

12-2007

Charge Effects in Bilayer Electric Transport

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CHARGE EFFECTS IN BILAYER ELECTRIC TRANSPORT

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Physics

by
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December 2007

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ABSTRACT

The mathematical isomorphism between the layer index in a bilayer system and the spin index in a spin-polarized system is used to draw an analogy between phenomena that occur in the two systems as a result of the electron-electron (e-e) interaction. First, the classical case of the Coulomb drag, a phenomenon resulting from momentum transfer through e-e collisions, is reviewed for both bilayer systems and spin-polarized systems. Then, the spin-backflow phenomenon, a result of the interaction-induced changes in the local quasiparticle energy that affects the charge transport in spin-polarized systems is presented. Finally, the iso-spin correlation is exploited to establish the equivalent charge backflow that appears in a bilayer system when an external electric field is applied to just a single layer. By accounting for interlayer e-e interactions, the backflow current in the otherwise dormant layer is shown to be proportional to the external field applied to the other layer.

DEDICATION

To my family.

ACKNOWLEDGEMENTS

I would like to acknowledge my committee members, Catalina Marinescu, Dick Manson, Chad Sosolik, and Dieter Hartmann, for their suggestions and comments.

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CHAPTER 1

INTRODUCTION

Understanding the macroscopic effects of the electron-electron interaction in solids has been a continuous effort for the past sixty years. A decisive step in this effort was marked by the creation of artificial semiconductor structures, where a controlled environment was made available for measuring the electron dynamics. The simplest such structure, a quantum well, is essentially an attractive potential zone that appears at the interface between two semiconductors whose gaps differ significantly, typically GaAs and AlGaAs. The electron liquid formed in a quantum well behave essentially as a collection two-dimensional (2D) plane wave states, a system referred to as a layer.[1]

The physics of 2D electrons inside a semiconductor layer has been a steady source of interesting problems since the confinement that quenches the motion perpendicular on the layer enhances the interaction driven effects. The experimental advantage of the setup allows the manipulation of the critical parameters of the interaction, such as the particle density and their effective mass. Reintroducing, in a controlled manner, a third degree of freedom for the electrons can be accomplished in a two-layer arrangement. A bilayer structure in which two spatially separated electrons sub-systems interact has been proven to be an ideal situation to explore various electron properties that are hard to measure in a homogeneous environment, such as the excited electron lifetime which was investigated by tunneling spectroscopy [2].

The unique experimental situation provided by a bilayer system made it clear that this setup can be used to successfully explore even more subtle feature of the interaction which otherwise would involve electrons of opposite spins in the same system. The fundamental idea behind this argument rests on the formal analogy that one can establish between the spin quantum number σ that differentiates the polarized electron populations in a homogeneous system and the layer index i of a bilayer system. A significant example of this spin-layer index isomorphism is represented by the Coulomb drag problem. In general, the drag phenomenon represents the transfer of momentum between the interacting components of a system that

occurs when only one component is driven out of equilibrium, making the response of the second component a direct measure of the strength of the interaction. The Coulomb drag appears as a result of the momentum transfer realized through particle collisions mediated by the Coulomb interaction.

The Coulomb drag effect in the bilayer system appears when an applied electric field in one layer induces an electric field in the other layer by a momentum transfer process that occurs between the layers, even in the absence of quantum mechanical tunneling of particles from one layer to the other. The experimental observation of the Coulomb drag [3] has been explained theoretically by Jauho and Smith [4] who calculate the rate of momentum transfer by estimating the rate of the electron-electron scattering in the bilayer system. D'Amico and Vignale [5] describe a similar effect in the spin-polarized system in which an injected spin σ current induces a spin $\bar{\sigma}$ current in the opposite spin channel. They attribute the spin-Coulomb drag effect to Coulomb collisions of electrons between the two separate spin channels. One can show that by applying an appropriate theory, the layer index and the spin index are mathematically isomorphic, and the theoretical treatment of the two systems becomes symmetric.

A different coupling effect, this time occurring in a spin-polarized system, has been proposed by Qian et al. [6] and derived in detail by Marinescu [7]. In this case, it was shown that the shifting induced by the Coulomb interaction with the electrons of opposite spins in the local energy of particles causes a backflow of spin-polarized current. This current can be characterized by an effective spin mass, different from the effective band mass that characterizes charge current. The purpose of this work is to explore a similar effect for the bilayer system by exploiting the mathematical symmetry of the spin-polarized and bilayer systems provided by their isomorphic quantum numbers. To illustrate the physics behind this effect, the Landau theory of Fermi liquids is employed as a way accounting for the e-e interaction.

CHAPTER 2

THEORIES OF COULOMB DRAG

In this chapter, the most important example of the spin/layer index isomorphism, the Coulomb drag effect, is discussed. This effect arises directly from particle collisions that transfer energy between otherwise disjoint distributions in each system, causing the appearance of a resistance between distributions to the current that drives the effect.

2.1 Coulomb Drag in a Bilayer System

Consider a pair of identical two-dimensional semiconducting slabs of width W and length l , placed parallel to each other separated by a perpendicular distance d , similar to the system described in [2] (see Fig. 2.1). The quantum mechanical tunneling of electrons from one layer to the other is made negligible by making d large enough to bring the interlayer tunneling probability very close to zero; however, this interlayer separation must still be small enough to allow for a measurable Coulomb interaction of the electrons between layers. A current density J_2 is driven through layer 2 with the boundary condition that no current flows in layer 1. The electric field present in layer 2 induces an electric field in layer 1. From Ohm's law, the applied electric field \vec{E}_2 in layer 2 and the response field \vec{E}_1 are given by

$$\vec{E}_1 = \rho_{11}\vec{J}_1 + \rho_{12}\vec{J}_2, \quad (2.1)$$

$$\vec{E}_2 = \rho_{21}\vec{J}_1 + \rho_{22}\vec{J}_2, \quad (2.2)$$

where ρ_{ij} represent the layer-indexed resistivities, and \vec{J}_i represent the current densities driven through the corresponding layers. According to the boundary condition, the current density through layer 1, \vec{J}_1 is zero. Thus, according to Eqs. (2.1) and (2.2), the applied field \vec{E}_2 in layer 2 is due to the applied current \vec{J}_2 , while the response field \vec{E}_1 in layer 1 is induced by a transresistivity ρ_{12} between the two layers, a phenomenon called Coulomb drag:

$$\vec{E}_1 = \rho_{12}\vec{J}_2, \quad (2.3)$$

$$\vec{E}_2 = \rho_{22}\vec{J}_2. \quad (2.4)$$

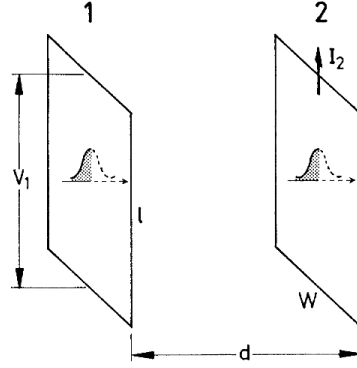


Figure 2.1 Schematic of the bilayer system: two 2D semiconductors of length l and width W are set at a perpendicular distance d large enough to forbid quantum tunneling [4]. The Coulomb interaction between the layers momentum transfer between the two layers.

