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Use of Orthogonal Arrays for Efficient Evaluation of Geometric Designs for Reducing Vibration of a Non-Pneumatic Wheel During High-Speed Rolling

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Use of Orthogonal Arrays for Efficient Evaluation of Geometric Designs for Reducing
Vibration of a Non-Pneumatic Wheel During High-Speed Rolling

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Mechanical Engineering

by
William Ladd Rutherford
August 2009

Accepted by:
Dr. Lonny L. Thompson, Committee Chair
Dr. John C. Ziegert
Dr. Gang Li

ABSTRACT

During high speed rolling of a non-pneumatic wheel, vibration may be produced by the interaction of collapsible spokes with a shear deformable ring as they enter the contact region, buckle and then snap back into a state of tension. In the present work, a 2D planar finite element model with geometric nonlinearity and explicit time-stepping is used to simulate rolling of the non-pneumatic wheel. Vibration characteristics are measured from the FFT frequency spectrum of the time-signals of perpendicular distance of marker nodes from the virtual plane of the spoke, thickness change in the ring between spokes, and ground reaction forces. Both maximum peak amplitudes and RMS measures are considered as measures of vibration.

In the present work, a systematic study of the effects of six key geometric design parameters is presented using Orthogonal Arrays. Orthogonal Arrays are part of a design process method developed by Taguchi which provides an efficient way to determine the effects of variable levels and a guide to optimal combinations of design variables. Two complementary Orthogonal Arrays are evaluated. The first is the L8 orthogonal array which considers the six geometric design variables evaluated at lower and higher limiting values for a total of eight experiments defined by statistically efficient variable combinations. Based on the results from the L8 orthogonal array, a second L9 orthogonal array experiment evaluates the nonlinear effects in the four parameters of greatest interest, (a) spoke length, (b) spoke curvature, (c) spoke thickness, and (d) shear beam thickness. The L9 array consists of nine experiments with efficient combinations of low,

intermediate, and high value levels. Results from the Orthogonal Array experiments were used to find combinations of parameters which significantly reduce peak and RMS amplitudes, and suggest which variables have the greatest effect on vibration amplitudes.

The results of orthogonal arrays indicate that spoke length and spoke curvature were the most influential parameters on the amplitude of vibration for all three vibration measures. The optimal configuration predicted for these two parameters is a wheel with short spokes with large curvature. The order of influence and optimal levels of the other four variables varies according to the measure of vibration. The results show that there was effectively no interaction between spoke length and spoke thickness. However there are interactions between other variables in the system, and this interaction is stronger when non-linear variable levels were considered from the L9-array. Geometries are presented that minimize vibration for each source, and an optimal geometry is suggested that significantly reduces vibration for all measures of vibration considered.

A study of natural frequency and mode shapes extracted from the operational state of the system suggest that geometries with high amplitude peaks in the FFT spectrum for spoke vibration show a correlation with spoke vibration mode shapes. For other geometries, spoke vibration amplitude peaks coincided with ring flower-pedal modes.

DEDICATION

This thesis is dedicated to my parents Jack and Virginia Rutherford for their continuous support during my two years of graduate school, as well as the 22 years before that.

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CHAPTER ONE

INTRODUCTION

The pneumatic tire has been the dominant wheel structure for automobiles for more than 100 years. Michelin engineers recently proposed a non-pneumatic wheel design named the Tweel™ [1]. A picture of recent Tweel™ design is shown in Figure 1-1.

Figure 1-1: Tweel™ Mounted on a Hub

The Tweel™ is molded from polyurethane (PU) material has three main parts: the shear beam, the hub, and a number of thin, deformable spokes. The axle of the vehicle is attached to the rigid hub. The shear beam is the layer in the outer ring between imbedded steel cables which form inextensible reinforcements. Under loading, the shear beam in the region of contact deforms almost entirely in pure shear. The shear beam enables the Tweel™ to have a large contact patch with relatively low and constant contact pressure.

Rubber tire tread is attached to the outer surface of the shear beam. The spokes connect the hub to the shear beam and support the weight of the vehicle, much like air pressure in a pneumatic tire. The spokes underneath the hub in the contact patch buckle and collapse when the Tweel™ is loaded, while the spokes above support the load through tension. The goal of the Tweel™ is to surpass the pneumatic tire's performance and functionality using a mechanical structure. The proposed advantages of the Tweel™ over the pneumatic tire will be discussed in the next section, but they include improved handling, rolling resistance, and resistance to flats.

1.1 Tweel™ Performance Characteristics

In order to design an improvement to the pneumatic tire, one must first understand what qualities make the tire better than its predecessor, the rigid wheel. The pneumatic tire exhibits four defining characteristics that were not present in the rigid wheel design. These four critical characteristics are [1]: 1) low energy loss from obstacle impact, 2) low stiffness, 3) low mass, and 4) low contact pressure. Low energy loss was the driving force behind the development of the pneumatic wheel. In 1888, John Boyd Dunlop conducted his famous barnyard experiment where he rolled a homemade pneumatic tire down the road alongside a prevalent rigid wheel design of the time. The pneumatic tire rolled farther before it lost its momentum and fell on its side. After some investigation and research, Dunlop concluded that the reason the pneumatic tire rolled farther was because it exhibited a lower energy loss over obstacles. The road the experiment was

conducted on was paved with cobblestones. It was Dunlop's experiments and observations that started the development of the pneumatic tire.

The second critical characteristic of the pneumatic tire is low stiffness. The pneumatic tire experiences much greater displacement during loading than the rigid wheel. As a result, the pneumatic tire exhibits a much lower vertical stiffness than the rigid wheel. Low vertical stiffness is important because it gives the vehicle and rider a smoother and more comfortable ride.

The third critical characteristic of the pneumatic tire is its relatively low mass. A lower mass gives better braking, acceleration, and handling characteristics. The rigid wheel is what is referred to as a "bottom loader" because the entire load of the vehicle is supported through compression by the spokes in between the hub and the ground [1]. As a result, the force is transmitted to a small area relative to the thickness and number of the spokes currently between the hub and the ground at any given time. In contrast, the pneumatic tire can be defined as a "top loader" [1]. In a top loader, the load is supported from above the hub and axle through tension provided by the inflation pressure. The load essentially hangs from the entire top of the tire. Therefore, the load is distributed over a larger area than the rigid wheel. The pneumatic wheel has a greater load carrying efficiency allowing for a smaller mass.

The pneumatic tire also exhibits relatively low contact pressure due to the lower stiffness and greater displacement under load. This elongated contact patch increases the area over which the force is applied between the ground and the wheel. An increase in area for a given load results in a decrease in contact pressure. This contact pressure is not

only low, but also uniformly distributed. A low magnitude and uniformly distributed contact pressure allows for the generation of high traction forces, uniform tire wear, and protection of the road surfaces from damage [1].

To be competitive with the pneumatic tire, any new wheel design needs to perform at least as good as, if not better, than the pneumatic tire in the four critical characteristics previously discussed. The Tweel™ is theorized to potentially outperform the pneumatic tire in each of the four critical characteristics previously mentioned. In consideration of the characteristic of low contact pressure, for the Tweel is made possible by the shear beam design. The shear beam consists of a relatively low modulus elastic layer sandwiched between two thin “inextensible membrane” layers. The thin inextensible layers have low bending stiffness but high stiffness in the circumferential direction. This composite structure will cause the shear beam to deform in nearly pure shear when passing through the contact region instead of bending. A classical beam deforms primarily in bending and would create high stress concentrations around the initial point of contact. The shear beam in the Tweel, when properly designed, will instead create an even contact pressure without high stress concentrations. As for the other desirable characteristics of a rolling wheel, the Tweel™ is predicted to have a comparably low energy loss over obstacle impact based on finite element simulation [1]. Since the Tweel™ is a top loader similar to the pneumatic tire, it has high load carrying efficiently and thus be potentially designed for low mass. Depending on the number and thickness of the spokes used, the Tweel™ could even have a lower mass than the tire because the Tweel™ does not need to maintain the high static inflation pressures that the

pneumatic tire uses. The last critical characteristic, low vertical stiffness is the characteristic in which the Tweel™ especially excels. The Tweel's stiffness is related to the size of the shear beam and spoke geometry. The number of spokes, length, thickness, curvature, and other characteristics can all be used to tune the stiffness of the Tweel. A pneumatic tire's vertical stiffness is defined by the air pressure inside the tire, and is coupled to many other design variables such as tire height, tire width, and lateral stiffness. An advantage of the Tweel is that the lateral stiffness is relatively decoupled from the vertical stiffness, allowing for optimal design for ride comfort and handling. A lower vertical stiffness improves ride comfort, while a high lateral stiffness improves handling. As an additional benefit, the absence of inflation pressure in the Tweel™ means it will always maintain ideal stiffness characteristics and never go flat.

During the initial testing of a Tweel™ design on a passenger vehicle, Michelin® discovered that during use the Tweel™ design produced unacceptably high levels of acoustic noise [2] at high rolling speeds. One of the goals of a recent project between Michelin and Clemson University on enabling the Tweel for automobile applications is to determine the source of the noise, and to design a Tweel™ that successfully reduces that noise.

1.2 Literature Review

Tire noise is not an issue unique to the Tweel™. In recent years, significant progress has been made in the diminishing the sound radiation from the power-train of automotive vehicles. Consequently, the rolling noise produced by the pneumatic tire itself has become the main source of noise for passenger cars traveling over 40 km/hr and

trucks traveling over 60 km/hr [3]. The noise from rolling wheels is especially important for the new class of electric powered vehicles which do not have noise problems characterized by combustible engines. Since decreasing tire noise is currently the biggest obstacle preventing a vehicle from emitting less noise, a large amount of research has recently been conducted in this area for pneumatic tires.

It has been found that the sound that radiates from a pneumatic tire can be classified into three components [4]. The first is whole-tire vibration produced by road roughness. This noise generally radiates from the tire's sidewall and is most remarkable in frequency band to 500 Hz. The other two sources of noise are tread block vibration and tread air groove resonance. These are remarkable in the frequency band up to 1000 Hz. However, optimizing tread patterns will not eliminate the noise as sound radiation is generated even in smooth tires with no tread [4]. Therefore, it is important to understand the characteristics of noise and vibration that arise from contact between a spinning tire and the road.

Brinkmeier *et al.* have proposed a method of using finite element analysis (FEA) tire models to simulate the noise up to 850 Hz that radiates from a pneumatic tire during rolling [3]. The FEA analysis procedure can be divided into six steps: (1) Computation of the nonlinear rolling process with a transport approximation, (2) Eigenvalue analysis for the steady state of the rolling tire, (3) Determination of the excitation due to the texture of the road, (4) Computation of the operational vibrations with modal superposition, and (5) Noise radiation analysis. While promising, the above procedure

makes approximations and assumptions based on the construction of a pneumatic tire which are not immediately transferable to the Tweel™.

While much research has been conducted on pneumatic tire noise, only a few engineers from Michelin® and Clemson University have studied vibration and noise emitted from the Tweel™. The initial hypothesis for the source of the noise was the effect of spokes buckling and snapping back into place at a high rate of speed causing spoke and interacting ring vibrations. The dynamics of the Tweel™ spokes were first studied by Cron [5] using ABAQUS finite element analysis software to analyze a single 3D Tweel™ spoke. The spoke was pre-tensioned and rotated around the hub using connector elements. The connectors were used to simulate enforced rotation of the spoke and buckling within the contact patch, then the snap-back into tension once outside the contact patch. This simulation was based on the assumption that the ring and hub control the motion of the spoke and the dynamics of the spoke has no effect on the hub or ring [5]. The motion of the spoke profile was based on an assumed path accounting for the local radius in the transition and contact regions.

In [2, 6] the actual motion of the spoke profile was predicted from a 2D planar finite element model of the entire Tweel™ to better understand the dynamics of the entire structure under operating conditions. The analysis procedure developed in [2, 6] was divided into three steps. First the entire model was cooled from 125 C to 25 C in ABAQUS Standard to introduce the appropriate amount of pretension into the spokes. In step two, the results from the cooling step, which included the stresses and strains, were

imported into ABAQUS Explicit using a restart file. A 60 kph rolling speed was induced in the Tweel™ while the load was dynamically simulated by a 12 mm upwards displacement of a rigid and frictionless ground. This loading step took 0.5 seconds. In step three the Tweel™ was allowed to roll for 0.5 seconds at steady-state with no change in temperature or ground displacement, corresponding to several revolutions of the Tweel.

Lateral spoke displacements during steady-state rolling were used to measure spoke vibration. An FFT frequency spectrum was conducted on this set of data. For the Tweel geometry studied, and a 2D plane stress explicit coupled-temperature finite element analysis, three amplitude peaks were discovered in the signal at frequencies of 190 Hz, 360 Hz, and 620 Hz. In addition to the 2D spoke vibration analysis, a single spoke 3D vibration analysis was conducted. The goal of the 3D analysis was to investigate any out-of-plane flapping that might be occurring in the spoke but was not able to be simulated by the 2D planar model. To insure realistic boundary conditions for the 3D spoke, the spoke length vs. time data from the 2D simulation was used as input to the 3D single spoke model. The results showed significant out-of-plane flapping behavior, with a frequency peak of 260 Hz that was not present in the 2D in-plane analysis.

In [7], the Tweel dynamic analysis was expanded to investigate the effect of edge scalloping, spoke thickness and taper, spoke width, and spoke curvature on the frequency and amplitude spoke vibrations during high-speed rolling. Results from this analysis showed that scalloping the spoke edges significantly reduces the amplitudes of the

vibration while keeping the frequencies at the peak amplitudes unchanged. An optimal amount of scalloping was determined. Results also suggested that changes in spoke thickness affect the amplitude, but not the frequencies of vibration. Changes in spoke curvature and spoke width did not significantly affect either frequency or amplitude of vibration.

1.3 Thesis Objective

A complete understanding of the effects of varying the Tweel™'s geometric parameters on the vibration properties of the spokes, ring, and interactions with ground is not currently available. The present work presents a systematic study, using orthogonal arrays, of the effects of six key geometric design parameters on Tweel™ vibration. Orthogonal Arrays are a tool used to provide an efficient way to determine the effects of variable levels on the amplitude of vibration and to guide the design to an optimal combination of design variables. Prior to this work, the orthogonal array process had not been applied to the objective of understanding Tweel™ vibration. Therefore, the details and theory of using the orthogonal array tool in conjunction with the Tweel™ vibration study will be discussed. Response measures will include RMS and maximum peak amplitudes obtained from FFT spectrum of marker nodes on both the spokes and ring. The effect of variable levels on shifting frequencies of amplitude peaks will also be studied. Based on deviations of variable levels from the mean, the relative effect of the geometric variables on reducing spoke and ring vibration amplitudes, and ground reaction amplitudes will be identified. Once the variables of highest influence are identified, optimal combinations of the geometric variables will be predicted. This geometry will

consider all three sources of vibration, as well as Tweel mass and vertical stiffness. Interactions between variables will also be investigated within the Orthogonal Array and from confirmation experiments. Two different orthogonal arrays will be used in the experiments. The first is a modified 2-level L_8 array which allows examination of potential interactions between two variables. The second is a 3-level L_9 array which enables investigation of nonlinear response to variable changes. Mode shapes of two Tweel™ geometries will be analyzed in order to identify a relationship between the ring flower-pedal modes, local spoke bending modes, and the amplitude peaks and frequency of vibration observed in the FFT spectrum.

