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Robust Geotechnical Design – A New Design Perspective

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Motivation and Research Objective

In routine geotechnical engineering practice, the engineer has to work with a small sample of data due to budget constraint. Because of complexity of soil deposits, it is often difficult to determine correctly the statistics of soil parameters that are required for reliability-based design of foundations. Furthermore, the traditional reliability-based design is sensitive to the variation of noise factors such as uncertain soil parameters. To address this dilemma, the authors present a new design methodology, termed Robust Geotechnical Design (RGD). The RGD aims to make the response of a geotechnical system immune to, or robust against, the variation of noise factors by carefully adjusting the design parameters. A multi-objective optimization is performed to identify designs that satisfy all the design requirements, safety, robustness, and cost efficiency.

Framework for Robust Geotechnical Design

The robust geotechnical design (RGD) methodology is outlined with three steps shown in Figure 1. *Step 1* is to quantify the uncertainty in sample statistics and specify the design domain. *Step 2* is to evaluate the variation of system response caused by uncertain sample statistics. *Step 3* involves a multi-objective optimization to identify the Pareto Front. As shown in Figure 2, when conflicting objectives are enforced, it is likely that no single best design exists. However, a set of designs may exist that are superior to all other designs in all objectives; but within the set, none of them is superior to others in all objectives. This set of optimal designs constitutes a Pareto Front. In this paper, the Non-dominated Sorting Genetic Algorithm (NSGA-II) as shown in Figure 3 is used for establishing the Pareto Front.

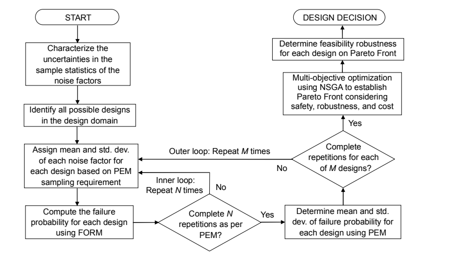


Figure 1. Flowchart illustrating robust geotechnical design

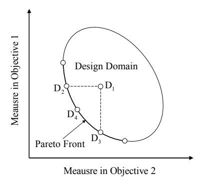


Figure 2. Illustration of Pareto Front

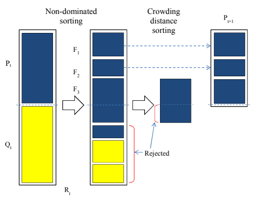
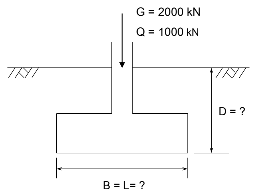


Figure 3. NSGA-II algorithm

Design Example of Shallow Foundations

An example of shallow foundation design is used to illustrate the proposed RGD approach. A square foundation, as shown in Figure 4, is to be designed to support vertical compressive loads with a permanent component of G with a COV of 10% and a transient component of Q with a COV of 18%. The soil profile at the site is a homogeneous dry sand. Ten effective friction angles are obtained from triaxial tests conducted on samples of this homogeneous sand.

The design parameters are the foundation width B and the embedment depth D. They are discretely distributed in the design domain. The ULS capacity adopts the Vesic model and SLS capacity adopts the method of normalized load-settlement curve.



Foundation Width:
$B \in \{1.0\text{m}, 1.1\text{m}, \dots, 5.0\text{m}\}$

Embedment Depth:
$D \in \{1.0\text{m}, 1.1\text{m}, \dots, 2.0\text{m}\}$

Ultimate Limit State:
$R_{ULS} = R_{Base} + R_{Side}$

Serviceability Limit State:
$R_{SLS} = \dfrac{R_{ULS}(s_i / B)}{a(s_i / B) + b}$

Figure 4. A shallow foundation design example

Characterization of Uncertainty in Sample Statistics

With a small sample of only 10 data of effective friction angles, there is uncertainty concerning the mean and standard deviation derived from this sample. In this paper, bootstrapping technique shown in Figure 5 is applied to evaluate the uncertainty of the sample mean and standard deviation.

The histograms of the sample mean and standard deviation of effective friction angles are obtained. As shown in Figure 6, COV of sample mean is estimated as 1.7% and COV of sample standard deviation is estimated as 17.9%. Similarly, the variation of the statistics of model parameters can also be obtained based on limited data from field load tests.

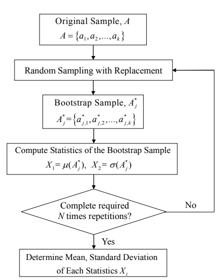


Figure 5. Illustration of bootstrap procedure

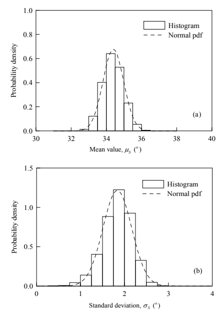


Figure 6. Histogram of sample statistics

Traditional Reliability-Based Geotechnical Design

For the shallow foundation example, if statistics of uncertain parameters are assumed with a fixed value (that is, taking only mean values of these statistics). The probability of SLS and ULS failure for each design is determined as shown in Figure 7. If the minimum cost is the only criteria for selecting the best design after screening with reliability requirements defined in Eurocode, then the design with *B = 1.9 m and D = 2.0 m* will be selected.