By writing $\rho_{12} = \rho_D$, Jauho and Smith [4] relate the response field and current density to the corresponding electrostatic potential V_1 , current I_2 , and the geometric parameters W and l of the semiconducting slabs:

$$\rho_D = \frac{E_1}{J_2} = \frac{WV_1}{I_2l} . \quad (2.5)$$

They relate the drag resistivity to a momentum relaxation rate $1/\tau_D$ by defining the drift velocity u_2

$$J_2 = N_2 e u_2 , \quad (2.6)$$

where N_2 is the number of charge carriers in layer 2. The electric field is related to the drag mobility μ_D experimentally according to

$$\mu_D = \frac{u_2}{E_1} , \quad (2.7)$$

which can be expressed in terms of the momentum relaxation rate according to

$$\mu_D = \frac{e}{m} \tau_D . \quad (2.8)$$

(m is the effective band mass of the electron, different from the actual electron mass). Thus, the momentum relaxation rate τ_D is related to the drag resistivity ρ_D according to

$$\rho_D = \frac{m}{N_2 e^2 \tau_D} . \quad (2.9)$$

Jauho and Smith [4] derive the momentum relaxation rate (and drag resistivity) by using the Boltzmann transport equation to balance the induced electric field in layer 1 with the drag caused by the current in layer 2. The transport equation for layer 1 is written

$$\frac{-e\vec{E}_1}{\hbar} \cdot \nabla_{\vec{k}_1} n_1^0 = \left(\frac{\partial n_1}{\partial t} \right)_{coll} , \quad (2.10)$$

where E_1 is the response field induced in layer 1 by the driving current in layer 2. The collision of electrons is responsible for the momentum relaxation and corresponding drag resistivity.

By considering only binary collisions, according to Fermi's Golden Rule the probability of being scattered out of state $|\vec{k}_1 \vec{k}_2\rangle$ into state $|\vec{k}'_1 \vec{k}'_2\rangle$ is

$$\wp_{\vec{k}_1 \vec{k}_2; \vec{k}'_1 \vec{k}'_2} = \frac{2\pi}{\hbar} |\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2 \rho_{\vec{k}_1 \vec{k}_2; \vec{k}'_1 \vec{k}'_2} , \quad (2.11)$$

In this case, $|\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2$ represents the quantum mechanical probability of the scattering event occurring while the statistical probability of scattering is

$$\rho_{\vec{k}_1 \vec{k}_2; \vec{k}'_1 \vec{k}'_2} = n_{\vec{k}_1} n_{\vec{k}_2} (1 - n_{\vec{k}'_1}) (1 - n_{\vec{k}'_2}) - n_{\vec{k}'_1} n_{\vec{k}'_2} (1 - n_{\vec{k}_1}) (1 - n_{\vec{k}_2}) , \quad (2.12)$$

as the occupancy statistics of both electron states must be considered. Applying the conservation of momentum and summing over \vec{k}'_1 and \vec{k}'_2 and the respective spins yields the change in particle distribution

$$\left(\frac{\partial n_{\vec{k}_1}}{\partial t} \right) = - \sum_{\vec{k}_2, \sigma_2} \sum_{\vec{k}'_1, \sigma_1} \wp_{\vec{k}_1 \vec{k}_2; \vec{k}'_1 \vec{k}'_2} . \quad (2.13)$$

In a linear approximation, the electron distribution function can be assumed to differ only weakly from its equilibrium value and thus can be written as:

$$n_k - n_k^0 = n_k^0 (1 - n_k^0) \psi_k , \quad (2.14)$$

with n_k^0 being the Fermi-Dirac distribution function written for an electron of momentum \vec{k} and energy $\epsilon_k = \hbar^2 k^2 / 2m$. ψ_k is the deviation of the distribution from equilibrium, obtained from

the solution of the Boltzmann equation written for the applied electric field in the corresponding layer. The *linearized* collision integral for the system is given by

$$\left(\frac{\partial n_1}{\partial t}\right)_{coll} = - \sum_{\sigma_2, \sigma_{1'}, \sigma_{2'}} \int \frac{d\mathbf{k}_2}{(2\pi)^2} \int \frac{d\mathbf{k}_1}{(2\pi)^2} |\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2 (\psi_1 + \psi_2 - \psi_{1'} - \psi_{2'}) \cdot n_1^0 n_2^0 (1 - n_{1'}^0) (1 - n_{2'}^0) \delta(\epsilon_1 + \epsilon_2 - \epsilon_{1'} - \epsilon_{2'}) \quad (2.15)$$

The deviation in layer 2 is obtained as the solution of the Boltzmann transport equation written for the electric field E_2 in the relaxation time approximation, as impurity scattering is the dominant mechanism. Writing τ_2 as the impurity scattering relaxation time, the deviation in layer 2 is

$$\psi_2 = -\frac{1}{k_B T} \tau_2 e v_2 E_2 . \quad (2.16)$$

Because no current flows through layer 1,

$$\psi_1 = \psi_{1'} = 0 . \quad (2.17)$$

Then the electric field E_1 balances the drag resulting from the nonzero ψ_2 and $\psi_{2'}$.

