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EXTINCT RADIOACTIVITIES: A THREE-PHASE MIXING MODEL

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ABSTRACT

A new class of models is advanced for interpreting the relationship of radioactive abundances in the early solar system to their average concentration in the interstellar medium. The model assumes that fresh radioactivities are ejected from supernovae into the hot interstellar medium, and that the time scales for changes of phase into molecular clouds determine how much survives for formation therein of our solar system. A more realistic and physically motivated understanding of the low observed concentrations of ^{129}I , ^{244}Pu , and ^{107}Pd may result.

Subject headings: abundances — meteors and meteorites — nucleosynthesis — solar system: general — stars: supernovae

I. INTRODUCTION

The interpretation of the extinct cosmochronologies (traditionally ^{129}I and ^{244}Pu) has always been a little vague. The major problem has not been the existence of ^{129}I ($\tau_{1/2} = 17$ Myr) in the early solar system, but rather that it was then and there less abundant than expected in the ISM. Various models of continuous galactic nucleosynthesis (Wasserburg, Schramm, and Huneke, 1969; Fowler 1972) lead one to expect a steady-state concentration $(^{129}\text{I}/^{127}\text{I})_{\text{ISM}} \approx 2-5 \times 10^{-3}$, whereas the meteoritic evidence suggests instead $(^{129}\text{I}/^{127}\text{I}) \approx 1 \times 10^{-4}$. The discrepancy of a factor 20 to 50 has been attributed to a "free-decay interval" $\Delta \approx 10^8$ yr between the end of nucleosynthesis and the formation of meteorites. The vagueness lies in the astrophysical interpretation of that free-decay interval. These two approaches have been postulated, but not modeled:

1. The presolar cloud was isolated without nucleosynthetic admixture for 10^8 yr before collapsing;
2. ^{129}I synthesis occurs each 10^8 yr as the solar neighborhood passes through a galactic density wave (Reeves 1972), in which case the last passage caused solar collapse whereas next-to-last passage created the youngest ^{129}I found in solar material.

My purpose here is not to criticize these ideas, but rather to set forth a variation based on phase changes of the ISM. Although the astrophysics must be speculative, an additional virtue of these ideas will be the mathematical solution of an interesting set of equations. Their own value may survive the detailed astrophysical assumption of the model.

A related problem has recently emerged with the careful determination (Hudson *et al.* 1982) of the abundance $^{244}\text{Pu} \approx 0.005^{238}\text{U}$ in the early solar system, an abundance some 3 to 10 times smaller than previously believed (e.g., Schramm 1973). This new determination of the ^{244}Pu abundance shows it to have been some 60 times smaller than that of ^{235}U , a result difficult to reconcile with nucleosynthesis over galactic times. I do

not wish to reenter that difficult subject here (e.g., Fowler 1972; Schramm 1973), but only to point to the evidence that this extinct radioactivity is also less abundant (by a factor of ~ 3) than one would expect from concordant models of continuous galactic nucleosynthesis over almost 10^{10} yr.

The specific mathematical model to be discussed here is motivated by the following assumptions, which are also illustrated in Figure 1. However, analogous mathematical treatment is easily constructed for other assumptions:

1. The mass of the ISM is dominated by three phases: molecular clouds, large H I clouds too massive to be evaporated by an external supernova shock wave, and average H I clouds plus the warm media.
2. Solar collapse occurred in a large molecular cloud.
3. Molecular clouds exchange matter with massive long-lived H I clouds on a time scale T_1 , but not appreciably with the warm medium itself.
4. Massive H I clouds exchange matter with average H I clouds on a time scale T_2 owing to competing processes of evaporation and coalescence (Kwan 1979).
5. Average H I clouds are equilibrated isotopically with the warm and hot media, with which they exchange on short time scales against evaporation, $\sim 10^6$ yr—here taken to be instantaneous (e.g., McKee and Ostriker 1977).

Figure 1 therefore shows effectively three phases. Their masses, M_1 , M_2 , and M_3 , will be later chosen for numerical example. What this model does is force fresh radioactivities to "diffuse" into the molecular clouds at a rate measured by the time scales for the successive phase changes. I am assuming that molecular clouds exist only in massive H I complexes, and therefore exchange only with massive H I clouds. I will later take $T_1 \approx 10^8$ yr.

