Elevated Neutral-to-Earth Voltage in Distribution Systems Including Harmonics

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ELEVATED NEUTRAL-TO-EARTH VOLTAGE IN DISTRIBUTION SYSTEMS INCLUDING HARMONICS

A Dissertation
Presented to
the Graduate School of
Clemson University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Electrical Engineering

by
Jian Jiang
December 2006

Accepted by:
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Dr. Michael A. Bridgwood
Dr. John J. Komo
Dr. Hyesuk K. Lee
The elevated neutral-to-earth voltage (NEV), and the related phenomenon called stray voltage, is analyzed in multigrounded distribution systems. Elevated NEV is typically caused by fundamental frequency currents returning to the source via the neutral conductor and earth. However, harmonic distortion is also found to contribute to elevated NEV. A multiphase harmonic load flow algorithm is developed to examine the effects of various factors on the NEV, including unsymmetrical system configuration, load unbalance and harmonic injection. To fulfill this objective, the system modeling is adapted to include the neutral conductor into the component equivalent circuit. The overhead transmission line is remodeled in detail based on the Carson’s line theory. The neutral and earth return paths are represented explicitly in the model. Additionally, the harmonic analysis, embedded in the load flow algorithm, is demonstrated using a single-phase uncontrolled capacitor-filtered rectifier model.

The algorithm and the associated models are tested on an IEEE example system. The load flows are performed under different system and load conditions, including both linear and non-linear loads. The accuracy of the
developed algorithm is verified by comparing the model predictions with field measurements on real multigrounded distribution feeders.

Unbalanced loading and system asymmetry are observed to be the important source of the elevated NEV. The magnitude is shown to be a function of the earth resistivity, residual return current, feeder length and the neutral conductor size. Additionally, the harmonic injection from nonlinear loads tends to deteriorate the NEV by injecting additive triplen harmonic current into the return path. Three-phasing of single-phase laterals, a common distribution system upgrade method, is examined for its effectiveness to mitigate elevated NEV when the system has harmonic loads. As expected, it is found that three-phasing is effective when the system has low distortion. However, three-phasing is less effective for alleviating NEV when the feeder is loaded with an appreciable amount of single-phase non-linear devices.
DEDICATION

I dedicate this work to my family. This dissertation exists because of their love and support.
ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Randy Collins, for all of his assistance and guidance. His high expectations and continuous support are greatly appreciated. I am grateful to the other committee members: Dr. Michael Bridgwood, Dr. John Komo and Dr. Hyesuk Lee. Additionally, I would like to thank the Duke Energy Corporation for their support of the research that is the topic of this dissertation.
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CHAPTER  I

INTRODUCTION TO ELEVATED NEUTRAL-TO-EARTH VOLTAGE

I. Voltage Potentials In The Earth

Elevated neutral-to-earth voltage (NEV) or so-called “stray voltage” has drawn increasing attention from the general public, regulatory organizations and utilities, for both technical and legal reasons [1]. Considerable controversy presently exists for the definition and usage of the term “stray voltage” when approaching the problem from different perspectives. Another related occurrence in power system is the ground potential rise (GPR). Hence, the background about the voltage potential in the earth accompanying grounding current is discussed briefly in order to clarify the concepts in this dissertation.

The nature of grounded power systems results in the fact that the neutral conductors are not always at the zero potential with respect to the earth underneath them. The concepts are easier to explain using the three-phase multigrounded power system in Fig. 1. The neutral conductor is grounded at multiple points along the transmission line conforming to the requirement of
the National Electrical Safety Code [2]. The NESC stipulates that, to be qualified as effectively grounded, transmission lines must be grounded at least four times per mile. This is one of the reasons why it is called a multigrounded power system.

A goal of power system designers and operators is to make the three-phase power systems as nearly balanced as possible. But practical power systems, especially distribution systems, are never perfectly balanced due to the unsymmetrical system configuration, numerous single-phase loads and unsymmetrical faults in the systems. Let’s look at an example. Assume that only 60 Hz currents are drawn by the loads. Thus when the load currents $I_a, I_b, I_c$ in
Fig. 1 are not balanced, there will be a residual $I_{\text{return}}$ returning to the source. Because of the multiple paths tracing back to the source neutral, the return current $I_{\text{return}}$, which equals the negative sum of the three phase currents, will return through the neutral conductor and the earth, dividing according to their respective impedance. (In this example, parallel utilities and non-radial geometries are ignored.)

Fig. 2. A three-phase multigrounded transmission line with earth return.
Since the neutral conductors are not perfect in practical systems, the return current is shared between the neutral and the earth. Some portion of the return current is always driven into the earth every time the neutral is grounded. As a result, the earth current will generate a series of voltage potentials around the grounding point as current flows away from the ground electrode through the non-zero earth resistivity. They are shown in Fig. 2 as the concentric rings at the foot of every pole.

Fig. 3. Voltage potentials in the earth due to the earth return current.
For simplicity, the earth is assumed to be a semi-infinite media with uniform resistivity. When the ground current enters the earth through the ground electrode, voltage potentials are generated with their magnitude determined by the ground electrode geometry, earth resistivity and the distance from the measuring point to the ground electrode. The exact values of the voltage potentials can be very complicated to compute due to the complexity of the ground electrode and the earth electrical characteristics [3]. But the basic trend is that the voltage potential decreases when moving away from the ground electrode relative to the remote earth. For two points around the ground electrode, e.g., A and B in Fig. 3, a voltage exists between these two points since they are at different distances from the ground electrode.

II. Definitions And Usage

At the time this dissertation is written, there is no unanimous definition on stray voltage. Since stray voltage was initially noticed in cow milking parlors, it has been explored extensively by engineers and researchers for improving the productivity in dairy farms [4]–[7]. Authorities in agriculture and public service have provided guidelines on defining the stray voltage problems [8]–[10]. Since they are all similar to each other, only the definition by U.S. Department of
Agriculture [8] is stated in this dissertation: “Stray voltage is a small voltage (less than 10V) that can be measured between two possible contact points.”

![Diagram](image-url)

**Fig. 4. Definition of stray voltage.**

The concept is best visualized using the earth voltage potential mechanism as shown in Fig. 4. As mentioned above, the two points, A and B possess different voltage potentials because of their different distances to the ground electrode. When a person or an animal contacts these two points at the
same time, the subjected voltage is the stray voltage. When the contact points are between the person’s or the animal’s feet with a separation distance of 3 feet (without any other contact to a grounded object), it is called “step voltage” [11] as illustrated in Fig. 4. If the contact points are the hand on a grounded object and the feet on the surface (with a separation distance of roughly 3 feet (1 meter), it is called “touch voltage” [11]. (Both of these terms are normally used for situations involving fault currents.)

The neutral-to-earth voltage is the voltage measured from the neutral conductor to the remote earth. Since the neutral is solidly connected to the ground electrode, the NEV is equal to the voltage potential difference from the ground electrode to a point at infinity. The NEV is thus the maximum voltage that can be measured in the earth. Considering the physical limit of a person or an animal, the stray voltage will always be smaller than NEV. Also it is usually the case that the voltage gradient is steeper close to the electrode, which causes higher stray voltage when a person or an animal approaching the ground electrode.

Although the NEV is not the stray voltage, the knowledge of NEV is extremely important in solving stray voltage problems. By analyzing the NEV profile of power system under different conditions, it is possible to locate the source of stray voltage and develop means to mitigate the problems.
Another phenomenon in the power system related to the earth return current is the ground potential rise. The GPR usually refers to the voltage differential measured at the substations between the neutral to remote earth when the ground fault current returns through the earth, creating high voltage drop on the substation’s ground grid [12]. Thus GPR is a measure of neutral voltage only when the system is undergoing a ground fault, while the relatively small NEV/stray voltage is the term for steady state. This dissertation focus on the steady-state elevated NEV analysis; GPR is not in the scope of this research project.

Because of the unbalance in a practical system, especially a multigrounded distribution system, the question about NEV is not if the NEV exists, but what is the safe level. As mentioned earlier in this section, there is no standard on stray voltage. It is only recommend by U.S. Department of Agriculture in [8] that actions should be taken to reduce neutral to earth voltage when the NEV at the service entrance or between contact points is higher than the 2 to 4 volts range.

The stray voltage problem concerns dairy farm owners because that current will flow through the cows’ body when they are subject to a portion of the NEV. It is widely accepted to apply the recommendation in [8] to simulate cow’s body resistance using a resistor of 500 Ω. Lefcourt performed extensive investigation on the response of farm animals to different body currents [13]–[15].
He discovered that animals can perceive body currents below 0.1 mA at 60 Hz under unusual circumstances. However, animals’ body currents below 0.3 mA often have no change in their behavior while temporary behavior changes were found with currents in the range of 0.3–0.6 mA at 60 Hz. Beside the academic research on animals’ response, authorities also provide guidelines for stray voltage monitoring and troubleshooting. For example, the “level of concern”, a conservative and pre-injury level is defined to be 2 mA by Public Service Commission of Wisconsin in [16].

Table 1. Estimated effects of 60 Hz AC current.

<table>
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<th>Current (mA)</th>
<th>Effect</th>
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<tr>
<td>1 mA</td>
<td>Barely perceptible</td>
</tr>
<tr>
<td>16 mA</td>
<td>Maximum current an average man can grasp and &quot;let go&quot;</td>
</tr>
<tr>
<td>20 mA</td>
<td>Paralysis of respiratory muscles</td>
</tr>
<tr>
<td>100 mA</td>
<td>Ventricular fibrillation threshold</td>
</tr>
<tr>
<td>2 A</td>
<td>Cardiac standstill and internal organ damage</td>
</tr>
<tr>
<td>15/20 A</td>
<td>Common fuse or breaker opens circuit</td>
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The estimated effect of 60 Hz AC current on human is summarized in [17] and shown in Table 1. Contact with currents of 20 mA can be lethal. The current through the human body is dependent on the human body’s impedance. However, the body impedance varies widely at different conditions. The body impedance is especially affected by the applied voltage. Under dry conditions, the resistance of the human body can be as high as 100,000 Ω. High-voltage electrical energy quickly breaks down human skin, reducing the human body resistance to about 500 Ω. The definition for high or low voltage changes in different scenarios. For example, a voltage of 1000 V is low voltage in power transmission systems, but it is not commonplace in a typical home or workplace. For the concern of electrical hazard to human, the “safe voltage” value found in International Electrotechnical Commission (IEC) is 42.4 V (peak) for AC voltage or 60 V for DC voltage [18].

III. Concerns And Alleviation Methods

Beside the interference with dairy farm animals, the unexpected touch voltage can also be annoying to utility customers. The varying NEV in some cases may affect the performance of sensitive electronic devices if they are using the neutral voltage as reference. Buried metallic pipes can experience extra
corrosion when the high NEV is present in the proximity, especially if it has become rectified.

Historically, the terms NEV and stray voltage are applied in the literatures interchangeably. After years of exploration, engineers and researchers have pinpointed the common origins for stray voltages at power frequency and developed mitigation methods accordingly. The major sources and some of the widely applied means for solving NEV-stray voltage problems are listed below.

Sources of NEV/stray voltage:

1. Power system grounding
2. Load unbalance
3. Transformer connections
4. Neutral conductor impedance

Mitigation methods for solving NEV/stray voltage:

1. Balancing the loads
2. Three-phasing single-phase laterals
3. Increasing the neutral conductor size
4. Improving grounding connection
5. Repairing bad neutral connections and splices

All of these methods are aimed at either reducing the unbalanced returning current or increasing the return paths' conductivity in the neutral
conductors. However, none of the work mentioned above considered the effect of harmonic distortion. The proliferation of power electronic devices, especially the single-phase nonlinear load on commercial/residential circuits, has raised new concerns for the amplified NEV level related to harmonic distortion [19].

In Fig. 5, the neutral voltage with respect to the nearby ground is measured on a distribution feeder using an oscilloscope. It is clear from the neutral voltage waveform that a fair amount of 9th harmonic is riding on the crest of the fundamental neutral voltage. This result cannot be seen if a standard RMS multi-meter is used. As the harmonic load currents increase with the utilization of nonlinear devices in power systems, it is very important to identify the additional NEV elevation due to harmonic distortion and assess its effect on the NEV profile and stray voltage mitigation methods.
Among the common means for tackling stray voltage problems, the load balancing method attempts to take advantage of the equal angle separation among the three balanced phase current. But the input current of the single-phase power electronic load is rich in triplen harmonics, which are of the additive zero sequence. Hence even if the single-phase power electronic loads are perfectly balanced among three phase, they still can produce considerable return current and thus develop elevated neutral-to-earth voltages.
CHAPTER II
RESEARCH OBJECTIVES

From the discussion presented in Chapter I, it is clear that conventional techniques and tools need to be reevaluated in the presence of new sources of NEV elevation and stray voltage problems. As the most frequently performed analysis on power systems, load flow is the best technique for computing the system’s voltage profile in steady state. Direct results of the systems’ NEV using a load flow technique are highly desirable in predicting and developing mitigation methods for stray voltage problems.

The first objective of this research project is to develop an appropriate load flow algorithm and the associated power system modeling technique for NEV profile calculation related to harmonic distortion. The new load flow algorithm and modeling technique are then tested on an IEEE example system for reliability evaluation. Also field measurements on real power systems are compared with algorithm calculation to verify its accuracy.

Based on the test results, application of the load flow algorithm is discussed for predicting and troubleshooting NEV elevation due to harmonic
distortion. The three–phasing method for distribution systems upgrading is evaluated for NEV alleviation with nonlinear devices connected in the system.

This dissertation will proceed in the following steps. The modern techniques in power system modeling, load flow and harmonic analysis are reviewed in the next chapter. Then the transmission line is modeled in Chapter IV using a new approach oriented to NEV analysis. A multiphase load flow is developed in Chapter V. The single–phase rectifier is analyzed and the load flow algorithm is expanded to include harmonic analysis in Chapter VI. After that, the algorithm is tested on an IEEE example system and actual distribution feeders to demonstrate its application in NEV analysis incorporating harmonic distortion. Then conclusions are drawn and future research work is suggested.
CHAPTER III

LITERATURE REVIEW

I. Power System Modeling

Most of the component models in power systems do not have the explicit neutral conductor. In their steady state models, the neutral variables are absorbed into the phase branch equations. The neutral grounding can be simply represented by a grounding resistance from the neutral to earth, if the neutral point is provided in the actual device. Mature techniques are available for steady state simulation in various literatures [20]–[22]. One exception is the transmission line model due to magnetic mutual coupling among phase, neutral and earth. More detailed examination of conventional modeling theories is required to accurately represent the transmission line for NEV analysis.

The transmission line is one of the most important components in power systems. Since the majority of power systems in North America are three–phase multigrounded, the effect of earth return path has to be considered for accurate transmission line modeling. The current distribution in the earth has been examined extensively in the literature. Three well-known modeling methods, i.e., Carson’s line, complex depth method and finite element method, are briefly
discussed below. In 1926, J.R. Carson (from Bell Laboratories) published a monumental paper [23] describing the calculation of the transmission line impedance incorporating the earth return effect. However, Carson’s formulas do not give a closed form solution. Instead, the impedances are expressed in improper integrations that need to be expanded into infinite series. Various approximation methods have been proposed based on series truncation [20]–[21] [24]. However, improper use of these approximation methods can cause considerable error at high frequency.

An alternative approximation method was proposed by A. Deri [25] using the concept of complex depth. In this method, the extensive earth is replaced by a set of earth return conductors located underneath the overhead lines with the depth of complex value. By assuming the complex depth for the earth return conductor, the problem of adding terms in the truncation approximation is eliminated when calculating high frequency impedance. The error of complex depth method increases with the ratio of the horizontal distances between conductors to their heights. Fortunately, this ratio in most realistic systems is too small to cause any practical problems in the impedance calculation.

In both Carson’s line and the complex depth methods, the earth is assumed to be a uniform semi-infinite media with non-ideal conductivity. The finite element method [26] is applied in the detailed analysis of the earth return
current distribution in soil with irregular terrain. Also the frequency-dependent impedance of transmission line can be calculated using the finite element method. This powerful method may not be preferable due to its high cost in implementation and long calculation time.

All the methods above assume perfect ground connection from the neutral conductors to the earth. Consequently, the variables related to neutral conductors can be eliminated from the final equivalent circuit. The earth impedance is first absorbed into the aerial conductors’ impedances according to KCL. With all of the voltages referring to the remote earth, the neutral voltage is always zero due to the ideal connection to earth. The neutral voltage equation is then eliminated by the Kron reduction method.

However, the current and voltage relative to earth of the neutral conductor is the goal of NEV analysis. The otherwise preferable elimination of neutral conductor equation is not desirable in NEV analysis. Furthermore, the non-ideal conductive earth presents impedance to the current flowing from neutral conductors to the earth. Recent works [27]–[28] have shown that improper application of these transmission line models can lead to serious error in transmission line impedance calculations.

Based on the above observations, a transmission line model needs to be developed dedicated for NEV analysis. In the new transmission line model, the
neutral conductor is represented explicitly for direct determination of the neutral-to-earth voltage. Since the Carson’s line is the standard method for transmission line modeling among power engineers, it will be applied as the foundation for the new model derivation.

II. Multiphase Load Flow

Load flow is the technique used in planning the future expansion of power systems as well as in determining the best operation of existing systems in steady state. The principle information obtained from a load flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line [22]. The first practical method for load flow calculation was formulated by Ward and Hale [29] in 1956. Since then the load flow techniques have been studied and documented extensively. For well-behaved systems like large scale transmission systems, the Newton-Raphson and fast decoupled load flow and their derivatives have been proven over years of successful application to be the most efficient solution techniques.

However these load flow methods fail when they are applied to ill-conditioned power systems. The distribution networks fall in the category of ill-conditioned systems for the following features found in the typical distribution system:
• Radial or near radial (weakly meshed) structure
• High R/X ratio
• Multiple phasing, unbalanced operation
• Unbalanced distributed loads
• High ratio of long-to-short line reactance of lines terminating on the same bus in rural areas

Special solution techniques dedicated for distribution systems load flow calculation have been developed by exploiting the radial structure of the distribution circuit. These different algorithms can be categorized into two basic types: Backward/forward sweep method and $Z_{bus}$ load flow method.

In the first category, the load flow algorithms are based on ladder network theory [30]–[32]. These methods take advantage of the radial nature of distribution system that the source reaches any node in the network via a unique path. The methods consist of two basic steps, i.e., backward sweep and forward sweep, which are repeated until convergence is achieved. The backward sweep is primarily a summation of currents or power tapped along the distribution feeders. The forward update is primarily a voltage drop calculation accompanied by the nodal voltage update.

In 1967, Berg et al. [30] presented a ladder theory based load flow algorithm which can be considered the start of the all of the backward/forward
sweep methods that followed. In this method, the driving point impedances are calculated from the last bus to the source, and are applied to update the currents and voltage forward from source to the last bus.

Among the variants developed over the years, the algorithm by Shirmohammadi et al. [33] is more intuitive to understand and implement. This method was initially proposed for single-phase load flow based on current calculation and expanded to power calculation [34] and three-phase load flow [35] later. The method starts with a flat voltage profile. Then the currents or powers are collected backward from the last bus to the source. After that the voltage drops are calculated forward from the source to the last bus, followed by the new voltage update.

The load flow solutions in the second category are the so-called $Z_{bus}$ load flow [36]–[37]. These methods use the sparse factorized $Y_{bus}$ and equivalent current injections to perform the load flow calculations. The $Z_{bus}$ method is based on the principle of superposition applied to the system bus voltages: the voltage of each bus is considered to arise from two contributions, the slack bus voltage and the equivalent current injections. The loads, cogenerators, line charging capacitors and any shunt elements are considered as current injections. The basic solution is outlined in the following steps:

- Take an initial guess on network voltage profile
- Optimally order and factorize $Y_{bus}$
- Compute equivalent current injections
- Compute voltage deviation due to current injections using the factorized $Y_{bus}$
- Update bus voltage
- Repeat the process until convergence is achieved
- Calculate power flow, current flow and system loss

The load flow techniques in both categories are tailored specifically for radial or weakly meshed systems. The experience of applying these techniques in distribution networks has shown different performance in different networks [38]. However, the neutral variables and earth return current are not available directly in both methods. A load flow algorithm dedicated for NEV analysis is required to analyze the neutral and grounding circuit. The backward/forward sweep method is applied as the basis for the new algorithm for its ease on implementation and efficiency in data storage.

III. Harmonic Analysis

Accompanying the increase of nonlinear devices in power system at various voltage levels, considerable progress has been achieved over the last two decades in harmonic analysis. Various methods have been developed to examine
the power system response to harmonic distortion, which can be classified in
three types.

The first step in harmonic analysis is to model the nonlinear devices by
computing their harmonic current spectrum as a function of the terminal voltage
and the nonlinear characteristics. Mature techniques are available to represent
the nonlinear devices for different requirement of details [39].

The simplest and most commonly used harmonic analysis technique is the
frequency scan [40]. It calculates the system response at a particular bus by
injecting harmonic current into the system at this bus and computing the voltage
response. Usually it is repeated within a range of frequencies. The voltage
responses are plotted vs. the corresponding frequencies to detect the possible
harmonic resonance at the buses of interest. It has been widely used in filter
design.

The second type harmonic analysis is the harmonic penetration study
which assume no harmonic interaction between the network and the nonlinear
devices [41]. The fundamental frequency load flow is performed by representing
the nonlinear devices as constant power loads. The fundamental bus voltages
obtained are used to determine harmonic currents from the nonlinear devices.
Finally, the harmonic bus voltages are calculated by injecting the harmonic
currents into the system.
Iterative harmonic load flow is the most comprehensive and accurate harmonic analysis technique. The harmonic interaction is included in nonlinear device models by expressing the harmonic current as a function of terminal voltage at all harmonic frequency of interest. The load flow calculations at harmonic frequencies are carried out similar to the fundamental frequency load flow. Convergence is then checked for all frequencies.

The conventional Newton-like load flow techniques [42] [43] have been developed to solve the harmonic–distorted system by expanding the fundamental frequency load flow calculation to harmonic frequencies. The same problems mentioned in last section will occur when these methods are applied to the ill-conditioned distribution systems. Instead of changing the Newton load flow techniques to suit the radial structure, a harmonic multiphase load flow algorithm is developed to expand the backward/forward sweep to harmonic frequencies.

IV. Summary

Various techniques have been developed for steady state modeling, load flow calculation and harmonic analysis. But NEV in an unbalanced distribution system with nonlinear devices is not directly available using the present analysis methods. Thus a multiphase harmonic load flow algorithm is developed in this
dissertation for NEV analysis. A multigrounded distribution line model is derived first in the next chapter.
I. Carson’s Line

A typical Carson’s line model is depicted in Fig. 6 for a section of a single-phase feeder. The neutral conductor runs parallel with the phase conductor and both conductors are ideally grounded at the receiving end. The branch currents, including the earth return current, satisfy the KCL, i.e. \( I_a + I_n + I_g = 0 \). The circuit loop voltage equation is

\[
\begin{bmatrix}
V_{aa} + V_{g's} \\
V_{nn} + V_{g's}
\end{bmatrix} =
\begin{bmatrix}
Z_{aa} & Z_{an} \\
Z_{an} & Z_{nn}
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_n
\end{bmatrix}
\] (1)

The self loop impedance \((Z_{aa} \text{ and } Z_{nn})\) in (1) is defined as the ratio of the voltage drop along the loop (as indicated in Fig. 6) to the current flowing through the conductor and returning via the earth. Similarly, the mutual loop impedance \((Z_{an} = Z_{na})\) is the ratio of voltage drop along one loop to the current flowing in the other loop.
Fig. 6. Carson’s line for single phase feeder.

For the general geometrical configuration of multiple aerial conductors shown in Fig. 7, Carson developed formulas to determine the self and mutual loop impedances. The conductors $a'$ and $b'$ are the images of the actual conductors $a$ and $b$, respectively. The earth resistivity is assumed to be uniform in the semi-infinite field underneath the earth surface. Originally, Carson derived the impedances using the c. g. s. (centimeter, gram and second) unit system. All distances were in centimeters and the resulting impedances in abohms/cm. The prefix “$ab$” means a multiplier $10^9$ in the SI system.
Fig. 7. Line geometrical spacing for two parallel conductors (a and b) with earth return.