By inserting the deviation functions into the linearized collision integral and summing over states $|\vec{k}_1 \sigma_1\rangle$, Eq. (2.15) becomes

$$2 \int \frac{d\mathbf{k}_1}{(2\pi)^2} k_1 \frac{-eE_1}{\hbar} \frac{\partial n_1^0}{\partial k_1} = \sum_{\sigma_1} \int \frac{d\mathbf{k}_1}{(2\pi)^2} k_1 \left(\frac{\partial n_1}{\partial t}\right)_{coll} . \quad (2.18)$$

It can be shown that, by writing \vec{k}_1 and $\vec{k}_{1'}$ in terms of momentum transfer \vec{q} according to

$$\vec{q} = \vec{k}_{1'} - \vec{k}_1 \quad (2.19)$$

and employing Eq. (2.15) self-consistently, the right hand side of Eq. (2.18) simplifies to

$$\sum_{\sigma_1} \int \frac{d\mathbf{k}_1}{(2\pi)^2} k_1 \left(\frac{\partial n_1}{\partial t}\right)_{coll} = -\frac{e\hbar E_2 \tau_2}{4mk_B T} \sum_{\sigma_1, \sigma_2, \sigma_{1'}, \sigma_{2'}} \int \frac{d\mathbf{k}_1}{(2\pi)^2} \int \frac{d\mathbf{k}_2}{(2\pi)^2} \int \frac{d\mathbf{k}'_1}{(2\pi)^2} |\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2 \cdot q^2 n_1^0 n_2^0 (1 - n_{1'}^0) (1 - n_{2'}^0) \delta(\epsilon_1 + \epsilon_2 - \epsilon_{1'} - \epsilon_{2'}) . \quad (2.20)$$

The susceptibility function is given by

$$\chi(\vec{q}, \omega) = - \int \frac{d\mathbf{k}_1}{(2\pi)^2} \frac{n^0(\epsilon_1) - n^0(\epsilon_{1'})}{\epsilon_1 - \epsilon_{1'} + \hbar\omega + i\delta} . \quad (2.21)$$

By inputting the imaginary part of the susceptibility function, Eq. (2.20) becomes

$$\sum_{\sigma_1} \int \frac{d\mathbf{k}_1}{(2\pi)^2} k_1 \left(\frac{\partial n_1}{\partial t} \right)_{coll} = -\frac{e\hbar^2 E_2 \tau_2}{8\pi^2 m k_B T} \sum_{\sigma_1, \sigma_2, \sigma_{1'}, \sigma_{2'}} \int \frac{d\mathbf{q}_1}{(2\pi)^2} \int_0^\infty d\omega [\text{Im}\chi(q, \omega)]^2 \cdot |\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2 \frac{q^2}{\sinh^2(\hbar\omega/2k_B T)} . \quad (2.22)$$

The scattering probability is given by the Born approximation using the Coulomb potential. This is given by

$$\sum_{\sigma_1, \sigma_2, \sigma_{1'}, \sigma_{2'}} |\langle \vec{k}'_1 \vec{k}'_2 | \hat{W} | \vec{k}_1 \vec{k}_2 \rangle|^2 = \frac{2\pi}{\hbar} 4|e\phi(q)|^2 , \quad (2.23)$$

where $e\phi(q)$ is the Fourier transform of the effective Coulomb interaction acting on the electrons. (The bare Coulomb interaction is divergent; however, other electrons nearby act to reduce the Coulomb interaction. This effective Coulomb interaction can be thought of from a field-theoretic method as the single electron plus a screening cloud produced by nearby electrons about the electron of interest. The presence of the screening cloud eliminates the divergence in the effective Coulomb interaction.)

Using this, they show the ratio of the electric fields is given by

$$\frac{E_1}{E_2} = \frac{\tau_2}{\tau_D} , \quad (2.24)$$

where E_2 is the driving field in layer 2, E_1 is the effective response field in layer 1, τ_2 is the momentum relaxation rate due to impurity scattering in layer 2, and τ_D is the momentum relaxation rate caused by momentum transfer between *interlayer* electrons:

$$\frac{1}{\tau_D} = \frac{\hbar^2}{2\pi^2 n_1 m k_B T} \int_0^\infty dq \int_0^\infty d\omega q^3 |e\phi(q)|^2 [|\text{Im}\chi(q, \omega)|^2] \frac{1}{\sinh^2(\hbar\omega/2k_B T)} . \quad (2.25)$$

It must be emphasized again that this effect is entirely due to interlayer collisions of electrons in the system, and is thus contained in the collision integral of the Boltzmann transport equation.

Upon comparing their results to the experimental results of Gramila et al. [3] in which the drag resistivity was measured in a bilayer system, Jauho and Smith find that only approximately one half of the momentum transfer in the system is accounted for by e-e collisions; there may be other mechanisms of momentum transfer in the system, one of which will be described at length in chapter four.

2.2 Spin Coulomb Drag in a Spin-polarized System

In the spin-polarized system, the electron distribution is split into two independent spin channels (in complete analogy to the independent layers of the bilayer system) with all spin-flip processes considered negligible such that two nearly independent distribution functions may be used to describe the system. In the Fermi liquid picture, the distributions act on each other only through the interaction energy. Through methods entirely symmetric to those described in the bilayer system, the spin-drag resistivity can be defined, making the two pictures isomorphic.

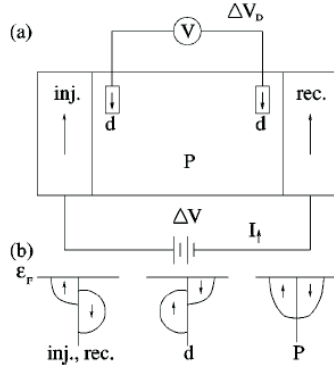


Figure 2.2 Experimental setup for a spin-polarized system: Part (a) shows the experimental setup, and part (b) shows the band structures of the injector (inj.), receiver (rec.), detectors (d) and the paramagnet (P) [5].

The spin-polarized system may be realized by injecting a spin polarized current into a two-dimensional paramagnetic material via a set of polarized injection and reception ferromagnets, measuring the resulting oppositely polarized current via a set of detection ferromagnets polarized opposite to the injector and receiver (see Fig. 2.2). In this experiment, D’Amico and Vignale [5] predict a spin Coulomb drag resistivity $\rho_{\uparrow\downarrow}$ that results from the interaction between oppositely polarized distributions. This effect is dictated entirely by the collisions of quasiparticles between the two distributions, containing the resulting spin drag effect entirely in the collision integral of the Boltzmann equation. Further, they predict the spin-drag effect is larger than the Coulomb drag effect for the bilayer system when similar parameters are used [8].

Weber et al. [9] observed the spin drag effect in spin polarized distributions contained in two dimensional quantum wells. At low temperatures ($T < 50K$), the transfer of momentum from one spin current to the other is observed, the strength of which is on the order predicted by D'Amico and Vignale.

