A Study of Estimation Techniques and of Mechatronic System Design and Testing

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A STUDY OF ESTIMATION TECHNIQUES AND OF MECHATRONIC SYSTEM DESIGN AND TESTING

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Electrical Engineering

by
Brett Castelloe
August 2007

Accepted by:
Dr. Darren Dawson, Committee Chair
Dr. John Wagner
Dr. Ian Walker
ABSTRACT

This thesis is a collection of two projects that the author worked on during his master’s studies at Clemson University. The first project—adaptive camera calibration—involves the design and simulation of an estimator for the calibration parameters of a camera. The second project—basket drive wear testing—includes the design of a test plan for measuring wear on a mechatronic system.

The first chapter serves to introduce both projects. Included is a literature review for the camera calibration project and an identification of the parties involved in the basket drive project.

In the second chapter, the models for the camera calibration cases—fixed camera (moving feature points) and moving camera (fixed feature points)—are presented. Also, the estimator for the calibration parameters is derived. Proof of stability for the estimator is offered, and simulation results are provided.

The third chapter explains the testing process for the basket drive project. First, information on the background and past issues are addressed. Next, pre-testing and testing procedures are outlined. Finally, the measurement methods are discussed.

The fourth chapter discusses the conclusions and future work for each project. For the camera calibration project, the performance of the simulation is evaluated
and future experimentation is described. For the basket drive system, difficulties with the plan are mentioned.
DEDICATION

This thesis is dedicated to my family. Mama and Daddy—thank you for your love and support throughout all of my schooling. Neil—you’re the world’s greatest brother; I wish you the best as you go to Iraq.
ACKNOWLEDGEMENTS

Completion of this thesis would not have been possible without the help of my advisors, colleges, family, and friends. I first would like to thank Dr. Darren Dawson for the direction that he provided during my stay at Clemson University. His guidance during my graduate projects was invaluable. In addition, I would like to thank Dr. John Wagner for advising me with the basket drive project and for reviewing my progress each week. I also want to say thank you to each of my committee members—Dr. Darren Dawson, Dr. John Wagner, and Dr. Ian Walker—for reviewing my work.

I would like to thank David Braganza, Hariprasad Kannan, Apoorva Kapadia, Nitendra Nath, and Enver Tatlicioglu for their direction in the adaptive camera calibration project. The theoretical background and the simulation of that theory would not have been completed without their involvement in the project.

From the Clemson Environmental Technologies Laboratory (CETL), I would like to thank Don Erich and Eileen Brown for their involvement in the basket drive project. In addition, I want to extend my gratitude to Carl Rathz for the assistance he provided and for performing the 3D scanning measurements. Also, I am grateful to Luis Aguilar for machining the parts that were necessary for the basket stand assembly and Gavin Wiggins for agreeing to continue the project after I graduate.
Finally, I would like to thank my family and friends for their support with these projects. First, to my parents who have always been available for direction when it was needed—I love you, and thank you for your faithfulness. Secondly, I want to say thank you to my girlfriend, Karen, for her patience with me as I worked on this thesis and for her help with revisions. Thirdly, I would like to thank my roommates for their encouragement and for all the good memories that we share. Lastly, I am greatly blessed and eternally grateful to my family and friends for their prayers that have sustained me throughout my graduate studies.
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CHAPTER 1
INTRODUCTION

1.1 Thesis Organization

This thesis is a collection of two separate projects—each of which required different research to solve the respective problems. The first problem, adaptive camera calibration, is introduced in Section 1.2 and addressed in Chapter 2. The second project, chain and sprocket reliability wear testing, is introduced in Section 1.3 and discussed in Chapter 3. Chapter 4 provides the conclusions and future work for each of the projects.

1.2 Adaptive Camera Calibration

The second chapter of this thesis presents an adaptive method for computing the calibration parameters of a camera. Previously, the intrinsic parameters of a camera have been estimated using a linear approach with motion restricted to translation along the optical axis of the camera [6]. More recently, a “visual servoing” approach has been used to find the intrinsic parameters [8]. In this work, the intrinsic and extrinsic parameters will be estimated according to Equation 2.16.

1.3 Chain and Sprocket Reliability Wear Testing

The third chapter of this thesis was written to specify the testing procedures involved with the Basket Drive Wear MOX-PDCF Support Task. It includes a
short introduction to the project, a testing procedure, and a description of the measuring techniques that will be used to analyze wear. The goal of this project is to analyze the component wear of a basket drive assembly in an abrasive, high temperature environment. The basket drive design that was provided by Los Alamos National Laboratory (LANL) has been fabricated by the Clemson University College of Engineering and Science machine shop. Experimentation will be conducted at the Clemson Environmental Technologies Laboratory (CETL) with specialized assistance provided by local academic research.
2.1 Overview

The work presented in this chapter was initiated in the booklet “Adaptive Camera Calibration,” written by Hariprasad Kannan [4]. The chapter presents an introduction and explanation of the problem, experimental setup, mathematical models, an estimator design, proof of stability for the estimator, and simulation results. In addition, Appendix A gives definitions of the terms used for the estimator. Appendix B shows the Simulink simulation that was used for this experiment.

2.2 Introduction to the Problem

2.2.1 Objective

To use an adaptive estimator to obtain the constant internal and external camera calibration parameters that are described later in the chapter. The objective will be achieved by moving feature points in front of a camera (or moving a camera about fixed feature points) and using the resulting image measurements to update the estimator.

2.2.2 Notation

The following convention is used throughout this chapter: for a variable $v$, $\hat{v}$ represents its estimated value and $\bar{v} = v - \hat{v}$ gives the estimator error.
2.2.3 Experimental Setup

The fixed camera system has a stationary camera looking at features attached to the end-effector. The robot is moved around in order to get enough images of the features at various positions and orientations. The location of the feature point with respect to the body (B) and world (W) frames is always known. This is a reasonable assumption because the link lengths are known and the current joint angles can be measured.

In the moving camera case, the feature is stationary and the camera is moved as it records the images. The assumption is made that the features are located at a known distance from B. This is reasonable as well. For example, suppose the camera fixed to the robotic arm of a space station needs to be calibrated. When the camera needs calibration, all it has to do is turn back to look at the space station and record images of some features on the space station. The location of the features will be known because the dimensions of the space station are known. Refer to Figure 2.3 for the fixed camera setup and Figure 2.5 for the moving camera setup.

2.3 Models

The camera is a mapping between the three dimensional world and a two dimensional image. Camera models are matrices with particular properties that represent the camera mapping. The simplest camera model is the pinhole camera model shown in Figure 2.1.
From Figure 2.1, it is noted that a feature that is bottom and left with respect to the camera’s point of view will be located in the top right portion of the image plane. From the similar triangles in the figure,

\[ \frac{l_x}{f} = \frac{-X_c}{Z_c - f} \approx \frac{-X_c}{Z_c} \]  

\[ \frac{l_y}{f} = \frac{-Y_c}{Z_c - f} \approx \frac{-Y_c}{Z_c} \]  

(2.1)
Figure 2.2: The usual shape of the sensor

The camera image may result in a parallelogram shape instead of a true rectangle.

From Figure 2.2 and Equation 2.1, the following relationship can be observed:

\[
\begin{bmatrix}
  m_x \\
  m_y
\end{bmatrix} = \begin{bmatrix}
  k_u & 0 \\
  0 & k_v
\end{bmatrix} \begin{bmatrix}
  l_x - l_y \cot \varphi \\
  \frac{l_y}{\sin \varphi}
\end{bmatrix} = \begin{bmatrix}
  -f k_u \frac{x_c}{Z_c} + f k_u \cot \varphi \frac{y_c}{Z_c} \\
  -f k_v \frac{y_c}{\sin \varphi Z_c}
\end{bmatrix}
\]  

(2.2)

Because the origin of the camera is located at the pixel coordinates

\([u_0 \quad v_0 \quad 1]^T\), the mapping from world coordinates \([X_c \quad Y_c \quad Z_c]^T\) to pixel coordinates \([u \quad v \quad 1]^T\) is

\[
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix} = \begin{bmatrix}
  u_0 - m_x \\
  v_0 - m_y \\
  1
\end{bmatrix} = \begin{bmatrix}
  f k_u & -f k_u \cot \varphi & u_0 \frac{x_c}{Z_c} \\
  0 & f k_v & v_0 \frac{y_c}{Z_c} \\
  0 & 0 & 1
\end{bmatrix}
\]

(2.3)
Calibration of a camera involves determining the parameters $f$, $k_u$, $k_v$, $\phi$, $u_0$, and $v_0$ and the (extrinsic) rotation and translation matrices. The matrix $A$ shown below is the intrinsic calibration matrix from Equation 2.3.

$$A = \begin{bmatrix} f k_u & -f k_u \cot \phi & u_0 \\ 0 & \frac{f k_v}{\sin \phi} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

### 2.3.1 Fixed Camera Case

Figures 2.3 and 2.4 show the fixed camera model that will be used for adaptive camera calibration. In the following figures, $W$ represents the world frame (fixed), $B$ the body frame (moving), $C$ the camera frame, and $F_i$ the $i^{th}$ feature point.

![Figure 2.3: The fixed camera setup](image)
Figure 2.4: The fixed camera model

In this model, the following are known or are measurable:

- \( x_B \in \mathbb{R}^3 \): position of B relative to W, expressed in W
- \( R_B \in \text{SO}(3) \): rotation from B to W (\( R_B : B \rightarrow W \)), expressed in W
- \( \bar{x}_{fi} \in \mathbb{R}^3 \): position of \( F_i \) relative to B, expressed in B
- \( x_{fi} \in \mathbb{R}^3 \): position of \( F_i \) relative to W, expressed in W

The following are the unknown extrinsic calibration parameters:

- \( R_C \in \text{SO}(3) \): rotation from C to W (\( R_C : C \rightarrow W \)), expressed in W
- \( x_C \in \mathbb{R}^3 \): position of C relative to W, expressed in W

The pixel coordinates \( (p_i) \) of the \( i^{th} \) feature point depend on its position with respect to the camera and the unknown intrinsic calibration matrix \( (A) \).

\[
p_i = [u_i \ v_i \ 1]^T
\]

\[
\bar{m}_i = [x_i \ y_i \ z_i]^T
\]

\[
A = \begin{bmatrix}
a_1 & a_2 & a_3 \\
0 & a_4 & a_5 \\
0 & 0 & 1
\end{bmatrix} \in \mathbb{R}^{3 \times 3}
\]
\begin{align*}
p_i &= \frac{1}{z_i} \cdot A \cdot \vec{m}_i \\
\end{align*}

From Figure 2.4,

\begin{align*}
R_B \vec{x}_{fi} + x_B &= x_{fi} \tag{2.5} \\
x_{fi} &= R_C \vec{m}_i + x_c \tag{2.6} \\
\vec{m}_i &= R_C^T (x_{fi} - x_c) \\
\vec{m}_i &= R_C^T (R_B \vec{x}_{fi} + x_B - x_c) \tag{2.7}
\end{align*}

Substituting Equation 2.7 into Equation 2.4 gives

\begin{align*}
p_i &= \frac{1}{z_i} \cdot A \cdot \left( R_C^T R_B \vec{x}_{fi} + R_C^T x_B - R_C^T x_c \right) \\
p_i &= \frac{1}{z_i} \cdot A \cdot \left[ R \quad t \right] \cdot \vec{x}_i \tag{2.8}
\end{align*}

where

\begin{align*}
R &\triangleq R_C^T \in SO(3) \\
t &\triangleq -R_C^T x_c \in \mathbb{R}^3 \\
\vec{x}_i &\triangleq \left[ (R_B \vec{x}_{fi} + x_B)^T \quad 1 \right]^T \in \mathbb{R}^4
\end{align*}

Therefore, the objective is to determine A, R, and t. Knowing these allows the intrinsic parameters \((f, k_u, f, k_v, \varphi, u_0, \text{ and } v_0)\) and the extrinsic parameters (\(R_c\) and \(x_c\)) to be found.