The self and mutual impedances for conductor $a$ are given below. The corresponding impedances for conductor $b$ can be obtained similarly.

$$Z_{aa-g} = z + j2\omega \ln\left(\frac{S_{aa}}{r_a}\right) + 4\omega \int_0^\infty \left(\sqrt{\mu^2 + j - \mu}\right) e^{-2h_\mu} d\mu \text{ abohms / cm}$$ (2)

$$Z_{ab-g} = j2\omega \ln\left(\frac{S_{ab}}{S_{ab}}\right) + 4\omega \int_0^\infty \left(\sqrt{\mu^2 + j - \mu}\right) e^{-2(h_\mu + h_\mu)} \cos x_{ab} d\mu \text{ abohms / cm}$$ (3)

where
\[ z = r + j \omega L_{\text{int}} = r + j \frac{\alpha}{2}, \text{ conductor internal impedance in } abohms/cm \]

\[ r \text{ conductor intrinsic resistance in } abohms/cm \]

\[ r_a \text{ conductor } a \text{ radius in centimeters} \]

\[ S \text{ distance from conductors to image conductors in centimeters} \]

\[ s \text{ distance between conductors in centimeters} \]

\[ h \text{ height from conductor to earth surface in centimeters} \]

\[ x \text{ horizontal distance between conductors in centimeters} \]

\[ h' = x\sqrt{\alpha}, \text{ dimensionless} \]

\[ x' = h\sqrt{\alpha}, \text{ dimensionless} \]

\[ \alpha = \omega \mu \sigma = 4\pi \sigma \omega \text{ in } cm^{-2} \text{ for } \mu = 4\pi \times 10^{-7} H/m = 4\pi \text{ abhenries/cm} \]

\( (\mu \text{ in this case represents the earth permeability}) \)

\[ \sigma \text{ earth conductivity in } abmho/cm \]

\[ \mu \text{ integration variable} \]

In both the self and mutual impedance formulas (2) and (3), the terms before the improper integrals represent the corresponding impedances when the earth is a perfect conductor. The integrals account for the effect of non–ideal earth conductivity on the self and mutual impedances, respectively.

For the self impedance, the first two terms can be combined together as follows:
\[ z + j2\omega \ln \frac{S_{aa}}{r_a} = r + j2\omega \frac{1}{2} + j2\omega \ln \frac{S_{aa}}{r_a} \]

\[ = r + j2\omega \left( \frac{1}{4} + \ln \frac{S_{aa}}{r_a} \right) \]

\[ = r + j2\omega \ln \frac{S_{aa}}{r_a} \text{ } \text{abohms/cm} \quad (4) \]

where

\[ r_a' = r_a e^{-\frac{1}{4}} \text{ cm} \quad (5) \]

The relation in (5) is actually the definition of Geometric Mean Radius (GMR) of solid cylinder conductors. The values for \( r \) and \( r_a' \) can be found in manufacturers' datasheets for standard conductors.

Next Carson solved the improper integrals in terms of infinite power series. Because of the similarity between the two integrals in the self and mutual impedances, a uniform solution was derived for the two integrals and the corresponding value can be evaluated by changing the values for the following two parameters accordingly.

\[ k = \begin{cases} S_{aa} \cdot \sqrt{\alpha} & \text{in the self impedance (dimensionless)} \\ S_{ab} \cdot \sqrt{\alpha} & \text{in the mutual impedance (dimensionless)} \end{cases} \quad (6) \]

\[ \theta = \begin{cases} 0 & \text{in the self impedance} \\ \theta_{ab} & \text{in the mutual impedance (radians)} \end{cases} \quad (7) \]
The improper integrals can be evaluated as follows:

$$4\omega\int_{0}^{\infty} \left( \sqrt{\mu^2 + j - \mu} \right) e^{-2kh_\omega \mu} d\mu = 4\omega \left( P + jQ \right) \text{ abohms} / \text{cm} \quad (8)$$

where

$$P = \frac{\pi}{8} - \frac{1}{3\sqrt{2}} \left( k \cos \theta \right) + \frac{k^2}{16} \left( 0.6728 + \ln \frac{2}{k} \right) \cos 2\theta$$

$$+ \frac{k^2 \cos 3\theta}{45\sqrt{2}} - \frac{\pi k^4 \cos 4\theta}{1536} \text{ abhenries} / \text{cm} \quad (9)$$

$$Q = -0.0386 + \frac{1}{2} \ln \frac{2}{k} + \frac{k \cos \theta}{3\sqrt{2}} - \frac{\pi k^2 \cos 2\theta}{64} + \frac{k^3 \cos 3\theta}{45\sqrt{2}}$$

$$- \frac{k^4 \theta \sin 4\theta}{384} - \frac{k^4 \cos 4\theta}{384} \left( \ln \frac{2}{k} + 1.0895 \right) \text{ abhenries} / \text{cm} \quad (10)$$

Note that $\mu$ in (8) is an integration variable, not the permeability. Since Carson solved the electromagnetic equations in terms of summation of infinite power series, truncation is required for engineering applications. Additionally, the unit system is cumbersome for power engineers, although it may be convenient in physics research area. Clarke [20] presented a very good approximation to the original solution including units transform for power systems analysis. The geometric parameters for conductors’ spacing and size are to be specified in feet conforming to the common practice among electrical
utilities in North America. Also, the calculated impedances need to be expressed in ohms per mile.

The change on geometric specification (i.e., c.g.s. units to conventional units) will not affect the terms before the improper integrals in (2) and (3), which give the values of self and mutual impedances assuming perfect earth conduction. As long as the units for spacing and conductor size are all the same, the logarithm values will not change. And the conductor intrinsic resistance $r$ is usually provided by manufactures in ohms per mile.

However, the new units of geometric parameters will affect the values for $k$, which consequently changes the earth return impedances in self and mutual impedances. If the earth conductivity $\sigma$ is replaced by $10^{-11}/\rho$ with $\rho$ equal the earth resistivity in $\Omega \cdot m$, then value of $k$ for self impedance equals

$$k = S_{aa} \times 30.48 \times \sqrt{\alpha}$$

$$= S_{aa} \times 30.48 \times \sqrt{\frac{4\pi \times 2\pi f \times 10^{-11}}{\rho}}$$

$$= 1.713 \times 10^{-3} \times \frac{S_{aa}}{2} \sqrt{\frac{f}{\rho}}$$

where

$S_{aa}$ (centimeters) = $S_{aa}$ (feet) $\times$ 30.48
The $k$ in mutual impedance can be evaluated similarly by changing $S_{aa}$ to $S_{ab}$. Since $1 \text{aboohms/cm} \cong 1.6093 \times 10^{-4} \ \Omega/\text{mi}$, the self and mutual impedance can be expressed in ohms per mile by introducing the constant $G = 1.6093 \times 10^{-4}$ as follows

$$Z_{aa-g} = r + \left[ j2\omega \ln \left( \frac{S_{aa}}{r_a^2} \right) + 4\omega (P + jQ) \right] G$$

$$= \left( r + j2\omega G \ln \frac{S_{aa}}{GMR_a} \right) + \left( 4G\omega P + j4G\omega Q \right) \ \Omega/\text{mi} \quad (11)$$

$$Z_{ab-g} = \left[ j2\omega \ln \left( \frac{S_{ab}}{s_{aa}} \right) + 4\omega (P + jQ) \right] G$$

$$= 4\omega PG + j \left[ 2\omega \ln \left( \frac{S_{ab}}{s_{aa}} \right) + 4\omega Q \right] G \ \Omega/\text{mi} \quad (12)$$

Note that names for $P$ and $Q$ are exactly the same for self and mutual impedance, but their values are different since they are calculated separately using corresponding $k$ and $\theta$ parameters according to (6) and (7). The letter $g$ in the subscripts will be dropped in the following derivation for simplicity.

Clarke pointed out in [20] that the power series for $P$ and $Q$ evaluation converge rapidly at the fundamental frequency and the order of four in series truncation will be sufficient for accuracy in the self and mutual impedance calculation. It is also shown in [44] that the truncation error with order of eight in
Carson’s model is only appreciable at frequency starting at 40 kHz, which is out of the range for steady state harmonic analysis. Thus the fourth order truncation suggested by Clarke is applied in this dissertation for transmission line modeling.

II. Practical Feeders with Non-Zero Grounding Resistance

For the self impedance $Z_{aa}$, the first two terms represent the line impedance if the earth is a perfect conductor, while the last term results from the non-zero earth resistivity. As mentioned at the beginning of this section, the Carson’s line model assumes that all the aerial conductors are perfectly connected to earth. Consequently these terms can be added together because the same current flowing through the aerial conductor returns through the earth. Same principle also applies to the mutual impedance $Z_{ab}$.

This assumption works well in transmission system analysis since the load currents in a transmission system are in most cases balanced and sinusoidal. The effect of neutral current can be ignored without losing generality. However, in distribution systems, the load currents can be quite unbalanced. Also the finite earth conductivity introduces grounding resistance to the portion of the current returning to the source through the ground electrode into the earth. A more general configuration is shown in Fig. 8 for a section of single phase feeder.
It should be noticed that the finite earth conductivity has a two-fold effect on the transmission line impedance [3]. In the proximity of the ground electrode, the electric field is predominant and the impedance against the returning current is mostly resistive. After entering the earth, the current distributes over the extensive field and the magnetic effect is the major factor determining the impedance along the path, which is described by the Carson’s model.

In Fig. 8, the ground electrodes are represented by the lumped resistance $R_g$ at the ends of the transmission line. The value of $R_g$ is determined by the
earth resistivity and the geometric configuration of grounding electrodes. Various works have been published to examine the calculation of $R_g$. Equations for some simple configurations [45] are listed in Table 2.

The Carson’s line model cannot be applied in the equivalent circuit in Fig. 8 due to the shunt branches at the terminals. It would be intuitive to decompose the loop equations into branch equations in order to add the shunt grounding branches into the model. However there are various ways to disassemble the loop impedances depending on the type of analysis. To avoid this uncertainty, an alternative method is developed by taking advantage of the basic circuit constraints while keeping the loop impedances in their entirety.
Table 2. Simple Grounding Electrodes Resistance [45].

<table>
<thead>
<tr>
<th>Electrode Type</th>
<th>Resistance Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemisphere</td>
<td>$R = \frac{\rho}{2\pi r}$</td>
</tr>
<tr>
<td>Disk</td>
<td>$R = \frac{\rho}{8r} + \frac{\rho}{8\pi} \left(1 - \frac{7r^2}{12(2z)^2} + \frac{33r^4}{40(2z)^4} + \ldots\right)$</td>
</tr>
<tr>
<td>Rod</td>
<td>$R = \frac{\rho}{2\pi L} \ln \left(\frac{4L}{r} - 1\right)$</td>
</tr>
<tr>
<td>Ring</td>
<td>$R = \frac{\rho}{4\pi^2 r} \left(\ln \frac{16r}{a} + \ln \frac{4r}{z}\right)$</td>
</tr>
<tr>
<td>Strip</td>
<td>$R = \frac{\rho}{4\pi L} \left(\ln \frac{4L}{a} + \frac{a^2 - \pi ab}{2(a + b)^2} + \ln \frac{2L}{z} - 1 + \frac{z}{L} \left(\frac{(2z)^2}{16L^2} + \frac{(2z)^4}{512L^4} + \ldots\right)\right)$</td>
</tr>
<tr>
<td>Buried Wire</td>
<td>$R = \frac{\rho}{4\pi L} \left(\ln \frac{4L}{r} + \ln \frac{2L}{z} - 2 + \frac{z}{L} \left(\frac{(2z)^2}{16L^2} + \frac{(2z)^4}{512L^4} + \ldots\right)\right)$</td>
</tr>
</tbody>
</table>
For the circuit in Fig. 6, the only assumption for the Carson’s model is that the branch currents have to satisfy the KCL, i.e. \( I_a + I_n + I_g = 0 \). Also the loop voltage drops can be rearranged as below

\[
V_{aa'} + V_{g,g} = V_a - V_{a'} + V_g - V_{g} = V_a - V_{g} - (V_{a'} - V_{g'}) = V_{ag} - V_{a'g'}.
\]  
\[
V_{nn'} + V_{g,g} = V_n - V_{n'} + V_g - V_{g} = V_n - V_{g} - (V_{n'} - V_{g'}) = V_{ng} - V_{n'g'}.
\]  

It is obvious that the loop voltage drop can interpreted as the voltage drop across the corresponding aerial branch within the loop if all the terminal voltages are referred to their own local earth.

The same interpretation also applies to the circuit in Fig. 8 after referring the node voltages to their local earth. In addition, the KCL holds for the branch currents within the dashed rectangle. Although the exact current division is yet unknown, the sum of the currents in the neutral conductor and the earth has to equal the negative phase current to complete the circuit.

Thus for the feeder shown in Fig. 8, all the basic constraints in the Carson’s line model are satisfied for the circuit inside of dashed rectangle independent of the terminal connection. Using the same voltage reference, the ground electrode can be represented as a shunt branch with admittance.
\[ [R]^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_g} \end{bmatrix} \] (15)

![Equivalent model for a multigrounded feeder](image)

Fig. 9. Equivalent model for a multigrounded feeder.

The complete model of the feeder including the grounding resistance is shown in Fig. 9. The series impedance \([Z_{se}]\) calculated from the Carson’s line model can be directly applied in the new model.

It is required by National Electrical Safety Code (NESC) that overhead lines must be grounded at least four times per mile to be qualified as effectively grounded [2]. The grounding resistance is not specified in NESC for multigrounded systems. A standard ground rod is 10 feet long with diameter of
5/8 inches. For a single ground rod driven vertically into the earth, the grounding resistance according to Table 2 is about 25 Ω with earth resistivity of 100 Ω·m. This resistance value will be applied in this dissertation unless otherwise specified.

According to the NESC grounding requirement, a practical distribution feeder may possess multiple Π segments. A single phase multigrounded feeder with two segments is illustrated in Fig. 10.

![Fig. 10. A single phase multigrounded feeder.](image-url)
The equivalent circuit in Fig. 10 represents two Π segments in series. The series admittance matrix \([Y]_i\) is calculated as the inverse the \([Z]_{se,i}\) for that segment. In power system analysis, the distributed loads connected to the transmission lines are usually aggregated at the nodes of interest. Thus for each feeder, the nodes in the middle need to be eliminated from the final equivalent circuit.

The Kron reduction method can be applied to simplify the above equivalent circuit with multiple Π segments. The procedure is illustrated for the feeder in Fig. 10. The admittance matrix \([Y]_{bus}\), including the middle node 2, is developed first using the admittance matrix assembling scheme [22]. For simplicity, the resistance of each ground electrode is assumed to have the same value. The resulting admittance matrix is given below.

\[
[Y]_{bus} = \begin{bmatrix}
[Y]_1 + [R]^{-1} & -[Y]_1 & 0 \\
-[Y]_1 & [Y]_1 + [Y]_2 + [R]^{-1} & -[Y]_2 \\
0 & -[Y]_2 & [Y]_2 + [R]^{-1}
\end{bmatrix}
\]  \hspace{1cm} (16)

Note that the dimension of the submatrixes, \([Y]_i\), \([R]^{-1}\) is two by two. The 0 terms represent a null matrix of the same dimension.

The nodal current injections are related to the node voltages as
\[
\begin{bmatrix}
[I_1] \\
[I_2] \\
[I_3]
\end{bmatrix} =
Y_{bus}
\begin{bmatrix}
[V_1] \\
[V_2] \\
[V_3]
\end{bmatrix}
\] (17)

where

\[
[I_i] =
\begin{bmatrix}
I_{axi} \\
I_{ai}
\end{bmatrix}
\text{ external current injections}
\]

\[
[V_i] =
\begin{bmatrix}
V_{axi} \\
V_{ai}
\end{bmatrix}
\text{ node voltages referred to local earth}
\]

As the loads are aggregated at the feeder terminals, the external current injections at the middle node are zero. Kron reduction is applied to \([Y_{bus}]\) by solving the second equation in (17) for \([V_2]\) and substitution in the first and the third equations. The new admittance matrix takes form of

\[
[Y_{bus-new}] =
\begin{bmatrix}
[A] & [B] \\
[B] & [A]
\end{bmatrix}
\] (18)

According to the admittance matrix assembling scheme, the off-diagonal terms equal the negative admittance connecting the corresponding nodes and the diagonal terms equal the sum of all admittance originating from that node. The resulting equivalent circuit is shown in Fig. 11.
The derivation for single phase feeder can be easily expanded to a three phase feeder by adapting the corresponding submatrixes to appropriate dimensions using the Carson’s line formulas. Hence, the model for transmission lines with multiple-grounds has been developed. The model in Fig. 11 is similar to the regular Π equivalent circuit, which makes it possible to be applied in various load flow algorithms. The current flowing in the neutral conductors and the neutral-to-earth voltage are represented explicitly for NEV analysis in multigrounded distribution systems.
CHAPTER V

MULTIPHASE LOAD FLOW FOR NEV ANALYSIS

The three phase load flow algorithm for distribution systems [35] is revised in this dissertation to analyze the NEV in an unbalanced network with various phasing configurations. Since the objective of this document is to determine the neutral to earth voltage, some adjustments are required during the application of the three phase load flow algorithm. The neutral conductor needs to be included in the load flow formulation in order to directly obtain information related to the network neutral conductors and the currents through earth. Also, the current division between the neutral conductor and the earth needs to be addressed for the stability of the load flow calculation. For simplicity, only the radial distribution network is discussed in this dissertation. The algorithm can be easily extended to a radial network with a few loops using the compensation theory in [34].

Since the load flow method is branch oriented, there is no need to construct either the nodal admittance matrix $Y_{bus}$ or the nodal impedance matrix $Z_{bus}$. Using the models developed in the previous chapter, the parameters for
each branch, including the transformers, in a given distribution system are calculated. The series primitive impedance matrixes are stored corresponding to the branch and the shunt admittance matrixes from neighboring branches are aggregated at each node.

To proceed in a branch-oriented load flow, the branches and nodes need to be numbered to describe the radial topology of the distribution systems. The procedure is better understood by using the following example network shown in Fig. 12. The source, usually representing the substation, is denoted as the root node, or node 0. The two nodes of each branch are labeled as $L_1$ and $L_2$, respectively, where the node closer to the root node is $L_1$ and the other node is $L_2$. The labeling procedure is shown in Fig. 12 for the several branches.

All of the branches within the network will be numbered in layers. The first layer consists of the branches directly connected with the root node. The branches in the first layer are numbered one by one. (Note $L_1$ and $L_2$ denote the node names and should not be confused with the layer number.) In the meantime, the $L_2$ node of the corresponding branch is assigned with a node number same as the branch number. Similarly, the next layer is composed of the branches whose $L_1$ node is connected to $L_2$ node of any branch in the first layer. The same procedure is carried out until all branches and nodes are numbered. The final result is depicted in Fig. 13.
Fig. 12. Topology plot of a radial network showing labeling hierarchy.

Fig. 13. The numbered radial distribution network.
The initial guess of the network static state starts the load flow calculation. The root node is chosen as the slack bus. If it happens to be the secondary terminal of the substation transformer, the tap setting is assumed for the root node phase voltages. Usually the voltages at the substation are well balanced and the neutral to earth voltage at the substation can be ignored as a result of the low grounding resistance there. For the rest of the nodes in the network, a flat voltage profile is assumed as the initial guess with the initial NEVs set to zero.

Note that the dimension of voltage vector for each node is four by one, even when none of the branches connected to the node is three-phase, four-wire. In the practical programming, a voltage vector with uniform dimension is easier to implement. A special indexing mechanism is required if using a vector with exact correspondence to the node phasing. The voltage for the non-existing phase or neutral conductor will follow the voltage of the node directly connected it and one layer higher. It will be shown that this arrangement does not affect the load flow results.

The iterative load flow algorithm consists of three steps. In iteration $k$,

1. **Node current injection.**

   The node current injections are calculated as function of node voltages. The loads at node $i$ can be represented as constant power, constant current and constant impedance. At node $i$,
\[
\begin{bmatrix}
I_{i,a} \\
I_{i,b} \\
I_{i,c}
\end{bmatrix}^k = \begin{bmatrix}
S_{i,a} \\
S_{i,b} \\
S_{i,c}
\end{bmatrix} \begin{bmatrix}
V_{i,a}^{(k-1)} - V_{i,n}^{(k-1)} \\
V_{i,b}^{(k-1)} - V_{i,n}^{(k-1)} \\
V_{i,c}^{(k-1)} - V_{i,n}^{(k-1)}
\end{bmatrix}^* - \begin{bmatrix}
Y_{i,a}^* \\
Y_{i,b}^* \\
Y_{i,c}^*
\end{bmatrix} \begin{bmatrix}
V_{i,a}^{(k-1)} - V_{i,n}^{(k-1)} \\
V_{i,b}^{(k-1)} - V_{i,n}^{(k-1)} \\
V_{i,c}^{(k-1)} - V_{i,n}^{(k-1)}
\end{bmatrix}
\]

+ \begin{bmatrix}
I_{i,a\_load} \\
I_{i,b\_load} \\
I_{i,c\_load}
\end{bmatrix}
\]

(19)

where

\( I_{i,a}, I_{i,b}, I_{i,c} \) are the total phase current injections using the generator convention,

\( S_{i,a}, S_{i,b}, S_{i,c} \) are the scheduled complex power injection including the load demand and the power delivery from the distributed generators,

\( V_{i,a}, V_{i,b}, V_{i,c}, V_{i,n} \) are the phase and/or neutral voltages referring to the local earth,

\( Y_{i,a}, Y_{i,b}, Y_{i,c} \) are the admittances of all shunt elements including the shunt capacitor, constant load impedance and any shunt branch in the branch equivalent circuit,

\( I_{i,a\_load}, I_{i,b\_load}, I_{i,c\_load} \) are the scheduled constant current loads.
2. Backward collection to obtain the branch current.

Starting from the outmost branch, calculate the branch current by summing the branch currents from lower layers if exist, plus the node current injection at the \( L_2 \) node of this branch. For branch \( L \),

\[
\begin{bmatrix}
I_{a}^L \\
I_{b}^L \\
I_{c}^L \\
\end{bmatrix}^k = -\begin{bmatrix}
I_{i,a}^L \\
I_{i,b}^L \\
I_{i,c}^L \\
\end{bmatrix}^k + \sum_{x \in X} \begin{bmatrix}
I_{x}^a \\
I_{x}^b \\
I_{x}^c \\
\end{bmatrix}^k
\]

(20)

where

\( X \) \quad \text{the set of branches whose } L_1 \text{ nodes are directly connected to the } L_2 \text{ of branch } L .

The first term in (20) is the local injections determined by (19). The negative sign results from the branch current reference directions where a current flowing from source to load is assumed positive.

Due to the shunt grounding branch, a portion of the return injection currents flow through \( Y_{sh} \). The detail of the current division will be discussed later in this chapter. It can be just assumed that the neutral conductor current in each branch has been collected and corrected to account for the current division between neutral conductor and earth return path.
3. Forward node voltage update.

Starting from the layer right below the root node, the voltage drop across each branch can be determined using the calculated branch currents. For branch \(L\), assume its \(L_1\) and \(L_2\) nodes equal \(j\) and \(i\), respectively. Then the node \(i\) voltages are updated as

\[
\begin{bmatrix} V_{i,a}^{(k)} \\ V_{i,b}^{(k)} \\ V_{i,c}^{(k)} \\ V_{i,n}^{(k)} \end{bmatrix} = \begin{bmatrix} V_{j,a}^{(k)} \\ V_{j,b}^{(k)} \\ V_{j,c}^{(k)} \end{bmatrix} - \begin{bmatrix} Z_{aa}^{L} & Z_{ab}^{L} & Z_{ac}^{L} & Z_{an}^{L} \\ & Z_{bb}^{L} & Z_{bc}^{L} & Z_{bn}^{L} \\ & & Z_{cc}^{L} & I_{cn}^{L} \\ & & & Z_{nn}^{L} \end{bmatrix} \begin{bmatrix} I_{a}^{L} \\ I_{b}^{L} \\ I_{c}^{L} \\ I_{n}^{L} \end{bmatrix}
\]

(21)

After the voltages are updated at all nodes, a convergence check is performed. Since constant power is not the only type of load in the system, the voltage error in (22) is checked instead of the usual power mismatch criterion. The maximum error of 0.001 p.u. is applied in this dissertation as the convergence criterion.

\[
\begin{bmatrix} \Delta V_{i,a}^{(k)} \\ \Delta V_{i,b}^{(k)} \\ \Delta V_{i,c}^{(k)} \\ \Delta V_{i,n}^{(k)} \end{bmatrix} = \begin{bmatrix} V_{i,a}^{(k)} \\ V_{i,b}^{(k)} \\ V_{i,c}^{(k)} \\ V_{i,n}^{(k)} \end{bmatrix} - \begin{bmatrix} V_{i,a}^{(k-1)} \\ V_{i,b}^{(k-1)} \\ V_{i,c}^{(k-1)} \\ V_{i,n}^{(k-1)} \end{bmatrix}
\]

(22)

As mentioned in step 2, the current division between the neutral conductor and the earth needs to be determined for correct neutral branch...
current $I^k_n$. A residual current $I_{\text{residual}}$ is defined for each branch, which is sum of the neutral injection at node $L_2$ of this branch and the sum of all the neutral branch currents from lower layers. The value of $I_{\text{residual}}$ can be determined by applying (20) to the neutral conductors. The residual current will return to the source via the neutral conductor and the earth path. The concept is illustrated for a single phase transmission line in Fig. 14. Note that the direction of $I_g$ is opposite to that in Carson’s model.

