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ASTRATION OF COSMOLOGICAL DEUTERIUM

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ABSTRACT

I reconsider the degree of astration of primordial deuterium by the continuous galactic processes of star formation and chemical evolution because of its importance to cosmology, because of its sharp dependence upon adopted values for the return fraction R and for today's gas fraction μ, and because I have found exact analytic solutions for galactic chemical evolution when infall of constant composition occurs at a rate $f(t) = kM_{G0}/\Delta[(t + \Delta)/\Delta]^{k-1} \exp -\omega t$. I present these solutions for the linear model with instantaneous recycling and with constant return fraction R. They suggest that big bang D/H was at least 3 times larger than the largest values observed in today's solar neighborhood and even larger yet if matter falling onto the disk is already astrated. The variations of this expectation with alterations of the model are described.

Subject headings: deuterium — nucleosynthesis

I. INTRODUCTION

The abundance of deuterium contains important constraints on any model of big bang nucleosynthesis. In isotropic models based on general relativity (the standard big bang), the larger the pregalactic D abundance the smaller the mass density must be, and the smaller the primordial ^{4}He abundance must be (Wagoner, Fowler, and Hoyle 1967). Because D is converted at least to ^{3}He upon incorporation into any star, the D abundance found in today's galactic interstellar medium must be less than the primordial D abundance if, as will be assumed in this paper, no galactic sources of D exist. The question I address here is: How much have the processes causing the galaxies to evolve in abundances lowered the D abundance if it is totally astrated within stars and if no significant production of D occurs in galaxies? This question has already been addressed in several papers (e.g., Audouze and Tinsley 1974; Ostriker and Tinsley 1975; Audouze *et al.* 1976). My reasons for returning to it are three: (1) its overwhelming importance to cosmology; (2) to detail its sensitivity to the parameters of galactic evolution; and (3) analytic models with time-dependent infall (Clayton 1984*a*, *b*).

The equations are simple. The mass of interstellar gas in a solar annulus changes at the rate

$$\frac{dM_G}{dt} = -(1 - R)\psi(t) + f(t)\,, \tag{1}$$

where $\psi(t)$ is the star formation rate, $f(t)$ is the infall rate, and R is the return fraction for instantaneous stellar evolution and is a constant if the initial mass function is constant. The instantaneous recycling approximation leads to somewhat erroneous results at the galactic center (Audouze *et al.* 1976) because the gas mass is so small there, but it is defensible for this problem in the disk portion of our Galaxy.

Assuming that all deuterium is destroyed when it is incorporated into a star, the rate of change of the D mass in the disk is

$$\frac{d}{dt}(\mathrm{D}M_G) = -\mathrm{D}\psi + \mathrm{D}_f f \tag{2}$$

where $\mathrm{D} \equiv X_\mathrm{D}$ is the mass fraction of D in the gas and D_f the mass fraction in the infall. Equation (2) can also be written, with the aid of equation (1), in the form

$$\frac{d\mathrm{D}}{dt} = -\mathrm{D}R\frac{\psi}{M_G} - (\mathrm{D} - \mathrm{D}_f)\frac{f}{M_G}\,. \tag{3}$$

If a fraction S_D of the returned deuterium actually survived the astration, the first terms on the right-hand sides would be multiplied by the factors $(1 - S_\mathrm{D} R)$ in equation (2) and by $(1 - S_\mathrm{D})$ in equation (3); however, because D is burned even in the pre–main-sequence surface convection zones, I will adhere to the simpler equations as written.

