Experimental Framework for the Position Control of Magnetic Microfibers

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EXPERIMENTAL FRAMEWORK FOR THE POSITION CONTROL OF MAGNETIC MICROFIBERS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Electrical Engineering

by
Sriram Ravindren
August 2008

Accepted by:
Dr. Richard E. Groff, Committee Chair
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Dr. Ian D. Walker
ABSTRACT

Magnetic microfibers have been developed in recent years with potential applications to a large number of fields, from MEMS to microfluidics to bio-chemical analyses. Magnetic microfibers based on polymer material like polypropylene and cellulose has been developed at Clemson University that are hollow cylindrical filaments internally coated with superparamagnetic nanoparticles. This internal coating provides the fiber with magnetic properties that allow them to be externally controlled using magnetic fields from a distance. These fibers can align with the external magnetic fields under constraints imposed by elastic properties and boundary conditions. Current work in this field is primarily focused on the use of microscale magnetic fibers under a constant homogenous magnetic field. The work presented in this thesis documents the experimental framework to provide precise position control under non-homogenous fields in a much larger scale. The fiber used is several centimeters long and is constrained at one end by being fixed to an acrylic base. Multiple solenoids are used to generate magnetic fields and cameras provide image information to the controller. The controller adjusts the current to the solenoids based on information obtained from the image. Current literature is surveyed to provide an overview of various models of magnetic fibers. Suggestions for improvements to the system are provided and future work that will aid in the transition to a model based control approach is explained.
DEDICATION

I would like to dedicate my work to my parents and my brother. Their constant encouragement, love and support through all the years of my studies, has helped me grab various opportunities in my life. Therefore, I dedicate this work in their honor.
ACKNOWLEDGMENTS

The success of any project would be incomplete without mentioning the people that made it possible and without whose guidance and advice, it would have been difficult to proceed with this project.

First and foremost, I would like to sincerely thank my advisor Dr. Richard Groff for taking me into his research team. His valuable advice and direction during the past eighteen months has enabled me to acquire a broader knowledge of engineering and mathematics than before. This project was inspired by a daring vision to use flexible magnetic filaments for propulsion in microrobots in fluids. By its very nature, it stands at the junction of several deeply mathematical fields, including but not limited to, Controls, Electromagnetics and Continuum mechanics. It takes someone with sufficient scientific grit to wade through the often esoteric literature and create a wake for other less mathematically inclined, but sufficiently motivated individuals to follow through. While surveying what has been a new field for me, Dr. Groff has helped me focus on developing the system at hand, while carefully setting aside literature that would have taken this research off in tangents. The driving idea behind this project and its enormous implications would continue to inspire several generations of his future graduate students.

I would like to thank Dr. Konstantin Kornev for providing us with material and advice during the course of this project. His constant encouragement and insistence that I continue my studies toward a higher degree and his advice that my scientific acuity would increase over time has been a key motivational factor through the final days of my thesis.

Special thanks goes to John Kelley for his help with the simulations. I would also like to thank John Kelley and Justin Mattlmore, who helped construct the base that has been a key component in this experiment.
My thanks also goes to several lab mates who, over the course of my degree here at Clemson University, have created a very positive and fun environment to do research. I would specifically like to mention Martin Hill and Matt Pepper for having helped made the atmosphere sufficiently conducive.

My final thanks goes out to all my friends in the Clemson community who have provided a stable social platform to enable me to focus on my studies.
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Chapter 1

Introduction

The idea of using magnetic actuators instead of electric devices at the sub-millimeter scale has been advocated over the past decade. It is very difficult to have direct contact and thus effective control over devices at this scale without introducing several practical overheads. It is evident that the most effective way to proceed is with practical implementations of remotely controlled, external actuation. It turns out that magnetic devices are more amenable to external actuation than electric devices. Single domain magnetic regions exist and can be manipulated using external magnetic fields. The key interest here is in the manipulation of the end-effector without direct physical contact.

Several practical implementations have been performed over the past decade. Magnetic fibers made of superparamagnetic beads coated with Streptavidin have been attached using DNA molecules and have been demonstrated to be sufficiently responsive to changing magnetic fields. The motion of these fibers has also been predicted theoretically and confirmed empirically. Unlike electric devices, the properties of magnetic devices are generally not hampered by chemical properties such as pH, temperature, etc. This is a tremendous advantage in microfluidics applications where magnetism has recently been introduced. Proposed applica-
tions for robotic and other mobile devices at the sub-millimeter scale include DNA unraveling, animal cell studies, microrobotics, etc.

Of special current interest in the field of magnetics is a new class of materials that are characterized as superparamagnetic. These are nanometer scale single magnetic domain crystals that exhibit behaviour similar to paramagnetism at low temperatures. These materials have ferromagnetic properties in bulk. They are used in biological analyses where they are individually enclosed in some polymer, like streptavidin, so that biomolecules may be attached. These particles require very low magnetic fields to magnetize them in some direction and they lack a magnetic memory in that they behave like non-magnetic materials once the external field is removed. Superparamagnetic beads range in size from m to nm. They are especially valuable in medical research as they can come coated with carboxyl or amino groups which make it easier to attach DNA strands and proteins to their surface. These particles are amongst the latest intellectual products in producing newer varieties of magnetic substances that are ever smaller, but retain the significant properties that characterize larger magnets.

By combining magnets in different orientations, predictable complex magnetic fields can be created on different size scales. It is necessary to differentiate between the two kinds of fields: Homogeneous and inhomogeneous magnetic fields [19]. Homogeneous magnetic fields are those fields that are uniform over a certain distance. The magnetic flux density of these fields remains constant over a certain distance, with very little or no gradient in the magnetic flux. These kinds of fields can be created using Helmholtz coils.

Inhomogeneous fields on the other hand, have a gradient in the magnetic flux density, i.e. the magnetic flux density decreases with distance from the surface of a magnet. This is typical of almost all magnets.

The ideal inhomogeneous field can be created using a Maxwell coil. Using this, we get a uniform field gradient that reduces to zero at the center.
1.1 Magnetic Microfibers

Magnetic microfibers are basically hollow cylindrical (sometimes ribbon) polymers coated with magnetic nanoparticles. These particles tend to be some variation of $Fe_3O_4$ in single domains. When the fiber is coated with these particles, it becomes responsive to external magnetic fields and shows great flexibility. It is important to note that the magnetic fiber itself does not generate a very strong magnetic field (when compared to the external field). The fiber is very light, weighing much less than a gram. The diameter of the magnetic fiber we use is around 83 m. (A flattened fiber can be around 260 m in width).

Applications of magnetic microfibers include:

1. Magnetic stirring in microfluidics

2. DNA separation

3. Droplet manipulation studies using fiber rails
4. Liquid transport in fluidic devices

5. Eventual application for propulsion of micro- and nano-devices in a fluidic channel

For microfluidics applications, there has been interest in using magnetic fields to transfer fluids by controlling the motion of the hollow fiber without direct physical contact between external controllers and the fiber.
1.2 Controlling the motion of the tip using magnetic fields

There are several different methods of controlling the motion of the tip by generating different magnetic fields. The simplest method is to use the inhomogeneous magnetic field of a solenoid electromagnet with an air core. The tip then moves in response to the local variation of the magnetic flux density, which is at its highest at the surface of the electromagnet. The
expression for the off-axis field of an air core solenoid magnet is given by

\[ B(r, z) = \frac{\mu_0 I k}{4\pi \sqrt{ar^3}} \left[ - (z - h) \left( K(k) - \frac{2-k^2}{2(1-k^2)} E(k) \right) \hat{r} + r \left( K(k) + \frac{k^2(r+a)-2r}{2r(1-k^2)} E(k) \right) \hat{z} \right] \]

where, \( k = \sqrt{\frac{4ar}{(r+a)^2+(z-h)^2}} \)

At the surface of the magnet \( B = \frac{\mu_0 NI}{L} \) where \( N \) is the number of turns, \( I \) is the current passing through the wire and \( L \) is the total length covered by the coil. The current \( I \) then is the programmable variable that determines the strength of the magnetic field. By varying the magnitude of the current, we can change the shape of the field. This results in the fiber moving to different positions along its workspace.