Fig. 14. Residual current division between neutral and earth.
An intuitive method to determine the neutral branch current \( I_n \) is to exploit the constraint that the voltage drop of the shunt grounding branch \( V_{n'g} \) results from the current flowing through it. With all voltage referred to local earth,

\[
I_g = \frac{V_{n'}}{R_g} \tag{23}
\]

\[
I_n = I_{\text{residual}} + I_g \tag{24}
\]

The current \( I_g \) in each shunt branch is first calculated using (23) and the neutral current \( I_n \) is obtained by (24). This neutral current is then inserted into step two of the iteration as a part of the results of the backward branch currents collection. However, unlike the phase currents, the earth current is less constrained by the network topology and load demand except for the calculated neutral voltage. Thus \( V_{n'} \) can converge to an incorrect value or fail to converge at all.

An improved means to ensure proper convergence is to take advantage of branch voltage equation and the actual circuit connection. In Fig. 14, the voltage drop across the neutral conductor is given as

\[
V_{n'n} = V_n - V_n' = Z_{nn} I_n + Z_{nn'} I_n \tag{25}
\]
The neutral current is obtained by solving (24) and (25) for $I_n$ simultaneously:

$$I_n = \frac{V_n - Z_{nn}I_a + R_g I_{\text{residual}}}{R_g + Z_{nn}}$$  \hspace{1cm} (26)

In the actual load flow calculation, $V_n$ is the source-end neutral voltage from the previous iteration, $I_a$ is the calculated phase current, and $R_g$ is replaced by the shunt admittance from the transmission line model. The residual current is forced to divide between neutral and earth according to the circuit configuration using the assumed node voltage, thus expediting the convergence.

The effectiveness of these two current division methods is demonstrated in the following simple example. In Fig. 15, a constant current load is fed by the substation through a single phase feeder. The substation is represented as an ideal voltage source. The feeder mutual impedance between the phase and neutral conductor is ignored for simplicity. The feeder series resistance is also neglected. The load current is assumed to be 0.1 p.u. The earth current $I_g$ in Fig. 15 is also assumed positive when it returns to the source.
Fig. 15. Example system to demonstrate current division between neutral and earth.

The following three sets of impedance values are tested:

Case 1: \( Z_n = 0.5 \, j \, p.u. \quad Z_g = 0.5 \, p.u. \)

Case 2: \( Z_n = 0.48 \, j \, p.u. \quad Z_g = 0.52 \, p.u. \)

Case 3: \( Z_n = 0.45 \, j \, p.u. \quad Z_g = 0.55 \, p.u. \)

Since the system is very simple, it can be manually solved for the actual neutral current. For a constant load current, the voltage drop across the phase feeder and consequently the load phase voltage is fixed. The substation is assumed to be perfectly grounded, thus the left shunt grounding branch is
shorted and $V_n$ is zero. Since there is only one load, the residual current $I_{\text{residual}}$ is equal to the load current $I_a$. The system can be solved after current division of $I_{\text{residual}}$ is determined between $Z_n$ and shunt grounding branch $R_g$ on the right.

From the above analysis it is clear that $Z_n$ and $R_g$ on the right are in parallel because of the shorted grounding branch on the left. $I_{\text{residual}}$ will simply return through the parallel combination of $Z_n$ and $R_g$ and the neutral current is determined as follows

$$I_n = \frac{R_g}{Z_n + R_g} I_{\text{residual}} = \frac{R_g}{Z_n + R_g} I_a \quad (27)$$

The obtained neutral currents in three test cases are applied to compare the performance of the two current division methods. Next, the system is solved by the load flow algorithm with different current division methods. The neutral voltage $V'_n$ is initially assumed to be zero. The results using the two methods above are compared with the actual values in Table 3. It is obvious from Table 3 that the second method utilizing forced current division is more accurate and much faster. In case 1 when the magnitudes of $Z_n$ and $R_g$ are equal, the results just bounce around two extreme values and never converge at all.
Table 3. Comparison of the two methods for current division.

<table>
<thead>
<tr>
<th>Case</th>
<th>$I_n$ (Actual value in p.u.)</th>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_n$ (p.u.)</td>
<td>Iteration #</td>
<td>$I_n$ (p.u.)</td>
</tr>
<tr>
<td>1</td>
<td>0.071∠−45°</td>
<td>diverge</td>
<td>0.071∠−45°</td>
</tr>
<tr>
<td>2</td>
<td>0.074∠−43°</td>
<td>0.075∠−43°</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>0.077∠−39°</td>
<td>0.076∠−39°</td>
<td>20</td>
</tr>
</tbody>
</table>

The convergence problem is the result of the close impedance magnitude of the neutral conductor and the shunt grounding branch. Normally, the neutral conductor impedance is much lower than that of the shunt grounding branch. The large difference in impedance magnitude helps the program converge to the correct neutral-to-earth voltage even if the initial value is just a random guess. However, the neutral conductor impedance increases when the load current contains high frequency harmonics or the feeder is long. The grounding impedance on the other hand is almost unaffected because the resistance is dominant in the grounding impedance. When the magnitude of $Z_n$ is close to
that of $R_g$, the first method cannot tell how much current should return through the earth unless the neutral-to-earth voltage value is already correct.

The second method improves the convergence by calculating the correct current division between neutral and earth every time the neutral-to-earth voltage is available. Thus the answer will get closer to the correct value in each iteration.

In the next chapter, this multiphase load flow algorithm is further extended to include harmonic analysis. The effect of nonlinear loads upon NEV in distribution systems can thus be analyzed using multiphase harmonic load flow calculation.
I. Modeling Of Single Phase Uncontrolled 
Capacitor-Filtered Rectifiers 
For Harmonic Load Flow

Single-phase power electronic loads, such as personal computers, are well known for their triplen-rich load currents. A typical diode bridge rectifier with capacitive load filtering is shown in Fig. 16. Most of the nonlinear loads in residential and commercial systems contain these types of rectifiers on their front-end. This device will be used for modeling nonlinear loads to demonstrate the harmonic multiphase load flow algorithm.

Fig. 16. Single-phase bridge rectifier.
A detailed model is proposed in [46], [47] to simulate this type of power electronic circuit. The nonlinear load current is calculated as a function of the distorted terminal voltage and device parameters. The general form of $V_{th}$ can be expressed as follows

$$V_{th} = \sqrt{2} \sum_{h} V_h \cos(h \theta + \phi_h)$$  \hspace{1cm} (28)$$

The terminal voltage $V_{th}$ provided to the model is obtained either from the initial guess or the intermediate result of the load flow calculation. A typical waveform for rectifier operation is shown in Fig. 17.

The angles $\theta_1$ and $\theta_2$ are the conduction and extinction angles, respectively. The detailed circuit analysis of the single phase rectifier is presented in the appendix. If the angles $\theta_1$ and $\theta_2$ are known, the closed form solution of the input current can be determined using the method in the appendix. The general form of the input current $i_x$ is given as follows

$$i_x = \sqrt{2} \sum_{h} I_h \cos(h \omega t + \phi_h)$$  \hspace{1cm} (29)$$
Fig. 17. The characteristic waveform of a single phase bridge rectifier.

For any values of $\theta_1$ and $\theta_2$, the following conditions are always true for the circuit in Fig. 17. First, the input current becomes zero at the extinction angle $\theta_2$. Second, the input voltage $V_{th}$ equals the output voltage $V_o$ at the conduction angle $\theta_1$, because of the zero initial input current and consequently the zero voltage drop across the input impedance. The forward voltage drops of the diodes are ignored for simplicity, but can be easily included in the modeling process.
The input current conduction angle $\theta_1$ and extinction angle $\theta_2$ can be calculated iteratively by applying the above boundary conditions

\[
i_s(\theta_2) = 0 \tag{30}
\]

\[
V_o(\theta_1) = V_{th}(\theta_1) \tag{31}
\]

First, an initial guess of $\theta_1$ is assumed for the conduction angle. The resulting input current and output voltage waveforms are calculated. Then the extinction angle $\theta_2$ is determined for the calculated input current waveform. The output voltage value $V_o(\theta_2)$ is evaluated and stored at this point using the calculated waveform. After $\theta_2$, the input current stops flowing and the capacitor starts discharging through the load resistor. The next conduction period starts when the instantaneous value of the input voltage increases to the output voltage

\[
V_o(\theta_2) e^{\alpha_4(\theta_2 - \theta_1 - \pi)} = V_{th}(\theta_1) \tag{32}
\]

where $\alpha_4$ is defined in the appendix. ($\alpha_4$ is the reciprocal of the circuit time constant when the diodes stop conducting.)
Fig. 18. Flow chart of the single phase rectifier modeling for harmonic load flow.
The new value of $\theta_i$ is updated by solving (32) for $\theta_i$. A convergence check is performed at this point to determine if the process should be continued. After the values of $\theta_1$ and $\theta_2$ settle down and converge, the Fourier series of the input current is calculated and fed back to the load flow algorithm. The procedure is illustrated by the flow chart in Fig. 18.

The single phase rectifier model is tested using a distorted input voltage. Let $R_i = 0.0085\Omega$, $L_i = 0.0513mH$, $C = 4200\mu F$, $R_{eq} = 3.71\Omega$. The input voltage harmonic content in RMS values are given as $V_1 = 240\angle 100^\circ V$, $V_3 = 15\angle 250^\circ V$, $V_5 = 10\angle 10^\circ V$, $V_7 = 10\angle 30^\circ V$. The even harmonics are negligible, and they usually do not exist in a properly designed power system with these types of loads. The resulting waveforms are plotted in Fig. 19.

II. Harmonic Multiphase Load Flow

With the single phase rectifier model just described, the harmonic current injection can be determined every time the nodal voltages are updated. Other types of nonlinear loads can be provided to the algorithm to analyze the interaction between the system distortion and the nonlinear load performance.
The nonlinear load models can be included in Step 1 of the multiphase load flow described in Chapter V as part of the nodal current injection calculation. After both linear and nonlinear load injections are calculated, they are summed together according to the frequency. The same current division procedure is applied to the harmonic currents to determine the neutral branch current at harmonic frequencies.
As the nonlinear loads are treated as the source of harmonic current, the root node voltages at harmonic frequencies are assumed to be zeros. The nodal voltages are updated at all frequencies of interest as in Step 3 of the multiphase load flow in the previous chapter.

The input current is injected into the network to calculate the branch harmonic currents and then the branch voltage drops. Following procedures similar to those outlined above, the new distorted bus voltages are found and compared to the criterion until they converge. A complete flow chart for the multiphase harmonic load flow in a distribution system is shown in Fig. 20.
Fig. 20. Multiphase harmonic load flow algorithm for distribution systems.
CHAPTER VII

NEV ANALYSIS USING THE MULTIPHASE HARMONIC LOAD FLOW ALGORITHM

I. IEEE Example System Tests

1. System description.

The IEEE 13 bus test distribution system [48] is widely applied to evaluate load flow algorithms dedicated to unbalanced distribution systems. The distribution feeder in this test system is highly loaded for its relatively short length. It features various unbalanced line configurations, wye and delta load connections, and voltage conditioning devices. This system is utilized in this dissertation to examine the reliability of the multiphase harmonic load flow algorithm developed in the previous chapters. Since this dissertation focuses on NEV analysis, certain simplifications are assumed for the system modeling for the scope of this dissertation:

1. All the underground cables are omitted since the emphasis for this research is on overhead (aerial) lines.

2. Line charging currents are negligible (all lines are short).
3. All power conditioning devices except the power factor correction capacitor banks are disabled.

4. The earth resistivity is assumed uniform and other parallel conducting paths connected to the neutral are absent.

Fig. 21 shows the network tested in this dissertation. The spot linear loads on the system are given in Table 4. The feeder phasing is indicated beside each branch. The missing branches and nodes are for the underground cables.

![IEEE TEST SYSTEM](image)

Fig. 21. IEEE test system with underground cable omitted.
Table 4. Spot linear loads on the IEEE test system.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Bus No.</th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kW</td>
<td>kvar</td>
<td>kW</td>
</tr>
<tr>
<td>Y</td>
<td>8</td>
<td>160</td>
<td>110</td>
<td>120</td>
</tr>
<tr>
<td>Y</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>170</td>
</tr>
<tr>
<td>Delta</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>230</td>
</tr>
<tr>
<td>Delta</td>
<td>3</td>
<td>385</td>
<td>220</td>
<td>385</td>
</tr>
<tr>
<td>Delta</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2. Earth Resistivity Effect

The effect of earth conductivity on the NEV is first examined using load flows of the feeder with just linear loads and three earth possible resistivities: 10, 100 and 1000 $\Omega \cdot m$. These represent the typical high, mean and low earth conductivities that might be found in the field. (100 $\Omega \cdot m$ with a standard 10 foot ground rod yields approximately 25 ohms of grounding resistance.) Figs. 22–25 show the phase-to-neutral voltages in per unit (p.u.) and the NEV in volts that result from the tests. These phase voltage plots provide an indication of the relative unbalance of the system.
Fig. 22. Phase voltages with linear loads and $\rho = 10 \Omega \cdot m$.

Fig. 23. Phase voltages with linear loads and $\rho = 100 \Omega \cdot m$. 
Fig. 24. Phase voltages with linear loads and $\rho = 1000 \Omega \cdot \text{m}$.

Fig. 25. NEV with linear loads at different earth resistivities.
From the plots in Fig. 22–24, it can be seen that the change in the earth conductivities has almost no effect on the phase voltages. The feeder’s series impedance is not high enough to make any difference in phase voltage drop. This result is reasonable considering the short length of the feeder. However, NEV is much more sensitive to the change in earth resistivity. The higher earth resistivity causes higher NEV, as is expected. It is shown Fig. 25 that NEV level is doubled when the earth resistivity increases from 10 Ω·m to 100 Ω·m. It is interesting to note that there is a significant difference between 10 Ω·m and 100 Ω·m, but above 100 Ω·m changes in the ground resistivity has little effect on the NEV.

In these linear loads tests, the higher NEV is incidental to unbalanced bus loads or single/two phase branches. Also the higher NEV on these buses can affect other buses if the affected buses are on the backward trace of the buses that cause NEV elevation, or they are at downstream of a bus with high NEV. For example, the trace of bus 7 back to the source consists of three phase branches. But unbalanced loads are connected at bus 3 upstream, and cause high NEV. So even no load current is drawn at bus 7, its NEV is still higher than the NEV at bus 8 where balanced loads are connected. These results show that besides the earth resistivity, the network topology and the location of unbalanced loads and branches are also important in an NEV investigation.
3. Effect of Nonlinear Loads

To examine the harmonic distortion effect on NEV, a 240V, 30kW single-phase rectifier with a 4200μF smoothing capacitor is applied to simulate the lumped nonlinear loads connected on the feeder from phase–to-neutral. The original linear loads on the feeder are kept unchanged. The earth resistivity is assumed to be 100 Ω⋅m in all of the following tests. Three scenarios are considered in this paper for illustration:

(i) A single rectifier on a single-phase line at bus 3
(ii) Three balanced rectifiers on a three-phase line at bus 3
(iii) Two identical rectifiers on two phases of a three-phase line at bus 3 to form an unbalanced load

Since the nonlinear loads are small compared with the linear loads in Table 4, the phase voltage profiles for all cases are essentially the same as the results in Figs. 22-24. The voltage profile for the Case (i) is shown in Fig. 26.
Fig. 26. Phase to neutral voltage with nonlinear loads.

The phase voltage distortion $\text{THD}_v$ in all cases is less than 2%, which should be quite acceptable. It is shown in Fig. 27 that the rectifier’s terminal voltage is almost sinusoidal for Case (i) conditions. An analysis of the voltage distortion alone might lead one to believe the harmonic NEV components would be small. The current injections and the $\text{THD}_i$ at the bus containing rectifier loads are listed in Table 5. It is obvious that the single-phase rectifier loads cause harmonic distortion in the phase current at the load terminal bus. Even higher distortion is expected in the neutral current due to the additive triplen harmonics and the cancellation among other frequency components. As a result, the NEV is also rich in triplen harmonics. The NEV spectrum at the bus with rectifier loads for the three cases is illustrated in Fig. 28.
Fig. 27. Waveforms at the rectifier terminal for Case (i).

Table 5. Current harmonics at the bus with rectifier loads.

<table>
<thead>
<tr>
<th>Case</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS (A)</td>
<td>THD (%)</td>
<td>RMS (A)</td>
</tr>
<tr>
<td>(i)</td>
<td>86.7</td>
<td>16.7</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>86.3</td>
<td>16.2</td>
<td>72.5</td>
</tr>
<tr>
<td>(iii)</td>
<td>86.3</td>
<td>16.4</td>
<td>72.9</td>
</tr>
</tbody>
</table>
As one would expect, the NEVs are elevated as the harmonic current returns through the neutral conductor. Triplen harmonics are additive in the neutral, and the other odd harmonic components are present in the neutral due to the unbalance among phases. Fig. 29 compares the neutral voltage in total RMS values with different loading scenarios including the case with only linear loads. Notice that the calculated NEV is higher than the stray voltage at the same bus since the NEV is the voltage from the ground electrode to the earth at infinity.
It is clear that the neutral voltages are higher in almost all nonlinear load cases except for Case (i), where the nonlinear load on phase $a$ helps balance the unbalanced load on phases $b$ and $c$ at bus 5. The balanced nonlinear loads in Case (ii) produce the highest NEV elevation due to the additive triplen harmonics. Notice that the NEV elevation in nonlinear load cases follows the similar profile obtained using linear loads, except for bus 3 where the rectifiers are connected. The nonlinear load current injected at bus 3 cause much higher NEV elevation than other buses with unbalanced loads. As observed in the linear load tests, the high NEV elevation at bus 3 also affected the NEV at bus 6, 7 and 9, which are at downstream of bus 3. For all the buses on other laterals (i.e., 2, 4, 5, and 8), the effect of nonlinear loads is much less. Bus 1 is the closest node to ideal ground of
the source. And all load currents in the network flow through this bus, which helps balance the phase currents. Consequently, the NEV at this bus is lowest among all the nodes on the feeder.

II. Comparison With Field Measurements

The test on the IEEE example system shows that the developed multiphase harmonic load flow algorithm developed in this dissertation is quite reliable on handling various unbalanced system configuration. The nonlinear model also works well for analyzing the effect of harmonic distortion on NEV elevation. In this section, the algorithm’s accuracy is verified by comparing the load flow calculations with field measurement.

A typical multigrounded three phase feeder with single phase lateral is chosen and the line configuration is shown in Fig. 30. The three phase feeder is fed through a 44/12.47 kV delta–wye grounded substation transformer. A typical impedance of 7% is assumed for this investigation. The series resistance of the transformer is ignored for simplicity. All conductors of the three phase primary and the single phase lateral, including the neutral conductors, are ACSR 336,400. The three phase primary and the single phase lateral are both 5 miles long.
Since the NEV value is the voltage from the ground conductor to the remote earth, it is hard to measure the total difference in voltage potentials using wires of finite length. The problems related to Electromagnetic Interference (EMI) in the field can also ruin the measured NEV data too. Thus the neutral current instead of NEV is used for comparison.

The examination is set up as follows. First the phase and neutral currents are measured using an oscilloscope. Only one current probe is available during the field test and the two currents have to be measured individually. To synchronize the two current waveforms, the phase voltage is taken as reference and measured with the current at the same time. It is assumed that the variation is voltage waveform is negligible during the field test, which is acceptable in a well-designed power system operating in steady state. After the waveforms are
taken, the two currents are synchronized to the same time frame and their harmonic components are computed. Then the phase current, including its harmonic components, is injected into the feeder and the neutral current is calculated using the developed multiphase harmonic load flow algorithm. The connection of field measurement is illustrated in Fig. 31 and the measured waveforms are shown in Fig. 32 and Fig. 33.

![Field measurement connection](image)

Fig. 31. Connection of the field measurement on the 7200 V phase conductor. (The neutral conductor is the lower wire.)
Fig. 32. Measured phase voltage and phase current.

Fig. 33. Measured phase voltage and neutral current.
In Fig. 32–33, the voltage waveforms are shown lighter by the lighter trace and scales are 5000 V/div and 10 A/div for voltage and current, respectively. Next, the current waveforms are shifted to synchronize with the phase voltage and the resulting waveforms are shown in Fig. 34.

The load currents on the three phase feeder are ignored for simplicity. It is assumed that the measured load current is the total load current on the single phase lateral, which are uniformly distributed along the length of the lateral. For a voltage drop calculation, the distributed loads can be lumped at the midpoint of the lateral [49], and the simulation is setup as in Fig. 35.

![Fig. 34. Synchronized waveforms, $V_a$ (solid) $I_a$ (dot) and $I_n$ (dashed).](image-url)
Fig. 35. Simulation setup for waveform verification.

Fig. 36. Waveform comparison, measurement (solid) and calculation (dashed).
The feeder is assumed to be grounded four times per mile using single ground rod of 10 foot length with a diameter of 5/8 inches. The earth resistivity of is assumed to be 100 $\Omega \cdot m$. The corresponding grounding resistance at each grounding point is thus 25 $\Omega$ according to Table 2. The neutral current spectrum is calculated with all the harmonic components. Then the waveform is generated using the calculated results and the comparison with the measured waveform is shown in Fig. 36. For the limited system information, the algorithm calculation is very accurate and matches the measurement closely.

With satisfactory results on reliability and accuracy tests, it is possible to apply the developed multiphase harmonic load flow algorithm in identifying the origin of NEV elevation in actual distribution systems. The system configuration data and load information are available to accurately model the distribution feeders. The NEV profiles can be computed for various operation conditions. After the power engineers take the field measurement, the discrepancy from the load flow calculation may indicate the possible location of the bad neutral connection or corroded splice on the feeder.
III. Evaluation Of Three-Phasing Method  
In The Presence Of Harmonic Distortion

The three-phasing method is commonly applied by electric utilities to upgrade their distribution feeders to meet the demand growth. The original single phase lateral is complemented by two other phase conductors running parallel with it. Ideally, the loads are distributed among the three phases as balanced as possible. The benefits of three-phasing a single phase lateral include reliability improvement, operational flexibility, lower line loss, voltage drop decrease, etc. Among these benefits, the cancellation in return currents is directly related to NEV mitigation. At the fundamental frequency (e.g. 60 Hz), the three phase currents can cancel each other at the neutral point if they are balanced or close to balanced. Consequently, almost no current needs to return the source and the associated NEV is largely reduced.

However, the situation is different if nonlinear loads, especially single phase electronic devices, are connected on the feeder. Most of these devices use uncontrolled capacitor filtering rectifiers as their front-end circuit. It is shown in previous analysis that single phase rectifiers input currents are rich in triplen harmonics and these triplen harmonic components are additive in neutral conductors. Thus the three-phasing method as an option for NEV alleviation needs to evaluated in the present of harmonic distortion.
The single phase lateral introduced in the last section is used to examine the effect of the three-phasing method on NEV. Assume the measured phase current on the single phase lateral is the original load that need to three-phased. The NEV profile along the feeder has been solved in the studies of the last section, and the harmonic component of the calculated currents and NEV are shown in Fig. 37–38. It can be seen that the original load contains only small amount of harmonic content. The resulting NEV is predominantly fundamental.

Fig. 37. Single phase lateral current spectrum (phase–neutral–earth currents are grouped at each harmonic frequency, in that order).
Fig. 38. Feeder NEV spectrum (RMS and harmonic components are grouped at each bus, in the order of RMS, fund., 3rd, 5th, 7th and 9th).

The single phase lateral is three-phased by adding two identical phase conductors. The spacing configuration is assumed to follow that of the three phase primary on the same feeder. (See Fig. 39) Then the measured single phase current is distributed among the three phases symmetrically. For the reasons mentioned in the last section, the three phase currents are then injected at midpoint on the new three phase branch as shown in Fig. 40.
Fig. 39 Three phase feeder spacing data.

Fig. 40. Three phasing test on the original system.
The network in Fig. 40 is solved using the multiphase harmonic load flow algorithm and the results are shown in Figs. 41–42. It is clear that in this case, the three phasing method considerably reduces the NEV profile along the feeder. On the single phase lateral, the NEV drops from 12.8 V to only 1 V. The reason for this satisfactory result can be found by examining the current spectrum after three phasing, depicted in Fig. 41.

![Current spectrum after three phasing](image)

Fig. 41. Current spectrum after three phasing (phase–neutral–earth currents are grouped at each harmonic frequency, in that order).
At the fundamental frequency, the phase current cancellation produces almost zero return current in either neutral conductor or the earth. The triplen harmonic components still exist in the neutral and the earth, but their contribution to the total NEV is negligible due to the small magnitude. The detailed values for NEV and currents’ harmonic content are compared in Table 6.

