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THE EFFECTS OF CONCRETE-REPRESENTATIONAL-ABSTRACT SEQUENCED INSTRUCTION ON STRUGGLING LEARNERS ACQUISITION, RETENTION, AND SELF-EFFICACY OF FRACTIONS

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THE EFFECTS OF CONCRETE-REPRESENTATIONAL-ABSTRACT SEQUENCED INSTRUCTION ON STRUGGLING LEARNERS ACQUISITION, RETENTION, AND SELF-EFFICACY OF FRACTIONS

A Dissertation
Presented to
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In Partial Fulfillment
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Doctor of Philosophy
Curriculum and Instruction

by
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Accepted by:
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ABSTRACT

Students in the United States are being outperformed by peers from other competitive nations on international mathematics assessments (TIMSS, 2007), lending to the national concern of the mathematics achievement of students within our country (NMAP, 2008). Success in mathematics is linked to graduation, higher education, and employment (Adelman, 1999; NMAP, 2008; NCTM 2010). A growing body of research supports instructional strategies, such as concrete-representational-abstract (CRA) sequence, to teach mathematics to students at risk of failure, including those with disabilities. This study extends the current body of CRA research by analyzing the effects of CRA sequence of instruction on student achievement, and retention, and self-efficacy of performance on computations of fractions. Thirty-five students participated in this study. Twenty students received CRA sequenced instruction, while 15 students received traditional instruction. Students’ performance and self-efficacy were assessed by two dependant variable measures, given prior to instruction, immediately after instruction, and four weeks after the completion of instruction. A series of repeated measures analysis of variances were performed to assess main effects and interaction effects for performance and self-efficacy of students in a control group who received traditional fractions instruction and a treatment group who received CRA sequenced instruction. Results revealed significant differences between the control and treatment groups on delayed-post measures of fractions computations. Students in the CRA group outperformed peers in the control group on the delayed-post assessment. No significant differences were detected on self-efficacy measures, with the exception of sources of
self-efficacy. Vicarious experiences as a source of self-efficacy significantly decreased on delayed-post measures. Students in the CRA group scored significantly higher than peer in the control group on the construct of negative psychological state (e.g., anxiety) as a source of self-efficacy on the presurvey, but not on the post- or delayed-post surveys, as a result of decrease in scores of negative psychological state. Conclusions and implications are discussed.
DEDICATION

To my husband, Michael, for all of his love and support- this degree is truly a shared accomplishment.

To my family, the Blurphy Bunch and Hughes, who encouraged me and continue to believe in me.

To my mother, Anne Marie Murphy Blatt, who inspired me as an educator and expected great things from me.

To my father, Robert F. Murphy, who instilled in me the value of education.
ACKNOWLEDGMENTS

My sincere appreciation to everyone who has helped me become the scholar I am and the scholar I have the potential to become. A heartfelt thank you to Dr. Linda B. Gambrell, who not only introduced me to educational research, but also remained with me on my journey and served as a co-advisor for my dissertation. Thank you to Dr. Paul J. Riccomini, my other co-advisor, whose guidance, knowledge, and dedication to mathematics and helping struggling learners is inspiring. Thank you to Dr. Pamela M. Stecker, who believed in my potential and continues to help me strive to excellence in scholarship. Thank you to Dr. Martha Thompson for providing guidance and helping me develop my skills as a researcher and statistician. You have all kept me grounded through this process.

Thank you to the Clemson University faculty, especially Drs. K, Ryan, and Hodge, who provided opportunities for me to develop as a researcher and writer and my NSF family, who provided encouragement and friendship. The friendly smiles and kind words shared in the hallways of Tillman were much appreciated.

Thank you to Dr. Renee Bradley for your mentorship.

Thank you to my family who provided encouragement throughout my journey.

Lastly, thank you to my husband, without you I would not be who I am today.
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CHAPTER ONE

DOCUMENTED NEED FOR EFFECTIVE MATHEMATICS INSTRUCTION

Mathematics achievement of students within the United States (US) has recently been under scrutiny. Student proficiency in mathematics is a concern among educators and educational stakeholders in the US based on scores from mathematics assessments (TIMSS, 2007). Students in the US continue to perform lower than students in many other industrialized nations on assessments of mathematics achievement (i.e., NMAP, 2008). Statistics from international assessments and reports suggest students from the US lack the skills to compete in an international arena, especially in fields requiring proficiency in mathematics (TIMSS, 2007; NMAP, 2008). Arguably, in order to advance in mathematics ranking at an international level, researchers, politicians, and educators must first address mathematics performance of students at state and local levels.

Within the US, proficiency in mathematics has grown as a concern among educators, parents, policy makers, and researchers. Success in mathematics is linked to graduation, higher education, and employment (Adelman, 1999; NMAP, 2008; NCTM 2010). Over the past decade, many experts have collaborated in efforts such as The Committee on Mathematics Learning (2001) and The National Mathematics Advisory Panel (NMAP, 2008) to address educational issues pertaining to mathematics proficiency. A growing consensus among experts indicates students need a strong curricula that provides a foundation for success as well as effective instruction, grounded in research, that is neither entirely ‘teacher centered’ nor ‘student centered’ (e.g., NMAP, 2008). In order to reach goals such as increased student graduation,
competitive access to higher education, and gainful employment, the NMAP emphasizes the need for student success in algebra. The NMAP therefore recommends that students develop competency in prerequisite skills to algebra prior to entering high school. For this reason, particular attention is being paid to acquisition of skills involving rational numbers, such as fractions (NMAP, 2008) for younger learners. Learning fractions is difficult for many students, but is especially challenging for students who have demonstrated persistent deficits in performance and those with special learning needs, such as a specific learning disability (SLD). Students with SLDs are at an even greater risk, than their peers without disabilities, to underachieve in mathematics and consequences resulting from underachievement.

The best way to ensure students who struggle with mathematics performance and those who have identified SLDs learn necessary prerequisite skills to algebra, such as operations involving fractions, is to teach such concepts using evidence-based instructional practices. Evidence-based practices in mathematics are those with documentations of success when implemented with similar student populations (e.g., educational needs, ages, gender) and have been shown to be effective when teaching mathematics concepts. An example of an evidence-based practice that has been successfully used to teach mathematics to students with SLDs is concrete-to-representational-to-abstract (CRA) sequence of instruction (NMAP, 2008).

Although CRA has a strong body of research that supports its use in mathematics, few studies have been conducted that look at proficient acquisition of computations involving fractions for middle school students who chronically struggle with mathematics. To date, only one study has been published that analyzed CRA sequence of teaching fractions to students who are at risk for failure in mathematics (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). The study looked at CRA sequenced instruction for a single skill (i.e., equivalent fractions) and
results of the study indicated that students who typically struggle with fractions and mathematics benefitted from the CRA instruction on equivalent fractions.

The need for improved student performance in mathematics, coupled with the substantial body of evidence to support the use of CRA to teach mathematical concepts, and the exceptional needs of struggling learners at risk of mathematics failure would suggest that more studies regarding the use of CRA instruction in mathematics for struggling learners, especially students who have an SLD would be available. However, few studies specifically target this instructional strategy as a means to support learning of prerequisite skills to algebra, such as fractions, for middle school students with deficits in mathematics. The small number of articles located suggests a need for quality research to be conducted in this area. The current study addresses the gap in research by examining student performance on operations involving fractions for middle school students who struggle with mathematics performance and are at risk for mathematics failure, such as those with SLDs, when taught by way of a CRA sequence as compared to traditional instructional practices.

**Components of Effective Mathematics Instruction**

It is accepted that the role of the teacher is pivotal in mathematics achievement of students. The teacher has influential role in how and what information is taught to students. Effective mathematics instruction is an appropriate balance that is neither entirely ‘teacher centered’ nor ‘student centered’ (NMAP, 2008; Kilpatrick, Swafford, & Findell et al., 2001). Instead, balance of instruction is determined by the needs of the students and the nature of the instructional content. Well-designed curricula in association with competent instructors aid student learning (NCTM, 2010). Stein and colleagues (1997, 2006) identified (a) instructional
design, (b) presentation techniques, and (c) organization of instruction as important components that contribute to student success in mathematics.

Steedly and colleagues (2007) and Stein and colleagues (2006) concluded that effective teaching is not haphazard, but instead requires purposeful and systematic sequence of instruction. They agree that students should be taught new skills after mastering prerequisite skills. Additionally, effective instruction includes explicit instruction of skills, that scaffold student learning. Student learning is enhanced through purposeful modeling, guided practice, and finally independent practice (Steedly et al., 2007; Stein et al., 2006).

Learning, naturally progresses from concrete to semiconcrete to abstract levels (Kamii, Kirkland, & Lewis, 2001). The NMAP (2008) recognizes concrete-representational-abstract (CRA) sequenced instruction as effective instruction and Siegler and colleagues (2010) recommend the use of visual representations to improve students’ understanding of fractions computations procedures. However, few studies analyze CRA sequenced instruction for middle school students (e.g., Witzel, 2005; Witzel, Mercer, & Miller, 2003) and even fewer on middle school students’ performance on fractions (e.g., Butler et al., 2003). Despite the understanding that motivation and self-efficacy contribute to student learning, no study to date assesses CRA sequence of instruction on both performance and self-efficacy of struggling learners.

**Purpose of the Study**

Empirical research documenting results of interventions aimed at improving student performance in mathematics is essential to improve the quality of instruction that students receive, which ultimately impacts student achievement. Individual research initiatives are important components that build a collective body of evidence of how to best instruct students. The study derives from the awareness of need for student proficiency in mathematics, while
considering the learning challenges experienced by students who struggle with mathematics and are at risk for mathematics failure. The research builds from an established body of research supporting CRA sequence of instruction, but uniquely adds to the body of research by extending the research to specifically teach operations involving fractions to students at risk of failure in mathematics and those with SLDs in mathematics. This study extends current knowledge of the effects of CRA sequenced instruction on student learning of mathematics, specifically fractions. Given the importance of student performance on mathematics, especially algebra and prerequisite to algebra skills (e.g., NMAP, 2008), the research is both timely and relevant.

**Research Questions**

The specific questions that guided the study were:

- *What are the effects of a concrete-representational-abstract sequence of instruction, as compared to traditional fractions instruction, on the mathematics achievement and knowledge retention of students with specific learning disabilities or those at risk of having a disability?*

- *What are the effects of a concrete-representational-abstract sequence of instruction, when compared to traditional fractions instruction, on the self-efficacy of students with specific learning disabilities or those at risk of having a disability?*

**Definition of Terms**

Key terms used in this research are defined below:

*Algebra*- A branch of mathematics where numbers are substituted with letters. In each problem, or equation, letters used represent a constant value. Example: \(5 + X = 27\). OR \(X/2 = 10-4\) OR \(A + B = C\).
At-risk (of having an SLD)- For the purpose of this research, students at risk of having an SLD are those who are chronically low performers in mathematics, as identified by their teachers and performance in mathematics class. Researchers historically use varying definitions for students who are low achieving or at-risk. As summarized by the NMAP (2008), factors that contribute to low achieving or at risk performance include: previous mathematics deficiencies, instruction by teachers with limited knowledge, limited informal mathematics support (e.g., home) or limited opportunities for incidental learning, deficiencies attending to academic tasks, and low motivation.

Concrete-Representational-Abstract: three part instructional strategy that builds students’ conceptual understanding by explicitly teaching meaningful connections from hand-on manipulatives, to representational pictures, to abstract concepts and symbols. Look up (NMAP, 2008).

Concrete: The initial instructional stage of CRA; the “doing” stage where students learn to use hand-on manipulatives to practice mathematics concepts. This stage includes visual, tactile and kinesthetic modalities (Witzel, Riccomini, & Schneider, 2008).

Representational: The second stage in CRA; the “seeing” stage where students learn and practice mathematics using pictures to represents of objects to model the concepts (Access Center, 2004).
Abstract: The final instructional stage in CRA; the “symbolic” stage, where students learn to use numbers and abstract symbols to model the mathematics concepts. (Access Center, 2004).

Evidence-based practice: (empirically supported practice) an educational practice that is supported by documented results from empirical research studying similar practices.

Fractions: a point on a number line that can be used to communicate ‘part of a whole’, a ration, quotient of a division problem, or part of a set (Gersten, Clark, & Witzel, 2009); rational numbers represented by a/b where b is not equal to 0.

Specific Learning Disability: In accordance with IDEA (2004), an SLD is

“a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which disorder may manifest itself in imperfect ability to listen, think, speak, read, write, spell or do mathematical calculations”

“such term includes such conditions as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.”

“such term down not include a learning problem that is primarily the result of visual, hearing, or motor disabilities, of mental retardation, of emotional disturbance, or of environmental, cultural, or economic disadvantage. “

Specific to this research, the student receives services in the area of mathematics, as indicated by the student’s IEP goals.
Explicit instruction: there is variability in how the term explicit instruction is operationally defined, however explicit instruction in mathematics involves three major parts including (a) demonstration by the teacher of a sequential, step-by-step plan to solve a problem, (b) strategy to meet the needs of a specific set of problems (as opposed to a general strategy) and (c) student demonstration of the same strategy during the course of the lesson (Gersten et al., 2009). More specific sequence of direct and explicit lessons will be provided in the paper.

Mathematics proficiency- adeptness in mathematics, (e.g., algebra, fractions, rational numbers, computations: addition, multiplication, subtraction, division).

Manipulatives- concrete, hands-on objects that individuals can move during a lesson to help learn the mathematics concepts.

Operations: Mathematical operations; calculation that uses mathematical methods; Specific to this study, operations involving fractions refers to calculations that use mathematical methods and include at least one number communicated as a fraction.
CHAPTER TWO

REVIEW OF RELATED LITERATURE

Currently, the importance of mathematics knowledge is receiving increased attention, spurring escalated efforts within the educational community to support student learning of mathematics with the goal to graduate competent mathematicians (e.g., NMAP, 2008). Educators rely on empirically supported instructional strategies to efficiently teach mathematics concepts to students. This chapter begins by outlining the importance of mathematics and highlighting the support for evidence-based practices within the classroom, especially for students who struggle with mathematics performance and are at risk for mathematics failure, such as those with specific learning disabilities (SLD).

Next, the chapter presents a review of literature for CRA instruction in mathematics for students who struggle in mathematics and those with SLDs, emphasizing the components of CRA that have established bodies of empirical evidence to support use for students with exceptional learning needs. Then the researcher anchors CRA instruction in Vygotsky’s social learning theory, emphasizing elements of social learning and the Zone of Proximal Development (1978). Lastly, the researcher extends discussion of student achievement by exploring the importance of student motivation and self-efficacy in mathematics learning and presents proper instruction, such as CRA, as an essential foundation for self-efficacy and successful acquisition of mathematics knowledge.

Importance of Mathematics

Today’s society requires individuals to acquire mathematics knowledge as a necessary means for productive citizenship and gainful employment (NCTM, 2010). Achievement in mathematics is linked to higher graduation rates (NMAP, 2008), completion of college
(Adelman, 1999), improved employment and higher income (Foegen, 2008), and success in technological career fields (NMAP, 2008). Despite awareness of these relationships and evidence of escalating consequences for ignoring these elements, mathematics achievement of students in the US remains less than desirable.

American students underperform in mathematics when compared to their international peers (NMAP, 2008). This underperformance threatens the US position as a leader in science and technology in a competitive, global economy. In addition, The Nation’s Report Card indicated that within the US, there are wide gaps between the performance of students who are considered ethnic minorities and their classmates (National Assessment of Educational Progress; NAEP, 2007, 2009); as well as students who have documented disabilities and their nondisabled peers (NAEP, 2007). Attempts have been made to address such discrepancies through legislation such as the Individuals with Disabilities Education Act and the Elementary and Secondary Education Act (IDEA, 2004; NCLB, 2001), competitive funding opportunities (e.g., grants), and ongoing research and professional development.

National efforts have been made to improve mathematics knowledge for students within the US. Over the past decade, numerous panels have been commissioned to determine best instructional practice in mathematics and encourage mathematics teachers to employ evidence-based practices in their classroom instruction. The work of the National Commission on Mathematics and Science Teaching for the 21st Century (2000), The Committee on Mathematics Learning, National Academy of Sciences (NAS; Kilpatrick et al., 2001), RAND Mathematics Study Panel (2003), and NMAP (2008) collectively indicate the complex and multifaceted nature of mathematics teaching and learning, while providing a comprehensive picture of the instructional supports necessary for student achievement in mathematics. Results yielded from
these expert panels should guide educators in developing curriculums and implementing
evidence-based practices to improve overall mathematics proficiency for students in the US.

**Understanding Problems in Mathematics**

In 1999, the Secretary of Education appointed the National Commission on Mathematics
and Science Teaching. The Commission sought to understand the problems in mathematics and
science education within the US and make recommendations accordingly. In their final report,
*Before It’s Too Late*, the Commission cautioned educational stakeholders about the
consequences of inadequate education in mathematics and science and established three
improvement goals (2000). The mathematics related goals of the Commission were to (a)
establish an ongoing system designed to improve the quality of mathematics instruction in grades
K-12, (b) increase the number of mathematics teachers and improve teacher preparation
programs, and (c) improve the working environment in order to make the teaching profession
more attractive for competitive professionals.

Following along the same mission as the Commission, The Committee on Mathematics
Learning was established by the National Research Council (Kilpatrick et al., 2001) with the aim
to address the educational needs of young mathematics learners. After thorough analysis of
current research, the results of the Committee yielded five major strands of mathematics
knowledge, including: (a) conceptual understanding, (b) procedural fluency, (c) strategic
competence, (d) adaptive reasoning, and (e) productive disposition. The Committee established
that each of the five strands have properties that encourage independent and interdependent of
themselves and each other (Kilpatrick et al., 2001). Each strand requires students reach a
proficient level in a variety of skills that build upon one another. The Committee concluded that
education in mathematics must develop in each of the five strands through a balance of explicit instruction and inquiry learning (Kilpatrick et al., 2001).

Two years later, the RAND (2003) report, which was conducted by the independent RAND agency, focused to develop a strategic research and development program and identify areas of weakness and needs in mathematics instruction. The results of the report generated three board areas identified as in needs of improvement. The first area described the need to develop teachers’ mathematical knowledge in ways that are useful for effective instruction. The second need emphasized the development of skills necessary for mathematical thinking and problem solving. The final area underscored the need for students to be engaged in learning algebra and prerequisite to algebra skills across their formal school years.

The most recent report published was from the NMAP (2008). NMAP was a result of an Executive Order from the President of the US to determine how to improve student success in mathematics. NMAP reviewed over 16,000 research publications and policy reports, heard public testimony from 110 individuals, reviewed commentary from over 150 organizations, and analyzed surveys from 743 algebra teachers over the period of 20 months. Their findings led to recommendations regarding six areas of interest, (a) curricular content, (b) learning processes, (c) teachers and teacher education, (d) instructional practices, (e) instructional materials, and (f) assessment. While improvement in all areas is necessary for collective improvement in mathematics achievement, the current research focuses primarily on the suggestions and findings regarding learning processes, instructional practices, and instructional materials. Ultimately, NMAP emphasized the importance of high-school students’ proficiency in algebra and the need to develop prerequisite skills to algebra in younger students.
The collective results yielded by the works of the expert panels mentioned above outlines the recent historic movement of mathematical needs in education while contributing to an overall picture of the current state of mathematics education in the US and suggestions for future mathematics improvement. While no panel of experts identifies a specific script or formula for success, they do present findings that demonstrate the use of certain instructional practices lead to higher rates of student success in mathematics. These instructional practices include empirically validated elements or strategies, which promote development of the skill sets necessary for student success in mathematics. Specific elements identified by the work of the experts are discussed later in the paper. Results are summarized in Table 2.1.

Table 2.1

**Expert Report Findings**

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<td>National Commission on Mathematics (1999).</td>
<td>Better understand and address areas of need regarding mathematics and science education.</td>
<td>Suggestions were made to focus efforts on system designed to (a) improve the quality of mathematics instruction, (b) improve teacher preparation programs and (c) establish competitive work environments for teachers.</td>
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<td>The Committee on Mathematics Learning (2001).</td>
<td>Address educational needs of young mathematics learners.</td>
<td>Results indicated students must be proficient in conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition and instruction in mathematics must be both explicit and inquiry based.</td>
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<tr>
<td>The RAND Report (2003).</td>
<td>Develop a strategic research and developmental program in mathematics.</td>
<td>Results indicated (a) teachers need to develop substantial mathematical knowledge, (b) emphasis should be placed on skill development, which is necessary for mathematical thinking and problem solving, (c) students need to be engaged in learning algebra and prerequisite skills through Grade 12.</td>
</tr>
<tr>
<td>National Mathematics Advisory Panel (2008).</td>
<td>A charge made by the President to assess the current educational system regarding mathematics and educational needs to ensure competitiveness in international arenas.</td>
<td>Extensive recommendations were given regarding (a) curricular content, (b) learning process, (c) teachers and teacher education, (d) instructional practices, (e) instructional materials, and (f) assessment. The report emphasized that students should reach proficiency in algebra in high school and prerequisite skills to algebra prior to high school.</td>
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Improving achievement in mathematics requires one to look at student acquisition of skills, but also requires a more broad understanding about the scope and sequence of mathematics learning. Structures must exist to properly guide student understanding of the sequential nature of mathematics, teach students the specific skills and strategies to be successful in mathematics, and provide additional supports for students who struggle or who demonstrate deficiencies or gaps in mathematics knowledge.

**Current Performance on Mathematics**

Mathematics achievement of students in the US is of concern as students in the US underperform in mathematics when compared to their international peers (NMAP, 2008; Provasnik, Gonzales, & Miller, 2009). For example, in an international comparison, the US ranked 24th out of 29 nations in student performance on problems solving and mathematics knowledge skills (Lemke et al., 2004). Deficits in mathematics knowledge has negative national economic repercussions, as many jobs that require a strong foundation in mathematics may be outsourced to other competitive countries. Mathematics knowledge is therefore necessary for success in a variety of professional fields including, engineering, economics, politics, accounting, banking, marketing, pharmaceutical and medical, education, retail, and construction jobs.

The Trends in International Mathematics and Science Study (TIMSS), sponsored by the International Association for the Evaluation of Educational Achievement (IEA), report international student achievement by comparing student performances from participating countries to established international benchmarks (2007). From this data researchers are able to compare the average performance of students in the US to students in other competitive
countries. According to results reported by the 2007 TIMSS report, average scores for students from the US in both Grades 4 and 8 were above the TIMSS scale average. However, fourth graders, on average, were outperformed by students from 8 of the 35 participating countries (i.e., Hong Kong, Singapore, Chinese Taipei, Japan, Kazakhstan, Russian Federation, England, and Latvia). Similarly, eighth graders from the US were outperformed, on average, by peers in 5 of the 47 participating countries (i.e., Chinese Taipei, Korea, Singapore, Hong Kong, and Japan). Despite improved mathematics performance of students from the US between 1995 to 2007 TIMSS reports, students from three competitive countries: Hong Kong, Japan, Singapore, consistently outperformed students from the US in mathematics. The disparities in mathematics performance between students from the U.S. and those from Hong Kong Japan, and Singapore suggest possible differences in the quality of instruction and training received by the students.

Another international cross-comparison assessment, the Programme for International Student Assessment (PISA), measures student performance of 15-year-olds in reading, mathematics, and science knowledge every three years. Data collected in 2006, indicated that, students from 23 of the 29 participating countries outperformed students from the US in mathematics. Results from the 2006 PISA situate ranking of performance for students in the US in the bottom quarter compared to performance for students from other participating countries (e.g., Czech Republic, France, Italy, Korea, United Kingdom). Students from the US scores statistically lower than the average scores on the mathematics scale. Average scores produced by students in the US were lower than average scores from 30 other participating countries on the mathematics assessments, including areas of China, Korea, Finland, Singapore, Canada, United Kingdom, New Zealand, Japan, and Germany (PISA, 2010). Current trajectories of performance for students from the US compared to their international peers as indicated by the
PISA surveys (2003, 2006, 2010) and TIMSS reports (1995, 2007) suggests a competitive turnaround in student performance is not anticipated unless improved instructional strategies are implemented in the US.

**Accountability for Student Performance**

While the need to improve student performance is evident, what is less clear is how to specifically define what constitutes successful mathematics outcomes and how best to achieve such outcomes. Regardless of specific clarity on how to advance mathematical performance, it is becoming increasingly evident that schools and teachers are being held accountable for student performance (NCLB, 2001). One effort to increase student performance in mathematics is to require students take more mathematics classes, therefore ensuring more access to mathematics curriculum. By 2004, 26 states within the US, required students have at least 2.5 credits in mathematics to graduate from high school (Council of Chief State School Officers, 2004). Supporters indicate the increased graduation requirements have led to higher student achievement (IES, 2008a), but dropout rates remain high. For example, the national average freshman graduation rate in the US is 75%, while South Carolina’s average falls at 61% (Stillwell, 2010). Increasing the number of courses required for graduation may increase the number of students who take mathematics courses, but results from national and international performances indicate increased number of graduation requirements does not ensure student achievement and increased mathematics knowledge. Effective mathematics instruction may, therefore, play a more important role in improving student performance than increasing graduation requirements for mathematics. As there are no scripted solutions to improve student proficiency in mathematics, educators must made decisions using the best guidelines and most accurate information available, which are guided by results from high-quality research. It is
important that educators provide effective instruction that incorporates empirically validated instructional approaches, most likely to effectively teach students the skills they need to be successful in high school mathematics courses.