### 2.3.2 Moving Camera Case

Figures 2.5 and 2.6 show the model that will be used with a moving camera.

Again, W represents the world frame (fixed), B the body frame (moving), C the camera frame, and \(F_i\) the \(i^{th}\) feature point.
In this model, the following are either known or measurable:

- $x_B \in \mathbb{R}^3$: position of B relative to W, expressed in W
- $R_B \in \text{SO}(3)$: rotation from B to W ($R_B: B \rightarrow W$), expressed in W
- $x_{fi} \in \mathbb{R}^3$: position of $F_i$ relative to W, expressed in W
The following are the unknown extrinsic calibration parameters:

- $R_C \in \text{SO}(3)$: rotation from C to B ($R_C: C \rightarrow B$), expressed in B
- $x_C \in \mathbb{R}^3$: position of C relative to B, expressed in B

Equation 2.1 still describes the location of $F_i$ in the image. From Figure 2.6,

$$R_B^T (x_{fi} - x_B) = R_C m_i + x_C$$  \hspace{1cm} (2.9)

$$\bar{m}_i = R_C^T (R_B^T (x_{fi} - x_B) - x_C)$$  \hspace{1cm} (2.10)

Substituting Equation 2.10 into Equation 2.4,

$$p_i = \frac{1}{z_i} \cdot A \cdot \left( R_C^T R_B^T (x_{fi} - x_B) - R_C^T x_C \right)$$

$$p_i = \frac{1}{z_i} \cdot A \cdot [R \ t] \cdot \bar{x}_i$$  \hspace{1cm} (2.11)

where

$$R \triangleq R_C^T \in \text{SO}(3)$$

$$t \triangleq -R_C^T x_C \in \mathbb{R}^3$$

$$\bar{x}_i \triangleq \left[ (R_B^T (x_{fi} - x_B))^T \ 1 \right]^T \in \mathbb{R}^4$$

As in the fixed camera case, the objective is to determine $A$, $R$, and $t$.

2.4 Estimator Design

2.4.1 Estimation Strategy

Notice the similarity between Equations 2.8 and 2.11. For either case, the estimator design will be the same. We can rewrite these two equations as follows:

$$p_i = \frac{1}{W_{xi} \theta_x} \cdot W_{xi} \theta_x$$  \hspace{1cm} (2.12)
where

\[ W_{xl} \cdot \theta_x = A \cdot [R \ t] \cdot \tilde{x}_l \in R^3 \]

\[ W_{zl} \cdot \theta_z = \{[R \ t] \cdot \tilde{x}_l\}_3 = z_l \in R \]

See Appendix A for definitions of \( W_{xl} \in R^{3 \times 12}, \theta_x \in R^{12}, W_{zl} \in R^{1 \times 4}, \) and \( \theta_z \in R^4. \)

Let \( \hat{p}_i \) be the estimate for the location of the \( i^{th} \) feature point \((F_i)\) in the image and \( \hat{\theta} \) be the estimate for \( \theta \), the calibration parameters. \( \hat{p}_i \) is found by

\[ \hat{p}_i = \frac{1}{w_{zl}\hat{\theta}_z} \cdot W_{xl}\hat{\theta}_x \quad (2.13) \]

Also,

\[ p_iW_{zl}\theta_z = W_{xl}\theta_x \]

\[ \hat{p}_iW_{zl}\hat{\theta}_z = W_{xl}\hat{\theta}_x \]

Subtracting,

\[ p_iW_{zl}\theta_z - \hat{p}_iW_{zl}\hat{\theta}_z = W_{xl}\theta_x - W_{xl}\hat{\theta}_x \]

\[ p_iW_{zl}\theta_z - \hat{p}_iW_{zl}\theta_z + \hat{p}_iW_{zl}\theta_z - \hat{p}_iW_{zl}\hat{\theta}_z = W_{xl}\theta_x - W_{xl}\hat{\theta}_x \]

Because \( \tilde{p}_i = p_i - \hat{p}_i \) and \( \tilde{\theta} = \theta - \hat{\theta} \),

\[ \tilde{p}_iW_{zl}\theta_z + \hat{p}_iW_{zl}\hat{\theta}_z = W_{xl}\tilde{\theta}_x \]

\[ \hat{p}_i = \frac{1}{w_{zl}\hat{\theta}_z} \cdot (W_{xl}\tilde{\theta}_x - \hat{p}_iW_{zl}\hat{\theta}_z) \]

\[ \hat{p}_i = \frac{1}{w_{zl}\hat{\theta}_z} \cdot [W_{xl} - \hat{p}_iW_{zl}] \begin{bmatrix} \tilde{\theta}_x \\ \tilde{\theta}_z \end{bmatrix} \]

\[ \hat{p}_i = \frac{1}{w_{zl}\hat{\theta}_z} \cdot W_{zl}\tilde{\theta} \quad (2.14) \]
where

\[
\tilde{W}_t = [W_{xt}, -\bar{\theta}_t W_{zt}] \in R^{3 \times 16}
\]

\[
\bar{\theta} = \begin{bmatrix} \bar{\theta}_x \\ \bar{\theta}_z \end{bmatrix} \in R^{16}
\]

For “n” features, Equation 2.14 can be rewritten as

\[
\bar{P} = B \cdot \tilde{W} \cdot \bar{\theta}
\]  \hspace{1cm} (2.15)

where

\[
\bar{P} = [\bar{p}_1^T, \bar{p}_2^T, \ldots, \bar{p}_n^T]^T \in R^{3n}
\]

\[
B = diag\left\{ \frac{1}{W_{z_1}\theta_z}, \frac{1}{W_{z_1}\theta_z}, \ldots, \frac{1}{W_{z_n}\theta_z}, \frac{1}{W_{z_n}\theta_z} \right\} \in R^{3n \times 3n}
\]

\[
\tilde{W} = \begin{bmatrix} \tilde{W}_1 \\ \tilde{W}_2 \\ \vdots \\ \tilde{W}_n \end{bmatrix} \in R^{3n \times 16}
\]

The objective of the estimation strategy is to exactly identify the unknown constant parameters \( \theta \) (i.e. \( \bar{\theta} \to 0 \)).

\subsection*{2.4.2 The Estimator}

\[
\dot{\theta} \triangleq Proj\{\alpha \Gamma \tilde{W} \bar{P}\} \hspace{1cm} (2.16)
\]

\[
\alpha \triangleq 1 + \sigma
\]

\( \alpha, \sigma \in R^+ \) are positive constants

\[
\frac{d}{dt} \{\Gamma^{-1}(t)\} = 2\tilde{W}^T\tilde{W} \hspace{1cm} (2.17)
\]

\( \Gamma^{-1}(t_0) \) is positive definite and symmetric
2.4.3 Calibration Parameters

The estimator in Equation 2.16 identifies the unknown constant parameters $\theta$.

$$\hat{\theta} \rightarrow 0 \text{ as } t \rightarrow \infty$$

Note from Equations 2.8 and 2.11 that $M$ can be defined as

$$M \triangleq A \cdot [R \quad t] \quad (2.18)$$

where $M \in R^{3 \times 4}$ contains all of the camera calibration parameters. Note also from Equation 2.12 and Section A.1 (in the appendices) that the unknown constant vector $\theta_x \in R^{12}$ contains all the elements of $M$.

$$M = \begin{bmatrix}
\theta_{x,1} & \theta_{x,2} & \theta_{x,3} & \theta_{x,4} \\
\theta_{x,5} & \theta_{x,6} & \theta_{x,7} & \theta_{x,8} \\
\theta_{x,9} & \theta_{x,10} & \theta_{x,11} & \theta_{x,12}
\end{bmatrix}$$

Therefore, $\hat{\theta}_x(t)$ provides an estimate of $M$. Now, define $D$ and $d$ so that

$$M = [D \quad d] = [AR \quad At] \quad (2.19)$$

$$\therefore D \triangleq AR$$

$$d \triangleq At$$

$$K \triangleq DD^T = (AR)(AR)^T = AA^T$$

$$K = AA^T = \begin{bmatrix}
a_1^2 + a_2^2 + a_3^2 & a_2a_4 + a_3a_5 & a_3 \\
a_2a_4 + a_3a_5 & a_4^2 + a_5^2 & a_5 \\
a_3 & a_5 & 1
\end{bmatrix} \quad (2.20)$$

Note that $\hat{\theta}_x$ may only be estimated up to a scale factor. To correct for the scaling factor, normalize $K$ so that $K_{33} = 1$. Divide $K$ by $K_{33}$. Appendix A.2 has the details of scale factor correction.

The entire intrinsic calibration matrix $(A)$ can now be determined by:

$$a_3 = K_{13} \quad (2.21)$$
\begin{align*}
a_5 &= K_{23} \\
a_4 &= \sqrt{K_{22} - a_5^2} \\
a_2 &= \frac{K_{21} - a_3 a_5}{a_4} \\
a_1 &= \sqrt{K_{11} - a_2^2 - a_3^2}
\end{align*}

It is straightforward to determine \( f \kappa_0, f \kappa_v, \varphi, u_0, \) and \( v_0 \) (parameters mentioned in Section 2.3) once the matrix \( A \) has been determined.