![Feeder NEV spectrum after three phasing](image)

Fig. 42. Feeder NEV spectrum after three phasing (RMS and harmonic components are grouped at each bus, in the order of RMS, fund., 3rd, 5th, 7th and 9th).
Table 6. Detailed results of harmonic components before and after three phasing using field measurement.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Single Phase</th>
<th>Three Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NEV (V)</td>
<td>Current (A)</td>
</tr>
<tr>
<td></td>
<td>I_{ph}</td>
<td>I_{n}</td>
</tr>
<tr>
<td>1</td>
<td>12.74</td>
<td>9.15</td>
</tr>
<tr>
<td>3</td>
<td>0.93</td>
<td>0.42</td>
</tr>
<tr>
<td>5</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>0.46</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>0.28</td>
<td>0.08</td>
</tr>
<tr>
<td>11</td>
<td>0.14</td>
<td>0.03</td>
</tr>
<tr>
<td>13</td>
<td>0.19</td>
<td>0.04</td>
</tr>
<tr>
<td>15</td>
<td>0.17</td>
<td>0.04</td>
</tr>
<tr>
<td>RMS_{TOT}</td>
<td>12.8</td>
<td></td>
</tr>
</tbody>
</table>

It is shown in Table 6 that after three phasing, the neutral and earth currents diminished at all frequencies except for triplen harmonic frequencies. The triplen harmonics do not change after three phasing although their magnitudes are small. As mentioned at the beginning of this section, three phasing is usually implemented due to the demand growth. It would be interesting to examine the three phasing method with increased load current, especially harmonic load current, on its effectiveness for NEV mitigation.
The measured single phase current in the last section represents a total load of 60 kW with a lagging power factor of 0.9. It is assumed that the system is experiencing a demand growth such that the load is doubled at the same power factor. For this experiment, 60 kW nonlinear loads will also be connected on the feeder. To maintain uniform nonlinear characteristics as used previously, the nonlinear loads are represented as three identical 20 kW rectifiers with 6000 $\mu$F smoothing capacitors. The feeder is simulated using the configuration in Fig. 35, except that the load current is increased as described above.

![Rectifier terminal waveforms](image)

*Fig. 43. Rectifier terminal waveforms on the single phase lateral with projected load growth.*
The calculated waveforms at the rectifiers’ terminals are shown in Fig. 43. Since the three rectifiers are identical and connected to the same phase, waveforms are shown only for one rectifier. The total rectifiers current can be obtained by multiplying the current shown by three. It is clear from Fig. 43 that the terminal voltage is seriously distorted with such high capacity of nonlinear loads connected on the same phase. This is also a sign calling for certain means for harmonic suppression. The currents and NEV harmonic spectrums are shown in Fig. 44–45.

Fig. 44. Current spectrum on the single phase lateral with projected load growth (phase–neutral–earth currents are grouped at each harmonic frequency, in that order).
As expected, the load current in Fig. 44 is rich in triplen harmonics and the 3rd harmonic component of the NEV in Fig. 45 is almost equal to the fundamental value. Next the single phase lateral is three-phased and the projected load growth is distributed among the three phases symmetrically. The three single-phase rectifiers are connected separately on each phase. Then feeder is simulated using the setup shown in Fig. 40. The waveforms at the terminals of the rectifier on the original phase are shown in Fig. 46. It is obvious that the terminal voltage waveform is improved and becomes nearly sinusoidal.
Fig. 46. Rectifier terminal waveforms after three phasing with projected load growth.

Fig. 47. Current spectrum after three phasing with projected load growth (phase–neutral–earth currents are grouped at each harmonic frequency, in that order).
The harmonic components of the currents and NEV are shown in Figs. 47–48. It is obvious from Fig. 48 that the triplen harmonics are the dominant components left after three phasing. Although the fundamental return current is diminished, as shown in Fig. 47, the triplen harmonics are still high in the neutral and earth. Actually, 3rd and 9th harmonic currents in the neutral and earth are even higher than phase current. The additive triplen harmonics are the major
factors that cause the NEV to only drop from 57 V to 37 V. The detailed values for NEV and currents’ harmonic components are compared in Table 7.

This section has demonstrated that three-phasing a feeder works well to mitigate NEV in systems only containing mostly linear loads. Its effectiveness is degraded in the presence of significant concentration of nonlinear loads in distribution systems. As shown in test results, the voltage harmonic distortion itself shows that harmonic suppression would be desirable. However, it is not clear how filtering for triplen harmonics might impact the NEV. From the perspective of NEV alleviation, diverting the phase harmonics into neutral conductor is clearly not the correct means of filtering.
Table 7. Detailed results of harmonic components before and after three phasing with project load growth.

| Harmonic Order | Single Phase | | | Three Phase | | |
| | NEV (V) | Current (A) | NEV (V) | Current (A) | | |
| | | $I_{ph}$ | $I_n$ | $I_e$ | | |
| 1 | 38.21 | 27.44 | 20.00 | 8.09 | 0.55 | 9.69 | 0.12 | 0.12 |
| 3 | 32.56 | 14.54 | 8.86 | 6.44 | 36.34 | 5.42 | 9.86 | 7.19 |
| 5 | 16.68 | 5.84 | 3.07 | 3.00 | 0.25 | 2.40 | 0.06 | 0.04 |
| 7 | 5.58 | 1.67 | 0.80 | 0.91 | 0.13 | 0.62 | 0.03 | 0.02 |
| 9 | 5.94 | 1.58 | 0.72 | 0.87 | 6.35 | 0.56 | 0.78 | 0.93 |
| 11 | 2.93 | 0.71 | 0.32 | 0.39 | 0.05 | 0.31 | 0.01 | 0.01 |
| 13 | 3.18 | 0.72 | 0.32 | 0.40 | 0.07 | 0.25 | 0.01 | 0.01 |
| 15 | 1.83 | 0.39 | 0.18 | 0.21 | 2.24 | 0.16 | 0.22 | 0.26 |
| RMS$_{tot}$ | 57.23 | | | | 36.96 | | | |
Neutral-to-earth voltage is caused, in part, because of the practice of grounding the neutral of power system distribution feeders at multiple points along their length for safety, fault clearing, and compliance with the codes. To estimate problems associated with elevated NEV, especially in the presence of harmonic distortion, several steps must be taken.

In this dissertation, multigrounded transmission lines are examined and a model dedicated for NEV analysis is derived based on Carson’s line theory. The neutral conductor is represented explicitly by considering the current distribution between neutral and earth paths. A multiphase harmonic load flow algorithm is developed to compute the NEV profile in radial distribution systems. Since it is a branch-oriented load flow technique, most of the distribution system components can be included by adjusting the model’s voltage to the potential from phase-to-local earth. The NEV profile throughout the system is available directly from the solution. Harmonic interaction between nonlinear devices and
distribution systems parameters is included by incorporating the nonlinear modeling into the multiphase load flow. A common single-phase uncontrolled rectifier is used to demonstrate the harmonic analysis.

The algorithm and the accompanying linear and nonlinear modeling techniques are tested on an IEEE example system as well as an actual distribution feeder model. Their reliability and accuracy are verified by satisfactory test results. Three phasing of feeders, one of the conventional means for distribution system upgrade, is evaluated using the developed algorithm from the perspective of NEV elevation.

It is observed from the test results that the harmonic injections from nonlinear devices, especially the single-phase power electronic loads, can considerably increase the NEV of the system. The location of nonlinear devices in distribution systems is also an important factor on elevated NEV. The highest elevation due to harmonic distortion is found on the bus where the nonlinear loads are connected. However, this elevation can affect the downstream buses as well as the buses on the path from the nonlinear load to source. The closer a bus is to the source, the lower the effect of nonlinear loads on NEV elevation, assuming the source has a nearly ideal grounding connection. The buses on other laterals are less related to the harmonic injection, except for the already connected linear loads.
The close match between field measurement and the load flow calculation suggests that the developed algorithm can be applied to identify the origin of NEV elevation in an actual distribution system. The evaluation of the three-phasing method shows that the method is not capable of reducing NEV profile on a distribution feeder with large amount of single phase nonlinear loads, although it works perfectly for a system with predominantly linear loads.

II. Future Work

Only one power electronic device, the uncontrolled capacitor filtered rectifier, is modeled in this document. However, the technique is not limited to the single-phase rectifier; various power electronic devices can be represented according to their nonlinear characteristics. Other non-linear devices, such as the saturated magnetic seen in single-phase induction motors and lighting ballasts, can be utilized with the load flow if modeled adequately.

Both linear and nonlinear loads are aggregated at the bus of interest in the harmonic load flow formulation. A recent trend in harmonic analysis is to study the distributed nonlinear loads throughout the system to account for the harmonic attenuation and diversity [50]. The simple structure of the load flow algorithm makes it feasible to improve the nonlinear load representation by distributing it for better accuracy.
Future enhancements of this model would be to incorporate the influence of the parallel utilities bonded to distribution system neutral, such as water pipes, gas pipes, and telecommunication wires, to analyze this often-encountered complex situation. Additionally, the geometry of the feeder (i.e. not constructed in straight line) and the use of non-overhead conductors (such as semiconductor jacketed underground cable) needs to be analyzed to produce a complete set of tools for engineers to use.

Stray voltage, or elevated NEV, is a serious problem for many electric utilities. This dissertation should provide a good first step in understanding the problems, the impact of nonlinear loads, and possible mitigation strategies.
APPENDICES
I. Circuit Analysis Of Single Phase Uncontrolled Capacitor Filtered Rectifier

A. Introduction

The circuit diagram for a typical single phase uncontrolled capacitor filtered rectifier is shown in Fig. A-1. The input voltage $V_{th}$ and input impedance $Z_r = R_r + j\omega L_r$ are provided by the load flow result and the system configuration. The capacitor $C$ and the resistor $R_{eq}$ represent the equivalent circuit of the rectifier. The goal of this analysis is to determine the closed form expressions of the input current $i_s$ in both time and frequency domains, for the given input voltage $V_{th}$.

Fig. A-1. Single phase uncontrolled capacitor filtered rectifier.
The diodes conduct in pairs to charge the load. When the diodes stop conduction, the input current is zero. The waveform for rectifier operation is shown in Fig. A-2. It is only of interest to examine the circuit performance during the interval $\theta_1 \leq \theta \leq \theta_2$ to determine the shape of $i_s$. In the following derivation, it is assumed that the conduction and extinction angles, $\theta_1$ and $\theta_2$, are known. Their actual values can be determined by the program in Chapter VI.

![Waveform Figure](image)

**Fig. A-2.** The typical terminal waveform at a single phase rectifier terminal.
For accurate modeling of rectifier response to harmonic distortion, the input voltage $V_{th}$ is not restricted to fundamental sinusoidal. The general expression for $V_{th}$ is given as $V_{th} = \sqrt{2} \sum h V_h \sin(h \theta + \phi_h)$. The subscript $h$ represents the harmonic order of interest. The highest harmonic frequency in a typical harmonic analysis could reach up to the 25th harmonic.

To simplify the analysis, the axis can be shifted by $\theta_1$ where the input current just starts flowing in the positive direction. The expression for $V_{th}$ is rewritten as follows

$$V_{th} = \sqrt{2} \sum h V_h \sin(h \theta + \phi_h + h \theta_1) = \sqrt{2} \sum h V_h \sin(h \theta + \delta_h) \quad (A-1)$$

At $\theta_1$, the circuit can be described by the following equations

$$V_{th} = R_i i_s + \omega L_i \frac{di_s}{d\theta} + V_o \quad (A-2)$$

$$i_s = \omega C \frac{dV_o}{d\theta} + \frac{V_o}{R_{eq}} \quad (A-3)$$

Solving (A-2) (A-3) simultaneously for $V_o$, the differential equation for $V_o$ is obtained as
\[
\omega^2 L_T C \frac{d^2 V_o}{d \theta^2} + \left( \omega R_T + \frac{\omega L_T}{R_{eq}} \right) \frac{d V_o}{d \theta} + \left( \frac{R_T}{R_{eq}} + 1 \right) V_o = V_{th} \tag{A-4}
\]

Let \( \alpha_1 = \frac{R_T}{\omega L_T} \), \( \alpha_2 = \frac{1}{\omega L_T} \), \( \alpha_3 = \frac{1}{\omega C} \), \( \alpha_4 = \frac{1}{\omega CR_{eq}} \), then (4) becomes

\[
\frac{d^2 V_o}{d \theta^2} + (\alpha_1 + \alpha_4) \frac{d V_o}{d \theta} + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) V_o = \alpha_2 \alpha_3 V_{th} \tag{A-5}
\]

Similarly, the equation for \( i_s \) can be obtained as flows

\[
\omega^2 L_T C \frac{d^2 i_s}{d \theta^2} + \left( \omega R_T + \frac{\omega L_T}{R_{eq}} \right) \frac{d i_s}{d \theta} + \left( \frac{R_T}{R_{eq}} + 1 \right) i_s = \frac{V_{th}}{R_{eq}} + \omega C \frac{d V_{th}}{d \theta} \tag{A-6}
\]

\[
\frac{d^2 i_s}{d \theta^2} + (\alpha_1 + \alpha_4) \frac{d i_s}{d \theta} + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) i_s = \alpha_2 \alpha_4 V_{th} + \alpha_2 \frac{d V_{th}}{d \theta} \tag{A-7}
\]

The differential equations (A-5) (A-7) for \( V_o \) and \( i_s \) have the same characteristic equation, which is what we expect. The only difference is the driving force on the right side of the equations. The characteristic roots are solved as follows

\[
s_{1,2} = a \pm b = -\frac{\alpha_1 + \alpha_4}{2} \pm \sqrt{\left(\frac{\alpha_1 - \alpha_4}{2}\right)^2 - \alpha_2 \alpha_3} \tag{A-8}
\]
B. The particular solution of $V_o$ and $i_s$

The particular solutions of $V_o$ and $i_s$ can be determined using the undetermined coefficients method. Since the forcing functions on the right of (A-5) (A-7) are summation of sinusoidal functions, the particular solutions should take similar form. The particular solution of $V_o$ and its first and second derivatives are written as follows

\begin{align}
V_o &= \sum_h M_h \cos h\theta + N_h \sin h\theta \quad \text{(A-9)} \\
V_o' &= \sum_h -M_h h \sin h\theta + N_h h \cos h\theta \quad \text{(A-10)} \\
V_o'' &= \sum_h -M_h h^2 \cos h\theta - N_h h^2 \sin h\theta \quad \text{(A-11)}
\end{align}

Substituting (A-9), (A-10) and (A-11) in (A-5), we have

\begin{align}
\sum_h \left( -M_h h^2 \cos h\theta - N_h h^2 \sin h\theta \right) + (\alpha_1 + \alpha_4) \sum_h (-M_h h \sin h\theta \\
+ N_h h \cos h\theta) + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) \sum_h (M_h \cos h\theta + N_h \sin h\theta) \quad \text{(A-12)}
\end{align}

\begin{align}
&= \sqrt{2} \alpha_2 \alpha_3 \sum_h V_h \left( \sin h\theta \cos \delta_h + \cos h\theta \sin \delta_h \right)
\end{align}

Collect similar terms for $\sin h\theta$ and $\cos h\theta$, and compare both sides of (A-12) at each harmonic, we have
\[-N_h h^2 + (\alpha_1 + \alpha_4)(-M_h h) + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3)N_h = \sqrt{2} \alpha_2 \alpha_3 V_h \cos \delta_h \quad (A-13)\]

\[-M_h h^2 + (\alpha_1 + \alpha_4)(N_h h) + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3)M_h = \sqrt{2} \alpha_2 \alpha_3 V_h \sin \delta_h \quad (A-14)\]

Put (A-13) and (A-14) in a matrix form

\[
\begin{bmatrix}
-h(\alpha_1 + \alpha_4) & -h^2 + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) \\
-h^2 + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) & h(\alpha_1 + \alpha_4)
\end{bmatrix}
\begin{bmatrix}
M_h \\
N_h
\end{bmatrix}
= \sqrt{2} \alpha_2 \alpha_3 V_h
\begin{bmatrix}
\cos \delta_h \\
\sin \delta_h
\end{bmatrix} \quad (A-15)
\]

The determinant of the square matrix on the left is calculated as

\[
\Delta = -h^2(\alpha_1 + \alpha_4)^2 - \left[-h^2 + (\alpha_1 \alpha_4 + \alpha_2 \alpha_3)\right]^2 \quad (A-16)
\]

Let \( B = \alpha_2 \alpha_3 = \frac{(\alpha_1 - \alpha_4)^2}{4} \), 
\( c_{1h} = \frac{1}{4a^2 h^2 + \left(h^2 - a^2 - B\right)^2} \), then \( \Delta = -\frac{1}{c_{1h}} \). The coefficients \( M_h \) and \( N_h \) can be calculated by inverting the coefficient matrix in (A-15)

\[
\begin{bmatrix}
M_h \\
N_h
\end{bmatrix}
= \sqrt{2} \alpha_2 \alpha_3 V_h
\begin{bmatrix}
h(\alpha_1 + \alpha_4) & h^2 - (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) \\
h^2 - (\alpha_1 \alpha_4 + \alpha_2 \alpha_3) & -h(\alpha_1 + \alpha_4)
\end{bmatrix}
\begin{bmatrix}
\cos \delta_h \\
\sin \delta_h
\end{bmatrix} \quad (A-17)
\]

Therefore

\[
M_h = -\sqrt{2} \alpha_2 V_h \left[\alpha_2 c_{1h} \left(h^2 - a^2 - B\right) \sin \delta_h - 2 a h \cos \delta_h \right] = -\sqrt{2} \alpha_2 V_h c_{sh} \quad (A-18)
\]
\[ N_h = \sqrt{2} \alpha_2 \frac{V_h}{h} \left[ \alpha_3 c_{1h} \left[ -2ah^2 \sin \delta_h + (a^2 + B - h^2)h \cos \delta_h \right] \right] = \sqrt{2} \alpha_2 \frac{V_h}{h} c_{7h} \quad (A-19) \]

where
\[
c_{5h} = \alpha_3 c_{1h} \left( h^2 - a^2 - B \right) \sin \delta_h - 2ah \cos \delta_h \]
\[
c_{7h} = \alpha_3 c_{1h} \left\{ -2ah^2 \sin \delta_h + (a^2 + B - h^2)h \cos \delta_h \right\} \]

Thus the particular solution of \( V_o \) becomes
\[
V_o = \sqrt{2} \sum_h V_h \left( -\alpha_3 c_{5h} \cos h \theta + \alpha_2 \frac{c_{7h}}{h} \sin h \theta \right) \quad (A-20) \]

Similar to \( V_o \), the particular solution of \( i_s \) is assumed to be as follows
\[
i_s = \sum_h M_h \cos h \theta + N_h \sin h \theta \quad (A-21) \]

Then its first and second order derivatives are calculated as
\[
i_s' = \sum_h -M_h \sin h \theta + N_h h \cos h \theta \quad (A-22) \]
\[
i_s'' = \sum_h -M_h h^2 \cos h \theta - N_h h^2 \sin h \theta \quad (A-23) \]

Substituting (A-21), (A-22) and (A-23) in (A-7), we have
\[
\sum_{h} \left( -M_{h} h^{2} \cos \theta - N_{h} h^{2} \sin \theta \right) + (\alpha_{1} + \alpha_{4}) \sum_{h} \left( -M_{h} h \sin \theta \right) \\
+ N_{h} h \cos \theta + (\alpha_{1} \alpha_{4} + \alpha_{2} \alpha_{3}) \sum_{h} \left( M_{h} \cos h \theta + N_{h} \sin h \theta \right) \\
= \sqrt{2} \alpha_{2} \sum_{h} V_{h} \left( \sin h \theta (\alpha_{4} \cos \delta_{h} - h \sin \delta_{h}) + \cos h \theta (\alpha_{4} \sin \delta_{h} + h \cos \delta_{h}) \right)
\]

or in a matrix form

\[
\begin{bmatrix}
-h(\alpha_{1} + \alpha_{4}) & -h^{2} + (\alpha_{1} \alpha_{4} + \alpha_{2} \alpha_{3}) \\
-h^{2} + (\alpha_{1} \alpha_{4} + \alpha_{2} \alpha_{3}) & h(\alpha_{1} + \alpha_{4})
\end{bmatrix}
\begin{bmatrix}
M_{h} \\
N_{h}
\end{bmatrix}
= \sqrt{2} \alpha_{2} V_{h} \begin{bmatrix}
\alpha_{4} \cos \delta_{h} - h \sin \delta_{h} \\
\alpha_{4} \sin \delta_{h} + h \cos \delta_{h}
\end{bmatrix}
\]

Note that the matrix on the left in (A-25) is the same as that in (A-15).

Solve (A-25) for \( M_{h} \) and \( N_{h} \)

\[
M_{h} = \sqrt{2} \alpha_{2} V_{h} (-c_{1h}) \left[ -2ah(\alpha_{4} \cos \delta_{h} - h \sin \delta_{h}) \\
+ (h^{2} - a^{2} - B)(\alpha_{4} \sin \delta_{h} + h \cos \delta_{h}) \right]
= \sqrt{2} \alpha_{2} V_{h} c_{2h}
\]

\[
N_{h} = \sqrt{2} \alpha_{2} V_{h} (-c_{1h}) \left[ (h^{2} - a^{2} - B)(\alpha_{4} \cos \delta_{h} - h \sin \delta_{h}) \\
+ 2ah(\alpha_{4} \sin \delta_{h} + h \cos \delta_{h}) \right]
= \sqrt{2} \alpha_{2} V_{h} c_{2h} \frac{c_{4h}}{h}
\]
where

\[ c_{2h} = c_{1h} \left\{ \sin \delta_h \left[ -2ah^2 + \alpha_4 \left( a^2 + B - h^2 \right) \right] + \cos \delta_h \left[ a^2 + B - h^2 + 2a\alpha_4 \right] \right\} \]

\[ c_{4h} = c_{1h} \left\{ \sin \delta_h \left[ -2a\alpha_4 + \left( h^2 - a^2 - B \right) \right] + h \cos \delta_h \left[ \left( a^2 + B - h^2 \right) \alpha_4 - 2ah^2 \right] \right\} \]

Thus the particular solution for \( i_s \) becomes

\[ i_s = \sqrt{2} \sum_h V_h \left( \alpha_2 c_{2h} \cosh \theta + \alpha_2 \frac{c_{4h}}{h} \sinh \theta \right) \]  \hspace{1cm} (A-28)

C. Complete solutions of \( V_o \) and \( i_s \)

After the particular solutions of \( V_o \) and \( i_s \) are determined, the next step is to find the general solutions for the output voltage and the input current. The characteristic roots in (A-8) can be either real or complex depending on the circuit parameters. Assume that the roots are real, then (A-8) is repeated as follows

\[ s_{1,2} = -\frac{\alpha_1 + \alpha_4}{2} \pm \sqrt{\left( \frac{\alpha_1 - \alpha_4}{4} - \alpha_2\alpha_3 \right)} = a \pm b \]  \hspace{1cm} (A-29)

Including the particular solution (A-20), the complete solution of \( V_o \) becomes
\[ V_o = A_1 e^{i \theta} + A_2 e^{i 2 \theta} + \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h \theta + \alpha_2 \frac{c_{7h}}{h} \sin h \theta \right) \] (A-30)

The parameters \( A_1 \) and \( A_2 \) in (A-30) can be determined using the initial conditions of the circuit. At \( \theta = 0 \), the rectifier starts conduction and the input voltage equals the output voltage, i.e.,

\[ V_o(0) = V_{th}(0) \] (A-31)

Substitute (A-1) and (A-30) in (A-31) for \( \theta = 0 \)

\[ V_o = V_{th}(0) = \sqrt{2} \sum_h V_h \sin \delta_h = A_1 + A_2 + \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \right) \] (A-32)

Therefore

\[ A_1 + A_2 = \sqrt{2} \sum_h V_h \left( \sin \delta_h + \alpha_2 c_{5h} \right) \] (A-33)

By examining the circuit, we have

\[ V_o = \frac{i_v}{\omega C} = \frac{1}{\omega C} \left( i_v - \frac{V_o}{R_{eq}} \right) \] (A-34)

At \( \theta = 0 \), the input current has to be zero, i.e.,
\[ i_s(0) = 0 \]  \hspace{1cm} (A-35)

Take derivative of (A-32) and evaluate at \( \theta = 0 \),

\[ V_o'(0) = s_1A_1 + s_2A_2 + \sqrt{2} \sum_h V_h \alpha_2 c_{\gamma_h} \]  \hspace{1cm} (A-36)

Substituting (A-32), (A-35) and (A-36) in (A-34), we

\[ V_o'(0) = -\frac{V_o(0)}{\omega C_{R_q}} = -\alpha_4 \sqrt{2} \sum_h V_h \sin \delta_h \]

\[ = s_1A_1 + s_2A_2 + \sqrt{2} \sum_h V_h \alpha_2 c_{\gamma_h} \]  \hspace{1cm} (A-37)

therefore

\[ s_1A_1 + s_2A_2 = -\sqrt{2} \sum_h V_h (\alpha_4 \sin \delta_h + \alpha_2 c_{\gamma_h}) \]  \hspace{1cm} (A-38)

Solve (A-33) and (A-38) for \( A_1 \) and \( A_2 \) simultaneously. Then for each harmonic, we have

\[ A_{1h} = \sqrt{2} \frac{V_h}{2b} \left[(s_1 + \alpha_4) \sin \delta_h + \alpha_2 (s_2 c_{\delta h} + c_{\delta b}) \right] \]

\[ A_{2h} = \sqrt{2} \frac{V_h}{2b} \left[-(s_2 + \alpha_4) \sin \delta_h - \alpha_2 (s_2 c_{\delta h} + c_{\delta b}) \right] \]

where
\[ c_{6h} = \alpha_2 c_{1h} \left[ 2a(a^2 + B)\sin\delta_h + (3a^2 - B + h^2)h\cos\delta_h \right] \]