An outline of the Thesis is as follows:

- Chapter 2 explains the two-dimensional Tweel™ finite element analysis modal created using ABAQUS. The reference geometry, material properties, and analysis procedure are all outlined.
- Chapter 3 describes the three potential sources of noise, spoke vibration, ring vibration, and ground interaction vibration, and how each will be measured. Examples from a baseline reference Tweel model are used to explain how the FFT spectrum is created and analyzed.
- Chapter 4 discusses the theory behind the orthogonal array experimental procedure. Discussions include the procedure for determining the variables of highest influence, calculating the recommended levels, and discovering and understanding interactions.
- Chapter 5 presents the results of the L_8 array, which analyzes six geometric variables at a high and low level. Amplitudes of vibrations are explained using ABAQUS

screen shots of the Tweel™ motion. Influence of each variable is discussed for each measurement of vibration. Recommended geometries and results of these geometries are presented. Effects of geometries on mass and vertical stiffness of the Tweel™ are discussed.

- Chapter 6 presents the results of the L₉ array, which analyzes four geometric variables at a high, medium, and low level. The purpose of this experiment is to confirm the results of the L₈ array and investigate any non-linear effects changing variable levels. Influence of each variable is discussed for each measurement of vibration. Recommended geometries and results of these geometries are presented. Effects of geometries on mass and vertical stiffness of the Tweel™ are discussed.
- Chapter 7 presents a discussion of mode shapes and natural frequencies for two Tweel™ geometries, one that results in high vibrations and one that results in low vibrations.
- Chapter 8 provides conclusions and suggestions for future work.

CHAPTER TWO

2D FEA TWEEL™ MODEL

The 2D Tweel™ finite element model used in the present study is similar to the previous 2D models used [2, 6, 7] with a few notable differences which will be explained. These advances improve the model's accuracy and efficiency and include changes in element type, material property definitions, and the analysis step procedure.

2.1 Tweel™ Geometry

The Tweel™ geometry was created using a plug-in supplied by Michelin. A plug-in is a piece of software that installs itself into another application to extend the capabilities of that application [8]. This plug-in utilizes a python module that creates a graphical user interface as shown in Figure 5-2. This graphical user interface allows the user to input different geometrical parameters and the plug-in will automatically generated a Tweel™ with the requested geometric properties.

Figure 2-1: Tweel™ Plug-in Interface showing reference BMW Mini Tweel parameters

The geometric parameters used in this study are designed for the BMW Mini and are different from the geometry used in previous studies [2, 6, 7] which were designed for the Audi. Figure 2-2 and Table 2-1 show a comparison between the geometric properties of the Audi and Mini models.

2D Audi Model

2D Mini Model

Figure 2-2: Audi and Mini Tweel™ Comparison

Table 2-1: Comparison of Audi and Mini Tweel™ Geometric Properties. All Units in mm Except Derad Orientation Parameters.

	Audi	Mini
Outside Diameter	603	593
Hub Diameter	400	410
Ring Thickness	13	19.5
Outside Coverage	2.5	7.0
Inside Coverage	2.5	2.5
Number of Spoke Pairs	20	25
Spoke Thickness	3.0	4.2
Spoke Curvature	5.0	8.0
Spoke Derad - Outer	0.10	0.15
Spoke Derad - Inner	0.66	0.60

The most significant difference between the Audi and Mini models are the increase in the number of spoke pairs, spoke thickness, thickness of the outside coverage, and the amount of spoke curvature. The shear beam thickness was also increased from 8

mm to 10 mm. The shear beam thickness is defined by the inside and outside coverage subtracted from the total ring thickness. The outer diameter was adapted for the smaller Mini vehicle and the hub diameter was increased for manufacturing purposes.

The procedure for modeling contact and interaction between the rigid ground and the Tweel™ is the same as used in [2, 6, 7]. For this model, zero friction is assigned between the contact of the outer ring coverage and rigid ground. This is a conservative assumption since it allows the ring to respond more freely than if it were constrained by frictional forces at the ground. No tread is included in the 2D plane model. Three-fourths of the Tweel™ is finely meshed leaving only one-fourth of the Tweel™ with an extra fine mesh. The extra fine mesh is used for data collection while the coarser mesh helps increase computational efficiency of the model. Detailed discussion of the interaction and meshing can be found in previous works [2, 6, 7]. The simulations in this work are modeled using plane strain elements, unlike the previous 2D Tweel™ simulations which were modeled with plane stress elements. From the geometry of the Tweel™, it is not clear which assumption best represents the full 3D model. Studies have shown that the simulation results for the plane strain elements more closely matched results from a 3D model. The plane strain models also required nearly half the computational time compared to plane stress models. The plane strain elements used in the present work are of type CPE4R, i.e., 4-node bilinear plane strain quadrilateral elements with reduced integration and hourglass control.

2.2 Tweel™ Material Properties

As discussed earlier, the Tweel™ is made up of two materials: an isotropic polyurethane (PU) material and an orthotropic elastic material. The orthotropic material makes up the two reinforcing layers on the top and bottom of the shear layer and is extremely stiff in the tangential (circumferential) direction. These two reinforcing layers ensure that the ring length is conserved, an important characteristic of wheel design. For the 2D planar model, the reinforcements are modeled with thin layers of 0.64 mm thickness. The isotropic polyurethane material makes up the rest of the Tweel™: the spokes, the shear layer, the inner coverage, and the outer coverage. The polyurethane material is modeled as a hyperelastic material using a geometric nonlinear analysis with relatively low modulus that allows for high strain with low stress. Figure 2-3 shows the components of the Tweel™ model. The components that are made of the PU material are shown in light blue while the orthotropic elastic reinforcement layers material are shown in red.

Figure 2-3: Tweel™ Components

2.2.1 Reinforcement Layer

The reinforcement layer is defined by the following linear elastic orthotropic properties where the stress-strain relations for the elastic orthotropic material are of the form [9]:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\ & D_{2222} & D_{2233} & 0 & 0 & 0 \\ & & D_{3333} & 0 & 0 & 0 \\ & & & D_{1212} & 0 & 0 \\ & sym & & & D_{1313} & 0 \\ & & & & & D_{2323} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}$$

where the subscript “11” corresponds to the radial direction, “22” corresponds to the tangential direction, and “33” is the out-of-plane direction. The elastic moduli are

defined in Table 2-2. The modulus D_{2222} , for the tangential direction, is a factor of 100 larger than the shear modulus, D_{1212} . The radial modulus D_{1111} is small indicating negligible bending stiffness.

Table 2-2: Elastic Moduli for Orthotropic Reinforcement material

Moduli	Value (daN / mm^2)
D1111	0.01
D1122	0
D2222	3217.75757575758
D1133	0
D2233	0
D3333	0.01
D1212	32.1775757575758
D1313	0
D2323	0

Other important material properties for the elastic orthotropic material include:

- Mass density $\rho = 1.54 \times 10^{-10} daN - sec^2 / mm^4$
- Thermal Expansion Coefficient $\alpha_{11} = \alpha_{33} = 0$ and $\alpha_{22} = 1.2 \times 10^{-5} / ^\circ C$
- Specific heat equal to $c_T = 4.75 \times 10^8 J / (Kg - K)$
- Conductivity $\kappa = 20 W / (m - K)$

2.2.2 Ring and Spokes

The hyperelastic properties of the spokes, shear layer, and both coverages are defined using the Marlow strain energy potential. Previous Tweel™ simulations [2, 6, 7] used the Mooney-Rivlin strain energy potential to define the hyperelastic material. The Marlow strain energy model is recommended when a set of uniaxial test data is available [8]. Recently, Michelin has supplied uniaxial stress-strain data which was utilized in this

model. The Marlow model should better represent the polyurethane material properties of the Tweel™.

For confidentiality reasons, the uniaxial test data will not be presented here, but Figure 2-4 and Figure 2-5 show a plot of the uniaxial stress-strain data used for the Marlow model. The Mooney-Rivlin stress-strain data included is also plotted for comparison. The stress-strain curve for the Mooney-Rivlin model was generated for uniaxial tension/compression by ABAQUS/CAE using the material properties defined in previous simulations.

Figure 2-4: Stress-Strain curves for Marlow and Mooney-Rivlin Hyperelastic Materials

Figure 2-5: Stress-Strain curves for Marlow and Mooney-Rivlin Hyperelastic Materials (Nominal strain ranges from -0.1 to 0.1)

From the above graphs, for large tensile and compressive strains, the Mooney-Rivlin model has a significantly different form than the Marlow model. From Figure 2-4, the Marlow property has overall less stress for a given strain. For the smaller nominal strains (less than 10%) shown in Figure 2-5, the stress-strain curves are more similar for the two models. Over this range, the Mooney-Rivlin model appears approximately linear, while the Marlow model shows a parabolic softening trend. The initial tangent modulus for the Marlow model, as defined by the slope on the stress-strain curve, is higher than the Mooney-Rivlin model, but as the nominal strains increases the Marlow material appears to be much softer with a lower tangent modulus compared with the Mooney-Rivlin model. As mentioned earlier, it is assumed that the Mooney-Rivlin model better

represents the stress-strain behavior of the polyurethane (PU) material used for construction of the Tweel™.

Other important material properties for the PU material include:

- Mass density $\rho = 1.1 \times 10^{-10} \text{ daN} \cdot \text{sec}^2 / \text{mm}^4$
- Thermal Expansion Coefficient $\alpha = 0.0002 / ^\circ \text{C}$
- Specific heat equal to $c_T = 2 \times 10^9 \text{ J} / (\text{Kg} \cdot \text{K})$
- Conductivity $\kappa = 0.3 \text{ W} / (\text{m} \cdot \text{K})$
- Poisson's Ratio $\nu = 0.45$

A Poisson's ratio $\nu = 0.45$ is defined to model a nearly incompressible hyperelastic material for explicit analysis.

2.3 Tweel™ Analysis Procedure

In the present work, a new step procedure has been adapted for the finite element simulations. For efficiency the cooling and static loading are combined in a common step for analysis. The vertical deflection is set to 15mm, minus small amount of thermal shrinkage due to cooling from 125 to 25 degrees Celsius, which corresponds to one-quarter static vehicle weight. The rotating speed is defined by 120 rad/sec which corresponds to 129.4 kph (80.4 mph). In the present work, all analysis steps are performed in Abaqus/Explicit. The procedure therefore requires no restart file which greatly increases efficiency. The steps used in the analysis procedure are as follows:

- **Predefined Field:** Initial condition, angular velocity 120 rad/sec, and Cooling with smooth step from 125 to 25 degrees, over period of 0.1 sec (in step 2)
- **Step 0:** Initial Condition: Hub Center is free to rotate

- **Step 1:** Establish Initial Rotation over period 0.005 sec.
- **Step 2:** Loading with 15mm pushup defined by instantaneous ground velocity of 150 mm/sec over 0.1 sec, and hub center rotational velocity set to 120 rad/sec (~130 km/hr).
- **Step 3:** Steady-State rolling period 0.3 sec

The Tweel™ procedure begins (Step 0) with the Tweel™ at 125 degrees C and rolling with a predefined rotation speed of 120 rad/sec distributed to all nodes in the mesh. Starting the Tweel™ with an initial velocity reduces the numerical noise that would be created if the Tweel™ were to start at zero and have to quickly ramp up to 120 rad/sec. The Tweel™ is allowed to rotate freely for 0.005 seconds while it becomes closer to steady state. During Step 2 the Tweel™ is cooled and loaded. The Tweel™ is cooled to 25 degrees C over a period of 0.1 seconds. This cooling procedure replicates the pre-stresses added to the Tweel™ during the manufacturing process. The Tweel™ is loaded 15 mm defined by instantaneous ground velocity of 150 mm/sec over 0.1 seconds. A boundary condition at the hub center of 120 rad/sec is enforced during this step to prevent the Tweel™ from decreasing speed while it is being cooled and loaded. During Step 3 the Tweel™ rolls in steady-state by modifying the ground to zero instantaneous speed. During Step 3 the majority of the test data is collected.

CHAPTER THREE

MEASURES OF TWEEL VIBRATION

It is assumed that a correlation exists between vibration in the Tweel™ during high-speed rolling and noise generation. In previous works [2, 6, 7, 10], only spoke vibrations were investigated as a possible source of noise from the Tweel™ during high-speed rolling. This work will investigate two other possible noise sources: ring vibrations, and the interaction between the Tweel™ and the ground. In the present work, vibration is measured during steady-state rolling for: (a) perpendicular distances of spoke marker nodes from the plane of the spoke, (b) ring thickness, and (c) ground vertical reactions. Changes in ring thickness and ground reactions forces are intended to measure spoke-to-spoke interactions and spoke passing frequencies. Time signals are processed using FFT for analysis of frequency response. FFT results are reported based on a zero padded FFT with Hamming window for improved resolution of harmonics [11]. All FFT results are reported with zero mean signals. The mean signal removes the static response at zero frequency in the FFT. Magnitudes of amplitude peaks and RMS of the FFT results will be used to quantify the level of vibration. Frequencies at peak amplitudes will also be measured. FFT magnitudes with Hamming window are computed for zero mean signals for the steady-state rolling step (total time from 0.105 sec. to 0.405 seconds). With the angular speed of 120 rad/sec (approximately 128 km/hr), the Tweel makes approximately

5 revolutions during the rolling step. All displacement data is collected at a sample rate of 5000 Hz during the rolling step, with Nyquist cutoff frequency of 2.5 kHz.

Figure 3- 1: Deformed geometry at end of rolling step (BMW Mini Reference Geometry)

Figure 3- 1 shows the deformed geometry of the BMW Mini Reference Tweel at the end of the Rolling Step. The geometric parameters for the BMW Mini Tweel are previously defined in Figure 2-2 and Table 2-1. The BMW Mini Tweel will serve as the reference benchmark and later be compared to the results of the geometric design variable studies. All example data used for explanation of the vibration measures in this chapter will be based on the BMW Mini Tweel™ reference geometry.

3.1 Spoke Vibration

It has been theorized that noise generated during Tweel™ operation is produced by vibration caused by the spokes buckling and snapping back into tension at a high rate of speed [2]. The magnitude of the lateral spoke vibration is measured using the three marker nodes seen in Figure 3-2. The marker nodes are located at the upper and lower quarter points, and middle of the spoke. The marker nodes are located on the inside edge of the right spoke in a spoke pair. It is assumed that the vibration on the other spoke in the shared spoke pair is similar. Figure 3- 3 shows a plot of the distance between the top and bottom marker node during the rolling step, which indicates change in spoke length. The profile of spoke length shows a transition from compression during buckling in the contact region to tension as it rotates around the sides and top. An imaginary line, shown in Figure 3-2, is created by connecting the top node and the bottom node of the spoke. The perpendicular distance of each of the three marker nodes from the line is recorded with respect to time. This procedure gives three sets of vibration data, one for each of the three marker nodes. These three sets of data can be seen in Figure 3-4.

