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Long-Wavelength Collective Excitations in a Two-Dimensional Spin-Polarized Electron Gas: A Phenomenological Theory

D C. Marinescu
Clemson University, dcm@clemson.edu

J J. Quinn
University of Tennessee

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D. C. Marinescu
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

J. J. Quinn
Department of Physics, University of Tennessee, Knoxville, Tennessee 37906
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The dispersion relations of the collective excitations induced in a 2D spin-polarized electron gas are derived within the framework of the Landau-Silin theory applied to a two-component Fermi liquid. The quasiparticle interaction is described by spin-dependent coefficients, which are parametric functions of the initial degree of polarization. The motion of the low-energy excitations satisfies a semiclassical transport equation solved consistently with the Maxwell equations for the electromagnetic field produced by spin and charge fluctuations. The long-wavelength limit of the self-sustained oscillations is analyzed as a function of the initial degree of polarization of the system, preset by a dc magnetic induction. [S0163-1829(98)01548-3]

I. INTRODUCTION

In an interacting system, collective excitations occur at those values of the frequency, ω, which are the poles of the response functions to an external electromagnetic perturbation. The Coulomb repulsion, in the case of an electron gas, conditions the existence of linearly independent spin and charge waves and determines the departure of the excitation spectra from the single electron transition energy. When a static magnetic field $\vec{B}=B\hat{z}$ is applied in equilibrium, the resulting spin population imbalance produces an enhancement of the collective-mode spectrum: it couples the dielectric and magnetic responses and induces an anisotropy of the spin excitations when the direction of the applied ac magnetic field is different from $\hat{z}$.[1,2]

In this paper we investigate the collective excitation spectrum of a two-dimensional spin-polarized electron gas (2D-SPEG) in the phenomenological framework of the Landau-Silin[3,4] theory of the degenerate Fermi liquid. The 2D electron gas, formed in the inversion layer of a metal-insulator-semiconductor structure, has traditionally served as a testing ground for the theoretical approximations of the many-body interaction. This system behaves fundamentally like a Fermi liquid,[5] whose density can be varied over many orders of magnitude by a static electric field.[6] Of course, a 2D-SPEG has only a single subband, and only intrasubband excitations. Previous phenomenological descriptions of the collective modes have provided remarkable agreement with experimental observations in simple metals[7–9] and in two-dimensional semiconductor structures.[10] In addition to the qualitative agreement, these models were easily generalized at all wave vectors and frequencies[9] and for arbitrarily shaped Fermi surfaces.[11]

The electron gas—n electrons per unit area, with charge $-e$, effective band mass m, and spin σ, imbedded in a positive background—is spin-polarized by a dc magnetic field B. Throughout this work, the spin polarization of the system, $\zeta=(n_\uparrow-n_\downarrow)/n$, is considered a parameter of the problem, taking on any value between zero and unity. The problem in which the spin imbalance is created by an external field should serve as guidance for the more complicated case of a self-consistent magnetic induction that locally spin-polarizes the electron gas. We expect that the many-body effects associated with the short-range Coulomb interaction are the same in the two cases, since the exchange and correlation local field corrections depend only on the particle density for each spin, and are independent of the cause of the spin polarization.

The interaction of the electrons with the dc field is determined by an effective gyromagnetic factor γ determined by the band structure. The spin splitting energy $2\gamma B$ is considered to be large compared to the cyclotron energy $\hbar\omega_c=\hbar eB/mc$; the latter is assumed to be small in respect to the Fermi energy ϵ_F. This assumption allows ζ its full variation range, without considering the Landau quantization of the electron orbits. One possible system for which this could serve as a model is a narrow quantum well of dilute magnetic semiconductor in a nonmagnetic host. At a very low temperature, the interaction of the conduction electrons with the paramagnetic ions leads to a huge enhancement of the effective γ value of the conduction electrons relative to the value in the absence of magnetic impurities. The assumption that the conduction electrons have an enhanced γ, in the investigation of spin excitations, is valid only if the spin excitation frequencies are not too close in value to the resonance frequency of the paramagnetic ions.

The method we adopt for determining the excitation frequencies of the collective modes is based on solving the transport equation for quasiparticles in the perturbative potential, self-consistently with Maxwell's equations for the fields associated with the induced charge and spin fluctuations. At resonance, the quasiparticle fluctuations satisfy a homogeneous system of equations which admits a nontrivial solution only for certain values of the frequency and wave vector. The spectrum of a 2D-SPEG in a magnetic field perpendicular to the electron layer consists of a multitude of collective excitations which reflect the superposition of the motion in the self-consistent electromagnetic field with the rotation in the applied dc magnetic field. When the wavelength of the perturbation is very long, the series of cyclotron harmonics can be decoupled from the excitations determined by the self-consistent field. The latter are spin- and charge-density waves generated by fluctuations in the number of electrons, and spin waves determined by spin-flip processes. Because of the initial spin imbalance, the charge and spin excitations are coupled through terms which are continuous functions of ζ. To illustrate these ideas, we analyze the simple case in which the quasiparticle interaction is the statically screened 2D Coulomb exchange interaction between quasiparticles of the same spin. For an unpolarized gas this approximation gives qualitatively correct results for a number of measured effects.[12] Our results for the dispersion laws are discussed as functions of the degree of spin polarization.

II. TRANSPORT EQUATION FOR QUASIPARTICLES

The 2D spin-polarized electron gas is treated like a two-component Fermi-liquid system: the electrons of momentum $\vec{k}$ and spin σ occupy states inside two Fermi discs of radii $p_{f\sigma}=(4\pi n_\sigma)^{1/2}$. The elementary excitations of the system are quasiparticles of energy $\epsilon_{k\sigma}$ and distribution function $n_{k\sigma}$. This picture is meaningful only in the vicinity of the Fermi surfaces, where the damping is negligible and a quasiparticle equilibrium state can be defined. For the low-energy excited states, the infinitesimal departure $\delta n_{k\sigma}$ from the ground-state distribution $n^0_{k\sigma}$ is the relevant function. $n^0_{k\sigma}$ is the Fermi function written for the quasiparticle energy with respect to the chemical potential of the system μ:

$$n^0_{k\sigma}=[1+e^{(\epsilon_{k\sigma}-\mu)/k_BT}]^{-1}. \tag{1}$$

The interaction between two quasiparticles in states $(\vec{k}\sigma)$ and $(\vec{k}'\sigma')$ is described by a symmetric function $\Phi_{k\sigma;k'\sigma'}$, whose most general form is[13]

$$\Phi_{\vec{k}\sigma;\vec{k}'\sigma'}=\phi_{\vec{k}\vec{k}'}+(\vec{\sigma}\cdot\vec{\sigma}')\psi_{kk'}. \tag{2}$$

In a system of charged particles, $\Phi_{\vec{k}\sigma;\vec{k}'\sigma'}$ is the screened Coulomb interaction.[13] This expression is valid if inversion symmetry and time reversal invariance are assumed. For a homogeneous, translationaly invariant system, $\Phi_{\vec{k}\sigma;\vec{k}'\sigma'}$ depends on the magnitude of $(\vec{k}-\vec{k}')$.

In the Landau theory of a Fermi liquid, the energy of a quasiparticle of momentum $\vec{k}$ and spin σ is determined by its interaction with the other excitations of the system,[3]

$$\epsilon_{k\sigma}=\epsilon^0_{k\sigma}+\sum_{k',\sigma'}\Phi_{\vec{k}\sigma,\vec{k}'\sigma'}\delta n_{k'\sigma'}-\gamma^*\sigma_zB, \tag{3}$$

with $\epsilon^0_{k\sigma}=\hbar^2k^2/2m$ the bare quasiparticle energy. The two possible states of the electronic spin in the static magnetic field are described as usual by the Pauli matrices σ_x, σ_y, and σ_z. The associated potential energy is $-\gamma^*\sigma_zB$, where the effective gyromagnetic factor γ^* is proportional to γ by a renormalization term which depend on the quasiparticle interaction, as shown in Eq. (46).