Hence the output voltage is determined for real characteristic roots and its complete solution is as follows

\[
V_o = \sqrt{2} \sum_h \frac{V_h}{2b} \left[ (s_1 + \alpha_1)\sin\delta_h + \alpha_2(s_1c_{5h} + c_{6h})e^{s_2\theta} - (s_2 + \alpha_1)\sin\delta_h \right. \\
+ \left. \alpha_2(s_1c_{5h} + c_{6h})e^{s_2\theta} \right] + \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h\theta + \alpha_2 \frac{c_{7h}}{h} \sin h\theta \right) \quad (A-39)
\]

Similarly, \( i_s \) is solved for real characteristic roots as follows

\[
i_s = \sqrt{2}\alpha_2 \sum_h \frac{V_h}{2b} \left[ -\sin\delta_h - s_1c_{2h} + c_{3h} \right] e^{s_2\theta} + \left( \sin\delta_h + s_2c_{2h} - c_{3h} \right) e^{s_2\theta} \\
+ \sqrt{2}\alpha_2 \sum_h V_h \left( c_{2h} \cos h\theta + \frac{c_{4h}}{h} \sin h\theta \right) \quad (A-40)
\]

where

\[
c_{3h} = c_{1h} \left\{ (a^2 + B)(a^2 + B - h^2 + 2a\alpha_4)\sin\delta_h + h(\alpha_4(3a^2 - B + h^2) + 2a(a^2 + B))\cos\delta_h \right\}
\]

If \( \frac{(\alpha_1 - \alpha_4)^2}{4} < \alpha_2\alpha_3 \), the characteristic roots of both (A-5) and (A-7) are complex, and the general solution of \( V_o \) are as follows

\[
V_o = e^{s_2\theta} \left( A_1 \cos b\theta + A_2 \sin b\theta \right) + \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h\theta + \alpha_2 \frac{c_{7h}}{h} \sin h\theta \right) \quad (A-41)
\]
where

\[ a = -\frac{\alpha_1 + \alpha_4}{2}, \quad b = \sqrt{\alpha_2\alpha_3 - \left(\frac{\alpha_1 - \alpha_4}{2}\right)^2} \]

Solve \( V_o \) first. When \( \theta = 0 \), (A-41) becomes

\[ V_o = V_{ih}(0) = \sqrt{2} \sum_h V_h \sin \delta_h \]

\[ = A_1 + \sqrt{2} \sum_h V_h (-\alpha_2 c_{2h}) \]

Therefore

\[ A_1 = \sqrt{2} \sum_h V_h (\sin \delta_h + \alpha_2 c_{2h}) \]  \( \text{(A-43)} \)

Similar to the solution of \( V_o \) with real characteristic roots, evaluate \( V_{ih} \) in (1) and the first derivative of \( V_o \) in (A-41) at \( \theta = 0 \), substitute the results and the initial input current, i.e., \( i_v(0) = 0 \), into (A-34). Then we have

\[ V_o'(0) = -\frac{V_o(0)}{\omega CR_{eq}} = -\alpha_4 \sqrt{2} \sum_h V_h \sin \delta_h \]

\[ = aA_1 + bA_2 + \sqrt{2} \sum_h V_h \alpha_2 c_{2h} \]
Solve for $A_2$

$$A_2 = \sqrt{2} \sum_h \frac{V_h}{b} \left[ (a + \alpha_1) \sin \delta_h + \alpha_2 (ac_{5h} + c_{6h}) \right]$$

The final solution for the output voltage is then

$$V_o = \sqrt{2} \sum_h \frac{V_h}{b} \left[ \left\{ (a + \alpha_1) \sin \delta_h + \alpha_2 (ac_{5h} + c_{6h}) \right\} \sin b \theta \right. \left. \right]$$

$$b \left[ \sin \delta_h + \alpha_2 c_{3h} \right] \cos b \theta e^{\alpha \theta}$$

$$+ \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h \theta + \alpha_2 \frac{c_{2h}}{h} \sin h \theta \right)$$

(A-44)

Similarly

$$i_s = \sqrt{2} \alpha_2 \sum_h \frac{V_h}{b} \left[ \left\{ -\sin \delta_h - ac_{2h} + c_{3h} \right\} \sin b \theta - bc_{2h} \cos b \theta \right] e^{\alpha \theta}$$

$$+ \sqrt{2} \alpha_2 \sum_h V_h \left( c_{2h} \cos h \theta + \frac{c_{4h}}{h} \sin h \theta \right)$$

(A-45)

Thus finish the solutions for the differential equations describing the circuit during positive conduction period. The equation for $i_s$ will exactly the same during the negative conduction period except that the phase angle is shifted by $\pi$ and the sign is negative. A simple summary of all solutions and the parameters is listed below.
When \( \frac{(\alpha_1 - \alpha_4)^2}{4} > \alpha_3 \alpha_5 \),

\[
i_s = \sqrt{2} \alpha_2 \sum_h V_h \left[ -\sin \delta_h - s_1 c_{2h} + c_{3h} \right] e^{\phi_i \theta} + \left\{ \sin \delta_h + s_2 c_{2h} - c_{3h} \right\} e^{\phi_j \theta} \\
+ \sqrt{2} \alpha_2 \sum_h V_h \left( c_{2h} \cos h \theta + \frac{c_{4h}}{h} \sin h \theta \right)
\]  
(A-46)

\[
V_o = \sqrt{2} \sum_h V_h \left[ (s_1 + \alpha_1) \sin \delta_h + \alpha_2 \left( s_1 c_{5h} + c_{6h} \right) \right] e^{\phi_i \theta} - \left( s_2 + \alpha_1 \right) \sin \delta_h \]
\[+ \alpha_2 \left( s_2 c_{5h} + c_{6h} \right) e^{\phi_j \theta} + \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h \theta + \alpha_2 \frac{c_{7h}}{h} \sin h \theta \right)
\]
(A-47)

where \( s_{1,2} = -\frac{\alpha_1 + \alpha_4}{2} \pm \sqrt{\frac{(\alpha_1 - \alpha_4)^2}{4} - \alpha_2 \alpha_3} = a \pm b; \)

When \( \frac{(\alpha_1 - \alpha_4)^2}{4} < \alpha_3 \alpha_5 \),

\[
i_s = \sqrt{2} \alpha_2 \sum_h V_h \left[ -\sin \delta_h - ac_{2h} + c_{3h} \right] \sin b \theta - bc_{2h} \cos b \theta \right] e^{\phi_i \theta} \\
+ \sqrt{2} \alpha_2 \sum_h V_h \left( c_{2h} \cos h \theta + \frac{c_{4h}}{h} \sin h \theta \right)
\]  
(A-48)

\[
V_o = \sqrt{2} \sum_h V_h \left[ \left( a + \alpha_1 \right) \sin \delta_h + \alpha_2 (ac_{5h} + c_{6h}) \right] \sin b \theta \]
\[b \left[ \sin \delta_h + \alpha_2 c_{5h} \right] \cos b \theta \right] e^{\phi_i \theta} \]
\[+ \sqrt{2} \sum_h V_h \left( -\alpha_2 c_{5h} \cos h \theta + \alpha_2 \frac{c_{7h}}{h} \sin h \theta \right)
\]
(A-49)

where \( a = -\frac{\alpha_1 + \alpha_4}{2}, \ b = \sqrt{\frac{\alpha_2 \alpha_3}{4} - \frac{(\alpha_1 - \alpha_4)^2}{4}} . \)
In both case, the \( c \) parameters' definition is the same. Let

\[
B = \alpha_2 \alpha_3 - \frac{(\alpha_1 - \alpha_4)^2}{4}, \text{ then }
\]

\[
c_{1h} = \frac{1}{4a^2 h^2 + (h^2 - a^2 - B)^2}
\]

\[
c_{2h} = c_{1h} \left\{ \sin \delta_h \left[ -2ah^2 + \alpha_4 \left( a^2 + B - h^2 \right) \right] + h \cos \delta_h \left( a^2 + B - h^2 + 2a \alpha_4 \right) \right\}
\]

\[
c_{3h} = \sin \delta_h + 2ac_{2h} - c_{4h}
\]

\[
= c_{1h} \left\{ \left( a^2 + B \right) \left( a^2 + B - h^2 + 2a \alpha_4 \right) \sin \delta_h \\
+ h \left( \alpha_4 \left( 3a^2 - B + h^2 \right) + 2a \left( a^2 + B \right) \right) \cos \delta_h \right\}
\]

\[
c_{4h} = c_{1h} \left\{ \sin \delta_h \left[ -2a \alpha_4 + \left( h^2 - a^2 - B \right) \right] h^2 + h \cos \delta_h \left( a^2 + B - h^2 \right) \left( \alpha_4 - 2ah^2 \right) \right\}
\]

\[
c_{5h} = \alpha_3 c_{1h} \left\{ \left( h^2 - a^2 - B \right) \sin \delta_h - 2ah \cos \delta_h \right\}
\]

\[
c_{6h} = -2ac_{5h} - c_{7h}
\]

\[
= \alpha_3 c_{1h} \left[ 2a \left( a^2 + B \right) \sin \delta_h + \left( 3a^2 - B + h^2 \right) h \cos \delta_h \right]
\]

\[
c_{7h} = \alpha_3 c_{1h} \left\{ -2ah^2 \sin \delta_h + \left( a^2 + B - h^2 \right) h \cos \delta_h \right\}
\]

\[D. \text{ Frequency domain expressions } i_s\]

With the closed form expressions of the input currents in time domain determined, it is possible to derive the Fourier series at various harmonics.
1. Real Roots

\[ i_s = \sqrt{2} \alpha_2 \sum_h \frac{V_h}{2b} \left[ -\sin \delta_h - s_4 c_{2h} + c_{3h} \right] e^{s_4 \theta} + \left[ \sin \delta_h + s_2 c_{2h} - c_{3h} \right] e^{s_2 \theta} \]

\[ + \sqrt{2} \alpha_2 \sum_h V_h \left( c_{2h} \cos h \theta + \frac{c_{4h}}{h} \sin h \theta \right) \]

where

\[ s_{1,2} = -\frac{\alpha_4 + \alpha_4}{2} \pm \sqrt{\frac{(\alpha_4 - \alpha_4)^2}{4} - \alpha_2 \alpha_4} = a \pm b; \]

Let

\[ c_{sh} = \sqrt{2} \alpha_2 \frac{V_h}{2b} \left( -\sin \delta_h - c_{2h} s_4 + c_{3h} \right) \]

\[ c_{sh} = \sqrt{2} \alpha_2 \frac{V_h}{2b} \left( \sin \delta_h + c_{2h} s_2 - c_{3h} \right) \]

The expression of input current is rewritten as follows

\[ i_s = \sum_h \left[ c_{sh} e^{s_4 \theta} + c_{sh} e^{s_2 \theta} \right] + \sqrt{2} \alpha_2 \sum_h V_h \left( c_{2h} \cos h \theta + \frac{c_{4h}}{h} \sin h \theta \right) \]

(A-50)

Since the expression of \( i_s \) is very complicated, the Fourier series are calculated for each term in (A-50) and add them together.

First term, \( \sum_h c_{sh} e^{s_4 \theta} \)
\[a_{k_1} = \frac{2}{\pi} \int_0^{\theta_1} \sum_h c_{8h} e^{s_1 \theta} \cos k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h c_{8h} \int_0^{\theta_1} e^{s_1 \theta} \cos k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h \frac{c_{8h}}{s_1^2 + k^2} e^{s_1 \theta} (k \sin k\theta + s_1 \cos k\theta)\bigg|_0^{\theta_1}\]
\[= \frac{2}{\pi} \sum_h \frac{c_{8h}}{s_1^2 + k^2} \left[e^{s_1 \theta_1} (k \sin k\theta_1 + s_1 \cos k\theta_1) - s_1 \right]
\]

\[b_{k_1} = \frac{2}{\pi} \int_0^{\theta_1} \sum_h c_{8h} e^{s_1 \theta} \sin k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h c_{8h} \int_0^{\theta_1} e^{s_1 \theta} \sin k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h \frac{c_{8h}}{s_1^2 + k^2} e^{s_1 \theta} (s_1 \sin k\theta - k \cos k\theta)\bigg|_0^{\theta_1}\]
\[= \frac{2}{\pi} \sum_h \frac{c_{8h}}{s_1^2 + k^2} \left[e^{s_1 \theta_1} (s_1 \sin k\theta_1 - k \cos k\theta_1) + k \right]
\]

Second term, \(\sum c_{9h} e^{s_2 \theta}\)

\[a_{k_2} = \frac{2}{\pi} \int_0^{\theta_2} \sum_h c_{9h} e^{s_2 \theta} \cos k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h c_{9h} \int_0^{\theta_2} e^{s_2 \theta} \cos k\theta \, d\theta\]
\[= \frac{2}{\pi} \sum_h \frac{c_{9h}}{s_2^2 + k^2} e^{s_2 \theta} (k \sin k\theta + s_2 \cos k\theta)\bigg|_0^{\theta_2}\]
\[= \frac{2}{\pi} \sum_h \frac{c_{9h}}{s_2^2 + k^2} \left[e^{s_2 \theta_2} (k \sin k\theta_2 + s_2 \cos k\theta_2) - s_2 \right]\]
\[ b_{h^2} = \frac{2}{\pi} \sum_{h} c_{0h} e^{\epsilon h^{2}} \sin k\theta \, d\theta \]

\[ = \frac{2}{\pi} \sum_{h} c_{0h} \int_{0}^{\theta_{0}} e^{\epsilon s^{2}} \sin k\theta \, d\theta \]

\[ = \frac{2}{\pi} \sum_{h} \frac{c_{0h}}{s^{2}_{2} + k^{2}} \left[ e^{\epsilon s^{2}} (s_{2} \sin k\theta_{2} - k \cos k\theta_{2}) \right]_{0}^{\theta_{0}} \]

\[ = \frac{2}{\pi} \sum_{h} \frac{c_{0h}}{s^{2}_{2} + k^{2}} \left[ e^{\epsilon s^{2}} (s_{2} \sin k\theta_{2} - k \cos k\theta_{2}) + k \right] \]

Third term, \( \sqrt{2} \alpha_{2} \sum_{h} V_{h} c_{2h} \cos h\theta \)

\[ a_{k3} = \frac{2}{\pi} \int_{0}^{\theta_{0}} \sqrt{2} \alpha_{2} \sum_{h} V_{h} c_{2h} \cos h\theta \cos k\theta \, d\theta \]

\[ = \frac{\sqrt{2} \alpha_{2}}{\pi} \sum_{h} V_{h} c_{2h} \int_{0}^{\theta_{0}} (\cos (h + k)\theta + \cos (h - k)\theta) \, d\theta \]

\[ = \frac{\sqrt{2} \alpha_{2}}{\pi} V_{k} c_{2k} \left( \frac{\sin 2k\theta_{2}}{2k} + \theta_{2} \right) + \frac{\sqrt{2} \alpha_{2}}{\pi} \sum_{h \neq k} V_{h} c_{2h} \left[ \frac{\sin (h + k)\theta_{2}}{h + k} + \frac{\sin (h - k)\theta_{2}}{h - k} \right] \]

\[ b_{k3} = \frac{2}{\pi} \int_{0}^{\theta_{0}} \sqrt{2} \alpha_{2} \sum_{h} V_{h} c_{2h} \cos h\theta \sin k\theta \, d\theta \]

\[ = \frac{\sqrt{2} \alpha_{2}}{\pi} \sum_{h} V_{h} c_{2h} \int_{0}^{\theta_{0}} (\sin (h + k)\theta - \sin (h - k)\theta) \, d\theta \]

\[ = \frac{\sqrt{2} \alpha_{2}}{2k\pi} V_{k} c_{2k} (1 - \cos 2k\theta_{2}) \]

\[ + \frac{\sqrt{2} \alpha_{2}}{\pi} \sum_{h \neq k} V_{h} c_{2h} \left[ \frac{\cos (h - k)\theta_{2}}{h - k} - \frac{\cos (h + k)\theta_{2}}{h + k} - \frac{2k}{h^{2} - k^{2}} \right] \]

Fourth term \( \sqrt{2} \alpha_{2} \sum_{h} V_{h} \frac{c_{4h}}{h} \sin h\theta \)
Finally, the total Fourier series coefficients are $a_k = a_{k1} + a_{k2} + a_{k3} + a_{k4}$, and

$$b_k = b_{k1} + b_{k2} + b_{k3} + b_{k4}$$

2. Complex Roots

$$i_s = \sqrt{2} \alpha_2 \sum_h V_h \left[ \delta_h \sin (b \theta) - c_{4h} \cos b \theta \right] e^{i \theta}$$

where $a = \frac{\alpha_1 + \alpha_4}{2}$, $b = \sqrt{\frac{\alpha_2 \alpha_3 - (\alpha_1 - \alpha_4)^2}{4}}$.
\[
\lambda_n = \frac{\sqrt{2} \alpha \sqrt{b h}}{b} \sqrt{\left( -\sin \delta - ac z h + c z h \right)^2 + \left( c z h \right)^2}
\]
\[
\tan \varphi = \frac{-bc z h}{-\sin \delta - ac z h + c z h}
\]

The expression of input current is rewritten as follows

\[
i_x = \sum \lambda_n \sin(b \theta + \varphi) e^{a \theta} + \sqrt{2} \alpha \sum h \left( c z h \cos h \theta + \frac{c h}{h} \sin h \theta \right)
\]

(A-51)

The \(a_k\) and \(b_k\) for the second and third terms will be the same as those for the third and fourth terms in the real roots case. Only the \(a_k\) and \(b_k\) for the first term need to be determined.

\[
a_k = \frac{2}{\pi} \int_0^{\delta_1} \sum \lambda_n \sin(b \theta + \varphi) e^{a \theta} \cos k \theta d\theta
\]
\[
= \sum \lambda_n \int_0^{\delta_1} \frac{e^{a \theta}}{\pi} \left[ \sin[(b + k) \theta + \varphi] + \sin[(b - k) \theta + \varphi] \right] d\theta
\]
\[
= \sum \lambda_n \left\{ \frac{e^{a \theta}}{\pi} \left[ \sin((b + k) \theta + \varphi) - (b + k) \cos((b + k) \theta + \varphi) \right]_{\delta_1}^{\delta_2} + \frac{e^{a \theta}}{a^2 + (b + k)^2} \left[ \sin((b - k) \theta + \varphi) - (b - k) \cos((b - k) \theta + \varphi) \right]_{\delta_1}^{\delta_2} \right\}
\]

Let \(\delta_1 = \frac{1}{\sqrt{a^2 + (b + k)^2}}\), \(\delta_2 = \frac{1}{\sqrt{a^2 + (b - k)^2}}\), \(\tan \phi_1 = \frac{b + k}{a}\), and \(\tan \phi_2 = \frac{b - k}{a}\)
\[ a_{k1} = \sum_{h} \frac{\lambda_h}{\pi} \delta_1 e^{a_{\theta}} \sin \left[(b + k)\theta + \varphi_h - \phi_1 \right]_0^{\theta_2} + \sum_{h} \frac{\lambda_h}{\pi} \delta_2 e^{a_{\theta}} \sin \left[(b - k)\theta + \varphi_h - \phi_2 \right]_0^{\theta_2} \]

\[ = \sum_{h} \frac{\lambda_h}{\pi} \delta_1 \left\{ e^{a_{\theta}} \sin \left[(b + k)\theta_2 + \varphi_h - \phi_1 \right] - \sin(\varphi_h - \phi_1) \right\} \\
+ \sum_{h} \frac{\lambda_h}{\pi} \delta_2 \left\{ e^{a_{\theta}} \sin \left[(b - k)\theta_2 + \varphi_h - \phi_2 \right] - \sin(\varphi_h - \phi_2) \right\} \]

\[ b_{k1} = \frac{2}{\pi} \sum_{h} \lambda_h \sin(b\theta + \varphi_h) e^{a_{\theta}} \sin k\theta d\theta \]

\[ = \sum_{h} - \frac{\lambda_h}{\pi} \left\{ \int_{0}^{\theta_2} e^{a_{\theta}} \left[ \cos((b + k)\theta + \varphi_h) - \cos((b - k)\theta + \varphi_h) \right] d\theta \right\} \]

\[ = \sum_{h} - \frac{\lambda_h}{\pi} \left\{ \frac{e^{a_{\theta}}}{a^2 + (b + k)^2} \left[ (b + k) \sin((b + k)\theta + \varphi_h) + a \cos((b + k)\theta + \varphi_h) \right]_0^{\theta_2} \right\} \]

\[ - \frac{e^{a_{\theta}}}{a^2 + (b - k)^2} \left[ (b - k) \sin((b - k)\theta + \varphi_h) + a \cos((b - k)\theta + \varphi_h) \right]_0^{\theta_2} \]

Using the same definitions for \( \delta_{1,2} \) and \( \tan \phi_{1,2} \), the equation for \( b_{k1} \) is rewritten as follows

\[ b_{k1} = \sum_{h} - \frac{\lambda_h}{\pi} \delta_1 e^{a_{\theta}} \cos((b + k)\theta + \varphi_h - \phi_1)_{0}^{\theta_2} + \sum_{h} \frac{\lambda_h}{\pi} \delta_2 e^{a_{\theta}} \cos((b - k)\theta + \varphi_h - \phi_2)_{0}^{\theta_2} \]

\[ = \sum_{h} \frac{\lambda_h}{\pi} \delta_2 \left\{ e^{a_{\theta}} \cos((b - k)\theta_2 + \varphi_h - \phi_2) - \cos(\varphi_h - \phi_2) \right\} \]

\[ - \sum_{h} \frac{\lambda_h}{\pi} \delta_1 \left\{ e^{a_{\theta}} \cos((b + k)\theta_2 + \varphi_h - \phi_1) - \cos(\varphi_h - \phi_1) \right\} \]

Finally, the total Fourier series coefficients are \( a_k = a_{k1} + a_{k3} + a_{k4} \), and

\[ b_k = b_{k1} + b_{k3} + b_{k4} \]
II. Multiphase Harmonic Load Flow Program Code

The multiphase harmonic load flow algorithm in this dissertation is developed using MATLAB® 5.3 (R11) from The MathWorks, Inc. A complete list of programs in this dissertation is presented below, followed by the detailed code for each program.