My first point is a physical one. The final D abundance, when the gas fraction in today's solar neighborhood is $\mu = M_G/M_\mathrm{tot} = \mu_\odot$, depends upon the time dependences for $\psi(t)$ and for $f(t)$. To see this clearly consider two limiting cases: (1) if $f = 0$ so that the gas declines as in a closed box, the final D will be much smaller than the initial D (see below), and (2) if the star formation rate ψ becomes much smaller than f at the present epoch, on the other hand, today's interstellar medium would be dominated by the infall having $\mathrm{D} \approx \mathrm{D}_f$, not astrated within the disk at all. A useful approximation to equation (3) in this regard can be obtained by estimating $d\mathrm{D}/dt \approx (\mathrm{D} - \mathrm{D}_f)/T$, as if deuterium has declined linearly from its initial value (taken to be equal to the infall D_f). Regrouping terms then gives the simple estimate

$$\frac{\mathrm{D}}{\mathrm{D}_f} \approx \frac{(1/T + f/M_G)}{(1/T + f/M_G + R\psi/M_G)}\,, \tag{4}$$

where all the quantities are evaluated at $t = T$. This approximation, which is pretty good for continuously evolving models, shows that arbitrary relations between f, M_G, and ψ allow many different results, as discussed immediately above.

It therefore seems to me that a nonrestrictive analytic class of models with physical continuity is the best for evaluating the expected degree of astration. I will present these in terms of the standard model family described by Clayton (1984*a*). These exact models, in which infall is today declining after an early maximum, span a useful space, allowing us to be somewhat less restrictive about the physical assumptions that are placed on analytic models: e.g., infall $f(t) =$ constant (Audouze *et al.* 1976, who call $f = \delta$ and who neglect to mention that their eq.

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[5] is restricted to linear models having constant infall); star formation rate $\psi(t) = \text{constant}$ (Tinsley 1981, who did not discuss D but who advocates constant ψ in her model); star formation rate $\psi(t)$ proportional to infall $f(t)$ (Ostriker and Tinsley 1975 in their "halo model"); and others. To avoid those simplifications of the relation between $\psi(t)$ and $f(t)$, Clayton (1984*a*, *b*) presented exact solutions for chosen families of time-dependent infall rates. To accomplish this he adopted the less restrictive assumption of the *linear model*, that the star formation rate be proportional to the gas available to form stars: $\psi(t) = \nu M_G(t)$, which he writes $(1 - R)\psi(t) = \omega M_G(t)$. This allows M_G to grow when early infall is strong and to decrease when infall abates, and to do so in a manner that seems physically plausible because it is the solution to a physical system. These analytic models will be chosen as the basis for estimating the astration of deuterium. Equations (1) and (3) then read

$$\frac{dM_G}{dt} = -\omega M_G + f(t) , \tag{5}$$

and

$$\frac{d\mathrm{D}}{dt} = -\frac{R\omega}{1-R}\,\mathrm{D} - (\mathrm{D} - \mathrm{D}_f)\,\frac{f}{M_G} . \tag{6}$$

My next point is another physical one. Equation (6) is exactly the same as the equation (e.g., eq. [10] of Clayton 1984*b*) for a radioactive species having half-life $\lambda = \omega R/(1 - R)$ and which is not synthesized by ongoing nucleosynthesis ($y_z = 0$). That is, the deutrium time dependence is identical to that of an imaginary nucleus whose initial abundance decays without regeneration by nucleosynthesis at rate λ, which will be of order $(5\ \mathrm{Gyr})^{-1}$ in infall models (see below), but which is regenerated by infall D_f. I will envision D_f as being constant in time, primarily because analytic solutions of equation (6) then are easily found but also because it is very unclear whether continuing astration of the infalling gas itself can sensibly lead to nonconstant D_f. It is the value of D_f during the last 80% or so of the galactic life that is the relevant value for equation (6), because early changes in D_f are obscured by a subsequent history of substantial infall. So I will assume constant D_f for this discussion. If the infall is not heavily astrated it may also be possible to identify D_f with the initial concentration in the disk, D_0, which might in turn be taken to measure big bang deuterium.