Undeformed Spoke

Deformed Spoke

Figure 3-2: Marker Nodes on Undeformed and Deformed Spoke in contact region

Table 3-1: Marker Node Number and Positions

10	Upper Quarter Node
163	Middle Node
11	Lower Quarter Node

Figure 3- 3: Spoke Length (BMW Mini Reference Geometry)

Figure 3-4: Perpendicular Distance of Marker Nodes from Plane of Spoke (BMW Mini Reference Geometry)

Figure 3-4 shows the perpendicular distance for the three middle nodes over all three steps. Step 1 (initial rolling) ends at 0.005 seconds and Step 2 (loading and cooling) ends at 0.105 seconds. The rolling step, Step 3, lasts from 0.105 to 0.405 seconds, and is where the majority of the vibration data will be collected. The large displacements seen every 0.052 seconds correspond to the spoke collapsing in the contact patch, with frequency $f = (120 \text{ rad/sec})/(2\pi) = 19.1 \text{ Hz}$. Note how they correspond to the steep drops in the spoke length in Figure 3- 3. Counting the spoke collapses, it can be determined that the Tweel™ rotates five full rotations during the rolling phase. Because of the curvature of the spoke, the middle node begins with a larger perpendicular distance from the plane of the spoke than the upper and lower quarter nodes. For the same reason, the middle node also has a larger peak displacement than the other two nodes. The upper quarter and lower quarter nodes have similar vibrational profiles, but are not identical.

The dominate frequency peaks and amplitudes of spoke vibration are found using a MATLAB program that performs a Fast Fourier Transform (FFT) on the spoke displacement data. The FFT results in this work will be reported using a Hamming Window. The use of a Hamming window is ideal for signal processing because it emphasizes the peak frequencies and minimizes the side lobes [11]. The Hamming window should give a clearer representation of the frequencies and amplitudes of vibration. An FFT plot of the upper quarter node displacement for the rolling step from Figure 3-4 is shown with Hamming window in Figure 3-5. The figure shows three amplitude peaks occurring at, 20 Hz, 270 Hz, and 540 Hz. The 20 Hz peak shows

overtones of the multiples of 20 Hz, until they diminish around 200 Hz. The 20 Hz peaks are a result of the rotational speed of the Tweel™, and correspond to the spoke collapses every 0.05 seconds shown in Figure 3-4. Because this 20 Hz peak is a result of the spoke collapse profile when transitioning from tension to collapse in the contact region, its amplitude will not be considered to generate any significant noise. Sound pressure levels (SPL) at frequencies below 100 Hz do not have significant impact on human perception of noise. It is common practice to significantly attenuate decibel (dB) levels of SPL at low frequencies as measured by ISO A-weighted dB curves. The two vibration amplitude peaks of interest which may result in significant noise perception are the 325 Hz peak and the 630 Hz peak. The 325 Hz peak could likely produce sound and it has the highest peak amplitude. The 630 Hz peak is likely an overtone of the 325 Hz peak. The 2nd peak does not appear at the middle marker node suggesting that a vibration mode is excited with zero amplitude at the middle node. The FFT plots are capable of finding peaks up to the Nyquist Frequency. Since the sampling rate of the data in Figure 3-4 is 5000 Hz, the Nyquist Frequency is 2500 Hz. Figure 3-5 shows the FFT up to 1000 Hz, because for this signal there are no significant amplitude peaks for frequencies greater than 1000 Hz.

Table 3-2 compares FFT results with Hamming Window for time signals of perpendicular distances of marker nodes from the plane of the spoke during rolling.

Figure 3-5: FFT plot of Perpendicular Distance of the Upper Quarter Node (BMW Mini Reference Geometry)

Figure 3-6: FFT plot of Perpendicular Distance of the Middle Quarter Node (BMW Mini Reference Geometry)

**Table 3-2: Peak Amplitudes of spoke vibration for marker nodes during steady rolling
(BMW Mini Reference Geometry)**

Marker Nodes	Peak Amplitudes	
	1 st Peak (325 HZ)	2 nd Peak (630 HZ)
Upper Quarter Node	130	150
Middle Node	190	
Lower Quarter Node	120	160

When comparing spoke vibration results for changes in Tweel™ geometry, both the magnitude of the maximum peak amplitude and RMS over the frequency range from 200 Hz to 1000 Hz will be recorded and used to compare the magnitude of vibration between experiments. RMS values are computed using the formula

$$RMS = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i)^2}$$

The RMS value gives a measure of overall vibration magnitude over the entire frequency range whereas the max peak amplitude measures the intensity of vibration at localized frequencies. Both measures are important in characterizing the amplitude of spoke vibration.

3.2 Ring Vibration

The magnitude of ring vibration will be determined using the four marker nodes shown in Figure 3-7. The distance between the top and bottom marker nodes of the ring for both inside the spoke pair and outside the spoke pair will be measured over time. The sampling rate and Nyquist Frequency for the ring vibration data is once again 5000 Hz and 2500 Hz, respectively. A set of ring thickness data inside a spoke pair defined by the distance between the top and bottom ring marker nodes for the reference Tweel geometry is shown in Figure 3-8 and Figure 3-9.

Figure 3-7: Marker Nodes Used for Measuring Changes in Ring Thickness in undeformed shape.

Table 3-3: Ring Marker Nodes Number and Position

1048	Inside Spoke Pair, Top Node
181	Inside Spoke Pair, Bottom Node
1041	Outside Spoke Pair, Top Node
188	Outside Spoke Pair, Bottom Node

Figure 3-8: Ring Thickness over all Steps (BMW Mini Reference Geometry)

Figure 3-9: Ring Thickness over rolling Step (BMW Mini Reference Geometry)

Figure 3-8 shows the ring thickness over all steps. The shrinking of the ring during the cooling step can clearly be seen by the decrease in average thickness from 19.5 mm to about 18.55 mm. The peaks in Figure 3-9 are a result of the ring traveling through the contact patch during rolling. While entering and leaving the contact patch the ring contracts and then expands at the center of the contact patch. A snapshot in time during rolling of the ring marker nodes in the contact region is shown in Figure 3-10. Also shown from this figure is the shear deformation developing in the shear beam in the transition region reaching a maximum near the boundary of the contact patch. The Hamming window FFT for this signal is shown in Figure 3-11.

Figure 3-10: Screen Shots of Ring Entering the Contact Patch

Figure 3-11: FFT of Ring Thickness (BMW Mini Reference Geometry)

The largest amplitude peak occurs is at about 95 Hz. All ring thickness data analyzed showed the highest peaks between 80 and 100 Hz. This peak is a result of the ring entering and leaving the contact patch. Since the goal of examining ring vibration is to find waves traveling thought the entire ring, this 100 Hz peak is not considered to be significant for noise. Once again, the max peak amplitude value and the RMS of the signal will be used to quantify the amount of vibration in the ring. For the ring data used in this study, the overtones of the 100 Hz peaks end roughly around 250 Hz and the last peaks occur around 1500 Hz. Therefore the peak amplitude and the RMS of the signal will be taken over a range of 250 Hz to 1500 Hz for all cases.

3.3 Ground Interaction Vibration

The biggest source of noise on a pneumatic tire is the interaction between the tire and the ground [4]. Therefore, the ground interaction on the Tweel™ is of interest. The reaction force of the Tweel™ on the rigid ground can be output by ABAQUS. The vertical reaction force is the resultant of all contact pressures at the contacting surface. Since the ground is frictionless, only the force in the vertical direction is significant. The vertical reaction force is recorded with a sampling rate 5000 Hz. Figure 3-12 and Figure 3-13 show the vertical reaction force over time.

Figure 3-12: Vertical Reaction Force over All Steps (BMW Mini Reference Geometry)

Figure 3-13: Vertical Reaction Force for Rolling Step (BMW Mini Reference Geometry)

Figure 3-14: FFT of Vertical Reaction Force (BMW Mini Reference Geometry)

Figure 3-12 shows a low vertical reaction force when the Tweel™ is not loaded, with the force increasing during the loading step. During the rolling-step the mean vertical reaction force is 4.16 daN. Figure 3-14 displays the FFT with Hamming window for the rolling step. This FFT shows clear peaks at 477 Hz and 954 Hz. Note that 954 is a multiple of 477 Hz. The spoke pair passing frequency, or frequency that the Tweel™ passes between each pair of spokes is $f = (120 \text{ rad/sec}) / (2\pi / 25 \text{ spoke pairs}) = 477 \text{ Hz}$. This indicates that there could be a vibration and noise problem due to the discrete stiffness of the Tweel™. Passing from a region of stiffness due to a spoke pair to the less stiff region between a spoke pair could create variation in contact pressure between the Tweel™ and ground resulting in noise. During the rolling step the area of the contact surface does not remain constant for the Tweel™. Due to the discrete stiffness of the spokes supporting the ring, the contact area increases and decreases at frequencies corresponding to spoke pair passing frequency of 477 Hz, and spoke-to-spoke passing frequency of 954 Hz. However, in Chapter 5, results will be presented that show not all geometries show a clear peak at 477 Hz, or any other distinct peaks. In general, the reaction force FFT spectrum shows more broadband response. The broadband peaks in reaction force may be due to flower-pedal standing waves in the ring. To quantify amplitude of vibration for the ground reaction, the RMS will be taken over the entire frequency band up to 2500 Hz.

CHAPTER FOUR

ORTHOGONAL ARRAY EXPERIMENTAL PROCEDURE

In the previous chapter it was discussed how spoke, ring, and ground interaction vibrations are measured using ABAQUS simulations. In this study, key geometrical variables will be studied to find their effect on the three types of vibration measures. The ABAQUS simulation used for modeling the response of the Tweel™ is computationally expensive and time consuming. In addition, due to the large number of geometrical variables that define the geometry of the Tweel, and the range of values that can be used for each variable, a large number of variable combinations would be needed to determine an optimal design and identify the parameters with the most effect on reducing vibration. To handle this problem, orthogonal arrays will be used to organize this design study in an efficient and effective way. This chapter details the orthogonal array procedure. From the orthogonal array results, optimal geometric parameters will be determined, and the variables with the greatest and least effect are identified.

4.1 Orthogonal Arrays

The orthogonal array procedure, also known as Taguchi Methods™, was invented by Dr. Genichi Taguchi in Japan in the 1950s. Due to the effects of WWII, resources and financial support in Japan at this time were scarce while industrial reconstruction was a priority. Dr. Taguchi used his background in engineering, statistics, and advanced mathematics to create an effective experimental tool for greatly reducing the size of experiments. The orthogonal array was developed to determine the influence of each

variable being studied on both the mean result and variation from that result. This efficient experimental technique was so successful that it was later implemented in many Japanese and American companies, such as AT&T and Ford Motor Company [12].

In order to most efficiently conduct and receive results from the ABAQUS simulations, the experimental process will be organized using orthogonal arrays.

Orthogonal arrays can be used to:

- Find and predict the optimal configuration of parameters
- Determine which parameters that have the greatest effect on the outputs
- Discover non-linear effects of parameter changes
- Discover any interactions between parameters
- Perform all of the above with the least amount of experimental runs

This chapter will describe how orthogonal arrays can be used to achieve these goals.

4.2 Orthogonal Array Theory

The first step of using orthogonal arrays in the experimental process is selecting the size and type array for the study. Arrays are named by the amount of experiments needed to complete the array. For example, an L_9 array requires 9 experiments to complete the study. The size and format of the arrays are found in orthogonal array resource books and usually range in size from an L_4 (four experiments) to an L_{81} (81 experiments) [13]. Determining the correct array for the study is based on the number of variables in the study and the number of factor levels for each variable. The number of factor levels is the number of values at which a design variable will be evaluated. A 2-

level factor will be evaluated at a low and a high value, while a 3-level factor will be evaluated at a low, medium, and high value. 2-level arrays can only detect linear result trends while 3-level arrays can detect non-linear result trends. An example of a small array is given in Table 4-1. This is an L_4 array with three variables that have two factor levels each (Low and High). The columns of the orthogonal array are pairwise orthogonal, i.e., for every pair of columns, all combinations of factor levels occur an equal number of times. The columns of the orthogonal array represent variables to be studied and the rows represent individual experiments.

Table 4-1: L_4 Array

<i>Experiment Number</i>	<i>Variable 1</i>	<i>Variable 2</i>	<i>Variable 3</i>
1	L	L	L
2	L	H	H
3	H	L	H
4	H	H	L

Using the L_4 orthogonal array, only 4 experiments are used to predict the effects of each variable on the output result. A complete analysis of all combinations of 3 design variables and 2 levels would require $2^3 = 8$ experiments. The effects of the variables are determined by computing averages. The estimates of the variable effects are then used to determine the optimal values which minimize or maximize the output result. The effect of a variable level is the deviation it causes from the overall mean response. If the effect of a variable is dependent on the level of another variable, then the two variables are said to have an interaction. One of the keys to conducting a successful orthogonal array study is the understanding of potential interactions between variables. The amount and degree

of the variable interactions will determine the quality and reliability of the results from the study.

In order to understand the interactions in the orthogonal array method, it will be compared to the classical experimental method. In the classical experimental method, only one variable is varied at a time from a reference level. Testing three variables at two levels each using the classical method and the reference based on a Low-level, would result in the experimental procedure shown in Table 4-2. This table shows a base case of all Low values, followed by each variable tested at a high level while the others remain constant.

Table 4-2: Classical Experiment Method

<i>Experiment Number</i>	<i>Variable 1</i>	<i>Variable 2</i>	<i>Variable 3</i>
1	L	L	L
2	H	L	L
3	L	H	L
4	L	L	H

To find the results of changing variable 1, the results of experiment 2 are compared with the base case. The results of changing variable 2 and 3 can be found in a similar manner. However, this procedure does not indicate what happens when two or three variables are changed at a time. The result of having two or three variables at a high level cannot be predicted by the classical experimental method, because that method does not consider interactions. The three types of interactions that are possible are, No interaction, Synergistic Interaction, and Anti-synergistic Interaction. The following

Figures show examples using short and long spoke length and thin and thick spoke thickness to illustrate potential interactions between variables.

Figure 4-1: No Interaction

Figure 4-2: Synergistic Interaction

Figure 4-3: Anti-synergistic Interaction

Figure 4-1, Figure 4-2, and Figure 4-3, show possible interactions between two variables, spoke length and spoke thickness, with two factor levels each. The amplitude values are for illustrative purposes only and do not represent actual values. Figure 4-1 shows the ideal case of no interaction, as indicated by the equivalent slopes of the two lines (parallel lines). If there are no interactions in an array, it is called an additive model. In an additive model, the change in results from increasing variable 1 and 2 is equal to the results of changing variable 1 plus the results of changing variable 2.

