity without considering the effect of $Re$. Kind (1986) suggested a method for doing such a correction. However, the correction does not work for blow-off from roof tops as the correction is based on data for surface shear stress driven motion initiation which is not the process by which debris is blown off roof tops. Rather, the blow-off is driven by vortices from flow separation and pressure variation over the roof surface.
6.5 Wind borne debris blow off rate and downwind flight distance

The particle blow off rate and particle flight distance was investigated in detail for different building geometry, particle properties and flow conditions.

A model was proposed for describing the mass blow off from the top of a roof. In the model, the whole blow off process was divided into two stages, an initial stage and a second stage. The initial stage refers to the initial time after the wind begins to blow during which particles leave the roof more rapidly. The second stage refers to the time period after initial stage in which the blow off process stabilizes and particles leave the roof at a lower rate. Each of these two stages is represented by straight fitted line as \( m = a_1 t \) and \( m = a_2 t + b \) for initial and second stage respectively. The coefficients \( a_1 \) and \( a_2 \) are the blow off rates for the initial and second stage, and \( m \) is the accumulated mass that has left the roof from the beginning of the process up to the time \( t \). The transition time from the initial to the second stage is denoted by \( t_e \). Based on those, dimensionless blow off rates for initial and second stage \( (A_1 \text{ and } A_2) \) and a dimensionless transition time \( T_e \) were calculated as functions of the wind speed and building geometry.

The Results show, both dimensionless blow off rates for initial and second stage \( (A_1 \text{ and } A_2) \) increase as the \( Fr_d^2 \) increases regardless of \( h/H \) ratio. That is, stronger winds will lead to higher lift and drag forces and higher particle removal rate. Also for constant \( Fr_d^2 \) value, except for very short parapets, as \( h/H \) is increased (parapet height increased), both \( A_1 \) and \( A_2 \) are decreasing due to the obstructing role of parapet. The shortest parapet height measured provides less protection compared to the zero parapet height. For relatively tall parapets, as the parapet height increases, it provides a better shelter for particles and reduces the rate at which
they leave the roof. Although $A_1$ and $A_2$ depend on $h/H$ and the Froude number, their ratio $A_2/A_1 \approx 0.21$ was found to be a constant.

The transition time $T_t$ is not an independent parameter, and its value and trend is absolutely defined based on the value and trend of $A_1$ and $A_2$. Dimensionless transition time $T_t$ exhibited a decreasing trend as the $Fr_d^2$ increases, but $T_t$ does not show any significant correlation with $h/H$.

As the particles leave the roof, it would be a matter of interest to know how far the particles are going to fly and where they are going to land. Experiments were conducted to assess the scale and distribution of the downwind debris field. The results show that much of the mass blow off is captured by the wake behind building and lands before the wake reattachment point. Also it shows that, some particles which escape the building wake can fly far away from the building.

The experimental data are compared with Monte Carlo simulations based on the debris flight equation for a sphere of Holmes (2004). Results show that the predictions of the debris flight model do not match the experimental results. The large discrepancy between the analytical model and experimental results is due to the debris flight model not considering the wake behind the building.

6.6 Future work

This research focused on conducting a comprehensive study on wind borne debris in severe storm experimentally, numerically and analytically. Although many of questions about this phenomenon were answered in this research, still there are some areas of interest which are beyond the current facility capability, and/or current research resources.
Although the effect of uncertainty in main input parameters i.e. particle size and wind turbulence intensity on windborne debris flight were studied through a series of Monte Carlo simulation, there are many other sources of uncertainty that need to be investigated. For example, the launch angle, location on the roof at which flight initiation occurs, and particle shape will all vary about some mean. Further, the wind velocity relative to the particle will change during flight as the particle accelerates resulting in a change in Reynolds number, and therefore drag coefficient. All of these parameters require further investigation. Also, further work is needed to accurately parameterize all input parameter distributions.

The experimental tests were conducted for a series of different parameters in order to cover a wide range of building geometry, particle properties and flow condition. Despite of that, tests were conducted for 2-dimensional buildings, without systematically considering the effects of turbulence and the atmospheric boundary layer. For future works, the current work should be extended into 3-dimensions. Also, the effect of wind turbulence and atmospheric boundary layer should be taken into account. Full scale tests are essential for calculating the critical condition, blow off rate and downwind flight field of wind borne debris. Conducting the experiments at full scale would eliminate the scaling issues and Reynolds number effects. Comprehensive test are needed for downwind debris field to provide flight maps of different particles under different wind and buildings conditions. In order to better understand the wind borne debris blow off mechanism and its downwind flight distance, it is important to investigate and understand the flow structure around the building and its effect on the flight path by using accurate equipment such as PIV or LDV.
### Table A: List of simulation cases for each initial height that used in captions and figure legends.

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<th>$I_z$</th>
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**$d_p \& l_x \& l_z$**

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