CHAPTER 3

A PHENOMENOLOGICAL DESCRIPTION OF THE INTERACTING ELECTRON SYSTEM

When describing interacting systems, two different pictures can be formulated to account for the interaction energies of particles in the systems. Microscopic theories utilize the knowledge of the quantum mechanical nature of the particle interaction, and through successive approximation predict the results of macroscopic parameters of the system from first principles. On the other hand, phenomenological theories simply account for the presence of interactions in the equations that determine the characteristics of the system without providing for their exact microscopic expression. The usefulness of phenomenological theories is made apparent, however, by their direct connection with experimental measurements. The charge backflow problem discussed in this work has a very simple description within the framework of the Landau theory of Fermi liquids, and some of the main points of this theory are presented below.

3.1 The Theory of Fermi Liquids

The phenomenological theory of Fermi liquids was proposed by Landau to provide a macroscopic description of the He^3 system, a fermionic liquid characterized by short-range interactions. By modifying the theory slightly to account for the long range nature of the Coulomb interaction, Silin was able to show the theory is equally applicable to interacting electron systems [10]. In the following discussion the most important properties of the electron liquid are reviewed by following the presentation of references [10] and [11].

Fermi liquid theory makes the basic assumption that the real eigenstates of the interacting electron system may be derived from the ideal eigenstates of the non-interacting electron system. This is done by very slowly (adiabatically) turning on the interactions of the electrons in the system such that the eigenstates of the non-interacting system map to the interacting eigenstates. This is particularly important for the mapping of an eigenstate of the non-interacting system to the ground state of the interacting system, as the ground state of the interacting system defines the Fermi surface at which the excitation of particles occurs. Fermi liquids in

which the ground state can be obtained from adiabatically turning on the interaction of some non-interacting state are called *normal* Fermi liquids, such as the electron systems in metals and semiconductors are normal. (This assumption breaks down in superconductors or in two-dimensional semiconductors in the presence of a strong magnetic field.)

This adiabatic switching process redefines the basic particle that is used to describe the macroscopic phenomena in the system, as the interaction between the electrons must be included. Thus, the basic unit is no longer just the electron, but rather the electron plus an interaction cloud that surrounds the electron, called a *quasiparticle* (QP). This is consistent with the difference between ideal and real eigenstates, as the ideal eigenstate describes the state of an electron, whereas the real eigenstate defines a QP state.

In phase space, the QPs are organized in increasing order of their momentum \vec{k} inside a symmetric distribution, called the Fermi sphere. The same state inside this sphere can be occupied by a maximum of two electrons of opposite spins. A single QP state is described by the usual plane wave function and its associated energy $\epsilon_{\vec{k}}$.

In equilibrium at temperature T , the QPs are described by the Fermi-Dirac distribution

$$n_{\vec{k}}^0 = \frac{1}{\exp\left(\frac{\epsilon_{\vec{k},i}^0 - \mu}{k_B T}\right) + 1}, \quad (3.1)$$

where μ , the chemical potential and k_B is the Boltzmann constant, is identified as the Fermi surface of the system:

$$\mu = \epsilon_F. \quad (3.2)$$

The Fermi surface represents the highest energy value attained by the QPs when the system is in the lowest possible energy configuration at zero temperature (Fig. 3.1).

The second assumption of Fermi liquid theory states that the energy of the system is a functional of the fluctuations in the QP distribution function, $\delta n_{\vec{k},i}$ defined by:

$$n_{\vec{k},i} = n_{\vec{k},i}^0 + \delta n_{\vec{k},i}, \quad (3.3)$$

Such an approximation is justified by the fact that at finite temperature, QPs can interact or scatter in the phase space only within a region of width $k_B T$ around the Fermi surface (Fig.

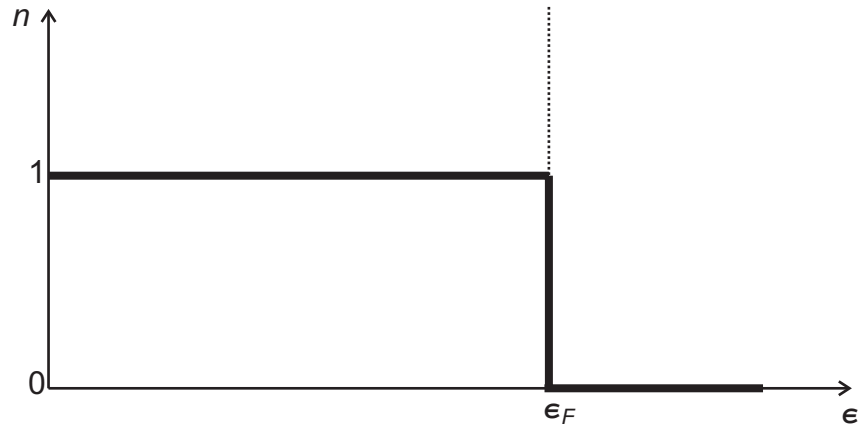


Figure 3.1 The equilibrium distribution is plotted as a function of quasiparticle energy. The Fermi surface ϵ_F occurs at the cutoff of occupation. Figure adapted from [11].

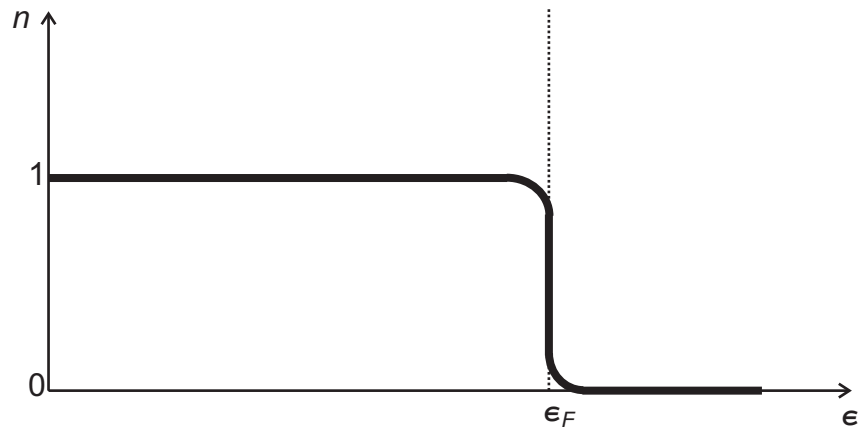


Figure 3.2 Equilibrium distribution of QPs at low temperatures: the distribution varies in a small width $\sim k_B T$ about the Fermi surface. Figure adapted from [11].