Figure 4-2 shows a Synergistic Interaction. In this case, the lines on the plot are not parallel, but an increase in spoke length and spoke thickness will still result in the highest amplitude. While these types of interactions do not yield additive models, they can still indicate what levels the variables should be set at to yield a maximum amplitude result.

An Anti-synergistic Interaction is shown in Figure 4-3. In this interaction, increasing spoke thickness and length separately increase the amplitude, but increasing both together reduces the amplitude, instead of obtaining maximum amplitude as would be the case for no-interaction. With an Anti-synergistic Interaction, it is difficult to predict how changing variables will affect the system and the Orthogonal Arrays will not predict the absolute optimal value. In this case, additional analysis and experimentation are needed.

The orthogonal array is arranged assuming that all interactions are additive. If this is true and there are no interactions, optimal factor levels can be accurately determined and optimal results can be predicted. If there are Synergistic interactions in the model, optimal levels can still be determined, but the predicted results may not be as accurate. Strong and large numbers of Anti-synergistic interactions can cause optimal levels identified by the model to be inaccurate or even misleading. If two variables are expected to have a strong interaction, the orthogonal arrays can be modified to expose the interaction between one or more variables.

4.3 Obtaining Results from an Orthogonal Array

As mentioned earlier, nonlinear behavior among the parameters can be determined only if more than two levels of parameters are used. For example the L_9 orthogonal array uses 3-levels (low, medium and high) to allow for potential nonlinear behavior in 4 variables. An example L_9 array is shown in Table 4-3 to aid in the discussion of obtaining results from an orthogonal array. This array investigates four 3-

level variables (V1, V2, V3, and V4) and the L, M, and H represent the low, medium, and high levels of each of the variables. In this example study, the first experiment is run with the lowest value of each parameter. The second experiment is conducted with the medium value of parameters V2, V3, and V4 and the lowest value of parameter V1, and so on. The values in the Amplitude Results column are output measures assigned arbitrary numbers for illustration of the method.

Table 4-3: Example L₉ Array

Experiment Number	V1	V2	V3	V4	Amplitude Results
1	L	L	L	L	200
2	L	M	M	M	100
3	L	H	H	H	300
4	M	L	M	H	250
5	M	M	H	L	450
6	M	H	L	M	650
7	H	L	H	M	450
8	H	M	L	H	650
9	H	H	M	L	700

A complete analysis of all combinations for 4 variables and 3 levels would require $3^4 = 81$ total experiments. Using the L₉ array, only 9 experiments are used to predict the effect of the variables on the amplitude result. Experiments 1, 2, and 3 all keep V1 at the low level. In the same three experiments, V2, V3, and V4 are each represented at each of the three levels exactly once. In experiments 4, 5, and 6, V1 is kept constant at a medium level. Once again V1, V2, and V3 are each represented at each of the three levels exactly once. The same is true with the last three experiments with V1 at a high level. Because

of this characteristic of orthogonal arrays, it is possible to quantify the effects of changing V1 on the overall system.

Since all three levels of every factor are equally represented, a balanced overall mean for the entire experimental region can be found by computing the average over the 9 experiments in the orthogonal array:

$$m = \frac{1}{9} \sum_{i=1}^9 \eta_i \quad (1)$$

In the above, m is the overall mean and η_i is the numerical result for each experiment. For this example, η is the amplitude result.

To determine the effect of a variable factor level, the mean equation can be modified to include the experiments corresponding to this variable factor. For example, to determine the effect of V1 it is necessary to find the deviation of V1 at each level from the overall mean. From Table 4-3, it can be seen that V1 is low for the first three experiments, medium for the next three experiments, and high for the last three experiments. Therefore, the means for each level of V1 would be found using equation 2-4.

$$m(V1_L) = \frac{1}{3}(\eta_1 + \eta_2 + \eta_3) \quad (2)$$

$$m(V1_M) = \frac{1}{3}(\eta_4 + \eta_5 + \eta_6) \quad (3)$$

$$m(V1_H) = \frac{1}{3}(\eta_7 + \eta_8 + \eta_9) \quad (4)$$

where $m(V1_L)$, $m(V1_M)$, and $m(V1_H)$ are the means of the three levels of V1. Variables V2, V3, and V4 follow the same pattern with different combinations of experiments. The means of each level for each variable, according to the results listed in Table 4-3 , can be seen in Table 4-4.

Table 4-4: Mean of Each Variable Level

Variable	Low	Medium	High
V1	200	450	600
V2	300	400	550
V3	500	350	400
V4	450	400	400
Total Mean:	416.67		

These individual means can be used to find the effect each variable has on the overall result by finding the deviation of each variable level from the overall mean. The effect of V1 at the low level can be found by the deviation $v1_L = m(V1_L) - m$. The effects of the other two levels can be found in the same way. The deviations of each variable level from the overall mean can be seen in Table 4-5. The variables that cause the greatest deviation from the mean have the strongest effect on the results and are therefore the most influential to the optimization of the design.

Table 4-5: Deviations from Overall Mean (Effects of Variable Levels)

Variable	Low	Medium	High
V1	-216.67	33.33	183.33
V2	-116.67	-16.67	133.33
V3	83.33	-66.67	-16.67
V4	33.33	-16.67	-16.67

From Table 4-5 it can be seen that V1 has the greatest effect on the results with a deviation range of -216.67 to 183.33. V2 has a smaller, but still significant influence on the results. Parameters V3 and V4 have significantly smaller deviation ranges than those of parameters V1 and V2.

To find the optimal level for each variable, the variable level effects from Table 4-5 are used. Assuming no interaction, and assuming an additive model, a predicted optimal combination is determined using the maximum or minimum deviation, depending on the goal, of each variable level effect. The results of the optimized case can be predicted using the following equation:

$$\eta(V1_i, V2_j, V3_k, V4_l) = m + v1_i + v2_j + v3_k + v4_l \quad (5)$$

where $v1_i$ is the deviation caused by setting variable V1 at level $V1_i$, $v2_j$ is the deviation caused by setting variable V2 at level $V2_j$, and so forth. The levels used should be the ones that yielded the maximum (or minimum) optimal value for each parameter. For example, if minimization is the goal, the predicted optimal variable levels for this data set would be $V1_L$, $V2_L$, $V3_M$, and $V4_M$ because they returned the lowest deviations. In order to discover the level of possible interactions it is necessary run a confirmation experiment with each variable at its optimal level ($V1_L$, $V2_L$, $V3_M$, and $V4_M$) and observe how well the results match the predicted optimal result obtained from equation (5). If the confirmation experimental results are the same as predicted by the additive model, there are no interactions and the true optimum has been found. If the two are not the same, there are interactions. These interactions can be either synergistic or anti-synergistic. If

the interactions are synergistic, the optimized case will still return the optimized results, just not the same results predicted by the additive model and equation (5). If the interactions are anti-synergistic, then the predicted optimal variable combination from orthogonal array and additive model may not be reliable. In this case, an expanded number of experiments are needed to determine a reliable optimal. As mentioned earlier, if two variables are expected to have a strong interaction, the orthogonal arrays can be expanded and modified to expose the interaction between one or more variables.

A summary of the key steps in analyzing data obtained from the Orthogonal Array experiments are: (1) Compute the response measure for each experiment, (2) Compute the effects of the variables using deviations from the mean, (3) Evaluate the relative importance of the variables, (4) Determine the predicted optimal level for each variable and predict the optimum combination, (5) Compare the results of the confirmation experiment with the prediction. If the results match the prediction, then the optimum conditions are considered confirmed; otherwise, additional analysis and experimentation are needed. If the predicted response under the predicted optimum does not match the observed response, then it implies that the interactions are important.

CHAPTER FIVE

L₈ ORTHOGONAL ARRAY STUDY

The six geometric variables hypothesized to have the greatest effect on Tweel™ vibration and the range of dimensions considered is shown in Table 5-1. Figure 1-1 is a diagram showing these dimensions. Three-levels for each of the six variables are considered: low, medium, and high. The middle values used are the base reference values for the BMW Mini Tweel™ model described earlier in Chapter 3. The low and high values of the parameters are 75% and 125% of the base values.

Figure 5-1: Diagram of Geometric Variables in Study

Table 5-1: Geometric Variables showing Low, Medium, and High Levels (Dimensions in mm)

	Middle (Base)	Low (75%)	High (125%)
Spoke Thickness (Ts)	4.2	3.15	5.25
Spoke Length ($0.5*Do - 0.5*Dh - Tr$)	72	54	90
Spoke Curvature (C)	8	6	10
Shear Beam Thickness ($Tr - Toc - Tic$)	10	7.5	12.5
Inside Coverage (Tic)	2.5	1.875	3.125
Outside Coverage (Toc)	7	5.25	8.75

In the first orthogonal array study; only the low and high levels will be considered. With 6 design variables and 2 levels, the smallest array which can accommodate this number of degrees-of-freedom is the L_8 array. The L_8 array experiment accommodates seven 2-level variables. Since only six variables will be analyzed, the third column will be left open in order to estimate the interaction between spoke thickness and spoke length. There are specific columns of the orthogonal arrays that can have an interaction attributed to them instead of a variable. Which columns can be left open to analyze interactions can be found in a book of orthogonal array tables [13]. The L_8 array used in this study is shown in Table 5-2.

Table 5-2: L₈ Array for Tweel study (Dimensions in mm)

Experiment Number	Spoke Thickness	Spoke Length	Spoke Length vs. Spoke Thickness	Spoke Curvature	Shear Beam Thickness	Inside Coverage	Outside Coverage
1	3.15	54		6	7.5	1.875	5.25
2	3.15	54		10	12.5	3.125	8.75
3	3.15	90		6	7.5	3.125	8.75
4	3.15	90		10	12.5	1.875	5.25
5	5.25	54		6	12.5	1.875	8.75
6	5.25	54		10	7.5	3.125	5.25
7	5.25	90		6	12.5	3.125	5.25
8	5.25	90		10	7.5	1.875	8.75

The following Sections discuss the results of the L₈ orthogonal array study that evaluates the six geometric variables at a high and low level each, and based on the vibration response measures described in Chapter 3: spoke vibration amplitudes, both RMS and peak amplitudes, frequencies of peak amplitudes, ring vibration amplitudes, and ground reaction force RMS amplitudes. Results include relative importance of each variable, the direction of improvement for each variable, and predictions on optimal variable combinations. Confirmation experiments are performed to assess the reliability of the predicted optimal values and a detailed analysis of possible interactions between spoke thickness and length is performed. Using the L₈ array, the effect of variable levels on mass and vertical stiffness properties is also evaluated. The stiffness is estimated from the mean value of the ground reaction force.

5.1 Amplitude of Spoke Vibration

As explained in Section 3-1, both the RMS and the peak amplitude of the FFT spectrum in the range 200 to 1000 Hz will be used to quantify the amplitude of vibration. Because no test data currently exists on the noise produced by the Tweel™, it is difficult to determine which of these response measures, RMS or peak amplitude, are most directly related to noise. The RMS data will be discussed first and the results for each of the eight experiments can be seen in Table 5-3. The upper quarter, middle, and lower quarter spoke marker nodes defined in Chapter 3 result in slightly different RMS values. In the present study, the maximum RMS of the three marker nodes is used for analysis.

Table 5-3: Response Measures for L₈ Spoke Vibration Experiments

Experiment	RMS Amplitude	Max Peak Amplitude	Frequency at Peak Amplitude (Hz)
1	24.7	150	350
2	10.1	50	495
3	145.4	1250	260
4	146	1050	210
5	9.6	65	350
6	6.1	42	955
7	208.1	1850	270
8	45.26	380	500

The effects of each variable level were found using the deviation from the overall mean as explained in Chapter 4. These results were then normalized by dividing each deviation by the overall mean. The resulting values will be referred to as the normalized deviations and will be used for comparing vibration results.

Table 5-4: RMS Amplitude Normalized Deviations for L_8 Spoke Vibration

Variables	Effect of Low Level	Effect of High Level
Spoke Length	-0.61	0.61
Spoke Curvature	0.20	-0.20
Outside Coverage	0.19	-0.19
Inside Coverage	-0.16	0.16
Shear Beam Thickness	-0.14	0.14
Spoke Thickness	0.07	-0.07

The variables are listed in order of greatest effect on the amplitude of vibration to least effect on amplitude of vibration. The level of each variable (low or high) that gives the smallest amplitude is indicated by the negative values and the yellow boxes. The deviations in the table shows that spoke length is predicted to, by far, have the greatest effect on the amplitude of vibration. A shorter spoke is predicted to reduce vibration significantly. This result is expected because a long spoke is less stiff in bending and can more easily deform under excitation. From structural mechanics, the bending stiffness of the spokes is proportional to the length cubed. Thus changing the length has a strong effect on the response. Interestingly, the spoke thickness is predicted to have the least effect on the amplitude. Table 5-4 shows that increasing the thickness of the spoke does reduce amplitude; however, the influence of this change is almost negligible. This result

could be important if a Tweel™ designer is concerned with weight reduction; thin spokes will have significantly less mass and little influence on amplitude of vibration.

Spoke curvature is predicted to be an influential variable on spoke amplitude. Larger spoke curvature is predicted to have smaller vibration amplitudes. Figure 5-2 and Figure 5-3 show the perpendicular distance results from two different experiments in the L_8 array. Both experiments have the same spoke length (the most influential variable), but the spokes in Experiment 1 have a small curvature while the spokes in experiment 6 have a large curvature. For the spokes with the large curvature in Figure 5-3, the middle node stays the farthest from the plane during the entire rolling process. For the smaller curvature spoke in Figure 5-2, the middle node does not stay the farthest from the plane during the entire rolling process. When the spokes are in tension and not in the contact patch, the middle node vibrates closer to the spoke plane than the upper and lower quarter nodes. This changes the overall shape of the spoke and shows increased oscillations. The RMS amplitude results from Experiment 1 are four times higher than the results from Experiment 6.

Figure 5-2: Perpendicular Spoke Distance of Experiment 1, Small Curvature

Figure 5-3: Perpendicular Spoke Distance of Experiment 6, Large Curvature

The normalized deviations of the three ring variables, shear beam thickness and outer/inner coverage all have nearly the same predicted effect on spoke vibration amplitude. The results indicate that increasing outside coverage thickness and reducing shear beam thickness produce less spoke vibration. This result was not expected, and requires investigation with further experiments. Increase in inside and outside coverage increases the bending stiffness in the ring. Note that this Tweel™ model does not include a tread, the addition of which could influence the effect of the outer coverage. In the present model, one or two elements are present through the thickness of the outer coverage. In future work, a finer mesh should be considered to confirm grid independence.

Table 5-3 shows that the geometry used in Experiment 6 results in the lowest amplitude of vibration for all of the L_8 geometries with a RMS response value of 6.1. Experiment 7 results in the highest RMS amplitude of 208.1. Experiment 6 has all variables except for the inner and outer coverages at the ideal levels as predicted by Table 5-3. The perpendicular distance from the spoke plane for Experiment 6 is shown in Figure 5-3; the FFT of the middle node is shown in Figure 5-4.