![Figure 1.5: Simulation of variation of magnetic flux lines by changing direction of currents.](image)

### 1.3 Organization of Thesis

Chapter 2 looks at models of magnetic filaments that have been published over the past decade. Magnetic fibers can be modeled as a 'string' of beads linked together that have very low resistance to bending [18] [5] [22] [21]. This particular model is based off of experiments where individual magnetic particles were linked together in a variety of different ways to form long flexible filaments. The bending forces computed are quite small while the string resists stretching. Other models, most notably the Cebers models [10] [9] [8] [7] [6] [11] [12], describe the fiber as a long flexible filament using Kirchoff’s elastic rod theory. This model has also been
used by Roper et al. [20] [15] who linked a series of streptavidin coated magnetite particles to form a magnetic string and attached the string to a human red blood cell to demonstrate propulsion of tiny cells using magnetic fields [13]. The elastica model describes the motion of the string under homogenous DC and AC magnetic fields quite well. These models can be used to model the motion of the magnetic microfiber that we use when the magnetic field applied across the fiber is homogenous. Some of these models consider the dipole interaction of the magnetic particles in the overall computation of forces. Finally, this chapter describes the process of manufacturing the magnetic microfiber. It explains how a hollow cylindrical fiber is filled with ferrofluid to capture magnetic particle. The super-paramagnetic nature of the individual ferrite particles provides the resulting magnetizable fiber with stronger than paramagnetic properties, without the hysteresis effects dominant in ferromagnetic structures.

Chapter 3 provides begins with an overview of magnetism in general. It describes the hysteresis effects that dominate the energy of magnetic materials. It then talks about various available magnets - specifically, permanent magnets and electromagnets. It goes on to describe the factors determining the selection of the electromagnet core - coercivity, remanent magnetization, magnetic permeability, commercial availability of magnets of the right shape and size. Finally, it describes the Fair-rite Material 61 ferrite core that we chose for our experiments. The chapter ends by looking at the various possible configurations that the electromagnets can be placed in. It looks at how the currents in two mutually perpendicular magnets can alter the magnetic potential in between them by providing multiple plots of the magnetic potential variation from the center of the surface of one of the magnets to the surface center of the other.

Chapter 4 looks at the experimental setup that constitutes the core objective of this thesis. The chapter describes the control loop that is responsible for the external actuation and control of the magnetic fiber. It describes the feedback loop wherein information regarding the magnetic fiber’s tip is provided through video analysis to a control loop in Simulink which in turn sends signals to the real time controller. The control signals from the hardware board is
then amplified by a linear amplifier and finally fed into a field coil. The effect of the control signal on the magnetic fiber is then monitored and the output adjusted accordingly.
Chapter 2

Models of Magnetic Fibers

2.1 Introduction

Although magnetic fibers have only been around quite recently, there have been several successful models that explain the motion of these fibers under different operating conditions. In most cases, the dynamics of the fibers have been explained under hydrodynamic conditions where the fiber is immersed in a fluid of some viscosity and is then stimulated using external magnetic fields. Many of the more popular models consider magnetostatic and hydrodynamic effects affecting magnetic fibers. A common trait observed across the models is the small size of the fiber, usually in the micrometer scale. The magnetic field applied is also homogeneous in that the flux density does not vary significantly across the medium in which the fiber is actuated. This field is easily obtained using the Helmholtz configuration. In other cases, the magnetic field applied may have a uniform field gradient across the medium that the fiber operates in. The latter, created using the Maxwell coil configuration, will not be considered for analysis here. A large amount of literature has gone into describing the motion of magnetic fibers in the micro-scale in the past five years because of their potential application for propulsion in microrobotics. Most of these robots are actuated using milli-tesla homogeneous
magnetic fields which are easy to create using small magnets. Here, we shall first briefly describe the various methods that are used in the analysis of the magnetic microfiber. We then proceed to describe the preparation process of magnetic microfibers.

Magnetic fibers bend under the influence of a **uniform** magnetic field in order to achieve equilibrium in the lowest energy state that is in the direction of the magnetic field. In order to achieve this final configuration, these fibers typically go through several easily identifiable transient states - with U- and S- metastable shapes being the most easily recognizable, if the fiber is free at both ends. The actual transient states can be determined quite readily from the initial state of the fibers. For example, if the magnetic fiber has a curved end (with both ends being free), then it will achieve a **hairpin** or U-configuration. If both the ends are bent in the direction of the magnetic field, the U-configuration is more readily identifiable and the fiber stays in this state for a longer period of time. The period of transition from this initial state to the final straight rod configuration can vary depending on a variety of factors - including how different from the equilibrium state the fiber initially was, the number of curves in the fiber in the initial state, the bending rigidity of the linkers, the viscosity of the surrounding fluid and the strength of the magnetic field applied to the environment. With more curves in the fiber during the initial state, the fiber’s intermediate stage can resemble an S-state. This has been observed experimentally Roper *et al.* [20]. The straight rod configuration is achieved only when the field is static long enough for the fiber to transition from its meta-stable state, or when multiple rotating fields interact to produce a single uniform plane where the straight line configuration is the most energy efficient state for the fiber.

**The models presented here differ from our setup primarily in terms of the size of the fiber and the nature of the magnetic field.** As explained in other chapters, the field that we use is non-homogenous with the flux density varying nonlinearity with the distance from the surface of the electromagnets and the potential wells vary in their location depending on the magnitude and direction of the currents in each of the electromagnets. The fiber itself is
several orders of magnitude larger than most of the fibers used in the models presented, but the underlying analyses remains the same otherwise. Being superparamagnetic in nature, the fibers do not retain any magnetization after the external field has been removed. As explained in the section on the manufacturing of the fibers, the individual magnetite nanoparticles only align in the direction of the magnetic field when the external field is applied. Once the external field is removed, the magnetite particles become randomly oriented and do not retain any net magnetization. Further, the presence of surfactants around the nanoparticles prevents the agglomeration of these particles thus stopping the formation of domains and as a result, the fiber does not exhibit ferromagnetic behavior. (It should be noted that the presence of the surfactant simply reduces the rate of agglomeration, but eventually the particles will start binding together once the surfactant wears off).

2.2 An Overview of Different Models

In this section, we look at the more popular models that have been published in the literature over the past decade. Many of the later models either build upon the previous models in terms of the different forces considered (stretching, bending, thermal effects) or present alternate ways of analysis of the fiber (either as a string of beads or as a long elastomer). All these models deal with free or cantilevered magnetic filaments that are subjected to a homogenous magnetic field that is either static or an AC field. The same models may not readily apply to cases with spatially varying magnetic fields.

2.2.1 Model 1

According to Hutter et al. [16], the buckling value or critical value of the magnetic induction $B_0$ for magnetoelastic buckling of a slender soft ferromagnetic (magnetic permeability $\mu \gg 1$) elastic beam of elliptic cross-section with semi-major axis $a_{\text{semi}}$ and semi-minor axis $b$ placed
in a uniform external magnetic field $B_0 = B_0 e_y$ is

$$
\left( \frac{B_0}{\sqrt{\mu_0 E}} \right)_{cr} = \frac{1}{2} \sqrt{\varepsilon \kappa \Lambda} \left( \frac{\pi b}{2l} \right)^{3/2}
$$

where $\mu_0$ is the magnetic permeability of vacuum, $E$ is the elastic modulus of the beam material, $\varepsilon = \frac{\pi a}{l} \ll 1$ for slender beams, $l$ is the length, $\Lambda = 1 + (2\beta \mu \varepsilon^2 \kappa)^{-1}$, $\mu \gg 1$

$$
\kappa = -\gamma - \ln \left( \frac{1 + \beta}{2} \right)
$$

$$
\gamma = 0.5772
$$

This analysis is based on a general formulation for a magnetoelastic body interacting with an external magnetic field. For a circle, with $a=b=R$,

$$
\left( \frac{B_0}{\sqrt{\mu_0 E}} \right)_{cr} = \frac{1}{2} \sqrt{\frac{\pi}{2\beta} + \frac{1}{\mu} \left( \frac{2l}{\pi b} \right)^2 \left( \frac{\pi b}{2l} \right)^2},
$$

where $\kappa = -\gamma - \ln \left( \frac{1 + \beta}{4\beta} \right) - \ln \left( \frac{\pi b}{2l} \right)$, $\beta = \frac{b}{a}$.

