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EMPIRICAL MODELING OF THE STRESS-STRAIN RELATIONSHIP FOR UPSETTING UNDER DIRECT ELECTRICAL CURRENT

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ABSTRACT
Present research involving Electrically-Assisted Manufacturing (EAM) has shown the advantageous effects that current provides on the mechanical properties with respect to part formation. Overall general results using this process include the reduction of the flow stress during forming, increased achievable deformation, and a decrease or elimination of springback. Specifically targeting the compression process, this paper evaluates the stress-strain relationship and develops from experimental results an initial stress predictor function that includes electrical current density. The derived empirical model is capable of estimating the flow stress for an EAM upsetting test. Once developed, the model is proven by testing it against various materials and electrical current densities.

INTRODUCTION
In industry, most manufactures use some method to increase the workability of the materials they process. For bulk material forming, there are several methods used to increase and improve the formability of the material. The following paragraphs describe these methods and highlight the benefits as well as the negative aspects. This paper focuses on the novel EAM process and the development of the electrical current’s effect on the stress-strain relationship. Thus, to better characterize the effects of electrical current applied during forming, this study presents a basic empirical model which relates the observed effects from experimental testing.

The first method commonly used to improve formability is hot working. This process is performed when the deformation of the material takes place at or above the recrystallization temperature of the material. With the increase in temperature, the workability of the material is increased, as a result of the reduced deformation forces required as well as the increased achievable deformation. Yet, there are several downsides associated with this process. Hot working leads to greater dimensional variability between parts and also leaves an increased number of residual stresses within the
material. Moreover, this process increases the possibility of workpiece/die adhesion, decreases the effectiveness of lubricant, and also decreases die strength (Groover, 2007).

Another alternative to improve formability is through incremental forming. In this process, the material is deformed to a certain strain level and then a heat treatment (i.e. process anneal) is performed to reduce the effects of the cold work that has been imposed on the material (Golovashchenko, 2005). This technique allows for increased deformation before the fracture strain is reached. However, with the constant need to stop deformation, it results in increased material processing time, thus limiting this process to lower-volume applications in industry. Also, with the requirement of refixturing in between heat treatments, this process introduces quality issues with respect to part dimensionality.

Finally, EAM is a process which combines the use of electrical current and a manufacturing process. In EAM, the workpiece is subject to electrical flow while it is being deformed, hence the deformation process does not have to be halted. This process is applicable to all conductive materials and can be applied to most major manufacturing processes. There are several additional factors that can be controlled to tune the process such as the application location, the form of electrical input (i.e. pulsed or continuous), and the amount of current. General results from this technique include decreased flow stress during deformation, increased fracture strain, and an overall reduction/elimination of springback. Currently, several works have concluded that for compressive loadings, the EAM technique is most beneficial when the electricity is applied continuously. In contrast, during tensile loading, pulsed electrical current has shown to be most beneficial in increasing the achievable elongation. To support these conclusions, in a study by Heigel et al., the application of electrical flow was examined for Al 6061 T6511 tensile specimens and the results showed that the flow stress of the material was reduced with the application of electricity (2005). More recently in 2008, Roth et al. developed an optimal parameter set (pulse duration, period, and current magnitude) for Al 5754 tensile specimens which led to achievable elongations of greater than 400% over the baseline. In contrast to these tensile papers, the application of electricity has been significantly beneficial in compression testing as well. Perkins et al. examined multiple materials and deduced that the forgeability could be significantly enhanced by increased fracture strain and the flow stress of the material could be significantly reduced in comparison to baseline testing (i.e. no electricity) (2007). In 2009, Green et al. utilized the EAM technique by applying single pulses of electrical current to completely eliminate springback in flattening and shape retention tests. Further discussion on the electrical current’s effects will be discussed in the Electrical Theory section.

DEFORMATION THEORY

When discussing metallic materials at an atomic level, the bonding consists of ion cores surrounded by valence electrons (electron cloud) which hold the ion cores together. When a material is plastically deformed, there is a stress imposed on the lattice and this causes dislocations to be created. The dislocation is a result of the material’s lattice being misaligned, thus creating tensile and compressive forces (lattice strain) around the dislocation. As plastic deformation occurs, the number of dislocations within the material’s lattice increases and they interact with other internal lattice flaws and dislocations, thus increasing the required force to continue deformation. This process is classified as cold work.