Figure 5- 4: FFT with Hamming Window for Spoke Vibration of the Middle Node, Experiment 6

As discussed earlier in Chapter 3, the frequency at which the spoke collapses is 20 Hz, with attenuated harmonics up to about 200 Hz and is not considered a significant source of noise. The highest peak in the frequency range after 200 Hz has maximum amplitude of 42. Figure 5-5 shows the perpendicular distance for the spoke marker nodes during rolling for Experiment 7.

Figure 5-5: Perpendicular Spoke Distance of Experiment 7

Experiment 7 shows a much higher spoke oscillations than Experiment 6. Not only is the displacement signal in Figure 5-5 much noisier, it also shows spoke displacement in both the negative and positive directions indicating that the spokes in this experiment are unstable and vibrating violently. Figure 5-6 shows the deformation of the spokes while they are in tension at the top of the Tweel™. For Experiment 7, the large vibration in the spokes is very pronounced and waves are visible especially in the middle pair of spokes. For Experiment 6, the spokes have almost completely maintained their shape while in tension with slight elongation and small vibration. The vibrating shapes of the spokes while in tension are related to the frequencies of the amplitude peaks in the FFT spectrum.

Figure 5-6: Spokes in Tension during Rolling Step.

(Top) Experiment 7, (Bottom) Experiment 6

The FFT spectrum for the middle marker node for Experiment 7 is shown Figure 5-7. For Experiment 7, a very large amplitude peak of 1850 occurs at 270 Hz. This maximum amplitude is over 300 times larger than the amplitude peak from Experiment 7.

Figure 5-7: FFT with Hamming Window for Spoke Vibration of Middle Node, Experiment 7

With each variable at its ideal level, as predicted by Table 5-3, an additional Tweel™ model was created for a confirmation experiment. If there are weak or no interactions between the variables, this additional model should result in the lowest amplitude of vibration among all the experiments. The geometry of this model is shown in Figure 5-8 with the FFT spectrum of the middle node in Figure 5-9. The max RMS value of 5.47 was smaller than any of the experiments in the Orthogonal Array, and confirms that the predicted ideal variable levels are reliable for spoke vibration reduction.

	Size (mm)
Spoke Length	54
Spoke Curvature	10
Inside Coverage	1.875
Outside Coverage	8.75
Shear Layer Thickness	7.5
Spoke Thickness	5.25

Figure 5-8: Tweel™ with Recommended Variable Levels for Reduction in Spoke Vibration Amplitude

Figure 5-9: FFT with Hamming Window for Spoke Vibration of Middle Node, Recommended Variable Levels

The third column in the L_8 array used in Table 5-2 was left blank in order to expose any interaction between spoke length and spoke thickness. Estimating this interaction can be done by investigating the non-parallelism of the curves in Figure 5-10. The data points on this plot were created by averaging the results of the experiments where thickness and length were both at the low level, thus creating the Short Spoke and Thick Spoke data point. The Long Spoke and Thick Spoke data point was created by averaging both variables at a high level, and the other two data points are the corresponding averages with combinations of the variables. From the plot it is clear that the lines are nearly parallel indicating that the two variables have little interaction.

Figure 5-10: Spoke Length and Spoke Thickness RMS Amplitude Interaction

The results for the max peak amplitudes of each experiment in Table 5-2 are also evaluated. The maximum peak values are the maximum values over all three marker nodes of investigation. The normalized deviations for each variable level showing the predicted effects on max peak amplitude are shown in

Table 5-5. The relative importance and predicted optimal variable levels for the max peak values are the same levels recommended from the RMS response measure. The main difference in the normalized deviations is the increased influence of all of the variables except spoke thickness. Also, in this array, the inner coverage is predicted to be slightly more influential than the outer coverage which is the opposite of the RMS case. The parameters which have the most relevance to the structural mechanics in order of importance are spoke vibration are spoke length, spoke curvature, and shear beam thickness.

Table 5-5: Max Peak Amplitude Normalized Deviations for L₈ Spoke Vibration

Variable	Effect of Low Level	Effect of High Level
Spoke Length	-0.87	0.87
Spoke Curvature	0.40	-0.40
Inside Coverage	-0.34	0.34
Outside Coverage	0.30	-0.30
Shear Beam Thickness	-0.27	0.27
Spoke Thickness	0.05	-0.05

5.2 Frequency of Spoke Vibration

Table 5-2 also shows the frequencies that correspond to the max peak amplitudes in the FFT spectrum across all spoke marker nodes. These frequencies may control the pitch of the sound from the vibrating spokes. The max amplitude frequencies for the 8 experiments range from 210 Hz to 955 Hz, showing that a wide variance in frequency can be achieved by different combinations of geometric variable levels in the Tweel™. Frequencies at other amplitude peaks which are not the maximum also shift, but not over such a large range.

Table 5-6 shows the normalized deviations of the variable levels in order of most influential to least influential on shifting the frequency corresponding to max peak amplitude.

Table 5-6: Peak Frequency Normalized Deviations for L₈ Spoke Vibration

Variable	Effect of Low Level	Effect of High Level
Spoke Curvature	-0.274	0.274
Spoke Length	0.268	-0.268
Spoke Thickness	-0.224	0.224
Shear Beam Thickness	0.218	-0.218
Inside Coverage	-0.168	0.168
Outside Coverage	0.053	-0.053

The normalized deviations indicate that spoke curvature, spoke length, spoke thickness, and shear beam thickness all have large effect on the frequency corresponding to max peak amplitude. The combination which is predicted to have the highest frequency at max peak amplitude is: high spoke curvature, low spoke length, high spoke thickness, and low shear beam thickness. The combination with the lowest frequency is just the opposite.

To illustrate the frequency behavior in spoke vibrations, results for Experiment 7 are examined in further detail since the amplitudes in this case are the largest and most clearly visible. The FFT spectrum for the spoke marker nodes at the upper quarter node and middle node are shown in Figure 5-11 and Figure 5-12. The FFT spectrum for the

lower quarter node is similar to the upper quarter node. Both the upper and lower quarter nodes have an amplitude peak at 270 Hz. However, the upper quarter node has an additional peak at 570 Hz. The middle node amplitude at 270 Hz is higher than the upper quarter node amplitude peak at 270 Hz. The physical motion of the spoke that corresponds to these peaks can be seen in Figure 5-13.

Figure 5-11: FFT with Hamming Window for Spoke Vibration of Upper Quarter Node, Experiment 7

Figure 5-12: FFT with Hamming Window for Spoke Vibration of Middle Node, Experiment 7

Figure 5-13: 270 Hz Vibration of Spokes while in Tension

Figure 5-13 shows three snapshots in time, collected from an ABAQUS animation, of the spoke with marker nodes as it travels in tension around the bottom quarter of the ring. The times that the snapshots were taken are displayed underneath the pictures. From the upper left picture to the lower middle picture is half the cycle. The nodes travel back to the starting point beneath the plane at 0.1706 seconds. The period of this motion corresponds to 270 Hz. Notice that the middle node displaces farther than the two quarter nodes, corresponding to the higher peak amplitude in the FFT spectrum. The two quarter nodes are displaced roughly the same distance from the plane. Ring

vibrations showing flower-pedal modes are also visible in Figure 5-13. Recall that these results are from Experiment 7 which also shows large spoke vibrations. The motion corresponding to the second peak amplitude in the FFT spectrum for the middle and upper quarter nodes at 570 Hz is described in Figure 5-14.

Figure 5-14: 570 Hz Vibration of Spokes While in Tension

These two screen shots show the motion with period that appears to correspond to the 570 Hz vibration. Timing this cycle is difficult due to the higher frequency and more complex motion, but from the screen shots it can be estimated between 500 and 600 Hz. This vibration does not cause as great a displacement as the lower frequency vibration, which explains the fact that the higher frequency has a smaller amplitude peak. The middle node does not displace at this frequency, which is why that peak is absent from the FFT spectrum of the middle node. It is interesting to note that both spokes in the pair do not show the same wave profile at the same time. This pattern of a single high peak in

the middle node FFT spectrum with two peaks in the quarter node FFT spectrums occurs for almost all the experiments, although the peaks are not well defined in the very low amplitude cases. The estimated interaction between spoke thickness and spoke length with regards to peak frequency is shown in Figure 5-15. Like the amplitude study, there is very little interaction between the two variables.

Figure 5-15: Spoke Length and Spoke Thickness Frequency Interaction

Because the sound emitted by the Tweel™ is function of both amplitude and frequency it would be incorrect to only consider them separately. For example, a low frequency vibration with high amplitude might be more desirable than a high frequency vibration with slightly lower amplitude, since human response to noise is more acute at 1000 Hz compared to 100 Hz. Though comparison of the frequency and amplitude variable level deviations, it was discovered that for four variables: spoke thickness, spoke

curvature, spoke length, and shear layer thickness, the levels that decrease amplitude will increase frequency of amplitudes peaks. While the frequency change is small compared to the amplitude change, these effects should still be investigated. One possible area for future analysis would be to consider an A-weighted filter for FFT or Power Spectral Density results which accounts for human response to noise, and reduces amplitude levels gradually from 1000 Hz.

MATLAB has a sound function that can convert the displacement signal into sound. This function was used to compare the different FFTs in terms of loudness to the human ear. The quarter node sound was compared to the middle node for each experiment. In each case the middle node created a lower sound than the quarter nodes, which indicates that the second peak in the quarter nodes is influencing the sound. The middle node also produced an audibly louder sound in most experiments; while in other experiments the amplitudes were too close to tell.

The sound output was also compared between experiments. The two experiments with the highest amplitudes, Experiments 3 and 7, do sound the loudest. Experiment 7 is extremely loud in comparison to the others. The experiments with the lowest amplitudes, Experiments 5 and 6, had the softest sound. The results from listening to the signals helps to justify not including the amplitude peaks under 200 Hz in the present analysis of vibration response measures in the present work.

5.3 Ring Vibration Amplitude

Like the spoke amplitude, both the RMS and maximum peak from the FFT spectrum will be used to quantify ring vibration amplitude. These results will be taken over the frequency range of 250 Hz to 1500 Hz at points both inside a spoke pair and outside a spoke pair (see Figure 3-7). As described in Chapter 3, the ring vibration is measured by the distance between points on the top and bottom of the ring. The marker nodes for monitoring ring vibration are attached to the inside and outside coverage's. The RMS and max peak amplitude response measures for the L₈ ring vibration experiments are given in Figure 5-8. Also shown is an additional experiment based on the recommended variables which optimized reduced spoke vibration. The variable level normalized deviations for the RMS inside a spoke pair and outside a spoke pair are shown in Figure 5-9.

Table 5- 7: Response Measures for L₈ Ring Vibration Experiments

Experiment	RMS Amplitude Inside	RMS Amplitude Outside	Max Value Inside	Max Value Outside
1	0.0956	0.1156	0.6984	0.6132
2	0.1966	0.212	0.7421	0.8012
3	0.6952	0.6563	2.2779	2.563
4	0.2677	0.2918	1.3676	1.2166
5	0.2477	0.265	1.7267	1.9561
6	0.0709	0.091	0.3109	0.574
7	0.4677	0.5571	2.3062	2.0818
8	0.2755	0.2598	1.3585	1.6832

Opt Spoke	0.1205	0.1113	0.7232	0.6976
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Table 5-8: Ring Vibration Normalized Deviations for RMS Amplitude, Inside and Outside

Inside Spoke Pair			Outside Spoke Pair		
Variables	Effect of Low Level	Effect of High Level	Variables	Effect of Low Level	Effect of High Level
Spoke Length	-0.473	0.473	Spoke Length	-0.442	0.442
Spoke Curvature	0.300	-0.300	Spoke Curvature	0.302	-0.302
Inside Coverage	-0.235	0.235	Inside Coverage	-0.239	0.239
Outside Coverage	-0.221	0.221	Outside Coverage	-0.138	0.138
Spoke Thickness	0.083	-0.083	Shear Beam Thickness	-0.083	0.083
Shear Beam Thickness	-0.018	0.018	Spoke Thickness	0.042	-0.042

The variable level deviations for the inside and outside points of measurement are nearly identical. The four most influential variables are in the same order between the inside and the outside. The results indicate that spoke length and curvature are the two most influential parameters on ring vibration, as they were in the spoke amplitude study. Also, like the spoke amplitude study, the variable deviations recommend a short spoke with a large curvature to decrease amplitude of vibration. Small inside and outside ring coverages are recommended as optimal for reducing ring vibrations. Results of the experiments show that the least influential parameters are the shear beam thickness and

spoke thickness. The low influence of shear beam thickness on ring vibration was not expected and will be investigated further in the next chapter.

Figure 5-16 and Figure 5-17 show the displacement signals of the experiments with the highest and lowest amplitudes of ring vibration, respectively. Figure 5-18 is from Experiment 3, while Figure 5-19 is taken from Experiment 6, both inside the spoke pair. Comparisons show the signal from Experiment 3 is visibly noisy compared to Experiment 6.

Figure 5-16: Ring thickness for Experiment 3

Figure 5-17: Ring thickness for Experiment 6

Figure 5-18 and Figure 5- 19 show the FFT spectrum for Experiment 3 and Experiment 6, respectively. The RMS amplitude for Experiment 3 is 0.695 with peak amplitude of 2.278. The RMS and peak amplitudes for Experiment 6 are much lower; RMS = 0.311, peak amplitude = 0.071.

Figure 5-18: FFT with Hamming Window of Experiment 3, Inside Spoke Pair

Figure 5-19: FFT with Hamming Window of Experiment 6, Inside Spoke Pair

Figure 5-18 shows that the ring thickness FFT spectrums do not always have defined frequency peaks and have more of a broadband frequency character. This makes it difficult to identify dominant frequencies of the signal. The maximum value of the FFT spectrum in the range 250 Hz to 1500 Hz for each experiment is shown in Table 5-7. Figure 5-8 displays the normalized variations determined using the maximum amplitude values over the frequency range from 250 Hz to 1500 Hz.

Table 5-9: Ring Vibration Normalized Deviations for Maximum Amplitude, Inside and Outside

Inside Spoke Pair			Outside Spoke Pair		
Variables	Effect of Low Level	Effect of High Level	Variables	Effect of Low Level	Effect of High Level
Spoke Length	-0.355	0.355	Spoke Length	-0.313	0.313
Spoke Curvature	0.299	-0.299	Spoke Curvature	0.256	-0.256
Outside Coverage	-0.132	0.132	Outside Coverage	-0.219	0.219
Shear Beam Thickness	-0.139	0.139	Spoke Thickness	-0.096	0.096
Spoke Thickness	-0.057	0.057	Shear Beam Thickness	-0.054	0.054
Inside Coverage	-0.045	0.045	Inside Coverage	-0.048	0.048

Once again the inside and outside measurement positions give nearly the same results between themselves with the only difference being that the spoke thickness and shear beam thickness have switched in order of influence between the two. With the max value measure, inside coverage has dropped to the least important variable and the shear beam thickness is now recommended at the lower level. These two changes are the only

differences between the RMS method and the max value response measures. Both sets of experimental results verify that there is no extra vibration occurring outside of the spoke pair that does not occur inside the spoke pair.