If $\mu$ is very large, then

$$
\left( \frac{B_0}{\sqrt{\mu_0 E}} \right)_{cr} = \frac{1}{2} \sqrt{\frac{\kappa}{2\beta} \left( \frac{\pi b}{2l} \right)^2},
$$

So, according to these results, the buckling value increases as the beam becomes more elliptical and decreases when it becomes more circular, provided the magnetic field is applied in the same direction.

2.2.2 Model 2: Modeling the magnetic fiber as a string of beads [14]

In order to numerically study the dynamics of the magnetic filament, a model is developed where the filament is represented as a set of $N$ rigidly connected beads forming a magnetic string of mass $m$. The inertia of the filament is considered negligible while the rigid connec-
tions of the filament mean that its total length remains constant. The forces are analyzed for each segment of length

\[ \Delta l = \frac{l}{N - 1} \]

assuming a total length.

The total stretching free energy is

\[ H^S = \frac{1}{2} k \sum_{i=2}^{N} (l_i - l_0)^2 \]

where \( N \) is the total number of beads and \( l_0 \) is the equilibrium value of the inter-bead distance.

The total stretching free energy on a bead \( i \) is then

\[ F^S_i = -\nabla r_i H^S = -k(l_i - l_0)\hat{t}_i + k(l_{i+1} - l_0)\hat{t}_{i+1}. \]

where \( \nabla r_i \) is the nabla operator with respect to \( r_i \).

The bending free energy for a worm-like chain is

\[ H^B = \frac{1}{2} A \int_0^L ds \left( \frac{d\hat{t}}{ds} \right)^2 \]

where \( L \) is the total length of the filament, \( s \) the arc length along it, and \( \hat{t} \) the unit tangent at location \( s \).

The discretized version of this is

\[ H_B = \frac{A}{l_0} \sum_{i=1}^{N} f_i (1 - \hat{t}_{i+1} \cdot \hat{t}_i), \]

where \( \hat{t}_i = \frac{\hat{t}_i}{l_i} \) and

\[ f_i = \begin{cases} 
1 & \text{for } 2 \leq i \leq N - 1 \\
0 & \text{for } i = 1, N 
\end{cases} \]

The total bending force acting on bead \( i \) is then

\[ F^B_i = -\nabla r_i H^B = \]
A single particle made from material with a magnetic susceptibility $\chi$ and subject to an external magnetic field $B$ has a dipole moment

$$p = \frac{4\pi a^3}{3\mu_0} \chi B$$

where

$$\mu_0 = 4\pi \times 10^{-7} N/A^2$$

and $a$ is the particle’s radius.

Thus, the filament is driven by an external magnetic field with magnitude $B$ and direction $\hat{p}$.

Here, if the field changes rapidly, the hydrodynamic friction of the string with its fluid medium increases and thus the string bends. However, if the field varies slowly, then the string can align itself in the direction of the magnetic field, thus acquiring a straight rod configuration most of the time.

For sufficiently large lengths $L$, the reduced energy terms become constants.

The authors proceed to derive, in a simple manner, the final reduced equations of motion of the bead $i$

$$\frac{d\tilde{r}_i}{dt} = S_p^{-4} \sum_j \tilde{\mu}_{ij} \tilde{F}_j$$

with

$$\tilde{F}_j = -\nabla_{\tilde{r}_i} \left( \tilde{H}^B + k_s \tilde{H}^s + B^2 \tilde{H}^D \right).$$

Here, the self-mobility is $\mu_{ii} = \mu_0 1$ with $\mu_0 = \frac{1}{(6\pi\eta a)}$ and the cross-mobility is

$$\mu_{ij} = \frac{1}{6\pi\eta r_{ij}} \left[ \frac{3}{4} (1 + \hat{r}_{ij} \otimes \hat{r}_{ij}) + \frac{1}{4} \frac{a_i^2 + a_j^2}{r_{ij}^2} (1 - 3\hat{r}_{ij} \otimes \hat{r}_{ij}) \right].$$
The Sperm number $Sp$ is $Sp = \left( \frac{6\pi \eta \omega L^4}{A} \right)^{1/3} = \frac{L}{l_h}$ and is a very important number that is the ratio of the frictional to bending forces. It determines the dynamics of the elastic filament in a viscous environment.

This model has the advantage of being simple and approximates the string in a very detailed manner considering the stretching, bending and most importantly the dipole-dipole interaction forces.

Using MATLAB, the simulation works well for any number of beads since the equations are properly described in a matrix format and does not required too much time to compute the effects over one time interval. It has the significant disadvantage of being computationally time consuming as it requires at least a million iterations to determine the motion of the string over one second since the time snapshots that we use to observe the string are quite small. Larger time intervals lead to serious errors in the simulated output. This disadvantage is characteristic of this type of mesoscale model [1]

### 2.2.3 Model 3 [22]

[Reference: (Shcherbakov and Winklhofer, 2004)]

Shcherbakov and Winklhofer (2004) consider the case of a long cantilevered magnetic fiber and attempt to derive the magnetoelastic equilibrium shapes for the one-dimensional magnetic elastomer.

They consider the magnetic energy of each element of the fiber to be

$$dW_{mag} = \frac{2\pi \chi^3 H_0^2 S}{1 + 4\pi \chi} \sin^2 [D - \psi(l)] dl$$

$H_0$ is the applied magnetic field at angle $D$, $\chi$ is the effective susceptibility of the material and is the angle that the fiber makes with respect to the abscissa.
They find the total free energy functional, based on the Kirchhoff model of elastic rods (from continuum theory) to be

\[
W = \int_0^L \left\{ \frac{E \cdot I}{2} \left( \frac{d\psi}{dl} \right)^2 + dW_{SP}^0 \sin^2 (D - \psi) \right\} \, dl
\]

The corresponding Euler equations after proper substitution is

\[
2 \frac{d^2 \varphi}{d\lambda^2} = \sin 2\varphi,
\]

with boundary conditions \( \varphi(0) = \varphi_0 = D - \psi_0 \) and \( \left( \frac{d\varphi}{d\lambda} \right)_{\lambda = \Lambda} = 0 \).

And, \( \Lambda = \frac{L}{l_0^{SP}} \), where \( l_0^{SP} = \frac{(1+4\pi\chi)EI}{2\pi\chi^3 H_0^2 S L} \).

The first boundary condition corresponds to the fixed end of the fiber, while the second boundary condition listed above is for the free end of the cantilevered fiber.