II. RETURN FRACTION AND GAS FRACTION

The astration of D depends sensitively on two quantities: the fraction R of gas that a generation of star formation returns to the interstellar medium and the gas fraction $\mu \equiv M_G/M_{\mathrm{tot}}$ that defines the present epoch. To see this sensitivity quickly and clearly, consider the closed-box model without infall, in which case (5) and (6) can be combined to read

$$\frac{1}{\mathrm{D}}\frac{d\mathrm{D}}{dt} = \frac{R}{1-R}\,\frac{1}{M_G}\,\frac{dM_G}{dt} \quad (f = 0) \tag{7}$$

with solution

$$\mathrm{D}/\mathrm{D}_0 = \mu^r \quad [r = R/(1 - R)] . \tag{8}$$

This solution was described by Ostriker and Tinsley (1975), who also showed that since metallicity $Z = -y_z \ln \mu$ in this simplified model, the anticorrelation of D gradient with metallicity gradient can be written (for primordial D) as

$$\frac{\mathrm{D}(Z)}{\mathrm{D}(Z_\odot)} = \mu_\odot{}^{r(Z - Z_\odot)/Z_\odot} . \tag{9}$$

But because they took the return fraction $R = 0.2$, giving $r = 0.25$, Ostriker and Tinsley (1975) may have underestimated both the μ dependence of the astration of D (eq. [8]) and its metallicity-gradient dependence (eq. [9]). With a choice $R = 0.5$ (see below), one would have $r = 1$ instead of 0.25, in which case $\mathrm{D}/\mathrm{D}_0 = \mu$, which constitutes a much greater degree of D astration in the solar neighborhood and elsewhere, and also a greater sensitivity to the correct value of $\mu_\odot$. Therefore we must briefly reconsider these quantities.

In her final paper Tinsley (1981) showed that stars below $0.1\ M_\odot$ do not contribute to the star formation rate. She also demonstrated that $R = 0.48$ for an initial mass function $\phi(m)$ favored by her, so I take $R = 0.5$ ($r = 1$) for mean numerical estimates. Although the R dependence of the models to be presented is not as great as in the absence of infall, equation (8) reminds us of the sensitive dependence to be expected in more realistic models. And with $R = 0.5$, equation (8) shows $\mathrm{D}/\mathrm{D}_0 = \mu_\odot$ in that simple model of the solar neighborhood, so that estimates of $\mu_\odot$ near 0.1 (e.g., Ostriker and Tinsley 1975) would show that 90% of primordial D has been astrated in a closed-box model of the solar neighborhood.

Tinsley (1981; see also 1980) has reconsidered the masses of the Population I system, to which disk evolution should apply and chosen $M_G = 8\ M_\odot\ \mathrm{pc}^{-2}$ (see also Sanders 1983) and $M_{\mathrm{tot}}\ (\mathrm{Pop\ I}) = 42\ M_\odot\ \mathrm{pc}^{-2}$, giving $\mu_\odot$ (today) = 0.19. This number is importantly larger than estimates based on total dynamic mass (Ostriker and Caldwell 1979), $\mu = 0.05$ to 0.10, because it means that D is less astrated in the solar neighborhood. Even so, in the simplified closed-box model the primordial D would have been concluded to have been 5 times greater than observed concentrations. Thus I will take $\mu_\odot = 0.19$ for best estimates in the analytic models. Should Tinsley's (1980, 1981) arguments prove incorrect, however, as they might if roughly $50\ M_\odot\ \mathrm{pc}^{-2}$ of dynamic mass represents hidden remnants of a larger early rate of star formation in the disk component, then the attendant lowering of $\mu_\odot$ to 0.1 or less would require a larger value for the primordial D abundance than the estimates that I will make in the following sections.

The dependence of D/D_f on the return fraction R is easily evaluated also in another extreme where constant infall f just matches a constant star-formation rate in such a way that the gas M_G is constant. Setting time derivatives in (5) and (6) equal to zero then gives immediately $\mathrm{D}/\mathrm{D}_f = 1 - R \approx 0.5$, some 2.5 times greater than in the closed box model.