A. Programs list

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assemble_config5 branch series impedance and shunt admittance at the given line length for line configuration type 605

**Single phase rectifier subroutine**

sphr single phase rectifier subprogram: import rectifier parameters, calculate input current Fourier series for given terminal voltage
c_cal calculate the constants used in the single phase rectifier model
is_comp input current waveform generation for complex characteristic roots
is_real input current waveform generation for real characteristic roots
isf_comp input current Fourier series for complex characteristic roots
isf_real input current Fourier series for real characteristic roots
Vo_comp output voltage waveform generation for complex characteristic roots during conduction
Vo_real output voltage waveform generation for real characteristic roots during conduction
Vdecay output voltage discharge waveform generation

**Load flow calculation**

main main program of load flow calculation
load_curr_inj calculate linear load current injection
sphr_curr combine the harmonic current from rectifier with the other linear load injection by frequency
branch_curr_cal collect injections at the receiving end of each branch and calculate the current division between neutral and earth
busout screen print the load flow result after convergence

**B. Program Codes**

assemble_config1_2
% Function [Zse,Ysh]=assemble_config1_2(H,Z,leng,Rg)
% Function for incremental model assembling for configuration #601 & #602 at all frequencies
% The input:
% Z series impedance Z for per unit length (mile)
% leng feeder length in miles
% Rg pole grounding resistance
% The output:
% Zse series admittance of the assembled model [4, 4, harm#]
% Ysh shunt admittance of the assembled model [4, 4, harm#]
% The function first calculates the total series impedance of the feeder
% Then the impedance is divided into increment of 200 feet long. The number of increments is rounded to the least value. The fractional length of the feeder is divided in two and included in the first and the last spans. The resulting spans are assembled using incidence matrix method. The nodes in the middle are eliminated by Kron reduction to obtain an equivalent pi model.
% The grounding resistances at the both ends are not included in this time.
% Since the phases are full in these config, no need to consider the phasing.

function [Zse,Ysh]=assemble_config1_2(H,Z,leng,Rg)
% Total series impedance
Ztot=Z*leng/5280;
% Impedance division among spans
span_num=ceil(leng/1320);
if span_num > 2
    for i=2:span_num-1
        span(i)=1320;
    end
    span(1)=(leng-1320*(span_num-2))/2; span(span_num)=span(1);
Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif span_num == 2
    span(1)=leng/2; span(2)=span(1);
Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif \text{span\_num} == 1
    \text{Z\_span}=\text{Ztot};
end

\% Define the dimension for the series impedance and shunt admittance matrix
\text{Zse}=\text{zeros}(4,4,\text{H});
\text{Yse}=\text{zeros}(4,4,\text{H});

\% Admittance matrix for each span
for \text{l}=1:\text{span\_num}
    for \text{m}=1:\text{H}
        \text{ZZ}=\text{Z\_span}((\text{m}-1)*4+1:(\text{m}-1)*4+4,:,\text{l});
        \text{Y\_span}((\text{m}-1)*4+1:(\text{m}-1)*4+4,:,\text{l})=\text{ZZ}^{-1};
    end
end

\% if \text{span\_num} == 1
    for \text{m}=1:\text{H}
        \text{Zse}(:,:,\text{m})=\text{Z\_span}((\text{m}-1)*4+1:(\text{m}-1)*4+4,:);
        \text{Ysh}(:,:,\text{m})=\text{zeros}(4,4);
    end
else
    \text{branch}=\text{span\_num}\times2-1;
    \text{node}=\text{span\_num}+1;

    \% Incidence matrix
    \text{In\_matrix}=\text{zeros}((\text{branch}\times4,\text{node}\times4));
    for \text{l}=1:\text{span\_num}
        \text{In\_matrix}((\text{l}-1)*4+1:(\text{l}-1)*4+4,\text{l}*4+1:(\text{l}-1)*4+4)=\text{eye}(4);
        \text{In\_matrix}((\text{l}-1)*4+1:(\text{l}-1)*4+4,\text{l}*4+4:1*4+4)=-\text{eye}(4);
    end
    for \text{l}=\text{span\_num}+1:\text{branch}
        \text{In\_matrix}((\text{l}-1)*4+1:(\text{l}-1)*4+4,\text{l}-\text{span\_num}*4+1:(\text{l}-\text{span\_num})*4+4)=\text{eye}(4);
    end

    \% Primitive branch impedance matrix
    \text{Y\_prim}=\text{zeros}((\text{branch}\times4,\text{branch}\times4));
    for \text{l}=1:\text{H}
        \% \text{l}\ =\ harmonic\ order
        for \text{m}=1:\text{span\_num}
            \text{Y\_prim}((\text{m}-1)*4+1:(\text{m}-1)*4+4,(\text{m}-1)*4+1:(\text{m}-1)*4+4)=\text{Y\_span}((\text{l}-1)*4+1:(\text{l}-1)*4+4,:,\text{m});
        end
        for \text{m}=(\text{span\_num}+1):\text{branch}
            \text{Y\_prim}((\text{m}-1)*4+1:(\text{m}-1)*4+4,(\text{m}-1)*4+1:(\text{m}-1)*4+4)=\text{Y\_span}((\text{l}-1)*4+1:(\text{l}-1)*4+4,:,\text{m});
        end
    end
end
Ybus assembled
Ybus=In_matrix*Y_prim*In_matrix;

Kron reduction
P=(node-1)*4; Q=node*4-3;
A=Ybus(1:4,1:4); B=Ybus(1:4,5:P); C=Ybus(1:4,Q:node*4);
D=Ybus(5:P,1:4); E=Ybus(5:P,5:P); F=Ybus(5:P,Q:node*4);
G=Ybus(Q:node*4,1:4); H=Ybus(Q:node*4,5:P); I=Ybus(Q:node*4,Q:node*4);
AA=A-(B*E^(-1)*D); BB=C-(B*E^(-1)*F);
CC=G-(H*E^(-1)*D); DD=I-(H*E^(-1)*F);
Ybus1=[AA BB; CC DD];
Zse(:,:,l)=(-BB)^(-1);
Ysh(:,:,l)=(AA+BB);

assemble_config3

function [Zse,Ysh]=assemble_config3(H,Z,leng,Rg)

Total series impedance
Ztot=Z*leng/5280;

The function first calculates the total series impedance of the feeder
Then the impedance is divided into increment of 200 feet long. The number
of increments is rounded to the least value. The fractional length of the
feeder is divided in two and included in the first and the last spans. The
resulting spans are assembled using incidence matrix method. The nodes in
the middle are eliminated by Kron reduction to obtain an equivalent pi model.
The grounding resistances at the both ends are not included in this time.
The feeder phasing is taken into account in the processs.
\% Impedance division among spans
span_num=ceil(leng/1320);
if span_num > 2
    for i=2:span_num-1
        span(i)=1320;
    end
    span(1)=(leng-1320*(span_num-2))/2; span(span_num)=span(1);
    Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif span_num == 2
    span(1)=leng/2; span(2)=span(1);
    Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif span_num == 1
    Z_span=Ztot; \% Z_span = [harm\#*4,4,span\#]
end
\% Define the dimension for the series impedance and shunt admittance matrix
Zse=zeros(4,4,H);
Yse=zeros(4,4,H);
\% Admittance matrix for each span
for l=1:span_num
    for m=1:H
        \% Extract nonzero components from the singular matrix B C N
        Z1_span=Z_span((m-1)*4+2:(m-1)*4+4,2:4,l);
        Y_span((m-1)*3+1:(m-1)*3+3,1:3,l)=Z1_span^-1; \% Y_span = [harm\#3,3,span\#]
    end
end
\%
\% if span_num == 1
for m=1:H
    Zse(:,:,m)=Z_span((m-1)*4+1:(m-1)*4+4,:);
    Ysh(:,:,m)=zeros(4,4);
end
else
    \% The admittance matrix for each span is now a 3 by 3 matrix
    branch=span_num*2-1;
node=span_num+1;
% Incidence matrix
In_matrix=zeros(branch*3,node*3);
for l=1:span_num
    In_matrix((l-1)*3+1:(l-1)*3+3,(l-1)*3+1:(l-1)*3+3)=eye(3);
    In_matrix((l-1)*3+1:(l-1)*3+3,(l-1)*3+1:(l-1)*3+3)=-eye(3);
end
for l=span_num+1:branch
    In_matrix((l-1)*3+1:(l-1)*3+3,(l-span_num)*3+1:(l-span_num)*3+3)=eye(3);
end
% Primitive branch impedance matrix
Y_prim=zeros(branch*3,branch*3);
Zse=zeros(4,4,H);
Ysh=zeros(4,4,H);
for l=1:H
    % harm order
    for m=1:span_num
        Y_prim((m-1)*3+1:(m-1)*3+3,(m-1)*3+1:(m-1)*3+3)=Y_span((l-1)*3+1:(l-1)*3+3,:)/Rg;
    end
    for m=(span_num+1):branch
        Y_prim(m*3,m*3)=1/Rg;
    end
    % Ybus assembled
    Ybus=In_matrix'*Y_prim*In_matrix;
    % Kron reduction
    P=(node-1)*3; Q=node*3-2;
    A=Ybus(1:3,1:3); B=Ybus(1:3,4:P); C=Ybus(1:3,Q:node*3);
    D=Ybus(4:P,1:3); E=Ybus(4:P,4:P); F=Ybus(4:P,Q:node*3);
    G=Ybus(Q:node*3,1:3); H=Ybus(Q:node*3,4:P); I=Ybus(Q:node*3,Q:node*3);
    AA=A-B*E^(-1)*D; BB=C-B*E^(-1)*F;
    CC=G-H*E^(-1)*D; DD=I-H*E^(-1)*F;
    Ybus1=[AA BB;CC DD];
    % Zero padding
    Zse(2:4,2:4,l)=-(BB)^(-1);
    Ysh(2:4,2:4,l)=(AA+BB);
end

assemble_config4

% Function [Yse,Ysh]=assemble_config4(H,Z,leng,Rg)
% Function for incremental model assembling for configuration #3
% The input:
% Z series impedance Z for per unit length (mile)
% leng feeder length in miles
% Rg pole grounding resistance
% The output:
% Zse series admittance of the assembled model [4, 4, harm#]
% Ysh shunt admittance of the assembled model [4, 4, harm#]
% The function first calculates the total series impedance of the feeder
% Then the impedance is divided into increment of 200 feet long. The number
% of increments is rounded to the least value. The fractional length of the
% feeder is divided in two and included in the first and the last spans. The
% resulting spans are assembled using incidence matrix method. The nodes in
% the middle are eliminated by Kron reduction to obtain an equivalent pi model.
% The grounding resistances at the both ends are not included in this time.
% The feeder phasing is taken into account in the process.

function [Zse,Ysh]=assemble_config4(H,Z,leng,Rg)
% Total series impedance
Ztot=Z*leng/5280; % Ztot = [harm#*4, 4]
% Impedance division among spans
span_num=ceil(leng/1320);
if span_num > 2
    for i=2:span_num-1
        span(i)=1320;
    end
    span(1)=(leng-1320*(span_num-2))/2; span(span_num)=span(1);
    Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif span_num == 2
    span(1)=leng/2; span(2)=span(1);
    Z_span=zeros(H*4,4,span_num);
    for l=1:span_num
        Z_span(:,:,l)=span(l)/leng*Ztot;
    end
elseif span_num == 1
    Z_span=Ztot; % Z_span = [harm#*4,4,span#]
% Define the dimension for the series impedance and shunt admittance matrix
Zse=zeros(4,4,H);
Yse=zeros(4,4,H);
% Admittance matrix for each span
for l=1:span_num
    for m=1:H
        % Extract nonzero components from the singular matrix
        Z1_span(1,1)=Z_span((m-1)*4+1,1,l);
        Z1_span(2:3,1)=Z_span((m-1)*4+3:(m-1)*4+4,1,l);
        Z1_span(1,2:3)=Z_span((m-1)*4+1,3:4,l);
        Z1_span(2:3,2:3)=Z_span((m-1)*4+3:(m-1)*4+4,3:4,l);
        %Z1_span
        pause
        Y_span((m-1)*3+1:(m-1)*3+3,1:3,l)=Z1_span^(-1);
        %Y_span=[harm#*3, 3, span#]
    end
end
% if span_num == 1
% for m=1:H
%     Zse(:,m)=Z_span((m-1)*4+1:(m-1)*4+4,:);
%     Ysh(:,m)=zeros(4,4);
% end
else
    branch=span_num*2-1;
    node=span_num+1;
    % Incidence matrix
    In_matrix=zeros(branch*3,node*3);
    for l=1:span_num
        In_matrix((l-1)*3+1:(l-1)*3+3,(l-1)*3+3+1:(l-1)*3+3)=eye(3);
        In_matrix((l-1)*3+1:(l-1)*3+3,l*3+1:l*3+3)=-eye(3);
    end
    for l=span_num+1:branch
        In_matrix((l-1)*3+1:(l-1)*3+3,(l-span_num)*3+1:(l-span_num)*3+3)=eye(3);
    end
    % Primitive branch impedance matrix
    Y_prim=zeros(branch*3,branch*3);
    for l=1:H
        % Harm order
        for m=1:span_num
Y\_prim((m-1)*3+1:(m-1)*3+3,(m-1)*3+1:(m-1)*3+3)=Y\_span((l-1)*3+1:(l-1)*3+3,:,:m);
end
for m=(span\_num+1):branch
    Y\_prim(m*3,m*3)=1/Rg;
end
% Ybus assembled
Ybus=In\_matrix\cdot Y\_prim\cdot In\_matrix;
% Kron reduction
P=(node-1)*3; Q=node*3-2;
A=Ybus(1:3,1:3); B=Ybus(1:3,4:P); C=Ybus(1:3,Q:node*3);
D=Ybus(4:P,1:3); E=Ybus(4:P,4:P); F=Ybus(4:P,Q:node*3);
G=Ybus(Q:node*3,1:3); H=Ybus(Q:node*3,4:P); I=Ybus(Q:node*3,Q:node*3);
AA=A-B\cdot E^{-1}\cdot D; BB=C-B\cdot E^{-1}\cdot F;
CC=G-H\cdot E^{-1}\cdot D; DD=I-H\cdot E^{-1}\cdot F;
Ybus1=[AA BB; CC DD];
% Zero padding
Zse1=(BB)^{-1}; Ysh1=(AA+BB);
Zse(1,1,l)=Zse1(1,1); Ysh(1,1,l)=Ysh1(1,1);
Zse(3:4,1,l)=Zse1(2:3,1); Ysh(3:4,1,l)=Ysh1(2:3,1);
Zse(1,3:4,l)=Zse1(1,2:3); Ysh(1,3:4,l)=Ysh1(1,2:3);
Zse(3:4,3:4,l)=Zse1(2:3,2:3); Ysh(3:4,3:4,l)=Ysh1(2:3,2:3);
end
end

assemble\_config5

% Function [Yse,Ysh]=assemble\_config5(H,Z,leng,Rg)
% % Function for incremental model assembling for configuration #605 at % all frequencies
% % The input:
% % Z series impedance Z for per unit length (mile)
% % leng feeder length in miles
% % Rg pole grounding resistance
% % The output:
% % Zse series admittance of the assembled model [4, 4, harm#]
% % Ysh shunt admittance of the assembled model [4, 4, harm#]
% The function first calculates the total series impedance of the feeder
% Then the impedance is divided into increment of 200 feet long. The number
% of increments is rounded to the least value. The fractional length of the
% feeder is divided in two and included in the first and the last spans. The
% resulting spans are assembled using incidence matrix method. The nodes in
% the middle are eliminated by Kron reduction to obtain an equivalent pi model.
% The grounding resistances at the both ends are not included in this time.
% The feeder phasing is taken into account in the processs.

function [Zse,Ysh]=assemble_config5(H,Z,leng,Rg)
% Total series impedance
Ztot=Z*leng/5280; % Ztot = [harm#*4, 4]
% Impedance division among spans
span_num=ceil(leng/1320);
if span_num > 2
   for i=2:span_num-1
      span(i)=1320;
   end
   span(1)=(leng-1320*(span_num-2))/2; span(span_num)=span(1);
   Z_span=zeros(H*4,4,span_num);
   for l=1:span_num
      Z_span(:,:,l)=span(l)/leng*Ztot;
   end
elseif span_num == 2
   span(1)=leng/2; span(2)=span(1);
   Z_span=zeros(H*4,4,span_num);
   for l=1:span_num
      Z_span(:,:,l)=span(l)/leng*Ztot;
   end
elseif span_num == 1
   Z_span=Ztot; % Z_span = [harm#*4,4,span#]
end
% Define the dimension for the series impedance and shunt admittance matrix
Zse=zeros(4,4,H);
Yse=zeros(4,4,H);
% Admittance matrix for each span
for l=1:span_num
   for m=1:H
      % Extract nonzero components from the singular matrix
      Z1_span=Z_span((m-1)*4+3:(m-1)*4+4,3:4,l);%pause
      Y_span((m-1)*2+1:(m-1)*2+2,1:2,l)=Z1_span^-1; % primitive admittance
      =\hbox{[harm#*2, 2, span#]}

end
end

% if span_num == 1
for m=1:H
    Zse(:,:,m)=Z_span((m-1)*4+1:(m-1)*4+4,:);
    Ysh(:,:,m)=zeros(4,4);
end
else
    branch=span_num*2-1;
    node=span_num+1;
    % Incidence matrix
    In_matrix=zeros(branch*2,node*2);
    for l=1:span_num
        In_matrix((l-1)*2+1:(l-1)*2+2,(l-1)*2+1:(l-1)*2+2)=eye(2);
        In_matrix((l-1)*2+1:(l-1)*2+2,l*2+1:l*2+2)=-eye(2);
    end
    for l=span_num+1:branch
        In_matrix((l-1)*2+1:(l-1)*2+2,(l-span_num)*2+1:(l-span_num)*2+2)=eye(2);
    end
    % Primitive branch impedance matrix
    Y_prim=zeros(branch*2,branch*2);
    for l=1:H
        % harm order
        for m=1:span_num
            Y_prim((m-1)*2+1:(m-1)*2+2,(m-1)*2+1:(m-1)*2+2)=Y_span((l-1)*2+1:(l-1)*2+2,:,m);
        end
        for m=(span_num+1):branch
            Y_prim(m*2,m*2)=1/Rg;
        end
    end
    % Ybus assembled
    Ybus=In_matrix*Y_prim*In_matrix;
    % Kron reduction
    P=(node-1)*2; Q=node*2-1;
    A=Ybus(1:2,1:2); B=Ybus(1:2,3:P); C=Ybus(1:2,Q:node*2);
    D=Ybus(3:P,1:2); E=Ybus(3:P,3:P); F=Ybus(3:P,Q:node*2);
    G=Ybus(Q:node*2,1:2); H=Ybus(Q:node*2,3:P); I=Ybus(Q:node*2,Q:node*2);
    AA=A-(B*E\*1*D); BB=C-(B*E\*1*F);
    CC=G-(H*E\*1*D); DD=I-(H*E\*1*F);
    Ybus1=[AA BB;CC DD];
% Zero padding
Zse(3:4,3:4,1)=(-BB)^-1; % The zero value in Zse does not mean short circuit, just not be given
Ysh(3:4,3:4,1)=(AA+BB); % any value, while the zero value in Ysh actually means zero
end
end

cfg_601

% Function cfg_601(H) Feeder Series Impedance Calculation for Config#601
% Load the feeder spacing data, calculate the series impedance per unit length
% then transmute the impedance matrix according the phasing specification.
% The earth return path is accounted for by using the Carson's line formula.
% Output: config_601_prim.wk1 = [real(H*4,4) \ imag(H*4,4)]

function config_601(H,p)
% Generate Spacing Matrix (Spacing 500)
Spacing=wk1read('spacing.wk1');
% S value
S500=zeros(4,4);
S500(1,1)=Spacing(1,1);
S500(1,2)=Spacing(2,1); S500(2,1)=S500(1,2);
S500(1,3)=Spacing(3,1); S500(3,1)=S500(1,3);
S500(1,4)=Spacing(4,1); S500(4,1)=S500(1,4);
S500(2,2)=Spacing(5,1);
S500(2,3)=Spacing(6,1); S500(3,2)=S500(2,3);
S500(2,4)=Spacing(7,1); S500(4,2)=S500(2,4);
S500(3,3)=Spacing(8,1);
S500(3,4)=Spacing(9,1); S500(4,3)=S500(3,4);
S500(4,4)=Spacing(10,1);
% D value
D500=zeros(4,4);
D500(1,2)=Spacing(11,1); D500(2,1)=D500(1,2);
D500(1,3)=Spacing(12,1); D500(3,1)=D500(1,3);
D500(1,4)=Spacing(13,1); D500(4,1)=D500(1,4);
D500(2,3)=Spacing(14,1); D500(3,2)=D500(2,3);
D500(2,4)=Spacing(15,1); D500(4,2)=D500(2,4);
D500(3,4)=Spacing(16,1); D500(4,3)=D500(3,4);
% Value specified for this configuration type
D500(1,1)=0.0244; D500(2,2)=0.0244; D500(3,3)=0.0244;
D500(4,4)=0.0244;
% theta value
theta=zeros(4,4);
theta(1,2)=Spacing(1,2);
theta(2,1)=theta(1,2);
theta(1,3)=Spacing(2,2);
theta(3,1)=theta(1,3);
theta(1,4)=Spacing(3,2);
theta(4,1)=theta(1,4);
theta(2,3)=Spacing(4,2);
theta(3,2)=theta(2,3);
theta(2,4)=Spacing(5,2);
theta(4,2)=theta(2,4);
theta(3,4)=Spacing(6,2);
theta(4,3)=theta(3,4);
% Primitive series impedances per unit length
G=1.6095e-4;
% Specified conductor resistance per unit length
R(1)=0.306; R(2)=R(1); R(3)=R(1); R(4)=0.306;
% Transmutation matrix
B=[0 1 0 0; 1 0 0 0; 0 0 1 0; 0 0 0 1]; % transmute 1 and 2
% Harmonics 1 ~ 2*H-1
for k=1:H
    w=2*pi*60*(2*k-1);
    freq=(2*k-1)*60;
    for i=1:4
        [P,Q]=pqfun(S500(i,i),theta(i,i),freq,p);
        Z1(i,i)=R(i)+4*w*P*G+j*(2*w*log(S500(i,i)/D500(i,i))+4*w*Q)*G;
    end
    for l=1:4
        for m=1:4
            if l ~= m
                [P,Q]=pqfun(S500(l,m),theta(l,m),freq,p);
                Z1(l,m)=4*w*P*G+j*(2*w*log(S500(l,m)/D500(l,m))+4*w*Q)*G;
            end
        end
    end
    % Transmutation according to the phasing configuration
    Z1=B*Z1*B;
end
% Assemble the sub-matrix in an increasing order of freqs
for l=1:4
    for m=1:4
        Z((k-1)*4+l,m)=Z1(l,m);
    end
end
end

% 
Z_real=real(Z); Z_imag=imag(Z);
ZZ=[Z_real Z_imag];
wk1write('config_601_prim.wk1',ZZ);

config_602

% Function config_602(H) Feeder Series Impedance Calculation for Config#602
% 
% Load the feeder spacing data, calculate the series impedance per unit length
% then transmute the impedance matrix according the phasing specification.
% The earth return path is accounted for by using the Carson's line formula.
% 
% Output: config_602_prim.wk1 = [real (H*4,4) | imag (H*4,4)]

function config_602(H,p)
% Generate Spacing Matrix (Spacing 500)
Spacing=wk1read('spacing.wk1');
% S value
S500=zeros(4,4);
S500(1,1)=Spacing(1,1);
S500(1,2)=Spacing(2,1); S500(2,1)=S500(1,2);
S500(1,3)=Spacing(3,1); S500(3,1)=S500(1,3);
S500(1,4)=Spacing(4,1); S500(4,1)=S500(1,4);
S500(2,2)=Spacing(5,1);
S500(2,3)=Spacing(6,1); S500(3,2)=S500(2,3);
S500(2,4)=Spacing(7,1); S500(4,2)=S500(2,4);
S500(3,3)=Spacing(8,1);
S500(3,4)=Spacing(9,1); S500(4,3)=S500(3,4);
S500(4,4)=Spacing(10,1);
% D value
D500=zeros(4,4);
D500(1,2)=Spacing(11,1); D500(2,1)=D500(1,2);
D500(1,3)=Spacing(12,1); D500(3,1)=D500(1,3);
D500(1,4)=Spacing(13,1); D500(4,1)=D500(1,4);
D500(2,3)=Spacing(14,1); D500(3,2)=D500(2,3);
D500(2,4)=Spacing(15,1); D500(4,2)=D500(2,4);
D500(3,4)=Spacing(16,1); D500(4,3)=D500(3,4);
% Value specified for this configuration type
D500_2=D500;
D500_2(1,1)=0.00814; D500_2(2,2)=0.00814; D500_2(3,3)=0.00814;
D500_2(4,4)=0.00814;
% theta value
theta=zeros(4,4);
theta(1,2)=Spacing(1,2); theta(2,1)=theta(1,2);
theta(1,3)=Spacing(2,2); theta(3,1)=theta(1,3);
theta(1,4)=Spacing(3,2); theta(4,1)=theta(1,4);
theta(2,3)=Spacing(4,2); theta(3,2)=theta(2,3);
theta(2,4)=Spacing(5,2); theta(4,2)=theta(2,4);
theta(3,4)=Spacing(6,2); theta(4,3)=theta(3,4);
% Primitive series impedances per unit length
G=1.6095e-4;
% Specified conductor resistance per unit length
R(1)=0.592; R(2)=R(1); R(3)=R(1); R(4)=0.592;
% Transmutation matrix
B1=[1 0 0 0;0 1 0 0;0 0 0 1]; % transmute 2 and 3
B2=[0 1 0 0;1 0 0 0;0 0 1 0;0 0 0 1]; % transmute 1 and 2
% Harmonics 1 2*H-1
for k=1:H
    w=2*pi*60*(2*k-1);
    freq=(2*k-1)*60;
    for i=1:4
        [P,Q]=pqfun(S500(i,i),theta(i,i),freq*60,p);
        Z1(i,i)=R(i)+4*w*P*G+j*(2*w*log(S500(i,i)/D500_2(i,i))+4*w*Q)*G;
    end
    for l=1:4
        for m=1:4
            if l ~= m
                [P,Q]=pqfun(S500(l,m),theta(l,m),freq,p);
                Z1(l,m)=4*w*P*G+j*(2*w*log(S500(l,m)/D500_2(l,m))+4*w*Q)*G;
            end
        end
    end
end
%Z1
Transmutation according to the phasing configuration

\[ Z_1 = B_1 Z_1 B_1 \]
\[ Z_1 = B_2 Z_1 B_2 \]