A weak electromagnetic perturbation, which consists of an electric field $\vec{E}e^{i(\vec{q}\cdot\vec{r}-\omega t)}$ and a magnetic induction $\vec{b}e^{i(\vec{q}\cdot\vec{r}-\omega t)}$ of arbitrary orientation, will create new quasiparticles with a position- and time-dependent distribution function $\tilde{n}_{k\sigma}(\vec{r},t)$,

$$\tilde{n}_{k\sigma}(\vec{r},t)=n^0_{k\sigma}+\delta\tilde{n}_{k\sigma}(\vec{r},t). \tag{4}$$

The deviation from the equilibrium, $\delta\tilde{n}_{k\sigma}(\vec{r},t)$, is considered infinitesimal and is usually written as a sum of spin-dependent and spin-independent parts,

$$\delta\tilde{n}_{k\sigma}(\vec{r},t)=\delta f_k(\vec{r},t)+\vec{\sigma}\cdot\delta\vec{g}_k(\vec{r},t). \tag{5}$$

The identity of $\delta f_k(\vec{r},t)$ and $\delta\vec{g}_k(\vec{r},t)$ is established by taking the trace of $\delta\tilde{n}_{k\sigma}$ multiplied by the Pauli matrices. The total density fluctuation is $\mathrm{Tr}[\delta\tilde{n}_{k\sigma}(\vec{r},t)]=2\delta f(\vec{r},t)$. The magnetization along the direction of the static field is proportional to $\mathrm{Tr}[\sigma_z\delta\tilde{n}_{k\sigma}(\vec{r},t)]=2\delta g^z_k(\vec{r},t)$. The off-diagonal elements of Eq. (5) describe spin-flip processes induced by the transverse components of $\vec{b}$, associated with the raising and lowering spin operators, $\sigma^{\pm}=\sigma_x\pm i\sigma_y$. Up-down $(-)$ and down-up $(+)$ electronic spin transitions generate a transverse magnetization proportional to $\delta\tilde{n}^{\pm}_k(\vec{r},t)=\mathrm{Tr}\{\sigma^{\pm}[\tilde{n}_{k\sigma}(\vec{r},t)]\}=2(\delta g^x_k\pm i\,\delta g^y_k)$.

Under the effect of the perturbation, the quasiparticle excitation energy $\tilde{\epsilon}_{k\sigma}(\vec{r},t)$ is changed from its equilibrium value $\epsilon_{k\sigma}$. Part of the deviation originates in the two-body interaction term in Eq. (3) which involves the new distribution function, Eq. (5). The rest is the potential energy of the bare electron in the local magnetic field, $\vec{b}$, which is the sum of the external ac applied field and the magnetic field associated with the spin-density fluctuations. As a result, the quasiparticle energy change $\delta\epsilon_{k\sigma}$ is

$$\delta\epsilon_{k\sigma}(\vec{r},t)=2\sum_{k'}\phi_{\vec{k}\vec{k}'}\delta f_{k'}(\vec{r},t)+2\vec{\sigma}\cdot\sum_{k'}\psi_{kk'}\delta\vec{g}_{k'}(\vec{r},t)-\gamma^*\vec{\sigma}\cdot\vec{b}. \tag{6}$$

This expression is valid for macroscopic variations in space and time, when $\delta n_{k\sigma}(\vec{r},t)$ can be considered constant over the range of the interaction and $\Phi_{\vec{k}\sigma;\vec{k}'\sigma'}$, averaged over the volume spanned by the two particles, depends only on their relative momentum. Then, $\tilde{\epsilon}_{k\sigma}$ is a local function of $\delta\tilde{n}_{k\sigma}(\vec{r},t)$.

The flow of the low-energy, noninteracting quasiparticles, driven by a weak electromagnetic field, in the phase space, can be described by a semiclassical transport equation. A detailed derivation of the transport equation for the quasiparticle in an electromagnetic field, which takes into account the spin degree of freedom, was given in Ref. 9, and we will quote here just the main results.

A solution $\delta\tilde{n}_{k\sigma}(\vec{r},t)$ of the transport equation should have the time and position dependence imposed by the external field, $\exp[i(\vec{q}\cdot\vec{r}-\omega t)]$. Since the lifetime of a quasiparticle is inversely proportional to the square of its energy in respect to the Fermi surface,[15] it is considered that the equilibrium distribution of the quasiparticles is nonzero only in the vicinity of the Fermi surface. In this perspective, $(-dn^0_{k\sigma}/d\epsilon_{k\sigma})$ behaves like a δ function at the Fermi surface. It is then convenient to express the solution in terms of a new function $\nu_\sigma(\vec{k})$ defined by

$$\delta\tilde{n}_\sigma(\vec{k})=\nu_\sigma(\vec{k},\omega)\left(-\frac{dn^0_{k\sigma}}{d\epsilon_{k\sigma}}\right)e^{i(\vec{q}\cdot\vec{r}-\omega t)}. \tag{7}$$

This choice imposes the restriction that all the functions in the transport equations be estimated at the Fermi surface corresponding to the spin σ electrons. The momentum dependence is completely specified by the angle made by $\vec{k}$ with $\hat{x}$ axis and by $p_{f\sigma}$. Because the interaction [Eq. (6)] couples the various amplitudes $\nu_\sigma(\vec{k})$, it is useful to obtain a solution in terms of a Fourier sum over independent components indexed by l, $\nu_{l\sigma}=(1/2\pi)\int_0^{2\pi}d\phi\nu_\sigma e^{-il\phi}$.

The charge and longitudinal spin-density fluctuations are induced by $\vec{E}$ and b_z, the components of the electromagnetic field which commute with the spin operator. These are local-field values, the sum of the external perturbation and the field generated by the spin and density fluctuations. In a collisionless regime (assumed for simplicity) the equation satisfied by $\nu_\sigma(\phi)$ is

$$-i\hbar\omega\nu_\sigma+\left(i\vec{q}\cdot\vec{v}_\sigma+\omega^*_{c\sigma}\frac{\partial}{\partial\phi}\right)[\nu_\sigma+\delta\epsilon_\sigma]-e\vec{v}_\sigma\cdot\vec{E}=0, \tag{8}$$

where $\omega^*_{c\sigma}=eB/m^*_\sigma c$ is the cyclotron frequency of an electron of spin σ. We have introduced m^*_σ for the mass of an electron of spin σ, which differs from the band effective mass by a factor determined by the interaction, as described by Eq. (44). In the case of the longitudinal spin and charge density fluctuations, $\delta\epsilon_\sigma$ is, from Eq. (6),

$$\delta\epsilon_{p_f\sigma}(\phi)=\sum_{\vec{k}'}(\phi_{p_{f\sigma};k'}+\psi_{p_{f\sigma};k'})\delta\tilde{n}_{k'\sigma}+\sum_{\vec{k}'}(\phi_{p_{f\sigma};k'}-\psi_{p_{f\sigma};k'})\delta\tilde{n}_{k'\bar{\sigma}}-\gamma^*\sigma b_z. \tag{9}$$

When Eq. (7) is employed, the summation over $\vec{k}'$ is constrained by the δ functions to the Fermi surface. One identifies $F_s(p_{f\sigma})=(\phi+\psi)(p_{f\sigma})$ as the interaction between two same-spin quasiparticles, with momenta of equal magnitude $p_{f\sigma}$ and a relative angular orientation $\theta=\phi-\phi'$. The corresponding Fourier sum is

$$F_s(2p_{f\sigma}\sin\,\theta/2)=\sum_l A_{l\sigma}(p_{f\sigma})e^{il\theta}, \tag{10}$$

where $A_{l\sigma}$'s depend parametrically on $p_{f\sigma}$. Since the interaction is a real, symmetric function in k and k', $A_l=A_{-l}$. The opposite-spin interaction is described by $F_a=\phi-\psi$ which has to be estimated for momenta on different Fermi surfaces, $k=p_{f\sigma}$ and $k'=p_{f\bar{\sigma}}$. In this case,

$$|\vec{k}-\vec{k}'|=\sqrt{p^2_{f\sigma}+p^2_{f\bar{\sigma}}-2p_{f\sigma}p_{f\bar{\sigma}}\cos\,\theta}. \tag{11}$$

The Fourier series introduces a new set of coefficients $B_l(p_{f\sigma};p_{f\bar{\sigma}})$, parametric functions of both Fermi momenta,

$$F_a\left(\sqrt{p^2_{f\sigma}+p^2_{f\bar{\sigma}}-2p_{f\sigma}p_{f\bar{\sigma}}\cos\,\theta}\right)=\sum_l B_l(p_{f\sigma};p_{f\bar{\sigma}})e^{il\theta}, \tag{12}$$

with $B_l=B_{-l}$, as imposed by the interaction being a real function symmetric in k and k'.

