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Quantum Hall Spherical Systems: The Filling Fraction

P Sitko
University of Tennessee

J J. Quinn
University of Tennessee

D C. Marinescu
Clemson University, dcm@clemson.edu

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P. Sitko
University of Tennessee, Knoxville, Tennessee 37996
and Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

J. J. Quinn
University of Tennessee, Knoxville, Tennessee 37996
and Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

D. C. Marinescu
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831
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Within the recently formulated composite fermion hierarchy the filling fraction of a spherical quantum Hall system is obtained when it can be expressed as an odd or even denominator fraction. A plot of $\nu(2S/N-1)$ as a function of $2S$ for a constant number of particles (up to $N=10\,001$) exhibits a structure of the fractional quantum Hall effect. It is confirmed that $\nu_e+\nu_h=1$ for all particle-hole conjugate systems, except systems with $N_e=N_h$ and $N_e=N_h\pm 1$. [S0163-1829(97)03132-9]

During the past 15 years, systems of the quantum Hall effect were intensively studied,[1] both for experimentally related situations (Laughlin droplet[2]) and for purely theoretical spherical systems.[3] The main problem of any quantum Hall related studies is the definition of the filling fraction. For infinite systems the filling fraction is defined as $\nu^{-1}=(B/\rho)[(h/e)]^{-1}$, i.e., the number of the flux quanta per electron. For finite systems, however, this definition can no longer be used. The only way of assigning the noninteger filling fraction to the system with a given number of particles and strength of the magnetic field is either intuitive[2,3] or based on the use of the composite fermion (CF) transformation.[4] We extend the formulation of the Jain filling fractions to all odd denominator hierarchy fractions. We make use of recently formulated[5] composite fermion hierarchy. The further extension is made by generalizing the composite fermion hierarchy to all even denominator fractions by introducing the idea of ''half-filled''[6] states of quasiparticles. In such a way we can obtain the filling fraction for almost all pairs of N (number of particles) and $2S$ (the strength of the magnetic monopole for the sphere).

We define the filling fraction in the following way. First, perform the CF transformation (if $\nu<1$) changing the value of $2p$ (the strength of the Chern-Simons field) to get at least one filled effective Landau shell. Hence,

$$\frac{1}{\nu}=2p+\frac{\alpha}{n+\nu_{\rm QE}}, \tag{1}$$

where α is the sign of the effective field (with respect to the real magnetic field), n is the number of filled effective Landau shells, and $\nu_{\rm QE}$ is the filling fraction for quasielectrons partially occupying the ''$n+1$'' effective shell. The process can be repeated on $\nu_{\rm QE}$ until at the mth step $\nu_{\rm QE}^m=0$ (the hierarchy odd denominator fraction) or $\alpha_{\rm QE}^m=0$ which corresponds to the ''half-filled'' $[\nu_{\rm QE}^m=(1/2p_{\rm QE}^m)]$ quasielectron case (even denominator fractions). Hence, we get all fractions, except the cases when one quasielectron is left (no fraction can be assigned to one particle system).

In Fig. 1 we plot the values of

$$\nu\ \frac{2S}{N-1} \tag{2}$$

for eight electrons for values of the filling starting at $\nu=1$ and going to $\nu=1/5$. The function (2) has the value of unity when $\nu=[(N-1)/2S]$, which occurs for the Laughlin states (1,1/3,1/5) and for the ''half-filled'' (1/2,1/4) states. For other fractions the function (2) varies from unity, but an abrupt change is clearly visible at ν close to 1/2 and 1/4. Such ''discontinuity'' can be explained already with introduction of Jain states. The integer filling (for real $2S$ or effective $2S^*$ field) is obtained when

$$N=n^2+n(2S), \tag{3}$$

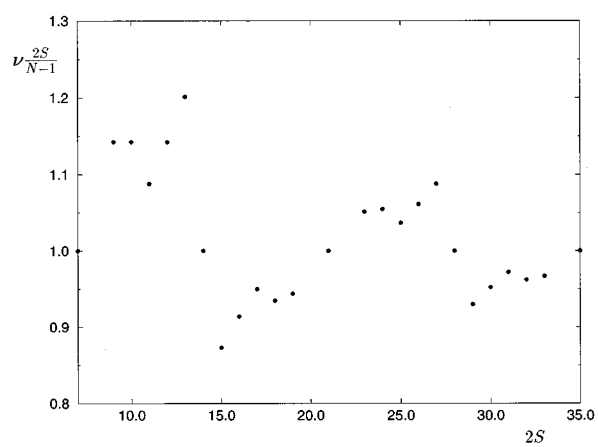

FIG. 1. The values of $\nu[2S/(N-1)]$ for $N=8$ within a range of $1\geqslant\nu\geqslant 1/5$.