Assemble the sub-matrix in an increasing order of freqs for \( l = 1:4 \)

\[
\text{for } m = 1:4 \\
Z((k-1)*4+l,m) = Z_1(l,m) \\
\text{end}
\]

\[
\text{end}
\]

\[
\text{end}
\]

\[
\%
\]

\[
Z_{\text{real}} = \text{real}(Z); Z_{\text{imag}} = \text{imag}(Z); \]

\[
\text{ZZ} = [Z_{\text{real}} \ Z_{\text{imag}}];
\]

\[
\text{wk1write('config_602_prim.wk1',ZZ);}
\]

\[
\text{config_603}
\]

\[
\%
\]

\[
\text{Function config_603(H) Feeder Series Impedance Calculation for Config#603}
\]

\[
\%
\]

\[
\text{Load the feeder spacing data, calculate the series impedance per unit length}
\]

\[
\%
\]

\[
\text{then transmute the impedance matrix according the phasing specification.}
\]

\[
\%
\]

\[
\text{The earth return path is accounted for by using the Carson's line formula.}
\]

\[
\%
\]

\[
\%
\]

\[
\text{Output: config_603_prim.wk1 = [real (H*4,4) \ | \ imag (H*4,4)]}
\]

\[
\text{function config_603(H,p)}
\]

\[
\%
\]

\[
\text{Generate Spacing Matrix}
\]

\[
\text{Spacing=wk1read('spacing.wk1');}
\]

\[
\%
\]

\[
\text{S value}
\]

\[
S505 = \text{zeros}(3,3);
\]

\[
S505(1,1) = \text{Spacing}(1,3);
\]

\[
S505(1,2) = \text{Spacing}(2,3); S505(2,1) = S505(1,2);
\]

\[
S505(1,3) = \text{Spacing}(3,3); S505(3,1) = S505(1,3);
\]

\[
S505(2,2) = \text{Spacing}(4,3);
\]

\[
S505(2,3) = \text{Spacing}(5,3); S505(3,2) = S505(2,3);
\]

\[
S505(3,3) = \text{Spacing}(6,3);
\]

\[
\%
\]

\[
\text{D value}
\]

\[
D505 = \text{zeros}(3,3);
\]

\[
D505(1,2) = \text{Spacing}(7,3); D505(2,1) = D505(1,2);
\]
D505(1,3)=Spacing(8,3); D505(3,1)=D505(1,3);
D505(2,3)=Spacing(9,3); D505(3,2)=D505(2,3);
% Value specified for this configuration type
D505(1,1)=0.00446; D505(2,2)=0.00446;
D505(3,3)=0.00446;
% theta value
theta=zeros(3,3);
theta(1,2)=Spacing(1,4); theta(2,1)=theta(1,2);
theta(1,3)=Spacing(2,4); theta(3,1)=theta(1,3);
theta(2,3)=Spacing(3,4); theta(3,2)=theta(2,3);
% Primitive series impedances per unit length
G=1.6095e-4;
% Specified conductor resistance per unit length
R(1)=1.12; R(2)=R(1); R(3)=1.12;
% Transmutation matrix
B1=[1 0 0;0 1 0;0 0 1]; % transmute 3 and 4
B2=[0 0 1;0 1 0;1 0 0]; % transmute 1 and 3
% Harmonics 1 ~ 2*H-1
for k=1:H
    w=2*pi*60*(2*k-1);
    freq=(2*k-1)*60;
    Z1=zeros(4,4); % initialize Z1 and set the dimension
    for i=1:3
        [P,Q]=pqfun(S505(i,i),theta(i,i),freq,p);
        Z1(i,i)=R(i)+4*w*P*G+j*(2*w*log(S505(i,i)/D505(i,i))+4*w*Q)*G;
    end
    for l=1:3
        for m=1:3
            if l ~= m
                [P,Q]=pqfun(S505(l,m),theta(l,m),freq,p);
                Z1(l,m)=4*w*P*G+j*(2*w*log(S505(l,m)/D505(l,m))+4*w*Q)*G;
            end
        end
    end
    % Transmutation according to the phasing configuration (Missing phase is left as zeros)
    Z1=B1*Z1*B1;
    Z1=B2*Z1*B2;
% Assemble the sub-matrix in an increasing order of freqs
for l=1:4
for m=1:4
    Z((k-1)*4+l,m)=Z1(l,m);
end
end

% Z_real=real(Z); Z_imag=imag(Z);
ZZ=[Z_real Z_imag];
wk1write('config_603_prim.wk1',ZZ);

config_604

% Function config_604(H) Feeder Series Impedance Calculation for Config#604
% Load the feeder spacing data, calculate the series impedance per unit length
% then transmute the impedance matrix according the phasing specification.
% The earth return path is accounted for by using the Carson's line formula.
%
% Output: config_604_prim.wk1 = [real (H*4,4) | imag (H*4,4)]

function config_604(H,p)
    % Generate Spacing Matrix
    Spacing=wk1read('spacing.wk1');
    % S value
    S505=zeros(3,3);
    S505(1,1)=Spacing(1,3);
    S505(1,2)=Spacing(2,3); S505(2,1)=S505(1,2);
    S505(1,3)=Spacing(3,3); S505(3,1)=S505(1,3);
    S505(2,2)=Spacing(4,3);
    S505(2,3)=Spacing(5,3); S505(3,2)=S505(2,3);
    S505(3,3)=Spacing(6,3);
    % D value
    D505=zeros(3,3);
    D505(1,2)=Spacing(7,3); D505(2,1)=D505(1,2);
    D505(1,3)=Spacing(8,3); D505(3,1)=D505(1,3);
    D505(2,3)=Spacing(9,3); D505(3,2)=D505(2,3);
    % Value specified for this configuration type
    D505(1,1)=0.00446; D505(2,2)=0.00446;
    D505(3,3)=0.00446;
\% theta value
theta=zeros(3,3);
theta(1,2)=Spacing(1,4); theta(2,1)=theta(1,2);
theta(1,3)=Spacing(2,4); theta(3,1)=theta(1,3);
theta(2,3)=Spacing(3,4); theta(3,2)=theta(2,3);
\%
% Primitive series impedances per unit length
G=1.6095e-4;
\%
% Specified conductor resistance per unit length
R(1)=1.12; R(2)=R(1); R(3)=1.12;
\%
% Transmutation matrix
B1=[1 0 0 0; 0 1 0 0; 0 0 0 1; 0 0 1 0]; \%
\% transmute between 3 and 4
B2=[1 0 0 0; 0 0 1 0; 0 1 0 0; 0 0 0 1]; \%
\% transmute between 2 and 3
\%
% Harmonics 1 2*H-1
for k=1:H
w=2*pi*60*(2*k-1);
freq=(2*k-1)*60;
Z1=zeros(4,4);
for i=1:3
\[P,Q\]=pqfun(S505(i,i),theta(i,i),freq,p);
Z1(i,i)=R(i)+4*w*P*G+j*(2*w*log(S505(i,i)/D505(i,i))+4*w*Q)*G;
end
for l=1:3
for m=1:3
if l ~= m
\[P,Q\]=pqfun(S505(l,m),theta(l,m),freq,p);
Z1(l,m)=4*w*P*G+j*(2*w*log(S505(l,m)/D505(l,m))+4*w*Q)*G;
end
end
end
\%
% Transmutation according to the phasing configuration (the missing phase is
left as zeros)
Z1=B1*Z1*B1;
Z1=B2*Z1*B2;
\%
% Assemble the sub-matrix in an increasing order of freqs
for l=1:4
for m=1:4
Z((k-1)*4+l,m)=Z1(l,m);
end
end
\%pause
% Function config_605(H) Feeder Series Impedance Calculation for Config#605
% Load the feeder spacing data, calculate the series impedance per unit length
% then transmute the impedance matrix according the phasing specification.
% The earth return path is accounted for by using the Carson’s line formula.
% Output: config_605_prim.wk1 = [real (H*4,4) | imag (H*4,4)]

function config_605(H,p)
% Generate Spacing Matrix
Spacing=wk1read('spacing.wk1');
% S value
S510=zeros(2,2);
S510(1,1)=Spacing(1,5);
S510(1,2)=Spacing(2,5);
S510(2,1)=S510(1,2);
S510(2,2)=Spacing(3,5);
% D value
D510=zeros(2,2);
D510(1,2)=Spacing(4,5);
D510(2,1)=D510(1,2);
% Value specified for this configuration type
D510(1,1)=0.00446;
D510(2,2)=0.00446;
% theta value
theta=zeros(2,2);
% Primitive series impedances per unit length
G=1.6095e-4;
% Specified conductor resistance per unit length
R(1)=1.12; R(2)=1.12;
% Transmutation matrix
B1=[1 0 0 0;0 0 0 1;0 0 1 0;0 1 0 0]; % transmute between 2 and 4
B2=[0 0 1 0;0 1 0 0;1 0 0 0;0 0 0 1]; % transmute between 1 and 3
% Harmonics 1 ~ 2*H-1
for k=1:H
    w=2*pi*60*(2*k-1);
    freq=(2*k-1)*60;
    Z1=zeros(4,4);
    for i=1:2
        [P,Q]=pqfun(S510(i,i),theta(i,i),freq,p);
        Z1(i,i)=R(i)+4*w*P*G+j*(2*w*log(S510(i,i)/D510(i,i))+4*w*Q)*G;
    end
    for l=1:2
        for m=1:2
            if l ~= m
                [P,Q]=pqfun(S510(l,m),theta(l,m),freq,p);
                Z1(l,m)=4*w*P*G+j*(2*w*log(S510(l,m)/D510(l,m))+4*w*Q)*G;
            end
        end
    end
% Transmutation according to the phasing configuration (the missing phase is left as zeros)
    Z1=B1*Z1*B1;
    Z1=B2*Z1*B2;
    for l=1:4
        for m=1:4
            Z((k-1)*4+l,m)=Z1(l,m);
        end
    end
% pause
end

% Z_real=real(Z); Z_imag=imag(Z);
ZZ=[Z_real Z_imag];
wk1write('config_605_prim.wk1',ZZ);

    config_702

% Function config_702 Distribution transformer impedance for config #702
%
% Calculate the impedance for the transformer config #702
% Output: config_702_prim.wk1 = [real (4*4) | imag (4*4)]
function config_702
Sb=5;
S=10; V1=44; V2=12.47;
R=0.; X=.07;
% per unit impedance on its own base
ZZ=(R+j*X)*Sb/S;
Z=ZZ*eye(4);
Z(4,4)=0;    % the neutral is specified as zero
%
config_702_data=[real(Z) imag(Z)];
wk1write('config_702.wk1',config_702_data)

% Spacing Calculation
%  
%  Feeder spacing parameters are calculated according to the spacing  type.
%  The output data would include the distances between conductors and the
%  distances between the conductors to the images. The corresponding angles
%  between the impedances for the P,Q functions are also calculated. The
%  output is then exported to the wk1 file "spacing.wk1" for impedance
%  calculation.
%
% spacing.wk1 = [16*6]

clear all
Spacing=zeros(16,6);
% Spacing data for each spacing type is output in two columns, with the first
% column for distances and the second column for the angles.
%
% spacing #500
% Distance column is of length of 16 numbers. 1~10 for S parameters (distances
% between conductors and images), 11~16 for D parameters (distances between
% conductors, not including the diagonal terms).
    h=24; a=4; b=3.5; c=1;d=0.5;
%
S(10)=2*h;       %S44
S(1)=2*(h+a);    %S11
S(5)=S(1);S(8)=S(1);  %S22 & S33
S(2)=sqrt(S(1)^2+(2*b)^2);    %S12
S(3)=sqrt(S(1)^2+(b-c)^2);    %S13
\[ S(4) = \sqrt{(2h+a)^2 + (b+d)^2}; \quad \%S14 \]
\[ S(6) = \sqrt{S(1)^2 + (b+c)^2}; \quad \%S23 \]
\[ S(7) = \sqrt{(2h+a)^2 + (b-d)^2}; \quad \%S24 \]
\[ S(9) = \sqrt{(2h+a)^2 + (c+d)^2}; \quad \%S34 \]
\%
\[ D(1) = 2b; \quad \%D12 \]
\[ D(2) = b - c; \quad \%D13 \]
\[ D(3) = \sqrt{(b+d)^2 + a^2}; \quad \%D14 \]
\[ D(4) = b + c; \quad \%D23 \]
\[ D(5) = \sqrt{(b-d)^2 + a^2}; \quad \%D24 \]
\[ D(6) = \sqrt{(c+d)^2 + a^2}; \quad \%D34 \]
\%
Angle column is of length of 6. All the angles are in radian.
\[ \theta(1) = \arccos\left(\frac{S(1)}{S(2)}\right); \quad \%\theta12 \]
\[ \theta(2) = \arccos\left(\frac{S(1)}{S(3)}\right); \quad \%\theta13 \]
\[ \theta(3) = \arccos\left(\frac{(2h+a)}{S(4)}\right); \quad \%\theta14 \]
\[ \theta(4) = \arccos\left(\frac{S(1)}{S(6)}\right); \quad \%\theta23 \]
\[ \theta(5) = \arccos\left(\frac{(2h+a)}{S(7)}\right); \quad \%\theta24 \]
\[ \theta(6) = \arccos\left(\frac{(2h+a)}{S(9)}\right); \quad \%\theta34 \]
\%
\[ \text{Spacing}(;1) = [S, D]'; \]
\[ \text{Spacing}(1:6,2) = \theta'; \]
\[ \text{clear} \ S \ D \ \theta \]
\%
Distance column is of length of 9 numbers with 7 zeros padded at the end
\%
1~6 for S parameters, 7~9 for D parameters
\[ h = 24; \quad a = 4; \quad b = 3.5; d = 0.5; \]
\%
\[ S(1) = 2(h+a); \quad \%S11 \]
\[ S(4) = S(1); \quad \%S22 \]
\[ S(6) = 2h; \quad \%S44 \]
\[ S(2) = \sqrt{S(1)^2 + (2b)^2}; \quad \%S12 \]
\[ S(3) = \sqrt{(2h+a)^2 + (b+d)^2}; \quad \%S14 \]
\[ S(5) = \sqrt{(2h+a)^2 + (b-d)^2}; \quad \%S24 \]
\%
\[ D(1) = 2b; \quad \%D12 \]
\[ D(2) = \sqrt{a^2 + (b+d)^2}; \quad \%D14 \]
\[ D(3) = \sqrt{a^2 + (b-d)^2}; \quad \%D24 \]
\%
Angle column is of length of 3. All the angles are in radian.
\[ \theta(1) = \arccos\left(\frac{S(1)}{S(2)}\right); \quad \%\theta12 \]
\[ \theta(2) = \arccos\left(\frac{(2h+a)}{S(3)}\right); \quad \%\theta14 \]
\[ \text{theta}(3) = \cos((2h+a)/S(5)); \]  % theta24
\[ \text{Spacing}(1:6,3)=S'; \text{Spacing}(7:9,3)=D'; \text{Spacing}(1:3,4)=\text{theta'}; \]
clear S D theta

\% Spacing #510
\% Distance column is of length of 4 numbers with 12 zeros padded at the end
\% h=24; a=5; d=0.5;
\%
\[ S(1)=2*(h+a); \]  % S11
\[ S(2)=(2*h+a); \]  % S14
\[ S(3)=2*h; \]  % S44
\[ D=a; \]  % D14
\%
\[ \text{Spacing}(1:3,5)=S'; \text{Spacing}(4,5)=D; \]
\%
\[ \text{wk1write('spacing.wk1',Spacing);} \]

pqfun

\% Function: P & Q Constants Evaluation
\%
\% [P,Q]=pqfun(S,theta,f,pp) calculate the P&Q constants according to the
\% Carson's line formulas for both self and mutual impedances.
\%
\% f = frequency
\% pp = earth resistivity
\% For self impedance,
\% Sii = twice the aerial conductor height
\% theta = 0
\%
\% For mutual impedance,
\% Sij = distance between the aerial conductor to the image of the other
\% conductor
\% theta = angle between Sii and Sij

function [P,Q]=pqfun(S,theta,f,pp)
K=8.565e-4*S*sqrt(f/pp);
P=pi/8-1/(3*sqrt(2))*K*cos(theta)+K^2/16*cos(2*theta)*(0.6728+log(2/K));
Q=−0.0386+1/2*log(2/K)+1/(3*sqrt(2))*K*cos(theta);

sphr

% Single Phase Rectifier Model
function
[Iamp,Iang,theta1,theta2,alpha,delta,a,b,c,B]=sphr1(Vth,theta,Rt,Lt,C,Req);
H=length(Vth);
w=2*pi*60;
% Convert the input angle for cosine oriented to sine oriented
for l=1:H
    theta(l)=theta(l)+90;
end
% Shift the waveform by angle of fund. voltage
theta_shift=theta(1);
for l=1:H
    theta_prim=theta(l)-(2*l-1)*theta_shift;
    theta(l)=theta_prim-floor(theta_prim/360)*360;
end
% Calculate circuit parameters
alpha(1)=Rt/(w*Lt);alpha(2)=1/(w*Lt);alpha(3)=1/(w*C);alpha(4)=1/(w*C*Req);
a=−(alpha(1)+alpha(4))/2;
B=alpha(2)*alpha(3)-(alpha(1)-alpha(4))^2/4;
% Determine real or complex routine
if B < 0
    Vo_fun='Vo_real';
    is_fun='is_real';
    isf_fun='isf_real';
    b=sqrt(-B);
elseif B > 0
    Vo_fun='Vo_comp';
    is_fun='is_comp';
    isf_fun='isf_comp';
    b=sqrt(B);
end
% Initial guess 80 degree after the positive going zero-crossing for a sine wave
theta1=70;
% Initialize iteration related variables
maxiter=100;iter=0;
accuracy=0.01;error=100;
del=0.0001;
options=optimset('TolX',del,'Display','off');
sphr_err=0;
% Iterations start
while iter < maxiter & error > accuracy
    iter=iter+1;
    % c & delta parameters related to the theta1
    for l=1:H
        h=2*l-1;
        delta(l)=theta(l)+h*theta1;
    end
    c=c_cal(a,B,alpha,delta);
    % find the first positive point on the current waveform
    for i=1:20
        is_test=feval(is_fun,i,Vth,delta,alpha,a,b,c);
        if is_test > 0
            break
        end
    end
    % calculate the theta2 with axis shifted to theta1
    err=0;
    while err ~=1
        theta2=fzero(is_fun,i,options,Vth,delta,alpha,a,b,c);%theta1,theta2
        if theta2 > i
            err=1;
        end
        i=i+1;
    end
    theta2=theta2-floor(theta2/360)*360;
    % calculate the output voltage at the end of conduction using calculated theta2
    Vo=feval(Vo_fun,theta2,Vth,delta,alpha,a,b,c);
    % shift it back to the source voltage reference
    theta2=theta2+theta1;
    % calculate the new theta1
    theta1_new=fzero('Vdecay',theta1,options,theta2,Vo,Vth,theta,alpha);
    theta1_new=theta1_new-floor(theta1_new/360)*360;
    error=abs(theta1-theta1_new);
    theta1=theta1_new;
    if iter == maxiter
        sphr_err=1;
fprintf(' Error in single phase rectifier routine \n');
break
end
end

% Calculate the Fourier series terms
if sphr_err ~= 1

% Update the values for all parameters related to theta1
for l=1:H
   h=2*l-1;
   delta(l)=theta(l)+h*theta1;
end

c=c_cal(a,B,alpha,delta);
[Iamp,Iang]=feval(isf_fun,theta1,theta2,Vth,delta,alpha,a,b,c);
end

% Back shift phase angle to the system original reference
for l=1:H
   Iang(l)=Iang(l)+(2*l-1)*theta_shift-90;
   Iang(l)=Iang(l)-floor(Iang(l)/360)*360;
end

t=0:.1:359.9;
Vs=zeros(size(t));
is=Vs;Vo=Vs;
theta1=theta1-floor(theta1/360)*360;
theta2=theta2-floor(theta2/360)*360;

c_cal

% Calculation of the value for parameter c
function c=c_cal(a,B,alpha,delta)
H=length(delta);
for l=1:H
   h=2*l-1;
   c(1,l)=1/(4*a^2*h^2+(h^2-a^2-B)^2);
   c(2,l)=c(1,l)*((-2*a*h^2+alpha(4)*(a^2+B-h^2))*sin(delta(l)/180*pi)...
      +(a^2+B-h^2+2*a*alpha(4))^h*cos(delta(l)/180*pi));
   c(3,l)=c(1,l)*((a^2+B)*a^2+B-h^2+2*a*alpha(4))^h*sin(delta(l)/180*pi)...
      +((3*a^2-B-h^2)*alpha(4)+2*a*(a^2+B))^h*cos(delta(l)/180*pi));
   c(4,l)=c(1,l)*((h^2-a^2-B)*a*alpha(4))^h*sin(delta(l)/180*pi)...
      +((a^2+B-h^2)*alpha(4)-2*a*h^2)^h*cos(delta(l)/180*pi));
   c(5,l)=alpha(3)*c(1,l)*((h^2-a^2-B)^h*sin(delta(l)/180*pi)-2*a*h^2*cos(delta(l)/180*pi));
\[
c(6,l) = \alpha(3) c(1,l) \left( 2a (a^2 + B) \sin(\delta(l)/180\pi) + (3a^2 + h^2 - B)h \cos(\delta(l)/180\pi) \right);
\]
\[
c(7,l) = \alpha(3) c(1,l) \left( -2a h^2 \sin(\delta(l)/180\pi) + (a^2 + B - h^2)h \cos(\delta(l)/180\pi) \right);
\]
\]
\end{verbatim}

\texttt{is\_comp}

\% is function with real characteristic roots
function is = is\_comp(ang, V, delta, alpha, a, b, c);
H = length(V);
is = 0;
ang = ang/180*pi;
A1 = 0; A2 = 0;
for \( l = 1: H \)
    \[ h = 2l - 1; \]
    \[ A1 = -\sin(\delta(l)/180\pi) - a c(2,l) + c(3,l); \]
    \[ A2 = -b c(2,l); \]
    isp = sqrt(2)*V(l)*alpha(2)*(c(2,l)*cos(h*ang)+c(4,l)/h*sin(h*ang));
    is1 = sqrt(2)*alpha(2)*V(l)/(2*b)*(A1*exp(s1*ang)+A2*exp(s2*ang))+isp;
    is = is + is1;
end

\texttt{is\_real}

\% is function with real characteristic roots
function is = is\_real(ang, V, delta, alpha, a, b, c);
H = length(V);
is = 0;
s1 = a+b; s2 = a-b;
ang = ang/180*pi;
A1 = 0; A2 = 0;
for \( l = 1: H \)
    \[ h = l^2 - 1; \]
    \[ A1 = -\sin(\delta(l)/180\pi) - s1 c(2,l) + c(3,l); \]
    \[ A2 = \sin(\delta(l)/180\pi) + s2 c(2,l) - c(3,l); \]
    isp = sqrt(2)*V(l)*alpha(2)*(c(2,l)*cos(h*ang)+c(4,l)/h*sin(h*ang));
    is1 = sqrt(2)*alpha(2)*V(l)/(2*b)*(A1*exp(s1*ang)+A2*exp(s2*ang))+isp;
    is = is + is1;
end
isf_comp

%Fourier series closed-form solution for real-roots case
function [Iamp,Iang,aa,bb]=isf_comp(theta1,theta2,V,delta,alpha,a,b,c);
H=length(V);
s1=a+b; s2=a-b;
theta=(theta2-theta1)/180*pi;
delta=delta/180*pi;
for m=1:H
    k=2*m-1;
a1=0; b1=0;
a3=a1; a4=a1;
b3=b1; b4=b1;
T1=1/sqrt(a^2+(b+k)^2); Tang1=angle(a+j*(b+k));
T2=1/sqrt(a^2+(b-k)^2); Tang2=angle(a+j*(b-k));
for n=1:H
    h=2*n-1;
lamida=sqrt(2)*alpha(2)*V(n)/b*sqrt((-sin(delta(n))-
a*c(2,n)+c(3,n))^2+(b*c(2,n))^2);
p=-sin(delta(n))-a*c(2,n)+c(3,n); q=-b*c(2,n);
phi=angle(p+j*q);
a1=a1+lamida/pi*T1*(exp(a*theta)*sin((b+k)*theta+phi-
Tang1)-
    sin(phi-
    Tang1))...
    -lamida/pi*T1*(exp(a*theta)*cos((b+k)*theta+phi-
    Tang1)-
    cos(phi-
    Tang1));
    b1=b1+lamida/pi*T2*(exp(a*theta)*sin((b-k)*theta+phi-
    Tang2)-
    sin(phi-
    Tang2))...
    -lamida/pi*T2*(exp(a*theta)*cos((b-k)*theta+phi-
    Tang2)-
    cos(phi-
    Tang2));
    if n ~= m
        a3=a3+lamida/pi*T1*(exp(a*theta)*sin((b+k)*theta+phi-
                        Tang1)-
                        sin((b+k)*theta))/((h+k)*theta)/(h-k));
        b3=b3+lamida/pi*T2*(exp(a*theta)*cos((b+k)*theta+phi-
                        Tang1)-
                        cos((b+k)*theta))/((h+k)*theta)/(h-k)-
                        2*sin((h+k)*theta)/(h^2-k^2));
        a4=a4+lamida/pi*T3*(exp(a*theta)*sin((b-k)*theta+phi-
                        Tang2)-
                        sin((b-k)*theta))/((h+k)*theta)/(h-k));
        b4=b4-sqrt(2)*alpha(2)/pi*V(n)*c(4,n)/h*(sin((h-k)*theta)/(h-k)-
                        sin((h+k)*theta)/(h+k));
    end
end
a3=a3+sqrt(2)*alpha(2)/pi*V(m)*c(2,m)*sin(2*k*theta)/(2*k+theta);
b3=b3+sqrt(2)*alpha(2)/(2*k*pi)*V(m)*c(2,m)*(1-cos(2*k*theta));
\[
a_4 = a_4 + \sqrt{2} \cdot \alpha(2)/(2 \cdot k^2 \cdot \pi) \cdot V(m) \cdot c(4, m) \cdot (1 - \cos(2 \cdot k \cdot \theta));
\]
\[
b_4 = b_4 + \sqrt{2} \cdot \alpha(2)/(k \cdot \pi) \cdot V(m) \cdot c(4, m) \cdot (\theta - \sin(2 \cdot k \cdot \theta)/(2 \cdot k));
\]
\[
aa(m) = a_1 + a_3 + a_4;
bb(m) = b_1 + b_3 + b_4;
I_{amp}(m) = \frac{\text{abs}(aa(m) + j \cdot bb(m))}{\sqrt{2}};
\]
\[
\text{ang} = \text{angle}(bb(m) + j \cdot aa(m))/\pi \cdot 180 - \theta_{1} \cdot k;
\]
\[
\text{I}_{\text{ang}}(m) = \text{ang} - \text{floor}(\text{ang}/360) \cdot 360;
\]
\[
\text{if } I_{\text{ang}}(m) > 180 \quad \text{then}
\]
\[
I_{\text{ang}}(m) = I_{\text{ang}}(m) - 360;
\]
\[
\text{end}
\]
\[
\text{end}
\]