Because neither of these simplified models is very realistic, I now present exact solutions for the time-dependent problem for a congenial parameterization of $f(t)$.

III. STANDARD MODEL WITH ASTRATION

Exact solutions with instantaneous recycling and a constant initial mass function were described by Clayton (1984*a*) for infall rates described by

$$f(t) = \frac{kM_{GO}}{\Delta}\left(\frac{t + \Delta}{\Delta}\right)^{k-1} e^{-\omega t} , \tag{10}$$

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where k is any positive integer, ω is as in equation (5), and Δ is a time parameter. The gas mass is then

$$M_G(t) = M_{GO}\left(\frac{t+\Delta}{\Delta}\right)^k e^{-\omega t}\,, \tag{11}$$

and the solution of (6) is, with D_f = constant and $D(0) = D_0$,

$$\frac{D}{D_f} = 1 - e^{-\lambda t}\left(\frac{\Delta}{t+\Delta}\right)^k\left[\lambda I_k(t,\lambda) - \frac{D_0}{D_f} + 1\right], \tag{12}$$

where $\lambda = \omega R/(1-R)$ and where the function

$$I_k(t,\lambda) \equiv \int_0^t \left(\frac{t'+\Delta}{\Delta}\right)^k e^{\lambda t'}dt'\,.$$

And the gas fraction follows from

$$M_{\text{tot}} = M_{GO} + \int_0^t f(t')dt'$$

and from equation (11) to be

$$\mu = \frac{M_G}{M_{\text{tot}}} = \left(\frac{t+\Delta}{\Delta}\right)^k e^{-\omega t}\left[1 + \frac{k}{\Delta}I_{k-1}(t,-\omega)\right]^{-1} \tag{13}$$

as described also by equation (20) and the subsequent recursion relation in Clayton (1984*a*, *b*). For this discussion I will take $k = 1$ corresponding to an exponentially declining infall rate. Larger values of k do not give significantly different results. For the sake of being specific I will also take $D_f = D_0$, as if predisk astration of the infalling gas is unimportant. This will give a lower limit to the total amount of astration within these models. Then equation (12) simplifies to

$$\left(\frac{D}{D_f}\right)_{k=1} = 1 - \frac{\lambda}{t+\Delta}\left[\frac{t+\Delta}{\lambda} - \frac{1}{\lambda^2} - e^{-\lambda t}\left(\frac{\Delta}{\lambda} - \frac{1}{\lambda^2}\right)\right] \tag{14}$$

and

$$\mu = (t+\Delta)\omega e^{-\omega t}[\Delta\omega + 1 - e^{-\omega t}]^{-1}\,. \tag{15}$$

A small survey of the values obtained with $R = 0.5$ are shown in Table 1, which displays results for two values of $\Delta = 0.1, 1.0$, for two values of gas fraction $\mu = 0.091, 0.19$, and for two values of the present galactic age $T = 10, 15$ Gyr. The values of ω shown are those needed to achieve the desired value of μ at time T for that choice of Δ. It is immediately obvious that for $\mu = 0.19$, the value $d \equiv D/D_f = 0.34$ independent of the value of Δ and of the galactic age T. This is reminiscent of the closed-model equation (8) where $d = \mu$, also independent of T, except that the astration is only about half as great. For $\mu = 0.091$, on the other hand, $d = 0.26$. Thus the $k = 1$ standard model with exponental infall gives substantial astration of cosmic D, though not as thoroughly as the simple closed box would indicate. These results are in fair agreement with approximation (4).

