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Extrapolation of Time and Frequency Responses of Resonant Antennas using Damped Sinusoids and Orthogonal Polynomials

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EXTRAPOLATION OF TIME AND FREQUENCY RESPONSES OF
RESONANT ANTENNAS USING DAMPED SINUSOIDS
AND ORTHOGONAL POLYNOMIALS

A Thesis
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
Electrical Engineering

by
James Michael Frye, Jr.
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Accepted by:
Dr. Anthony Q. Martin, Committee Chair
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ABSTRACT

An extrapolation technique is presented which reduces the computational demands of obtaining a wide-band electromagnetic response from a resonant antenna using traditional computational electromagnetic (CEM) methods. It has been shown that a wide-band response can be extrapolated by fitting early-time and low-frequency data as a finite summation of orthogonal polynomials. However, because the orthogonal polynomials provide compact time support, representation of the long time responses intrinsic to resonant structures in practice proves computationally inefficient and can lead to numerical instabilities. This paper outlines the incorporation of damped sinusoids to efficiently, accurately, and reliably extrapolate both time- and frequency-domain responses of resonant antennas due to a wide-band source. A genetic algorithm (GA) is used to select the necessary extrapolation parameters. The wide-band driving-point current response of several resonant antennas is accurately extrapolated. The transmission-line matrix (TLM) method and the method of moments (MoM) are used to compute early-time and low-frequency data, respectively. Fundamentally different discretizations of the structure of interest are used, illustrating in principle the independence of the technique and the choice of computational methods employed to provide the necessary input data.

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PREFACE

The numerical technique presented in this thesis aims to significantly reduce the computational burdens of computing a wide-band response from a resonant antenna or structure. The response can be any electromagnetic quantity of interest defined in both the time- and frequency-domain, such as the driving-point current at the feed point of an antenna or the electric field at a point in space near the structure. Using traditional computational methods, computing such a response from a resonant structure can be extremely time-consuming and often requires immense computing resources.

The proposed technique simultaneously extrapolates both time and frequency representations of the complete response by processing only early-time and low-frequency data. Due to the nature of computational electromagnetics methods, early-time and low-frequency data are typically relatively easy to obtain, whereas computing accurate late-time and high-frequency data can be arduous. Thus, the extrapolation discussed can potentially provide significant computational advantages over traditional approaches.

Potential applications include the rapid wide-band characterization of multi-band antennas, such as those designed for wireless communications applications. Additionally, the technique can be used to efficiently analyze cavity-type structures over a wide range of frequencies. These structures are of interest, for example, when numerically modeling devices containing electronics embedded inside metallic enclosures.

EXTRAPOLATION OF TIME AND FREQUENCY RESPONSES OF RESONANT ANTENNAS USING DAMPED SINUSOIDS AND ORTHOGONAL POLYNOMIALS

I. Introduction

Traditionally, to determine a wide-band electromagnetic (EM) response of an antenna or structure (e.g. driving-point current or EM field value) Maxwell's equations are solved exclusively in either the time or frequency domain using one of the many computational electromagnetic (CEM) methods available today. If the structure has strong, or high-Q, resonances in the desired frequency range, the numerical solution in both time and frequency domains is often computationally burdensome. Using time-domain methods, wide-band information can be obtained by exciting the structure with a narrow pulse, time-stepping the transient response to zero, and Fourier transforming the result. However, for resonant structures energy dissipation can be exceedingly slow. Thus, obtaining the complete response can require thousands of time steps and hence significant computation times, especially for complex structures. Frequency-domain methods require a separate evaluation at each frequency which for wide-band characterization can be burdensome. Maintaining accuracy at higher frequencies generally requires increased spatial discretization, leading to larger system matrices and greater computational expense. Additionally, sharp spikes near resonances necessitate fine sampling over the frequency range of interest.

In [1]-[5], it is shown a time- and frequency-domain response due to a wide-band source can be generated from early-time and low-frequency data by fitting the response as a summation of N orthogonal polynomials. The representation obtained provides a “closed-form approximation” that can be evaluated at time and/or frequency points of interest, including the late-time and high-frequency ranges where the respective responses are unknown. The frequency response can be evaluated at small intervals to characterize resonances and the time response can be evaluated for later times until steady-state is reached. However, because the polynomials used are time-compact, accurately representing the long time responses intrinsic to resonant structures in practice proves inefficient and numerically difficult.

This paper extends the extrapolation technique in [1]-[5] by using M damped sinusoids in addition to N orthogonal polynomials to represent a response. Damped sinusoids provide the support to efficiently and accurately model resonances. Thus, the number of functions required is reduced, computation times are decreased, and the accuracy of the extrapolation is improved [6], [7]. Furthermore, by including M damped sinusoids, multiple resonances can be represented. Consequently, wide-band responses from multi-resonant structures, such as multi-band or cavity-type antennas can be extrapolated.

The stability and accuracy of the extrapolation critically depends on the non-trivial selection of several parameters which define the polynomials and damped sinusoids used. In this work, optimal parameters are determined using a genetic

algorithm (GA). This approach effectively automates the technique and provides confidence in the accuracy of the extrapolation.

The wide-band driving-point current response of several resonant antennas is extrapolated using both damped sinusoids and polynomials. The performance is compared to extrapolation using only polynomials. The transmission-line matrix (TLM) method is used to compute the desired time-domain response, whereas the frequency-domain response is computed using the Method of Moments (MoM) to solve the electrical field integral equation (EFIE). These computed responses provide early-time and low-frequency data, respectively, and are used to check the accuracy of the extrapolation. The two methods used employ fundamentally different discretizations the structure of interest; however, it is demonstrated that accurate extrapolations are still obtained. In [1]-[5], the EFIE is solved in the time and frequency domain using an identical triangular surface patch mesh for both models. In this work, different discretizations are used, illustrating in principle the independence of the technique and the numerical methods used to compute the necessary input data.

This paper is organized as follows. Section II presents an overview of extrapolation using orthogonal polynomials. The incorporation of damped sinusoids with polynomials is formulated in Section III. In Section IV, the use of a GA to select the necessary extrapolation parameters is discussed. Section V presents several numerical examples, and conclusions are discussed in Section VI.