Evidence indicates that employing empirically validated instructional strategies is the most efficient means to increase mathematics knowledge and improve performance. In accordance with the most recent reauthorization of the Elementary and Secondary Education Act (ESEA), formally known as the No Child Left Behind Act (NCLB, 2001), teachers must instruct students using evidence-based practices and are accountable for the improved achievement of their students. Benefits of instruction are most often monitored by documented performance on high-stakes tests, which are reported and scored for annual yearly progress (AYP). Student performance on high-stakes assessments as predominate measures of school accountability has received much criticism. For example, using results from high-stakes assessments as an accountability system to evaluate efficiency of teachers’ instruction has been criticized as isolating for teachers, by disregarding collaborative efforts of instruction, and imposing increased instructional control on teachers (Cobb & McCain, 2006; Valli, Croninger, & Walters, 2007).

While ESEA (NCLB, 2001) focuses on student achievement for all students, the Individuals with Disabilities Education Improvement Act (IDEA, 2004) focus solely on achievement of students with exceptionalities. Historically, students with disabilities were educated separately from their peers without disabilities. Education reform made access to general education available for students with disabilities; however, while it was once believed that providing physical access to general education classrooms was adequate as a way to educate students with disabilities with their nondisabled peers, this is no longer the case. Merely
admitting students with disabilities to the physical location of a general education classroom is no longer an accepted practice. It is now recognized that students with disabilities must also have access to grade-level curriculum and appropriate instruction that benefits the individual students (i.e., results in learning). IDEA (2004) mandated educators take responsibility for student progress by providing evidence-based instruction and tracking student progress towards individual, educational goals.

Similar to ESEA, IDEA emphasized the need to use instructional strategies that have empirical evidence that support use with students who have disabilities. All students who qualify for services under IDEA have individualized education plans (IEP) that set academic and behavioral objectives for the student based on individualized student strengths and weaknesses as a result of the disability. IDEA stipulates that IEPs for students with disabilities must include measurable goals, and educators must monitor student progress toward these individualized student goals. The IEP is considered the heart of special education, as it requires teachers to instruct based on the child’s unique learning needs. Emphasis on monitoring of student performance within the IEP highlights the need for student growth and movement toward achieving grade level educational goals. Focus on student progress, as opposed to academic access, requires teachers monitor student advancement toward goals and change instructional techniques if student performance is stagnant or inadequate progression toward educational goals.

As indicated earlier, access to academics is not enough if students do not have the opportunity to achieve and improve within their academic programs. The fundamental shift within special education to emphasize increased student achievement, not just physical access, is
yet another element of educational reform that heightens the importance of effective teaching and instruction.

**Academic Performance in General Education**

Poor mathematics achievement of students within the US relates to students both with and without disabilities. Lower performance in mathematics has been correlated to race and ethnicity, poverty status, education of the mother, and English as a second language (IES, 2008b). Despite consistent improvement in scores over the past six-years in mathematics achievement, significant discrepancies remain between the performance of students of ethnic minority (e.g., black, Hispanic) and their Caucasian peers (National Research Council, NRC, 2007) and students who speak English as a second language and English speaking peers. A similar gap remains between students who qualify for free or reduced lunch and their peers who do not qualify for free or reduced lunch (NRC). It is the responsibility of the educational system to meet the needs of diverse learners and learners from diverse backgrounds, and best prepare them for success in school. To accomplish this task, researchers have looked at factors that predict and indicate success in mathematics and consequently retention and success in school (NMAP, 2008).

Success in mathematics while students are in school may have direct and indirect consequences for them throughout their lifetimes. Research indicates that performance in ninth grade algebra is a strong indicator of likeliness to drop out of high school (Balfanz & Legters, 2004). Chronic low performance in early years of school is likely to have severe long-term consequences for the student, especially in mathematics where acquisition of new skills relies on mastery of prerequisite skills. In order for students to be successful in higher-level mathematics courses such as algebra as emphasized by the NMAP (2008), students must thoroughly
understand underlying mathematics concepts and master previously taught skills. Based on international reports, performance of students within the US depicts a bleak picture of international competitiveness (e.g., TIMSS, 2007). International comparisons provide an expansive scope of student performance and mathematics knowledge, but within the US, the performance of each individual state may provide more accurate insight on local and regional performance as well as trends and areas of national need.

**Current Performance in General Education within South Carolina**

The need for mathematics knowledge within the state of South Carolina is apparent from recent trends and reports. South Carolina has recently slipped in rankings based on achievement in mathematics for students in Grades 4 and 8 on the Nation’s Report Card (NAEP, 2005, 2007, 2009). In 2005, out of the 50 US and the District of Columbia, South Carolina was ranked 28th for students in Grade 4 and 21st for students in Grade 8. Despite policy changes and a national call for achievement accountability, in 2007 South Carolina’s ranking drastically dropped to 38th and 33rd, respectively, in mathematics (NAEP, 2007) a 10-placement drop for Grade 4 and a 12-placement drop for Grade 8. Approximately 70% of all eighth graders received a score of Basic or Below Basic on the mathematics assessment (NAEP, 2009) suggesting lack of mastery. Low performance by students from South Carolina has contributed to State initiatives aimed at improving student performance in mathematics.

The South Carolina Education Oversight Committee (2010) released its 2020 vision that “all students will graduate with the knowledge and skills necessary to compete successfully in the global economy, participate in a democratic society, and contribute positively as members of families and communities”, however specific mathematics goals to reach the 2020 vision were not specified by the committee on their website or mailings. When considering that 30% of the
questions on the NEAP assessment taken by students in Grade 8 cover algebra content, and an additional 20% of the questions are on number properties and operations (NAEP, 2009); special attention may need to be given to teaching algebra skills and prerequisite skills to Algebra (NMAP, 2008). Sanders, Riccomini, and Witzel (2005) reported that high-school students in South Carolina who were enrolled in algebra courses were not adequately prepared to learn the material. That is, many students lacked the prerequisite knowledge necessary for success in algebra courses. As indicated by state rankings and assessments of student performance, students in South Carolina currently lack skills necessary to be successful in higher mathematics courses, such as algebra, which may result in short-term and long-term consequences for students in the State.

Academic Performance in Special Education

Many students struggle learning mathematics concepts. Mathematics performance for all students has recently received attention, this is especially true for students who are at risk of failure, especially those with a documented SLDs. National outcomes for students with SLDs are bleak, as students with disabilities consistently produce scores lower than their nondisabled peers on high-stakes assessments (NAEP, 2007). Prior to IDEA (2004), many students with disabilities were excluded from high-stakes testing and overall reports of state progress. IDEA (2004) implemented guidelines that included more students with disabilities in high-stakes assessments. Advocates for children with disabilities applauded the inclusion of students in state assessments. However, accessibility is not sufficient if students who are included in the assessments do not receive the quality instruction necessary to improve academic performance, especially for the majority of students with SLDs whose performance is at a basic or below basic level.
**Graduation rates.** States are responsible to set up graduation requirements for students within the state. Some states have multiple graduation tracks and allow students with disabilities to obtain alternative diplomas. Nationally, the number of students receiving services for an SLD who graduate with a regular school diploma has improved over the past decade. In 1997, only 51% of students with an SLD graduated with a regular diploma (Cortiella, 2009). In 2007, the percent of students who graduated from high school increased to 61%. The trend is positive, however the number of students with an SLD who graduate from high school with a regular diploma is still low, constituting only three of every five students with an identified SLD. These numbers do not consider those who chronically struggle with academics or those who are at risk of having a disability, but were never diagnosed.

In 2007, approximately 14% of students with an SLD received an alternative certificate of high-school completion. This number had doubled in the past 10 years. South Carolina is one of many states that require students to pass an exit exam prior to graduation. Considering that many students with an SLD have difficulty with test taking, an exit exam may add another obstacle and is another challenge for students with SLDs. In 2004-2005, the average freshman graduation rate for students with and without disabilities in South Carolina was 60.1% (Cataldi, Laird, & KewalRamani, 2009), slightly lower than the 2007 US average.

**Drop-out rates.** Students who chronically struggle with academics and are consistent low performers are at a greater risk to drop out of school than students who experience academic success while in school. Students with an SLD are more likely to dropout of school than their peers without a disability, placing them at a disadvantage in the competitive workplace and suggesting a lower trajectory of job and career salary. In 2007, 25% of students with an SLD dropped out of high school. This is a significant improvement from the 41% it was a decade ago,
but much progress is still needed (Cortiella, 2009), especially when considering the long-term consequences dropping out of high school may have on an individual’s livelihood and the larger economy. South Carolina did not submit dropout data for the 2005-2006 school year (Cataldi et al., 2009).

**Grades.** It may come as no surprise when considering the exceptional learning needs of students with SLDs, that students with an SLD often receive lower grades than peers without a disability. It might be argued that grades are arbitrary records of student performance on classroom assessments, but grades that students receive consequently affect graduation, scholarships, acceptance into college, as well as a variety of other areas. The impact that grades have on such influential factors also places students with an SLD at a disadvantage in areas with long-term consequences. Compounding the issue that students with SLDs may struggle to achieve competitive grades in courses, is the fact that students with SLD often have deficits or gaps in academic skills. According to the National Longitudinal Transition Study 2, only 13% of students with an SLD are above, at, or less than one grade level behind the enrolled grade (Cortiella, 2009). Sixty-four percent of students are at least three grade levels behind, with a remarkable 20% of students with an SLD performing five or more grade levels behind their enrolled grade. Deficits or gaps in skills and strategies make future success in academics even more challenging for students with SLDs to achieve.

**Employment.** Employment is necessary for independent living and autonomous functioning within society. Gainful employment allows one to improve economical status. Only about half (55%) of adults with an SDL were employed in 2005. This is compared to 76% of adults without an SLD who are employed. Adults with an SLD were twice as likely as their peers without an SLD to experience unemployment (Cortiella, 2009), which may be a result of
failure to complete high school or college or failure to learn the skills necessary to be successful in the workplace. These discouraging employment statistics coupled with the educational statistics imply that the negative employment consequences for adults SLDs may stem from success, or lack-there-of, during the individuals educational career as a student and that educational outcomes of students with SLDs and may influence career choices as well as long-term employment outcomes.

Reports indicate approximately 6% of school age children are being served for an SLD (U.S. Department of Education, National Center for Education Statistics [NCES], 2010), however it has been estimated that as much as 10% of school age children may have an SLD. Students who receive services for SLDs make up almost half of all students who receive special education services (NCES). Over 50% of those students identified with an SLD have mathematics goals on their IEP (NCES; Hallahan, Kauffman, & Pullen, 2007). These statistics do not include students who have not yet been identified with a disability and those who do not qualify for special educational services, but are at risk of failure in mathematics. It is often difficult for educators to differentiate between what students have an SLD and what students are at-risk, as both groups of students often demonstrate similar behaviors and poor academic achievement. Weighing these considerations, students with low performance in mathematics as well as students with an identified SLD were included in this study.

The Need for Proficiency in Mathematics

In an effort to improve quality of instruction, legal obligations require teachers to use evidence-based teaching strategies to meet the learning needs of their students (IDEA, 2004; NCLB, 2001). According to the latest advisory panel on mathematics, improved student performance in mathematics requires changes across several major categories including
curricular content and learning process (NMAP, 2008). The NMAP report that curriculum should logically support the development of mathematics acquisition by carefully sequencing content for learners. Mathematics concepts naturally increase in complexity as students progress through school and therefore learning more complex aspects of mathematics require an understanding of less complex aspects. For example, students must understand the concept of numbers and the value associated with numbers before the students can be expected to comprehend and perform addition, which requires combining values together to create new values. A firm understanding of simple addition is essential before learning multiplication, which is a form of repeated addition (e.g., $6 \times 3 = 6 + 6 + 6$). It can be communicated that mathematics concepts build from prerequisite skills. Essentially, developing student proficiency requires that educators work backward from the educational goals to determine proficiency in which skills are necessary for success in the ultimate educational goal.

The NMAP (2008) emphasizes the importance of student proficiency in algebra prior to high school as necessary for students to achieve with higher-order mathematics (e.g., Calculus, Trigonometry) Almost one-third of the questions on the 8th grade NEAP assessment covered topics pertaining to algebra (2009). Therefore, curricula that teach algebra and prerequisite skills to algebra are necessary to prepare students for success in mathematics, as measured by the NEAP assessment (NMAP, 2008). In order for adequate student proficiency in algebra by eighth grade, the curriculum must support student acquisition of prerequisite skills throughout elementary and middle school years. Prerequisite skills to algebra, such as a strong foundation of rational numbers and fractions, build a foundation from which students can successfully master algebraic concepts, and become prepared to achieve success in higher mathematics. Yet many students fail to demonstrate proficiency in algebra or prerequisite to algebra skills.
Many students without an identified SLD who struggle with mathematics often demonstrate chronic behaviors of failure and deficiencies in performance similar to peers identified with a disability, making it challenging for general and special education teachers to meet the critical needs of struggling students. Prerequisite skills for algebra (e.g., rational numbers, computation, basic facts, and fractions) are often problematic for students (NMAP, 2008). Although students at risk of mathematics failure and students with SLDs often experience problems in multiple areas, fractions appear especially difficult for students with an SLD (NMAP, 2008; Sanders et al., 2005). Educational emphasis on teaching fractions in curricula has increased as acquisition of fractions is now understood to be an area critical to mathematics proficiency and success in algebra (NMAP, 2008). Placement in an inclusive classroom with a standard curriculum may offer students with disabilities access to grade level content along side peers without disabilities, but that alone does not ensure that students, especially struggling students at risk of failure, adequately learn necessary mathematics material.

When considering appropriate education, it is important to not only address curricular context, but also effective teaching and implementation of strategies. A strong curriculum is necessary to introduce the content objectives in an efficient and sustainable order, as well as support effective presentation of curricular content but is not effective without proper instruction. The best chance of students attaining proficiency in mathematics is through the application of evidence-based instructional practices.

**What Research Says About Effective Mathematics Instruction**

A considerable body of evidence establishes the powerful role of the teacher in best instruction practices of mathematics. Proper instruction is neither teacher centered nor student centered (NMAP, 2008; Kilpatrick et al., 2001), but instead an appropriate balance of
instruction, determined by the needs of the students and the nature of the content. Teachers require support from well-designed curricula, but must possess competencies to engage students through quality instruction (NCTM, 2010). The work of Stein and colleagues (1997, 2006) identify three instructional variables central to student acquisition of mathematics knowledge. According to the Stein and colleagues, (a) instructional design, (b) presentation techniques, and (c) organization of instruction each play an important role in skill acquisition, and all three components contribute to student success in mathematics. The next section addresses each of Stein and colleagues’ essential elements and provide current research on the topic, such as the meta-analysis of effective instructional strategies in mathematics conducted by Gersten and colleagues (2009).

**What Research Says About Instructional Design**

While there is no dispute that design of instruction is fundamental to quality instruction, researchers have proposed different elements identified as necessary for quality instruction. For example, after reviewing the finding of the four most recent panels on mathematics, Steedly and colleagues (2007) concluded that effective teaching consists of (a) systematic, (b) explicit, and (c) sequential instruction. Stein, Kinder, Silbert, and Carnine (2006) identified similar design elements necessary for effective instruction which included (a) proper sequence of skills, (b) explicit design, (c) teaching of preskills, (d) careful selection of samples, and (e) ample opportunities for students practice and review. Although communicated in different ways, both formulas for effective instructional design emphasized a systematic, sequence that introduces new skills after mastery of prerequisite skills, and incorporate explicit instruction. Skills are introduced and taught in an explicit way that scaffold student learning through purposeful modeling, guided practice, and independent practice. Research suggests that when these
elements are present, students can expect to build knowledge in a scaffold and efficient manner (e.g., Steedly et al., 2007; Stein et al., 2006).

**Objectives.** Identifying the objectives that students need to learn is an essential component of instructional design. Objectives that clearly outline specific and observable behaviors allow both the teacher and the student to know what is expected from the student during the lesson. Also, clearly defined objectives communicate to the teacher when students meet the lesson objectives. Vague objectives that lack precise measurement of skills do not identify what specific behaviors the teacher is looking for, nor when the teacher has the information necessary to determine the student has achieved mastery of the objective. Well-defined objectives clearly describe what behaviors the student is expected to demonstrate and when the student is expected to achieve mastery of the skills.

**Order.** The order in which information is presented and taught is critical to student learning in mathematics, a subject where preskills and easy skills are necessary to complete more complex and difficult concepts (NMAP, 2008). By teaching easy skills first, students are able to build knowledge and self-efficacy that they can be successful in mathematics. Many complex skills are actually combinations of more simple skills. Learning easier skills allows the student to apply learned strategies to the more complex situations with ease. For example, students must learn subtraction before they learn long division (which requires the preskill of subtraction), but students must also learn subtraction without regrouping before they learn subtraction with regrouping. Subtracting without regrouping is an easier skill to learn, and logically have priority subtraction with regrouping. Carefully sequencing instruction and purposefully demonstrate examples or non-examples allows students to learn when and how to apply the strategies (NMAP, 2008).
**Sequence.** It is also important for teachers to consider the sequence in which they teach skills, paying special attention to separating the introduction of skills that are confusing. Embedded in the emphasis of sequencing of skills is the necessity to teach preskills prior to more complex skills. Mastery of component skills builds a strong foundation from which mathematics strategies are learned (Stein et al., 2006). For example, long division requires students to multiply, subtract, compare numbers, and add; therefore mastery of specific prerequisite skills are required before a student learns the strategies to divide larger numbers. Additionally, teaching multiples of 6 and multiples of 4 may be confusing, if introduced in succession, as both sets include some of the same numbers (e.g., 12, 24). To support success with algebra, it is suggested that students in earlier grades develop strong number sense (NMAP, 2008). Well developed understanding of number sense assists students in the process of developing necessary geometry skills as well as better understanding of concepts related to fractions, such as percents, decimals, and negative numbers (NMAP). It stands to reason that students who have strong mathematical skills upon entering algebra classes, are better equipped to perform the necessary algebraic tasks and therefore have a better chance of experiencing success with algebra.

**Explicit instruction.** There is considerable evidence to suggest students who struggle with mathematics learning often benefit from explicit instruction (e.g., Goeke, 2009; Fuchs, Fuchs, Powell, Seethaler, Cirino, & Fletcher, 2008; Miller & Hudson, 2007; 2006). Explicit instruction is clear, systematic, and presented in a direct way that decreases (or eliminates) the learners need to construct knowledge (Goeke, 2009).

There are six main components present in direct and explicit instruction: (a) daily review, (b) presentation of new information, (c) guided practice, (d) independent practice, (e)
corrective feedback, and (f) weekly and monthly monitoring. Each of the components is structured to maximize student benefit from lesson presentation and instruction.

Daily review allows teachers to go over necessary information and encourage students to use background knowledge. In mathematics, procedures and concepts build in complexity, making it important for students to recall the previously mastered material and apply it to new lessons. While many advanced learners may automatically make the connections from learned information to new information, students who struggle to learn mathematics concepts and those with an SLD often lack skills fluency do not automatically make those connections. Daily review allows the teachers to clearly and precisely make the connections for the students. A review is generally followed by the presentation of new information that systematically uses the skills presented in the review.

During the presentation of new information, the teacher clearly shares the information in a clear and concise manner that fosters student understanding. The language, procedures, and examples used are carefully and systematically chosen to best meet the needs of the students and communicate the objective of the lesson. Teachers model what the students are expected to perform and learn during the lesson. The modeling provides a guide, allowing the students to see the correct application of the skill or concept in a scaffold and supportive environment.

Students practice the new skill with the guidance of the teacher who purposefully removes support as the students move toward independent practice of the information. The progression of removing scaffolding is targeted and purposeful. If support is removed too soon, students may be unsuccessful at the task or skill and if the supports are removed too late, students may become bored and apathetic during the lesson. The work expected during a lesson
should be challenging, yet not so challenging that the students cannot experience success with
the support and guidance of the teacher.

At any point during the lesson, the teacher is able to provide corrective feedback, which
may require reteaching of a skill or additional scaffolding of the present skill. Perfect practice of
a skill or concept helps to develop mastery of a skill. Repeated success not only develops
mastery, but also helps the student develop fluency with the skills. Fluency with skills allows
students to recall info in a quick and efficient manner, thus decreasing cognitive load and
allowing students to apply to more complex or difficult skills.

During explicit instruction, students are not only continuously monitored throughout the
lesson, but also across lessons. Monitoring student progress is important to determine
improvement in student performance over time. Monitoring student progress allows teachers to
make instructional decisions based on student learning needs. Success of a lesson is determined
by students’ ability to meet lesson objectives.

Stein and colleagues (2006) assert that explicit instruction is unambiguous and teaches
concepts that are generalizable and applicable to implementation with a variety of different
problems. For example, when teaching multiplication, it may not be beneficial for struggling
learners to learn a finger shortcut for multiples of nine if that strategy will confuse them in the
future when learning multiplication including numbers larger than nine, where the strategy is no
longer applicable and conceptual understanding of multiplication is necessary for success. This
‘shortcut’ cannot be used to find multiples of any other numbers and it can only be used through
9x10. The limited success struggling students experience may lead to confusion with more
complex multiplication. Beneficial strategies are ones that can be applied to multiple
mathematical situations. Guided practice during strategy instruction allows teachers to scaffold
student learning and help students to develop permanency and perfect practice. Scaffold support throughout the lesson is systematically and purposefully taken away as students are able to succeed at the task independently. Independent practice allows the student to work individually, while under the close observation of the teacher. Throughout the lesson, skillful teachers are alert of student performance and provide corrective feedback as needed. Corrective feedback is specific to the student’s needs and based on student errors. Immediate feedback and monitoring during the lesson is important, however that assesses student performance at that moment, not growth over time.

**Importance of examples.** Constructing and choosing appropriate examples is a key element to instructional design. There are two major rules when deciding what examples to include with instruction (Stein et al., 2006). First, examples should include problems from previously taught lessons, eliminating confusion that may result from seeing examples of unfamiliar concepts (NMAP, 2008). Also, it is important to add examples from previously mastered material. This second component helps the student learn to discriminate between problem types and determine when to use the new rule and when to apply previously mastered strategies (Stein et al., 2006). By including problems from previously taught strategies, students are able to review material and maintain mastery. Research supports including examples and non-examples (Miller & Hudson, 2007) in the lesson to assist learning appropriate application of a strategy and reduce overgeneralization of a strategy.

**Practice and review.** The last consideration for instructional design is practice and review. Students should have the opportunity to practice using conspicuous learning strategies (Stein, Carnine, & Dixon, 1998; Miller & Hudson, 2007; NMAP, 2008). That is, the steps and strategies the student will use to accomplish the task must be made apparent in a clear way. It is
only after students show mastery that they should have the opportunity to practice independently (Miller & Hudson, 2007). By allowing students to work independently only after they have proven they have mastered the skill, students are given the opportunity to consistently and correctly practice the skill, building permanency and fluency.

**What Research Says About Instructional Delivery**

The second area addressed by Stein and colleagues was instructional delivery. Initial assessment and progress monitoring of student growth contributes to effective instruction. The techniques used by a teacher during a lesson may influence rate of learning and student self-concept (Stein et al., 2006). The teacher directly provides portions of the lesson, commanding responsibility for the pace and direction of the lesson. The teacher must maintain student attention, which is influenced by the length, pacing, and signals used by the teacher. Effective explanations are brief, concise, and allow individual opportunities for student response as well as group or choral opportunities for response.

**Variety of strategies.** Well-designed lessons utilize a variety of strategies to engage students in learning. Effective instruction is based on students’ learning needs. Examples of effective strategies include graphic organizers, learning strategies, mnemonics, examples and non-examples, peer-mediated strategies, and visual representations (Miller & Hudson, 2007; Sanders, 2007; Kunsch, Jitendra, & Sood, 2007; NMAP, 2008). Research supports teaching mathematics concepts using representations that move the learner from the concrete to representational to abstract demonstrations of the strategy (Witzel, 2005; Witzel & Riccomini, 2009).

**Error analysis.** During an explicitly taught lesson, when a teacher encounters a student error, it is in the best interest of the student that the teacher correct the error by modeling the
correct answer, testing the student, and testing the student again after a delay. This type of interaction allows the student to see the problem solved correctly and properly mirror the behavior. Sometimes an error is a symptom of a more chronic problem. Research suggests teachers who are more proficient in the content being taught are better at analyzing the errors made by the student, and consequently more efficiently help the student fix the source of the error. Error diagnosis and remediation allow the teacher to determine the cause of a pattern of errors and may require the teacher to analyze worksheets, interview the student, and provide reteaching as necessary (Ashlock, 2002; Riccomini, 2005).