It is worth comparing at least one of these models with the model advocated by Tinsley (1981). Consider the case $\mu = 0.19$, $T = 15$, $\Delta = 1$. This model achieves a final mass 6.37 times greater than its initial disk mass owing to the infall, and the average star formation rate is $\bar{\psi} = 1.61\ \psi(15)$. Thus from Tinsley's $\bar{\psi}t = 67\ M_\odot\ \text{pc}^{-2}$ we get $\psi = 2.8\ M_\odot\ \text{pc}^{-2}$ Gyr today (at $T = 15$). Since $(1-R)\psi = \omega M_G$, this requires $\omega M_G = 1.4\ M_\odot\ \text{pc}^{-2}$ Gyr, which, with Tinsley's choice $M_g = 8\ M_\odot\ \text{pc}^{-2}$, gives $\omega = 1.4/8 = 0.175$, almost exactly the value $\omega = 0.172$ needed to give $\mu = 0.19$ in Table 1. This illustrates that these linear physical models are just as easy to use as more restrictive prescriptions for $\psi(t)$ and $f(t)$. And astration has cut D to 34% of its cosmic abundance. The approximation (4) for the astration is evaluated in this model by noting that, in units of initial disk mass, $M_G(15) = 1.21$, $f(15) = 0.076$, and $\psi(15) = \omega M_G/(1-R) = 0.42$, so that equation (4) gives $D/D_f = 0.42$, somewhat greater than the exact value 0.34 because $(D - D_f)/T$ has overestimated the time derivative dD/dt at $T = 15$. It is also worth noting from Clayton's (1984*a*) equation (19) that a primary nucleosynthesis product grows in $k = 1$ models according to

$$Z - Z_f = \frac{y_z\omega\Delta}{2}\left[\frac{t+\Delta}{\Delta} - \frac{\Delta}{t+\Delta}\right] \tag{16}$$

because it allows a simple analysis of gradient effects. Table 1 reveals that for $R = 0.5$ in $k = 1$ models $d \approx \mu^{0.6}$ as a rough rule of thumb. And Table 1 also shows that, at $T = 15$ say, a 37% larger value for ω, at an interior galactocentric distance, say, leads to a halving of μ and to a 37% increase, by equation (16), of the Population I metallicity. That is, an increase in metallicity by about $\frac{1}{3}$ reduces D/H to $\frac{1}{2}$ its local value. The gradient $d \log \text{O/H}/dr = -0.07\ \text{kpc}^{-1}$ inferred for the Galaxy by Shaver *et al.* (1983) would reach this condition at a distance of only 2 kpc toward the galactic center. An important challenge to astronomy is to measure this D/H gradient, as discussed by Ostriker and Tinsley (1975), in order to confirm that it is appropriately astrated for a cosmological relic.

The sensitive dependence of the astration on the crucial value of the return fraction R can be seen from evaluating equation (14) for a few values of R. In the $k = 1$ model having $\Delta = 1$, $\omega = 0.172$, so that $\mu_\odot = 0.19$, the solar astration $d_\odot = D_\odot/D_f$ assumes these values: $(R, d_\odot) = (0.6, 0.238)$, $(0.5, 0.340)$, $(0.4, 0.459)$. These values are also roughly $d = \mu^{0.65r}$. The point is that the range $R = 0.4$ to 0.6 causes $d_\odot$ to change by a factor of 2; therefore, precise evaluation of the return fraction is quite important.

a) Truncated Infall

The Clayton (1984*a*) standard model has the property that infall continues today. This in turn reduces the degree of astration within the disk of the interstellar gas, since much of it has just fallen in. If infall stopped at some time T_f, however, the

TABLE 1

STANDARD LINEAR MODELS ($k = 1$, $R = 0.5$)

Parameters	$T = 15$	$T = 10$
$\Delta = 0.1$, $\mu = 0.091$.....	$\omega = 0.248$, $d = 0.26$	$\omega = 0.370$, $d = 0.26$
$\mu = 0.19$......	$\omega = 0.181$, $d = 0.34$	$\omega = 0.271$, $d = 0.34$
$\Delta = 1.0$, $\mu = 0.091$.....	$\omega = 0.236$, $d = 0.26$	$\omega = 0.346$, $d = 0.26$
$\mu = 0.19$......	$\omega = 0.172$, $d = 0.34$	$\omega = 0.252$, $d = 0.34$

