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On the Association of Gamma-Ray Bursts with Supernovae

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ABSTRACT

The recent discovery of a supernova (SN 1998bw) seemingly associated with GRB 980425 adds a new twist to the decades-old debate over the origin of gamma-ray bursts. To investigate the possibility that some (or all) bursts are associated with supernovae, we performed a systematic search for temporal/angular correlations using catalogs of BATSE and BATSE/*Ulysses* burst locations. We find no associations with any of the precise BATSE/*Ulysses* locations, which allows us to conclude that the fraction of high-fluence gamma-ray bursts from known supernovae is small (<0.2%). For the more numerous weaker bursts, the corresponding limiting fraction of 1.5% is less constraining due to the imprecise locations of these events. This limit (1.5% $\simeq$ 18 bursts) allows that a large fraction of the recent supernovae used as a comparison data set (18 supernovae $\simeq$ 20%) could have associated gamma-ray bursts. Thus, although we find no significant evidence to support a burst/supernova association, the possibility cannot be excluded for weak bursts.

Subject headings: gamma rays: bursts — supernovae: general

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1. Introduction

Ever since the discovery of gamma-ray bursts (GRBs) more than 30 years ago, researchers have sought to associate them with known astrophysical objects. This effort was fruitless until recently when accurate and timely GRB locations provided by the *Beppo*SAX Wide Field Cameras led to the discovery of X-ray/optical/radio transients and associated host galaxies (Costa et al. 1997; van Paradijs et al. 1997; Frail et al. 1997). While the objects causing the bursts, themselves, still remain a mystery, it is now possible to directly identify and study their host galaxies. In particular, optical spectroscopy of two host objects (presumably dim galaxies) has provided direct measurement of their redshift/distance: for GRB 970508 the redshift $z = 0.835$ (Metzger et al. 1997) and for GRB 971214 $z = 3.42$ (Kulkarni et al. 1998). These measurements provide the first direct evidence of the cosmological origin of GRBs—something which was strongly suggested by many years of *Compton*-BATSE measurements indicating an isotropic, inhomogeneous spatial distribution of GRB sources (e.g., Briggs et al. 1996; Pendleton et al. 1996).

An apparent discrepancy with the now “standard” picture of GRBs originating in distant galaxies is the recent discovery of a bright supernova (SN 1998bw) within the $8'$ (radius) *Beppo*SAX Wide Field Camera error circle of GRB 980425 (Galama et al. 1998a). Galama et al. (1998b) estimated, with conservative *a posteriori* assumptions, that the chance probability of such an occurrence is $\sim 10^{-4}$. It is thus difficult to reject the hypothesis that SN 1998bw is related to GRB 980425, even though the optical lightcurve is markedly different from those of other GRB optical afterglows. This conflicts with the standard view of GRB origin because SN 1998bw resides in a relatively nearby galaxy (ESO 184-G82) with redshift $z = 0.0085$ (Tinney et al. 1998) and the burst was average in its gamma-ray properties (e.g., peak intensity, spectrum, fluence, duration; Kippen et al. 1998a) compared to other BATSE bursts. The notion that an average burst could result from a nearby supernova event is difficult to reconcile with a population of bursts originating at cosmological distances.

In this *Letter* we report on a systematic search for similar GRB/Supernova coincidences that may have occurred since 1991 April, when BATSE began observing bursts. We utilize numerous precise GRB locations provided by BATSE/*Ulysses* Interplanetary Network (IPN) timing analysis, in addition to the less precise (but more numerous) events localized with BATSE alone. At the end of the *Letter*, we compare our results to those of an independent study (Wang & Wheeler 1998) which reported a correlation between GRBs and type Ib/c supernovae.