Since there are no defined frequency peaks for some of the experiments, a full orthogonal array study cannot be conducted for the frequency of the peaks. However, there are interesting patterns in the FFT spectrums that can be examined. The first pattern is the signals that show well defined peaks, which can be seen in Figure 5- 19 and Figure 5-20. These signals come from Experiments 5 and 6; both of which have thick and short spokes. In contrast, Experiments 3 and 7 have the least defined frequency peaks and more of a broadband frequency response. These two experiments also have the two highest RMS and max amplitude values.

Figure 5-20: FFT with Hamming Window of Experiment 5, Outside Spoke Pair

Figure 5-21: FFT with Hamming Window of Experiment 7, Outside Spoke Pair

Experiment 2 has an interesting broadband frequency FFT spectrum which can be seen in Figure 5-22.

Figure 5-22: FFT with Hamming Window of Experiment 2, Inside Spoke Pair

An alternative response measure of ring vibration is obtained by tracking the radial distance between the hub center and points on the inside and outside of the ring. Figure 5-23 through Figure 5-26 shows the radial distance between the hub center and the marker node on the inside of the ring between spoke pairs for Experiment 3 and Experiment 6 together with their corresponding FFT spectrums. Results on the outside of the ring are very similar. Comparisons of results between the experiments confirm that Experiment 3 has much higher vibration than Experiment 6.

Figure 5-23: Experiment 3

Figure 5- 24: FFT Spectrum for Experiment 3

Figure 5- 25: Experiment 6

Figure 5-26: FFT Spectrum for Experiment 6

5.4 Ground Interaction Vibration Amplitude

For the ground interaction amplitude study, only the RMS amplitude will be considered. The RMS values for each experiment are listed in Table 5-10. Table 5-11 shows the normalized deviations for each variable level.

Table 5-10: RMS Amplitude Results for L₈ Ground Interaction

Experiment	RMS Amplitude
1	6.95
2	6.26
3	32.17
4	13.47
5	6.91
6	6.24
7	44.09
8	12.36

Table 5-11: RMS Amplitude Normalized Deviations for L₈ Ground Interaction

Variable	Effect of Low Level	Effect of High Level
Spoke Length	-0.590	0.590
Spoke Curvature	0.403	-0.403
Inside Coverage	-0.382	0.382
Outside Coverage	0.102	-0.102
Shear Beam Thickness	-0.101	0.101
Spoke Thickness	-0.084	0.084

From the table of variations it is apparent that there are three variables that are predicted to be heavily influential to ground interaction amplitude, and three that are only slightly influential. Spoke length, spoke curvature and shear beam thickness are also highly influential parameters for spoke vibration, with the same levels recommended to decrease the amplitude of vibration. Experiment 7 has the highest ground interaction vibration, just as it has the highest spoke vibration. Experiment 6 has the lowest amplitude of vibration for both sources of vibration as well. For these reasons, it is suggested that spoke vibration and ground interaction are highly related. Figure 5-27 and Figure 5- 28 show the reaction forces about the zero mean for Experiment 6 and 7, respectively.

Figure 5-27: Zero Mean Signal of Ground Reaction Force, Experiment 6

Figure 5- 28: Zero Mean Signal of Ground Reaction Force, Experiment 7

While both data sets appear very noisy, the peaks in Experiment 6 range between 0.6 to 0.8 daN. In contrast, the peaks in Experiment 7 are much larger and range between 5 to 15 daN. The FFT spectrums for these two signals are shown in Figure 5-29 and Figure 5-30, respectively. While a clear amplitude peak at frequency 477 Hz appears in Figure 5-30 for Experiment 6, the amplitude is still much lower than most of the peaks in Experiment 7, shown in Figure 5-30. Figure 5-30 does not show a clear dominant 477 Hz peak.

Figure 5-29: FFT with Hamming Window of Ground Interaction, Experiment 6

Figure 5-30: FFT with Hamming Window of Ground Interaction, Experiment 7

The lack of dominate frequency peaks in Figure 5-30 demonstrates that not all of the ground vibration signals show the spoke pair passing frequency discussed in Section 3-3. Four of the eight experiments did not show strong peaks at 477 Hz. Three of the experiments that did not clearly show the spoke pair's passing frequency were the cases with the 3 highest amplitudes. This suggests the excessive vibration in these signals masks the 477 Hz peak. Experiment 2 is the one case with low amplitude which does not show a strong peak at 477 Hz; instead it has a peak at 2000 Hz. The FFT spectrum of Experiment 2 is shown in Figure 5-31. Experiment 2 has geometry with all thick ring parameters with a short and thin spoke.

Figure 5-31: FFT with Hamming Window of Ground Interaction, Experiment 2

To take a closer look at what motion is causing the 477 Hz interaction with the ground, screen shots of an ABAQUS animation of Experiment 8 were taken. Experiment 8 was used for this analysis because it showed the highest and best defined peaks at 477 Hz. Figure 5-32, Figure 5-33, and Figure 5-34 contain three screen shots that show how the ring is vibrating with the ground as it passes between spoke pairs. All three screen shots were made with the same window. Figure 5-32 shows a red line that will designate the starting position. This line is following the bottom node on the ring. Notice that the bottom node is currently touching the ground and the red line is intersecting the first spoke in a pair. The shear deformation angle in the shear beam layer is also clearly visible.

Figure 5-32: Screen shot showing ground interaction, 0.2632 sec

Figure 5-33: Screen shot showing ground interaction, 0.2638 sec

Figure 5-34: Screen shot showing ground interaction, 0.2644 sec

After the Tweel™ has rolled for 0.0006 seconds, another screen shot was taken. At this point in the simulation, the bottom node (red dotted line) is no longer in contact with the ground and a ring flower-pedal like mode is forming. The black dotted line marks the starting position of the node being tracked. Now this point is off the ground as

well. The next screen shot was taken after another 0.0006 seconds has passed. This time the ring is back in contact with the ground. This sequence only shows half of the ring mode cycle, as the second spoke in the pair is now where the red line started. At 0.2652 the ring touches the ground again the first spoke in a new pair. This time interval (0.2632 to 0.2652) corresponds to a frequency of 500 Hz which is approximately the same as the 477 Hz peak shown in the FFT spectrum of the ground interaction. Recall that the spoke pair passing frequency, or frequency that the Tweel™ passes between each pair of spokes is $f = (120 \text{ rad/sec}) / (2 \pi / 25 \text{ spoke pairs}) = 477 \text{ Hz}$. These results indicate that the contact area is changing with the spoke passing frequency due to the discrete changes in ring stiffness due to the attached spokes. Due to the discrete stiffness of the spokes supporting the ring, the contact area increases and decreases at frequencies corresponding to the spoke pair passing frequency of 477 Hz and to spoke-to-spoke passing frequency of 954 Hz.

The MATLAB sound function was used to compare results between experiments. The 477 Hz peak can be heard in the cases with defined FFT peaks. The two cases with the highest amplitudes, Experiment 3 and 7, are very loud and sound more like random noise than any specific peaks. It is hypothesized that large amplitudes and lack of 477 Hz peaks in these cases is due to excessive flower-petal vibration modes occurring in the ring and making contact with the ground. The experiment that sounds the quietest is Experiment 2, which has the low amplitude peaks at the high 2000 Hz frequency.

5.5 Tweel™ Mass

Each time the size of a geometric variable is changed, it results in a change of the mass of the Tweel™. These changes in mass, along with what variables affect mass the most, are of importance to a Tweel™ designer. The masses of each Tweel™ experiment were calculation based on multiplying the mass density with the estimated area of the geometry created by the Python plug-in in Abaqus/CAE. The variable effect levels were determined using the same variation from the mean procedure used throughout this work. Table 5-12 shows the mass of the elastomer for each experiment and

Table 5-13 shows the deviation from the mean for each variable. It is assumed that lower mass is beneficial, so the recommended levels are highlighted in yellow.

Table 5-12: Elastomer Masses of L₉ Experiments

Experiment	Tweel™ Mass (kg / 100 mm)
1	4.34
2	6.18
3	5.79
4	5.84
5	6.89
6	5.53
7	7.38
8	6.86
Mean	6.10

Table 5-13: Effect of Changing Geometric Variables on Elastomer Mass,

Units: kg/100 mm Tweel™ Width

Variable	Effect of Low Level	Effect of High Level
Spoke Thickness	-0.56	0.57
Shear Beam Thickness	-0.47	0.47
Spoke Length	-0.36	0.37
Outside Coverage	-0.33	0.33
Inside Coverage	-0.12	0.12

Because total mass is simply the sum of the masses of all the parts, there are no interactions and the model is additive. Since the model is additive, any combination of parameters can be predicted. For example, the mass of a tire with all low levels would be the mean plus the negative value of each variable. The analysis shows that spoke thickness and shear beam thickness, the parameters that had the least effect on most vibration amplitude response measures, had the greatest effect on mass.

5.6 Tweel™ Stiffness

It is useful to know how the changes in geometry will affect the vertical stiffness of the Tweel™. Vertical stiffness is defined as the vertical load divided by the ground displacement. Recall that a ground displacement of 15 mm is enforced for all Tweel™ geometries, so the load on the Tweel™ is directly related to the vertical stiffness. The

load on the tire is estimated by taking the mean of the ground reaction force. Table 5-14 gives the mean ground reaction force for all experiments. Table 5-15 shows the effect of each variable level on the load as measured by the deviations from the mean. The variable levels that result in lower vertical stiffness are highlighted.

Table 5-14: Stiffness Indication of L₈ Experiments

Experiment	Mean Ground Reaction Force (daN)
1	3.13
2	2.96
3	3.78
4	3.54
5	7.15
6	3.97
7	6.76
8	3.77
Mean	4.38

Table 5-15: Effect of Variable Levels of Stiffness (Units: daN)

Variable	Effect of Low Level	Effect of High Level
Spoke Thickness	-1.031	1.031
Spoke Curvature	0.823	-0.823
Shear Beam Thickness	-0.720	0.720
Spoke Length	-0.080	0.080
Outside Coverage	-0.032	0.032

Inside Coverage	0.016	-0.016
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Spoke thickness, spoke curvature, and shear beam thickness all have substantial influences on the vertical stiffness. The Tweel™ increases stiffness as the spokes get thicker with less curvature. Stiffness also increases with an increase in shear beam thickness. This result is consistent with the equations that define the properties of the shear beam detailed by Rhyne and Cron [1]. Spoke length and inside/outside coverage do not have a significant change on the Tweel mass.

5.7 Optimal Levels

All three noise sources recommend nearly the same levels for each variable, with only a one level change between each one. Table 5-16 shows the recommend level for each variable for each vibration response measure base on the RMS amplitude results. As previously discussed, a model was created with the recommended values to minimize spoke vibration amplitude. This model gave the lowest amplitude (compared to all other experiments) for spoke vibration. The recommended levels for spoke vibration are nearly the same as the recommended levels for ground and ring vibration, and also result in the lowest values for ground interaction (4.95) and close to the lowest values for ring vibration.

Table 5-16: Comparison of Recommended Levels for each Vibration Response Measure

	Spoke Level	Ground Level	Ring Level
Spoke Length	Low	Low	Low
Spoke Curvature	High	High	High
Inside Coverage	Low	Low	Low
Outside Coverage	High	High	Low
Shear Beam Thickness	Low	Low	Low
Spoke Thickness	High	Low	High

In the present study, each geometric variable was varied by high and low levels derived from a 25% deviation from a baseline BMW Mini Tweel. The results show that spoke length change has the greatest influence on vibrations by a large factor for all measures. The spoke curvature also was an important parameter effecting vibration. Spoke thickness was generally the least influential geometric variable effecting vibration, yet has the greatest influence on Tweel mass and vertical stiffness.

The confirmation experiment did result in the lowest vibrations, but the results did not match the predicted results based on an additive model. Therefore, none of the three vibration results form a perfect additive model. As a result, there is some level of interactions between the variables. Using the L_8 array, it was estimated that there was no significant interaction between spoke thicknesses and spoke length.

CHAPTER SIX

L₉ ORTHOGONAL ARRAY STUDY

In order to investigate nonlinear behavior in the design variable levels, an L₉ array is studied using four key geometric variables at a low, medium, and high level each. A secondary goal of the study is to confirm the results obtained from the L₈ array. To conduct this array study it is necessary to decrease the number of variables investigated from six to four. The four variables chosen are spoke length, spoke curvature, spoke thickness, and shear beam thickness. Spoke length and spoke curvature were chosen because they were the two most influential parameters on amplitude of vibration from the L₈ study. Spoke thickness was chosen because changing thickness had shown to have non-linear effects on the results of a previous study [14]. Of the three ring parameters, shear beam was chosen because it is the most crucial to the structural mechanics of the Tweel™. The L₉ array used for this study is shown in Table 6-1.

Table 6-1: L₉ Array for Tweel (Dimensions in mm)

Experiment Number	Spoke Thickness	Spoke Length	Spoke Curvature	Shear Beam Thickness
1	3.15	54	6	7.5
2	3.15	72	8	10
3	3.15	90	10	12.5
4	4.2	54	8	12.5
5	4.2	72	10	7.5
6	4.2	90	6	10
7	5.25	54	10	10
8	5.25	72	6	12.5
9	5.25	90	8	7.5

6.1 Spoke Vibration

The spoke vibration amplitude and frequency results are shown in Table 6-2. The normalized deviations for the RMS amplitude comparison for each variable are shown in Table 6-3. The yellow boxes are used to highlight the negative values which are the most influential in reducing vibration amplitude. The variables in Table 6-3 are sorted in order of most influential to least influential; for a three level parameter the magnitude of influence is the range of the greatest positive and negative values. With a three level array, the change between a low to medium variable can be distinguished from the change between a medium and high variable. For example, the change from low to medium spoke length results in a normalized deviation change of 0.3, but the change from the medium level to the high level spoke length results in a normalized deviation change of 1.98. That means the effect of change from a 72 mm spoke to a 90 mm spoke will increase the amplitude much more than the change from a 54 mm spoke to a 72 mm spoke.

Table 6-2: RMS, Peak Amplitude, and Peak Frequency Spoke Vibration Results from L₉

Experiment	RMS Amplitude	Peak Amplitude	Peak Frequency
1	16.14	96	440
2	42.35	375	265
3	104.7	910	215
4	5.16	30	345
5	12.75	78	573
6	154.88	980	268
7	5.79	42	936
8	33.54	235	325
9	222.29	1575	270
Mean	66.40	480.11	

Table 6-3: RMS Amplitude Normalized Deviations for L₉ Spoke Vibration

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.86	-0.56	1.42
Spoke Curvature	0.03	0.35	-0.38
Shear Beam Thickness	0.26	0.02	-0.28
Spoke Thickness	-0.18	-0.13	0.31

The results from the L₉ experiments predict that spoke length and spoke curvature are the two most influential parameters, with spoke length being much more dominant than any other variable. To minimize vibration, a short spoke with a large curvature is recommended. Shear beam thickness and spoke thickness are less influential. A thick shear beam is recommended and a thin spoke is recommended. The deviations from Table 6-3 indicate that spoke length, spoke curvature, and spoke thickness show non-linear behavior when changing levels. Shear beam thickness does not show much non-linearity.