II. Overview of Polynomial Extrapolation

The fundamental basis of the orthogonal polynomial extrapolation technique developed in [1]-[5] is that the early-time and low-frequency portions of a wide-band response contain mutually complimentary information that can be used to generate the remaining late-time and high-frequency information. Early-time data contain high-frequency information whereas low-frequency data contain late-time information. Consequently, early-time and low-frequency data can be used to fit the response as a summation of orthogonal polynomials which accurately represents the remaining late-time and high-frequency behavior. No new information is created, but rather known data are cleverly processed to yield an approximation of the entire response [3]. This approach is computationally efficient because low-frequency and early-time data are relatively easy to obtain. Early-time data points are computed first in time-stepping formulations and low-frequency evaluations are generally less demanding because the required level of spatial discretization increases with frequency.

Let $x(t)$ and $X(f)$ denote the time and frequency representations of a wide-band response of the structure of interest, respectively. Again, the response can be any electromagnetic quantity defined in both domains, such as the driving-point current at the feed point of an antenna or the electric field at a point in space near the structure. Time- and frequency-domain CEM methods are used to generate early-time and low-frequency portions of $x(t)$ and $X(f)$, respectively. A discretized version of the actual structure geometry is used to compute each

response. Each model is determined by the underlying numerical formulation utilized and may differ depending on the specific CEM method.

The discrete time-domain response $\mathbf{x}$ contains P data points sampled at Δt intervals and is partitioned into early-time and late-time data, $\mathbf{x}_{ET}$ and $\mathbf{x}_{LT}$. The spatial discretization of the time-domain model typically determines Δt . The discrete frequency response $\mathbf{X}$ contains Q data points sampled at Δf intervals and is partitioned into low-frequency and high-frequency data, $\mathbf{X}_{LF}$ and $\mathbf{X}_{HF}$. The complete responses can be written as

$$\mathbf{x} = \{ \mathbf{x}_{ET}, \mathbf{x}_{LT} \}, \quad \mathbf{X} = \{ \mathbf{X}_{LF}, \mathbf{X}_{HF} \}. \quad (1)$$

A time-domain CEM method is used to compute $\mathbf{x}_{ET}$ which holds the first p time samples of $\mathbf{x}$. A frequency-domain CEM method is used to compute $\mathbf{X}_{LF}$ which contains the first q frequency points of $\mathbf{X}$. The data vectors $\mathbf{x}_{ET}$, $\mathbf{x}_{LT}$, $\mathbf{X}_{LF}$, and $\mathbf{X}_{HF}$ can be explicitly expressed as

$$\begin{aligned} \mathbf{x}_{ET} &= \{ x(t_0), x(t_1), \dots, x(t_{p-1}) \} \\ \mathbf{x}_{LT} &= \{ x(t_p), x(t_{p+1}), \dots, x(t_{P-1}) \} \\ \mathbf{X}_{LF} &= \{ X(f_0), X(f_1), \dots, X(f_{q-1}) \} \\ \mathbf{X}_{HF} &= \{ X(f_q), X(f_{q+1}), \dots, X(f_{Q-1}) \} \end{aligned} \quad (2)$$

where $t_i = i\Delta t$ and $f_j = (j+1)\Delta f$.

The goal of the extrapolation is to use early-time and low-frequency data, $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$, to determine representations of the complete response, $\mathbf{x}$ and $\mathbf{X}$, which accurately approximate late-time and high-frequency data, $\mathbf{x}_{LT}$ and $\mathbf{X}_{HF}$.

The functions $\hat{x}(t)$ and $\hat{X}(f)$ denote the approximations of $x(t)$ and $X(f)$, respectively. To determine $\hat{x}(t)$ and $\hat{X}(f)$, both $x(t)$ and $X(f)$ are fitted as N polynomials:

$$\begin{aligned} x(t) &\approx \hat{x}(t) = \sum_{n=0}^{N-1} a_n \phi_n(t/l_1) \\ X(f) &\approx \hat{X}(f) = \sum_{n=0}^{N-1} a_n \Phi_n(f/l_2). \end{aligned} \quad (3)$$

The functions $\phi_n(t/l_1)$ and $\Phi_n(f/l_2)$ in (3) represent the pair of n th-order orthogonal polynomials [5]. Each pair is related by the Fourier transform and scaled by l_1 and l_2 , where $l_1 = 1/(2\pi l_2)$. As indicated in (3), a single set of N unknown weighting coefficients a_n is used for both time and frequency functions. These coefficients can be determined by solving a system of linear equations using only $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$. This system can be written in matrix form as

$$\begin{bmatrix} \phi_0(t_0/l_1) & \cdots & \phi_{N-1}(t_0/l_1) \\ \vdots & \ddots & \vdots \\ \phi_0(t_{p-1}/l_1) & \cdots & \phi_{N-1}(t_{p-1}/l_1) \\ \text{Re} \left\{ \begin{bmatrix} \Phi_0(f_0/l_2) & \cdots & \Phi_{N-1}(f_0/l_2) \\ \vdots & \ddots & \vdots \\ \Phi_0(f_{q-1}/l_2) & \cdots & \Phi_{N-1}(f_{q-1}/l_2) \end{bmatrix} \right\} \\ \text{Im} \left\{ \begin{bmatrix} \Phi_0(f_0/l_2) & \cdots & \Phi_{N-1}(f_0/l_2) \\ \vdots & \ddots & \vdots \\ \Phi_0(f_{q-1}/l_2) & \cdots & \Phi_{N-1}(f_{q-1}/l_2) \end{bmatrix} \right\} \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_{N-1} \end{bmatrix} = \begin{bmatrix} x(t_0) \\ \vdots \\ x(t_{p-1}) \\ \text{Re} \left\{ \begin{bmatrix} X(f_0) \\ \vdots \\ X(f_{q-1}) \end{bmatrix} \right\} \\ \text{Im} \left\{ \begin{bmatrix} X(f_0) \\ \vdots \\ X(f_{q-1}) \end{bmatrix} \right\} \end{bmatrix} \quad (4)$$

where $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts of the complex argument, respectively. The matrix equation of (4) can also be expressed as $\mathbf{B}\mathbf{a}_n = \mathbf{c}$ where $\mathbf{B}$, $\mathbf{a}_n$, and $\mathbf{c}$ have dimensions $(p+2q) \times N$, $N \times 1$, and $(p+2q) \times 1$, respectively. The N polynomials in (3) are evaluated to fill $\mathbf{B}$, data computed with CEM methods are used to fill $\mathbf{c}$, and $\mathbf{a}_n$ contains the N unknown coefficients. In practical cases $(p+2q) > N$, so the system of equations in (4) is overdetermined. A least squares solution for $\mathbf{a}_n$ can be found using the singular value decomposition (SVD) method [8]. Once determined, the values of $\mathbf{a}_n$ are used in (3) to generate $\hat{x}(t)$ and $\hat{X}(f)$. If the extrapolation is successful, these functions closely approximate of $x(t)$ and $X(f)$, including in the late-time and high-frequency ranges [1]-[5].

