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Under appropriate conditions electron-hole symmetry should apply to a partially filled Landau level of a two-dimensional electron gas. This suggests that the application of Jain's composite fermion (CF) picture to either electrons or holes should lead to equivalent results. Surprisingly, for a system of N_e electrons on a Haldane sphere, this is not true for three values of the Landau level degeneracy $2S+1$. When $N_e-1\leqslant S\leqslant N_e$, the sum of the electron filling factor ν and the hole filling factor μ, as determined from Jain's picture, is smaller than unity. Because of this, use of the relation $\nu=1-\mu$ can lead to ''twin'' or ''alias'' states having different values of ν for the same N_e and $2S+1$. One example is the ''half-filled'' state. It is determined by requiring the effective (mean-field) flux $2S^*$ ''seen'' by one CF to vanish. Different results are obtained when $S_e^*=S_e-(N_e-1)$ and $S_h^*=S-(N_h-1)$ are set equal to zero. The same problem arises in the CF hierarchy picture when the number of quasielectrons n_{QE} is related to the effective flux $2S^*$ by $2(n_{\rm QE}-1)\leqslant 2S^*\leqslant 2n_{\rm QE}$. [S0163-1829(97)00647-4]

I. INTRODUCTION

A strongly correlated two-dimensional electron gas, subjected to a magnetic field B, behaves like an incompressible quantum fluid generating fractional quantum Hall states when the ratio of the particle density n to the magnetic flux density, expressed in flux quanta, is a simple fraction with an odd denominator $\nu=n\phi_0/B$ ($\phi_0=hc/e$, the flux quantum). This property of the interacting electron system can be explained in terms of the composite fermion (CF) picture, proposed by Jain[1] to describe the sequence of Laughlin states with $\nu=1/3,2/5,3/7,\ldots$. The composite fermion transformation, as defined by Jain, attaches to each electron an even number $2p$ of flux quanta oriented opposite to the applied magnetic field, such that the effective mean field flux per particle is $\nu^{*-1}=\nu^{-1}-2p$. Fractional fillings correspond then to integer quantum Hall states of the weakly interacting CF system, described by ν^*. This theoretical approach was extended by Halperin, Lee, and Read[2] to the properties of the compressible $\nu=1/2$ state, the accumulation point of the odd-denominator sequences. For an infinite number of particles in the electron gas, it has been established that the half-filled state occurs when the effective magnetic field ''seen'' by a composite fermion is zero.

Important insight into the nature of the fractional filling states has been obtained by studying a system of N_e electrons on a sphere of radius R that contains at its center a magnetic monopole of strength $2S\phi_0$ generating a radial field $B=(\hbar c/e)SR^{-2}$.[3] The single electron energy $\xi=(\hbar\omega_c/2S)[l(l+1)-S^2]$ depends on the angular momentum $l=S+n$ ($n=0,1,2,\ldots$), but not on its z component. Therefore, the nth Landau level (or nth angular momentum shell) has a degeneracy $g_n=2S+(2n+1)$.

When the composite fermion transformation is applied, the effective monopole strength S^* experienced by one CF electron is

$$2S_e^*=2S-2p(N_e-1). \tag{1}$$

For simplicity, henceforth we consider $p=1$. The single CF states are quantized in the same way as the electron states if S_e^* replaces S and ω_c^* replaces ω_c. If N_e composite fermions fill exactly an integer number ν^* of shells, then

$$\nu=\frac{\nu^*}{1+2\nu^*}, \tag{2}$$

where $\nu^*=\pm1,\pm2,\pm3,\ldots$ describes the condensed states in the principal Jain sequence. This leads to a unique relationship between the value of S and N_e at which a particular filling occurs. At $S_e^*=0$, the effective magnetic field seen by a CF is zero. By analogy to the infinite system, this criterion has been used to define a half-filled Landau level.[4]

For magnetic fields large enough that $\hbar\omega_c\gg e^2/l$, all but the lowest electron Landau level can be neglected. One can then invoke particle-hole symmetry and apply the composite fermion transformation to holes instead of electrons. Obviously,

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$$N_e+N_h=2S+1. \quad (3)$$

The effective monopole strength seen by a CF hole is

$$2S_h^*=2S-2(N_h-1). \quad (4)$$

The requirement that the CF holes fill an integer number μ^* of shells gives a fractional filling

$$\mu=\frac{\mu^*}{1+2\mu^*}, \quad (5)$$

where $\mu^*=\pm1, \pm2, \ldots$ describes the CF ''hole'' filling of the condensed liquid states. The analog of the half-filled state in this case is obtained for $S_h^*=0$. The half-filled state should be obtained for the same values of the monopole flux in the two representations. However, $S_e^*=0$ when $2S=2(N_e-1)$, whereas $S_h^*=0$ when $2S=2N_e$. In other words, for the same value of N_e the half-filled state occurs for an electron CF at values of $2S$ different from that for a CF hole.