The extrinsic parameters can also be identified by

\begin{align*}
R &= A^{-1} D \\
t &= A^{-1} d
\end{align*}

(2.22)

2.5 **Proof of Stability**

Choose the Lyapunov function [11]

\[ V = \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \bar{\theta} \]  

(2.23)

Since \( \Gamma^{-1} \) is positive definite, \( V \geq 0 \). Then the derivative of this function is

\[ \dot{V} = \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} + \frac{1}{2} \dot{\bar{\theta}}^T \frac{d}{dt} (\Gamma^{-1}) \bar{\theta} \]

From Equations 2.16 and 2.17,

\[ \dot{\bar{\theta}} = \dot{\theta} - \dot{\hat{\theta}} = -\hat{\theta} = -\text{Proj} \{ \alpha \Gamma \bar{W}^T \bar{P} \} \]

\[ \frac{d}{dt} (\Gamma^{-1}) = 2 \bar{W}^T \bar{W} \]

Note that \( \text{Proj}[\tau_l] \leq \tau_l \). Therefore, by substituting,

\[ \dot{V} \leq -\alpha \bar{\theta}^T \Gamma^{-1} \Gamma \bar{W}^T \bar{P} + \bar{\theta}^T \bar{W}^T \bar{W} \bar{\theta} \]
Also, from Equation 2.15,

\[ B^{-1}\tilde{p} = \bar{W}\bar{\theta} \]

\[ \dot{V} \leq -\alpha(\bar{W}\bar{\theta})^T\tilde{p} + (\bar{W}\bar{\theta})^T(\bar{W}\bar{\theta}) \]

\[ \dot{V} \leq -\alpha(B^{-1}\tilde{p})^T\tilde{p} + (B^{-1}\tilde{p})^T(B^{-1}\tilde{p}) \]

Since B is a diagonal matrix, \((B^{-1})^T = B^{-1}\).

\[ \dot{V} \leq -\alpha\tilde{p}^TB^{-1}\tilde{p} + \tilde{p}^T(B^{-1})^2\tilde{p} \]

Now,

\[ \exists \delta_1, ..., \delta_n \in R^+ \quad \text{such that} \quad \delta_1 \geq W_{z1}\theta_z \geq \epsilon_1 > 0 \]

\[ \epsilon_1, ..., \epsilon_n \in R^+ \quad \text{such that} \quad \delta_n \geq W_{zn}\theta_z \geq \epsilon_n > 0 \]

Choose \(\delta, \epsilon\) such that

\[ \delta \in R = \max\{\delta_1, ..., \delta_n\} \]
\[ \epsilon \in R = \min\{\epsilon_1, ..., \epsilon_n\} \]

Therefore,

\[ \tilde{p}^TB^{-1}\tilde{p} \geq \epsilon\|\tilde{p}\|^2 \]

\[ \tilde{p}^T(B^{-1})^2\tilde{p} \leq \delta^2\|\tilde{p}\|^2 \]

\[ \dot{V} \leq -\alpha\epsilon\|\tilde{p}\|^2 + \delta^2\|\tilde{p}\|^2 \]

Choose \(\alpha \geq 1 + \frac{\delta^2}{\epsilon}\). Therefore,

\[ \dot{V} \leq -\epsilon\|\tilde{p}\|^2 \leq 0 \quad (2.24) \]

\[ 0 \leq \int_{t_0}^{\infty} \epsilon\|\tilde{p}\|^2 dt < V(t_0) - V(\infty) \quad (2.25) \]
Therefore, \( V(t) < V(t_0) \ \forall t \in R^+ \Rightarrow V(t) \in L_\infty \) (i.e. \( V(t) \) is bounded). From Equation 2.25, \( \tilde{P}(t) \in L_2 \). \( B^{-1} \) is bounded (from the assumptions on \( W_{xi}\theta_z \)).

Therefore, \( B^{-1}\tilde{P} = \tilde{W}\tilde{\theta} \Rightarrow \tilde{W}(t)\tilde{\theta}(t) \in L_2 \). From Equation 2.23, \( \tilde{\theta}(t)^T\Gamma^{-1}(t)\tilde{\theta}(t) \in L_\infty \).

The persistent excitation condition is assumed:

\[
\gamma_1 I_n \leq \int_0^T \tilde{W}(\tau)^T \tilde{W}(\tau) d\tau \leq \gamma_2 I_n
\]

where \( I_n \in R^{n \times n} \) is the n x n identity matrix and \( \gamma_1, \gamma_2 \in R^+ \) are positive constants.

\( \Gamma^{-1}(0) \) is positive definite, and \( \frac{d}{dt}(\Gamma^{-1}) = 2\tilde{W}^T\tilde{W} \geq 0 \). Therefore, \( \Gamma^{-1}(t) \) is positive definite for all \( t \in R^+ \). Because \( \Gamma^{-1}(t) \) is bounded, \( \tilde{\theta}(t) \in L_\infty \).

Because \( W_x \) and \( W_z \) are composed only of bounded, measurable signals, \( W_x(t), W_z(t) \in L_\infty \). Also, \( \theta_x, \theta_z, \theta \in L_\infty \) because they are composed of constant, physical quantities. Therefore, \( \tilde{\theta}(t) = \theta - \tilde{\theta}(t) \Rightarrow \tilde{\theta}(t) \in L_\infty \). Substituting these bounded quantities into Equation 2.13 shows that \( \dot{\rho}_i(t) \in L_\infty \ \forall i \iff \dot{\rho}(t) \in L_\infty \). \( \tilde{W}(t) \in L_\infty \) because it is composed of bounded signals.

By Equation 2.16, it is clear that \( \dot{\tilde{\theta}}(t) \in L_\infty \). Therefore, \( \dot{\tilde{\theta}}(t) \in L_\infty \). Because \( \dot{W}_x \) and \( \dot{W}_z \) are composed of bounded rigid body motion velocities (bounded for the motions of this system), \( \dot{W}_x(t), \dot{W}_z(t) \in L_\infty \). By taking the derivative of the equation \( \tilde{W}_i = [W_{xi} \ -\dot{\rho}_iW_{zi}] \), it is straightforward to show that \( \dot{\tilde{W}}(t) \) is composed of bounded quantities. Therefore, \( \dot{\tilde{W}}(t) \in L_\infty \).
\( \dot{W}(t), \dot{\theta}(t) \in \mathcal{L}_\infty \Rightarrow \frac{d}{dt}(\dot{W}(t)\dot{\theta}(t)) \in \mathcal{L}_\infty \Rightarrow \dot{W}(t)\dot{\theta}(t) \) is uniformly continuous. Therefore, \( \dot{W}(t)\dot{\theta}(t) \in \mathcal{L}_2 \Rightarrow \dot{W}(t)\dot{\theta}(t) \to 0 \) as \( t \to \infty \). Therefore, Equation 2.15 shows that \( \tilde{P}(t) \to 0 \) because \( B \) is positive definite. From Equation 2.16, \( \tilde{P}(t) \to 0 \Rightarrow \dot{\theta}(t) \to 0 \). Because \( \theta \) is constant, \( \dot{\theta}(t) \to 0 \) as \( t \to \infty \) (assuming the satisfaction of the persistent excitation condition from Equation 2.26).

2.6 Simulation Results
The estimator given in Equation 2.16 was simulated for both the fixed and moving camera cases, and the simulation is included in Appendix B. Stability was confirmed for the system when valid inputs were given. The results in this section compare the performances of the moving and fixed camera cases along with the results from changing other system characteristics: varying the number of feature points, changing the gain, differing fixed body inputs, and adding noise.

Unless otherwise noted, for each of the simulations in Section 2.6, the following parameters are used for each:

\[ \alpha = 1000 \]

\[ n = 10 \text{ (feature points)} \]

\[ T = 240 \text{ s (simulation length)} \]
2.6.1 Fixed Camera Results

Simulation 1: Calibration matrices are close to the initial guess (see Section B.1).

\[ f k_u = 822, \ f k_v = 813, \ \varphi = 87^\circ, \ u_0 = 321, \ v_0 = 239 \]

\[ R_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

\[ x_c = [0.4 \ -2.3 \ 0.47]^T \]

\[ v_b = [0.05 \cos t \ .003 \ .15 \sin t]^T \]

\[ \omega_b = [0 \ 0 \ 0]^T \]
Results: The estimation is accurate to within .0001.

\[ f_k u = 822, f_k v = 813, \varphi = 87^\circ, u_0 = 321, v_0 = 239 \]

\[ R_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

\[ x_c = [0.4 \quad -2.3 \quad 0.47]^T \]

Figure 2.7: Fixed camera estimation output
Simulation 2: Calibration matrices are very different from the initial guess.

\[ f k_u = 745, f k_v = 852, \varphi = 73^o, u_0 = 160, v_0 = 120 \]

\[ R_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \]

\[ x_c = [1.4 \quad -4 \quad -0.3]^T \]

\[ v_b = [0.05 \cos t \quad 0.003 \quad 0.15 \sin t]^T \]

\[ \omega_b = [0 \quad 0 \quad 0]^T \]

Results: The estimation is accurate to within .0001.

\[ f k_u = 745, f k_v = 852, \varphi = 73^o, u_0 = 160, v_0 = 120 \]

\[ R_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \]

\[ x_c = [1.4 \quad -4 \quad -0.3]^T \]

Figure 2.8: Another fixed camera estimation output
2.6.2 Moving Camera Results

Simulation 3: Calibration matrices are close to the initial guess.

\[ f k_u = 822, \, f k_v = 813, \, \varphi = 87^\circ, \, u_0 = 321, \, v_0 = 239 \]

\[ R_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

\[ x_c = [0.4 \quad -2.3 \quad 0.47]^T \]

\[ v_c = [-0.05 \, \cos \, t \quad -0.003 \quad -0.15 \, \sin \, t]^T \]

\[ \omega_c = [0 \quad 0 \quad 0]^T \]

Results: The estimation is accurate to within .0001.

\[ f k_u = 822, \, f k_v = 813, \, \varphi = 87^\circ, \, u_0 = 321, \, v_0 = 239 \]

\[ R_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

\[ x_c = [0.4 \quad -2.3 \quad 0.47]^T \]

![Figure 2.9: Moving camera estimation output](image)
Simulation 4: Calibration matrices are very different from the initial guess.

\[ f k_u = 745, \ f k_v = 852, \ \varphi = 73^\circ, \ u_o = 160, \ v_o = 120 \]

\[
R_c = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
x_c = [1.4 \ -4 \ -0.3]^T
\]

\[
v_c = [-.05 \cos t \ -0.03 \ -15 \sin t]^T
\]

\[
\omega_c = [0 \ 0 \ 0]^T
\]

Results: The estimation is accurate to within .0001.

\[ f k_u = 745, \ f k_v = 852, \ \varphi = 73^\circ, \ u_o = 160, \ v_o = 120 \]

\[
R_c = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
x_c = [1.4 \ -4 \ -0.3]^T
\]

Figure 2.10: Another moving camera estimation output
Note: Because the fixed and moving camera simulations have the same estimator, the remaining subsections will use only the moving camera simulation. The given values for \( \alpha, n, \) and \( T \) still hold unless otherwise noted.

### 2.6.3 Quantity of Feature Points Needed

In the previous two sections, 10 feature points have been used for each simulation. This section will explore the results of using fewer feature points.