III. Incorporation of Damped Sinusoids

The polynomials discussed in Section II provide compact time support which increases with their order n [9]. Therefore, to represent the inherently long time responses and resonances of resonant antennas, potentially hundreds or thousands of polynomials are required. The total number of polynomials N dictates the number of equations in (4), so computationally it is desirable to minimize N while maintaining accuracy. Furthermore, rapid oscillations in higher-order polynomials can lead to computation errors and numerical instabilities [1]-[3], [5], [9]. For these reasons an additional type of function, the damped sinusoid, is

incorporated into the summations of (3) to accurately and efficiently represent resonances in the response.

The responses $x(t)$ and $X(f)$ can each be viewed as a superposition of two partial responses with distinct characteristics:

$$\begin{aligned} x(t) &= x^R(t) + x^T(t) \\ X(f) &= X^R(f) + X^T(f). \end{aligned} \tag{5}$$

In (5), $x^R(t)$ and $X^R(f)$ denote functions containing the resonant behavior of $x(t)$ and $X(f)$. Oscillation and exponential decay characterize $x^R(t)$ whereas sharp spikes around any resonant frequencies distinguish $X^R(f)$. Damped sinusoids and their frequency representations exhibit this behavior and thus are selected to represent $x^R(t)$ and $X^R(f)$, respectively. The remaining transient behavior of $x(t)$ and $X(f)$ is contained in $x^T(t)$ and $X^T(f)$. These functions decay rapidly in the time domain and are relatively smooth in the frequency domain. Accordingly, $x^T(t)$ and $X^T(f)$ are fitted by polynomials whose compact time support can effectively represent this behavior. By using two types of functions with distinct characteristics well suited for a specific behavior, the accuracy of the extrapolation is improved and the computation time decreased because fewer terms are needed.

The responses $x(t)$ and $X(f)$ are approximated by $\hat{x}(t)$ and $\hat{X}(f)$ which are each comprised of a superposition of N orthogonal polynomials and M damped sinusoids:

$$\begin{aligned} x(t) &\approx \hat{x}(t) = \sum_{n=0}^{N-1} a_n \phi_n(t/l_1) + \sum_{m=0}^{M-1} g_m(t) \\ X(f) &\approx \hat{X}(f) = \sum_{n=0}^{N-1} a_n \Phi_n(f/l_2) + \sum_{m=0}^{M-1} G_m(f). \end{aligned} \quad (6)$$

In (6), the functions $g_m(t)$ and $G_m(f)$ represent the time and frequency forms of the m th damped sinusoid and are related by the Fourier transform. In general, each pair is defined by amplitude constants, a resonant frequency, and an exponential decay or damping factor. The addition of a single damped sinusoid with N polynomials has been shown to yield significant computational advantages when used to extrapolate a response dominated by one strong resonance [7]. The form of (6), however, maintains more generality by using M damped sinusoids, allowing multiple resonances to be efficiently fit.

In (6), a_n can be determined by isolating the part of the complete response represented by polynomials alone. To do so, the damped sinusoids are subtracted in (6) to yield an expression similar to (3) which equates the summation of polynomials directly with the response it approximates:

$$\begin{aligned} x(t) - \sum_{m=0}^{M-1} g_m(t) &\approx \sum_{n=0}^{N-1} a_n \phi_n(t/l_1) \\ X(f) - \sum_{m=0}^{M-1} G_m(f) &\approx \sum_{n=0}^{N-1} a_n \Phi_n(f/l_2). \end{aligned} \quad (7)$$

The result in (7) can be interpreted as approximating $x(t)$ and $X(f)$ with the resonances removed as a summation of N polynomials. Removing the resonances allows the remaining response to be represented by time-compact polynomials alone.

Observing the similarity between (3) and (7) reveals $\mathbf{a}_n$ in (7) can be determined using the matrix formulation in (4) with $\mathbf{c}$, replaced by

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix} \quad (8)$$

where $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are defined as

$$\mathbf{c}_1 = \begin{bmatrix} x(t_0) - \sum_{m=0}^{M-1} g_m(t_0) \\ \vdots \\ x(t_{p-1}) - \sum_{m=0}^{M-1} g_m(t_{p-1}) \end{bmatrix}, \quad (9)$$

$$\mathbf{c}_2 = \left[\text{Re} \left\{ \begin{bmatrix} X(f_0) - \sum_{m=0}^{M-1} G_m(f_0) \\ \vdots \\ X(f_{q-1}) - \sum_{m=0}^{M-1} G_m(f_{q-1}) \end{bmatrix} \right\} \right], \quad \mathbf{c}_3 = \left[\text{Im} \left\{ \begin{bmatrix} X(f_0) - \sum_{m=0}^{M-1} G_m(f_0) \\ \vdots \\ X(f_{q-1}) - \sum_{m=0}^{M-1} G_m(f_{q-1}) \end{bmatrix} \right\} \right].$$

The definition of $\mathbf{c}$ in (8) and (9) incorporates the subtraction of the damped sinusoids as indicated in (7). A least-squares solution for $\mathbf{a}_n$ can then be determined and used in (6) to construct $\hat{x}(t)$ and $\hat{X}(f)$ which are accurate approximations of $x(t)$ and $X(f)$ if the extrapolation is successful. Constants

characterizing each damped sinusoid must be determined prior to the computation of $\mathbf{a}_n$ because these functions are needed to fill $\mathbf{c}$ in (8) and (9). For an accurate extrapolation, the damped sinusoids must precisely represent the resonances of the response.

IV. GA-Based Selection of Extrapolation Parameters

To reliably extrapolate a wide-band response, the proper selection of several parameters is critical. Polynomial parameters must be carefully chosen which yield accurate and stable results. When using damped sinusoids, constants defining each term must also be determined. In this work, GA optimization is used to automate the selection process. Previously, the Powell method was used to optimize the nonlinear problem of determining polynomial parameters [10].