In the light of the above considerations, one could think of three possible descriptions of the half-filled Landau level (or the half-filled shell): $2N_e = 2N_h=2S+1$, which occurs only when $2S$ is odd (because N_e and N_h are integers) and corresponds to having half as many electrons as there are single-particle states $2S+1$; $2S_e^*=0$, which occurs at $2S=2(N_e-1)$ or at $2S=2N_h$ and corresponds to CF electrons seeing an effective magnetic field $B_e^*=0$; and $2S_h^*=0$, which occurs at $2S=2(N_h-1)$ or at $2S=N_e$ and corresponds to an effective magnetic field B_h^* seen by the CF holes equal to zero. In this paper we show how this dilemma leads to ''twin'' or ''alias'' states[6] and we discuss the appropriate description of the half-filled Landau level for systems with small number of particles.

II. COMPOSITE FERMION TRANSFORMATION AND ELECTRON-HOLE SYMMETRY

The states of the Jain sequence occur when $2S_e$ is such that an integral number of shells is filled with CF electrons. If n shells are filled, then the number of electrons satisfies

$$\sum_i^n g_i=N_e. \quad (6)$$

When the summation is performed this is written

$$(2|S_e^*|+n)n=N_e. \quad (7)$$

If we identify n with the magnitude of ν^*, the inverse of the effective flux seen by a CF electron, and impose the condition that ν^* have the same sign as S_e^*, then $\nu^*=\nu_+^*$ if $S>(N_e-1)$, $\nu^*=\nu_-^*$ if $S<(N_e-1)$, and ν^* is undefined if $S=N_e-1$. Here $\nu_\pm^*$ are given by

$$\nu_\pm^*=-S_e^*\pm[(S_e^*)^2+N_e]^{1/2}. \quad (8)$$

The same analysis applied to CF holes filling an integral number of shells gives $\mu^*=\mu_+^*$ if $S<N_e$, $\mu^*=\mu_-^*$ if $S>N_e$, and μ^* is undefined if $S=N_e$. Here $\mu_\pm^*=-S_h^*\pm[(S_h^*)^2+N_h]^{1/2}$. From their definition, $S_e^*+S_h^*=1$. This implies that $(S_e^*)^2+N_e=(S_h^*)^2+N_h$ and $\nu_+^*+\mu_-^*=\nu_-^*+\mu_+^*=-(S_e^*+S_h^*)=-1$. From this one can easily demonstrate that for values of S outside the range $N_e-1\leqslant S\leqslant N_e$, the sum of ν and μ [as defined by Eqs. (2) and (5), respectively] is equal to unity. For the three values in the range $N_e-1\leqslant S\leqslant N_e$, the sum $\nu+\mu$ is always less than unity.

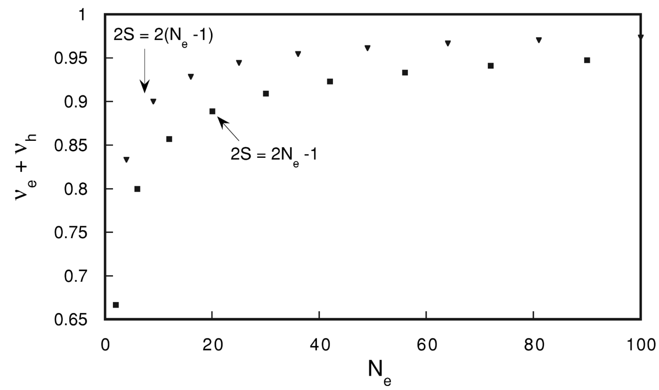

FIG. 1. Plot of $\nu_e+\nu_h$ as determined from the Jain construction for $2S=2(N_e-1)$ and $2S=2N_e-1$.

In Fig. 1 we plot $\nu+\mu$ vs N_e for states in the principal Jain sequence that have $S=N_e-1$ and $S=N_e-1/2$. It has been common to assume that $\nu+\mu=1$ for all values of $2S$. We have just demonstrated that for finite-size systems in a spherical surface this is not true if S is in the range $N_e-1<S<N_e$. We believe that use of $\nu+\mu=1$ in situations where it is not valid and defining values of ν^* (or μ^*) when $S_e^*=0$ (or $S_h^*=0$) leads to twin or alias states discussed by other authors.[5,6] The electron-hole symmetry can be employed to resolve alias states obtained for $(N_e,2S)=(n^2,2n^2-2)$. In these cases, $|\nu^*|=n$ and two fractional fillings, both corresponding to incompressible states, can be envisioned, $\nu=\nu^*/(2p\nu^*\pm1)$, depending on the sign chosen for ν^*. The $N_h=n^2-1$, CF holes, however, experience a nonzero effective field $S_h^*=1$ and occupy exactly $\mu^*=n-1$ shells. From Eq. (5), μ is $(n-1)/(2n-1)$. ν is then simply determined, as $1-\mu$, leading to the only possible electron fractional filling $\nu=n/(2n-1)$.

