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INHOMOGENEOUS CHEMICAL EVOLUTION OF THE GALACTIC DISK

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ABSTRACT

We present analytical models for inhomogeneous chemical evolution (ICE) of systems in which the star formation history resembles a series of bursts, localized in space and/or time, with intermittent periods of remixing. The additional parameter of this model is the metallicity increment of bursting subsystems, but this parameter is constrained by the spread in the age-metallicity relation. We apply this model to the solar annulus in the Galactic disk and show that ICE models yield an improved fit to the observed shape of the stellar abundance distribution function (ADF). The G-dwarf problem can be alleviated with ICE models, but infall of metal poor gas and/or some preenrichment of the disk during the epoch of protogalactic evolution is still required to explain the paucity of low-metallicity dwarfs. ICE models also suggest an explanation of the reduced frequency of metal-rich G-dwarfs relative to the predictions of the simple model. It does not seem likely that chemical evolution of the solar annulus proceeded in a medium that was well-mixed at all times. So, while the simple model may be used to describe the evolution of bulk abundances in a well-mixed system, we promote the use of simple ICE models to describe the evolution of systems that locally undergo enhanced star-forming bursts, but in which the resulting chemical enrichments are only slowly remixed and homogenized within the global system.

Subject headings: stars: abundances — stars: evolution — stars: statistics

1. INTRODUCTION

The steady increase of data quantity and quality in the study of gaseous and stellar components of galaxies has in recent years encouraged theorists to develop numerical models of galactic evolution in which the coupled chemodynamical equations of the interacting gas-star system are solved consistently (Hensler 1987; Burkert, Truran, & Hensler 1992; Malinie, Hartmann, & Mathews 1991; Theis, Burkert, & Hensler 1992). However, the solutions of these equations must still be considered with caution because they are based on poorly known initial conditions as well as uncertain prescriptions for a number of important "microprocesses" such as star formation and supernova heating of the interstellar medium (ISM). The challenge of developing a comprehensive theory of galactic chemical and dynamical evolution (GCD) has yet to be met, but significant progress toward that goal has been made for the subproblem of chemical evolution. The evolution of bulk abundances in the Galaxy is arguably the best-understood aspect of GCD. Most studies of chemical evolution utilize all or some of the approximations of the so-called "simple model" (one zone, instantaneous recycling, instantaneous mixing, closed box, homogeneity, etc.; Searle & Sargent 1972; Pagel & Patchett 1975; Tinsley 1980). Because of its simplicity, this model has been used extensively to gain insight into the bulk chemical evolution of the major stellar components of the Galaxy; halo, bulge, and disk (e.g., Pagel 1993). While shortcomings introduced by the instantaneous recycling approximation (IRA) have long been recognized and avoided in realistic models (Talbot & Arnett 1971; Matteucci & Francois 1989), the effects of inhomogeneities in the ISM have also been recognized (Schmidt 1963; Dixon 1965, 1966; Hearnshaw 1972; Talbot & Arnett 1973; Pagel & Patchett 1975; Tinsley 1975; Nissen 1988) but have not been implemented in realistic chemical evolution models. To investigate this problem without resorting to detailed numerical simulations, and to develop a simple benchmark against which more elaborate models can be compared, we derive a modified simple model of the evolution of the solar annulus of the Galactic disk. This modified model takes spatial and/or temporal inhomogeneities into account, but otherwise retains all of the approximations that make the simple model so useful.

There are many observational features that have to be included in GCD studies of the solar neighborhood, but if one is concerned only with chemical evolution, then the two most important observational constraints are the age-metallicity relation (AMR) (Twarog 1980; Twarog & Wheeler 1982; Carlberg et al. 1985; Schuster & Nissen 1989) and the abundance distribution function (ADF) of long-lived stars formed in the vicinity of the solar annulus (e.g., van den Bergh 1962; Schmidt 1963; Pagel & Patchett 1975; Pagel 1989a). Most simulations address the temporal evolution of the mean metallicity; but Gilmore (1989) emphasized that the AMR derived from F stars in the solar vicinity exhibits scatter much in excess of the experimental uncertainties. There is clearly a real intrinsic scatter in the enrichment history of the ISM (see Mayor 1976 for an early review). This spread in metallicities at a given age, which is also noticeable in nearby clusters and groups (Nissen 1988; Boesgaard 1989; Lambert 1989), is not described in the framework of simple models of chemical evolution.

The observed scatter seems to imply that the disk contains

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chemical inhomogeneities that can avoid remixing long enough to allow star formation in both the enriched and unenriched portions, say 10^{8-9} yr. Gilmore (1989) suggested that the scatter could be a consequence of self-enrichment in giant molecular cloud (GMC) complexes. The lifetime of GMCs is short compared to the time scale of galactic evolution, but much larger than the lifetime of the massive stars which produce the bulk of the Galactic chemical enrichment ΔZ. Therefore self-enrichment could occur inside a GMC. Even after the disruption of the GMC, the ΔZ inhomogeneity remains. It spreads only slowly through the annulus. The time for the disrupted medium to again enter a new GMC is less than 10^9 yr, so the enriched region can produce stars somewhat later that have ΔZ relative to those forming elsewhere at that time. So we do not restrict our thinking to Gilmore's self-enriched GMC. A series of enrichment/dispersal phases would lead to a sawtooth progression of metallicity with age (Gilmore 1989; Pilyugin 1992), thus providing a natural explanation to the observed scatter in the abundance distribution of stars at a given age. Here, we derive a simple model for the present-day ADF that results from this kind of sawtooth evolution.