A confirmation experiment was created to using the levels recommended by Table 6-3. The RMS amplitude value of the spoke vibration in this case was 13.22. Comparing this value to Table 6-2, it can be seen that this value is not the lowest amplitude. Because 13.22 is not the lowest amplitude, it is clear that the model is not additive, and there are

interactions that influence the data. A hypothesis is that spoke curvature and length have an interaction that is affecting the results of the shear beam thickness and the spoke thickness. It is hypothesized that the recommended levels for spoke curvature and spoke length are the optimal levels because they are the two most influential parameters and have the same recommended levels shown the L_8 array results. This interaction suggests that Table 6-3 may be recommending non-optimal variable levels for shear beam thickness and spoke thickness. In addition to the model with the recommended levels, two more models were created to investigate the effects of shear beam and spoke thickness. Each model has the shortest spoke length and largest spoke curvature, as recommended by Table 6-3. The shear beam and thickness were adjusted and the results can be seen in Table 6-4. Model 1 is the model with the recommended levels and Model 2 and 3 are variations of that model.

Table 6-4: Shear Beam and Spoke Thickness Study on Spoke Vibration

	Spoke Length	Spoke Curvature	Spoke Thickness	Shear Beam Thickness	Max RMS Amplitude	Peak Amplitude
Model 1	Low	High	Low	High	13.22	72
Model 2	Low	High	High	High	4.96	28
Model 3	Low	High	High	Low	5.85	38

Table 6-4 shows that both shear beam thickness and spoke thickness at a high level gives the lowest amplitude of all combinations investigated. This does not mean that this combination is necessarily the optimal combination, but it does show that there are interactions that are giving incorrect recommended levels of improvement. The other

important result from Table 6-4 is that this finer adjustment of the variables of less influence has little effect on the amplitudes. All three models in Table 6-4 have low amplitudes compared to the mean of values in Table 6-2.

Table 6-5 shows the normalized deviations of the peaks amplitudes for spoke vibration. The difference between this table and the RMS deviation table is that this table recommends a medium spoke length instead of a small spoke length. The recommended levels of this table more closely match the lowest amplitude case listed in Table 6-4.

Table 6-5: Peak Amplitude Normalized Deviations for L_9 Spoke Vibration

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.88	-0.52	1.41
Spoke Curvature	-0.09	0.37	-0.28
Spoke Thickness	-0.04	-0.24	0.29
Shear Beam Thickness	0.21	-0.03	-0.18

The normalized deviations for the frequencies of the peak amplitudes are listed in Table 6-6. The highlighted boxes designate the parameters that will shift the frequencies of the peak amplitudes to lower frequency values. Spoke curvature and shear beam thickness both show strong non-linear effects of change.

Table 6-6: Frequency Normalized Deviations for L₉ Spoke Vibration

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	0.42	-0.04	-0.38
Spoke Curvature	-0.15	-0.27	0.42
Spoke Thickness	-0.24	-0.02	0.26
Shear Beam Thickness	0.06	0.21	-0.27

6.2 Ring Vibration

Results of all nine experiments for RMS and peak amplitudes are listed in Table 6-7. The RMS sensitivity indexes are listed in Table 6-8 and Table 6-9. All of the variables in the deviation tables show non-linear effects of changing factor levels. Both the inside and outside RMS deviation tables have the same order of importance and recommended levels. Both tables also have the same recommended variable levels as the RMS deviation tables for spoke vibration. The difference between ring vibration and spoke vibration is the magnitude of the shear beam deviations.

Table 6-7: RMS and Peak Amplitudes for Ring Vibration Results from L₉

Experiment	Inside RMS Amplitude	Outside RMS Amplitude	Inside Peak Amplitude	Outside Peak Amplitude
1	0.1051	0.1037	0.6069	0.5127
2	0.1409	0.1502	0.5335	0.5629
3	0.233	0.2717	1.0218	1.2651
4	0.1828	0.192	0.9582	0.9583

5	0.0723	0.0785	0.313	0.2852
6	0.679	0.7008	2.2509	2.479
7	0.1074	0.1099	0.5455	0.6877
8	0.2666	0.2711	1.5887	1.4826
9	0.7116	0.7478	2.8186	2.861
Mean	0.278	0.292	1.182	1.233

Table 6-8: RMS Amplitude Normalized Deviations for L₉ Ring Vibration, Inside Spoke

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.525	-0.424	0.949
Spoke Curvature	0.261	0.243	-0.505
Spoke Thickness	-0.425	0.122	0.303
Shear Beam Thickness	0.067	0.113	-0.181

Table 6-9: RMS Amplitude Normalized Deviations for L₉ Ring Vibration, Outside Spoke

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.537	-0.429	0.966
Spoke Curvature	0.229	0.245	-0.474
Spoke Thickness	-0.399	0.110	0.290
Shear Beam Thickness	0.063	0.098	-0.160

Since the levels recommended to minimize ring vibration were the same levels that were recommended to minimize spoke vibration, the same model could be used for

both confirmation experiments. The RMS amplitude value of the ring vibration from the confirmation experiment was 0.1887 inside the spoke pair and 0.1893 for outside the spoke pair. Comparing these values to Table 6-7, it can be seen that this value is not the lowest amplitude. This indicates that the ring vibration data also has interactions. Therefore, the same shear beam and spoke thickness study that was done with spoke vibration was repeated for ring vibration. This study is shown in

Table 6-10. For ring vibration Model 3 with thick spokes and a thin shear beam produces the lowest amplitudes. While all three values are lower than the mean, Model 1 and 2 give nearly the same result. It has yet to be determined how substantial this ring vibration is to the noise produced by the Tweel™. The amplitude of this vibration is on the order of .1 mm, while the spoke vibration can have amplitudes up to 5-10 mm.

Table 6-10: Shear Beam and Spoke Thickness Study on Ring Vibration

	Spoke Length	Spoke Curvature	Spoke Thickness	Shear Beam Thickness	Inside RMS	Outside RMS
Model 1	Low	High	Low	High	0.1887	0.1893
Model 2	Low	High	High	High	0.182	0.1921
Model 3	Low	High	High	Low	0.0858	0.0959

The next two tables show the normalized deviations of the peak amplitude values. There are a couple of small differences between the peak value deviations and the RMS deviations. The first is that spoke thickness becomes more influential than spoke curvature. However, the ranges of the spoke curvature and spoke thickness variations are very close in magnitude, so this is not of great interest. Also the recommended level of

shear beam thickness has changed, but shear beam thickness shows little influence for this response measure, and thus the change is not a significant factor.

Table 6-11: Peak Amplitude Normalized Deviations for L₉ Ring Vibration, Inside Spoke Pair

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.405	-0.313	0.718
Spoke Thickness	-0.390	-0.007	0.397
Spoke Curvature	0.254	0.216	-0.470
Shear Beam Thickness	0.054	-0.061	0.006

Table 6-12: Peak Amplitude Normalized Deviations for L₉ Ring Vibration, Inside Spoke Pair

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.416	-0.370	0.786
Spoke Thickness	-0.367	0.007	0.360
Spoke Curvature	0.210	0.185	-0.395
Shear Beam Thickness	-0.011	0.008	0.002

6.3 Ground Interaction Vibration

Results for RMS ground vibrations are listed in Table 6-13 and the RMS normalized deviations are listed in Table 6-14. The spoke length and spoke curvature deviations show non-linear effects of changing factor levels. Table 6-14 shows the same

recommended levels as the RMS deviation tables for spoke and ring vibration. Also, with the exception of the spoke length, all variables show comparable ranges for deviations from the mean.

Table 6-13: RMS Ground Interaction Vibration Results from L₉

Experiment	RMS Amplitude
1	6.96
2	10.02
3	13.82
4	7.23
5	6.46
6	43.35
7	5.84
8	11.31
9	54.38
Mean	17.708

Table 6-14: RMS Amplitude Normalized Deviations for L₉ Ground Interaction Vibration

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Length	-0.62	-0.48	1.10
Spoke Curvature	0.16	0.35	-0.51
Spoke Thickness	-0.42	0.07	0.35
Shear Beam Thickness	0.28	0.11	-0.39

Since the levels recommended to minimize ring vibration were the same levels recommended to minimize spoke vibration and ring vibration, the same confirmation

experiment can be used again to check interactions. The RMS amplitude value from the confirmation experiment was 9.03. Comparing this value to Table 6-7, it can be seen that this value is not the lowest amplitude. Like the other two sources of vibration, the ground vibration data also has interactions. Additional experiments are again used to further study shear beam and spoke thickness levels ground vibration amplitudes. This study is shown in Table 6-15. For ground vibration Model 3 with thick spokes and a thin shear beam produces the lowest amplitudes. Model 3 also produced the lowest ring vibrations. However, all three results in Table 6-15 are similar, and all three values are low in comparison to the rest of the results in Table 6-13.

Table 6-15: Shear Beam and Spoke Thickness Study on Ground Interaction Vibration

	Spoke Length	Spoke Curvature	Spoke Thickness	Shear Beam Thickness	RMS Amplitude
Model 1	Low	High	Low	High	9.0345
Model 2	Low	High	High	High	7.2944
Model 3	Low	High	High	Low	6.864

6.4 Comparison of L₉ Array and L₈ Array Variable Level Deviations

This section includes a comparison of the normalized deviations between the L₈ array and the L₉ array. For all three sources of vibration, spoke thickness is the most influential variable followed by spoke curvature. Spoke curvature is recommended at a high level and spoke length is recommended at a low level for all cases. These conclusions are true for both the L₈ and L₉ array. The L₈ and L₉ array also agree almost completely on the recommended levels of the frequencies of the peak amplitudes for

spoke vibration. The only difference is that a medium value of spoke curvature is recommended which was not available in the L₈ array.

The unexpected results from the L₉ array are that the shear beam thickness and spoke thickness recommended levels have changed from the L₈ Array. In the case of the L₉ array, the thick shear beam and a thin spoke are now recommended to decrease amplitude of vibration. This is true for all sources of vibration, which is the opposite of the L₈ results where a thin shear beam and a thick spoke were recommended. An explanation for these could be that there is a strong interaction that is identified when non-linear effects are considered. This L₉ array was created assuming there were no interactions, and no columns were left empty to investigate interactions, so each variable in the array is confounded with the others. This is not a problem if there are no strong interactions. However, in a three-level array, if two variables have an interaction it will affect the results of two other variables in the array. The other condition that could cause these contradictory results is that in the L₉ array, both inner and outer coverage are set at a value (medium level) that they were not tested at in the L₈ array.

6.5 Tweel™ Mass and Stiffness

Like the L₈ array, the L₉ array results will be analyzed to determine how changes in geometry affect the mass and vertical stiffness of the Tweel™. Table 6- 16 shows the changes in mass. This array is additive, so the mass of the Tweel™ can be predicted for any combination of these parameters. In this case, note the linearity of the three variable levels. These results confirm the L₈ results that spoke thickness is the most important

geometric parameter affecting the mass, followed by shear beam thickness and spoke length.

Table 6- 16: Masses of L₉ Experiments

Experiment	Tweel™ Mass (kg / 100 mm)
1	4.8
2	5.54
3	6.29
4	6.22
5	5.64
6	6.47
7	6.21
8	7.15
9	6.66
Mean	6.11

**Table 6-17: Effect of Changing Geometric Variables on Elastomer Mass,
Units: kg/100 mm Tweel™ Width**

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Thickness	-0.566	0.001	0.564
Shear Beam Thickness	-0.409	-0.036	0.444
Spoke Length	-0.366	0.001	0.364

Table 6-19 shows the effects of the variable levels on the vertical stiffness. Spoke thickness, shear beam thickness, and spoke curvature all have large effects on the stiffness. Spoke length has little effect. These results are consistent with the stiffness

results from the L_8 array. Also, the results of this experiment are nearly additive, so the stiffness of any geometry can be estimated by using the deviations from the mean listed in Table 6-19.

Table 6-18: Stiffness Indication of L_9 Experiments

Experiment	Mean Ground Reaction Force (daN)
1	3.20
2	3.35
3	3.58
4	4.75
5	2.89
6	5.25
7	4.63
8	6.68
9	4.69
Mean	4.34

Table 6-19: Effect of Variable Levels for Stiffness

Variable	Effect of Low Level	Effect of Medium Level	Effect of High Level
Spoke Thickness	-0.96	-0.04	1.00
Shear Beam Thickness	-0.74	0.07	0.67
Spoke Curvature	0.71	-0.07	-0.63
Spoke Length	-0.14	-0.03	0.17

6.6 Optimized Tweel™ Geometry

The vibration results from Chapter 5 and 6 show that no one model gives the lowest values for all measure of vibration. The results of the mass and stiffness studies indicate that the geometries that result in low vibration may also be undesirably high in Tweel mass or vertical stiffness. Five models will be considered in this chapter in regards to all three sources of vibration, as well as mass, and vertical stiffness. The five models with geometric properties are shown in Table 6-20. The first model is the optimal spoke model predicted by the L_8 array and discussed in Chapter 5. The next three models are the three additional models discussed in this chapter in Sections 6.1, 6.2, and 6.3. The 5th model was created as an additional experiment after observing the L_9 spoke vibration results that suggested a thick ring might reduce vibration. This model has high thickness levels for all three of the ring variables and thick spokes, and thus has very high Tweel™ mass and vertical stiffness. Each of the models considered in Table 6-20 have the same short spoke length and large spoke curvature as recommended by both L_8 and L_9 arrays.

Table 6-20: Five Geometries Considered for Optimal Case (all dimensions in mm)

	Spoke Length	Spoke Curvature	Spoke Thickness	Shear Beam Thickness	Inside Coverage	Outside Coverage
L8 Optimal	54	10	5.25	7.5	1.875	8.75
L9 Optimal 1	54	10	3.15	12.5	2.5	7
L9 Optimal 2	54	10	5.25	12.5	2.5	7
L9 Optimal 3	54	10	5.25	7.5	2.5	7
All Thick	54	10	5.25	12.5	3.125	8.75

Table 6-21 shows the vibration, mass, and stiffness results for the models in Table 6-20. The ring vibration listed in the table is the average value for the inside and outside ring vibration. All five geometries in Table 6-21 have relatively low amplitudes of vibration, and each of the geometries will result in a Tweel™ with low vibration. Which geometry is chosen depends on what characteristics are important to the designer. If minimizing spoke and ground vibration is the most important criteria, the “All Thick” geometry is suggested. However, this model is the heaviest and stiffest geometry, with a mass of 1 kg /100 mm more than the reference case (17% increase). If decreasing the mass and stiffness of the Tweel™ while still resulting in low vibration is important, the “L₉ Optimal 1” geometry is suggested. However, this geometry has the highest spoke and ground vibration of the five geometries compared in Table 6-21. The lowest ring vibration occurs with the “L₉ Optimal 3” geometry and this geometry is a good choice for reducing all vibrations, mass, and stiffness. The “L₈ Optimal” geometry, while not the

lowest value in any of the measures, it is always close to the minimum value for all measures. The “L8 Optimal” geometry gives the combination of variables with the best overall reduction in vibration amplitude with relatively low mass and vertical stiffness. Based on this result, the L₈ Optimal geometry is recommended for the final Tweel™ design.