Table 1 shows the least cost designs that satisfy the target failure probability requirement are sensitive to the assumed statistics of noise factors. Thus, an acceptable design (for example, *B = 1.9 m and D = 2.0 m*) obtained with the traditional reliability-based design method may no longer be satisfactory if the standard deviation of noise factors is underestimated by a certain margin as shown in Table 2.

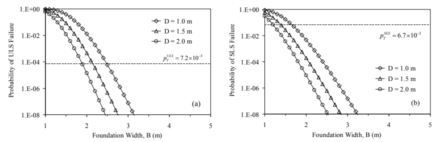


Figure 7. Probabilities of failure with fixed statistics of noise factors

Table 1. Least-cost designs under various uncertainty levels

σ_S (°)	σ_{BF}	B (m)	D (m)	Cost (USD)
1.12	0.148	1.6	2.0	769.4
1.12	0.203	1.8	1.8	910.8
1.12	0.260	1.9	2.0	1026.0
1.84	0.148	1.8	2.0	936.5
1.84	0.203	1.9	2.0	1026.0
1.84	0.260	2.1	1.9	1200.1
2.43	0.148	2.0	1.9	1104.0
2.43	0.203	2.1	2.0	1216.9
2.43	0.260	2.3	1.9	1404.0

Table 2. Failure probability of a given design under various uncertainty levels

σ_S (°)	σ_{BF}	B (m)	D (m)	p_f^{ULS}
1.12	0.148	1.9	2.0	2.01E-08
1.12	0.203	1.9	2.0	1.95E-06
1.12	0.260	1.9	2.0	4.68E-05
1.84	0.148	1.9	2.0	6.83E-06
1.84	0.203	1.9	2.0	6.36E-05
1.84	0.260	1.9	2.0	3.83E-04
2.43	0.148	1.9	2.0	1.30E-04
2.43	0.203	1.9	2.0	4.77E-04
2.43	0.260	1.9	2.0	1.50E-03

Robust Geotechnical Design of Shallow Foundations

As per the flowchart in Figure 1, the mean and standard deviation of the failure probability caused by uncertain sample statistics of noise factors can be obtained for all possible designs in the design space using point estimate method (PEM) integrated with first-order reliability method (FORM). Figure 8 shows the mean ULS failure probability for selected designs and the standard deviation of the ULS failure probability of selected acceptable designs.

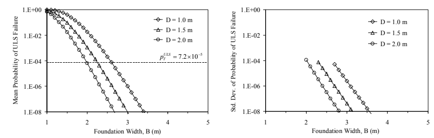


Figure 8. Mean and std. dev. of ULS failure probabilities for selected designs

The RGD approach considers safety, cost, and robustness in a design. In this study, the design is optimized with two objectives, robustness and cost, subjected to the safety constraint. In the shallow foundation design, this optimization with NSGA-II can be set up as shown in Figure 9.

For the shallow foundation example, non-dominated designs are selected into the Pareto Front, as shown in Figure 10. It can be observed that there is an obvious trade-off relationship between cost and robustness. The obtained Pareto Front can be used as a design aid for the decision maker to select the best design based on the desired target cost or robustness level.

Find $\mathbf{d} = [B, D]$

Subject to: $B \in \{1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \dots, 5.0\text{m}\}$

$D \in \{1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \dots, 2.0\text{m}\}$

$\mu_p^{ULS} < p_T^{ULS} = 7.2 \times 10^{-5}$

Objectives: Minimizing the std dev of ULS failure probability (σ_p)

Minimizing the cost for shallow foundation.

Figure 9. Set-up of optimization

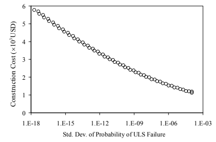


Figure 10. Pareto Front in a bi-objective space

Selection of Best Design Based on Feasibility Robustness

The Pareto Front specifies a trade-off relationship between cost and robustness. To facilitate its use, the concept of feasibility robustness is further introduced. The feasibility robustness is the degree at which a design can remain feasible (acceptable in terms of satisfying the safety and serviceability requirements) with respect to a pre-defined constraint even when its input parameters undergo variations. Symbolically, feasibility robustness can be formulated as follows:

$$\Pr[(p_f^{ULS} - p_T^{ULS}) < 0] = \Phi(\beta_\beta) \geq P_0$$

The term β_β may be used as an index of feasibility robustness. The relationship between β_β and the cost for the 62 designs on the Pareto Front is shown in Figure 11. By selecting a target feasibility robustness level, the least cost designs corresponding to different target feasibility robustness levels can be identified (Table 3).

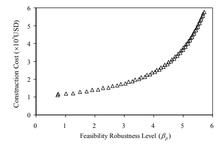


Figure 11. Cost versus feasibility robustness

Table 3. Selected final designs at various feasibility robustness levels

β_β	P_0	B (m)	D (m)	Cost (USD)
1	84.13%	2.1	1.9	1200.1
2	97.72%	2.3	2.0	1423.7
3	99.87%	2.6	2.0	1763.7
4	99.997%	3.1	2.0	2409.8

Concluding Remarks

1. A new design methodology, termed Robust Geotechnical Design (RGD), is developed in this paper. The RGD is realized through a multi-objective optimization, considering safety, robustness, and cost. The RGD is an innovative geotechnical design tool, which resolves the problems with the traditional reliability-based design.
2. The significance of the RGD methodology is demonstrated with an example of shallow foundation design.

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