After summarizing the basic features of the observed AMR (§ 2) and the well-known G-dwarf problem (§ 3), we employ a simple model describing a series of enrichment/dispersal phases to derive the abundance distribution of stars resulting from such inhomogeneous chemical evolution (§ 4). We show in § 5 that ICE models do not solve the G-dwarf problem, but that the resulting theoretical ADF can provide a much better fit to the overall shape of the observed ADF. Conclusions and applications of our modified simple model are presented in § 6.

2. THE AGE-METALLICITY RELATION

Ultraviolet color excesses are correlated with stellar metal content (Wallerstein 1962), so that a systematic change in the metal content of the ISM should result in an observable correlation between stellar metallicities and ages. This color-age relationship has been noted (Sandage 1962, 1982), and Twarog (1980) used four-color and Hβ photometry of a large sample of southern F dwarfs in conjunction with theoretical stellar evolution isochrones to derive the first convincing estimate of the AMR in the solar neighborhood. The metallicity- and age dependence of stellar luminosity leads to a spread of sampling distances in a magnitude limited sample. Twarog (1980) corrected for this selection effect by calculating $V/V_{\max}$ weighted averages, but he did not correct for vertical metallicity gradients or for an increase of scale height with age. The results of this study, based on a final sample of 329 stars, suggests a significant rate of increase of the disk's metal content, δ[Fe/H] ~ 0.3 per 10^{10} yr. Carlberg et al. (1985) used a revised metallicity calibration and updated isochrones to re-derive the AMR from a sample of 255 stars. These authors confirmed the basic trends in the AMR found by Twarog, but obtained a somewhat slower rate of change. However, the analysis of Nissen et al. (1985) tends to confirm the Twarog data. Figure 1 shows the results for both of these data sets. Barry (1988) used the correlation between stellar chromospheric activity and age to derive an AMR for 44 F and G stars within 25 pc of the Sun. His results show a still smaller rate of metal enrichment, if any, but the uncertainties in the activity-age relationship are still rather large. Assuming a disk age of 12 Gyr, Rana (1991) suggests the following fit to the data

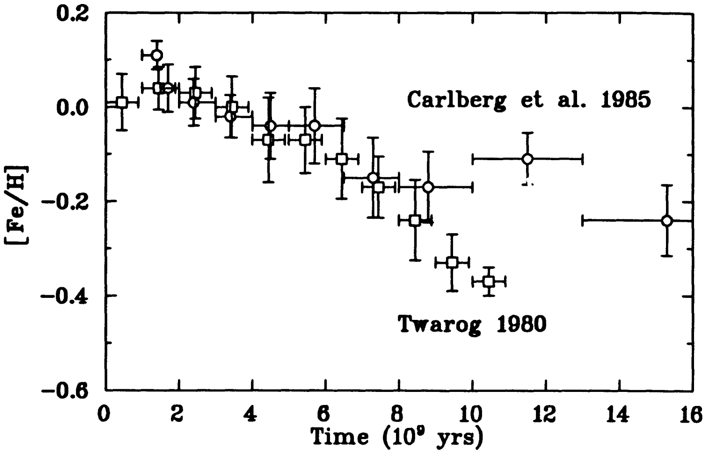

FIG. 1.—Age metallicity relation (AMR) of F dwarfs in the solar neighborhood. The data of Twarog (1980) are derived from four-color and Hβ photometry of 329 stars (*circles*). Carlberg et al. (1985) reanalyzed the Twarog data including information on the age-velocity-dispersion relation (*squares*). Both investigations suggest a rising metal content of the gas in the solar annulus with time, but the slope is different in the two studies. Shown is the evolution of the mean metallicity. At any time the spread around the mean is ~ 0.2, comparable to the total increase over the age of the disk.

$$\log\left(\frac{Z}{Z_\odot}\right) = \left[\frac{\mathrm{Fe}}{\mathrm{H}}\right] = A - B(C + t)^{-1} , \qquad (2.1)$$

with $A = 0.68$, $B = 11.2$ Gyr, and $C = 8$ Gyr. According to this fit, the initial disk metallicity was [Fe/H]$(t = 0) = -0.72$, or $Z(0) = 0.19\ Z_\odot$. At the birth of the solar system, $\sim$4.5 Gyr ago, the mean metallicity in the ISM was $\sim$90% solar, and the present metallicity in the ISM is $\sim$30% above solar. The slightly elevated metal content of the solar nebula with respect to the average ISM suggests that perhaps the birthplace of the Sun was located inside one of the regions that had evolved a little further than the average material near the solar neighborhood.

The slope in the AMR at the beginning of star formation in the disk ($t = 0$) according to equation (2.1) is

$$\frac{dZ}{dt}(0) \sim 7 \times 10^{-4}\ \mathrm{Gyr}^{-1} . \qquad (2.2)$$

This value is close to the slope

$$\frac{dZ}{dt} \sim 0.001\ \mathrm{Gyr}^{-1} \qquad (2.3)$$

found for the AMR of halo stars (Schuster & Nissen 1989), suggesting that the chemical evolution of disk and halo were perhaps connected in a continuous fashion (Sandage & Fouts 1987). It is quite clear from the data of Schuster & Nissen (1989; their Fig. 7) that the observed spread of ages at a given metallicity is intrinsic to the stars and not just due to measurement errors. Similarly, the spread in metallicites in a given age bin for the data of Carlberg et al. (1985, their Fig. 3) also appears to be well in excess of instrumental uncertainties (although the authors do not discuss this point explicitly). Additional support for an intrinsic spread comes from recent high precision abundance determinations of nearby clusters and groups (Boesgaard 1989; Lambert 1989). The dispersion of [Fe/H] in these data is $\sigma \sim 0.1$–0.2, and corresponds to a dispersion in metallicity (Edmunds 1975) of

$$\delta Z \sim 3\ \sigma\ Z_\odot \sim 0.01 . \qquad (2.4)$$

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The dispersion estimates of Twarog (1980) indicate that the spread roughly decreases linearly with increasing metallicity. Therefore it is reasonable to take δZ to be approximately constant.