The objective is to determine the “optimal” parameters which minimize the difference between extrapolated responses, $\hat{x}(t)$ and $\hat{X}(f)$, and those directly computed, $x(t)$ and $X(f)$. This difference, E , can be quantified using a normalized root-mean-square comparison as

$$E = \frac{1}{2} \left\{ \frac{\|\mathbf{x} - \hat{\mathbf{x}}\|}{\|\mathbf{x}\|} + \frac{\|\mathbf{X} - \hat{\mathbf{X}}\|}{\|\mathbf{X}\|} \right\} \quad (10)$$

where $\|\square\|$ denotes the l^2 -norm. The leading $1/2$ is included to average time- and frequency-domain differences. Computing E in (10) requires complete knowledge of $\mathbf{x}$ and $\mathbf{X}$; however, in practice only $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$ are available

because $\mathbf{x}_{LT}$ and $\mathbf{X}_{HF}$ are unknowns to be extrapolated. Therefore, an estimate of E using only $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$ is desired.

It has been found in the present work that E can be estimated by $\hat{E}$, defined as

$$\hat{E} = \frac{1}{2} \left\{ \frac{\|\mathbf{x}_{ET} - \hat{\mathbf{x}}_{ET}\| + \|\text{IFFT}\{\hat{\mathbf{X}}\}_{LT} - \hat{\mathbf{x}}_{LT}\|}{\|\mathbf{x}_{ET}\|} + \frac{\|\mathbf{X}_{LF} - \hat{\mathbf{X}}_{LF}\| + \|\text{FFT}\{\hat{\mathbf{x}}\}_{HF} - \hat{\mathbf{X}}_{HF}\|}{\|\mathbf{X}_{LF}\|} \right\} \quad (11)$$

where FFT and $IFFT$ denote the fast Fourier transform and its inverse, respectively. At early-time and low-frequency points, agreement is directly calculated by computing the norm of the difference between the extrapolated and directly computed data. These comparisons correspond to the first expressions in the numerator of both terms in (11). In late-time and high-frequency ranges, no directly computed data are available to compare with. Consequently, to estimate accuracy the FFT and $IFFT$ are utilized. The late-time portion of the extrapolated time data $\hat{\mathbf{x}}_{LT}$ is compared to the late-time portion of the $IFFT$ of the complete extrapolated frequency response $\hat{\mathbf{X}}$. Similarly, the high-frequency portion of the extrapolated frequency data $\hat{\mathbf{X}}_{HF}$ is compared to the high-frequency portion of the FFT of the complete extrapolated time response $\hat{\mathbf{x}}$. The known data $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$ are used to normalize the two terms in $\hat{E}$, respectively. To optimize the extrapolation, a GA is used to minimize $\hat{E}$ in (11) as a function of the necessary parameters. Sections A and B explicitly describe the variables optimized.

Optimization times can be decreased by reducing the computations required to solve the matrix equation of (4) for $\mathbf{a}_n$. In addition to SVD, a least squares solution to an overdetermined system of linear equations can be obtained several other ways [8]. One efficient alternative is computing the pseudo-inverse of $\mathbf{B}$. The coefficients are computed with $\mathbf{a}_n = (\mathbf{B}^* \mathbf{B})^{-1} \mathbf{B}^* \mathbf{c}$, where $\mathbf{B}^*$ denotes the conjugate transpose of $\mathbf{B}$ and $(\mathbf{B}^* \mathbf{B})^{-1}$ denotes the inverse of the square matrix, $\mathbf{B}^* \mathbf{B}$. In the numerical results obtained, the use of the SVD or pseudo-inverse techniques does not affect the value of E ultimately obtained by the GA; however, using the pseudo-inverse obtains the least-squares solution significantly faster, though either may be used. Optimization times can be further reduced with the pseudo-inverse by checking the condition number of the square matrix $\mathbf{B}^* \mathbf{B}$. If this condition number is found to be large for a given set of parameters, then the least-squares solution obtained will likely be inaccurate and the extrapolation unstable. Therefore, no further calculations are needed and the GA fitness value can be automatically penalized. For that parameter set, the quantities $\hat{\mathbf{x}}$, $\hat{\mathbf{X}}$, and $\hat{E}$ need not be determined and thus the computation time is reduced. The additional burden of checking the condition number is offset by the time savings it produces, given that the matrix $\mathbf{B}^* \mathbf{B}$ must be determined in the pseudo-inverse solution [7].

A. Orthogonal Polynomial Parameters

The accuracy and stability of the polynomial extrapolation depends critically on the selection of the quantity N (number of terms in (3)) and the scaling factor l_1 [1]-[5]. Empirical stability bounds for N and l_1 have been presented for several polynomial types [9]; however, there remain no precise criteria to select their optimal values for an arbitrary response. Consequently, in this work, both N and l_1 are set as GA optimization variables whose range can be generally defined using the bounds in [9]. For the numerical examples in Section V, associate Hermite (AH) functions are used as polynomials in the extrapolation. Laguerre and Bessel-Chebyshev functions could also have been used [5], [9]; however, like AH functions, these polynomials also provide compact time support. Additionally, for a convergent extrapolation, the performance is found to be nearly the same for each of polynomials studied [5].

Because AH functions provide equal support around their origin and it is assumed that $x(t) = 0$ for $t \leq 0$, it is beneficial to center each polynomial around $t = t_0$, rather than at $t = 0$ [1], [2]. Typically, the optimal time center is around half of the time support of the response [1], [2], but again no precise selection criteria exists, so t_0 is selected as a third optimization variable. Thus, when using only polynomials, the GA is used to minimize $\hat{E}$ in (11) as a function of three variables: N , l_1 , and t_0 .

B. Damped Sinusoid Parameters

In addition to selecting polynomial parameters, representing a response as damped sinusoids and polynomials in as (6) also requires the determination of parameters defining the M damped sinusoids used. Their determination is paramount to the success of the extrapolation and, in practice, must be accomplished using only $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$. In this work, the necessary damped sinusoid parameters are determined using the matrix pencil method (MPM) [11]. Other techniques, such as Prony's method [12], could have potentially been used; however, MPM was chosen for its computationally efficiency and numerical stability [11].