2. Supernova Catalog & Selection Criteria

The working hypothesis in this study, based on the possibility that GRB 980425 and SN 1998bw are associated, is that some fraction of GRBs may be related to supernova explosions. We therefore search for coincidences, both in time and direction, between known GRBs and known supernovae. As a comparison dataset, we use a list of all reported supernovae obtained from the Central Bureau of Astronomical Telegrams [10]. This list includes the date of discovery, the SN position and/or the host galaxy position, the optical magnitude at the time of discovery, and the SN type (if it has been determined). At faint magnitudes, the supernova sample is highly biased by deep search campaigns that scan small regions of the sky. The bias is less severe for brighter events that are more easily detected and we therefore begin by limiting the sample to magnitudes $M \leq 17$ ($\sim$3 mag fainter than SN 1998bw at the time of its discovery). We also reject the few SN with unspecified magnitudes and use the host galaxy position when the SN position is not specified. These selections yield a list of 160 supernovae with discovery dates from 1991 Feb. through 1998 May—including 58 events classified as SN type Ia, 11 as type Ib/Ic and 91 others (either type II, or unclassified). The only other selection criterion is the choice of a time window ΔT (SN discovery time minus the GRB time) wherein GRBs will be considered as possibly related to supernovae. Given the considerable spread ($\sim$days to weeks) in the time from a SN explosion to its optical discovery, we allow a generous window of $0 \leq \Delta T \leq 30$ days as our baseline selection.

3. GRB/Supernova Correlation Analysis

We employ a standard approach for examining the GRB/SN correlation. When comparing a sample of GRBs to the SN list described above, the number of GRB/SN pairs within the ΔT search window are counted to provide a measure of the temporal association. Directional association is measured by only including those pairs where the SN position is within the corresponding GRB error box (we use the term "error box" to describe the region of uncertainty around a GRB location, which can be of arbitrary shape). The exact definition of what constitutes a GRB error box depends on the GRB catalog being considered as described below. The total number of GRB/SN pairs $N^{\rm P}$ meeting these criteria is used as a correlation statistic.

To assess the significance of the measured correlation statistic we employ Monte Carlo simulations wherein 10^4 random GRB catalogs are generated under the null-hypotheses that GRBs are distributed uniformly in time; distributed isotropically on the sky (corrected for the observing biases of the particular GRB catalog being considered); and are uncorrelated with supernovae. The correlation statistic is computed between each random catalog and the supernovae list—thereby providing a measure of the statistical distribution of $N^{\rm P}$ values. The significance P of the observed correlation $N^{\rm P}_{\rm obs}$ is given by the fraction of simulated catalogs having $N^{\rm P} \geq N^{\rm P}_{\rm obs}$.

In the case of GRB catalogs with imprecise locations (where many chance coincidences are probable), an alternative test is used that considers the entire distribution of GRB/SN angular deviations, rather than simply testing if supernovae locations are inside or outside pre-defined GRB error boxes. In this test the Kolmogorov-Smirnov technique (see Press et al. 1989) is applied between the cumulative distribution of the measured GRB/SN angular deviations and that from

[10] http://cfa-www.harvard.edu/iau/lists/Supernovae.html

the average of many simulated GRB catalogs. The parameter of interest is the maximum absolute deviation $D_{\rm KS}$ between the two distributions. The significance $P_{\rm KS}$ of a measured correlation is given by the fraction of simulated catalogs having $D_{\rm KS} \geq$ the measured value.

3.1. BATSE/*Ulysses* Results

The BATSE 4B (revised) catalog contains locations for 1637 GRBs that were detected from 1991 April through 1996 Sept. (Paciesas et al. 1998). Unfortunately, BATSE locations are not particularly useful for correlation studies due to their large uncertainties (several degrees, on average). One way to improve the precision is to combine the BATSE data with BATSE/*Ulysses* (hereafter B/U) IPN timing annuli (see e.g., Kippen, Hurley & Pendleton 1998). Since the annuli are very precise (typically ∼arc-minutes, in one dimension), the intersection of an annulus with its corresponding BATSE localization results in an error box typically 25 times smaller in area than the BATSE localization alone. IPN annuli are available for the 415 BATSE 4B (revised) bursts also detected by *Ulysses* (Hurley et al. 1998a, 1998b; Laros et al. 1998). In general, these are the ∼25% most fluent BATSE bursts—a reflection of the difference in sensitivity between *Ulysses* and BATSE. IPN data for bursts beyond the 4B catalog interval are available, but we do not use them since they have not undergone final processing and checking.

In the correlation analysis, we define a combined B/U error box to be the intersection of the 3σ timing annulus (Hurley et al. 1998a) with the corresponding 99.73% confidence (i.e., 3σ) BATSE error circle (as computed with the "core-plus-tail" error distribution model of Briggs et al. 1998a, b; hereafter the CPT model). The use of such a large BATSE error circle does not cause significant loss of correlation sensitivity because the timing annuli are so precise. In performing the required Monte Carlo simulations, burst positions are sampled according to the BATSE 4B exposure function (Hakkila et al. 1998) and then randomly displaced according to the real B/U burst location uncertainty distributions. Location displacement is important, for the uncertainty in a B/U location is highly correlated with its position on the sky—a result of the fact the *Ulysses* spacecraft follows a non-random trajectory.