With respect to modeling the behavior of materials, the most common constitutive equation for a strain hardening and strain rate sensitive material can be defined by:

$$\sigma = C'\varepsilon^n\dot{\varepsilon}^m$$

(1)

where, $\sigma$ is the flow stress, $C'$ is the strength coefficient, $\varepsilon$ is the strain, $n$ is the strain hardening exponent, $\dot{\varepsilon}$ is the strain rate, and $m$ is the strain rate sensitivity exponent (Kalpakjian, 1997). This relationship is used as the basis for the presented empirical model to characterize plastic deformation with the addition of electricity.

ELECTRICAL THEORY

In the following subsections, a brief historical review is presented which details the research
that has been explored to examine the effects of electricity on the properties of materials. A combined postulated theory which describes the microscopic interaction of electrical current and metals is also discussed.

**Electrical Field Effect**

Beginning in the 1950’s, the effects of electrical flow in materials have been under investigation. In 1959, Machlin et al. began examining the effect of current in NaCl and concluded that the ductility, flow stress, and yield strength were affected. Shortly thereafter, in 1969, a very detailed review of the effects of electrical current in materials was presented in a book by Nabarro. Later, Troitskii et al. examined the effects of electrical current flow in various materials and concluded that the addition of electricity influenced the flow stress of the material (1969). In 1988, Xu et al. began examining the microstructural alterations due to the addition of electrical flow. From this study, it was established that the electricity caused the grain size and the recrystallization rate to increase. In 2000, Conrad concluded that the application of short duration/high current density electrical pulses could affect the plasticity and phase transformation in materials. Most recently in 2007, Andrawes et al., through tensile testing of Al 6061, showed that the addition of electricity could reduce process energy requirements.

Hence, many researchers have been developing and examining the effects of electricity within materials, and this paper plans to increase the understanding by developing an experimental relationship between an applied electrical current and the flow stress of the material.

**Electrical Theory**

Electric current is classified as the flow of electrons through a conductive material where the flow is restricted by the resistivity of the material. When considering the resistance that limits the flow, in metals, the resistivity is determined by the atomic structure, bonding, and spacing. Though, the resistivity is also increased by the number of defects (interfacial and point) and the number of dislocations present within the material. Hence, as the dislocation density increases (i.e. material deforms) the resistance for electrical flow through the material increases. This concept of electron interaction with lattice flaws is the main driving force for the increased workability.

It is postulated that the increase in formability is caused by three main attributes as electrical current passes through a material during deformation. These include the influence of localized resistive heating, kinetic energy, and a great quantity of additional electrons in the material.

The first concept uses the idea that the electrons flowing through the material scatter off of internal lattice flaws (grain boundaries, voids, cracks, impurity atoms, etc.), thus releasing energy. This energy is released as heat within the material and can be considered localized resistive heating. This local temperature is much greater compared to the overall observed global temperature of the material. The rise in temperature creates local lattice expansion around flaws, weakens the metallic bonds, and allows for easier lattice distortion such that the dislocations are able to pass by lattice obstacles much easier. Additionally, as dislocations are also defects in the material, the electrons scatter of them as well, even while moving through the material. Thus, this heated region around the moving dislocation will also expand the lattice and weaken the surrounding bonds. The second portion is based upon the fact that, as the electricity is flowing through the material, the electrons will impact the dislocation line, thus applying a force on the dislocation. This force aids in allowing the dislocation to pass by obstacles in the lattice more easily. This concept has been noted by Kravchenko when discussing his theory of electroplasticity (1966). Last, the flow of electricity adds an abundance of extra electrons to the material’s lattice which aids in the diffusion-based bonding occurring during deformation. Specifically, the extra amount of electrons in the metal’s electron cloud will lower the bonding forces between the ion cores and their original valence electrons since there is sufficiently more. This lower bonding force lowers the required energy to break and reform bonds, hence easing dislocation motion.

In short, these three concepts collectively increase the workability of a material through improvements such as increased elongation and decreased flow stress.
PREVIOUS RESEARCH

For the development of the empirical model, uniaxial compression data was examined from previous research that examined the forgeability of Magnesium AZ31B-O when subjected to continuous electrical current (Jones, 2009). In this study, achievements of 447% increase in fracture position were observed through the utilization of EAM. Below is a brief description of the experimental setup, results, and findings from this research.