3.2). For low temperatures (such that $k_B T \ll \mu$), the availability of scattering states is created only when the distribution function changes infinitesimally from its equilibrium value, Eq. (3.1).

$$E[n_{\vec{k},i}] = E_0 + \sum_{\vec{k},i} \epsilon_{\vec{k},i}^0 \delta n_{\vec{k},i} + \sum_{\vec{k},i} \sum_{\vec{k}',j} f_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k},i} \delta n_{\vec{k}',j} + O(\delta n^3). \quad (3.4)$$

Here, the index i is employed to designate the additional quantum numbers that can be used to describe the QP states, such as spin or, as we discuss later, the layer index. Mathematically, $\epsilon_{\vec{k},i}^0$ is the first variational derivative of the system energy E , and $f_{\vec{k},i;\vec{k}',j}$ is the second variational derivative of the distribution with respect to the energy. By writing the local energy of a QP as

$$\epsilon_{\vec{k},i} = \epsilon_{\vec{k},i}^0 + \sum_{\vec{k}',j} f_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k}',j}, \quad (3.5)$$

it becomes apparent that $\epsilon_{\vec{k},i}^0$ is the non-interacting energy of a QP, and $\sum_{\vec{k}',j} f_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k}',j}$ represents the interaction energy. Thus, the first-order shift in QP energy $\delta \epsilon_{\vec{k},i}$ is determined by

$$\delta \epsilon_{\vec{k},i} = \sum_{\vec{k}',j} f_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k}',j}, \quad (3.6)$$

implying

$$\epsilon_{\vec{k},i} = \epsilon_{\vec{k},i}^0 + \delta \epsilon_{\vec{k},i}. \quad (3.7)$$

The interaction parameter $f_{\vec{k},i;\vec{k}',j}$ is introduced as a phenomenological function that cannot be obtained within the confines of Fermi liquid theory and needs to be calculated microscopically from first principles. System symmetries and other properties are exploited to simplify the form of this function as much as possible. Reference [11] provides a few important examples.

For systems that exhibit translational invariance (this includes electron liquids found in semiconductors and metals), the interaction parameter is not a function of the individual positions of each QP, but rather just a function of the distance between the QPs. Thus, a Fourier transformed interaction term is just a function of the difference in momenta between QPs:

$$f_{\vec{k},i;\vec{k}',j} = f_{i,j}(|\vec{k} - \vec{k}'|). \quad (3.8)$$

In another common treatment, to account for the spin-dependent effects imposed by the Pauli exclusion principle, the interaction parameter is separated into a spin-independent and a spin-dependent part,

$$f_{\vec{k},\sigma;\vec{k}',\sigma'} = \phi_{\vec{k},\vec{k}'} + \sigma \cdot \sigma' \psi_{\vec{k},\vec{k}'} . \quad (3.9)$$

This polarization effect may also be expressed in terms of spin-symmetric and spin-antisymmetric interactions,

$$\begin{aligned} f_{\vec{k},\vec{k}'}^{\uparrow\uparrow} &= f_{\vec{k},\vec{k}'}^S + f_{\vec{k},\vec{k}'}^A , \\ f_{\vec{k},\vec{k}'}^{\uparrow\downarrow} &= f_{\vec{k},\vec{k}'}^S - f_{\vec{k},\vec{k}'}^A . \end{aligned} \quad (3.10)$$

A further simplification comes from noting that all excited QP states are very close (within $\sim k_B T$) of the Fermi surface. Thus, $|\vec{k}| \approx |\vec{k}'|$, and $f_{\vec{k},i;\vec{k}',j}$ depends only upon the relative angle θ of the two vectors. $f_{\vec{k},i;\vec{k}',j}$ may be expanded in terms of a series of Legendre polynomials in three dimensions or angle-dependent exponentials in two dimensions,

$$f_{\vec{k},\vec{k}'}^{S(A)} = \begin{cases} \sum_{\ell=0}^{\infty} f_{\ell}^{S(A)} P_{\ell}(\cos \theta) , & a = 3 \\ \sum_{\ell=0}^{\infty} f_{\ell}^{S(A)} e^{i\ell\theta} . & a = 2 \end{cases} \quad (3.11)$$

3.2 Transport in Fermi Liquids

Describing QP transport in Fermi liquid theory is a consequence of Liouville's theorem, which expresses the conservation of particle number density in a volume of phase space. For a particle distribution $n_{\vec{k},i}$ containing particles with the single-particle equilibrium Hamiltonian $\epsilon_{\vec{k},i}$, it is written as

$$\frac{\partial n_{\vec{k},i}}{\partial t} + \frac{1}{\hbar} \{n_{\vec{k},i}, \epsilon_{\vec{k},i}\}_{PB} = 0 . \quad (3.12)$$

This semiclassical equation is written in terms of a Poisson bracket such that the inclusion of explicit spin-dependent terms and other non-commuting variables to the Hamiltonian may be treated with ease by adding a quantum mechanical commutator to the left hand side. When the Poisson bracket is expanded, Eq. (3.12) becomes

$$\frac{\partial n_{\vec{k},i}}{\partial t} + \frac{1}{\hbar} \nabla_{\vec{k}} \epsilon_{\vec{k},i} \cdot \nabla_{\vec{r}} n_{\vec{k},i} - \frac{1}{\hbar} \nabla_{\vec{k}} n_{\vec{k},i} \cdot \nabla_{\vec{r}} \epsilon_{\vec{k},i} = 0 . \quad (3.13)$$

In reality, particle collisions will always occur, and these collisions may act to disrupt the conservation of particle number density in a given cell in phase space. To accommodate for this effect, a source/sink term called the collision integral is added to the right hand side to provide the Boltzmann transport equation:

$$\frac{\partial n_{\vec{k},i}}{\partial t} + \frac{1}{\hbar} \{n_{\vec{k},i}, \epsilon_{\vec{k},i}\}_{PB} = \left(\frac{\partial n_{\vec{k},i}}{\partial t} \right)_{coll} . \quad (3.14)$$

By identifying the local QP energy as the equilibrium Hamiltonian $\epsilon_{\vec{k},i}$ for particles in the system, this equation is now directly applicable to Fermi liquids with the exception of the collision integral; this is treated outside the confines of the Fermi liquid theory.