Solving for \( \varphi_0 \) (angle at the fixed end) and \( \varphi_1 \) (angle at the other end) and applying numerical methods, this method computes the angles for various points of the fiber using techniques involving elliptic integrals and thus determines the shape of the fiber for different values of
total length and different magnetic field strengths according to the expressions shown below in Eq(2.1).

\[
\frac{x(l)}{\lambda_0} = \int_0^\lambda \cos \psi (\tilde{\lambda}) \, d\tilde{\lambda}
\]

and

\[
\frac{y(l)}{\lambda_0} = \int_0^\lambda \sin \psi (\tilde{\lambda}) \, d\tilde{\lambda}
\]  

(2.1)

### 2.2.4 Model 4: Cebers-Javaitis Model

A. Cebers in several papers considers the equilibrium shapes of superparamagnetic and ferromagnetic filaments based on the extended Kirchhoff model of elastic rods, taking into account the spontaneous magnetization and finite values of magnetic susceptibility of the fiber. The energy of the magnetic filament with superparamagnetic properties (without regard to the manner of distribution of the particles) is

\[
E = \frac{1}{2} C_b \int \left( \frac{d\vartheta}{dl} \right)^2 \, dl - \frac{b^2(\mu - 1)^2 H_0^2}{8(\mu + 1)} \int \cos^2 \vartheta \, dl + \int MH_0 \cos \vartheta \, dl
\]

Here, \( C_b \) is the bending modulus, \( \vartheta \) is the angle that the tangent \( \vec{t} = (\cos \vartheta, \sin \vartheta) \) makes with the magnetic field, \(-M\vec{t}\) is the spontaneous magnetization of the filament per unit length, \( b \) is the radius of the filament and \( \mu \) is the magnetic permeability of the filament. Minimizing the energy term with respect to \( \Theta \), the Euler-Lagrange equation gives

\[
-C_b \frac{d^2\vartheta}{dl^2} + \frac{b^2(\mu - 1)^2 H_0^2}{8(\mu + 1)} \sin(2\vartheta) - MH_0 \sin \vartheta = 0
\]
Introducing \( C_{mp} = \frac{b^2 (\mu - 1)^2 H^2 L^2}{8 (\mu + 1) C_b} \) and \( C_{mf} = \frac{MH_b L^2}{C_b} \), and then setting \( \frac{1}{2a^2} = \frac{C_{mp}}{C_{mf}} \) and \( z = \cos \vartheta \), we get

\[
a^2 z'^2 = \left(1 - z^2\right) \left[C^2 - (z - a^2)^2\right]
\]

Here, \( C \) is the constant of integration. Solving for this equation gives 4 roots for \( z \): \( a^2 \pm C \) and \( \pm a \). Setting \( \alpha_1 = -1 \), \( \alpha_2 = a^2 - C \), \( \alpha_3 = 1 \), \( \alpha_4 = a^2 + C \) gives two different solutions for the shapes of the fiber corresponding to the two values of the constant of integration: \( C < 1 \) and \( C > 1 \). \( C \) is obtained from

\[
C = \frac{2K^2 \left(\frac{1}{h}\right)}{B^2}
\]

where \( K \) is the Complete Elliptic Integral of the First kind. The angle is determined from the expression

\[
\cos \vartheta = \alpha_3 + \frac{(\alpha_3 - \alpha_2) \left(\text{sn} \left(\frac{\beta}{\pi}, \frac{1}{h}\right) - 1\right)}{(p-1) \text{sn} \left(\frac{\beta}{\pi}, \frac{1}{h}\right) + p + 1}
\]

Integrating the equations for the tangent

\[
\frac{dx}{dl} = \cos \vartheta
\]

\[
\frac{dy}{dl} = \sin \vartheta
\]

we get the different shapes for the filament at different values of the magnetoelastic numbers \( C_{mp} \) and \( C_{mf} \).

### 2.2.5 Observations of Various Models

1. For a unconstrained fiber in a uniform magnetic field, the global energy minima corresponds to a straight fiber in the direction of the magnetic field.
2. S-like characteristic shapes are formed when the frequency of the magnetic field is large enough. These shapes are temporary (but sometimes long lasting) transient (intermediate) stages that stay until the final energy minima configuration is reached.

3. U-like metastable shapes with lengths larger than the radius of curvature can also be formed.

4. The shape that the fiber takes varies with the initial configuration of the fiber. Many different metastable states of equilibrium are possible.

5. Hairpin shape of the filament exists even when the length of the arms of the filament are different.

6. If the length of the shorter arm is small enough, the filament may relax to a straight configuration.

7. The equilibrium shape of the hairpin results from the interaction between the magnetic field trying to orient the filament in its direction and the elastic forces resisting the bending.

8. Magnetic filaments can be used as force sensors, since their stiffness depends on the magnetic field.

### 2.3 Creating the Magnetic Microfiber

This section provides an overview of the manufacture of the paramagnetic fibers using commercially available ferrofluids.
2.3.1 Description of the fiber

The magnetic microfiber is a hollow cylindrical shell made of either Polypropylene \((C_3H_6)_n\) or Cellulose \((C_6H_{10}O_5)_n\). The fibers are also available with pores across the cylinder’s surface. The fiber is immersed in a water-based ferrofluid. Due to capillary action, the ferrofluid initially enters the hollow tube at one end and is then made to flow completely through the tube. The fiber and the ferrofluid are both available commercially. The aggregation of the particles needs to be controlled as this would determine strength of magnetic domains. If particles agglomerate to form clumps when the magnetic field is not applied, the magnetic nature of iron oxide particles would make cause the magnetic behavior of the fiber to be ferromagnetic, rather than paramagnetic. Encapsulating these individual paramagnetic nanoparticles into the tube produces the required paramagnetic fiber. The prevention of this clustering is done at the selection of the ferrofluid. These fluids are available in a variety of specifications. The ferrofluid used to fill the tube is water based (EMG 508) from Ferrotec Corporation, which contains magnetite \((Fe_3O_4)\) particles with a characteristic diameter of 10 nm.

2.3.2 Principle

The procedure followed here is described in the paper by Korneva et al.[17] where the authors describe how they use the phenomenon of spontaneous penetration of fluids into wettable capillaries as a guide to filling their nanotubes with magnetic nanoparticles. The same principle works in the macro scale with our fibers. They suggest how using melting of ferrous metals to penetrate the fiber would be unsuitable because the high melting point (around 1535°C) of Fe would cause its melt to react with the carbon of the fiber. Additionally, they apply a magnetic field of around 0.4T in their setup to promote the rate of penetration, which is quite efficient even without the field. The field serves as an addition to the capillary action by directing the magnetic nanoparticles into the tubes by pulling the particles towards it. Although the effect of
magnetic fields is quite insignificant for their experiments with carbon nanotubes because of instantaneous penetration, it becomes an extremely efficient method for tubes in the centimeter scale, such as ours. (The statistical analysis that they perform show that the nanoparticles fill the tubes equally well for tubes with diameters in the nanometer scale, with and without a magnetic field, which suggests that capillary forces are primarily responsible for the filling). In our case where the fiber is several orders of magnitude larger than the carbon nanotubes, a faster filling rate can only be achieved by using an external magnet as a driving force.

Figure 2.2: TEM images showing the tubes filled with nanoparticles. A similar process occurs in the magnetic microfibers that we use. (Source: Korneva et al.[17]).

2.3.3 Process

After the ferrofluid has entered the fiber through capillary action, it is then drawn across the length of the fiber through any number of ways. One method would be to pull the leading ferrofluids across the tube using a permanent magnet (say, an Nd magnet) while the rest of the fluid continues to enter at the other end.

This is done until the entire space of the cylinder is filled with the ferrofluid. After this, the fiber is left to ‘dry’. The solvent (i.e. the water carrier) in the ferrofluid subsequently evaporates over time, leaving behind the ferrofluid particles. The separation may also be induced manually to speed up the process. Under ideal circumstances, using a ferrofluid
containing a uniform distribution of the paramagnetic iron oxide nanoparticles and a smooth progress of the fluid itself across the length of the fiber would deposit the particles roughly uniformly along the entire fiber. Practically, there would be a mild discontinuity in the particle distribution at the ends of the fiber. This would be an effect of the evaporation of the solvent which occurs through the open ends. The surface adhesion of the fiber also needs to be tuned carefully to cause the particles to attach themselves all around the inner surface area of the cylinder, rather than only along some particular patches (which may occur).

The particles remain attached to the fiber after the deposition process because of strong adhesion forces, while other particles - if any - on the outside are loose and can be washed away.