Average H I clouds are evaporated by supernova shocks (e.g., McKee and Ostriker 1977). Therefore, fresh ^{129}I is uniformly mixed over a medium containing the

FIG. 1.—Schematic diagram for the transport of fresh radioactivities made in supernova explosions into the collapsing solar system. The concentration X_1 in molecular clouds is determined by mass exchange with surrounding large H I clouds on a time scale T_1 . The concentration X_2 in the large H I clouds is governed by the time scale T_2 with which they exchange matter with average H I clouds. Because supernova shocks evaporate average H I clouds, their mass is included with that of the hot media in the initial reservoir for fresh radioactivities.

hot medium, the average clouds, and their surrounding warm media. It is not known how the large H I clouds form, but I take it that they do so by collisions of smaller clouds (Kwan 1979). Because such collisions occur each 10^7 yr (Spitzer 1978), at least 10^8 yr is needed to build them up. Once a cloud is large, I assume it no longer mixes with the hot medium instantaneously, but rather on a time scale T_2 . The mixing is no longer fast because supernova shock waves cannot evaporate a large cloud. Time scale T_2 may also be 10^8 yr, both from the time required to grow a massive cloud by collisions, and from the 10^8 yr between spiral crossings that may have something to do with that growth.

I do not pretend to justify these assumptions because they lie on the forefront of astrophysical questions. Once having postulated them, however, a simple mathematics that is interesting in its own right follows, as does a potential interpretation of the low abundance of ^{129}I , ^{244}Pu , and other extinct radioactivities. The mathematical description and the basic idea may point the way to the proper interpretation of the very old puzzles involved, but that will require a more precise and better justified astrophysical model of both the phase changes and of the actual circumstances of the birth of the Sun. If the physical lifetimes of the phases are actually less than the mixing times defined below, the model will have to be reformulated appropriately. And if the Sun was not born in a molecular cloud, the model is probably irrelevant. Third, it might be that internal supernovae disperse molecular clouds into warm media rather than fragmenting them into large H I clouds, as assumed here. Appropriate equations can be formulated along lines advanced here for any alternative. So let us take the idea that molecular clouds differ isotopically from the bulk ISM owing to incomplete mixing (Townes 1977) and treat it for the special case of radioactive nuclei.

II. A SIMPLE MATHEMATICAL MODEL

Let X_i designate the fraction by mass of ^{129}I in each phase (using Fig. 1 as a tentative guide). For simplicity take each phase to have a well mixed and uniform value of X_i . Turbulence probably provides such mixing (as well as the diffusion across the boundaries between the phases). The total mass of ^{129}I in the molecular cloud phase is then $X_1 M_1$, where M_1 is the mass of that phase. That ^{129}I mass changes owing to radioactive decay and to interchange with phase 2 at a rate defined as

$$\left(\frac{dM_1}{dt}\right)_{\text{exchanged}} \equiv \frac{M_1}{T_1}, \quad (1)$$

so that the actual change in ^{129}I mass is

$$\frac{d}{dt}(X_1 M_1) = X_1 \frac{dM_1}{dt} + M_1 \frac{dX_1}{dt}. \quad (2)$$

For simplicity consider an independent steady state. Then the rate (1) at which mass moves from phase 1 to phase 2 is balanced by the inverse flow from 2 to 1, and $dM_1/dt = 0$ in equation (2). Its second term becomes

$$\begin{aligned} M_1 \frac{dX_1}{dt} &= -M_1 \frac{X_1}{\tau} + (X_2 - X_1) \left(\frac{dM}{dt}\right)_{\text{ex}} \\ &= M_1 \left(-\frac{X_1}{\tau} + \frac{X_2 - X_1}{T_1}\right), \end{aligned} \quad (3)$$

where τ is the radioactive mean lifetime.

Suppressing the time dependence of equation (2) in favor of the steady-state solution wherein the time derivatives vanish then yields

$$X_1 = (1 + T_1/\tau)^{-1} X_2 \equiv \beta_1 X_2, \quad (4)$$

a simple relation between isotopic concentrations in the

adjacent phases. Similarly the differential equation for the second zone becomes

$$\frac{d}{dt}(X_2 M_2) = -M_2 \frac{X_2}{\tau} - \frac{M_1}{T_1}(X_2 - X_1) + \frac{M_2}{T_2}(X_3 - X_2), \quad (5)$$

whose solution depends on the relative rates at which zone 2 exchanges mass with both zone 1 and zone 3. Introducing a parameter α to measure these relative rates,