CHAPTER 4
ISOMORPHIC BACKFLOW CURRENTS IN SPIN-POLARIZED AND BILAYER
SYSTEMS

4.1 Spin Backflow in a Spin-polarized System

Another effect that relies on the spin-spin interactions occurs when introducing an electric field in the spin system was described by Qian et. al [6]. When an electric field drives a spin system out of equilibrium, it produces a current density given by

$$\vec{J} = \sum_{\vec{k}, \sigma} \frac{\hbar \vec{k}}{m} n_{\sigma} . \quad (4.1)$$

The effective band mass m that appears in the current density is shown to differ from the effective mass m^* that describes the total spin current density given by

$$\vec{J}_s = \sum_{\vec{k}} \frac{\hbar \vec{k}}{m^*} [n_{\uparrow} - n_{\downarrow}] . \quad (4.2)$$

The reason for this is that the state that contains a QP of momentum \vec{k} and spin σ , $|\vec{k}, \sigma\rangle$ is an eigenstate of the total current operator \hat{J} but *not* an eigenstate of the spin-current operators \hat{J}_{σ} or the total spin-current operator \hat{J}_s . Thus, the total current is dictated by the total current operator and the effective band mass, but the total spin current appears smaller than one would first expect, making the effective spin mass larger than the band mass.

Marinescu [7] shows that this lessened spin current is a result of a transfer of QP momentum from the injection channel to the opposing spin channel through QP interactions. The momentum transfer, however, is not a result of collision mechanisms contained explicitly in the collision integral on the right hand side of the Boltzmann transport equation; instead, it arises from the modification of the QP distribution through the changes in their local energy. The difference between the two is the drag current density that results from the interaction that transfers some momentum from the injection channel to the opposing spin channel, and it is

proportional to the relative drift velocity of the two spin channels:

$$\vec{J}_d = -\alpha(\vec{v}_\sigma - \vec{v}_{\bar{\sigma}}) . \quad (4.3)$$

She shows the constant of proportionality α is simply the angular average of the interchannel interaction using Fermi liquid theory:

$$\alpha = \frac{e\hbar^2}{m^*} \sum_{\vec{k}} \sum_{\vec{k}'} \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},\sigma}} \right) \left(-\frac{\partial n_{\vec{k}',\bar{\sigma}}^0}{\partial \epsilon_{\vec{k}',\bar{\sigma}}} \right) f_{\vec{k}',\sigma;\vec{k},\bar{\sigma}}(\vec{k} \cdot \vec{k}') , \quad (4.4)$$

where

$$f_{\vec{k}',\sigma;\vec{k},\bar{\sigma}} = \begin{cases} \sum_{\ell=0}^{\infty} A_\ell P_\ell(\cos \theta) , & a = 3 \\ \sum_{\ell=0}^{\infty} B_\ell e^{i\ell\theta} , & a = 2 \end{cases} . \quad (4.5)$$

The A_ℓ and B_ℓ are related to the spin symmetric and antisymmetric Landau parameters given in Eqs. (3.10) and (3.11).

The presence of another form of momentum transfer in the spin system suggests an isomorphic process may occur in the bilayer system. This process is discussed in detail in the next subsection.

4.2 Charge Backflow in a Bilayer System

The goal of the following calculation is to create an isomorphic picture for the bilayer system to that of the spin mass problem. The fundamental mechanism behind this problem is the change in the local energy of a QP in layer one due to the interaction with QPs from layer two. This in turn affects the local distribution function, thus distorting the expected value of the QP current.

Consider a system of two 2D layers, sufficiently close for a significant interaction to develop without substantial tunneling so that the electron densities in each layer remain the same. The layer index $i = 1, 2$ is introduced, and the Landau theory of Fermi liquids is used to describe the local energy of each QP in the system. Since the spin degree of freedom is not relevant to the calculation, is it not explicitly declared, but the degree of spin degeneracy is considered implicitly when electron states are counted. By applying an external electrostatic

potential V_i to layer i , the energy of an electron is

$$\epsilon_{\vec{k},i}^0 = \frac{\hbar^2 k^2}{2m} - eV_i, \quad (4.6)$$

where m is the effective band mass, and the total energy of the QP is

$$\begin{aligned} \epsilon_{\vec{k},i} &= \epsilon_{\vec{k},i}^0 + \delta\epsilon_{\vec{k},i}, \\ &= \frac{\hbar^2 k^2}{2m} - eV_i + \sum_{\vec{k}',j} f_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k}',j}. \end{aligned} \quad (4.7)$$

The interaction parameter $f_{\vec{k},i;\vec{k}',j}$ is expanded into interlayer and intralayer terms,

$$f_{\vec{k},i;\vec{k}',j} = \phi_{\vec{k},i;\vec{k}',j} \delta_{i,j} + \psi_{\vec{k},i;\vec{k}',j} \quad (4.8)$$

By using Eq. (3.14) and identifying

$$\vec{v}_{\vec{k},i} = \frac{1}{\hbar} \nabla_{\vec{k}} \epsilon_{\vec{k},i}, \quad (4.9)$$

$$n_{\vec{k},i} = n_{\vec{k},i}^0 + \delta n_{\vec{k},i}, \quad (4.10)$$

$$\epsilon_{\vec{k},i} = \epsilon_{\vec{k},i}^0 + \delta\epsilon_{\vec{k},i}, \quad (4.11)$$

$$\delta\epsilon_{\vec{k},i} = \sum_{\vec{k}'} \phi_{\vec{k},i;\vec{k}',i} \delta n_{\vec{k}',i} + \sum_{\vec{k}'} \psi_{\vec{k},i;\vec{k}',j} \delta n_{\vec{k}',j} \quad (4.12)$$

the Boltzmann transport equation for layer i is written

$$\frac{\partial n_{\vec{k},i}}{\partial t} + \nabla_{\vec{r}}(n_{\vec{k},i}^0 + \delta n_{\vec{k},i}) \cdot \vec{v}_{\vec{k},i} - \frac{1}{\hbar} \nabla_{\vec{r}}(\epsilon_{\vec{k},i}^0 + \delta\epsilon_{\vec{k},i}) \cdot \nabla_{\vec{k}}(n_{\vec{k},i}^0 + \delta n_{\vec{k},i}) = \left(\frac{\partial n_{\vec{k},i}}{\partial t} \right)_{coll}. \quad (4.13)$$