### 2.3.4 Ferrofluids

Ferrofluids are stable colloidal suspensions of sub-domain magnetic nanoparticles in a non-polar liquid carrier - water, organic, etc. The particles have an average size of around 100 Angstroms (10 nm) and are coated with a stabilizing dispersing agent (a surfactant) that prevents particle agglomeration when strong magnetic field gradients are applied to the ferrofluid. Typical ferrofluids have about 5% magnetic material, 10% surfactant and 85% carrier, by volume. The magnetic material is typically a compound of iron such as Magnetite or Hematite. The surfactant is typically a carbon-based fluid or long chain fatty acid molecules such as Oleic
acid, Citric acid, tetramethylammonium hydroxide, etc that can form micelles. The polar head of the surfactant molecule adsorbs to the magnetite particle to which they are attracted to by ion-dipole forces. The non-polar tail is attracted to the liquid carrier. The resulting structure is an inverse-micelle as shown in the figure[1.

![Figure 2.4](http://jchemed.chem.wisc.edu/jcesoft/cca/cca2/MAIN/FEFLUID/CD2R1.HTM)

**Figure 2.4:** Multiple surfactant molecules forming inverse micelles around the central magnetite particle in the carrier fluid prevents agglomeration. Source (re-edited from): 1

Several surfactant molecules may adsorb to a single magnetic particle. When the surfactants attach to the magnetic particle, the separation between neighboring magnetic nanoparticles is increased. This prevents the particles from agglomerating to form clusters which will settle out because of their large mass. Thus, the particle-surfactant combination is small enough that Brownian effects are significant and the distribution of the magnetic particles is thus mostly uniform throughout. The surfactant thus makes the ferrofluid stable and prolongs the settling rate of the nanoparticles. The ferromagnetic fluids are different from magnetorheological fluids where the particle size is in the micron scale which causes them to settle down faster as Brownian motion cannot keep them suspended.

---

2.3.5 Conclusion

Motion of the magnetic fiber occurs when the magnetite nanoparticles in the hollow fiber, deposited along the inner surface of the fiber after evaporation of the carrier from the ferrofluid, are attracted by the net magnetic field due to external electromagnets and overcome the forces resisting the bending of the fiber. The fiber does not stretch significantly during this application of the external field, as the stretching forces are not very high. Various models seek to explain this process as a result of the various interacting forces, such as the magnetic moments of the magnetite nanoparticles, the bending force and stretching force of the fiber, in the presence of an external field. These different analyses model the fiber as a continuum elastic fragment or a string of magnetized particles, characterizing the same forces in a different manner. Most of the existing models deal with fibers at a much smaller scale than the one that we are using. Also, the external field has a gradient that varies in every direction, unlike the uniform magnetic field considered in most models.
Chapter 3

Selection of Magnets

3.1 Introduction

The weakly paramagnetic nature of the magnetic fiber demands strong magnetic forces to actuate it. This requires us to design strong magnets with high magnetic flux densities that can generate the required forces. Also, the magnetic fiber responds strongly to the potential gradient of the magnetic field, thus adding an extra consideration into the design of the system in terms of the orientation of the magnets. Some choices such as permanent magnets can automatically be ruled out because of the difficulty associated with operating them remotely, while other remotely operable magnets such as the powerful iron-core electromagnets are infeasible for fast efficient operation because of physical effects such as magnetic hysteresis and their associated time lags. Further, the placement of the chosen magnets determines the manner in which the fiber responds to changes in the field because of its intrinsic physical tendency to respond to the difference in magnetic potential. This chapter looks at different types of magnets that could be used to actuate the fiber as well as the different possible orientations and their associated pros and cons.
3.2 Relevant Magnetostatic Theory

The vector $H$ is a field vector in an infinitesimally small needle-shaped cavity that satisfies the following relation: $\oint H \cdot ds = 0$, which mathematically states that the work done in bringing a test pole from infinity to a point $(x, y, z)$ is independent of the path taken as the work done depends on the scalar potential $\varphi$ at the start and end points respectively. $H$ is related to $\varphi$ as its negative gradient: $H = -\nabla \varphi$

When the cavity is a disk of infinitesimally small height, the field vector is called the magnetic induction $B$. $B$ then satisfies Maxwell’s equations $\nabla \cdot B = 0$. $B$ and $H$ are related by $B = H + 4\pi M$ with $B$ in Gauss (C.G.S) or Tesla (S.I).

The magnetic flux $\Phi$ the flux of the vector $B$ through a surface of area $A$:

$$\Phi = \int_A (B \cdot n) dS$$

with $\Phi$ in Maxwell (C.G.S) or Weber (S.I).

The graphical meaning of flux is a group of field lines whose density is equal to $B$ and whose direction is along $B$. If $B$, $H$ and $M$ are parallel, then the permeability is defined by $B = \mu H$ and the susceptibility $\chi$ is given by $M = \chi H$. Also from the above relations, $\mu = 1 + 4\pi \chi$.

If $\chi < 0$, the material is diamagnetic and is repelled by magnetic fields towards to minima of the magnetic field strength. Most materials are intrinsically diamagnetic. This includes water, wood, protein, DNA, etc. If $\chi > 0$, the material is paramagnetic and is attracted by a small force toward the magnetic field maxima. This includes Oxygen, Platinum, etc. Ferromagnetic materials are strongly attracted to magnetic fields as their susceptibility $\chi \gg 0$. Diamagnetic and paramagnetic materials lose their magnetism after the imposing magnetic field has been removed. Ferromagnetic materials retain at least some magnetization after the
Figure 3.1: Comparison of the three main magnetic effects and their M-H curves (diamagnetism not included).
Source: Microsystems Laboratory, ETH Lausanne

external magnetizing field has ceased. Another class of magnetism is super-paramagnetism. Superparamagnetic materials have a core which can be as small as 10 nm. These are usually single domain magnetic particles that have ferromagnetic properties in bulk, but are prevented from clumping together with other ferrite particles (to form large ferromagnetic domains), by being encased in polymer shells or surrounded by inverse micelle fatty acid structures. They respond to the magnetic fields just like ferromagnetic particles and are more strongly attracted to them than paramagnetic substances, but unlike the former, these particles do not retain a remnant magnetic field once the external magnetizing field has ceased.

The force $F$ on a magnetic particle inside a magnetic field depends on the particle’s volume $V$, the difference $\Delta \chi$ between the susceptibilities of the particle $\chi_p$ and the medium $\chi_m$, and the strength and gradient of the magnetic flux density $B$ [19]:

$$F = \frac{V \cdot \Delta \chi}{\mu_0} \left( B \cdot \nabla \right) B$$

For diamagnetic particles in a diamagnetic medium like water, $\Delta \chi$ can be positive or negative causing the field to attract (negative $F$ causing the field to attract the particles towards the
local maxima) or repel (positive $F$ causing repulsion towards the field minima) the particles. In the case of a super-paramagnetic fiber in air, this force is positive, causing the fiber to move towards the field maxima. For homogenous fields, the field gradient is zero. The particle is then magnetized but not pulled or pushed in any direction, i.e. it does not experience any force.

### 3.3 Permanent Magnets

Permanent magnets are magnetic materials that produce a large magnetic flux with a very low mass. The ideal permanent magnet would have a square magnetic hysteresis loop with a large coercive field so that it does not lose its flux completely after it has been magnetized nor can it be easily demagnetized. Many ferromagnetic materials have this property and tend to stay magnetized long after the external magnetizing field has been removed.

Spatially non-uniform fields exert a force on the magnetic moment given by the energy gradient $F = \nabla (m \cdot B)$. They exert non-uniform forces on magnetic materials. These magnets are capable of producing very complex flux patterns with rapid spatial variation ($\nabla B > 100$ T/m). The equivalent solenoid electromagnets need to be several centimeters in diameter.

Time-varying fields can be produced by displacing or rotating the magnets. Varying fields can generate emfs according to Faraday’s law and produce Eddy currents and associated forces in a nearby conductor. Permanent magnets have a near rectangular $B$-$H$ curve due to their high permeability.
3.4 Hysteresis Effects

The magnetic characteristics of magnets are typically specified in terms of the magnet’s hysteresis loop. Specifically, the particular factors to be observed are the Coercivity and the Remanence of the magnet.