$$\frac{M_1}{T_1} \equiv \alpha \frac{M_2}{T_2}, \quad (6)$$

the steady-state solution becomes

$$X_2 = \beta_2 X_3, \quad \text{where} \quad \beta_2 = \left(\alpha + 1 - \alpha \beta_1 + \frac{T_2}{\tau} \right)^{-1}, \quad (7)$$

which, along with equation (4), completes the ratio among the three steady-state values of X_i , viz., $X_1:X_2:X_3 = \beta_1:\beta_2:1$. Because ratios are involved, one can equally well interpret X as the ratio of the radioactive species to a stable one; e.g., $^{129}\text{I}/^{127}\text{I}$. This is possible because the production of elements is so slow in comparison with T_1 and T_2 that stable nuclei have the same concentration in each of the three phases. An abundance gradient exists only when the radioactive mean life is comparable to or less than the time scales for mass exchange.

To solve for the absolute concentrations it would be necessary to solve the equation for the radioactive mass in zone 3 in the face of both decay and supernova nucleosynthesis:

$$\frac{d}{dt}(X_3 M_3) = -M_3 \frac{X_3}{\tau} - \frac{M_2}{T_2}(X_3 - X_2) + M_3 p(t), \quad (8)$$

where $p(t)$ is the supernova production rate per unit mass. Equation (8) can be solved for many forms of $p(t)$; however, the problem is almost identical to the traditional one for solving for concentrations in a well-mixed ISM (Fowler 1972; Schramm 1973). Because that problem introduces so many other uncertainties, it seems preferable to attempt to add nothing to its literature at this time. To do so is not necessary, because each concentration X_i can be expressed in terms of the value $\langle X \rangle$ it would have had with the same production rate $p(t)$ if the ISM were homogeneous (single phase). That result is easily demonstrated to be

$$X_1 = \langle X \rangle \frac{M_1 + M_2 + M_3}{M_1 + (M_2/\beta_1) + (M_3/\beta_1\beta_2)}. \quad (9)$$

This result, along with the definitions of β_1 and β_2 as ratios of concentrations (eqs. [4] and [7]), is the major result of this work. Because $\beta < 1$, it shows how much

smaller is the value of X_1 in the molecular cloud than is its average in the entire ISM. The values of X in the other zones then follow also from β_1 and β_2 .

As an explicit example from the literature, recall that Fowler (1972) illustrated that $\langle ^{129}\text{I}/^{127}\text{I} \rangle \approx 2 \times 10^{-3}$ for exponential models of galactic nucleosynthesis that produce an appropriate concordance for other cosmochronological pairs. That result may be used for $\langle X \rangle$ in equation (9) without need of solving equation (8). Then from a choice for M_i , T_i the quantities β_i and X_i can be computed for that model of nucleosynthesis.

Note that this three-phase model is easily reduced to two phases. Letting $T_1 \rightarrow 0$, which instantaneously mixes M_1 and M_2 , yields $\beta_1 = 1$ from equation (4). Hence, $X_1 = X_2$, and, because M_1 and M_2 are then simply additive in equation (9), they are merged into a single zone of mass $M_1 + M_2$. And the terms containing α cancel in equation (7). Were one to also let T_2 be small, then β_2 would also approach unity, and all $X_i = \langle X \rangle$, the average concentrations for that choice of production rate $p(t)$.

It is conceptually convenient to factor the problem in the way I have indicated. Deducing the history of galactic nucleosynthesis [the form of $p(t)$] from radioactive abundances is a problem with many complexities. That calculation is based usually on $\langle X \rangle$, with the initial solar abundances being used to indicate values for $\langle X \rangle$. Introducing a multiphase medium appears to make the problem much harder to pin down. But we have shown that the relationship of the initial solar abundances X_1 to $\langle X \rangle$ is, in the model evaluated, simply parameterized. This interpretation is, therefore, no more of a limitation than was the $\Delta \approx 10^8$ yr "waiting period," which is merely replaced by a new suggested interpretation. But the two interpretations differ quantitatively, as will be seen below.