Using MPM, a time-domain function $y(t)$ is approximated with K complex exponentials as

$$y(t) \approx \sum_{k=0}^{K-1} (A_k + jB_k) \exp\{(-\alpha_k + j\omega_k)t\}. \quad (12)$$

In (12), A_k and B_k signify the real and imaginary parts of the complex amplitude of the k th term whereas α_k and ω_k denote the damping factor and angular frequency, respectively. Using an early-time portion of $y(t)$ sampled at Δt intervals, the MPM algorithm computes A_k , B_k , α_k , ω_k , and K . The details of this computation can be found in [11]. For the extrapolation damped sinusoids are used, but the terms of (12) are complex exponentials. However, the sum of two terms of (12) which are complex conjugates yields a single damped sinusoid. In this work, complex conjugate pairs in (12) determined by MPM are used to define the desired M damped sinusoids. In (6), the m th damped sinusoid in time and

frequency is denoted as $g_m(t)$ and $G_m(\omega)$, respectively. When using MPM as outlined, these terms can be expressed as

$$g_m^{MPM}(t) = e^{-\alpha_m t} \{ A_m \cos \omega_m t - B_m \sin \omega_m t \} \quad (13)$$

and

$$G_m^{MPM}(\omega) = \frac{A_m \alpha_m - B_m \omega_m + j A_m \omega}{(\omega_m)^2 + (\alpha_m + j \omega)^2}. \quad (14)$$

In (13) and (14), A_m , B_m , α_m , and ω_m correspond to the same quantities as defined for (12), however, each is now associated with the m th damped sinusoid, rather than the k th complex exponential. The damped sinusoids determined are comparable to the “singularities” of the response as defined in the singularity expansion method (SEM). A discussion of SEM is given in chapter 3 of [13].

Because (13) does not in general have the property $g_m^{MPM}(t) = 0$ for $t \leq 0$, a ramping envelope of the form $(1 - e^{-\beta_m t})$ is appended to enforce this condition. The constant β_m is selected such that the m th term has smoothly ramped up to 99% of its peak value after five periods, or at $t = 10\pi/\omega_m$. This modification allows the polynomials to more accurately represent the response after removing each damped sinusoid. With the ramping envelop included, (13) and (14) become

$$g_m(t) = (1 - e^{-\beta_m t}) e^{-\alpha_m t} \{ A_m \cos(\omega_m t) - B_m \sin(\omega_m t) \} \quad (15)$$

and

$$G_m(\omega) = \frac{A_m \alpha_m - B_m \omega_m + j A_m \omega}{(\omega_m)^2 + (\alpha_m + j \omega)^2} - \frac{A_m (\alpha_m + \beta_m) - B_m \omega_m + j A_m \omega}{(\omega_m)^2 + (\alpha_m + \beta_m + j \omega)^2}. \quad (16)$$

Terms of the form in (15) and (16) are used to represent resonances and define $g_m(t)$ and $G_m(\omega)$ in (6).

While excellent results have been obtained using MPM, its accurate application in practice requires the selection a “time window” which defines the subset of sampled time data to be operated on. Three parameters specify this time window: the beginning time sample p_1 , the ending time sample p_2 , and the decimation factor d . Some decimation is generally required because Δt is selected based the spatial discretization of the time-domain model and is often not the optimal time sample for MPM. While some guidelines have been presented, e.g. [14], [15], no exact criteria exists for optimal selection of p_1 , p_2 , and d . Therefore, in this work these variables are selected using the GA. When applying MPM alone to extrapolate a time-domain response, it is unclear how one would optimize these parameters in practice using only early-time data. For the proposed technique, however, $\hat{E}$ in (11) provides a measure which can be minimized to reliably select the necessary parameters. As discussed, $\hat{E}$ is computed using early-time and low-frequency data. Because these data sets contain mutually complimentary information, $\hat{E}$ provides a good estimate of the actual agreement between the directly computed and extrapolated responses. Consequently, when extrapolating using both polynomials and damped sinusoids, the GA is used to minimize $\hat{E}$ in (11) as a function of six variables:

p_1 , p_2 , d , N , l_1 , and t_0 .

V. Numerical Results

For three resonant antennas, the driving-point current responses due to a wide-band voltage source, denoted as $x(t)$ and $X(f)$, are extrapolated using early-time and low-frequency data $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$. To compute $x(t)$, TLM [16] is employed using MicroStripes [17]. With TLM, a volume surrounding the structure is discretized as cubical nodes interconnected by virtual transmission lines. To compute $X(f)$, MoM is used to solve the EFIE for the surface current on the structure with triangular patch discretization and Rao-Wilton-Glisson (RWG) edge elements [18], [19]. The surface current on each patch is determined by solving a linear system of equations. Not only does the discretization of each numerical model differ, but whereas MoM calculates current values directly, TLM uses adjacent fields to determine current values at a point. Thus, it is expected that these computed responses will differ slightly due to “modeling” or “discretization” differences. However, here it is shown that an accurate extrapolation can still be achieved. Therefore, the choice of numerical tools utilized is not unique. Alternative formulations such as the finite-difference time-domain method (FDTD) [20] or the finite element method (FEM) [21] could have also been utilized, in principle, to compute $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$, respectively. It also should be noted that the structure must be modeled with two independent CEM methods which alone provides a valuable check on the validity of the numerically computed responses.

A differentiated Gaussian pulse (DGP) is used to provide wide-band voltage excitation. The pulse can be expressed in the time domain as

$$v_0(t) = -\frac{(t-t_d)}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(t-t_d)^2}{2\sigma^2}\right\} \quad (17)$$

and in the frequency domain as

$$V_0(\omega) = j\omega\sigma^2 \exp\{-j\omega t_d\} \exp\left\{-\left(\omega^2\sigma^2\right)/2\right\}. \quad (18)$$

In (17) and (18), σ specifies the time width of the pulse and t_d represents a time delay. The maximum value of the magnitude of $V_0(\omega)$ occurs at $\omega = 1/\sigma$ and the pulse has no DC content. The parameter σ is selected as $\sigma = 3.5/(2\pi f_{\max})$ where $f_{\max}$ specifies the highest frequency of interest. This choice ensures that the magnitude of $V_0(\omega)$ has approximately decayed 1% of its maximum value at $f_{\max}$ and defines the effective spectral content of the pulse. The time delay t_d is selected as $t_d = 10\sigma$ so that $v_0(t) \approx 0$ for $t \leq 0$.