As shown in the figure, the non-magnetized material initially starts at the origin when the applied magnetizing force $H$ is zero. It then traces the path dashed path and eventually reaches saturation at around point $a$. Hysteresis refers to the fact that the magnetization curve cannot be retraced in the same cycle. This is due to the existence of magnetic domains which store energy when magnetized to some direction and require some extra energy to reorient them. Permanent magnets permanently retain this magnetization.
3.5 Choice of field/Magnet orientation

The electromagnets can be arranged in several different configurations to get a desired net field, whether in terms of the magnetic flux distribution, the magnetic potential or the magnetic force. The fiber, being paramagnetic in nature, responds most strongly to the change in magnetic potential, i.e. the magnetic force. The manipulation of the location of the magnetic force maxima and minima is thus required to move the fiber from point to point. Determining this analytically requires devising a new model that takes into account the various shapes and configurations of the fiber along with the other parameters mentioned earlier. The model desired would have a unique, invertible mapping of the currents in the solenoids to the orientation of the fiber, i.e. the various shapes it may take. This model may take into account various other parameters such as the permeabilities, the orientation of the magnets, the various distances involved, etc. The fiber also generates a small field around itself due to its magnetic nature and this factor needs to be taken into consideration. The constrained nature of the fiber (being cantilevered at one end) makes it different from models that consider a free unconstrained fiber. The nonlinear nature of the magnetic field along the length of the fiber would make this model different from existing models that consider only uniform magnetic fields that have a constant value in the region that the fiber operates in.

We take the approach of using a simple linear controller to control the currents flowing through the field. For example, one approach that we take is to use a single integrator in the system which, after in determines the position of the fiber, sends an appropriate amount of current to the field coils. The system continuously monitors the tip’s pixel coordinates (obtained from the video data) and constantly adjusts the current until the difference between the actual and desired coordinates has been reduced completely (we provide a small margin of error to account for the noise that the video carries with it due to the resolution of the microscope).
Another point to note is that the tip of the sting is not necessarily a tapered point at its free end. Due to the manufacturing process, the fibers have been cut at several points to get a shorter fiber for each experiment. This chopping of the fiber, results in a cross sectional area that appears as a straight line. We assume the last pixel to be one of a possible series of points. And this particular factor is also responsible for the constant variation in the tip position as read by the algorithm, despite checks provided in the software side to correct the noise in the images.

While the signals sent by the Quanser controller are very small, the actual current requirement is determined by the specifications of the solenoid. The current amplifier can amplify the input current it receives from the Quanser board (which is in the mA range) to several Amps. This though, may damage the electromagnet due to gradual build up of heat. Also, it is unfeasible to have a system that requires several amps of current to run to be made compact in the long run. The electromagnet chosen needs to have a small amount of current flowing through it. This in turn requires that the number of turns of the magnet wire around the electromagnet -having determined the magnetic flux requirements before hand - be proportionally large. A large number of turns thus requires that the wire diameter be small in order that the system is made compact and the core is small. This increases the resistance per unit length of the total wire wound around the magnet as the resistance is inversely proportional to the cross-sectional area of the wire. Additionally, the large number of turns requires that the wires be wound continuously over multiple layers so that a large amount of flux is confined to a smaller volume.

To summarize, we require a large number of turns of magnet wire over multiple layers around the electromagnetic core, with a small amount of current running through it. Another key factor in increasing the strength of the magnetic force is the permeability of the magnetic material. The higher the permeability, the stronger the magnetic flux density at the surface of the electromagnet.
Unlike metal alloys, Composite Powdered-metal cores store considerable energy in the non-magnetic regions between the high permeability magnetic particles, i.e. within the air gap or the binding material. These non-magnetic regions are distributed throughout the core during the manufacturing process. By varying the particle size and amount of magnetically inert material in the initial mix, the effective permeability of the material can be varied.

3.5.1 Electromagnet Cores

Various electromagnet cores are available, as detailed in the accompanying chart below. There are two general classes of ferrite cores: Soft Ferrites and Hard Ferrites. The terms soft and hard relate to the coercive force required to demagnetize the material from their remanent magnetization field down to zero. Hard ferrites are usually permanent or near permanent magnets (like iron) with a coercive force of around 2000 Oersted (159155 A/m), while soft ferrite cores require only around 0.5 to 4 Oersted (39.78 to 318 A/m). Remanent magnetic flux density is the flux density remaining in the material after the applied magnetic field strength has reduced to zero. The low remanent magnetization of soft ferrite cores makes them much more attractive than the hard ferrite alternative because we require the magnetic field to be turned off as soon as the magnetizing currents are removed. Otherwise, we will have to supply a large amount of current quickly in the opposite direction to cancel the existing field, which would lead to inductive and heating effects. We will also have to take into account the time delay that this correction would introduce will creating appropriate controllers for the system, thus adding to system complexity.

Soft ferrites have the relative disadvantage of producing weaker fields compared to hard ferrites. Yet these ferrites can be made to produce strong fields. Several soft ferrite cores are available commercially. These include Moly-permalloy (MPP), Kool-mu, High-flux and
regular ferrite cores. These are available in a large variety of shapes and sizes and are used in a large number of applications ranging from transformers to chokes in computer cables.

The above mentioned soft ferrites are powdered cores with distributed air gaps. Each has been designed according to their intended applications. MPP has a heavy content of Nickel (79%) along with 17% Iron and 4% Molybdenum. This alloy has the lowest core loss among all powdered core materials. High flux powder cores have a higher content of iron (50%) along with 50% Nickel giving it a higher flux capacity among the powdered cores. The Kool-Mu is used primarily for high frequency applications because of its low (almost zero) magnetostriction, especially at higher frequencies.

These powdered cores are used in power inductor applications like SMPS output filters, differential inductors and flyback transformers. Each material has its own advantage: High Flux has the highest flux capacity, MPP has the lowest core loss, and Kool-Mu has the lowest material costs.

### 3.5.2 Ferrite Cores

Ferrites are dark grey or black ceramic materials\(^2\). The general composition of ferrites is \(\text{MeFe}_2\text{O}_4\) where \(\text{Me}\) represents one or several divalent metals such as Manganese (Mn), Zinc (Zn), Nickel (Ni), Cobalt (Co), Copper (Cu), Iron (Fe) or Magnesium (Mg). Ferrites are made by sintering iron oxide with oxides and carbonates of Mg and Zn or Ni and Zn\(^3\). A sintered ferrite consists of crystals that are \(\approx 10\text{-}20\) micrometers in size. The domains in these crystals are already aligned due to ferrimagnetism. When an external magnetic field is applied to the material, the domains align with the field.

The relative permeability of power ferrite materials is in the range of 1500 to 3000. Regular ferrite cores have a wide range of permeability from 20 (Material 33) (at an initial perme-

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\(^3\)http://focus.ti.com/lit/ml/slup123/slup123.pdf
ability at less than 10 Gauss) to 2500 (Material 73) to 15000 (Material H). When an air-gap is introduced in the material, magnetic polarization becomes more difficult. The flux density then becomes lower. The effective permeability is given by

\[ \mu_e = \frac{\mu_i}{1 + \frac{G\mu_i}{l_e}} \]

where \( \mu_i \) is the initial permeability, \( G \) is the air-gap length and \( l_e \) is the effective length of the magnetic circuit. As the air-gap gets longer, the effective permeability will increase.

Ferrites are also available in a wide variety of shapes and sizes, including those not available with other powdered metal cores. These cores are also considerably cheaper than MPP or Sendust. The main disadvantage of the ferrite core is that it is a ceramic and shatters very easily in high shock environments. Also, care must be taken when handling the ferrites as they can easily split when dropped. The saturation flux density of ferrites is also generally lower than the powdered metal cores. Despite these disadvantages, the ferrite core was chosen for this experiment because it was available in the rod form which would enable the winding.
of a solenoid. The rod shape was not readily available for the other cores which were usually available as toroids or E-rings.