There are three types of mathematics errors that occur, including (a) factual errors, (b) component skill errors, and (c) strategy errors. For example, a student who is missing most of the addition practice problems on an assignment for fractions with like and unlike denominator, the teacher must determine what errors are being made and why. The teacher may determine that the student doesn’t know their basic addition facts (e.g., factual errors, $3 + 5 = 8$) or that the student is unaware if the denominators are common or uncommon (e.g., strategy error, when to find like denominators). Because the teacher knows the nature of the errors, the teacher is able to reteach the necessary skills with more precision. By pinpointing the source of the error, the teacher does not have to waste precious teaching time on elements the student has already mastered, but rather focus on the elements of most difficulty for the student (Riccomini, 2005). Identifying and addressing errors early can save instructional time for teachers who would otherwise have to help the student unlearn an incorrect strategy in order to master the correct strategy.
Classroom Organization and Management

The last major component identified by Stein and colleagues (2006) is classroom organization and management. During an explicitly taught lesson the teacher is required to make effective use of classroom resources and instructional time. Elements of a daily lesson often include teacher directed instruction, independent work, and time to review newly learned material. Students that are engaged in learning are more likely to attend to the information being taught.

Proper teacher directed instruction encourages student engagement through a quick pace lesson that allows ample opportunities for student-teacher interaction. The quick pace and multiple opportunities for interactions encourage student to remain attentive and engaged for the duration of the lesson. During instruction, the teacher demonstrates the new skills and students are able to practice under the supervision of an expert teacher. Lessons are scaffolded based on the specific needs of the students (Stein et al., 1998). Skillful teachers make instructional decisions based on perceived student performance and ongoing assessment. Teachers have the responsibility to reteach and model how to correctly solve problems as well as provide additional examples if the students are struggling with the corrective use of the strategy and can succeed with the task independently. Additional supports must be systematically removed as the students demonstrate greater understanding of the strategy. As mentioned earlier, students should not be expected to work independently, or without assistance, until they have demonstrated approximately 90% proficiency for the skill (Stein et al., 2006). Even after students demonstrate proficiency and are encouraged to work independently, performance should remain closely monitored by the teacher. Ample opportunities to successfully perform a skill or practice a
strategy are necessary before a student can achieve mastery. Skillful teachers therefore must monitor and support student mastery of mathematics.

Meeting the educational needs of students with disabilities requires structured design of the lessons, including instructional design, instructional delivery, and attention to classroom management. Attending to the components of instructional design allows teachers to make instructional decision grounded in evidence-based practices aimed to help students learn and progress toward academic goals. The teacher has control over the instructional design elements included in the lesson and plays an important role in student success. Elements of instructional design may include systematic and explicit introduction of new materials, ample opportunities to practice, guided feedback and error correction. Well-designed approach to instruction allows the teacher to set the student up for academic success in mathematics.

**CRA as Empirically Supported Instruction**

There are many elements to consider when choosing which evidence-based instructional strategy to implement in the classroom; some instructional strategies naturally include more of the identified elements than others. The CRA sequence of teaching mathematics concepts embodies many of elements of instructional design and delivery suggested by experts (e.g., Stein et al., 2006; NMAP, 2008). The following section will explain what elements of effective instruction are parts of the CRA sequence of instruction.

**Instructional Design**

**Objectives.** Objectives must be clearly defined before the lesson. The objectives of the lesson clearly state the purpose, behaviors, and outcomes of the lesson. The CRA sequence of instruction incorporates expected student outcomes during independent practice. Teachers should move forward with the lesson only if students meet the objectives in the lesson. For
example, if students are required to do 10 addition problems independently, the teacher should not introduce a new lesson until students have demonstrated mastery by meeting the objective of correctly completing 9 of the 10 addition problems.

**Order and sequence.** By the nature of the intervention, concepts are presented in a natural order that builds on more difficult material and moves the student’s learning from the physical, concrete understanding to the more abstract use of symbols and numbers. Mathematics instruction must progress from concrete to semiconcrete (representational) to abstract symbols (Kamii et al., 2001). Students learn a concept by manipulating concrete objects that demonstrate what is happening in the mathematics lesson. After they achieved mastery and manipulated the material in the physical state, students learn to complete the same operations using picture (semiconcrete/representational) pictures instead of manipulatives. The pictorial illustrations mirror the concrete models to provide consistency for student learning. The same concepts are then connected to the abstract state, as students learn to successfully complete the same operation using symbols and numbers. In the CRA sequence, easier skills are taught before more difficult skills, so students are able to use previously learned information to make sense of new information.

**Explicit instruction.** The NMAP (2008) clearly states that instruction should not be entirely teacher centered, nor should it be entirely student centered. Instead balanced teaching includes both teacher and student centered instruction, based on the background knowledge and instructional needs of the child. Students who demonstrate persistent failure in mathematics and those with disabilities may require more teacher-centered instruction during the introduction of new material. Explicit instruction allows the teacher to control and lead the lesson at the beginning, introducing new material in a clear and concise format, and gradually removed
scaffold supports, shifting the responsibility of the learning to the student. During explicit CRA instruction, students are not required to construct new knowledge, which consequently decreases the chances that students will construct incorrect knowledge. Instead, students are expected to follow along as the teacher models how to successfully perform the new mathematics task and demonstrate the tasks or strategies under the supervision of the teacher. For example, when teaching four-halves the teacher would first demonstrate how to divide four manipulatives (e.g., popsicle sticks) into two groups (e.g., cups). After the teacher modeled the lesson by physically moving the manipulatives and concurrently describing what is happening in the lesson, students have the opportunity to practice the new material under the guidance of the teacher.

**Importance of examples.** Examples are structured into every level of the CRA sequence. Each mathematical concept is presented in a series of three lessons. Examples for the first lesson are completed with the use of concrete manipulatives. During the second lesson, the examples are given with picture representations. The third lesson demonstrates mathematics concepts with symbols and numbers. All examples are chosen prior to implementation of the lesson in order to ensure cohesive progression of the lesson and eliminate the need for the teacher to create examples spontaneously.

**Practice and review.** Students are given the opportunity to practice new concepts under the close guidance of the teacher and then again more independently. As suggested by Gersten and colleagues (2009), review is embedded into the lesson, aimed to increase student mastery of new concepts and emphasize elements from previously taught lessons.

First, CRA is effective in instructional design because it explicitly teaches new information through a systematic and cohesive sequence of events. Teachers demonstrate why the mathematics concepts occur by physically manipulating tangible objects. Concepts are first
presented by the teachers through a modality that allow the students to physically manipulate objects; the manipulation of objects supports student learning of physical attributes of complex concepts. The physical demonstration is followed by opportunities for students to practice the constructs, first with the teacher, and then under supervision of the teacher. The order in which mathematical concepts are introduced foster student understanding of the underlying reasons of the mathematics concepts. For example, students in the concrete level of CRA may practice division by physically manipulating object to simulate the concept of division. Only after students demonstrate mastery do they advance to the next, more complex lessons. Instruction develops student understanding of visual representations of mathematics concepts, and eventually mathematics symbols and complex aspects. Once students have mastered the concept in the concrete form, are they able to progress to the more difficult, representational level of learning. Visual representations of mathematics help students build mathematics knowledge (NMAP, 2008; Siegler et al., 2010). Mastery is once again expected before students learn mathematics in the abstract form of symbols and numbers.

**Instructional Delivery**

**Varieties of strategies.** Student learning is supported through a variety of strategies. CRA allows students to develop skills through multiple modalities. Concrete manipulatives are hands-on and allow students to learn the properties of the fundamental concepts. The properties remain the same, but the *doing* embedded in concrete level of instruction is replaced with *seeing* pictures that represent the original physical manipulative. The representational level acts as a bridge between the concrete form of mathematics and the abstract form that consists of numbers and symbols and may benefit learners (NMAP, 2008). Visual representations may aid students’ understanding of fractions computations procedures (Siegler et al., 2010).
**Error analysis.** In CRA, student learning is under the supervision of the instructor. The instructor plays an important role by ensuring student performance is accurate during guided instruction and again during independent practice. Teacher and student interaction allow the students to demonstrate perfect practice of the mathematics constructs, promoting mastery and fluency.

**Classroom Organization and Management**

Lastly, CRA addresses essential elements of classroom organization and management. Effective CRA physically is engaging, quick paced, and appropriately challenging. Essentially students are actively involved in the learning process. As with explicit instruction, after the teacher models the activity, students actively participate in their learning. The teacher is responsible for keeping the pace of the lesson quick, but appropriate for students’ needs. The teacher is also responsible for having the materials ready for the lesson (e.g., manipulatives, scripted lesson).

Mathematics instruction naturally progress from concrete representation, to semiconcrete representation, to abstract symbols (Kamii et al., 2001). During the concrete phase, the concrete materials used may not mirror the mathematical problem (e.g., using a pie chart to represent slices in a pizza), and therefore must be given meaning by the instructor and the learner. Throughout modeling, common examples, such as using a pizza to describe parts of a whole, may lack generalizability of strategies and skills. While it is easy to conceptualize eating one-fourth of a pizza, it is more difficult to understand eating nine-fifths of a pizza, or eleven-fifths of a pizza. The image of the whole may become distorted by the same background knowledge that gave meaning it in the first place. Therefore, it may benefit student acquisition of fractions if the
concepts are introduced in a concrete way that allows students to generalize the strategy across the content of learning fractions.

In review, CRA sequence of instruction includes many elements of effective instruction, including elements of instructional design, instructional delivery, and classroom organization and management. Students work toward clear objectives, by participating in an ordered and sequential lesson, explicitly taught and directed by the teacher. Examples purposefully provide students ample opportunities to review and practice skills in different modalities. During instructional delivery, students are taught a variety of strategies. Teachers take responsibility of the lesson by analyzing student errors and monitoring student progress. Teachers engage students in a quick paced lesson and gradually remove supports as students are able to successfully work more independently, thus shifting the responsibility to the student.

**Foundation for Current Study**

This chapter reviews research, legislation, and theory, which present need for the proposed research. The need for quality research to be conducted regarding student achievement mathematics, especially fractions as a prerequisite skill for algebra, is evident (e.g., NMAP, 2008). While there is much information available about quality instruction in mathematics and the importance of student success in mathematics, much more needs to be examined regarding mathematics acquisition. The chapter builds the theoretical framework that supports the proposed study and reviews the current body of research relevant to the research. The purpose of the literature review was to determine the current research for a CRA sequence of teaching mathematics to students with specific learning disabilities in mathematics and students at risk of a specific learning disability in mathematics. As described in this chapter, the use of a CRA
sequence to teach mathematics is supported and complemented by the works of Vygotsky (1979) and Bandura (1997, 2006a, 2006b).

**Manipulatives to Teach Mathematics**

The use of manipulatives or physical representations to teach mathematics is considered commonplace in mathematics instruction as it follows a natural progression of student learning (Kamii et al., 2001). Physical representations connect mathematics concepts to real-world experiences (NMAP, 2008). For example, one way to introduce the concept of counting or the procedures of dividing is to use blocks.

**Use of Manipulative to Teach Mathematics for Students with Disabilities**

Manipulatives are often used to introduce mathematics concepts. However, a body of empirical research supporting for the use of manipulatives with instruction for students with disabilities is still growing (Witzel & Allsopp, 2007). CRA sequence of instruction has been used to teach mathematics facts to elementary students with SLD (Mercer & Miller, 1993), multiplication and related problem solving for students with intellectual disabilities (Morin & Miller, 1998), and area and perimeter skills for middle and high school students with SLD (Cass, Cates, Smith, & Jackson, 2003). CRA sequence of instruction has also been integrated with other empirically supported teaching strategies to teach middle school students with SLD or emotional and behavioral difficulties (Allsopp, Kyger, Loving, Gerretson, Carson, & Ray, 2008), and may contribute to long-term maintenance of skills (Cass et al.)

Evidence supports that as students learn mathematics their understanding must progress from the physical representation to the abstract concepts necessary to be successful with mathematics. However transitioning from concrete, representations of mathematics to abstract symbolism of mathematics may be too broad a conceptual leap for some students. Without a
thorough understanding of the foundational concepts on which abstract understandings develop and scaffolded learning supports, students are not able to become critical and literate mathematicians. Confusions or misconceptions that occur during the fundamental learning of underlying mathematics concepts, may lead to greater difficulties throughout school and beyond. It is important that students develop correct mathematics concepts early in their mathematics careers, as delays in effective instruction may be detrimental to students’ academic success. Additional scaffold instruction using CRA sequence may benefit students who have gaps in their understanding of mathematics. Instruction that sequences from concrete to representational to abstract concepts of mathematics (CRA) may be used to bridge the gap between concrete and abstract representations. The representational stage allows students to refine understanding of mathematics with pictures that represent physical objects prior to working with number and symbols that also represent physical objects.

The CRA sequence has been used to teach mathematics concepts to students for decades, however more research needs to be conducted that focuses specifically on teaching prerequisite to algebra skills, such as fractions, to students with SLD. To date, little research has been conducted on this topic for this specific population. Table 2.2 communicates how CRA sequenced instruction addresses concerns and suggestions of recent panels of experts.
Table 2.2

_CRA as a Potential Solution to Expert Suggestions_

<table>
<thead>
<tr>
<th>Panel</th>
<th>Recommendation</th>
<th>CRA as potential solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>National Commission on Mathematics (1999).</td>
<td>One recommendation was to focus efforts on system designed to improve the quality of mathematics instruction.</td>
<td>CRA provides evidence-supported instruction that teaches procedural and conceptual understanding of fractions.</td>
</tr>
<tr>
<td>The Committee on Mathematics Learning (2001).</td>
<td>Results indicated students must be proficient in conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition and instruction in mathematics must be both explicit and inquiry based.</td>
<td>CRA is explicit instruction that aids students’ conceptual understanding and therefore procedural fluency in fractions.</td>
</tr>
<tr>
<td>The RAND Report (2003).</td>
<td>One outcome is that instructional emphasis should be placed on skill development, which is necessary for mathematical thinking and problem solving. Additionally students need to be engaged in learning algebra and prerequisite skills from Kindergarten through Grade 12.</td>
<td>CRA helps students develop skills in a scaffolded format, that gradually releases control to the students. CRA sequenced instruction to teach fractions helps students learn prerequisite to algebra skills, such as rational numbers and fractions.</td>
</tr>
<tr>
<td>National Mathematics Advisory Panel (2008).</td>
<td>The Panel suggests that mathematics instruction should be neither entirely teacher centered nor student centered. The report emphasized that students should reach proficiency in Algebra in high school and prerequisite skills to algebra prior to high school.</td>
<td>CRA gradually releases responsibility of learning to the students, only after students demonstrate readiness. CRA to teach fractions helps students develop and master prerequisite to algebra skills.</td>
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</tbody>
</table>

_Current CRA Literature for Students who Struggle with Mathematics_

I used the key terms *CRA, learning disability*, and *mathematics* to search for articles in Academic Search Premier, Education Research Complete, ERIC, Psychology and Behavioral Sciences Collection, PsycINFO, and Teacher Reference Center. Results yielded five articles. Due to the small number of articles found in the initial search, I manipulated the search words to include the following combinations: *CRA, fractions and learning disabilities*, and *CRA and mathematics* and *CRA, struggling learners, or at-risk*. An additional 25 articles were located.
Abstracts were screened for relevant articles. I then searched the reference sections of the articles found for more articles. Lastly, I searched by last name of researchers (i.e., Riccomini, Witzel, Flores, Mercer, Miller, Gersten). To be included in the review of literature the studies (a) included students with SLD, (b) examined a dependent variable that addressed mathematics, (c) used CRA sequence to teach the learned concepts, and were (d) published between 1999 and 2009. Studies prior to 1999 are addressed in a literature review conducted by Witzel (2003). Only four studies were located that met all inclusion criteria and were therefore included in the review.

Flores (2009) used single subject design across with a multiple baseline across students to study six third-graders with SLD or unidentified mathematics difficulties response to CRA instruction of subtraction with regrouping. During the first three lessons, students used manipulatives such as foam blocks to learn and practice two-digit subtraction. Lessons four through six were taught using representations. During lesson seven, students were taught a mnemonic designed to help remember sequence of steps. For the last three lessons, the teachers explicitly taught the students to subtract using abstract mathematics symbols. Results indicated improved student achievement as a result of the CRA sequence of instruction. Limitations of the study included a small number of participants in the intervention and instruction on only one mathematics concept.

A study conducted by Butler, Miller, Crehan, Babbit, and Pierce (2003) was the only published account found of a CRA study that specifically looked at student acquisition of fractions. In their article Butler and colleagues documented the importance of fractions and the chronic failure of students with SDL in mathematics. They compared student achievement on concepts and procedures for equivalent fractions of sixth, seventh, and eighth graders identified
Table 2.3

*Articles Included in Literature Review*

<table>
<thead>
<tr>
<th>Author, (Date)</th>
<th>Participants</th>
<th>Methodology</th>
<th>Results</th>
<th>Weaknesses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butler, Miller, Crehan, Babbitt, &amp; Pierce (2003)</td>
<td>50, middle school students</td>
<td>Pre-post design</td>
<td>Both groups improved, higher overall mean scores for CRA group</td>
<td>Intervention is limited to 10 lessons</td>
</tr>
<tr>
<td>Flores (2009)</td>
<td>6, 3rd graders</td>
<td>SSRD, Multiple baseline across students</td>
<td>Student achievement increased at implementation of CRA instruction</td>
<td>Small n, intervention was limited to 10 lessons, only one maintenance point was collected.</td>
</tr>
<tr>
<td>Witzel (2005)</td>
<td>231, middle school students</td>
<td>Pre-post-follow up design with random assignment of clusters.</td>
<td>Students in CRA treatment group scored higher on post and follow up tests than students receiving multisensory algebra model.</td>
<td>The assessment tool was designed specifically for this study and needs further evaluation.</td>
</tr>
<tr>
<td>Witzel, Mercer, &amp; Miller (2003)</td>
<td>34, Grade 6 and Grade 7 students with SLD or at risk for an SLD.</td>
<td>Pre-post-follow up design with random assignment of clusters.</td>
<td>Both groups improved in overall performance on the post assessment. CRA group outperformed control group on post and follow-up assessments. Fewer procedural errors in CRA group.</td>
<td>The assessment tool was designed specifically for this study and needs further evaluation. Assessments only tested performance on abstract knowledge. Lesson sequence reflected the sequence of common curriculums.</td>
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</table>

with SDL or other disabilities across a control group receiving representational to abstract (RA) sequence of instruction and an intervention group receiving CRA sequence of instruction.

Students learned about fraction equivalency and nonequivalency across 10 scripted lessons. For the intervention group, the first three lessons were taught using concrete manipulatives. During the third lesson, students were also taught to express mixed numbers and improper fractions.
using manipulatives. Lessons four to six required students to use representational drawings to communicate and develop the skills learned in the first three lessons. During lessons 7 through 10, students were taught how to use algorithms to solve similar problems. Students in the control group used representational drawings instead of hands-on manipulatives during the first three lessons. Data indicated both the RA control groups and the CRA treatment groups improved, although overall mean scores for the CRA group were higher than scores for the RA group. Despite promising results, no other published CRA intervention for fractions could be located. It is important to note that the study only consisted of 10 scripted lessons, one third the number of scripted lessons proposed in the current research. Additionally, the study of fractions focused on basic functions related to equivalency and did not teach multiplying, dividing, adding, or subtracting fractions.

Witzel (2005) used a CRA sequence to teach algebra to students with SLDs in inclusive settings. The study included a pre assessment, post assessment, and a delayed-post assessment. Participating teachers taught both the CRA sequence and the standard abstract sequence of teaching. Data from students who had complete data sets (n = 231) were analyzed. While both the treatment and the comparison group improved, the treatment group yielded greater gains from the pretest and posttest, outscoring the control group. This is a considerable achievement because the control group significantly outperformed the treatment group on the pretest.

Witzel, Mercer, and Miller (2003) conducted a study with sixth and seventh graders who were match-paired by performance on the study pretest. Students were match based on pretest score, age, achievement score, and class performance. Students in the treatment group received explicit CRA sequenced instruction. Student in the comparison group received typical instruction. Students in both the experimental and comparison groups received instruction from
the same teacher. The CRA group outperformed control group on post and follow-up assessments. Analysis indicated students in the CRA group produced fewer procedural errors supporting the use of an explicit CRA sequence to teach algebra to sixth and seventh graders with and without disabilities.

Of the four studies found that met the inclusionary criteria, two of the studies (Butler et al., 2003; Witzel et al., 2003) were also included in the final report created by the NMAP (2008). Only one study examined CRA sequence of instruction to teach fractions (Butler et al.). The study examined performance of middle school students, however the limitations of the study (e.g., short treatment length) reinforce the potential of CRA instruction for sixth and seventh graders with SLD in mathematics and therefore the need for further investigation into CRA in this area.

**Social Development Theory and Zone of Proximal Development Framework for Study**

CRA is grounded in theories of Vygotsky, addressing student learning, and Bandura’s theories on motivation to learn. Vygotsky asserted that individuals learn in social context and acquisitions of new skills are supported through social learning (1978). According to Vygotsky, learning takes place at the social level before it takes place on the individual level. Consequently, learners benefit from guided instruction that incorporates both modeling and corrective feedback provided by knowledgeable adults or capable peers within the learners’ society, which in this case is their current educational environment (i.e., teachers and advanced students). The mentors are expected to model the behavioral expectations and demonstrate that specific procedures lead to desired outcomes. The culture of the classroom fosters student learning with the cycle of modeling and feedback. The vicarious experiences provide a model for students’ learning.
In the second part of his theory, Vygotsky suggests students acquire knowledge best within their zone of proximal development (ZPD). The ZPD is the distance between what a student can do independently versus the potential of what the student can do under guidance and/or with collaboration of a mentor. The ZPD is the area where students most benefit from instruction because they are introduced to skills that may be too difficult for them to successfully complete by themselves, but can achieve with the scaffold guidance of an expert individual. The ZPD recognizes the sequence of development learning and addresses the need to guide students as they learn more complex and abstract concepts. During instruction, the teacher initially has the responsibility of monitoring the student and identifying what factors are necessary for students’ unique learning needs, such as setting goals, planning, evaluating, and focusing attention on relevant information (Baker, 2002). By assuming responsibility of students’ learning, the teacher is able to model the learning expectations and procedures. The teacher must therefore choose to introduce strategies that are challenging for the student (i.e., the student cannot perform independently) while making instructional goals attainable (i.e., not too difficult for the child to practice the strategies with teacher assistance).

During the modeling, Vygotsky (1978) asserts that children must first question the material attempting to be processed in order to accept and internalize the new knowledge as a useful strategy that assimilates with their existing knowledge. The learning, therefore, is not passive, but rather active on the part of the learner. This must take place before the student can independently utilize the knowledge. The teacher can aid in this process by providing appropriate and purposeful feedback while attending to students’ metacognition. For example, when students learn multiplication as a form of repeated addition, they internally question if that makes sense based on what they know about addition and rational numbers. After the child
accepts the new knowledge, the child is able to internalize the new information and eventually transfer the knowledge into independent practice. The teacher aids in this process by modeling the behaviors (e.g., with manipulatives) in ways that are familiar to the student, but also extend student knowledge and promote mastery.

The sequence of instruction requires teachers to lead the learning of the student and provide assistance with skills that are currently too difficult for the child to do independently, with the recognition that purposeful withdrawal of support will allow the student to successfully internalize the learning and apply the new knowledge unaided. Over time, the student will become more autonomous, and consequently self-guide and self-regulate desired learning.

Vygotsky’s theories are embedded within multiple elements of CRA instruction. First, CRA instruction uses explicit teaching where the instructor models the strategy, providing social learning with a competent mentor. Within the same explicit format, student learning is scaffold as teachers shift from models to monitors of student learning. The role of the learning shifts from the teacher to the student, under the constant supervision and guidance of the mentor (i.e., teacher). Additionally, CRA instruction sequences from a hands-on learning with concrete manipulatives, where students make sense of physical objects; to representational pictures that model the manipulatives, eventually to an abstract sequence where numbers and symbols represent the objects. Students are introduced to more abstract and difficult learning only after they have mastered and internalized easier and more concrete representations of the strategy.

Both explicit format and sequencing incorporated in CRA instruction complement social learning and ZPD components of Vygotsky’s theory by incorporating modeling from a mentor assisted to self-regulated acquisition of knowledge, as well as a sequence of lessons that increase in complexity and challenge (1978).
For the reasons stated above, the theories of Vygotsky (e.g., 1978) that form the foundation for CRA instruction are beneficial for students with exceptional learning needs, such as those with an SLD. Due to emphasizes of student mastery within CRA sequenced instruction, progression of the lesson is based on the needs of the students, as opposed to mere exposure to the content. Scaffold releases of supports are based on how responsive the students are to the CRA sequence of teaching. Within the lesson, teachers are responsible for providing more or less scaffolding based on student performance. For example, if students are struggling during the guided practice component of the lesson, teachers must provide more modeling and additional opportunities for students to practice with extra support. Vigilant monitoring by the teacher allows them to assess student performance and gradually releasing supports as the students demonstrate they are able to take on more challenging learning tasks. Instruction within the ZPD is important to student learning, but is incomplete without student motivation to learn new and sometimes challenging material.