For each simulation in this section, the calibration parameters and velocity inputs are:

\[
f k_u = 810, \quad f k_v = 820, \quad \phi = 86^\circ, \quad u_0 = 320, \quad v_0 = 240
\]

\[
R_c = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}
\]

\[
x_c = [1 \quad -4 \quad -2]^T
\]

\[
v_c = [-.05 \cos t \quad -.003 \quad .15 \sin t]^T
\]

\[
\omega_c = [0 \quad 0 \quad 0]^T
\]

The matrix of feature points is shown below. When fewer than 10 points are used, the points on the right are dropped.

\[
\begin{bmatrix}
3 & -3 & 5 & -4 & 0 & 0 & 12 & -5 & 4 & 5 \\
22 & 35 & 24 & 37 & 25 & 32 & 35 & 42 & 23 & 26 \\
-4 & 6 & 7 & -1 & 0 & 1 & 12 & -1 & -7 & -8
\end{bmatrix}
\]
Simulation 5: $n = 1$

Results:

\[
f_{k_u} = 82.5, f_{k_v} = 77.2, \varphi = 29.7^\circ, u_0 = 245.0, v_0 = 165.1
\]

\[
R_c = \begin{bmatrix}
0.0812 & 0.7814 & -0.6187 \\
0.8441 & 0.2762 & 0.4596 \\
-0.5300 & 0.5595 & 0.6372
\end{bmatrix}
\]

\[
x_c = [0.8553 \quad 0.2932 \quad 0.4417]^T
\]
Simulation 6: \( n = 2 \)

Results:

\[
f_k u = 758.7, \ f_k v = 741.6, \ \varphi = 80.1^\circ, \ u_0 = 330.0, \ v_0 = 200.7
\]

\[
R_c = \begin{bmatrix}
0.0014 & 0.9994 & -0.0344 \\
-0.0659 & 0.0344 & 0.9972 \\
-0.9976 & -0.0008 & -0.0659
\end{bmatrix}
\]

\[
x_c = [0.4880 \ -2.4192 \ -0.2080]^T
\]

Figure 2.12: Estimation with 2 feature points
Simulation 7: $n = 5$

Results:

\[ f_k = 868.0, \ f_k = 875.7, \ \varphi = 86.7^\circ, \ u_0 = 310.2, \ v_0 = 133.6 \]

\[ R_c = \begin{bmatrix}
-0.0028 & 0.9923 & -0.1239 \\
0.0239 & 0.1240 & 0.9920 \\
-0.9997 & 0.0002 & 0.0241 \\
\end{bmatrix} \]

\[ x_c = [1.1135 \ -6.4418 \ -2.1917]^T \]

Figure 2.13: Estimation with 5 feature points
Simulation 8: $n = 7$

Results:

\[ f k_u = 810, f k_v = 820, \varphi = 86^\circ, u_0 = 320, v_0 = 240 \]

\[ R_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \]

\[ x_c = [1 \quad -4 \quad -2]^T \]

Figure 2.14: Estimation with 7 feature points
2.6.4 Changing the Gain

This section will explore the response of the system when smaller values of $\alpha$ (the estimator’s gain) are used. In each of the previous simulations, $\alpha = 1000$. When a smaller gain is used, the estimator will converge more slowly. However, a large gain will amplify noise and the error resulting from it. For each simulation in this section, the initial inputs are:

$$f k_u = 810, f k_v = 820, \varphi = 86^\circ, u_0 = 320, v_0 = 240$$

$$R_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$x_c = \begin{bmatrix} 1 \\ -4 \\ -2 \end{bmatrix}^T$$

$$n = 10$$

$$v_c = [-.05 \cos t \ - .003 \ - .15 \sin t]^T$$

$$\omega_c = [0 \ 0 \ 0]^T$$
Simulation 9: $\alpha = 5$

Results:

\[ f_{k_1} = 837.8, \; f_{k_2} = 845.2, \; \varphi = 85.9^\circ, \; u_0 = 307.4, \; v_0 = 249.5 \]

\[
R_c = \begin{bmatrix}
-0.0005 & 1.0000 & 0.0071 \\
0.0175 & -0.0071 & 0.9998 \\
-0.9998 & -0.0006 & 0.0175 \\
\end{bmatrix}
\]

\[
x_c = [1.1159 \; -4.9223 \; -2.0984]^T
\]

Figure 2.15: Estimation gain $\alpha = 5$
Simulation 10: $\alpha = 50$

Results:

\[ f_k u = 811.0, f_k v = 820.9, \varphi = 86.0^\circ, u_0 = 319.7, v_0 = 240.1 \]

\[
R_c = \begin{bmatrix}
0.0000 & 1.0000 & 0.0002 \\
0.0004 & -0.0002 & 1.0000 \\
-1.0000 & 0.0000 & 0.0004
\end{bmatrix}
\]

\[
x_c = [0.9994 \quad -4.0369 \quad -2.0010]^T
\]

Figure 2.16: Estimation gain $\alpha = 50$
2.6.5 Types of Fixed Body Inputs

The movement of the robot or camera will also have an effect on the estimator.

Note from Section 2.6.3 that 5 feature points did not force the estimator to converge where $\tilde{\theta} = 0$. This section will explore the response to different input motions using that same number of points.

$$f k_u = 810, f k_v = 820, \varphi = 86^\circ, u_0 = 320, v_0 = 240$$

$$R_c = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}$$

$$x_c = [1 \quad -4 \quad -2]^T$$

Simulation 11: Larger velocity.

$$v_c = [-.5 \cos t \quad -.13 \quad -.65 \sin t]^T$$

$$\omega_c = [0 \quad 0 \quad 0]^T$$

Results:

$$f k_u = 814.6, f k_v = 824.8, \varphi = 86.1^\circ, u_0 = 320.2, v_0 = 232.6$$

$$R_c = \begin{bmatrix}
-0.0001 & 1.0000 & -0.0091 \\
0.0008 & 0.0091 & 1.0000 \\
-1.0000 & -0.0001 & 0.0008
\end{bmatrix}$$

$$x_c = [1.0090 \quad -4.3451 \quad -2.0282]^T$$
Figure 2.17: Estimation with a translational velocity input
Simulation 12: Rotational velocity (larger feature point displacement than translational velocity from the previous simulation).

\[ v_c = [0 \quad 0 \quad 0]^T \]

\[ \omega_c = [0.1 \quad 0.2 \quad -0.1]^T \]

Results:

\[ f k_u = 810.1, f k_v = 820.2, \varphi = 86.0^\circ, u_0 = 319.9, v_0 = 240.1 \]

\[ R_c = \begin{bmatrix} -0.0002 & 1.0000 & 0.0000 \\ 0.0002 & -0.0000 & 1.0000 \\ -1.0000 & -0.0002 & 0.0002 \end{bmatrix} \]

\[ x_c = [1.0022 \quad -4.0009 \quad -2.0039]^T \]

Figure 2.18: Estimation with a rotational velocity input
Simulation 13: Both translational and rotational velocities.

\[ \nu_c = [\begin{array}{c} -0.05 \cos t \\ -0.003 \\ -0.15 \sin t \end{array}]^T \]

\[ \omega_c = [\begin{array}{c} 0.1 \\ 0.2 \\ -0.1 \end{array}]^T \]

Results:

\[ f_{k_u} = 810.1, \ f_{k_v} = 820.2, \ \varphi = 86.0^\circ, \ \nu_0 = 319.9, \ \nu_0 = 240.1 \]

\[ R_c = \begin{bmatrix} -0.0002 & 1.0000 & 0.0001 \\ 0.0002 & -0.0001 & 1.0000 \\ -1.0000 & -0.0002 & 0.0002 \end{bmatrix} \]

\[ x_c = [1.0025 \ -4.0006 \ -2.0044]^T \]

Figure 2.19: Estimation with translational and rotational velocity inputs
Simulation 14: No motion.

\[ v_c = [0 \ 0 \ 0]^T \]

\[ \omega_c = [0 \ 0 \ 0]^T \]

Results:

\[ f k_u = 875.1, \ f k_v = 881.6, \ \varphi = 86.8^\circ, \ u_0 = 308.0, \ v_0 = 117.5 \]

\[ R_c = \begin{bmatrix} -0.0033 & 0.9899 & -0.1415 \\ 0.0277 & 0.1415 & 0.9895 \\ -0.9996 & 0.0007 & 0.0278 \end{bmatrix} \]

\[ x_c = [1.1316 \ -6.7342 \ -2.2052]^T \]

Figure 2.20: Estimation with no input motion
2.6.6 Addition of Noise

This section will study the effect of adding noise to the inputs. Noise was added to both $\bar{x}_t$ and $P$ as described in Section B.2.3.

$$f k_u = 810, f k_v = 820, \varphi = 86^\circ, u_0 = 320, v_0 = 240$$

$$R_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$x_c = [1 \ -4 \ -2]^T$$

$$v_c = [-.5 \cos t \ -13 \ -65 \sin t]^T$$

$$\omega_c = [0 \ 0 \ 0]^T$$
Simulation 15: noise variance = .001, \( \alpha = 1000 \), \( n = 10 \)

Results:

\[ f_k = 798.2, f_k' = 807.3, \phi = 85.7^\circ, u_0 = 323.5, v_0 = 242.6 \]

\[
R_c = \begin{bmatrix}
0.0020 & 1.0000 & 0.0073 \\
-0.0094 & -0.0072 & 0.9999 \\
-1.0000 & 0.0021 & -0.0094
\end{bmatrix}
\]

\[
x_c = [0.8053 \ -3.4613 \ -1.7729]^T
\]

Figure 2.21: Medium noise
Simulation 16: noise variance = .01, $\alpha = 1000$, $n = 10$

Results:

\[
fk_u = 874.7, \; fk_v = 875.6, \; \phi = 87.4^o, \; u_0 = 329.2, \; v_0 = 222.2
\]

\[
R_c = \begin{bmatrix}
-0.0124 & 0.9993 & -0.0356 \\
0.0152 & 0.0358 & 0.9992 \\
-0.9998 & -0.0118 & 0.0157
\end{bmatrix}
\]

\[
x_c = [1.4695 \; -6.4144 \; -2.9601]^T
\]

Figure 2.22: Large noise
Simulation 17: noise variance = .01, \( \alpha = 100 \), \( n = 10 \)

Results:

\[
\begin{align*}
    f_k^u &= 823.7, f_k^v = 823.8, \varphi = 86.2^\circ, u_0 = 316.3, v_0 = 251.8 \\
    R_c &= \begin{bmatrix} -0.0068 & 0.9999 & 0.0119 \\ 0.0074 & -0.0119 & 0.9999 \\ -0.9999 & -0.0068 & 0.0073 \end{bmatrix} \\
    x_c &= [1.0247 \quad -4.2809 \quad -2.0756]^T
\end{align*}
\]

Figure 2.23: Large noise, smaller gain
Simulation 18: noise variance = .01, $\alpha = 1000$, $n = 15$

Results:

\[ f_k^u = 716.3, f_k^v = 700.3, \varphi = 84.1^\circ, u_0 = 286.6, v_0 = 264.0 \]

\[
R_c = \begin{bmatrix}
0.0000 & 0.9970 & 0.0771 \\
-0.0108 & -0.0771 & 0.9970 \\
-0.9999 & 0.0009 & -0.0107
\end{bmatrix}
\]

\[ x_c = [-0.8785 \quad 1.4955 \quad 0.3180]^T \]

Figure 2.24: Large noise, more feature points
CHAPTER 3
CHAIN AND SPROCKET RELIABILITY WEAR TESTING

3.1 Overview
A large portion of the test plan presented in this chapter is based on a test plan that was written by Chris Simoson [9]. As part of the author’s thesis work, the original test plan has been revised and modified.