3. THE G-DWARF PROBLEM

A fossil record of the nucleosynthetic history of the solar neighborhood is preserved in the relative freqency of long-lived stars as a function of metallicity, usually taken to be represented by [Fe/H]. Early estimates of this abundance distribution function (ADF) clearly suggested that there are fewer low-metallicity stars than predicted by simple chemical evolution models (van den Bergh 1962; Schmidt 1963). This deficit is now known as the G-dwarf problem. Based on a volume-limited sample of 132 G-dwarfs within ~25 pc of the Sun, Pagel & Patchett (1975) derived a reliable cumulative ADF, confirming the G-dwarf problem. Using the improved metallicity–UV-excess calibrations of Cameron (1985) and convolving the data with 0.2 dex spread to take into account the observed spread, Pagel (1989a) reanalyzed the ADF of Pagel & Patchett. He advocates the use of the differential ADF (number of stars per unit metallicity) to fit models and suggests a variable transformation from [Fe/H] to [O/H], because of the IRA assumption in simple models. We follow his prescription

$$\log \Phi = [\mathrm{O/H}] = [\mathrm{Fe/H}]/2 \tag{3.2a}$$

for $[\mathrm{Fe/H}] \geq -1.2$ and

$$\log \Phi = [\mathrm{Fe/H}] + 0.6 \tag{3.2b}$$

for $[\mathrm{Fe/H}] \leq -1.2$. Figure 2 shows the differential ADF as a function of the oxygen abundance. The simple model of chemical evolution yields a straight line in this representation of the data (see below). The relative paucity of low-metallicity G-dwarfs is evident. We also note the deviations from a straight line for high-metallicity stars. Although the statistics of the highest metallicity bin is poor, this paucity of high-metallicity stars (relative to simple model fits), if real, could represent a high-metallicity analog of the G-dwarf problem. We emphasize that the detailed shape of the ADF at the high metallicity end could provide significant constraints on the models (see below) and we urge observers to focus some attention on that part of the ADF as well.

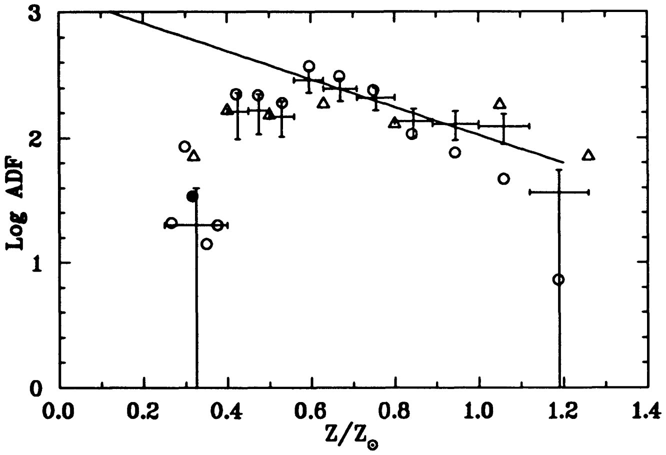

FIG. 2.—The differential Abundance Distribution Function (ADF) of G-dwarfs in the solar neighborhood. Shown is the number of stars per unit metallicity (oxygen). The data of Pagel & Patchett (1975), revised by Pagel (1989a), are shown with error bars. Correcting for the increase in velocity dispersion with time, Rana (1991) determined a modified G-dwarf ADF (*circles*). At low-metallicities four points given in Rana (1991) are averaged into one data point (*filled circle*). Also shown are the estimates of Sommer-Larsen (1991), who also applies some corrections for the vertical height distribution of dwarfs (*triangles*). To within the error bars all three methods appear to agree well with each other. The simple closed box model of chemical evolution (*straight line*) does not fit the low-metallicity part of the observed ADF. This paucity of low-metallicity stars relative to simple model predictions is referred to as the G-dwarf problem.

The ADF of Pagel & Patchett (1975) is derived from a volume-limited sample of 132 G dwarfs. According to the AMR (§ 2), low-metallicity stars are, on average, older than those with higher metallicity. G-dwarf masses are of order 1 $M_\odot$, so that their velocity distribution has increased with time due to encounters with giant molecular clouds and other scattering agents in the disk. Consequently, the scale height of G-dwarfs increases with their age, leading to reduced numbers of low-metallicity dwarfs in a volume-limited sample. Rana (1991) used a simple fit to the AMR

$$\log\left(\frac{t}{1\ \mathrm{Gyr}}\right) = 0.93 + 1.30\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right] - 0.04\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right]^2 \tag{3.3}$$