To quantify the performance of the technique, extrapolated and directly computed responses are compared in late-time and high-frequency ranges only by defining E_{ext} as

$$E_{ext} = \frac{1}{2} \left\{ \frac{\|\mathbf{x}_{LT} - \hat{\mathbf{x}}_{LT}\|}{\|\mathbf{x}_{LT}\|} + \frac{\|\mathbf{X}_{HF} - \hat{\mathbf{X}}_{HF}\|}{\|\mathbf{X}_{HF}\|} \right\}. \quad (19)$$

The expression in (19) is an effective measure of the performance of the extrapolation. The leading 1/2 is included to average time- and frequency-domain differences. Computing E_{ext} in (19) requires $\mathbf{x}_{LT}$ and $\mathbf{X}_{HF}$ and therefore cannot

be directly minimized in practice; however, it provides a valuable measure to compare against performance of the extrapolation when minimizing $\hat{E}$ in (11).

Example 1: Monopole with Four Parasitic Elements

The first antenna considered is a monopole with height $h_1 = 16$ cm centered on a finite ground plane and loaded by four parasitic elements (Fig. 1). Each element is cylindrical with radius $a = 1$ mm and assumed to be perfectly conducting. The ground plane has dimensions 26 cm $\times$ 26 cm and is assumed to be vanishingly thin and perfectly conducting. Two parasitic elements with height $h_1 = 16$ cm are aligned along the x -axis and equally offset from center element by $d_1 = 10$ cm. The remaining two parasitic elements have height $h_2 = 10$ cm and are equally offset from the center element by $d_2 = 7$ cm along the y -axis. Compared with a single-element monopole, the parasitic elements significantly increase the strength of the resonance at 440 MHz where the height of the center element is approximately $\lambda/4$.

Fig. 1. Geometry of monopole with four parasitic elements: $h_1 = 16$ cm , $h_2 = 10$ cm , $d_1 = 10$ cm , $d_2 = 7$ cm , $L = 26$ cm .

The antenna is driven by a DGP voltage source with $f_{\max} = 3.5 \text{ GHz}$ (Fig. 2) applied at the base of the center element. The time-domain current response at the feed point was computed at $\Delta t = 10 \text{ ps}$ intervals up to $T = 248 \text{ ns}$. The frequency-domain response was computed at $\Delta f = 2 \text{ MHz}$ intervals up to $W = 3.5 \text{ GHz}$. The frequency response was multiplied by the spectrum of the DGP to obtain the response due to the pulse.

Fig. 2. DGP voltage excitation: (a) time domain, (b) frequency domain.

Fig. 3 compares the directly computed responses with extrapolated versions using polynomials and damped sinusoids. Excellent agreement is seen. The vertical dashed lines indicate the partition between early-time/late-time and low-frequency/high-frequency. All data to the left are assumed to be known while data to the right is assumed to be unknown and must be extrapolated. Only 10% of the time-domain data and 25% of the frequency-domain data was used to accurately extrapolate the response (11% of total number of data points). All necessary parameters were selected by minimizing $\hat{E}$ in (11) with a GA. The response is efficiently represented with only $N=49$ polynomials and $M=14$ damped sinusoids. As seen in Fig. 4, extrapolation using only polynomials does not accurately represent the time and frequency responses.

Fig. 3. Extrapolation of driving-point current response of the monopole with parasitic elements using damped sinusoids and polynomials: (a) time domain, (b) frequency domain.

Fig. 4. Extrapolation of driving-point current response of the monopole with parasitic elements using only polynomials: (a) time domain, (b) frequency domain.

To evaluate the performance of the GA optimization, E_{ext} in (19) is computed as a function of the percentage of the complete time response used in the extrapolation (Fig. 5). As in the Fig. 3 and Fig. 4, 25% of the complete frequency response is also used. The solid lines indicate extrapolation using both polynomials and damped sinusoids (DS/Poly.) whereas the dotted lines signify using only polynomials (Poly.). Accuracy is seen to be significantly improved with the addition of damped sinusoids. The curves with circle markers were determined using a GA to directly minimize E_{ext} . The quantity E_{ext} effectively defines a “lower bound” on the performance of the procedure for the given amount of input data. The curves with square markers are obtained by minimizing $\hat{E}$ in (11) which requires only $\mathbf{x}_{ET}$ and $\mathbf{X}_{LF}$ to compute and thus can be used in practice. As seen in Fig. 5, with both damped sinusoids and polynomials, nearly the same accuracy is achieved by minimizing either $\hat{E}$ or E_{ext} using at least 10% of the time response.

Fig. 5. GA-optimized results for extrapolation of response from monopole with parasitic elements. Extrapolation performance is plotted versus percentage of time-domain data used (25% of frequency data used).

Example 2: E-shaped Patch Antenna

Next, the driving-point current response of a dual-band E-shaped patch antenna (Fig. 6) is extrapolated. The antenna is designed to radiate effectively at 1.9 GHz and 2.4 GHz for use with wireless communications applications [22]. The base of the probe feed is driven with a DGP voltage source with $f_{\max} = 4$ GHz. The time-domain response was computed at $\Delta t = 10$ ps intervals up to $T = 211$ ns and the frequency response was computed at $\Delta f = 2$ MHz up to $W = 4$ GHz.

Fig. 6. Geometry of E-shaped patch antenna.

Using 5% of the time-domain data and 25% of the frequency-domain data (7% of total data), the complete response is accurately extrapolated (Fig. 7) with $N = 32$ polynomials and $M = 9$ damped sinusoids. As evident in Fig. 8, the response is not accurately extrapolated using polynomials alone. As before, all necessary parameters are selected by minimizing $\hat{E}$ with a GA. By extrapolating

the response, no direct frequency-domain computations are required at the designed frequencies of operation, 1.9 GHz and 2.4 GHz. Instead, this information is determined from early-time data and thus the computational burden of directly modeling the structure at high frequencies is eased. However, care must be taken to sufficiently discretize the time-domain model for accuracy at the highest frequency of interest.

Fig. 7. Extrapolation of driving-point current response of E-shaped patch antenna using damped sinusoids and polynomials: (a) time domain, (b) frequency domain.