3.5.3 Rounding of the $B-H$ Characteristic

In composite metal cores and ferrite cores, the non-magnetic gaps (air or binder) that hold the metal or ferrite particles together store energy within the core. This is significant enough to cause the $B-H$ characteristics of the core to become rounded all over and thus be less than ideal. The ideal loop is a square loop with a high magnetic permeability and very little stored energy, called a Sharp Saturation characteristic. The presence of the non-magnetic gaps (also
Figure 3.5: Flux lines tend to take the path with more magnetic particles, avoiding the gaps as much as possible. The gaps become the preferred medium when the denser regions have reached saturation referred to as magnetic hard spots) will cause the gradual decrease of incremental permeability until saturation is reached. This is known as the Soft Saturation characteristic.

Unlike ideal metal alloy cores, the flux change is not discrete from inside to outside. The flux change and the flux are distributed across the entire core because of the soft saturation. At low magnetic flux densities, the flux lines tend to take the path where the magnetic particles are in highest concentration as these are much more permeable than nearby gapped areas. Further, as the magnetic flux density increases, the regions where magnetic particles are in close proximity tend to saturate first. These regions are then not readily available for the magnetic flux lines to pass through. A further increment in the flux causes them to go to nearby unsaturated areas where the density of magnetic particles is much less and the gap is much wider.
Thus, as the flux density increases in this manner, the overall permeability of the material progressively reduces, causing the softening of the $B-H$ curve.

### 3.6 Core used in the experiment

The core used in the experiment is Material 61 ferrite core (CWS ByteMark, Orange, CA) which is a high frequency NiZn ferrite used for inductive applications up to 25 MHz and EMI noise suppression above 200 MHz.

### 3.7 Electromagnet orientation and potential curves at various currents

Below, we examine some graphs showing the variation of magnetic flux density for different values of currents when two magnets are turned on. The plots show the variation in the magnetic flux between the East and North field coils as we set the current in the East coil at a constant 1 Amp in the North and East magnets.

As can be seen from the plots, a minima is created between the two magnets when measured from the East to the North magnet. This minima tends to stay roughly in one location when the currents on either magnet are changed.

### 3.8 Conclusion

The 61 material ferrite core used in the experiments has a low hysteresis. It is highly controllable as it responds quickly to the change in current without showing significant time lag afterwards. The field generated by the solenoid arrangement is sufficient to properly actuate the magnetic fiber.
Figure 3.6: Contour plot of magnetic flux density B when the East and North magnets are charged and attract.

Figure 3.7: Variation of magnetic flux density with distance from surface of a solenoid. Graph shows variation along each red segment from East to North magnet.
Chapter 4

Experimental Framework

4.1 Introduction

This chapter describes the design and construction of a new experimental setup to control the position and orientation of the tip of a magnetic fiber. The magnetic manipulator is designed to generate typical magnetic fields between 0.06-0.02 Teslas at 1 Amp is presented in this chapter. The solenoid electromagnets generating the fields are controlled independently by closed feedback loops. Video microscopy, data acquisition boards and computerized image analysis are used to control and guide the motion of the magnetic fiber for two-dimensional translations and rotations.

This setup offers a simple two-dimensional control of the tip of the magnetic fiber. The current setup has 4 solenoids, but only two are used in the experiment to actuate the magnetic fiber. The experimental setup explained here provides a new method to measure the magnetic properties of the fiber as well as provide a framework for future experiments using the magnetic fiber. The framework built provides a highly flexible way to replace or add new magnets in a manner that does not compromise the existing structure.
4.2 Design Requirements

4.2.1 Range of forces

The key objective in this experiment is to generate sufficiently strong magnetic fields that can actuate the fiber from a distance with precise control of the position of the fiber. Based on the experiments conducted to gauge the response of the fiber to magnetic forces, the minimum magnetic field strength required to induce a noticeable response in the field is determined to be roughly 0.05 Tesla. While this level of magnetic flux density is simple to produce, a stronger field is required to produce significant 'useful' motion in the fiber. This value is anywhere above 0.2 Tesla with the magnetic field gradients varying more strongly at higher flux values. This requires the electromagnets used to have more turns or higher amperage. Thus, what is required is the generation of magnetic fields of around 0.2 Tesla, varying over a few centimeters, with the magnetic field gradient changing rather smoothly.

4.2.2 Controlling the magnetic field

The control of the magnetic field is necessary in order to precisely control the movement of the fiber within the distances considered. One method of doing this is to use electromagnetic solenoids whose currents can be controlled through a computer. The use of soft magnetic material as the magnet core is preferred over hard magnetic material because the latter is capable of producing strong hysteresis effects with remnant magnetic fields that are difficult to offset without some penalty in the timely control of the target or the use of rapidly changing currents. Soft magnetic materials are preferred because they tend to have a lower residual magnetic field and also because a wide range of such material with different desirable magnetic and electric properties is available commercially at low cost. Examples of commonly used soft magnetic materials include Molypermalloy (MPP), Hi-Flux, Kool-Mu and soft ferrite cores. The
solenoids employed here use soft ferrite cores because of their desirable magnetic properties and their commercial availability in easy to use rod form.

4.2.3 2D manipulation of the fiber

The actual control of the fiber was achieved using a setup that consists of a microscope placed underneath the magnetic fiber that records the motion of the fiber and simultaneously transmitted the images to a computer. Image analysis was then performed on a computer, from which the position of the fiber tip was determined. This position has errors which causes the current tip position to fluctuate as the fiber moves. The position was then fed into a software model of the feedback loop. The model was responsible for controlling the signals to the individual magnets. This model was designed to be run in real-time on special hardware - a real time controller. Signals from the controller were then fed to each of the individual electromagnets independently after being amplified to sufficiently strong amounts. The entire feedback loop from the image acquisition to the generation of the magnetic field controlling the fiber takes only a few milliseconds - a time period that can be sped up further by improving on the efficiency of the code used, the computer performing the analysis and by lowering the noise of the image acquired from the microscope camera. Another factor that could possibly speed up the final control is the hysteresis performance of the magnet cores themselves. The control of the field generated currently assumes that the hysteresis is low enough for the present purpose that time lags can be ignored. The stable control of the fields is still possible using the feedback loop as the controllers used in the loop constantly change their output based on the position of the fiber, which is the net result of among other things, the elastic forces within the fiber and all magnetic effects, not all of which accurately modeled into the system.
Figure 4.1: Contour Plot of magnetic field in a plane. All four electromagnets are placed as shown in this simulation. Here, all the field coils have 1 Amp current running through them.

Figure 4.2: Flux lines with alternate field coils having 1 Amp and -1 Amp current flowing through them.

4.3 Apparatus

4.3.1 Principle

The magnetic fields produced need to have a uniform variation in field gradient. The magnetic setup used in the experiment consists of four mutually perpendicular soft ferrite magnets.
placed around the magnetic fiber. The distances of the front face of the individual magnets from the fiber can be varied by moving the magnets forward or backward with respect to the fiber. This may become essential when there is a need to retain a strong field but keep the fiber in a more controllable region of the magnetic field where the variation of the magnetic potential is not as severe as near the solenoids’ faces. The magnetic fiber is held downwards from an acrylic base such that it becomes cantilevered with one end immersed in the field and the other attached to the base. The field is controlled by a setup that regulates the current flowing through the solenoids. The feedback loop controls the generation of the field from all four magnets independently based on the positions of the tip as observed from the microscope placed directly underneath the fiber.

**Magnetic Coils and Pole Pieces**

The magnetic field is produced by four individual electromagnets which are solenoids made from soft ferrite core with copper wire turns around it. The core used is low permeability 61 Material soft ferrite rods purchased from McMaster-Carr. Each core has an outer diameter of 0.375”(9.525 mm) weighing 8.6g for rods 1” (25 mm) long, with a magnetic permeability of
Figure 4.4: Variation of magnetic flux density from the surface of the active electromagnet to the surface of the opposite electromagnet is shown. This simulation is for a solenoid with 1350 turns of 28 AWG coil over 6 layers wound continuously.