Comparing the 415 combined B/U burst locations with the SN catalog, we find 585 GRB/SN pairs within the 30 day search window. In *none* of these pairs is the SN location within the GRB error box. This measured value of $N^{\rm P}_{\rm obs} = 0$ is consistent with the number $\langle N^{\rm P} \rangle = 0.06 \pm 0.24$ expected by chance and nearly independent of the size of the ΔT search window. (In fact, the only 2 spatially coincident GRB/SN pairs meeting our criteria have $\Delta T = 826$ days and 844 days, respectively). We performed further simulations wherein a fraction of the bursts $F_{\rm B}$ were forced to originate (before applying the location displacement process) at a cataloged SN location. The value of $N^{\rm P}_{\rm obs} = 0$ is inconsistent (at the 99.5% confidence level) with even a single B/U burst originating at any of the reported supernovae locations. We thus have a conservative limit that $F_{\rm B} < 1/415$. However, there is the possibility that $F_{\rm B}$ could be larger for low-fluence bursts not included in the B/U sample.

3.2. BATSE-only Results

To investigate the possible correlation of weaker GRBs with supernovae, we must use the BATSE locations alone. This significantly reduces the correlation sensitivity—especially for weak bursts, which have larger-than-average location uncertainties. Due to the imprecise locations, and the large number of BATSE bursts, many chance GRB/SN coincidences are likely. The correlation sensitivity is thus a strong function of the GRB error circle definition used in computing the $N^{\rm P}$ statistic. We performed simulations varying the confidence level of the BATSE error circle and find that using ∼68.27% confidence (i.e., $\sim 1\sigma$) BATSE error circles nearly optimizes the sensitivity, with the ability to detect $F_{\rm B} \sim 1\%$. Since we have already found that none of the B/U bursts are associated with a SN, including them in the BATSE-alone analysis will only add statistical noise. We therefore consider only the 1222 BATSE 4B (revised) bursts that are not included in the B/U sample (referred to as the 4B−IPN sample). For these bursts, $N^{\rm P}_{\rm obs} = 9$, whereas $\langle N^{\rm P} \rangle = 6.9 \pm 2.6$ are expected by chance. This is an insignificant excess, found to occur with a probability $P = 0.260$. The alternative correlation test confirms this result, yielding a probability value of $P_{\rm KS} = 0.650$. It is possible that other SN selection criteria could yield different results. We therefore repeated the analysis, varying the ΔT temporal window and SN magnitude limit. As shown in Figure 1, no significant correlation was found for any selection. The lowest probability is at the marginal ∼7% level, which we judge to be insignificant given the many trials of different selection parameters.

We are fortunate to have two large, independent samples of weak GRB locations to examine: BATSE post-4B bursts (Meegan et al. 1998) and untriggered BATSE bursts (Kommers et al. 1997, 1998). As indicated in Table 1, there are no significant excess correlations for either of these data sets. Table 1 also includes results using separate SN types, for which there are also no significant correlations. Lacking any significant detections, we performed additional simulations varying $F_{\rm B}$. For the 4B−IPN sample, we find that the data require $F_{\rm B} < 1.5\%$ at the 99.9% confidence level.