3.1.1 Introduction
Testing a system for reliability includes analyzing the wear of components within that system. Specifically, rotation transmitted by chain and sprocket within a high temperature and abrasive environment is expected to cause a large amount of wear. Many methods will be used in this project to measure the wear on such a system.

3.1.2 Background
The basket drive project began in June 2005 with an expected completion date of August 2006. However, the failure of a bearing during the pre-testing stages of the project necessitated the redesign of the entire base stand. The base, end, and side plates, both shafts (drive and idler), and the bearings were all changed. Much effort was spent in an attempt to prevent a failure from occurring in the newly designed system.
3.1.3 Schedule

Assembly of the new base stand has been completed. The only remaining assembly task for the basket drive system is the design and fabrication of a coupler to connect the motor shaft and the drive shaft. Pre-testing tasks will begin after the coupler has been machined, and these tasks should be finished by the end of May 2007. The testing schedule is planned as follows: 2 cycles per day that testing is possible (25 weekdays = 5 weeks for each phase of 50 cycles) and about 5 weeks of downtime while measurements are made between each cycle. With 350 cycles and 8 measurement periods, the testing phase should last about 75 weeks.

3.2 Experimental Configuration

3.2.1 System Configuration

A douser containing aluminum oxide, a very abrasive powder, resides inside a basket whose surface contains thousands of small holes through which the powder may sift. A chain and thin rod have been attached to the basket at each end around the outer surface. The basket is supported by two shafts. The idler shaft contains two idle rollers while the drive shaft contains two sprockets that transfer drive to the basket. This entire assembly is contained within a furnace that features a door to seal and insulate the inner environment. In this manner, all of the dispensed powder will sift through the holes in the basket, creating a dusty atmosphere and coating all surfaces within the furnace.
An electric motor located outside of the furnace door can be lowered into position and coupled to the drive shaft. The motor delivers the torque necessary to turn the sprocket and chain assembly. Once the furnace has been heated to 650 °C, the basket is rotated at one revolution per minute. Assuming a “gear ratio” between the different radii in contact, friction compels the douser to spin at a rate of approximately 7.33 times that of the basket.

This test plan utilizes the douser method for testing. Alternate modes of testing may require modification of the basket to incorporate flights or metal fins within
the inner surface of the basket. Should such a method be employed, the douser could no longer be used because it would not be allowed to rotate freely. The current testing mode relies on this condition in order to induce the douser’s motion to dispense powder.

3.2.2 Materials

The basket and chain are made of Inconel 600, an anti-corrosive, high strength nickel chromium alloy with exceptional resistive properties to heat and oxidation. The gear sprockets, idle rollers, and douser are Stainless Steel 304. Each inconel chain consists of 148 links with 74 rollers that contact the ten teeth of each stainless steel sprocket. CAD drawings of the basket, sprockets, idle rollers, shafts, and base have been included in Appendix C.

The furnace utilized in the experiment is produced by Thermolyne. It is controlled via a Furnatrol Type 53700 controller. A Leeson Speedmaster motor controller regulates a Leeson Model 985-661D variable speed electric motor at the prescribed angular velocity.

3.3 Pre-testing Procedures

Before testing can begin, it is important to confirm that each component of the basket drive system operates properly. In addition, benchmarking data must be acquired. Due to the expectations for wear, this data must be complete and precise. The following pre-testing procedure will serve to qualify the experiment and to establish a set of original measurements for each basket drive component.
1. Complete the assembly of the basket system.
2. Complete the revised test plan and get it approved.
3. Test the basket without powder or temperature by placing a video camera inside the furnace to observe as the motor turns the basket. It is important to ensure smooth operation of the system.
4. Conduct all measurements for baseline values. (Refer to Section 3.5 for a description of each method that will be used for wear analysis).
5. Test at temperature but without powder to verify that the furnace functions properly.
6. Determine and mark the motor speed required to turn the basket at 1 rpm.
7. Determine the duration of time required to dispense the powder from the douser. A time trial of about 30 minutes with powder (without temperature) will yield a rough flow rate. From this flow rate, the total time for all powder to be dispensed may be calculated.
8. Clean the parts as described in Section 3.4.4.
9. Begin the testing as described in Section 3.4.

3.4 Testing Procedures

3.4.1 Testing Overview

Each testing cycle begins with a warm-up period to allow the furnace to reach the specified temperature (650 °C). When operational temperature is achieved, the motor will turn the basket drive for twice the amount of time necessary for a full douser to dispense all of its powder. Once the allotted time has elapsed, the motor will stop and the furnace will be allowed to cool down. After collecting the powder required for the Inductively Coupled Plasma (ICP) test, a single test cycle will be complete.

In order to catch the beginnings of wear, the measurements which are described in Section 3.5 will be conducted after each of the first two sets of 25 cycles. Testing will then continue with measurements taken in 50 cycle increments until the goal of 350 cycles is met as shown in Table 3.1.
Table 3.1: Testing cycle increments between measurements

<table>
<thead>
<tr>
<th>Baseline</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
</tr>
</thead>
</table>

3.4.2 Procedure for a Single Test Cycle

1. Fill the douser with aluminum oxide powder.
2. Assemble the basket drive system within the furnace.
3. Heat the furnace to the specified temperature.
4. Run the motor for the specified time so that the douser is completely emptied.
5. Stop the motor and furnace so that the system is allowed to cool.
6. Thoroughly mix the powder, and then collect 2 mg for ICP.
7. Replace the powder that was removed for ICP so that the douser can be refilled for the next cycle.

3.4.3 Procedure for a Measurement Cycle

1. Follow normal testing steps including collecting the aluminum oxide powder and preparing the douser for the next test.
2. Coordinate measurement scanning should be run whenever the CMS machine is available.
3. Give the necessary parts to Carl for non-contact 3D scanning. (Note: give Carl about 3 days notice before scanning should begin.)
4. Run ICP on the powder samples collected since the last measurement cycle.
5. Clean the basket and give the smaller parts a sonic bath as described in Section 3.4.4.
6. Conduct weight loss and diameter measurements.
7. Complete the digital imaging of the basket.
8. Use the scanning electron microscope for the sprockets and idle rollers.
9. Run non-contact surface profilometry on the sprockets and idle rollers.

3.4.4 Cleaning Procedure

Upon completion of each series of test cycles, the components will be cleaned to remove aluminum oxide and any other unwanted matter. In order to produce reliable and repeatable cleaning, the smaller parts (sprockets and idle rollers) will be placed in a sonic bath, with a small amount of mild dishwashing detergent, for
twenty minutes of sonic activity with no heat. The parts will then be brushed with a clean toothbrush that has soft bristles and rinsed three times with deionized (DI) water. Because the basket is too large for the sonic bath, it will be washed in a mild detergent solution using a soft bristle toothbrush and then rinsed three times with DI water.

### 3.5 Measurement Methods

Contact between the sprockets and chain will likely be the primary location for wear. Due to the gear ratio between the sprockets and the chain, the sprockets are expected to see at least 3.7 times more wear than the harder inconel chain rollers [9]. In order to track the wear for repeatable results, the gear sprockets and idler rollers have been marked. The front sprocket and idler roller have been labeled with an “F” while the rear sprocket and idle roller have been labeled with an “R.” Symmetrically opposing teeth on the sprockets have been marked to designate the measurement that will be conducted. Similarly, the idle rollers have been marked to assign locations for the scanning electron microscope, coordinate measurement scanning, and surface profilometry.

As the idle rollers on the second shaft will be in continuous contact with the inconel rod around the circumference of the basket, wear in the form of a groove in the idle rollers is expected. The powder passing through the basket holes and the contact between the douser and the inside surface of the basket will also be a source of wear. Tests have been devised in an effort to account for wear from all these locations. The eight techniques shown in Table 3.2 are described in the following subsections.
Table 3.2: Methods for measuring wear

<table>
<thead>
<tr>
<th>Type of Analysis</th>
<th>Components</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Contact 3D Scanning</td>
<td>Sprockets, idle rollers, chain, rod, and outside of basket</td>
<td>Quantitative and qualitative 3D comparisons</td>
</tr>
<tr>
<td>Chemical Analysis</td>
<td>Powder sample (all parts)</td>
<td>Quantitative evidence for wear</td>
</tr>
<tr>
<td>Weight Loss</td>
<td>Sprockets, idle rollers, and basket</td>
<td>Quantitative evidence for wear</td>
</tr>
<tr>
<td>Specific Diameter</td>
<td>Idle and chain rollers</td>
<td>Quantitative comparison</td>
</tr>
<tr>
<td>Digital Imaging</td>
<td>Basket holes</td>
<td>Quantitative and qualitative analysis</td>
</tr>
<tr>
<td>Scanning Electron Microscope</td>
<td>Sprockets and idle rollers</td>
<td>Qualitative images</td>
</tr>
<tr>
<td>Coordinate Measurement Scanning</td>
<td>Sprockets and idle rollers</td>
<td>Quantitative and qualitative 3D comparison</td>
</tr>
<tr>
<td>Non-Contact Surface Profilometry</td>
<td>Sprockets and idle rollers</td>
<td>Quantitative and qualitative 3D scans</td>
</tr>
</tbody>
</table>

3.5.1 Non-Contact 3D Scanning

Located at CETL, a Konica Minolta 910 non-contact 3D digitizer (3D scanner) is capable of merging multiple viewpoints into a three dimensional model via the Raindrop Geomagic Studio 6 software package. The scanner is utilized with every component: sprockets, idle rollers, chain, circumferential rod, and basket surface.

With an accuracy of approximately 0.006 inch, the 3D scanner works well for larger objects, but it cannot always pick up sharp edges. Therefore, some
rounding of corners is to be expected. Scanning is limited to the outer portion of the basket because the camera cannot fit inside.

In order to scan the basket surface in a repeatable fashion, small pins are inserted into marked holes. These marks are then aligned and merged to produce a single, more complete image. Software post-processing utilizing Qualify 7 allows for the comparison of images, and a detailed report may also be composed with color coded areas indicating the amount of wear that has occurred relative to the original surface. Refer to Appendix D for results of the baseline scans.

3.5.2 Chemical Analysis

Chemical analysis is available on site at CETL. Inductively Coupled Plasma (ICP) excites atoms from a small sample of the aluminum oxide in order to find unique traces of inorganic substances. Thus, knowing the composition of the inconel and stainless steel utilized, it will be possible to determine the source of any trace materials found. Unfortunately, while the chemical analysis process is able to provide evidence of which contacting surfaces have worn, it is incapable of pinpointing the exact location of the wear.