125 (at Flux density $B < 0.001$ T (10 Gauss))$^4$. The individual cores are wound with 1350 turns of 28 AWG (0.014” or 0.0037 mm O.D.) copper wire solid conductor over 10 layers. The wire has a resistance of 0.214 ohms/meter and a current capacity of 0.378 A/sq. cm. This gives the wire a net resistance of approximately 17.3 ohms for a total length of 80 meters. The wire is coated with oil-based protective enamel to provide better heat dissipation and to immobilize the lower layers during the coil winding stage.

4.3.2 Electronic and Computer Control

The key feature of this experimental setup is the precise control of the fiber across the required region. This is achieved by regulating the magnetic field in the region of interest by controlling the current using images fed into the feedback loop from the microscope. The data is used to determine the position of the fiber and the controller generates appropriate currents to control the magnetic field and thus, the tip of the fiber. The figure shows a block diagram of the entire loop.

Feedback Circuitry

The feedback circuitry consists of four solenoid electromagnets with 1350 turns each. Two Techron linear amplifiers with a combined 4 output channels supply currents to each electromagnet. A Quanser Q4 control board supplies signals to the input channels of the linear amplifiers. The signal sent is received by the Q4 board from a PCI board on the PC. The data is processed in a MATLAB environment running the software developed to control the whole setup - from the controller to the image analyzer. The controller has an integrator which constantly compares the difference between desired and actually position of the fiber and accordingly adjusts its output to the Quanser board. Currently, two mutually perpendicular solenoids are actively being used to control the motion of the fiber in a section of its workspace.

Computer Control

The process starts with the program written on MATLAB. This program activates the camera in the microscope which replies with the first frame. The camera is interfaced with Matlab using a free program called VCAPG2 \(^5\) which is a ’MATLAB resident video capture program’ that uses Microsoft’s DirectShow Libraries and runs on Matlab version 7.2.0.232 (R2006a). (The program is available for free download from the MATLAB Central File Exchange as a dll file). The program captures an image frame and feeds it to the Matlab program that calls it. The Frame is then analyzed through an image analysis program that looks for the

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\(^5\)http://www.ikko.k.hosei.ac.jp/matlab/matkatuyo/vcapg2.htm
fiber in the image within certain specified limits. After identifying the fiber, it then looks for the tip of the fiber which it then returns to the calling program. The base program then uses this fiber tip coordinate to set the initial values in the Simulink model shown. After this, the program asks for the user to set a desired destination point (or a series of points in case of the multiple path program) using mouse clicks on the figure. Another alternative is to enter the desired coordinates at the command line. The desired coordinates are then set in the Simulink model. The basic model is previously built to check for errors. Once the current and desired parameters are set, the controller is then activated, which downloads the model from the server to the client. The application used is a real-time Windows application called WinCon that runs code generated from Simulink in real-time on the local PC. The real-time code is generated as a stand-alone controller that runs independently of Simulink. The parameters in the Simulink model (the coordinates in this case) may be changed on the fly through Simulink. The Simulink model consists of an Integral controller which compares the actual and desired coordinates and accordingly adjusts the output based on the difference.

Figure 4.6: Experimental Setup.
between the two, until a final setpoint for the magnetic field is reached or a preset cap on the maximum output is reached - whichever comes first. The discrete time integral controller has a gain of 0.01 which was determined to be sufficient to drive the individual coordinates to their setpoint. The dc loop gain is a product of this gain and the gain obtained from the amplifier. The output from the WinCon controller is sent to a Quanser Q4 board (from Quanser Inc., Ontario, Canada). The Q4 is a hardware-in-the-loop control board that supports up to 4 (analog or digital) inputs and 4 outputs. The output from the Q4 board is then amplified by a linear amplifier that finally feeds the current into the individual electromagnets.

Each coordinate in the four quadrants (planar) that the fiber moves across is represented by a single solenoid. Thus the four magnets (N-S-E-W) representing the Positive/Negative abscissa and ordinates are used to guide the fiber across a range of coordinates. The four magnets are independent of each other and receive their signals from the Quanser board (through the linear amplifier) individually. In the experiments conducted, presently only two of these solenoids are being used simultaneously to actuate the fiber.
**Coil Current Generation**

The four electromagnets receive their currents from an AE Techron 5530 Linear Amplifier (AE Techron Inc., Elkhart, IN) which can output a maximum of 150 watts per channel. The currents passing through the electromagnets are measured using a Tektronix TDS3034B 300 MHz 4 Channel Digital Phosphor Oscilloscope. The loop is designed such that the cap on the PC output is reached at a value of 4 from the Simulink model, which corresponds to a maximum of 2.68 Amps for any given electromagnet. If left unchecked, this amount of current over a short period of time (about a few minutes) can damage the electromagnets. If repeated several times, the heat generated can permanently damage the coil winding and the coils would then have to be replaced. A recommended time period for currents above 2 Amps would be 1 minute, after which a cooling down period would have to be observed. One reason for this is the coil coating, which can be replaced with better epoxy that can dissipate higher amounts of heat without damaging the inner coils.

**4.3.3 Image Acquisition and Analysis**

The images of the magnetic fiber are acquired using a QX5 computer microscope (Digital Blue Corporation, Marietta, GA). The camera has a 640 X 480 pixel VGA resolution and is set at 10X magnification. The camera is USB connected and is powered by it. It generates a maximum of 15 frames per second. The camera is held pointing upwards towards the fiber that is held downwards from the base as shown in the figure. It can be separated from its holder, thus making it possible to hold it in the desired orientation.

The camera is orientated in such a way that it points to the fiber with the fiber’s base at the center of the image. The individual frames are analyzed by the fiber detection program developed in MATLAB. The program takes a sub-image of the frame to reduce computation time and converts the image from RGB to Black and White after setting a gray threshold.
suitable for the lighting. This is threshold value is hardcoded into the program when the initial test is performed to gauge the lighting requirements. The threshold is performed to eliminate the large amount of noise that is a result of the camera’s specifications.

From the resulting image, the program follows the fiber pixels and locates the very last pixel and sets this as the tip pixel. This value is then stored as two separate variables available for use to any calling program. The program is set up as a function in MATLAB that when called, returns the fiber’s tip coordinates, as well as a processed image that consists of a highlighted portion that represents the fiber that the code has identified.

The code assumes that

1. The lighting is sufficient for detecting the fiber
2. The preset sub-image coordinates contains the fiber

The function is called repeatedly with each successive frame that the base program receives. The return rate of the code is dependent on the processing speed of the machine, but the
code generally processes about 5 frames per second. This speed is sufficient for low speed movement of the fiber.

The image produced has the pixel coordinates printed at the tip pixel as shown in the figure. The frame number is also listed at the top of the image. The fiber itself is indicated by a blue color and the fiber tip is indicated by a yellow pixel, while the destination of the fiber is indicated by a small green square a few pixels thick. Finally, the path that the fiber’s tip takes is indicated by white as shown in the image.

The coordinates of every pixel that the fiber’s tip follows is continuously recorded for later analysis. An example of this is shown in the figure which shows a plot of the difference coordinates (abscissa and ordinate) of the desired and actual positions. With an integral controller, the following effect is observed.
4.4 Conclusion

The experimental framework thus provides us with a method to conveniently analyze different motions of the fiber and its response to different kinds of magnetic fields. The framework can be further improved by improving the power requirements of the field coils. This can be done by increasing the number of turns in the solenoid. Additional magnets need to be positioned outside the current plane to make the magnet move in a close surface within its workspace. An
additional camera needs to be provided in another plane that will provide full 3 dimensional control.

The video analysis currently identifies the fiber's motion in the region of the workspace where the two active magnets cause the fiber to move. This needs to be extended to include all four visible quadrants. The image analysis can be improved further to reduce noise and improve the detection of the fiber tip and making the image analysis robust.
Bibliography