4. Comparison with Other Results

Our analysis yields no significant evidence of an association between GRBs and known supernovae. This is in contradiction with the results of an independent study performed by Wang & Wheeler (1998, hereafter WW98), where a significant correlation with type Ib/c supernovae was reported. To examine whether the two studies are discrepant, we applied our correlation analysis technique to the list of 21 supernovae used by WW98 and the full "current" BATSE catalog, which includes all bursts detected through 1998 May. We find 229 GRB/SN pairs within the supernovae temporal windows specified by WW98. None of these pairs has the SN position within the corresponding BATSE 68.27% confidence GRB error radius—consistent with the number $\langle N^{\rm P} \rangle = 0.68 \pm 0.85$ expected by chance. The alternate correlation technique, which considers the full distribution of GRB/SN angular separations, yields $P_{\rm KS} = 0.1702$. If, like WW98, we limit the sample to only consider the 6 type Ib/c supernovae, the value becomes $P_{\rm KS} = 0.0946$. We thus conclude that there is no significant correlation—in agreement with our original results. It appears that the WW98 result is an artifact of their use of an inappropriate or errant definition of BATSE location errors. As indicated in Table 2, the 6 GRB/SN Ib/c associations proposed by WW98 are not very likely based upon the angular separations and the CPT model of the BATSE error distribution. In fact, only four of the proposed associations have separations within the BATSE 99.73% confidence (i.e., 3σ) limits. If a constant 1.6° BATSE systematic error model is used instead of the more appropriate CPT model, only three of the pairs are left within the BATSE 3σ confidence circles. It is unclear what BATSE error model WW98 used to find their 6 proposed associations. Further evidence against the reality of the SN Ib/c correlation is given by the fact that 2 of the 6 proposed associations (GRB 960221/SN 1996N and GRB 980218/SN 1998T) are completely ruled-out by BATSE/*Ulysses* IPN timing annuli. The remaining 4 bursts were not detected by *Ulysses*.

5. Discussion

Our study shows that no GRB with a precise BATSE/*Ulysses* location is associated with any known supernova (within our search constraints). We are thus able to conclude, with high confidence, that the fraction of high-fluence bursts associated with *known* supernovae is small ($F_{\rm B} < 0.2\%$). The result for weaker bursts that $F_{\rm B} < 1.5\%$ is far less constraining, due to the greater number and much larger location uncertainties of these events. Thus, although we found no significant evidence for a GRB/SN association, this possibility still exists for weak bursts. In fact, the $F_{\rm B} < 1.5\%$ limit still allows that $\sim$20% of the reported supernovae we considered could have an associated BATSE GRB. The incompleteness of the supernova sample makes the result for weak events even less constraining. The expected number of supernovae reaching a *peak* with $m_{\rm B}$ brighter than 16 is estimated to be 120–150 yr^{-1} (Galama et al. 1998b), whereas the detected rate for these bright events is about 12 per year from the start of BATSE. Therefore, the number of undetected bright events is about 90% of the total number of supernovae. Our limits on the fraction of GRBs from known supernovae must therefore be increased by $\sim$10 times when applied all supernovae.

An independent limit on the fraction of GRBs related to non-cosmological supernovae is given by the shape of the burst V/V_{max} distribution. This distribution should be uniform between 0 and 1 for bursts of such local origin (Schmidt, Higdon, & Heuter 1988). In contrast, the measured distribution is highly non-uniform, with most bursts concentrating at $V/V_{max} < 0.5$. The smallest measured values of V/V_{max} thus set an upper limit on the normalization of any uniformly distributed component in the full distribution. The most sensitive search so far for faint GRBs is that carried out by Kommers et al. (1997, 1998), which can detect bursts that have peak fluxes lower by a factor of 2 than those detected with the onboard BATSE burst trigger. In their sample of 2267 GRBs, only 26 have $0.8 < V/V_{max} < 1.0$; so at most only $5 \times (26 \pm 5) = 130 \pm 25$ of the 2267 bursts could come from a spatially homogeneous population (Kommers et al. 1998). This corresponds to just 6 ± 1 percent of all bursts that could be associated with "nearby" supernovae.

This brings us back to the instigator of this study—the GRB 980425/SN 1998bw association. If the association is real and (as the BATSE data allow, but do not indicate) a small fraction of weak GRBs are associated with supernovae, we are left to explain why the association does not also exist for strong bursts. The hypothesis put forth by Wang & Wheeler (1998) is that all GRBs are related to supernovae, but only for some nearby events (presumably those like SN 1998bw) are we able observe a weak isotropic emission. For the more numerous cosmologically distant sources, the isotropic component is unobservable, and the bursts are due to highly collimated emission. The other possibility is that bursts like GRB 980425 are the result of rare SN events and are completely unrelated to the bulk of gamma-ray bursts (Woosley, Eastman & Schmidt 1998; Iwamoto et al. 1998). This would imply that different mechanisms can produce bursts with similar gamma-ray properties. Woosley, Eastman & Schmidt (1998) suggested that GRB 970514 may be another example of this phenomena, since the BATSE location and time of this burst are coincident with SN 1997cy—a particularly luminous SN with strange spectral properties. (Unfortunately, an IPN annulus is not available). It is possible that studies of the spectral and temporal properties of GRB sub-sets could provide some further insight (see e.g., Bloom et al. 1998). However, without a large sample of accurate locations for weak bursts, it is unlikely that either of these scenarios can be ruled out.