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degree of astration is almost as complete as in the closed box model. This can be easily seen in the standard models with continuous but truncated infall presented by Clayton (1984*b*); however, within the present context it is simpler to consider models in which $f(t)$ and $M_G(t)$ are as in equations (10) and (11) until time T_f, when f discontinuously goes to zero. The continuations for $t > T_f$ are then

$$D(t) = D(T_f)e^{-r\omega(t-T_f)}\ , \quad f = 0 \tag{17}$$

and

$$M_G(t) = M_G(T_f)e^{-\omega(t-T_f)}\ , \quad f = 0\ , \tag{18}$$

and the total mass M_{tot} retains a constant value for $t > T_f$. It is then a simple matter to choose, for each T_f, the appropriate value of ω to achieve $\mu_\odot = 0.19$ at $T = 15$. Utilization again of the $k = 1$ standard model prior to truncation then yields the results shown in Table 2. For truncation at $T_f = 10$, for example, in a 15 Gyr galaxy, the value of ω must be smaller, $\omega = 0.149$ as opposed to 0.172 for the untruncated infall, in order that the final gas fraction not deplete beyond $\mu_\odot = 0.19$, with the result that the astration is greater in the truncated model [$d(15) = 0.234$ as opposed to 0.340 in the untruncated case]. That is, the total astration can easily be a factor of 4 if infall occurred for no more than $\frac{2}{3}$ of the Galactic lifetime. The question of whether infall continues into the Galaxy today thus has a large impact on our interpretation of the deuterium abundance. Be at this point reminded of the basic assumption of this discussion, that the star formation rate is linear, $\psi = \nu M_G$, and that ν does not change when infall ceases. More arbitrary prescriptions for star formation cannot be so easily pinned down nor (unfortunately) disproven.

b) Quadratic Star Formation

For a given rate of infall, the final astration will be less if the physics of star formation enhances early star formation over the rate in the linear model. One good example is the quadratic star formation rate, $\psi = \omega_2 m^2$. Clayton (1985) has considered this model to show the ways in which the standard tests of galactic evolution are altered from the expectations of the linear model. We extend that comparison here to the D abundance. To make a long story short, the astration becomes $d = D/D_f = 0.57$ instead of 0.34 for the same infall rate (eq. [10]), with either $\Delta = 0.1$ or $\Delta = 1$. The reason is simple. For the quadratic model the final ($t = 15$) star formation rate is smaller than in the linear model, even though the infall rate and final gas fraction, $\mu_\odot = 0.19$, are identical. This result matches physical intuition, and the degree of change is in rough agreement with the simple approximation (4). Unfortunately, a (nonexistent) realistic theory will have to be very firmly established to argue that the star formation rate varies either more or less steeply than linearly in the total mass available to form stars. The differences highlighted by Clayton (1985) will help.

TABLE 2

TRUNCATED LINEAR MODELS ($k = 1$, $\Delta = 1$, $R = 0.5$)

T_f	ω[a]	$d(T_f)$[b]	$d(t = 15)$[c]
10......	0.149	0.493	0.234
11......	0.153	0.458	0.248
12......	0.158	0.424	0.264
13......	0.163	0.394	0.284
14......	0.168	0.365	0.309
15......	0.172	0.3430	0.340

[a] Chosen to give $\mu(15) = 0.19$.
[b] $D(t = T_f)/D_f$ at end of infall.
[c] D/D_f at $T = 15$ when $\mu = 0.19$.