With Eqs. (10) and (12), the summation over $\vec{k}'$ in $\delta\epsilon_{k\sigma}(\phi)$ [Eq. (9)], is easily performed assuming a constant density of states at the Fermi surface, $N(\epsilon_{f\sigma})=m^*_\sigma/2\pi\hbar^2$. The Fourier component of the interaction energy is then

$$\mathcal{E}_{l\sigma}=\frac{m^*_\sigma}{2\pi\hbar^2}A_{l\sigma}\nu_{l\sigma}+\frac{m^*_{\bar{\sigma}}}{2\pi\hbar^2}B_l\nu_{l\bar{\sigma}}. \tag{13}$$

The effective mass of the quasiparticles, m^*, is different from the effective band mass on account of the interaction, as it is derived in Eq. (44). We introduce the following notation:

$$\alpha_{l\sigma}=1+\frac{m^*_\sigma}{2\pi\hbar^2}A_{l\sigma}, \tag{14}$$

$$\beta_{l\sigma}=\frac{m^*_\sigma}{2\pi\hbar^2}B_l. \tag{15}$$

The standard equation of motion satisfied by the lth Fourier component of ν_σ for $\vec{q}=q\hat{y}$ is, from Eqs. (8) and (13):

$$(-i\omega+il\omega^*_{c\sigma}\alpha_\sigma)\nu_{l\sigma}+il\omega^*_{c\sigma}\beta_{\bar{\sigma}}\nu_{l\bar{\sigma}}+\frac{qv_{f\sigma}}{2}[\alpha_{(l-1)\sigma}\nu_{(l-1)\sigma}+\beta_{(l-1)\bar{\sigma}}\nu_{(l-1)\bar{\sigma}}-\alpha_{(l+1)\sigma}\nu_{(l+1)\sigma}-\beta_{(l+1)\bar{\sigma}}\nu_{(l+1)\bar{\sigma}}]$$
$$=\frac{ev_{f\sigma}}{2}(E_-\delta_{l,1}+E_+\delta_{l,-1}). \tag{16}$$

The $(\pm)$ index is used for the linear combinations of the $\hat{x}$ and $\hat{y}$ vectorial components: $E_\pm=E_x\pm iE_y$. The right-hand side of Eq. (16) is the Fourier transform of $\vec{v}\cdot\vec{E}$; it involves just the $l=\pm1$ components associated with the projections of the quasiparticle velocity on the $\hat{x}$ and $\hat{y}$ axes.

Spin-flip processes are driven by b_x and b_y, and as a result a nonzero spin-density fluctuation is induced perpendicular on $\hat{z}$. Associated with these density fluctuations, spin waves propagate for those frequencies which are poles of the transverse magnetization response functions. A solution to a transport equation satisfied by such a density fluctuation, δn^-, can be written as in Eq. (7), with the spin index chosen appropriately. In the case of spin-flip processes, the interaction $\delta\epsilon_{k\sigma}$ is, from Eq. (6),

$$\delta\epsilon_{k\sigma}=2\sum_{\vec{k}'}\psi_{\vec{k}\vec{k}'}\nu^-(k')\left(-\frac{dn^0_{k\uparrow}}{d\epsilon_{k\uparrow}}\right)-2\gamma^*b^-. \qquad (17)$$

$\psi_{\vec{k}\vec{k}'}$ has to be estimated at the spin-up Fermi surface for two momenta of equal magnitude, $p_{f\uparrow}$ and relative angle $\theta=\phi-\phi'$. (The spins whose projection along $\hat{z}$ was 1 are the ones which flip to -1 under the influence of b^-.) We recognize that $\psi_{\vec{k}\vec{k}'}=(F_s-F_a)/2$, and from Eqs. (10) and (12) write directly:

$$\psi(2p_{f\uparrow}\sin\,\theta/2)=\tfrac{1}{2}\sum_l\,[A_l(p_{f\uparrow})-B_l(p_{f\uparrow};p_{f\uparrow})]e^{il\theta}. \qquad (18)$$

Finally,

$$\mathcal{E}_l^-=\frac{m^*_\uparrow}{2\pi\hbar^2}[A_l(p_{f\uparrow})-B_l(p_{f\uparrow};p_{f\uparrow})]-\gamma^*b^-\delta_{l,0}. \qquad (19)$$

Therefore, the transport equation satisfied by the lth Fourier component of the transverse spin fluctuations for propagation along the $\hat{y}$ axis is

$$[-i\omega\nu_l^-+it_l(l\omega^*_{c\uparrow}+2\gamma^*B)]\nu_l^- +\frac{v_{f\uparrow}}{2}q[t_{l-1}\nu^-_{l-1}-t_{l+1}\nu^-_{l+1}] =\frac{\gamma^*\beta}{2}[b_-\delta_{l,1}+b_+\delta_{l,-1}]. \qquad (20)$$

The phenomenological coefficient t_l is defined by

$$t_l=1+(m^*_\uparrow/2\pi\hbar^2)[A_l(p_{f\uparrow})-B_l(p_{f\uparrow};p_{f\uparrow})]. \qquad (21)$$

ν_l^+, the Fourier coefficient of the complementary spin-flip processes, satisfies the complex conjugate equation written for $(-\vec{B})$, with all the coefficients calculated at the down-spin Fermi surface.

III. CHARGE AND SPIN-DENSITY COLLECTIVE EXCITATIONS

The self-sustained oscillations of the system occur in the absence of the external perturbation. However, the induced spin and charge fluctuations generate an electric current and a spin magnetization, which, in accordance with Maxwell equations, create local electric and magnetic fields. The latter can be set equal to zero, since it is much weaker than the electrostatic interaction.[11] The local electric field is determined by the electric current of the quasiparticle flow, and depends implicitly on the quasiparticle fluctuations $\delta\tilde{n}_{k\sigma}$. Under these conditions, the transport equation for the longitudinal density fluctuations [Eq. (16)], generates an infinite homogeneous system in $\nu_{l\sigma}$. A nontrivial solution exists only for those values of ω and $\vec{q}$ for which the determinant multiplying the column vector $\nu_{l\sigma}$ vanishes. The relation between ω and $\vec{q}$ obtained this way is equivalent to finding the poles of the response function.[9]

In Eq. (16), the $l=\pm1$ components are coupled to the local electric field produced by the electric current. For a propagation vector $\vec{q}$ along the y direction, and a similar geometry, it was established that the the electric field is connected to the current, $\vec{j}$, through[16]

$$i\omega\left(\frac{\beta c^2}{2\pi\omega^2}E_x,-\frac{\epsilon_0}{2\pi\beta}E_y\right)=\vec{j}. \qquad (22)$$

$\beta^2=q^2-\epsilon_0\omega^2/c^2$ is a positive quantity. In the Fermi-liquid theory, the electric current is the sum over the $\vec{k}$ space of the the bare electron velocity weighted by the deviation from equilibrium of the quasiparticle distribution functions:[14]

$$\vec{j}=-e\sum_{k,\sigma}\frac{\hbar\vec{k}}{m}\delta\tilde{n}_{k\sigma}. \qquad (23)$$

For a translationally invariant system, this is equivalent to[13]

$$\vec{j}=\sum_{k,\sigma}\vec{v}_{k\sigma}\left(-\frac{dn^0_{k\sigma}}{d\epsilon_{k\sigma}}\right)(\nu_{k\sigma}+\delta\epsilon_{k\sigma}). \qquad (24)$$

When the Fourier expansions for $\nu_{k\sigma}$ and $\delta\epsilon_{k\sigma}$ are inserted into Eq. (24), the only components selected correspond to $l=\pm1$. Hence

$$j_y^x=-e\sum_\sigma\frac{m^*_\sigma v_{f\sigma}}{4\pi\hbar^2}[(\nu_{-1\sigma}+\mathcal{E}_{-1\sigma})\pm(\nu_{1\sigma}+\mathcal{E}_{1\sigma})], \qquad (25)$$

The electric field is, from Eqs. (25) and (22),

$$E_\pm=\frac{2e\pi i\omega}{\beta c^2}\sum_\sigma\frac{m^*_\sigma v_{f\sigma}}{4\pi\hbar^2}\times\left[(\nu_{-1\sigma}+\mathcal{E}_{-1\sigma})\left(1\mp\frac{\beta^2c^2}{\omega^2\epsilon_0}\right)+(\nu_{1\sigma}+\mathcal{E}_{1\sigma})\left(1\pm\frac{\beta^2c^2}{\omega^2\epsilon_0}\right)\right]. \qquad (26)$$