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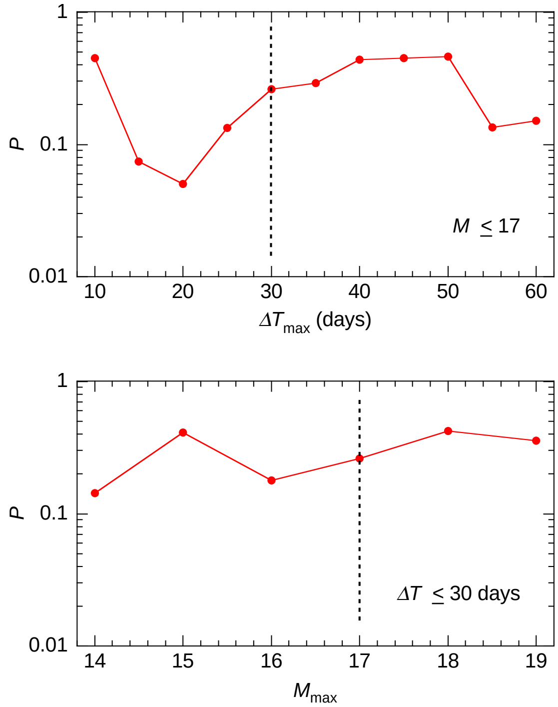

Fig. 1.— Gamma-ray burst/ supernova correlation significance P (small number indicates higher significance) as a function of the supernova magnitude selection $M \leq M_{\rm max}$ and temporal window $0 \leq \Delta T \leq \Delta T_{\rm max}$. Dashed lines indicate the baseline search criteria.

Table 1. GRB/SN Correlation Statistics

GRB Catalog	No. GRBs	SN Type	$N^{\rm P}_{\rm obs}$	$\langle N^{\rm P} \rangle$	P	$P_{\rm KS}$
B/U IPN	415	All	0	0.06 ± 0.24	1.0000	···
BATSE 4B−IPN	1222	All	12	6.9 ± 2.6	0.2598	0.6504
		Ia	1	2.3 ± 1.5	0.8967	0.7172
		Ib/c	0	0.4 ± 0.6	1.0000	0.3527
		Other	8	4.3 ± 2.1	0.0684	0.4197
BATSE post 4B	496	All	5	3.9 ± 2.0	0.3568	0.0761
		Ia	0	1.8 ± 1.4	1.0000	0.0730
		Ib/c	2	0.6 ± 0.6	0.0669	0.0840
		Other	3	1.3 ± 1.3	0.2492	0.8665
BATSE untriggered	876	All	44	42.7 ± 7.2	0.4395	0.5761
		Ia	22	17.0 ± 4.4	0.1532	0.4214
		Ib/c	4	3.0 ± 1.8	0.3645	0.7526
		Other	18	22.7 ± 5.1	0.8502	0.4963

Note: These data were obtained using the baseline search criteria of $M \leq 17$ and $0 \leq \Delta T \leq 30$ days.

Table 2. GRB/SN Associations Proposed by WW98

Supernova	GRB	BATSE No.	$\Delta\theta$[a] (°)	$\sigma_{\rm stat}$[b] (°)	$N\sigma$[c]	$N\sigma$[d]
1992ad	920609	1641	17.5	9.99	2.0	2.1
1994I	940331	2900	28.6	7.89	4.4	5.0
1996N	960221	4959	7.8	4.01	1.8	2.3
1997X	970103	5740	13.4	3.87	3.1	4.5
1997ei	971120	6488	19.7	9.94	2.4	2.5
1998T	980218	6605	6.3	0.93	2.0	4.8

[a]Angular separation between supernova and GRB location centroid.

[b]BATSE statistical uncertainty radius.

[c]BATSE confidence level at $\Delta\theta$ (converted to units of Gaussian standard deviations) based on the CPT error model.

[d]BATSE confidence level at $\Delta\theta$ based on a 1.6° systematic error.