**Motivation and Self-Efficacy**

Motivation is hallmark to learning and an integral component for student success. Positive student perceptions of mathematics have been linked to higher student achievement (House & Telese, 2008), strategic problem solving (Hoffman & Schraw, 2009), as well as execution of metacognitive strategies (Klassen, 2006). The concept of motivation is multifaceted and has been widely interpreted, however few studies have examined the influence of motivation on specific problem solving outcomes (Hoffman & Schraw), especially outcomes on tasks involving fractions. According to Thorndike (2005), measuring psychological constructs of students, such as motivation, may be problematic and inconsistent for researchers. Contributing to the complexity of measuring psychological constructs are (a) multiple theoretical lenses
Motivation to learn has been a topic of interest for decades (e.g., Deci, Benware, & Landy, 1974; Pintrich & De Groot, 1990). Collectively research suggests students’ content specific motivation to learn decreases as students progress through school (e.g., Wigfield, Eccles, Mac Iver, Reuman, & Midgley, 1991). This motivational decline has been attributed to physical changes in self or school and classroom environments (e.g., Wigfield et al.), increased peer awareness and instruction that emphasize competition over curiosity (Guthrie & Wigfield, 1997), or potential history of failure (Pajares, 2006). While there is a considerable body of research regarding motivation of typical learners, data regarding certain topics are fragmented. For example, there are more studies conducted on general academic motivation or motivation to read than motivation to engage in mathematics. Of the studies found related to mathematics, most addressed the motivation of secondary and college students. This is not surprising given the complexity and increased difficulty of mathematics at those levels. Less often examined was motivation in the early middle grades.

As children mature into adolescents, they confront new challenges and increasingly difficult tasks (Bandura, 2006a). External influences on adolescents also change, as behaviors are seen in the larger scope of the social community (Bandura). Adolescents are expected to be more independent and self-reliant while navigating through society norms. Therefore, self-initiation and goal setting become increasingly important as adolescents are expected to work toward fulfilling their potential in adulthood.
Defining Motivation

According to Bandura (2000a), individuals who set goals recognize the relationship between working toward a goal and achieving the goal. They are able to anticipate the possibility of success based on personal actions. On the other hand, if individuals do not believe they can successfully accomplish a task, there is (a) no intrinsic motivation to set the initial goal, and (b) belief that it is wasteful to work toward an unattainable outcome. Goal setting, therefore, requires an internal locus of control and the self-confidence that one’s actions influence outcomes. Consequently, the belief of self as a change agent fuels the self-motivation of adolescents to set and achieve a desired goal.

Currently, there are numerous theoretical perspectives that have implications for motivation. Variables studied to evaluate student motivation differ by study such as, (a) engagement, academic self-efficacy, and intrinsic motivation (Fan & Williams, 2010), (b) anxiety, interest, task value, self-efficacy, and goal orientation (Linder & Smart, under review), and (c) self-efficacy, instrumentality and goals (Greene, Miller, Crowson, Duke, & Akey, 2004). Reliability was reported in two of the studies (Linder & Smart; Greene et al.) as Cronbach’s alpha 0.83 and 0.76-0.92, respectfully; indicating that the instruments measured the anticipated number of constructs. These results, while reliable, do not verifying that the variables addressed encompassed the complete construct of motivation, as each measure of motivation included different variables. Particularly salient across all constructs of motivation is self-efficacy.

Present in the conceptual representations of motivation described by Fan and Williams (2010), Smart and Linder (under review), and Green and colleagues (2004) are the construct of self-efficacy. Bandura defined self-efficacy as “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (1997, p. 3). Essentially,
self-efficacy is perceived awareness of one’s ability to produce the desired results for a particular task (Zimmerman, 2000). Bandura (1997) suggests that self-efficacy is content driven and relies on the difficulty of the identified task. Self-efficacy appears to be content specific, indicating one’s perceived ability to accomplish a task. This perception may differ with familiarity and comfort with the task, therefore one’s perceived ability may change as task requirements change. For example, a student may expect a higher rate of success on a mathematics task that requires the student to add single-digit numbers than on a more complex mathematics task such as reducing algebraic equations.

Sources of Self-efficacy

Emphasizing the social conditions of learning, Bandura (1997) hypothesized self-efficacy derived from four sources: (a) personal achievement and past experiences, (b) vicarious learning, (c) social persuasion, and (d) emotional or psychological states. Measures created support Bandura’s theory and suggest the sources of self-efficacy may differ for groups or individuals (Hampton, 1998; Hampton & Mason, 2003; Nielsen & Moore, 2003). For example, students with SLD reported lower achievement and less positive feedback than peers without disabilities (Hampton & Mason). Despite investigations into the sources of self-efficacy, most research examines self-efficacy as one construct and does not differentiate between potential sources.

What Research Says About Self-efficacy

Studies have shown that student self-efficacy decreases as the difficulty of desired tasks increase (e.g., Chen & Zimmerman, 2007) and factors such as gender, age, and culture influence self-efficacy. For example, by middle school, boys reported higher self-efficacy in mathematics than girls (Pintrich & de Groot, 1990). As students progress in school, academic tasks naturally become more complex, anticipated success consequently decreases, especially for students who
have acquired a history of failure. Intrapersonal factors such as cultural differences have also been explored. Cross-national comparison has reported that American students are more motivated by positive reinforcement (as opposed to negative reinforcement) from teachers (Tsao, 2005) and reported less accurate self-efficacy beliefs on mathematics than their Taiwanese peers (Chen & Zimmerman). Both Japanese and American students who earn higher test scores indicate more positive beliefs about mathematics ability (House & Telese, 2008).

Positive self-efficacy has been linked to achievement (Pajares, 1996), even after controlling for mental ability (Pajares & Kranzler, 1995). Students who do not believe they can accomplish a task, such as mathematics, are more likely to avoid the task; on the other hand, students who believe they can successfully accomplish the task may be more likely to work harder toward that goal and persist through obstacles or adverse encounters (Bandura, 1986; Pajares & Kranzler). Strong self-efficacy suggests success is obtainable, if the student works hard enough and puts forth the effort necessary to accomplish the challenging task. Students with strong self-efficacy are therefore more likely to attempt more challenging tasks.

Assuming it is true, that (a) self-efficacy is content driven, and (b) positive self-efficacy relies on the difficulty of the task; it seems logical to assume self-efficacy may be influenced by personal performance and past successes or failures. For example, a student may have a history of success in gymnastics (e.g., make the cheerleading squad, placing in competitions), increasing the student’s belief that the student can be successful in those athletic tasks. The same student may not have experienced success in mathematics (e.g., poor grades, chronic low test scores), consequently decreasing the student’s belief that the student can be successful at similar types of tasks. In effect, positive self-efficacy may be grounded by cyclical experiences of success on similar content. Researchers have sought to identify other variables that may contribute to
increased mathematics motivation, including the use of technology, such as interactive whiteboards (Torff & Tirotto, 2009) or specialized software (Isiksal & Askar, 2005), parental involvement (Fan & Williams, 2010), or by learning through authentic tasks (Maki et al., 2006).

**The Influence of Self-Efficacy on Student Performance**

Research suggests that the impact of self-efficacy on performance may be stronger than the impact of actual knowledge. That is, two students may achieve the same outcomes on a task even though Student A was more proficient on the materials, Student B was more confident that he/she would succeed at the task. This is not to suggest that Student B’s high self-efficacy made up for lack of knowledge, but that Student A did not achieve maximum performance due in part to a lack of self-confidence (e.g., anxiety). Bandura (1997) suggests that an optimistically inflated belief of self-efficacy may benefit a student, however that is not always the case. Inflated perception of success may not be beneficial to the student if the student lacks the basic skills necessary to complete the task. Referred to as judgment disparity, the disconnect between ability and perceived ability may not have the same beneficial results for a student with “deficient academic preparation and achievement” (Bandura, p. 65). A student’s strong self-efficacious belief for success will not allow the student to perform beyond personal capabilities (Pajares, 2006).

Students, such as those with specific learning disabilities (SLD), who display unexpected underachievement in specific areas may display unrealistic expectations of success that do not benefit student performance (Klassen, 2006). This may be due in part to the metacognition required to accurately perceive self-ability or lack of skill. Research indicates students with SLD are less likely to effectively use metacognitive strategies and therefore may not only have difficulty performing the required task (i.e., lack of skill) but also determining what skills are
necessary to be successful at the required task. As the academic demands of adolescent students become increasingly more difficult, students who do not have the metacognitive strategies of skills necessary to perform a task especially students with SLD, or those at risk of having a SLD may display lower levels of self-efficacy.

**The Role of Skill in Self-Efficacy**

It is essential to self-efficacy that the individual has the skills necessary to perform the desired task. As suggested by Bandura (1997), self-efficacy cannot improve behavior outcomes if fundamental skills necessary to perform the task are absent. For example, an individual may believe he will successfully perform a task that requires addition of numbers; however if the individual does not know how to add and successfully performs the task, success is due to coincidence and not personal belief of success. In other words, skill must be present for self-efficacy of an outcome, and therefore the outcome can improve the self-efficacy and subsequently self-efficacy can increase likelihood of success of the behavior.

**Directions for Future Research on Student’s Motivation to Engage in Mathematics**

Building from the fundamental notion that the link between student motivation and student performance is contingent upon content proficiency and develops from experiences of success, the primary way to increase a student’s self-efficacy is through enhancing skill development. Emphasis on increasing the self-efficacious beliefs of a student naturally relies on increasing the student’s history of success on the required task (Pajaras, 2006). It is not enough, then to expect an educational intervention that does not focus on skill development to effectively increase a student’s belief of future success. Interventions that seek to increase student self-efficacy in mathematics, especially for struggling students, must focus on empirical evidence that supports mathematics learning.
Self-Efficacy for Students with SLD

It has been estimated that between 6 and 10% of school age children may have a SLD (National Center for Educational Statistics, 2009; Fuchs, 2002) and over 50% of those students identified with a SLD have difficulties in mathematics (NCES, 2010; Hallahan, Kauffman, & Pullen, 2007). Although we know a good deal about motivation, research to date exploring mathematics motivation of students with SLD is fragmented, at best. Early research indicates, students who have a SLD reported lower self-efficacy beliefs about academics than peers without a disability (Clever, Bear, & Juvonen, 1992) and among students who have been diagnosed with a SLD, students with higher IQ scores reported lower self-efficacy beliefs than peers with a lower IQ scores (Baum & Owen, 1988). The academic struggles associated with students with SLD likely contribute to limited self-perceptions of success in academic subjects such as mathematics. Persistent failure, perceptions of peer success, changing social demands, and emotional factors, identified by Bandura as sources of self-efficacy (1997), may compound with academic struggles of students with SDL to decrease expectations of success on a task and consequently likelihood of actual success and future performance, as well. Additional research is needed to examine self-efficacy and achievement on mathematics tasks of students with SLD.

In conclusion, a variety of interventions have been attempted to motivate students with and without disabilities in mathematics (e.g., use of a technology; Torff & Tirotto, 2009) as well as ways to give purpose to mathematics (e.g., authentic and problem solving tasks; Maki et al., 2006), however fewer studies addressed improving student skills of students as a way to increase self-efficacy on mathematics tasks, specifically fractions. The proposed research addresses both areas of weakness in the current body of research by examining the role of self-efficacy for
students with SLD and those at risk of having an SLD and examining the role of improved skills in mathematics as on self-efficacy.

The purpose of the current research was to explore mathematics achievement on self-efficacy. Specifically, *What are the effects of a CRA sequence of instruction, when compared to traditional fractions instruction, on the self-efficacy of students with SLDs?* The research examined the self-efficacy of students on a mathematics task involving fractions. Both a control group and a treatment group received a self-efficacy survey prior to implementing the intervention, after the treatment group has received the six-week CRA intervention, and four weeks after the completion of fractions instruction.

For the purposes of the investigation, the motivational construct of self-efficacy is defined as a student’s perceived awareness of personal ability to perform a given mathematics task on fractions. As self-efficacy is content specific, an instrument was created to assess the self-efficacy of adolescent students on mathematical tasks regarding fractions. The measure was created by modifying previously established surveys (e.g., Smart & Linder, unpublished) as well as following the guidelines prescribed by Bandura (2006b). The measure will address sources of self-efficacy (Bandura, 1997) through a survey adapted from Usher and Pajaras (2009) assessing sources of self-efficacy for mathematics.

Building from the fundamental notion that the link between student motivation and student performance is contingent upon student’s content proficiency, it is necessary to develop student knowledge. The best way to develop content proficiency is to effectively teach the concepts, through evidence-supported instruction. CRA is an effective instructional sequence that is supported by research for many mathematics topics.
The purpose of this research was to compare the effects of CRA sequence of instruction with traditional instruction on acquisition and retention of mathematics knowledge involving fractions. Considering the educational needs of students with disabilities, the study specifically looked at skill acquisition of students who have been identified as having an SLD. A growing body of research on mathematics motivation supports the invested interest in analyzing student self-efficacy of fractions in addition to actual performance. Therefore, the study also analyzed self-efficacy of students during the intervention and after the intervention. The sources of self-efficacy were measured to assess differences between groups and changes within students over time.
CHAPTER THREE

METHODS AND PROCEDURES

The demand for efficient mathematical instruction, especially for algebra and prerequisite to algebra skills is evident. To evaluate the effectiveness of CRA sequence of instruction, the proposed study analyzed two groups of students randomly assigned to a control or treatment condition. The aim of this research was to compare the effects of CRA sequence of instruction with traditional instruction on acquisition and retention of mathematical operations and student self-efficacy involving fractions. The study specifically looked at the learning of students who have been identified as at risk for mathematics failure, including those students who have been identified with an SLD. Chapter Three outlines the research methods including the hypotheses, subjects, procedures, experimental design, and experimental analysis.

Hypotheses

The research was conducted to determine the effects of the sequence of CRA instruction on the performance of students with SLDs or students who are at risk of having a SLD, regarding the acquisition and retention of operations involving fractions as well as self-efficacy toward fractions. Teachers who participated in the study received training in CRA instructions and training in the use of scripted lessons to teach concepts and procedures involving fractions (e.g., addition, subtraction, multiplication, division, reducing fractions) from the researcher. Descriptive and inferential statistics were conducted to discover the effects of CRA instruction on fractions performance and self-efficacy. As generally recognized in educational research, the level of statistical significance was set at $p = .05$, recognizing that there is a 95% confidence interval the results of the study were not due to chance.
The proposed research aims to analyze the following research questions: (a) What are the effects of a CRA sequence of instruction, as compared to traditional fractions instruction, on the mathematics achievement and knowledge retention of students with SLDs and those at risk of failure in mathematics? and (b) What are the effects of a CRA sequence of instruction, when compared to traditional fractions instruction, on the self-efficacy of students with SLDs and those at risk of failure in mathematics?

**Methods**

Implementation of empirically supported instruction may be the most efficient way to improve student performance in mathematics for struggling learners; consequently providing a foundation for positive self-efficacy and future success. CRA sequence of instruction has a growing body of research that supports its use with students who have an SLD. The nature of CRA instruction explicitly scaffolds learning based on student needs, allowing them to learn through a variety of modalities. Research has demonstrated that students learning mathematics in this way (e.g., CRA) builds skills (Witzel, Riccomini, & Schneider, 2008). This study extends the current body of research to examine impact of CRA instruction fraction performance and self-efficacy for students in Grades 6, 7, and 8 who have an established SLD, or are at risk of having an SLD.

**Setting**

The study took place at an urban middle school in the southeastern part of the U.S. The school district serves over 12,000 students in kindergarten through Grade 12. The middle school is considered to be a Title I school that serves over 480 students in Grades 6, 7, and 8.

The school selected for this study is located in the northwestern part of a southeastern state. District wide, 58.72% of students are Caucasian, and 41.28% are of Ethic origin. Over
60% of students qualify for free or reduced lunch. Just over 17% of students qualify for special education. Four female teachers agreed to participate in the study. One special education teacher taught a self-contained classroom of students in Grades 6, 7, and 8. The other special education teacher team-taught with the 6th grade mathematics teachers in inclusive 6th grade mathematics classrooms. Two general education teachers taught the 6th grade mathematics. The two general education mathematics teachers and the second special education teacher that team-taught with them were currently enrolled in a mathematics masters program at a local university.

Participants

Students in Grades 6, 7, and 8 both with and without disabilities were included in the study. The students who participated in this study demonstrated low performance in mathematics, as indicated by their service placement in a self-contained mathematics class or a heterogeneous, inclusive classroom not on an honors track. The teachers identified both inclusive classrooms in this study as low performing. One teacher noted that the students in these two classrooms were the, “lowest (performing) students (she has) seen in many years”. Students who struggle with mathematics manifest similar behaviors and low performance, regardless of whether the students have been formally identified with a disability or are considered to be at risk of having a disability. Low mathematics performance is an issue for many students, apart from eligibility for special education services (NMAP, 2008). Over 62% of eighth graders in the district scored scores at Basic or Below Basic levels on end-of-year mathematics assessments (Annual Report, 2008). Scores were similar for students in seventh grade (over 60%) and students in sixth grade (55%) who produced scores at Basic or Below Basic levels. Many instructional strategies with empirical support may be effective for struggling students, regardless of special education identification.
At 77%, the majority of students who participated in this study were male (n = 27). Just less than 42% were Caucasian, 52% were African American, 9% were Hispanic. One student opted out of participating in the study. Seven students moved out of the district during the time of the intervention, three students had incomplete data sets due to extended suspensions or illnesses, three students were absent on the day on the delayed-post assessment and were unable to make-up the assessment before winter break. Students with missing data were excluded from statistical analysis.

Power was determined to identify how many students were necessary to include in the study to recognize significant change, if significant change had indeed occurred. With \( \alpha \) set at .05, and 1-beta at a conservative 0.90, effect was set at .245, as determined by Witzel, Mercer, and Miller (2005), a CRA intervention that assessed student performance on algebra. G-power, a statistical program, was used to determine the minimum number of participants necessary to detect statistical differences in group performances. It was determined that a total number of 38 students was necessary to best detect statistical differences, when present. A total of 48 students were initially included in the present study. Due to attrition and incomplete data sets, data from 35 students were included in final analyses.

Prior to making the decision to use data from 35 students, a one-way ANOVA was performed to compare computations scores on pre assessments for all students who took the pre assessment (n = 48) and students who completed the series of assessments (n = 35). Results indicated there were no significant differences between the two groups \( F(1,81) = 0.32, p = .576 \). Statistical analyses were also performed on the complete 35-student data set and a data set with scores from 38 students, which included predicted scores for three students who were only missing the delayed-post assessment scores. Results from a one-way ANOVA indicated there
were no statistical differences between the two groups on the pre assessments $F(1,71) = 0.002, p = .963$. Due to the results from these analyses, the researcher confidently used the un-biased data set ($n = 35$) to communicate results of the study.

Forty-eight, 6th, 7th, and 8th grade students with SLD in mathematics or those at risk of having an SLD, initially agreed to participate in this study, resulting in complete data sets for 35 students included in analysis. Inclusive criteria for participation in the study included: (a) recipient of instruction in a resource room mathematics classroom or inclusive classroom setting, (b) chronic low performance in mathematics, and (c) IEP goal(s) in mathematics or teacher identification of student’s low performance. One student joined the CRA group in the inclusive setting, but was not included in the study, as there was not a complete data set for the student.

Students who participated in this study received instruction in one of three classes: two inclusive 6th grade classes and one self-contained 6th, 7th, and 8th grade class. Students in the self-contained classroom were randomly assigned to a treatment condition. Students in the inclusive classrooms were randomly assigned to treatment by classroom, that is one classroom received CRA sequenced instruction and one classroom received traditional instruction. Student descriptives are depicted in Table 3.1.

Four teachers agreed to participate in this study. Two of the teachers are general education mathematics teachers for Grade 6. Two of the teachers are special education teachers, one coteaches with the general education teachers, the other works in a self-contained classroom.
Table 3.1

Demographic Characteristics of Sample

<table>
<thead>
<tr>
<th></th>
<th>CRA Group</th>
<th>Control Group</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>20</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>Male</td>
<td>15</td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>Female</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Caucasian</td>
<td>8</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>African American</td>
<td>10</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>Hispanic</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>SPED</td>
<td>13</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Learning Disability</td>
<td>12</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>Deaf or Hard of Hearing</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Research Design

To evaluate the effect of CRA sequence of instruction on acquisition and retention of mathematics operations involving fractions and student self-efficacy, an experimental two-by-three mixed-model design was implemented. The design analyzed potential main effects and interaction effect on mathematics acquisition and retention on operations involving fractions and self-efficacy. Data was analyzed to determine if there were differential changes by condition (i.e., control and CRA treatment).

The special education teachers instructed students in the both the treatment and control conditions. The teachers taught both the treatment and control groups to minimize effects for teacher differences. Students who received the treatment condition were taught using the CRA sequence of teaching computations involving as set forth by Witzel and Riccomini (2009), while students in the control condition were instructed by way of traditional mathematics instruction.
The scripted lessons used for this intervention were developed by Witzel and Riccomini (2009). Across the 30-lesson series, students were taught to (a) divide with fractional answers, (b) multiply, (c) divide, (d) find equivalent fractions, (e) reduce and compare fractions, (f) add and subtract fractions with like denominators, and (g) add and subtract fractions with unlike denominators. Each operational construct, with the exception of dividing, was first taught using concrete manipulatives (i.e., sticks, cups, and paper signs). During the concrete lesson, the teacher first modeled how to perform the operation. Having a teacher model the desired task allowed the students to observe what was expected of them during guided and independent practice. After the teacher modeled how to solve the operations using manipulatives, the teacher and students participated in the guided practice portion of the lesson. This section consisted of call and response between the teacher and students and allowed the students to practice solving the equations with the scaffolded support of the teacher. The third section of each lesson consisted of independent practice. During independent practice, the teachers circulated to ensure students were solving the equations correctly, as modeled and practiced.

During the second lesson of each operation series, students were then taught to do the same operations by drawing pictures that were similar to the concrete manipulatives. This step was intended to bridge and scaffold student learning from concrete to abstract. The format of each lesson (i.e., modeling, guided practice, independent practice) was repeated during the representations lessons.

In the third lesson in each operation series, students were taught to solve the operations using abstract numbers and symbols. Just as with the concrete and representational lessons, the scripted lesson included modeling, guided practice, and independent practice. Essentially, each of the lessons included modeling, guided practice, and independent practice.
Embedded within the 30 lessons were five lessons that provided opportunities for students to practice the operations and generalize the information. These lessons followed the same internal structure (i.e., modeling, guided practice, independent practice) but were not scripted.

The special education teachers who provided the instruction in the research were given the list of fractions operations topics that were taught during the intervention and were asked to teach those topics for approximately 20 minutes a day for the 30 days of the intervention. For example, there were four CRA lessons that taught multiplication of fractions. The teachers were asked to teach multiplication of fractions for four, 20 minutes lessons using traditional instruction. For the traditional instruction, the researcher did not tell the teachers how to teach the topics. Instead the researcher instructed the teachers to teach the fractions operations as they typically would. The researcher then observed the lessons to operationally define typical fractions instruction in the mathematics classrooms.

Subjects were situated in three classrooms, two inclusionary settings of students in Grade 6 and one self-contained classroom for students in Grades 6, 7, and 8. Each of the inclusionary classrooms was randomly assigned to a condition. Students in the self-contained classroom were randomly assigned to conditions to control for group differences. Additional analyses were conducted to ensure the groups were not statistically prior to the intervention period. An ANOVA was performed after the pre assessment to determine there were no significant differences between the two groups prior to the implementation of the intervention on measures of performance or self-efficacy. At .05 level of significance, it was concluded that there were no statistical differences between the two groups.
Results from the student achievement test on mathematic operations involving fractions (e.g., pre assessments, post assessments, and delayed-post assessments) were analyzed to determine if there were variations between achievement, growth, and retention between the two conditions. Interaction of the intervention on student performance was analyzed using a series of ANOVAs were performed to determine differences in student performance from the pretest to the post and delayed-post intervention tests.

A second ANOVA was performed to measure interaction of the intervention on differences for student self-efficacy through pre and post surveys. The statistical analyses and interaction contrasts were conducted using SPSS 19.

Research Instrumentation

Computations of fractions assessment. The researcher developed the fractions computations assessment for the purpose of this research. The content included in the assessment measured student performance on basic computations (e.g., addition, subtraction, multiplication, division, reducing, equivalency) of fractions with like and unlike denominators, as well as mixed numbers. The assessment was originally developed based on assessments and lessons created by the Rational Number Project. The Rational Number Project is located at the University of Minnesota and was originally supported by grants from the National Science Foundation. As part of the assessment creation process, the researcher looked at a variety of curricula material, web-based teacher resources, and released questions from state high-stakes assessments.

The final 25-item product was a result of an effort to pool together all of the resources described above. Prior to the research, data from 26 subjects were used to determine internal consistency of the measures, Spearman-Brown (.95), Cronbach's alpha (.86), split-half
correlations (.90), and KR20 (.86). Based on observation and performance of students on the reliability measures of the assessments collected prior to the start of the current research, the researcher made the decision to add three computation problems to the assessment to ensure results were not limited by a ceiling effect.

Analysis of internal reliability of the dependent variable was performed on the new assessments. Data from 48 subjects were used to determine Spearman-Brown (.91), Cronbach's alpha (.78), split-half correlations (.83), and KR20 (.78). Each reliability measure has benefits, however for the duration of this research, Cronbach’s alpha, which calculates pairwise correlations between items, were used to determine internal consistency of the fractions computations measure. Using Cronbach’s alpha, internal reliability on the 28-item preassessment for the 35 students who completed the study was determined to be acceptable (.77). Although the internal consistency decreased on the longer assessment and with fewer subjects, finding indicates measures are within an acceptable range (α = .60 -.70) encroaching good reliability (α > .80).

