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COULOMB DE-EXCITATION OF C^{12} IN A HELIUM GAS AT HIGH DENSITY AND TEMPERATURE

The radiative de-excitation of an excited residual nucleus is a familiar mechanism for the completion of nuclear reactions in both the laboratory and the stellar interior. In particular, the rate for the helium-burning reaction

$$He^4 + Be^8 \rightarrow C^{12*}\,(0+) \rightarrow C^{12*}\,(2+) + \gamma \tag{1}$$

is proportional to the electric quadrupole radiative width for the transition from the 7.65 MeV 0+ state to the 4.43 MeV 2+ state of C^{12}. In addition to radiative transitions, certain electromagnetic (but non-radiative) processes have been observed in the laboratory in which electromagnetic transitions between nuclear states are induced by inelastic scattering with particles. In the present Note we are interested specifically in the electromagnetic de-excitation of C^{12*} (0+) by the scattering with a helium gas. In a separate paper we shall consider more generally the subject of nuclear electromagnetic de-excitation induced by both nuclei and electrons.

The process of nuclear Coulomb de-excitation (actually *excitation* in the case of laboratory physics) has been thoroughly discussed in reviews by Alder, Bohr, Huus, Mottelson, and Winther (1956; henceforth referred to as "ABHMW") and Breit and Gluckstern (1959). Physically a time-varying electric field, produced at an excited "target" nucleus (carbon, 0+) by the nearby passage of a "projectile" nucleus (helium), causes a transition in the internal state of the carbon nucleus. Unlike the spontaneous radiative transition rate, the helium-induced $E2$ transition rate depends linearly on the helium density and increases with temperature, being proportional to $T^{1/2}$ at high temperature. When stimulated emission is included, the radiative transition rate is also an increasing function of temperature, becoming linear at high temperature. In the temperature range considered in this Note (10^{10} ° K and less), however, the helium-induced transition rate increases monotonically with temperature relative to the radiative transition rate. In consequence of the above considerations, the helium-induced electromagnetic transition rate should become competitive with the radiative transition rate at sufficiently high density and temperature.

The most extreme densities encountered in stellar nucleosynthesis are found in the imploded matter of supernovae. In this terminal stage of stellar evolution the stellar core collapses due to an instability triggered either by inverse beta decay or by photodisintegration. The overlying layers initially follow the implosion, until a core stiffening leads to subsequent ejection of the outer portions of the imploded star. The matter near the mass cut dividing the ultimate collapsed remnant from the redispersed matter achieves densities in the range 10^{12}–10^{13} gm/cm^3 (Colgate and White 1966; see also Arnett 1966) before the expansion. The constituents, predominantly neutrons and alpha particles, reinitiate fusion reactions during the expansion with densities and temperatures in the gross range of 10^9–10^{12} gm/cm^3 and 10^9–10^{10} ° K, respectively.[1] It is our present purpose to examine the relative importance of helium-induced electromagnetic transitions and radiative transitions in C^{12} for this range of density and temperature.

The cross-section for a nuclear electric quadrupole transition, induced in a nucleus of charge Z by a nucleus of charge Z_i, is given in Preston (1962) as

$$\sigma(E2) = \left(\frac{Z_i e}{\hbar v}\right)^2 \left(\frac{M v v'}{Z Z_i e^2}\right)^2 B(E2)\, f_{E2}(\xi, \eta')\,, \tag{2}$$

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[1] The photodisintegration rate of C^{12} will be enhanced at these high temperatures, but this rate is estimated to be still much smaller than neutron- and alpha-capture rates in C^{12} at the relevant densities.

where M is the reduced mass of the system, v and v' are the initial and final relative velocities, respectively, $B(E2)$ is a purely *nuclear* quantity, proportional to the absolute square of the nuclear transition matrix element, and $f_{E2}(\xi,\eta')$ is a function that depends on the Coulomb force and, in general, on the multipole order, but is independent of nuclear structure. The parameters ξ and η' are given by

$$\eta' = Z_1 Z \alpha c/v' , \qquad \eta = Z_1 Z \alpha c/v , \qquad \xi = \eta - \eta' , \tag{3}$$

where $\alpha = \frac{1}{137}$ and c is the velocity of light. In the definition of the parameter ξ, and in the function f_{E2} (regarded as a function of η and η'), the parameters η and η' are interchanged in comparison with the laboratory (*excitation*) expressions given in Preston (1962). In the limit $v' \rightarrow 0$ ($\eta' \rightarrow \infty$), the Coulomb de-excitation function $f_{E2}(\xi) = f_{E2}(\xi,\infty)$ can be calculated from a semi-classical theory in which the orbit of the projectile nucleus is assumed to be hyperbolic and is unperturbed by the target nucleus.