and an age-dependent scale height

$$h_z = h_z(0)\left(1 + \frac{t}{t_0}\right)^{2/3}, \tag{3.4}$$

with $t_0 = 0.5$ Gyr and $h_z(0) = 95$ pc, derived from stellar dynamics simulations of Wielen (1977) and Villumsen (1983), to derive corrections factors for the transformation between the number of G-dwarfs born in the solar neighborhood and their currently observed density. In Figure 2 we show the Rana (1991) ADF (corrected for a few typographical errors in 1 and renormalized using the average enhancement factor of 5.83). The scale height corrections do alter the ADF in some parts, but the paucity of stars on both ends of the distribution and the linear midsection remain salient features. Significant changes in the ADF due to a change from volume densities to surface densities, which is the relevant quantity in the context of chemical evolution models, were also obtained by Sommer-Larsen (1991), who considered several models for the vertical disk structure of the solar neighborhood. Their KG-corrected ADF, arbitrarily renormalized by adding 3 to the logarithm of their relative number, is also shown in Figure 2. The data of Pagel (1989a) appear to be intermediate between the estimates of Rana and Sommer-Larsen. An alternative approach was taken by Rana & Basu (1990), who compiled data from a heterogeneous set of sources of magnitude-limited samples. Arguing that the increased sampling depth for low-metallicity stars (due to the metallicity dependence of the luminosity) compensates, to first order, the increased scale heights discussed above, they derive an ADF for 214 G-dwarfs. Although the authors do not think that there exists at present absolutely reliable data and a good sample to confront chemical evolution theories, their ADF appears to be similar to that of Pagel (1989a). However, the authors also note that the Pagel ADF appears to be broader and shifted to lower metallicities. Clearly more work is needed to clarify these observational issues. Rana & Basu (1990) also note that the ADF is deficient in both the extreme metal poor and extreme metal rich stars. They attribute the bulk of both deficits to the particular data selection and analysis techniques employed by Pagel and his co-workers. From the model presented in the following section it will be clear that these observational questions must be

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resolved before a rigorous test of chemical evolution of the solar annulus can be attempted.

There have been many proposed solutions to the G-dwarf problem, and here is not the place to review all of these attempts. We merely mention generic features of possible solutions and present a few models to have a benchmark for later comparison with our results derived below. One solution to the G-dwarf problem (Truran & Cameron 1971) suggests prompt initial enrichment of the disk, i.e., some metal contamination operating prior to the onset of star formation in the disk. This solution roughly accounts for the sharp drop of the ADF at low metallicity, but clearly does not fit the observed shape. The data (Fig. 2) suggest that $\Phi(0) \sim 0.3$, or log $[Z(0)/Z_\odot] = [\mathrm{Fe/H}](0) \sim -1$. Although this assumption may appear somewhat ad hoc, the value implied by the disk data coincides with the transition between the observed disk and halo globular cluster population (Zinn 1985; Armandroff & Zinn 1988). Also, recent calculations of Galaxy collapse (Burkert, Truran, & Hensler 1992; Mathews & Schramm 1993) suggest that the protodisk would have been enriched to $[\mathrm{Fe/H}] \sim -1.5$ in the halo formation phase, thereafter increasing to $[\mathrm{Fe/H}] \sim -0.5$ during the thick disk stage lasting ~ 0.5 Gyr. Combined halo-disk chemical evolution models (Pagel 1989b; Ferrini et al. 1992) have shown that good fits to the G-dwarf ADF can be obtained for reasonable halo formation parameters. On the other hand, the presence of local disk stars with metallicities as low as $[\mathrm{Fe/H}] = -1.6$ (Morrison, Flynn, & Freeman 1990) suggests that the prompt initial enrichment picture is an oversimplification of the early chemical evolution of the disk.

Another class of solutions to the G-dwarf problem invokes infall of metal-poor gas from the halo (Larson 1972; Tosi 1988; Matteucci & Francois 1989; Pagel 1989b). Radial flows through the solar annulus can also be important for establishing the present-day ADF (Tinsley & Larson 1978; Mayor & Vigroux 1981; Lacey & Fall 1985; Pitts & Taylor 1989; Clarke 1989, 1991; Sommer-Larsen & Yoshii 1989). Employing the IRA and a linear dependence of the star formation rate on gas density Clayton (1984, 1985, 1986, 1988) and Clayton & Pantelaki (1986) developed analytic inflow models for the synthesis of primary and secondary species, respectively. The rate of inflow of metal-poor gas in Clayton's (1985, 1986) "standard model" is expressed by

$$f(t) = \frac{kM_g(0)}{\Delta}\left(\frac{t+\Delta}{\Delta}\right)^{k-1} \exp(-\omega t)\,, \tag{3.5a}$$

with k an integer and Δ and ω are fitting parameters. All quantities of interest can then be expressed analytically, in analogy to the closed-box model. The total gas mass then evolves according to

$$M_g(t) = M_g(0)\left(\frac{t+\Delta}{\Delta}\right)^{k} \exp(-\omega t)\,, \tag{3.5b}$$

and the total disk mass grows with time. The initial disk, prior to the onset of star formation, is assumed to be completely gaseous, i.e., $M_{\mathrm{tot}}(0) = M_g(0)$. We introduce the dimensionless time $\zeta = t/T_D$, where T_D is the age of the disk. Since T_D is uncertain and Δ is a fitting parameter to be selected, we combine them into the parameter $\eta = T_D/\Delta$. For the $k = 1$ family a value of $\eta \sim 10$ gives an acceptable fit to the observed AMR (Clayton 1986), but the AMR is not very sensitive to the parameter Δ so that η could substantially differ from the nominal value 10 used in this study. With $x = 1 + \eta\zeta$ the metallicity of the gas evolves according to

$$Z(\zeta) = y(k+1)^{-1}\Omega\eta^{-1}(x - x^{-k})\,, \tag{3.6}$$

where the parameter $\Omega = \omega\Delta\eta$ can be determined from the present-day metallicity $Z_p = Z(1)$. The differential ADF (dN/dZ) of Clayton's standard model is given by

$$\Upsilon_C(Z) = \frac{1}{y}(k+1)x^k[1 + kx^{-(k+1)}]^{-1} \exp(-\Omega\zeta)\,. \tag{3.7}$$