The computations of fractions assessment consisted of 25 problems, however question five required three answers (i.e., equivalent fractions) and therefore were scored as three separate answers yielding a 28-item assessment. One point was given for each correct answer resulting in a possible score of 28. Answers were not required to be renamed to simplest form to be marked as correct unless the directions specifically stated, “reduce the fraction to the simplest form” (n = 4). The assessment was divided into sections based on the directions necessary to explain to the students what is required of them in the section. For example, the first set of equations require the students to “reduce the fractions to the simplest form”, the second equation required that the students “find equivalent fractions” and the third section required students to “add subtract,
multiply, or divide to solve the equation.” The operations expected for the students to perform were randomly mixed in the third section to prevent students from getting into a pattern of solving problems and require that students understand the operation and what is necessary to solve the operation. (See Appendix A).

**Simplify fractions.** Students were expected to rename four fractions in their simplest form. Of the four problems, two of the fractions were mixed fractions.

**Find equivalent fractions.** Students were expected to provide fractions equivalent to two-thirds. There were four spaces available for equivalent fractions.

**Add, subtract, multiply, or divide.** The third section of the computations assessment included a variety of computations. Included on the assessment were (a) six addition problems, two with like denominators and four with unlike denominators; (b) four subtraction problems, two with like denominators and two with unlike denominators; (c) six multiplication problems, two with like denominators and four with unlike denominators; and (d) four division problems, all with unlike denominators.

The computation problems were intended to assess a variety of skills involving fractions. The difficulty of the computation problems varied. In order to minimize repetitive and routine procedures to answer problems, the types of problems (e.g., addition and multiplication) appeared in various orders. The length of the assessment limited students completing the assessment within the time limit, therefore minimizing the chance of students reaching a ceiling effect.

To control for students’ reading levels, teachers read the directions aloud, “Solve the following problems. *Please show all of your work.*” The researcher proactively indicated the
date, pre assessment, and teacher name on the assessments. The children were responsible to put their first name and on the assessment where it said “First name”.

**Self-efficacy dependent variable.** Self-efficacy is content specific (Bandura, 1997); therefore, a tool intended to measure students’ beliefs that they will be successful on tasks involving fractions must pose questions pertaining to students’ anticipated performance on fractions. It has been hypothesized that several motivational sources contribute to the construct of self-efficacy. Research suggests motivational sources of self-efficacy may be diverse for different populations (Hampton, 1998; Hampton & Mason, 2003; Nielsen & Moore, 2003; Clever et al., 1992). The self-efficacy measure for this study was adapted from a variety of established motivational surveys (e.g., Fan & Williams, 2010; Smart & Linder, under review; Greene et al., 2004), as well as a source of student self-efficacy survey (Usher & Pajares, 2009).

The assessment survey evaluated the self-efficacy on mathematics involving fractions of students in Grades 6, 7, and 8. The wording of the survey was targeted for students in Grades 6, 7, and 8 and specifically assessed students’ perceived performance on computations involving fractions. The self-efficacy survey was divided into two parts. The first page analyzed student belief that they may successfully solve fractions operations and the second page analyzed source of self-efficacy (i.e., mastery experiences, vicarious experiences, social persuasions, and psychological state). Analysis of internal consistency was conducted separately for the two different constructs on the presurvey. Data from 47 subjects were used to determine Spearman-Brown (0.96), Cronbach's alpha (0.95), and split-half correlations (0.92) for the construct of perceived self-efficacy. Measures of internal consistency indicating good reliability (α > 0.8) indicating the measure accurately measures one construct (i.e., self-efficacy).
The same analyses were conducted to determine internal consistency for the portion of the survey that assessed sources of self-efficacy. Data from 47 subjects were used to determine Spearman-Brown (.90), Cronbach's alpha (.85), and split-half correlations (.82) for the source self-efficacy. Cronbach’s alpha was determined for each of the four constructs identified as a source of self-efficacy. Acceptable internal consistency was determined for all four constructs, mastery experiences ($\alpha = .75$), vicarious experiences ($\alpha = .83$), social persuasions ($\alpha = .85$); and psychological state ($\alpha = .90$) as sources of self-efficacy. For the complete survey, see Appendix B.

Students are required to put their first name on the self-efficacy survey. Due to the vocabulary and content of the self-efficacy surveys, the teachers read the directions and content of the survey aloud. Teachers read the directions as followed:

“Mathematics Inventory: This questionnaire is designed to help us get a better understanding of the kinds of things that are difficult for students. Please rate how certain you are that you can do each of the things described below by writing the appropriate number. Your answers will be kept strictly confidential and will not be identified by name.

Rate your degree of confidence by recording a number from 0 to 10 given the scale below.

0 1 2 3 4 5 6 7 8 9 10

Cannot Moderately Highly certain
do it at all can do can do”

and,

“Rate your degree of agreement to each sentence below by recording a number from 0 to 10 given the scale below.

0 1 2 3 4 5 6 7 8 9 10
After each statement was read aloud, students write number from 0 to 10 according to their agreement with the statement. Self-efficacy surveys were given prior to the computations fractions assessment. The self-efficacy survey consists of 55 questions that were rated on a Likert 10 point scale. The survey was in two parts. The first part assesses students’ belief that they can successfully perform mathematics operations involving fractions. The second portion of the self-efficacy survey assessed students’ source of self-efficacy beliefs.

**Procedures**

In this section, the instructional procedures of the research are described. See Figure 3.1 for calendar of events.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Pre assessments</th>
<th>Intervention</th>
<th>Post assessments</th>
<th>Delayed-post assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>September 27-29, 2010</td>
<td>CRA fraction instruction</td>
<td>November 22-23, 2010</td>
<td>December 15-16, 2010</td>
</tr>
</tbody>
</table>

Figure 3.1: Calendar of Assessments.

**Pre-intervention measurement.** Pre-intervention measures included (a) fraction performance on computations assessment (see Appendix A) and (b) self-efficacy survey (see
Appendix B). They were collected during the mathematics class time on September 27th to 29th, 2010.

**Post-intervention measurement.** Post-intervention assessments for fraction performance on computations and self-efficacy and delayed-post intervention assessment for fraction knowledge were the same as the pre-intervention measures. Test-retest effect should not be an issue because the tests were administered at least four weeks apart. Post assessments were given between November 22nd and 23rd, 2010, and delayed-post assessments were given between December 15th and 16th, 2010.

**Reliability.** Statistical analyses were performed to determine reliability of measures. Results indicated all measures were reliable. Results are discussed.

**Teacher training.** Teacher’s knowledge of mathematics has impact on student learning (Jerald, 2006); therefore, it was important that the teachers received adequate training prior to implementing the CRA intervention. While four teachers agreed to cooperate with the study, essentially the two special education teachers actively participated as instructors for the study. Both special education teachers taught a control condition (i.e., traditional instruction) and a treatment condition (i.e., CRA) to minimize any teacher effects. The participating teachers received at least four hours of training, conducted by the researcher, prior to implementation of the intervention.

Both teachers are certified as special educators, as required by State standards. The first special education teacher is the lead teacher in a self-contained special education classroom for students in Grades 6, 7, and 8. She is responsible for teaching all subjects to all students as well as meet their IEP goals. Certain students join or leave her classroom for different subjects. In
addition to her regular students, two 6th grade boys join her classroom for mathematics. She was responsible for teaching a CRA treatment group and a control group.

The second special education teacher worked with general education teachers across grade levels and subject areas to support students with disabilities in inclusive settings. She co-taught with both general education teachers who allowed me to collect data in their classrooms. Again, the special educator taught both the CRA lessons and traditional fractions lessons to the students.

The two general education teachers taught 6th grade mathematics. Their main roles during the intervention period were as support in their classrooms. While the (second) special education teacher taught the lesson, the general education teachers circled the classroom and attended to students who required additional behavioral or academic support. The general education teachers were responsible for teaching on the one day the special education teacher was absent.

During the initial training, both special education teachers and the general education teacher in the CRA treatment classroom demonstrated declarative, procedural, and conditional knowledge pertaining to the different modalities inherent in the CRA intervention (See Appendix C for sample scripted lessons). In order to perform fidelity checks, the researcher followed the following criteria set forth in a study conducted by Butler and colleagues (2003): (a) thorough explanation of concepts during the lesson, (b) appropriate student feedback, (c) monitoring of student work throughout guided and independent practice, (d) sequential implementation of the lesson, and (e) adequate time allocation for each section of the lesson. (See Appendix D for an example of a fidelity checklist).
It was noted that the use of concrete manipulatives (i.e., popsicle sticks, cups, and mathematics symbols) were present in 25% of the lessons for the CRA group. This percent is to be expected as only 25% of the scripted lessons required the teachers to use manipulatives. The use of concrete manipulatives (i.e., fraction bars and egg cartons) were present in 6% of the observed lessons. In accordance with what was expected during the treatment, representations and models were used in 25% of the CRA lessons. On contrary, representations and models were present in 17% of observed lessons. It should be noted, however, that although representations and models were used in these lessons, they may have been used or demonstrated by the teacher and were not necessarily used by the students.

Modeling procedures and expected student behaviors was presented in 91% of CRA lessons, in stark contrast to the 19% of observed traditional (i.e., control) lessons. Guided practice and independent practice were observed in 100% of the CRA lessons, while only seen in 46% and 59% of traditional lessons. When considering these latter descriptive statistics, it should be noted that the guided practice in the CRA treatment group purposefully included opportunities for call and response, which was less consistent in the guided practice observed in the control group. Additionally, CRA lessons incorporated modeling, guided practice, and independent practice in each lesson, while traditional (i.e., control) lessons may have been entirely guided practice or entirely independent practice in conjunction with the teacher correction.

The researcher recorded start and stop times for observed fractions lessons during CRA and traditional instruction fractions instruction. Due to a few minutes overlap in schedules, the researcher occasionally walked into the classroom when the instruction had already started. Consequently, start and stop times were recorded for 50 lessons (26 CRA lessons and 24 control
lessons). Results from a one-way ANOVA ($F(1,47) = 0.29, p = .596$) indicated no significant differences in the amount of fractions instructional time.

During the first week of intervention, the researcher met with each teacher individually to discuss any comments or concerns about the implementation of the intervention. For the remaining five weeks of the intervention, the teachers received additional training regarding CRA instruction. Due to time constraints, the researcher met with each teacher individually throughout the intervention and frequently communicated via e-mail. In addition to the professional development and scheduled meetings, the researcher was available at the school to answer teachers’ questions and address teachers’ concerns.

Students at the middle school were dismissed at 3:25pm. Teacher training took place in the self-contained special education teacher’s classroom from 3:30 until 4:30 for training. All teachers were present and sat at a rectangular table at the front of the classroom. At the training, I provided an overview of research demonstrating the need for effective fraction instruction. I continued with a brief overview of effective instruction. The CRA intervention was described in detail followed by modeling of the scripted lessons. The teachers watched a clip that demonstrated the modeling aspects of Lesson 1. Samples from Lessons 2 and 3 were modeled for the teachers. The teachers followed along with the script and practiced the script. The researcher monitored the teacher’s demonstration of understanding.

The following materials were given to the teachers at the meeting:

1. Gallon baggies for each teacher and each student participating in the CRA intervention with (8) cups, (60) popsicle sticks, (2 of each) paper operations symbols. See Figure 3.2.
2. Teacher binders with copies of the first 10 lessons. (Additional lessons were provided after meeting with the teachers during the intervention). The gradual sharing of lesson plans allowed the researcher to make sure the teachers understood the requirements of the lessons.

The following materials were given to the teachers at on a different date and collected by the researcher:

1. Student assent forms (See Appendix E).
2. Parent permission forms (See Appendix F).
3. Teacher information letters (See Appendix G).

The following materials were given to the teachers after the initial training:

1. Student worksheet packets in orange folders
2. Additional orange folders.
3. Additional manipulative materials
4. Self-efficacy assessment

5. Fractions computations assessment

6. Additional lesson plans

The researcher noted that after the first three lessons there was concern regarding fidelity of treatment due to lack of modeling. The researcher taught Lesson 4 to both CRA treatment classrooms. The purpose of this was to model the expectations of the lesson to the teachers. Upon reflection of notes, the researchers is confident that the lower fidelity did not occur due to lack of training, but rather teachers reverting to the traditional and comfortable styles of teaching. The traditional instruction received by both control groups will be discussed in greater detail later in the paper.

**Intervention Procedure**

The intervention was designed to incorporate widely accepted practices of effective instruction, such as systematic and explicit instruction (Goeke, 2009; Fuchs et al., 2008; Miller & Hudson, 2007, 2006), the use of CRA (Witzel et al., 2003, 2009), consistent and immediate instructor feedback, and ample opportunities for student success (Stein et al., 2006) provided in scripted lessons. The intervention targeted specific skills necessary to develop mathematics knowledge and be successful in computations regarding fractions in order to prepare the students for future high school algebra courses (NMAP, 2008). Scripted lessons from the book *Computation of Fractions: Math Intervention for Elementary and Middle Grades Students* (Witzel & Riccomini, 2009) were used for the intervention.

During the professional development, teachers were taught to use the scripted lessons outlined in Witzel and Riccomini’s (2009) publication of the computation of fractions. However, because explicit instruction requires students to demonstrate mastery of a skill before
achieving independent practice as a means for advancement to a new lesson; teachers were given the liberty to diverge from the script as necessary to provide extra support, answer student questions, and ensure student understanding. The researcher was present for 70% of the lessons and documented fidelity of classroom instruction. Researcher notes indicated any deviations from the original script. For example, teachers often read the script verbatim, however if a student asked a clarifying question, the teacher would deviate from the script to answer the question. These precautions were taken to ensure students reached proficiency and were prepared for advancement to the next level in order to provide additional exemplary examples.

**Delivery format of intervention.** The intervention lasted for approximately six weeks (30, approximately 20-minute lessons) and each mathematics concept was taught in at least three phases. It was noted by the researcher and teachers that the concrete lessons took more time to deliver than the representational and abstract lessons. This may be due to the fact that concrete lessons introduced new topics or that passing out the materials and physically manipulating the objects required more time to complete.

**Organization of lessons.** During each lesson, the teacher progressed through an explicit sequence of steps, which required the teacher to (a) introduce the material, (b) model the desired behavior, (c) guide students through practice, and (d) monitor students’ independent practice. The instructional steps within each lesson allowed the students opportunities to practice the computation of operations involving fractions and receive corrective feedback as necessary.

**Step 1: Introduced the lesson.** Teachers briefly introduced the lesson topic and shares with the students their expectations for the day.

**Step 2: Modeled the lesson.** Teachers modeled the correct procedures necessary to complete the desired task. As scripted, teachers practiced using metacognitive strategies out-
loud to model the reasoning and thinking behind the behaviors. During this step, the teachers were the only individuals using the concrete manipulatives, representational pictures, or numbers and symbols. Teachers demonstrated the behaviors necessary to successfully complete the computation of fractions.

**Step 3: Guided students through practice.** During this step, the teacher scaffolded student learning by guiding students through the steps of the desired tasks. The structured practice allowed students to correctly display the desired behaviors and successfully complete the task with the support of the teacher. During the third step, both the teacher and the students participated by using the concrete manipulatives, representational pictures, or numbers and symbols.

**Step 4: Provided opportunities for independent practice and feedback.** After students had demonstrated that they were successful in step three (i.e., by showing approximately 90% accuracy), they were able to practice independently under the observation of their teacher. This step allowed students the opportunity to become fluent with the mastered material. The teacher(s) circulated around the classroom and were available to work with students who did not reach approximately 90% accuracy during the guided practice.

**Step 5: Provided opportunities for practice and review.** As students learned the strategies, it was important that they had opportunities to practice discriminating between when to use and when not to use the strategy (i.e., conditional knowledge). Teachers had the opportunity to provide feedback and determine mastery of previously taught strategies.

**Organization of lesson series.** Lessons progressed in a sequential fashion, building up to more difficult concepts from prerequisite concepts. During the first phase of instruction the teachers and students used concrete manipulatives to physically practice the mechanics of the
construct. The second phase acted as a transition from concrete to abstract, offering instruction and practice at a semiconcrete, representational level. The pictures mirrored the concrete manipulatives used in the first phase of the lesson series, aiding student understanding as a scaffold step between the concrete instruction and abstract instruction. The last instructional phase was the abstract phase. In the last phase of the intervention, students were introduced to the computation of fractions at the abstract level, where numbers and symbols were used to represent computations involving fractions. For example, two (sticks) divided by three (groups/cups) would be represented as 2/3.

**Phase 1: Concrete.** Students used hands-on manipulatives to develop a conceptual understanding of the fraction computations. The teacher modeled the learning objectives in the lesson by physically moving objects and concurrently describing the process of solving the computation problems. Over the course of the lesson, students were taught how to use the manipulatives to solve similar computations and eventually achieved independent practice. Popsicle sticks, small plastic cups, and paper symbols (e.g., addition, subtraction, multiplication, division) were used as physical manipulatives. Each student had their own gallon-sized plastic bag filled with manipulatives. See Figure 3.3 for example.
**Phase 2: Representational.** Students bridged their knowledge from concrete manipulatives to abstract numbers and symbols through the use of picture representations as a means to develop a more thorough understanding of the concepts. Similar to the first phase of the lesson, the teacher models how to solve the computation problems with the use of pictures that were visually similar to the manipulatives used in the first phase. Representational symbols (i.e., drawings of sticks and cups) were introduced during the second phase of the lessons. See Figure 3.4 for example.
Phase 3: Abstract. Students used numbers and symbols to learn and practice computations with fractions. The problems in this phase were similar to those in the first two phases, the major difference being that numbers and symbols replaced the manipulatives and picture representations. Students were required to solve the problems with the conceptual understandings developed in the two previous phases. Students were taught to solve operations involving fractions using abstract numbers and symbols during the third phase of the lessons.

Descriptions of Classrooms

The study took place in three classrooms, one self-contained classroom and two inclusive classrooms. Students in the self-contained classroom were randomly assigned to treatment groups. Students in the inclusive classrooms were randomly assigned to a treatment by classroom.
**Self-contained.** Instruction in the self-contained SLD classroom took place in small groups that consisted of six to nine students. The special educator taught both the control and treatment groups. During the instructional time the treatment group received the CRA instruction, the control group worked independently on Daily Math problems. The same was true when the control group received instruction, the treatment group worked on the same independent math problems. The paraprofessional was present in the classroom and reminded the students to stay on task. The paraprofessional provided minimum assistance to the students with their mathematics assignments.

Small group instruction took place in one of two settings. On ‘A’ days at the middle school, an adjoining classroom was vacant and the teacher was able to provide instruction in the classroom. Students either sat at two hexagonal tables, or independent desks during instruction. On ‘B’ days the adjoining room was occupied and instruction took place at the back of the classroom, at a large rectangular table. To further prevent contamination, an easel that was used during instruction was placed between the students who received instruction and those who completed independent seatwork. The easel provided a visual barrier so neither group was able to see how the other group was instructed. The paraprofessional was responsible for overseeing the students when they were not receiving instruction from the special education teacher. Additionally, all students, regardless of condition, had orange folders that contained their small group fraction instruction work. For the CRA intervention group, the folders contained a stapled packed of the 30 handouts that accompanied the lessons.

**Inclusive setting.** In the inclusive settings, one classroom of student received CRA instruction and one classroom of students received traditional instruction. Both classrooms were cotaught and the special educator took the leading role in teaching the fractions in both
classrooms, in both classrooms the special educator was absent for two lessons, in which case the general education teacher taught the lessons.

**Traditional Instruction**

The traditional instruction observed during the period of the intervention varied from the CRA sequence of instruction in a variety of ways. On a typical day, students in the inclusive classroom entered the classroom and sat at their assigned desk. Desks were arranged in rows, each desk paired with another desk. As a result of the setting, some students sat next to other students while some students say by themselves. Upon arrival to the class, students were expected to work on a weekly assignment that included daily review of previously taught materials. The work was done independently. At the end of the week, the teacher performed the daily mathematics problems on the board. Students were required to fix their personal errors and turn in the sheet for a credit grade. Occasionally students were required to finish their daily problems as homework.

Lessons typically started by the teacher asking students to take out paper and pencil. Students copied a problem that the teacher wrote on the board. Students were either required to work the problems with the teacher, or independently with the expectation that the teacher would later work out the problem as students checked their answers.

The concrete or hands-on elements pivotal to the CRA intervention were absent from all traditional instruction. More common was the presence of representational instruction using models required by the curriculum. Two-dimensional models were drawn on the board or using the SMART Board for the children to copy. It is stated in the 6th grade curriculum that these two-dimensional representational models should be used to teach the students about multiplication with fractions. The models were overlapping graphs used to demonstrate
<table>
<thead>
<tr>
<th>Skill</th>
<th>Grade 6 Standard</th>
<th>Grade 7 Standard</th>
<th>Grade 8 Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding / subtracting fractions</td>
<td>6-2 The student will demonstrate through the mathematical processes an understanding of the concepts of whole-number percentages, integers, and ration and rate; the addition and subtraction fo fractions; accurate, efficient, and generalizable methods of multiplying and dividing fractions and decimals; and the use of exponential notation to represent who numbers. 6-2.4 Apply an algorithm to add and subtract fractions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding equivalent fractions/ comparing fractions</td>
<td></td>
<td></td>
<td>8-2.4 Compare rational and irrational numbers by using the symbols ≤, ≥, &lt;, &gt;, and =.</td>
</tr>
<tr>
<td>Dividing with fractional answers</td>
<td>6-2.6 Understand the relationship between ratio/rate and multiplication/division.</td>
<td>7-2.1 Understand fractional percentages and percentages greater than one hundred.</td>
<td>8-2.7 Apply ratios, rates, and proportions.</td>
</tr>
<tr>
<td>Multiplying fractions</td>
<td>6-2.5 Generate strategies to multiply and divide fractions and decimals.</td>
<td>7-2.9 Apply an algorithm to multiply and divide fractions and decimals.</td>
<td>8-2.2 Understand the effect of multiplying and dividing a rational number by another rational number.</td>
</tr>
<tr>
<td>Dividing fractions</td>
<td>6-2</td>
<td>7-2.9</td>
<td>8-2.2 Understand the effect of multiplying and dividing a rational number by another rational number.</td>
</tr>
</tbody>
</table>

Figure 3.5: Fractions Skills Coded According to 2010-1011 South Carolina Standards
multiplication and division of fractions. Students are also taught the algorithm, however the algorithms are not taught for mastery until students are in the seventh grade. See Figure 3.5 for South Carolina Educational Standards pertaining to fraction operations taught during this intervention.

Another element that is critical to the CRA intervention that was absent in the traditional instruction was modeling. The traditional instruction often consisted of guided practice followed by independent practice, or exclusively independent practice. After students completed the assigned problems during independent practice, the teacher completed the computation problem on the SMART Board or front board. The teacher solving the problem on the board resembled ‘modeling’, however it happened after the students had already solved the problem, either correctly or incorrectly, therefore suggesting the actions were corrective and not initial instruction.

**Fidelity of Treatment**

To ensure the steps of the intervention were implemented consistently and accurately, fidelity checks were collected using multiple measures. First the researcher completed a checklist for 21 (70%) of the 30 lessons for the treatment condition. Both the control and treatment groups were observed by the researcher to (a) assess fidelity of treatment and (b) document procedures and experiences in the classroom. The researcher observed approximately 70% of the lessons (n = 21) and kept a written account of observations during lesson. Additionally, selected lessons were audio-recorded. The observations and selected lesson transcripts were used to determine the operational definition of traditional instruction in the control group as well as support the researcher’s accuracy for fidelity of treatment.
Fidelity checklist was changed to include observations that can be present in the control group. A continuum of observed behavior replaced a dichotomous response to observed behavior. Furthermore, the researcher observed 21 (70%) of the 30 lessons for the control group lessons to ensure traditional instruction were being implemented.
CHAPTER FOUR

RESULTS

This study investigated the effects of a CRA sequence of instruction compared to traditional instruction, on acquisition, retention, and self-efficacy of fractions computations for middle school students at risk for failure. The experimental study analyzed student performance and self-efficacy in a series of repeated measures ANOVAs with additional simple comparisons. Chapter Four presents the data findings, including (a) research questions, (b) summary of overall findings, (c) description of quantitative findings for student performance on computations of operations involving fractions, and (d) description of quantitative findings for students’ self-efficacy.

Research Questions

The research sought to analyze instructional interventions, specifically CRA sequence of instruction, on student acquisition, retention, and self-efficacy of students’ performance on computations of operations involving fractions. A series of general linear models for repeated measures were performed to detect changes over time on a fractions computations assessment and self-efficacy survey. Four teachers participated in this study. Complete data sets were collected for 35 students in Grades 6, 7, and 8 who were identified by their teachers as being at risk for failure in mathematics. The treatment group (n = 20) received CRA sequence of instruction over 30 scripted lessons, taught by the special education teacher. The control group (n = 15) received traditional instruction on the same topics taught to the CRA treatment group for the same duration of time. Treatment fidelity measures were collected for both groups and indicated teachers followed procedures adequately to be included in the study.
Summary of Overall Findings

There were significant differences on computations assessment scores for both instructional groups on dependent variable measures given after fractions instruction. There were no significant differences on post assessment computations measures between the two instructional groups. However, the CRA group produced significantly different scores on delayed-post computations assessments, outperforming peers who received traditional fractions instruction. There were significant differences detected on self-efficacy for students from pre to post surveys for both groups, however there were no differences between groups. The only significantly different scores on sources of self-efficacy (main effect) were with vicarious experiences, from pre survey to delayed-post survey. There were no differences between groups in regards to sources of self-efficacy, with the exception of psychological state. Anxiety as a source of self-efficacy, as measured by the psychological state, decreased for the group who received CRA instruction, but not for the group that received traditional instruction.