Fig. 8. Extrapolation of driving-point current response of E-shaped patch antenna using only polynomials: (a) time domain, (b) frequency domain.

Fig. 9 depicts E_{ext} computed as a function of the percentage of time response used in the extrapolation. As in the Fig. 7 and Fig. 8, 25% of the complete frequency response is also used. Similar conclusions to those made for Fig. 5 are drawn. The use of damped sinusoids again significantly improves the performance of the extrapolation. Also, it is observed that an accurate extrapolation is achieved using only 4% of the time-domain data combined with 25% of the frequency-domain data. The curves representing minimization using $\hat{E}$ and E_{ext} converge for cases using more than 4% of the time-domain data and thus at these points optimal parameters can be determined using only early-time and low-frequency data by minimizing $\hat{E}$.

Fig. 9. GA-optimized results for extrapolation of response from E-shaped patch antenna. Extrapolation agreement plotted versus percentage of time-domain data used (25% of frequency data used).

Example 3: Cavity-Backed Slot Antenna with Monopole

The final example is that of a rectangular cavity-backed slot antenna with an interior monopole [23] as depicted in Fig. 10. The antenna is designed to operate at 2.45 GHz where the length of the slot is approximately $\lambda/2$ and the monopole height is approximately $\lambda/4$. The walls of the cavity and surface of the cylindrical monopole are assumed to be perfectly conducting. The antenna is driven by a DGP voltage source with $f_{\max} = 15$ GHz applied at the base of the monopole. The desired response is the resulting current at the feed point. The wide-band pulse excites numerous cavity modes and many resonances of both the monopole and slot. Consequently, the effects of these multiple resonances are present in the driving-point current. Each resonance must be accurately represented to successfully extrapolate the response. The results obtained illustrate that by using M damped sinusoids, multiple resonances can be effectively represented.

Fig. 10. Geometry of cavity-backed slot antenna with monopole.

TLM is again used to compute the response in the time-domain. The response provides early-time data and is used to check the accuracy of the extrapolation. The frequency-domain data is obtained by taking the Fourier transform of computed time-domain response [10]. This response provides the low-frequency data used in the extrapolation. While this approach would not be used in practice, it allows the technique to be tested and removes any effects of discretization differences between the time and frequency responses. The same method of checking the technique is utilized in [10].

The time response was sampled at $\Delta t = 30\text{ps}$ intervals whereas the frequency response was sampled at $\Delta f = 1\text{MHz}$ up to $W = 15\text{GHz}$. Using damped sinusoids and polynomials, the response was accurately extrapolated using only 2.5% of the time-domain data and 25% of the frequency-domain data. Agreement between extrapolated and directly calculated responses is good as seen in Fig. 11 and Fig. 12. The response was represented using $N = 484$ polynomials and $M = 44$ damped sinusoids. Conversely, the response is not accurately extrapolated using only polynomials as in Fig. 13 and Fig. 14.

(a)

(b)

Fig. 11. Extrapolation of the time-domain driving-point current response of cavity-backed slot antenna using both damped sinusoids and polynomials: (a) 0-400 ns, (b) 22-32 ns.

Fig. 12. Extrapolation of the frequency-domain driving-point current response of cavity-backed slot antenna using both damped sinusoids and polynomials.

(a)

(b)

Fig. 13. Extrapolation of the time-domain driving-point current response of cavity-backed slot antenna using only polynomials: (a) 0-400 ns, (b) 22-32 ns.

Fig. 14. Extrapolation of the frequency-domain driving-point current response of cavity-backed slot antenna using only polynomials.

Determining the wide-band response of the cavity-backed slot antenna using traditional CEM techniques is extremely computationally demanding. The transient current response decays very slowly because energy must bounce around inside the cavity and finally propagate through slot to be radiated outside the structure. Tens of thousands of time steps are consequently needed to compute the complete time response. Additionally, frequency-domain formulations require hundreds or thousands of separate evaluations to sufficiently characterize the structure over the wide frequency range of interest. Therefore, extrapolation offers substantial computational benefits. The complete response is obtained without time-stepping the transient response to steady-state or sweeping the structure over countless frequency points, which at higher frequencies may also be limited by the computer resources available.

VI. Conclusions

This paper presents a technique to simultaneously extrapolate time- and frequency-domain responses due to wide-band excitation of a resonant antenna using only early-time and low-frequency data. The response is accurately and efficiently represented by a superposition of N orthogonal polynomials and M damped sinusoids. Including damped sinusoids allows multiple resonances to be effectively modeled. The resulting representation of the response can be evaluated at time and/or frequency points of interest and provides a single set of parameters which characterizes the response in both time and frequency.

A method of incorporating damped sinusoids with orthogonal polynomials and determining parameters defining each function is presented. Significant

improvements on the accuracy and efficiency of extrapolating wide-band responses from resonant antennas are observed compared to using polynomials alone. The wide-band driving-point current response of three resonant antennas due to a DGP voltage source has been accurately extrapolated. Additionally, a GA has been used to reliably select necessary parameters. An approximation of the extrapolation accuracy $\hat{E}$ is developed which requires only known early-time and low-frequency data and thus can be utilized in practice. By minimizing this quantity, one can gain confidence in the accuracy of the extrapolation and determine if adequate amounts of early-time and low-frequency data are available to yield an accurate extrapolation. Thus, the technique can be periodically applied with available data while additional data is concurrently computed. When the extrapolation is found to converge, the procedure can be stopped.

The extrapolation technique is shown to operate successfully even when applied using time and frequency responses computed from CEM methods using radically different discretizations of the modeled structure, such as TLM and MoM. Thus, the technique can be applied independently of the choice of the CEM methods used, provided each is a full-wave solver of Maxwell's equations. Therefore, the choice may be made based on the user's familiarity with a particular method, selected based on the modeling method best suited for the given application, or based on simply what is available to the user. Another potential application is the characterization of electrically large resonant structures whose simulation at high frequencies may be limited by the computing resources available. The desired

high-frequency information can be accurately extrapolated without requiring direct computations using frequency-domain formulations.