Upon inclusion in the transport equation of the electric field, the generic equations satisfied self-consistently by $\nu_{l\sigma}$ can be written. We define $\Omega_{l\sigma}=[-i\omega+il\omega^*_{c\sigma}\alpha_{l\sigma}]/\omega^*_{c\sigma}$, $\mu_{l\sigma}=il\beta_{l\sigma}$, and introduce $X_\sigma=v_{f\sigma}q/\omega^*_{c\sigma}$. The equation satisfied by the lth Fourier component is

$$\Omega_{l\sigma}\nu_{l\sigma}+\mu_{l\bar{\sigma}}\nu_{l\bar{\sigma}} =\frac{X_\sigma}{2}[\alpha_{(l-1)\sigma}\nu_{(l-1)\sigma}+\beta_{(l-1)\bar{\sigma}}\nu_{(l-1)\bar{\sigma}} -\alpha_{(l+1)\sigma}\nu_{(l+1)\sigma}-\beta_{(l+1)\bar{\sigma}}\nu_{(l+1)\bar{\sigma}}] +\frac{ev_{f\sigma}}{2}(E_-\delta_{l,1}+E_+\delta_{l,-1}), \qquad (27)$$

with $E_\pm$ determined by Eq. (26). For $|l|>2$, the nonzero elements of the determinant are concentrated in a 6×6 block about the diagonal, generated by the interdependence between the Fourier components of opposite spins corresponding to $l,l-1$, and $l+1$. For $l=0$ the conservation of charge for each spin population is obtained:

$$\Omega_{0\sigma}\nu_{0\sigma}+\mu_{0\bar{\sigma}}\nu_{0\bar{\sigma}} = \left(\frac{X_\sigma}{2}\right)^2 [\alpha_{-1\sigma}\nu_{-1\sigma}+\beta_{-1\bar{\sigma}}\nu_{-1\bar{\sigma}} - \alpha_{1\sigma}\nu_{1\sigma}-\beta_{1\bar{\sigma}}\nu_{1\bar{\sigma}}]. \quad (28)$$

The dispersion relation $\omega(\vec{q})$ is the solution to the determinantal equation obtained from the requirement that the homogeneous system satisfied by $\nu_{l\sigma}$ have a nonzero solution for all l. This very complicated expression describes all the coupled modes of longitudinal spin and charge oscillations, along with a series of cyclotron harmonics generated for $|l|>2$. As an example, we have derived the equation satisfied by the $l=2$ cyclotron harmonics and the magnetoplasma modes, in the long-wavelength limit, when the coupling constant $X_\sigma = qv_{f\sigma}/\omega^*_{c\sigma}$ is small compared to the unity in Eq. (47). Higher-order harmonics are also involved, but with smaller contributions. In the lowest-order approximation we will assume that the cyclotron harmonics and the plasma waves are linearly independent, and obtain analytic results for $\omega(q)$ up to terms quadratic in X.[15]

The modes that propagate in the system under the effect of the self-consistent electric field are magnetoplasma oscillations, and they correspond to $|l|\leqslant 1$. Their excitation frequencies are much larger than ω^*_s, and we approximate $\Omega_{1\sigma}\approx -i\omega/\omega^*_{c\sigma}$ and $\mu_{1\sigma}\approx 0$ in Eq. (27). Under this assumption analytic solutions are obtained for $\omega(q)$. The low-frequency mode is a spin-density wave:

$$\omega^2_{SDW}(q)=\frac{q^2 v_{f\sigma}v_{f\bar{\sigma}}}{2}(\alpha_{1\sigma}\alpha_{1\bar{\sigma}}-\beta_{1\sigma}\beta_{1\bar{\sigma}})\frac{\left[\left(\sqrt{\frac{m^*_\sigma}{m^*_{\bar{\sigma}}}}\,\alpha_{0\sigma}-\sqrt{\frac{m^*_{\bar{\sigma}}}{m^*_\sigma}}\,\beta_{0\sigma}\right)+\left(\sqrt{\frac{m^*_{\bar{\sigma}}}{m^*_\sigma}}\,2\alpha_{0\bar{\sigma}}-\sqrt{\frac{m^*_\sigma}{m^*_{\bar{\sigma}}}}\,\beta_{0\bar{\sigma}}\right)\right]}{\left[\left(\sqrt{\frac{n_\sigma}{n_{\bar{\sigma}}}\frac{m^*_{\bar{\sigma}}}{m^*_\sigma}}\,\alpha_{1\sigma}+\sqrt{\frac{n_{\bar{\sigma}}}{n_\sigma}}\,\beta_{1\sigma}\right)+\sqrt{\frac{n_{\bar{\sigma}}}{n_{bs}}\frac{m^*_\sigma}{m^*_{\bar{\sigma}}}}\,\alpha_{1\bar{\sigma}}+\sqrt{\frac{n_\sigma}{n_{\bar{\sigma}}}}\,\beta_{1\bar{\sigma}}\right)\right]}, \quad (29)$$

This density excitation originates in the spin-antisymmetric fluctuation in the number of electrons whose spin is parallel to the $\hat{z}$ axis. At small wave vectors ω_- is proportional to q and the geometric mean average of the Fermi velocities. Even though this oscillation is generated by density variations, the coupling with the spin degree of freedom determines the linear dependence on q, as an intermediary between a plasma wave, proportional to $\sqrt{q}$ and a spin wave $\sim q^2$.

The high-frequency solution is a superposition of plasma waves associated with each spin population:

$$\omega^2_+(q)=\bar{\omega}^2_\sigma+\bar{\omega}^2_{\bar{\sigma}}. \quad (30)$$

$\bar{\omega}_\sigma$ is the plasma frequency for a spin-σ electron gas in a dielectric medium of permittivity ϵ_s,

$$\bar{\omega}^2_\sigma=\frac{2\pi e^2 n_\sigma}{\epsilon_s m^*_\sigma}q\left(\alpha_{1\sigma}+\beta_{1\sigma}\sqrt{\frac{n_{\bar{\sigma}}}{n_\sigma}}\right) + \frac{q^2 v^2_{f\sigma}}{2}\left[\alpha_{1\sigma}\alpha_{0\sigma}+\left(\frac{m^*_\sigma}{m^*_{\bar{\sigma}}}\right)\sqrt{\frac{n_{\bar{\sigma}}}{n_\sigma}}\,\beta_{1\sigma}\beta_{0\bar{\sigma}}\right]. \quad (31)$$

In this spin-symmetric mode, the effect of the spin polarization is described by the terms proportional to $\beta_{1\sigma}$ and $\beta_{1\bar{\sigma}}$, the Fourier coefficients of the interaction between opposite spins. The dependence on ζ is quadratic since these are charge fluctuations, invariant under the change $\vec{B}\rightarrow(-\vec{B})$.

In addition to the modes described above, for $|l|>2$, in the system propagate coupled cyclotron harmonics associated with the electron motion in the static magnetic field. These excitations begin at

$$\omega^*_{c\pm}=\frac{l}{2}[\omega^*_{c\sigma}\alpha_{l\sigma}+\omega^*_{c\bar{\sigma}}\alpha_{l\bar{\sigma}} \pm\sqrt{4\omega^*_{c\sigma}\omega^*_{c\bar{\sigma}}\beta_{l\sigma}\beta_{l\bar{\sigma}}+(\omega^*_{c\sigma}\alpha_{l\sigma}-\omega^*_{c\bar{\sigma}}\alpha_{l\bar{\sigma}})^2}]. \quad (32)$$

The coupling between the two waves is measured by $\omega^*_{c\sigma}\omega^*_{c\bar{\sigma}}\beta_{l\sigma}\beta_{l\bar{\sigma}}$, which reflects the interaction between the opposite spin electrons. At low polarization values, $(\omega^*_{c\sigma}\alpha_\sigma - \omega^*_{c\bar{\sigma}}\alpha_{\bar{\sigma}})^2 \ll 4\omega^*_{c\sigma}\omega^*_{c\bar{\sigma}}\beta_\sigma\beta_{\bar{\sigma}}$. In this approximation, the two cyclotron harmonics are

$$\omega^0_{c\pm}=\frac{l}{2}\left[\omega^*_{c\sigma}(\alpha_{l\sigma}\pm\beta_{l\sigma})+\omega^*_{c\bar{\sigma}}(\alpha_{l\bar{\sigma}}\pm\beta_{l\bar{\sigma}}) \pm\frac{(\omega^*_{c\sigma}\alpha_{l\sigma}-\omega^*_{c\bar{\sigma}}\alpha_{l\bar{\sigma}})^2}{\omega_{c\sigma}\omega_{c\bar{\sigma}}\beta_{l\sigma}\beta_{l\bar{\sigma}}}\right]. \quad (33)$$

The two solutions correspond to a charge mode (+), determined by $(\alpha_l+\beta_l)$, the Fourier coefficient of the spin independent part of the interaction, and a spin mode (−), driven by $(\alpha_l-\beta_l)$. To the lowest order in ζ, the spin symmetric oscillation is a linear superposition of cyclotron harmonics:

$$\omega^*_{c\pm}=\frac{l}{2}[\omega^*_{c\sigma}(\alpha_{l\sigma}\pm\beta_{l\sigma})+\omega^*_{c\bar{\sigma}}(\alpha_{l\bar{\sigma}}\pm\beta_{l\bar{\sigma}})]. \quad (34)$$