Results of Quantitative Analysis Computations of Fractions and Self-efficacy

Computations Dependent Variable

A 28-item assessment ($\alpha = .78$) required students to produce answers to computations of operations involving fractions (i.e., reducing fractions, finding equivalent fractions, adding, subtracting, multiplying, or dividing fractions). Data were collected on student performance at three points of time, (a) a pre assessment given prior to receiving instruction on fractions, (b) a post assessment given immediately at the completion of the instruction on fractions, and (c) a delayed-post assessment given four-weeks after completion of instruction on fractions. Students had nine minutes to complete each assessment. The researcher collected the assessments and scored them accordingly. A two-by-three repeated measures ANOVA with follow up simple
comparisons were conducted to assess for main effect and interaction effect for the two instructional groups. Means and standard deviations are provided in Table 4.1. Visual representation of findings can be found in Figure 4.1.
### Table 4.1

*Scores for Assessment of Operations Involving Fractions*

<table>
<thead>
<tr>
<th>Condition</th>
<th>n</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Delayed-Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>15</td>
<td>4.40 (3.07)</td>
<td>8.47 (2.61)</td>
<td>7.20 (3.03)</td>
</tr>
<tr>
<td>Treatment</td>
<td>20</td>
<td>4.30 (3.56)</td>
<td>9.00 (4.47)</td>
<td>10.60 (3.56)</td>
</tr>
</tbody>
</table>

Figure 4.1: Visual Representation of Fractions Computations
**Homogeneity of groups.** Although students were randomly assigned to treatment or condition groups either individually (i.e., self-contained classroom) or nested in a classroom (i.e., inclusive classrooms), additional analyses were conducted to determine normal distribution across groups. A one-way ANOVA was performed to determine there were no significant differences between CRA treatment and control groups prior to receiving instruction on fractions $F(1,33) = 0.01, p = 0.931$. There were no significant differences between the control and treatment group, $t(33) = .09, p = .93$. Levene’s test of equality of variances was conducted within the ANOVA; lack of significance for pre ($F(1,33) = 0.77, \alpha = .387$), post ($F(1, 33) = 2.89, \alpha = .099$), and delayed-post assessments ($F(1,33) = 0.51, \alpha = .480$) indicating homogeneity of variance within groups.

**Mean improvement by groups.** A repeated measures ANOVA was performed to answer the research question: *What are the effects of a concrete-representational-abstract sequence of instruction, as compared to traditional fractions instruction, on the mathematics achievement and knowledge retention of students with specific learning disabilities or those at risk of having a disability?*

A repeated measures ANOVA was conducted to investigate differences in performance on a measure of fractions knowledge. Results presented in Table 4.2, showed a significant main effect for instruction based on simple comparisons from the pre assessment to the post assessment $F(1,34) = 51.80, p < .001$ and delayed-post measures, $F(1,34) = 60.98, p < .001$. 
Table 4.2

*Mean Improvement for Group with Simple Interactions*

<table>
<thead>
<tr>
<th>Comparison</th>
<th>n</th>
<th>df</th>
<th>F</th>
<th>α</th>
<th>μ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre to post</td>
<td>35</td>
<td>1</td>
<td>51.798</td>
<td>0.000</td>
<td>0.604</td>
</tr>
<tr>
<td>Pre to delayed-post</td>
<td>35</td>
<td>1</td>
<td>60.982</td>
<td>0.000</td>
<td>0.642</td>
</tr>
</tbody>
</table>

**Acquisition of skills.** The post assessment was given at the conclusion of fractions instruction. It was the same assessment used for the pretest.

**Main effect.** Results from simple comparisons in the ANOVA yielded significant improvement in performance for students in the traditional instruction group, as indicated by performance on the post assessment $F(1,14) = 30.75$, $p < .001$. The same comparison measures for the CRA performance group also produced significant gains from the pre assessment to the post assessment $F(1,19) = 25.12$, $p < .001$. Both groups performed significantly different on pre and post assessment measures, producing higher scores on the post assessment measure. Results are presented in Table 4.3.
Main Effects for Pre-to-Post Assessments

<table>
<thead>
<tr>
<th>Comparison</th>
<th>n</th>
<th>df</th>
<th>F</th>
<th>α</th>
<th>μ^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>15</td>
<td>1</td>
<td>30.752</td>
<td>0.000</td>
<td>0.687</td>
</tr>
<tr>
<td>CRA</td>
<td>20</td>
<td>1</td>
<td>25.117</td>
<td>0.000</td>
<td>0.569</td>
</tr>
</tbody>
</table>

Interaction effect. Results from the repeated measures ANOVA and simple comparisons yielded no significant interaction effect between the control group and the CRA treatment group on performance on the post assessment, $F(1,34) = 0.25, p = .618$.

Retention of fractions. The delayed-post assessment was the same assessment used for the pre and posttest measures. The purpose of the delayed-post assessment was to assess retention of fractions skills.

Main effect. Results from simple comparisons in the ANOVA (see Table 4.4) indicated students produced significantly higher scores on delayed-post measures than on pretest measures. These differences in student scores were apparent for the control group $F(1,14) = 8.83, p = .010$ and CRA treatment group $F(1,19) = 95.34, p < .001$. 

Table 4.3
Table 4.4

*Main Effects for Pre-to-Delayed-post Assessments*

<table>
<thead>
<tr>
<th>Comparison</th>
<th>n</th>
<th>df</th>
<th>F</th>
<th>α</th>
<th>μ^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>15</td>
<td>1</td>
<td>8.883</td>
<td>0.010</td>
<td>0.387</td>
</tr>
<tr>
<td>CRA</td>
<td>20</td>
<td>1</td>
<td>95.336</td>
<td>0.000</td>
<td>0.834</td>
</tr>
</tbody>
</table>

**Interaction effect.** An interaction effect were detected for performance on the delayed-post assessment, $F(1,33) = 10.06, p = .003, \eta^2 = .234$, (see Table 4.5) where students in the CRA treatment group significantly outperformed the control group on measures of retention. The purpose of the delayed-post assessment, given four weeks after the completion of instruction, was to test for retention of skills. Students in the CRA treatment group produced scores that were higher and significantly different than those of peers in the control group.

An ANOVA with follow-up simple comparisons indicated that although mean scores from the post assessment to the delayed-post assessment decreased for the control group, there were no significant differences between the assessment scores. $F(1,33) = 2.07, p = .172$. The same analysis was performed for the CRA treatment group. Although the mean score for the CRA treatment group increased from the post assessment to the delayed-post assessment, there were no significant differences between the scores $F(1,33) = 3.67, p = .070$. It should be noted that the increase in scores for the CRA treatment group approached significance. However, the differences between the three groups did not reach significance. Despite the event that the changes in mean scores independently did not reach significance, a one-way ANOVA
determined that there were significant differences between groups on the delayed-post assessment scores $F(1,33) = 8.86, p = .005$.

Table 4.5

*Interaction Effects for Intervention on Computation Scores*

<table>
<thead>
<tr>
<th>Comparison</th>
<th>n</th>
<th>df</th>
<th>F</th>
<th>alpha</th>
<th>partial eta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre to post</td>
<td>35</td>
<td>1</td>
<td>0.254</td>
<td>0.618</td>
<td>0.008</td>
</tr>
<tr>
<td>Pre to delayed-post</td>
<td>35</td>
<td>1</td>
<td>10.055</td>
<td>0.003</td>
<td>0.234</td>
</tr>
</tbody>
</table>

**Self-efficacy Dependent Variable**

A 31-item assessment ($\alpha = .95$) required students to provide degree to which they agreed with a given statement or computation problem using a 10-point Likert scale. As provided in the directions, a lower number indicated student belief that the student “cannot do it at all”, a middle number expressed “moderately can do”, with a high number communicating, “highly certain can do”. Higher student scores communicated student self-perception that the student would be highly successful on operations involving fractions. Data were collected on student performance at three points of time, (a) a presurvey given prior to receiving instruction on fractions, (b) a postsurvey given immediately at the completion of the instruction on fractions, and (c) a delayed-post survey given four-weeks after completion of instruction on fractions. The teacher read the survey aloud while students responded to survey items. The researcher collected the assessments and entered the scores into SPSS 19 as reported by the students. A two-by-three
repeated measures ANOVA was conducted to assess for main effect and interaction effect for the two instructional groups. Means and standard deviations are provided in Table 4.6

Table 4.6

*Scores for Self-efficacy*

<table>
<thead>
<tr>
<th>Condition</th>
<th>n</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Delayed-Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>(15)</td>
<td>29.89 (13.70)</td>
<td>32.63 (8.94)</td>
<td>40.89 (9.77)</td>
</tr>
<tr>
<td>Treatment</td>
<td>(20)</td>
<td>27.73 (11.65)</td>
<td>31.72 (10.83)</td>
<td>36.70 (12.70)</td>
</tr>
</tbody>
</table>

A series of repeated measures ANOVAs were performed to answer the research question:

*What are the effects of a concrete-representational-abstract sequence of instruction, when compared to traditional fractions instruction, on the self-efficacy of students with specific learning disabilities or those at risk of having a disability?* Results yielded no significant interaction effect between the control group and the CRA treatment group on performance on the post assessment.

**Homogeneity of groups.** Students were randomly assigned to treatment or condition groups either individually (self-contained classroom) or nested in a classroom (inclusive classrooms), however additional analyses were conducted to determine normal distribution across groups. An independent sample t-test was performed to determine if there were significant differences of self-efficacy between CRA treatment and control groups prior to
receiving instruction on fractions. Levene’s test of equality of variances was conducted to
determine lack of significance for presurveys ($F(1,33) = .937, \alpha = .340$) indicating homogeneity
of variance within groups.

**Self-efficacy postsurvey.** The self-efficacy postsurvey was given to students at the
completion of instruction on fractions. The postsurvey was identical to the presurvey and given
to the students by the same procedures.

**Main effect.** A series of repeated measure ANOVAs were performed to analyze change
in self-efficacy over time, however change in self-efficacy was not determined between the
presurvey and the postsurvey $F(1,32) = 2.83, p = .102$.

**Interaction effect.** The interaction effect determines if there were performance
differences between the two instructional groups over time. No statistical differences were
detected between the treatment group and the control group for self-efficacy of performance on
either the postsurvey $F(1,32) = 0.03, p = .866$.

**Self-efficacy delayed-post assessment.** The self-efficacy survey was given four weeks
after the conclusion of teaching fractions to students. The survey was identical to the pre and
post survey.

**Main effect.** A series of repeated measure ANOVAs were performed to analyze change
in self-efficacy over time yielding the presence of a main effect from scores on the presurvey to
the delayed postsurvey $F(1,32) = 19.20, p < .001, \eta^2 = .375$

**Interaction effect.** The interaction effect measures if there were differences between
performance of the two instructional groups over time. No statistical differences were detected
between the treatment group and the control group for self-efficacy of performance on the
delayed-post survey $F(1, 32) = .34, p = .562$. 
Source of Self-efficacy Dependant Variable

Homogeneity of groups. The second portion of the self-efficacy survey consisted of four constructs that measured sources of self-efficacy. A series of t-tests were performed on data from the presurvey to determine if there were differences between groups prior to the start of the intervention. There were no significant differences between the control group and the CRA treatment group for mastery experiences \( t(33) = 0.38, p = .705 \), vicarious experiences \( t(33) = -0.06, p = .955 \), or social persuasions \( t(33) = -0.31, p = .758 \). There was, however, a significant difference between the control and CRA treatment group on the construct of psychological state \( t(33) = -2.34, p = .010 \), with students from the CRA group posting higher scores on measures given prior to the intervention, indicating greater anxiety of psychological state. It should be noted that the questions assessing students’ psychological state were written in the negative form, therefore a higher score would communicate a lower psychological state and a higher score would suggest less anxiety and a higher psychological state toward fractions.

Mean improvement by group. A series of repeated measures ANOVAs were performed to assess changes in the sources of student self-efficacy over time. There were no main effects for mastery experience \( F(1,33) = 0.21, p = .803 \), social persuasions \( F(1,33) = 0.39, p = .655 \); or psychological state \( F(1,33) = 2.08, p = .134 \). There was a main effect for vicarious experiences \( F(1,33) = 3.71, p = .030 \). There were also no interaction effects detected for mastery experience \( F(1,33) = 0.80, p = .450 \); vicarious experiences \( F(1,33) = 0.37, p = .690 \); or social persuasions \( F(1,33) = 0.25, p = .756 \).

There was however an interaction effect for students’ psychological state pertaining to self-efficacy to perform fractions \( F(1,33) = 5.39, p = .007 \) suggesting that there were differences detected in students’ scores in relation to the type of instruction the students received.
A repeated measures ANOVA with simple contrasts was performed to determine significant changes in student scores. Although analyses were performed for all four constructs, only results from constructs that posed main or interaction effects are shared.

**Sources of self-efficacy postsurvey.** As indicated previously, the postsurveys were given after the completion of fractions instruction.

**Main effect.** Significant differences in students’ reported value of vicarious experiences were not detected between the pre and postsurveys $F(1,33) = 1.12, p = .300$.

**Interaction effect.** As indicated by the evidence of an interaction effect between the CRA treatment and control groups, there were significant differences between student scores over time. Significant changes over time in psychological state were detected between the two groups from the pre- to post- survey, $F(1,33) = 10.62, p = .003$;

**Sources of self-efficacy delayed-post survey.** The delayed-post survey was given four weeks after the completion of fractions instruction.

**Main effect.** Significant differences in students’ reported value of vicarious experiences were detected between the pre and delayed-post surveys $F(1,33) = 8.13, p = .007$,

**Interaction effect.** Significant changes over time in psychological state were not detected between the two groups on pre and delayed-post surveys $F(1,33) = 3.70, p = .063$. Students in the CRA treatment group scored significantly higher than students in the control group on the psychological state construct during the presurvey, indicating higher psychological distress toward fractions, but scored similarly to students in the control group on both the post- and delayed-post assessments. Scores from students in the CRA treatment group decreased significantly from the pre to postsurveys, on measures of psychological distress toward fractions.
CHAPTER FIVE

CONCLUSION

Chapter Five discusses the conclusions indicated by the data analyses presented in Chapter Four. Chapter Five includes (a) Purpose and Methods, (b) Data Analysis, (c) Discussion, and (d) Summary and Conclusions.

Purpose and Methods

Given the current mathematics performance of students within the US (PISA, 2010, 2006; NMAP, 2008), there is emphasis on identifying effective instructional strategies to improve students’ understanding of fractions computations procedures (NMAP, 2008; Siegler et al., 2010). The purpose of this research was to analyze the effects of a CRA sequence of instruction compared to traditional instruction on acquisition and retention of mathematic operations and self-efficacy of success on fractions for middle school students at risk for failure.

The CRA sequence of instruction teaches following a natural progression of student learning from concrete to semiconcrete to abstract (Kamii et al., 2001). It incorporates students’ learning through multiple modalities, allowing students to develop a fundamental understanding of fractions through representations (Siegler et al., 2010). The scripted lessons incorporated essential elements of design and instruction including systematic and sequenced introduction of skills, mastery of prerequisite skills, and explicit instruction (Stein et al., 2006; Steedley et al., 2007).

The experimental study analyzed student performance and self-efficacy in a series of repeated measures ANOVAs. Only compete data sets were included in the analyses. Thirty-five students from three classrooms completed the intervention and assessments.
Each of the two inclusive 6th grade classrooms was randomly assigned to the CRA or traditional instruction group. Students in the self-contained classroom were randomly assigned to the CRA or traditional instruction group. The same special education teacher taught the scripted CRA lessons to the treatment group and traditional instructional lessons to the control groups in the inclusive classrooms. A special education teacher taught both the CRA and traditional instruction groups in the self-contained settings. This design purposefully controlled for teacher effects between groups.

For the group that received CRA instruction, the teachers taught 30 scripted lessons that instructed students how to (a) divide with fractional answers, (b) rename and simplify fractions, (c) find equivalent fractions, (d) add and subtract, and (e) multiply and divide fractions. Addition, subtraction, multiplication, and division included equations with like and unlike denominators. Each topic was taught with a CRA sequence, with the exception of division, which was only taught in the abstract sequence. Observations were made for 70% of the lessons taught, resulting in 95% fidelity of treatment.

The group that received traditional instruction was taught with strategies and sequence chosen by the teachers. Prior to the intervention, the teachers received a list of topics to be taught during the 30-lesson intervention time. Observations were made by the researcher to document and operationally define traditional instruction in the context of this study and ensure the topics taught to the CRA group were also taught to the traditional instruction group.

Student performance on operations involving fractions as well as self-efficacy were assessed at three points in time (a) prior to instruction, (b) immediately after fractions instruction (i.e., CRA or traditional instruction), and (c) four weeks after the completion of fractions instruction. The dependent variables aimed to assess knowledge and retention of fractions as
well as students’ expectancy of success or failure on fractions. Student performance on computations of fractions are discussed. Next, findings regarding students’ self-efficacy are discussed.

**Data Analysis**

**Computations of Fractions**

A repeated measures ANOVA was performed to test for significance between two instructional groups on pre assessment, post assessment, and delayed-post assessment scores assessing computations involving fractions. Follow-up analysis included simple comparisons. The analysis was conducted to answer the following research question:

> What are the effects of a concrete-representational-abstract sequence of instruction, as compared to traditional fractions instruction, on the mathematics achievement and knowledge retention of students with specific learning disabilities or those at risk of having a disability?

The research question inherently addresses two separate queries: (a) was there a difference between two instructional groups from the pre assessment to the post assessment (i.e., initial mathematics achievement) and (b) was there a difference between two instructional groups from the pre assessment to the post assessment (i.e., retention)? Simple comparisons were performed to accurately assess the data. The questions must be answered separately.

There were no significant differences in student performance on assessment prior to the intervention and immediately following the intervention. To simply answer to the first part of the research question: no, there were no significant differences in initial student performance as indicated by student scores on pre and posttest measures. In this study, student from both the control and CRA treatment groups performed better on post assessments than on pre
assessments. Essentially, students from both groups produced more correct answers on the post assessment, resulting in higher scores, after classroom instruction on fractions. The scores between the two groups on post assessments were not statistically different, indicating student performance on the post assessments were similar. It appears that both groups were able to learn information necessary to produce correct answers on computations involving fractions on the post assessment. These results differ from previous findings (e.g., Witzel et al., 2003) which showed students in CRA instructional groups outperformed students in other instructional groups on initial measures of performance. The results from the post assessments, however provide a limited picture of student performance.

What may be more interesting are the results of student performance on the delayed post assessment collected four weeks after the completion of the fractions instruction. There was a significant interaction effect between groups on the delayed-post assessment. Despite four weeks without instruction on fractions, descriptive statistics indicate students from the treatment group produced higher scores on the delayed-post assessment than on the post assessments. On the same assessment taken the same day, students in the control group produced scores that were lower than their scores on the post assessment from four weeks earlier. Results indicate four weeks after the completion of fractions instruction, students in this study that received CRA instruction demonstrated higher rates of retention and produced more correct answers than peers who received traditional instruction. Additional analyses determined there was no significant differences between the post and delayed-post assessments for the control group. The same was true for the CRA treatment group. The lack of significant differences between the post and delayed-post assessments might lead one to believe that there were no interaction effects, however a one-way ANOVA indicated there were significant differences between the two
instructional group on delayed-post measures. Essentially, the differences between scores on the post assessment, although not significant, were large enough that even though it could not be concluded that there were significant changes in student performance between the post and delayed-post measures, it could be concluded that students in the CRA treatment group significantly outperformed students in the control group. The answer to the second part of the research question would ultimately suggest that, yes, there were significant differences in retention of learned material as indicated by performance on the delayed-post assessments.

The interaction effect on the delayed-post measure demonstrated that students in the CRA treatment group significantly outperformed students on the measures of knowledge retention. While the effect size of the intervention is considered small by conventionally acceptable standards (i.e., .20 as small, .50 as medium, and .80 as large; Cohen, 1992), the small sample size must be also considered. Conservative analyses were used for effect size, and it is reasonable to consider that if the same findings were yielded from a larger sample, the effect size may be larger effect size. Replication would be needed to determine if this is the case.

One possible explanation for the differences in student performances on the delayed-post measures, resulting in the interaction effect from instruction, is that students in the control group memorized the algorithms necessary to perform on the post assessment, but did not thoroughly learn the material. Memorizing algorithms does not require a learner to understand why or how mathematical operations works, instead the algorithms themselves are an abstract series of steps performed to produce a correct answer. One can essentially use an algorithm to produce a correct answer without understanding the basic principles that allow the algorithm to work. This may be what students in the control group essentially did, learn the abstract steps necessary to solve a problem without anchored understanding. The post assessment occurred immediately
after students received instruction on fractions. The material may have been fresh in students’ minds resulting in increased performance. This explanation would also lend itself to make sense of the differences in performance on the delayed-post assessments that occurred weeks after the completion of instruction on fractions. Students in the control group (i.e., traditional instruction) were taught algorithms to perform fractions, without thorough instruction on why the algorithms can be used to solve a problem (e.g., concrete or representational instruction). Without a working or conceptual understanding of why the operations work and what is happening to the numbers, students may not be able to anchor the algorithms to a deeper understanding. Therefore the algorithms were easily forgotten after students no longer need to immediately recall information. Students in the CRA group learned the mathematics concepts through a series of scaffold steps aimed to help students develop a conceptual and procedural understanding of the fractions operations. Rather than memorizing an algorithm that might be easily forgotten or mistakenly performed, students in the CRA group may have been able to make sense of the computations and why the mathematics procedure worked, allowing them to make sense of procedural steps even if they were able unable to recall abstract steps of an algorithm.

Another explanation is that students in the control group stored correct algorithms in their short-term memory but never moved the information to their long-term memory, and therefore were not able to retrieve necessary information at a later date. In contrast, the CRA group may have moved the information from short-term memory to long-term memory where they were able to store and successfully recall the information at a later point in time. This may be in part to the instruction teaching the method through students’ different modalities (i.e., concrete, semiconcrete, abstract). The procedure of actually doing the fractions operations during the
concrete lessons and scaffolding the information during the representational lessons to the abstract lessons allowed the students to make sense of the information and thoroughly learn the operations, therefore being able to solve the operations, even after the completion of instruction.

A third explanation is that students in the CRA instructional group benefited from direct and explicit instruction associated with the CRA sequenced instruction. Student learning may have been scaffolded as the teachers released control to the students. The scripted lessons provided purposeful modeling of skills before students interacted with the teacher with guided practice. As students performed the computations correctly, more instructional release was given to the students as they participated in independent practice. It may have been the acts of perfect practice performed by students in the CRA group that contributed to mastery and fluency of skills. In contrast, students in the traditional instruction group received less modeling and guided practice, requiring students to create meaning and potentially learn through trial by error instead of mastery by accurate practice. Accordingly, students from both groups might be able to perform better on immediate measures of performance but unable to produce equal results without mastery of skills.

Results from this research corroborate findings from previous research that support CRA sequenced instruction for students who are at risk of failure in mathematics. The results of this study add to a growing body of research analyzing intervention supports for student learning of prerequisite skills to algebra, specifically the CRA sequence of teaching middle school students fractions.

The results from this study differ from previous research, however, in that results from previous research demonstrated changes in initial student learning of material and does not address students’ retention of learned information. In this study, initial student performance did
not differ on computations of operations involving fraction, however did show significant
differences in retention of learned information. This difference may be of particular interest in
light of the NMAP (2008) emphasis on student mastery of prerequisite to algebra skills.
Considering mathematics skills and concepts increase in difficulty as students progress through
school, retention of prerequisite and foundational skills (e.g., rational numbers and fractions) are
essential to build a foundation from which students can succeed in mathematics. Students who
accurately understand concepts and procedures and retain fractions information for longer
periods of time may subsequently build for success on future mathematics performance.

**Self-efficacy of Fractions**

A series of repeated measures ANOVAs were performed to test for significance between
two instructional groups on presurvey, postsurvey, and delayed-post survey scores assessing self-
efficacy of students on mathematics involving fractions. The analysis was conducted to answer
the following research question.