IV. ^{3}He

The case of ^{3}He is much like that of D in that it is produced in the standard big bang nucleosynthesis model and it is astrated in stars. But there are two differences that should be explicitly noted in its differential equation: (1) since not *all* ^{3}He need be destroyed upon astration, let S_3 be the fraction of initial ^{3}He that survives in the return fraction R; and (2) because ^{3}He is also synthesized in stars, there must be a yield term. One part of the ^{3}He yield results from the proton–proton chain and is reasonably well understood (Iben 1967); however, a second part comes from the proton capture by the remaining primordial D, and thus decreases in proportion to D. Because much of the *p-p* chain ^{3}He comes from low-mass stars it is considerably delayed in its ejection (Iben 1967; Rood, Steigman, and Tinsley 1976). Therefore instantaneous recycling is not really a good approximation for that part of the ^{3}He yield, which effectively increases with time (per unit mass of star formation). For the yield from D, on the other hand, instantaneous recycling is a good approximation, but that portion of the yield decreases with time owing to the diminishing D abundance. It is not yet clear which part of the ^{3}He yield dominates. Denoting the yield by y_3 we have

$$\frac{d}{dt}(X_3 M_G) = -X_3\psi + X_3 RS_3\psi + X_{3f}f + y_3(1-R)\psi \tag{19}$$

reduces in the linear model $\psi = \omega M_G/(1-R)$ to

$$\frac{dX_3}{dt} = y_3\omega - (X_3 - X_{3f})\frac{f}{M_G} - \frac{R\omega}{1-R}(1-S_3)X_3\ . \tag{20}$$

This equation was solved for a constant y_3 in his standard model by Clayton (1984*a*), who showed (see his eq. [18]) that if $X_3(0) = X_{3f}$ = constant, then

$$X_3 - X_{3f} = (y_3\omega - \lambda X_{3f})e^{-\lambda t}\left(\frac{\Delta}{t+\Delta}\right)^k I_k(t, \lambda)\ , \tag{21}$$

where the pseudodecay rate owing to astration is $\lambda = (1-S_3)R\omega/(1-R)$. This exact analytic result for this family of infall rates is easily evaluated (Clayton 1984*a*). If y_3 cannot be reasonably approximated by a constant, there is little choice but the numerical integration of (20). Another reason for displaying equation (21) is that the same result obtains for D if it also has a source proportional to the star formation rate. I will not evaluate equation (21) at this time because both the fraction S_3 surviving return from stars and the stellar nucleosynthesis yield y_3 per unit increase of star mass are quite in need of better determination. But to those calculating these quantities, equation (21) and the standard family of models with infall are recommended as a comparison against numerical integrations.

V. CONCLUSIONS

I have reconsidered the problem of the destruction of cosmologically primordial deuterium owing to stellar astration. Chief results are:

1. The present gas fraction μ and the return fraction R (taken to be instantaneous) should be chosen more carefully

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than in previous works because the astration depends so strongly upon their values. This is illustrated by showing that $D/D_f = \mu^r$ in the closed box model and is equal to $(1 - R)$ in the steady-state model in which constant infall rate balances a constant star formation rate.

2. In linear star formation models with infall, astration appears as a pseudodecay rate, and the abundance equation can therefore be solved exactly within the families of time-dependent infall described by Clayton (1948*a*, *b*) in which infall composition is constant.

3. For $k = 1$ standard models (exponential infall), the surviving fraction of deuterium is $D/D_f = 0.34$ for final $\mu = 0.19$, independent of galactic age and of infall magnitude (Δ). Because this is, I think, close to the most probable case, we should probably interpret cosmological D/H as being about 3 times greater than the *largest* value seen in the solar neighborhood (because local astration inhomogeneities could produce a smaller D/H than average whenever a larger than average fraction of the matter viewed has been returned from stars).

4. The dependence of D/D_f on R for final $\mu = 0.19$ is listed. For the $k = 1$ standard models it is roughly equal to $\mu^{0.65r}$.

5. The way in which truncated infall demands that the degree of astration is greater (at fixed final μ) is explained and an exact example is evaluated. For standard $k = 1$ models truncated at $t = T_f$ and that have $\mu = 0.19$ today the results are roughly approximated by $D/D_f = 0.34 - 0.024(T - T_f)$.

6. The exact solution for the standard model is also given when stellar nucleosynthesis competes with astration (as for ^{3}He).

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