For the spin-symmetric mode, ω_{c+}^*, the fundamental absorption $l=1$ occurs at the bare cyclotron frequency ω_c, as required by Kohn's theorem. This is possible because of the renormalization of the effective mass [Eq. (44)]. If the opposite-spin interaction is neglected, i.e., $\beta_{l\sigma}$ is set equal to zero in Eq. (32), the spin and charge excitations start at the same frequency. At large polarizations, $1-|\zeta|\ll 1$, and the interaction between quasiparticles of opposite spin, described by β_l, becomes very small. Then $(l\omega_{c\sigma}^*\alpha_{l\sigma}-l\omega_{c\bar{\sigma}}^*\alpha_{l\bar{\sigma}})^2 \gg 4l^2\omega_{c\sigma}^*\omega_{c\bar{\sigma}}^*\beta_{l\sigma}\beta_{l\bar{\sigma}}$. In this limit, the cyclotron motion of electrons σ and $\bar{\sigma}$ decouples, and each mode is excited independently at a frequency

$$\omega_c^*=l\omega_{c\sigma}^*\left[\alpha_{l\sigma}+\frac{\beta_{l\sigma}}{(\omega_\sigma^*\alpha_{l\sigma}-\omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}})}\right]. \tag{35}$$

IV. SPIN WAVES

Under the effect of the transverse components of the ac magnetic field, $b^{\pm}$, some electrons change their spin state. These spin-flip processes determine magnetic fluctuations in the $\hat{x}-\hat{y}$ plane, perpendicular on the polarizing dc field. At the resonance of the transverse magnetization response functions, spin waves propagate.

The dynamics of the spin-flip processes is described by a transport equation [Eq. (20)]. The excitation frequencies for the collective modes associated to the up-down transition, are those values of ω for which the determinant of the homogeneous system generated by Eq. (20) vanishes. The only nonzero elements of the matrix formed with the Fourier component of the fluctuations ν_l^- are those along the diagonal and those adjacent to it. In the long-wavelength limit, when $X_\uparrow=qv_{f\uparrow}/\omega_\uparrow^*\ll 1$ the excitation frequency is

$$\hbar\omega_l^-=(l\omega_{c\uparrow}^*+2\gamma^*B)t_{l\uparrow}+\left(\frac{X_\uparrow}{2}\right)^2 t_{l\uparrow}\omega_{c\uparrow}^*\left(\frac{t_{l+1\uparrow}}{\Omega_{l+1\uparrow}}+\frac{t_{l-1\uparrow}}{\Omega_{l-1\uparrow}}\right), \tag{36}$$

where $\Omega_{(l\pm 1)\uparrow}=\{-(\omega_{c\uparrow}^*+2\gamma^*B)t_{l\uparrow}+t_{(l\pm 1)}[(l\pm 1)\omega_{c\uparrow}^*+2\gamma^*B]\}$.

Equation (36) describes a spin wave associated with the poles of the transverse magnetization response function. ω_l^- starts at the Zeeman spin splitting energy, the minimum energy for a spin flip, and increases proportionally to $\vec{q}^2$ through a term solely generated by the spin-dependent part of the quasiparticle interaction. The effect of the spin polarization on these values is determined by the change in the effective mass, as well as by the change in the interaction parameters. The fundamental resonance occurs at $\omega_0^-=2\gamma^*t_0B$. Since $t_0=1+m^*(A_0-B_0)/2\pi\hbar^2$, the renormalization of γ^* because of the interaction [Eq. (46)], is completely canceled, and we regain the excitation frequency of a non-interacting electron system.

The description of a spin wave at high magnetic fields becomes difficult in terms of a Fermi-liquid theory. At $q=0$, a spin excitation requires an energy comparable to the difference between the Fermi energies of the up- and down-spin quasiparticles. When $\zeta\sim 1$, this difference is almost equal to $\epsilon_{f\uparrow}$. For such a large excitation energy, the quasiparticle concept is not well defined (its lifetime is extremely small when the excitation energy is of the order of ϵ_F).

V. DISCUSSION

The excitation frequencies for the various collective modes of a spin-polarized electron gas derived in the previous sections depend on the Fourier coefficients of the quasiparticle interaction $\Phi_{k\sigma,k'\sigma,}$. In the Landau theory of a Fermi liquid, the function $\Phi_{k\sigma,k'\sigma'}$ is not specified. An analytic expression for it requires a microscopic theory of the many-body interaction in the electron gas. To illustrate some of our results, we choose the very simple case in which $\Phi_{k\sigma,k'\sigma'}$ is the screened Coulomb interaction in two dimensions, in the Thomas-Fermi approximation. In the lowest order, the only process which contributes is same-spin exchange and $\Phi_{k\sigma,k'\sigma'}$ is[17]

$$\Phi_{k\sigma,k'\sigma'}=-\frac{2\pi e^2}{|\vec{k}-\vec{k}'|\epsilon_s(|\vec{k}-\vec{k}'|,\epsilon_k-\epsilon_{k'})}. \tag{37}$$

For quasiparticles at the same Fermi surface, such that $\epsilon_k=\epsilon_{k'}$, the dielectric function ϵ_s is equal to its static value, fairly well approximated by

$$\epsilon_s(\vec{q},0)=\epsilon_s(1+q/q_{TF}), \tag{38}$$

where ϵ_s is the dielectric constant of the semiconductor and q_{TF} is the screening length of the interaction in the Thomas-Fermi (TF) approximation, $q_{TF}=2\pi ne^2/\epsilon_F$.

We consider numerical values typical of inversion layer structures: electronic density per unit area $n=5\times 10^{11}$ cm^2, band effective mass $m=0.2m_e$ (m_e is the bare electron mass), $\epsilon_s=11.9$, and band effective gyromagnetic factor $\gamma=100\mu_B$ ($\mu_B=e\hbar/2mc$ is the Bohr magneton).

The effective mass of the interacting quasiparticles varies in respect to the bulk value on account on the exchange interaction, as we show in Eq. (44). This renormalization is modified by the applied dc magnetic field which creates an imbalance in the number of electrons of each spin. In Fig. 1, the variation of m_σ with ζ reflects the proportionality on $1/\sqrt{n_\sigma}$, characteristic of a 2D system. The depletion in the minority spins (considered $\bar{\sigma}$) determines a fast divergence of their effective mass as $1/\sqrt{(1-\zeta)}$. Of course, one needs to consider higher-order approximations for the quasiparticle interaction to put a limit on the increase in $m_{\bar{\sigma}}^*$. In the inset, we plot γ/γ^*. It is expected that the picture is accurate only at low values of polarization, when only effects linear in B occur.

In the exchange approximation, the spin-symmetric charge-density mode ω_+, which is described in Eq. (30) as a linear superposition of plasma waves associated independently with electrons of spin σ and $\bar{\sigma}$, is constant. The dependence on ζ is canceled between the renormalization of the effective mass and the variation of the Fourier coefficient of

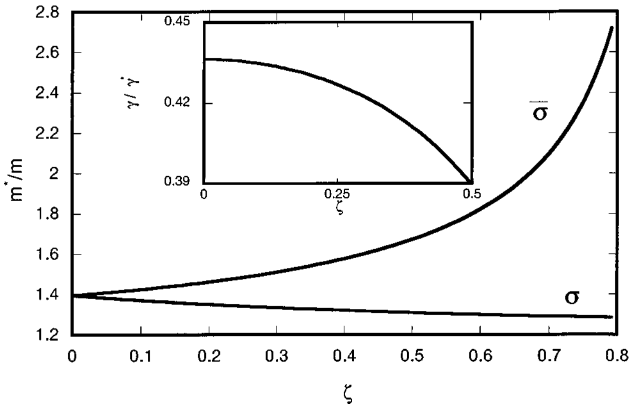

FIG. 1. The variation of the effective mass with the degree of spin polarization is studied assuming a screened exchange interaction among the same-spin electrons. The dependence $m^*_\sigma \sim \sqrt{1/(1\pm\zeta)}$ is characteristic of 2D electron systems. In the inset, at small polarization values γ/γ^* has a very slow decrease with ζ.