The thermonuclear reaction rate per target nucleus for an $E2$ Coulomb de-excitation is given by the standard integral over a Maxwellian distribution of ion velocities:

$$R(E2) = 4\pi n(Z)\left(\frac{M}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^2 \exp(-Mv^2/2k_B T)\, \sigma(E2)\, v\, dv , \tag{4}$$

where $n(Z)$ is the projectile number density, k_B is the Boltzmann constant, and T is the absolute temperature. To obtain a dimensionless expression independent of nuclear structure we divide equation (4) by the rate per target nucleus for a *radiative* $E2$ transition

$$R_\gamma(E2) = \frac{8\pi}{150}\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^5 \frac{B(E2)}{1-\exp(-\hbar\omega/k_B T)} , \tag{5}$$

where $R_\gamma(E2)$ includes both spontaneous and stimulated emission, and $\hbar\omega$ is the transition energy. The relative $E2$ transition rate for C^{12} is then given by

$$R(E2)/R_\gamma(E2) = 4.63 \times 10^{-5} \rho(\mathrm{He})\beta^{3/2}[1 - \exp(-\hbar\omega\beta)]\, I_{E2}(\omega,\beta)/(\hbar\omega)^5 , \tag{6}$$

where

$$I_{E2}(\omega,\beta) = \int_0^\infty W(W+\hbar\omega) f_{E2}(\xi,\eta')\exp(-\beta W)\, dW \tag{7}$$

and W is the kinetic energy in the center-of-mass system. In equations (6) and (7) all energies are measured in units of the electron rest energy, and $\beta = 1/k_B T$. The integral in equation (7) was evaluated numerically using the Gauss-Laguerre quadrature technique together with a two-dimensional Lagrangian interpolation of the tabulated values of f_{E2} given in ABHMW. The error in this procedure is estimated to be on the order of 1 per cent. In Figure 1 we have plotted $A(\mathrm{He;C}) = \log_{10}[R(E2)/R_\gamma(E2)]$ against temperature for various values of helium density. An empirical fit to the calculated curves is given by

$$R(E2)/R_\gamma(E2) = 2.22 \times 10^3 \rho_9(\mathrm{He}) \exp(-19.36/T_9) , \tag{8}$$

where $\rho_9(\mathrm{He}) = 10^{-9}\rho(\mathrm{He})$ and $T_9 = 10^{-9}T$. The fit is good to about 5 per cent for $1 \lesssim T_9 \lesssim 4$ and to about 1–2 per cent for $4 \lesssim T_9 \lesssim 10$. From the results of Seeger and Kavanagh (1962) we obtain a total electromagnetic width

$$\Gamma_{\mathrm{e\,m}}(T,\rho) = (2.4 \pm 1.5)\frac{1 + (2.22 \times 10^3)\rho_9(\mathrm{He})\exp(-19.36/T_9)}{1-\exp(-37.42/T_9)} 10^{-3}\,\mathrm{eV} , \tag{9}$$

which includes helium-induced transitions and both spontaneous and stimulated emission of radiation.

We have considered only the *electromagnetic* width of the 7.65 MeV state of C^{12} in the presence of a dense hot helium gas. But we note that the state may also possess nuclear widths that are unobservable in the laboratory but which may be important in the context of this Note. The most promising of these stems from the dense neutron gas which generally accompanies the dense helium gas. The inelastic scattering of neutrons from the C^{12} state may provide the most rapid de-excitation of the state. We intentionally exclude the nuclear interactions from this Note but acknowledge their possible importance to the supernova problem.

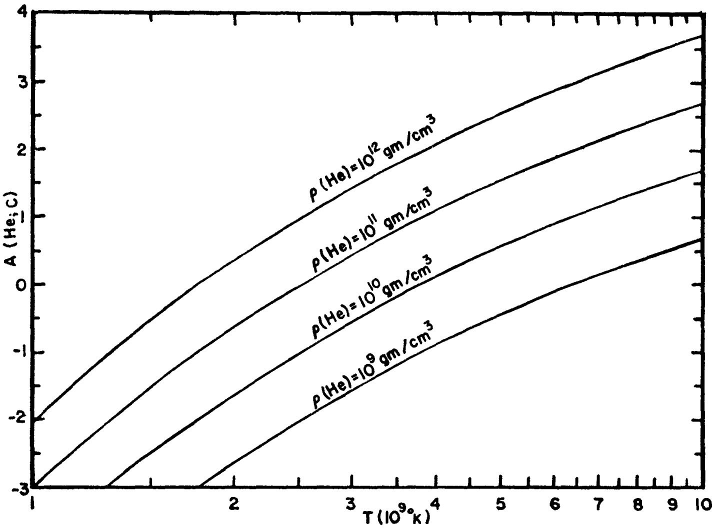

FIG. 1.—$A(\mathrm{He;C}) = \log_{10}[R(E2)/R_\gamma(E2)]$ as a function of temperature for different values of helium density. $R(E2)$ and $R_\gamma(E2)$ are the helium-induced and radiative electric quadrupole transition rates, respectively, for the 7.65 MeV $(0+) \rightarrow$ 4.43 MeV $(2+)$ transition in C^{12}. $R_\gamma(E2)$ includes both spontaneous and stimulated radiative transitions.

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