These infall models alleviate the G-dwarf problem because the disk mass is build up over an extended period, so that the birthrate of low-metallicity stars is reduced with respect to models in which all the gas is available for star formation from the beginning. We assume that the metallicity of the infalling matter is zero. The growth factor of the disk is determined by the present-day mass fraction of gas in the disk, μ_p, via

$$A_m = \frac{M_{\mathrm{tot}}(\zeta = 1)}{M_{\mathrm{tot}}(\zeta = 0)} = (1+\eta)^k \mu_p^{-1} \exp(-\Omega)\,. \tag{3.8}$$

In Figure 3 we show $k = 0, 1, 2, 3, 4$ models for $\eta = 10$, $\mu_p = 0.1$ and $y = 0.52\ Z_\odot$ as a benchmark for comparison with ICE models. The curves are normalized to log $\Upsilon = 0.2(10 - k)$ at $Z_p = 1.2$. The disk mass of these models grows by factors $A_M = 1.0, 1.6, 2.2, 3.0$, and 4.2, respectively. The $k = 0$ model is the closed box model. Clearly, these infall models can alleviate the G-dwarf problem, although a good fit to the observed shape of the ADF does not seem possible. Similar inflow models were derived by Lynden-Bell (1975). Although a simple model with initial disk enrichment would have a low-z turnoff that is much too sharp, and the shape of the infall models does not quite match the data, one can imagine that a combination of both models could provide good fits of the ADF for reasonable choices in parameter space. The drop in the ADF is smooth for both the Clayton and Lynden-Bell models, but both have extended low-z tails that may imply too many stars

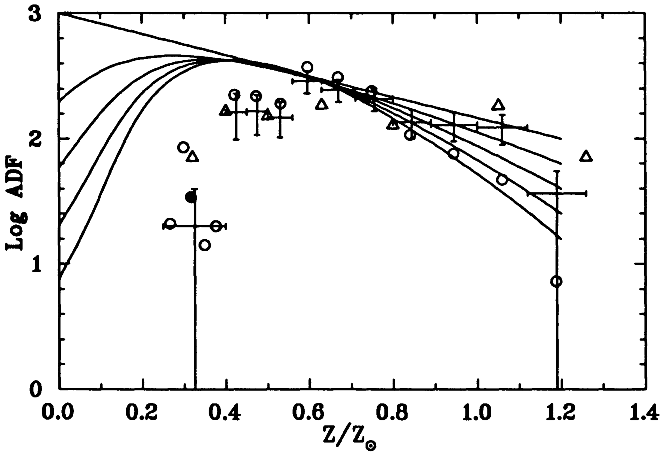

FIG. 3.—The G-dwarf problem can be addressed with infall models. Shown are several such models using the analytic standard prescription of Clayton (1985). The parameter (see text) are $\eta = 10$ and $k = 0, 1, 2, 3, 4$. The $k = 0$ curve is identical to the closed box simple model shown in Fig. 2, which fixes the yield to $\sim 0.5\ Z_\odot$. There is no initial enrichment in these models, and the infalling gas has zero metallicity as well. Although these curves yield improved approximations to the observed ADF, their detailed shapes do not seem to match the observations well.

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at extremely low metallicities. Furthermore, none of these models explains the drop in the ADF at high metallicities. The effective yield of Clayton's models is $y/(k+1)$ so that a steepening of the ADF at higher metallicities is observed (Fig. 3), but the models behave almost like closed box models at late times, and thus one obtains straight lines instead of a smooth rollover. The ICE models developed below yield satisfactory fits at both ends of the ADF.

One of the reasons for chosing G-dwarf observations is the assumption that all G-dwarfs ever formed are still present. Since stellar lifetimes become shorter with decreasing metal content, this assumption could break down, leaving fewer low-z dwarfs. Inclusion of metallicity dependent main-sequence lifetimes was shown to slightly alleviate but not solve the G-dwarf problem (Bazan & Mathews 1990). A similar reduction of low-z low-mass stars can be achieved if a metal-dependent initial mass function (IMF) favors high-mass star formation at lower metallicities (Schmidt 1963; Pagel & Patchett 1975; many others). Alternatively, if the metallicity of the ISM does not bias the IMF but the overall star formation efficiency, then the presence of large scale inhomogeneities in the ISM could solve the G-dwarf problem if stars were preferentially formed in regions with above-average metallicity (Talbot & Arnett 1973). In this picture of metal enhanced star formation one needs to achieve the average $\delta Z = 0.01$ for a system in which only a fraction of the total gas mass, f, undergoes star formation. Single supernovae or even multiple supernovae in OB associations fall short of this requirement by several orders of magnitude, leading to the suggestion that either supernova-triggered star formation or differentiation processes within the star forming regions need to be invoked to enhance the metallicity spread (Edmunds 1975). Alternatively, Grenon (1973) suggested that if the solar neighborhood G-dwarf sample contains significant contributions from stars formed well outside the sampling volume, any significant metallicity gradient, known to be present in our Galaxy (Shaver et al. 1983; Grenon 1987; Rubin et al. 1988) could provide a source for part of the observed spread. In that case we would expect the oldest stars to sample the widest range in galactocentric radius and thus the dispersion should increase as the metallicity decreases. Obviously, the correct treatment of all the effects mentioned here require a full GCD approach. In this work we do not consider Galactic metallicity gradients and dynamical considerations, but focus on a simple model that accounts for enhanced evolution in subsets of the system, which thereafter are mixed with the remaining system.