Experimental Setup

With respect to the testing, cylindrical Mg AZ31B-O specimens underwent uniaxial compression by first preloading the specimen to 222.5N to ensure sufficient contact, and then the tests were performed with varying current densities (current divided by initial cross-sectional area) at a platen speed of 25.4mm/min until specimen fracture or the limits of the fixtures were reached. With these results, they were compared to a baseline specimen (i.e. 0A/mm²) to determine the effectiveness of the electricity in increasing the forgeability.

Results

For this experimental analysis, a baseline test as well as a 10, 20, 30, and 40A/mm² test run at 25.4mm/min were examined. The results are shown in Figure 1. It can be observed that the flow stress decreased and the deformation position increased with the addition of electrical current. However, to gain significant improvement, larger current densities were required to appreciably influence the flow stress and achievable deformation. This result of a defined threshold has been noted in other studies involving compression testing with the addition of electricity (Siropis, 2009). In this study, it was proposed that the threshold exists for materials with larger grains as the energy required to push the more evenly distributed dislocations at the grain boundaries increases. Also, for the 40A/mm² test, it can be seen that the material actually exhibited a decrease in the flow stress with an increasing strain (i.e. strain weakening). To examine the visual results from this test set, Figure 2 displays the final specimen geometries. As displayed, the Baseline, 10, and 20A/mm² tests failed due to shear fracturing, where as the 30A/mm² test had significant cracking, and finally, the 40A/mm² did not fracture and reached the predetermined load limit.

With respect to the current density throughout the testing, the setup in this work did not have the capability to continuously vary the current during the test to allow for a constant density.
As a result, the current decreased as a function of specimen strain. The decrease in current density as a function of strain is displayed in Figure 3 for each electrical test.

EMPIRICAL MODELING

For the development of the electrical current and flow stress model, previously performed continuously applied stress vs. strain results was analyzed. The following subsections discuss the methodology utilized in creating the stress predictor function and the observed comparison between empirical results and actual testing.

Relationship Development

When examining the relationship between the flow stress and applied strain with the addition of current, a range of the strain equivalent to the baseline fracture strain was examined. This is shown in Figure 4 for the different current densities.

The next step was to determine the difference or ratio between the baseline and each of the electrical tests. To perform this, a ratio was determined which would relate the electrical test back to the baseline. The stress ratio with respect to strain is shown in Figure 5. After seeing the relationship for each current density, an exponential fit (dashed line) was utilized as the transfer function between the electrical test and the baseline. To better characterize the relationship, the different current density’s stress ratio converged to one at an approximate strain value of 0.018, thus an additional offset parameter in the function was included. This is mainly a result of the elastic limit of the material and fixture compliance from the machine. Likewise, the exponential decay values (0.175, 1.605, 7.431, and 17.528), were plotted against their respective nominal current density to determine a relationship between the individual current density tests. The results from this are shown in Figure 6 and a power function trend line (dashed line) was fitted to this observed relation. Using the two values (0.000075 and 3.36), these simply characterize the given material. Combining the developed relationship with the nominal mathematical descriptor in (1) is postulated to characterize the electrically-deformed material.

\[
y = 0.000075x^{3.36}
\]

Empirical Relationship

Using the methodology from the Relationship Development section, the resulting model for the upsetting flow stress is predicted using:
\[ \sigma_{\text{predicted}} = K' \epsilon^n \dot{\varepsilon}^m \exp \left( - \left( \varepsilon - C \right) \Phi \right) \]  

(2)

where, \( \sigma_{\text{predicted}} \) is the predicted flow stress with the application of electricity, \( K' \) is the strength coefficient, \( \epsilon \) is the strain, \( n \) is the strain hardening exponent, \( \dot{\varepsilon} \) is the strain rate, \( m \) is the strain rate sensitivity exponent, \( \Phi \) is the current density, and \( a, B, \) and \( C \) are material specific constants for this model. Also, when considering the current density, since the current varies as a function of strain, the current density in this model was considered to be decreasing. Neglecting barreling, the instantaneous current density during the test can be described by:

\[ \Phi = \frac{l}{A_o h_o} \]  

(3)

where, \( \Phi \) is the current density, \( l \) is the current, \( A_o \) is the initial area of the specimen, \( h_o \) is the initial height of the specimen, and \( h \) is the instantaneous height of the specimen during the testing. Thus, as the test progresses, the current density decreases as the specimen decreases in height. One important fact to note is that the current \( (l) \) during the test is assumed to be constant. However, since the current provider for this setup supplies a constant voltage, the current is dependent on the resistance of the system, simply using Ohm’s Law. As a result, the resistance of the system is dependent on the temperature, geometrical changes, and microstructural changes of the specimen. As the temperature is increased, this causes the resistance to increase due to the increased material resistivity. Also, as the specimen is compressed, the area increases and the length decreases, thus reducing the resistance. Last, as the specimen is deformed, the microstructure (number of vacant lattice sites, interstitials present, the number of dislocations, and the number of stacking faults) may increase or decrease the resistance of the material. Hence, the overall effect is not fully mapped in this research; thus the current is assumed to be constant.