\textit{isf\_real}

\%Fourier series closed-form solution for real-roots case
function \[I_{\text{amp}}, I_{\text{ang}}, aa, bb\] = isf\_real(\theta_{1}, \theta_{2}, V, \delta_{\text{eta}}, \alpha, a, b, c);

H1 = 900;
H = length(V);
Vth = zeros(1, H1);
delta1 = Vth;
c = zeros(7, H1);

for l = 1: H
    Vth(l) = V(l);
delta1(l) = delta(l);
    for m = 1: 7
        cc(m, l) = c(m, l);
    end
end

V = Vth; delta = delta1; c = cc;
s1 = a + b; s2 = a - b;
a1 = 0; b1 = 0;
a2 = a1; a3 = a1; a4 = a1;
b2 = b1; b3 = b1; b4 = b1;
theta = (theta2 - theta1)/180*\pi;
delta = delta/180*\pi;
for m = 1: H1
    k = 2*m - 1;
a1 = 0; b1 = 0;
a2 = a1; a3 = a1; a4 = a1;
b2=b1;b3=b1;b4=b1;
for \( l=1:H \)
    \( h=2*l-1; \)
    \[ c8=\sqrt{2}\alpha(2)V(l)/(2b)(-\sin(delta(l))-c(2,l)s1+c(3,l)); \]
    \[ c9=\sqrt{2}\alpha(2)V(l)/(2b)(\sin(delta(l))+c(2,l)s2-c(3,l)); \]
    \[ a1=a1+2/\pi c8/(s1^2+k^2)(\exp(s1*theta)(k\sin(k*theta)+s1\cos(k*theta))-s1); \]
    \[ b1=b1+2/\pi c8/(s1^2+k^2)(\exp(s1*theta)(s1\sin(k*theta)-k\cos(k*theta))+k); \]
    \[ a2=a2+2/\pi c9/(s2^2+k^2)(\exp(s2*theta)(k\sin(k*theta)+s2\cos(k*theta))-s2); \]
    \[ b2=b2+2/\pi c9/(s2^2+k^2)(\exp(s2*theta)(s2\sin(k*theta)-k\cos(k*theta))+k); \]
    if \( l \neq m \)
        \[ a3=a3+\sqrt{2}\alpha(2)/\pi V(l)c(2,l)(\sin((h+k)*theta)/(h+k)+\sin((h-k)*theta)/(h-k)); \]
        \[ b3=b3+\sqrt{2}\alpha(2)/\pi V(l)c(2,l)(\cos((h-k)*theta)/(h-k)-\cos((h+k)*theta)/(h+k)-2k/(h^2-k^2)); \]
        \[ a4=a4+\sqrt{2}\alpha(2)/\pi V(l)c(4,l)/h(2h/(h^2-k^2)-\cos((h+k)*theta)/(h+k)-\cos((h-k)*theta)/(h-k)); \]
        \[ b4=b4+\sqrt{2}\alpha(2)/\pi V(l)c(4,l)/h(\sin((h-k)*theta)/(h-k)-\sin((h+k)*theta)/(h+k)); \]
    end
end
a3=a3+\sqrt{2}\alpha(2)/\pi V(m)c(2,m)(\sin(2k*theta)/(2k)+\theta);
b3=b3+\sqrt{2}\alpha(2)/(2k\pi)\sqrt{2}\alpha(2)/\pi V(l)c(2,m)(1-\cos(2k*theta));
a4=a4+\sqrt{2}\alpha(2)/(2k^2\pi)\sqrt{2}\alpha(2)/\pi V(l)c(4,m)(1-\cos(2k*theta));
b4=b4+\sqrt{2}\alpha(2)/(k\pi)\sqrt{2}\alpha(2)/\pi V(l)c(4,m)(\theta-\sin(2k*theta)/(2k));
aa(m)=a1+a2+a3+a4;
bb(m)=b1+b2+b3+b4;
Iamp(m)=abs(aa(m)+j*bb(m))/sqrt(2);
ang=angle(bb(m)+j*aa(m))/\pi*180-theta1*k;
lang(m)=ang-floor(ang/360)*360;
if lang(m) > 180
    lang(m)=lang(m)-360;
end
end

Vo_comp

\% is function with real characteristic roots
function Vo=Vo_comp(ang,V,delta,alpha,a,b,c);
H=length(V);
Vo=0;
ang=ang/180*pi;
A1=0;A2=0;
for l=1:H
    h=2*l-1;
    A1=(a+alpha(1))*sin(delta(l)/180*pi)+alpha(2)*(a*c(5,l)+c(6,l));
    A2=b*sin(delta(l)/180*pi)+b*alpha(2)*c(5,l);
    Vop=sqrt(2)*V(l)*alpha(2)*(-c(5,l)*cos(h*ang)+c(7,l)/h*sin(h*ang));
    V1=sqrt(2)*V(l)/b*(A1*sin(b*ang)+A2*cos(b*ang))*exp(a*ang)+Vop;
    Vo=Vo+V1;
end

    V0_real

% is function with real characteristic roots
function Vo=V0_real(ang,V,delta,alpha,a,b,c);
H=length(V);
Vo=0;
s1=a+b;s2=a-b;
ang=ang/180*pi;
A1=0;A2=0;
for l=1:H
    h=2*l-1;
    A1=(s1+alpha(1))*sin(delta(l)/180*pi)+alpha(2)*(s1*c(5,l)+c(6,l));
    A2=(s2+alpha(1))*sin(delta(l)/180*pi)+alpha(2)*(s2*c(5,l)+c(6,l));
    Vop=sqrt(2)*V(l)*alpha(2)*(-c(5,l)*cos(h*ang)+c(7,l)/h*sin(h*ang));
    V1=sqrt(2)*V(l)/(2*b)*(A1*exp(s1*ang)-A2*exp(s2*ang))+Vop;
    Vo=Vo+V1;
end

    Vdecay

% Voltage free-decaying function
function Y=Vdecay(theta1,theta2,Vo,Vth,theta,alpha)
H=length(Vth);
YY=Vo*exp(alpha(4)*((theta2-theta1)/180*pi-pi));
Y11=0;
for l=1:H
    h=2*l-1;
    Y1=sqrt(2)*Vth(l)*sin(h*theta1/180*pi+theta(l)/180*pi);
    Y11=Y11+Y1;
end
Y=YY+Y11;

main

% Multiphase load flow program       Main Program
%
% The load flow solution is obtained through the following iteration scheme.
% The system voltages are assumed before the iteration started. In the iteration,
% the nodal injection currents are first calculated using the assumed node
% voltages and the given load data. Then the branch currents are calculated by
% gathering the load injection from the lowest level nodes (branches) toward
% the root node (substation). As the currents in the branches direct connected
% the root node are obtained, the new node voltages are calculated using the
% voltage drops due to the branch currents, from the higher level to lower level.
% Thus finishes one iteration. The max power mismatch is calculated within
% each iteration. The iteration stops when the mismatch is below the accuracy
% requirement or the iteration number is larger than the specification, which
% implicates divergence.
    clear all
    % System topology
    line_conf=csvread('Line_data.csv',3);
    % L1 and L2 are specified in 'Line_data1.csv', where the L2 node is the further
end
    L1=line_conf(:,2); L2=line_conf(:,3);
    bus=max(L2);
    branch=line_conf(:,1); branch_num=max(branch);
    level=line_conf(:,8); level_num=max(level);
    bus_type wk1read('bus_type.wk1');
    In=wk1read('In.wk1');
    % Line impedance & Nodal admittance
    Line_se_data=wk1read('Line_se.wk1'); Node_sh_data=wk1read('Node_sh.wk1');
    H=round(length(Line_se_data(:,1))/4);
    % Change layout of the line impedance and nodal admittance to one
    for l=1:H
        inde=(l-1)*4;
        Line_se(:,:,l)=Line_se_data((inde+1):(inde+4),1:branch_num*4)+j*Line_se_data((inde+1):(inde+4),branch_num*4+1:branch_num*8);
Node_sh(:,:,l)=Node_sh_data((inde+1):(inde+4),1:bus*4)+j*Node_sh_data((inde+1):(inde+4),bus*4+1:bus*8);
end

% Network Ybus
ybus_load;

% Load data
load_data=csvread('Load_data.csv',4,1); % load information of bus#, load type, number in the same type
load_num=length(load_data(:,1)); % load number, may not equal bus number
load_p_n=0; load_c_n=0; load_imp_n=0; load_conv_n=0; load_sphr_n=0;
for l=1:load_num
    if load_data(l,3) == 1 || load_data(l,3) == 2
        load_p_n=load_p_n+1;
        load_power=csvread('Load_power.csv');
    elseif load_data(l,3) == 3 || load_data(l,3) == 4
        load_c_n=load_c_n+1;
        load_curr=csvread('Load_curr.csv');
    elseif load_data(l,3) == 5 || load_data(l,3) == 6
        load_imp_n=load_imp_n+1;
        load_imp=csvread('Load_imp.csv');
    elseif load_data(l,3) == 7
        load_conv_n=load_conv_n+1;
    elseif load_data(l,3) == 8
        load_sphr_n=load_sphr_n+1;
    end
end

% Base variables
Sb=5000; % Base power in kVA
Vb=4.16; % Base voltage in kV
Ib=Sb/sqrt(3)/Vb; % Base current
Zb=Vb^2/(Sb/1000); % Base impedance

% Normalize load data
if load_p_n >= 1
    load_power=load_power/Sb;
end
if load_c_n >= 1
    for l=1:load_c_n
        inde=(l-1)*6;
        load_curr(:,inde+1)=load_curr(:,inde+1)/Ib;
        load_curr(:,inde+3)=load_curr(:,inde+3)/Ib;
    end
```matlab
load_curr(:,inde+5)=load_curr(:,inde+5)/Ib;
end
if load_imp_n >= 1
    load_imp=load_imp/Zb;
end
%   Converter data
if load_conv_n > 0
    converter_data_load
end
%   Single phase rectifier data
if load_sphr_n > 0
    sphr_data_load;
end
%   The specified root node voltage
Vroot=zeros(4,H);  
Vroot(:,1)=[1.0; 1.0*exp(-2*pi*j/3); 1.0*exp(2*pi*j/3); 0];  
%   The initial assumption for the nodal voltage
Vnode=zeros(4,bus,H);  
Vnode(1,:,1)=1; Vnode(2,:,1)=exp(-2*pi*j/3); Vnode(3,:,1)=exp(2*pi*j/3);  
%   iteration control variables
iter=0; maxiter=1000;
fund_accuracy=0.001; harm_accuracy=1e-6;
converge=1;
%   iteration start
while iter <= maxiter & converge == 1
    iter=iter+1;
    %Collect load currents at the load terminal bus
    load_curr_inj;
    %Collect converter currents if exist
    if load_conv_n > 0
        conv_curr;
    end
    %Collect sphr current if exist
    if load_sphr_n > 0
        sphr_curr;
    end
    %Branch current calculation
    branch_curr_cal;
```
%Nodal voltage forward sweep
Vnode1=Vnode;
%first level
for B=1:branch_num
    if level(B) == 1
        for h=1:H
            Vnode(:,L2(B),h)=Vroot(:,h)-Line_se(:,((B-1)*4+1):((B-1)*4+4))*branch_curr(:,B,h);
        end
    end
end %higher level
for l=2:level_num
    for B=1:branch_num
        if level(B) == l
            for h=1:H
                Vnode(:,L2(B),h)=Vnode(:,L1(B),h)-Line_se(:,((B-1)*4+1):((B-1)*4+4),h)*branch_curr(:,B,h);
            end
        end
    end
end %Convergence check
dV=Vnode-Vnode1;
%fundamental
err_fund=max(abs(dV(:,:,1)));
maxerr_fund=max(err_fund);
%harmonic
err_harm=zeros(1,H-1);
for h=1:H-1
    err_harm_bus=max(abs(dV(:,:,h+1))); %max error among all phases at same freq. at same bus
    err_harm(h)=max(err_harm_bus); %max error among all buses at same freq.
end
maxerr_harm=max(err_harm);
Node_vol_mag=abs(Vnode);
Node_vol_ang=angle(Vnode)/pi*180;
%busout,pause if iter == maxiter
if maxerr_fund > fund_accuracy | maxerr_harm > harm_accuracy
fprintf(‘\nWARNING: Iterative solution did not converge after ’)
fprintf(‘%g’, iter),
fprintf(‘ iterations. \n\n’)
fprintf(’Press Enter to terminate the iterations and print the results \n’)
converge = -1; pause,
break
end
elseif iter < maxiter & maxerr_fund < fund_accuracy & maxerr_harm < harm_accuracy
    converge = 0;
end
if iter > 2
    Vnode=Vnode1+.6*dV;
end
end
if converge ~= 0
    tech=’
    ITERATIVE SOLUTION DID NOT CONVERGE’);
else
    tech=’
    Power Flow Solution For The IEEE 13-Bus Test System’);
end
Node_vol_mag=abs(Vnode);
Node_vol_ang=angle(Vnode)/pi*180;
busout

load_curr_inj

% Subprogram: Regular load current injection
%
% Const. power, current and impedance load current injection are calculated and
% collected at each bus
load_inj=zeros(4,bus,H);
for b=1:bus
    for l=1:load_num
        if load_data(l,1) == b
            if load_data(l,3) == 1 | load_data(l,3) == 2
                S1=load_power(load_data(l,4),1)+j*load_power(load_data(l,4),2);
                S2=load_power(load_data(l,4),3)+j*load_power(load_data(l,4),4);
                S3=load_power(load_data(l,4),5)+j*load_power(load_data(l,4),6);
                if load_data(l,3) == 1
                    % do something
                end
            end
        end
    end
end
load_inj(1,b,1)=load_inj(1,b,1)+conj(S1/(Vnode(1,b,1)-Vnode(4,b,1)));
load_inj(2,b,1)=load_inj(2,b,1)+conj(S2/(Vnode(2,b,1)-Vnode(4,b,1)));
load_inj(3,b,1)=load_inj(3,b,1)+conj(S3/(Vnode(3,b,1)-Vnode(4,b,1)));
load_inj(4,b,1)=load_inj(4,b,1)-(load_inj(1,b,1)+load_inj(2,b,1)+load_inj(3,b,1));
elseif load_data(l,3) == 2
  inj1=conj(S1/(Vnode(1,b,1)-Vnode(2,b,1)));
  inj2=conj(S2/(Vnode(2,b,1)-Vnode(3,b,1)));
  inj3=conj(S3/(Vnode(3,b,1)-Vnode(1,b,1)));
  load_inj(1,b,1)=load_inj(1,b,1)+inj1-inj3;
  load_inj(2,b,1)=load_inj(2,b,1)+inj2-inj1;
  load_inj(3,b,1)=load_inj(3,b,1)+inj3-inj2;
end
elseif load_data(l,3) == 3
  inde=(load_data(l,4)-1)*6;
  for h=1:H
    load_inj(1,b,h)=load_inj(1,b,h)+load_curr(h,inde+1)*exp(j*load_curr(h,inde+2)/180*pi);
    load_inj(2,b,h)=load_inj(2,b,h)+load_curr(h,inde+3)*exp(j*load_curr(h,inde+4)/180*pi);
    load_inj(3,b,h)=load_inj(3,b,h)+load_curr(h,inde+5)*exp(j*load_curr(h,inde+6)/180*pi);
    load_inj(4,b,h)=load_inj(4,b,h)-(load_inj(1,b,h)+load_inj(2,b,h)+load_inj(3,b,h));
  end
elseif load_data(l,3) == 4
  inde=(load_data(l,4)-1)*6;
  for h=1:H
    inj1=load_curr(h,inde+1)*exp(j*load_curr(h,inde+2)/180*pi);
    inj2=load_curr(h,inde+3)*exp(j*load_curr(h,inde+4)/180*pi);
    inj3=load_curr(h,inde+5)*exp(j*load_curr(h,inde+6)/180*pi);
    load_inj(1,b,h)=load_inj(1,b,h)+inj1-inj3;
    load_inj(2,b,h)=load_inj(2,b,h)+inj2-inj1;
    load_inj(3,b,h)=load_inj(3,b,h)+inj3-inj2;
  end
elseif load_data(l,3) == 5
  for h=1:H
    load_inj(1,b,h)=load_inj(1,b,h)+(Vnode(1,b,h)-Vnode(4,b,h))/...
      (real(load_imp(l,1))+j*(2*h-1)*imag(load_imp(l,1)));
    load_inj(2,b,h)=load_inj(2,b,h)+(Vnode(2,b,h)-Vnode(4,b,h))/...
(real(load_imp(l,2))+j*(2*h-1)*imag(load_imp(l,2)));
load_inj(2,b,h)=load_inj(3,b,h)+(Vnode(3,b,h)-Vnode(4,b,h))/...
(real(load_imp(l,3))+j*(2*h-1)*imag(load_imp(l,3)));
load_inj(4,b,h)=load_inj(4,b,h)-
(load_inj(1,b,h)+load_inj(2,b,h)+load_inj(3,b,h));
end
elseif load_data(l,3) == 6
for h=1:H
  inj1=(Vnode(1,b,h)-Vnode(2,b,h))/(real(load_imp(l,1))+j*(2*h-1)*imag(load_imp(l,1)));  
  inj2=(Vnode(2,b,h)-Vnode(3,b,h))/(real(load_imp(l,2))+j*(2*h-1)*imag(load_imp(l,2)));  
  inj1=(Vnode(3,b,h)-Vnode(1,b,h))/(real(load_imp(l,3))+j*(2*h-1)*imag(load_imp(l,3)));  
  load_inj(1,b,h)=load_inj(1,b,h)+inj1-inj3;  
  load_inj(2,b,h)=load_inj(2,b,h)+inj2-inj1;  
  load_inj(3,b,h)=load_inj(3,b,h)+inj3-inj2;  
end
end
end
end
nam

sphr_curr

% Collect sphr load currents if called
for l=1:load_sphr_n
  sphr_bus=sphr_data(l,1);
  if sphr_type(l,1) == 1
    V1=Vnode(sphr_type(l,2),sphr_bus,:); V2=Vnode(4,sphr_bus,:);
    Vth=abs(V1-V2)*240; theta=angle(V1-V2)/pi*180;
    [Iamp,Iang,theta1,theta2,alpha,delta,a,b,c,B]=sphr1(Vth,theta,sphr_rth(l),sphr_Lth(l),sphr_cap(l),sphr_cap(l),sphr_req(l));
    lamp=Iamp/Ib/10;
    for m=1:H
      load_inj(sphr_type(l,2),sphr_bus,m)=load_inj(sphr_type(l,2),sphr_bus,m)+lamp(m)*exp(j*Iang(m)/180*pi);
      load_inj(4,sphr_bus,m)=load_inj(4,sphr_bus,m)-lamp(m)*exp(j*Iang(m)/180*pi);
    end
  end
elseif sphr_type(l,1) == 2
    V1=Vnode(sphr_type(l,2),sphr_bus,:);
    V2=Vnode(rem(sphr_type(l,2),3)+1,sphr_bus,:);
    Vth=abs(V1-V2)*240; theta=angle(V1-V2)/pi*180;
    [Iamp,lang,theta1,theta2]=sphr1(Vth,theta,sphr_rth(l),sphr_Lth(l),sphr_cap(l),sphr_req(l));
    Iamp=Iamp/Ib/10;
    for m=1:H
        load_inj(sphr_type(l,2),sphr_bus,m)=load_inj(sphr_type(l,2),sphr_bus,m)+Iamp(m)*exp(j*lang(m)/180*pi);
        load_inj(rem(sphr_type(l,2),3)+1,sphr_bus,m)=load_inj(rem(sphr_type(l,2),3)+1,sphr_bus,m)...
            -Iamp(m)*exp(j*lang(m)/180*pi);
    end
end
for m=1:H
    I_sphr(m,:,l)=[Iamp(m) lang(m)];
end
end

branch_curr_cal

% Subprogram: Calculated branch currents starting from the lowest level
%
% Total return currents are splitted between neutral and earth according to the
% voltage equation
branch_curr=zeros(4,branch_num,H);
curr_res=zeros(branch_num,H);
% last level
for B=1:branch_num
    if level(B)==level_num
        branch_curr(1:3,B,:)=load_inj(1:3,L2(B,:));
        if L1(B)==0
            V_up(:)=Vroot(4,:);
        else
            for h=1:H
                V_up(h)=Vnode(4,L1(B),h);
            end
        end
    end
    for m=1:H
        if level(B)==level_num
            branch_curr(1:3,B,:)=load_inj(1:3,L2(B,:));
        else
            for h=1:H
                V_up(h)=Vnode(4,L1(B),h);
            end
        end
    end
end
branch_curr(4,B,h)=(V_up(h)-Line_se(4,(B-1)*4+1:(B-1)*4+3,h)*branch_curr(1:3,B,h)... +load_inj(4,L2(B),h)/Node_sh(4,L2(B)*4,h))/(Line_se(4,B*4,h)+1/Node_sh(4,L2(B)*4,h));

% earth current in the last level
curr_res(B,h)=branch_curr(1,B,h)+branch_curr(2,B,h)+branch_curr(3,B,h)+branch_curr(4,B,h);

end
end
end

% higher level
for l=1:level_num-1
    for B=1:branch_num
        if level(B) == (level_num-l)
            return_curr=zeros(1,H);
            for n=1:branch_num
                if L1(n) == L2(B)
                    branch_curr(1:3,B,:)=branch_curr(1:3,B,:)+branch_curr(1:3,n,:);
                    for h=1:H
                        return_curr(h)=return_curr(h)+branch_curr(4,n,h);
                    end
                end
            end
            branch_curr(1:3,B,:)=branch_curr(1:3,B,:)+load_inj(1:3,L2(B),:);
            for h=1:H
                return_curr(h)=return_curr(h)+load_inj(4,L2(B),h);
            end
            if L1(B) == 0
                V_up(:)=Vroot(4,:);
            else
                for h=1:H
                    V_up(h)=Vnode(4,L1(B),h);
                end
            end
            for h=1:H
                branch_curr(4,B,h)=(V_up(h)-Line_se(4,(B-1)*4+1:(B-1)*4+3,h)*branch_curr(1:3,B,h)... +return_curr(1,h)/Node_sh(4,L2(B)*4,h))/(Line_se(4,B*4,h)+1/Node_sh(4,L2(B)*4,h));
            end
        end
    end
end

% earth current
curr_res(B,h)=branch_curr(1,B,h)+branch_curr(2,B,h)+branch_curr(3,B,h)+branch_curr(4,B,h);
end
end
eND
end
branch_curr_mag=abs(branch_curr);
branch_curr_ang=angle(branch_curr)/pi*180;

busout

% This program prints the power flow solution in a tabulated form
% on the screen.
fprintf('

');
%disp(tech)
fprintf(' Maximum Voltage Mismatch = %g \n', maxerr_fund)
fprintf(' Maximum Harmonics Mismatch = %g \n', maxerr_harm)
fprintf(' No. of Iterations = %g \n', iter)
head =[' Bus Voltage THD Voltage THD Voltage THD Voltage THD
' ' No. TOT RMS % TOT RMS % TOT RMS % TOT RMS % TOT RMS % TOT RMS
', ' A B C N ' ']

disp(head)
for n=1:bus
fprintf(' %5g', n),
[RMS(n,1),THD(n,1)]=harm(Node_vol_mag(1,n,:));
fprintf(' %.8f', RMS(n,1)), fprintf(' %.9f', THD(n,1)),
[RMS(n,2),THD(n,2)]=harm(Node_vol_mag(2,n,:));
fprintf(' %.8f', RMS(n,2)), fprintf(' %.9f', THD(n,2)),
[RMS(n,3),THD(n,3)]=harm(Node_vol_mag(3,n,:));
fprintf(' %.8f', RMS(n,3)), fprintf(' %.9f', THD(n,3)),
[RMS(n,4),THD(n,4)]=harm(Node_vol_mag(4,n,:));
fprintf(' %.8f', RMS(n,4)), fprintf(' %.9f', THD(n,4))
end
fprintf('
');
wk1write('Node_tot.wk1',RMS)
wk1write('THD.wk1',THD);
for l=1:H
loadnode(:,l)=Node_vol_mag(:,2,l);
end
wk1write('loadfunvol.wk1',abs(Node_vol_mag(:,;1)));
REFERENCES