4. INHOMOGENEOUS CHEMICAL EVOLUTION

We argue that the chemical evolution of the disk does not proceed smoothly, but rather in discrete star formation episodes, localized in space and/or time, followed by period of remixing with the disk. The detailed evolution could have been quite complex, but we seek a generalization of the simple model that can describe the main features of such inhomogeneous chemical evolution (ICE) while retaining the analytic power of the simple model. Thus, to derive the ADF for ICE models we make the following simplifying assumptions:

1. The ADF of the solar neighborhood follows from the chemical evolution of a globally closed system (of unit mass). Initially the mass fraction of stars is assumed to be zero, but the initial metallicity, Z_0, is assumed to be nonzero to account for possible preenrichment.

2. The IMF and the effective yield, $y_{\rm eff}$, are assumed to be constant throughout the evolution. The value of $y_{\rm eff}$ is determined from the observed present-day metal abundance in the interstellar gas, $Z_p \sim Z_\odot$, and the present-day gas fraction in the solar neighborhood, $\mu_p \sim 0.2 \pm 0.1$ (e.g., Binney & Tremaine 1987).

3. As in the simple model we assume that the IRA holds, although this approximation is not likely to be useful if one wishes to study very early and very late phases in Galactic evolution (e.g., Clayton & Pantelaki 1986).

4. Star formation does not proceed homogeneously throughout the whole system, but occurs in a series of events that take place in subsystems with fraction f of the total Galactic mass. Each subsystem is assumed to evolve like a closed-box simple model until star formation terminates. The enriched gas of the subsystem is then remixed into the global system. The dilution is assumed to homogenize efficiently the gas in the global system at the end of each star-forming epoch. The disruption is assumed to occur when the gas inside the subsystem has been enriched by an amount δZ. This increment of the metallicity is assumed to be constant and is obtained from the intrinsic scatter in the AMR (§ 2). This assumption of constant δZ is, perhaps, resonable if one imagines a self-regulation of star-forming regions whereby once there is sufficient supernova heating per unit mass the region is dispersed. It is also consistent with star formation induced by cloud-cloud collisions or star formation induced by passage of material through a spiral arm.

Assumptions (1)–(3) are standard assumptions of the simple model. Assumption (4) accomplishes the desired minimal generalization of the simple model for systems with an inhomogeneous star formation and mixing history. The chemical evolution of this system (self-enrichment in many subsystems, followed by dispersal into the global medium) is expected to lead to a sawtooth progression of mean metallicity with age (Gilmore 1989), while the observed spread of metallicity at all ages is a build-in feature of the ICE model.

Let Z_{n-1} and μ_{n-1} be the gas metallicity and gas mass fraction before the nth mixing event, respectively. We assume $\mu_0 = 1$. During every chemical enhancement period the metallicity in the star-forming subregion of mass f increases to $Z_{n-1} + \delta Z$, and the gas fraction decreases to $\mu_{n-1} \exp(-\delta Z/y)$. Metallicity and gas fraction in the remaining part of the whole system do not vary. In the subsequent dispersal phase, the gas fraction in the system is assumed to reach its new average level before the next burst, i.e.,

$$\mu_n = f\mu_{n-1} \exp(-\delta Z/y) + (1-f)\mu_{n-1} = a\mu_{n-1} , \quad (4.1a)$$

where we have defined

$$a = 1 - f(1 - e^{-\delta Z/y}) . \quad (4.1b)$$

The fraction of the total mass in heavy elements is then

$$\begin{aligned} M_h &= f\mu_{n-1} e^{-\delta Z/y}(Z_{n-1} + \delta Z) + (1-f)\mu_{n-1} Z_{n-1} \\ &= \mu_{n-1}(aZ_{n-1} + b\,\delta Z) , \end{aligned} \quad (4.2a)$$

with

$$b = f\,e^{-\delta Z/y} = a + f - 1 . \quad (4.2b)$$

The average metallicity after homogenization of the system is then

$$Z_n = \frac{M_h}{\mu_n} = Z_{n-1} + \frac{b}{a}\,\delta Z . \quad (4.3)$$

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The above equations are consistent with the familiar relation between Z_n and μ_n,

$$Z_n = Z_0 + y_{\rm eff} \ln\left(\frac{1}{\mu_n}\right) , \quad (4.4)$$

but with $y_{\rm eff}$ the "effective yield" now given by

$$y_{\rm eff} = \frac{f \delta Z e^{-\delta Z/y}}{-a \ln a} = \frac{b \delta Z}{-a \ln a} . \quad (4.5)$$

The metallicity distribution resulting from this simple ICE model is the sum of the distributions of stars formed in every stage

$$\Upsilon(Z) = \sum_{n \geq 0} \Upsilon_n(Z) , \quad (4.6a)$$

where

$$\Upsilon_n(Z) = \begin{cases} (\mu_n/y) \exp\left[-(Z - Z_n)/y\right] & \text{if } Z_n \leq Z \leq Z_n + \delta Z \\ 0 & \text{otherwise} . \end{cases} \quad (4.6b)$$

5. RESULTS

We have computed the metallicity distribution $\Upsilon(Z)$ from equations (4.6a) and (4.6b) using $Z_0 = 0.3\ Z_\odot$ and an effective yield determined from

$$Z_p = Z_0 + y_{\rm eff} \ln\left(\frac{1}{\mu_p}\right) . \quad (5.1)$$

We have chosen $Z_p = 1.2\ Z_\odot$ (eq. [2.1]) and μ_p is the present-day mass fraction of the interstellar gas in the solar vicinity.