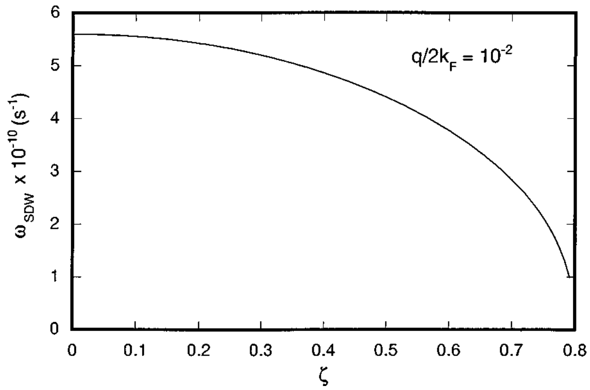

FIG. 2. The excitation frequency of the longitudinal spin-density waves, estimated at $q/2k_F=10^{-2}$, has a quadratic dependence on ζ. At larger polarization values, the increase in the effective mass of the minority spins determines a rapid decline.

the spin-symmetric part of the interaction, $\alpha_{l\sigma}$. The excitation frequency of the spin-antisymmetric mode is presented as a function of ζ in Fig. 2. The proportionality of ω_{SDW} [Eq. (29)], to the geometric mean of the Fermi velocities of the quasiparticles determines its variation as $\sqrt{1-\zeta^2}$ (SDW stands for spin-density wave). The quadratic dependence on ζ is implied by the origin of these oscillations, spin-antisymmetric fluctuations in the density of electrons whose spin remains parallel to the $\hat{z}$ axis. Toward large values of the polarization, the decrease in ω_{SDW} is accelerated by the abrupt variation of $m^*_{\bar{\sigma}}$.

The model considered here generates a simplified description of the cyclotronic modes as $\omega_{cl\sigma}=l\alpha_{l\sigma}\omega^*_\sigma$. The $l=1$ mode, the fundamental absorption, occurs at the noninteracting frequency value implied by the of the effective mass [Eq. (44)]. The inset of Fig. 3 presents the linear dependence of ω^*_{c1} on ζ. For $l=2$, the quasiparticle interaction, embodied by $\alpha_{2\sigma}$, changes dramatically the linearity, especially for the minority spins, whose reduced Fermi momentum gives rise to a large same spin interaction. The coupling between magnetoplasmons and the cyclotron modes is determined by terms proportional to q^2 and higher powers of the wave vectors. This crossover becomes important at values of the frequency ω which are poles of the magnetic-susceptibility function.

The spin waves [Eq. (36)], are excited at a frequency $\omega_l^-=2\gamma^*B\alpha_{l\sigma}$. The fundamental absorption $l=0$ and the first harmonic $l=1$ are presented in Fig. 4. Of course, the $l=0$ mode occurs at the noninteracting frequency value. At low values of ζ, where this model is expected to be correct for spin waves, the difference in the excitation frequencies of the two spin-flip processes is produced by the change in the effective mass of the corresponding quasiparticles.

The phenomenological model we present can be easily modified to describe the long wavelength limit, or arbitrarily shaped Fermi surfaces. A comparison between the excitation frequencies of the collective modes predicted in this framework and spectroscopical measurements in spin-polarized 2D electron system would be most relevant for determining an accurate approximation for the quasiparticle interaction.

VI. EFFECTIVE MASSES AND γ FACTOR IN A SPIN-POLARIZED ELECTRON SYSTEM

In the Landau theory of a Fermi liquid, the effective mass is determined from the ratio of the Fermi momentum of a quasiparticle and its velocity calculated at the Fermi surface, $v_{p_{f\sigma}}=p_{f\sigma}/m^*$. The velocity of a quasiparticle, $v_{k\sigma}$, is defined as the gradient in respect with momentum of the quasiparticle energy,

$$\vec{v}_{k\sigma}=\nabla_k\epsilon_{ks}. \tag{39}$$

When the expression for the equilibrium quasiparticle energy [Eq. (6)] is introduced into Eq. (39), the velocity of a quasiparticle of spin σ is obtained to be

$$\vec{v}_\sigma=\nabla_{\vec{k}}\epsilon^0_k+\sum_{k',\sigma'}\nabla_{\vec{k}}\Phi_{k\sigma;k'\sigma'}\delta n_{k'\sigma'}. \tag{40}$$

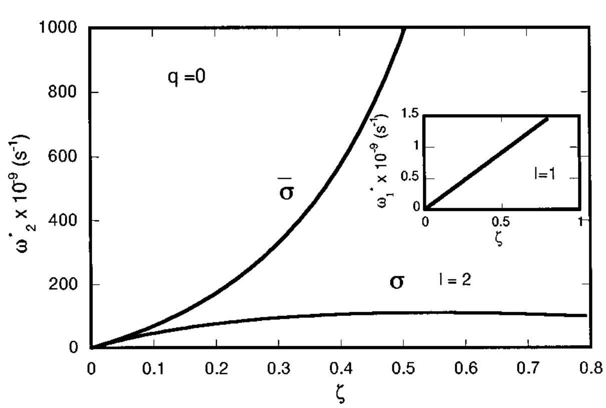

FIG. 3. The cyclotron harmonics for $l=1$ and are dependent on ζ through the effective mass and $\alpha_{l\sigma}$. The $l=1$ mode is excited at the same frequency as in the non interacting system.

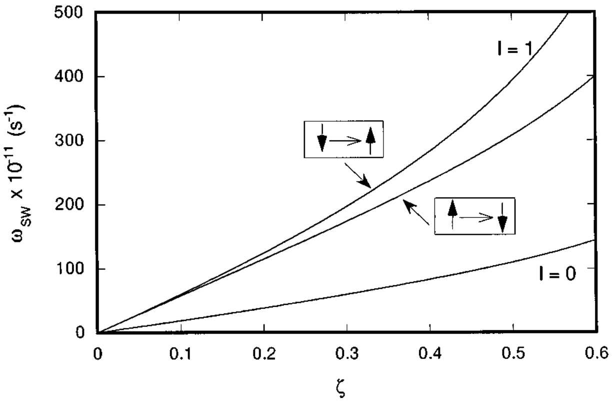

FIG. 4. Spin-flip processes are excited at harmonics of the Zeeman spin splitting energy $2\gamma^*B$. The $l=0$ resonance is not changed by the interaction.

The gradient of the noninteracting energy with respect to $\vec{k}$ is the momentum of the particle divided by the bare mass, $\hbar\vec{k}/m$. For a translationally invariant system, $\Phi_{k\sigma;k'\sigma'}$ depends on $|\vec{k}-\vec{k}'|$, and $\nabla_{\vec{k}}\Phi_{k\sigma;k'\sigma'}=-\nabla_{\vec{k}'}\Phi_{k\sigma;k'\sigma'}$. Using this property, after some integration by parts in the sum after $\vec{k}'$, we can write

$$\vec{v}_{\sigma}=\frac{\hbar\vec{k}}{m}-\sum_{k'\sigma'}\Phi_{k\sigma;k'\sigma'}\nabla_{\vec{k}'}\delta n_{k'\sigma'}. \qquad (41)$$

The gradient in respect to $\vec{k}$ of the equilibrium quasiparticle distribution function is $\nabla_{\vec{k}}\delta n_{k\sigma}=-(dn^0_{k\sigma}/d\epsilon_{k\sigma})\vec{v}_{\sigma}$. With this result, Eq. (41) becomes

$$\vec{v}_{\sigma}=\frac{\hbar\vec{k}}{m}+\sum_{k'}(\Phi_{kk'}+\Psi_{kk'})\frac{dn_{k'\sigma'}}{d\epsilon_{k'}}\vec{v}_{\sigma'}+\sum_{k'}(\Phi_{kk'}-\Psi_{kk'})\frac{dn_{k'\bar{\sigma}'}}{d\epsilon_{k'}}\vec{v}_{\sigma'}. \qquad (42)$$

Equation (42) is a condition imposed on $(\Phi_{kk'}\pm\Psi_{kk'})$ by Galilean invariance. In the case of an isotropic system, $\vec{v}_{\sigma}$ is parallel to $\vec{k}$. Under the assumption that $-(dn^0_{k\sigma}/d\epsilon_{k\sigma})$ behaves like a δ function at the Fermi surface, the summation over $\vec{k}'$ can be easily performed. Employing a constant density of states at the Fermi surface, $N(0)=m^*_{\sigma}/2\pi\hbar^2$, and introducing the Fourier components of the interaction energy [Eq. (13)], we obtain

$$v_{\sigma}=\frac{p_{f\sigma}}{m}-p_{f\sigma}\left(\frac{A_{1\sigma}}{2\pi\hbar^2}\right)-p_{f\bar{\sigma}}\left(\frac{B_{1\sigma}}{2\pi\hbar^2}\right). \qquad (43)$$

Then the renormalized effective mass for an electron of spin σ is

$$\frac{1}{m^*_{\sigma}}=\frac{1}{m}\left[1-\frac{m}{2\pi\hbar^2}A_{1\sigma}-\frac{mB_{1\sigma}}{2\pi\hbar^2}(n_{\bar{\sigma}}/n_{\sigma})^{1/2}\right]. \qquad (44)$$