Table 6-21: Results for Five Geometries Considered for Optimal Case

	Spoke	Ground	Ring	Mass	Stiffness
L8 Optimal	5.47	4.95	.116	5.95	4.05
L9 Optimal 1	13.22	9.03	0.189	5.75	2.99
L9 Optimal 2	4.96	7.29	0.187	6.69	5.22
L9 Optimal 3	5.85	6.86	.091	5.88	4.00
All Thick	4.80	4.48	0.190	7.12	5.29

Table 6-22 shows the benchmark reference, average and experiment with the highest vibration amplitudes for comparison. The averages in Table 6-22 are the average values for all the L₈ and L₉ experiments. Experiment 9 from the L₉ array is presented because it has the highest amplitudes for all measure of vibration. An important result observed from

Table 6-22 is the amount of improvement that can be made, especially in the amplitude of spoke vibration. The L8 optimal recommended geometry shows a significant reduction in vibration amplitude compared to the Reference geometry with some reduction in mass and stiffness. Another important result is how intense the vibrations can become if a Tweel™ designer chooses certain combinations of geometric variable levels. The geometry in Experiment 9 results in 40 times the amplitude of spoke vibration, 10 times the amplitude of ground vibration, and 7 times the amplitude of ring vibration when compared to the “L8 Optimal” geometry.

Table 6-22: Comparison of Recommended Optimal, Reference, Average, and Worst Case

	Spoke	Ground	Ring	Mass	Stiffness
L8 Optimal	5.47	4.95	.116	5.95	4.05
Reference	30.6	8.59	.153	6.10	4.16
Average Values	70.4	16.88	.291	6.1	4.36
L₉: Experiment 9	222.3	54.38	.730	6.66	4.69

CHAPTER SEVEN

TWEEL MODE SHAPES

This chapter will analyze the mode shapes and natural frequencies of two different Tweel™ geometries: Experiment 7 and Experiment 5 from the L₈ array defined in Table 5-2. Experiment 7 has a geometry that results in large vibration amplitudes with peak amplitude at a relatively low frequency of 270 Hz for spoke frequency. Experiment 5 has relatively low vibration amplitudes with a maximum frequency peak at 350 Hz for spoke vibration.

The mode shapes and natural frequencies are computed using ABAQUS Standard. The procedure has two steps. The first step is a loading and cooling step. The ground is moved vertically with a velocity of 150 mm over 0.1 seconds producing a 15 mm displacement. At the same time, cooling takes place from 125 to 25 degrees Celsius. The combined cooling and loading in Step 1 is the same as for the dynamic rolling case described in Chapter 2. Step 2 is a linear perturbation. ABAQUS uses a Lanczos eigensolver to extract all mode shapes at frequencies less than 600 Hz for Experiment 7 and 700 Hz for experiment 5. The hub center is left free to rotate during this Step. The interaction properties and element types are the same that were used in the dynamic rolling procedure. Since the eigenvalue extraction is performed after a steady-state cooling and loading step, dynamic inertia effects are not considered in the computed mode shapes and natural frequencies.

Figure 5-11 and Figure 5-12 show the FFT spectrum of the upper quarter node and middle spoke marker nodes for Experiment 7 obtained during rolling. Amplitude

peaks occur at 270 Hz for the middle marker nodes and 270 Hz and 570 Hz for the upper and lower quarter nodes. Figure 7-1 and Figure 7-2 show the FFT spectrums for Experiment 5. Experiment 5 geometry results in low amplitude of vibration with higher frequency peaks at 340 Hz and 630 Hz. Note how the upper quarter and middle node FFT spectrums look similar and there does not appear to be any extra peaks in the upper quarter node signal.

Figure 7-1: FFT with Hamming Window for Spoke Vibration in Experiment 5, Upper Quarter Node

Figure 7-2: FFT with Hamming Window for Spoke Vibration in Experiment 5, Middle Node

The mode shapes from the geometry in Experiment 7 (high amplitude, lower frequency peaks) are discussed first. The first 25 mode shapes correspond to ring modes. The ring modes take the form of a triangle at 225 Hz, a square at 232 Hz, a pentagon at 241 Hz, and the number of sides continues to increase as the frequency increases. Figure 7-3 shows a square mode shape and Figure 7-4 shows an octagonal mode shape. At mode 25, corresponding to a natural frequency of 265 Hz, the ring no longer shows deformation and the spokes begin to deform. Between 265 Hz and 273 Hz there are 22 modes that excited the spokes. These modes shapes take the form of local bending modes in the spokes. The mode shape at 270 Hz corresponding to the first frequency

peak in the FFT spectrum is shown in Figure 7-5 and appears as the first bending mode in a spoke pair, similar to the motion shown earlier in Figure 5-12.

Figure 7-3: Experiment 7: Mode 11, 236 Hz (Ring Mode)

Figure 7-4: Experiment 7: Mode 18, 258 Hz (Ring Mode)

Figure 7-5: Experiment 7: Mode 42, 270 Hz (Local Spoke Mode)

As the frequency of the mode shapes continues to increase, the ring starts to get excited again. From 276 Hz to 544 Hz there are 35 mode shapes that excite the ring into the flower petal shape like the one shown in Figure 7-6. The number of petals increases as the frequency increases. Flower petal ring mode shapes are also present in pneumatic tires [15]. From 564 Hz to 584 Hz there are 32 mode shapes that excite the spokes in different shapes than the lower frequency spoke mode shapes. Figure 7-7 shows the mode shape that corresponds to the 570 Hz peak in the FFT spectrum. This mode shape takes the form of a second bending mode for spoke pairs with no movement of the middle node, similar to the motion described earlier in Figure 5-14.

Figure 7-6: Experiment 7: Mode 61, 334 Hz (Ring Mode)

Figure 7-7: Experiment 7: Mode 101, 569 Hz (Local Spoke Mode)

Figure 7-8 thru Figure 7-10 show four mode shapes from the geometry for Experiment 5. This geometry results in small amplitude peaks at higher frequencies than

Experiment 7. Figure 7-8 shows a square ring mode shape at 263 Hz. The ring mode shapes for this geometry also increase in sides with an increase in frequency. Figure 7-9 is the mode shape at 340 Hz, which corresponds to the frequency peak in Figure 7-1. This mode shape does not show a spoke-only deformation mode, unlike the mode shape that corresponds to the frequency peaks in Experiment 7. For Experiment 5, the spoke-only mode shapes do not occur until 528 Hz. Figure 7-10 shows the modes shape that corresponds to the frequency of the second peak. The spokes are not of the same shape that causes the second peak in Experiment 7. This result could explain why the FFT spectrum for the middle node in Experiment 5 does not show an absence of a second peak. There are less mode shapes for Experiment 5 in the lower frequency range. Experiment 5 has 49 mode shapes less than 600 Hz. Experiment 7 has 122 in the same range.

Figure 7-8: Experiment 5: Mode 8, 263 Hz

Figure 7-9: Experiment 5: Mode 14, 340 Hz

Figure 7-10: Experiment 5: Mode 57, 631 Hz

In summary, the natural frequencies and mode shapes suggest that geometries with high amplitude peaks in the FFT spectrum for spoke vibration show a correlation with spoke vibration mode shapes. For other geometries with low vibration amplitudes, there was not as strong a correlation between spoke vibration mode shapes and spoke vibration amplitude peaks in the FFT spectrums. Ring flower-pedal modes were present in all experiments. Experiment 7 had more modes in the frequency range below 600 Hz where the largest amplitude peaks occur in the FFT spectrums.

CHAPTER EIGHT

CONCLUSION

8.1 Conclusions

Two Orthogonal Array studies were conducted, an L_8 array with six geometric variables at two variable levels and a L_9 array with four geometric variables at three variable levels. Each geometry created was tested for spoke, ring and ground interaction vibration. The mass and vertical stiffness of each geometry was also measured. Both the L_8 and the L_9 arrays experienced interactions, meaning that the recommended levels for some variables are dependent on the levels of other variables. Further investigation of this dependence will be a subject of future work. The conclusive and important results from both studies are as follows:

- Spoke length is the most influential variable on the amplitude of spoke, ring, and ground interaction vibration. A short spoke will result in the smallest amplitude of vibration for all three measurements.
- Spoke curvature is the second most influential variable on the amplitude of spoke, ring, and ground interaction vibration. A large spoke curvature will result in the smallest amplitude of vibration for all three measurements.
- Spoke thickness has a small influence on the amplitude of spoke vibration. A thick spoke results in a small decrease in amplitude of vibration.

- Shear beam, outside coverage, and inside coverage generally have similar levels of importance for all three levels of vibration. Recommended levels vary with vibration measurement.
- A wide variance in frequencies of peak amplitudes for spoke vibration can be achieved by different combinations of geometric variable levels in the Tweel™.
- There is effectively no interaction between spoke thickness and spoke length.
- Spoke thickness is the most influential variable on the mass and vertical stiffness of the Tweel™. A thicker spoke increases both.
- Shear beam thickness and spoke curvature are the second and third most influential parameters on vertical stiffness. A thicker shear beam and smaller spoke curvature will result in a stiffer Tweel™.
- The relative influence between the different geometric variables effecting vibration amplitudes in the Tweel™ are not strongly correlated with the relative effects on total mass or vertical stiffness. For example, spoke thickness has a large effect on total mass and vertical stiffness, yet small effect on vibration amplitudes. Spoke length has a large influence on vibration amplitude and small influence on vertical stiffness. Spoke curvature was highly influential on both amplitude and vertical stiffness.
- There are strong non-linear effects of changing variable levels.

With recommended levels obtained from the arrays, five very low vibration geometries were created. Each geometry has advantages, but the geometry named “L8 Optimal” in Chapter 6 was considered the best combination for low vibration, mass, and

vertical stiffness. This geometry included short and thick spokes with a large curvature, a thick outer coverage, and a thin shear beam and inner coverage. This “L8 Optimal” geometry resulted in an 82% decrease in spoke vibration, 42% decrease in ground vibration, and 24% decrease in ring vibration when compared to the BMW Mini reference geometry. This geometry also has less mass and stiffness than the reference geometry.

Perhaps as important as the recommended levels of geometric variables for reduced vibration are the variable levels a Tweel™ designer should avoid. Designing a Tweel™ with the same variable levels as Experiment 9 in the L₉ array will result in roughly 7 times the spoke vibration, 6 times the ground vibration, and 5 times the ring vibration when compared to the reference case.

The variety of experimental geometries resulted in a large variation of amplitude of vibration for each measure of vibration. Through analyzing these results, a more thorough and physical understanding of the amplitude peaks was acquired. The spoke motion that seems to correlate with the FFT amplitude peaks has been identified with ABAQUS animations. These peaks seem to correspond to mode shapes that excite the spokes. The motion that causes the peaks in the ground interaction FFT spectrum have also been investigated with ABAQUS animations. These motions were identified and compared by viewing extreme cases.

8.2 Future Work

Further orthogonal array experiments and studies are suggested for future work. One suggestion would be to determine a measure to better clarify flower petal ring

vibration. The current ring vibration measurement calculates ring thickness from changes in distance between points between the inner and outer coverage. The radial distance between the hub center and points on the inner and outer coverage have also been investigated in a few cases, which gave similar results. These measures are influenced by both radial components and tangential components resulting from the shear deformation in the shear beam. A new measure could be created that measures only the radial component to give a more accurate measure to determine how out-of-round the ring becomes during rolling. Direct calculation of shear deformation in the shear beam could also be of interest. Another suggestion would be a thorough mesh convergence study, which may result in a finer mesh being applied to the Tweel™ FEA model.

As stated earlier in this Chapter, further orthogonal array experiments should be conducted in order to gain a greater understanding of the magnitude of interactions between variables. For future work, two additional L_8 arrays are suggested. The first suggested array shown in Table 8-1 is designed to investigate interactions in the ring variables. The L_8 Array in the present work showed unexpected levels of influence for the shear beam, inner coverage, and outer coverage. Because the shear beam is crucial to the operation of the Tweel™, interactions between it and the inner and outer coverage should be examined further. The ground contact pressure is proportional to the shear beam thickness. Increase in the inside and outside coverage increases the bending stiffness in the ring which may result in changes in contact area and non-uniform pressure distribution with areas of high pressure concentrations. The array in Table 8-1 can account for interaction of the shear beam between the inner coverage, outer coverage, and

spoke thickness. The interaction between shear beam and spoke thickness is investigated because spoke thickness has the greatest effect on Tweel™ stiffness.

Table 8-1: L₈ Array to Investigate Interaction Between Ring Variables

Experiment Number	Shear Beam	Inner Coverage	Shear Beam vs. Inner Coverage	Outer Coverage	Shear Beam vs. Outer Coverage	Shear Beam vs. Spoke Thickness	Spoke Thickness
1	Low	Low		Low			Low
2	Low	Low		High			High
3	Low	High		Low			High
4	Low	High		High			Low
5	High	Low		Low			High
6	High	Low		High			Low
7	High	High		Low			Low
8	High	High		High			High

The second suggested array shown in Table 8-2 investigates the relationship between the spoke variables. The L₉ results in the present study suggest that there may be an interaction between spoke curvature and spoke length. This relationship can be investigated further by using Table 8-2. In order to fully understand the effect of spoke curvature on vibration, the inner and outer Derad values should also be considered. The Derad values were not analyzed in this study, but they control the orientation of the spoke with respect to radial lines. The array in Table 8-2 accounts for interaction of the spoke curvature between spoke length, inner derad, and outer derad.

Table 8-2: L₈ Array to Investigate Interaction Between Spoke Variables

Experiment Number	Spoke Curvature	Spoke Length	Spoke Curvature vs. Spoke Length	Outer Derad	Spoke Curvature vs. Outer Derad	Spoke Curvature vs. Inner Derad	Inner Derad
1	Low	Low		Low			Low
2	Low	Low		High			High
3	Low	High		Low			High
4	Low	High		High			Low
5	High	Low		Low			High
6	High	Low		High			Low
7	High	High		Low			Low
8	High	High		High			High

While these L₈ arrays only evaluate two levels, they should give a first-order indication of which variables have interactions, and how strong the interactions are. The results from these additional experiments should aid in the further understanding of how each variable affects vibration.

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