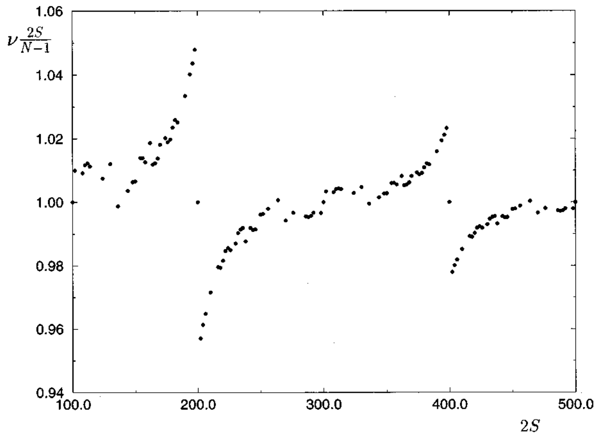

FIG. 2. The same as Fig. 1 for $N=101$.

and $N=n^2+n(2S^*)$ for Jain states. Since $2S^*=2S-2p(N-1)$, we get

$$\frac{2S}{N-1}=\frac{2pn+1}{n}-\frac{n^2-1}{n(N-1)}, \quad \text{for } 2S^*>0, \tag{4}$$

and

$$\frac{2S}{N-1}=\frac{2pn-1}{n}+\frac{n^2-1}{n(N-1)}, \quad \text{for } 2S^*<0. \tag{5}$$

Hence, approaching the $1/2p$ states from both sides we find the corrections to the filling fractions to be of opposite sign, with a maximum correction for maximum n.

We plot similar results for $N=101$ (Fig. 2). The discontinuity at $1/2p$ states is not only confirmed by the Jain states but also by other hierarchy states. Additionally, similar discontinuities can be seen at each $1/2p_{\mathrm{QE}}$ state of quasielectrons (even denominator fractions). In the range of $300<2S<500$ the curve clearly repeats the results for $100<2S<300$, because in the formulation of the fraction we change only the value of $2p$. We confirm this by plotting the results for $N=10001$ within the range $10\,000<2S<30\,000$ (Fig. 3), and within the range $30\,000<2S<50\,000$ (Fig. 4) with an adjusted scale for the function (2). Such a repeating structure can be also seen when looking at quasielectron filling, i.e., $\nu=4/5$ ($\nu_{\mathrm{QE}}=1/3$) down to $\nu=2/3$ ($\nu_{\mathrm{QE}}=1$). We plot the results for $N=10\,001$ and the range $12\,500<2S<15\,000$ in Fig. 5. The curves are not exactly the same, however, due to the fact that the number of quasielectrons changes with $2S$, in contrast to all other figures where $N=\text{const}$.

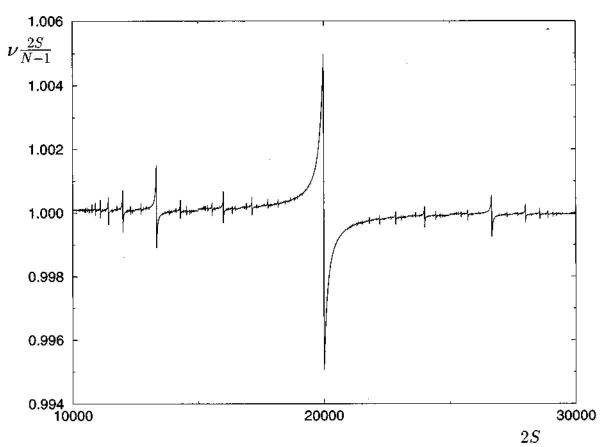

FIG. 3. The values of $\nu[2S/(N-1)]$ for $N=10\,001$ within a range of $1\geqslant\nu\geqslant 1/3$.

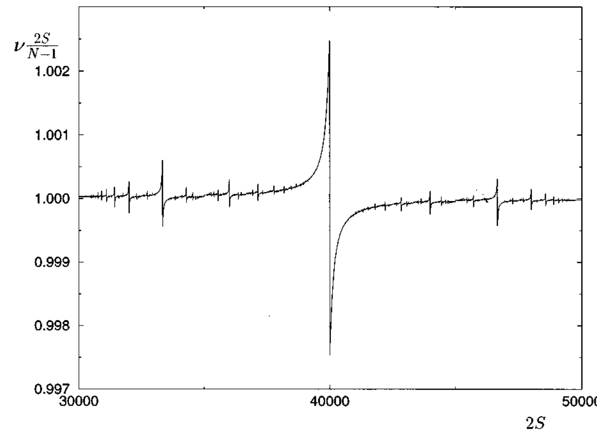

FIG. 4. The same as Fig. 3 for $1/3\geqslant\nu\geqslant 1/5$.