As a result of the interaction, the effective gyromagnetic factor varies also from its band value. The infinitesimal change in the total energy of the system produced by a spin flip, from up to down for an electron of momentum k, is, from Eq. (3),

$$\delta\epsilon_k=-2\gamma^*B-\gamma^*B\times\left[\sum_{k'_\uparrow}\Psi_{kk'}\left(-\frac{dn^0_{k\uparrow}}{d\epsilon_{k\uparrow}}\right)+\sum_{k'_\downarrow}\Psi_{kk'}\left(-\frac{dn^0_{k\downarrow}}{d\epsilon_{k\downarrow}}\right)\right]. \qquad (45)$$

However, the electron-electron interactions do not change the fundamental excitation frequency for the spin-flip processes. Consequently, $\delta\epsilon_k=-2\gamma B$, as for a noninteracting system. Then in terms of the γ factor for the noninteracting system, we have

$$\gamma/\gamma^*=1+\frac{m^*_\uparrow}{4\pi\hbar^2}(A^0_\uparrow-B^0_\uparrow)+\frac{m^*_\downarrow}{4\pi\hbar^2}(A^0_\downarrow-B^0_\downarrow). \qquad (46)$$

In the limit of weak polarization, the densities of states at the two Fermi surfaces are equal, and one obtains exactly the result of the Landau theory. At large magnetic fields, the problem is not well defined, because the energy of excitation for a spin-flip process becomes comparable to the Fermi energy. Consequently, Eq. (45) is valid only for magnetic fields for which the splitting energy, $2\gamma^*B$ remains much lower than the Fermi energy.

VII. THE DISPERSION LAW FOR THE MAGNETOPLASMA MODES

In Sec. III of this paper we derived the general equations of motion satisfied by the lth Fourier component of the quasiparticle fluctuations [Eq. (27)]. The dispersion law $\omega(q)$ for the collective excitations is obtained by imposing the condition that the infinite homogeneous system generated by Eq. (27) has a nontrivial solution. For $|l|>2$, the interaction couples three consecutive cyclotron harmonics $(l-1)$, l, and $(l+1)$ only. For $|l|\leqslant 2$, the second cyclotron harmonic is also coupled to the plasma oscillations driven by the self-consistent electromagnetic field. The coupling constant is proportional to $X=qv_f/\omega^*$, a small parameter in the long-wavelength limit. Up to the second order in X, the secular equation satisfied by ω is

$$
\begin{aligned}
&\Bigg\{\Omega_{2\sigma}\Omega_{2\bar{\sigma}}\Omega_{-2\sigma}\Omega_{-2\bar{\sigma}}\Bigg[1+\sum_{l}{}'\left(\frac{X_{\sigma}}{2}\right)^{2}\frac{\alpha_{l\sigma}\alpha_{(l-1)\sigma}}{\Omega_{l\sigma}\Omega_{(l-1)\sigma}}+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}\frac{\alpha_{l\bar{\sigma}}\alpha_{(l-1)\bar{\sigma}}}{\Omega_{l\bar{\sigma}}\Omega_{(l-1)\bar{\sigma}}}+\frac{X_{\sigma}X_{\bar{\sigma}}}{4}\left(\frac{\beta_{l\bar{\sigma}}\beta_{(l-1)\sigma}}{\Omega_{l\bar{\sigma}}\Omega_{(l-1)\sigma}}+\frac{\beta_{l\sigma}\beta_{(l-1)\bar{\sigma}}}{\Omega_{l\sigma}\Omega_{(l-1)\bar{\sigma}}}\right)\Bigg]\\
&\quad+\mu_{2\sigma}\mu_{2\bar{\sigma}}\mu_{-2\sigma}\mu_{-2\bar{\sigma}}\Bigg[1+\sum_{l}{}'\left(\frac{X_{\sigma}}{2}\right)^{2}\frac{\beta_{l\bar{\sigma}}\beta_{(l-1)\bar{\sigma}}}{\mu_{l\bar{\sigma}}\mu_{(l-1)\bar{\sigma}}}+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}\frac{\beta_{l\sigma}\beta_{(l-1)\sigma}}{\mu_{l\sigma}\mu_{(l-1)\sigma}}+\frac{X_{\sigma}X_{\bar{\sigma}}}{4}\left(\frac{\alpha_{l\bar{\sigma}}\alpha_{(l-1)\sigma}}{\mu_{l\bar{\sigma}}\mu_{(l-1)\sigma}}+\frac{\alpha_{l\sigma}\alpha_{(l-1)\bar{\sigma}}}{\mu_{l\sigma}\mu_{(l-1)\bar{\sigma}}}\right)\Bigg]\\
&\quad-(\Omega_{2\sigma}\Omega_{2\bar{\sigma}}\mu_{-2\sigma}\mu_{-2\bar{\sigma}}+\Omega_{-2\sigma}\Omega_{-2\bar{\sigma}}\mu_{2\sigma}\mu_{2\bar{\sigma}})\Bigg[1+\sum_{l}{}'\left(\frac{X_{\sigma}}{2}\right)^{2}\frac{\beta_{l\bar{\sigma}}\alpha_{(l-1)\sigma}}{\Omega_{l\bar{\sigma}}\Omega_{(l-1)\sigma}}+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}\frac{\beta_{l\sigma}\alpha_{(l-1)\bar{\sigma}}}{\mu_{l\sigma}\Omega_{(l-1)\bar{\sigma}}}\\
&\quad+\frac{X_{\sigma}X_{\bar{\sigma}}}{4}\left(\frac{\alpha_{l\bar{\sigma}}\beta_{(l-1)\bar{\sigma}}}{\mu_{l\bar{\sigma}}\Omega_{(l-1)\bar{\sigma}}}+\frac{\alpha_{l\sigma}\beta_{(l-1)\sigma}}{\mu_{l\sigma}\Omega_{(l-1)\sigma}}\right)\Bigg]\Bigg\}E+(\Omega_{2\sigma}\Omega_{2\bar{\sigma}}-\mu_{2\sigma}\mu_{2\bar{\sigma}})\Bigg\{\left(\frac{X_{\sigma}}{2}\right)^{2}(E_{22}\alpha_{-1\sigma}-E_{21}\beta_{-1\bar{\sigma}})\\
&\quad\times(\alpha_{-2\sigma}\Omega_{-2\bar{\sigma}}-\mu_{-2\sigma}\beta_{-2\bar{\sigma}})+\left(\frac{X_{\sigma}}{2}\right)^{2}(E_{11}\alpha_{-1\sigma}-E_{12}\beta_{-1\sigma})(\alpha_{-2\bar{\sigma}}\Omega_{-2\sigma}-\beta_{-2\sigma}\mu_{-2\bar{\sigma}})-\frac{X_{\sigma}X_{\bar{\sigma}}}{4}\\
&\quad\times(E_{22}\beta_{-1\sigma}-E_{21}\alpha_{-1\bar{\sigma}})(\alpha_{-2\bar{\sigma}}\Omega_{-2\sigma}-\beta_{-2\sigma}\mu_{-2\bar{\sigma}})+\frac{X_{\sigma}X_{\bar{\sigma}}}{4}(E_{11}\beta_{-1\bar{\sigma}}-E_{12}\alpha_{-1\sigma})(\alpha_{-2\bar{\sigma}}\mu_{-2\sigma}-\beta_{-2\sigma}\Omega_{-2\bar{\sigma}})\Bigg\}\\
&\quad+(\Omega_{-2\sigma}\Omega_{-2\bar{\sigma}}-\mu_{-2\sigma}\mu_{-2\bar{\sigma}})\Bigg\{\left(\frac{X_{\sigma}}{2}\right)^{2}(E_{44}\alpha_{1\sigma}-E_{43}\beta_{1\bar{\sigma}})(\alpha_{2\sigma}\Omega_{2\bar{\sigma}}-\mu_{2\sigma}\beta_{2\bar{\sigma}})+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}(E_{33}\alpha_{1\sigma}-E_{34}\beta_{1\sigma})\\
&\quad\times(\alpha_{2\bar{\sigma}}\Omega_{2\sigma}-\beta_{2\sigma}\mu_{2\bar{\sigma}})-\frac{X_{\sigma}X_{\bar{\sigma}}}{4}(E_{44}\beta_{1\sigma}-E_{43}\alpha_{1\bar{\sigma}})(\alpha_{2\bar{\sigma}}\Omega_{2\sigma}-\beta_{2\sigma}\mu_{2\bar{\sigma}})+\frac{X_{\sigma}X_{\bar{\sigma}}}{4}(E_{33}\beta_{1\bar{\sigma}}-E_{34}\alpha_{1\sigma})\\
&\quad\times(\alpha_{2\bar{\sigma}}\mu_{2\sigma}-\beta_{2\sigma}\Omega_{2\bar{\sigma}})\Bigg\}=0. \qquad (47)
\end{aligned}
$$