The equation is linearized by eliminating all second-order terms in the distribution change,

$$\begin{aligned} \frac{\partial n_{\vec{k},i}}{\partial t} + \nabla_{\vec{r}} n_{\vec{k},i}^0 \cdot \vec{v}_{\vec{k},i} + \nabla_{\vec{r}} \delta n_{\vec{k},i} \cdot \vec{v}_{\vec{k},i} - \frac{1}{\hbar} \nabla_{\vec{r}} \epsilon_{\vec{k},i}^0 \cdot \nabla_{\vec{k}} n_{\vec{k},i}^0 \\ - \frac{1}{\hbar} \nabla_{\vec{r}} \delta\epsilon_{\vec{k},i} \cdot \nabla_{\vec{k}} n_{\vec{k},i}^0 - \frac{1}{\hbar} \nabla_{\vec{r}} \epsilon_{\vec{k},i}^0 \cdot \nabla_{\vec{k}} \delta n_{\vec{k},i} = \left(\frac{\partial n_{\vec{k},i}}{\partial t} \right)_{coll}. \end{aligned} \quad (4.14)$$

By identifying

$$\nabla_{\vec{r}} n_{\vec{k},i}^0 = \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \nabla_{\vec{r}} \epsilon_{\vec{k},i} \approx \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \nabla_{\vec{r}} \epsilon_{\vec{k},i}^0, \quad (4.15)$$

$$\nabla_{\vec{r}} \epsilon_{\vec{k},i}^0 = \nabla_{\vec{r}} \left(\frac{\hbar^2 k^2}{2m} - eV_i \right) = e\vec{E}_i, \quad (4.16)$$

$$\nabla_{\vec{k}} n_{\vec{k},i}^0 = \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \nabla_{\vec{k}} \epsilon_{\vec{k},i} = \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \hbar \vec{v}_{\vec{k},i}, \quad (4.17)$$

$$\nabla_{\vec{k}} \delta n_{\vec{k},i} = \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \nabla_{\vec{k}} \epsilon_{\vec{k},i} = \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \hbar \vec{v}_{\vec{k},i}, \quad (4.18)$$

the transport equation takes the form

$$\frac{\partial n_{\vec{k},i}}{\partial t} + \vec{v}_{\vec{k},i} \cdot \nabla_{\vec{r}} \delta n_{\vec{k},i} + e \vec{v}_{\vec{k},i} \cdot \vec{E}_i \left(-\frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \right) - \frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \vec{v}_{\vec{k},i} \cdot \nabla_{\vec{r}} \delta \epsilon_{\vec{k},i} = \left(\frac{\partial n_{\vec{k},i}}{\partial t} \right)_{coll}, \quad (4.19)$$

or equivalently

$$\frac{\partial n_{\vec{k},i}}{\partial t} + e \vec{v}_{\vec{k},i} \cdot \vec{E}_i \left(-\frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \right) + \vec{v}_{\vec{k},i} \cdot \nabla_{\vec{r}} \left[\delta n_{\vec{k},i} + \delta \epsilon_{\vec{k},i} \left(-\frac{\partial n_{\vec{k},i}^0}{\partial \epsilon_{\vec{k},i}} \right) \right] = \left(\frac{\partial n_{\vec{k},i}}{\partial t} \right)_{coll}. \quad (4.20)$$

In the absence of collisions or of an electric field, by summing Eq. (4.20) over \vec{k}_1 , the conservation law for the electrons in layer 1

$$\frac{\partial(-en_1)}{\partial t} + \nabla_{\vec{r}} \cdot \vec{J}_1 = 0 \quad (4.21)$$

is reconstituted, where the current density in layer 1 \vec{J}_1 is found to be

$$\vec{J}_1 = -e \sum_{\vec{k}} \vec{v}_{\vec{k},1} \left[\delta n_{\vec{k},1} + \sum_{\vec{k}'} \phi_{\vec{k},1;\vec{k}',1} \delta n_{\vec{k}',1} \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) + \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \delta n_{\vec{k}',2} \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) \right]. \quad (4.22)$$

Eq. (4.22) expresses the conservation of charge in layer 1, when the interaction with the electrons in layer 2 is also considered. This result, however, needs to be compared with the usual current operator in layer 1 that is introduced by the Landau theory of Fermi liquids,

$$\vec{J}_1 \equiv -e \sum_{\vec{k}} \frac{\hbar \vec{k}}{m} \delta n_{\vec{k},1}. \quad (4.23)$$

that defines the charge current in terms of the bare particle momentum.

The relationship between the velocity of a bare particle and the group velocity of a QP in layer 1, is obtained from the definition of the latter

$$\begin{aligned}\vec{v}_{\vec{k},1} &\equiv \frac{1}{\hbar} \nabla_{\vec{k}} \epsilon_{\vec{k},1} , \\ &= \frac{\hbar \vec{k}}{m} + \frac{1}{\hbar} \sum_{\vec{k}',j} \nabla_{\vec{k}} (f_{\vec{k},1;\vec{k}',j}) \delta n_{\vec{k}',j} .\end{aligned}\quad (4.24)$$

Because $f(\vec{k}, \vec{k}')$ embodies the Coulomb interaction, making it a function of $|\vec{k} - \vec{k}'|$, the momentum \vec{k} gradient of the interaction energy is simply related to the momentum \vec{k}' gradient of the interaction energy,

$$\nabla_{\vec{k}} (f_{\vec{k},1;\vec{k}',j}) \delta n_{\vec{k}',j} = -\nabla_{\vec{k}'} (f_{\vec{k},1;\vec{k}',j}) \delta n_{\vec{k}',j} .\quad (4.25)$$

Then the QP velocity is

$$\vec{v}_{\vec{k},1} = \frac{\hbar \vec{k}}{m} - \frac{1}{\hbar} \sum_{\vec{k}',j} \nabla_{\vec{k}'} (f_{\vec{k},1;\vec{k}',j}) \delta n_{\vec{k}',j} .\quad (4.26)$$

The infinite sum may be treated as an integral over momentum k -space, as there are a thermodynamically large number of states available for a QP to occupy. Thus, by integrating by parts, taking into account that the Coulomb interaction must die off at very large distances, the QP velocity is written

$$\vec{v}_{\vec{k},1} = \frac{\hbar \vec{k}}{m} + \frac{1}{\hbar} \sum_{\vec{k}',j} f_{\vec{k},1;\vec{k}',j} \nabla_{\vec{k}'} \delta n_{\vec{k}',j} .\quad (4.27)$$