The subject of powder sampling for analysis has received some attention in the past because of concerns about heterogeneous wear. However, the physical dimensions of the current basket drive base pose a problem: only a small gap exists between the base plate and the lower portion of the rotating basket. As the basket turns and powder is dispensed, the powder will accumulate below the basket to a point where newly dispensed power will be scraped off, making it
difficult for a partition to represent a particular component. A larger catch tray will encompass the whole basket assembly. In an effort to standardize the analysis, the powder will be mixed uniformly before a 2 mg sample is removed. The powder will then be reused except for the sample that is taken for chemical analysis. The displaced powder used for analysis will be replaced with new powder. Since the douser holds over 3 kg of powder, the replacement of 2 mg with new powder is assumed to be negligible.

The detection limits for the target material components are approximately 10 (ppm) (mg/kg) for Ni, Cr, Fe, Mn, and Cu. None of the target materials are expected in the baseline analysis of clean powder.

<table>
<thead>
<tr>
<th>Metals targeted using ICP [3], [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inconel 600</strong></td>
</tr>
<tr>
<td>Ni – 75%</td>
</tr>
<tr>
<td>Cr – 15.5%</td>
</tr>
<tr>
<td>Fe – 8%</td>
</tr>
<tr>
<td>Mn – 0.5%</td>
</tr>
</tbody>
</table>

3.5.3 Weight Loss Analysis

After performing the cleaning procedure described in Section 3.4.4, the sprockets, idle rollers, and basket will be weighed using digital scales. An A&D EP-41KA (calibrated on 10.19.06 by Greenville Scale Co., Inc.) scale is used for the basket and a Sartorius (calibrated on 10.19.06 by Greenville Scale Co., Inc.) will be used for the idle rollers and sprockets.
While all of the related components can be subjected to this test procedure, weight analysis will only provide data as to the quantity of material lost and will not be capable of locating the exact position of the wear. Table 3.4 and Appendix D currently display the baseline data.

Table 3.4: Masses of the basket drive components

<table>
<thead>
<tr>
<th>Number of Cycles, mass recorded in grams (g)</th>
<th>Baseline</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basket</td>
<td>8067</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sprocket F</td>
<td>114.753</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sprocket R</td>
<td>113.966</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Idler F</td>
<td>152.516</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Idler R</td>
<td>154.080</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.5.4 *Specific Diameter*

In order to track the wear of the chain rollers and the idle rollers, their diameters will be recorded after each set of fifty cycles. Two symmetrically opposing chain links on the front chain (each link containing two rollers) have been marked for observation. Using calipers, measurements will be taken to the nearest 0.001 of an inch. This data will be recorded in Table 3.5.
Table 3.5: Specific diameter of the rollers

<table>
<thead>
<tr>
<th></th>
<th>Number of Cycles, diameter recorded in inches (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline  25  50  100  150  200  250  300  350</td>
</tr>
<tr>
<td>Idler F</td>
<td>1.251</td>
</tr>
<tr>
<td>Idler R</td>
<td>1.223</td>
</tr>
<tr>
<td>Chain Roller A1</td>
<td>0.3135</td>
</tr>
<tr>
<td>Chain Roller A2</td>
<td>0.3135</td>
</tr>
<tr>
<td>Chain Roller B1</td>
<td>0.3130</td>
</tr>
<tr>
<td>Chain Roller B2</td>
<td>0.3145</td>
</tr>
</tbody>
</table>

3.5.5 Digital Imaging

In order to obtain quantitative wear information on the inner surface of the basket, digital imaging will be employed. The interior surface of the basket will be sampled at multiple points using a 0.25 inch CCD camera that incorporates a 50x zoom lens. Once the images have been captured, they will be post-processed using the imaging toolbox capabilities of MATLAB. MATLAB will overlay the images, which allows a representation of the variation between two concurrent images to be produced. Thus, the capability exists to compare data from the current cycle with that of any of the previous cycles and also with the original data set.
The camera is to be mounted to the basket via a fixture that is positioned using the holes for the binding screws that fix the end cover in place. The fixture design allows the camera to rotate to multiple locations around the inner radius of the basket and to move along the length of the basket.

3.5.6 Scanning Electron Microscope

The Hitachi S-3500N Scanning Electron Microscope (SEM) is located in the SEM lab of the Clemson Research Park. It provides 2D images that show the crystalline structure of the specimen. Due to size limitations, the sprockets and idle rollers are the only components capable of being placed within the SEM vacuum chamber. The extremely detailed images provide impressive visual surface representation, but comparative wear analysis is difficult to quantify as the depth of the surface disparities cannot be determined.

The gear sprockets and idle rollers have received a 0.3mm wide mark as a reference point for using the SEM and non-contact surface profilometer. As stated in Table 3.2, the SEM will primarily serve as a qualitative visual observation of wear over time. However, it may be possible to quantitatively
monitor the change in surface features because the SEM post-processing software can measure the distance between points.

The sprockets are positioned in a small aluminum fixture which is placed on a pedestal inside the SEM chamber. The magnification and intensity may be selected to reach the desired view. A collection of the base scans are located in Appendix D. Each location was scanned with three different magnifications: 70x, 350x, and 1000x.

3.5.7 Coordinate Measurement Scanning

A coordinate measuring machine (CMM) is available through the Mechanical Engineering department at Clemson University. It is slow and must touch the object being scanned, but it has very good precision. A scanning program must be written instructing the scanner how to inspect the specimen. The software Rapidform 2004 allows merging of multiple scans and comparative analysis. Again, only the gear sprockets and idle rollers fit within the confines of the machine. The software is also able to filter the scan appropriately and compare successive scans with detailed reports. The imperfections in the material may be seen by using the software to zoom in; this feature seems promising for comparative wear analysis.

3.5.8 Non-Contact Surface Profilometry

A non-contact surface profilometer is fundamentally more accurate than the contact profilometer. Under the supervision of the Biotribology department at Clemson University, the use of a Wyko/Veeco non-contact surface profilometer
provides a very detailed (on the order of a nanometer) three dimensional surface scan. Regrettably, it is difficult to obtain a repeatable image of the same location for comparative wear analysis. Additionally the surface profilometer is restricted to small work pieces. Only the gear sprockets and idle rollers are small enough to be scanned.

Table 3.6: Measurement methods summary

<table>
<thead>
<tr>
<th>Type of Analysis</th>
<th>Parts</th>
<th>Location</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Contact 3D Scanning</td>
<td>Sprockets Idle rollers Chain</td>
<td>CETL – Carl Rathz</td>
<td>Carl runs the tests: give him about 3 days notice</td>
</tr>
<tr>
<td></td>
<td>Circumferential rod Basket</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemical Analysis</td>
<td>Powder Sample</td>
<td>CETL – Steve Hoeffner</td>
<td>Contact Steve for training to run the tests</td>
</tr>
<tr>
<td>Weight Loss</td>
<td>Sprockets Idle rollers Basket</td>
<td>CETL</td>
<td>Scales located in lower floor at CETL</td>
</tr>
<tr>
<td>Specific Diameter</td>
<td>Idle rollers Chain rollers</td>
<td>CETL</td>
<td>Calipers located in Carl’s office</td>
</tr>
<tr>
<td>Digital Imaging</td>
<td>Basket holes</td>
<td>CETL/Clemson</td>
<td>None</td>
</tr>
<tr>
<td>Scanning Electron Microscope</td>
<td>Sprockets Idle rollers</td>
<td>SEM lab</td>
<td>Need an IDO and then training to run the SEM</td>
</tr>
<tr>
<td>Coordinate Measurement Scanning</td>
<td>Sprockets Idle rollers</td>
<td>EIB – David Moline</td>
<td>None</td>
</tr>
<tr>
<td>Non-Contact Surface Profilometry</td>
<td>Sprockets Idle rollers</td>
<td>Biotribology lab – Martine Laberge</td>
<td>Machine is currently being repaired</td>
</tr>
</tbody>
</table>
CHAPTER 4
CONCLUSIONS AND FUTURE WORK

4.1 Adaptive Camera Calibration

4.1.1 Conclusions

Under reasonable input conditions, the estimator worked as expected. For the inputs given in Sections 2.6.1 through 2.6.3, at least 7 feature points were required for convergence. A smaller number of points did not yield sufficient information for \( \bar{\theta} \to 0 \) even though the estimator drove \( \bar{P} \to 0 \). A different velocity input from the moving body or camera also affects the final estimate for \( \theta \). From the results in Section 2.6.5, the rotational velocity yielded a better estimate than the translational velocity. This difference came about because the rotational velocity that was given created larger displacements of feature point positions and not because rotational velocities are always better. The \( \bar{x}_i \) input was more varied; therefore, convergence was improved despite having a reduced number of feature points.

Increasing the gain \( \alpha \) will force the estimator to converge more quickly. However, the large gain struggles when noise is added. Notice from Section 2.6.4 that the smaller gains did not force \( \bar{\theta} \to 0 \) quickly. A gain of at least \( \alpha = 100 \) should be used. Also, notice in Section 2.6.6 that the smaller gain was a better compensation for noise than adding more feature points.
4.1.2 Future Work

Once simulation has been completed, the experimental phase is set to begin. Both the fixed and moving camera cases will be set up and run according to Figures 2.3 and 2.5. The camera’s calibration parameters may be obtained through the use of a Matlab toolbox, and the experimental results will be compared to these parameters [1]. As with the simulation, the experimental phase of the project will compare the results of using various numbers of feature points. It is likely that a range from 1 to 20 points will be tested. The open source computer vision library has a feature tracker that can be used for the project [7].

4.2 Chain and Sprocket Reliability Wear Testing

4.2.1 Conclusions

A great amount of time has been devoted to finding the tools that will be necessary to observe the wear that will occur during the basket drive testing. The capabilities of each measuring device vary as to their ability to measure the wear on the components. Fortunately, the components of greatest interest (the sprockets and the idle rollers) are able to fit within the confines of the profilometer, SEM, and CMM. Three dimensional scanning will view those parts as well as the outer components of the basket—surface, circumferential rod, and chain. Additionally, while some of the testing methods may or may not prove to be analytically beneficial, most will still provide a visual illustration of wear. Testing progress alone will show how well each analysis technique will reveal the effects of multiple bodies in frictional contact.
The anticipated downtime for conducting the eight measurements may be longer than desired. The basket drive system must be disassembled and the sprockets and idle rollers must be tested at multiple locations where each party needs sufficient time to scan the components. While most of the measuring devices described take a few hours to perform their scan, the CMS takes as long as 16 hours for one scan. Four CMS scans are necessary to cover sprocket teeth 1 and 3, teeth 2 and 4, and the two idle rollers.

Due to the time and money invested in this project, the test plan has been reviewed numerous times so the testing procedure will be perfected before heat and powder begin to cause wear in the basket drive parts. Before testing starts, this updated plan must be approved by all parties concerned.