Using this model, the material behavior of this magnesium alloy was then plotted for different current densities. The results are depicted in Figure 7. As shown, the empirical model predicts the relationship of the flow stress when the process is subject to a continuous current. There are some more noticeable differences in the empirical model and the actual test results at higher current densities, however, it still provides a fair approximation when considering the range of current density covered. The final material specific constants which yielded the minimum least-squares error are: \( a = 0.00009 \), \( B = 3.38 \), and \( C = 0.018 \). These values differ slightly from the derived values in the Relationship Development section, as the original values were used along with the least squares method to reduce any error present in the curve fitting (Figure 5/Figure 6). The average error for all of the curves is 5.82MPa, with the largest difference being 27.48MPa.

![FIGURE 7. COMPARISON OF EMPIRICAL MODEL FOR MAGNESIUM OVER SMALL STRAIN VALUES.](image)

**TESTING/VERIFICATION OF MODEL**

To further the ability of this predictor to map the flow stress during an EAM compressive process, the model was used to look at larger strain regions for different materials. Once again, the model results will be compared to the experimental results of the testing.

**Copper C11000**

The empirical model was tested and compared to previous experimental tests of Copper C11000 under an EAM upsetting process (Perkins, 2007). Shown in Figure 8 are the results of the experimental testing over the deformation region of the test. As can be observed, the empirical model is capable of providing a close approximation of the flow stress throughout the duration of the test for a given current density. Also, in comparison to the magnesium results presented in Figure 7,
empirical model is utilized over a larger strain area, thus proving the estimation ability of the stress predictor function. For this material, the material constants were once again fit to the data using the least squares regression to minimize the error. The values for $a$, $B$, and $C$ are 2.23, 0.00015, and 0.0, respectively. Overall, the average error present for this material is 6.81MPa, with a maximum error of 25.32MPa, occurring during the initial region of the 60.9A/mm$^2$ test.

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Again, the empirical model was tested with 304 Steel compression data (Perkins, 2007). The model and experimental results are shown in Figure 9. This test once more examines larger strains in comparison to the initial magnesium results. To determine the final constants, the least square regression was utilized and produced the values: $a = 1.46$, $B = 0.015$, and $C = 0.0$. When examining these results, the empirical relationship was only able to provide a rough estimate in comparison to the previously discussed figures which more successfully mapped the experimental results over the strain. For this material, the average error was 50.90MPa with a maximum error of 141.76MPa.

### Overall Observations

The inconsistencies, mostly in the steel model, may be a result of the simplicity of the model, such that the values $a$, $B$, and $C$ cannot fully represent the material’s behavior. Thus, a more accurate model may involve the addition and relationship of grain size or material resistivity (vacant lattice sites/ interstitial ions/ dislocation density/ stacking faults). Another aspect that should be considered is the field density throughout the specimen, thus barreling compensation (especially for larger strains) may increase the accuracy. An added factor that may also play a crucial role in modeling the flow stress is the occurrence of an electrical threshold (Perkins, 2007). This threshold causes a non-uniform stress distribution between current densities for a given strain.

### CONCLUSION

This paper developed an initial empirical relationship which provides a representation for the flow stress during an EAM compression test. The preliminary results from this study show positive results for the classification of flow stress over short and larger ranges of strain. Also, the results obtained from this study have shown and discussed the many aspects that affect the flow stress in relation to the addition of electrical current.

### FUTURE WORK

This work presented a simplistic model which was capable of modeling the relationship between stress and an applied electrical current. Future work will involve deriving an energy-based model which will also help to determine the relationship between flow stress and electrical current. This will be based on the total energy into and out of the system to determine the influence of the electrical energy on the dislocation motion. Also, the adaptation of a control system which would allow for a constant
current density to be applied to the specimen, thus isolating another variable in the model may be another approach. Moreover, the study of more materials subject to EAM processes will provide more insight to the relationship. Last, the extension of the empirical model to tensile processes will help to understand the effects of an applied current for this process.

REFERENCES