To determine this present-day gas fraction, one needs to estimate the surface mass densities in the solar neighborhood. From the kinematic properties of stellar tracer populations, a dynamic estimate of the total surface mass density, Σ, can be derived. Kuijken & Gilmore (1989a, b, c; hereafter KG) find $\Sigma = 46 \pm 9\ M_\odot\ {\rm pc}^{-2}$, revised slightly in a more recent study; $\Sigma = 48 \pm 9\ M_\odot\ {\rm pc}^{-2}$ (Kuijken & Gilmore 1991). However, there is substantial debate about the Galactic z-force at large distances above the plane, and the integrated disk surface density of KG could be as much as 33% higher (Bahcall, Flynn, & Gould 1992). This range coincides with that suggested by Gould (1990). Values derived from the present-day mass function of main-sequence stars in the solar neighborhood also agree with these estimates (e.g., Basu & Rana 1992a). The gas density, estimated by Bahcall et al. (1992) with a four-component model for molecular, atomic, and ionized gas, is $13 \pm 4\ M_\odot\ {\rm pc}^{-2}$. A much smaller value of $6.6\ M_\odot\ {\rm pc}^{-2}$ was employed by Basu & Rana (1992a). Obviously, the present-day gas fraction is rather uncertain, $\mu_p = 0.2 \pm 0.1$ appears to be a reasonable range (Pagel & Patchett 1975), but we will use the mean value hereafter.

From equation (5.1) we thus obtain $y_{\rm eff} = 0.55\ Z_\odot$. The abundance distribution $\Upsilon_h(Z)$ resulting from the (homogeneous) simple model with the same initial metallicity Z_0 and gas fraction μ_p is

$$\Upsilon_h(Z) = \frac{1}{y} \exp\left(-\frac{Z - Z_0}{y}\right) , \quad (5.2)$$

and the total mass of stars (including remnants) ever formed is

$$\int_{Z_0}^{Z_p} \Upsilon_h(Z) dZ = 1 - \exp\left(-\frac{Z_p - Z_0}{y}\right) = 1 - \mu_p . \quad (5.3)$$

If the systems undergoes N mixing events, the parameter a can be determined from equation (4.1a) via

$$\ln \mu_p = \ln \mu_0 + N \ln a . \quad (5.4)$$

We will assume $\mu_0 = 1$. From equations (4.1b) and (4.5) we obtain

$$L = f \exp(-x) = -\frac{y_{\rm eff}\, a \ln a}{\delta Z} , \quad (5.5)$$

where $x = \delta Z/y$, and

$$f = 1 - a + L . \quad (5.6)$$

The star-forming average mass fraction f is thus determined from the effective yield and the metal enhancement δZ (which we obtain from the AMR spread). The real yield then follows from equation (5.5). If one considers only a small number of events, then the fraction of the system that undergoes separate evolution is large (e.g., $N = 10$ leads to $f \sim 0.5$). Since we envision that only a few percent of the system is "active" at any one time, we prefer to employ $N \sim 100$ (which gives $f = 0.035$). In that case, the true yield is $y = 0.72\ Z_\odot$.

The resulting ICE-ADF for the parameters discussed above is shown in Figure 4. It is obvious from this figure that the ICE model yields a smooth curve (N large) that matches the observations very well. Note that this is not really a fit, because the distribution follows from parameters that were justified on separate grounds. The only free parameter is the overall normalization, just as in the case of the infall models discussed in § 3. We have normalized the curve so that the maximum of the ADF is equal to its observed peak value. Although improved "fits" are conceivable, we do not pursue this avenue here. The

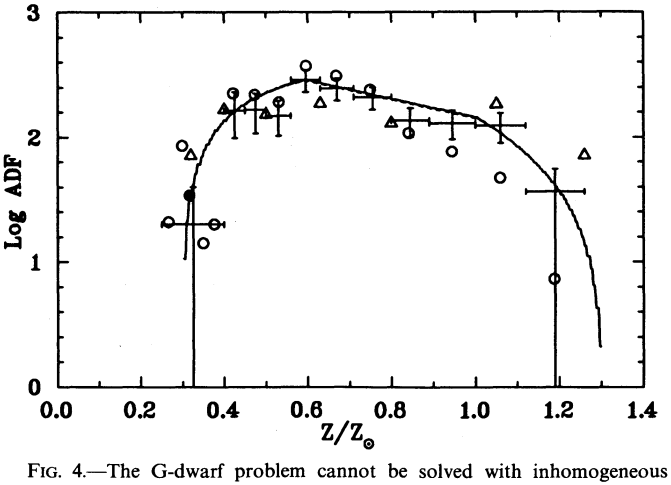

FIG. 4.—The G-dwarf problem cannot be solved with inhomogeneous models but if one assumes an initial enrichment $Z_i = 0.3\ Z_\odot$ the prescription of the ICE models (see text) give an effective yield of 0.58 $Z_\odot$ (real yield is ~25% higher) for $N = 100$ starburst episodes each involving ~3.5% of the total gas mass of the system. The chemical inhomogeneities in the model (0.3 $Z_\odot$) are taken from the spread in the AMR. The resulting ICE ADF (*solid curve*) appears to "fit" the data very well. In particular, the shape at the low-Z end of the ADF is matched far better than in any oher model invented to solve the G-dwarf problem. Also, note that the observed decrease of stellar frequency at the high-Z end is naturally explained by inhomogeneous models of chemical evolution of the Galactic disk.