*What are the effects of a concrete-representational-abstract sequence of instruction,
when compared to traditional fractions instruction, on the self-efficacy of students with
specific learning disabilities or those at risk of having a disability?*

**Perceived success on performance.** The first part of the self-efficacy measure assessed
students’ beliefs that they may be successful on computations of operations involving fractions.
Results indicate there were no differences between the control group and treatment group over
time on self-efficacy measures, indicating that the type of instruction (i.e., CRA or traditional)
did not significantly impact student self-efficacy over time. Research links self-efficacy and
performance (House & Telese, 2008; Hoffman & Schraw, 2009; Klassen, 2006), in efforts to
change self-efficacy as a result of effective instruction and behaviors of success during
instruction, the researcher analyzed student self-efficacy toward fractions before and after receiving instruction (CRA or traditional) on fractions. The lack of significance in change from pre to postsurveys is not surprising, as attitudes are historically difficult to change, especially over such a short period of time. It should be noted, however that there was a significant difference between pre and delayed-post survey scores. The delayed-post survey scores were significantly higher than presurvey scores indicating students reported greater confidence and higher self-efficacy toward fractions. This finding is peculiar, and deviates from logic that would reason the change would occur after instruction, not four weeks after the completion of instruction, or not at all. The increase in self-efficacy for students, regardless of instructional type, on the delayed-post survey may reflect inflated perceptions that developed from not having to solve fractions, but remembering success in mathematics. Another possible explanation is that because mathematics builds from prerequisite skills, students are working on more difficult mathematics computations four weeks after fractions instruction and therefore project feelings of success on prerequisite and previously learned skills. Because one limitation of the study is that it took place at one middle school, it is not inconceivable that recent events at the school or in the community may have contributed to students’ self-reported self-efficacy scores. Additional research pertaining to self-efficacy and instruction over time is needed to shed light on these findings.

**Sources of self-efficacy.** The second portion of the self-efficacy survey explored the sources of students’ self-efficacy beliefs (Usher & Pajares, 2009). Consistent with findings from the first portion of the self-efficacy survey, there were few changes in students’ reported sources of self-efficacy, however the changes and differences are worth examination. Measures that analyze potential sources of self-efficacy suggested source of self-efficacy (i.e., mastery
experience and social persuasion) remained relatively consistent, with the exception of vicarious experience, which yielded a main effect and psychological state, which yielded an interaction effect. Students’ emphasis on mastery experiences and social persuasions as sources of self-efficacy did not change significantly, nor were there differences between the two instructional groups. Regardless of type of instruction students received on fractions, their perspectives of personal experience and social pressures as an influence of their anticipation of success or failure on fractions remained relatively consistent. Perhaps the time differences between assessments were not large enough to detect differences, or students’ experiences were not extreme enough to warrant change of an established belief.

Unlike grounding self-efficacy in personal experience and social persuasions, the emphasis on success through vicarious experiences (i.e., adults, peers, and self) decreased four weeks after the completion of fractions instruction for both instructional groups. This may be, in part, because students from both groups were no longer academically focusing on fractions instruction and therefore had less opportunities to watch others (e.g., teacher, peers) perform operations involving fractions. Four weeks after the conclusion of fractions instruction, student may have therefore valued envisioning the success of others as a viable contribution or prediction of their own success. Without the immediacy and persistency of teacher instruction, student perceptions of success by watching others correctly perform tasks was less evident and perceived as less important as a contributing factor to personal self-efficacy. It is important to note that there were no differences between the instructional groups with regard to the decrease of vicarious experiences as a contributing factor to self-efficacy, indicating that the decrease in emphasized success of others as a source of self-perceptions of success was similar for both instructional groups.
The most interesting finding, perhaps, was the decrease in anxious psychological state for students in the CRA group immediately after the completion of fractions instruction, which then remained consistent four weeks after the completion of instruction. Students in the CRA treatment group scored significantly higher than peers who were in the control group on measures that assess psychological state (i.e., anxiety) as a source of self-efficacy. The statements that assessed psychological state were written in the negative; therefore higher scores resulted in reports of higher anxiety and lower mental attitudes of success on fractions. Prior to instruction, students in the CRA group reported lower scores for psychological state than peers in the control group. At the completion of the intervention, the psychological state of students in the CRA group significantly decreased (indicating increase in mental state) to scores similar to those posed by the control group. While it is not determined why the scores were significantly different prior to instruction, the decrease in scores for the CRA group suggest the source of anxiety was either undetected by the researcher or CRA sequence of instruction provided students reason to be less anxious toward fractions. The decrease in anxiety may have resulted because students developed a conceptual understanding of fractions contributing to demystifying mathematics involved with fractions. Anxiety toward fractions decreased for students in the CRA group at the same time they received evidence-based instruction on fractions. The significant change in scores for students in the CRA treatment group, but not the control group, suggests the differences were a result of the intervention (i.e., CRA sequenced instruction) and not haphazard chance. Results imply that CRA instruction may have contributed to decreased student anxiety toward fractions after instruction. It should be noted that as assessments indicated a decrease in anxiety as a contributing factor to students’ self-efficacy, student performance on computations involving fractions increased.
The decrease in student anxiety coincides with instruction that addressed procedural and conceptual understanding of fractions. The sequenced nature of CRA instruction scaffolds learning from concrete to semiconcrete (i.e., representational) to abstract, teaching students through different modalities. Siegler and colleagues (2010) recommend visual representations be used during instruction to improve students’ understanding of fractions computations procedures. The scaffolded instruction or interaction with manipulative and visual (picture) representations may have helped demystify fractions and help students better understand fractions (as supported by student scores on the computations assessment), consequently lessening their anxiety when working with fractions. More research is needed to determine why these changes were present in the data and if findings can be replicated.

**Limitations and Implication for Future Research**

There are two major threats to external validity, which jeopardize incorrect generalization of study findings. First, the study took place in one school, in one district in the Southeastern part of the US. Replication is needed to determine that these results were not specific to this particular place and setting. Additionally, the teachers who participated in this study volunteered to be a part of the research, which may reflect their willingness and interest in improving student performance, consequently threatening the external validity across participants.

There are also threats to reliability present in this study. First, the researcher created all of the instruments used in this study. The researcher created the measures due to lack of measures with established reliability available on fractions. Precautions were taken to limit threats, including assessing the internal consistency of performance measures and modeling measures from previously established measures. Internal consistency measures indicated the measure was valid, although future research may use these researcher-created measures as well
as other fractions assessments to calculate changes in students’ performance. An interesting study would be to correlate such measures and include the State’s high-stakes assessment as a dependent variable.

The researcher observed 70% of the lessons, to ensure teachers followed scripted lessons for the CRA instruction as well as measures to document and operationally define traditional instruction. The researcher sat in the back of the classroom with a laptop and recorded observational notes during the lessons. However, it should be noted that the researcher conducted all fidelity checks for both the treatment and control groups. The researcher was aware of limitations and bias and made efforts to record observations without bias. Replication studies may employ multiple researchers to observe and compare notes in an effort to reduce bias and ensure accurate portrayal of behaviors.

After the first three lessons, the researcher documented that the teachers were not modeling during the first part of the lesson. The researcher consulted advisors and intervened in the research by teaching Lesson Four to the two CRA treatment groups. Lesson Four was the first lesson in the second series of CRA sequence. The topic of Lesson Four was dividing with fractional answers. Fidelity of treatment for modeling improved after the researcher intervention.

In addition to a fidelity checklist, the researcher typed observational notes to document and operationally define classroom instruction. The purpose of these observations and notes was to document what types of instruction the control group received. Qualitative analysis of notes was not included in the present study, however future analyses may use qualitative data to explore classroom teaching and learning experiences, such as documentation of student
engagement or disengagement (e.g., calling out, time off task), teacher behaviors (e.g., frequency of praise and reinforcement), and classroom environment.

Another threat to internal validity was a practice effect. When developing measures, the researcher considered making alternative formats of the assessments, however decided instead to assess students with the same measure. It is customary to give assessments at least three weeks apart to minimize the practice effect. For the purpose of this research, the post assessments were given seven weeks after the pre assessment. The delayed-post assessments were given four weeks after the post assessment, therefore minimizing the test-retest effect on student performance. These precautions were taken to minimize inflated scores on the post or delayed-post assessments due to student familiarity with the measure.

There was a considerable amount of attrition (n = 10), as a number of students moved out of district. Additionally, three students were unable to take the delayed-post assessments on the day it was given. The three students did not return to class before the school district’s winter break and therefore were not able to make up the assessment within a reasonable time. Although unfortunate, attrition naturally occurs in research, especially in more transient areas. The researcher performed a series of ANOVAs on the pre assessments to determine if there were differences between the students who left the study and students who remained in the study. No significant differences were detected between the students who agreed to participate in the study (n = 48) and those who completed the study (n = 35), $F(1,81) = 0.32, p = .576$ or between the group of students who completed the study with the three students who did not complete a delayed-post assessment (n = 38) and the students who completed all assessment measures (n = 35), $F(1,71) = 0.002, p = .963$. Due to the results from these analyses, the researcher confidently used the un-biased data set (n = 35) to communicate results of the study.
The CRA sequence of instruction implemented in this study yielded promising findings that suggest CRA sequenced instruction may help students who are at risk for mathematics failure retain learned fractions materials. The results of individual studies are finite, however this research adds to a growing body of research that supports CRA instruction for struggling learners. Given current educational emphasis on mathematics achievement of students, this area is ripe for study. More research is needed to determine if CRA effectively aids student learning and retention of fractions. Additionally, future research is necessary to determine how instruction, such as CRA, may influence student self-efficacy with regard to student performance. Longitudinal studies may assess trends in student self-efficacy in relation to instruction and achievement.

This particular research study assessed student performance on fractions computations (i.e., reducing, equivalency, addition, subtraction, multiplication, division) as one construct of student performance. It did not examine the role of CRA instruction on individual fractions concepts such as the work by Flores (2009) who found CRA instruction beneficial when teaching subtraction, or Witzel and colleagues (2003) who analyzed CRA sequenced instruction on students’ acquisition of algebra. Research is needed to determine if CRA sequence of instruction is more effective for students’ learning of particular fractions concepts as opposed to others (e.g., finding equivalent fractions vs. adding fractions with unlike denominators) as well as the most effective proportion of concrete, representational, and abstract instruction on different concepts. For example, students performed poorly on addition and subtraction with unlike denominators; however, would additional instruction at the representational level provide necessary scaffolding and strategies practice that would improve students’ performance at the abstract level? Flores (2009) looked at CRA instruction where teachers taught the concrete lesson for three days,
followed by three days of representational instruction, before teaching for three days using abstract numbers and symbols, while this study taught using each modality for only one day. Future research should assess amount of time spent teaching each fractions concept for mastery.

Findings from this study suggest CRA sequenced instruction may help students retain learned fractions material. Replication of this study is needed to assess retention of fractions and student performance after the conclusion of instruction. This research compared CRA sequenced instruction to traditional instruction, however another interesting comparison would be CRA sequenced instruction, another empirically supported intervention, and traditional instruction. Future research might explore the effectiveness of CRA instruction to other evidence-based practices. Additional research is needed to assess retention of skills over longer periods of time. Currently, there is a lack of empirical evidence that supports instructional techniques that support long-term performance on mathematics.

The researcher did not observe instruction during the four weeks between the post assessment and delayed-post assessment. The strict pacing guide set forth by the school district, mathematics curriculum taught immediately after daily lessons, incidental conversations with the teachers, and timing of Thanksgiving break and district assessments suggest that fractions instruction did not occur during this time. According to the district’s pacing guide (see Appendix H), fractions were taught at the beginning of the academic year, followed by multiplication and division of decimals, percentages, and exponents. This information was corroborated by the second special education teacher who told the researcher the 6th grade mathematics classes already taught the fractions units and researcher observations of instruction on decimals and percentages after daily lessons. Although confident that neither instructional group received additional classroom fractions instruction between the post assessment and
delayed-post assessment, there is no researcher documentation of what lessons were taught during this time.

This research used researcher-made assessments, however understanding the global importance of students’ performance on mathematics as well as current legislation that emphasizes high-stakes assessments, research is needed to analyze effects of CRA instruction of fractions on students’ performance on high-stakes assessments. Current legislation requires students with and without disabilities participate in high-stakes assessments (IDEA, 2004; ESEA, 2001), which contribute to school reports and ratings (ESEA), making this research timely and purposeful.

The computations assessments were timed. This was done as a precaution to reduce the chances of a ceiling effect (i.e., students reach the maximum possible score reducing ability to detect changes in student performance). Future studies may examine student performance on untimed measures. It may be that students in one instructional group might outperform students in the other instructional group, but not on measures that require students to perform in a finite period of time. Knowing an algorithm may help student perform at a quicker pace than students who may still rely on other strategies to produce the correct answer on a problem. Additional post-hoc analyses may be conducted on data collected from this study. For example, the researcher may assess how scoring assessments by correct digit as opposed to correct answer may communicate student performance. The researcher may also assess the number of correct responses in relationship to the number of attempted responses to determine if there were basic differences in performance between the two instructional groups.

Follow-up research may look at the types of errors made by student in the two different instructional groups. Were errors made at the same rates? For example, were the ratios of
problems attempted and problems solved correctly similar between the two instructional groups or did one group attempt to solve more problems? Valuable information about student learning can be deducted from student errors. Were the same type of errors made by students who received different instruction (i.e., factual, component skills, or strategy errors)?

As fractions are indicated as a prerequisite skill to algebra (NMAP, 2008), therefore it would be interesting to assess potential benefits of CRA instruction of fractions on student acquisition of algebra. This would be a difficult task, as many factors influence student learning and the research design would be longitudinal, however one worth exploring. If results from this study may provide small insight, certain instructional strategies may help students retain information for longer periods of time or potentially influence student fluency of skills. It is logical to inquire how these results may influence students learning of algebra. Any conclusions drawn from this research regarding future achievement on algebra is speculative and future research is necessary to make more definitive claims regarding the topic.

The purpose of this study was to assess the effects of a scripted CRA sequence of instruction on student performance. Observational notes from the researcher suggest teachers’ unfamiliarity or resistance to the modeling aspect of the lessons. This may be the traditional nature of middle school instruction, which emphasizes independent student performance. Future research should explore teacher behaviors during CRA lessons and teacher attitudes to CRA sequenced instruction. Emphasis on professional development may influence the findings and provide another area ripe for study.

Conclusions

Recent comparisons between students within the US and international peers indicate students from the US are being outperformed on mathematics assessments (TIMSS, 2007).
These international findings, pooled with statistics linking mathematics to graduation, higher education, and gainful employment, Adelman, 1999; Foegen, 2008; NMAP, 2008; NCTM 2010) add to current educational emphasis on mathematics achievement in our country (e.g., NMAP, 2008; TIMSS, 2007). Culminating findings from recent panels of experts summarize historical educational movements in mathematics, current performance, and future needs (e.g., NMAP, 2008). Additionally, researchers such as Stein and colleagues and (1997, 2006) established components they deemed critical to successful mathematics curricula and instruction, such as instructional design and delivery.

One area of critical need is algebra, as completion of algebra by high school allows students to take higher-ordered mathematics courses. In order for students to be proficient in algebra by high school, they need to learn prerequisite to algebra skills, such as rational numbers and fractions, prior to enrolling in an algebra course (NMAP, 2008). One instructional strategy recognized by the NMAP as an effective sequence of instruction is CRA. The NMAP (2008) reported research supported CRA sequenced instruction for struggling students. Additionally, CRA sequenced instruction has a growing body of research that supports its use for struggling learners, such as those with SLDs. The research presented in this paper adds to that body of research to use CRA to teach fractions to middle school students who struggle in mathematics (Flores, 2009; Witzel et al., 2003).

Results of this research suggest CRA sequenced instruction, as described in this study, is an effective instructional strategy to teach fractions to students who struggle with mathematics. More interestingly, findings from this study suggest that, for the students who participated in the research, the CRA sequenced instruction was more effective than traditional instruction on students’ retention of fractions knowledge. Students in this study who received CRA instruction
retained fractions information better than students who received traditional instruction, as indicated by performance on an assessment given four weeks after the completion of fractions instruction.

No significant changes were detected in students’ self-efficacy for fractions over the course of this research. Changes were detected regarding the reported source of students’ self-efficacy, as reported scores for students in both instructional groups decreased on the delayed-post survey for vicarious experiences. Additionally, during the presurvey, students in the CRA treatment group reported scores significantly lower than the control group.

A major limitation to this study was that data were collected from students in one school at one point in time. Replication of the study is needed to determine if results can be repeated. Another limitation is that assessments were researcher-made for the purpose of this study. Statistical analyses were conducted to determine the internal reliability of the measures, which all yielded good reliability.

It is clear, from the findings of this study, that more research is needed on the effects of CRA sequenced instruction on student learning. Performance of students in both instructional groups (CRA sequenced and traditional) improved after instruction on fractions. Students who received CRA-sequenced instruction outperformed peers who received traditional instruction on delayed-post assessments designed to measure student retention of fractions. These findings add to previous research that supports CRA, while contributing unique findings, and warrants the need for more research on CRA sequenced instruction.
APPENDICES

Appendix A

Computations Assessment

First name: ____________________________________________
Class: _____________________________________________

Please Circle: Pre  Post  Follow-up

Date: __________________________________________

Solve the following problems. Please show all of your work

Reduce the following fractions.

1. $\frac{4}{8} = ________$

2. $\frac{6}{10} = ________$

3. $\frac{9}{3} = ________$

4. $\frac{8}{4} = ________$

Find equivalent fractions.

5. $\frac{2}{3} = \frac{______}{______} = \frac{______}{______} = \frac{______}{______}$

Add, Subtract, Multiply, or Divide.

6. $\frac{4}{6} + \frac{1}{6} = \frac{______}{______}$

7. $\frac{5}{8} - \frac{1}{8} = \frac{______}{______}$

8. $\frac{1}{2} \times \frac{2}{5} = \frac{______}{______}$

9. $\frac{2}{3} + \frac{1}{4} = \frac{______}{______}$

10. $\frac{2}{3} \times \frac{1}{2} = \frac{______}{______}$

11. $\frac{2}{9} + \frac{2}{9} = \frac{______}{______}$

12. $\frac{4}{4} - \frac{3}{4} = \frac{______}{______}$

13. $\frac{1}{3} + \frac{1}{2} = \frac{______}{______}$

14. $\frac{3}{2} + \frac{2}{6} = \frac{______}{______}$

15. $\frac{2}{5} + \frac{4}{3} = \frac{______}{______}$

16. $\frac{1}{4} \times \frac{2}{4} = \frac{______}{______}$

17. $\frac{7}{8} - \frac{1}{4} = \frac{______}{______}$

18. $\frac{4}{3} - \frac{5}{6} = \frac{______}{______}$

19. $\frac{2}{3} \times \frac{3}{3} = \frac{______}{______}$
Appendix B

Self-efficacy Survey

Mathematics Inventory- Part A

This questionnaire is designed to help us get a better understanding of the kinds of things that are difficult for students. Please rate how certain you are that you can do each of the things described below by writing the appropriate number. Your answers will be kept strictly confidential and will not be identified by name.

*Rate your degree of confidence by recording a number from 0 to 10 given the scale below.*

<table>
<thead>
<tr>
<th>Cannot do it at all</th>
<th>Moderately can do</th>
<th>Highly certain can do</th>
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<td>0</td>
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</table>

1. Add fractions __________
2. Subtract fractions __________
3. Multiply fractions __________
4. Divide fractions __________

5. Find equivalent fractions __________
6. Add two numbers that are both fractions __________
7. Subtract two numbers that are both fractions __________
8. Multiply two numbers that are both fractions __________

9. Divide two numbers that are both fractions __________
10. Find two fractions that are equal __________
11. Reduce a fraction to its simplest form __________
12. Add two fractions with the same denominator __________

13. Add two fractions with different denominators __________
14. Subtract two fractions with the same denominator __________
15. Subtract two fractions with different denominators __________
16. Multiply two fractions with the same denominator __________

17. Multiply two fractions with different denominators __________
18. Divide two fractions with the same denominator __________
19. Divide two fractions with different denominators __________
20. Compare two fractions with the same denominator

21. Compare two fractions with different denominators

22. Place fractions where they belong on a number line

23. Order fractions on a number line

Mathematics Inventory- Part B

Rate your degree of confidence that you can solve each problem by recording a number from 0 to 10 given the scale below.

<table>
<thead>
<tr>
<th>Cannot do it at all</th>
<th>Moderately can do</th>
<th>Highly certain can do</th>
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</table>

1. $\frac{5}{8} + \frac{2}{8}$
2. $\frac{1}{6} + \frac{5}{9}$
3. $\frac{4}{7} - \frac{2}{3}$
4. $\frac{9}{11} - \frac{4}{11}$
5. $\frac{3}{5} \times \frac{1}{5}$
6. $\frac{4}{3} \times \frac{1}{2}$
7. $\frac{8}{9} \div \frac{2}{9}$
8. $\frac{4}{7} \div \frac{2}{3}$
Mathematics Inventory - Part B

*Rate your degree of agreement to each sentence below by recording a number from 0 to 10 given the scale below.*

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<th>10</th>
</tr>
</thead>
</table>
| Completely Disagree | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10
| Moderately Agree | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Completely Agree

1. I make excellent grades on math tests that involve fractions. __________
2. I have always been successful with fractions. __________
3. Even when I study very hard, I do poorly in math involving fractions. __________
4. I got good grades in math on my last report card. __________
5. I do well on math assignments that require I complete problems with fractions. __________
6. I do well on even the most difficult math assignments involving fractions. __________
7. Seeing adults do well on fractions pushes me to do better. __________
8. When I see how my math teacher solves a fractions problem, I can picture myself solving the problem in the same way. __________
9. Seeing kids do better than me in math involving fractions pushes me to do better. __________
10. When I see how another student solves a fractions problem, I can see myself solving the problem in the same way. __________
11. I imagine myself working through challenging math problems successfully. __________
12. I compete with myself in math involving fractions. __________
13. My math teachers have told me that I am good at learning fractions. __________
14. People have told me that I have talent for math involving fractions. __________
15. Adults in my family have told me what a good math student I am. __________
16. I have been praised for my ability in math involving fractions. __________
17. Other students have told me that I’m good at learning fractions. __________
18. My classmates like to work with me with fractions because they think I’m good at it. __________
19. Just being in math class when we do fractions makes me feel stressed and nervous. __________
20. Doing math work with fractions takes all of my energy. __________
21. I start to feel stressed-out as soon as I begin working with fractions in my math work. __________
22. My mind goes blank and I am unable to think clearly when doing math work with fractions. __________
23. I get depressed when I think about learning about fractions. __________
24. My whole body becomes tense when I have to do math involving fractions. __________
Appendix C

Sample Scripted Lessons (Witzel & Riccomini, 2009)

Lesson # 1  Division with fractional answers  1.C – (sticks and cups method)

Describe / Model (Answer is in sticks per cup)

note. \( \frac{8}{4} \) may also be represented as \( 8 \div 4 \)

a) 8 sticks  
   4 cups

b) 5 sticks  
   3 cups

c) 3 sticks  
   4 cups

d) 6 sticks  
   4 cups

Guided Practice

e) 3 sticks  
   3 cups

f) 9 sticks  
   2 cups

Word problem

g) A mother gave 10 dollars (sticks) to her 4 children (cups) to split up (÷) evenly. How many (=) dollars will go to each child? Show your answer using the objects provided.

What does the fraction in the answer mean?

Independent Practice

h) 4 sticks  
   3 cups

i) 7 sticks  
   3 cups

j) 6 sticks  
   3 cups

k) 3 sticks  
   1 cup
Lesson 1: Division with fractional answers

1.C – (sticks and cups method)

Describe / Model (Answer is in sticks per cup)
Note: $\frac{8}{4}$ may also be represented as $8 \div 4$

(Problem A: 8 sticks/4 cups) Eight-fourths or eight sticks per four cups. How many sticks per one cup? Let’s divide the eight sticks per the four cups. Place one stick in the first cup below, the next stick in the next cup and so on. In the end, you should have two sticks in each of the cups. Are there even amounts of sticks in each cup? Good. Remove the divisor line as it is no longer needed since the division is complete. Now that we are done with the division, we don’t need the symbol telling us to divide. What do we have left? How many in each cup? Yes, there are two sticks per each cup. Write 2 sticks / 1 cup.

(Problem B: 5 sticks/3 cups) There are 5 sticks per 3 cups. How many sticks per one cup? Let’s divide the five sticks per the 3 cups. Place one stick in the first cup below, the next stick in the next cup and so on. In the end, you should have two sticks in 2 cups and one stick in the last cup. Are there even amounts of sticks in each cup? No, then let’s back up one stick. Remove the last stick moved from the second cup. Are there even amounts of sticks in each cup now? No, then let’s back up one stick again. Are there even amounts of sticks in each cup now? Yes? Good. Remove the divisor line as it is no longer needed since the division is complete. Now that we are done with the division, we don’t need the symbol telling us to divide. What do we have left? How many in each cup? Yes, there is one stick per each cup and two sticks that need to be divided into 3 cups. Write 1 sticks / 1 cup and 2 sticks / 3 cups = 1 2/3.

(Problem C: 3 sticks/4 cups) There are 3 sticks per 4 cups. How many sticks per one cup? Let’s divide the three sticks per the 4 cups. Place one stick in the first cup below, the next stick in the next cup and so on. In the end, you should have three sticks in first 3 cups and no sticks in the last cup. Are there even amounts of sticks in each cup? No, then let’s back up one stick. Remove the last stick moved from the second cup. Are there even amounts of sticks in each cup now? No, then let’s back up one stick again. Are there even amounts of sticks in each cup now? No then let’s back up one stick again. Are there an even amount of sticks in each cup now? Yes? But what, they are all empty. The answer is going to be less than one. Remove the divisor line as it is no longer needed since the division is complete. Now that we are done with the division, we don’t need the symbol telling us to divide. What do we have left? How many in each cup? Yes, there are no sticks per each cup and three sticks that need to be divided into 4 cups. Write 0 sticks / 1 cup and 3 sticks / 4 cups = 3/4.