III. A NUMERICAL EXAMPLE

As an example, suppose that the masses M_1 , M_2 , and M_3 stand in proportion to their total masses at our galactocentric distance. These are not precisely known, but take $M_1 = 30\%$ of ISM in molecular clouds, $M_2 = 30\%$ in massive H I clouds, and $M_3 = 40\%$ in average H I clouds plus their warm surrounding media. Then equation (9) reads simply, as an average expectation,

$$X_1 = \langle X \rangle (0.30 + 0.30/\beta_1 + 0.40/\beta_1\beta_2)^{-1}. \quad (10)$$

I arbitrarily take $T_1 = 10^8$ yr $= T_2$, giving $\alpha = 1$, to evaluate β_1 and β_2 . For ^{129}I , these have then the values $\beta_1(^{129}\text{I}) = 0.20$ and $\beta_2(^{129}\text{I}) = 0.17$. From them one finds $X_1(^{129}\text{I}) = 0.074\langle X \rangle$, $X_2(^{129}\text{I}) = 0.37\langle X \rangle$, and $X_3(^{129}\text{I}) = 2.15\langle X \rangle$. This arbitrary example has the interesting feature of reducing $\langle X(^{129}\text{I}) \rangle \approx 2 \times 10^{-3}$ to $X_1(^{129}\text{I}) = 1.5 \times 10^{-4}$, comparable to the value inferred from meteorites. The other two phases then have substantially larger values X_2 and X_3 . Any inhomogeneity in the actual mixing of these separate reservoirs then affords the possibility of generating "isotopic anomalies" in the $^{129}\text{I}/^{127}\text{I}$ ratio. Such inhomogeneities need not

be spatial at the time of aggregation, but could also represent differential depletion of ^{129}I onto different parts of a grain-size spectrum (Clayton 1980), which then “remembers” the prior spatial inhomogeneity. These ideas may enable one to understand why $^{129}\text{I}/^{127}\text{I}$ isotopic anomalies are much larger than found in other elements if the apparently irrelevant “ ^{129}I ages” (Jordan, Kirsten, and Richter 1979; Caffee *et al.* 1982; Crabb, Lewis, and Anders 1982) do in fact reflect isotopic variations unrelated to the relative times of aggregation. The astrophysical ideas for these interpretations were first sketched in prior publications (Clayton 1980, 1981) of the writer.

The situation for ^{244}Pu ($\tau_{1/2} = 81$ Myr) is also interesting. Using the same parameters, we find $\beta_1(^{244}\text{Pu}) = 0.539$ and $\beta_2(^{244}\text{Pu}) = 0.432$, giving $X_1(^{244}\text{Pu}) = 0.39\langle X(^{244}\text{Pu}) \rangle$. In this case the observed $X_1 = 0.005$ from Hudson *et al.* (1982) corresponds to $\langle ^{244}\text{Pu}/^{238}\text{U} \rangle = 0.013$, a number more easily reconciled with the known $\langle ^{235}\text{U}/^{238}\text{U} \rangle = 0.31$ in terms of continuous nucleosynthesis.

For shorter half-lives the expected concentrations of this model are even more reduced from $\langle X \rangle$, but, interestingly, not as reduced as in a “free-decay waiting period.” A good case in point is ^{107}Pd ($\tau = 9.4$ Myr), and which yields from equation (10) the expectation $X_1(^{107}\text{Pd}) = 0.016\langle X(^{107}\text{Pd}) \rangle$. This concentration is small, but it is 670 times larger than the value $\exp(-100/9.4)\langle X \rangle = 2.4 \times 10^{-5}\langle X \rangle$ expected from a

free-decay interval $\Delta = 10^8$ yr. Such distinctions may ultimately be necessary for interpretation of the ^{107}Pd concentrations inferred from ^{107}Ag excesses in iron meteorites (Kelly and Wasserburg 1978).

For ^{26}Al the difference is still greater. However, the ^{26}Al concentrations expected on any model of continuous nucleosynthesis without a final spike (not considered here) are too small if ^{26}Al was really *live* in the meteorites as claimed (Lee, Papanastassiou, and Wasserburg 1977). Either there was a final spike producing fresh ^{26}Al , or the ^{26}Mg -Al correlation in SUNOCON cores has been remembered (Clayton 1977, 1981, 1982) during their later sintering into CaAl-rich minerals.

In conclusion, it seems that models of this kind may be useful to interpretation of extinct radioactivities. Equation (9) might usefully be considered as an alternative to free-decay models, provided that astronomical research can better define the mixing parameters and the actual circumstances of the solar birth. It has the advantages (if correct) of eliminating a rather arbitrary 10^8 yr waiting period and of laying a framework for $^{129}\text{I}/^{127}\text{I}$ isotopic anomalies of large size.

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