ADF of the modified simple model using "standard parameters" is already good enough to match most data points to better than one standard deviation.

Let us summarize the most salient features of the ICE models.

1. Relative to the predictions of the simple model, the effects of inhomogeneous chemical evolution appear to improve the fits to the G-dwarf data (Pagel 1989a; Bazan & Mathews 1990; Rana 1991). In particular, the homogeneous model does not fit the data in the region $0.3\ Z_\odot < Z < 0.6\ Z_\odot$ and is unable to explain the possible smooth drop at metallicities above $\sim Z_\odot$. Considerations of inhomogeneous chemical evolution of the disk lead to improved fits to the shape of the metallicity distribution in exactly these regimes. This improvement is obtained with no free parameters as long as one interpretes the observed scatter in the AMR as a measure of the extent to which chemical evolution inside the inhomogeneities deviates from that of the mean annulus.

2. Since the present approach was carried out in the framework of the instantaneous recycling approximation, an interesting next step will be to check the consequences of finite stellar lifetimes (although the derivation would no longer be analytic) and more complex mixing histories. In addition, nuclear chronometers (which are explicitly time dependent) would also have been affected by ICE because the shorter lived chronometers, like ^{129}I and ^{24}Pu, will be sensitive to any recent bursts of star formation while the longer lived chronometers, like ^{235}U, ^{238}U, and ^{232}Th, will only be sensitive to the average star formation rate.

3. Our inhomogeneous model asserts that there is a link between the shape of the differential distribution of stellar metallicities and the scatter in the age-metallicity relation. Further data on both the ADF and the AMR are, therefore, urgently needed to support this connection in detail.

6. CONCLUSIONS

We have derived a modified simple model that describes the chemical evolution of closed systems in which a fraction of the mass becomes more metal enriched than the average before it is mixed with the rest of the system. The motivation for our approach was the extension of the simple model to include the effects of the observed metallicity spread for stars of a given age, while preserving the mathematical simplicity of the original model. We have shown that the resulting ICE models improve the fits to the shape of the observed ADF of G-dwarfs in the solar neighborhood. Although pregalactic enrichment, or some other traditional solution, is still needed to "solve" the G-dwarf problem, the additional feature of a metallicity dispersion improves the overall fit to the observed shape of the ADF at both extremes of the observed metallicity range. We note that no additional parameters are needed in ICE models if the metallicity dispersion due to inhomogeneities in the ISM is equated with the observed spread in the AMR. Basu & Rana (1992b) also noted that the inclusion of chemical inhomogeneities can improve the fits to the G-dwarf abundance distribution. In that work a convolution of the simple chemical evolution equations with a gaussian spread about the evolving mean metallicity (see also Rana & Basu 1990) was considered.

With respect to the ADF as a tracer of chemical evolution in the solar annulus, our results were based on $N \sim 100$ "starburst episodes" each involving a mass fraction of ~ 0.1. This yields a smooth "fit" to the observed ADF. This number of nucleosynthesis events is consistent with the idea that the nucleosynthesis history in the annulus is density-wave driven. Nuclear cosmochronology in such regular episodic star formation scenarios for the solar neighborhood was studied by Trivedy (1977) who concluded that discrete nucleosynthesis reproduces observed meteoritic isotope ratios consistent with short-, intermediate-, and long-lived chronometers. We will discuss nuclear chronometry for ICE models in a later paper.

Another application of the model discussed here are the gas-rich blue compact dwarf (BCD) galaxies which perhaps evolve globally as a closed box, but in which a few percent of the mass contributes to short episodes of starburst activity that are followed by longer periods of "quiesence" in which the enriched H II regions are remixed in the system (Kunth & Sargent 1986; Pantelaki & Clayton 1987; Matteucci & Tosi 1985; Pilyugin 1992; Clayton & Pantelaki 1993). The large spreads in the observed [N/O] and [O/H] abundance ratios in BCDs is consistent with ICE parameters of $f \sim 0.02$ and $N \leq 100$ (Pilyugin 1992), which is surprisingly similar to the parameters for the solar annulus considering the rather different environments found in BCD galaxies and late-type spirals.

The results of this investigation suggest that future chemical evolution models should attempt to take inhomogeneities into account. The modified simple model developed here suggests that refined fits to the observed ADF of long-lived stars are possible when inhomogeneities are included. Whether the scenario employed here is a realistic representation of the star formation history of the solar annulus is not clear, but the main conclusions reached in this study do not depend on the particulars of the "microprocesses" that drive the chemical (and dynamical) evolution of the Galactic disk. It seems inevitable that the chemical enrichment history was different for different parts of the Galaxy. Simple models for the evolution of average properties are of great importance for guiding our understanding of the global historic record as it is written in the atmospheres of low-mass stars. This study suggests that the stellar record contains retrievable information about the deviations from the mean evolutionary path.

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