Expanding the interaction parameter in terms of intralayer and interlayer interaction terms according to Eq. (4.8) and applying Eq. (4.18), a *self-consistent* expression for the QP velocity is obtained,

$$\vec{v}_{\vec{k},1} = \frac{\hbar \vec{k}}{m} - \sum_{\vec{k}'} \phi_{\vec{k},1;\vec{k}',1} \left(-\frac{\partial n_{\vec{k}',1}^0}{\partial \epsilon_{\vec{k}',1}} \right) \vec{v}_{\vec{k}',1} - \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \left(-\frac{\partial n_{\vec{k}',2}^0}{\partial \epsilon_{\vec{k}',2}} \right) \vec{v}_{\vec{k}',2} .\quad (4.28)$$

For isotropic systems, the QP velocity is parallel with the momentum, and the effective mass of the QP may be redefined according to

$$\vec{v}_{\vec{k},i} = \frac{\hbar \vec{k}}{m_i^*} .\quad (4.29)$$

Inserting Eq. (4.28) into Eq. (4.23) yields

$$\vec{J}_1 = -e \sum_{\vec{k}} \left[\vec{v}_{\vec{k}',1} + \sum_{\vec{k}'} \phi_{\vec{k},1;\vec{k}',1} \left(-\frac{\partial n_{\vec{k}',1}^0}{\partial \epsilon_{\vec{k}',1}} \right) \vec{v}_{\vec{k}',1} + \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \left(-\frac{\partial n_{\vec{k}',2}^0}{\partial \epsilon_{\vec{k}',2}} \right) \vec{v}_{\vec{k}',2} \right] \delta n_{\vec{k},1}. \quad (4.30)$$

By changing the summation order in the last two terms, the current density in layer 1 takes the final form

$$\begin{aligned} \vec{J}_1 = & -e \sum_{\vec{k}} \vec{v}_{\vec{k},1} \left[\delta n_{\vec{k},1} + \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) \sum_{\vec{k}'} \phi_{\vec{k},1;\vec{k}',1} \delta n_{\vec{k}',1} \right] \\ & -e \sum_{\vec{k}} \vec{v}_{\vec{k},2} \left(-\frac{\partial n_{\vec{k},2}^0}{\partial \epsilon_{\vec{k},2}} \right) \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \delta n_{\vec{k}',1}. \end{aligned} \quad (4.31)$$

4.3 Differential Current Density in a Bilayer System

Noting that the QP current density given in Eq. (4.22) by the continuity equation is different from that given by the QP velocity in Eq. (4.31), a differential current density \vec{J}_D can be defined,

$$\vec{J}_1^\dagger \equiv \vec{J}_D + \vec{J}_1, \quad (4.32)$$

or

$$\vec{J}_D = \vec{J}_1^\dagger - \vec{J}_1. \quad (4.33)$$

Because the current density \vec{J}_1^\dagger is that which appears in the continuity equation, it is this current density that determines the flow of current through phase space. This means that \vec{J}_1 is the current density carried by just the QPs in layer 1, while the differential current density \vec{J}_D represents a *drag current* between the two layers that contributes to the overall current density of the system. Thus, by placing an electric field through layer 2, a current is driven in layer 1 simply by the action of the Coulomb potential, as given by the left-hand side of the Boltzmann transport equation. Similar to the spin system, this effect is entirely different from the problem investigated by [4] in which a current driven through layer 2 drives a response current through layer 1 entirely through interlayer QP collisions, an effect contained exclusively in the collision integral of the right-hand side of the Boltzmann transport equation. The differential current

given by Eq. (4.33) is

$$\vec{J}_D = -e \sum_{\vec{k}} \vec{v}_{\vec{k},1} \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \delta n_{\vec{k}',2} + e \sum_{\vec{k}} \vec{v}_{\vec{k},2} \left(-\frac{\partial n_{\vec{k},2}^0}{\partial \epsilon_{\vec{k},2}} \right) \sum_{\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} \delta n_{\vec{k}',1}. \quad (4.34)$$

Assume now that an electric field is established in layer 2, such that the motion of the electrons in the second layer is limited only by impurity scattering. Consequently, in the relaxation time approximation, a solution is obtained for the Boltzmann transport equation

$$\delta n_{\vec{k},2} = -e\tau \vec{v}_{\vec{k},2} \cdot \vec{E}_2 \left(-\frac{\partial n_{\vec{k},2}^0}{\partial \epsilon_{\vec{k},2}} \right). \quad (4.35)$$

With this, Eq. (4.34) indicates that, even in the absence of an electric field in layer 1, a current \vec{J}_D will be established in layer 1 given by

$$\vec{J}_D = e^2\tau \left[\sum_{\vec{k},\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} (\vec{v}_{\vec{k},1} \cdot \vec{v}_{\vec{k}',2}) \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) \left(-\frac{\partial n_{\vec{k},2}^0}{\partial \epsilon_{\vec{k},2}} \right) \right] \vec{E}_2. \quad (4.36)$$

Moreover, by introducing α_{12} as

$$\alpha_{12} = e^2\tau \left[\sum_{\vec{k},\vec{k}'} \psi_{\vec{k},1;\vec{k}',2} (\vec{v}_{\vec{k},1} \cdot \vec{v}_{\vec{k}',2}) \left(-\frac{\partial n_{\vec{k},1}^0}{\partial \epsilon_{\vec{k},1}} \right) \left(-\frac{\partial n_{\vec{k},2}^0}{\partial \epsilon_{\vec{k},2}} \right) \right], \quad (4.37)$$

the current density is written

$$\vec{J}_D = \alpha_{12} \vec{E}_2. \quad (4.38)$$

It thus becomes apparent that \vec{J}_D represents a response current required by the conservation of charge in layer 1 that results from the modification of the local energy of QPs in layer 1 as a result of their interaction with the QPs in layer 2. It is worth emphasizing again that this effect is entirely different from the Coulomb drag effect that results from the collisions of electrons described in detail in [4] contained in the collision integral. The bilayer backflow current is proportional to the Fermi parameters according to Eq. (4.36).

While this effect has yet to be verified experimentally, the bilayer system presents the advantage that the two layers can be independently coupled to external field sources, compared with the case of the spin-polarized system where *both* spin populations are subjected to an electric field.

CHAPTER 5

CONCLUSIONS

A backflow current in an interacting bilayer system has been predicted by exploiting the known quantum number isomorphism between spin and layer index between the spin-polarized and bilayer systems, respectively. By applying an external electric field to only a single layer, a response current in the other layer results from interlayer Coulomb interaction. This interaction is a result of the modification of the local quasiparticle energies in each layer. This additional momentum transfer mechanism is a possible candidate for the missing momentum transfer noted by Jauho and Smith [4].

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