The fractional fillings of states belonging to the principal Jain sequence for an $N_e=12$ electron system calculated for values of $2S$ going from 11 to 33 are given in Table I. Equally good candidates for the ''half-filled'' Landau level are the states occurring at $2S=22$ when $S_e^*=0$ and at $2S=24$ when $S_h^*=0$. The former is represented as being half filled with electrons, whereas the latter is half filled with holes. The CF electron state with $2S=22$ can be thought of as containing three quasielectrons of the $\nu=3/5$ state (or of the $\nu=3/7$) state. Since each quasielectron has angular momentum $l_{\rm QE}=3$, the allowed multiplets of the mean-field ground state have $L=0\oplus2\oplus3\oplus4\oplus6$ and the ground state is highly degenerate. This is expected for a half-filled state.[4] For $2S=23$, both descriptions give a 3/7-filled state, since this state is less than half filled for both CF electrons and CF holes. It has $\nu^*=\mu^*=3$. Notice that for $2S<22$ and $2S>24$, $\nu^*+\mu^*=-1$ and $\nu+\mu=1$.

However, whenever N_e is the square of an integer $N_e=n^2=2^2,3^2,\ldots$, the value $S_e^*=0$ occurs at an integral CF filling $|\nu^*|=n$. This means that even though $S_e^*=0$ and

TABLE I. Values of $2S_e^*$, the effective flux seen by a composite fermion electron, and $2S_h^*$, that seen by a CF hole, are given for various values of $2S$. g_{ei} (g_{hi}) are the degeneracies of the ith electron (hole) CF level. ν_e^* and ν_h^* are the effective integral CF fillings and $\nu_e = \nu_e^*/(1+2\nu_e^*)$ and $\nu_h = \nu_h^*/(1+2\nu_h^*)$. $N_e = 12$.

$2S$	$2S_e^*$	g_{e1}	g_{e2}	g_{e3}	ν^*	ν	N_h	$2S_h^*$	g_{h1}	g_{h2}	g_{h3}	μ^*	μ
11	−11	12			−1	1	0						
18	−4	5	7		−2	2/3	7	6	7			1	1/3
21	−1	2	4	6	−3	3/5	10	3	4	6		2	2/5
22	0	1				1/2		11	2	2			
23	1	2	4	6	3	3/7	12	1	2	4	6	3	3/7
24	2	3					13	0	1				1/2
26	4	5	7		2	2/5	15	−2	3	5	7	−3	3/5
33	11	12			1	1/3	22	−9	10	12		−2	2/3

the effective magnetic field $\omega_c^* = 0$, there is a gap in the spectrum produced by the kinetic energy $(\hbar^2/2mR^2)n(n+1)$. The ground state is nondegenerate and it does not appear to have the properties expected for a half-filled state. Thus the cancellation of the effective field experienced by a CF is a rather necessary but not sufficient condition for a compressible state. This argument is supported by the results of exact numerical diagonalization of the Hamiltonian for a system $(N_e, 2S) = (9,16)$, which show that the ground state of total angular momentum $L = 0$ is clearly separated from the first excited states. Furthermore, the energy spectrum is identical, up to an overall constant, to that of $(N_e, 2S) = (8,16)$. In this latter case, $2S_e^* = 2$ and $\nu = 2/5$. It seems that the description of $(N_e, 2S) = (9,16)$ as a half-filled state using as an argument just the cancellation of the effective field is ill advised. Some care must be exercised to be sure that the candidate half-filled state has a highly degenerate ground state.

III. SUMMARY

In this paper we have demonstrated that due to finite-size effects $\nu + \mu$ as defined by Jain's CF picture is not equal to unity for values of $2S$ in the range $N_e - 1 \leqslant S \leqslant N_e$. We believe that inappropriate use of $\nu + \mu = 1$ leads to twin or alias states. In addition, when $2S_e^* = 0$ (or $2S_h^* = 0$) a half-filled state does not necessarily result since the actual energy spectrum can be identified with that of a well-defined condensed state of the Jain sequence. In the CF hierarchy picture[7] in which flux tubes are attached to quasiparticle excitations (CF in an otherwise empty shell) to give new mean-field composite fermions, this same ambiguity arises. Whenever the effective flux value $2S^*$ at a given value of the hierarchy is related to the number of quasielectrons by $2(n_{\mathrm{QE}} - 1) \leqslant 2S^* \leqslant 2n_{\mathrm{QE}}$, attaching flux quanta to quasiholes instead of quasielectrons gives fillings ν_i and μ_i that do not sum to unity.

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