VII. References

- [1] M. M. Rao, T. K. Sarkar, T. Anjali, and R. S. Adve, "Simultaneous extrapolation in time and frequency domains using Hermite expansions," *IEEE Trans. Antennas Propag.*, vol. 47, pp. 1108-1115, Jun. 1999.
- [2] M. M. Rao, T. K. Sarkar, R. S. Adve, T. Anjali, and J. F. Callejon, "Extrapolation of electromagnetic responses from conducting objects in time and frequency domains," *IEEE Trans. Microw. Theory Tech.*, vol. 47, pp. 1964-1974, Oct. 1999.
- [3] T. K. Sarkar, J. Koh, W. Lee, and M. Salazar-Palma, "Analysis of electromagnetic systems irradiated by ultra-short ultra-wide-band pulse," *Meas. Sci. Technol.*, vol. 12, pp. 1757-1768, Oct. 2001.
- [4] T. K. Sarkar and J. Koh, "Generation of a wide-band electromagnetic response through a Laguerre expansion using early-time and low-frequency data," *IEEE Trans. Microw. Theory Tech.*, vol. 50, pp. 1408-1416, May 2002.
- [5] M. Yuan, J. Koh, T. K. Sarkar, W. Lee, and M. Salazar-Palma, "A comparison of performance of three orthogonal polynomials in extraction of wide-band response using early time and low frequency data," *IEEE Trans. Antennas Propag.*, vol. 53, pp. 785-792, Feb. 2005.
- [6] J. M. Frye and A. Q. Martin, "Extrapolation of electromagnetic responses from resonant structures," *2007 IEEE AP-S Society International Symposium*, Honolulu, HI.
- [7] J. M. Frye and A. Q. Martin, "Efficient extrapolation of wideband responses of highly resonant antennas and structures via a damped sinusoid and orthogonal polynomial expansion technique," presented at *2006 IEEE AP-S Society International Symposium and USNC/URSI National Radio Science Meeting*, Albuquerque, NM.
- [8] G. H. Golub and C. F. Van Loan, *Matrix Computations*. Baltimore, MD: Johns Hopkins Univ. Press, 1991.
- [9] M. Yuan, A. De, T. K. Sarkar, J. Koh, and B. H. Jung, "Conditions for generation of stable and accurate hybrid TD-FD MoM solutions," *IEEE Trans. Microw. Theory Tech.*, vol. 54, pp. 2552-2563, June 2006.

- [10] J.-L. Hu, C. H. Chan, and T. K. Sarkar, "Optimal simultaneous interpolation/extrapolation algorithm of electromagnetic responses in time and frequency domains," *IEEE Trans. Microw. Theory Tech.*, vol. 49, pp. 1725-1732, Oct. 2001.
- [11] T. K. Sarkar and O. Pereira, "Using the matrix pencil method to estimate the parameters of a sum of complex exponentials," *IEEE Antenna Propag. Mag.*, vol. 3, pp. 48-55, Feb. 1995.
- [12] W. L. Ko and R. Mittra, "A Combination of FD-TD and Prony's Methods for Analyzing Microwave Integrated Circuits," *IEEE Trans. Microw. Theory Tech.*, vol. 39, pp. 2176-2181, Dec. 1991.
- [13] L. B. Felson, Ed., *Transient Electromagnetic Fields*. New York: Springer-Verlag, 1975.
- [14] X. Luo, M. Li., and J.L. Drewniak, "Time history extrapolation for FDTD modeling of shielding enclosure designs and EMI antenna geometries," *Proc. IEEE Electromagn. Compat. Symp.*, Denver, CO, Aug. 1998, pp. 1172-1177.
- [15] P. Kozakowski and M. Mrozowski, "Low-order models from FDTD time samples," *IEEE Microw. Wireless Compon. Lett.*, vol. 12, pp. 438-440, Nov. 2002.
- [16] C. Christopoulos, *The Transmission-Line Modeling Method*. New York, NY: IEEE Press, 1995.
- [17] MicroStripes Electromagnetic Simulation Software: www.flomerics.com.
- [18] S. Makarov, *Antenna and EM Modeling with MATLAB*. New York, NY: Wiley, 2002.
- [19] S. M. Rao, D. R. Wilton, and A. W. Glisson. "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. AP-30, pp. 409-418, May 1982.
- [20] Taflove and S. Hagness, *Computation Electrodynamics: The Finite-Difference Time-Domain Method*. Boston, MA: Artech House, 2000.
- [21] J. Jin, *The Finite Element Method in Electromagnetics*. New York, NY: Wiley, 2002.

- [22] F. Yang, X.-X. Zhang, X. Ye, and Y. Rahmat-Samii, "Wide-band E-shaped patch antennas for wireless communications," *IEEE Trans. Antennas Propag.*, vol. 45, pp. 1094-1100, July 2001.
- [23] T. Hikage, M. Omiya, and K. Itoh, "Considerations on performance evaluation of cavity-backed slot antenna using the FDTD technique," *IEEE Trans. Antennas Propag.*, vol. 49, pp. 1712-1717, Dec. 2001.

CONCLUSIONS

This thesis presents a technique to efficiently compute a complete wide-band response using only early-time and low-frequency data. Efforts have been made to develop this technique to allow for its reliable and accurate application in practice and provide an efficient method to numerically compute wide-band responses from resonant structures, which have many practical applications.

One application is the efficient wide-band characterization of multi-band or multi-resonance antennas commonly used to meet the increasing demands of wireless communication applications. Example 2 illustrates the technique applied to extrapolate a response from such an antenna, the E-shaped patch. The proposed technique can potentially be utilized in the design process to rapidly analyze the antenna's performance. Typically, to achieve performance goals, many designs must be analyzed until specifications are adequately met; therefore, the technique can provide a means to speed up this process or allow significantly more designs to be considered to determine the optimal parameters to meet given specifications.

In example 3, a response inside a cavity structure is extrapolated over a wide frequency range. Such a problem is of great interest when assessing electromagnetic interference (EMI) and susceptibility issues, which are becoming increasingly important as the widespread reliance of digital electronics continues to swell. Electronics embedded inside a metallic enclosure can be disrupted or permanently damaged by electromagnetic energy propagating into the device from an external source or by interactions of energy interior to the device.

Effectively addressing and further understanding EMI issues will likely require developing numerical techniques to effectively analyze the complex geometries of realistic electronic systems and enclosures. Hybrid techniques, such as the one presented in this thesis, can potentially help provide the methods needed to aid in the efficient numerical simulation of such structures and further the understanding of signal penetration into enclosures and their interaction with complex environments inside.