4.2.2 Future Work

Once assembly of the system is complete and the revision of the test plan presented in Chapter 3 is approved, the pre-testing and testing procedures will be performed. Gavin Wiggins, a Ph.D. student in Mechanical Engineering at Clemson University, will perform the testing phase of the project. As directed in the test plan, he will oversee the testing cycles and make the necessary measurements at 50 cycle increments. Should another part failure occur, future work would also include redesign of the system to accommodate for the problem.
APPENDICES
A.1 Definitions

A.1.1 Constant Calibration Matrices

Intrinsic Matrix:

\[
A = \begin{bmatrix}
a_1 & a_2 & a_3 \\
0 & a_4 & a_5 \\
0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
fk_u & -fk_u \cot \varphi & u_0 \\
0 & fk_v \sin \varphi & v_0 \\
0 & 0 & 1
\end{bmatrix}
\]

Extrinsic Rotation and Translation Matrices:

\[
[R \ t] = \begin{bmatrix}
r_{11} & r_{12} & r_{13} & t_1 \\
0 & f & 0 & t_2 \\
0 & 0 & f & t_3
\end{bmatrix}
\]

A.1.2 Estimator Variables

The matrices $W_{xi}$ and $W_{zi}$ are derived from the input $\tilde{x}_i = [\tilde{x}_{i1} \quad \tilde{x}_{i2} \quad \tilde{x}_{i3} \quad 1]^T$ as it is defined for the specific case being used (fixed or moving camera). Therefore, the W matrices contain the measurable data. As noted in Sections 2.3.1 and 2.3.2, $\tilde{x}_i$ is found by:

\[
\tilde{x}_i = [(R_B \tilde{x}_{fi} + x_B)^T \quad 1]^T
\]

for the fixed camera case and

\[
\tilde{x}_i = [(R_B^T (x_{fi} - x_B))^T \quad 1]^T
\]

for the moving camera case.
For the $i^{th}$ feature point, $W_{xi}$ and $W_{zi}$ are

$$
W_{xi} = \begin{bmatrix}
\bar{x}_{i1} & \bar{x}_{i2} & \bar{x}_{i3} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \in \mathbb{R}^{3\times12}
$$

$$
W_{zi} = \begin{bmatrix}
\bar{x}_{i1} & \bar{x}_{i2} & \bar{x}_{i3} & 1
\end{bmatrix} \in \mathbb{R}^{1\times4}
$$

The matrices $\theta_x$ and $\theta_z$ contain the unknowns that are to be estimated. These matrices contain the data that cannot be measured. $\hat{\theta}_x$ and $\hat{\theta}_z$ are the estimates for these unknowns.

$$
\theta_x = \begin{bmatrix}
a_{1}r_{11} + a_{2}r_{21} + a_{3}r_{31} \\
a_{1}r_{12} + a_{2}r_{22} + a_{3}r_{32} \\
a_{1}r_{13} + a_{2}r_{23} + a_{3}r_{33} \\
a_{1}t_{1} + a_{2}t_{2} + a_{3}t_{3} \\
a_{4}r_{21} + a_{5}r_{31} \\
a_{4}r_{22} + a_{5}r_{32} \\
a_{4}r_{23} + a_{5}r_{33} \\
a_{4}t_{2} + a_{5}t_{3} \\
r_{31} \\
r_{32} \\
r_{33} \\
t_{3}
\end{bmatrix} \in \mathbb{R}^{12}
$$

$$
\theta_z = \begin{bmatrix}
r_{31} \\
r_{32} \\
r_{33} \\
t_{3}
\end{bmatrix} \in \mathbb{R}^{4}
$$

A.2 Scale Factor Correction

Since $\hat{\theta}_x$ is estimated within a scale factor $\lambda \in \mathbb{R}$, $M$ is also estimated to that factor.

$$
M = \lambda \cdot A \cdot [R \ t]
$$

$$
M = A \cdot [\lambda R \ \lambda t]
$$

$$
D = \lambda AR
$$

$$
K = DD^T = \lambda^2(AR)(AR)^T = \lambda^2 AA^T
$$

66
However, $K_{33}$ must be a 1. Therefore, the scale factor can be determined as

$$\lambda^2 = K_{33}$$

$$\lambda = \sqrt{K_{33}}$$

Note that $K_{33}$ will always be positive since $K = DD^T$. 
Appendix B
Simulink Model

B.1 Overview

The Simulink model used for this experiment is shown in Figure B.1. The model has three major subsystems which are discussed in the following sections:

- Inputs (measurements)
- Estimator
- Error Calculation

![Simulation model for adaptive camera calibration.](image)

The model loops the output of the estimator (\( \hat{\theta} \)) through an integrator and back as an input.
The integrator is initialized for the intrinsic parameters $k_u = 820$, $k_v = 810$, $\varphi = 90^\circ$, $u_0 = 320$, $v_0 = 240$ and for extrinsic parameters

$$R_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$x_c = [0 \ -2 \ 0.5]^T$$

Therefore, the matrices $A$, $R$, and $t$ are initially guessed to be

$$A = \begin{bmatrix} 820 & 0 & 320 \\ 0 & 810 & 240 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R \ t] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

According to Section A.1.2, $\hat{\theta}$ is initially guessed to be

$$\hat{\theta}_x = \begin{bmatrix} -820 \\ 320 \\ 0 \\ 640 \\ 0 \\ 240 \\ 810 \\ 75 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\hat{\theta}_z = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

The integrator can be initialized for any intrinsic and extrinsic parameters desired.

In a typical experiment, intelligent guesses may be made according to the environment given. (The extrinsic rotation and translation matrices may be
completely different than those used in this simulation.) Such guesses should not affect the results for convergence—only the speed at which the estimator converges. For each of the simulations that are recorded in Chapter 2 of this work, the matrices shown above were used as initial data.

### B.2 Input Measurements

The outputs of this subsystem depend on which case (fixed or moving camera) is being simulated. The estimator needs \( \bar{x} \) as one of its inputs.

\[
\bar{x} = [\bar{x}_1^T \quad \cdots \quad \bar{x}_n^T]^T
\]

Sections 2.3.1 and 2.3.2 show that for the fixed camera case, \( \bar{x}_i \) is found by

\[
\bar{x}_i = \left( (R_B \bar{x}_{f,i} + x_B)^T \quad 1 \right)^T
\]

and for the moving camera case,

\[
\bar{x}_i = \left( (R_B^T (x_{f,i} - x_B))^T \quad 1 \right)^T
\]

In addition, the estimator needs to know the location of each feature point in the image. In an experiment, each of these would be measured. For this simulation, the initial positions of the feature points are given. Also, the rotation and translation of the rigid body (or camera) is given. The physical position of each feature point and the corresponding image locations are then calculated. One special consideration must be made: the feature points must always be positioned where the \( z_i \) component is positive (feature points must be located in the positive direction of the camera’s optical axis). This is not an issue for a physical system because the camera will not capture positions behind it.
Figure B.2: The inputs (measurements) subsystem

One of the inputs to the system is the actual calibration parameters. From these parameters, $\theta$ is calculated for use in the error subsystem. Also, the real calibration parameters are used in calculation of the positions of the feature points. Another input to the system is the initial feature points. In the figure above, 10 feature points are given.
B.2.1 Feature Point Positions in the Real World

This function calculates the velocities of each feature point according to the given velocity of the body. A translational and a rotational velocity are chosen in the function.

For the fixed camera case, $v_b$ and $\omega_b$ represent the velocity of the moving rigid body containing the feature points. The initial feature points given are the feature point coordinates with respect to the camera frame. The velocity of each feature point is then individually found by

$$\dot{x}_i = v + (\omega \times \vec{x}_i)$$

where $v$ is the translational velocity and $\omega$ is the rotational velocity.

```matlab
function x_dot = fcn(t, x)
    dim = size(x,2);
    vb = [.05*cos(t), .003, .15*sin(t)]'; % robot's translational velocity
    wb = [0,0,0]'; % robot's rotational velocity
    x_dot = zeros(3,dim);
    for n = 1:dim
        x_dot(1:3,n) = vb + cross(wb, x(1:3,n));
    end
```

Figure B.3: Feature point velocities for the fixed camera case
For the moving camera case, $v_c$ and $\omega_c$ represent the velocity of the moving rigid body to which the camera is attached. The initial points given are the feature point coordinates with respect to the camera frame. The velocity of each feature point is then individually found by

$$\dot{x}_i = -v - (\omega \times \dot{x}_i)$$

where $v$ is the translational velocity and $\omega$ is the rotational velocity.

```matlab
function x_dot = fcn(t, x)
    dim = size(x,2);
    vc = [.05*cos(t), -.7, .15*sin(t)]'; % camera's translational velocity
    wc = [.05,.25,.1]'; % camera's rotational velocity
    x_dot = zeros(3,dim);
    for n = 1:dim
        x_dot(1:3,n) = -vc - cross(wc, x(1:3,n));
    end
```

Figure B.4: Feature point velocities for the moving camera case

The positions of the feature points ($\bar{x}$) are then determined by integrating the velocities that were calculated.
B.2.2 Feature Point Positions in the Image

The positions of the feature points in the real world are easy to calculate once the real world positions are known. The camera’s intrinsic and extrinsic calibration matrices are necessary as well. Equations 2.8 and 2.11 show where the points will fall on the image plane.

```
function p = fcn(x, parameters)

    dim = size(x,2);
    u0 = parameters(1);
    v0 = parameters(2);
    fku = parameters(3);
    fkv = parameters(4);
    phi = parameters(5);

    r11 = parameters(6);
    r12 = parameters(7);
    r13 = parameters(8);
    r21 = parameters(9);
    r22 = parameters(10);
    r23 = parameters(11);
    r31 = parameters(12);
    r32 = parameters(13);
    r33 = parameters(14);
    p1 = parameters(15);
    p2 = parameters(16);
    p3 = parameters(17);

    R = [r11,r12,r13;r21,r22,r23;r31,r32,r33]';
    t = -R * [p1; p2; p3];

    p = zeros(3,dim);
    for n = 1:dim
        xc = [R, t] * [x(1,n); x(2,n); x(3,n); 1];
        p(1:2,n) = [fku, -fku / tan(phi * pi/180), u0; 0, fkv / sin(phi * pi/180), v0] * [xc(1) / xc(3); xc(2) / xc(3); 1];
        p(3,n) = 1;
    end
```

Figure B.5: Simulating the location of the feature points in the image
B.2.3 Adding Noise

One test that was performed was adding noise to the inputs. The data gathered in an experiment will not be ideal, so concern must be made for the validity of the estimator with noisy data. Gaussian noise was added to each of the estimator’s inputs (x and p) as shown in Figure B.6. The noise can easily be removed by setting its variance to 0.

![Diagram of noise added to the inputs]

Figure B.6: Noise added to the inputs
B.3 Estimator

As mentioned in Section 2.4, the same estimator design is used for both the fixed and moving camera cases. That design is shown in Figure B.7. The estimator subsystem has three inputs, one output, and five major operational blocks.