(Problem D: 4 sticks/4 cups) There are four sticks per four cups. How many sticks per one cup? Let’s divide the four sticks per the four cups. Place one stick in the first cup below, the next stick in the next cup and so on. In the end there should be one stick in each of the four cups. Are there even amounts of sticks in each cup? Yes, very good. Now, we are finished with the division so we can move the divisor line. How many sticks are left in each cup? Yes, there is one stick per one cup. Write 1 stick/1 cup.
**Guided Practice**

Let’s try some together.

(Problem E: 9 sticks/2 cups)  
How do we set this up? Let’s complete it with the objects on your desks.  
Ask the students a series of questions and have them repeat the questions.  
What do we lay down in the numerator?  9 sticks  
What do we lay down in the denominator?  2 cups  
What separates the two?  Divisor line  
Now how do we separate the sticks into the cups?  Should end up with four sticks in the first cup and five in the second.

Now that all the sticks have been moved into cups, what do we ask ourselves?  
Are there even amounts of sticks in each cup now?  No  
What do we remove?  The last stick put into the second cup. (move back to numerator)  
Are there even amounts of sticks in each cup now?  Yes.  
How many sticks per cup?  Four  
How many sticks were not put into a cup?  One  
Our answer will be four sticks per one cup and one stick per two cups which is equal to 4 ½.

(Problem F: 3 sticks/3 cups)  
How do we set this up? Let’s complete it with the objects on your desks.  
Ask the students a series of questions and have them repeat the questions.  
What do we lay down in the numerator?  3 sticks  
What do we lay down in the denominator?  3 cups  
What separates the two?  Divisor line  
Now, how do we separate the sticks into the cups?  Should end up with one stick in each cup.

Now that all sticks have been moved into cups what should we ask ourselves?  
Are there even amounts of sticks in each cup?  Yes.  
Are we finished dividing?  Yes  
Then, what can we remove?  Divisor line  
How many sticks per cup?  1 stick  
We write our answer as 1 stick/1 cup; 1/1; or 1.

Let’s look at this scenario: (g) A mother gave 10 dollars (sticks) to her four children (cups) to split up (÷) evenly. How many (=) dollars will go to each child? We are going to show our answer using the objects provided.  
Let’s begin by setting this up.  
What are we trying to divide evenly?  10 dollars.  
Among who/what?  4 children  
So, let’s let 10 sticks represent 10 dollars in the numerator and 4 cups represent 4 children in the denominator.

What separates the two?  Divisor line  
Now let’s split the “dollars” per the “children”.  
Start by giving one dollar to each child. Have all the dollars been given out?  No  
Let’s continue. Give all of the children another dollar. (2 sticks per each cup)  
Have all of the dollars been given out?  No.  Let’s continue. (2 sticks in first 2 cups)  
Have all of the dollars been given out?  Yes.
Now, what can we ask ourselves?
Does each child have an even amount of dollars? No.
What should we do now? Remove the last two dollars from the first two cups and place them back in the numerator.
Now, do all of the children have an even amount of dollars? Yes.
How many dollars per child? Two dollars/1 child
How many left over? Two dollars/four children
So our answer is 2 dollars and 2 dollars/4 children OR 2 2/4 which is equal to 2 1/2.
If we are dealing with change then we could evenly give each child two dollars and fifty cents.

Independent Practice
It’s your turn to try some practice problems on your own. Use your sticks, cups, and divisor line to work the problems. Be sure to draw a picture of your answer, and the number answer on paper.
Lesson #2  Division with fractional answers  2.R – (tallies and groups method)

Describe / Model (answer is in tallies per group)

a) 12 tallies  
   4 groups

b) 8 tallies  
   3 groups

c) 13 tallies  
   5 groups

d) 15 tallies  
   2 groups

Guided Practice

e) 5 tallies  
   5 groups

f) 19 tallies  
   4 groups

Word problem

g) 3 dogs (groups) were sharing dinner. The owner placed 8 scoops (tallies) of food in the one feeding dish. If each dog ate the same amount of food, how many scoops of food would each (÷) dog eat? Show your answer using the same picture format in this lesson.

What does the fraction in the answer mean?

Independent Practice

h) 9 tallies  
   3 groups

i) 6 tallies  
   1 group

j) 14 tallies  
   3 groups

k) 18 tallies  
   5 groups
Lesson 2: Division with fractional answers 2.R – (tallies and groups method)

Describe / Model (Answer is in tallies per group)

Today we are going to work some division problems much like the problems we worked yesterday. Yesterday we worked these problems using sticks and cups. Today we will be using tally marks to represent the sticks and groups (circles) to represent the cups.

(Problem A: 12 tallies/4 groups) Twelve tallies per four groups. How many tallies per one group? Let’s divide the twelve tallies per the four groups. Problem should be set up with twelve tally marks in the numerator, a divisor line, and four circles (or groups) in the denominator. Place one tally in each of the four groups. Cross out four tallies from the numerator. Place one tally in each of the four groups again. Cross out four more tallies from the numerator. Repeat until tallies are evenly divided among the four groups. You should end with three tally marks per four groups. Are there even amounts of tallies in each group? Good. Write the tallies per group after the previous step without the divisor line as it is no longer needed since the division is complete. Now that we are done with the division, we don’t need the symbol telling us to divide. What do we have left? How tallies in each group? Yes, there are three tallies per group. Write 3 tallies/1 group.

(Problem B: 8 tallies/3 groups) There are 8 tallies per 3 groups. How many tallies per one group? Let’s divide the eight per the 3 groups. Place one tally in each of the 3 groups until all tallies are distributed. In the end, you should have three tallies in the first two groups and two in the last group. Are there even amounts of tallies in each group? No, then let’s back up. Remove the last tally put in the second group (move back to numerator). Are there even amounts of tallies in each group now? No, then let’s back up one tally again (move tally back to numerator). Are there even amounts of tallies in each group now? Yes? Good. What do we have left? How many in each group? Yes, there are two tallies per each group and two tallies that need to be divided into 3 groups. Write 2 tallies/1 group and 2 tallies/3 groups = 2 2/3.

(Problem C: 13 tallies/5 groups) There are 13 tallies per 5 groups. How many tallies per one group? Let’s divide the 13 tallies per the 5 groups. Place one tally into each group until all tallies are distributed. You should end up with 3 tallies in the first three groups and two tallies in the last two groups. Are there even amounts of tallies in each group? No, then let’s back up one tally. Cross out the last tally from the third group (add it back to the numerator). Are there even amounts of tallies in each group now? No, then let’s back up one tally again (cross it out of the group and add it back to the numerator). Are there even amounts of tallies in each group now? No then let’s back up another tally. Are there an even amount of tallies in each group now? Yes. Now that we are finished we do not need the divisor line. What do we have left? How many in each group? Yes, there are two tallies per each group and three tallies that need to be divided into five groups. Write 2 tallies/1 group and 3 tallies/5 groups OR 2 3/5.

(Problem D: 15 tallies/2 groups) There are 15 tallies per two groups. How many tallies per one group? Let’s divide the 15 tallies per the 2 groups. Place one tally in each group and so on until all the tallies are distributed. You should end up with 8 tallies in the first group and
seven tallies in the second. Are there even amounts of tallies in each group? No. What should we do now? Let’s back up one tally. Cross out the last tally placed in the first group (add it back to the numerator). Now, are there even amounts of tallies in each group? Yes, very good. What do we have left? How many tallies in each group? Seven. How many tallies left to be divided into two groups? One. Write 7 tallies/1 group and 1 tally/2 groups OR 7 ½.

**Guided Practice**
Let’s try some together.
(Problem E: 5 tallies/5 groups) How do we set this up? Let’s complete it with paper, pencil, tally marks, and groups.
Ask the students a series of questions and have them repeat the questions.
What do we draw in the numerator? 5 tallies
What do we draw in the denominator? 5 groups (or circles)
What separates the two? Divisor line
Now how do we separate the tallies into the groups? One tally in each group.
Now that all the tallies have been moved into groups, what do we ask ourselves? Are there even amounts of tallies in each group? Yes.
How many tallies per group? One
Our answer will be 1 tally per 1 group; 1/1 = 1.

(Problem F: 19 tallies/4 groups) How do we set this up? Let’s complete it with paper, pencil, tally marks, and groups.
Ask the students a series of questions and have them repeat the questions.
What do we draw in the numerator? 19 tallies
What do we draw in the denominator? 4 groups (or circles)
What separates the two? Divisor line
Now how do we separate the tallies into the groups? 5 tallies in the first 3 groups and 4 in the last group.
Now that all the tallies have been moved into groups, what do we ask ourselves? Are there even amounts of tallies in each group? No.
What should we do now? Back up a tally.
Are there even amounts of tallies in each group? No.
What should we do? Back up another tally.
Are there even amounts of tallies in each group? No.
What should we do? Back up another tally.
Are there even amounts of tallies in each group? Yes.
How many tallies per group? Four.
How many tallies in the numerator to be divided into four groups? 3
Our answer will be 4 tallies/1 group and 3 tallies/4 groups; 4 3/4.

Let’s look at this scenario: (g) Three dogs were sharing dinner. The owner placed 8 scoops (tallies) of food in the one feeding dish. If each dog ate the same amount of food, how many
scoops of food would each (÷) eat? We will show are answer using pictures with tallies and groups.
Let’s begin by setting this up.
What are we trying to divide evenly? 8 scoops of dog food.
Among who/what? 3 dogs
So, let’s let 8 tallies represent 8 scoops in the numerator and 3 groups represent 3 dogs in the denominator.
What separates the two? Divisor line
Now let’s split the “scoops” per the “dogs”.
Start by giving one scoop to each dog. Have all the scoops been given out? No
Let’s continue. Give all of the dogs another scoop. (2 tallies per each group)
Have all of the scoops been given out? No. Let’s continue. (2 tallies in first 2 groups)
Have all of the scoops been given out? Yes.
Now, what can we ask ourselves?
Does each dog have an even amount of scoops? No.
What should we do now? Remove the last two scoops from the first two dogs and place them back in the numerator.
Now, do all of the dogs have an even amount of scoops? Yes.
How many scoops per dog? 2 scoops/1 dog
How many left over? 2 scoops/3 dogs
So our answer is 2 scoops per 1 dog and 2 more scoops to be divided amongst 3 dogs. We write our answer as 2 2/3.

Independent Practice
It’s your turn to try some practice problems on your own. Use your pencil and paper to draw pictures using tallies and groups. Be sure to draw a picture of your answer and give the number answer on your paper.
Lesson #3  Division with fractional answers  3.A – (abstract only)

Describe / Model

a) \( \frac{23}{4} \)

b) \( \frac{14}{5} \)

Guided Practice

c) \( \frac{32}{6} \)

d) \( \frac{17}{3} \)

e) \( \frac{16}{4} \)

f) \( \frac{60}{8} \)

Word problem

g) 18 slices of pizzas were bought for 5 basketball players. The slices were to be shared equally. How many slices of pizza would each player eat? Show your answer using the same abstract steps in this lesson.

What does the fraction in the answer mean?

Independent Practice

h) \( \frac{19}{7} \)

i) \( \frac{55}{9} \)

j) \( \frac{42}{8} \)

k) \( \frac{21}{4} \)
Lesson 3: Division with fractional answers

Describe/ Model

We are now going to begin working some division problems with fractional answers just as we have been with sticks and tallies; however, now we are going to learn to find our answers without using those sticks and tallies. I am going to work a couple of problems for you...

(Problem A: 23/4) Our first problem is twenty-three fourths or twenty-three divided by four. This problem is just like asking to separate 23 tallies amongst 4 groups. We want to find how we can put 23 into 4 groups. We know that 4 will go into 23 evenly 5 times (5 x 4 = 20). So, we have 5 “tallies” per each “group” as of now. We have used 20 tallies and we have 3 left over because 23-20=3. So our answer is going to be 5/1 and ¾ OR 5 ¾.

(Problem B: 14/5) Our next problem is fourteen fifths or 14 divided by 5. We want to know how we can separate 14 into 5. The first thing we need to decide is how many times 5 will evenly go into 14. We can use our multiplication facts. Let’s try 5 x 3. Five times three is equal to what? 15. We know this can’t work because we only have 14 in our numerator. Let’s back up one number to 5 x 2. Five times two equals what number? 10. So far we have 2 “tallies” in each of our 5 “groups” which means we have used 10 of our “tallies”. How many do we have left over? Very good, 4. So we have 2/1 and 4/5. We write our answer as 2 4/5.

Guided Practice

Let’s try some of these together…

(Problem C: 32/6) How do we read this problem? Thirty-two sixths
What is another way to read this problem? 32 divided by 6
What number are we trying to separate into groups? 32
How many groups are we working with? 6
What do we need to do first? Find out how many times 6 will go into 32 evenly (5).
How many “tallies” have we used in this case? 6 x 5 = 30
How many are left over? 32-30=2 Great job.
So, the first part of our answer will be? 5/1
The second part of our answer will be? 2/6
So we can write our answer as 5 2/6.

(Problem D: 17/3) How do we read this problem? Seventeen thirds
What is another way to read this problem? 17 divided by 3
What number are we trying to separate into groups? 17
How many groups are we working with? 3
What do we need to do first? Find out how many times 3 will go into 17 evenly (5).
How many “tallies” have we used in this case? 3 x 5 = 15
How many are left over? 17-15 = 2 Great job.
So, the first part of our answer will be? 5/1
The second part of our answer will be? 2/3
So we can write our answer as 5 2/3.

(Problem E: 16/4) How do we read this problem? Sixteen fourths
What is another way to read this problem? 16 divided by 4
What number are we trying to separate into groups? 16
How many groups are we working with? 4
What do we need to do first? Find out how many times 4 will go into 16 evenly (4).
How many “tallies” have we used in this case? 4 x 4 = 16
How many are left over? 16-16 = 0 Great job.
So, the first part of our answer will be? 4/1
The second part of our answer will be? There is no second part because there are no tallies left over.
So we can write our answer as 4/1 or 4.

(Problem F: 60/8) How do we read this problem? Sixty eighths
What is another way to read this problem? 60 divided by 8
What number are we trying to separate into groups? 60
How many groups are we working with? 8
What do we need to do first? Find out how many times 8 will go into 60 evenly (7).
How many “tallies” have we used in this case? 8 x 7 = 56
How many are left over? 60-56 = 4 Great job.
So, the first part of our answer will be? 7/1
The second part of our answer will be? 4/8
So we can write our answer as 7 4/8.

Let’s take a look at this scenario…

(Problem G) 18 slices of pizza were bought for 5 basketball players. The slices were to be shared equally. How many slices of pizza would each player eat? We will show our answer using the same abstract steps in this lesson.

What is this problem asking us to do? Divide
So, let’s think about how we would write this division problem as a fraction.
What are we trying to divide? 18 slices of pizza
Who are we dividing the pizza among? 5 basketball players
So, we can write this problem as 18/5.
How do we read this problem? Eighteen fifths
What is another way to read this problem? 18 divided by 5
What number are we trying to separate into groups? 18
How many groups are we working with? 5
What do we need to do first? Find out how many times 5 will go into 18 evenly (3).
How many slices of pizza have we used in this case? 5 x 3 = 15
How many are left over? 18-15 = 3 Great job.
So, the first part of our answer will be? 3/1
The second part of our answer will be? 3/5
So we can write our answer as 3 3/5. Each basketball player will eat 3 3/5 slices of pizza.

**Independent Practice**

Now you are going to try some of these problems on your own. Be sure to use the abstract method just as we have been doing today. If you become stuck on a problem you can use the tally method to help you.

Figure B-1: The caption for figure B-1 goes here.
Appendix D

Fidelity Checklist Example

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<tr>
<th>Teacher/Interventionist: XXXX</th>
<th>School: XXX</th>
<th>Grade: XXX</th>
<th>Date(s) of Observation: 10-14-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Time: 1:19</td>
<td>Math Skill(s): Lesson # if appropriate: 9</td>
<td>Group Size: 9</td>
<td></td>
</tr>
<tr>
<td>End Time: 1:50</td>
<td>Intervention/Strategy: CRA- Representational</td>
<td>Observer: XXXXX</td>
<td></td>
</tr>
</tbody>
</table>

Materials used in lesson: SmartBoard,

Directions: During the observation, place a “0” for Never, “1” for sometimes, and a “2” for Always in the column for each step observed (or not observed). Tally the numbers to calculate the overall fidelity of the intervention. Calculate the fidelity score/percentage at the bottom of the form.

*Note: If the step is not observable, write N/A in the appropriate column and do not include in the calculation of fidelity. However, essential items must be scored or fidelity cannot be established. The observer may have to do a follow-up visit to score the item(s). Fidelity is considered established when a score of 90% or higher is achieved.*

<table>
<thead>
<tr>
<th>NA</th>
<th>A</th>
<th>S</th>
<th>N</th>
<th>Checklist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduce lesson</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1. Topic/objectives of lessons are introduced.</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2. Instruction is explicit (teacher-directed)</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3. Teacher introduces skill/strategy (e.g., discusses why the skill/strategy is important)</td>
</tr>
</tbody>
</table>

Body of Lesson | | | | |
| NA | 2 | 1 | 0 | 4. Teacher models skill/strategy more than one time |
| NA | 2 | 1 | 0 | 5. Teacher follows script (optional for treatment group) |
| NA | 2 | 1 | 0 | 6. Teacher models metacognitive strategies |
| NA | 2 | 1 | 0 | 7. Teacher provides guided practice (e.g., reteaches when necessary, models skill, asks frequent questions to group and individuals to check for understanding, solve problems together) |
| NA | 2 | 1 | 0 | 8. Teacher provides visual model/representation of skill/concept |

Materials used: | | | | |
| NA | 2 | 1 | 0 | 9. Teacher provides practice on worksheet with “correctly worked” examples throughout |
| NA | 2 | 1 | 0 | 10. Teacher follows procedure for error correction (e.g., corrects all errors immediately using appropriate correction procedures such as |
modeling the correct response/behavior, asking the student to perform the correct way, reinforcing correct performance)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 11. Students are engaged and responsive during teacher-led instruction (e.g., group responses and individual turns are offered)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Independent practice</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 12. Students are given ample opportunity to respond or practice new skill/strategy independently before moving to new skill or next part of lesson</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 13. Independent practice is monitored</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 14. Students are provided with positive feedback</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 15. Skills are retaught as necessary (optional)</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 16.</td>
</tr>
<tr>
<td>NA</td>
<td>2</td>
<td>1</td>
<td>0 17.</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>30 / 30 = 100% Fidelity Score</td>
</tr>
</tbody>
</table>

- Fidelity established (= or > 90%)
- Fidelity established with recommendations
- Fidelity not met (request follow-up observation(s) and/or coaching visit)
- Other (explain)________________________________________ __________________________________________

Observer’s Comments

Students are sitting at their own desks today, the teacher is working on the SmartBoard. The students are all paying attention. One student calls out that this is easy. L uses mathematics language…
Appendix E

Student Assent Forms

Child/Minor Assent to Participate in a Research Study
Clemson University

The Effects of Sequenced Instruction on Mathematics Achievement

You are being invited to participate in a research study. Below you will find answers to some of the questions that you may have.

Who Are We?
- We are education researchers at Clemson University.

What Is It For?
- We want to learn more about how the sequence of instruction in mathematics influences student learning.

Why You?
- Learning fractions is included in the South Carolina state curriculum for students in Grades 6, 7, and 8.
- Participation is voluntary, there are no negative consequences if you decide that you don’t want to participate.

What Will You Have To Do?
- You will have to take assessments at the beginning and end of the research, as well as six weeks after the research has been completed. The scores on these assessments are for the purpose of assessing how effective the instruction was on your learning, not for a grade.
- You will participate in class, just as you normally do, as your teacher instructs you and your classmates on how to do mathematical operations involving fractions.
- You may be asked to be interviewed by the researcher after the completion of the study. Your participation in the interview is voluntary and will take approximately 10 minutes.

What Are The Good Things And Bad Things That May Happen To You If You Are In The Study?
- The instruction may help you learn how to solve operations involving fractions.
- Your participation in the study will also help us learn more about how instruction influences student learning.
What If You Want To Stop? Will You Get In Trouble?
• Participation in this study is voluntary.
• Participation in the research will not be used to positively or negatively impact grades, participation in programs, etc.

Are There Any Other Choices?
• If you choose not to participate, the researchers will not collect your work to analyze.

Do You Have Any Questions?
• You can ask questions at any time. You can ask them now. You can ask later. You can talk to me or you can talk to someone else at any time during the study. Here are the telephone numbers to reach us: XXX-XXX-XXXX (Elizabeth Hughes, Curriculum and Instruction, Clemson University).

By participating in this study, I am saying that I have read this form and have asked any questions that I may have. All of my questions have been answered so that I understand what I am being asked to do. I am willing and would like to participate in this study.

A copy of this form will be given to you.
Appendix F

Parent Consent Form

Parental Consent Form for Participation of a Child in a Research Study
Clemson University

The Effects of Sequenced Instruction on Mathematics Achievement

Description of the Research and Your Child’s Participation
Your child has been invited to participate in a research study conducted by Dr. Linda B. Gambrell and Mrs. Elizabeth M. Hughes. The purpose of this research is to examine the influence of instruction on student learning of fractions.

Your child’s participation will involve your child completing a one-time assessment on computations of fractions. The amount of time required for your child’s participation will be approximately 10-15 minutes.

Risks and Discomforts
There are no known risks associated with this research.

Potential Benefits
Experts have noted the importance of students competency in mathematics involving fractions, the instruction may help your child learn important skills involving fractions. Additionally, this research may help us to understand a fractions assessment.

Protection of Confidentiality
We will do everything we can to protect your child’s privacy. The assessment will be anonymous. Your child’s name will not appear on the assessment.

Voluntary Participation
Participation in this research study is voluntary. You may refuse to allow your child to participate or withdraw your child from the study at any time. Your child will not be penalized in any way should you decide to withdraw your child from this study or not to allow your child to participate.

Contact Information
If you have any questions or concerns about this study or if any problems arise, please contact Dr. Linda B Gambrell at Clemson University at XXX-XXX-XXXX. If you have any questions or concerns about your child’s rights as a research participant, please contact the Clemson University Office of Research Compliance (ORC) at XXX-XXX-XXXX or irb@clemson.edu. If you are outside of the Upstate South Carolina area, please use the ORC’s toll-free number, XXX-XXX-XXXX.

Consent
I have read this parental permission form and have been given the opportunity to ask questions. I give my permission for my child to participate in this study.

Parent’s signature: _______________________________  Date: ______________

Child’s Name: ______________________________________

A copy of this parental permission form will be given to you.
Appendix G

Teacher Information Letter

Information Concerning Participation in a Research Study
Clemson University

The Effects of Sequenced Instruction on Mathematics Achievement

Description of the Research and Your Participation

You are invited to participate in a research study conducted by Dr. Linda B. Gambrell and Mrs. Elizabeth M. Hughes. The purpose of this research is to examine the influence of sequenced instruction on student learning of fractions.

Your participation will involve giving your students a pre-assessment, post-assessment, and delayed post-assessment. Additionally, you will be required to implement 30 (approximately 15 minute) lessons that teach students how to complete operations involving fractions (e.g., multiplication with unlike denominators, finding equivalent fractions) either using a scripted lesson provided by the researchers or traditional method of instruction. Professional development on lesson delivery will be provided by the researchers.

The amount of time required for your participation will be 1 hour professional development on lesson implementation, time required to give the assessments (approximately 30 minutes each time), and time required to implement 30 scripted or traditional lessons (approximately 15 minutes each). Selected lessons may be audio-recorded to ensure fidelity of treatment and that the scripted lessons are being implemented.

Risks and Discomforts

There are no known risks associated with this research.

Potential Benefits

Students may benefit from instruction on fractions. This research may help us to understand how sequence of instruction influences student learning.
Alternatives to Research Participation

You may choose not to participate.

Protection of Confidentiality

We will do everything we can to protect your privacy and the privacy of your students. Students will be identified by their first name and a unique identification number. Your identity, or the identity of your school and district will not be revealed in any publication that might result from this study.

Voluntary Participation

Your participation in this research study is voluntary. You may choose not to participate and you may withdraw your consent to participate at any time. You will not be penalized in any way should you decide not to participate or to withdraw from this study.

Contact Information

If you have any questions or concerns about this study or if any problems arise, please contact Dr. Linda B Gambrell at Clemson University at XXX-XXX-XXXX. If you have any questions or concerns about your rights as a research participant, please contact the Clemson University Office of Research Compliance (ORC) at XXX-XXX-XXXX or irb@clemson.edu. If you are outside of the Upstate South Carolina area, please use the ORC’s toll-free number, XXX-XXX-XXXX.

A copy of this informational letter will be given to you.
Appendix H

Excerpt from District 6th Grade Pacing Guide

**Suggested Order of Instruction:**
Fractions: 6-2.4, 6-2.5
Multiplying and Dividing Decimals: 6-2.5
Percentages: 6-2.1, 6-2.3
Exponents: 6-2.7, 6-2.8

**UNIT: Exponents**
Indicators: 6-2.7, 6-2.8
REFERENCES


Linder, S. M. & Smart, J. (under review).