Above, E_{ij} is the 3×3 determinant obtained from determinant E, given below, by striking out row i and column j. The index of summation l' can take all the integer values except 0 and ±1:

$$
E=\begin{vmatrix} W_{-1\bar{\sigma}} & -P_{1\bar{\sigma}} & Z_{-1\sigma} & -R_{1\sigma}\\ -P_{1\bar{\sigma}} & W_{-1\sigma} & -R_{1\bar{\sigma}} & Z_{1\sigma}\\ Z_{-1\bar{\sigma}} & -R_{-1\sigma} & W_{1\bar{\sigma}} & -P_{1\sigma}\\ -R_{-1\bar{\sigma}} & Z_{1\bar{\sigma}} & -P_{-1\sigma} & W_{1\sigma}\end{vmatrix}, \qquad (48)
$$

where the following notations were introduced:

$$
\gamma_{\sigma}^{\pm}=-\frac{i\omega}{\beta c^{2}}\left(1\pm\frac{\beta^{2}c^{2}}{\epsilon_{0}\omega^{2}}\right)p_{f\sigma}, \qquad (49)
$$

$$
W_{1\sigma}=\Omega_{1\sigma}-\frac{ev_{f\sigma}}{2\omega_{\sigma}^{*}}(\alpha_{1\sigma}\gamma_{\sigma}^{-}+\beta_{1\sigma}\gamma_{\bar{\sigma}}^{-})+\left(\frac{X_{\sigma}}{2}\right)^{2}\alpha_{1\sigma}U_{0\sigma}+\frac{X_{\sigma}X_{\bar{\sigma}}}{2}\beta_{1\sigma}V_{0\bar{\sigma}}, \qquad (50)
$$

$$
P_{1\sigma}=\frac{ev_{f\sigma}}{2\omega_{\sigma}^{*}}(\alpha_{1\sigma}\gamma_{\sigma}^{+}+\beta_{1\sigma}\gamma_{\bar{\sigma}}^{+})+\left(\frac{X_{\sigma}}{2}\right)^{2}\alpha_{1\sigma}U_{0\sigma}+\frac{X_{\sigma}X_{\bar{\sigma}}}{2}\beta_{1\sigma}V_{0\bar{\sigma}}, \qquad (51)
$$

$$
Z_{1\bar{\sigma}}=\mu_{1\bar{\sigma}}-\frac{ev_{f\sigma}}{2\omega_{\sigma}^{*}}(\alpha_{1\bar{\sigma}}\gamma_{\bar{\sigma}}^{-}+\beta_{1\bar{\sigma}}\gamma_{\sigma}^{-})+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}\beta_{1\bar{\sigma}}U_{0\sigma}+\frac{X_{\sigma}X_{\bar{\sigma}}}{2}\alpha_{1\bar{\sigma}}V_{0\bar{\sigma}}, \qquad (52)
$$

$$
R_{1\bar{\sigma}}=\frac{ev_{f\sigma}}{2\omega_{\sigma}^{*}}(\alpha_{1\bar{\sigma}}\gamma_{\bar{\sigma}}^{+}+\beta_{1\bar{\sigma}}\gamma_{\sigma}^{+})+\left(\frac{X_{\bar{\sigma}}}{2}\right)^{2}\beta_{1\bar{\sigma}}U_{0\sigma}+\frac{X_{\sigma}X_{\bar{\sigma}}}{2}\alpha_{1\bar{\sigma}}V_{0\bar{\sigma}}, \qquad (53)
$$

$$
U_{0\sigma}=\frac{\Omega_{0\bar{\sigma}}\alpha_{0\sigma}-\mu_{0\sigma}\beta_{0\bar{\sigma}}}{\Omega_{0\sigma}\Omega_{0\bar{\sigma}}-\mu_{0\sigma}\mu_{0\bar{\sigma}}}, \qquad (54)
$$

$$
V_{0\sigma}=\frac{\Omega_{0\bar{\sigma}}\beta_{0\sigma}-\mu_{0\sigma}\alpha_{0\bar{\sigma}}}{\Omega_{0\sigma}\Omega_{0\bar{\sigma}}-\mu_{0\sigma}\mu_{0\bar{\sigma}}}. \qquad (55)
$$

Finding a solution to the secular equation is not possible, unless some simplifying assumptions are made. We argue that, in the small-q limit, the cyclotron harmonics can be decoupled from the plasma waves. In this limit, the equation satisfied by the cyclotron harmonics of index l is

$$[\Omega_{(l-1)\sigma}\Omega_{(l-1)\bar{\sigma}}-\mu_{(l-1)\sigma}\mu_{(l-1)\bar{\sigma}}](\Omega_{l\sigma}\Omega_{l\bar{\sigma}}-\mu_{l\sigma}\mu_{l\bar{\sigma}})[\Omega_{(l+1)\sigma}\Omega_{(l+1)\bar{\sigma}}-\mu_{(l+1)\sigma}\mu_{(l+1)\bar{\sigma}}]$$
$$+[\Omega_{(l+1)\sigma}\Omega_{(l+1)\bar{\sigma}}-\mu_{(l+1)\sigma}\mu_{(l+1)\bar{\sigma}}]T_{l-1}+[\Omega_{(l-1)\sigma}\Omega_{(l-1)\bar{\sigma}}-\mu_{(l-1)\sigma}\mu_{(l-1)\bar{\sigma}}]T_{l+1}=0, \tag{56}$$

where

$$T_{l\pm1}=\left(\frac{X_\sigma}{2}\right)^2[\Omega_{(l\pm1)\bar{\sigma}}\alpha_{(l\pm1)\sigma}-\mu_{(l\pm1)\sigma}\beta_{(l\pm1)\bar{\sigma}}][\Omega_{l\bar{\sigma}}\alpha_{l\sigma}-\beta_{l\bar{\sigma}}\mu_{l\sigma}]+\left(\frac{X_{\bar{\sigma}}}{2}\right)^2[\Omega_{(l\pm1)\sigma}\alpha_{(l\pm1)\bar{\sigma}}-\mu_{(l\pm1)\bar{\sigma}}\beta_{(l\pm1)\sigma}]$$
$$\times(\Omega_{l\sigma}\alpha_{l\bar{\sigma}}-\mu_{l\bar{\sigma}}\beta_{l\sigma})+\left(\frac{X_\sigma X_{\bar{\sigma}}}{4}\right)[\Omega_{(l\pm1)\bar{\sigma}}\beta_{(l\pm1)\sigma}-\mu_{(l\pm1)\sigma}\alpha_{(l\pm1)\bar{\sigma}}](\Omega_{l\sigma}\beta_{l\bar{\sigma}}-\mu_{l\bar{\sigma}}\alpha_{l\sigma})$$
$$+\left(\frac{X_\sigma X_{\bar{\sigma}}}{4}\right)[\Omega_{(l\pm1)\sigma}\beta_{(l\pm1)\bar{\sigma}}-\mu_{(l\pm1)\bar{\sigma}}\alpha_{(l\pm1)\sigma}](\Omega_{l\bar{\sigma}}\beta_{l\sigma}-\mu_{l\sigma}\alpha_{l\bar{\sigma}}). \tag{57}$$

Up to the second order in q, the excitation frequencies of the cyclotron harmonics are

$$\omega_\pm^*=\omega_\pm^0+\frac{T_{(l+1)}}{[\Omega_{(l+1)\sigma}\Omega_{(l+1)\bar{\sigma}}-\mu_{(l+1)\sigma}\mu_{(l+1)\bar{\sigma}}]}+\frac{T_{(l-1)}}{[\Omega_{(l-1)\sigma}\Omega_{(l-1)\bar{\sigma}}-\mu_{(l-1)\sigma}\mu_{(l-1)\bar{\sigma}}]}, \tag{58}$$

where $\omega_\pm^0$ are given by

$$\omega_\pm^*=\frac{l}{2}[\omega_{c\sigma}^*\alpha_{l\sigma}+\omega_{c\bar{\sigma}}^*\alpha_{l\bar{\sigma}}\pm\sqrt{4\omega_{c\sigma}^*\omega_{c\bar{\sigma}}^*\beta_{l\sigma}\beta_{l\bar{\sigma}}+(\omega_{c\sigma}^*\alpha_{l\sigma}-\omega_{c\bar{\sigma}}^*\alpha_{l\bar{\sigma}})^2}]. \tag{59}$$

ACKNOWLEDGMENT

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