![Figure B.7: The estimator subsystem](image)

Figure B.7: The estimator subsystem
**B.3.1 Inputs and Outputs**

The inputs to the estimator are:

- \( x = \bar{x}_i \) which is defined in Sections 2.3.1 and 2.3.2 for the two different cases. This input comes directly from the input measurements block.

- \( p = [p_1^T \quad p_2^T \quad ... \quad p_n^T]^T \) which are the locations of the feature points in the image. No consideration is given in the simulation for whether the points lie within the image or whether they are out of the physical bounds.

- \( \hat{\theta} \) comes from the output of the integrator. This is the input that completes the estimation loop.

The output is:

- \( \hat{\theta} \) is found from Equation 2.16.

In addition, although \( \bar{p} \) is not an output from the subsystem, it is displayed on a scope so that it can be viewed.
B.3.2 Calculating $W_x$ and $W_z$

This block computes $W_x$ and $W_z$ from the input $x$ according to the equations given in Section A.1.2.

$$W_{xl} = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{i1} & x_{i2} & x_{i3} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{i1} & x_{i2} & x_{i3} & 1 \end{bmatrix} \in R^{3 \times 12}$$

$$W_x = \begin{bmatrix} W_{x1} \\ \vdots \\ W_{xn} \end{bmatrix} \in R^{3n \times 12}$$

$$W_{zl} = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & 1 \end{bmatrix} \in R^{1 \times 4}$$

$$W_z = \begin{bmatrix} W_{z1} \\ \vdots \\ W_{zn} \end{bmatrix} \in R^{n \times 4}$$

The Embedded Matlab code shown in Figure B.8 implements this calculation for any number of feature points.

```
function [Wx, Wz] = fcn(x)

dim = size(x,2);

Wx = zeros(3 * dim, 12);
Wz = zeros(dim, 4);
for n = 1:dim
    Wx(3*n - 2, 1:12) = [x(1,n), x(2,n), x(3,n), 1, 0, 0, 0, 0, 0, 0, 0, 0];
    Wx(3*n - 1, 1:12) = [0, 0, 0, 0, x(1,n), x(2,n), x(3,n), 1, 0, 0, 0, 0];
    Wx(3*n, 1:12) = [0, 0, 0, 0, 0, 0, 0, 0, x(1,n), x(2,n), x(3,n), 1];
    Wz(n, 1:4) = [x(1,n), x(2,n), x(3,n), 1];
end
```

Figure B.8: Calculation of $W_x$ and $W_z$
B.3.3 Calculating $\hat{P}$ and $\bar{P}$

This block computes $\hat{P}$ from $W_x$, $W_z$, and $\hat{\theta}$ according to Equation 2.13. Next, $\bar{P}$ is calculated from $\bar{P} = P - \hat{P}$.

```
function [p_hat, p_tilda] = fcn(Wx, Wz, p, theta_hat)

dim = size(p,2);
thetaX_hat = theta_hat(1:12);
thetaZ_hat = theta_hat(13:16);

v = zeros(3 * dim, 1);
for n = 1:dim
    temp = 1 / (Wz(n, 1:4) * thetaZ_hat);
    v(3*n - 2) = temp;
    v(3*n - 1) = temp;
    v(3*n) = temp;
end

B = diag(v);
p_hat = B * Wx * thetaX_hat;

% reshape the p matrix as a (2n x 1) from a (2 x n)
newP = zeros(3 * dim, 1);
for n = 1:dim
    newP(3*n - 2) = p(1,n);
    newP(3*n - 1) = p(2,n);
    newP(3*n) = p(3,n);
end

p_tilda = newP - p_hat;
```

Figure B.9: Calculation of $\hat{P}$ and $\bar{P}$
B.3.4 Calculating $\bar{W}$

This block computes $\bar{W}$ from $W_x$, $W_z$, and $\tilde{\beta}$ according to the following equations that were given in Section 2.4.1:

$$\bar{W}_i = [W_{xi} \quad -\tilde{\beta}_i W_{zi}] \in R^{3 \times 16}$$

$$\bar{W} = \begin{bmatrix} \bar{W}_1 \\ \bar{W}_2 \\ \vdots \\ \bar{W}_n \end{bmatrix} \in R^{3n \times 16}$$

```matlab
function W_bar = fcn(Wx, Wz, p_hat)
    dim = size(Wz, 1);
    temp = zeros(3 * dim, 4);
    for n = 1:dim
        temp(3*n - 2, 1:4) = -1 * p_hat(3*n - 2) * Wz(n, 1:4);
        temp(3*n - 1, 1:4) = -1 * p_hat(3*n - 1) * Wz(n, 1:4);
        temp(3 * n, 1:4) = -1 * p_hat(3*n) * Wz(n, 1:4);
    end
end
```

Figure B.10: Calculation of $\bar{W}$

B.3.5 Calculating $\Gamma$

This subsystem computes the matrix $\Gamma$ according to Equation 2.17. The integrator is initialized with the 16 x 16 identity matrix (which is positive definite).

Figure B.11: Calculation of $\Gamma$
B.3.6 Calculating $\hat{\theta}$

This block computes $\hat{\theta}$ as specified in equation 2.16. The projection algorithm is ignored as the inputs are controlled so that the feature points are always located on the positive $z$ axis of the camera.

$$\hat{\theta} = \alpha \Gamma \hat{W} \hat{P}$$

The gain constant, $\alpha$, is varied throughout the experiment and its value is recorded in each case. In the figure below, the gain is 1000.

```matlab
function theta_hat_dot = fcn(p_tilda, W_bar, gamma)
    alpha = 1000;
    theta_hat_dot = alpha * gamma * W_bar' * p_tilda;

Figure B.12: Calculation of $\dot{\hat{\theta}}$
B.4 Error Calculation

Section 2.3 concludes by stating that the goals of both the fixed and moving camera cases are to determine the calibration matrices A, R, and t. The error calculation subsystem is the same for either case. It is shown in Figure B.13. This subsystem has three main operational blocks. The inputs to the error calculation subsystem are the estimated and actual matrices $\hat{\theta}$ and $\theta$.

Figure B.13: The error calculation subsystem
B.4.1 Calculating $A$, $R$, and $t$

Figure B.14 shows the Matlab code used to calculate the calibration matrices $A$, $R$, and $t$. $A$ is the intrinsic matrix for either of the two cases. The calculations follow those given in Section 2.4.3.

```matlab
function [A, R, t, scale] = fcn(theta_hat)

D = [theta_hat(1), theta_hat(2), theta_hat(3); theta_hat(5), theta_hat(6), theta_hat(7); theta_hat(9), theta_hat(10), theta_hat(11)];
K = D * D';

scale = sqrt(K(3,3));
D = D / sqrt(K(3,3));
d = [theta_hat(4), theta_hat(8), theta_hat(12)] / sqrt(K(3,3));
K = K / K(3,3);

a3 = K(1,3);
a5 = K(2,3);
a4 = sqrt(K(2,2) - a5^2);
a2 = (K(2,1) - a3 * a5) / a4;
a1 = sqrt(K(1,1) - a2^2 - a3^2);
A = [a1, a2, a3; 0, a4, a5; 0, 0, 1];
R = inv(A) * D;
t = inv(A) * d;
```

Figure B.14: Calculating $A$, $R$, and $t$

B.4.2 Calculating $\bar{\theta}$

Note from Figure B.14 in the previous section that the scaling factor can be determined. The estimate $\hat{\theta}$ must be scaled before $\bar{\theta}$ can be calculated. Therefore, for the unadjusted $\hat{\theta}$ and scaling factor $\lambda$,

$$\bar{\theta} = \theta - \frac{\hat{\theta}}{\lambda}$$

The scaled estimate and the norm of the error are both displayed in scopes.
B.4.3 Calculating Calibration Parameters

As mentioned previously, $A$ is the intrinsic calibration matrix. Calculation of the parameters $f_k u$, $f_k v$, $\varphi$, $u_0$, and $v_0$ will be the same for either case. In addition, the rotation and translation matrices will be found in the same way for either case.

For both cases, they are given by

$$R_c = R^T$$

$$x_c = -R_c t$$

Figure B.15 shows the above calculations. For simplicity, it is set to display only one parameter at a time (right now, $f_k u$ is being displayed).

```matlab
function fku = fcn(A,R,t)

% intrinsic parameters
fku = A(1,1);
phi = atan(-fku / A(1,2));
fkv = A(2,2) * sin(phi);
u0 = A(1,3);
v0 = A(2,3);
phi = phi * 180/pi;

% extrinsic parameters
Rc = R';
x_c = -Rc * t;

Figure B.15: Calculating calibration parameters
Appendix C

Basket Drive CAD Drawings

Figure C.1: Basket
Figure C.2: Basket assembly
Figure C.3: Chain
Figure C.5: Idle rollers
Figure C.6: Drive shaft
Figure C.7: Idler shaft
Figure C.8: Base plate
Figure C.9: End plates
Figure C.10: Side plates
Figure C.11: Inner bearings
Figure C.12: Outer bearings
Appendix D

Baseline Measurements for Wear Testing

D.1 Non-Contact 3D Scanning

As was mentioned previously, Carl Rathz (CETL) runs the 3D scans for the basket drive project. Figure D.1 shows a portion of one of those scans.

![Figure D.1: Part of the basket surface 3D scan](image)

D.2 Chemical Analysis

None of the metals listed in Table 3.3 were found in the original sample of powder. Also note that a sample of unused powder will be tested again with the 25 samples from the first set of cycles.
D.3 Weight Loss Analysis

Basket: 8067 g
Sprocket F: 114.753 g
Sprocket R: 113.966 g
Idler F: 152.516 g
Idler R: 154.080 g

D.4 Specific Diameter

Idler F: 1.251 in
Idler R: 1.223 in
Chain roller A1: 0.3135 in
Chain roller A2: 0.3135 in
Chain roller B1: 0.3130 in
Chain roller B2: 0.3145 in

D.5 Digital Imaging

The scans shown below are samples taken from Chris Simoson’s baseline scans. They will be redone but are included here to show typical results of the imaging.
D.6 Scanning Electron Microscope

Also from Chris’s baseline scans, the sample SEM images in Figures D.3, D.4, and D.5 show the abilities of the scanning electron microscope. There are three different zooms. For each set of images, the left image is from idler F and the right image is from sprocket F.
D.7 Coordinate Measurement Scanning

Again from Chris’s scans, the figure below gives two outputs from the CMS.

Figure D.4: SEM images at zoom 350x

Figure D.5: SEM images at zoom 1000x

Figure D.6: Sample CMS results
D.8 Non-Contact Surface Profilometry

Like the scans in Sections D.5, D.6, and D.7, the profilometry scans included in this report were made by Chris Simoson and are included only as examples.

These baseline tests will be rerun before testing begins. The figure below gives the surface plot and its corresponding 3D plot.

Figure D.7: Profilometry sample of a surface (top) and its 3D plot (bottom)
REFERENCES