In order to see the structure for $\nu>1$ we plot the results for $N=101$ for $0<2S<500$ in Fig. 6. A better view is obtained when $N=10\,001$ and $2000<2S<10\,000$ in Fig. 7. The discontinuities again come from analogous ''half-filled'' states at higher Landau levels. The integer fillings are seen as little jumps at the curve. Such jumps are also seen in Fig. 2 for Laughlin and in Figs. 3, 4 for Jain states. In fact, all odd denominator hierarchy states lead to such jumps (each of them can be seen if the number of particles is large enough) and all even denominator fractions give discontinuities when resolution of the curves (the scale and the number of particles) increases. Thus, for an infinite number of particles the curve exhibits fractal structure.

In order to confirm our method of defining the filling fraction we calculate the sum of $\nu_e+\nu_h$ for particle-hole conjugate states ($\nu_e<1$). The sum is always one, as expected, except for the three cases when $N_e=N_h$, and $N_e=N_h\pm 1$,

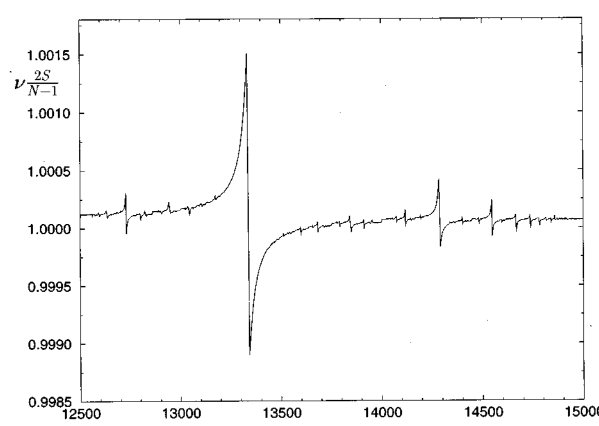

FIG. 5. The spectrum of the filling fraction (multiplied by $[2S/(N-1)]$) for $N=10\,001$ and $12\,500\geqslant 2S\geqslant 15\,000$. This represents the systems of quasielectrons within a range of $1\geqslant\nu_{\mathrm{QE}}\geqslant 1/3$ (note that ν_{QE} increases with $2S$).

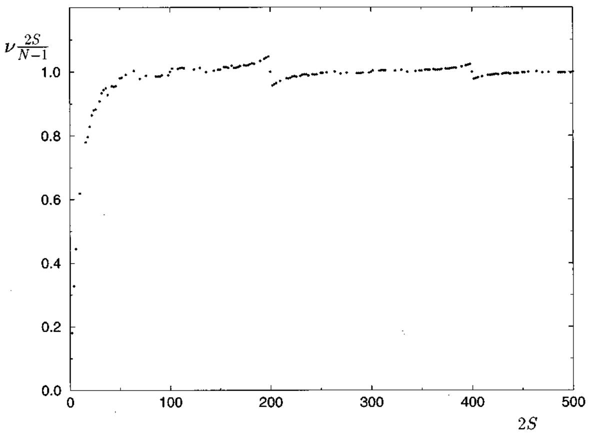

FIG. 6. The whole spectrum $0<2S\leqslant 500$ for $N=101$.

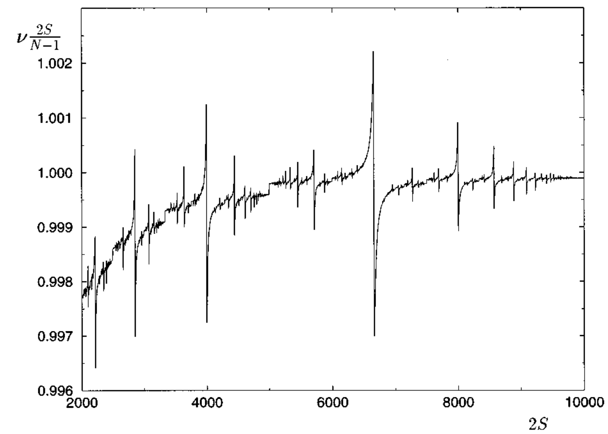

FIG. 7. The range of integer filling for $N=10\,001$, $2000\leqslant 2S\leqslant 10\,000$.

which correspond to difficulty in determing the filling fraction of 1/2. The filling $\nu_e=1/2$ is obtained when $N_e=N_h+1$, hence, the filling fraction ν_h for N_h is necessarily less than 1/2. A similar problem is for $N_e=N_h$ when the CF hierarchy fraction is less than 1/2. It is worth noting, however, that no problems occur at other fillings $1/2p$ $(1/4,1/6,\ldots)$ which represent exactly the same problem in terms of composite fermions.

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